A New Classification Framework for the Economic Lot Scheduling Problem

TESI DI LAUREA MAGISTRALE IN
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“The management Engineer as one of its responsibilities is safeguarding the great resources of capital available in its country for industrial purposes and of guiding the equally great resources of human labor so as to transform raw materials into articles which may be universally distributed among our people for their common good. “

Fairfield E. Raymond

Abstract

The Economic Lot Scheduling Problem (ELSP) is a pivotal challenge in the field of manufacturing, characterized by the quest for optimal batch sizes to produce multiple products efficiently on a single machine. This problem entails the development of diverse strategies, encompassing independent solutions, adjustments to production parameters, setup times, and responses to varying demand characteristics. Central to the lot scheduling and sizing decisions is the critical trade-off between inventory holding costs and setup costs. This thesis embarks on an extensive journey through the historical evolution of lot sizing models, tracing its roots from the foundational Economic Order Quantity (EOQ) model introduced by Ford W. Harris. This progression ultimately leads to the derivation of the Economic Production Quantity model based on the E.W. Taft model, paving the way for the transition to the ELSP. A vital component of this exploration is a comprehensive literature survey, providing insight into the historical and contemporary landscape of the field. Furthermore, this research introduces a new classification framework, designed to accommodate the multifaceted aspects influencing problem complexity, thus aligning more closely with industrial practices, comparing the different resolution methods to be able to decide which method to use or which is the most appropriate. The fusion of historical insights and modern approaches underscores the significance of this work in bridging the gap between theoretical knowledge and practical implementation in the complex domain of lot scheduling. The ELSP holds substantial academic and industrial significance, promising enhanced manufacturing efficiency, demand management, and cost optimization. By integrating a thorough literature survey, this thesis strives to illuminate the past, present, and future of lot scheduling in contemporary manufacturing contexts.

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Key-words: Economic Lot Scheduling Problem (ELSP), Lot Sizing, Manufacturing Efficiency, Inventory Management, Cost Optimization, Literature Survey

Abstract in italiano

Il Problema Economico della Programmazione dei Lotti (ELSP) rappresenta una sfida cruciale nel campo della produzione, caratterizzata dalla ricerca delle dimensioni ottimali del lotto per produrre efficientemente più prodotti su una singola macchina. Questo problema comporta lo sviluppo di diverse strategie, che comprendono soluzioni indipendenti, regolazioni dei parametri di produzione, tempi di allestimento e risposte alle variazioni delle caratteristiche della domanda. Al centro delle decisioni di programmazione e dimensionamento del lotto c'è il trade-off critico tra i costi di mantenimento dell'inventario e i costi di allestimento. Questa tesi intraprende un ampio percorso attraverso l'evoluzione storica dei modelli di dimensionamento del lotto, seguendone le radici dal fondamentale modello Economic Order Quantity (EOQ) introdotto da Ford W. Harris. Questa progressione porta alla derivazione del modello Economic Production Quantity basato sul modello di E.W. Taft, aprendo la strada al passaggio all'ELSP. Un componente vitale di questa esplorazione è una completa ricerca bibliografica, fornendo un'analisi del panorama storico e contemporaneo del settore. Inoltre, questa ricerca introduce un nuovo quadro di classificazione, progettato per ospitare gli aspetti sfaccettati che influenzano la complessità del problema, allineandosi così più strettamente con le pratiche industriali. confrontando i diversi metodi di risoluzione per poter decidere quale metodo utilizzare o quale sia il più appropriato. La fusione di intuizioni storiche e approcci moderni sottolinea l'importanza di questo lavoro nel colmare il divario tra la conoscenza teorica e l'implementazione pratica nel complesso settore della programmazione del lotto. L'ELSP ha una significativa rilevanza accademica e industriale, promettendo un miglioramento dell'efficienza produttiva, della gestione della domanda e dell'ottimizzazione dei costi. Integrando una completa ricerca bibliografica, questa tesi si propone di illuminare il passato, il presente e il futuro della programmazione del lotto nei contesti produttivi contemporanei.

Parole chiave: Problema Economico della Programmazione dei Lotti (ELSP), Dimensionamento del Lotto, Efficienza di Produzione, Gestione dell'Inventario, Ottimizzazione dei Costi, Ricerca Bibliografica

Contents

Introduction

The economic lot scheduling problem has been one of the most important issues that have been studied in recent years in manufacturing, the problem arises from the challenge of finding the most economical batch size that allows the production of various products with a single machine, more specifically to determine the schedules of production cycles with the aim of minimizing the total costs that are incurred per unit of time and at the same time, meet the demand for each product. There are different strategies to address this problem such as an independent solution, a single cycle solution, variation of production parameters, setup times, and other approaches have also been studied in which deterministic demand and stochastic demand are studied. The limitation of resources brings us to the problem of interdependence between products. If the utilization of resources that are commonly shared is low, it is possible to use the economic quantity order model for each product and obtain an optimal solution. The EOQ model was a major breakthrough in inventory management. It was the first model that provided a systematic way to minimize the total cost of inventory. The EOQ model is still used today, and it is one of the most widely used inventory management models in the world.

The limited availability of resources leads to complex coordination problems when resource utilization is high. It is in this latter context that the Economic Lot Scheduling Problem (ELSP) was born.

The cost of items that are involved in this problem usually includes the inventory and holding costs, the setup costs and sometimes other variables like lot scheduling and lot sizing. These are all important concepts in production and operations management, and by the fact that they are all interrelated is important to understand how they interact with each other in order to help companies to optimize their production process. The inventory holding costs are the costs associated with storing unsold inventory and can embrace, storage costs, labor costs, insurance costs, taxes, depreciation, shrinkage, and opportunity costs. Setup costs are the costs associated with preparing a machine or production line to produce a new product. These costs can include the cost of changing over tools and dies, cleaning and sanitizing the equipment, and training employees on the new product. The lot scheduling is the process of determining when and how much of each product to produce. Lot sizing is the process of determining the quantity of each product to produce in each lot.

The trade-off between inventory holding costs and setup costs is a key consideration in lot scheduling and sizing. If companies produce large lots, they will have lower setup costs per unit, but they will also have higher inventory holding costs. If companies produce small lots, they will have lower inventory holding costs, but they will also have higher setup costs per unit.

The following graphs shows a visual representation of the ELSP system.

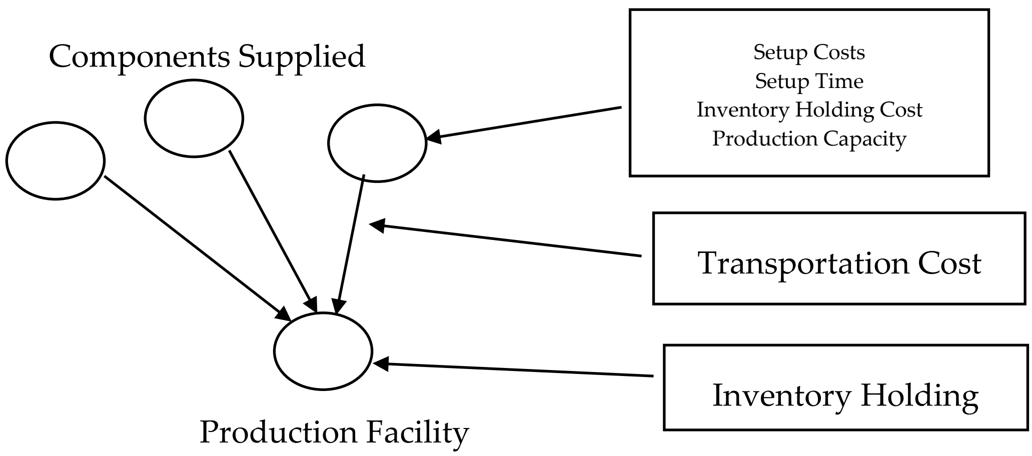

Figure 1. Visual Representation of the ELSP Problem

In occasions there is more cost effective to use a single high-speed machine that can produce multiple items rather that a dedicated machine for each item. This is because economies of scale are the cost advantages that manufacturing companies obtain due to their size, as a business grows, it can produce goods more efficient to produce goods, which leads to lower costs per unit. For example, a company that produces widgets, cogs and sprockets may choose to use a single computer numerical control machine to produce all three items. This machine is more efficient and productive than 3 machines because take advantages of bulk discounts on raw materials.

However as previously mentioned there are some potential drawbacks to using a single machine, like will be more difficult to change the machine over to produce different items and if the machine breaks down, it will disrupt the production of all three items.

The ELSP problem is presented in several applications in the industry for example in milling of gear houses, painting for metal rolls, welding of rear axles, painting of truck components, paper production. Holmbom (2014)

According to Boctor (1987) in the next situations the ELSP can be useful:

1. Facilities for metal stamping and plastic extrusion, where various items necessitate distinct molds for machine configuration.
2. Production lines responsible for a variety of products and/or diverse product variants (including electrical appliances, automobiles, etc.).
3. Mixing and blending plants (such as those for paints, beverages, and animal feed) where various items are packaged into separate containers.
4. Textile or carpet weaving assembly lines that create primary products in diverse shades, widths, or qualities.

Beck and Glock (2019) conducted research on the articles they mention or are applied at an industrial level and showed the following practical applications of the ELSP: Automotive, Chemical, Fiberglas and Food industry, also Metal and plastic production, pharmaceutical industry, sanitary and household industry, semiconductor, and the service industry.

Thesis scope

The objective of this thesis is to initially present a historical review of the different approaches that have been presented to date for this problem starting from the "Economic Order Quantity" model presented by Ford W. Harris which consists of a simple square equation that was the beginning of a large number of contributions and modifications to the formula as well as instruments, graphics and manuals that were proposed throughout the twentieth century.

After this, I'll explain the Harris model in depth as well as the E.W. Taft model that led to the derivation of the" Economic Production Quantity". In Chapter 3 I'll start the transition from the first Economic lot models to the Economic Lot Scheduling problem, developing a mathematical example where the ELSP is derived in a concise and simple way, and later in after as a result of the analysis of the arduous existing literature, a new classification is proposed that aims to consider new factors that affect the complexity of the problem and that try to get closer to the industrial reality. Finally, after a comparative analysis of the different methods I present the comparative conclusion of the research and answer the question. What method to use?

1 Chapter one. Historical Aspects

1.1. The Evolution of Economic Lot Scheduling

A frequent management problem in the first half of the 20th century was the lack of a method to coordinate sales demand, working capital requirements, raw material purchase, and the nature of the process. All these factors are fundamental to production control, and if they are not properly correlated, it will cancel all the methods developed to eliminate all possible waste in time, labor, and material. Similarly, the profit expected in return for the sale of each product depends on the quantity produced in any single lot.

Not long after the systematic production control methods based on the principles of Frederick W. Taylor demonstrated their usefulness, many of their early supporters discovered that the final cost of a unit of production depended on the size of the production lot when a repetitive manufacturing process was involved. Raymond, (1931) This includes the fact that each production process requires a setup of a sequence of machines, which would be dismantled each time there was an opportunity to produce an additional supply of a particular unit of production, and that in the intervals between these manufacturing periods, a certain number of units must be held in inventory in anticipation of future orders. The eventual cost of production and storage can be minimized when there is a balance between the "unit allotment of the machine changeover cost and the charges incurred by storage including the cost of capital, through the selection of an appropriate lot size."

The optimal production lot size is the one that minimizes the total cost of production and storage. The total production cost includes the cost of raw materials, the cost of labor, and the cost of machinery. The storage cost includes the cost of land, the cost of construction, and the cost of inventory.

The first formal definition of an economic production quantity was given by Fairfield E. Raymond in his book Quantity and Economic in Manufacture, his definition was:

"An economic-production quantity is that quantity which can be produced at the lowest total unit cost consistent with an economical use of capital in its manufacture, taking into consideration not only the preparation charges against each process order and investment charges on the capital involved, but also rental charges on the space occupied by the article when carried in stock, losses due to deterioration and obsolescence, and the nature of the process."

The discovery of these relationships resulted in a new policy of reducing costs, reducing inventories, and having a balance between production schedules and demand for vents. At first the so-called "lot size economy" was too elementary, because none had the opportunity to study the factors involved.

Just as it is obvious that mass production helps the progress of an industry, it is important however to find coordination between scheduled sales and production schedules that allows a more economical use of capital and a reduction in inventories.

From here the question arises: what quantity should be produced? Before the introduction of the economic lot scheduling problem there were two main approaches. The first one focused on lower manufacturing costs due to economies of scale. A second one in lower inventory costs due to smaller order quantities Raymond (1931).

The first approach states that a large quantity can be produced in each lot for a single setting up of the manufacturing equipment, in order that the lowest possible manufacturing cost can be attained through a distribution of the preparation charges over the greatest number of units that can be conveniently placed in production.

The second approach defines a condition where is more important the control of inventories is more important that of cost, in this situation a small quantity can be produced that will be just sufficient to meet the immediate sales demand and cover the time required to replenish the stocks of finished products. The production minded company will produce the largest quantity to obtain the lowest unit cost, and the financial minded will produce the smallest quantity possible, because he realizes that the invest capital can be more rapidly turned over.

Both types of strategies are correct in their judgment. However, the first policy will increase inventories and consume capital while the second will incur an unnecessary large allotment of the set-up costs in proportion to the costs of direct labor, material, and overhead for each unit processed.

From these two different mindsets the problem of finding an economic balance between production lot size and unit cost arises with the need to minimize total manufacturing costs. This balance is achieved when the unit allotment of the total machine set up and production control charges, incurred by the preparation for the manufacture of that lot, is equal to the unit allotment of the cost of carrying the average number of pieces in storage for the time that any one of the pieces produced in that lot still remains in stock. The desired economic balance can be achieved, and the unit cost will become at minimum.

The first recorded study of the development of an economic lot was firstly attributed to George D. Babcock in 1912, creator of the Water-tube boiler, during its connection with the installation of a Taylor System in the plant f the H.H. Frankling

Manufacturing company in Syracuse, New York. When he was analyzing the production problems of this plant. He formerly recorded its work in 1917 in his book *The Taylor System in Franklin Management Application and Results* page 126 published by The Engineering Magazine Company in New York. He classified the following types of processes. Babcock (1917)

1. One order for one piece never to be reproduced.
2. One order for several pieces never to be reproduced.
3. Repeat orders at irregular intervals for one or a few pieces.
4. Repeat orders at irregular intervals for many pieces.
5. Repeat orders at uniform intervals for one or a few pieces.
6. Repeat orders at uniform intervals for many pieces.
7. Continuous orders for the same piece.

These types of processes are classified by orders and work, so a clear conclusion is that in a situation where it is necessary to repeat orders the economical state of production will depend upon the lot size in order to regulate the frequency of production according to the number of pieces required.

George D. Babcock, who worked at the Frankling plant, developed a mathematical basis for establishing the correct lot size for a given set of conditions. However, he decided that the use of formulas was not practical for a planning department because cubic equations had to be employed and it was too complex to be used by planners on a day-to-day basis. As a result, the Franklin plant simply established standard lot sizes based on their experience and intuition. Raymond (1931)

Although Babcock made the first approximation to the problem, it was necessary to find a mathematical relation or formula that allows to obtain an approximate value in a quantitative way to the economic lot.

The first attempt to find a mathematical relation to the economic lot size was in 1913, by engineer Ford W. Harris a former Westing manufacturing engineer.

While working at Westinghouse Electric and Manufacturing Company in East Pittsburgh that he developed the first formula. His work was published in *Factory*, The Magazine of Management of A.W. Shaw Company of Chicago the chapter was called *"How Many Parts to Make at Once"* In this document, he manifested the problem of finding the most economical quantity to produce of each lotus of a product to meet the required demand. Harris (1913)

Despite being highly publicized, Ford Whitman Harris's work on the economic order quantity (EOQ) formula was not recognized until 75 years after its publication, due to errors in future citations. The formula was rediscovered in 1989 and 1996 by Donald Erlenkotter. Erlenkotter (1990)

Although more than 100 papers were published in the next 20 years to the publication of Harrys, on many occasions some authors added their own embellishments creating many confusions, so the ASME asked Professor Raymond a literature survey with the aim of comparing and combining the different studies in a publication called Quantity and Economy in Manufacturing in 1931. From the literary survey of Professor Raymond was possible to manifest the need to modify the EOQ for more complex cases such as non-constant demand and this was a fundamental basis for the ELSP problem.

The mathematical expression presented by Harris consisted of a simple formula where the economic lot depended only on 3 variables; lot size, setup cost and the unit cost for this development Harris correctly assumed that the best lotus size was one for which the final cost of each unit produced was the minimum. Harris's conclusions avoided using high-level mathematics. He assumed that the economical production can be attained when the unit allotment of the set-up costs are equal to the carrying charges of the stock of the specific article in completed form. Harris (1913)

The model for economic lot scheduling was formally established by R.H. Wilson in 1934 from the work of Harris and in this work the first mention was made to the term "Economic Order Quantity" which has the following assumptions. Raymond (1931)

1. There is a constant demand for the product.
2. The lead time for the product is constant.
3. The ordering costs are fixed.
4. The holding costs are proportional to the amount of inventory held.
5. The lead time for the product is constant.
6. The ordering costs are fixed.
7. The holding costs are proportional to the amount of inventory held.

1.1. First Attempts to find the Economic Lot Size

There was a wide proliferation of work in this area. In his work, Raymond made a very complete summary of all the formulas and strategies that existed until the time he wrote the book, each author and each proposed formula considered more and more factors than the elementary sales, setup cost and the capital invested. These formulas produced acceptable economic lots and manuals and instruments were created from these formulas that companies used throughout the first half of the twentieth century and some of the second half, until the works of Rogers in 1958, and Bomberger in 1966. They opened a new paradigm by studying the so-called ELSP today where the solution is not so trivial but more real cases could be covered at the industry levell.

Below is the table that summarizes Raymond's work on the methods to calculate the economic lot. The table reports the year of the first appearance in the literature of the formula and the approximate year of registration, as well as the authors, more than one author may be associated with a single formula due to the fact that several authors derived the same formula or reported the formula with various variations, the uncertainty regarding this situation and authors is due to the problems and methods of reference that existed at that time, the most conclusive fact that demonstrates this is that Harris's authorship of the formula was not given to him until almost 75 years and as a detail Harris' formula was initially called Wilson's formula, because he did similar work in 1934. Raymond (1931)

Table 1a – EOQ Summary of formulas

First Appearance	Form	Authors	Approximate date of record
1912	Cubic Equation not published	G.D. Babcock	1912
1915	$Q = \sqrt{\left(P.\frac{S}{c}\right)k}$	F.W. Harris D.B. Carter General Electric Co. J. A. Bennie P.E. Holden K.W. Stilman Benning and Littlefield J.M. Christman G.H. Mellen	1913* 1915 1922 1922 1923 1924 1925 1925 1925
1917	Special adaptation $Q = \sqrt{\left(\frac{P.S}{c.i}\right)k}$	Eli Lilly & Co. S.A. Morse W.E. Camp Holtzer Cabot Co. N.R. Richardson	1917 1917
1917	$Q = \sqrt{\frac{P.S.k}{c.(i+f_i)}}$ fi = allowances for insurance, storage costs, etc.	H.T. Stock R.C. Davis B.Cooper G. Pennington E.T. Philips W.L. Jones J.W. Hallock	1923 1925 1926 1927 1927 1929 1929
1918	$Q = \sqrt{\frac{P.S.D'}{c.i(D'-S)}k}$	E.W. Taft F.H. Thompson R.C. Davis J.M. Christman	1918 1923 1925 1925

Table 1b – EOQ Summary of formulas

First Appearance	Form	Authors	Approximate date of record
1918	$Q = \sqrt{\frac{P.S.D'}{c.i(D'-S)}k}$	B. Cooper G. Pennington E.T. Philips W.L. Jones J.W. Hallock	1926 1927 1927 1929 1929
1918	$Q = -\frac{P}{C} + \sqrt{\frac{P^2}{c^2} + \frac{P.S.D'}{c.i(D'-S)}k}$	E.W. Taft G. Pennington F.H. Thompson	1918 1927 1923
1923	$Q = \sqrt{\frac{P.S.k}{c.i.f_a + \frac{s.b}{h}.k'_b}}$	F.H. Thompson R.C. Davis C.N. Neklutin	1923 1926 1929
1924	$Q = \sqrt{\frac{P.S.k}{c.i + (m+c)t_P.S.i}}$	A.C. Brungardt P.N. Lehoczky	1923 1927

* *Raymond erroneously gave the year 1915 as the date of publication of the formula, it was published in 1913.*

Where:

Q: *The lot size*

P: *The total cost of preparing for the manufacture of a lot*

S: *The daily rate of consumption or sales*

c: *Average capital invested in any phase of manufacture*

k: *Any constant factor*

i: *The interest rate charged on borrowed capital expressed decimally*

D': *The uniformal daily rate of withdrawal of articles from work in process*

f_a: *The sales factor for correcting the lot size for changes in the rate of consumption*

h: *The height to which storage permissible on any swuare foot of floor space*

b: *The overall volume or bulk of a unit of production*

k'_b: *The bin factor*

M: *The toal cost of machine changeover for the lot as a whole*

t_p: *The unit − process time for average piece*

1.1.1. The general form

Introduced in 1913 by F.W.Harris, this formula will be explained more in detail in chapter 2. The variable k represents a constant that includes the interest rate i normally assumed to be 6 per cent.

$$Q = \sqrt{\left(P.\frac{S}{c}\right)k}$$

By 1918 became more practical to not include in the constant k but introduce i in the denominator and its value decided by executive decision

$$Q = \sqrt{\left(\frac{P.S}{c.i}\right)k}$$

After 1923 was suggested to give another approach to the rate of interest proposing that not only the employee can determine the cost of capital, but also include an allowance for depreciation, this means that allowances for insurance, rent, taxes, and storage costs appear.

In 1926 B.F. Cooper from the General Electric in addition to previous variables, decided to add the allowance for obsolescence which at the end is a right consideration that interest rate is not the only factor that affect the cost of capital.

1.1.2. Correction for semicontinuous production:

E.W. Taft, while working for the Winchester Repeating Arms company, introduced a method of correcting the quantity produced for units that were diverted to current orders. This was done to account for the savings that resulted from the overlapping of the manufacturing period and the subsequent sales period. The investment charges were then computed only on the basis of the quantity that actually reached stores. In other words, Taft realized that the company was saving money by manufacturing products during the same time period that they were being sold. This was because the company did not have to carry as much inventory as it would have if it manufactured products well in advance of sales.

$$Q = \sqrt{\frac{P.S.D'}{c.i(D'-S)}k}$$

But in this case D′ was erroneously considered as the rate of production and not the rate of delivery to stores D.

1.1.3. E.W. Taft second model:

Also introduced by E.W. Taft where the method of approach is to compound the cost of capital in each step of manufacturing process and in so doing the formula for the economic lot size becomes.

$$Q = -\frac{P}{C} + \sqrt{\frac{P^2}{c^2} + \frac{P.S.D'}{c.i(D' - S)}k}$$

1.1.4. Introduction of the space charge element:

F.H. Thompson of Dennison Manufacturing Company developed a formula with a new cost factor that recognized the fact that the cost of the storage spaces should depend upon the bulk of the product and not upon its value.

$$Q = \sqrt{\frac{P.S.k}{c.i.f_a + \frac{s.b}{h}.k_b'}}$$

Prior to Harris's work there was some resistance to developing a mathematical technique that would allow graphically, or with a formula, to represent the problem.

Although the formula was very efficient at various times there are many cases in which it will not be useful, for example when the demand is variable. Successful in cases where there is variable demand, product deterioration, style changes, rapidly occurring improvements in design, and other factors.

1.1.5. Introduction of the work in process

Developed by A. O. Brungardt of the Walworth Company, described a method to determine the economic lot size including the cost of capital invested in work in process as well as that derived from articles in stores. It was evident that if inventories of work in process were as large as inventories of finished parts or products, the equality between the preparation costs and the total cost of capital must take into consideration the investment charges on both classes of inventories.

$$Q = \sqrt{\frac{P.S.k}{c.i.+(c+m) * t_p * S - i}}$$

1.2. Practical adoption of the general form

During the first half of the 20th century, manufacturing companies would prefer to use the general form of the equation so as not to engage in methods with complicated techniques, most manufacturing problems at that time allowed the simplest formula to be used to find the economic lot, however in some specific situations the more mathematically complex methods were used.

$$Q = \sqrt{\left(P.\frac{Sa}{c}\right)k}$$

To determine the constant k, after the interest rate was decided, the following formula was used, where r is the rate of return.

$$k_r = \frac{2}{i\left(1+\frac{r}{i}\right)^2}$$

1.3. First Instruments to determine the economic lot

In many situations it was more convenient to specify the rate of consumption in terms of year, sometimes in months, weeks, or days. To do this it was necessary to have a standard base, because if the formula had to be introduced in another term that is different from years, a correction factor had to be entered in the numerator, because the base time used for the interest rate and the rate of return was in years, so the factor was going to be equal to 12 if the consumption rate Sa was expressed in months, 52 if Sa in weeks, and 300 if expressed in days, The following figure shows a nomographic slide rule used for determining the economic lot sizes, developed by Benjamin Cooper for the General Electric Company. Raymond (1931)

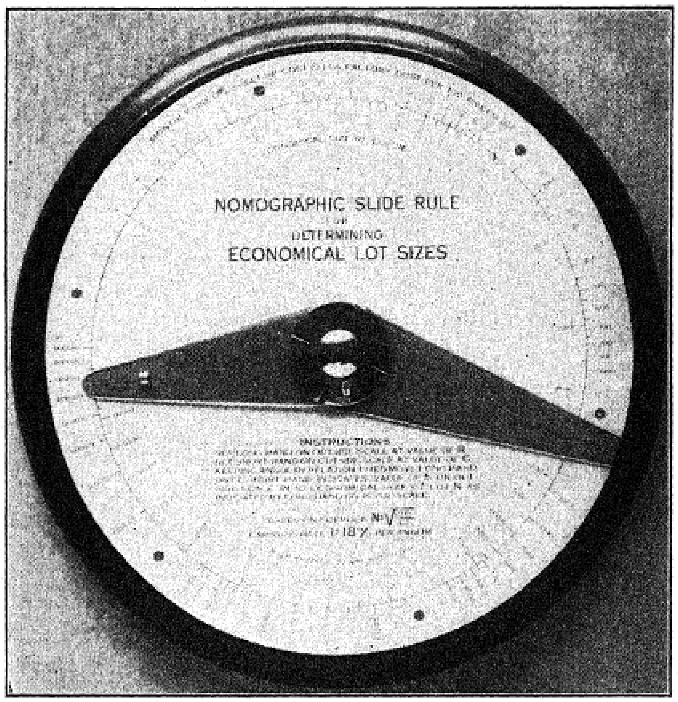

Figure 2 – Nomographic slide rule for the economic lot

The figure 3 shows a design for an economic order calculator build by Western Electric and the figure 4 a graph for determining the economic lot used in other contexts.

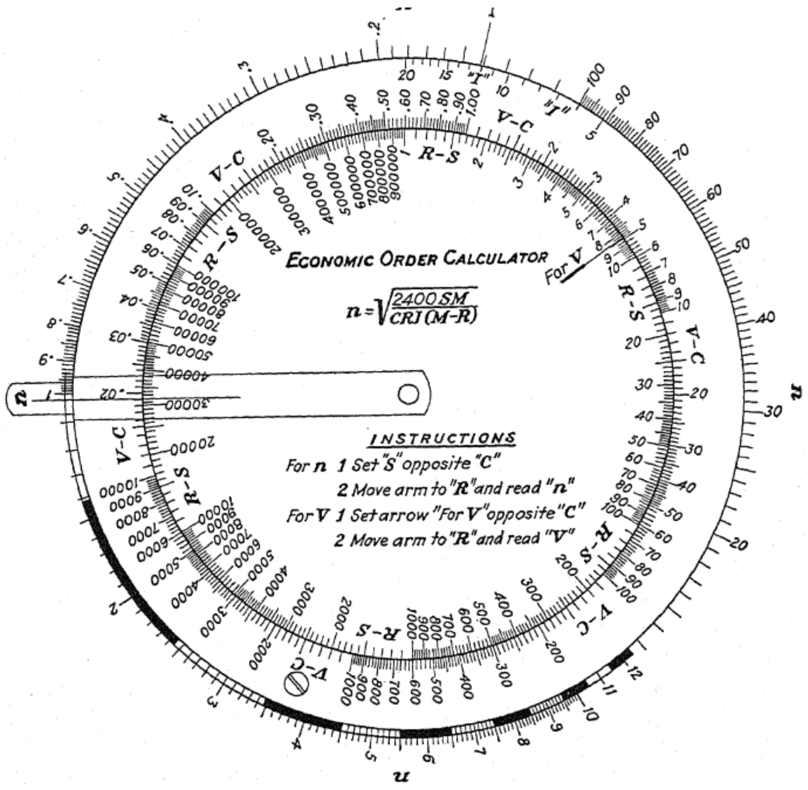

Figure 3. Western Electric Economic Order Calculator

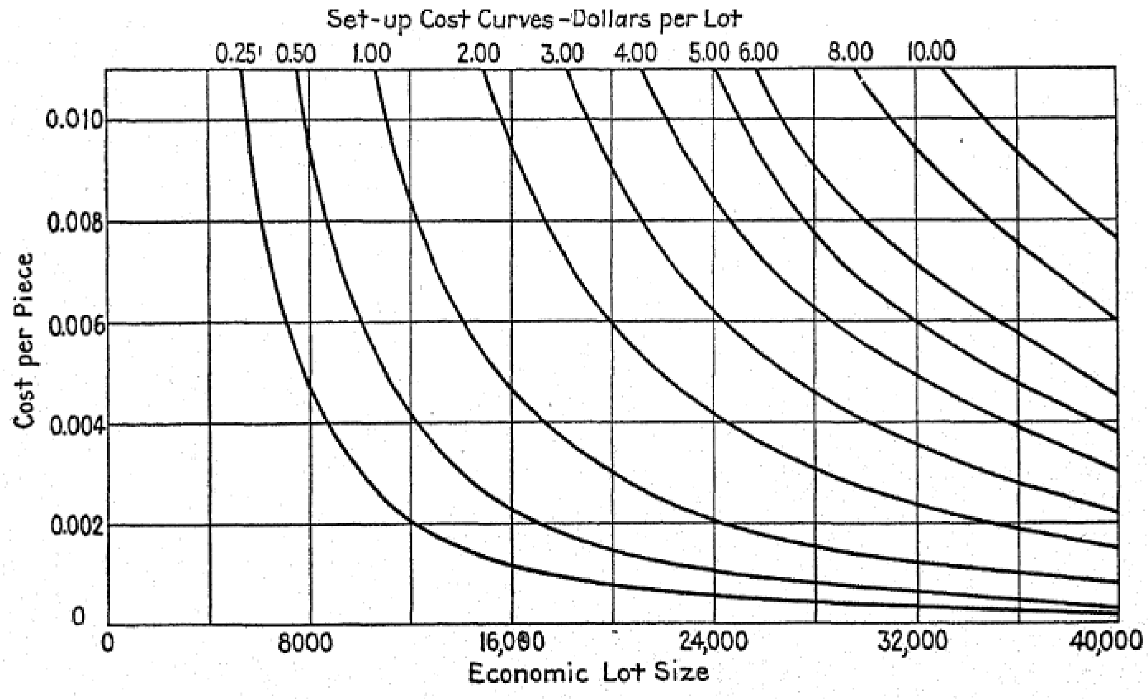

Figure. 4 – Economic Lot Size Graph

It is important to study the instruments that were used in the past to determine the economic lot, despite being based only on the general formula, they were widely disseminated and used by companies, there were handbooks and manuals to determine the economic lot because it was a common problem in manufacturing. Nowadays the methods used are more sophisticated and most require more precise methods such as dynamic programming, algorithms or through heuristics or analytical methods which will be explained in detail later.

The most valuable contribution of Raymond was, in addition to the unification of methodologies made to date, the realization of various graphs and methods very well explained and with examples that allowed the reader to find the most economical lot.

Raymond made the first allusion to ELSP by stating that many of the formulas used so far did not consider many situations where demand is variable, where setups times and inventory and holding costs have to be considered.

2 Chapter Two. EOQ Model and Transition to ELSP

1.4. Ford Whitman Harris's economical lot size model

Is not possible discuss the Economic Lot Scheduling problem without understand the basis of the **EOQ model**.

Although Harris's discovery was widely publicized and was even used in the Air Force by that time. ref Due to erroneous citation and other factors, the author was not known until many years later, Harris remained in obscurity for several years.

In his paper, Harrys defined the following variables that have appeared countless times in other publications to this day.

Unit Cost(C). *Cost in dollars per unit of output under continuous production.*

Set-up Cost (S). *Cost of getting the materials and tools ready to start work on an order. It involves also, the cost of handling the order in the office and throughout the factory.*

Interest and Depreciation on Stock (/). *Large orders in the shop mean large deliveries to the storeroom, and large deliveries mean carrying a large stock. Carrying a large stock means a lot of money tied up and a heavy depreciation. It will here be assumed that a charge of ten per cent on stock is a fair one to cover both interest and depreciation. It is probable that double this would be fairer in many instances.*

Movement (M). *It is evident that the greater the movement of the stock the larger can be the quantities manufactured on an order. This, then, is a vital factor.*

Manufacturing Interval (T). *This is the time required to make up and deliver to the storeroom an order, and, while it seldom is a vital factor, it is of value in the discussion.*

This document defines for the first time the variable to be solved in the problem in question. X that Harris defines as "Unknown size of order, or lot size, which is the most economical" This is the most economic quantity to be produced of a product.

The average stock, if the movement is regular, is one half X because the stock will spend equal amounts of time in both the up and down movements. For example, if the stock moves up 1 unit and then down 1 unit, the average stock will be 0.5 units. So evidently the cost of having the average stock will be CX/2

As the set-up cost per order is S, the average set up cost of the stock will be.

$$½(CS + S)$$

And with a depreciation of 10% by year we have

$$1/20(CS + S)$$

With an annual production of 12 M:

$$(1/240M)(CX + S)$$

The set-up cost to x units will be s/x so the total cost by unit with the interest rate will be:

$$Y = \frac{1}{240M}(CX + S) + \frac{S}{X} + C$$

So, the problem is summarized in finding the value of X that gives me the minimum Y.

The solution to this problem is a typical optimization problem where Y reduces to 240MS divided by C.

$$Y = \sqrt{\frac{240MS}{C}} = \sqrt{\frac{2x12MS}{0.1C}}$$

Erlenkotter (1990) presented a compilation of the early literature and described Harris's career as a production engineer and discovered that he was the creator of the formula that some authors believed Wilson to be because of mis references. Erlenkotter developed a scheme based on Harrys' model but did not consider more recent work in that area in his work.

The answer given by Harrys was later called **Economic Order Quantity Formula.** The following graph shows the solution of the Harris formula. Harris (1913)

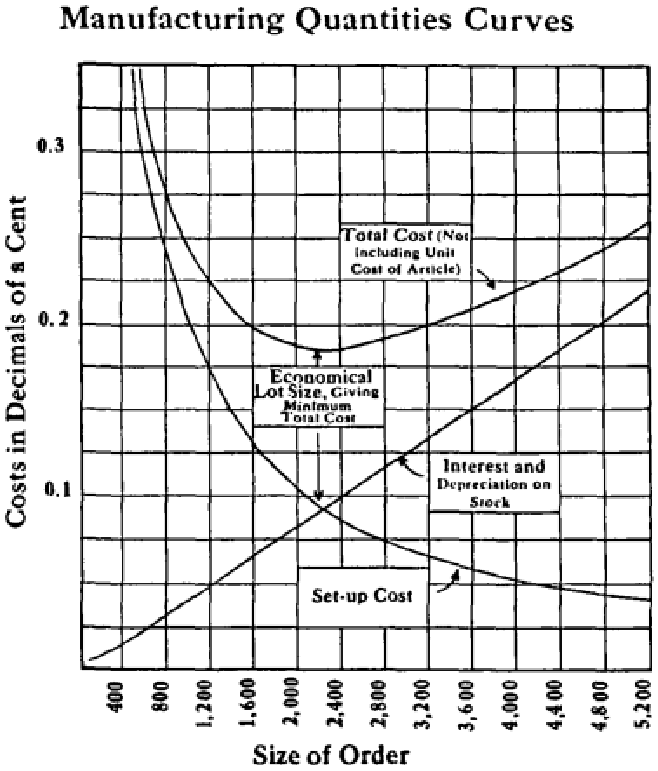

Figure. 5. Economic Order Quantity Graph

The following figure shows the value of the cost per set up part for various production quantities and the interest and depreciation under the same conditions.

As mentioned above, there was a particularly wide distribution of Harry's EOQ formula but without certainty about the author, and this caused them to be made. Initially it was called Wilson lot size formula, Then Camps Formula,

In conclusion, it may be well to say that the method given is not rigorously accurate, for many minor factors have purposely been left out of consideration. It may be objected that interest and depreciation should be figured, not only on original cost, but also on the set-up cost, since that must be incurred before the parts can be stocked. Such refinements, however, while interesting, are too fine spun to be practical.

The general theory as developed here is reasonably correct and will be found to give good results.

Here Harris states that the formula is not perfectly accurate, because it does not consider all the factors that can affect inventory levels. However, the author argues that the EOQ is still useful and is the starting point for setting inventory levels.

The minor factors to which Harry made references could include interest, depreciation, and the cost of setting up production.

2.2. Economic Production Lot

So far it has been assumed that all replenishments happen infinitely and infinitely production rate, however if the production rate is finite, assume that the average inventory (q/2) changes since the maximum inventory is less than the order quantity. This contribution was made by Taft in 1918 in his work "The most economical production lot" obtaining the renowned Economic Production Lot Formula, the derivation of which will be presented below.

$$C(q) = \frac{dA}{Q} + \frac{hq}{2k}(k - d)$$

So, the Economic quantity that gives the minimum cost according to E.W. Taft is

$$q = \sqrt{\frac{2dA}{h} * \frac{1}{1 - \frac{d}{k}}}$$

This formula was called the Economic Production Lot Erlenkotter (1990)

In 1958 Rogers studied the problem of inventory control in a single machine with multi system and applied the ELP formula to each item individually, according to his findings is usually impossible to build a practical production schedule from these order quantities since 2 or more items must be produced at the same time, in this context the ELSP rises.

In his work Rogers described a procedure for the solution in which each lot size of each product is aisled from the formula of the economic lot size EOQ. EOQ is an independent solution because ignores the principle that several products can be produced at the same time, this implies shares the capacity (utilization) of it. The value obtained with this method is always a lower bound in the calculus of the minimum cost of the problem is barely feasible.

Rogers was the first person to officially studied and mention the economic lot scheduling problem, he even mentioned the problem in the title of its work called: *"A computational approach to the economic lot scheduling problem"*. The Roger method allowed to find the minimum cost discarding the constraint capacity of the machine and solving each lot sizing in an independent way.

In 1966 Bombereg improve the Rogers method by making it simpler by stablishing a constraint in the time available for setups at the machine. In addition to the already mentioned benefits of repetitive schedules this method offers the less obvious, but possibly more important, benefits that come from being able to evaluate the effects on operating costs of the use of overtime in the plant, and the increased production capacity through capital acquisition.

2.3. Transition from EOQ to ELSP

The EOQ model revolutionized inventory management by introducing a methodology to determine the optimal order quantity that minimizes the total cost of inventory, considering holding and ordering costs. For example, a retailer might utilize the EOQ model to determine the optimal number of items to order, ensuring that the accumulated costs of storing and reordering inventory are minimized.

This model formulated by F.W. Harris in 1913, laid the foundation for inventory management by optimizing the order quantity that minimizes total inventory costs. However, the EOQ model's focus remained on a single-item scenario, wherein it did not address the complexities of scheduling multiple items in a manufacturing context, leading to the need for a more encompassing model. the advent of manufacturing environments producing multiple items on a single machine necessitated a transition towards a more comprehensive model, thereby leading to the emergence of the ELSP.

The limitation of the EOQ model in addressing single-item scenarios paved the way for the ELSP, aiming to optimize production schedules in a multi-product, single-machine environment. Rogers, in his pioneering work on the ELSP, introduced methodologies to determine the optimal production quantity and sequence for multiple items sharing a single machine, thereby extending the principles of the EOQ to a multi-product manufacturing context.

The ELSP, unlike the EOQ, had to contend with additional complexities, such as the sequencing of various products and ensuring that the production schedule aligns with varying demand patterns for different items. Here, the objective extended beyond merely minimizing costs to also ensuring that production schedules were feasible and met diverse demand requirements effectively.

The ELSP gradually evolved to incorporate various real-world complexities and methodologies to enhance its applicability in diverse manufacturing environments. Scholars like Bomberger and Rogers introduced dynamic programming approaches to navigate through the complexities of the ELSP.

In the realm of inventory management and production scheduling, the transition from the EOQ to the ELSP reflects an evolutionary path that not only encapsulated the fundamental principles of cost minimization but also expanded to encompass the complexities and demands of multi-product manufacturing environments. This journey, enriched by numerous scholars and practitioners, illustrates the progressive nature of operations research in adapting to the changing landscapes and demands of the industrial sector.

From the singular focus of the EOQ to the multi-dimensional complexities of the ELSP, the transition embodies the adaptability and progression of operations research methodologies in meeting the evolving demands of the industry. The EOQ, with its foundational principles of cost minimization, provided the steppingstone for the ELSP, which expanded these principles to navigate through the intricacies of multi-product scheduling, thereby shaping the landscape of modern production management and inventory control. The contributions of various scholars and the practical applications of these models stand testament to their pivotal role in shaping inventory management and production scheduling practices across various industries. There are other authors who make important contributions to this problem, Elmagraby (1978) made a review of the problem up to 1978, then there were Axsater (1983), Boctor (1982), Dobson (1988), Delporte and Thomas (1978), Fujita (1978), Graves and Haessler (1978), Graves (1979), Haessler (19799; Hsu (1983), Maxwell and Singh (1983), Park and Yun (1984), Roundy (1985), Vemunganti (1978) and Zipkin (1978),

2.4. ELSP Problem Description

In the classic Economic Lot Quantity case the problem simply is one of finding the lot size which minimizes the sum of the unit costs of setting up to manufacture or buy the item and of holding the resulting stock as shown. When additional variables (price as function of lot size, set-up costs as function of lot size, or production costs as function of production rate) are taken in consideration, the cost minimization expression becomes so involved that solutions are too hard. Producing multiple products on a single machine is very common in industry, continued improvements in production equipment productivity and increasing product options leads to high volume machines and facilities that produce a wide number of products.

From the in-depth reading of the literature that I have done for this work, I can clearly say that the ELSP have been studied in both academia and industry, but there are not many results that are well know, there is also a lack of common terminology, despite the fact that have been studied from the second half of the XX century.

In this section, I developed an easy method to derive the ELSP problem, I made the general assumptions used in the literature for this subject and choose simple but reasonable values in order to keep the mathematics simple.

The ELSP problem is to find the best way to produce multiple products on a single production line in a repeating cycle, such that the total cost per unit time is minimized and the demand for each product is met. The costs considered in the ELSP problem include setup costs, inventory holding costs, production costs, and other costs related to scheduling and sizing batches. Beranek (1967) cautioned decision-makers to be aware of the financial implications of lot size inventory models.

The following graph was created using Excel to show graphically the basis of the problem.

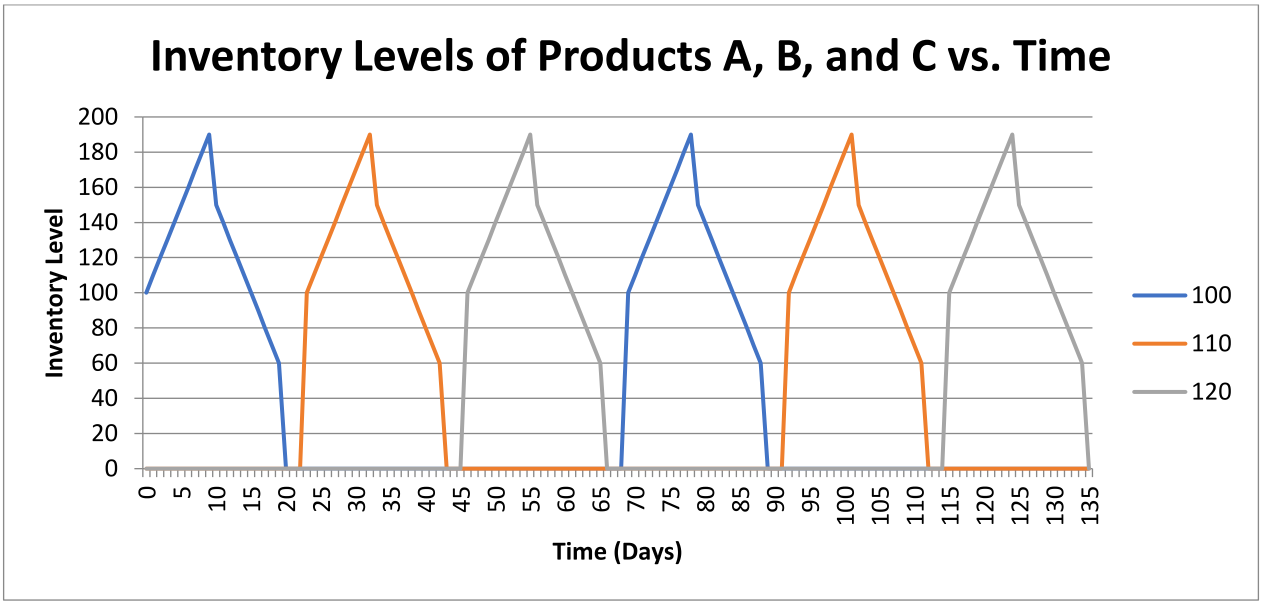

Figure 6. Graph of Inventory Levels vs Days

In the graph 1 there is a visualization of the inventory levels of three products (A, B, and C) over time, up to day 135, with a setup time of 3 days between the production shifts of these products. The x-axis represents time (in days), and the y-axis represents the inventory levels of the products. Many companies follow cyclic schedule because from a setup perspective is easier to go from one product to another.

The following part shows the mathematical and graphical demonstration of the ELSP problem, to use it to adopt the assumptions most used in the literature.

1. Only one product can be produced at a time.
2. The planning horizon is infinite.
3. The demand rate for each product is constant.
4. The production rate for each product is constant.
5. The inventory holding cost for each product is constant.
6. The time to setup for production of product is sequence independent.
7. The cost to setup for production of product is sequence independent.
8. Production will be according to a cyclic schedule with each product produced only once in a cycle.
9. Inventory will be zero when production of a product begins.
10. The following table represent the parameters used to solve the problem.

Table 2. Initial values for ELSP problem

Product	di (units/day)	pi units/day	hi ($/units/day)	Si (days)	Ci ($)
1	10	100	1	2	1000
2	20	100	2	3	1500
3	30	100	3	3	1500
			Sum	8	4000

Parameters:

$di: demand\ rate\ for\ product\ i, untis/day$

$pi: production\ rate\ for\ product\ i, \frac{units}{day}$

$\rho_i: utilization\ of\ product\ i, \rho_i = \frac{di}{pi}$

$h_i = inventory\ holding\ cost\ for\ product\ i, \frac{\frac{\$}{unit}}{day}$

$S_i = setup\ time\ for\ product\ i, days$

$C_i = setup\ cost\ for\ product\ i, \$$

2.4.1. Feasibility

Using the definitions presented above, there are several rules that can be applied in this case. For example, the utilization for each product will be less than one, $\rho_i < 1$ while for all products summed, it will be equal to or less than one. $\sum_i \rho_i \leq 1$ Otherwise, I will have too many products required and so much production time and I won't be able to meet all demand.

Then I proceed to determine the utilization for each product

Product	di/pi
1	0.1
2	0.2
3	0.3

Where the total utilization for all products is

$$\sum_i \rho_i = 0.1 + 0.2 + 0.3 = 0.6$$

It can be seen from the last table that the utilizations are less than one and it is also checked for the sum of utilization of all products. The latter value means that products take 10% ,20% and 30% of capacity individually and the total production line will be producing at **60%** of its rate capacity, this allows 40% of time available for setups and for preventative maintenance and avoid unexpected shutdowns that can dramatically disrupt the supply chain.

2.4.2. Cycle length

The first thing to determine after verifying that the utilization calculation is correct is to the Cycle Length, or in the industry more commonly known as campaign length.

The Cycle Length, t_c is the time it takes in time to perform an entire production cycle.

From the table 2 I have:

Product	**Si (days)**
1	2
2	3
3	3

And the total time of setup is 2+3+3 = **8 days**, is not possible to have 8 days of setup so t_c must be greater than 8.

If I had an 8-length cycle it means that I spend the first 2 days setting up for product 1, then I wouldn't have time to product 1 because I would have to immediately begin to setup product 2, and so on, and at the end there won't be time for production at the end.

So, there should be a **minimum cycle length** where I will have enough effective capacity to be able to produce all products.

To determine the campaign length first I will calculate the fraction of the cycle available for setups. It is given by the formula:

$$1 - \sum\rho_i$$

In this example I have 1-0.6 = 0.4 that is a 40% of the cycle time is available for setups

2.4.3. Minimum cycle length

The time available for setups depend on cycle t_C and is qual to:

$$(1 - \sum\rho_i)t_c$$

Using the values of this case we have:

$$(1 - 0.6)t_c$$

If I suppose that the cycle length is $t_c = 10$ then.

$$(1 - 0.6) * 10 = 4$$

Which is not enough time, as previously demonstrated. To determine the minimum cycle time, I have to meet the requirement:

$$(1 - \sum\rho_i)t_c \geq \sum S_i$$

From the table 2 I have:

$$\sum S_i = 8$$

And using a little algebra and rearranging:

$$t_c \geq \frac{\sum S_i}{(1 - \sum\rho_i)}$$

The last expression, the minimum cycle time, is the first thing I must calculate about the **economic lot scheduling problem.**

For this particular problem I have:

$$t_c \geq \frac{\sum S_i}{(1 - \sum\rho_i)} = \frac{8}{1 - 0.6} = \frac{0.8}{0.4} = 20$$

If add more products to the line production the numerator term sum of setups will increase. The harder problem here is what will happen to the utilizations.

Some authors have stablished the term of **cannibalization**, that will be better explained with the following example.

San Carlo Gruppo Alimentare S.p.A is an Italian company that produces the popular classical chips. Then let's suppose that San Carlo group will add to the production line the limon flavor product.

Figure 7. Two different chips products

Obviously, each chips flavor has a different demand, the total setup time will increase. The cannibalization will be presented when some consumer shifts from classical flavor to lemon flavor so the sum of utilization can increase or remain the same. If there is a lot of cannibalizations the whole denominator will decrease because I am subtracting the sum of utilizations from one.

$$t_c \geq \frac{\sum s_i}{(1 - \sum \rho_i)}$$

If I'm increasing the total setups and decreasing the denominators i.e., increasing the demands. The minimum cycle length will increase, increasing inventory levels and increasing costs.

When the total utilization is higher than 80% 85% is very likely to have problems related to increase lead times, increase delays or breakdown problems.

2.4.4. Minimum cost cycle

Now I will introduce the Minimum Cost cycle term, the next important question to make is how long the campaign length should be to minimize the total costs, in other words the costs that we are looking at total setup is Total inventory costs plus holding costs.

By setting t_c we can determine t_p that is total production time for each product.

From the formula EPQ, economic production quantity, previously mentioned in this work the average inventory for a single product is:

$$I_i = \frac{(p_i - d_i) * t_P}{2}$$

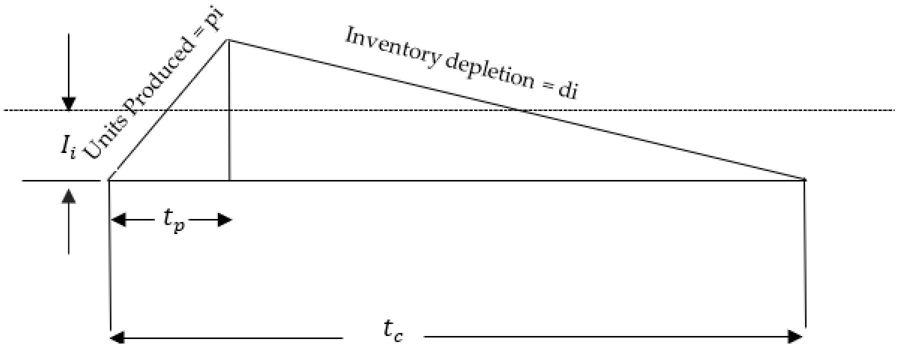

Figure 8. Graphical representation of inventory levels

To calculate the average inventory, I subtract the demand rate from the production rate. This gives us the net rate at which we are adding to or subtracting from inventory. Then multiply this value for the production time that is the length of time that takes to run a piece of equipment to produce a batch of products.

In the figure 6, the increasing part shows how long the inventory is being building up, and the decreasing part shows how inventory previously built up is being depleted. I want the cycle length, not the production time, to determine how long it takes to build up inventory. Therefore, I need to find a substitute for t_p. I can use the identity of the graph to do this.

$$p_i t_P = d_I t_c$$

Using the formula of average inventory levels, I have

$$I_i = \frac{(p_i - d_i) * t_P}{2}$$

$$p_i t_P = d_I t_c$$

$$\frac{p_i t_P}{p_i} = \frac{d_I t_c}{p_i}$$

$$t_P = \frac{d_i t_c}{p_i}$$

And now making the substitution

$$I_i = \frac{(p_i - d_i) * t_P}{2} = \frac{(p_i - d_i) * d_i}{p_i * 2} t_c = \left(\frac{p_i}{p_i} - \frac{d_i}{p_i}\right)\left(\frac{d_i}{2}\right) t_c = \frac{(1 - \rho_i) d_i t_c}{2}$$

$$I_i = \frac{(1-\rho_i)d_i t_c}{2}$$

And with the latter expression I can now determine the average inventory for each product, and it is in terms of t_c

Now I want to insert another term that is the daily holding cost, the daily holding cost for a product is given by the following equation.

$$\frac{h_i(1-\rho_i)d_i t_c}{2}$$

And the daily setup cost would be:

$$\frac{C_i}{t_c}$$

If my cycle length is 20 days and the setup cost is 2000$ then the daily setup by day will be 200$/day if the setup last longer, the daily setup cost will increase, and I must hold more inventory for a longer period of time.

For all products I have total costs:

$$T_c = \frac{\sum_i C_i}{t_c} + \left[\sum_i \frac{h_i(1-\rho_i)d_i}{2}\right] t_c$$

In this expression I add all terms together and sum up for all products and all the variables are known except t_c

From the last expression for total costs, I can see as expected that as the cycle length gets bigger the setup cost is going to be smaller per day, but the inventory holding cost is going up, and what I am looking for, is a balance that minimize the total cost and trying to find a balance between setup costs and holding costs.

So, using the last expressions is possible to extend the table.

Table 3. Final ELSP parameters

Product	di (units/day)	pi units/day	hi ($/units/day)	Si (days)	Ci ($)	ρ_i	$(1-\rho_i)d_i$	$\frac{h_i(1-\rho_i)d_i}{2}$
1	10	100	1	2	1000	0.1	0.9	4.5
2	20	100	2	3	1500	0.2	0.8	16
3	30	100	3	3	1500	0.3	0.7	31.5
			Sum	8	*4000*	*0.6*	*2.4*	*52*

And the Total costs

$$T_c = \frac{4000}{t_c} + 52t_c$$

Where the $\frac{4000}{t_c}$ component are the order costs and the $52t_c$ part correspond to holding costs

Graphing the total cost function, I have

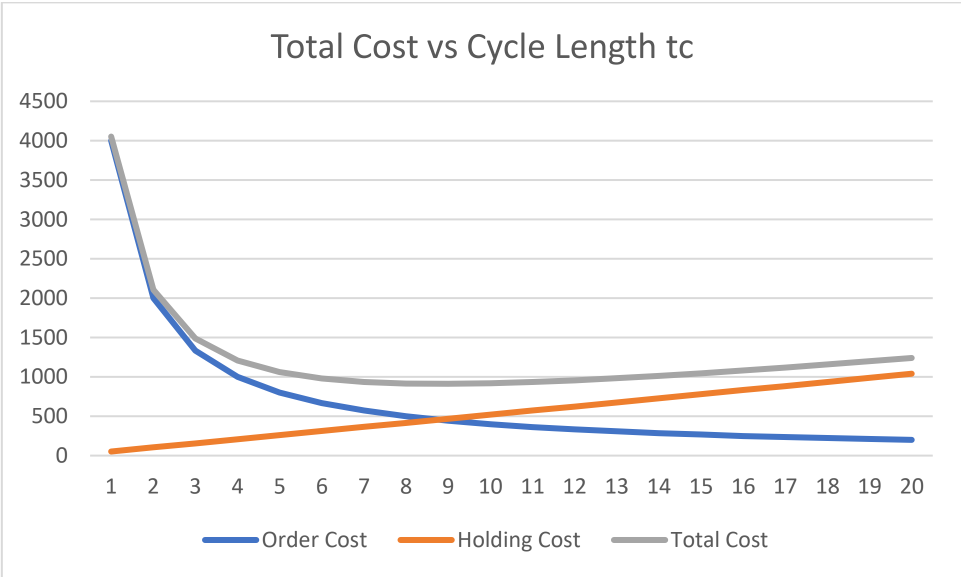

Figure 9a. Graphical representation of total cost and cycle length

This seems the EOQ formula and essentially it is, but the only difference is that the x axis is the number of days of the cycle or campaign length and not the lot size.

Now it looks that the minimum cost is in this point where the holding cost and ordering cost are equal and in fact it is.

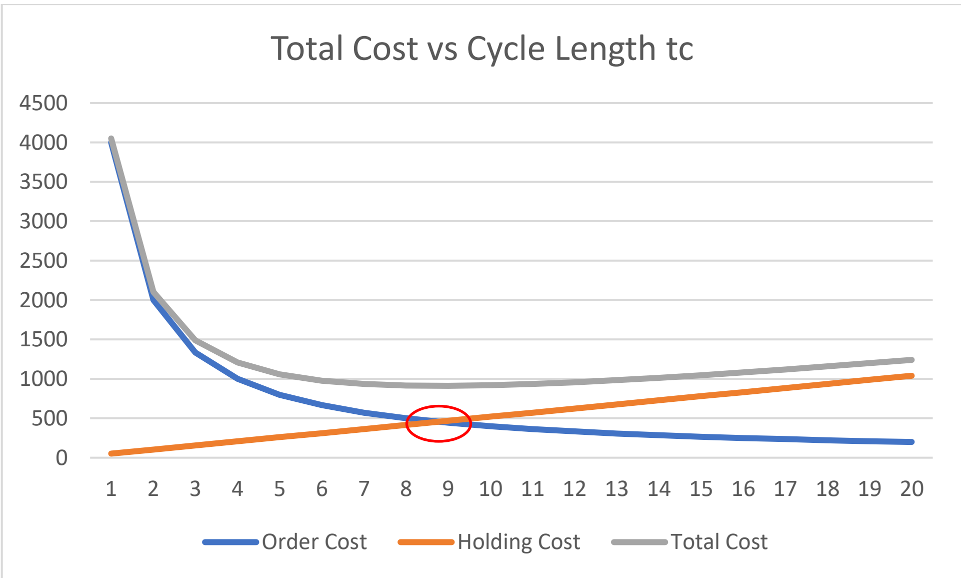

Figure 9b. Graphical cost minimization

But if I use some calculus using the formula for the cycle length that minimizes the total cost

$$t_c = \sqrt{\frac{\sum_i C_i}{\sum_i \frac{h_i(1-\rho_i)d_i}{2}}} = \sqrt{\frac{4000}{52}} = 8.77\ days$$

And this is ideal in terms of cost. But I previously calculated that we need at least a 20-day cycle.

$$t_c \leq \frac{\sum_i S_i\, n}{(1 - \sum_i \rho_i)} = \frac{8}{1-0.6} = \frac{8}{0.4} = 20$$

Without a 20-day cycle I cannot make the product because from the 8.77 days' time cycle 8 days will be consumed by setups and only have ¾ of day to make the products with is practical impossible.

As a summary: The scheduling problem is scheduling the production of a set of items in a single machine minimizing the long run average holding up and set up costs under the assumptions of know constant demand and production rates.

Is evident that when the number of items is large of two, and the schedule is "tight" much computation will be involved in going through numerous rounds of the steps and that the use of more sophisticates' methods will be needed. For values of holding and setup costs and if there would be restrictions on lot sizes, no feasible schedule may result. The methods have the restriction that only will give a minimum cost schedule when there is a feasible schedule of specified accuracy in a finite number of steps.

This last example shows how the ELSP problem is presented in the literature. This approach is usually called single-stage ELSP, and it applies to manufacturing systems that produce products with a single level of structure, meaning that the end products are directly produced from raw materials without any intermediate products or subassemblies.

When one or more of the assumptions stated at the beginning of this chapter are not met, the problem becomes a multi-stage ELSP problem. Multi-stage ELSP problems are more complex and require heuristic solutions or more robust mathematical models.

2.5. NP-hard classification

In 1997 Gallego and Shaw proved that the ELSP problem is a NP-hard. This classification belongs to a class of problems that are at least as difficult to solve as any problem in NP, which is a class of problems that can be solved in a polynomial time by a non-deterministic Turing machine. In other words, any NP-hard problem can be reduced to any NP problem, but the other way around is not necessarily true. So, there is no known polynomial time algorithm for solving it. This means that the time it takes to solve the ELSP problem grows exponentially with the size of the problem. However, there are good heuristics that have been published and where is possible to obtain feasible production schedules whose average cost is often close to optimal one. Dobson (1987)

To understand better what a NP-hard problem is consider the following example:

If a person is trying to find the shortest route between two cities. This person is allowed to try every possible route, then will eventually find the shortest route. However, the number of possible routes is very large, so this could take a very long time.

If, on the other hand, the same person is only allowed to try a limited number of routes, then may not find the shortest route. However, can still find a good route that is close to the shortest route.

Heuristic algorithms for the ELSP problem are like trying to find the shortest route between two cities using a limited number of routes. They can help you to find a good solution to the problem, but they may not find the optimal solution.

3 Chapter Three. Classification Proposal:

Several significant works propose classifications for the ELSP, including literature reviews that date back to Fairfield E. Raymond's 1931 contribution to the EOQ problem. Notably, Elmaghraby in 1978 made a crucial contribution by suggesting a classification into two categories: analytical solutions and heuristic approaches. In 2015, Santander-Mercado and Jubiz-Diaz conducted a comprehensive literature review, delving into articles produced up to 1998. They presented a classification based on scheduling policy and solving methodologies, which significantly influenced subsequent research. Their classification is depicted in the following graph.

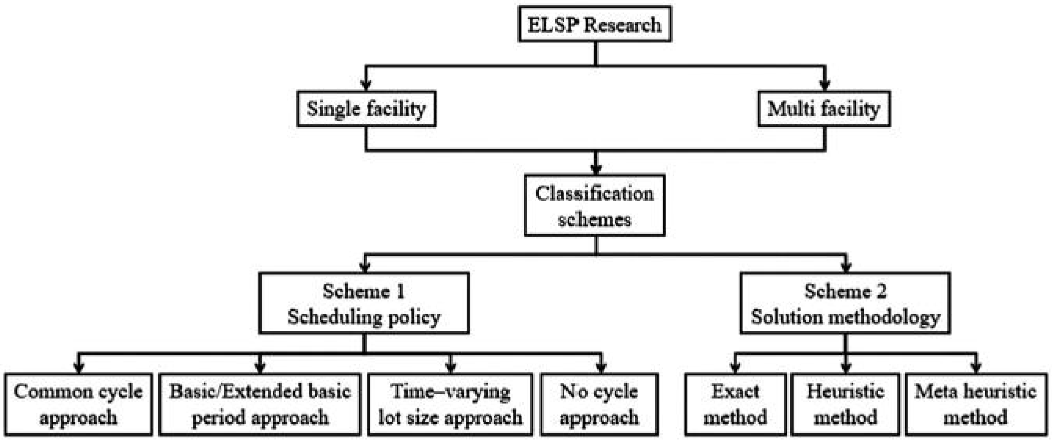

Figure 10. Santander-Mercado and Jubiz-Diaz ELSP classification

Beck and Glock (2019) conducted a study involving 242 articles obtained through a systematic literature search. Employing a content analysis methodology, they scrutinized the literature sample. The findings indicate a predominant focus within the ELSP literature on the development of solution methodologies and specific assumptions associated with setup and production rates. Stochastic and dynamic problems have also garnered considerable attention. Conversely, areas such as energy cost, sustainability, and practical applications have not been thoroughly explored. The paper posits potential avenues for future research, including the incorporation of energy efficiency and sustainability criteria into ELSP models and an examination of emerging research trends over time.

3.1. New classification framework

I will present a comparative analysis of the different solutions proposed so far and I will write down the strengths and weaknesses of each method, explaining in what circumstances it would be better to use one method or another.

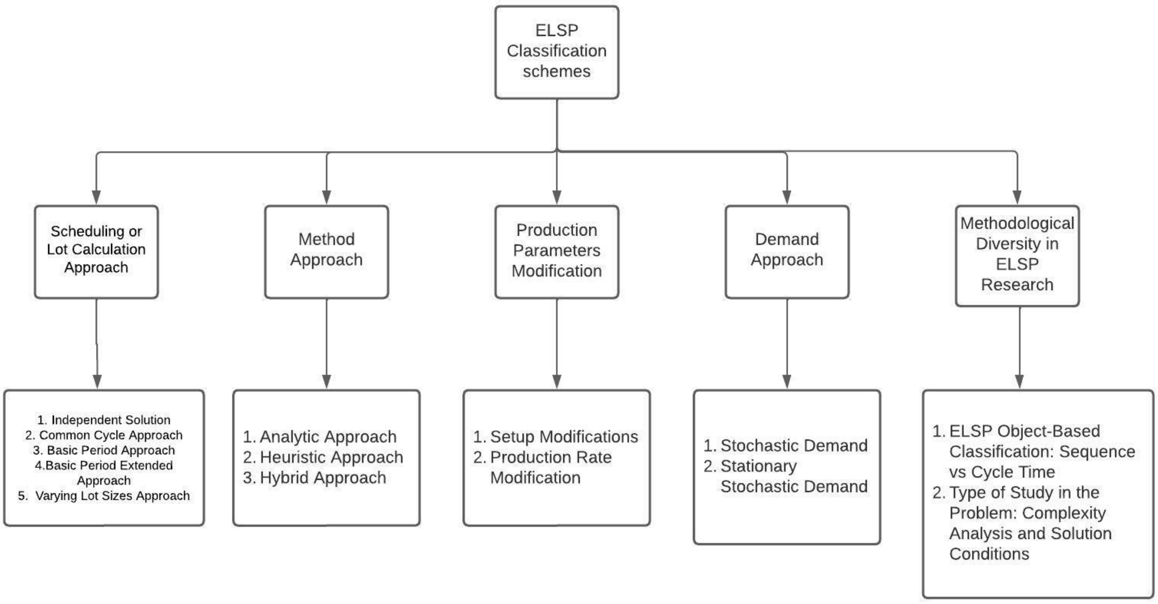

Figure 11. New Classification Framework

3.1.1. Scheduling or Lot Calculation Approach

Is the most elemental approach for the ELSP problem, consist in stablish some restrictions in cycle time in order to optimize the duration of the individual cycles constrained to the restrictions imposed. The solution in this say is feasible and optimal for this group of solutions. 5 taxonomic classifications are proposed.

3.1.1.1. Independent Solution:

The solution proposed by Harris in 1913, explained extensively in Chapter 2, is commonly called the Independent Solution. And it consists of calculating the size of the lotus in isolation, based on the formula EOQ, Economical Order Quantity. It is independent since it ignores the fact that several products can be produced by the same machine. This solution is rarely feasible because each product is considered in isolation from the real problem.

The following function again represents the total costs where C_i in this case I represent the cycle length for each item as T_i

$$CT = \frac{1}{T_i} C_i + \frac{h_i d_i T_i}{2}(1 - \frac{d_i}{p_i})$$

By deriving from time, I get

$$\frac{\partial C}{\partial T_i} = \frac{\partial \left(\frac{1}{T_i} C_i + \frac{h_i d_i T_I}{2}\left(1 - \frac{d_i}{p_i}\right)\right)}{\partial T_i} = 0$$

$$-\frac{1}{T_i^2} = \frac{\partial \left(\frac{1}{T_i} C_i + \frac{h_i d_i T_I}{2}\left(1 - \frac{d_i}{p_i}\right)\right)}{\partial T_i} = 0$$

$$T_i = \sqrt{\frac{2C_i}{h_i d_i (1 - \frac{d_i}{p_i})}}$$

The equation above is the independent solution that represents the cycle time that minimizes the total costs.

- ***Strengths***: Simplicity and ease of application.
- ***Weaknesses:*** Ignores interdependencies between products, limiting real-world feasibility.

While providing a straightforward solution, its impracticality in dynamic manufacturing environments raises concerns.

3.1.1.2. Common Cycle Approach

This model was proposed by Hanssmann in 1962, the aim of which is to obtain cyclical co-production programs of a given duration, this given duration is the common cycle, these cycles are repeated periodically for a multi-item production system. During scheduling each product will be produced only once in each cycle. The aim is to obtain a common co-production cycle that, being able to include the set-up times of each product, has a global cost.

Jones and Inman in 1989 showed that if the ratio of setup cost to inventory cost is equivalent or very similar for all products, the program proposed by this method is very close in the common cycle. Below is the mathematical relationship of this solution.

$$CT = \frac{1}{T_i} \sum_{i=1}^{g} C_i + T \sum_{i=1}^{g} h_i \frac{d_i}{2}(1 - \frac{d_i}{p_i})$$

$$\frac{\partial C}{\partial T_i} = \frac{\partial\left(\frac{1}{T_i}\sum_{i=1}^{g} C_i + T\sum_{i=1}^{g} h_i \frac{d_i}{2}\left(1 - \frac{d_i}{p_i}\right)\right)}{\partial T} = 0$$

$$-\frac{1}{T^2}\sum_{i=1}^{g} C_i + T\sum_{i=1}^{g} h_i \frac{d_i}{2}\left(1 - \frac{d_i}{p_i}\right) = 0$$

$$T = \sqrt{\frac{2\sum_{i=1}^{g} C_i}{\sum_{i=1}^{g} h_i\, d_i (1 - \frac{d_i}{p_i})}}$$

This solution represents an upper bound of the problem.

- ***Strengths**:* Global cost optimization, upper bound solution
- ***Weaknesses**:* Assumes uniform setup-to-inventory cost ratios, limiting adaptability, may lack practicality in diverse scenarios.

3.1.1.3. Basic Period Approach

This method was proposed by Bomberger in 1966 and consists of taking different cycles for each of the items or projects, but with the consideration that each independent cycle time of each product is an integer multiple of the basic period that is called T_{pb} This basic period is long enough to accommodate the production of all products. The method was also proposed by Dobson in 1987 and Gallego and Moon in 1992

$$T^e = k_i T_{pb}$$

Again, the total cost function consists of the number of setups performed and the cost of storing the products within the cycle.

$$CT = \frac{1}{T_{pb}}\sum_{i=1}^{g}\frac{C_i}{k_i} + T_{pb}\sum_{i=1}^{g} k_i h_i \frac{d_i}{2}(1 - \frac{d_i}{p_i})$$

- ***Strengths**:* Integer multiples provide structure, and feasibility enhancements in Dobson's extension.
- ***Weaknesses**:* May limit adaptability in dynamic manufacturing environments.

3.1.1.4. Basic Period Extended Approach

This method was proposed by Elmaghraby in 1978, which addresses the problem as an extension of Bomberger's method. A basic period T_{pb} is assumed and the cycle time is represented by integer multiples of T_{pb}. The difference is that two consecutive base periods each of T_{pb} duration are taken, and the items are loaded into these two base periods.

From this method, the feasibility constraint of the common cycle is relaxed. The extended basic period method is considered superior to the basic period method, but its resolution is more complex in terms of the quality of the solution. Haessler 1979 sun 2010

- ***Strengths**:* Feasibility constraint of the common cycle is relaxed.
- ***Weaknesses**:* Resolution is more complex

3.1.1.5. Varying Lot Sizes Approach

The first to write about this problem was Maxwell in 1964, in his solution he proposed different batch sizes for any product during a long cycle. The aim is to overcome the feasibility problems that may occur to the other approaches, PB, PBE. This formulation allows you to modify production orders over time and specifically includes setup times in the problem formulation.

Zipkin in 1991 states that the advantage of this method is the presentation of a more feasible algorithm. Gallego and Shaw in 1997 showed that even with this approximation it is still NP-Hard

***Strengths**:* Addresses feasibility issues by allowing different batch sizes.

***Weaknesses**:* NP-Hard nature, limiting practicality.

3.1.2. Method Approach

According to the methodology used, it is possible to classify this category into the following 3 categories. Analytical methods with which the optimum can be obtained from a limited version of the original problem, methods that sometimes achieve good and sometimes very good solutions to the problem through heuristics. and the combination of the two methodologies using a mix of computational techniques and heuristics with the aim of improving the solution.

3.1.2.1. Analytic Approach

Analytical methods consisting of mathematical problems solved by programming, for example Bomberger developed a Dynamic programming approach to solve the

ELSP based on the Basic Period method. Madigan in 1966 presented a method of simple resolution with infinite planning horizon under BP policy.

Fujita in 1978 reduced the problem to a simple systematic calculation and feasibility check. Other important solutions were those of Elmaghraby in 1978 and Park and Yun in 1984 where they proposed an enumeration algorithm based on feasibility tests of schedules.

Hodgson and Nuttel in 1986 developed a linear programming model that presents inventory cost as a linear function of the cycle.

- ***Strengths:*** Analytical methods, such as linear programming or dynamic programming, exhibit several strengths when applied to the Economic Lot Scheduling Problem (ELSP). These methods provide optimality assurance, ensuring the derivation of optimal solutions, which is particularly advantageous in scenarios where achieving the best possible solution is critical. Additionally, analytical methods are rooted in rigorous mathematical and theoretical foundations, offering a clear understanding of the problem and enabling insights into optimal scheduling and lot sizing. Moreover, their adaptability to different constraints allows for a nuanced exploration of various manufacturing scenarios, enhancing their versatility in addressing complex real-world constraints.

- ***Weaknesses*** Solving large instances may demand substantial computational resources and time. Sensitivity to model assumptions is another limitation, with the effectiveness of these methods contingent on the accuracy of underlying assumptions and parameters, making them vulnerable to deviations from real-world conditions. Furthermore, the applicability of analytical methods diminishes as the size of ELSP instances increases, rendering them impractical for scenarios involving numerous products and machines. Challenges also arise in handling dynamic environments, where the assumption of static parameters may limit their adaptability to rapidly changing manufacturing conditions. Additionally, the complexity of implementation poses a hurdle, requiring specialized knowledge in mathematical optimization and potentially excluding practitioners without advanced analytical skills from leveraging these methods effectively.

3.1.2.2. Heuristic Approach

Heuristic methods sometimes lead to good and very good solutions to the problem.

- ***Strengths**:* Heuristic approaches showcase remarkable efficiency in addressing the intricate challenges of the Economic Lot Scheduling Problem (ELSP) within practical timeframes, particularly excelling in the complex realms of scheduling and optimization inherent in manufacturing. The flexibility and robustness exhibited by heuristic approaches proves instrumental in seamlessly adapting to the diverse variations and constraints inherent in ELSP. This adaptability is especially vital in dynamic manufacturing environments, where heuristics prove adept at navigating through complex solution spaces. Leveraging their renowned computational speed, heuristics offer a significant advantage by swiftly generating feasible schedules, thereby enhancing their adaptability to the evolving requirements of production scenarios.

- ***Weaknesses**:* Their weakness lies in the absence of a guarantee for optimality, potentially yielding suboptimal results with varying quality. The challenge of solution quality becomes pronounced in scenarios where optimal outcomes are paramount. Heuristics also exhibit a high sensitivity to initial conditions or parameter choices, and suboptimal selections at the outset can lead to less effective solutions. Additionally, the limited depth of problem understanding offered by heuristics presents a constraint, impeding the identification of optimal strategies and exploration of intricate production relationships within the Economic Lot Scheduling Problem (ELSP). Generalization challenges further arise, as heuristics may struggle to extend their effectiveness consistently across different instances or variations of ELSP, influenced by specific problem characteristics. Moreover, the inherent trade-off challenges in balancing solution quality and computational speed are prominent, requiring a judicious consideration of this delicate equilibrium in specific heuristic applications.

3.1.2.3. Hybrid Approach

The complexity of the ELSP, especially for the multi-facility version, has led some authors to propose solutions that include computation techniques with heuristics, obtaining an improvement of the solutions. Chang and Yao (2008) proposed a threepass approach using Carreno's (1990) heuristic of the genetic algorithm and a binary search algorithm, generating optimal schedules.

Chang and Yao (2010) introduced a two-phase scheme employing Carreno's heuristic to simultaneously allocate products to facilities. Santander-Mercado (2016). Following the assignment, Dobson's (1987) heuristic was applied to address the Economic Lot Scheduling Problem (ELSP) for individual facilities, generating feasible schedules using the Total Variable Lot Size approach. In an alternate strategy, Chan, Chang, and Chan (2012) advocated a division of the problem into a master problem and subproblems. The master problem optimizes item allocation to facilities, while lot sizing and sequencing decisions for each facility are determined through integer programming and Genetic Algorithms, respectively. Chan (2012).

- ***Strengths****:* Leverages the strengths of both analytical and heuristic methods.
- ***Weaknesses****:* Potential increased complexity.

A promising avenue for achieving a balance between optimality and practicality, particularly in complex ELSP scenarios.

3.1.3. Production Parameters Modification

A general classification of the contributions identified in the literature review is proposed, based on the main variable that can be modified. Two major groups emerge: those in which the production ratio can be modified and those in which concepts related to the setup are modified.

3.1.3.1. Setup Modifications

The setup modification approach offers a practical approach to enhance the adaptability of solutions to real-world manufacturing scenarios. There are different ways to modify the setup of a production process, the following were found in the literature:

Cost of Setup and Inventory: Analyze the trade-off between setup costs and inventory holding costs.

Time of Setup: The time required for setup can be a crucial factor in scheduling. Some authors proposed modifications that aim to minimize the setup time or explore the effects of varying setup times on the overall production schedule.

Setup and Quality: Some studies proposed how modifications to the setup process influence the quality of the produced items. This could include considerations for ensuring a smooth setup to maintain product quality.

Stabilization of the Period: Some authors addressed how setup modifications contribute to stabilizing production periods. This could involve strategies to minimize disruptions caused by setup changes and enhance the overall stability of the manufacturing process.

- ***Strengths**:* Addresses real-world constraints and variations.
- ***Weaknesses**:* Complexity in modeling and implementation.

3.1.3.2. Production Rate Modification

The production rate modification classification consists basically of installations with low utilization and Shelf Life that refers to the duration or time span during which a produced item remains viable or suitable for use or sale. This approach enables optimization beyond lot sizing and sequencing, addressing the broader dynamics of the manufacturing process.

- ***Strengths**:* Offers flexibility in adjusting production rates to optimize efficiency.
- ***Weaknesses**:* Complexity in balancing production rate modifications with other constraints.

3.1.4. Demand Approach

This approach acknowledges the dynamic nature of demand patterns and seeks to optimize production schedules in response to fluctuating or stochastic demand scenarios. A typology of articles can be distinguished, focusing on the evaluation of various well-known heuristics through simulation, introducing modifications to some fundamental characteristics of the ELSP problem. In some instances, the same heuristics are repeated across different articles, such as those explored by Bomberger (1966), Doll and Whylbark (1973), Fransoo and others (1995), Segerstedt (1999), and Vergin (1978). For all cases presented in this work classified into the demand approach method, the classical feature of deterministic demand has been modified.

3.1.4.1. Stochastic Demand

In the stochastic demand approach, the ELSP accounts for the inherent uncertainty in demand. Stochastic demand implies that the quantity of products needed within a given time frame follows a probability distribution rather than being fixed. This approach recognizes the unpredictable nature of customer orders and aims to develop scheduling strategies that can adapt to varying demand levels. By incorporating probabilistic models or stochastic processes, the ELSP aims to create production schedules that are robust and resilient in the face of demand fluctuations. This approach provides resilience against demand fluctuations, especially in unpredictable markets, but requires sophisticated modeling.

- ***Strengths**:* Accounts for inherent uncertainty in demand patterns.
- ***Weaknesses**:* Increased complexity in modeling stochastic processes.

3.1.4.2. Stationary Stochastic Demand

Within the broader demand approach, stationary stochastic demand specifically refers to a type of stochastic demand where the statistical properties of the demand distribution remain constant over time. While the demand levels may vary, the underlying characteristics of the stochastic process governing these variations remain unchanged. This allows for the application of time-independent probabilistic models to predict and manage demand fluctuations. Strategies under the stationary stochastic demand approach may involve setting lot sizes and production schedules that balance the costs associated with inventory holding and setup against the uncertainty posed by stochastic demand.

- ***Strengths**:* Time-independent probabilistic models for predicting demand fluctuations.
- ***Weaknesses**:* Limited adaptability to changing market conditions.

3.1.5. Methodological diversity in ELSP research

In this classification scheme instead of presenting solutions I present a classification system that delves into the intricate landscape of the Economic Lot Scheduling Problem (ELSP) by categorizing methodologies based on diverse dimensions. It encompasses two primary branches: one focusing on the sequence and/or cycle time objectives within the ELSP problem and the other addressing the complexity analysis and solution conditions. The first branch emphasizes the critical interplay between scheduling and lot sizing, highlighting the equivalence between lot size calculation and cycle time determination. Notably, it underscores the historical evolution of sequencing rules and heuristics. The second branch explores the problem's complexity, establishing its NP-hard nature and providing insights into optimal and feasible conditions. The inclusion of various algorithms, both analytical and heuristic, enriches the comprehensive framework, offering researchers a nuanced understanding of ELSP challenges and diverse solution approaches. The title encapsulates the essence of this classification, emphasizing the methodological richness present in ELSP research.

3.1.5.1. ELSP object-based classification: sequence vs cycle time

The comprehensive ELSP problem involves determining both the manufacturing sequence and the quantity to produce for each item, commonly referred to as calculating the lot size. As outlined in the section on basic concepts of the ELSP problem.

The demand in the classical problem is known and deterministic, it can be asserted that calculating the lot size is equivalent to determining either the cycle time or the production frequency of product.

However, in the existing literature, the number of references pertaining to devising new methods for calculating the optimal cycle size surpasses those aimed at introducing novel approaches to establishing the production sequence. One of the earliest sequencing rules is credited to Delporte (1977), who formulated a set of heuristics for determining the manufacturing order of different items. Another sequencing heuristic, included in Dobson's study in 1987 has been utilized by various authors such as Gallego (1992) and Zipkin (1991). Despite this, one of the most straightforward and widely adopted sequencing rules, noted by several authors.

3.1.5.2. Type of study addressed in the problem: complexity analysis and solution conditions.

There have been found two articles that study the complexity of the problem. In the first instance, Hsu (1983) demonstrates that even a highly restrictive version of the original Economic Lot Scheduling Problem (ELSP) transforms into an NP-hard problem. His study introduces an implicit enumeration procedure to test feasibility. Subsequently, Gallego (1997) establish that ELSP is strictly NP-hard under general cyclic programs, cyclic programs with zero inventories, cyclic programs with invariant times, and cyclic programs with basic periods.

Moreover, a variety of algorithms proposed by different researchers, covering both analytical and heuristic approaches, establish optimality conditions in certain instances. Examples include Bourland and Yano (1997), Carstensen (1999), Gallego (1992), On the contrary, alternate algorithms focus on feasibility conditions, as observed in the works of Bomberger (1966), Davis (1990), Dobson (1987), Doll and Whylbark (1973), Elmaghraby (1978), Goyal (1997), Haessler (1976), Hanssmann (1962), Hsu (1983), Madigan (1968) Vemuganti (1978)

3.2. Comparative conclusions

The comparative analysis of various methods for addressing the Economic Lot Scheduling Problem (ELSP) reveals key considerations related to feasibility, optimality, flexibility, and practical applicability. The trade-off between feasibility and optimality proves critical, as methods like Independent Solution and Common Cycle Approaches offer simplicity but may lack adaptability. In contrast, advanced methods such as the Varying Lot Sizes Approach and hybrid solutions prioritize feasibility, addressing real-world complexities but potentially falling short of achieving global optimality. Flexibility is highlighted as crucial, with methods like the Basic Period and its extensions striking a commendable balance between structural robustness and adaptability, suitable for a broader range of manufacturing scenarios. Practical applicability underscores the significance of adaptability in dynamic manufacturing environments, with heuristic approaches and hybrid solutions emerging as practical choices for efficiently navigating complex solution spaces.

The incorporation of production parameters modification methods, including setup modifications, production rate adjustments, and the demand approach, enriches the comparative landscape. Setup modifications offer practical means to enhance adaptability to real-world constraints but pose challenges in modeling complexity. Production rate modification, coupled with setup changes, introduces flexibility but requires careful consideration in balancing with other constraints. The demand approach, particularly the stochastic demand approach, addresses uncertainty but demands sophisticated modeling tools, with the stationary stochastic demand approach providing a compromise between simplicity and accuracy for relatively stable demand characteristics.

In conclusion, there is no one-size-fits-all solution for ELSP, emphasizing the need for a nuanced understanding of each method's strengths and limitations. The choice of method depends on the specific manufacturing context, requiring careful consideration of trade-offs and priorities. The continuous evolution of manufacturing dynamics necessitates ongoing exploration and refinement of methods to meet the evolving demands of ELSP. Hybrid approaches, integrating analytical rigor with heuristic adaptability, show promise in providing holistic solutions to the multifaceted challenges inherent in the modern manufacturing landscape.

3.3. What method to use?

One of the objectives of this study is to facilitate the selection of a method that can be implemented for planning in real-world situations. As indicated earlier, there is no simple solution to the ELSP. It is necessary to employ procedures with heuristic algorithms. For example, the methods classified in this study as Scheduling or Lot Calculation Approach assume that the optimal solution takes the form of a basic cycle. On the other hand, some authors propose non-cyclical solutions. Boctor (1987) studied a special case with only two products that must be produced and demonstrated that a necessary condition for the production of N products in ELSP is that each product cycle must be an integer multiple of some basic period.

However, the performance of a heuristic method depends on the particularities of the problem and the properties and variables in the analyzed situation. Many papers in the literature tend to present results for a single problem, making it important to conduct a comparative analysis of various methods. It is desirable for the selected method to minimize the use of ad hoc methods or trial and error, as it will be used by production planners who may not be familiar with complex mathematics.

That being stated the desirable characteristics for any method intended for real-world implementation include:

1. Viability must be ensured without exception from the outset.
2. The solution should be straightforward without requiring ad hoc steps.
3. A structured computational approach is preferred, emphasizing efficiency, and minimizing the need for trial and error.
4. No external educated guesses for specific parameters should be necessary, and there should be no need for initial assumptions in the planning process at an infinite horizon.

Studying which of the many methods meet these characteristics is an extensive task beyond the scope of this thesis. However, after a superficial analysis of some common methods, the following observations can be made:

Bomberger's method, a type of analytical solution using the Basic Period method, presents a formal algorithm that can be replicated. However, it is not a feasible solution due to its computational cost and impracticality for large ELSP problems. The Bomberger method is sensitive to the initial estimate of the basic period and is not very accurate for ELSP with multiple basic periods.

On the other hand, Rogers (1958) presents a non-cyclical, heuristic solution. However, it is neither independent nor viable, efficient, or suitable for infinite horizon planning.

Contrarily, the Elmaghraby (1978) method is feasible for computing the basic period of the ELSP but lacks efficiency. It is a modified version of the Bomberger method that is more robust to noise and less sensitive to the initial estimate of the basic period. However, it is also more computationally expensive than the Bomberger method.

In contrast, the Haessler Extended Basic Period (1979) algorithm, present viable, independent solutions with infinite horizon planning and general solutions. Achieving cost-effective and realistic schedules compared to other EBP approaches is credited to its integration of analytical calculation and constrained enumeration. This effectiveness is further bolstered by an explicit built-in procedure for generating feasible schedules, as detailed by Lopez and Kingsman (1991).

3.4. Classification of literature reviewed

Finally, in this section I present the main references for the different methods described in this chapter. The criteria for selecting authors are based on two main factors. The first criterion involves authors who were pioneers in studying the problem, contributing to the initial definition of ELSP (Earliest Late Start Problem) and some of the described methods. There is a common agreement on the significance of these authors in the field, and they are consistently cited, such as Rogers, Bomberger, Gallego, Moon, Elmaghraby, and others.

The second criterion considers authors who are repeatedly cited in the literature and introduce a new method or a novel approach to addressing the problem. Given the extensive nature of the literature, it is impractical to mention all authors, but by referencing these individuals, one can gain a clear understanding of the various approaches available for solving the ELSP problem.

Table 4. Scheduling or Lot Calculation Approach

Scheduling or Lot Calculation Approach	
Method	**Authors**
Independent Solution	Gallego (1990), Madigan (1968), Mallya (1992), Rogers (1958)
Common Cycle Approach	Eilon (1957), Madigan (1968), Hansman (1962), Jones (1989), Hwang (1964), Gallego (1990), Khoury (2001), Maxwell (1964)
Basic Period Approach	Doll and Whybark (1973), Haessler and Hogue (1976), Hahm and Yano (1998), Khouja and others (1998), Leachman and Gascon (1988), Soman and Others (2004)
Basic Period Extended Approach	Elmaghraby (1978), Fujita (1978), Geng and Vickson (1988), Haessler (1978), Larrañeta and Onieva (1988)
Varying Lot Sizes Approach	Carstensen (1999), Chang and Chan (2012) Chang and Others (2006), Chan and Chung (2010), Delporte and Thomas (1977), Dobson (1987), Gallego and Roundy (1992); Gallego and Shaw (1997), Liu, Wu and Zhou (2008); Luo (2010); Maxwell (1964), Moon and others (1998), Moon, Hahm and Lee (1998), Zipkin (1991)

Table 5. Method Approach

Method Approach	
Method	**Authors**
Heuristic Approach	Dobson (1987), Doll and others (1973) Geng and Vickson (1998), Giri and Moon (2004) Giri, Moon and Yun (2003) Haessler (1979) Haessler and others (1976) Khouja (1997) Larreñeta and Onieva (1988) Madigan (1968) Maxwell (1964) Moon, Giri and Choi (2002) Moon and Christy (1998) Soman and others (1969) Wagner and Davis (2002) Yao and Elmagraby (2001)
Analytic Approach	Boomberger (1966) Dobson (1987) Elmaghraby and others (1970) Fujita (1978) Gallego (1990) Gallego and Joneja (1994) Gallego and others (1997) Goyal (1973) Haessler (1979) Hahm and others (1995) Hodgson (1970) Hwang and others (1993) Khoury and others (2001) Krone (1964) Larsen (2005) Madigan (1968) Maxwell (1964) Park and Yun (1984) Roundy (1989) Silver (1990)
Hybrid Approach	Carstensen (1999) Chang and Others (2006) Chang and Yao (2009) Delporte and Thomas (1977) Dobson (1987) Dobson (1987) Elmaghraby (1978) Elmaghraby (1978) Khouja (2000) Moon and others (2002) Rogers (1958) Yao and Huang (2005) Zipkin (1991)

Table 6. Demand Approach

Production Parameters Modification	
Method	**Authors**
Setup Modification	Allen (1990), Gallego (1993), Khouja (1997), Moon and others (1991), Moon and Christy (1998), Silver (1990), Silver (1995), Viswanathan and Goyal (1997)
Production Rate Modification	Allen (1990) Silver (1990) Moon and others (1991) Gallego (1993) Khouja (1997) Moon and Christy (1998)

Table 7. Demand Approach

Demand Approach	
Method	**Authors**
Stochastic Demand	Brander (2004), Gascon (1994), Leachmand and others (1988); Soman and others (2004)
Stationary Stochastic Demand	Brander and others (2005)

4 Chapter Four. Conclusion and Future Developments

The historical context of this problem provides insight into how engineers of the time confronted the challenges of producing the most economical lot, considering variables such as inventory costs, setup costs, demand, and others. Although almost 100 years have passed and the methods and instruments used back then might be considered obsolete, the variables to be optimized remain the same. Understanding how they addressed and found solutions at that time can undoubtedly offer perspectives on resolving contemporary manufacturing problems.

The confirmation that the problem is classified as NP-hard implies that finding a precise solution for all cases is not possible. The solutions and approaches presented in this study will always be either a lower bound or an upper bound of the optimal solution, relaxing some variables or using heuristic methods to achieve a satisfactory solution for certain production schedules. Some solutions require complex mathematical methods and additional effort beyond the standard production processes of a company. Therefore, it is proposed as an additional study to assess the cost-benefit for companies in determining the most accurate ELSP.

This study has introduced a classification considering new factors do not present in the available state-of-the-art literature. Numerous references have been found that deviate from the classical conditions of the ELSP, aiming to align more closely with industrial reality and offering interesting research avenues.

However, contradictions have been identified in different types of classifications where authors assigned varying classifications to the same solution. This, in some specific cases, is due to existing agreements in the literature regarding certain classifications. For instance, the solutions referred to in this study as Scheduling or Lot Calculation Approach are consistently grouped into the same category of solutions, establishing a standard in the literature.

In addition, as I attempt to respond to the question of which solution is more suitable, it becomes evident that the appropriateness depends heavily on the context of the specific manufacturing environment, its intricacies, and the goals of optimization. This underscores the importance of tailoring solutions to the unique challenges faced by each industrial setting.

Furthermore, it is noteworthy that the ELSP has undergone extensive examination in both academic and industrial spheres. However, despite decades of research, the results are not as widely disseminated or recognized as one might expect. This lack of widespread acknowledgment may be attributed to the intricate nature of the problem and the varied approaches undertaken by researchers and practitioners.

Considering the plethora of references and diverse methodologies discovered in the literature, it becomes evident that the ELSP is a complex issue that demands continuous exploration. The absence of a unified terminology further complicates the understanding and comparison of different studies.

One of the peculiarities that I found in this research was that some authors did not verify the original sources and based its work in the mentions of other. For example one of the sources used in this work, describe the work of Rogers as follows: "*According to Chan and Chung, Rogers is considered as the first researcher in ELSP"* The objective of the historical account made at the beginning of this work was to find the pioneers in these works due to the fact that, as happened with Harris himself, these actions can lead to reference errors. As I showed Rogers was the first person ever to mention the ELSP.

Therefore, as a potential avenue for future research, an in-depth analysis of the number of mentions in the literature by various authors and their respective contributions could shed light on emerging trends, prevalent methodologies, and potential gaps in the existing body of knowledge. Such an exploration might contribute to establishing a more cohesive understanding of ELSP, fostering collaboration, and guiding future research endeavors in this intricate domain.

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