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A Comprehensive Analysis of Genetic Algorithms and Metaheuristics for the Economic Lot Scheduling Problem

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ABSTRACT

This thesis presents a critical and methodical analysis of the existing literature on the Economic Lot Scheduling Problem, with a specific focus on metaheuristic solution methods, particularly genetic algorithms. While the study begins with a broad overview of this complex scheduling issue, it quickly narrows down to scrutinize the application of genetic algorithms in this context. Through a comparative analysis of various authors' works, the aim is to enrich the understanding of the effectiveness of genetic algorithms in solving this problem. By doing so, this work intends to contribute to the existing literature by providing an in-depth evaluation of genetic algorithms as a solution method for the Economic Lot Scheduling Problem.

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1. INTRODUCTION

The Economic Lot Scheduling Problem (ELSP) has been widely studied, with extensive literature reviews already available. There are comprehensive analyses that take a broad view of the subject, such as the taxonomic classification based on scheduling policies and solving methodologies proposed by Santander-Mercado & Jubiz-Diaz (2016), and the study by Chan et al. (2012) aimed at identifying key research themes and organizing a roadmap through recent trends. Given this extensive coverage, it would be redundant to address the problem in a general sense. Instead, this thesis aims to delve deeper into a specific aspect that has been less explored. The primary focus here is on metaheuristic solution methods, specifically genetic algorithms, in the context of the ELSP. The ultimate goal is to provide a more profound understanding of the effectiveness of genetic algorithms in solving the ELSP, comparing the works of various authors and their approaches. This work is not intended to be another comprehensive review, but rather an in-depth investigation into a critical area of the ELSP question.

The Economic Lot Scheduling Problem is a common practical issue that arises when multiple products need to be scheduled for production on a single facility. This scenario is frequently encountered in various industries, such as food processing where a variety of products are made on the same equipment, or in electronics manufacturing where a single assembly line is used for multiple products. This cost-conscious approach of utilizing a single, high-speed machine capable of manufacturing a range of products is often more economical than investing in multiple machines, each dedicated to a particular product. However, this strategy prompts an operational challenge: how to efficiently plan the production schedule for different products on this multifunctional machine. This planning involves not only determining the sequence in which the products should be produced, but also defining the quantity to be manufactured in each production cycle for every product.

This thesis begins with an outline of the research methodology employed in this study, presented in Section 2. The subsequent section, Section 3, provides an extensive literature review, laying the groundwork for the subsequent exploration of the ELSP. Section 4 delves into the details of the ELSP, beginning with a clear problem definition, followed by an exploration of scheduling policies and solution methodologies. This section also discusses the specific assumptions made in the ELSP and its various extensions in the field. In Section 5, the focus shifts to meta-heuristics, providing an overview of the tool and its applications. The section particularly highlights the use of Genetic Algorithms, presenting a comparative analysis of various Genetic Algorithm methods. The performance evaluation of these methods, providing an understanding of their efficacy in solving the ELSP, is also covered in this section. The thesis concludes with Section 6, synthesizing the findings of the study and presenting the conclusions drawn.

2. RESEARCH METHODOLOGY

This chapter provides a detailed exposition of the research methodology adopted to conduct the literature review related to the Economic Lot Scheduling Problem. The core of the research methodology used for this thesis is based on the examination and analysis of various academic sources. These include scholarly articles and conferences from high-profile journals such as "Operations Research," "Management Science," "Production and Operations Management," and "Journal of Operations Management." In addition, lecture notes directly related to ELSP were consulted. The search for relevant material was conducted using the keywords "ELSP" and "Economic Lot Scheduling Problem" on various academic databases and search engines. These included Google Scholar, JSTOR, ScienceDirect, IEEE Xplore, and Scopus. Despite the limited access to all free resources, the available scholarly literature on ELSP is comprehensive and diverse. The relevance and breadth of this topic is mainly due to two factors. First, ELSP is a field of study that has been developed and deepened over many decades, with the first formulation of the problem dating back to Jack Rogers in 1958. Second, ELSP comes with many aspects resulting from different starting assumptions, each of which can lead to a different and unique solution to the problem.

It is important to note that when the topic of discussion is as broad as in the case of ELSP, reviews of the literature may not be able to go into detail or delve into specific approaches and solutions. This further substantiates the aim of this thesis, which does not merely summarize existing research, but seeks to conduct a deeper and more detailed analysis. The goal is to explore and better understand metaheuristic methods, with a focus on genetic algorithms, as applied to ELSP. This approach is guided both by a personal interest in the topic and by the recognition that, as evidenced by the existing literature, genetic algorithms are among the most effective solutions for ELSP.

The initial literature review encompassed a collection of more than 60 articles. In an effort to refine the analysis and align it with the research objectives, a selection of the most pertinent papers was executed, while less relevant ones were excluded. The focus was rigorously confined to articles that addressed the Economic Lot Scheduling Problem within the context of single-machine scenarios. Consequently, the review intentionally excluded studies discussing job shop, multi-machine, and multi-factory contexts. These contexts were disregarded as they encompass dynamics and challenges that substantially diverge from the scope of this research. Additionally, the multi-factory context was specifically excluded due to the insufficiency of available literature, which would impede a comprehensive analysis. In addition, the review deliberately excluded stochastic scheduling problems, including the Stochastic Lot Scheduling Problem and its variants such as the Stochastic Economic Lot Scheduling Problem. Although these problems provide valuable insights into scheduling under uncertainty, they introduce complexities that make them substantially different from the classic ELSP. The focus of this research is strictly on the traditional deterministic form of ELSP, as the stochastic and deterministic versions of the problem are not directly comparable due to their different underlying assumptions and methods of resolution.

3. LITERATURE REVIEW

The study of the Economic Lot-Scheduling Problem has a long and rich history in the field of operations research and management science. The problem was first formulated in 1958 by Rogers, marking a significant milestone in the understanding of production scheduling. He introduced the basic problem of scheduling multiple items on a single machine, aiming to minimize the setup costs and inventory holding costs. His work laid the foundation for the many studies that followed, by identifying the primary trade-off in the ELSP between setup and holding costs.

After Rogers, an important contribution to the field was made by Hanssman (1962), who introduced the idea of a cycle strategy where each item is produced exactly once in each cycle. This approach greatly simplifies the problem and provides an upper bound on the optimal solution.

Since then, this problem has been addressed in the literature from a multitude of approaches, and multiple variants of the problem have been defined. The most widespread and traditional definition of the problem was established by Bomberger (1966); for further details, please refer to the following chapter. Notably, Bomberger introduced the Basic Period approach. This approach pivots on the idea that the cycle times for each of the items should be integral multiples of a fundamental minimum cycle time. This work is among the most well-known within the ELSP literature.

A significant advancement in the field was made by Elmaghraby (1978), who developed a comprehensive framework to solve the ELSP that includes a series of heuristics to generate and evaluate potential schedules. His work provided a more practical approach to solve the ELSP in real-world situations. Moreover, Elmaghraby introduced the concept of an 'extended basic period'. This concept was developed to resolve issues encountered with the feasibility condition of the basic period, which required it to be long enough to accommodate the production and setup times of all items. In 1979 Haessler introduced a heuristic procedure utilizing the concept of the extended basic period. His method included a mechanism for verifying the feasibility of the solution, thereby adding a layer of practicality and reliability to the problem-solving process in ELSP.

The Economic Lot-Scheduling Problem was first identified as NP-hard by Hsu (1983), a classification later expanded by Gallego & Shaw (1997) to include diverse scheduling schemes. This complexity has led researchers to explore various strategies for addressing the problem.

4. THE ECONOMIC LOT SCHEDULING PROBLEM

The Economic Lot-Scheduling Problem integrates two primary challenges in the field of operations management: the lot-sizing problem and the scheduling problem. In the context of lot-sizing, the goal is to determine the optimal volume of units of a product to be produced or purchased in a single batch or lot. This decision requires a delicate balance between setup or ordering costs — which are constant for each lot and thus can be mitigated by producing larger lots — and inventory holding costs, which escalate in relation to the size of the lot. Simultaneously, the scheduling problem focuses on the choice of when and in what sequence various tasks or jobs should be executed. This decision can be complex, primarily when dealing with constraints such as limited availability of resources or strict deadlines to be respected. Within the context of the ELSP, these two problems are combined. Consider, for example, a single production resource such as a machine or production line, which can be utilized to produce multiple products. The objective then becomes to decide both the lot size for each product — that is, how many units of each product should be produced in a single setup — and the scheduling — that is, the order in which the products should be produced — in order to minimize the total costs (Rogers 1958).

This context may encompass a range of costs, including setup costs, inventory holding costs, and potentially also stockout costs in the event that production fails to meet demand. Therefore, the ELSP represents a complex and notoriously difficult problem to solve, but its relevance is unquestionable in many real-world applications in the field of production and supply chain management.

The concept of batching becomes crucial here, mainly because there is usually a cost or time required when the machine transitions from producing one product to another. This switch-over expense could be associated with the cleaning process, or the scrap losses incurred when adjusting the machine for the next product. The system experiences a downtime during these adjustments, implying a non-productive period.

Consequently, it necessitates maintaining a higher inventory level to cover for this pause in production.

Therefore, the ELSP represents a complex and notoriously difficult problem to solve, but its relevance is unquestionable in many real-world applications in the field of production and supply chain management. Boctor (1987) describes diverse applications of the ELSP. It is especially apt for assembly lines producing multiple products or models, like appliances or vehicles, and metal forming or plastics production lines where each product requires a unique die. ELSP is also applicable to blending and mixing facilities where different products are filled into distinct containers, and to weaving production lines where the primary product is manufactured in varying colors, widths, and grades.

4.1 Problem Definition

In the traditional and most widely accepted formulation of the Economic Lot Scheduling Problem, by Bomberger (1966), the following conditions are observed:

- A single machine for production is present, which is designed to process one product at a time.
- The production capacity, although limited, is adequate to fulfill demand requirements.
- The production and demand rates for each product are deterministic, known and constant.
- Each product comes with a fixed setup time and setup cost, both of which are independent of the production sequence.
- The holding cost of the inventory is directly proportional to the quantity of product kept in stock.

Given these parameters, the aim is to determine the most efficient production sequence in order to minimize total costs, which encompass both setup and inventory holding costs.

Two additional assumptions typically found in the traditional model of the Economic Lot-Scheduling Problem are the absence of backorders and shortages. These conditions are theoretically allowed by the premise that the machine's capacity surpasses demand. Given these assumptions and conditions, the Economic Lot Scheduling Problem can be mathematically formulated to precisely depict the production sequence optimization.
Since the beginning of ELSP research, many adaptations of the problem have been studied. These variations are founded on a range of assumptions and hypotheses including considerations for returns, remanufacturing, allowance for backorders and shortages, deteriorating production, and accounting for shelf life. The aim of these adaptations is to tailor the models to align more closely with the specialized conditions encountered in the industrial landscape.

4.2 Scheduling Policy

In the Economic Lot Scheduling Problem the main goal is to develop efficient production schedules that can satisfy demand while minimizing costs. This process typically involves selecting a "scheduling policy" approach. As stated by Santander-Mercado & Jubiz-Diaz (2016), the scheduling policy provides a framework for constructing production schedules, with different policies leading to different schedule types and performance outcomes.
These scheduling policies are broadly grouped into four main categories:

- the Common Cycle (CC),
- the Basic Period (BP),
- the Extended Basic Period (EBP),
- and the Time-Varying Lot Sizes (TVLS).

Introduced by Hanssmann in 1967, the **Common Cycle Approach** assumes an identical cycle time for each product. This means that the length of the total cycle has to accommodate the production of all products, including setup times. This approach simplifies scheduling by creating a uniform production rhythm, always yielding a feasible schedule. However, its cost can be significantly higher compared to the lower bound (LB), primarily due to its lack of flexibility in scenarios where products have different demand rates or production times.

The **Basic Period Approach**, proposed by Bomberger in 1966, provides more flexibility by allowing different cycle times for different products. It defines a basic time period serving as a reference for scheduling various items production, provided that the cycle times are an integer multiple (n_i) of the basic period T. Although generally providing better solutions to ELSP than the CC approach, formulating a feasible schedule using the BP approach is also NP-hard.

Bomberger's BP approach was later refined by Elmaghraby in 1978 with the introduction of the **Extended Basic Period** Approach. In particular, the EBP approach relaxed the constraint requiring the basic period to be long enough to cover the production runs and setup times of all items. Instead, it utilizes two consecutive fundamental cycles of duration W. The items are loaded according to specific rules: if the multiplier for the product i (n_i) is odd, the item is loaded on both periods; if instead it is even, the item is loaded on only one period. This arrangement ensures the two periods do not interfere with each other. The feasibility of the schedule is determined by checking if all items assigned to each period can be produced within the time W. This approach provides greater flexibility and is particularly suitable for situations where demand or production times vary significantly among different products.

The **Power of Two** (PoT) restriction on multipliers, as applied in the Extended Basic Period policy, offers significant advantages in production scheduling. The key advantage is that, since all multipliers are even, item processing can be divided over two different cyclic periods without overlap. This mechanism substantially reduces potential conflicts and simplifies interference testing. As pointed out by Sun et al. (2009), the length of the complete repeating cycle is confined to the value of the largest multiplier, rather than the least common multiple of all the multipliers. Chatfield (2007) further emphasizes the benefits of the PoT policy in facilitating the development of effective and straightforward heuristics. This approach typically eliminates the need for a lengthy fundamental cycle time for scheduling production, making its implementation on the factory floor both practical and uncomplicated. It's worth noting that the PoT policy has been recognized in literature for consistently delivering high-quality solutions.

Introduced by Dobson in 1987, the **Time-Varying Lot Sizes** approach allows for different lot sizes for different products within a cycle. Consistently produces better-quality, feasible schedules, particularly in scenarios with significant variations in demand or production times across different products. Providing the most general model among the approaches, it has the unique advantage of not requiring the production runs of an item to be of the same length. However, compared to other methods, it is the most challenging to solve.

Each of these approaches offers distinct advantages and is suitable for specific types of scheduling problems. The choice of approach often depends on the specific constraints and objectives of the problem at hand.

Most research on the Economic Lot Scheduling Problem tends to rely on at least one of two assumptions: the Equal-Lot-Size (ELS) assumption or the Zero-Switch (ZS) assumption, (also known as Zero Inventory Production (ZIP)). The ELS assumption mandates that the production batch sizes for each item be the same. In other words, if we are manufacturing multiple items, each batch of each item will contain the same quantity. On the other hand, the ZS assumption dictates that a new production cycle for a particular item can only start when the inventory level of that item reaches zero. Thus, we do not start producing more units of an item until we have completely exhausted the existing stock of that item.

Bulut and Tasgetiren (2014) offer an interesting categorization of scheduling policies. They divide them into two different model approaches: the Fundamental Cycle (FC) approach and the Cyclic Schedule (CS) approach.

The FC approach, which is grounded in both ELS and ZS assumptions, mandates that the cycle time for any item must be an integer multiple of a base period, known as the Fundamental Cycle. Under this approach, the length of the main cycle is determined by the least common multiple of these integer multipliers. This schedule within the main cycle is then repeated indefinitely. Models operating under the Fundamental Cycle approach are three: the Common Cycle approach, the Basic Period approach, and the Extended Basic Period approach. On the other hand, models developed under the CS approach do not adhere to the ELS assumption, and they permit lot sizes to vary over time. However, these models still uphold the ZS assumption. They provide complete schedules, but often result in solutions with extended cycle times, given their flexibility in accommodating varying lot sizes.

4.3 Solution methodology

Before delving into the specific methodologies employed to tackle the ELSP, it's important to set the stage with an understanding of the problem's inherent complexity and the implications this has for potential solutions.

The classification NP-hard refers to optimization problems that cannot be reduced to a solvable polynomial form. In other words, it is challenging to verify every potential solution in a reasonable computational time. Given its basic assumptions, the ELSP present a complexity that has been proven to be NP-hard by both Hsu (1983) and Gallego and Shaw (1997). Hsu (1983) demonstrated that even a simplified version of the ELSP is NP-hard and suggested that an enumerative procedure is sensible for solving ELSP. Gallego and Shaw (1997) further extended Hsu's (1983) study by proving that ELSP is NP-hard under various scheduling schemes, including general cycle, zero-inventory cycle, time-invariant cycle, lot-invariant cycle, and basic period cycle. Therefore, it can be concluded that ELSP is NP-hard in general. In other words, it is not possible to transform the ELSP into a solvable polynomial form, making the search for an analytical solution for ELSP unfeasible without relaxing some assumptions.

This NP-hardness of ELSP has led to a branching in research, with two distinct approaches emerging: simplified versions that involve significant relaxation of restrictions, and complex versions that incorporate sophisticated mathematical models. The former can be tackled using traditional programming methods, especially when employing the common cycle policy. On the other hand, heuristic approaches have gained popularity as the preferred research direction for addressing the more complex versions of the ELSP problems.

The solution methodology for the Economic Lot Scheduling Problem can be divided into various categories. Among these, **exact methods** are commonly used to solve restricted versions of the original problem. These exact methods include techniques such as branch and bound, dynamic programming, enumeration, exact algorithms, linear programming, marginal analysis, and integer linear programming, to name a few (Beck & Glock, 2020). However, due to the complexity of the ELSP, **heuristic methods** are often preferred. These heuristic methods encompass various strategies and techniques like dispatch rules, priority rules, g-group heuristic, Johnson's algorithm, mixed integer nonlinear programming (MINLP), pt heuristic, and two-group heuristic, among others (Beck & Glock, 2020). In addition, **meta-heuristic methods** have gained popularity for their ability to find high-quality solutions in a reasonable time. Examples of these meta-heuristic methods include local search, neighborhood search, artificial bee colony algorithm, simulated annealing, ant colony algorithm, binary search, cuckoo search, and genetic algorithm. Particularly popular among these are the genetic algorithm, tabu search, simulated annealing, and the hybrid genetic algorithm (Beck & Glock, 2020). While exact methods were once dominant, there has been a shift in recent years towards using more heuristic and meta-heuristic methods to solve the ELSP, reflecting the ongoing efforts to address the complex nature of the ELSP more effectively, efficiently, and robustly (Beck & Glock, 2020).

4.4 Specific assumptions

This chapter includes a selection of examples where assumptions have been adjusted to allow the traditional ELSP problem to align more effectively with specific real-world situations. Please note that the list provided is not exhaustive and does not delve into an in-depth exploration of each topic. The integration of specific assumptions, or the implementation of variants of the traditional ELSP problem - whether through the addition of variables or the relaxation of certain assumptions - typically results in increased complexity of the mathematical model.

In response to envinronmental pressures, many businesses are now adopting remanufacturing alongside traditional manufacturing. Remanufacturing restores used products to like-new condition, while returns indicate products sent back to the firm, often serving as input for remanufacturing. The return/demand ratio links manufacturing and remanufacturing lot sizes. This leads to a complex variant of the scheduling problem known as the Economic Lot Scheduling Problem with Returns (ELSPR), where the sequencing of production lots influences inventory costs, making sequencing decisions vital when determining lot sizes (Tang and Teunter, 2009).

As businesses incorporate the concept of returns into their operations, they need systems to effectively manage these returned items. A crucial aspect of dealing with returns involves distinguishing between remanufacturable and non-remanufacturable items. This is where the concept of a sorting line, as studied by Ferretti (2020), comes into play. A sorting line allows a company to separate remanufacturable returns from those that are not. Of course, when implementing such a system, it's important to consider the associated costs of the sorting line and how disposal options for scrap items can impact the total costs of the system. This aspect of handling returns, and remanufacturing adds another layer of complexity to the ELSPR, making its study and solution even more challenging.

Alle et al. (2004) studied a Mixed Integer Linear Programming (MILP) model for the ELSP, taking into account performance decay. This phenomenon, prevalent in many industrial processes such as chemical processing, involves decreased efficiency over time due to factors like catalyst deactivation or heat-exchanger fouling. Regular maintenance is required to restore performance, creating an optimization problem: the balance between continuing at reduced yields or stopping for maintenance. The optimal solution balances production losses from performance decay with gains from continuous operation.

Giri et al. (2003) studied the ELSP in which the production facility is assumed to deteriorate due to aging, transitioning from an "in-control" state to an "out-of-control" state, leading to the production of defective items. To manage this issue, their strategy is the following: if the process is found to be in an "out-of-control" state, then corrective maintenance is performed to restore it to an "in-control" state before the start of the next production run; otherwise, preventive maintenance is carried out to enhance system reliability. Therefore, the problem's formulation should consider the quality-related costs due to the possible production of nonconforming items, as well as inspection and maintenance costs.

In 2011, Goncalves and Sousa explored the ELSP with the inclusion of backorders, a scenario often encountered in practical settings, which can contribute to a reduction in inventory costs. Indeed, their model was specifically developed to address the challenges of a can filling company. The company struggled with maintaining excessive inventory levels to prevent backorders. The complexity of their scheduling was due not just by the wide range of products being manufactured, but also by the volatility in demand rates and the resultant unbalanced inventories.

4.5 Extension of ELSP

Variations in basic assumptions and constraints can dramatically alter the nature and structure of the problem. These variations become unique problems in themselves. While they maintain a relationship with the original ELSP, they are not considered within the same category. Instead, they are often treated separately, each with its unique characteristics and challenges. Among these related but distinct problems, some have attracted significant research interest due to their practical implications and theoretical complexity. The following discussion will delve into these categories, highlighting their distinct features and the research trends surrounding them.

Stochastic Economic Lot Scheduling Problem

The deterministic nature of the Economic Lot Scheduling Problem may limit its applicability in scenarios where demand is uncertain. To address this, the Stochastic Lot

Scheduling Problem (SLSP) and, more specifically, the Stochastic Economic Lot Scheduling Problem (SELSP) have been introduced as natural extensions of the ELSP within a stochastic context (Winands et al., 2005; Sox et al., 1999). The SELSP accounts for uncertainty and variability of parameters, offering a model which more accurately mirrors practical industrial scenarios. However, this increased adherence to industrial reality brings with it greater analytical complexity. Indeed, until the late 70s, attention to the SELSP was limited, despite its undeniable practical value (Winands et al., 2005). In a stochastic environment, the traditional rigid cyclic production plan of the ELSP is no longer sufficient. A more flexible approach is needed, capable of responding to the dynamic changes that characterize these contexts. Furthermore, stocks for individual products take on a more significant role in the SELSP compared to the ELSP. They not only reduce the number of setups in a cycle but also serve as a protection against stockouts and scheduling conflicts due to variations in demand, production, or setup times (Winands et al., 2005). Despite being more complex, the SELSP does not entirely abandon the insights derived from the ELSP. In fact, a deterministic production plan is often used as a basis for solving the stochastic problem, providing a starting point to tackle its increased complexity (Sox et al., 1999). Research on the SELSP continues to be an area of great interest due to its practical relevance and theoretical complexity. The open questions, both from a theoretical and practical point of view, stimulate further research in this field, making it a topic of fundamental importance for industrial production management (Winands et al., 2005; Sox et al., 1999).

Multi-facility ELSP

The multi-facility version of the Economic Lot Scheduling Problem, such as the Flow Shop ELSP (FS-ELSP), introduces a range of additional complexities compared to the single-machine version. In FS-ELSP, each item must go through several stages of production, each with specific setup and processing times. This introduces additional constraints, such as the fact that a machine can't process more than one product at a time and that a lot can't be transferred to the next stage until it's completely processed at the current stage. These constraints can generate waiting times between stages, increasing total costs (Huang & Yao, 2008). Although these complexities are not present in the single-machine ELSP, the objective remains the same: to determine optimal lot sizes and generate a feasible production schedule that minimizes average total costs while satisfying the demand for each product. Despite the increased complexity, the multi-facility ELSP has attracted research attention due to its application in various industrial sectors, such as metal forming, plastic extrusion, assembly lines, pharmaceutical, and biochemical companies. These production systems usually operate with multiple facilities, necessitating different problem-solving approaches than those used for the single-facility environment (Santander-Mercado & Jubiz-Diaz, 2016). It's important to underscore that the ELSP is already NP-hard, making its flow shop or multi-facility versions even more complex. Analytical solutions for such problems seem unmanageable. Chan, Chung, and Lim (2013), in their paper "Recent research trend of economic-lot scheduling problems," suggest that to derive valuable managerial insights, a mix of computational techniques, heuristics, and potentially, simulation approaches would be beneficial. Additionally, they propose that Genetic Algorithms, or computational intelligence approaches in general, can address the complexity of the ELSP problem, especially in its flow shop or multi-facility configurations, which are not uncommon in real-world applications. This perspective provides a research roadmap for future explorations in the field. However, for the purpose of this study, the focus will remain exclusively on the single-machine ELSP, leaving the exploration of multi-facility ELSP complexities and solutions for future research.

Capacitated Lot Scheduling Problem

The ELSP assumes that production capacity is sufficient to meet all demand. In the Capacitated Lot Scheduling Problem (CLSP), this assumption is modified by introducing a constraint of limited production capacity. This change implies that demand may not be entirely satisfied, affecting production scheduling and thereby increasing problem complexity, even though it brings the model closer to many real-world production situations.

5. META-HEURISTICS METHODS

5.1 Overview of meta-heuristics

Blum and Roli (2001) define a metaheuristic as a high-level process that iteratively guides and adjusts heuristic methods to generate high-quality solutions for complex problems. They often employ probabilistic decisions, differentiating from pure random search by using intelligent bias based on the objective function, previous decisions, or prior performance. In essence, a metaheuristic is a flexible algorithmic framework adaptable to a wide array of optimization problems.

Key properties of metaheuristics, as outlined by Blum and Roli (2001), include:

- Strategies guiding the search process.
- Aim for efficient exploration to find near-optimal solutions.
- Range from simple to complex processes.
- Operate in an approximate, non-deterministic manner.
- Incorporate mechanisms to avoid search confinement.
- Abstract level descriptions due to their non-problem-specific nature.
- Use of domain-specific knowledge through controlled heuristics.
- Utilize search experience to guide the process.

To better visualize the process of metaheuristics, Figure 1 presents a generalized flowchart that captures the main steps through which metaheuristic algorithms progress. This schematic representation underscores the iterative nature of these methods in their pursuit of the best feasible solution.

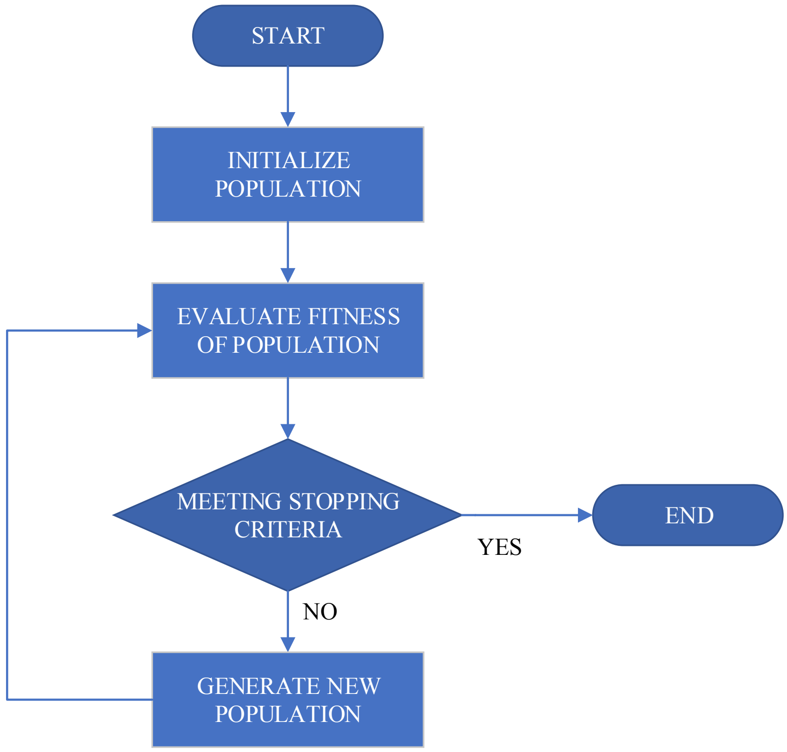

Figure 1. Basic flowchart of a metaheuristic process

The process begins with the initialization of a population of potential solutions. In the context of the ELSP, these solutions represent different production sequences. Each solution's fitness is then evaluated, providing a measure of quality that guides the search process towards promising solution areas. For ELSP, the fitness function typically aims to minimize the total costs associated with the production sequence, such as inventory and setup costs. The algorithm subsequently checks whether a specific stopping condition, such as a maximum number of iterations or the achievement of a target solution quality, has been met. If the stopping criteria are not met, the algorithm generates a new set of solutions. This generation process, which often involves strategies like crossover and mutation in genetic algorithms or neighborhood search in methods like simulated annealing or tabu search, is a critical step in the metaheuristic process. The algorithm continues to iterate through these steps, constantly evaluating and refining the production sequences, until the stopping criteria are satisfied.

In the context of the NP-hard Economic Lot Scheduling Problem, research has often pursued two main solution approaches: simplified versions with many relaxations, and more sophisticated versions. As noted by Chan Chung and Lim (2012), metaheuristics have been particularly valuable for the latter, thanks to their ability to effectively navigate complex search spaces and efficiently find near-optimal solutions. Their unique properties make them well-suited for tackling sophisticated versions of NP-hard problems such as the ELSP, especially when dealing with large-scale problems that cannot be solved exactly within a reasonable timeframe. This has led to their wide use in the literature. In a content analysis conducted on a sample of 242 articles, Back and Glock (2020) revealed interesting trends in the methodology used for solving the ELSP. They classified the solution methodologies into five categories: exact methods, heuristic methods, meta-heuristic methods, artificial intelligence, and simulation. From 1958 to 1997, the term "metaheuristics" was a recording hit in only 11% of the articles within the "solution methodology" category. This trend changed significantly after 1998, when Khouja et al. published the first study using a genetic algorithm. From 1998 to 2019, the recording hits for "metaheuristics" increased to 47%, indicating a growing interest in this approach in the field of ELSP.

Blum and Roli (2001) propose several ways of classifying metaheuristic algorithms. For instance, they distinguish between nature-inspired algorithms, such as Genetic Algorithms and Differential Evolution, and non-nature-inspired algorithms, like Tabu Search and Simulated Annealing. Another key classification is based on the type of search approach employed: single point search methods, exemplified by Simulated Annealing and Tabu Search, versus population-based methods, such as Genetic Algorithms and Differential Evolution. They also highlight the usage of a static versus a dynamic objective function, and the employment of one or multiple neighborhood structures. The use of memory during the search process is also emphasized, with methods like Tabu Search employing memory, in contrast to memory-less approaches like Simulated Annealing. According to Blum and Roli, the use of memory is a fundamental feature of a powerful metaheuristic. The authors also note that the distinction between single point search methods and population-based methods provides a clear description of the algorithms and point out a trend towards the hybridization of metaheuristics, integrating single point search methods into population-based ones (Blum & Roli, 2001).

In the following sections, we shall embark on a brief exploration of various metaheuristics, elucidating their operational mechanisms and presenting specific instances of their application within the context of the ELSP. This comprehensive examination serves a dual purpose: not only does it aid in enhancing our understanding of these diverse techniques, but it also provides a broader comparative framework that allows for a more nuanced evaluation of the effectiveness of GAs across the wide spectrum of metaheuristics.

1. **Differential Evolution Algorithm**

To introduce Differential Evolution (DE), our reference is the study by Tasgetiren et al. (2011), which marked the first application of DE to the ELSP. DE is a population-based evolutionary optimization method that functions as a stochastic global optimizer. As Tasgetiren et al. (2011) report, DE was originally conceived by Storn and Price (1997) as a solution to the Chebychev polynomial fitting problem, and it has since proven successful in a multitude of applications. Although there are several mutation variations in traditional DEs, in their study Tasgetiren et al. (2011) follow the DE/rand/1/bin scheme proposed by Storn and Price (1997). This scheme, "DE/rand/1/bin", represents a Differential Evolution (DE) algorithm in which an individual is randomly ("rand") selected for mutation, the mutation is based on the difference between a single ("1") pair of individuals, and a binary ("bin") crossover is used to create the new individuals. A key distinction of DE is this unique differential mutation approach. According to Tasgetiren et al. (2011), mutant individuals are generated by adding a weighted difference between two randomly selected population vectors to a third member, thus perturbing the original vector and effectively exploring the solution space. Tasgetiren et al. (2011) address the ELSP using a BP policy and employ the DE to solve the non-linear and integer optimization problem. They draw inspiration from the model of Khouja et al. (1998) for solution representation. In fact, each individual in their model is composed of a floating-point value for the fundamental cycle T and integer multipliers k_i of T for each item. The encoding is similar to that of Khouja's, despite the DE algorithm operating solely within a continuous domain. Indeed, the values of the multipliers are truncated to handle this issue.

2. Simulated Annealing

In their 2008 study, Raza and Akgunduz implemented a Simulated Annealing (SA) algorithm to address the ELSP, using Dobson's (1987) Time Varying Lot Size formulation. This brief overview will provide insights into the general functioning of their approach. Simulated Annealing is a stochastic search algorithm inspired by the annealing process of slowly cooling metals to minimize their energy states. In optimization terms, it's designed to avoid getting trapped in local minima within a solution space by occasionally allowing "uphill moves," i.e., transitions to worse solutions (Blum and Roli, 2003). Motivated by the recent success of other meta-heuristics in solving the ELSP, Raza and Akgunduz (2008) introduced their SA approach. They capitalized on SA's adaptability and its ability to explore a larger solution space to avoid local minima, thus efficiently solving the ELSP.

5.2 Genetic Algorithms

In recent years, Genetic Algorithms (GAs) have significantly gained traction within the Production and Operations Management (POM) field as a problem-solving tool, demonstrating their potential to tackle a broad spectrum of problems. Their heuristic nature provides a versatile and effective alternative to traditional methods like hill climbing, Tabu search, and simulated annealing. Even at the time of the review by Aytug et al. (2003), GAs were already considered valuable tools, and since then, the scientific literature has further expanded, particularly concerning the Economic Lot Scheduling Problem. This growth is underscored by Chung and Chan (2012), who recognize GAs as an optimal approach for the ELSP. GAs are a robust and flexible class of optimization techniques that operate over iterative generations. They are based on principles of genetic inheritance and Darwinian 'survival of the fittest', aiming to find near-optimal solutions to complex problems (Khouja et al., 1998). As Raza and Akgunduz (2008) note, the distinctly stochastic nature of GAs, including the initial random generation of solutions and the probabilistic selection of individuals for reproduction, makes them well suited to problems that are complex and have a large search space, making them impossible to search exhaustively. Before we delve into the details of these components, let's consider the following flowchart (Figure 2), which provides a visual overview of the Genetic Algorithm process. This will help us better understand how these components interact within the algorithm.

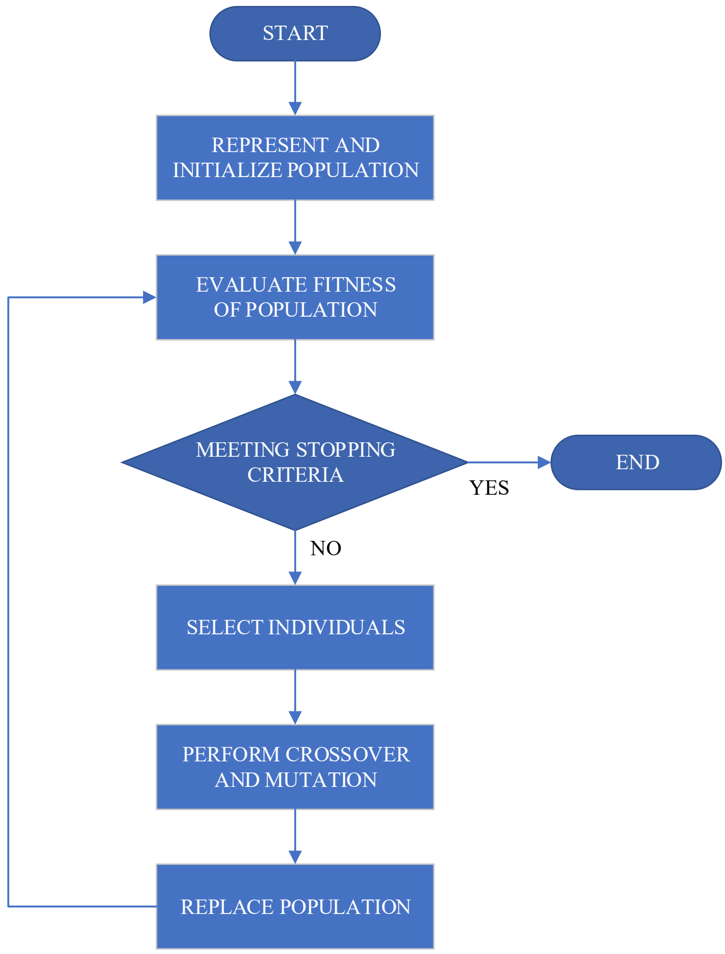

Figure 2. Overview of the Genetic Algorithm process

In the following analysis, we will explore the eight basic components that a GA must have, as defined by Aytug et al. (2003).

1. **Genetic Representation for Potential Solutions**

In GAs, each potential solution to the problem at hand is represented as an individual, often referred to as a chromosome. Depending on the nature of the problem, chromosomes can be represented using various data structures. As Aytug et al. (2003) and Raza and Akgunduz (2008) explain, it's common to use binary, integer, or floating-point representations. These various representations are designed to suit the problem domain. In the specific case of the ELSP, the solution can be represented as a discrete sequence of production lots. This makes GAs a particularly suitable choice of algorithm; as stated in the work of Aytug et al. (2003), there is empirical evidence of their success in tackling computationally intractable problems, especially those with discrete solution spaces. The representation of the solution further depends on the chosen solution method, such as Basic Period (BP), Extended Basic Period (EBP), or Time Varying Lot Sizes (TVLS), while the general objective is to effectively represent a production sequence for optimization.

2. Creating an Initial Population of Solutions

The genetic algorithm process begins with the creation of an initial population of individuals. These individuals, each representing a potential solution, are typically generated randomly within the problem's search space (Raza and Akgunduz, 2008). This initial population is the starting point for the search for optimal solutions (Khouja et al., 1998).

3. Evaluation Function to Rate 'Fitness'

Each chromosome in the population is evaluated using a fitness function, which measures how well it solves the problem. Higher fitness values correspond to better solutions. As per the principle of "survival of the fittest," individuals with higher fitness are more likely to be selected for the next generation (Aytug et al., 2003). This evaluation process includes a step called fitness scaling, which is used to adjust the fitness values of individuals in the population (Raza and Akgunduz, 2008). The fitness function for ELSP typically aims to minimize the total costs associated with the production sequence, such as inventory and setup costs. At each iteration, every new generation of production sequences is evaluated based on this fitness function. Often, a penalty term is incorporated into the fitness function to discourage infeasible solutions that might appear in the new generations.

4. Scheme for Selecting Individuals

Selection is a vital step in the genetic algorithm process. Selection mechanisms determine which individuals are chosen to create the next generation. The selection scheme is typically probabilistic and biased towards individuals with higher fitness. There are various selection methods, but all aim to balance the exploration of new areas in the solution space with the exploitation of already discovered effective solutions (Khouja et al., 1998). The selection scheme for ELSP could be a technique like roulette wheel selection or tournament selection, where individuals (production sequences) are selected probabilistically based on their fitness values. Other selection strategies such as rank selection or steady-state selection can also be utilized, each offering different balances between exploration and exploitation. Elitism, which guarantees the survival of the most fit individuals, is another method often incorporated to maintain high-quality solutions in the population. The choice of selection scheme significantly influences the performance of the Genetic Algorithm and should be tailored to the specific characteristics of the problem.

5. **Operators to Alter Genetic Composition**

Genetic operators, such as mutation and crossover, are used to alter the genetic makeup of individuals. Crossover combines parts of two or more individuals to create new offspring. Mutation, on the other hand, introduces small variations in an individual. These operators play a crucial role in exploring the search space for optimal solutions (Aytug et al., 2003). The crossover operator could be applied by swapping portions of two production sequences to create new offspring. Mutation can be introduced by randomly altering a position in the production sequence.

6. **Replacement Scheme for Creating New Generations**

The selected and genetically altered chromosomes form the next generation. This replacement scheme allows the genetic algorithm to iteratively improve the population, ideally leading to increasingly better solutions over time. The specific replacement scheme used can vary, but the primary goal is always to maintain or improve the overall fitness of the population (Aytug et al., 2003). The new generation of production sequences is formed from the offspring (created by crossover and mutation). Often, an elitist strategy is applied, where some of the better-performing individuals from the current generation are retained. However, it's vital to maintain a degree of exploration in the population to avoid convergence to local optima. This is typically achieved by ensuring a portion of the new generation is composed of diverse or mutated solutions.

7. **Stopping Criteria**

The iterative process of selection, crossover, and replacement continues until certain termination criteria are met. According to Aytug et al. (2003), these criteria might include reaching a fixed number of generations, a lack of diversity in the population, or a predefined threshold of solution quality.

8. Selection of GA Parameters

The performance of genetic algorithms can be significantly influenced by the choice of parameters, such as population size, crossover rate, mutation rate, and selection strategy. The optimal parameter settings often depend on the specific problem being solved, and it can be a complex task to select the best parameters (Khouja et al., 1998). The importance of parameter selection in genetic algorithms is highlighted in Chatfield's (2007) study, which dedicates an entire section to applying an "offline performance" metric (described in another of his papers), which tracks a genetic algorithm's progress toward finding the best solution. Through this function, he is able to identify the optimal values for crossover and mutation applied to the Bomberger problem. The study also includes graphs depicting the impact of these parameters through curves.

After having outlined the fundamental aspects of a genetic algorithm, the focus now shifts to an analysis of how the authors, who are the subjects of this thesis, have developed their genetic algorithms to solve the Economic Lot Scheduling Problem. Table 1 provides a brief description of the method and summarizes the benchmarks used for the experimental validation of their proposed models. Table 2, on the other hand, offers a succinct overview of the specific features of these genetic algorithms. A more detailed examination of the information summarized in these tables will be undertaken in the subsequent chapter, which will focus on analyzing and comparing the various approaches.

Table 1. Comparisons on Genetic Algorithms and benchmark problems

Author	Method description	Benchmark problems	Compared methods
Khouja et al. (1998)	GA for solving the ELSP using the BP approach	Bomberger's 10 items problem	Dynamic Programming by Bomberger (1966)
Moon et al. (2002)	Hybrid Genetic Algorithm based on the time-varying lot sizes approach	Bomberger's 10 items problem Mallya's 5 items problem	Heuristic by Dobson (1987) GA by Khouja et al. (1998)
Chatfield (2007)	Genetic Lot Scheduling (GLS) is a genetic algorithm for ELSP that utilizes an approach similar to EBP, but with an enhanced item-to-period loading scheme and integer multipliers for the cycle time.	Bomberger's 10 items problem Six benchmark ELSP problems	GA by Khouja et al. (1998) Heuristic by Haessler (1979)
Sun et al. (2009)	GA for solving ELSP under the EBP and PoT policy	Bomberger's 10 items problem Six benchmark ELSP problems	GA by Chatfield (2007) Heuristic by Haessler (1979) GA by Khouja et al. (1998)
Qiu and Chang (2009)	Hybrid Genetic Algorithm with BP approach that incorporates specific heuristics to expedite the discovery of feasible solutions and mitigate the risk of converging to local optima.	Bomberger's 10 items problem, with 66% and 88% utilization rate	GA by Khouja et al. (1998)
Goncalves and Sousa (2011)	Genetic Algorithm for Lot Sizing (GALS): hybrid approach combining a GA and a surrogate LP formulation for the ELSP, with a Rolling Horizons policy that allows backorders.	Bomberger's 10 items problem Mallya's 5 items problem	Heuristic by Dobson (1987) GA by Moon et al. (2002) Simulated Annealing by Raza and Akgunduz (2008)
Chung and Chan (2012)	Two-level GA using either the nearest integer or power-of-two methods for a time-varying lot size ELSP	Bomberger's 10 items problem Mallya's 5 items problem	Heuristic by Dobson (1987) GA by Moon et al. (2002) Simulated Annealing by Raza and Akgunduz (2008)

Table 2. Comparisons of GA's characteristics

Author	Encoding and representation	Population parameters and termination condition applied	Genetic operators 1. selection, 2. replacement, 3. crossover, 4. mutation	Feasibility and fitness
Khouja et al. (1998)	Chromosomes consist of one floating-point number for cycle time and n integers indicating the cycle count for each item production $(T, k_1, k_2, \dots, k_n)$, represented in binary format.	Population size: 200. Random initialization. Runs up to 1 000 generations; stops if no improvement for 150 gens or < 0.1 improvement in last 10 best solutions.	1. Stochastic tournament, 2. Generational replacement with elitism, 3. Combination of 1-point, 2-points, and uniform crossover, probability = 0,7 4. Probability of mutation: 1/(chromosome length).	Objective fitness function with increasing penalization of infeasible solutions
Moon et al. (2002)	Real integer string representation. Two parts of chromosome: $Part\ A = (X_1, X_2, \dots, X_n)$, X_i is index of product. $Part\ B = (Y_1, Y_2, \dots, Y_n)$, Y_i is absolute location of genes in Part A.	Population size: 100. Random initialization. Runs until 1000 generations or no improvement over 150 generations.	1. Stochastic tournament, 2. Generational replacement with elitism 3. Partial Matched Crossover, with a 0,9 rate, 4. Mutation rate: 1/ (string length of chromosome).	Objective Fitness Function, scaled with sigma truncation
Chatfield (2007)	Solutions for production scheduling are encoded in binary strings. Each chromosome includes a fundamental cycle W, multipliers $\{n_1, n_2, \dots, n_N\}$, and start periods $\{b_1, b_2, \dots, b_N\}$.	Population size: 30 Random initialization. 10 000 generations.	1. Fitness-weighted roulette wheel selection 2. Generational replacement with elitism 3. One-point crossover $p_{cross} = 0{,}8$ 4. Bit flipping (0 becomes 1 and vice versa), $p_{mut} = 0{,}01$	Cost-based fitness function, scaled in a range between C_{min} and C_{max}, with infeasibility penalty

Sun et al. (2009)	Solutions for production scheduling are encoded in integer strings. Each chromosome consists of a set of power-of-two multipliers $\{n_1, n_2, \dots, n_I\}$,, and production positions $\{j_1, j_2, \dots, j_I\}$,.	Population size: 100. Run for 10 000 generations or until no improvement over 1 000 generations.	1. Fitness Proportional Selection 2. Elitist strategy (10%) 3. Two-point crossover 4. Mutation rate = 0,1	Objective fitness function with increasing penalization of infeasible solutions
Qiu and Chang (2009)	Each individual, encoded as a binary string, represents a production schedule with the fundamental cycle T and the multipliers $(k_1, k_2, \dots, k_n)$ for each item's production.	Initialization: assign each $k_1 = 1$ and sets T to a random value between its LB and UB.	Not specified	Feasibility is guided by 3 heuristic theorems to find viable solutions and avoid local optima.
Goncalves and Sousa (2011)	Random-Keys alphabet used for encoding. Indirect representation generates the Master Production Sequence (MPS) and the maximum number of setups for the solution. Final solution (PS and production times/quantities) obtained via Linear Programming.	Population size: 100. Initialization with a Random-key vectors uniformly sampled from [0,1]. Stop after 100 generations	1. Elitist (top 15% copied) 2. Elitist strategy 3. Parameterized uniform crossover, between top 15% chromosomes and others $p_{cross} = 0{,}7$ 4. Generation of new chromosomes (bottom 10%)	Feasibility: All offspring formed by crossover are feasible solutions. Fitness: Exact total cost function
Chung and Chan (2012)	Two-level GA, with two integer chromosomes. Type α: each gene represents product's production frequency rounded off to the nearest integer or to the power of two of T. Type β: an ordered string representing the corresponding sequence of product production.	Population size: 20 + 50. Random initialization (in α two randomly chosen chromosomes are recoded: one to the nearest integer frequency, and another to the nearest power of two frequency)	1. Roulette wheel 2. Elitist strategy (low fit 20% replaced with a random generation) 3. $cr^1 = cr^2 = 0{,}2$ 4. $mr^1 = mr^2 = 0{,}2$	Fitness is calculated using Dobson's ELSP formulation (cost and durations), first on chromosome β, then on α.

5.3 A comparative analysis of Genetic Algorithms

Commencing our analysis, it becomes apparent that different authors have adopted unique approaches in applying Genetic Algorithms to the Economic Lot Scheduling Problem, starting with different scheduling policies.

Khouja (1998) introduced a Genetic Algorithm as a solution to the ELSP, formulating it based on the Basic Period approach. Also Qiu and Chang (2009) opted for the Basic Period approach in their genetic algorithm, due to its ability to create economically viable lot sizes and ensure production feasibility. Despite its inability to guarantee feasible production schedules, they chose it for its practicality and widespread use in production facilities, aiming to address its shortcomings through their research of an efficient algorithm.

Sun et al.'s (2009) genetic algorithm employs the EBP policy with PoT multipliers, where each cycle time is a power of two of the basic period. Chatfield (2007), instead, utilizes an approach similar to EBP, that enhanced item-to-period loading scheme and integer cycle time multipliers, aiming to construct a fully defined and simplified solution structure. The identification of a fundamental cycle and a set of multipliers alone isn't sufficient to fully define a solution: by incorporating a set of start periods along with basic item loading and sequencing rules, this approach creates a production schedule that can be feasibly and efficiently checked.

Moon et al. (2002), building upon Dobson's (1987) formulation for the Time-Varying Lot Sizes approach, proposed a hybrid GA. However, their approach utilized the GA solely for determining the production sequence, without extending it to the determination of production run lengths, frequencies, or cycle lengths. In a similar vein, Chung and Chang (2012) also started with Dobson's formulation for the Time-Varying Lot Size, and proposed a modified hybrid GA designed to address the rounding off of the production frequency method. Their GA uniquely combined the procedures of rounding off the production frequencies, optimizing the production schedule, and calculating the production time into a single GA.

As pointed out by Chatfield (2007), Basic Period approach, despite being well-suited to chromosomal representation and genetic search, imposes limitations that prevent it from producing solutions as competitive as those from less restrictive ELSP formulations. This point will be further emphasized by the forthcoming results of the comparison experiments. However, shifting to a formulation like the EBP is not without its challenges, particularly due to the complexity brought about by feasibility issues, which call for a more intricate representation of solutions. One possible middle ground could be the adoption of the Power of Two policy, which provides a balance between solution flexibility and feasibility constraints. This policy restricts the feasible production cycles to powers of two, reducing the complexity of the search space while allowing for a wider array of potential solutions compared to the Basic Period approach.

A separate discussion is required for the work of Goncalves and Sousa (2011), particularly because they also consider backorders. This is not a common assumption within the scope of the classical ELSP, as the hypothesis of production capacity satisfying all demand typically makes the use of backorders unnecessary (though it could be more effective for high utilization rates). Indeed, the inclusion of backorders significantly complicates the resolution model: the paper employs a rolling horizon policy with a non-linear and not positive-definite objective function. This model is subsequently simplified by applying an upper bound on inventory costs, transforming it into a mixed integer quadratic programming model. Finally, the quadratic objective is linearized into a linear programming (LP) model.

This added complexity has implications for the performance evaluation of their study. Specifically, Goncalves and Sousa assessed the effectiveness of their Genetic Algorithm for Lot Sizing by benchmarking it against other classical ELSP approaches. They enforced a cyclical production schedule to minimize both setup and inventory holding costs by equalizing the initial and final inventory levels and setting the planning horizon to the sum of the setups in the production schedule. To ensure fair comparisons and prevent backorders in the optimal solutions, they increased the backorder cost per unit until backorders were eliminated, while also monitoring for any backorder occurrences.

The genetic algorithm introduced by Khouja et al. (1998) represents the first application of this metaheuristic for solving the ELSP. The Basic Period formulation of the ELSP is ideally suited for genetic algorithms and offers significant advantages over the traditional dynamic programming method proposed by Bomberger (1966). Indeed, finding the optimal solution for the fundamental cycle T and multipliers with dynamic programming requires numerous iterations, resulting in a process that is both computationally intensive and time-consuming. However, through Khouja et al.'s representation in the chromosomes, the cycle time and multipliers are simultaneously optimized using the genetic algorithm. This approach significantly simplifies the process. Qiu and Chang (2009) also use the BP approach and follow Khouja's genetic algorithm representation. However, their introduces some innovations as it is a hybrid GA that also uses heuristics. For instance, they expedite the search for the optimal solution by initializing T within its upper and lower bounds and setting each multiplier equal to 1 for the first generation of members. This approach, inspired by the Common Cycle method, ensures the creation of some feasible individuals in the initial generation. This increases the possibility of finding a feasible search space, leading to a faster convergence rate towards the optimal solution.

Sun et al. (2009) applied a GA to the ELSP under the Extended Basic Period policy with Power of Two multipliers. Their innovative approach uses a chromosome, composed of two arrays of integer numbers, to represent a solution. This chromosome comprises a set of PoT multipliers n_i and a corresponding set of production positions j_i. The cycle length (W) is not explicitly stated in this chromosome, instead, its optimal value is analytically determined based on the given multipliers (n_i) and positions (j_i). To ensure the feasibility of the production schedule represented by the chromosome, they set constraints on its integers. Specifically, the multiplier for each product is limited to the range $1 \leq n_i < 1/\rho_i$ and the production position for each product is constrained to $1 \leq j_i < n_i$.

Chatfield (2007) developed the Genetic Lot Scheduling (GLS), an innovative approach to the ELSP. In this approach, solutions are represented by binary strings encoding a fundamental cycle (W), its multipliers $\{n_1, n_2, \dots, n_N\}$ and start periods $\{b_1, b_2, \dots, b_N\}$. The inclusion of start periods allows the algorithm to construct a fully defined solution structure, enhancing the item-to-period loading scheme and allowing integer multipliers for the cycle time. Unlike traditional ELSP methods that restrict multipliers to Powers of Two, the GLS approach permits multipliers to be distributed across various periods. Items are assigned to periods based on their multiplier (n_i) and start period (b_i) values, with the condition that $1 \leq b_i \leq n_i$. A standout feature of this approach is how it manages items with non-PoT multipliers. It reserves machine time for these items in an 'odd/even' style, similar to the EBP approach, which prevents sequence feasibility issues and helps maintain schedule stability. Moreover, GLS addresses sequence interference problems by sequencing items within a period based on their multiplier value. This effectively extends the nested schedule properties to all integer multiplier values, significantly enhancing the feasibility and efficiency of the production schedule.

Both studies, Moon et al. (2002) and Chung and Chan (2012), apply a genetic algorithm based on the TVLS approach to solve the ELSP. However, they differ significantly in chromosome representation and problem-solving structure. Moon et al. (2002) use a hybrid GA and present a composite and unique chromosome representation. Their chromosome is split into two parts: Part A, which contains the indices of the products, and Part B, which indicates the absolute positions of the genes in Part A. This model combines the production frequencies and production sequence into a single chromosome, allowing the GA to operate on both aspects simultaneously. On the other hand, Chung and Chan (2012) propose a nested two-level GA with a distinct chromosome representation at each level. At the first level, the α-type chromosome optimizes the production frequencies, rounded to the nearest integer or the power of two. At the second level, the β-type chromosome, derived from the optimized α-type chromosome, is used to optimize the production sequence. This nested approach allows for more specific and focused optimization, addressing each aspect of the problem separately.

Selection and replacement operators play a crucial role in determining the effectiveness of genetic algorithms, but there's no universally superior approach. The choice of operators largely depends on the specific problem the algorithm needs to solve.

In terms of selection, a common approach is fitness-proportional selection, often termed "roulette wheel selection". This method, used by researchers like Chatfield (2007) and Sun et al. (2009), assigns each individual a selection probability proportional to their fitness. Thus, individuals with a high fitness score have a higher chance of being selected for crossover. This approach tends to favor high-quality solutions, accelerating algorithm convergence, but risks premature convergence to a local optimum. Another common selection approach is tournament selection, used by Khouja et al. (1998) and Moon et al. (2002). In this method, a subset of individuals is randomly selected, and the one with the best fitness is chosen. This strategy tends to maintain greater population diversity, reducing the risk of premature convergence.

Regarding replacement, the most widespread approach seems to be elitism, where the best solutions are preserved from one generation to the next. This method has been used by Khouja et al., Moon et al., Chatfield, and Sun et al. Elitism helps ensure the algorithm doesn't lose its found best solutions, but if applied too rigidly, can limit population diversity and hinder the exploration of new solutions.

Fitness-proportional selection and elitism are popular techniques, but there's no "one-size-fits-all" solution. The optimal choice of genetic operators depends on the specific problem to be solved and requires a careful balance between preserving found best solutions and exploring new solution space areas.

A similar discussion can be made for genetic operators, where there is no universally "best" genetic operator. The choice of genetic operators is heavily dependent on the nature of the problem and the specific needs of the algorithm. It requires a balance between preserving genetic material that contributes to high fitness and introducing new genetic material to explore a wider range of potential solutions.

Single-point crossover is the most commonly used among the authors studied, being employed by Khouja et al. (1998) and Moon et al. (2002). However, two-point crossover and uniform crossover are also used when the authors believe they are more suitable to their specific problem. For instance, Chatfield (2007) opts for two-point crossover, while Sun et al. (2009) use uniform crossover.

Bit-flip mutation is the most common mutation operator due to its simplicity and ability to introduce new diversity. This operator is used by Chatfield, Sun et al., and Khouja et al. However, when a higher level of diversity is necessary, multi-point mutation is employed, as is the case with Goncalves and Sousa (2011).
Every iteration of the genetic algorithm involves the evaluation of solutions in terms of feasibility and fitness. Both of these aspects depend on the specific ELSP formulation being used, whether it's the Basic Period approach, Extended Basic Period approach, or Time Varying Lot Sizes approach. Initially, the feasibility of a solution is assessed. This involves checking if the solution meets all of the problem's constraints, such as adhering to production capacities. Next, the fitness of the solution is evaluated. This process gauges the quality of the solution using the objective cost function as a reference. After the genetic operators have been applied, the algorithm may generate infeasible solutions. To handle these cases, a penalty function is typically incorporated into the fitness evaluation. Instead of outright discarding infeasible solutions, they are assigned a penalty to indicate their deviation from feasibility. This approach is crucial as it helps prevent premature convergence to local optima and encourages the exploration towards a globally optimal solution. This penalty approach in fitness evaluations has been utilized by various researchers in the field, including Khouja et al. (1998), Chatfield (2007), Sun et al. (2009), and Qiu and Chang (2009). Qiu and Chang (2009), in particular, employed heuristics to further enhance the fitness of solutions, directing the algorithm towards feasible and high-quality solutions.

5.4 Evaluation of performances

The objective of this research is to explore the recent advancements and enhancements in the application of Genetic Algorithms to solve the Economic Lot Scheduling Problem. The optimal approach to evaluate performance involves implementing a series of simulations to validate the efficacy of the proposed solution method. As indicated in Table 1, all authors have used the classic 10-items problem, originally proposed by

Bomberger in 1966, as a common ground for their experiments. This benchmark has been extensively utilized to test new approaches to the ELSP, providing a consistent basis of comparison for advancements in this field. The data for Bomberger's stamping problem are detailed in Table 3. Costs are computed based on 240 operational days per year, and production is estimated for an eight-hour workday. An annual interest rate of 10% is also accounted for. The desired utilizations are obtained by multiplying the base demands by a constant.

Table 3. Data from Bomberger's (1966) 10-items problem

Item i	Prod. rate p_i (units/day)	Demand rate d_i (units/day)	Setup time s_i (days)	Setup cost A_i ($)	Standard cost c_i ($/unit)
1	30 000	100	1	15	0,006 5
2	8 000	100	1	20	0,177 5
3	9 500	200	2	30	0,127 5
4	7 500	400	1	10	0,100 0
5	2 000	20	4	110	2,785 0
6	6 000	20	2	50	0,267 5
7	2 400	6	8	310	1,500 0
8	1 300	85	4	130	5,900 0
9	2 000	85	6	200	0,900 0
10	15 000	100	1	5	0,040 0

In order to gauge the quality of the models, it's critical to consider the utilization of the machines (or the sum of densities), defined as: $\rho = \sum_i \frac{d_i}{p_i}$ (where: i = item index; d_i = demand rate for item i; p_i = production rate for item i). Some authors, such as Moon et al. (2002), and Chung and Chan (2012), refer to its complementary: $\kappa = 1 - \sum_i \frac{d_i}{p_i}$, which is defined by Moon et al. (2002) as the long-run proportion of time available for setups. Utilization measure is critical for evaluating results as it provides a broader perspective. In the literature, reference utilization values such as 66% and 88% have been used.

However, it's clear that solving the ELSP is more challenging and meaningful for high values of utilization (small values of κ). In fact, values of 92%, 95%, and 98% are often considered as well. Moon et al. (2002), Goncalves and Sousa (2011), and Chung and Chan (2012) have calculated the total cost only for $\kappa = 0{,}01$, representing a highly loaded facility. These latter authors, by the way, are the only ones who have calculated the total cost on a daily basis, considering 240 working days in a year. For ease of comparison, in this study, the annual costs will be reported for all models. Table 4 provides annual costs expressed in dollars for different levels of utilization, with each row representing a different author's model. Furthermore, lower bound values have also been included as a point of comparison.

The simplest approach that comes to mind when finding a lower bound to the ELSP is the Independent Solution (IS). This method applies basic lot-sizing techniques to each product independently, treating them as if they were the only item being produced. By calculating the Economic Production Quantity (EPQ) for each product individually, the IS essentially disregards the capacity constraints imposed by the shared use of the machine among several products. This oversimplified approach has limited its application as a benchmark, with only a few studies, such as those by Khouja et al. (1998) and Chatfield (2007), considering it. In contrast, the majority of papers reviewed in this thesis adopt a tight lower bound scheme for benchmarking. This scheme, initially suggested by Bomberger (1966) and later rediscovered by researchers starting with Dobson (1987), calculates the EPQ under the machine's capacity constraints. This ensures a sufficient amount of time is available for setups, although it does not account for the synchronization constraint preventing simultaneous production of two items. Despite not being a closed-form expression like the IS, the solution to this nonlinear programming problem, which can be obtained via a line search algorithm, serves as a more accurate and practical lower bound on the total cost for the general ELSP due to its consideration of setup capacity. The common cycle approach, assuming identical cycle times for all products, has been proposed as an upper bound by some researchers. However, for benchmarking purposes in this study, this approach may not be considered useful.

Table 4. Annual cost (in USD) for different level of utilization

Utilization	66%	88%	92%	95%	98%	99%
Independent Solution		7 589	7 715	7 812	7 906	7 936
Tight Lower Bound	6 739	7 589	7 715	8 420	15 683	29 508
Dynamic Programming by Bomberger (1966)	7 178	8 796				
Heuristic by Haessler (1979)		7 697	7 972	11 962	22 526	
Heuristic by Dobson (1987)		7 697				30 802
Khouja et al. (1998)	7 024	8 782	9 746	12 018	24 534	55 545
Moon et al. (2002)						30 269
Chatfield (2007)		7 697	7 947	9 140	20 500	
Simulated Annealing by Raza and Akgunduz (2008)						30 034
Sun et al. (2009)		7 697	7 947	9 097	19 004	
Qiu and Chang (2009)	6 998	8 738				
Goncalves and Sousa (2011)						30 058
Differential Evolution algorithm by Tasgetiren et al. (2011)	7 024	8 782	9 746	11 952	24 477	47 633
Chung and Chan (2012)						29 698

One intriguing observation from the data is that the performance of several algorithms is highly dependent on the level of utilization. While certain algorithms excel in scenarios with low utilization, their performance can dramatically decrease when faced with high utilization problems. In some instances, identifying an optimal feasible solution becomes increasingly challenging as the utilization level reaches its peak. This pattern is particularly evident when examining the maximum level of utilization. The most successful model, as shown in the table, was devised by Chung and Chang (2012), utilizing a two-level genetic algorithm. This model comes remarkably close to the tight lower bound value, indicating its high efficiency. Similarly, the genetic algorithms developed by Moon et al. (2002) and Goncalves and Sousa (2011), along with the Simulated Annealing method by Raza and Akgunduz (2008), demonstrate commendable performance. On the contrary, the model proposed by Khouja seems to falter when dealing with high levels of utilization, indicating a possible limitation in its application.

The authors who employ the Time Varying Lot Size scheduling policy, including Moon et al. (2002), Raza and Akgunduz (2008), and Chung and Chan (2012), incorporate production times into their solutions. These sequences of production times are derived from the production sequences that are presented as solutions from their respective algorithms. In their solution for Bomberger's 10-items problem, all of them considered a utilization rate of 99%, which is representative of a highly loaded facility. Under this assumption, it's feasible to approximate idle time as zero. By applying the constraints of the ELSP in Dobson's (1987) formulation, it's possible to estimate the production times. This method is referred to as the "quick-and-dirty" heuristic by Moon et al. (2002). However, these production time calculations are not included in this study because they do not provide a basis for comparison, as they are not a standard feature across all the analyzed models, and they do not offer any significant insight for comparing models or identifying a superior one.

The experimental outcomes clearly indicate that the time-varying lot size approach is the most effective. In their 1998 study, Khouja et al. developed a genetic algorithm based on the Basic Period approach. While they recognized that this model might not surpass Dobson's Time-Varying Lot-Size method, proposed in 1987 and known for its superior performance, they nonetheless chose to use the Basic Period approach in their pioneering application of genetic algorithms to the ELSP.

6. CONCLUSIONS

A key aspect of this study lies in the substantial differences between scheduling policies. The time-varying lot size method has proven to be the best both in terms of total cost performance and inherent flexibility. This method allows for variable production run lengths, catering to the dynamic needs of production processes. However, it's also the most complex model to implement. Therefore, it's imperative to compare the solutions proposed by various authors based on their adopted scheduling policy. Moreover, when applied to real-world scenarios, the chosen model should best align with the case study, considering factors such as implementation difficulty and calculation speed. This ensures that the model not only theoretically fits but also practically adapts to the specific circumstances of each case.

The study of genetic algorithms applied to the Economic Lot Scheduling Problem has involved a comprehensive review of papers produced over a long-time span. This provides a unique perspective on the evolution of this field, both in terms of the models used and the results achieved. Throughout the thesis, we trace this evolution, highlighting key developments and shifts in approach.

As the sphere of production scheduling evolves, numerous opportunities arise for further investigation and refinement. One prominent area of interest is the study of the effects of specific assumptions on the efficacy of scheduling methods. The incorporation of these assumptions into the models not only opens up new possibilities for refining and improving scheduling methods, but also increases model complexity. This poses a significant challenge in developing models that accurately represent the evolving realities of production while remaining manageable and efficient in practice. By carefully selecting and implementing these assumptions, we could tailor our models more closely to the realities of production. This could enable us to better address issues like waste reduction, energy efficiency, and other sustainability goals. In this way, the ongoing refinement of these models could contribute significantly to the broader transition towards more sustainable production practices. The continuous evolution, development, and adaptation of scheduling methods, particularly as applied to the

ELSP, are crucial for improving both production processes and sustainability outcomes. The challenges are significant, but so too are the opportunities for innovation and improvement. As we move forward, the evolution of these methods will likely play a key role in shaping the future of sustainable production.

7. BIBLIOGRAPHY

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