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Methods and models for index tracking optimization

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Preface

I am immensely grateful to my professor, Fabio Salassa, for his guidance and support throughout my thesis journey. A heartfelt thank you to my friends for their encouragement and to my family for their unwavering belief in me. This accomplishment reflects the collective care and commitment of everyone who has supported me along the way.

Thank you.

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Abstract
In the ever-evolving landscape of finance, the quest for optimal portfolio management has been a continuous pursuit. The process of selecting the right mix of assets to maximize returns while minimizing risk has long been at the core of investment strategy. With the advent of computational tools and the availability of financial data, this endeavor has witnessed a paradigm shift. Today, investors and financial analysts have at their disposal a plethora of quantitative techniques and software applications for portfolio optimization, each with its own strengths and limitations.
This thesis delves into the realm of portfolio optimization by exploring and comparing the efficacy of two distinct methodologies—Monte Carlo simulation and Sequential Least Squares Quadratic Programming (SLSQP)—in the context of stock portfolio management. By examining and contrasting these three approaches, we aim to shed light on the advantages, drawbacks, and real-world applicability of each. Furthermore, we will assess the performance of these optimization models by comparing them with the Italian stock index IT40.
The primary objectives of this study are to assess the performance of these optimization models, analyze their outcomes, and provide insights into their practical implications for financial decision-makers. Additionally, we seek to determine how diversification, a fundamental concept in portfolio theory, influences the effectiveness of these models in constructing portfolios that achieve the delicate balance between risk and return.

Keywords: portfolio management, portfolio optimization, modern portfolio theory

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1 Introduction

1.1 Portfolio Management

Portfolio management is a dynamic and multifaceted discipline that serves as the keystone of modern finance. At its core, it is a strategic approach to selecting and overseeing a group of investments that align with the long-term financial objectives and risk tolerance of individuals, corporations, and institutions. In essence, it is the art and science of managing financial resources, a practice that has evolved into an essential component of financial decision-making.
Portfolio management presents two distinct approaches: passive and active strategies. These strategies cater to diverse investment philosophies and financial objectives.

Passive Management: Passive portfolio management represents a hands-off, long-term investment approach, often referred to as a "set-and-forget" strategy. It typically involves the allocation of resources to one or more exchange-traded (ETF) index funds. This approach, commonly known as indexing or index investing, is shaped by the principles of modern portfolio theory (MPT), enabling investors to optimize their asset allocation.

Active Management: In contrast, active portfolio management is characterized by a dynamic approach, where the primary goal is to surpass the performance of benchmark indices. It involves proactive buying and selling of individual stocks and other assets.
Generally, actively managed portfolios include closed-end funds. Active managers employ a wide array of quantitative and qualitative models to enhance their evaluations of potential investments.

1.2 Problem Discussion

This thesis embarks on an exploration of investment portfolio optimization, with a specific focus on assessing the effectiveness of various portfolio optimization models developed through Python. These models encompass quadratic optimization strategies, coupled with the implementation of Monte Carlo simulation. The study scrutinizes how these diverse models can be utilized to strike a balance between risk and return, particularly concerning the Italian index IT40, without using an active management approach.

In today's economy, investors seek to find the optimal equilibrium between risk and return while confronted with an enormous amount of alternatives, the inherent complexity and volatility of financial markets necessitate the application of advanced quantitative tools and optimization models. Portfolio optimization becomes crucial in this context.

The purpose of this thesis was to simulate the experience of an average investor, lacking of active management strategies and complex quantitative tools, with a central inquiry: Can a portfolio constructed by randomly selecting stocks and subsequently optimizing it using efficient models outperform the market?

The rationale for selecting this particular problem is particularly resonant. Because of my Italian heritage and because it mirrors Italy's distinctive financial dynamics, the Italian index IT40 holds significant relevance as a benchmark. Italy’s financial landscape presents challenges, and evaluating portfolio performance in relation to IT40 not only offers profound academic insights but also holds tangible practical value.

This thesis seeks to make a contribution to the evolving field of portfolio management by meticulously evaluating the applicability and effectiveness of diverse optimization models.
The objective is not only to assess how these models perform in isolation but also to provide a comparative analysis of their performance in relation to one another. These models vary not only in their underlying principles but also in complexity.
This comparative approach allows us to gain a clear understanding of the real-world effectiveness of these models and their practicality in the context of the financial landscape.

1.3 Research Questions

Building upon the foundational discussions in the preceding sections, this research is underpinned by the following fundamental questions:

- Can the Italian index IT40 be considered a benchmark for an optimal portfolio?

- To what extent can Modern Portfolio Theory (MPT) harness the potential of Python-based portfolio optimization models and Monte Carlo simulation in transforming our portfolio into an optimal portfolio?

- Do these diverse optimization models outperform the selected Italian index IT40 in terms of generating superior returns?

- Do these diverse optimization models outperform the selected Italian index IT40 in terms of generating inferior volatility?

- Can these optimization models provide investors with an elevated risk-adjusted return compared to the Italian index IT40?

- Which of these optimization models proves to be more efficient in terms of returns and volatility?

- Which of these optimization models is simpler to construct and comprehend?

- Do these optimization models outperform more stable and efficient indices as the S&P500, NASDAQ and NIFTY?

1.4 Thesis Objectives

The overarching purpose of this thesis is to investigate the potential for students and professionals to utilize different Python-based portfolio optimization models, including quadratic and Monte Carlo simulation, to attain a higher return and/or lower volatility compared to investing in preselected indices as the IT40 and S&P500. This research endeavors to shed light on the versatility and efficacy of these models in portfolio management and to provide insights into their practical application in the dynamic world of investment.

1.5 Research Methodology

The key components of the research methodology are as follows:

Literature Review: The study starts with a comprehensive review of academic literature, focusing on gaining deep insights into the theories, concepts, and best practices that underlie stock portfolio optimization. This review forms the theoretical foundation for the research and provides valuable insights into the academic discussions and practical approaches, particularly centered around Modern Portfolio Theory (MPT).

Model Implementation: The core of this research lies in the implementation of portfolio optimization models. Python will serve as the primary tool for constructing and applying different models, including quadratic optimization, and Monte Carlo simulation. These models will be fed with real-world data, mirroring practical investment scenarios.

Performance Evaluation: The results obtained from the diverse optimization models will undergo rigorous performance evaluation. This assessment encompasses a comprehensive analysis of their effectiveness in risk management, return generation, and overall portfolio performance. Specifically, these models will be compared in terms of their ability to generate returns and manage volatility, as measured by the Sharpe ratio. The objective is to uncover the strengths and limitations of each model through practical application and quantitative assessment. This comparative analysis is pivotal in understanding how these models perform concerning risk-adjusted returns, offering insights into their practical utility within the investment landscape.

Comparative Analysis and Sensitivity Testing: After the performance evaluation between the various portfolio optimization models, the study transitions to a comprehensive comparison between our models and selected indices, extending beyond the IT40. This broader analysis encompasses prominent market benchmarks, shedding light on how our models perform in comparison to established indices.
Additionally, the research delves into a series of controlled experiments to investigate how altering certain constraints and characteristics of the optimization models impacts portfolio outcomes. These experiments involve varying factors such as the initial asset allocation of the models and the time frame used to estimate the expected metrics. Through these experiments, the study aims to explore the sensitivity of portfolio performance to changes in these key parameters and, in turn, provide insights into adaptable strategies for optimizing portfolios under varying conditions.

2 Literature Review

In this chapter, we'll delve into the fundamentals of portfolio optimization. We will explore concepts like expected return, standard deviation, portfolio risk, diversification, covariance, Modern Portfolio Theory (MPT), the efficient frontier, the Capital Allocation Line, and the Sharpe Ratio. These principles are essential for building and managing well-balanced investment portfolios that effectively balance risk and return.

2.1 Fundamentals of Portfolio Optimization

2.1.1 Expected Return

A portfolio's expected return is the weighted average of the expected return of the individual assets. Depending on the weight of an individual asset this asset will have a larger or smaller impact on the return of the portfolio. Alternative assets differ in their terms of expected return, but the expected return is only a part of the asset's future performance. What may influence the expected return is how volatile the asset is. There are different approaches to estimate the expected return of an asset. One approach is to estimate the probability of different return outcomes, opposed to making estimates based on historical data. In our case we made estimates based on the historical closing prices of the last 5 years of each stock in the portfolio.
If an accurate measurement of the return of each asset can be made, the return of the whole portfolio can be predicted with the same accuracy. Unfortunately it is not possible to state the rate of return of an asset with certainty. The objective is to make a prediction about each asset in order to produce predictions about the whole portfolio.
The following equation (Equation 1) shows the expected return of the portfolio.

Equation 1

$$E(Rp) = \sum_{i=1}^{N} X_i \cdot E_i$$

Where:

$E(Rp)$ = expected return of the portfolio
X_i = proportion of security i
E_i = expected return of asset i

2.1.2 Standard Deviation

The standard deviation is the measurement of uncertainty associated with the asset, a measure of the dispersion of a set of data from its mean. As with the expected return, it is a measurement needed to estimate the standard deviation of the portfolio. The standard deviation is considered to be the risk measurement of an investment. It can be provided by logic estimation or by setting up a probability distribution. The standard deviation of the portfolio's rate of return depends on the standard deviations of return for its component securities, their correlation coefficients and the proportions invested.

It is calculated with the following equation (Equation 2):

Equation 2

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \rho_{ij} \sigma_i \sigma_j}$$

Where:

$X_i X_j$ = proportions invested in each asset
ρ_{ij} = correlation coefficients between i and j
$\sigma_i \sigma_j$ = standard deviation of each asset

2.2 Diversification & Covariance

Markowitz, inventor of the Modern Portfolio Theory, emphasized that the number of securities in a portfolio isn't the sole determinant of its effectiveness. What truly matters is the correlation between those securities. In more technical terms, this concept underscores the importance of diversification. Modern Portfolio Theory (MPT) formalized diversification by introducing the statistical concepts of covariance and correlation.
In essence, this principle suggests that concentrating all investments in assets with highly correlated returns, even if the chance of individual asset failure is low, isn't a prudent strategy. This is because highly correlated assets tend to move in the same direction. If one asset fails, it's likely that others will follow, potentially leading to a significant loss in the overall portfolio.
The idea of diversification is so intuitive and influential that it has found applications across various areas in finance. Many financial innovations are built on the foundation of diversification, either applying the concept itself or improving the estimation of variances and covariances, enabling more precise risk assessment.
Markowitz's significant contribution was devising a method to determine the overall risk of a portfolio. He introduced the concept of covariance, which measures the direction in which a group of stocks moves. High covariance indicates that two stocks tend to move together, while low covariance describes stocks moving in opposite directions. Markowitz argued that portfolio risk isn't solely about individual stock variances but is strongly influenced by the covariance of the entire portfolio. The more stocks move in the same direction, the greater the risk of simultaneous declines due to economic shifts.
Portfolios with assets that exhibit low covariance are less likely to move in sync, resulting in reduced portfolio volatility. Ideally, one seeks securities with minimal covariance. The covariance between the returns of two securities is calculated as the product of their correlation coefficient and the standard deviations of their returns.
This approach, by Markowitz, revolutionized portfolio management by introducing a quantitative way to account for the interplay between assets, and remains a fundamental concept in modern finance.

The equation is the following (Equation 3):

Equation 3

$$C_{jk} = \rho_{jk} \cdot \sigma_j \cdot \sigma_k$$

Where:

ρ_{jk} = correlation of coefficients
$\sigma_j \, \sigma_k$ = Standard deviation of each asset

The chart below (Figure 1) displays the risk-return relationship when transitioning from a 100% bond portfolio (marked by the red dot) to a 100% equity portfolio (marked by the blue dot). Along this trajectory, the green dot represents a portfolio with a 40% allocation to equities and 60% to bonds.

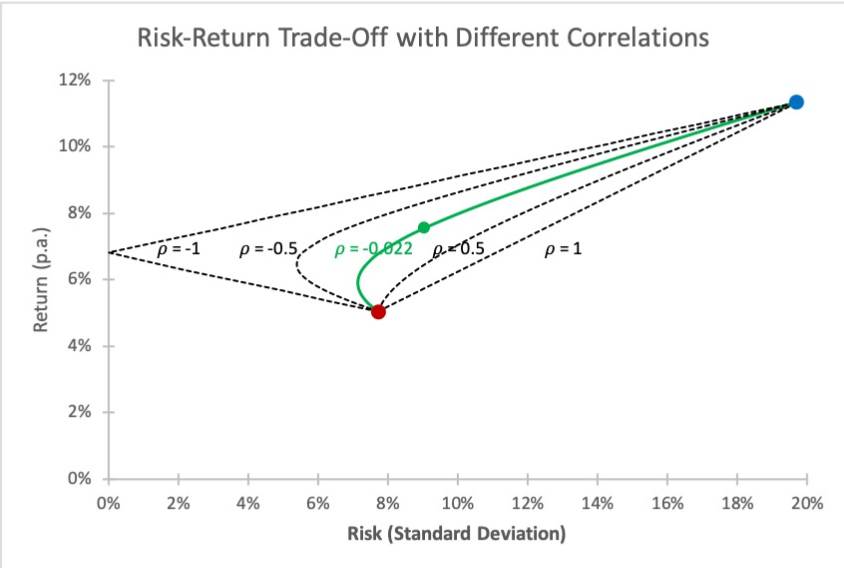


Figure 1 [1]

Evidently, introducing a modest proportion of equities to a bond portfolio significantly enhances the risk-return profile. Initially, as one shifts away from a bond-heavy portfolio, risk decreases while returns increase. These portfolios outperform a pure bond portfolio, offering higher returns with lower associated risk.
However, it's crucial to recognize the classical risk-return trade-off. There comes a point where additional returns can only be achieved by accepting higher levels of risk.
This graph also underscores the pivotal role of correlation. The dashed lines on the chart illustrate the risk-return relationship under various assumed correlations, ranging from a perfect positive correlation ($\rho=1$) to a perfect negative correlation ($\rho=-1$). Notably:

- In the case of perfect positive correlation ($\rho=1$), diversification has limited effect, resulting in a linear relationship between risk and return for the portfolio constituents.
- On the opposite extreme, with perfect negative correlation ($\rho=-1$), we achieve perfect diversification, virtually eliminating risk. In our example, this would require an allocation of approximately 28% to equities and 72% to bonds.

2.3 Modern Portfolio Theory

Before the advent of Modern Portfolio Theory (MPT) by Markowitz, portfolio management and risk considerations received little attention from investors. Portfolios were essentially assembled almost randomly; if an investor believed a stock was on an upward trajectory, it found its way into the portfolio.

Imagine an investor arranging a table of portfolios with identical risk levels but varying returns. In this scenario, the choice of the best portfolio becomes clear: one would naturally select the portfolio with the highest return when risk is kept constant, and conversely, the one with the lowest risk when seeking a particular level of expected return.
The theoretical best portfolio embodies the concept of achieving the lowest risk for a given expected return or the highest expected return for a given risk level. MPT's essence lies in optimizing the relationship between risk and return by constructing portfolios based on asset returns, risks, and their correlations with other assets. It creates a framework where every expected return is associated with a level of risk, and this risk-return relationship can be optimized through diversification. Such portfolios meeting these criteria are known as efficient portfolios. No other portfolio can offer a higher return at the same risk level. Conversely, a portfolio is deemed inefficient if it's possible to achieve a higher expected return without taking on more risk, or to reduce risk without sacrificing the expected return.

Investors have a valuable tool at their disposal to mitigate portfolio risk: diversification through holding combinations of assets that are not entirely positively correlated. In simpler terms, by holding a diversified portfolio of assets, investors can reduce their exposure to the risks associated with individual assets. Diversification has the potential to maintain the same expected return for a portfolio while reducing overall risk. The mean-variance framework for constructing optimal investment portfolios, pioneered by Markowitz, has laid the foundation for this approach and has been further refined by subsequent economists and mathematicians who have addressed its limitations.
In a scenario where all asset pairs exhibit zero correlations, meaning they are entirely uncorrelated, the portfolio's return variance can be calculated by summing the squares of the fractions allocated to each asset multiplied by the individual asset's return variance. The portfolio's standard deviation is then determined as the square root of this summation.
Conversely, if all asset pairs display a correlation coefficient of 1, indicating perfect positive correlation, the portfolio's return standard deviation becomes the sum of each asset's standard deviation weighted by the fractions allocated to them. In cases where portfolio weights and asset return standard deviations are predetermined, a correlation coefficient of 1 results in the highest possible standard deviation of portfolio return.
Two key principles within MPT underpin the formulation of the efficient frontier: maintaining a constant level of variance while maximizing expected return and holding the expected return constant while minimizing variance.

2.3.1 Efficient Frontier

Markowitz also introduced the concept of the efficient frontier, which is essentially a graphical representation with expected return on one axis and risk on the other. It's a curve that encompasses all portfolios optimized for expected return at a particular level of risk. This curve stretches from the bottom-left corner to the top-right corner, with each point along the line denoting a specific combination of potential reward and its associated level of risk.

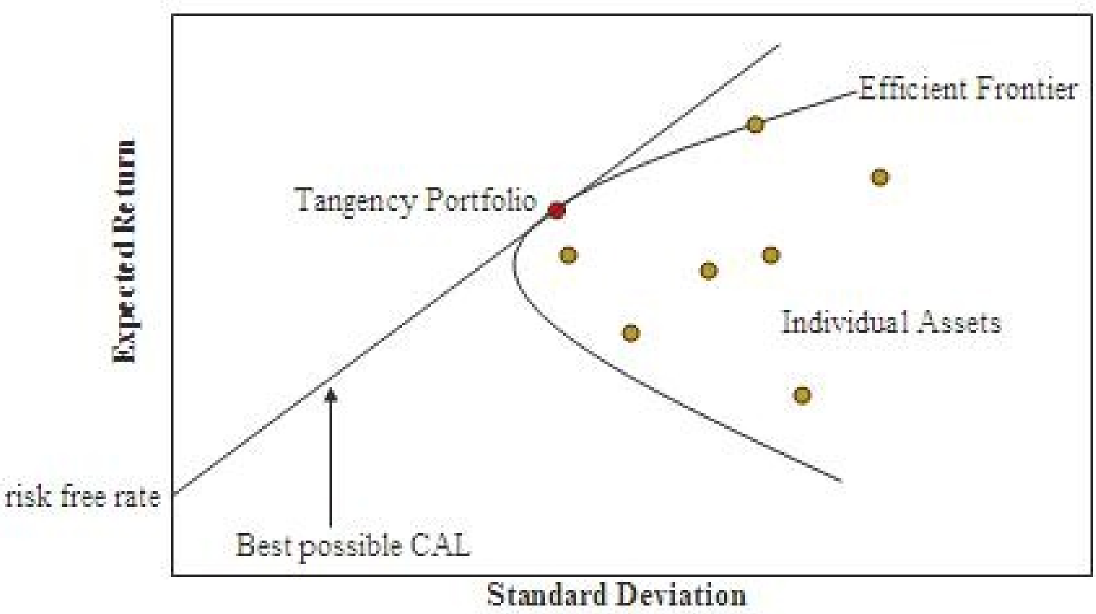


Figure 2 [2]

The graph above (Figure 2) shows the efficient frontier, the yellow dots represent the different combinations of the portfolio while the red dot represents the optimized portfolio.
In other words, it delineates the boundary of attainable risk-return combinations that an investor can achieve through different asset allocations. Portfolios lying on the Efficient Frontier are considered optimal because they offer the best risk-return trade-offs.

The concept of the efficient frontier provides a solution for determining the optimal level of diversification. It can be applied in various contexts and essentially represents a curve on a graph illustrating the connection between return and risk across a range of portfolios. To reside on the efficient frontier, a portfolio must maximize its return for a given level of risk.
In essence, it underscores the basic notion that risk and return are intertwined, suggesting that it is possible to quantify the degree of risk required to attain varying levels of return.

The most efficient portfolio is one that offers the highest return for a given level of risk. Achieving a tangent portfolio on the efficient frontier necessitates precise weighting of each asset in the overall portfolio. Using a single asset or equally distributing fractions of each asset won't connect with the efficient frontier. This weighting process is crucial to achieve a tangent portfolio on the efficient frontier.
For every level of return, there is a portfolio offering the lowest achievable risk, and conversely, for every risk level, there exists a portfolio offering the highest return. Any portfolio situated in the upper part of the curve is considered efficient, delivering the maximum expected return for a given risk level.
Investors invariably seek a portfolio along the efficient frontier, providing the highest possible return for a given level of volatility.
Due to changing returns over time and the influence of macroeconomic factors on the market and specific industry sectors, the allocation of portfolio weights must be periodically adjusted.

An inefficient portfolio exposes an investor to undue risk without a commensurate level of return.

2.3.2 Capital Allocation Line and Sharpe Ratio

The capital allocation line (CAL), shows the risk-return combination available by varying asset allocation, by choosing a different point on the CAL (see Figure 3). It plots the risk/return ratio by varying the allocation between risk-free assets to the risky assets.
The CAL-slope of the line is equal the increase in expected return that an investor can obtain per unit of additional risk, that is, extra return per unit of risk.

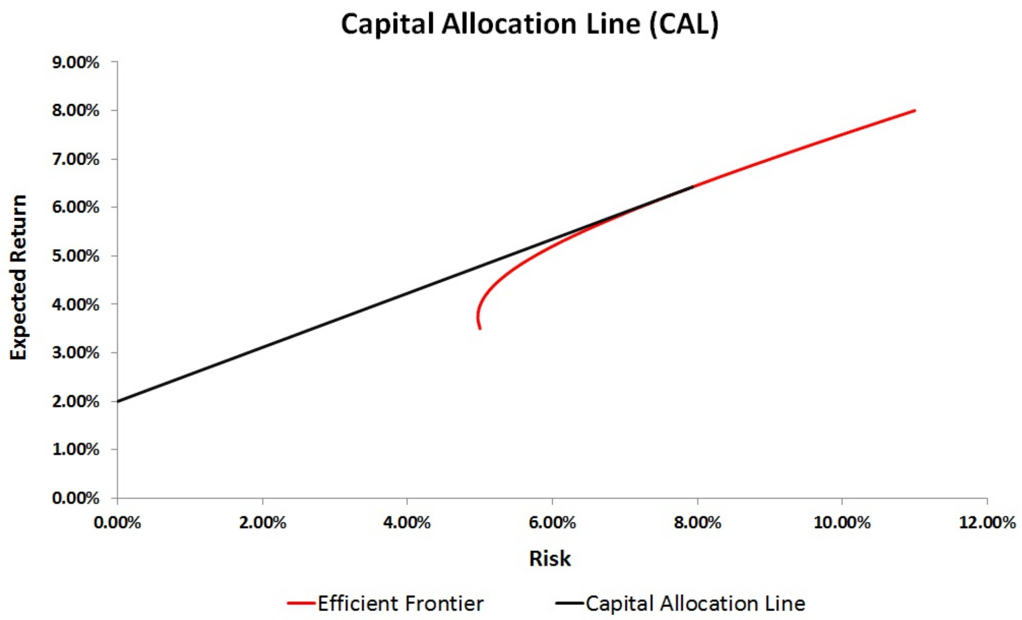


Figure 3 [3]

That is why the CAL-slope can also be interpreted as the Sharpe-ratio which is used to compare the expected returns of an investment to the amount of undertaken risk. The Sharpe-ratio is more appropriate when analyzing an entire portfolio then for example a single security. It tells us whether the returns from a portfolio come from good investments or as a result of excess risk. The Sharpe-ratio is used to compare it to a benchmark or another index, as in our case. The higher the ratio is, the better its risk adjusted performance has been. It is calculated with the following equation (Equation 4):

Equation 4

$$S = \frac{E(r) - r_f}{\sigma}$$

Where:

S = Sharpe Ratio
$E(r)$ = Expected Return of the Portfolio
r_f = Risk-free Rate
σ = Volatility of the Portfolio

2.3.3 Optimal Portfolio

The optimization principle should follow when the investor know the relation between risk and return. As illustrated graphically bellow (see Figure 4), the portfolio P (i.e., the CAL tangent to the efficient frontier) is the solution to the optimization problem of maximizing the slope of the CAL (maximizing the Sharpe Ratio)

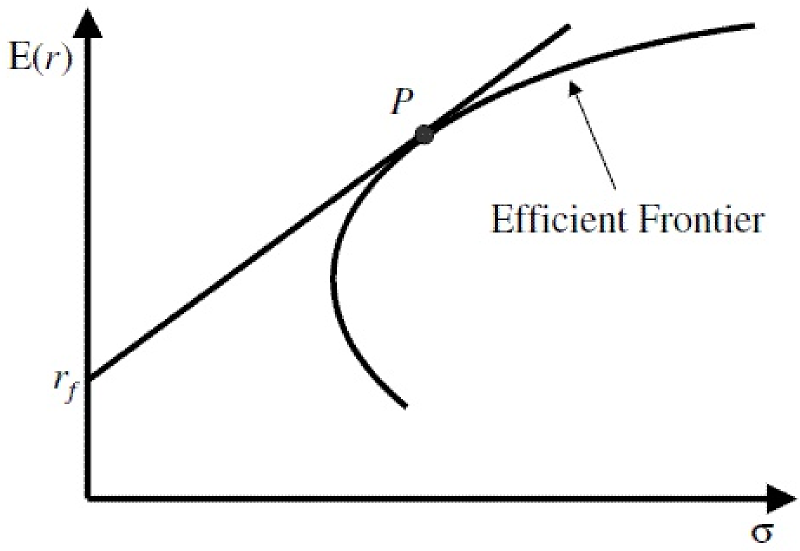


Figure 4 [4]

We can combine the efficient frontier and/or capital allocation line with indifference curves.

The optimal portfolio is the portfolio that gives the investor the greatest possible utility:

- Two investors will select the same portfolio from the efficient set only if their utility curves are identical.
- Utility curves to the right represent less risk-averse investors; utility curves to the left represent more risk-averse investors.

This is portfolio selection without a risk-free asset (see Figure 5):

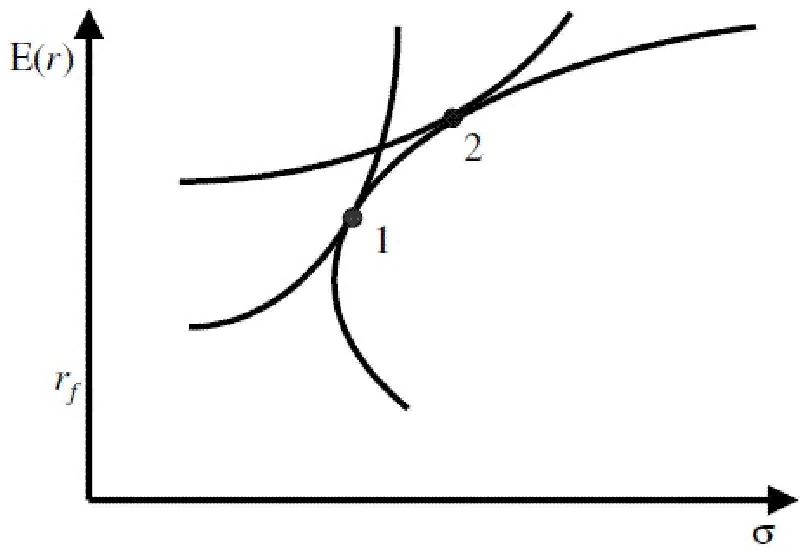


Figure 5 [5]

The optimal portfolio for each investor is the highest indifference curve that is tangent to the efficient frontier.

This is portfolio selection with a risk-free asset (see Figure 6):

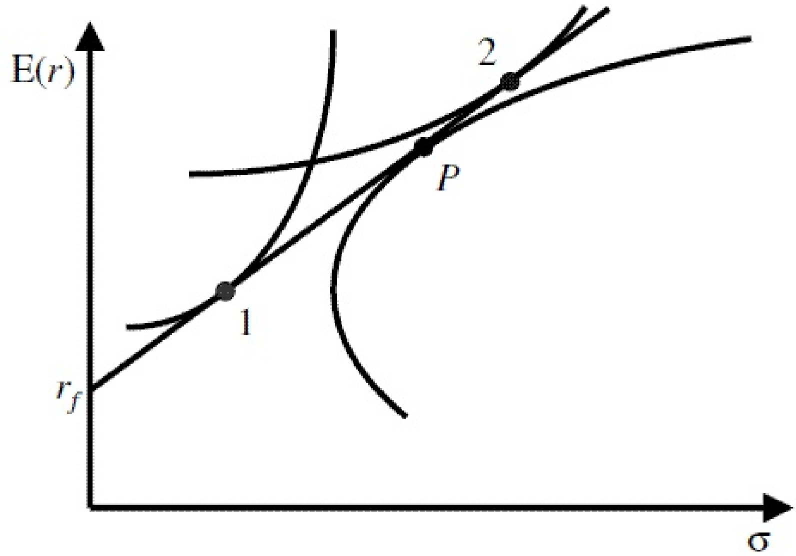


Figure 6 [6]

The optimal portfolio for each investor is the highest indifference curve that is tangent to the capital allocation line.

3 Research Approach

This chapter outlines the methodology for portfolio optimization, which involves selecting 40 random stocks and optimizing the portfolio to determine the optimal weights. The optimization process will be carried out using two different methods: the Efficient Frontier (utilizing Monte Carlo simulation) and Sequential Least Squares Quadratic Programming (SLSQP). Subsequently, a comparative analysis will be conducted to evaluate the performance of these portfolios based on return, volatility, Sharpe ratio, and their respective comparisons with many indices, considering two different timeframes.

3.1 Methodology

Portfolio Selection

The study begins with the random selection of 40 stocks. The selection process aims to ensure diversity within the portfolio and to create a representative sample of stocks from various sectors and industries. All the stocks chosen for this analysis are sourced from reputable and widely recognized companies, aligning with the objective of simulating the investment behavior of the average investor who may not engage in exhaustive research and analysis. These stocks will be bought and held, with no short positions.

Portfolio Optimization Methods

• The first method for portfolio optimization is the Efficient Frontier, constructed with the aid of Monte Carlo simulation. The Efficient Frontier represents a set of portfolios that offer the best trade-offs between expected return and risk (volatility). The goal is to construct a portfolio with optimal asset weights that maximize the Sharpe ratio, a measure of risk-adjusted return. The Monte Carlo simulation, run 3000 times, fine-tunes the allocation of assets to achieve the most favorable portfolio configuration.

• Sequential Least Squares Quadratic Programming (SLSQP) is the second optimization method. It is a numerical optimization technique used for solving nonlinear constrained optimization problems. SLSQP enables a more intricate assessment of stock characteristics, considering nonlinear relationships and constraints within the data. This optimization method aims to maximize the Sharpe ratio while implementing a constraint that prevents any single stock from dominating the portfolio by accounting for a maximum allocation of 30% to each stock. This constraint ensures that the portfolio isn't overly skewed towards a few large companies, promoting a more balanced and diversified investment approach.

Comparative Analysis

After applying the optimization methods, two distinct portfolios will be constructed. These portfolios will be evaluated based on key performance metrics like the expected return, volatility, and the Sharpe Ratio. This analysis will provide insights into the trade-offs and characteristics of each portfolio under different optimization methods.
Subsequently, the performance of these portfolios will be compared with each other to identify the strengths and weaknesses of each optimization method. The comparative analysis will also include a comparative evaluation of our portfolios mainly against the IT40 index but also against S&P 500, NASDAQ, DJI and NIFTY. This step allows for an assessment of how well the portfolios have performed in comparison to the broader market, providing valuable insights for investors and financial analysts.

3.2 The Choice of the Optimization Models and Building the Efficient Frontier

The Efficient Frontier is a graphical representation of all possible portfolios that an investor can construct with a given set of assets. These portfolios exhibit varying combinations of expected return and risk (measured by standard deviation). The Efficient Frontier curve plots these portfolios, showing the trade-off between risk and return.
Monte Carlo simulation generates a multitude of random portfolios, each residing at a different point on the risk-return spectrum. By running the simulation numerous times, you create a distribution of possible portfolios, which can be visualized as a cloud of points. The Efficient Frontier is then drawn by connecting the portfolios with the highest Sharpe ratios for each level of risk (volatility).
The construction of the Efficient Frontier involves several steps, and in this study, these steps will be carried out after the stock selection:

Asset Selection: The initial phase of the portfolio construction process involved a random selection of 40 stocks, guided solely by my foundational knowledge of the market without any preceding analysis or research. This approach was intentionally adopted to simulate an entry-level investment strategy, where choices are influenced primarily by general awareness rather than in-depth, data-driven decision-making.

Expected Returns and Risk: Once the 40 stocks are selected, their expected returns and risk, measured by standard deviation, will be estimated based on historical data and the optimization results. This will serve as the foundation for constructing the Efficient Frontier.

Correlation Analysis: The correlation analysis among the selected stocks will provide insights into their interdependencies, helping to understand how they move in relation to each other. This information is vital for creating an efficient and diversified portfolio.

Portfolio Simulations: Mathematical techniques, including Monte Carlo simulations, will be employed to simulate a wide range of portfolio combinations. These simulations will consider various weightings of the 40 chosen stocks, leveraging the outcomes of the two optimization models.

Efficiency Frontier Plot: The risk (standard deviation) will be plotted on the x-axis, and expected return on the y-axis to create the Efficient Frontier. This graph will illustrate the risk-return trade-off for different portfolio compositions based on the selected stocks and their optimized weights.

Mean-Variance Optimization:
Mean-Variance Optimization, a fundamental component of Efficient Frontier Theory, will play a pivotal role in determining the optimal allocation of assets within the portfolio. This technique considers not only the expected returns and standard deviations of the assets but also the correlations among them. By utilizing this optimization method, the study aims to identify the most efficient asset allocation that maximizes risk-adjusted returns based on the selected stocks.

Risk-Return Trade-Off:
The Efficient Frontier Theory underscores the critical concept of the risk-return trade-off. Portfolios situated on the Efficient Frontier exemplify this trade-off, illustrating that investors cannot achieve higher expected returns without accepting an increase in risk. In this study, the stocks chosen through the optimization models are strategically aligned to this trade-off, allowing investors to tailor their portfolios to match their specific risk tolerance and return objectives effectively.

The choice of Quadratic Programming (QP), specifically the SQP method, proves to be well-suited for this optimization problem. Quadratic Programming excels in handling quadratic objective functions subject to linear equality and inequality constraints.
In our case, the non-negativity constraints on asset weights align with the typical structure of QP problems. Moreover, the quadratic nature of the Sharpe ratio, involving variances and covariances of asset returns, harmonizes seamlessly with the inherent characteristics of QP methods.
SQP, as an iterative method for constrained nonlinear optimization, fits seamlessly into the portfolio optimization landscape.
One key advantage of SQP is its suitability for problems where the objective function and constraints are twice continuously differentiable. In the case of portfolio optimization, where the objective is to maximize the Sharpe ratio, the use of SQP ensures that the optimization process is both efficient and accurate.
SLSQP is an SQP method: it replaces the original problem with a sequence of Quadratic Problems (QP) whose objective are second-order approximations of the Lagrangian and whose constraints are the linearized original constraints. It then uses globalization techniques to guarantee convergence whatever the initial point.
The inclusion of SLSQP as a complementary optimization method offers a more robust approach. SLSQP is suitable for handling non-linear relationships, making it better equipped to capture the interactions among financial assets. This method is particularly valuable when dealing with more complex, nonlinear constraints and objectives. While it provides a more accurate representation of real-world financial market dynamics, SLSQP's drawback lies in its increased computational complexity, which may demand more computational resources and time.

3.3 The IT40 Index

The IT40 index, also known as the FTSE Italia All-Share, is a comprehensive stock market index that encompasses a diverse range of Italian companies. This index is designed to offer a holistic view of the Italian equity market and serves as a performance benchmark for investors and fund managers. It comprises 40 major Italian companies, each of which represents a distinct segment of the Italian economy. These companies span various industries, contributing to the diversity and representativeness of the index.
The chart below (Figure 7) represents the price trend of the index over the last 5 years.

Figure 7 [7]

The Constituent Stocks

The IT40 index is comprised of a select group of Italian companies, each with its unique characteristics and business focus. These constituent stocks represent a cross-section of Italian industry sectors, encompassing both established firms and up-and-coming enterprises. The constituent stocks include prominent players in sectors such as finance, manufacturing, energy, telecommunications, and consumer goods. They represent a microcosm of the Italian economy and offer valuable insights into its health and vitality. (See *Appendix B*)

Industry Landscape

To gain a more comprehensive understanding of the IT40 index, it is essential to appreciate the industries that these constituent stocks collectively represent. Italy boasts a diverse economic landscape, with a rich tapestry of industries contributing to its growth. The constituent stocks of the IT40 index participate in these key sectors:

- **Financial Services**: A significant proportion of the index comprises banks and financial institutions. These entities are instrumental in supporting Italy's economic activities and capital markets.

- **Manufacturing**: Italian manufacturing companies represent a substantial share of the index. These companies engage in the production of a wide array of goods, including automotive products, machinery, and industrial equipment.

- **Energy and Utilities**: The energy sector is another vital component of the Italian economy. Companies within the IT40 index are actively involved in the generation and distribution of energy, ensuring the nation's power needs are met.

- **Telecommunications**: In an increasingly connected world, telecommunications companies play a pivotal role in keeping Italy connected and accessible. This sector is well-represented within the IT40 index.

- **Consumer Goods**: Italian consumer goods companies cater to domestic and international markets, producing high-quality products that are recognized worldwide.

The pie chart represents the division of the index based on sector (Figure 8)

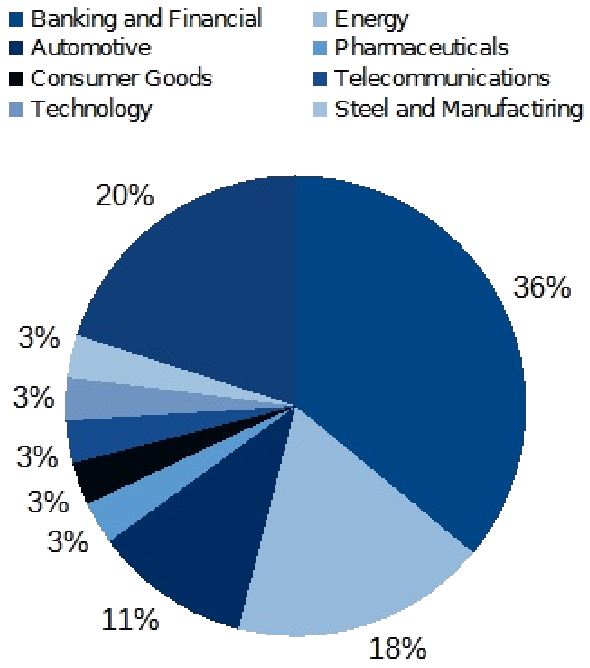


Figure 8

Rationale for Choosing the IT40 Index as a Benchmark

The selection of the IT40 index as a benchmark for this thesis is underpinned by several compelling reasons:

- **Representation**: The IT40 index offers a comprehensive representation of the Italian equity market. Its constituent stocks collectively span a wide range of industries, making it an ideal choice for assessing the performance of diverse portfolios.

- **Relevance**: The Italian market is of significant relevance within the European and global contexts. Understanding its dynamics and performance is pertinent for investors seeking to make informed investment decisions.

- **Diversity**: The diversity of the IT40 index's constituent stocks ensures that the thesis's findings and recommendations are broadly applicable to a variety of investment scenarios and portfolios.

- **Comparative Analysis**: By comparing various portfolio optimization models against the performance of the IT40 index, this research aims to offer practical insights and recommendations that can guide investment decisions within the Italian market.

3.4 S&P500, NASDAQ-100, NIFTY & DOW JONES INDUSTRIAL

By including these benchmarks, we aim to provide a well-rounded evaluation of our portfolios' performance considering international diversification.

S&P500:

The S&P500, often referred to as the Standard & Poor's 500, is a widely recognized stock market index that measures the performance of 500 of the largest publicly traded companies

in the United States. It encompasses companies from various industries and is regarded as a key indicator of the overall health of the U.S. stock market and worldwide. We have chosen the S&P 500 as a benchmark due to its global prominence and influence, providing a baseline for comparing our portfolios against a well-established and diversified market index.
The comparison with the S&P 500 could also be used as a measure of the beta of our portfolio, helping to assess its sensitivity to market fluctuations.

NDX (NASDAQ-100):
The NDX, or NASDAQ-100, is a stock market index composed of 100 of the largest non-financial companies listed on the NASDAQ stock exchange. It is heavily weighted towards technology and biotech sectors, making it a prime gauge for the performance of the industry leaders in these innovative and fast-paced fields. By including the NDX in our comparative analysis, we can measure our portfolio's performance against a benchmark that is representative of the high-growth potential sectors that are defining the new economy. This comparison is particularly useful for assessing the portfolio's alignment with technological advancement and its exposure to high-tech market volatility.

DJI (Dow Jones Industrial Average):
The DJI, more commonly known as the Dow Jones Industrial Average, is one of the oldest and most famous stock market indices. It tracks 30 large, publicly-owned companies based in the United States, covering all industries except for transportation and utilities. The DJI is often synonymous with the market to the general public and is used as a barometer for the overall performance of the U.S. economy and investor sentiment. Including the DJI in our portfolio comparison offers a perspective against a traditional, blue-chip index that has stood the test of time. It allows us to evaluate how well our portfolio might perform in a more conservative and established market environment, often preferred by risk-averse investors.

NIFTY:
NIFTY is an index that represents the National Stock Exchange of India (NSE). It comprises 50 of the largest and most liquid companies in the Indian equity market. Our selection of NIFTY as a benchmark extends our analysis to a different global market, allowing us to evaluate how our portfolios perform in diverse economic and market conditions. This choice offers valuable insights for investors with a global perspective, considering investments beyond the United States.

3.5 Implementation of the models

This chapter discusses the implementation of portfolio optimization models using Python and the utilization of the open source library SciPy. These models aim to construct an investment portfolio with the objective of maximizing returns while managing risk and adhering to various constraints.
The only bounds and constraints on both of these models are that the sum of all weights of the stocks in the portfolio must be equal to one and that the maximum weight that each stock could have in the portfolio cannot exceed the 30% of the total weights. (See *Appendix A* for the code)

Efficient Frontier

1) Data Retrieval:

- You start by defining a list of stock tickers that you want to consider for your portfolio.
- You specify the date range for data retrieval using the Yahoo Finance platform.
- Historical price data for the selected stock tickers is collected over the specified date range.

2) Random Portfolio Generation:

- You proceed to generate a random portfolio by assigning random weights to each stock in your selected list.
- To ensure that the weights sum up to 1 (representing a fully invested portfolio), you normalize the random weights.

3) Portfolio Metrics:

- Key portfolio metrics are calculated based on the randomly generated portfolio. These metrics include:
  * Expected Return: The expected annualized return of the portfolio.
  * Expected Volatility: The expected annualized volatility (standard deviation) of the portfolio.
  * Sharpe Ratio: A measure of the risk-adjusted return, which indicates how well the portfolio performs relative to its risk.
- The results of these calculations are stored in a DataFrame for further analysis.

4) Monte Carlo Simulation:

- You perform a Monte Carlo simulation, generating a large number of random portfolios with various asset allocations.
- For each of these portfolios, you calculate the expected return, volatility, and Sharpe ratio.
- The results are collected and stored in a DataFrame, providing insights into the risk-return trade-off across multiple portfolios.

5) Identifying Optimal Portfolios:

- From the set of simulated portfolios, you identify two critical portfolios:
  * Max Sharpe Ratio Portfolio: The portfolio with the highest Sharpe ratio, representing the optimal risk-adjusted return.
  * Min Volatility Portfolio: The portfolio with the lowest volatility, signifying the minimum risk portfolio.
- These portfolios are essential for understanding the trade-off between risk and return.

6) Plotting the Results:

- To visually illustrate the risk-return trade-off, you create a scatter plot.
- The plot showcases the relationship between expected returns and portfolio volatility for all the simulated portfolios.
- The Max Sharpe Ratio Portfolio and Min Volatility Portfolio are highlighted on the plot for reference.

Optimization with SLSQP

1) Objective Function for Optimization:

- You define an objective function for the optimization process, aiming to maximize the Sharpe ratio.
- Since the optimization method minimizes by default, you use the negative Sharpe ratio as the objective to maximize. This is equivalent to maximizing the Sharpe ratio.

2) Constraints:
- Specific constraints are set for guiding the optimization process:
* Sum of Portfolio Weights: The constraint ensures that the sum of portfolio weights equals 1, representing a fully invested portfolio.

3) Optimization:
- The Sequential Least Squares Quadratic Programming (SLSQP) optimization method is used to find the optimal portfolio allocation.
- The objective of the optimization is to maximize the Sharpe ratio while adhering to the defined constraints, including the limitation on individual stock allocations not exceeding 30% of the portfolio.
- The initial allocation of the assets is equal and not randomized as before (2,5% each stock)

4) Display Results:
- After the optimization process, you print and display the results, which include the following:
- Optimized Portfolio Weights: These are the portfolio weights for each selected stock that maximize the Sharpe ratio, while respecting the specified constraints.
- This optimized portfolio allocation represents the final outcome of the entire process, providing a strategy for constructing a portfolio that balances risk and return effectively.

3.6 Parameters and Assumptions of our Portfolios

This portfolio optimization study is underpinned by several key assumptions, each of which plays a pivotal role in shaping the research framework. Firstly, the "buy and hold" assumption implies that once assets are acquired within the portfolio, they are retained over the entire analysis period without active trading or rebalancing. This simplifies the portfolio management approach and allows for a focused assessment of the optimization models' performance. Secondly, the absence of transaction costs and taxes acknowledges that the study operates within a theoretical framework, detached from real-world financial constraints. This enables a more streamlined evaluation of the models' effectiveness in isolation. Lastly, the analysis and the expected metrics are based on historical data spanning the last year and 5 years. This temporal boundary serves as the foundation for assessing the models' performance and ensuring the relevance of the findings to contemporary investment scenarios.
The risk free rate used to calculate the Sharpe Ratio is approximated at 4%, similar to the yield of the U.S. 10 year treasury bond which is 4.5% as today.
Portfolio optimization is a complex task that often involves a multitude of variables, constraints, and objectives. These intricate interactions can sometimes lead to suboptimal solutions, as the algorithm may struggle to navigate through the vast landscape of possible portfolio combinations. While advanced optimization techniques aim to converge towards the best possible portfolio, real-world complexities and uncertainties can limit the assurance of absolute optimality.

3.7 Asset Allocation

Simulating the choices of the average investor who often gravitates toward these popular and well-recognized companies, in our quest for portfolio optimization, we curated a selection of 40 renowned global companies, primarily active in the United States market. These companies represent a diverse spectrum of industries, rendering them ideal candidates for constructing a diversified portfolio.

To establish our initial portfolio, the one built with the efficient frontier method, we adopted a randomized approach, allocating each stock a weight without any predefined bias or pattern. This random allocation introduced an element of chance into our portfolio construction, providing the foundation for subsequent refinement and optimization.
As part of our analysis, we will also explore the performance of equal-weighted portfolios, with the second model (SLSQP), where each stock within the portfolio carries an identical weight. This approach, characterized by its simplicity and fairness, serves as a benchmark against which the optimized portfolios can be compared. This broader perspective allows us to assess how our portfolio strategies measure up against both random and equal-weighted alternatives.

Companies:

- Apple Inc. (AAPL)
- Microsoft Corporation (MSFT)
- Amazon.com Inc. (AMZN)
- Alphabet Inc. (GOOGL)
- Meta Platforms, Inc. (formerly Facebook, Inc.) (META)
- Tesla, Inc. (TSLA)
- JPMorgan Chase & Co. (JPM)
- Johnson & Johnson (JNJ)
- Visa Inc. (V)
- Procter & Gamble Company (PG)
- Berkshire Hathaway Inc. (Class B) (BRK-B)
- Walmart Inc. (WMT)
- Mastercard Incorporated (MA)
- NVIDIA Corporation (NVDA)
- The Home Depot, Inc. (HD)
- The Walt Disney Company (DIS)
- Verizon Communications Inc. (VZ)
- Pfizer Inc. (PFE)
- Intel Corporation (INTC)
- Cisco Systems, Inc. (CSCO)
- AT&T Inc. (T)
- PepsiCo, Inc. (PEP)
- Salesforce.com, Inc. (CRM)
- The Coca-Cola Company (KO)
- Exxon Mobil Corporation (XOM)
- Netflix, Inc. (NFLX)
- NIKE, Inc. (NKE)
- Thermo Fisher Scientific Inc. (TMO)
- Adobe Inc. (ADBE)
- Bank of America Corporation (BAC)
- Abbott Laboratories (ABT)
- Novartis AG (NVS)
- Merck & Co., Inc. (MRK)
- Comcast Corporation (CMCSA)
- PayPal Holdings, Inc. (PYPL)
- UnitedHealth Group Incorporated (UNH)
- Honeywell International Inc. (HON)
- The Goldman Sachs Group, Inc. (GS)
- Amgen Inc. (AMGN)

Country Allocations:

The pie chart below represents the division of our portfolio based on countries (*Figure 9*)

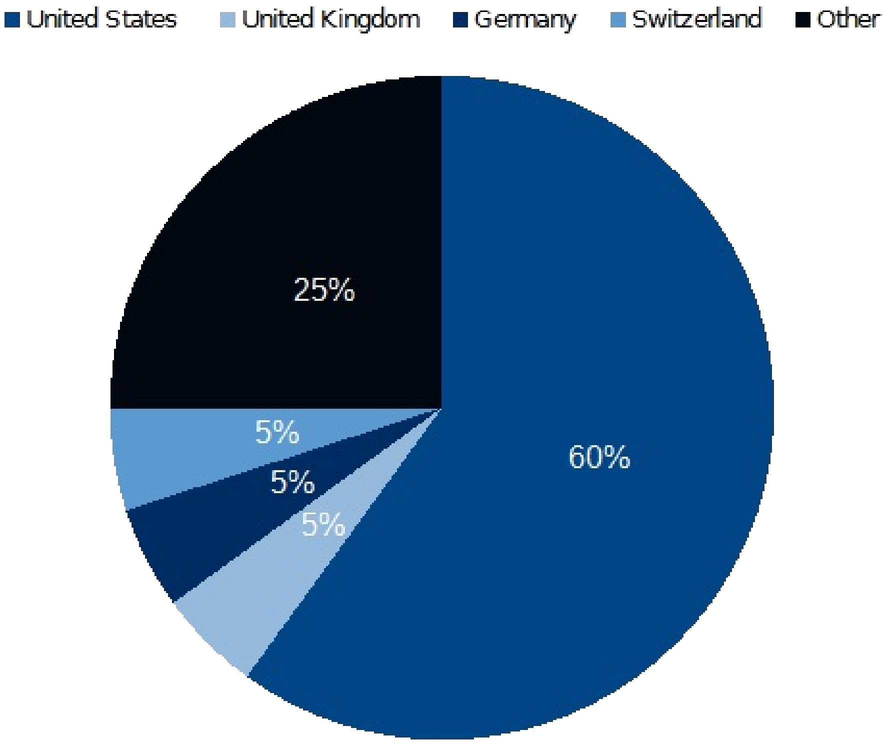


Figure 9

Industry Allocations:

The pie chart below represents the division of our portfolio based on industries (*Figure 10*)

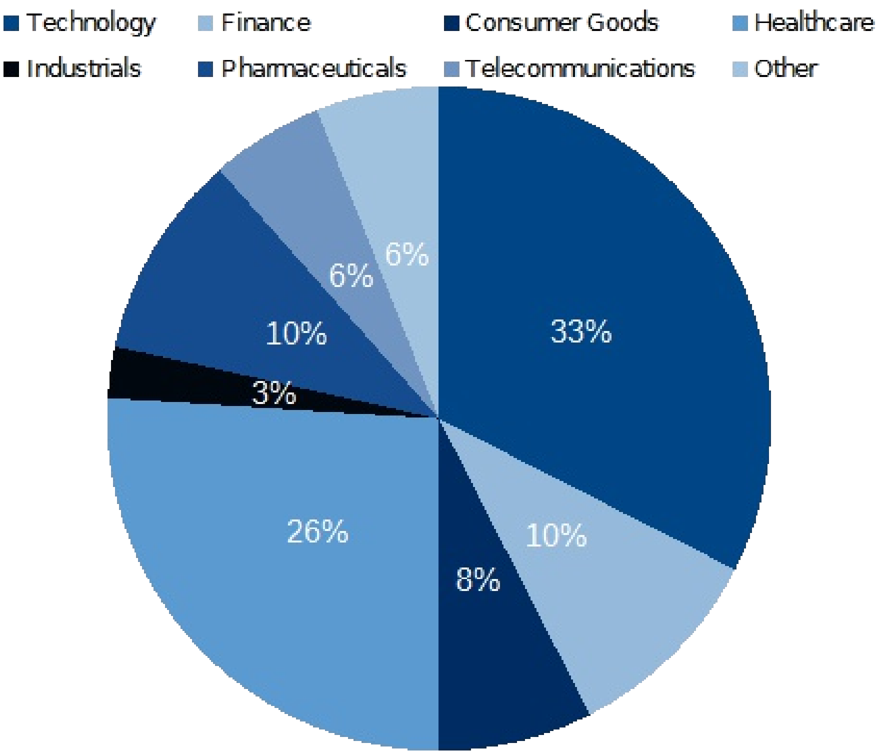


Figure 10

4 Discussion of the results

The primary objective of this chapter is to present a comprehensive and in-depth analysis of the results derived from the evaluation of two distinct investment portfolios, labeled as Portfolio 1 and Portfolio 2. These portfolios represent different investment strategies, each designed to cater to varying investor preferences and objectives. In the quest for meaningful insights, we will subject these portfolios to rigorous scrutiny, utilizing key performance metrics and characteristics.
Notably, this chapter extends its analysis beyond the Italian market represented by the IT40 index. We will also compare the performance of our portfolios with the S&P 500, NDX, DJI and NIFTY, for several compelling reasons.
It’s important to remind that the non-linearity of constraints and the objective function does not inherently guarantee optimality in portfolio optimization. In other words, the mere presence of non-linear relationships within the constraints and the optimization goal does not ensure that the algorithm will unfailingly deliver the perfectly optimized portfolio.

Expanding the Benchmark Universe
While the IT40 index serves as a valuable benchmark for our analysis, we recognize the need to provide a broader perspective on portfolio performance. In view of this, we have incorporated the S&P 500, NDX and DJI as one of the benchmarks. This U.S.-based index, comprising some of the world's largest and most influential companies, offers a contrasting market landscape compared to the IT40. By comparing our portfolios against the S&P 500, we aim to discern their resilience and adaptability across different market environments.

To further enhance the global perspective, we have introduced the NIFTY index as another benchmark. NIFTY represents the Indian equity market, a distinctive and dynamic landscape that adds a layer of complexity to our comparative analysis. By assessing portfolio performance against NIFTY, we gain insights into their adaptability across diverse global markets.

Altered Timeframes for Performance Evaluation
Our evaluation of portfolio performance will extend beyond the conventional one-year historical data. To provide a more robust assessment, we will calculate expected returns and volatility for both one-year and five-year timeframes. This adjustment allows us to capture short-term and long-term performance dynamics. The one-year horizon reflects the immediate responsiveness of the portfolios to market fluctuations, while the five-year perspective encompasses a more extended investment journey, shedding light on their consistency and stability.

Random and Equal-Weighted Portfolios
In our pursuit of a comprehensive analysis, we will delve into two additional portfolio strategies. The first strategy involves the introduction of randomness to the stocks within our portfolio before optimization. In the second strategy all stocks within the portfolio will be assigned an equal weight of 2.5% (as there are 40 stocks in total). This strategy revolves around equal-weighted portfolios, where every asset within the portfolio carries an equal weight. This approach is distinguished by its simplicity and fairness, making it an ideal benchmark against which the other portfolios can be measured.

Our objective in introducing these alternative portfolio strategies is twofold: first, to expand the breadth of our analysis, and second, to provide a well-rounded assessment of their performance in a spectrum of investment scenarios. By introducing randomness to the initial stock weights, we aim to simulate the challenges and uncertainties that investors may encounter in real-world investment decisions. This enhances the practical relevance of our analysis, offering insights into portfolio adaptability under varying and unpredictable conditions.

4.1 Performance Metrics

Before analyzing the individual portfolios, it is essential to understand the performance metrics employed in this assessment. We use the following key metrics:

- **Sharpe Ratio**: The Sharpe Ratio measures the risk-adjusted return of a portfolio. It provides insights into whether a portfolio's returns adequately compensate for the risk taken.

- **Expected Return**: The expected return is the mean return of the portfolio, indicating its historical performance.

- **Volatility (Standard Deviation)**: Volatility is a measure of risk, signifying the extent of variation in the portfolio's returns. Higher volatility implies greater risk.

4.2 Comparison of results from different models

As we delved into the optimization process, the two distinct approaches offered unique insights into risk reduction and return enhancement. The first approach, known as the efficient frontier, maintained the presence of all 40 stocks in our portfolio.
The second approach, the Sequential Least Squares Quadratic Programming (SLSQP) method, took a different path. It meticulously optimized the portfolio, resulting in a more focused selection of stocks. In this process, the SLSQP method effectively eliminated most of the stocks, retaining only seven of the original 40.
The rationale behind these divergent outcomes lies in the inherent nature of the methods. The efficient frontier prioritizes diversification across different sectors and industries, striving to capitalize on their independence to reduce risk. This approach seeks to emulate a global market with each industrial sector representing a national market. As a result, it preserves the presence of all stocks.
Conversely, the SLSQP method, while equally concerned with risk reduction, adopts a more concentrated approach. It identifies specific stocks that contribute most to maximizing the Sharpe ratio, effectively eliminating those that do not significantly enhance portfolio performance. The outcome is a leaner, yet more optimized, portfolio.

4.2.1 Optimized results - 5 year time frame

In our quest for portfolio optimization, we begin with a randomized allocation of 40 globally acclaimed companies, predominantly from the U.S. market. At the outset of the first method each stock's initial weight was chosen randomly, while the second one each stock’s initial weight was equal distributed. This randomness was introduced to underscore the unpredictability of market conditions, to challenge our optimization methods and to simulate the average investor, who usually doesn’t put enough calculus in the choice of his stocks.
The expected return and volatility presented in this analysis are derived from historical closing prices spanning the last 5 years.

Portfolio 0 - Not Optimized

The journey started with a random allocation of weights, deviating significantly from the concept of equal allocation. Over the 5-year period, this portfolio exhibited an expected return of 10.50% and a relatively low volatility of 21.45%. The Sharpe ratio, which measures risk-adjusted return, stood at 0.396, indicating a satisfactory performance. The high Sharpe ratio is likely an artifact of the random allocation and may not be sustainable or replicable.

Expected Return	Expected Volatility	Sharpe Ratio
0.104974	0.214493	0.396162

The pie chart below represents the initial allocation of the stocks in our portfolio (*Figure 11*)

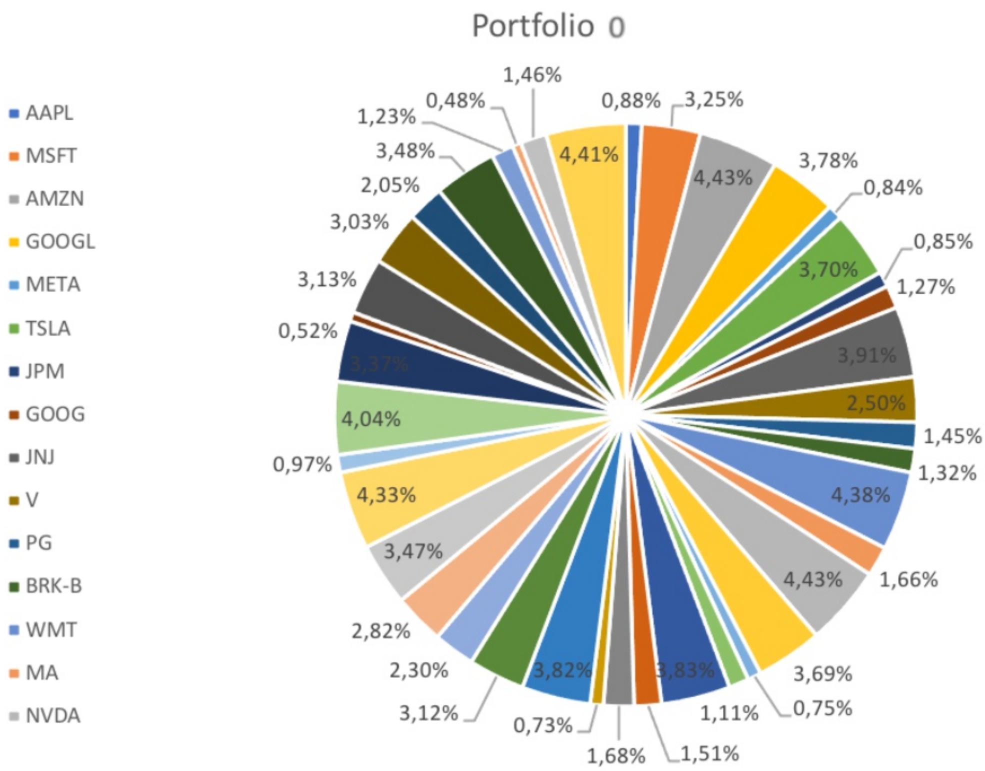


Figure 11

Portfolio 1 - Efficient Frontier

As we delved deeper into portfolio optimization, we explored the Efficient Frontier model. This approach aimed to identify the portfolio with the highest Sharpe ratio while retaining all 40 stocks in the mix. After the extensive analysis of the Monte Carlo simulations, we achieved a promising Sharpe ratio of 0.320. The expected return of 10.99% was slightly higher compared to the non-optimized portfolio, with a volatility of 21.86%. This Portfolio (Figure 12) is designed to be more stable and resilient over time and it's more likely to provide a better risk-adjusted return over the long term because it's designed with an understanding of asset correlations and historical performance, which random allocation lacks.

Expected Return	Expected Volatility	Sharpe Ratio
0.109975	0.218596	0.32011

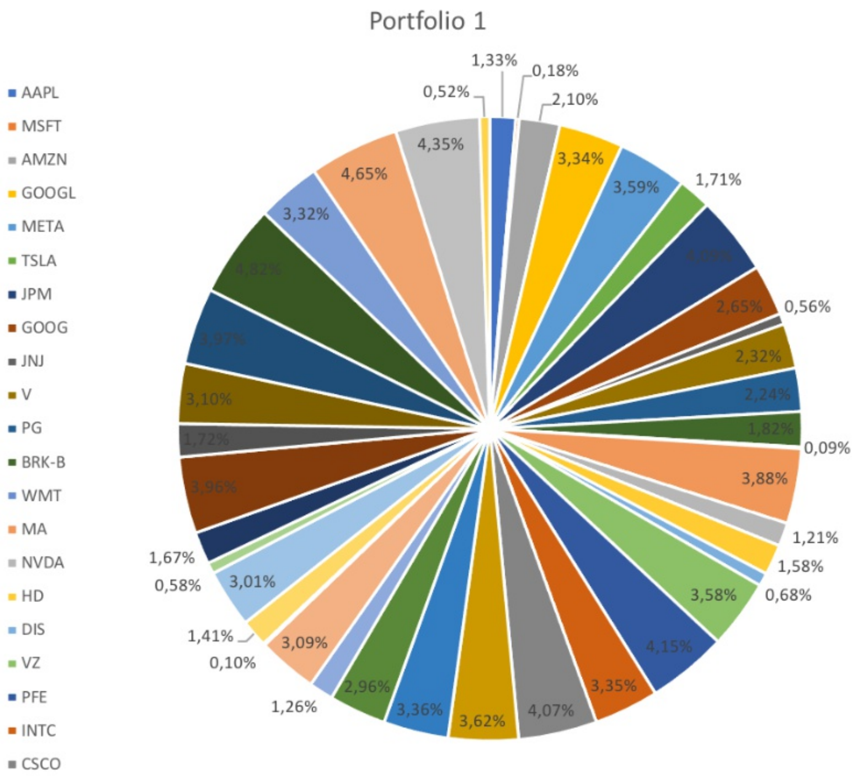


Figure 12

It's essential to emphasize that the Monte Carlo simulation was run 3000 times to enhance the robustness and reliability of our results.
By changing the number of iterations of the Monte Carlo simulation from 3000 to 5000 we obtain the results shown below:

Expected Return	Expected Volatility	Sharpe Ratio
0.113381	0.221925	0.420778

In the illustration below (See Figure 13), each red dot corresponds to one of the 3000 portfolio iterations conducted through a Monte Carlo simulation. The red star indicates the portfolio that was optimized to maximize the Sharpe ratio, aiming to achieve the best risk-adjusted return. In contrast, the blue star represents the portfolio optimized to minimize volatility, emphasizing stability and lower risk.

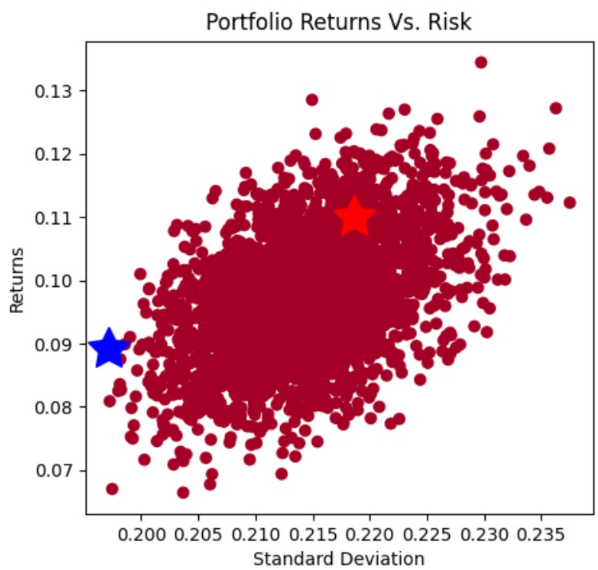


Figure 13

Portfolio 2 - SLSQP

In the final phase of our analysis, we embraced the Sequential Least Squares Quadratic Programming (SLSQP) method. This approach imposed constraints to ensure that no stock could represent more than 30% of the portfolio's allocation. As a result, we saw variations in portfolio weights, with only a subset of the original stocks included (Figure 14). For example, Procter&Gamble (PG), Tesla (TSLA), and NVIDIA (NVDA) carried substantial weight in the portfolio. This approach led to a remarkable Sharpe ratio of 0.799, significantly surpassing the other two portfolios. However, the expected return of 28.17% came at the cost of higher volatility (30.24%). The risk-adjusted return represented by the Sharpe ratio underscored the effectiveness of risk management.

Expected Return	Expected Volatility	Sharpe Ratio
0.2817	0.3024	0.7992

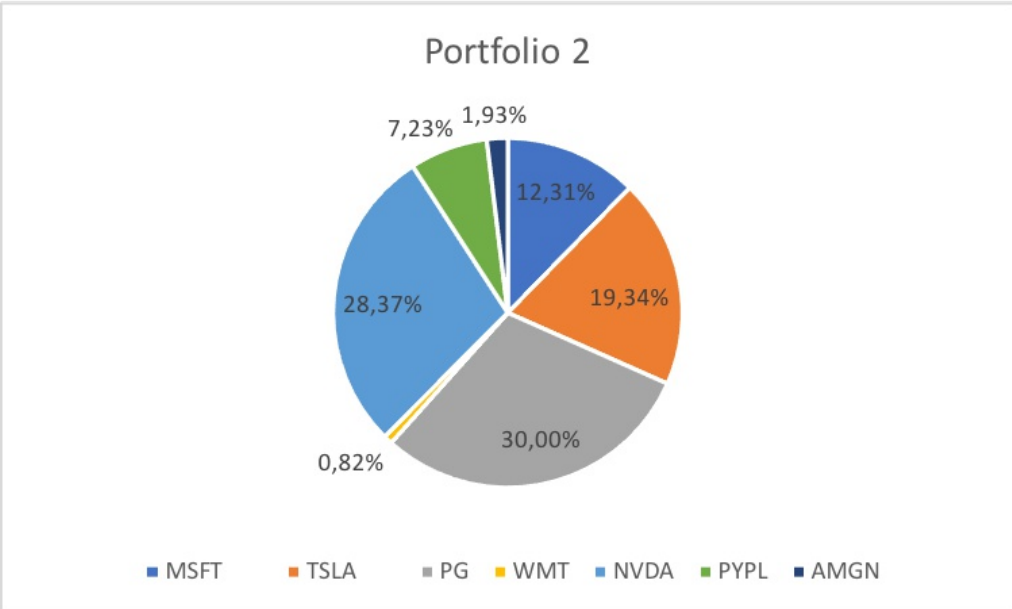


Figure 14

The Efficient Frontier (Portfolio 1) sought to identify the optimal balance between risk and return, resulting in a more favorable Sharpe ratio. By using a constrained approach, the SLSQP optimization (Portfolio 2) managed to harness the benefits of specific stocks while imposing limitations on the maximum allocation of any single stock to mitigate risk. As a consequence, the SLSQP portfolio achieved higher returns at the expense of greater volatility. This trade-off between risk and return was evident in the remarkable Sharpe ratio.

4.2.2 Optimized results - 1 year time frame

In this section the portfolios were also initialized with random allocations in the first case and equal allocation in the second case, and the optimization aimed to maximize the Sharpe ratio, constrained by a 30% upper limit on individual stock allocations. The significant difference between these two time frames is the historical price data used to evaluate expected results.The expected return and volatility presented in this analysis are derived from historical closing prices spanning the last year.

Portfolio 0 - Not optimized

The first portfolio, "Not Optimized," represents an initial allocation of stocks without any systematic optimization. Over the one-year period, this portfolio achieved a respectable Sharpe ratio of 0.298123. However, it's important to note that this difference in Sharpe ratio is primarily attributed to the one-year data used for returns. The one-year historical prices for the stock returns contributed to the higher volatility observed in this period.

Expected Return	Expected Volatility	Sharpe Ratio
0.0677	0.1600	0.2981

Portfolio 1 - Efficient Frontier

The "Efficient Frontier" portfolio presents a Sharpe ratio of 0.105346, which is notably lower compared to the 5-year time frame. This difference in performance is predominantly a result of the shorter historical price data used for the one-year returns. The optimization process adapts to this shorter data series by favoring a more conservative allocation with a focus on risk management.

Expected Return	Expected Volatility	Sharpe Ratio
0.0557	0.1495	0.1053

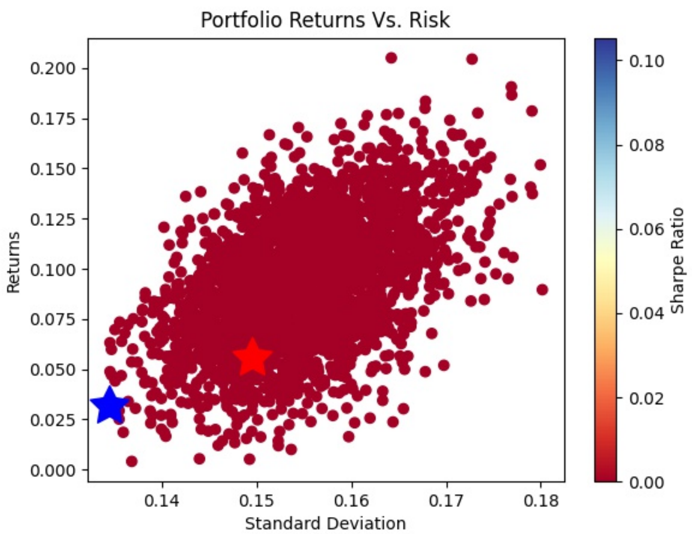


Figure 15

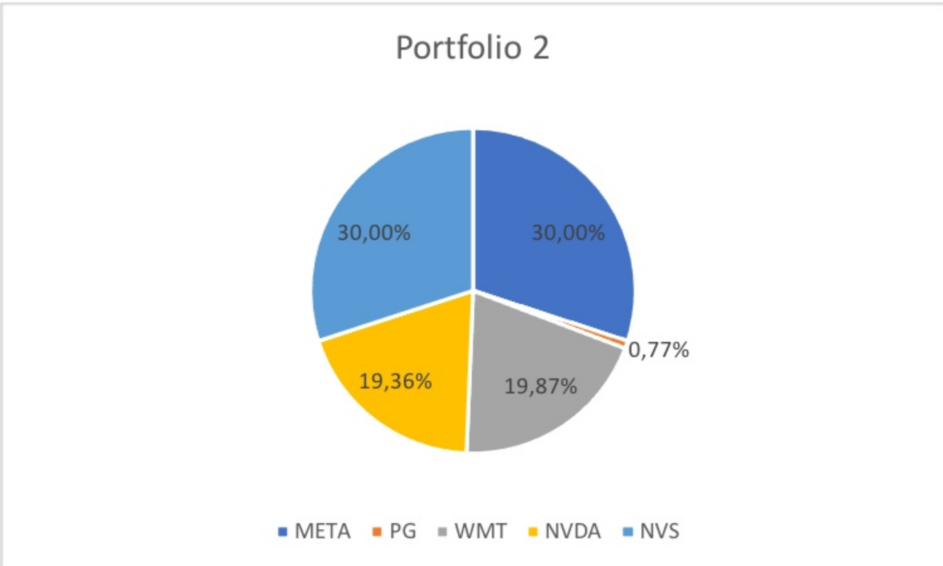
In the illustration above (Figure 15), each red dot corresponds to one of the 3000 portfolio iterations conducted through a Monte Carlo simulation. The red star indicates the portfolio that was optimized to maximize the Sharpe ratio, aiming to achieve the best risk-adjusted return. In contrast, the blue star represents the portfolio optimized to minimize volatility, emphasizing stability and lower risk.

Portfolio 2 - SLSQP

The "SLSQP" portfolio, optimized for maximum Sharpe ratio, notably achieves a much higher Sharpe ratio of 2.8572. However, it's important to understand that this remarkable result is influenced by the one-year data series, leading to a different risk-return profile. The one-year prices for stock returns can be more susceptible to short-term market fluctuations, influencing the optimization towards a potentially riskier allocation.
While these metrics may appear impressive, their reliability is called into question due to the absence of a more comprehensive historical perspective.

Expected Return	Expected Volatility	Sharpe Ratio
0.6573	0.2160	2.8572

An intriguing observation is the shift in the "SLSQP" portfolio (Figure 16). In the 1-year context, this portfolio included stocks Meta Platforms and Novartis AG (NVS) as key components. This highlights the impact of the time frame, where different stocks may be chosen based on shorter-term performance expectations.



It is essential to recognize that utilizing a significantly shorter historical data set, such as one year, can yield notably different and potentially less reliable results. The reason for this discrepancy lies in the limited data points available to capture the nuances of market dynamics.

4.2.3 In depth study of the SLSQP method

Portfolio only stocks - risk profile = 20%

In addition to exploring different initial allocation strategies, enhancing the optimization process by introducing an additional constraint has proven to be instrumental in refining the portfolio management approach. A critical revelation emerged during this analysis, prompting the inclusion of a constraint within the Sequential Least Squares Quadratic Programming (SLSQP) method. This new constraint strategically caps the portfolio's risk at 20%, addressing a significant challenge encountered in the optimization process.
The initial algorithm, driven solely by the objective of maximizing returns, exhibited a tendency to prioritize higher returns at the expense of increased volatility. This unbridled pursuit of returns often led to portfolios with elevated risk levels, potentially exposing investors to unforeseen market fluctuations. By imposing a constraint to limit the risk to a predefined threshold of 20%, the revised optimization algorithm strikes a delicate balance between returns and volatility. This adjustment ensures that the resulting portfolio remains efficient and aligned with risk tolerance, mitigating the inherent trade-off between maximizing returns and managing risk (Figure 17).
The impact of this additional constraint is reflected in the optimization results, where the portfolio's Sharpe Ratio, a key metric gauging risk-adjusted performance, attains a level of 0.7749 which is similar to the last one but the volatility dropped down of almost 10%.

Expected Return	Expected Volatility	Sharpe Ratio
0.1950	0.200	0.7749

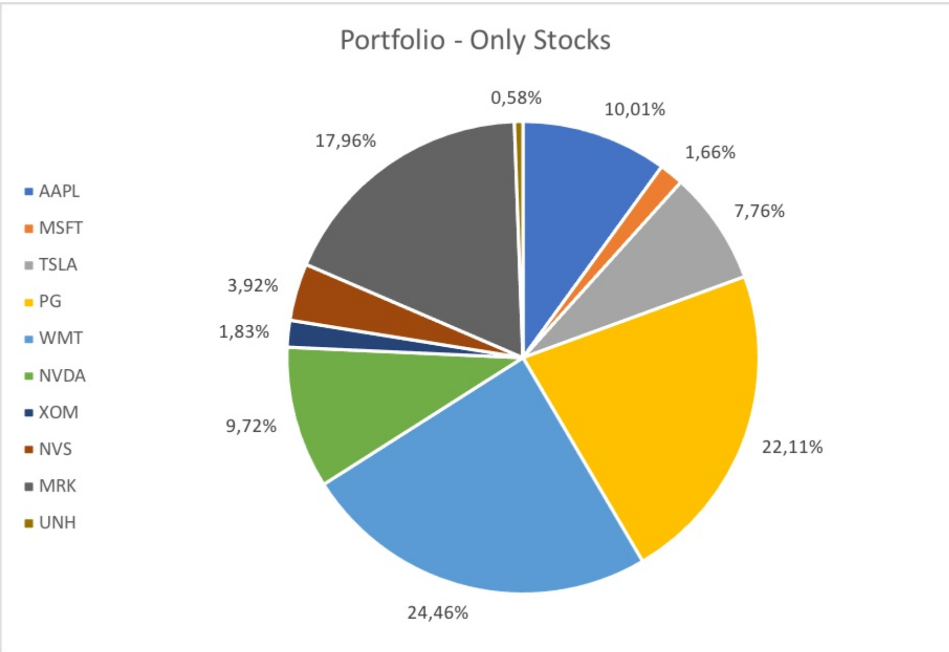


Figure 17

This signifies a notable improvement in the portfolio's efficiency, showcasing the algorithm's ability to achieve a favorable balance between returns and volatility while adhering to the stipulated risk constraint.

Post-Optimization Analysis

Following the optimization process, we conducted a comprehensive post-optimization analysis to gain deeper insights into the portfolio's characteristics and performance. This analysis included the following key components:

HHI Index (Herfindahl-Hirschman Index): We assessed the portfolio's concentration by calculating the HHI Index. This index measures the distribution of investments among different assets, helping us identify potential over-concentration or diversification gaps.

Our portfolio has HHI = 0.16861

Top Correlation Matrix: We analyzed the correlation matrix of the portfolio's assets to identify any significant correlations or interdependencies among the chosen investments. This information aids in understanding how assets move in relation to one another.

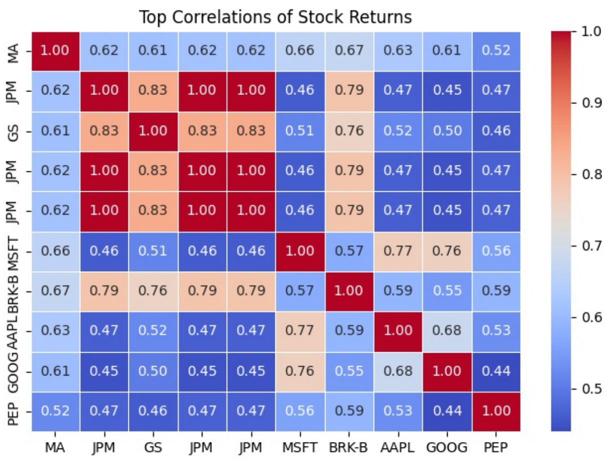


Figure 18

This heatmap (See Figure 18) illustrates the pairwise correlation coefficients between the returns of selected stocks, providing a visual representation of the strength of linear relationships between them. Each square corresponds to the correlation between the stock returns listed along the vertical and horizontal axes, which include prominent companies across various sectors. For instance, a correlation of 1.00 is naturally observed between identical stocks, as seen along the diagonal from the top left to bottom right, which acts as a reference for maximum correlation. The heatmap reveals varying degrees of correlation among the stocks, with some pairs exhibiting higher correlation (darker red shades) and others lower (tending towards blue), which is critical for portfolio diversification. Notable high correlations are evident between pairs such as JPM and GS, suggesting similar market behavior, while lower correlations, such as between AAPL and PEP, indicate more divergent price movements that could be beneficial for risk reduction in a diversified portfolio. This matrix is a fundamental tool in portfolio optimization, as it allows investors to understand and exploit the relationships between different assets to optimize the risk-return profile of their investment portfolio.

Value at Risk (VaR) and Conditional Value at Risk (CVaR): VaR and CVaR analyses were performed to quantify the potential losses the portfolio might face under adverse market conditions. These risk metrics provide valuable insights into the downside risk associated with the portfolio.

- Value at Risk (VaR) at 5% level: -0.0187
- Conditional Value at Risk (CVaR) at 5% level: -0.0296

Robustness Analysis: To assess the robustness of our portfolio, we conducted a sensitivity analysis, varying key parameters and observing their impact on the Sharpe Ratio. This analysis helps us understand how changes in market conditions or portfolio constraints may affect our investment strategy's performance.
The plot (See Figure 19) effectively conveys the trade-off between risk and return, demonstrating that while stricter risk constraints can enhance the Sharpe ratio, increasing return expectations without proportionate risk compensation may reduce it.

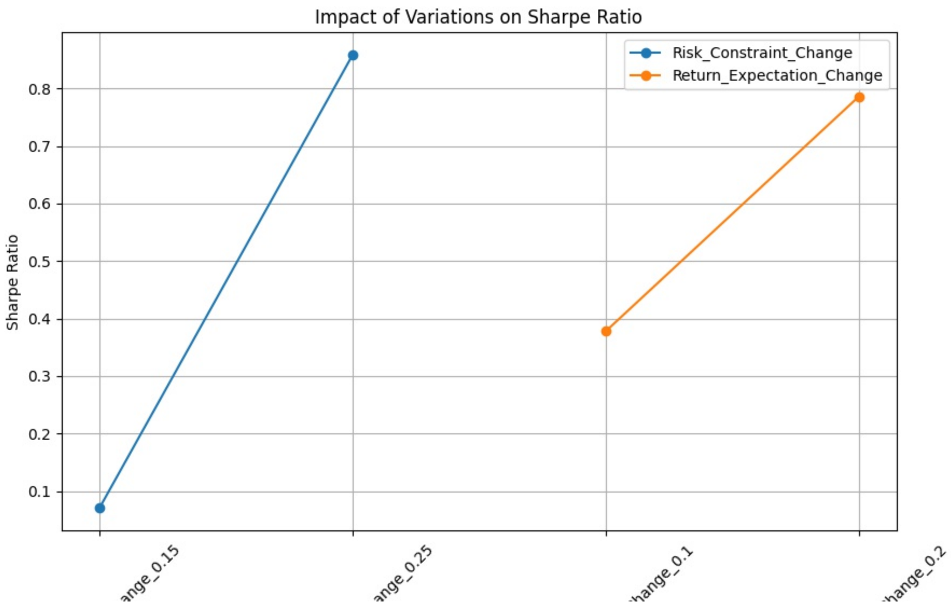


Figure 19

Portfolio Stocks , Bonds, Commodities - Risk Profile 20%

In our pursuit of refining portfolio optimization techniques and gaining a broader perspective on asset allocation, we ventured into an intriguing experiment. Beyond our initial focus on stocks, we sought to understand how the optimization process would perform when confronted with a diverse set of asset classes. This included not only equities but also bonds, commodities, and investments in producers of lithium materials—an essential component in the manufacturing of chips and batteries.
Our decision to expand the asset universe stemmed from the recognition that a well-diversified portfolio could potentially offer investors a more balanced risk-return profile. We believed that incorporating assets beyond equities could provide opportunities for risk mitigation and enhanced portfolio stability.

Bonds: Including bonds in our portfolio allowed us to introduce fixed-income assets known for their relative stability and income generation. Bonds serve as a counterbalance to the potentially higher volatility of equities, contributing to a more diversified investment mix.

Commodities: The inclusion of commodities brought exposure to an asset class that tends to perform differently from traditional financial instruments. Commodities, such as precious metals or energy resources, can serve as hedges against inflation or geopolitical risks.

Producers of Lithium Materials: Recognizing the increasing demand for lithium materials in the technology sector, particularly in the production of chips and batteries, we strategically invested in companies engaged in lithium mining and processing. This investment aimed to capture growth potential in a sector poised for expansion.

We applied the same optimization methodology, leveraging the Sequential Least Squares Quadratic Programming (SLSQP) approach, to create diversified portfolios. However, this time, the challenge was more complex due to the inclusion of multiple asset classes with varying risk-return profiles.
The objective remained consistent: to maximize returns while managing risk. Yet, the optimization process had to adapt to accommodate a broader set of constraints and correlations among different asset types. The goal was to create portfolios that not only sought to maximize returns but also optimized diversification across asset classes.
Below it is shown the list of the stocks, bonds and commodities added:

- SQM: Sociedad Química y Minera - Industry: Chemicals (specializes in nutrient and lithium supply)
- ALB: Albemarle Corporation - Industry: Specialty Chemicals (specifically lithium production for electric vehicle batteries)
- LTHM: Livent Corporation - Industry: Chemicals (focuses on lithium for electric vehicles and energy storage)
- GNENF: Giga Metals Corporation - Industry: Mining (nickel and cobalt for electric vehicle batteries)
- TIANF: Tianqi Lithium - Industry: Chemicals (lithium products for electric vehicles and energy storage systems)
- FCX: Freeport-McMoRan Inc. - Industry: Mining (principal products include copper, gold, molybdenum)
- CMCLF: China Molybdenum Co., Ltd. - Industry: Mining (molybdenum, tungsten, copper, cobalt, niobium, and phosphate)
- ERG: Eurasian Resources Group - Industry: Diversified Natural Resources (varied portfolio including mining and energy)
- STMNF: Standard Lithium Ltd. - Industry: Chemicals (development of lithium brine properties)
- OREAF: Orocobre Limited - Industry: Chemicals (specializes in lithium mining and processing)
- WMLLF: Wealth Minerals Ltd. - Industry: Mining (focuses on acquiring and developing lithium projects)
- VGLT: Vanguard Long-Term Treasury ETF - Industry: Finance (investment fund focused on long-term U.S. Treasury securities)
- TLT: iShares 20+ Year Treasury Bond ETF - Industry: Finance (investment fund in long-term U.S. Treasury bonds)
- GLD: SPDR Gold Trust - Industry: Finance (exchange-traded fund focused on gold)
- SLV: iShares Silver Trust - Industry: Finance (exchange-traded fund focused on silver)
- PALL: Aberdeen Standard Physical Palladium Shares ETF - Industry: Finance (exchange-traded fund backed by physical palladium)

The impact of this additional constraint and the new assets (bonds and commodities) reflected in the optimization results, where the portfolio's Sharpe Ratio attains a level of 1.0627.

Expected Return	Expected Volatility	Sharpe Ratio
0.2525	0.200	1.0627

The largest segment (See Figure 20), constituting 30% of the portfolio, is invested in Gold. This is followed by TIANF (Tianqi Lithium.) at 17.16% and AAPL (Apple, Inc) at 14.42%, demonstrating substantial investments in both the technology sector and commodities (gold, in this case).
The presence of GLD as a non-equity asset class provides a contrast to the previous data, which focused solely on stock correlations. The inclusion of gold as a commodity can be a strategic choice for diversification and hedging against market volatility or inflation.
The remainder of the portfolio is spread across various sectors and includes PG (Procter & Gamble Co.) ,WMT (Walmart Inc.), NVDA (NVIDIA Corporation) and TSLA (Tesla, Inc).

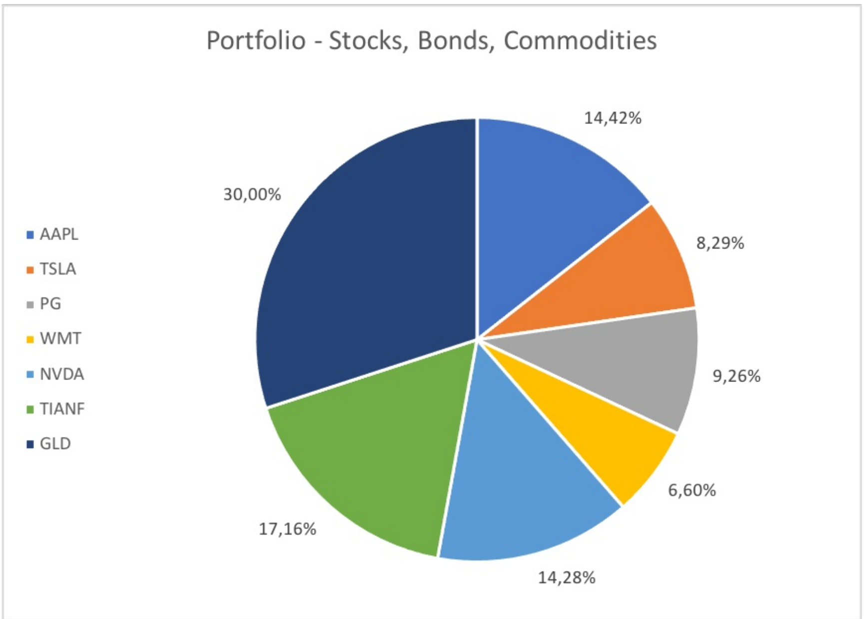


Figure 20

Post-Optimization Analysis

The HHI Index is equal to: 0.1804053
Value at Risk (VaR) at 5% level: -0.0159
Conditional Value at Risk (CVaR) at 5% level: -0.0284

The correlation matrix shows high correlation (0,69) between V and BRK-B and between GOOG and AMZN

Robustness Analysis (Figure 21):

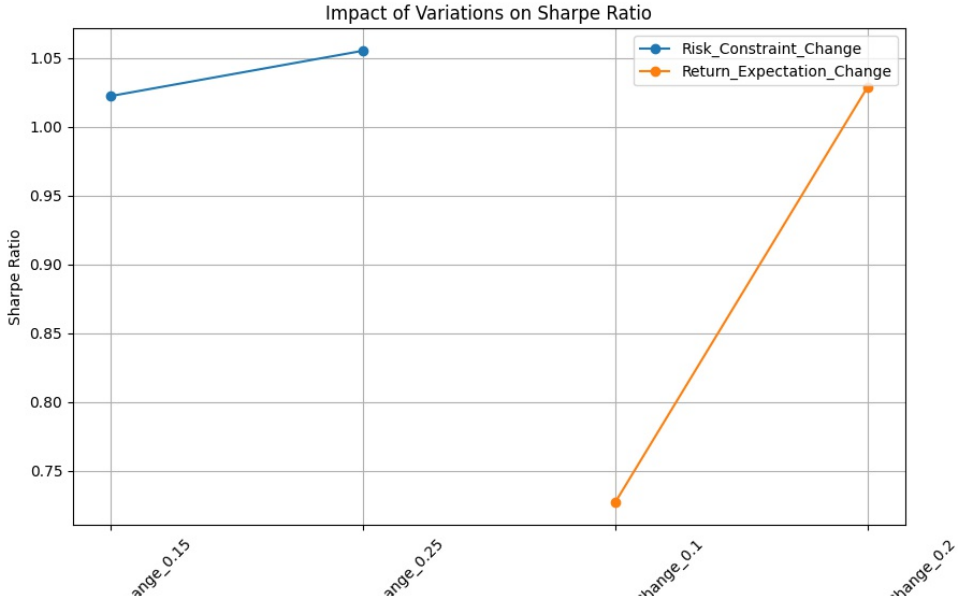


Figure 21

Portfolio Stocks , Bonds, Commodities - Risk Profile 15%

As part of our ongoing quest for portfolio optimization, we recognized the importance of fine-tuning the risk constraint to align with varying investor preferences and market conditions. In this endeavor, we decided to lower the risk constraint from the previous 20% to 15%. This adjustment aimed to explore the potential impact on our portfolio optimization model, specifically in terms of risk management and asset allocation.

Expected Return	Expected Volatility	Sharpe Ratio
0.1981	0.150	1.0542

The reduction in the portfolio's risk profile from 20% to 15% implies that the portfolio manager has strategically repositioned assets to balance growth with stability (Figure 22). This might involve reducing positions in more volatile stocks or sectors and increasing holdings in defensive stocks or alternative assets like gold. The presence of GLD (30%) and ERG (17,77%) could be part of the strategy to lower the overall volatility and risk of the portfolio.

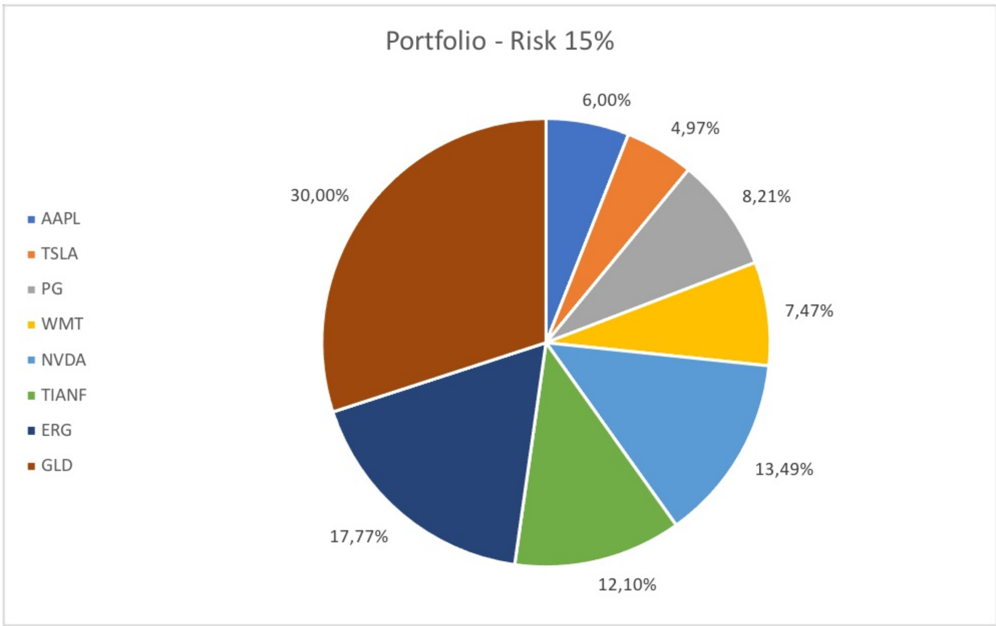


Figure 22

Post-Optimization Analysis

HHI Index: 0.172796
Top Correlation Matrix (Figure 23)

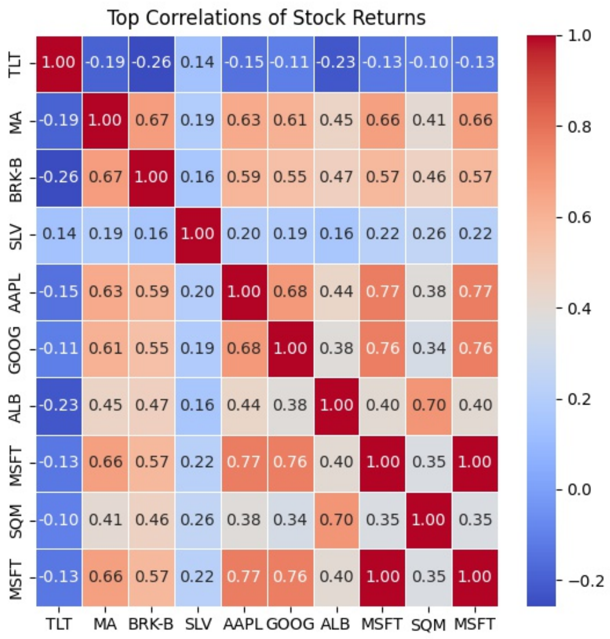


Figure 23

Value at Risk (VaR) and Conditional Value at Risk (CVaR):

- Value at Risk (VaR) at 5% level: -0.0123
- Conditional Value at Risk (CVaR) at 5% level: -0.0212

Robustness Analysis (Figure 24):

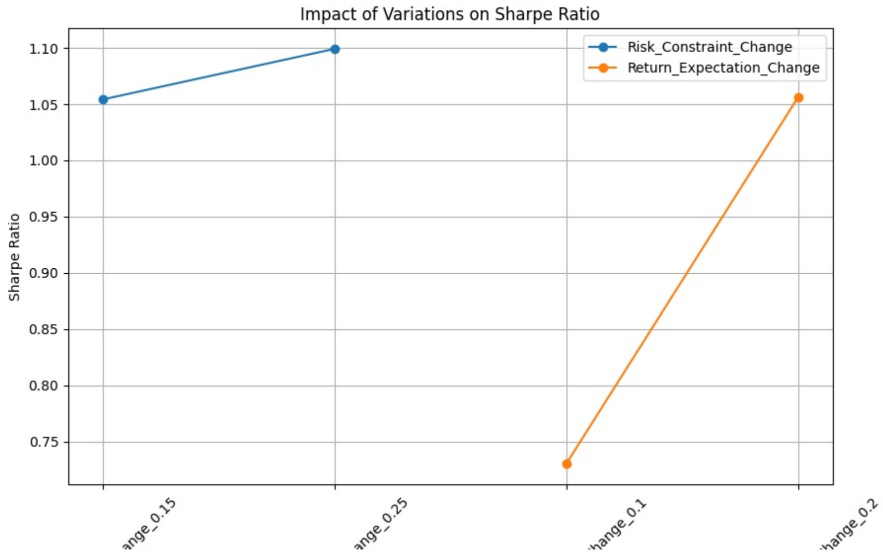


Figure 24

Portfolio Stocks , Bonds, Commodities - Risk Profile 15% - TimeFrame 2015-2020

In our continuous pursuit of refining our portfolio optimization model, we recognized the significance of examining the influence of historical time frames on our investment strategies. To gain a deeper understanding of how changing economic and market conditions might impact our approach, we made a deliberate shift in the time frame of our data. Specifically, we transitioned from using closing prices spanning the period from 2018 to 2023 (the last five years) to closing prices covering the interval from 2015 to 2020. This adjustment allowed us to contrast the dynamics of more stable times, enabling us to assess how different historical periods might influence our portfolio optimization model. By embracing this historical perspective, we aimed to refine our strategies, enhance our risk management, and adapt to the ever-evolving investment landscape more effectively.

Expected Return	Expected Volatility	Sharpe Ratio
0.2922	0.150	1.6814

The portfolio's reduction in risk from 20% to 15%, evidenced by the period from 2015 to 2020, suggests a strategic pivot towards stability without forgoing growth. The substantial allocations to VGLT (21%) and gold (18%) highlight a defensive stance, with these assets likely serving to mitigate market volatility. The investment in NVDA (8.13%) and MA (Mastercard) at 14.23%, which is represented by the grey slice, indicates a balanced approach, coupling growth potential from the tech and consumer finance sectors with the protective elements of traditional safe-haven assets (figure 25).

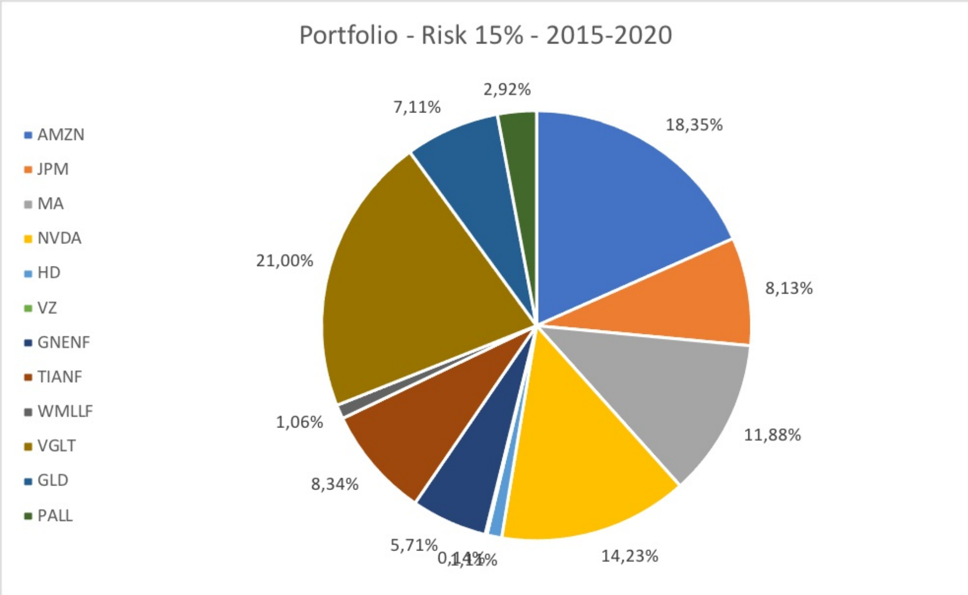


Figure 25

Post-Optimization analysis

HHI Index: 0.135143
Value at Risk (VaR) and Conditional Value at Risk (CVaR):

- Value at Risk (VaR) at 5% level: -0.0105
- Conditional Value at Risk (CVaR) at 5% level: -0.0152

Portfolio Stocks , Bonds, Commodities - Risk Profile 15% - TimeFrame 2013-2023

As we continue to evolve our portfolio optimization strategies, the final adjustment we've made is extending the time frame of our historical data. We transitioned from the previous time frame of 2018-2023 to a more comprehensive and robust data set, covering the period from 2013 to 2023. This transition stems from our commitment to delivering the most reliable and complete investment solutions. The decision to include a more extensive historical time frame is underpinned by several compelling reasons. Firstly, it provides a broader view of market cycles and economic conditions, allowing us to capture a more diverse range of market scenarios. This increased historical context empowers our optimization model to adapt to different phases of market volatility, economic expansion, and contraction. Secondly, the extended time frame enables us to assess the long-term performance and stability of our investment strategies. It allows us to identify patterns, trends, and correlations that might not be apparent in shorter time frames, thus enhancing the reliability of our optimization model's forecasts.
The portfolio composition based on data from 2013-2023 showcases robust results, Sharpe ratio equal to 1.1, with an emphasis on diversification and risk management, reflecting a decade of market dynamics including periods of rapid growth and significant downturns.

Expected Return	Expected Volatility	Sharpe Ratio
0.2067	0.150	1.111

Notably, the highest allocations are in GLD (18.83%) and VGLT (17.58%), which are gold and long-term treasury ETFs, respectively. These allocations are typically indicative of a conservative investment approach, prioritizing capital preservation and hedging against market volatility and inflation, which could be particularly relevant given the economic uncertainties and fluctuations over the past decade.
NVDA (NVIDIA Corporation) holds the largest equity position at 16.47%, underscoring a conviction in the growth trajectory of the technology sector, particularly in areas such as gaming, data centers, and artificial intelligence. NVIDIA's stock has seen significant appreciation over this period, benefiting from these industry trends.
TIANF (Tianqi Lithium) at 10.39% represents a strategic bet on the burgeoning electric vehicle and energy storage markets, which have seen exponential growth due to the global shift towards renewable energy and sustainable technologies.
The portfolio also holds meaningful positions in foundational tech and consumer stocks like AAPL (Apple Inc.) at 9.18%, which has consistently expanded its ecosystem over the past decade, and WMT (Walmart Inc.) at 6.38%, a staple in the consumer retail space.
The presence of JPM (JPMorgan Chase) at 2.83% and PG (Procter & Gamble) at 3.00% provides exposure to the financial services and consumer goods sectors, both of which add stability and potential for steady dividends.
The investment in TSLA (Tesla, Inc.) at 1.80% and STMNF (Standard Lithium Ltd.) at 3.73% may reflect a more speculative component, with Tesla leading the electric vehicle market and lithium being crucial for battery production.
The absence of allocations in major tech companies like AMZN (Amazon), GOOG (Alphabet), and META (Meta Platforms) suggests a potential underweighting in some of the major growth drivers of the last decade, possibly in favor of more balanced risk exposure or due to strategic selections based on the portfolio's risk-return objectives (Figure 26).

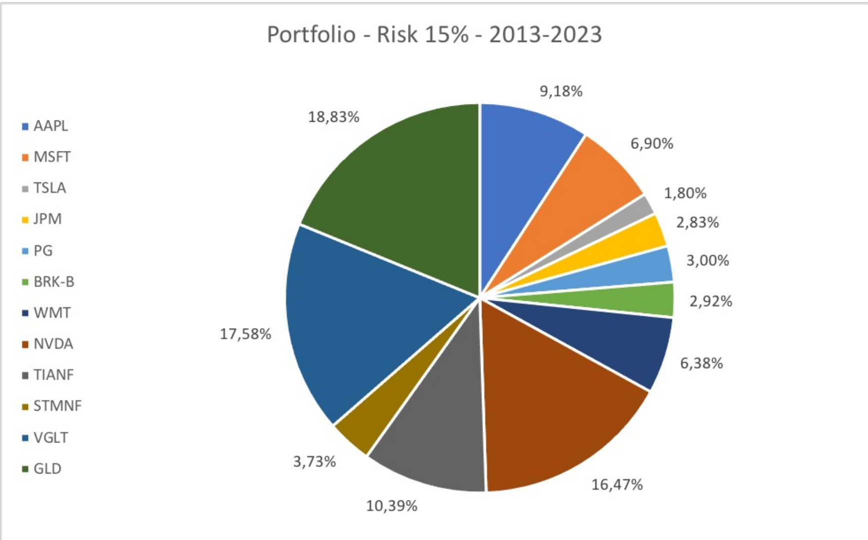


Figure 26

Overall, the portfolio's allocations over the last ten years indicate a careful balance between growth-oriented investments and defensive positioning. This balance seems designed to navigate a complex investment landscape characterized by both disruptive innovation and significant economic and geopolitical risks. The leaning towards gold and treasuries suggests a cautious view of market conditions, while investments in technology and lithium signal a focus on sectors anticipated to grow due to technological and environmental trend.

Post-Optimization analysis

HHI Index: 0.125797

Value at Risk (VaR) and Conditional Value at Risk (CVaR):

- Value at Risk (VaR) at 5% level: -0.0155
- Conditional Value at Risk (CVaR) at 5% level: -0.0239

The top correlation matrix (Figure 27) for the portfolio reveals several trends in asset behavior from 2013-2023. TLT, representing long-term treasuries, generally moves inversely to stock market assets like JPM and MA, highlighting its potential as a diversifying agent within the portfolio. MA and JPM, both in the financial sector, alongside tech giant GOOG, exhibit high positive correlations, indicating that their returns are often aligned, reflective of broader economic trends.GLD, an investment in gold, shows a negative correlation with these stocks, suggesting its role as a stabilizing asset when equities face downward pressure. Similarly, tech stocks like NVDA, GOOG, and AMZN display strong positive correlations with each other, pointing to their collective sensitivity to tech sector dynamics.

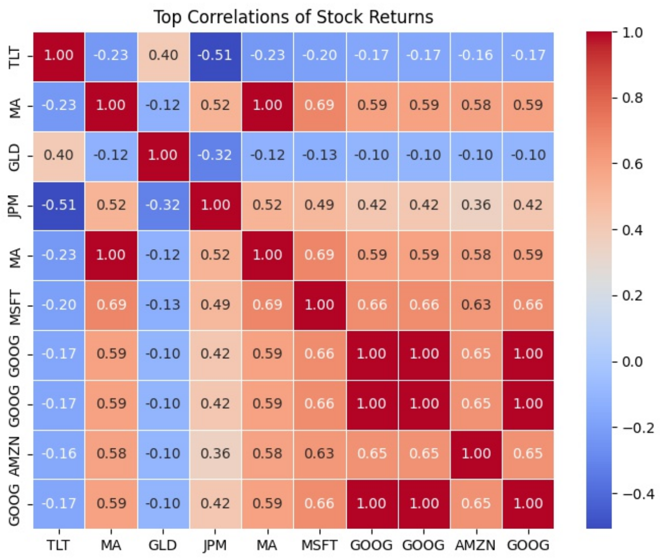


Figure 27

Robustness analysis (Figure 28)

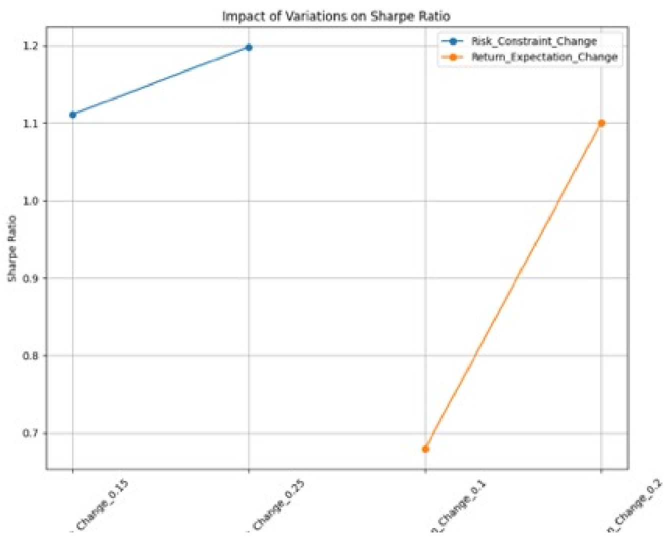


Figure 28

4.3 Comparison between our Portfolios and the Indexes (2018-2023)

In the conclusion of this thesis, we have examined the performance of various portfolios optimized through different methods and constraints against widely recognized indices (Figure 29).

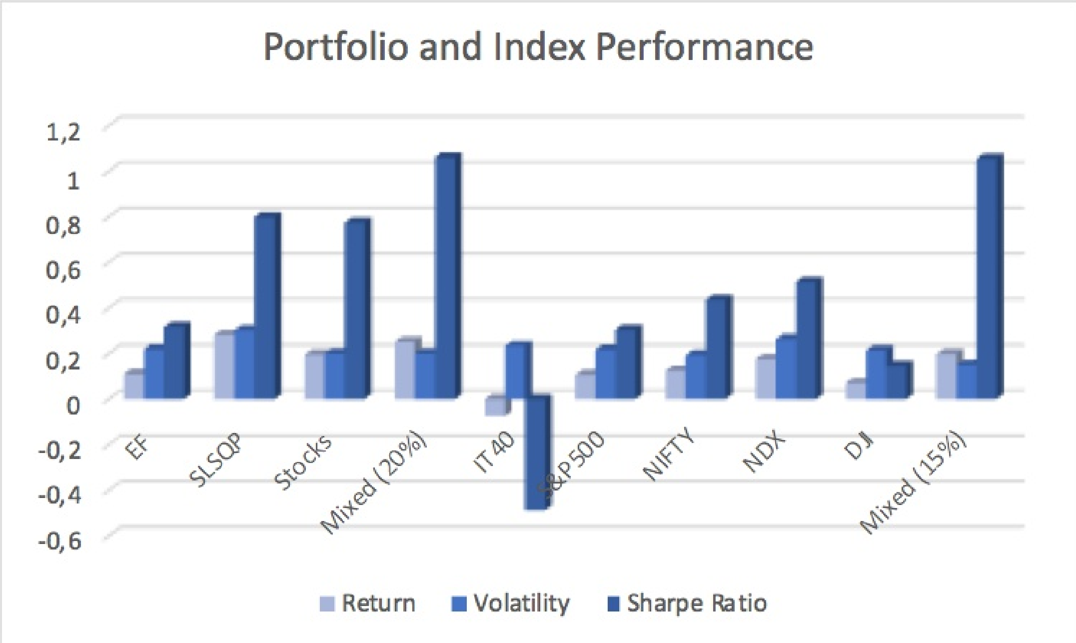


Figure 29

The Efficient Frontier (EF) portfolio, constructed using Monte Carlo simulation to find the optimal risk-return balance, shows modest returns with moderate volatility, resulting in a relatively low Sharpe Ratio. This indicates that while the EF method aims to achieve efficiency in theory, it does not necessarily lead to the highest risk-adjusted returns in practice.

The portfolio optimized using the SLSQP algorithm without any risk constraints and comprising only stocks demonstrates a substantially higher return and Sharpe Ratio compared to the EF portfolio. This suggests that the SLSQP optimization method, which is more focused on local convexity rather than global diversification, can potentially achieve more favorable outcomes in certain market conditions.

When a risk constraint is introduced at 20% for the SLSQP optimized stocks portfolio, the return diminishes slightly, yet the Sharpe Ratio remains high. This showcases the impact of risk management strategies in preserving the quality of returns when adjusting for volatility.

The inclusion of bonds and commodities (Mixed 20%) with the same risk constraint elevates the Sharpe Ratio even further, evidencing the benefits of diversification across asset classes. This diversification appears to stabilize returns while maintaining a low volatility, enhancing the risk-adjusted return.

Remarkably, when the risk constraint is tightened to 15% in the mixed portfolio, the Sharpe Ratio remains comparably high, illustrating that a more stringent risk management approach can still yield highly efficient results, especially in a diversified portfolio that includes stocks, bonds, and commodities. This could imply that there is an optimal level of risk management that aligns well with diversified asset allocation to maximize risk-adjusted returns.

In contrast to these optimized portfolios, the performance of the famous indices (IT40, S&P500, NIFTY, NDX, and DJI) presents a spectrum of results. The IT40 index, for instance, exhibits a negative return and Sharpe Ratio, suggesting a period of underperformance. On the other hand, the S&P500, NIFTY, and NDX indices show positive returns, but with Sharpe Ratios that are less impressive when compared to the optimized portfolios. The DJI index, while offering low volatility, also provides a low return, which translates to a low Sharpe Ratio, indicating a conservative risk-return profile.
The results of the optimized portfolios underscore the effectiveness of portfolio optimization techniques in enhancing returns and managing risk, particularly when contrasted with traditional market indices. The impact of risk constraints and asset diversification on the optimized portfolios is clear: they not only improve the risk-adjusted returns but also demonstrate the potential to outperform market benchmarks, which is a pivotal finding for strategic portfolio management. These insights validate the utility of quantitative optimization methods in constructing portfolios that can potentially exceed the performance of passive investment strategies, especially in times of market volatility and uncertainty.

Comparison between our Portfolios and the Indexes (2013-2023)
The final graph (Figure 30) presents a compelling narrative for the performance of a diversified optimized portfolio compared to major market indices over a more extended period of 10 years.

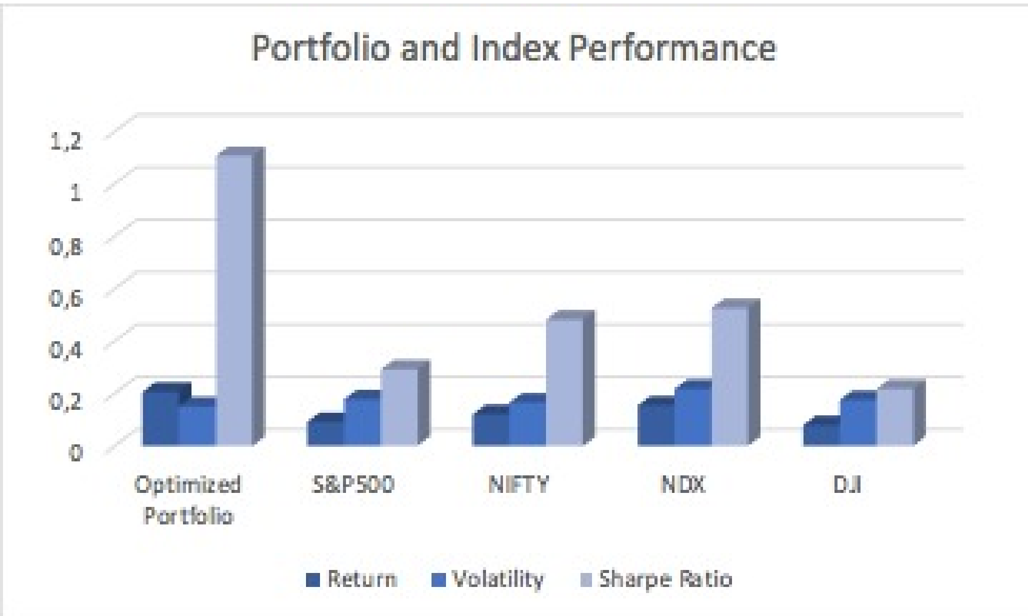


Figure 30

The optimized portfolio, with a risk constraint set at 15%, has a return of 20.6% and a Sharpe Ratio of 1.107, significantly outpacing the S&P 500, NIFTY, NDX, and DJI in terms of risk-adjusted returns. Notably, the optimized portfolio achieves this superior performance with a volatility profile very similar to the indices, underscoring the effectiveness of its risk management.

The S&P 500 index, while a staple of market performance benchmarks, offers less than half the return of the optimized portfolio and a Sharpe Ratio that is nearly four times lower.
This stark contrast emphasizes the potential limitations of passively holding a broad market index, particularly over a period that includes significant market upheavals.
NIFTY and NDX, representing Indian and tech-heavy U.S. markets respectively, deliver higher returns than the S&P 500 and DJI, but still fall short of the optimized portfolio's performance. The NDX does achieve the highest return among the indices at 15.4%, yet it also accompanies higher volatility and a Sharpe Ratio that is less than half that of the optimized portfolio, indicating a less efficient risk-return trade-off.
The DJI, traditionally composed of blue-chip companies, shows the lowest return and Sharpe Ratio among all compared investments, perhaps reflecting its composition of more established and less growth-oriented companies, which might not have captured the full extent of the bull market in the past decade.
In conclusion, the data supports the thesis that a well-constructed optimized portfolio, composed of stocks, bonds, and commodities with a disciplined risk constraint, can achieve superior risk-adjusted returns over a significant period. The 10-year horizon provides a robust basis for this conclusion, suggesting that the optimization strategy employed is not merely capitalizing on recent market conditions but is instead a sound approach to long-term investment. The findings illustrate the advantages of active portfolio management and the application of optimization algorithms, which can be tailored to meet specific risk tolerances while still pursuing aggressive growth targets, a balance that is not typically achievable through index investing alone.

5 Conclusions

5.1 Summary of Findings

In light of these findings, it becomes evident that systematic portfolio optimization plays a pivotal role in managing risk and achieving superior risk-adjusted returns. The extended analysis over a 10-year horizon reinforces the critical role of systematic portfolio optimization in enhancing risk-adjusted returns and navigating market volatility effectively. The data clearly shows that a diversified portfolio, optimized with a constraint of 15% risk and encompassing stocks, bonds, and commodities, can significantly outperform traditional market indices such as the S&P 500, NIFTY, NDX, and DJI. This superior performance is particularly noteworthy given the comparable levels of volatility, highlighting the strategic value of asset allocation and risk management.

Moreover, the findings underscore the importance of the historical context in portfolio optimization. The last decade, with its mix of bull markets, financial crises, a global pandemic, and geopolitical shifts, has provided a robust testing ground for the resilience and adaptability of investment strategies. The optimized portfolio's success over this period indicates a robustness that suggests its strategy is well-suited to different market phases.

The implications of these findings are twofold. Firstly, they confirm the potential of optimized portfolios to adapt to and capitalize on long-term market trends, rather than short-term fluctuations. Secondly, they demonstrate the necessity of continuous research and development in the field of portfolio management, especially in the creation of dynamic, responsive investment strategies.

5.2 Suggestions for Further Research

Future research should focus on the development of an active portfolio management algorithm that can dynamically adjust its composition in response to changing market conditions. The goal would be to enable the portfolio to execute buy and sell decisions autonomously, optimizing for both immediate market opportunities and long-term investment objectives.

Additionally, it would be valuable to explore the creation of a user-oriented algorithm that can assess an individual's risk profile through a series of targeted questions about age, investment preferences, financial goals, etc. This would move away from a one-size-fits-all approach and towards a more personalized, automated risk assessment process, thereby eliminating the need for manual risk profile adjustments.

Such advancements could mark a significant step forward in personal finance, making sophisticated investment strategies accessible to a broader audience and allowing for more personalized, goal-oriented investing. These endeavors would not only enhance individual financial empowerment but also contribute to the overall stability and efficiency of financial markets.

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8 Appendices

[Appendix A: Portfolio Optimization Code]

```
import pathlib
import yfinance as yf
import pandas as pd
from datetime import datetime, timedelta
import numpy as np
from scipy.optimize import minimize
import matplotlib.pyplot as plt

tickers = ["AAPL", "MSFT", "AMZN", "META", "TSLA", "JPM", "GOOG", "JNJ", "V"
"PG", "BRK-B", "WMT", "MA", "NVDA", "HD", "DIS", "VZ", "PFE", "INTC", "CSCO",
"T", "PEP", "CRM", "KO", "XOM", "NFLX", "NKE", "TMO", "ADBE","BAC", "ABT",
"NVS", "MRK", "CMCSA", "PYPL", "UNH", "HON", "GS", "AMGN"
]

number_of_stocks = len(tickers)
end_date = datetime.today()
start_date = end_date - timedelta(days=5 * 365)
print(start_date)

adj_close_df = pd.DataFrame()

for ticker in tickers:
    data = yf.download(ticker, start=start_date, end=end_date)
    adj_close_df[ticker] = data["Adj Close"]

log_returns = np.log(1 + adj_close_df.pct_change())
cov_matrix = log_returns.cov() * 252

random_weights = np.array(np.random.random(number_of_stocks))  # generate random
weights and rebalance it to make the sum = 1

rebalance_weights = random_weights / np.sum(random_weights)

exp_ret = np.sum((log_returns.mean() * rebalance_weights) * 252)

exp_vol = np.sqrt(
    np.dot(
        rebalance_weights.T,
        np.dot(
            log_returns.cov() * 252,
            rebalance_weights
        )
    )
)
```

```
sharpe_ratio = (exp_ret - .04) / exp_vol
weights_df = pd.DataFrame(data={
    'random_weights': random_weights,
    'rebalance_weights': rebalance_weights
})

print(weights_df)
print('')

metrics_df = pd.DataFrame(data={
    'Expected Portfolio Returns': exp_ret,
    'Expected Portfolio Volatility': exp_vol,
    'Portfolio Sharpe Ratio': sharpe_ratio
}, index=[0])

print(metrics_df)


num_of_portfolios = 3000

all_weights = np.zeros((num_of_portfolios, number_of_stocks))
ret_array = np.zeros(num_of_portfolios)
vol_array = np.zeros(num_of_portfolios)
sharpe_array= np.zeros(num_of_portfolios)

for ind in range(num_of_portfolios):
    weights = np.array(np.random.random(number_of_stocks))
    weights = weights / np.sum(weights)

    all_weights[ind, :] = weights
    ret_array[ind] = np.sum((log_returns.mean() * weights) * 252)
    vol_array[ind] = np.sqrt(
        np.dot(weights.T, np.dot(log_returns.cov() * 252, weights))
    )

sharpe_array[ind] = (ret_arr[ind] - .04) / vol_arr[ind]

simulations_data = [ret_array, vol_array, sharpe_array, all_weights]

simulations_df = pd.DataFrame(data=simulations_data).T

simulations_df.columns = [
    'Returns',
    'Volatility',
    'Sharpe Ratio',
    'Portfolio Weights'
]
simulations_df = simulations_df.infer_objects()
```

```
print('')
print('=' * 80)
print('SIMULATION RESULT:')
print('=' * 80)
print(simulations_df.head())
print('=' * 80)

max_sharpe_ratio = simulations_df.loc[simulations_df['Sharpe Ratio'].idxmax()]
min_volatility = simulations_df.loc[simulations_df['Volatility'].idxmin()]

print('')
print('=' * 80)
print('MAX SHARPE RATIO:')
print('-' * 80)
print(max_sharpe_ratio)
print('')
print('=' * 80)
print('MIN VOLATILITY:')
print('-' * 80)
print(min_volatility)

plt.scatter(
    y=simulations_df['Returns'],
    x=simulations_df['Volatility'],
    c=simulations_df['Sharpe Ratio'],
    cmap='RdYlBu'
)

plt.title('Portfolio Returns Vs. Risk')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Standard Deviation')
plt.ylabel('Returns')

plt.scatter(
    max_sharpe_ratio[1],
    max_sharpe_ratio[0],
    marker=(5, 1, 0),
    color='r',
    s=600
)


plt.show()
def get_metrics(weights: list) -> np.array:

    weights = np.array(weights)

   ret = np.sum(log_returns.mean() * weights) * 252
```

```
    vol = np.sqrt(
        np.dot(weights.T, np.dot(log_returns.cov() * 252, weights))
    )

    sr = (ret - .04) / vol

    return np.array([ret, vol, sr])


def neg_sharpe(weights: list) -> np.array:
    return get_metrics(weights)[2] * -1


def check_sum(weights: list) -> float:
    return np.sum(weights) - 1


bounds = tuple((0, 0.3) for symbol in range(number_of_stocks))

constraints = ({'type': 'eq', 'fun': check_sum})

init_guess = number_of_stocks * [1 / number_of_stocks]

optimized_sharpe = minimize(
    neg_sharpe,
    init_guess,
    method='SLSQP',
    bounds=bounds,
    constraints=constraints
)

print('')
print('=' * 80)
print('OPTIMIZED SHARPE RATIO:')
print('=' * 80)
print(optimized_sharpe)
print('=' * 80)

optimized_metrics = get_metrics(weights=optimized_sharpe.x)

print('')
print('=' * 80)
print('OPTIMIZED WEIGHTS:')
print('-' * 80)
print(optimized_sharpe.x)
print('=' * 80)
```

```
print('')
print('=' * 80)
print('OPTIMIZED METRICS:')
print('-' * 80)
print(optimized_metrics)
print('=' * 80)



NEW CONSTRAINTS AND POST-OPTIMIZATION ANALYSIS
risk_constraint = 0.20  # Set an upper limit on portfolio risk (e.g., 15%)

def risk_constraint_fn(weights):
    portfolio_risk = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights)))
    return risk_constraint - portfolio_risk

constraints = [{'type': 'eq', 'fun': check_sum}, {'type': 'eq', 'fun': risk_constraint_fn}]

top_corr_pairs =
correlation_matrix.unstack().sort_values(ascending=False).drop_duplicates()
top_corr_pairs = top_corr_pairs[top_corr_pairs != 1]  # Exclude self-correlations
top_corr_pairs = top_corr_pairs.head(10)  # Display top 10 correlations
top_corr_matrix = log_returns[list(top_corr_pairs.index.get_level_values(0))].corr()

plt.figure(figsize=(10, 8))
sns.heatmap(top_corr_matrix, annot=True, cmap="coolwarm", fmt=".2f", linewidths=.5)
plt.title("Top Correlations of Stock Returns")
plt.show()

hhi = np.sum(np.square(optimized_weights))
print(f"Herfindahl-Hirschman Index (HHI): {hhi}")

variations = {
    'Risk_Constraint_Change': [0.15, 0.25],  # Change in risk tolerance
    'Return_Expectation_Change': [0.10, 0.60],  # Adjusting expected return requirements
}

def robustness_analysis(variations):
    results = []
    for var_name, var_values in variations.items():
        for var_value in var_values:
            if var_name == 'Risk_Constraint_Change':
            modified_risk_constraint = var_value
            def risk_constraint_fn(weights):
                portfolio_risk = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights)))
                return modified_risk_constraint - portfolio_risk
```

```
modified_constraints = [{'type': 'eq', 'fun': check_sum}, {'type': 'eq', 'fun':
risk_constraint_fn}]
        elif var_name == 'Return_Expectation_Change':
            # Adjust expected return requirement
            modified_return_expectation = var_value
            def neg_sharpe_with_return_expectation(weights):
                return get_metrics(weights)[0] - modified_return_expectation

            modified_constraints = [{'type': 'eq', 'fun': check_sum}, {'type': 'eq', 'fun':
neg_sharpe_with_return_expectation}]
        else:
            # Handle other variations...
            modified_constraints = []

def calculate_var_cvar(weights, alpha=0.05):
    portfolio_returns = np.dot(log_returns, weights)
    portfolio_returns = portfolio_returns[~np.isnan(portfolio_returns)]
    portfolio_returns_sorted = np.sort(portfolio_returns)

    # Calculate VaR at alpha level
    var = np.percentile(portfolio_returns_sorted, alpha * 100)

    # Calculate CVaR at alpha level
    mask = portfolio_returns_sorted <= var
    cvar = portfolio_returns_sorted[mask].mean()

    return var, cvar




# Calculate VaR and CVaR for the optimized portfolio
optimized_weights = optimized_sharpe.x
var, cvar = calculate_var_cvar(optimized_weights)
```

[Appendix B: Companies of the index IT40]

companies =
"A2A",
"Amplifon",
"Assicurazioni Generali",
"Azimut Holding",
"Banca Generali",
"Banca Mediolanum",
"Banco Bpm",
"BCA MPS",
"Bper Banca",
"Campari",
"CNH Industrial NV",
"Diasorin",
"Enel",
"Eni SpA",
"ERG",
"Ferrari NV",
"FinecoBank",
"Hera",
"Interpump",
"Intesa Sanpaolo",
"Inwit",
"Italgas",
"Iveco NV",
"Leonardo",
"Mediobanca",
"Moncler SpA",
"Nexi",
"Pirelli & C",
"Poste Italiane",
"Prysmian",
"Recordati",
"Saipem",
"Snam",
"Stellantis NV",
"STMicroelectronics",
"Telecom Italia",
"Tenaris",
"Terna",
"UniCredit",
"Unipol Gruppo"