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# The impact of adoption processes on the Bass Model

Relatore:
Prof.ssa Francesca Montagna

Candidata:
Kawtar Fajri, 288932

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*To my mother, Aicha.*

*“Paradise is at the feet of mothers.”*

# INTRODUCTION

Innovation adoption and diffusion are crucial aspects for the understanding of the acceptance and spread of innovative products in the market. Various models have been proposed to study the dynamics of adoption and diffusion processes, providing valuable insights into the factors that influence consumer behavior and market trends.

Within the past few years, the rapid growth of the Internet and the spread of the latest technologies raise important questions about the existing Bass-type diffusion models. The new environment we live in nowadays together with the unique features of digital products indeed introduce new factors to be included when modelling diffusion processes, significantly impacting the ones already embrace in the literature. Therefore, this thesis aims to propose a new analytical model to properly conceptualize the diffusion framework of digital products.

The first chapter begins with an overview of adoption and diffusion models, highlighting their significance in understanding innovation dynamics. It provides a comprehensive review of recent literature, focusing on advancements in modeling innovation diffusion. Several prominent models are discussed, including the Bass model, Bemmaor & Lee model, Guseo, Mortarino & Darda model, Guseo & Guidolin model, Bass model with network externalities, Norton & Bass model, generalized Norton & Bass model, and generalized Norton & Bass model with marketing mix. Each model's key concepts and contributions to the field are examined.

Chapter 2 delves into the specific characteristics of digital products that have transformed traditional adoption and diffusion patterns. It explores the main drivers of adoption and diffusion, with a specific focus on product adaptability, switching costs, and marketing mix. The chapter proposes a new analytical model tailored for digital products, taking into account these novel factors and their impact on the adoption and diffusion dynamics.

Finally, chapter 3 focuses on the mathematical implementation and data analysis of the proposed analytical model, using smartphones and automated driving as case studies to validate the model's effectiveness comparing the results with real-world data.

# Chapter 1: ADOPTION AND DIFFUSION MODELS OF INNOVATIVE PRODUCTS

Adoption and diffusion refer to the processes through which individuals or groups accept and integrate new innovations, ideas, or products into their lives or society at large.

Adoption refers to the individual decision and action of accepting and using an innovation or product. It involves the evaluation of the innovation's perceived benefits, compatibility with existing practices, and the individual's willingness to try and use it. Adoption can be influenced by factors such as perceived usefulness, ease of use, social influence, and personal preferences.

Diffusion, on the other hand, pertains to the spread of an innovation across a population or social system. It encompasses the process by which the innovation is communicated, accepted, and adopted by a larger group or society. Diffusion is influenced by various factors, including communication channels, social networks, opinion leaders, and the characteristics of the innovation itself.

The adoption and diffusion of innovations are dynamic and complex phenomena that can be studied and modeled to understand the factors that influence their success and impact. By examining the patterns of adoption and diffusion, researchers and practitioners can gain insights into the rate of acceptance, the factors that facilitate or hinder adoption, and the strategies that can be employed to accelerate diffusion and maximize the impact of innovations on individuals and society.

Introductory and advanced microeconomic models often simplify the adoption of new technology as an immediate and universal process. However, historical evidence show that the diffusion of technology is often slow and gradual, with most paths of diffusion showing a sigmoidal pattern. Additionally, different innovations diffuse at different speeds, and similar innovations can diffuse at different speeds across various sectors or countries. Therefore, the historical data requires us to explain three recurring phenomena: the slow and gradual diffusion of new technologies, the sigmoidal pattern of diffusion, and the varying speeds at which innovations diffuse.

The diffusion curve, also known as the adoption curve or S-curve, is a graphical representation that depicts the spread of an innovation or product through a population over time. It illustrates the cumulative number or percentage of adopters on the y-axis and the passage of time on the x-axis. The diffusion curve illustrates the rate of adoption over time, showing how the innovation reaches saturation or the point where the majority of the population has adopted it. Understanding the diffusion curve helps researchers and businesses forecast adoption rates, identify barriers and catalysts for adoption, and develop targeted strategies to accelerate diffusion and maximize the reach and impact of the innovation.

Related to the diffusion curve, there is the Rogers' curve developed by Everett Rogers [1], is a model that categorizes adopters based on their time of adoption. It divides the population into different groups or segments, depending on when they adopt the innovation. The adoption curve is typically divided into five categories:

Innovators: At the beginning of the curve, a small percentage of individuals, known as innovators, embrace the innovation. Innovators are often risk-takers and enthusiasts who are eager to try new things.

Early Adopters: Following the innovators, early adopters begin to adopt the innovation. They tend to be opinion leaders and influencers within their social networks. Their adoption sets the stage for broader acceptance.

Early Majority: As the innovation gains traction, the early majority, which constitutes a larger portion of the population, starts adopting it. They tend to be more cautious and rely on the experiences and opinions of early adopters before making their own decisions.

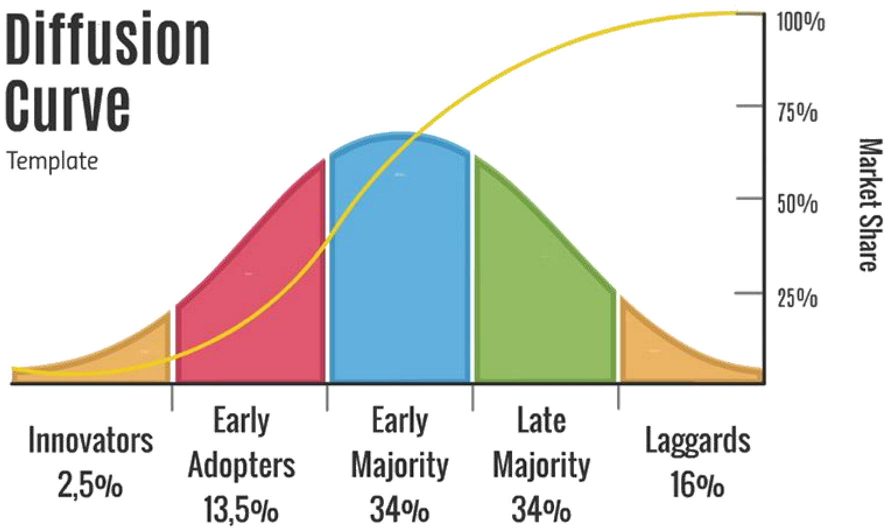
Late Majority: The late majority represents the next wave of adopters. They are more skeptical and cautious than previous groups but eventually adopt the innovation due to social pressures, competitive necessity, or other external factors.

Laggards: Laggards are the final segment of the diffusion curve. They are the last to adopt the innovation, often due to resistance to change or lack of awareness. Laggards may eventually adopt the innovation but at a slower pace.

The Rogers' curve helps to understand the diffusion process by highlighting the different segments of adopters and their characteristics. It provides insights into the pace of adoption, the factors influencing adoption, and the potential barriers faced by different adopter groups. The S-curve and Rogers' curve are related in the sense that the S-curve represents the cumulative adoption rate over time, while the Rogers' curve classifies the adopters into different segments based on when they adopt. The S-curve shows the overall growth of adoption, while the Rogers' curve shows the distribution of adopters across different categories.

Typically, the early adopters and innovators are the ones responsible for the initial rapid growth phase seen in the S-curve. As the adoption progresses, the early and late majority contribute to the middle section of the S-curve. The laggards, who adopt much later, contribute to the flattening and saturation phase of the S-curve.

To conclude, both models are useful for understanding the diffusion of innovations and the adoption process within a population.




## BASS MODEL

The **Bass model** holds immense importance in the study of adoption and diffusion processes, as it provides valuable insights into how innovations spread among populations. It serves as a foundational framework for understanding the dynamics of adoption and predicting the future market penetration of new products or technologies. The Bass model's significance lies in its ability to estimate the rate of adoption and forecast adoption patterns, aiding strategic decision-making and marketing planning. It is a mathematical model developed by Frank Bass in 1969 [2] based on the idea that a product's adoption rate is determined by two factors: the number of people who are exposed to the product, and the number of people who decide to purchase the product. So, the model considers the influence of the number of early adopters, the rate at which they influence subsequent adopters, and the rate of imitation by other consumers. It assumes that the rate of adoption is a function of the proportion of the population that have already adopted the product, the rate at which these early adopters influence subsequent adopters, and the rate of imitation by other consumers. The model can also be used to measure the effectiveness of promotional campaigns and marketing strategies that are aimed at increasing the rate of adoption of a new product or service. The model is widely used by businesses and marketers to analyze the diffusion of new products and services, and to predict their rate of adoption within a given market. The model is widely used in marketing and business analytics and has been used to model the diffusion of various products, including new technology and online services. It is expressed as a differential equation, which can be solved to find the rate of adoption over time. The equation considers four parameters: p, q, M, and q/p. The parameter p is the proportion of the population that have already adopted the product. The parameter q is the rate at which these early adopters influence subsequent adopters. The parameter M is the potential market size for the product, and q/p is the rate of imitation by other consumers. The model is used to predict the rate of adoption of a new product or service by consumers, and to identify factors that influence that rate of adoption.

The equation for the number of adopters in time t is as follows:

$$z'(t) = \left(p + q * \frac{z}{m}\right) * (m - z)$$

The equation for the cumulative adoption rate in time t is as follows:

$$z(t) = m * \frac{1 - exp(-(p+q)*t)}{1 + \frac{q}{p} * \exp(-(p+q)*t)}$$

However, it is crucial to acknowledge the Bass model's limitations and the subsequent development of various extensions to relax these limitations.

One of the limitations of the Bass model is the assumption of homogeneity within the considered population. To address this limitation, researchers have proposed models that incorporate heterogeneity among adopters. Bemmaor and Lee, as well as Guseo, Mortarino, and Darda, have developed extensions to the Bass model that account for different adoption behaviors and characteristics among individuals. By incorporating heterogeneity, these models offer a more nuanced understanding of the diffusion process.

Another limitation of the Bass model is its assumption of a constant market potential. In reality, the market potential can vary due to external factors. Researchers like Guseo and Guidolin, as well as those considering network externalities, have proposed models that incorporate a variable market potential. These models recognize that the size and composition of the potential adopter pool can change over time, leading to a more accurate representation of the diffusion process.

The Bass model does not include the influence of the marketing mix, which consists of factors such as price, advertising, and other interventions. However, the general Bass model can be expanded to incorporate the marketing mix variables. By considering the impact of these variables, such as through pricing and advertising strategies, researchers gain a deeper understanding of how marketing efforts affect the diffusion process. This extension provides valuable insights for designing effective marketing campaigns.

Furthermore, the Bass model solely focuses on the diffusion of innovations and does not account for substitution effects. Researchers such as Norton and Bass have proposed models that incorporate the replacement of successive generations of innovations. These models recognize that the adoption of new innovations can be influenced not only by their inherent characteristics but also by the substitution of existing products or technologies. By considering replacement effects, these models offer a more comprehensive understanding of the diffusion process.

Now, each model will be explored in detail. The assumptions, mathematical formulation, and application in predicting adoption and diffusion patterns will be examined. Following that, the extensions and alternative models that relax the limitations of the Bass model will be explored, enabling a more comprehensive understanding of the complexities involved in the adoption and diffusion of innovations.

## BEMMAOR & LEE MODEL

The **Bemmaor and Lee Model** [3] is an extension of the Bass Model that was designed to address the limitations of the original model. This extension adds two additional terms to the Bass Model, which are the innovator coefficient and the late majority coefficient. The innovator coefficient represents the rate at which innovators adopt the new product or idea, while the late majority coefficient represents the rate at which laggards adopt the new product or idea. This extension of the Bass Model allows for a more accurate prediction of the rate of adoption of the new product or idea.

The Bemmoar and Lee model is an extension of the Bass model that considers the effect of heterogeneous imitators on the diffusion process. It assumes that the rate of adoption is a function of the number of adopters, the rate of innovation, the rate of imitation, and the level of heterogeneity among adopters. It also includes a parameter, α, which can be used to control the rate of adoption. By varying α, the speed of diffusion can be adjusted both at the start and after the start.

It is expressed as:

$$F(t) = \frac{1 - exp(-(p+q)*t)}{(1+\frac{q}{p}*\exp(-(p+q)*t))\hat{}(\alpha)}$$

Where p is the rate of adoption of the product by innovators, q is the rate of adoption of the product by imitators, and α is the speed at which the innovation diffuses among the imitators. α is a parameter of η which represents the individual propensity to purchase: η ~ G (1/β, α), where β = q/p.

η ~ G (1/β, α) is a probability distribution function which describes the individual propensity to purchase. η is a random variable which is distributed according to a Gamma distribution with shape parameter 1/β and scale parameter α. The parameters β and α depend on the rate of adoption of the product by innovators and imitators, respectively.

## GUSEO, MORTARINO & DARDA MODEL

The **Guseo, Mortarino & Darda model** [4] is an extension of this model that considers the impact of different marketing activities on the diffusion of the product or idea. The model looks at how marketing activities can influence the speed and rate of adoption of the product or idea. It also looks at how certain marketing activities can be more effective at different stages of the product's lifecycle. This model can be used to better understand how marketing activities can help to increase the diffusion of a new product or idea over time.

In this case, heterogeneity is considered for both imitators, as in Bemmaor & Lee, and for innovators. It is a more general model, both of Bass and of B&L, in fact for δ=1, it reduces to the B&L model and if also α=1, then it is reduced to the standard version of the Bass model.

$$F(t) = \frac{(1 - exp(-(p+q)*t))^{\wedge}(\delta)}{(1 + \frac{q}{p} * \exp(-(p+q)*t))^{\wedge}(\alpha)}$$

δ=Rate of innovation diffusion among innovators

$\alpha$=Rate of innovation diffusion among imitators

It is a more general model that considers for both imitators and innovators, and it can be reduced to the Bemmaor & Lee model and the standard Bass model when certain parameters are set to specific values.

## GUSEO & GUIDOLIN MODEL

The **Guseo & Guidolin Model** [5] attempts to better explain the adoption of innovations by adding two additional parameters to the Bass Model: the influence of a product's price on its diffusion and the influence of promotional campaigns. The Guseo & Guidolin Model suggests that the demand for an innovation is influenced by the price of the product, the promotional campaigns, and the characteristics of the innovation itself. The model also proposes that the diffusion of an innovation will be greater if it is perceived as having a higher quality and lower price. This model is often used by marketers to better understand how the diffusion of an innovation is affected by various factors.

This model allows for two phases of the diffusion process to be identified – communication and adoption – and for the temporal positioning of these phases to be specified. Additionally, it does not assume that the communication phase must always precede the adoption phase.

$$z(t) = m(t) * \frac{1 - exp(-(p+q)*t)}{1 + \frac{q}{p} * \exp(-(p+q)*t)}$$

$$z(t) = K * \sqrt{\frac{1 - exp(-(p_c+q_c)*t)}{1 + \frac{q_c}{p_c} * \exp(-(p_c+q_c)*t)} * \frac{1 - exp(-(p_s+q_s)*t)}{1 + \frac{q_s}{p_s} * \exp(-(p_s+q_s)*t)}}$$

In m(t), m is a variable over time that depends on the "communication process"; whereas in Bass model, m= constant. Whereas, in General Bass Model which, on the other hand allows only to modify the temporal structure of the diffusion and not its size.

## BASS MODEL WITH NETWORK EXTERNALITIES

The **Bass model with network externalities** [6] describes how an innovative product is adopted by consumers over time. It considers the fact that the adoption of a product by one consumer increases the likelihood of other consumers adopting it. This makes the adoption process a cumulative one, as the more consumers who adopt the product, the more likely other consumers will adopt it. The model also considers the fact that the rate of adoption depends upon the size of the existing user base, as the larger it is, the more likely it is that a new consumer will adopt the product. This makes network externalities an important factor to consider when predicting the rate of adoption of a product.

The threshold levels of adoption H~$\mathcal{N}$ ($\mu$, $\vartheta$^2) are used to determine the dynamic structure of the model, where the mean value of the thresholds ($\mu$) decreases as the adoption rate (y(t)) decreases. Additionally, the Guseo Guidolin Model approach is used to recreate the model.

$$z(t) = m(t) * \frac{1 - exp(-(p+q)*t)}{1 + \frac{q}{p} * \exp(-(p+q)*t)}$$

$$z(t) = K * P(H \leq y(t)) * \frac{1 - exp(-(p+q)*t)}{1 + \frac{q}{p} * \exp(-(p+q)*t)}$$

$$\mathrm{m(t)} = K * P(H \leq y(t))$$

$$\mathrm{m(t)} = K * \Phi(\frac{y(t) - \mu}{\vartheta})$$

## GENERAL BASS MODEL

The **General Bass Model** [7] is a more general model that applies to the diffusion of any type of innovation in fact it provides the ability to simulate the adoption and diffusion of products, services, and ideas. It can be used to evaluate the potential success of an innovation by predicting the rate of adoption, the duration of the diffusion process, and the ultimate size of the market and it can also be used to identify the potential for marketing campaigns to accelerate the adoption of an innovation. It looks at the adoption of an innovation over time, looking at the four components of the diffusion process: innovation decision, communication channels, time, and social system.

$$z'(t) = \left(p + q * \frac{z}{m}\right) * (m - z) * x(t)$$

$$z(t) = m * \frac{1 - exp\left(-(p+q) * \int_0^t x(\tau)d\tau\right)}{1 + \frac{q}{p} * \exp\left(-(p+q) * \int_0^t x(\tau)d\tau\right)}$$

x(t) can be considered a marketing mix intervention, as it is a function of external factors that could be manipulated by marketing mix strategies. It modifies the temporal structure of the diffusion but not the value of its internal parameters (m, p, q). This means it anticipates or delays adoptions but does not increase or decrease them.

General Formula:

$$x(t) = 1 + \beta 1 \frac{Pr'(t)}{Pr(t)} + \beta 2 \frac{A'(t)}{A(t)}$$

Exponential Function- drastic perturbation and strong and fast effects:

$$x(t) = 1 + \sum_{i=1}^{3} c_i * e^{b_i*(t-a_i)} * I_{t \geq a_i}$$

Rectangular Function- stable perturbations for a long period:

$$x(t) = 1 + \sum_{i=1}^{3} c_i * I_{t \geq a_i} * I_{t \leq b_i}$$

## NORTON & BASS MODEL

The **Norton & Bass model** [8] was developed to account for the fact that the product diffusion process does not always follow the same pattern. The Bass model assumes that the diffusion of a product is a function of the product's inherent characteristics and the marketing activities used to promote it. The Norton & Bass model, on the other hand, takes this a step further by incorporating external factors such as consumer behavior and the competitive landscape into the equation. This allows the model to predict the rate of adoption of the product more accurately.

The Norton-Bass model does not differentiate those who have already adopted the old generation from those who have not.

$F_G(t)$= diffusion of adoption concerning generation G:

$$F_g(t) = \begin{cases} 0, \ t < 0 \\ \dfrac{1 - exp\left(-\left(p_g + q_g\right) * t\right)}{1 + \dfrac{q_g}{p_g} * \exp\left(-\left(p_g + q_g\right) * t\right)}, \ t \geq 0 \end{cases}$$

$S_G(t)$= the number of units in use for generation G:

$$S_1(t) = F_1(t) * m_1 - F_2(t - \tau_2) * F_1(t) * m_1$$
$$= F_1(t) * m_1[1 - F_2(t - \tau 2)] \ for \ t > 0$$

$$S_2(t) = F_2(t - \tau_2) * [m_2 + F_1(t) * m_1] \ for \ t > \tau_2$$

## GENERALIZED NORTON & BASS MODEL

The **Generalized Norton & Bass model** [9] is a more comprehensive version of the Norton-Bass model and the Bass model of the diffusion of an innovative product. The Generalized Norton & Bass model considers the influence of adoption rates on the diffusion process, which the other two models do not. It also considers how the size of the adopter population, rate of imitation, and the rate of innovation can all impact the rate of diffusion of an innovative product. Additionally, the Generalized Norton & Bass model considers how marketing efforts can accelerate the diffusion process. In contrast, the Bass and Norton-Bass models only consider how marketing efforts can affect the rate of adoption, and not the rate of diffusion as a whole.

The GNB model separates the two different types of substitutions.

$U_2(t)$ = The number of leapfrogging adoptions:

$$U_2(t) = \int_{\tau_2}^{t} u_2(\theta)d\theta = m_1 \int_{\tau_2}^{t} f_1(\theta)F_2(\theta - \tau_2)d\theta$$

$W_2(t)$= The number of leapfrogging adoptions:

$$W_2(t) = \int_{\tau_2}^{t} w_2(\theta)d\theta = m_1 \int_{\tau_2}^{t} F_1(\theta)f_2(\theta - \tau_2)d\theta$$

$Y_G(t)$ =the cumulative number of adoptions of G:

$$S_1(t) = Y_1(t) - W_2(t) = m_1(t)F_1(t)[1 - F_2(t)(t - \tau_2)]\ for\ t > 0$$

$S_2(t) = Y_2(t) = [m_2 + m_1F_1(t)]F_2(t)(t - \tau_2)\ for\ t < \tau_2$

## GENERALIZED NORTON & BASS MODEL WITH MARKETING MIX

The **Generalized Norton & Bass model with Marketing Mix** is an analytical tool used to simulate the relationship between sales and a company's marketing mix, which includes price, promotion, product, and place. It has been generalized to include additional elements of marketing mix, such as branding and advertising. The model is based on the idea that sales can be predicted by looking at the effect of different marketing mix elements and how they interact with each other. The model can be used to assess the impact of marketing strategies and to forecast sales.

The marketing mix can be introduced as in the GBM: multiplicative factors. $x_G(t)$= marketing effort of generation G. $X_G(t)$= cumulative marketing effort of generation G:

$$X_G(t) = \int_0^t x_g(\theta)d\theta$$

While the rest would be modified consequently:

$$F_G(t) = \frac{1 - exp\left(-(p_G + q_G) * \int_0^t x_G(\theta)d\theta\right)}{1 + \frac{q_G}{p_G} * \exp\left(-(p_G + q_G) * \int_0^t x_G(\theta)d\theta\right)}$$

## BOEHNER & GOLD MODEL

The **Boehner & Gold model** [10] considers five factors: innovativeness, trialability, observability, complexity, and relative advantage. The model predicts the rate of adoption of an innovation based on these factors and how they interact. The model is useful in understanding how consumers make decisions about adopting new products and can be used to inform marketing strategies.

It starts from the GBM:

z'(t)= number of new adopters in period t

z(t-1) = cumulative number of previous buyers.

z(t)= total number of adopters or total demand at time t.

$$z'(t) = \left(p + q * \frac{z}{m}\right) * (m - z) * x(t)$$

$$z\,(t) = z'(t) + z(t-1)$$

$$z\,(t) = pmx(t) + z(t-1) * [1 + (q-p)x(t)] - \frac{qx(t)}{m}(z(t-1))^2$$

Market potential as a Cobb-Douglas function where m is function of price and advertising.

m = market potential (size)

s = scaling factor

P = price

A = advertising expenditure

e = coefficient of sensitivity for price

f = coefficient of sensitivity for advertising

$$m = sP^{-e}A^{f}$$

$$z\,(t) = p * sP^{-e}A^{f} + z(t-1) * [1 + (q-p)] - \frac{q}{sP^{-e}A^{f}}(z(t-1))^2$$

# Chapter 2: A NEW ANALYTICAL MODEL FOR DIGITAL PRODUCTS

The previous literature review has laid the foundation for identifying new elements that can be integrated into an enhanced version of the Bass Model. In this chapter, an overview of digital products is presented, emphasizing the emerging need for new models. Subsequently, the parameters to be included and the optimal approach for incorporating them are identified. Finally, the chapter introduces the novel model that incorporates these advancements. Within this context, the chapter explores the concept of innovative products and delves into the models used to understand their adoption and diffusion. Specifically, it examines smartphones and automated driving as prime examples of innovative products, highlighting the need for incorporating new factors in the adoption and diffusion models.

## DIGITAL PRODUCTS HAVE CHANGED ADOPTION AND DIFFUSION PROCESSES

In today's rapidly evolving technological landscape, innovative products play a pivotal role in shaping society and transforming various industries.

Innovative products represent groundbreaking solutions that bring about substantial advancements or enhancements compared to existing offerings. These products possess the power to disrupt established markets, reshape consumer behaviors, and open up new avenues for businesses. The successful adoption and diffusion of innovative products serve as pivotal determinants of their ultimate triumph and their profound impact on society.

By their very nature, innovative products introduce fresh approaches, features, or functionalities that challenge the status quo. They transcend conventional boundaries, pushing the limits of what was previously deemed possible. Through their introduction, they redefine consumer expectations, demands, and preferences, prompting shifts in behavior and consumption patterns.

Moreover, innovative products often act as catalysts for industry-wide transformations, triggering a ripple effect across the entire business ecosystem. They foster the emergence of new markets, create opportunities for entrepreneurial ventures, and propel economic growth. The adoption and diffusion of these products serve as crucial markers of their acceptance, indicating the level of demand, market penetration, and long-term sustainability.

Understanding the adoption and diffusion dynamics of innovative products is of paramount importance [11]. These processes determine whether the innovation will be embraced by consumers, integrated into their lives, and eventually become a mainstream phenomenon. By comprehending the factors that drive or hinder adoption, businesses and researchers can refine their strategies, tailor their marketing efforts, and identify potential obstacles to overcome.

Furthermore, the impact of innovative products extends beyond individual consumers and businesses. Their widespread adoption can have far-reaching societal implications, influencing the way we live, work, and interact with one another. They have the potential to reshape entire industries, revolutionize established practices, and address pressing societal challenges.

In conclusion, innovative products represent pivotal drivers of progress, instigating transformative changes across various domains. The adoption and diffusion of these products act as critical barometers of their success and influence on society. By studying the intricate dynamics of adoption and diffusion, stakeholders can harness valuable insights to navigate the complex landscape of innovation and maximize the potential impact of these game-changing solutions.

Several models have been developed to understand the process of adoption and diffusion of innovations. The most well-known model is the Diffusion of Innovations theory by Everett Rogers. This theory identifies five stages of adoption: knowledge, persuasion, decision, implementation, and confirmation. Other models include the Technology Acceptance Model (TAM), the Unified Theory of Acceptance and Use of Technology (UTAUT), and the Innovation Diffusion Model (IDM).

Traditional models have proven effective in explaining the adoption and diffusion of many innovative products. However, in today's dynamic environment, new factors need to be considered to enhance the accuracy and comprehensiveness of these models. Two areas that warrant attention are smartphones and automated driving.

Smartphones have revolutionized the way people communicate, access information, and engage with various services. They are a quintessential example of an innovative product with widespread adoption. To model the adoption and diffusion of smartphones, new factors must be considered, such as:

- Technological Convergence: Smartphones have evolved into multifunctional devices, integrating features like cameras, GPS, music players, and more. The convergence of technologies within smartphones influences adoption and diffusion patterns, as consumers consider the breadth of functionalities and their relevance to their daily lives.
- Mobile Ecosystems: The emergence of mobile app ecosystems, such as Apple's App Store and Google Play, has significantly contributed to smartphone adoption and diffusion. These ecosystems provide a vast array of applications, creating value for users and fostering a network effect that drives adoption.

Automated driving, also known as autonomous vehicles or self-driving cars, represents another revolutionary innovation. Modeling the adoption and diffusion of automated driving requires the consideration of new factors:

- Safety and Trust: The adoption of automated driving heavily relies on the perception of safety and trust. Consumers must feel confident that the technology can navigate roads reliably and mitigate potential risks. Factors like regulatory frameworks, infrastructure, and public awareness play crucial roles in the adoption process.
- Ethical and Legal Considerations: Automated driving raises complex ethical and legal questions, such as liability in accidents and decision-making algorithms in critical situations. These considerations must be incorporated into adoption and diffusion models to understand the public's acceptance and address potential barriers.

While traditional adoption and diffusion models have been valuable, they must adapt to the evolving landscape of innovative products. Incorporating new factors into these models can provide a more comprehensive understanding of the adoption and diffusion processes. Three important factors that warrant consideration are product adaptability, marketing mix, and switching costs.

Product adaptability refers to the ability of an innovative product to meet the diverse needs and preferences of consumers. In today's market, consumers expect products that can be customized or personalized to suit their specific requirements. The level of adaptability offered by an innovative product influences its adoption and diffusion. Models should account for factors such as modularity, upgradability, and compatibility with existing technologies to capture the impact of product adaptability on adoption rates.

The marketing mix encompasses various elements such as product, price, place, and promotion. Traditional adoption and diffusion models often focus on the product itself, but the marketing mix plays a crucial role in influencing consumer perceptions, attitudes, and behaviors. Factors such as effective communication strategies, targeted marketing campaigns, distribution channels, and pricing strategies should be integrated into adoption and diffusion models to better understand their impact on the adoption process.

Switching costs refer to the expenses, effort, or risks associated with changing from one product or service to another. When evaluating the adoption of innovative products, consumers consider not only the benefits but also the costs associated with switching from their existing solutions. These costs can be financial, psychological, or related to learning and adaptation. Incorporating switching costs into adoption and diffusion models provides insights into the barriers consumers face when considering a new product and helps identify strategies to mitigate these barriers.

As the landscape of innovation continues to evolve rapidly, it is crucial to refine adoption and diffusion models to account for new factors. Product adaptability, marketing mix, and switching costs are three important considerations that need to be integrated into these models. By doing so, researchers and practitioners can gain a more comprehensive understanding of the adoption and diffusion processes, ultimately informing effective strategies to drive the successful adoption of innovative products.

## FACTORS INFLUENCING INNOVATION ADOPTION AND DIFFUSION

The adoption and diffusion of innovations are influenced by a variety of factors that have been studied extensively in the literature. These factors shed light on the dynamics of innovation adoption and provide valuable insights into the drivers of successful diffusion.

One key finding from the literature is the role of network externalities and switching costs in determining the relationship between product quality and new product performance [6]. Network externalities positively impact new product performance, while switching costs have a negative influence. This suggests that the presence of a strong network effect can enhance the adoption and diffusion of innovations, while high switching costs can impede the process.

Another important finding is related to the impact of sequential market entries and competition modeling on multi-innovation diffusions [12]. The timing and order of product introductions can significantly affect market outcomes. Understanding the interaction between multiple innovations and their impact on consumer preferences over time is crucial for strategic decision-making and managing innovation releases effectively.

Studies examining the acceptance of specific innovations, such as e-book readers [13] and wearable technologies [14], have highlighted the influence of perceived innovative attributes and switching costs on adoption behavior. Factors such as relative advantage, compatibility, complexity, and social influence play a role in shaping consumers' acceptance and intention to switch to new technologies.

The impact of promotional sales in new product life cycles and their relationship with competition and forecasting has also been explored [15]. Promotional sales can boost market share and sales volume in the short term but may not be sustainable in the long run. The optimal pricing strategy for new products depends on the level of competition and the timing of the product launch, emphasizing the importance of considering both competition and forecasting in pricing decisions.

Switching costs and network effects have been found to significantly impact buyer behavior and market outcomes in IT markets [16]. The interplay between these factors can lead to lock-in effects or increased competition, depending on the strength of network effects. Understanding the dynamics between switching costs and network effects is crucial for understanding buyer behavior and designing effective marketing strategies. [17]

The relationship between innovation attributes, switching costs, and consumers' switching intentions has also been investigated in various contexts, such as traditional wristwatches to smartwatches [18]. Factors like relative advantage, complexity, and different types of switching costs influence consumers' intention to switch [19]. Considering these factors can help businesses design and market products that better meet consumers' needs and preferences, ultimately increasing adoption rates.

Finally, an empirical study based on expectation-disconfirmation theory highlighted the role of perceived benefits, expectations, and confirmation in users' intention to switch to disruptive technologies [20]. Understanding the factors that drive users to switch from existing dominant technologies to disruptive ones provides valuable insights into user behavior and the factors that influence successful diffusion.

In conclusion, the literature on innovation adoption and diffusion has provided valuable insights into the factors that influence the process. These factors include network effects, switching costs, market timing, perceived innovation attributes, pricing strategies, and buyer behavior. Understanding these factors and their interplay is crucial for organizations aiming to effectively introduce and diffuse innovations in the market. By considering these factors and leveraging the findings from the literature, businesses can enhance their innovation strategies and increase the likelihood of successful adoption and diffusion.

## NEW FACTORS INFLUENCING INNOVATION ADOPTION AND DIFFUSION: PROPOSED MODEL

### PRODUCT ADAPTABILITY

Initially, by considering **Adaptability** as a technical aspect of the product, it can increase the number of market potentials, since it would include not only the potential adopters of such technologies, but also the adopters of preceding technologies that are compatible with this new one, therefore not belonging to the same product category (unlike the affordance hypothesis). We could consider as part of the PRODUCT element in the marketing mix, expenditures on product to increase its compatibility and by considering elasticity of product compatibility on demand too. This would also allow to model the switching cost associated with the adoption of the new product, since the customer would be able to switch from the old product to the new one without a too high financial cost. Furthermore, the mathematical function of this model could consider the different levels of adaptability, to make the model more precise and reliable.

$$m = sP^{-e}A^{f}Prod^{a}$$

$prod$= expenditure on product to increase its compatibility.

$a$= elasticity of product compatibility on demand

The growth of consumption of technological products has made the need for adaptability of such products essential for their compatibility. Therefore, adaptability could be defined as a technological intervention and introduced into the intervention function of the GBM x(t) as a drastic and fast perturbation and thus as an exponential function. By defining the depth and sign of intervention, the persistency of the induced effects and whether they are positive or negative if the memory of these interventions is decaying to the stationary position (mean reverting), the start time of the intervention and the adaptability effort, it is possible to model the switching cost. These considerations allow to implement a mathematical function that considers the adaptability/complementarity of the product. In this way, the effects of the intervention can be evaluated and the desired outcomes from the intervention can be achieved.

$$x(t) = 1 + c * e^{b*(t-a)} * I_{t\geq a}$$

c= depth and sign of intervention

b= persistency of the induced effects and are negative if the memory of these interventions is decaying to the stationary position (mean reverting)

a= start time of the intervention.

I= AD= adaptability effort

Consequently, the **Adaptability** has been considered as a concept that is used to explain how a system reacts to drastic and sudden changes to its environment. It is often used in the context of digital products, where it is used to measure the importance of complementarity between complementary digital products, as well as the degree of complementarity between the digital product for which diffusion is to be estimated and the technologies that are complementary to it. In other words, adaptability measures how well a digital product or system can adjust to changes in its environment and respond efficiently. This concept can be used to assess the potential success of a digital product or system in the market and ensure that it is able to remain competitive in a rapidly changing environment.

The exponential form of the intervention function can be used to model the diffusion of technological interventions. The diffusion usually follows an S-shaped curve, with a steep increase in the first phase, followed by slowing down and, eventually, by a plateau. The parameters of this function, namely c, b, and a, are used to represent the diffusion of the technology. c is a scaling factor that indicates the effect of complementarity between different technologies on diffusion. b is the persistence of the introduced effect; if the complementarity generates negative externalities, such as a lack of differentiation, b will be negative. Finally, a is the trigger, or the moment when the intervention is introduced. By using this function, it is possible to estimate the effect of technological interventions on the diffusion of technologies.

$$y(t,d) = 1 + c \ e^{b(t-a)} \ * d_{t \geq a}$$

$$d = \begin{cases} 1 & if \ \ t \geq a \\ 0 & otherwise \end{cases}$$

By expanding the intervention function y (t, d) to a mixed form, it is possible to better understand the effects of technological features on the diffusion of digital products. This is achieved by analyzing the phenomena of interest and distinguishing between exponential and mixed shocks. As such, y (t, d) can be used to assess the impact of adaptability, complementarity, and performance on the diffusion model.

## SWITCHING COSTS

**Switching costs** play an important role when implementing a mathematical function to evaluate customer retention and churn rate. The cost of switching indicates the amount of money, time, and effort needed to switch from one system to another. As the cost of switching increases, the desire to switch and the value of switching decreases. Research has shown that user switching intention is determined by the expectation regarding disruptive technology and the dissatisfaction with the incumbent technology [19], while switching costs are not significant in the decision to acquire a disruptive technology. Switching costs can be divided into three categories: procedural, financial, and relational. Procedural switching costs refer to the time and resources associated with changing, while financial switching costs refer to the loss of financially measurable resources. Relational switching costs refer to the emotional inconvenience from breaking of bonds and the loss of identity. Studies have found that procedural switching costs do not affect switching intention, while financial switching costs are negatively associated with switching intention. Additionally, procedural, and relational switching costs have been found to mediate the relationship fully or partially between the perceived innovation attributes and the use of the innovation, but not financial switching costs. Relational switching costs have been found to have a negative effect on customers' switching behavior, but a positive effect on share of wallet, cross-buying across products categories and cross-buying across services categories. On the other hand, procedural switching costs have been found to have a positive effect on share of wallet and financial switching costs have been found to have a positive effect on cross-buying across products categories and cross-buying across services categories.

Switching cost can be modeled by taking into consideration the propensity to purchase of both innovators and imitators, as in the case of Bemmaor & Lee and Guseo, Mortarino & Darda. The distribution of the propensity to purchase has two parameters, δ and α, and thus the speed of diffusion of the innovation among the innovators and imitators. A is a parameter of η which represents the individual propensity of the imitators to purchase: η~G $(1/\beta,\ \alpha)$, $\beta = q/p$ δ is a parameter of η which represents the individual propensity of the innovators to purchase: η~G$(1/\beta$,"δ"), $\beta = q/p$ In order to model switching cost mathematically, we need to consider the aforementioned parameters δ and α, which represent the individual propensity to purchase. By analyzing the distribution of the propensity to purchase, we can estimate the speed of diffusion of the innovation among the innovators and imitators. Additionally, the parameter $\beta$ $(q/p)$ can be used to calculate the cost of switching from one product to another, as it indicates the probability of a consumer to switch from one product to another.

Switching costs can be modeled as either a multiplicative factor or an exponential factor. An intervention characterized by this factor does not change the market potential, however it alters the speed of the adoption curve. Switching costs have a greater impact on the beginning of the diffusion, but their influence decreases over time. It is important to consider the type of technology to be adopted and the significance of the switching costs.

In first considerations **Affordance** has been considered to be linked to the communication phase, in order to teach potential adopters how to use the product and therefore increase the market potential at each period of time, as in Guseo-Guidolin, $q_c =$ an internal component of communication.

Otherwise, Affordance effect can be linked to the marketing mix under PRODUCT, dedicating an initial test service to increase the affordability of potential adopters, considering that this will involve a special expense.

First option: $m = sP^{-e}A^{f}Prod^{a}$, Prod=expenditure on trial period of the product., a= elasticity of product trial period on demand.

Second option: $x(t) = 1 + \beta 1 \frac{Pr\prime(t)}{Pr(t)} + \beta 2 \frac{A\prime(t)}{A(t)} + \beta 3 \frac{Prod\prime(t)}{Prod(t)}$ ,prod=expenditure on trial period of the product.

As it would be shown in the final considerations the aspect of the affordances would be embedded into a more impacting factor, such as the switching cost till this one would be impacted by the affordance itself, so it would be better to not extract it separately.

Finally, this elaborate explores the factors that influence the diffusion of digital products with respect to **switching costs**. According on Antonelli's theory of sunk cost, it argues that the size of sunk costs incurred in previous technologies has a significant effect on the adoption of a new technology at time t_0. Specifically, the higher the sunk costs, the lower the likelihood of adoption. However, the new technology will be adopted when its performance levels exceed that of the previous one, which implies that there is a temporal delay before adoption. Overall, this study suggests that switching costs can play a pivotal role in the diffusion of digital products.

The study of switching costs is a critical component of assessing their impact on the industry. Historical evidence suggests that the introduction of the first smartphone was not hindered by user-side switching cost phenomena. However, when evaluating the effects of switching costs in the present day, it is necessary to consider the number of investments that have been made by potential adopters, such as the purchase of the smartphone and any peripheral devices connected to it, as these can influence decision making. Therefore, it is essential to conduct a study into the effects of switching costs to gain a better understanding of their impact on the industry. [21]

The importance of brand image and usability of digital products in terms of establishing consumer lock-in, thus increasing switching costs, should not be underestimated. Brand image, usability, and the sector itself can have a significant impact on consumer switching costs, as measured in terms of time, money and effort invested in the product by the user. Additionally, these factors can be studied in terms of the value of the goods and services provided, the degree of user satisfaction, the loyalty of customers and the sustainability of the business model [22]. Ultimately, understanding the effects of brand image, usability and sector on consumer switching costs can help organizations to create more successful digital products and better manage their customer base.

$$SC(t) = \begin{cases} \xi(t) & se\ t_{ingresso} > t_{ingressoCompet.} \\ \gamma(brand,\ usabilità) * t + s(settore,\ sunk\ cost) + Im(\xi) & per\ t > t_{ingresso} \end{cases}$$

ξ(t): estimate of the potential impact of market entry delays compared to competitors; Im(ξ): image of the function it represents SC (0) at the launch of the digital product; s (sector, sunk cost): probability that the sector influences the Switching Costs and the effect of sunk costs according to Antonelli's theory; γ (brand, usability): estimates the impact of the brand (number and attractiveness) on Switching Costs; also estimates the impact of affordance on Switching Costs;

Supposing that the aim is to estimate the impact of switching costs on the new market entry assuming that there is no story such that ξ(t)=0

Digital Calculator: the sector is irrelevant and there are no relevant sunk costs. Assuming that the choice of adoption is not influenced by the brand and that the usability is almost identical compared to competing products, we will have that:

$$\text{For } s \to 1,\ SC(t) = 1 \to \gamma t = 1 - s$$

Smartphones: The sector is relevant with medium range sunk cost. Also, suppose you are in a newcomer position and consequently in the presence of established competitors:

$$\text{For } 0 < s < 1, SC\ (t) > 1 \rightarrow \gamma t > 1 - s$$

Self-driving cars: +the sector and sunk cost are extremely impactful. In this case, suppose you are a well-known company in the sector that competes with little-known companies with few resources, you will have that:

$$\text{For } s \rightarrow 0,\ 0 < SC\ (t) < 1 \rightarrow \gamma t < 1 - s$$

Switching costs initial considerations during the analytical formulation took into consideration the different time periods in which the competitors enter the market. This consideration had the goal to put evidence the aspect that the initial switching cost would assume different behavior basing on the specific actor and how the market is performing with the actual competitors already present in it. Then the behavior of the switching cost of such actor would depend both on the precedent function and new elements not considered before.

Therefore, even if such considerations were considered in the analytical version of the model, successively it was preferred to simplify the hypothesis by considering the option that all the competitors are entering the market at the same time and that the behavior of the switching costs will depend into the impact of the brand and the affordance and still the general sector and sunk cost.

At this point, the potential impact of the market entry delays compared to competitors would be neglected; instead, this would be translated into the consideration of assuming that the competitors could enter the market without their switching costs being affected by the others.

In addition, the reflection assumed would also allow the implementation of the function of the impact of switching costs decreasing the variables that may affect the global effect and so, consent to focus on more impacting variables derived by the effect of the other parameters introduced in the study.

Consequently, the function of the impact of the switching costs would be simplified by being as follows, where still γ (brand, usability): estimates the impact of the brand (number and attractiveness) on Switching Costs; also estimates the impact of affordance on Switching Costs; s (sector, sunk cost): probability that the sector influences the Switching Costs and the effect of sunk costs according to Antonelli's theory.

$$SC(t) = \gamma(brand,\ usabilit\grave{a}) * t + s(settore,\ sunk\ cost)$$

## MARKETING MIX

The empirical findings of Boehner and Gold [10] suggest that the Cobb-Douglas function can be used to effectively model the impact of **Marketing mix** on the diffusion of digital products. This result is supported by the hypothesis that marketing plays a key role in the early stages of diffusion, allowing the product to reach its critical mass, beyond which other factors (e.g., word-of-mouth, networking, etc.) become more influential [23]. Such evidence highlights the importance of marketing in the process of product adoption and suggests that the Cobb-Douglas function may be a useful tool for accurately predicting the diffusion of digital products.

It is essential to understand that distribution and promotion strategies should not be considered as a corporate strategy, but rather as the effects that these policies have on potential adopters. Therefore, promotion must be seen as a method to increase awareness of the digital product, while distribution should be regarded as a method to ensure the availability of the digital product. Additionally, the concept of omnichannel has become increasingly important in the field of diffusion studies, as it allows small businesses to access marketing tools which were formerly only available to large corporations.

In accordance with the General Bass Model, the impact of marketing on the probability of the residual market embracing novel technology is evident. Upon achieving a certain level of awareness and availability, it is assumed that the **tipping point** is reached, leading to the proliferation of other adoption phenomena associated with other forces (e.g., q's "amplified" effect due to WOM and Social Networking). Accordingly, marketing's influence on the residual market's adoption of new technology is an important factor to consider.

The effect of marketing levers on z'(t) can be represented by a Cobb-Douglas with an exponential impulse, with β_i representing the marginal returns: where β_i = 1 implies constant returns, β_i > 1 implies increasing returns, and β_i < 1 implies decreasing returns. The parameters k, d, $\alpha$, pr, aw, and av represent the marketing effort necessary to maintain the achieved results, a dummy variable, the scale factor estimating the efficiency with which the marketing levers are used, the price, awareness, and availability, respectively. Furthermore, $\beta$ represents the market elasticity with respect to the respective marketing levers.

$$x(t,d) = k + d\,[\alpha\, pr^{-\beta_1 t}\, aw^{\beta_2 t}\, av^{\beta_3 t}]$$

$$d = \begin{cases} 1 & if\;\; z(t-1) \leq \dfrac{m}{2} \\ 0 & otherwise \end{cases}$$

$$\beta_i = \begin{cases} = 1 & \rightarrow constant\; marginal\; returns; \\ > 1 & \rightarrow increasing\; marginal\; returns; \\ < 1 & \rightarrow descreasing\; marginal\; returns; \end{cases}$$

## ANALYTICAL MODEL

The **analytical model** for the diffusion of digital products on the GBM framework can be represented as a function with the following components:

$$z'(t,d) = \left(p + \frac{q\, z(t)}{m}\right) SC(t)\, \big(m - z(t)\big)\;\, x(t,d)\;\, y(t,d)$$

The analytical model proposed for the diffusion of digital products within the GBM (General Bass Model) framework presents a comprehensive representation of the factors influencing the adoption and diffusion process. This model is formulated as a function that combines various components to capture the dynamics of diffusion.

The function incorporates the concept of Switching Costs (SC(t)), which plays a crucial role in shaping the hazard function of the diffusion process. Switching Costs are influenced by historical periods, sector-specific factors, sunk costs, brand value, and affordance, and their impact on the adoption decision is taken into account. This allows for a more nuanced understanding of the barriers and incentives that affect consumer behavior during the diffusion process.

The component x(t, d) within the function represents the factors that influence the adoption of digital products. This includes elements such as pricing strategies, awareness campaigns, and the availability of the product. By incorporating these factors, the model recognizes the importance of marketing and market dynamics in driving adoption rates.

Furthermore, the component y(t, d) in the function introduces a technological intervention function. This takes into consideration various technological characteristics that impact the diffusion of digital products, such as adaptability, complementarity, and performance. This recognizes that the inherent features and capabilities of the product itself can significantly influence its adoption and diffusion.

It is important to note that the proposed model builds upon the foundational framework of the General Bass Model, which provides a solid basis for understanding the diffusion process. The model extends this base by incorporating specific effect functions for each factor studied, analyzing them individually based on their unique nature and characteristics.

In the model, the parameters p, q, and m retain their original definitions from the General Bass Model. The parameter m represents the market size and provides a scale for demand forecasting. The coefficients p and q determine the shape of the adoption curve, with p representing the rate or probability of innovation adoption at a given time, and q accounting for the effects of imitation or social contagion.

By combining these components and parameters, the analytical model offers a more comprehensive and nuanced understanding of the diffusion of digital products within the GBM framework. It provides a valuable tool for researchers and practitioners to analyze and predict the adoption and diffusion patterns of digital innovations, taking into account various factors that influence the process.

# Chapter 3: MATHEMATICAL IMPLEMENTATION AND DATA ANALYSIS

This chapter focuses on the crucial step of mathematical implementation and data analysis in the study of adoption and diffusion models proposed for digital products. This step serves as a means to provide concrete evidence for the assumptions and reasoning made in the previous stages of the analytical model. By representing the effects believed to impact adoption as functions within the main function, it becomes possible to describe the rate of adoption and diffusion over time and make predictions accordingly.

Mathematical implementation enables the testing of various scenarios and assumptions embedded in the analytical model. This process ensures the reasonableness and accuracy of the previously made assumptions and allows for the identification of potential obstacles and limitations in representing the adoption and diffusion of digital products. By recognizing these obstacles, effective measures can be devised to overcome them.

This chapter begins by formulating hypotheses based on specific parameters relevant to the type of digital product under study. A thorough study of the product's domain and a review of scientific papers serve as the foundation for these hypotheses. The implementation of individual functions and the main function is carried out using MATLAB, providing a computational framework for conducting simulations and analyzing data.

Simulations are conducted based on the values assigned to the variables and assumptions made in the model. These simulations can be performed both individually and in combination, enabling a comprehensive exploration of the model's behavior and predictions. Graphical representations of the simulation results add visual clarity to the analysis.

Through the analysis of these simulations, parameters whose variations significantly affect the adoption and diffusion process of digital products are identified. This identification allows for subsequent simulations to focus solely on the significant parameters, leading to more refined and targeted analysis. Graphics representing these simulations provide valuable insights into the dynamics and outcomes of the adoption and diffusion process for digital products.

In summary, this chapter emphasizes the importance of mathematical implementation and data analysis in validating the assumptions and reasonings made in the analytical model for digital products. By testing various scenarios and assessing the impact of different parameters, a more accurate understanding of the adoption and diffusion dynamics of digital products can be achieved.

## SMARTPHONES CASE STUDY

Smartphones have emerged as a groundbreaking technological phenomenon that has fundamentally reshaped the fabric of human communication, information access, and service engagement. Their impact on society is nothing short of revolutionary, solidifying their status as a quintessential example of an innovative product with unparalleled widespread adoption.

In the era before smartphones, communication was confined to traditional methods such as landline telephones and basic mobile devices. However, with the advent of smartphones, a paradigm shift occurred. These handheld marvels seamlessly integrated multiple functionalities and technologies, consolidating telephony, computing power, internet connectivity, and multimedia capabilities into a single, portable device.

The transformative power of smartphones lies in their ability to connect people instantaneously, transcending geographical boundaries and time zones. They have become indispensable tools for interpersonal communication, offering a myriad of communication channels such as voice calls, text messaging, video conferencing, and social media platforms. Smartphones have fostered an era of constant connectivity, enabling individuals to stay in touch, share experiences, and engage with others effortlessly.

Beyond communication, smartphones have become gateways to vast repositories of information. With internet access at their fingertips, users can retrieve news, browse websites, access educational resources, and explore a world of knowledge with unprecedented ease and speed. The information revolution brought about by smartphones has democratized access to information, empowering individuals to stay informed, expand their horizons, and participate actively in the digital age.

Additionally, smartphones have revolutionized the way people engage with various services, transcending traditional boundaries and transforming numerous industries. From e-commerce to transportation, entertainment to healthcare, smartphones have disrupted established business models, creating new opportunities and channels for interaction. Mobile applications have flourished, offering users a plethora of services and conveniences at their fingertips, ranging from food delivery and ride-hailing to online banking and streaming media.

The extraordinary rate of adoption of smartphones reflects the profound impact they have had on society. From individuals of all age groups and backgrounds to businesses of all sizes and sectors, smartphones have become ubiquitous tools that have seamlessly integrated into our daily lives. Their intuitive user interfaces, versatility, and ever-expanding capabilities have captured the imaginations of billions worldwide, driving their rapid adoption and solidifying their position as a transformative force in the modern world.

In conclusion, smartphones epitomize the epitome of innovative products, forever altering the way we communicate, access information, and engage with services. Their widespread adoption is a testament to their immense influence on society, ushering in a new era of interconnectedness, knowledge sharing, and convenience. As smartphones continue to evolve, their impact will undoubtedly shape the future of technology and human interaction, cementing their place in history as one of the most significant innovations of our time.

## HYPOTHESIS

Derived from extensive literature and research, the following approximations and ranges offer valuable insights into the parameters and market dynamics inherent in the smartphone industry. By shedding light on these factors, they greatly contribute to our understanding and analysis of adoption and diffusion patterns.

The parameter ranges and market potential estimates for smartphones are subject to variation based on specific market conditions, technological advancements, and regional factors, as evidenced by research and literature. The coefficient of innovation (p) in the context of smartphone adoption has been estimated to fall within the range of approximately 0.01 to 0.10, while the coefficient of imitation (q) is estimated to range from 0.10 to 0.50. Estimating the market potential (m) for smartphones on a global scale is complex due to factors such as population size, market saturation, and regional disparities. However, the market potential for smartphones is significant, with the potential number of adopters ranging from hundreds of millions to billions of users worldwide. [24]

Similarly, the values for price elasticity, advertising elasticity, and distribution elasticity in the smartphone industry can vary based on market conditions, consumer behavior, and specific product or brand considerations. In general, the price elasticity of smartphones is considered relatively elastic, with estimates ranging from -1.0 to -2.0 or higher. This indicates that a 1% increase in price could result in a more than 1% decrease in demand. Advertising elasticity, on the other hand, typically falls within the range of 0.1 to 0.5 or higher, suggesting that a 1% increase in advertising spending could lead to a 0.1% to 0.5% or higher increase in demand. While specific estimates for distribution elasticity may be less readily available, it is generally recognized that widespread availability and efficient supply chains positively influence smartphone demand.

In terms of price, the approximate mean ranges for high-end smartphones, mid-range smartphones, and budget-friendly smartphones fall within $800 to $1,500, $300 to $600, and $100 to $300, respectively. Advertising budgets for prominent smartphone brands typically range from around 10% to 20% of their overall revenue, while emerging brands may allocate a lower proportion of their revenue, ranging from 5% to 10%. Distribution channels play a crucial role, with physical retail stores accounting for approximately 50% of total sales, online sales channels contributing 30% to 40%, and collaborations with telecom providers contributing 10% to 20% of smartphone distribution.

The hypotheses formulated for the parameters are rooted in a comprehensive analysis that draws from both literatures, as well as the principles of graph theory. By combining insights from these diverse sources, a more holistic understanding of the adoption and diffusion process is achieved. Literature provides a foundation of knowledge regarding consumer behavior, market conditions, and industry trends. These sources offer valuable empirical evidence and theoretical frameworks for parameter estimation. Additionally, the application of graph theory allows for analyzing and understanding the properties and behaviors of the considered function, used to represent each of the drivers. This integration of literature and graph theory ensures a multidimensional approach to parameter hypothesis, enhancing the accuracy and robustness of the analysis.

### Product Adaptability

$$y(t,d) = 1 + c\ \ e^{b(t-a)}\ \ * d_{t \geq a}$$

$$d = \begin{cases} 1 & if\ \ t \geq a \\ 0 & otherwise \end{cases}$$

Domain : $\mathbb{R}\ (all\ real\ numbers)$

Range: $\{y \in \mathbb{R}: b \neq 0, c > 0, y > 1$

Limit: $\lim_{t \to 1} y(t) = 1\ \ for\ (c,a) \in \mathbb{R}^2\ b > 0$

Marketing Mix [7]

$$x(t,d) = k + d\ [\alpha\ pr^{-\beta_1 t}\ aw^{\beta_2 t}\ av^{\beta_3 t}]$$

$$d = \begin{cases} 1 & if\ \ z(t-1) \leq \dfrac{m}{2} \\ 0 & otherwise \end{cases}$$

$$\beta_i = \begin{cases} = 1\ \ \to constant\ marginal\ returns; \\ > 1\ \ \to increasing\ marginal\ returns; \\ < 1\ \ \to descreasing\ marginal\ returns; \end{cases}$$

$$k \cong 1$$

$$\beta_1 = \begin{cases} 0.35\ \ \to Low \\ 1.00\ \ \to Medium\ \ \ \to Chosen\ case \\ 3.00\ \ \to High \end{cases}$$

$$\beta_2 = \begin{cases} 0.25\ \ \to Low \\ 0.50\ \ \to Medium\ \ \ \to Chosen\ case \\ 0.70\ \ \to High \end{cases}$$

$$\beta_3 = \begin{cases} 0.25\ \ \to Low \\ 0.50\ \ \to Medium\ \ \ \to Chosen\ case \\ 0.70\ \ \to High \end{cases}$$

$$pr = \begin{cases} 2 \div 3\ \ \to Low \\ 4 \div 6\ \ \to Medium\ \ \ \to Chosen\ case \\ 7 \div 10\ \ \to High \end{cases}$$

$$aw = \begin{cases} 1 \div 2\ \ \to Low \\ 3 \div 5\ \ \to Medium\ \ \ \to Chosen\ case \\ 6 \div 10\ \ \to High \end{cases}$$

$$av = \begin{cases} 1 \div 2\ \ \to Low \\ 3 \div 5\ \ \to Medium\ \ \ \to Chosen\ case \\ 6 \div 10\ \ \to High \end{cases}$$

Switching Costs

$$SC(t) = \gamma(brand,\ usabilit\grave{a}) * t + s(settore,\ sunk\ cost)$$

Roots: $t \neq 0,\ \gamma = -\frac{s}{t}$ → chosen case

$s = 0, t = 0$ → not chosen case

Domain: $\mathbb{R}$ $(all\ real\ numbers)$

Range: $\{SC\ \in \mathbb{R}: SC = s\ or\ \gamma \neq 0\}$

Smartphones: The sector is relevant with medium range sunk cost. Also, suppose you are in a newcomer position and consequently in the presence of established competitors:

3For 0 < $s$ < 1, $SC$ $(t)$ > 1 → $\gamma t$ > 1 − $s$

## IMPLEMENTATION

The implementation has been conducted using MATLAB, with the aim of presenting a comprehensive analysis of the adoption and diffusion process of a digital product. By incorporating functions for product adaptability, marketing mix effectiveness, and switching cost, the scripts point to evaluate the key factors influencing the adoption of the product. Through simulation and visualization, the scripts explore the impact of product adaptability intervention, marketing mix efforts, and switching cost components on the adoption dynamics. The simulations are compared with the well-known Bass model to validate the results and provide insights into the effectiveness of the proposed approach. By examining the punctual adoption in each time period and the cumulative adoptions, the scripts enable a detailed assessment of the diffusion level at different stages. This comprehensive analysis provides valuable insights into the adoption and diffusion patterns of digital products, allowing researchers and practitioners to understand and optimize the factors driving successful adoption strategies.

The first function, called "Product Adaptability" calculates the adaptability of the digital product at a given time (t) based on the intervention moment (a). It defines constants bb (persistence of the introduced effect) and cc (scale factor) and includes an if-else statement to determine the value of the dummy variable (d) based on the time and intervention moment. The adaptability (y) is then calculated using an exponential function with the persistence factor, time difference, and the dummy variable.

The second function, named "Marketing Mix" calculates the effectiveness of marketing efforts for the digital product at a given time (t) based on various marketing factors and variables. It defines constants and variables such as beta1, beta2, beta3, alfa, kk, pr, aw, av, and mm. It includes an if-else statement to determine the value of the dummy variable (dd) based on a control variable (cont) and a threshold value. The marketing effectiveness (x) is calculated using an algebraic expression that combines price, awareness, availability, time, and the marketing elasticities. The effectiveness is then scaled by the alfa factor and added to the baseline marketing effectiveness level (k).

While, the third function, "switching cost" calculates the switching cost (SC) for the digital product based on the time (T), the sunk cost (S), and a constant value (k). It computes the variable Y by subtracting the ratio of SS to T from the constant k. The switching cost (SC) is then calculated by multiplying Y by T and adding S.

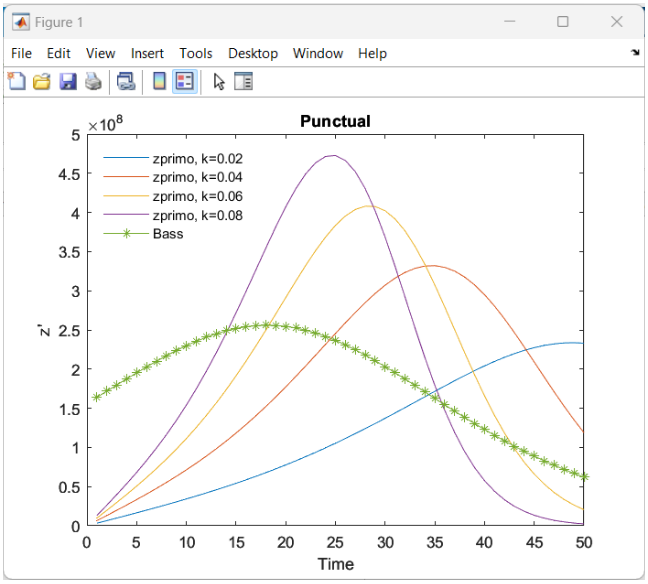
In the other hand, the main simulation code performs a series of iterations using a while loop and a for loop. It simulates the adoption and diffusion process of the digital product over a specific time period represented by the variable 'time'. It calculates and updates the adoption variable (z) and its derivative (z') based on various factors, including market potential, marketing efforts, adaptability, and switching costs. It also implements the Bass model for comparison. The simulation results are visualized using two figures: one for the adoption derivative (z') and another for the adoption variable (z). The figures display the results for different values of the marketing constant (k) or marketing factors levels (pr, aw, av) and include the legend indicating the specific values.

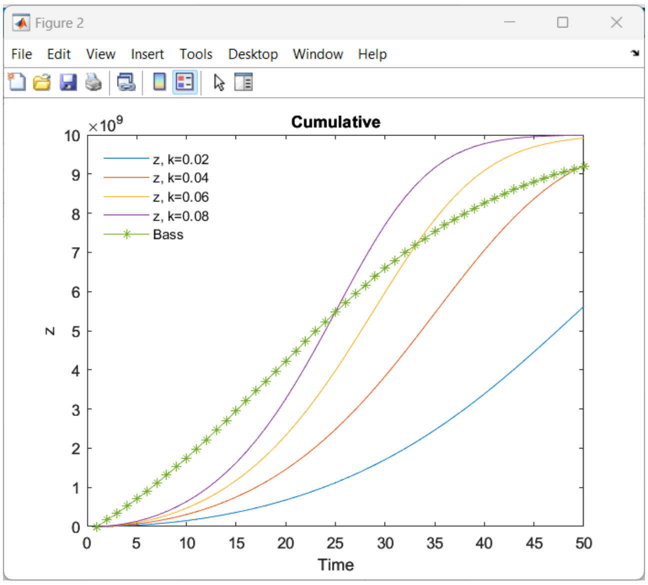
## SIMULATIONS

In the mathematical implementation and data analysis stage, the adoption and diffusion dynamics of digital products are assessed through the use of simulations and graphical representations. Two key graphs are generated to evaluate the adoption patterns and diffusion levels over time.

In the upcoming section, the results of simulations incorporating the **Switching Costs** component into the base Bass model will be showcased. This component represented by the following formula:

$$SC(t) = \gamma(brand,\ usabilità) * t + s(settore,\ sunk\ cost)$$






The first graph illustrates the punctual adoption in each time period. It showcases the number of adoptions that occur at specific time points, providing insights into the rate of adoption at different stages. Switching cost takes into consideration various factors that influence adoption, such as sunk costs, technology performance, usability, and brand. Sunk costs have a dampening effect on adoption likelihood, as they increase the perceived risk and commitment associated with adopting the new technology. However, as the performance of the new technology surpasses that of the previous one, adoption levels increase. This suggests that customers are more willing to adopt when the benefits in terms of usability and brand superiority outweigh the sunk cost.

The second graph focuses on the cumulative adoptions, which allows for the assessment of the diffusion level at each time period. It takes into account the cumulative number of adoptions that have taken place up to a specific point in time. This graph reflects the overall spread and acceptance of the digital product among the target audience. The adoption curve in this graph is influenced by the interplay between sunk costs, switching costs, technology performance, usability, and brand. In the initial phase, when the technology is relatively new, the sunk costs act as a deterrent to adoption as customers may perceive commitment as intimidating. During this phase, the most influential factor is the sunk cost effect. As the technology gradually spreads, customers are more likely to adopt due to the network effects and the advantages of being part of a larger user base. Towards the later stages, as the market becomes saturated, the adoption curve starts to decrease. During this phase, the brand and usability effects become more relevant as customers look for differentiation and enhanced user experience.

In the forthcoming section, the showcased results will revolve around simulations integrating the Marketing Mix component into the foundational Bass model. This component is represented by the following formula:

$$x(t,d) = k + d\ [\alpha\ pr^{-\beta_1 t}\ aw^{\beta_2 t}\ av^{\beta_3 t}]$$

$$d = \begin{cases} 1 & if\ \ z(t-1) \leq \frac{m}{2} \\ 0 & otherwise \end{cases}$$

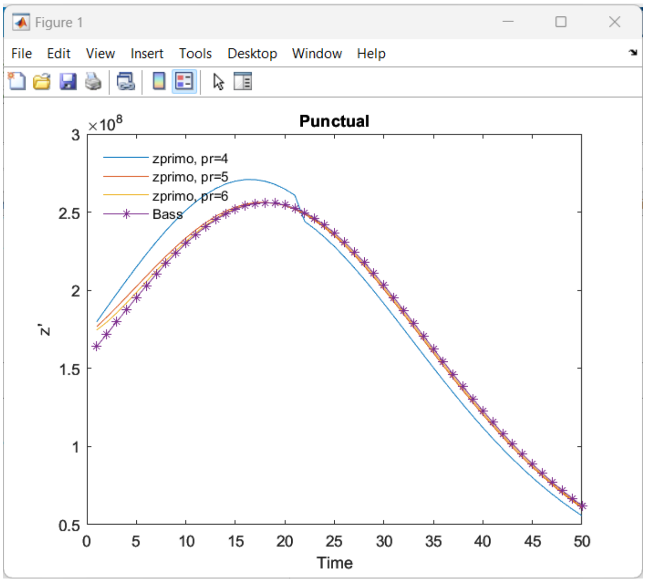
The graph on the left represents the punctual adoption in each time period, showcasing the number of adoptions occurring at specific points in time. This graph allows for the evaluation of the rate of adoption and provides insights into the role of marketing in driving the residual market's adoption during the early stages of diffusion.

To study the influence of marketing on the adoption process, the effect of marketing levers is considered as an exponential impulse with suitable parameters representing the necessary marketing efforts to achieve and maintain results. This graph demonstrates the effectiveness and efficiency of the marketing levers used to promote the new technology. The impact of marketing efforts can be observed in the adoption curve, reflecting the successful reach and engagement of the target market, ultimately contributing to the product's attainment of critical mass.

The graph on the right represents the cumulative adoptions, providing a measure of the diffusion level at each time period. It considers the overall number of adoptions that have occurred up to a specific point in time. This graph allows for the assessment of the impact of various marketing factors on the diffusion process. Specifically, it highlights the influence of price, availability, and awareness on the diffusion dynamics.

The impact of the price factor is observed when the price is at its lower bound. A lower price stimulates adoption, attracting more customers to adopt the new technology. Availability, represented by its high values, also plays a significant role in driving adoption. When the product is widely available, customers have greater access and are more likely to adopt. The impact of awareness, on the other hand, becomes relevant when it is at its high values. Higher awareness levels increase the visibility and understanding of the new technology, leading to a higher adoption rate.

These factors interact in complex ways, and the graph showcases their combined influence on the diffusion process. For instance, awareness is impactful when availability is at its lower bound and price is at its low-medium values. This combination highlights the importance of building awareness and generating interest in the early stages of diffusion, even when the product may not be widely available or at the lowest price point.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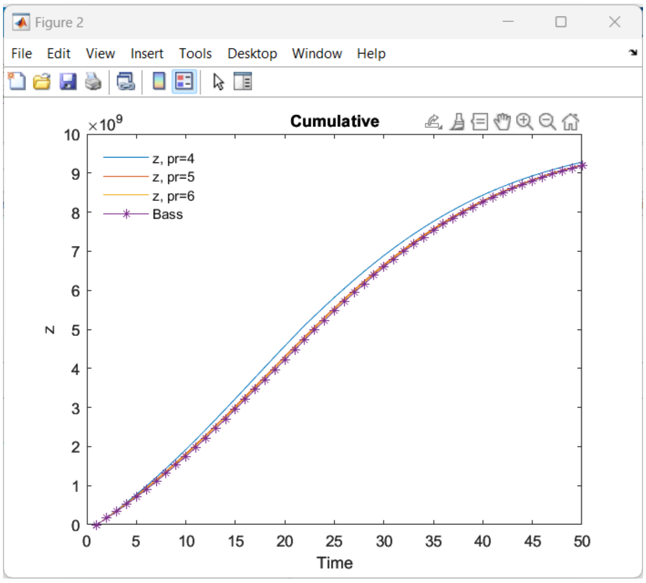
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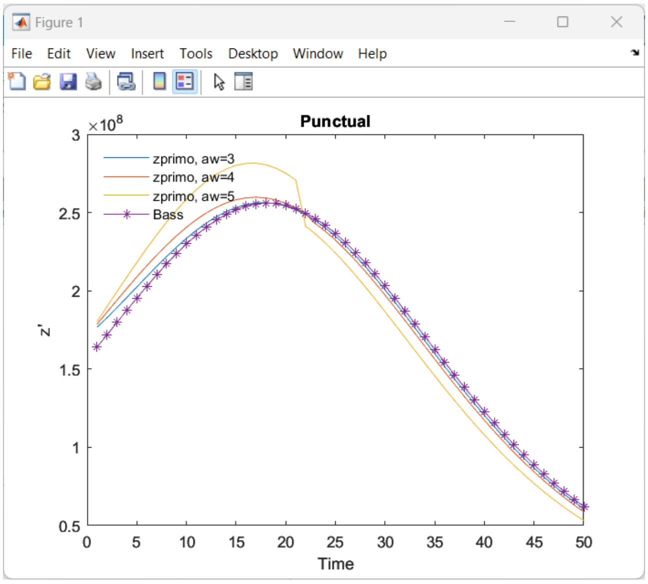
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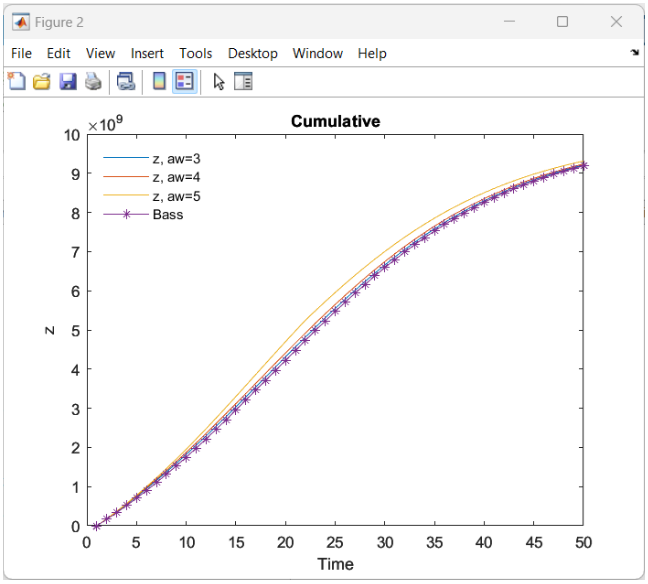
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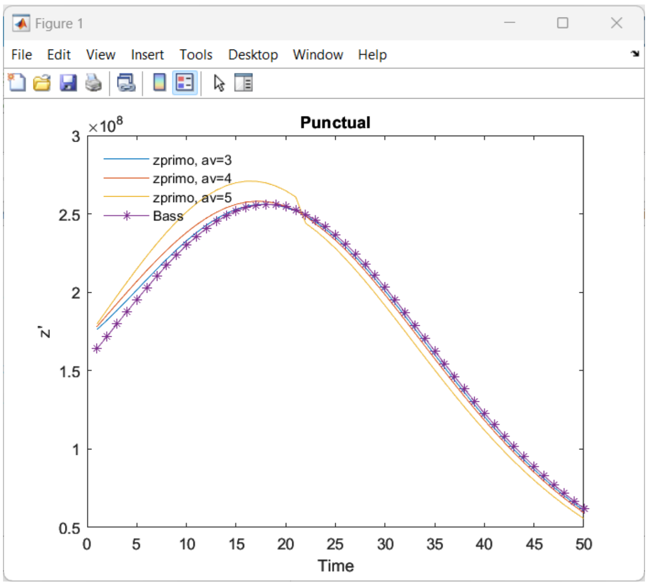
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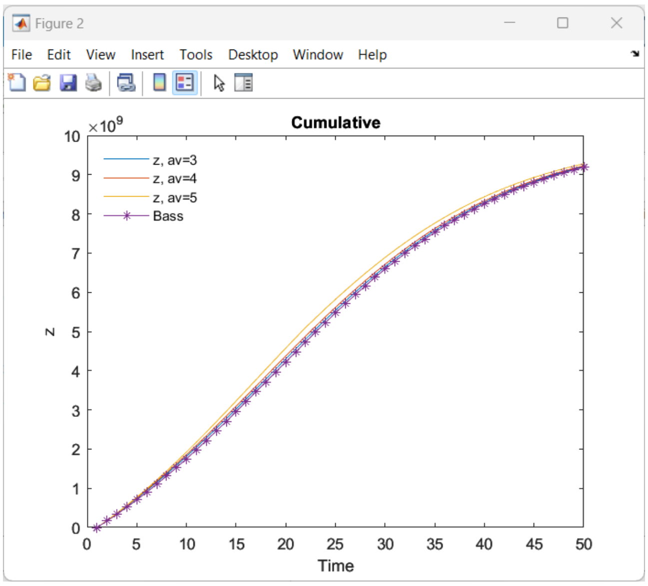
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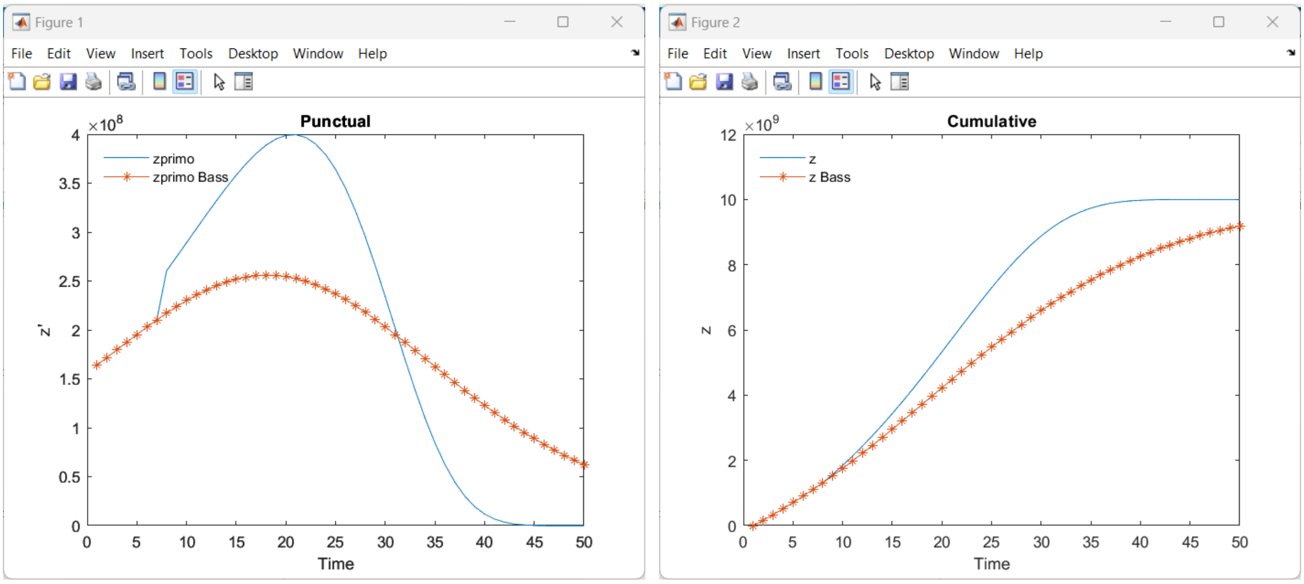
In the following segment, the outcomes of simulations incorporating the Product Adaptability component into the base Bass model will be unveiled, highlighting the results expressed by the following formula:

$$y(t,d) = 1 + c\ e^{b(t-a)}\ * d_{t \geq a}$$

$$d = \begin{cases} 1 & if\ \ t \geq a \\ 0 & otherwise \end{cases}$$




On the left the graph represents the punctual adoption in each time period, showcasing the number of adoptions occurring at specific points in time. This graph allows for the evaluation of how well the digital product can adjust to drastic and sudden changes in its environment and respond efficiently.

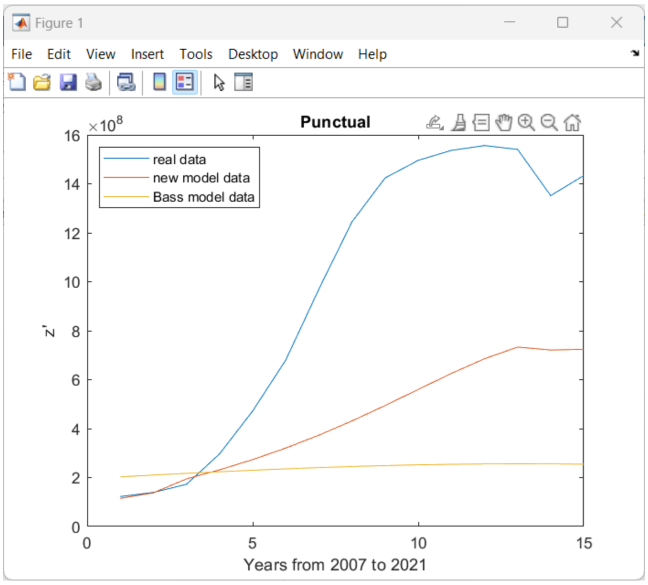
To model the diffusion of technological interventions, the exponential form of the intervention function is utilized. This function captures the impact of interventions on the adoption process, reflecting how the digital product responds to external changes or interventions. In the first graph, the adoption curve demonstrates the diffusion process and the effect of the intervention, which, in this case, is represented by the product adaptability. Typically, diffusion follows an S-shaped curve, characterized by a steep increase in adoption during the initial phase, followed by a gradual slowing down and eventually reaching a plateau. More precisely, the first graph exhibits the steep increase in adoption during the time period in which the intervention of product adaptability has been implemented.

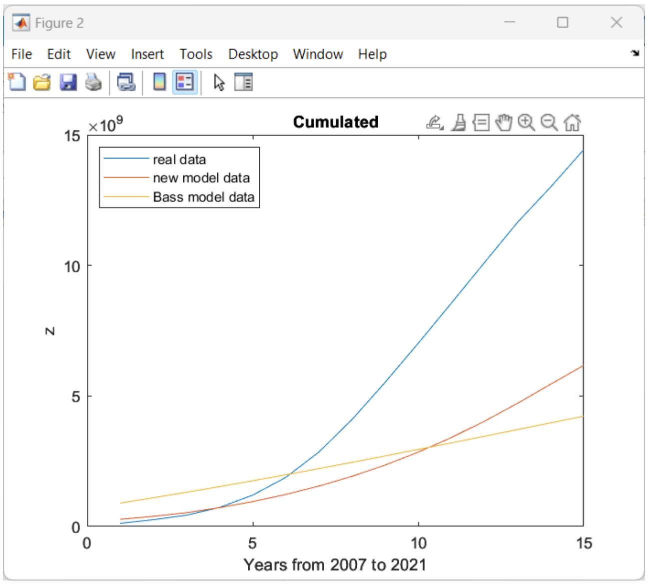
While, the right graph represents the cumulative adoptions, providing a measure of the diffusion level at each time period. It showcases the overall number of adoptions that have occurred up to a specific point in time. This graph allows for the evaluation of the overall diffusion pattern and the effectiveness of the digital product's response to the intervention of product adaptability.

By analyzing these two graphs, researchers can gain insights into the adoption patterns and diffusion levels of the digital product in response to the intervention of product adaptability. The steep increase in adoption observed in the first graph indicates that the digital product adjusts well to drastic and sudden changes in its environment. The S-shaped curve in diffusion, commonly observed, signifies the rapid growth in adoption initially, followed by a gradual saturation as the product reaches its market potential. The cumulative adoption graph provides a comprehensive view of the diffusion process, allowing researchers to assess the overall effectiveness of the digital product's response to the intervention of product adaptability.

## DATA COMPARISION

Comparing the output of both the Bass and the proposed model for smartphone adoption with real data [25], several differences became evident, highlighting some limitations of the models.
Initially, both the proposed model and the widely used Bass model predicted a higher number of adopters than what was observed in the real data. Conversely, after a certain period of time, the curve of real data surpassed the predictions of the two models. These disparities can be attributed to inherent limitations within the models. One notable limitation is the assumption of a static target market, disregarding factors such as population growth and changes in market potential over time. Additionally, the models often overlook the influence of the replacement cycle, wherein individuals upgrade their smartphones, leading to a higher adoption rate than initially predicted. Furthermore, the models do not account for the possibility of multiple device ownership by individuals, resulting in an underestimation of the actual number of devices in use. To improve the accuracy of such models, it is crucial to incorporate dynamic target market sizes, consider the impact of replacement behavior, and account for the possibility of multiple device ownership. The new model represents better but not perfectly real data when compared to Bass model. To address these limitations in relation to real-world data on smartphone adoption it is essential to analyze real-world data and adjust model parameters based on empirical observations to refine the predictions and align them more closely with the complex dynamics of smartphone adoption and diffusion.



## AUTOMATED-DRIVING CASE STUDY

Automated driving, colloquially known as autonomous vehicles or self-driving cars, stands as an exemplification of an unparalleled revolutionary innovation that has the potential to redefine transportation as we know it. This groundbreaking technology represents a significant leap forward in the automotive industry, promising a future where vehicles navigate roads and transport passengers with minimal human intervention.

Automated driving signifies a paradigm shift in the way we perceive and interact with automobiles. By combining cutting-edge technologies such as artificial intelligence, sensor systems, and advanced computing, self-driving cars possess the capability to perceive their surroundings, analyze data, make decisions, and execute maneuvers without direct human control.

This innovation carries profound implications for various aspects of society, ranging from transportation efficiency and safety to urban planning and environmental sustainability. Automated driving has the potential to revolutionize mobility by mitigating traffic congestion, reducing accidents caused by human error, and optimizing road capacity through efficient vehicle coordination.

Furthermore, the advent of self-driving cars introduces a transformative shift in consumer behavior and travel experiences. Passengers can reclaim their time during commutes, engaging in productive tasks, entertainment, or relaxation while the vehicle autonomously handles the driving responsibilities. This paradigm shift challenges traditional notions of personal transportation and opens possibilities for reimagining vehicle interiors, services, and business models.

The potential benefits of automated driving extend beyond individual convenience. This innovation holds the promise of enhancing accessibility for people with mobility limitations, providing transportation solutions for underserved communities, and improving overall transportation equity. By reducing the reliance on human drivers, self-driving cars have the potential to disrupt traditional employment models in the transportation sector, necessitating the consideration of societal and economic implications.

However, the adoption and implementation of automated driving are accompanied by a myriad of challenges. Technical hurdles, including ensuring robust and reliable sensing and decision-making capabilities, as well as addressing ethical considerations and liability frameworks, demand comprehensive solutions. Furthermore, public trust and acceptance play crucial roles in determining the success and widespread adoption of self-driving cars.

In conclusion, automated driving represents a truly revolutionary innovation in the automotive industry. With the potential to reshape transportation, enhance safety, redefine travel experiences, and address societal challenges, self-driving cars stand at the forefront of technological advancements. While significant challenges and considerations lie ahead, the transformative potential of automated driving is undeniable, making it a critical area of research and development as we navigate the future of transportation.

## HYPOTHESIS

The ranges and approximations, learnt from literature, offer valuable insights into parameter hypotheses, market potential, elasticity considerations, and pricing dynamics within the realms of automated driving, ADAS, and EV sectors. By considering these findings, a deeper understanding of these areas can be achieved, enabling informed decision-making and strategic planning in the context of these technologies. [24]

The parameter ranges and market potential for automated driving and related technologies, such as electric vehicles (EVs) and advanced driver-assistance systems (ADAS), have been investigated based on literature and research [26]. The estimated ranges for the coefficient of innovation (p) and coefficient of imitation (q) in the context of automated driving and similar technologies fall approximately between 0.01 to 0.20 for both parameters. Estimating the market potential (m) for these technologies is challenging due to various factors. For automated driving, the market potential could range from several hundred million to potentially billions of vehicles worldwide, considering factors such as the global vehicle fleet size, infrastructure development, and regulatory frameworks. Estimates for the market potential of EVs range from tens of millions to potentially hundreds of millions of vehicles globally. The market potential for ADAS technologies ranges from hundreds of millions to potentially billions of vehicles worldwide. These ranges provide approximations based on literature and studies related to the adoption of similar technologies.

Regarding elasticity values, specific values for automated driving may be limited at this stage of development. However, general ranges based on observations and studies related to ADAS and EVs can be considered. The price elasticity for ADAS components ranges from -1.0 to -2.0 or higher, indicating relatively elastic demand. For EVs, the price elasticity typically falls within -1.0 to -2.5 or higher, suggesting a relatively elastic demand as well. Advertising elasticity for ADAS technologies ranges from 0.1 to 0.5 or lower, while for EVs, it typically ranges from 0.1 to 0.5 or higher. Specific studies on distribution elasticity for ADAS technologies are limited, but increased availability and accessibility through various channels contribute to higher demand. Distribution elasticity for EVs depends on factors such as charging infrastructure and dealership availability. While exact ranges may vary, increased accessibility is expected to positively impact EV demand. These ranges provide approximate observations based on literature, considering market dynamics, regional variations, and product attributes within the ADAS and EV sectors.

In terms of price, the mean ranges for automated driving technologies, ADAS features, and EVs vary based on factors such as the level of autonomy, complexity, integration, and model types. For fully automated driving technologies, prices can range from tens of thousands to hundreds of thousands of dollars due to advanced hardware and software components. ADAS feature prices typically range from a few hundred to a few thousand dollars, while EV prices vary significantly depending on the model, ranging from around $30,000 to over $100,000. Advertising and distribution budgets also vary depending on the technology and market. For instance, advertising budgets for fully automated driving technologies range from a few million to tens of millions of dollars, while advertising expenditure for ADAS and EVs can range from a few million to hundreds of millions of dollars, depending on brand presence and market reach. Distribution costs for automated driving technologies can range from tens of millions to hundreds of millions of dollars, while distribution expenditure for ADAS and EVs may vary depending on factors such as retrofitting, partnerships, and market volume.

The hypotheses formulated for the parameters are based on a comprehensive analysis that draws from a synthesis of relevant literature and the principles of graph theory. This amalgamation of diverse sources results in a more comprehensive understanding of the adoption and diffusion process. Literature serves as a valuable foundation for understanding consumer behavior, market conditions, and industry trends. It provides empirical evidence and theoretical frameworks that contribute to accurate parameter estimation. Furthermore, the application of graph theory enables the examination and comprehension of the properties and behaviors of the considered function, which represents each of the drivers. This integration of literature and graph theory facilitates a multidimensional approach to parameter hypothesis, ultimately enhancing the accuracy and robustness of the analysis.

Product Adaptability

$$y(t,d) = 1 + c\ \ e^{b(t-a)}\ * d_{t \geq a}$$

$$d = \begin{cases} 1 & if\ \ t \geq a \\ 0 & otherwise \end{cases}$$

Domain : $\mathbb{R}\ (all\ real\ numbers)$

Range: $\{y \in \mathbb{R}: b \neq 0, c > 0, y > 1$

Limit: $\lim_{t \to 1} y(t) = 1\ \ for\ (c,a) \in \mathbb{R}^2\ b > 0$

Marketing Mix [7]

$$x(t,d) = k + d\ [\alpha\ pr^{-\beta_1 t}\ aw^{\beta_2 t}\ av^{\beta_3 t}]$$

$$d = \begin{cases} 1 & if\ \ z(t-1) \leq \dfrac{m}{2} \\ 0 & otherwise \end{cases}$$

$$\beta_i = \begin{cases} = 1 \ \rightarrow constant\ marginal\ returns; \\ > 1 \ \rightarrow increasing\ marginal\ returns; \\ < 1 \ \rightarrow descreasing\ marginal\ returns; \end{cases}$$

$$k \cong 1$$

$$\beta_1 = \begin{cases} 0.35\ \rightarrow Low \\ 1.00\ \rightarrow Medium\ \rightarrow choosen\ one \\ 3.00\ \rightarrow High \end{cases}$$

$$\beta_2 = \begin{cases} 0.25 \ \rightarrow Low \\ 0.50 \ \rightarrow Medium \rightarrow choosen\ one \\ 0.70 \ \rightarrow High \end{cases}$$

$$\beta_3 = \begin{cases} 0.25 \ \rightarrow Low \\ 0.50 \ \rightarrow Medium \ \rightarrow choosen\ one \\ 0.70 \ \rightarrow High \end{cases}$$

$$pr = \begin{cases} 2 \div 3 \ \rightarrow Low \\ 4 \div 6 \ \rightarrow Medium \\ 7 \div 10 \ \rightarrow High \rightarrow choosen\ one \end{cases}$$

$$av = \begin{cases} 1 \div 2 \ \rightarrow Low \ \rightarrow choosen\ one \\ 3 \div 5 \ \rightarrow Medium \\ 6 \div 10 \ \rightarrow High \end{cases}$$

$$aw = \begin{cases} 1 \div 2 \ \rightarrow Low \ \rightarrow choosen\ one \\ 3 \div 5 \ \rightarrow Medium \\ 6 \div 10 \ \rightarrow High \end{cases}$$

Switching Costs

$$SC(t) = \gamma(brand,\ usabilità) * t + s(settore,\ sunk\ cost)$$

Roots: $t \neq 0,\ \gamma = -\frac{s}{t}$ → chosen case

$s = 0, t = 0$ → not chosen case

Domain: $\mathbb{R}\ (all\ real\ numbers)$

Range: $\{SC \ \in \mathbb{R}: SC = s\ or\ \gamma \neq 0\}$

Self-driving cars: +the sector and sunk cost are extremely impactful. In this case, suppose you are a well-known company in the sector that competes with little-known companies with few resources, you will have that:

For s → 0, 0 < SC (t) < 1 → γt < 1 – s

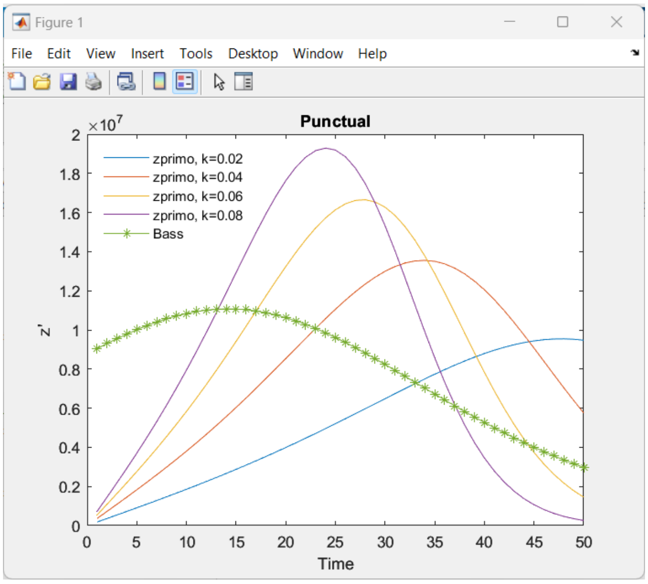
## IMPLEMENTATION

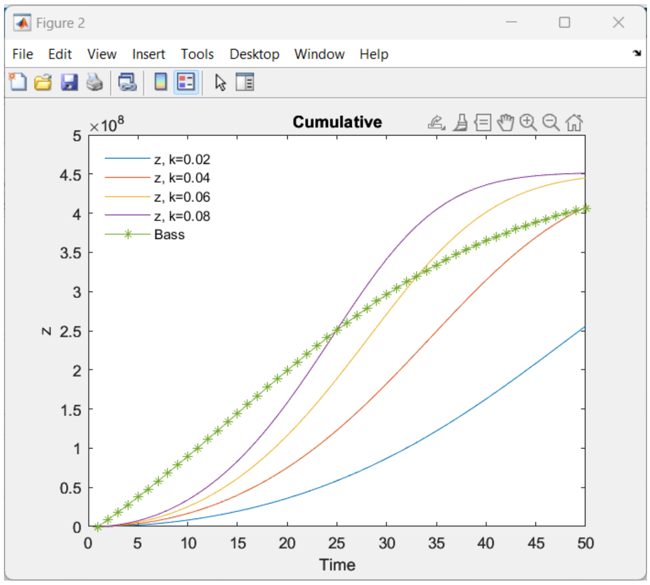
In a similar manner, the same implementation procedure shown for the study case of smartphones is applied to study the adoption and diffusion process of automated driving. However, it is important to note that the hypothesis regarding certain parameters is at an early stage due to the limited availability of studies and information in the field of adoptions and diffusion for both automated driving and similar products. Despite this challenge, the MATLAB script takes into account the relative parameters, constants, and variables specific to the study case of automated driving. By adapting the functions for adaptability, marketing effectiveness, and switching cost, the script aims to capture the unique characteristics and challenges associated with the adoption of automated driving technology. The simulation and visualization processes are carried out over a specified time period, enabling a detailed analysis of the punctual adoption and cumulative adoptions of automated driving. Furthermore, the results obtained from the simulations are compared with the corresponding Bass model to validate the findings and provide insights into the effectiveness of the implemented approach. This comprehensive analysis, although still in its early stages, allows researchers and practitioners to gain a deeper understanding of the adoption patterns and diffusion dynamics of automated driving, facilitating the development of effective strategies and interventions in this rapidly evolving domain. As further research and data become available, it is anticipated that the accuracy and precision of the hypothesis regarding the parameters will improve, contributing to a more robust understanding of the adoption and diffusion process for automated driving and similar technologies.

## SIMULATIONS

In the following section, we will unveil the results of simulations that incorporate the **Switching Costs** component into the base Bass model. This component, represented by a specific formula, plays a crucial role in capturing the impact of switching costs within the model.

$$SC(t) = \gamma(brand,\ usabilità) * t + s(settore,\ sunk\ cost)$$



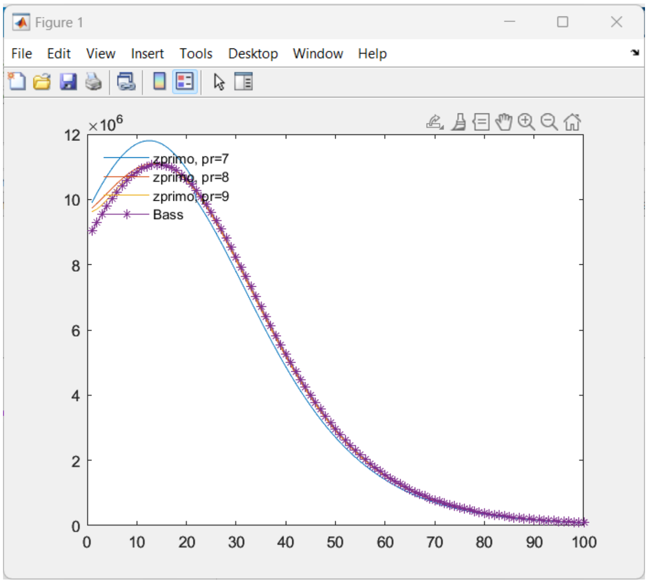

The on right graph depicts the temporal adoption pattern, showcasing the number of adoptions occurring at specific time intervals. It offers insights into the rate of adoption at different stages. The incorporation of switching costs in the model accounts for various influential factors, including sunk costs, technology performance, usability, and brand. Sunk costs increase perceived risk and commitment, dampening adoption likelihood. However, as the new technology outperforms its predecessor, adoption levels rise, indicating that customers are more inclined to adopt when the benefits in terms of usability and brand superiority outweigh the sunk costs.

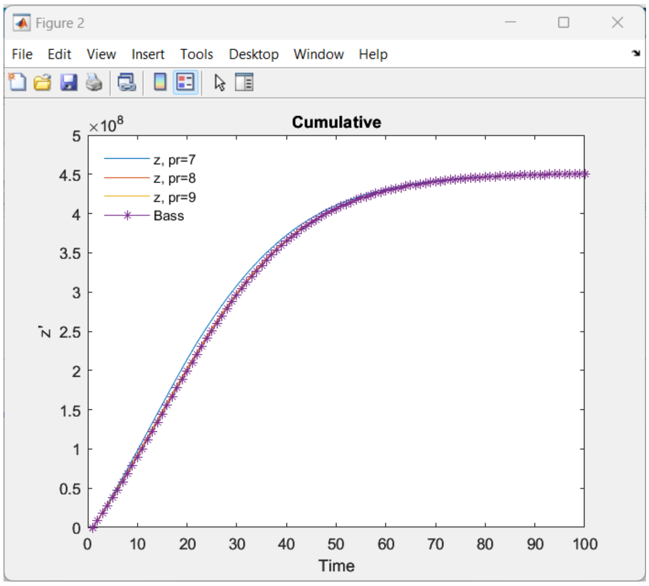
While, other one graph emphasizes cumulative adoptions, providing an assessment of the diffusion level over time. It considers the total number of adoptions up to a specific point, reflecting the overall spread and acceptance of the digital product among the target audience. This graph's adoption curve is shaped by the interplay of sunk costs, switching costs, technology performance, usability, and brand. In the early phase, when the technology is new, sunk costs act as a deterrent due to perceived commitment. At this stage, the most influential factor is the sunk cost effect. As the technology gradually spreads, network effects and the advantages of being part of a larger user base drive adoption. In the later stages, as the market saturates, the adoption curve starts to decline. During this phase, the brand and usability effects become more significant as customers seek differentiation and enhanced user experience.

In the other hand, the showcased results in the upcoming section will revolve around simulations that integrate the **Marketing Mix** component into the foundational Bass model. It is represented by the following formula:

$$x(t, d) = k + d\,[\alpha\, pr^{-\beta_1 t}\, aw^{\beta_2 t}\, av^{\beta_3 t}]$$

$$d = \begin{cases} 1 & if\ \ z(t-1) \leq \dfrac{m}{2} \\ 0 & otherwise \end{cases}$$




As in the case study of smartphones, the graph on the left illustrates the punctual adoption in each time period, displaying the number of adoptions occurring at specific time points. This graph provides insights into the rate of adoption and emphasizes the role of marketing in driving the residual market's adoption during the early stages of diffusion.

To examine the influence of marketing on the adoption process, the impact of marketing levers is considered as an exponential impulse with appropriate parameters that represent the necessary marketing efforts to achieve and sustain results. The effectiveness and efficiency of the marketing levers employed to promote the new technology are demonstrated in this graph. The adoption curve reflects the impact of marketing efforts, showcasing successful reach and engagement with the target market, ultimately contributing to the attainment of critical mass for the product.

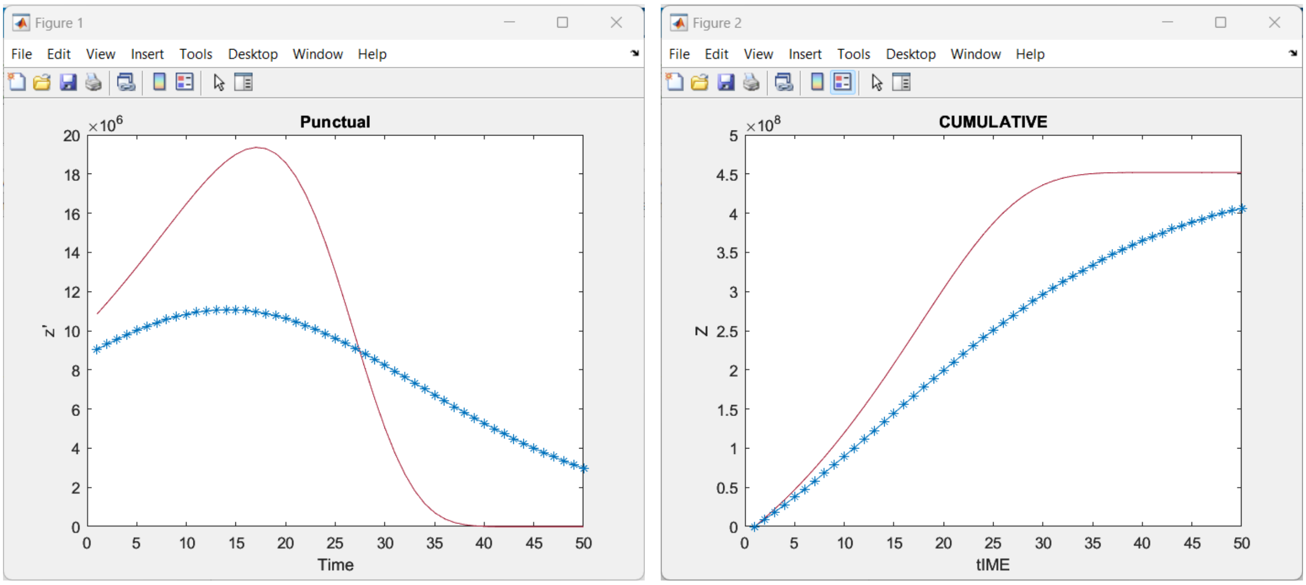
On the right, the graph represents cumulative adoptions, offering a measure of the diffusion level at different time periods. It takes into account the overall number of adoptions that have taken place up to a specific point in time. This graph enables an assessment of the influence of various marketing factors on the diffusion process, specifically highlighting the effects of price, availability, and awareness.

In the case of automated driving, considering a higher price range and lower levels of awareness and availability, the most influential driver would be the price, outweighing the impact of awareness and availability on the adoption dynamics.

The results of the **Product Adaptability** component's impact will be presented next, following the formula that represents it:

$$y(t,d) = 1 + c\ e^{b(t-a)}\ * d_{t\geq a}$$

$$d = \begin{cases} 1 & if\ \ t \geq a \\ 0 & otherwise \end{cases}$$




The graph on the left illustrates the punctual adoption in each time period, highlighting the specific points in time where adoptions occur. This graph facilitates the evaluation of the digital product's ability to adapt to sudden and significant changes in its environment and respond effectively.

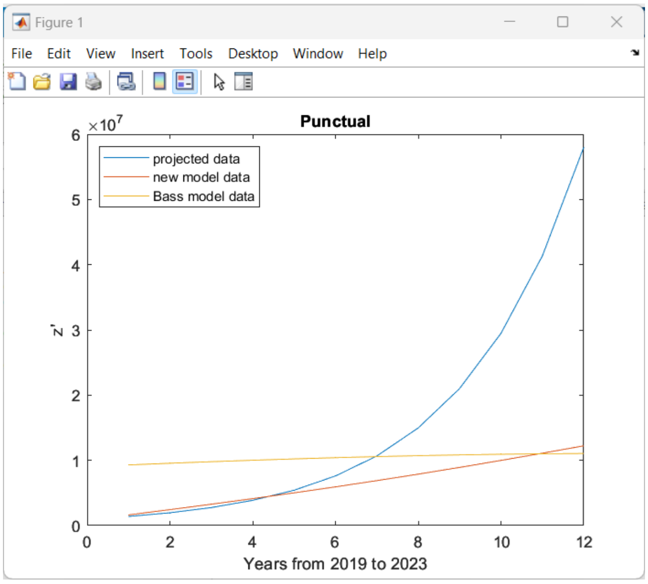
To model the diffusion of technological interventions, an exponential form of the intervention function is employed. This function captures the impact of interventions on the adoption process, reflecting how the digital product adjusts in response to external changes or interventions. In the first graph, the adoption curve demonstrates the diffusion process and the influence of the intervention, which is represented by the product adaptability. Typically, diffusion follows an S-shaped curve, characterized by an initial rapid increase in adoption, followed by a gradual slowing down and eventual plateau. Specifically, the first graph depicts the steep rise in adoption during the period when the intervention of product adaptability is implemented.

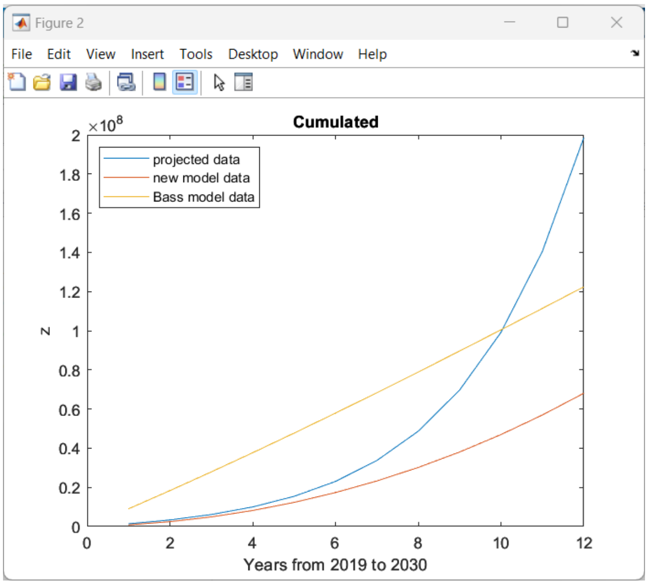
On the right, the graph portrays the cumulative adoptions, providing a measure of the overall diffusion level at each time period. It showcases the cumulative number of adoptions that have taken place up to a specific point in time. This graph enables the assessment of the overall diffusion pattern and the effectiveness of the digital product's response to the intervention of product adaptability.

In the context of the automated driving case study, an assumption is made regarding the occurrence of an intervention. This intervention is envisioned to take place several decades after the initial diffusion, with the purpose of encompassing potential technological advancements and interventions that are not presently taken into account. By considering this extended timeframe, the analysis aims to capture the potential future developments and their potential impact on the adoption and diffusion dynamics of automated driving technologies.

## DATA COMPARISION

In the case of automated driving, where real-world data is not yet available, the models' outputs will be compared to projections based on empirical observations and market trends [27], [28]. Due to the lack of real data, the comparison will be made against projected data, which may have certain limitations. However, this approach allows for an assessment of the models' accuracy and their ability to capture the adoption and diffusion dynamics of automated driving technologies. By analyzing the disparities between the model predictions and the projected data, insights can be gained regarding the limitations of the models and potential areas for improvement. The aim is to refine the models and align them more closely with the complex dynamics of automated driving adoption and diffusion by incorporating factors. In comparing the projected data with the output of both the proposed model and the Bass model for automated driving, it becomes apparent that there are significant differences in their alignment. The projected data shows that the proposed model output is relatively well-aligned with the initial stages of the diffusion process, indicating its ability to capture the early adoption patterns. However, as the diffusion progresses, the projected data diverges from the output of both models. On the other hand, the Bass model output shows a complete lack of alignment with both the projected data and the proposed model output throughout the entire diffusion process. This discrepancy highlights the limitations of the Bass model in accurately representing the complex dynamics of automated driving adoption and diffusion. It emphasizes the need for the proposed model, which takes into account additional factors and complexities, to better capture the real-world patterns and improve the accuracy of future projections.



# CONCLUSIONS

Talking about digital innovation, the processes of diffusion and adoption play crucial roles in shaping the success and impact of digital products. Diffusion refers to the spread and adoption of new technologies, ideas, or products within a particular social system. It involves the transfer of information and knowledge from early adopters to the wider population, gradually influencing their acceptance and usage. Digital products, with their rapid development and evolving capabilities, have become a focal point in the study of diffusion and adoption.

The proposed model aims to understand and predict the diffusion and adoption of digital products. By analyzing data on taking into account three key factors: switching costs, product adaptability, and marketing mix. the model seeks to provide insights into the factors that influence the adoption process. It can help businesses and researchers identify the key determinants that drive or hinder the acceptance of digital products, enabling them to devise effective strategies for product development, marketing, and user engagement.

To validate the proposed model, it was tested using smartphones and automated driving as case studies. While Smartphones represent a widely adopted digital product, automated driving, in the other hand, as an emerging technology, presents unique challenges and opportunities for adoption. So, the simulations provide valuable insights into the impact of various factors on the adoption process, namely the effectiveness of marketing efforts, the influence of product adaptability, and the presence of switching costs.

In particular, the focus on the marketing mix highlights that they are more impactful in the case of smartphones than in the case of automated driving. This is because the ranges in which price, awareness and availability factors are allowed are different for each of the two considered technologies. In fact, the combination of driver ranges shows how the same factors can be relevant or not based on the hypothesis done and on the type of the digital product considered.

In any case, while in smartphones analysis the impact was evident for all the three marketing mix drivers, in the automated driving the impact was more relevant for the price. Instead, the awareness and the availability were less impactful, since the range considered for the price is the highest one, while for the other two drivers is the lowest one. These findings are crucial for developing effective marketing strategies to promote the adoption and diffusion of new technology, ultimately driving the product towards its critical mass and widespread acceptance.

The simulations conducted in response to the intervention of product adaptability allow to understand the resulting dynamics of adoption and diffusion, as well as for informing strategies to optimize the response of the digital product to interventions, these results might help companies in refining their product development strategies to drive successful adoption and maximize market penetration. As a matter of fact, these simulations enable to evaluate the success of adoption strategies.

Switching costs have a significant impact on the adoption of digital products, this is shown in the two study cases: smartphones and automated driving. In fact, by considering smartphones, factors like sunk costs, usability, and brand loyalty create barriers to switching to a different product.

Familiarity with the interface and ecosystem of their device further increases the perceived cost of switching. Loyalty also plays a role, as consumers become attached to the brand.

Similarly, in the context of automated driving, switching costs could arise from investments in knowledge, training, and infrastructure. Usability factors and the learning curve associated with autonomous vehicles add to the cost of switching. Brand reputation and trust impact the adoption of automated driving technology. Taking into account these switching cost components is crucial for researchers studying the adoption process and understanding the rate of adoption, diffusion levels, and the influence of various factors in these domains.

When comparing the output of the proposed model and the widely used Bass model with real-world data, several differences become evident. Both models initially predict a higher number of adopters than what is observed in reality. However, over time, the curve of real data surpasses the predictions of the models. These disparities can be attributed to inherent limitations within the models, such as the assumption of a static target market and the oversight of factors like population growth, changes in market potential, replacement behavior, and multiple device ownership.

To improve the accuracy of these models, it will be essential to incorporate dynamic target market sizes, consider the impact of replacement behavior, and account for the possibility of multiple device ownership. By analyzing real-world data and adjusting model parameters based on empirical observations, we can refine the predictions and align them more closely with the complex dynamics of smartphone adoption and diffusion.

In conclusion, the case study of smartphones provides valuable insights into the transformative impact of these devices on society. The analysis of adoption and diffusion patterns, along with the comparison of model predictions to real-world data, highlights the need for continuous refinement and improvement of adoption models. By considering the dynamic nature of the market and accounting for various influencing factors, it will be possible to enhance the accuracy of these models and better understand the adoption and diffusion dynamics of smartphones.

on the other hand, the case study on automated driving showcases its immense potential to revolutionize transportation and reshape our relationship with vehicles. Furthermore, the introduction of self-driving cars brings about a transformative change in consumer behavior and travel experiences. This implicate that the adoption and implementation of automated driving come with various challenges.

In fact, in the analysis of simulations and data comparison, we gain valuable insights into the adoption and diffusion dynamics of automated driving technologies. While real-world data is not yet available for automated driving, projections based on empirical observations and market trends serve as a benchmark for evaluating the accuracy of the models.

The proposed model incorporates additional factors and complexities with the aim to better capture real-world patterns and enhance the accuracy of future projections. By integrating factors such as product adaptability, marketing mix, and switching costs, the proposed model aims to provide a more comprehensive understanding of the adoption and diffusion dynamics of automated driving technologies.

# BIBLIOGRAPHY


[1] E. M. Rogers, Diffusion of Innovations, 5th ed. Free Press., 2003.

[2] F. M. Bass, A New Product Growth Model for Consumer Durables. Management Science,15(5), 215-227, 1969.

[3] A. C. Bemmaor and J. Lee, "The Impact of Heterogeneity and Ill-Conditioning on Diffusion Model Parameter Estimates," *Marketing Science,* vol. 21, pp. 209-220, 200.

[4] R. Guseo, C. Mortarino and M. A. Darda, "Homogeneous and heterogeneous diffusion models: Algerian natural gas production, Technological Forecasting and Social Change," vol. 90, pp. 366-378, 2015.

[5] M. Guidolin and R. Guseo , "Modelling seasonality in innovation diffusion, Technological Forecasting and Social Change," vol. 86, pp. 33-40, 2014.

[6] F.-J. Molina-Castillo, J.-L. Munuera-Alemán and R. Calantone, "Product Quality and New Product Performance: The Role of Network Externalities and Switching Costs," *Journal of Product Innovation Management,* vol. 28, pp. 915 - 929, 2011.

[7] R. Boehner and S. C. Gold, "MODELING THE IMPACT OF MARKETING MIX ON THE DIFFUSION OF INNOVATION IN THE GENERALIZED BASS MODEL OF FIRM DEMAND," *Developments in Business Simulation and Experiential Learning,* vol. 39, 2012.

[8] J. A. Norton and F. M. Bass, "A Diffusion Theory Model of Adoption and Substitution for Successive Generations of High-Technology Products.," *Management Science,* vol. 33, pp. 1069-1086, 1987.

[9] Z. Jiang and C. J. Dipak, "A Generalized Norton-Bass Model for Multigeneration Diffusion," *Management Science,* vol. 58, no. 10, pp. 1887-1897, 2012.

[10] R. Boehner and S. C. Gold, "The Influence of the Marketing Mix on the Diffusion of Innovation: Bass Model Redux," 2015.

[11] R. Guseo and M. Guidolin, "Market potential dynamics in innovation diffusion: Modelling the synergy between two driving forces, Technological Forecasting and Social Change," vol. 78, pp. 13-24, 2011.

[12] R. Guseo and C. Mortarino, "Sequential market entries and competition modelling in multi-innovation diffusions, European Journal of Operational Research," vol. 216, pp. 658-667, 2012.

[13] L.-Y. Huang and Y.-J. Hsieh, "Consumer electronics acceptance based on innovation attributes," *Electronic Commerce Research and Applications,* vol. 11, pp. 218-228, 2012.

[14] M. Kalantari, Consumers' adoption of wearable technologies: literature review, synthesis, and future research agenda. International Journal of Technology Marketing, 2017.

[15] M. Guidolin, R. Guseo and C. Mortarino, "Regular and promotional sales in new product life cycles: Competition and forecasting, Computers & Industrial Engineering," vol. 130, pp. Pages 250-257, 2019.

[16] p.-y. Chen and C. Forman, , "Switching costs, network effects, and buyer behavior in IT markets.," 2008.

[17] P.-y. Chen and C. Forman, "Network Effects and Switching Costs in the Market for Routers and Switches," vol. Working Papers, no. 03-03, 2003.

[18] M. C. Bolen, "From traditional wristwatch to smartwatch: Understanding the relationship between innovation attributes, switching costs and consumers' switching intention," *Technology in Society,* vol. 63, 2020.

[19] C. Msaed, O. S. Al-Kwifi and A. Zafar, "Building a comprehensive model to investigate factors behind switching intention of high-technology products. Journal of Product and Brand Management," vol. 26, pp. 102-119, 2017.

[20] L. Fan and Y.-h. Suh, "why do users switch to a disruptive technology? An empirical study based on expectation-disconfirmation theory," in *Information & Management* , 2014, pp. Pages 240-248.

[21] P. Nie, C. Wang, Y. Chen and Y. Yang, "Effects of switching costs on innovative investment. Technological and Economic Development of Economy," vol. 24, pp. 933-949, 2018.

[22] M. Cantamessa and F. Montagna, "Management of Innovation and Product Development, Integrating Business and Technological Perspectives," Springer-Verlag London 2016, p. Chapter 13.

[23] F. Lissoni, "La diffusione delle innovazioni," in *Economia dell'innovazione*, Carocci, 2000, p. DISPENSA 9.

[24] Y. Orbach, "Parametric analysis of the Bass model.," *"Innovative Marketing (hybrid)",* vol. 12, 2016.

[25] study_id10490_smartphones-statista-dossier.pdf.

[26] J. MASSIANI and A. GOHS, "The choice of Bass model coefficients to forecast diffusion for innovative products: An empirical investigation for new automotive technologies," *Research in Transportation Economics,* vol. 50, pp. 17-28, 2015.

[27] study_id69417_autonomous-vehicles.pdf.

[28] study_id28221_driverless-cars-statista-dossier.pdf.

[29] N. Kumar, "Review of Innovation Diffusion Models," 2015.