POLITECNICO DI TORINO

Dipartimento di Ingegneria Strutturale, Edile e Geotecnica

Master of Science in Civil Engineering

Master's degree thesis

Structural optimization by solving CSP via metaheuristic algorithm.

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APRIL 2023 - A.Y. 2022/2023

Alle persone che amo,
che ho amato e che amerò.

Abstract

The Edisonian method is the initial method, which is still often used by most structural design professionals. Instead of following a precise theoretical procedure, the Edisonian method of reaching the optimum solution is characterized by trial and error discovery. The design iteration process was automated as a result of the need for a more effective design approach, which brings the structure closer to almost the optimum in less time. The techniques used to automatically arrive at the optimal answer are known as structural optimization (SO) techniques. This method at first used Hard Computing (HC) approaches based on rigorous mathematical, sequential and analytical models. Then, more fast algorithms are used called metaheuristics algorithms, these techniques don't reach an exact optimum but the near-best solution with a tolerable imprecision. In the past decades, several SO was developed which have different goals such as cost minimization, Structural performance improvement and environmental impact minimization. In particular, in the first half of the thesis, a review of the scientific literature concerning the various approaches to SO related to the minimization of the cost is made.

Afterwards, at the core of this study, a new optimization method to design simple steel truss structures for the evaluation of the optimal stock of existing elements is introduced. To achieve this goal, the well-known bin packing problem (BPP) will be implemented within the structural optimization procedure. The (one-dimensional) BPP has been studied since the sixties by Kantorovich (1960). Specifically, among all the variants and generalizations of the BPP, one of the most common applications in real-world cases is the Cutting Stock Problem (CSP) in which the objective is to produce d_j copies of each item type j (i.e. to cut/pack them) by employing the minimum number of bins (frequently called rolls) so that the total weight in any bin does not exceed the capacity. In the civil engineering field, structural optimization

is often employed aiming to improve the load-bearing capacity and the global performance of the structure itself. This includes, for instance, the maximization of the performance ratio through the minimization of the structure weight. However, this goal does not guarantee maximum efficiency in terms of structural elements reusing and minimization of waste during the industrial production phase. To overcome these limits, authors propose a stock-constrained structural optimization in which a heuristic search technique is adopted and the best arrangement of bars whit the lowest cut-off waste is obtained for a 10-bar-truss and Warren truss (studied in two grouping strategies) case studies. For completeness reasons, a comparison between the solution obtained by the classic minimum weight optimization problem and the stock-constrained one is discussed.

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Chapter 1

Introduction

The first way, which is still commonly employed by most structural design specialists, is the Edisonian approach. The Edisonian approach to the invention is defined by trial and error discovery rather than a systematic theoretical process. Edison employed this method to create the incandescent light bulb despite having no understanding of the electrical resistance of the materials. The distinction between Edison and today's engineers is that they frequently have substantial technical knowledge and skill in their field. This allows them to generate solid first ideas based on intuition before analyzing them using collapse criteria or design guidelines. Design problems uncovered during testing are then addressed using the so-called make-or-break technique. Advances in computer science enable the creation of software capable of creating virtual prototypes that imitate the structural behaviour of the building and allow the evaluation of the structural performances of the buildings digitally. Despite this significant advancement that results in an Edisonian computer-aided approach, there are two major drawbacks: the manual change of the computer model each time until the appropriate structural performances are achieved, and the fact that the design is strongly bonded with the limited human intuition. The end result is frequently a far-optimal solution that is a design of the team's total experience.[52]

This necessity to have a more efficient design procedure, which means structure much closer to the optimum and in less time, led to automating the design iteration procedure. The methods used for reaching the best solution in an automated way are called Optimization methods. The act of attaining the optimal result under given conditions is known as optimization. The ultimate objective of all such judgments is to either reduce the amount of work necessary or maximize the desired reward. Because the effort or benefit wanted in each practical scenario may be described as a function of specific choice factors, optimization can be defined as the process of determining the circumstances that provide the highest or least value of a function. [66]

Optimization methods have been around since Newton, Lagrange, and Cauchy. Newton and Leibnitz's contributions to calculus paved the way for the creation of differential calculus approaches to optimization. The calculus of variations, which deals with the minimizing of a function, was founded by Bernoulli, Euler, Lagrange, and Weierstrass. To have an actual application of these methods it needs to waiting the mid-twentieth century when digital computers enabled the application of optimization algorithms and motivated further research, which has as result a lot of new optimization techniques. [66]

A significant first classification of the optimization methodologies is between Hard Computing techniques and Soft Computing (SC) techniques. Hard Computing procedures are the ones that are based on rigorous mathematical, sequential and analytical models. That methods give precise and exact results but have a very high computational cost and not works for non-derivative functions. Some Hard Computing techniques are ones related to Lagrange's multiplier and Operational Research. Soft Computing, on the other hand, is made up of two technological disciplines: approximation reasoning and randomized search. Approximate reasoning is a collection of knowledge-driven procedures that trade soundness or completeness in order to obtain considerable reasoning speedup. Furthermore, randomized search is a family of numerical optimization methods that function by iteratively advancing in the search space towards better positions, which are sampled from a hypersphere around the present position. The SC methods don't reach an exact optimum but the near-best solution with a tolerable imprecision. Examples of SC procedures are all the metaheuristic algorithm that imitate some natural pattern as the swarm behaviour or the Darwinian Evolutionary selection, the Artificial Neural Network (ANN) that was inspired by the brain function and Fuzzy Logic that use degrees of truth instead of the binary "true" and "false" of Boolean logic. For completeness, it is proper to cite that there are also some hybrid approaches that join the Hard

Computing techniques and the Soft Computing ones. [26]
The basic formulation of an optimization problem is as follows:

$$min f(x) \tag{1.1}$$

Where $f(x)$ is the objective function that should be minimised to reach the optimum, for example in civil engineering can be the cost or the weight. Meanwhile, x is the set of design variables that can be discrete if assumes a finite number of isolated values or continuous if assumes an infinite number of different values.
The problem is always subjected to a certain number of constraints:

$$\begin{aligned} g_j &\leq 0, \quad j = 1, ..., n \\ l_j &= 0, \quad j = 1, ..., p \end{aligned} \tag{1.2}$$

$g_j(X)$ and $l_j(X)$ are known as inequality and equality constraints, respectively. These constraints represent the domain limits of the objective function. For example, civil engineering can be the Structural performance requirements according to the technical regulations.
Optimization problems can have one or more objective functions, and in this case, the optimization problem is called single-objective (SO) and multi-objective (MO) optimization, respectively. While SO problems give a single result that is the minimum of the entire function, for MOs the resolution is more complicated. The main problem is that the different objective functions usually conflict, which means that as one function is minimized, the other can increase and vice versa. For solving this kind of problem in a simple way basically, there are two methods which are Pareto and scalarization. In the Pareto method, there is a dominated solution and a non-dominated solution obtained by a continuously updated algorithm. The Pareto front gives a set of optimal solutions that are non-dominating in nature. Meanwhile, the scalarization approach uses weights to combine multi-objective functions into a single solution. [34]
The thesis is articulated in 4 chapters excluded the current introduction. The first one makes a detailed review of the literature on cost structural optimization in the

last decades. The second one exhibits the mathematical formulation of the cutting stock problem according to the column generation procedure. The third one is the core of the thesis and treats the structural optimization via CSP developed in this thesis. At the end of this chapter three case studies are proposed as a test to endorse the CSP optimization (10 bar truss, Symmetric Warren and 4 cross-sections Warren). Finally, the last chapter collects the conclusion and the future works of the research in this field.

Chapter 2

Cost Optimization Procedures Review

The optimization methods have a wide application in many fields like informatics, medicine, economy, etc. Among these, civil engineering represents one of the latest branches of engineering in which optimization procedures have been successfully experienced. Several aspects of the design process were considered within the optimization process as the maximization or minimization of single or multi-objective functions expressed in terms of structural performances, environment and economics. In this chapter, an overview of the most promising techniques adopted for the cost optimization of steel structures.

The various optimization methodologies can be implemented at three different levels correlated with the design variables involved in the process. Topological optimization, size optimization, and shape optimization are three kinds of structural optimization issues that can be performed separately or integrated into the same procedure at the same time (depicted in the figure 2.1.

Topological optimization aims to optimize the structural layout within a given design space, for a given set of loads and boundary conditions, and with the best possible system performance. It can be thought of as a procedure for optimizing the topological arrangement of material in the design domain, eliminating unnecessary material volume. It can be obtained by managing, in a different way, the connectivity matrix of the nodes composing the structure or by modifying either the connection stiffness value and/or the type of the connection scheme. Each of the mentioned-above approaches aims to detect the optimal load path and, generally, the most efficient stress distribution among the bars.

Shape optimization is concerned with optimizing the overall shape or contour of a fixed-topology structural system. The design variables are either some of the coordinates of the important points in the structure's border or other characteristics that affect the outer structure's shape.

Sizing optimization is aimed at optimizing geometrical parameters, such as the width or thickness of members in a structural system whose topology and shape are fixed. The dimensions or the areas of the cross-sections of the structural parts are typically used as design variables. [26][52]

Although the first optimization problems were entirely formulated to achieve efficient and safe structural systems, already at the beginning of the century, researchers started to appreciate the potentiality of cost-optimized procedures as a powerful tool to reduce the cost during the production, construction and assembly phase.

In the first applications, the economic cost is indirectly evaluated as the amount of steel employed for the construction (e.g. [16],[85],[45], [38]). This rudimentary approach is based on the link between the structural aspect (e.g. material saving) and the economical convenience (e.g. cost saving) of the structural proposal (e.g. [28], [6]), [18], [58]). In this respect, the authors supposed that performing a minimization of the weight (or volume) results in a minimization of the employed material cost also in special structure (i.e. steel structure for waste incineration steam generator [17] and steel truss arches strengthening in prestressed girder bridges [21]). The weight optimization correlated with cost was used also as a way to take into account the cost in a more wide structural performance optimization. For instance, this kind of resolution was implemented for coupling the material minimization with the improvement of seismic performances. The seismic performances were taken into consideration in terms of uniform ductility demand in all stories by controlling the inter-story drift (e.g. [88], [33]). Other papers treat the seismic performances of buckling restrained braces (BRB) structural systems (e.g. [67], [68], [27]) or seismic shear walls (e.g. [71]).

Specifically, two studies are relevant in this sense despite they did not take into account a defined cost function. The first study of Liu et al. in [56] lead not only

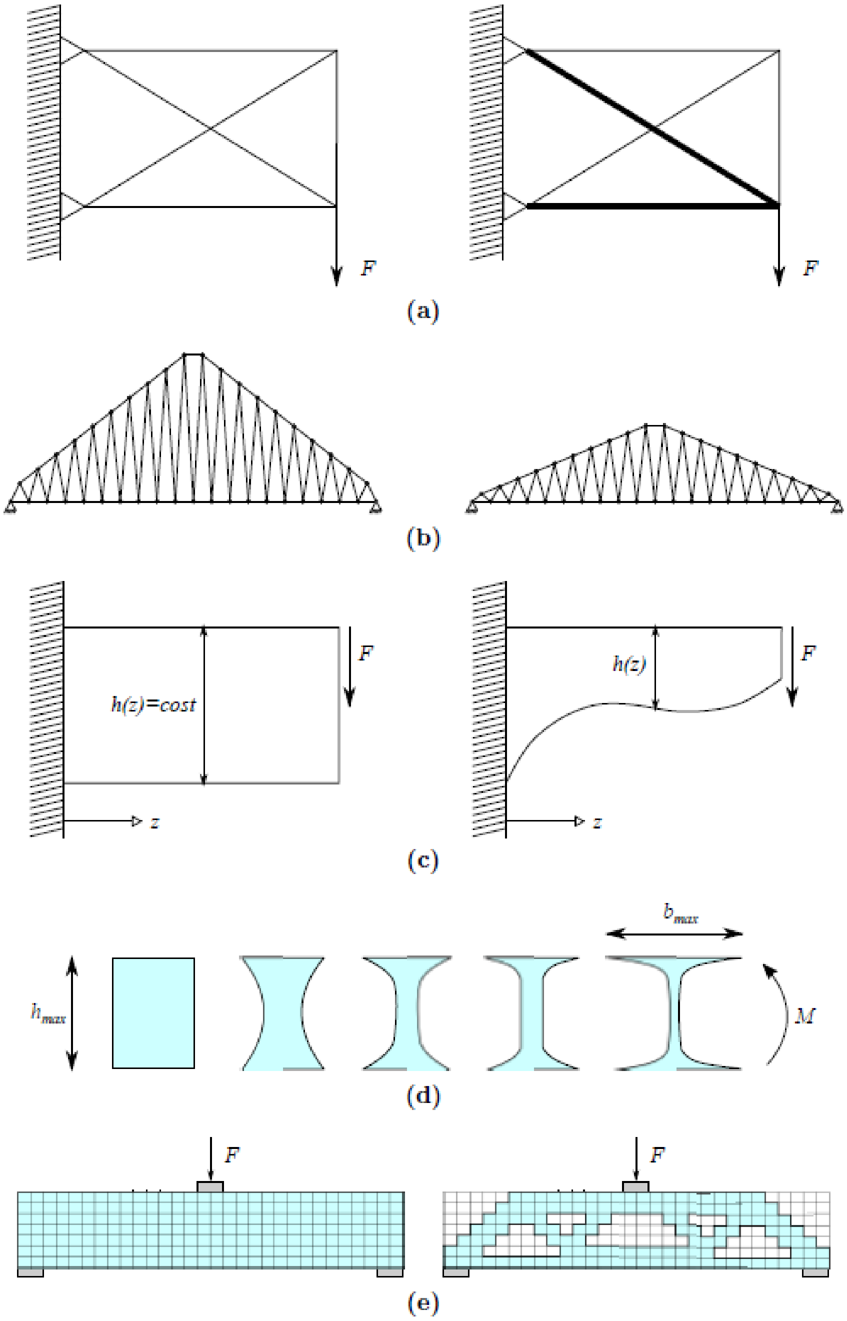

Figure 2.1: On the left column, starting design situations are presented; on the right column, optimized designs are depicted. (a) Size optimization; (b) Shape optimization of a Warren truss beam; (c) Shape optimization with a parametric function h(z); (d) Continuum Topology optimization of a rectangular cross section under a bending moment in the elastic stage; (e) Topology optimization on a discrete grid (image from [69]).

to provide a design based on the structural strength but also by taking into account the global robustness of the structure subjected to seismic actions. To achieve this goal, the sensibility of the problem is affected by “hard-to-control” input parameters (called “noise factors”) for the evaluation of the structural robustness. In order to overcome this limit, the authors adopted carefully adjusting “easy-to-control” procedures with suitable input parameters (e.g. a maximum inter-story drift). For simplicity, only the ground motion variability was used as a noise factor of the robust design. The problem was structured as a multi-objective optimization with three objective functions: the weight of the structure correlated with the total cost of the structure, the mean value of the maximum inter-story drift considered for seismic safety measure and, at least, the standard deviation of the maximum inter-story drift that was linked with the robustness. The second, in spite of the use of only the weight as a cost indicator, shows a different approach with respect to the other optimization. In this study ([25]), Drewniok et al. developed a tool called Lightest Beam Method (LBM) studied for non-composite universal beam (UB) members in a structure. The aim of the study was to show how this tool works and its effectiveness in economical and environmental terms. The method works by computing the SLS requirement according to the loading state in terms of deflection and natural frequency and by choosing the lightest standard element that fulfils the structural requirements. In compliance with European design requirements, the LBM selects the lightest beam from a library of UBs including the "Blue Book” of the Steel Construction Institute (SCI). LBM enables the user to determine steel savings given assumed topology and geometry, and so structural engineers may utilize it as an accessible form of individual member optimization. The described method, in comparison to the others, was not a stochastic search of the optimum but a comparison of the whole cross-section list with the structural requirements, and afterwards among the cross-section that fulfils the constraints the lightest was chosen. Using the Eurocodes to select the lightest part, we can save 26.5% of steel by mass, with half of the beams determined by serviceability limit states (SLS). When deflection is calculated using variable loads, the fraction of beams regulated by the SLS falls to 31.1%, resulting in an additional 2.2% mass reduction. Steel savings of 34.5% may be obtained by adopting lower natural frequency assumptions (3 Hz) and the average steel yield strength rather than the characteristic steel yield strength. Based on case studies analysis, it was discovered that 1/3 of the steel in the frames could have been saved, representing 36% of the original embodied carbon or 5% of the building's whole-life carbon over 60 years.

Successively, an actual but elementary cost function was created by multiplying the weight by the unitary material cost. After that, other components were directly implemented in the cost objective function as connection, fabrication, transportation and erection costs [60]. The new cost function called *total initial cost*, results to be more representative of the effective price of the entire structure and an increasing level of accuracy in the evaluation of each detailed cost item can be obtained [4]. Further developments in cost optimization were to evaluate the cost along the entire service life of the structure by including the maintenance, demolition and disposal *Life Cycle Cost* (LCC). The procedure called *Life Cycle Assessment* (LCA) was used to evaluate not only the cost but also the structure's environmental impact for the entire duration of the construction service.[73]

The following sections aim to compose a review of different cost optimization problems over the years. All the scientific papers are organized according to the specific main class of optimization performed by the authors (e.g. size, shape and topology). For the sake of completeness, optimization conducted when size, shape and topology act simultaneously have been included in the bibliography analysis.

2.1 Size optimization

The most common typology of optimization was the one that takes into consideration only the cross-sectional size of each element. The design variables in these problems are generally the dimensions of the structural elements and/or their cross-sectional mechanical properties (e.g. total area, flexural inertia etc.). As described before, while the weight of a steel building is a crucial component of the total cost, cost reduction should be the ultimate goal for making the best use of available resources.

The first development in this direction consists in multiply the weight or the volume of each member by the unitary cost of material such that an actual cost function in monetary terms is derived. Undoubtedly, the outcomes obtained by this simplified approach represent a crucial part of the overall cost of construction and they can be adopted in a preliminary estimation phase. However, it is not sufficient when a real estimation is required. Specific production procedures or activities realized on the construction site may represent up to 30% of the total cost. The main problem associated with using a unit cost for materials, fabrication, assembly and erection is that the various procedures are closely related to local market prices. That issue entails the difficulty to translate these methods into different geographical and economical regions.

To overcome this problem Jamai et al. in [40] propose a solution for considering the fabrication cost, with specific regard to the role of welding as a percentage of the material cost. This method computes the fabrication cost as a specific cost ratio of k_f/k_m, in a range between 0 and 2kg/min, where k_f and k_m represent the fabrication and the material cost respectively. In this way, it is possible to adapt the minimization of the cost function, as directly dependents on the time needed to accomplish that specific work in different economic conditions. The paper advances some extension of this method for other fabrication costs like flattening plates, surface preparation, cutting and edge grinding.

Another study that considers a very detailed cost function was developed by Pavlovcic et al. in [60] that considers both manufacturing and material costs. The cost function was a sum of the prices due to steel elements, welding, cutting, painting, surface preparation, flange aligning, joints, transportation and erection based on the Slovenian market. The variables, handled as discrete variables, used in the computation were related to the dimension of the cross-section (i.e. the H section, the height of the section, the length of the flange, the web and the flange thickness).

In particular, an approach to consider the welding cost in the objective function was developed in [42] where the welding cost was the sum of the factory welding cost (welding that was done in the factory before the installation on site) and the site welding cost (welding that was done directly in construction site). For the computation of the welding cost with different through heights, it was necessary to convert all the fillets into standard 6 mm welding fillets. Specifically, each welding cost is computed by multiplying the unit length price of a 6 mm fillet weld by each length which is converted to a 6 mm weld considering the conversion ratio of the welding. For allowing a correct design of the welding joint constructability constraints were added to guarantee the feasibility of the connection.

Another interesting application of the size optimization was for the Modular building system (MSB) like in [29]. The paper performs a comparative study between three types of cross-sections that are the lipped channel, folded flange and super-sigma. In this case, the size optimization has the function to find the best solution for each member typology (lipped channel, folded flange and super-sigma) in order to make the results comparable and to have a reliable comparison between the cross-section types. For the reason exhibited previously a size optimization was performed on the elements cross-section of each typology in order to obtain the best solution between the three different element typologies in analysis

Even if the considerations of the fabrication and erection cost were a great improvement in the cost optimization and allow to achieve more suitable results this approach did not take into account other important factors which were introduced by Sarma et al. (e.g. [74]), through the concept of life cycle cost (LCC) where the maintenance cost was evaluated. This optimization was based on four criteria which are the selection of the discrete commercially available sections with the lowest cost, the selection of the standard section with the lightest weight, the search for the minimum number of different types of commercially available sections and, finally, the selection of the section with the minimum total perimeter length. Those four criteria guarantee not only a low initial cost but also a low maintenance cost since, for instance, the least perimeter of the element cross-section assures a low painting cost during the building life.

LCC analysis was also widely used to evaluate the cheapest solution when seismic forces are applied to the structure. Specifically, the seismic damage cost and repair cost has been evaluated in different manners with the expression of the total cost function as a single-objective function or through a multi-objective function where conflicting target functions have been simultaneously optimized.

To achieve this goal, the single-objective function, in general, is formulated as the sum of the initial cost function and new contributions related to seismic risk. Li, Jiang et al.([54]) developed an objective cost function which is the summation of the initial material cost and the future expected damage loss. In this paper, the initial cost was simply computed as the addition of the columns and beams cost. Meanwhile, the damage loss was calculated as a function of the inter-story index evaluated as proportional to the inter-story drift obtained with the modified pushover analysis

according to the Chinese regulation and expressed by the fuzzy decision theory.
On the other hand, Sarcheshmehpou et al.([72]) substituted the future expected loss with an LCC. To achieve this goal three different approaches were performed: code-based design, cost-based design and fixed-weight design. The code-based approach aims at minimizing the construction cost of the building while two other approaches result in a design with the least cost in the lifetime of the building. Finally, the fixed weight approach redistributes the total structural material of the code-based design to attain a new design with less LCC. For the dynamic analysis and the computation life cycle, the Endurance Time (ET) method was performed.
Moreover, for the computation of the LCC, Ghaderi ([30]) proposed the use of Wen and Kang's formulation applied to multiple damage states. For each damage state, the exceedance cost is determined as a percentage of the initial cost.
As mentioned above, there are also multi-objective approaches to solving the LCC optimization. Liu et al. ([55]), for instance, used as objective functions the initial construction cost, the degree of complexity of the structure in terms of the number of different standard steel section types and the lifetime seismic damage cost calculated with a confidence level of limit state probabilities. Each confidence level has a drift ratio limit that was the constraint of the cost damage objective function. While the initial cost was simply the weight of the structure multiplied by the unitary cost, the damage cost function, for each damage state, was formulated as the sum between the direct structural and non-structural damage and repairing cost, cost due to loss of contents, relocation cost, direct and indirect economic loss, human injury cost and human fatality cost.
Conversely, Saadat et al. ([70]) implemented a multi-objective optimization problem where the two objective functions were the initial construction cost associated with the weight of the building and the expected annual loss (EAL) considering direct economic losses. Seismic performance and loss estimation of a structure can be organized into four steps: probabilistic seismic hazard analysis (PSHA), probabilistic seismic demand analysis (structural analysis), probabilistic capacity analysis and probabilistic loss analysis. PSHA's purpose is to quantify the uncertainties in the location, magnitude, and resultant shaking intensity of a hypothetical future earthquake at a specific site and integrate the data to generate an explicit description of the distribution of future shaking at that site. The second phase in a structure's

loss assessment procedure is to find the optimal engineering demand parameters (EDPs) that best represent its reaction at various hazard levels. The EDPs acquired through a nonlinear time-history analysis were the inter-story drift and peak floor acceleration, which were utilized to calculate the building's direct economic loss. Afterwards, the probabilistic capacity analysis provides the fragility curve of the structure which is the distribution of the probability of damage exceedance for given EDPs. The last point was the probabilistic loss analysis that aimed to convert the damage estimate in the previous point into direct dollar losses, downtime (or restoration time) and deaths.

Another way to evaluate seismic damage is to implement in the multi-objective optimization problem, in addition to the initial cost, the overall damage index (ODI) that quantifies structural damage during an earthquake according to Gholizade and Fattahi ([31]).

This approach require an huge computational effort due to the complexity of the seismic analysis like a pushover, time-history analysis end direct dynamic integration of ground motion. To achieve time competitive techniques, Kaveh et al. ([48]) developed a multi-objective problem where the two objective functions were the initial cost of the structure simply computed as the weight function (linked with the unit initial cost of the building) and the life cycle cost computed with the Wen and Kang's formula as a percentage of the initial cost. The importance of this research lies in the advances implemented into the optimization algorithm that was based on a non-dominated sorting genetic algorithm (NSGA-II) with the aid of a specific meta-model utilized for reducing the number of fitness function evaluations. The meta-model was a neural network, which can be trained and utilized for predicting the response of a function. In this study, a radial basis function (RBF) network is constructed and trained for predicting the response of the considered solution in the optimization process.

When the buildings are subjected to horizontal loads, the traditional approach developed by researchers aimed to estimate the fitness of the objective function by evaluating the violation of seismic constraints according to the various technical regulations. However, several authors demonstrated that hybrid optimization techniques for cost and the seismic performances evaluation must be preferred in order to improve the accuracy of the optimal solution and the computational time too.

Following this trail, Tu et al. in [84] created a multi-objective collaborative optimal design procedure for steel frames equipped with buckling tension bracing (BRB) to minimize seismic damage and material cost. For this purpose, two objective functions were built in order to create a Pareto front. The first one was simply the material cost, while the second objective function taken the maximum energy dissipation ratio referred to a story among all stories, where the dissipation coefficient was evaluated as the ratio of the hysteretic energy dissipated by the BRB at the level of the retrofitted story and the hysteretic energy dissipated by the entire frame (columns, beams and BRB system) at the level of the same story.
Moreover, in 2020 Barg et al. in [9], for the evaluation of the inter-story drift limitation and the optimum seismic design, used a multi-objective solution where in addition to a material cost function they added the displacement participation factor (DPF) as a second objective function. The DPF was a function that is the virtual internal work. The loaded structure is subjected to a unit virtual force applied in the direction and position of the drift limit of interest. For these hypothetical truss elements, the virtual internal work (or displacement participation factor, DPF) of each element is calculated using a formula derived from the virtual work principle, where all stresses and strains are assumed to be constant throughout the length of the element. As a result, without reanalyzing the structure, we can compute the influence on any global drift for all scaling alternatives.
And again, regarding the performance optimization coupled with an economic design, Xu et al. in [87] performed a size optimization for a supertall structure subjected to wind load. This study first addresses the optimization formulation of a complex structure system which includes concrete-filled steel tube (CFST) frame members and shear wall members. The objective function of the problem was a cost function. The cost function was simply the volume of the structural elements multiplied by the unit volume material cost of concrete and steel. The objective function was expressed in terms of the problem design variables that are linked with the size of the element (thickness of the shear wall and external diameter and thickness of the CFST element). The optimization method was tested on a real-life structural system: the Guangzhou West Tower. The results, regarding this structure, reveal that the overall material cost consistently converged before 9 design cycles and a significative decrease of 20.56% from 136.8 to 108.7 million yuan was

obtained. Meanwhile, wind-induced reactions such as displacement response at the top of the structure, inter-story drift response, and acceleration response drastically decreased.

2.2 Shape optimization

The goal of shape or geometry optimization is to obtain an ideal shape by defining certain critical points whose motions condition the structure's overall shape with a given size, topology and/or some set boundary conditions. In the case of a truss construction, for example, the key points are the nodes where the members converge. When a node is moved, all the linked elements must change their length and inclination, affecting the overall geometry of the structure. This type of optimization is not used alone in cost optimization but is often implemented as an aid to cross-sectional size optimization (which is more efficient for the minimization of the material cost) and topological optimization.

Size and Shape optimization As mentioned above shape optimization is mostly used as an improvement of the already efficient size optimization.
Two studies were developed by Aydin et al. in [8] and [7] where was applied combined shape and size optimization on the prestressed steel trusses. Since the raw material cost of the steel structure is higher than that of the reinforced concrete structure, to reduce the material used in the steel structure, an external prestressing system applied to the steel truss was developed. This particular process makes it possible to reduce the amount of steel used in construction. While the first paper is referred only to Warren's prestressed truss, the second investigated a more generic problem in which the objective function was a cost function that summed up the price contribution of steel elements and tendons by multiplying the weight with the unitary cost of the steel and prestressed tendons respectively. Size variables are the cross-sections of the truss system's bars and deviators, while shape variables are the height of the beam and the eccentricity of the prestressing tendon. The study used both discrete and continuous optimization to perform size and shape optimization respectively. The goal of these studies is to develop the lowest-cost structure while taking into account strength, stability, and displacement limits under various load-

ing conditions and prestressed system losses.
As observed in [7], prestressing can cut the overall cost of steel truss beams by up to 25%. Based on the material weight utilized, the reduction rate can reach up to 28%. Furthermore, it was proved that up to 12% more economical results could be achieved without increasing the total beam height.
Another relevant work, in the field of truss optimization, was done by Tiainen et al. in 2017 [81], in which different size-shape Warren truss configurations were obtained through a parametric analysis. Furthermore, steel grades were adopted as design variables of the optimization problem. For this purpose, two types of Warren trusses were investigated with and without vertical members. A cost and separately a size and shape optimization was implemented in order to have comparable trusses for each steel grade (from S355 to S960). The design variables were the truss height, the locations of joints, the gap width at the joints (continuous variables) and the member sections (from a catalogue of cold-formed square tubes as discrete variables). The constraints were derived from the Eurocodes and consisted of strength, buckling, serviceability and geometrical limitations. Two different optimizations were performed: at first a traditional weight minimization was performed; Secondly, a more complex cost function was developed by considering material cost, member cutting cost, member welding cost and truss painting cost. The results analysis indicate considerable weight savings when utilizing high-strength steel (HSS), up to 50% for S960. For the larger of the two load values examined, the cost reduction was smaller, but still around 20 percent
One of the most comprehensive and complex size-shape optimizations was advanced by De Santana Gomez et al. in [23]. This article covers the difficulties of addressing real-world structural optimization issues while considering the repercussions of failure. The optimization consists of a risk optimization problem that was implemented by adding the expected cost of failure to the other terms that constitute the total life-cycle expected to cost. The terms of the total expected cost function were the manufacturing cost, the operation cost, the inspection and maintenance cost, the disposal cost and finally the expected cost of failure. The expected cost of failure was computed as the sum of the expected cost for each failure mode. For each failure mode, the expected cost of failure was given as the product of failure cost by the failure probability. To decrease the computational cost due to the FEM analysis the

ANN meta-models were implemented. Another way to implement a shape optimization on a simple size optimization is to make some holes in the elements in such a way as to reduce the mass of the members. These kinds of elements are called castellated beams. The evaluation of the fabrication cost became a crucial item in representing floor system (which are the set of structural elements that supports a floor) with castellated beams where the cutting, welding and, generally, assembly procedures represent an important component of the total price as evidenced by Kaveh et al. in [46]. The castellated beams are perforated elements that allow the improvement of the structural performances by using the same amount of material as in the normal full beams. The best options are the hexagonal emptying function that permits to have zero waste of material. The formulation of the problem was equipped with an advanced cost function constituted by the material and fabrication price of floor system constituents. The material cost depends on the weight of the steel beam or reinforced concrete slab while the fabrication is directly related to the length of the manufacturing operation (cutting, welding, etc. . .). The entire problem was broken down into multiple sub-problems that are treated independently which are the composite deck-slab optimization and composite castellated beams optimization. The variables were related to the section dimension of the beam, holes characteristics of the castellated beams, concrete thickness and the other deck slab geometric value.

2.3 Topology optimization

Topology optimization not only seeks the optimal spatial arrangement of structural elements within a given domain that affects structural layout, but it also examines how the available material might be structured to achieve the greatest structural performance. For instance, the purpose of topology optimization for a fixed span is to specify the type of truss to utilize for the specified loads and boundary conditions. Another way to perform topology optimization in continuous design space is to remove the less stressed material while adding material in the more strained locations. Actually, topology optimization is not simply the change in the arrangement of the elements but also every modification that varies the internal load path in the structure. Therefore, the change in the connection typology (pinned or fixed joint) and stiffness (semi-rigid connection) should be considered as a specific case of topology optimization since variations of the stress distribution occur in the structures. As demonstrated by the researches reported in the future section, topology optimization results in a refined optimal strategy in which novel efficient load paths can be recognized. Subsequently, though size optimization brings a simple reduction of the overall weight, changing the topology of the structures permits to achieve of innovative arrangements with a higher cost-saving impact. For these reasons, once a preliminary weight cutting is performed by using size optimization, topology can be adopted as a second-level strategy to achieve a more refined solution. This aspect is demonstrated by the fact that any pure topology optimization has been recognized in the literature. Most frequently, authors preferred a hybrid optimization by coupling size and topology or size, shape and topology.

Size and Topology optimization Without any doubt, coupled size and topology represent the most common optimization adopted by researchers due to their adaptability to different engineering problems. Specifically, this approach is chosen when the topology of the structures is changed by varying the properties connections at the level of each node. The most relevant and used topological improvement to a simple size optimization was to consider the connection inside the original problem. In general, the connections to be considered in optimization can be the joint between the columns and beams (called simply connection in the following) and the column base connection that are the connection between the columns and the foundation (called column base connections).
Historically, Simoes et al. in 1996 introduced the concept of the semi-rigid connection ([78]). The authors noticed that traditional approaches to steel frame design ignore connection behaviour. Instead, the idealizations pinned and totally rigid were utilized. Although these models make analysis and design methods easier, the expected frame response may not be practical. In real-world structures, most connections are designed in order to transfer moments and rotations, which can contribute significantly to the final stress and displacement distribution within the structure. The term semi-rigid is widely used to describe the intermediate behaviour of stiffness connections between extreme cases (double pinned, double fixed or pinned-fixed joints). In this way, a more reliable prediction of frame behaviour can be achieved

by optimizing the mechanical properties of connections aiming to investigate how these changes affect the overall structural behaviour.

To involve economic consideration taking into account trivial details related to connections, Simoes et al. modelled the semi-rigid connections like springs with a certain stiffness. The objective function was composed of the members' weight and the cost of the connection which is the product between the fixity factor (usually chosen into the range $[0, 1]$ and representative of the stiffness grade of the connection) and a cost coefficient. In this way, the cost associated to a semi-rigid connection is evaluated as an extra cost to be summed to the base cost of a generic connection which is assumed be equal to a pinned constraint scheme.

Further improvement in semi-rigid connection optimization was accomplished by Hayalioglu et al. ([37]). In this study, the design of the connections was modelled with both Frye and Morris polynomial model and linear spring model for the standard connection and column base respectively. In particular, the cost function was given by the sum of the cost of the elements (that are simply the weight of the member multiplied by the cost for unit weight), the cost of the beam-column connection and the column base connection. As discrete design variables, a set of available steel sections (AISC wide flange forms) is employed. At each iteration, either the connection cost and column base, are upgraded proportionally to the rigidity of the joint stiffness through some cost coefficients. Different coefficients were employed for column-beam and column-base connections.

Moreover, another procedure in which the semi-rigid connection was included in the design was developed by Truong et al. ([83]). The suggested approach for optimizing semi-rigid steel frames was built in such a way that the cost function considers both the steel frame weight and semi-rigid joints, while structural constraints as member stresses and nodal displacements and/or inter-story drifts were adopted. If the topology of the structure was changed by optimizing the stiffness values of the semi-rigid connections, the sizing of each member was performed by choosing the optimal cross-sectional areas. While the members' (columns and beams) cost was simply the weight for the unitary cost, the complexity of the connection was evaluated by adopting a proportional trend between cost and the increasing level of joint rigidity. In order to investigate the effect of the intermediate value of stiffness within the structure, the authors chose as pinned and fixed joints the value of $2.26 \cdot 108$ Nmm/rad

and $5.65 \cdot 1011$ Nmm/rad respectively. Once the extreme boundary configuration was fixed, the stiffness values of the semi-rigid connections were derived by interpolating between the fixed lower and upper bound. The structural analysis of the structure was performed through the Practical Advanced Analysis (PAA) and, at each iteration, the distribution of stress into the structure was upgraded according to the corresponding attempting value of each connection. This method allows to perform of a nonlinear inelastic analysis with a low computational cost.

Meanwhile, the previous studies treat only the material and connection cost taking into account only the boundary static scheme of the connection, Prendes et al. in [64] tried to investigate the effect of welded connections into the global optimum of the structure. In this way, the optimization was able to include all the technical procedures required by each phase of the welding operations. In general, the cost function was constituted by four terms referred to the cost of vertical elements, horizontal members, semi-rigid connection and welding cost. The first two terms were obtained by multiplying the weight mass of these items by the steel market price, while the semi-rigid connection cost was obtained as a percentage of the element price multiplied by a suitable value of stiffness grade of joints (starting from 12.5% up to 22.5%, from the more flexible to the stiffer one). Lastly, the welding cost is simply the sum of the filler material, gas, machines and operation cost of the work. The discrete design variables involved in the process are the moment of inertia (strictly bounded with the standard profile typology) and the joint stiffness.

Basically, the mentioned above approaches to consider semi-rigid connections were widely used by various authors for developing different optimization algorithms thanks to its reliability and simplicity in implementation (e.g. [65], [35], [76]). Some authors added a specific constraint to guarantee the constructability of the connection (e.g. [36], [82]). The constructability constraints assure the correct allocation of the beam to the columns in a connection by checking the correct matching of the members (i.e. the web of the beam connected to the flange of the column should be shorter or equal to the flange length).

While so far only work associating connection cost with an optimized grade of joint's stiffness has been presented, other authors focused on involving specific types of connections with a well-defined degree of constraints into a cost function.

For instance, Kaveh et al. in [47] arranged an objective function which was a simple cost function composed of the cost of the structural members (computed as usual like weight by unit cost) and the cost of the connections. The connections were assumed pinned or rigid, labelled as 0 and 1 respectively. While the latter is associated with a cost equal to 900$/connection, the former cost was assumed to be equal to zero based on the authors' practical evaluation. As a set of variables of Design Vector, all cross-sections and beam-end connection types are chosen. The components' cross-sections are taken from the AISC steel construction guidebook.

The effect of different types of static schemes for each connection was investigated by Laberdi et al. in [2]. In this paper, the Design variable was constituted by two vectors: the first one considers all the possible W-shape cross sections assigned to each structural element through thumb criteria of grouping, instead, the second one effectively considers the type of connections at each frame's joints. In this study, the connection can be either a pin or a moment connection, in particular, the pin connection does not transmit a bending moment while the moment connection transmits a percentage of the moment. Combining the information derived by coupling beam sizes and connection types, four types of beams were defined: fully moment-connected, fully pinned, left-end moment-connected and right-end moment-connected.

Other authors used a combined approach where only some specific connections typologies were taken into consideration. In particular, it was taken into account only some specific connections and not the entire spectrum of stiffness and transmitted moment. One of these was Jarmai et al. in [41] which takes into account only four different connection types. In the mentioned research, the optimum design of a structure was performed on a simplified model focused only to the central part of a three-story building frame. The structure was subjected both to vertical and horizontal (seismic) loading. For the structural analysis of the building, the framework was decomposed into three parts and only the central part was analysed through a fish-bone model. For the columns and the beams was used a welded square box cross-section and a rolled universal beam I cross-section respectively and their dimensions was adopted as design variables of the optimization problem. Four types of assembled connections are considered and the most economical one was entered into the model. The objective function was a cost function with a detailed computation of the costs (fabrication, connection and material cost) that contribute to the total price of the building. Specifically, material, design, assembly, inspection, cutting and welding according to Japanese calculation was considered.

An ulterior improvement was done by Ali et al. in [4] which evaluated all the fabrication and erection phases by considering the most used connection typologies in common practice. A realistic optimization of frame design should include the effective costs of various phases of production, including manufacturing and erection, for this reason, a multi-stage production cost optimization was developed. The optimization problem had as discrete design variables three vectors that contained: the cross-sectional sizes of structural members, the type of beam-to-column connections and the type of column bases. The optimal size of the structural member was selected from available steel profiles instead, the most adopted technical solutions by engineers of internal connections and column base connections were considered. Buckling and serviceability constraints were applied to the optimization problem. Finally, the objective function was a production cost function expressed as the sum of the superstructure cost and the foundation cost. Superstructure cost comprises the material, fabrication and erection costs. The material price was simply determined as the unit price for different section types and different steel grades multiplied by the effective element length. On the other hand, the fabrication cost was the set of the elementary manufacturing operations evaluated as the time required to build structural parts. By adding a cost per hour of workshop labour, the time is transformed into a price. Lastly, the erection was simply computed by multiplying the total mass of the structure by a cost factor that converts man labour and machine power involved into the erection in economical terms. The second term of the objective function, which is the foundation cost, was the sum of the volume of excavated soil and concrete volume footing times the relative cost factor.

Up to now, topological optimization has been discussed with only connections in mind. Other applications rely into change the arrangement or the connectivity matrix of structural elements when size optimization is performed simultaneously. The most classic optimization that falls under these characteristics adopts a steel portal frame as a case of interest for several optimization strategies.

In this regard, Phan et al. make a notable contribution with three in which several optimization strategies and interesting real-world applications were proposed. In the first ([61]) the authors studied the rigid-jointed cold-formed steel portal frames

used for light industrial and agricultural buildings with a span of up to 20 m. One of the advantages is the competitive cost compared to the hot-rolled systems. The variables were taken into account where the spacing and pitch of the frame as continuous variables and the section size as discrete ones. The optimization of the minimum cost was performed with a real-coded genetic algorithm that minimizes a cost function considering the unit length cost multiplied by the length of the main structural members and the frame spacing. The greater the spacing of the frames, the lower the total cost of the structure, hence, fewer steel portals are required.
The second paper ([62]) include also joint effects and secondary members. The cost function was taken into account the portal frames' cost and their spacing and pitch, the secondary members (purlins) and the brackets used in the bolted joint. Four distinct discrete variables were used, namely, the cross-section sizes of the columns and rafters, and both the length and depth of the eaves haunch. The analysis was performed by employing both rigid and semi-rigid joints (in the second case the semi-rigid connections are modelled like zero-dimensional springs with a certain stiffness).
Finally, Phan et al. in the third research ([63]) for ulterior reduce the cost of the industrial building consider the stressed skin action of the roof profiles that increase the global strength and allows more economic solution for the structural members. For this research, a frame with a span of 12 m, height to eaves of 3 m and roof pitch of 10° was adopted. Once the frame layout is fixed, 6 building configurations were investigated. The building configurations were constructed by 3, 4, and 5 portal frames respectively, with two variations with frame spacings of 4 and 6 m for each of these models. The stressed-skin was considered in order to evaluate the additional rigidity to the overall structure thanks to the shear stiffness of these panels. This resulted in a global cost-saving of the structure thanks to stress limitation at the level of connections. The objective function was a cost function per square meter of floor area which takes into account the cost of the members and the cost of the angle brackets divided by the span length and the frame spacing. The discrete decision variables were the size of the column and rafters by choosing from a list of sections available in the UK. Moreover, grouping strategies were performed to minimize the length of the bolt connections which range in a continuum interval of 200 up to 2000 mm.

The stressed skin action was taken into account by other authors like Wrzesien et al. ([86]). In this work, the effect of stressed skin action on cost optimization of cold-formed portal frame buildings was studied by comparing different structures with a different number of bays. The paper studies buildings with a span of 6 m, height-to-eaves of 3 m and frame spacing of 3 m. The objective function is the sum of the cost of members and angle brackets per unit floor area. The design variables were the size of the columns and rafters (discrete variables) selected from the standard cross-section available in the UK, the length and the width of the bolt group (continuous variables) used in the connections and the centre of the bolt group (that was assumed approximately be equal to the Centre of Rotation (ICR)). The constraints are related to the strength and deflection specifications by taking into account the beneficial effect of stressed skin action acting on the roof diaphragms. The optimization was performed by a real-coded Genetic Algorithm for a different number of bays. The comparative study shows that the stressed skin action is relevant for buildings with few bays, less evident for structures with more than 9, meanwhile the effect can be considered almost negligible for more than 12 bays.
However, with the increasing demand for sustainable design procedures by society, several authors tried to couple the economical aspect with the environmental one into optimization.
To reach this purpose Mavrokapnidis et al. in [57] analysed the environmental performances of the cost-optimized tall buildings on five different topologies. The five structural systems involved in the analysis were: the concrete *Tube in Tube*, the concrete *Braced Tube*, the steel *Braced Tube*, the steel Diagrid and the steel Outrigger. The previous structural systems were optimized by the Optimization Computing Platform (OCP) design tool developed by the Institute of Structural Analysis and Antiseismic Research of the National Technical University of Athens which uses a probabilistic and deterministic combined search. The structural elements have been classified into columns, beams, walls and braces depending on their role in the structural systems, while their cross-sectional dimensions constitute the design variables of the optimization problem. The objective function was a cost function that takes into account the material cost and the construction cost. Structural verification was conducted according to Eurocode specifications. On the five optimized results was performed a Life Cycle Assessment (LCA) "cradle to grave" that computes the

environmental impact deriving by the acquiring of row material for disposal and recycling in order to make a comparison within the different environmental behaviours among the tall structural typologies. Generally, the results showed that the minimization of the construction materials used for buildings' erection alleviates the environmental problems, that arise during the construction phase.

As has been seen before, cost optimization can be used also for more particular applications like environmental ones. One other utilization of cost optimization was in the seismic field.

The application can be various and focused on optimizing several aspects related to the seismic behaviour of a structure. One of these can be the design of bracing systems like in the Braga et al. study ([11]). The proposed procedure allows the design of a bracing system by minimizing the intervention cost. The procedure allows obtaining the minimum cost through a dimensional and topological optimization of the bracing. The structural behaviour of the system was obtained by a linear modal analysis that gave the IDR (inter-story drift ratio) and braces displacement values which are constrained by serviceability limitation expressed in terms of IDR. The independent variables of the problem are the ones required for a bracing design like the area of each steel truss, the yielding force and the yielding displacement. The objective function was the sum of material and works price for the realization of the intervention and was composed of the cost of steel elements, dissipative devices, masonry works (removal and reconstruction or drilling and traces of the infills) and foundation system improvement). The optimization aims to minimize the dimension of the global stiffness matrix of the consolidated system through an iterative algorithm. Once the minimum size of the stiffness matrix is obtained, the optimal retrofitted configuration and the optimal number of bracings are detected.

Another more specific application of cost optimization on the seismic devices was relative to the yielding metallic devices for seismic loads [77]. Yielding metallic devices are very useful to reduce inter-story drifts and enhance ductile behaviour during seismic events. This article treats the cost optimization of a specific type of dissipators which are realized by assembling a determinate number of triangular plates that, through their deformation, are able to dissipate seismic energy. The design parameters that concur in the cost minimization are the thickness of the triangular plates and their number. The least cost is reached by a Genetic algorithm that minimizes a cost function that is the sum of the total failure cost and the cost of the devices including their replacement cost. The failure cost is linked to the story drift and floor acceleration responses computed by assigning to the devices a proper stiffness. The randomness of the seismic events with various probabilities of occurrence is included in the optimization by a seismic hazard curve.

2.4 Size, Shape and Topology optimization

The most effective and powerful approach to achieving the minimum cost of a structure is a combination of all three levels described above (size, shape, and topology). In this section, these three approaches are integrated simultaneously in the same procedure. The advantage of this type of technique is to achieve a design that is simultaneously well-compliant with all three design aspects.
Historically, the first studies that used combined optimization were performed by Lee et al. in 1975 ([53]) and Thomas et al. in 1977 ([80]).
In the first paper, the minimum cost design of a steel portal framed building was achieved through the determination of the optimum shape and topology of the structure that is a function of material and fabrication cost only. The material costs were taken from the "British steel corporation home trade price list" which gives the unit weight cost and surface preparation cost meanwhile the fabrication costs were computed for the main construction procedure (welding, cutting and drilling) with the Standard Minute Value technique. The S.M.V. technique gives a preliminary estimation of the time-specific cost for a single task. Aiming to accomplish the optimization task the design variables were formulated into primary variables, normally chosen at the beginning of the design process (building length, building width, number of bays, eaves height, roof pitch and frame spacing), and secondary variables, which are selected after the previous one, like the length and size of the various elements (stanchion, rafter, wind braces, etc. . .).
On the other hand, Thomas et al. proposed an algorithm that allows a nonlinear optimization technique (SUMT nonlinear programming or by transforming the constrained problem into a series of non-constrained ones by the introduction of penalties functions), for the least cost elastic design of roof systems composed of rigid steel trusses, web joists and steel roof deck. This procedure is used for various

grades of steel and standard sections. The method is a recursive search for the combination of the design variables which safely supports the design load for the least cost based on the design specification of AISC. Combined stresses due to axial forces, secondary bending moments and buckling effective length are calculated. In particular, it considers changes in the mechanical properties of the members, geometric variation and topology. The independent design variables are the cross-sectional area of each member, the plane coordinate of each joint and the number of trusses in the system. Conversely, the dependent variables are the variables associated with the previous one (for example the cross-sectional inertia dependent to the geometry, section modulus and radius of gyration). All variables are considered to be continuous due to the nature of the adopted optimization strategy. The criteria to evaluate the cost function is based on the overall material and fabrication cost computed by simply multiplying the area/weight of the element by the unit area/weight material and fabrication cost of the specific element. In this study, an important limitation was highlighted by the same authors related to the total absence of constructability constraints which would allow the discarding of unfeasible solutions. Lower and upper bound and/or employed structural constraints can not guarantee that no realistic element sizing (negligible cross-section areas) and/or unfeasible connections between elements of the structure appear in the optimal solution.

More recent was the contribution of Kaveh et al. in [44] where the authors determine the cost optimization of a composite floor system, made-up of steel beams and a reinforced concrete slab. The cost function was determined by the minimum cost of the concrete, steel beams and shear studs (the connectors between the steel girder and the concrete slab). The design variables, that were assumed discrete, were the steel beam spacing, the beam size and the concrete slab thickness.

Since considering all the necessary design variables for each optimization level in a single objective function result in a heavy computational effort and in a lack of control of each parameter related to the single optimization strategies, many authors made use of parametric optimizations with specific regard to topology and/or shape. In this regard, Kripka et al. in [51] develop a size and shape optimization by ranging between 7 common different topologies of steel trusses with parallel chords. The 7 truss typologies were analysed through a simple cost function composed of the total weight of the structure multiplied by the unit cost of the steel. The design variables are the cross-sections of the truss members (treated as a discrete variable that comes from the standard cross-sectional list) and the coordinate of the top chord (continuous variable) in order to find the best span-height relationship for this kind of beam. Because the same profile is used for a group of pieces, the number of design variables is drastically decreased in comparison to the total number of elements. Regarding the examples presented, we observed great variation in the final weight of the structure as the number of groups of elements increased. By including truss height in the set of design variables, significant additional cost savings were obtained. For the cases considered, the suggestion found in the technical literature for the relation between height and span was not valid, because it led to structures with a higher cost than those obtained with optimization.

Along this trail, Alhendi et al. in [3] implemented another parametric optimization regarding parallel chord trusses. The aim of this study was to investigate the performance and the cost-effectiveness of three parallel chord composite floor trusses (Prat, Howe and Warren) with 4 different composite floor panels and load intensities. The analysis was made by making 165 models with different spans between the trusses and different truss depths and panel typologies. The variables which come from this analysis are the span range and span and depth ratio of the composite floor truss system. In addition to that, a cost estimation function was implemented as the sum of the material cost, fabrication cost and painting cost. The models were computed through RFEM software and the minimum was reached by the mathematical regression found from the 165 models results. By observing the results pointed out by the optimization, the authors provided useful technical specifications for the composite tabular-floor trusses design.

Moreover, Mensinger et al. performed a full-complete optimization on floor systems by coupling the size search technique with parametric shape and topology analysis ([59]). A cost and environmental impact multi-objective optimization was implemented into the preliminary structural design for given rectangular boundary shapes by using the Sustainable Office Designer (SOD) which was developed as a SketchUp plug-in. The cost function was simply the masses of the various element (steel members, reinforced concrete elements and profiled sheeting) multiplied by the unitary price of the material. The other objective function was the German Environmental Product Declaration (EDP) which is a coefficient that takes into account many environmental indexes such as the global warming potential (GWP), ozone depletion potential (ODP), photochemical ozone creation potential (POCP), acidification potential (AP), eutrophication potential (EP), non-renewable primary energy (PEne) and renewable primary Energy (PEe). The design variables are discrete and related to the primary beam, secondary beam and column cross-section and also the slab thickness. To permit a more efficient computation effort, only 1 slab thickness and 2 column cross-sections per floor were allowed. All the secondary beam and main beam have been assumed to be the same cross-section resulting in a reduction of a number of Design variables involved in the optimization. Nine different floor layouts were examined by changing the dimensions of the rectangular plant and the arrangement of secondary and main beams.

The combined size, shape and topology approach lead to the resolution of very sophisticated problems. In this sense, Cicconi et al. ([20]) elaborate on a special parametric procedure to accomplish a specific task. This paper proposes a sequential multi-objective procedure for the structural optimization of modular industrial towers such as the steel structure of the chimney used in oil and gas power plants. The optimization method is articulated in three stages of optimization: preliminary design, embodiment design and detail design. Objective functions focus on weight and cost reduction. The first optimization level uses a 1-D model (pole model), whereas the second uses a 3-D shell model. Finally, the third stage entails thorough design, which includes simulations and analysis based on 3-D solid models. Thanks to this approach, all the manufacturing, logistical, non-destructive testing and assembly cost aspect were investigated in different stages of optimization aiming to reduce the complexity of the entire problem into easier-to-handled problems.

Review table of the application of optimization strategies to steel structures

Ref.	Year	**S**tatic **D**ynamic	Size	Shape	Topology	**S**ingle/**M**ulti Objective	ID-OF	Design criteria	Design Variables
[53]	1975	**S**	☑	☑	☑	**S**	(2) Material cost (2) Fabrication cost	Stresses Geometry boundaries	Cross-section Building geometry Elements' number
[80]	1977	**S/D**	☑	☑	☑	**S**	(2) Material cost (2) Fabrication cost	Stresses Buckling	Cross-section Elemnts' number Nodal coordinates
[33]	1989	**S/D**	☑	☐	☐	**S**	(1) Weight (1) Inter-story drift	Geometry Inter-story drift	Cross-section
[38]	1992	**S**	☑	☐	☐	**S**	(1) Weight	buckling Geometry Serviceability constraits	Cross-section
[16]	1995	**S/D**	☑	☐	☐	**S**	(1) Weight	stresses Inter-story drift	Cross-section

[78]	1996	**S**	☑	☐	☑	**S**	(1) Weight (2) Connection cost	stresses Serviceability constraints	cross-section Connection type
[40]	1999	**S**	☑	☐	☐	**S**	(1) Weight	Stresses Inter-story drift	Cross-section
[74]	2002	**S/D**	☑	☐	☐	**S**	(2) Initial cost (2) Painting cost	Stresses	Cross-section
[28]	2003	**S/D**	☑	☐	☑	**S**	(1) Weight	Stresses Serviceability constraints	Cross-section Connection type
[55]	2004	**S/D**	☑	☐	☐	**M**	(2) Material cost (1) Complexity (2) Damage cost	Inter-story drift	Cross-section Connection type
[60]	2004	**S**	☑	☐	☐	**S**	(2) Material cost (2) Manufacturing cost	Stresses	Cross-section

[37]	2005	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Serviceability Geometrical boundaries	Cross-section Connection type
[41]	2006	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost (2) Fabrication cost	Stresses Inter-story drift	Cross-section Connection type
[88]	2006	**S/D**	☑	☐	☐	**S**	(1) Weight (1) Uniform inter-story drift	Inter-story drift Geometrical boundaries	Cross-section
[4]	2009	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Fabrication cost (2) Erection cost (2) Foundation cost	Stresses Buckling Serviceability	Cross-section Connection type
[27]	2009	**S/D**	☑	☐	☐	**S/M**	(2) Material cost (1) Damage	Stresses Buckling Serviceability Seismic	Cross-section

[44]	2010	**S**	☑	☑	☑	**S**	(2) Material cost	Stresses Serviceability	Cross-section Beam spacing Slab thickness
[54]	2012	**S/D**	☑	☐	☐	**S**	(1) Initial cost (2) Damage (EDL)	Serviceability Interstory-drift	Cross-section
[48]	2012	**S/D**	☑	☐	☐	**M**	(1) Weight (2) Life Cycle Cost	Stresses Serviceability	Cross-section
[18]	2013	**S**	☑	☑	☐	**S**	(1) Weight	Stresses Serviceability	Cross-section Shape ratio
[23]	2013	**S/D**	☑	☑	☐	**S**	(2) Life Cycle Cost	Stresses Serviceability Buckling	Cross-section Nodes' coordinates
[45]	2013	**S**	☑	☐	☐	**S**	(1) Weight	Stresses Serviceability	Cross-section

[56]	2013	**S/D**	☑	☐	☐	**M**	(1) Weight (1) Mean value inter-story drift (1) Dispersion inter-story drift	Stresses Serviceability Seismic	Cross-section
[61]	2013	**S**	☑	☐	☑	**S**	(2) Material cost	Stresses Serviceability	Cross-section
[65]	2013	**S/D**	☑	☐	☑	**S**	(2) Material cost	Stresses Serviceability Geometrical boundaries	Cross-section Connection type
[35]	2014	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Serviceability	Cross-section Connection type
[2]	2015	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Inter-story drift Constructability	Cross-section Connection type

[6]	2015	**S**	☑	☐	☑	**S/M**	(1) Weight	Stresses Geometric boundary Buckling Serviceability	Cross-section Connection type
[8]	2015	**S**	☑	☑	☐	**S**	(2) Material cost	Stresses Serviceability Buckling	Cross-section Truss height
[36]	2015	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Serviceability Inter-story drift	Cross-section Connection stiffness
[63]	2015	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Serviceability	Cross-section Connection type Elements number
[58]	2016	**S**	☑	☑	☑	**S**	(1) Weight	Stresses Serviceability Geometric boundaries	Cross-section

[64]	2016	**S**	☑	☐	☑	**S**	(2) Material cost (2) Welding cost (2) Connection cost	Stresses Serviceability Buckling	Cross-section Connection stiffness
[70]	2016	**S/D**	☑	☐	☐	**M**	(1) Weight (1) Damage (EAL)	Stresses Serviceability Seismic	Cross-section
[85]	2016	**S/D**	☑	☐	☐	**S**	(1) Weight	Stresses Serviceability Joint strength	Cross-section Connection type
[86]	2016	**S/D**	☑	☐	☑	**S**	(2) Material cost	Stresses Serviceability	Cross-section Connection type
[47]	2017	**S**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Serviceability	Cross-section Connection type
[42]	2017	**S/D**	☑	☐	☐	**S/M**	(2) Fabrication cost (2) Welding cost (1) Structural performances	Stresses Drift angle Story stiffness Serviceability Constructability	Cross-section

[59]	2017	**S**	☑	☑	☑	**M**	(2) Material cost (3) Environmental impact	Stresses Serviceability	Cross-section
[62]	2017	**S**	☑	☐	☑	**S**	(2) Material cost	Stresses Serviceability	Cross-section Connection type Elements' number
[77]	2017	**S/D**	☑	☐	☑	**S**	(2) Failure cost (2) Replacement cost	Seismic	Dissipators' thickness number
[81]	2017	**S**	☑	☑	☐	**S**	(1) Weight (2) Material cost (2) Fabrication cost	Stresses Serviceability Buckling Geometrical boundaries	Truss height Joints location Cross-section
[83]	2017	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Serviceability Inter-story drift	Cross-section Connection type

[51]	2018	**S**	☑	☑	☑	**S**	(2) Material cost	Stresses Serviceability Buckling	cross-section Shape parameters Topology
[67]	2018	**S/D**	☑	☐	☐	**M**	(1) Weight (1) Seismic energy	Seismic Inter-story drift	Cross-section
[76]	2018	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Serviceability Geometric boundaries	Cross-section Connection type
[82]	2018	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Connection cost	Stresses Inter-story drift Constructability	Cross-section Connection type
[11]	2019	**S/D**	☑	☐	☑	**S**	(2) Material cost (2) Work price	Serviceability Inter-story drift	Cross-section Yielding force Yielding displacement
[57]	2019	**S**	☑	☐	☑	**S**	(2) Material cost (2) Work price	Stresses Serviceability	Cross-section

[9]	2020	**S**	☑	☐	☐	**M**	(2) Material cost (1) Displacement (DPF)	Stresses Serviceability Buckling	Cross-section
[20]	2020	**S**	☑	☑	☑	**M**	(1) Weight (2) Material cost	Stresses Serviceability	
[29]	2020	**S**	☑	☐	☐	**S**	(1) Weight (2) Material cost	Stresses Serviceability	Cross-section
[25]	2020	**S/D**	☑	☐	☐	**S**	(1) Lightest Beam	Stresses Serviceability Natural frequency	Cross-section
[68]	2020	**S/D**	☑	☐	☐	**M**	(1) Weight (1) Seismic energy	seismic Inter-story drift	Cross-section
[84]	2020	**S/D**	☑	☐	☐	**M**	(1) Energy dissipation (2) Material cost	Stresses Serviceability Inter-story drift	Cross-section

[87]	2020	**S/D**	☑	☐	☐	**S**	(2) Material cost	Serviceability Inter-story drift	Cross-section
[3]	2021	**S**	☑	☑	☑	**S**	(2) Material cost (2) Fabrication cost (2) Painting cost	Stresses Serviceability Constructability	Cross-section
[17]	2021	**S**	☑	☐	☑	**S**	(1) Weight	Stresses Buckling	Slenderness Element number
[21]	2021	**S**	☑	☐	☐	**S**	(1) Weight (1) Global stiffness	Stresses Buckling	Cross-section
[30]	2021	**S/D**	☑	☐	☐	**S**	(1) Weight (2) Life Cycle Cost	Stresses Geometric boundaries Seismic	Cross-section
[31]	2021	**S/D**	☑	☐	☐	**M**	(2) Initial cost (2) Damage (ODI)	Stresses Geometric boundaries Seismic	Cross-section

[46]	2021	**S**	☑	☐	☐	**S**	(2) Material cost (2) Fabrication cost	Stresses Serviceability Geometric boundaries	Cross-section Holes features Slab thickness
[71]	2021	**S/D**	☑	☐	☑	**S**	(1) Weight	Stresses Serviceability Geometric boundaries	Cross-section Wall arrangement
[72]	2021	**S/D**	☑	☐	☐	**S**	(2) Initial cost (2) Life Cycle Cost	Stresses Serviceability Geometric boundaries	Cross-section
[7]	2022	**S**	☑	☑	☐	**S**	(2) Material cost	Stresses Serviceability Prestress losses Buckling	Cross-section Truss height Prestressing eccentricity

Chapter 3

Cutting Stock Problem

The previous chapter discusses how to optimize the structures' price by manipulating material, fabrication and maintenance costs. Specifically, materials cost minimization considers only those elements involved in the construction process.

As will be seen in the current chapter, a significant part of the expenses is also the waste of material from the cutting process. In other words, minimising the amount of material involved in the construction process without a carefully cutting design, which diminishes the waste, leads to inefficient cost optimization.

Construction and demolition wastes were expected to account for around 23% of the overall solid waste stream. This waste ratio equates to more than 100 million tonnes every year. Other nations' percentages corroborate the estimates from the United States. A percentage of the waste created by stock reduction is preventable, which means it is generated as a result of improper material utilisation. The quantity of superfluous acquired materials, needless workmanship, wastes and trucking and tipping fees required to discharge the garbage would be reduced if the supplies were used more efficiently [49].

Indeed, efficient resource use is not just in the interests of the industrialist, but also of the world at large. The disposal of trash from a stock-cutting operation may cause pollution, and excessive wasting may deplete our planet's precious supplies [19].

Cutting losses is possibly the most major source of steel waste. Cutting losses arise when normal steel lengths are shortened to fit the project's required lengths. A significant amount of the created steel waste according to Adham et al. ([75]) is related to cutting losses, which are mostly caused by:

- dividing an order into separate, smaller orders typically results in more waste due to fewer cutting alternatives
- using inefficient cutting patterns in the cutting schedule results in the generation of avoidable waste that could be avoided through better stock-cutting planning
- using the optimum cutting patterns may result in unavoidable waste that is the minimum waste generated if the optimum cutting patterns are used

In order to minimise the waste the cutting stock problem (CSP) is a significant source of one-dimensional stock waste in the construction industry.
The following part of this chapter is organized into three subsections. The first section shows the state of art about cutting stock problems. Afterwards, the following section treats the cutting stock problem applied in the structural optimization of trusses. Lastly, a mathematical formulation of the cutting stock problem applied in the following chapters was developed.

3.1 State of art

Cutting and packing (C&P) problems of concrete and abstract objects appear under various specifications (i.e. cutting problems, knapsack problems, container and vehicle loading problems, pallet loading, bin packing, assembly line balancing, capital budgeting, changing coins, etc.) within disciplines such as Management Science, Information and Computer Science, Engineering, Mathematics, and Operations Research. All of these problems have essentially the same logical structure [10].
The most famous of these problems is the bin packing problem (BPP) which determines how to pack as many items as possible into a container or, in other words, minimize the number of containers (bins) used for the same stock of goods. Informally, the BPP can be stated as follows. There are given n *items*, each with an integer *weight* $w_j (j = 1..., n)$ and an infinite number of identical bins with integer *capacity* c. The goal is to load all the products into as few bins as possible so that the total weight packed in each bin does not exceed the limit. [24]

Almost all the other C&P problems are variants (e.g. pallet loading problem) or generalizations (e.g. cutting stock problem) of the BPP.
In particular, in civil engineering, the most common problem of diminishing waste due to the steel element cutting process can be solved with the cutting stock problem. In summary, the cutting stock problem (CSP) tackles the practical question of cutting off needed pieces from stock material with the least trim loss. In more technical terms, the CSP can be defined starting from the BPP definition as follows. There are m *item* kinds, each with an integer *weight* w_j and an integer *demand* $d_j(j = 1, ..., m)$, as well as a huge number of identical integer *capacity* c bins. In the CSP literature, the bins are typically referred to as rolls, a word derived from early implementations in the paper industry, and "cutting" is commonly used rather than "packing". The goal is to manufacture d_j copies of each item type j (i.e., cut/pack them) using the fewest number of bins possible while ensuring that the total weight in every bin does not exceed the capacity [24].
Moreover, the cutting stock problem can be classified as a one-dimensional and two-dimensional problem. A specified set of order lengths must be extracted from stock rods of a defined length in order to solve the one-dimensional cutting stock problem (1D-CSP). Usually, the goal is to use the fewest amount of rods possible (material input). The two-dimensional two-stage constrained cutting problem (2D-2CP) aims to select the most valuable group of rectangular objects from a single rectangular plate. Furthermore, the two-dimensional CSP can involve regular or irregular shapes, in the second case, the problem is called nesting and have a more difficult solution.[10]
These kinds of problems are complex combinatorial optimization, which in mathematical terms is a strongly NP-hard problem. For this reason, many linear programming, heuristic and metaheuristic approaches were proposed over the years.
The first approach to the C&P problems dates back to the thirties with Kantorovich in [43]. Although his approach is poor and only handles small-scale cases, it aids in understanding the problem structure.
Numerous heuristic approaches (such as first solving the linear programming LP problem and then converting the LP result to an integer solution) take advantage of this problem's linear programming (LP) relaxation.

This problem is often formulated as an integer programming (IP) problem, and its linear programming (LP) relaxation is exploited in many heuristic algorithms. In mathematics, the relaxation of a (mixed) integer linear program is the problem that arises by removing the integrality constraint of each variable and allows solving the integer programming (IP) problem as a linear programming one. This relaxing technique converts an NP-hard optimization issue (integer programming) into a similar problem that can be solved in polynomial time.

However, this method makes it impractical to take into account all cutting patterns that are practical and correspond to the columns in the LP formulation, especially when the length of a single item is much smaller than the roll length. By resolving the related knapsack issue, Gilmore and Gomory provided an inventive method to identify the cutting patterns required to enhance the LP solution.

Gilmore and Gomory ([32]) proposed a column generation approach inspired by Dantzig and Wolfe ([22]) for decreasing stock and bin packing concerns (BBP). Because enumerating all possible cutting patterns would take an inordinate amount of time, it reduces valid patterns repeatedly and adds them to the issue based on their contribution to the objective function. The column generation approach made large-scale cutting stock issues solvable in a reasonable amount of time.

In the following years, many algorithms were developed to solve the problem. While the most precise but computational and time consuming are the more rigorous procedure based on the integer linear programming, in more recent times several metaheuristic procedures have been implemented (e.g. Genetic Algorithm, Simulated annealing and Tabu search).

3.2 Use of cutting stock problem in truss solutions

In the previous section, it was seen the general formulation of the Cutting Stock problem. In recent years, the civil engineering sector has become increasingly interested in implementing the reuse of construction materials. The researcher notices that the cost and environmental optimization, by themselves, in not sufficient for reaching the best outcome. The building sector is a major contributor to material consumption, energy use, greenhouse gas emission, and waste production [1] [5].

This problem can be solved basically in two ways: by minimizing the waste in the fabrication phase or by reusing stock materials from other structures. The first

way wasn't so predicated and this was one of the aims of this thesis and the entire research. The second path was deeply explored by Brütting et al. in various publications that treated the reuse of the construction stock elements. In this section, it is possible to see the highlights of this specific topic.
The idea of Brütting et al. is to use the principle of circular economy in order to reduce the cost and the environmental impact of structures. In a circular economy, manufactured goods are kept in use as long as possible through closed loops, which consist of repair, reuse, and recycling. In particular, they were concerned about reuse because less energy is spent on reprocessing with respect to recycling.
A first way to approach the reuse optimization problem it can be seen in [13]. In this study, structural optimization with stock constraints was shown.
For clarity, the term *member* is used for a position or bar in a reticulated structure and *member length* is the distance between nodes at this position. The term *element* is used for the individual component of a stock. The stock was reused materials which have different dimensions. In Figure 3.1 it is possible to see the two ways for approaching the problem: the 1-to-1 *assignment* of elements to positions in the truss (as in Stock A), and a *cutting stock approach*, where multiple members can be cut from individual elements(as in Stock B).

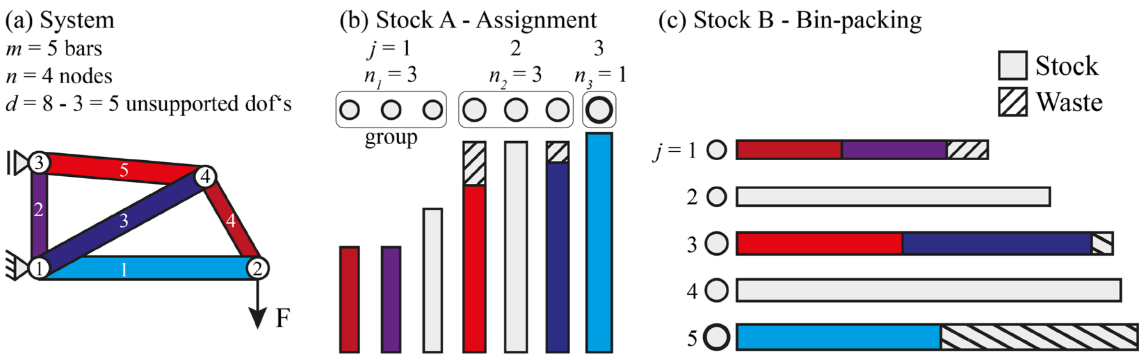

Figure 3.1: (a) Cantilever truss, (b) stock A and assignment, (c) stock B and cutting stock configuration (image taken from [13]

In the first case, which is the *assignment* problem, the objective of the optimization is to avoid waste by minimizing the long distance between members and stock elements. The second one is to find a cutting pattern which minimizes global waste. Both cases were solved through a MILP (mixed-integer programming) procedure.
Another problem addressed in the paper is that the lengths of the chosen elements

may not correspond exactly to the lengths of the structure's members once both problems have been solved. For this reason, shape optimization is then used to reduce cut-off waste for the globally optimal assignment or bin-packing solutions by changing the placements of the structure nodes (coordinates).

Finally, Brütting et al. exposed also a procedure to optimize the configuration of a stock or kit-of-parts such that its elements can be reused in various structures. This last consideration allowed to spread of the stock of reusing items in many structures and the outcome is an ulterior minimization of the waste.

Ulterior amendments, only for the assignment problem, were done to this basic formulation in [12] where the assignment was coupled with a topological optimization. After that, the truss was subjected to shape optimization. Is also noticeable the introduction of an absolute *buffer* that allows the allocation of also the very short items.

Moreover, in 2020 still Brütting et al. ([15]) report an entire structural optimization based on the principles set out above. In this work it is possible to see the simultaneous analysis and design approach, structural analysis is part of the optimization formulation by treating member end forces as well as nodal displacements and rotations as continuous state variables. Furthermore, designing a custom kit of parts (which is a group of discrete building parts that have been pre-engineered and are created to be put together in many ways to define a finished building) whose components are prepared to be combined in various structural configurations, serving diverse purposes, is an alternate approach to component reuse.

In one of the more recent papers, Brütting et al. ([14]) use the assignment and CSP for creating a kit of parts used for three different construction types. The kit-of-parts bars are tubular components joined with bolts at spherical joints. The structures' original topology and geometry are provided as input. The process consists of two steps. To allow for the reuse of similar bars in different structures, the structural geometries and kit-of-parts bars' length and cross-section dimensions are optimised in the initial stage. The second stage optimises the hole pattern for the spherical joints' connection details, allowing each joint to be reused in many constructions.

3.3 Mathematical formulation of the problem

Now the following section analyzes the mathematical formulation of the cutting stock problem by using the column generation technique to reduce the problem's computational cost.

3.3.1 Bin Packing formulation

In order to understand how the cutting stock problem works it is necessary to exhibit the mathematical formulation of the one-dimensional bin packing problem. This problem aims to allocate a set of items into the minimum number of bins.
For initializing the problem are necessary the following parameters:

- I: set of items, indexed by i
- B: set of bins, indexed by b
- L_i: length of item i
- L: length of each bin
- $x_{i,b} \in \{0,1\}$: unitary if item i is allocated to bin k, 0 otherwise
- $y_b \in \{0,1\}$: unitary if bin b is used, 0 otherwise

The mathematical formulation of the problem is:

$$min \sum_{b \in B} y_b \tag{3.1}$$

Subjected to the following constraints:

$$\sum_{b \in B} x_{i,b} = 1 \quad \forall i \in I \tag{3.2}$$

$$\sum_{i \in I} L_i x_{i,b} \leq L y_b \quad \forall b \in B \tag{3.3}$$

$$x_{i,b} \leq y_b \quad \forall i \in I, \quad b \in B \tag{3.4}$$

$$x_{i,b} \in \{0,1\} \quad \forall i \in I, \quad b \in B \tag{3.5}$$

$$y_b \in \{0,1\} \quad \forall b \in B \tag{3.6}$$

The principal equation of the bin packing problem (3.1) simply minimizes the number of bins used to obtain the requested items. The mathematical equation was subjected to some constraints. In particular, the (3.2) equation means that each item must be assigned to a bin (i.e. each item should be cut from one of the paper rolls available). Additionally, the second condition (3.3) assures that the length of all items associated with a bin should not exceed the length of the bin and the third (3.4) entails that an item can be assigned to a bin if and only if that bin is used. Finally, the following two equation ((3.5), (3.6)) express the domains of the two decision variable $x_{i,b}$ and y_b.

3.3.2 Column generation

The bin packing problem is a very complex combinatorial problem. For simplifying this problem the Column Generation formulation is used in this thesis. In this formulation, the main element is no longer the bin, but the feasible cutting pattern, that is, the possible arrangement of items in a bin. Since enumerating all feasible cutting patterns is prohibitively time-consuming, it generates valid patterns iteratively and adds them to the problem according to their contribution to the objective function.

The first step is to set up the restrained master problem (RMP).

The parameters involved into the column generation model are:

- I: set of unique items (subset of items with unique distinct lengths), indexed by i
- P: set of paths, indexed by p
- L_i: length of item i

- Q_i: quantity needed for item i
- L: length of each bin
- $M_{i,p}$: matrix whose element (i, p) defines the number of times item i is included in path p
- $x_p \in \mathbb{Z}$: number of times path p is chosen

The mathematical formulation is:

$$min \sum_{p \in P} x_p \tag{3.7}$$

Subjected to:

$$\sum_{p \in P} M_{i,p} x_p \geq Q_i \quad \forall i \in I \tag{3.8}$$

$$x_p \in \mathbb{Z} \tag{3.9}$$

The objective function of the RMP (3.7) is the minimization of the number of paths used which is strictly correlated with the minimization of the number of bins. The objective function is subjected to two constraints. First, it needs to select the number of paths in a way such that every unique item appears at least as many times as needed and this is what is done in (3.8). The second constraint defines the x_p domain.

The next step is to write the dual problem. The dual problem is a formulation correlated with the principal problem exposed above which is the primal. The dual problem is write in such a way that:

- A mostly horizontal constraint matrix becomes a mostly vertical constraint matrix
- A minimization problem (3.7) becomes a maximization problem (3.10)
- The objective value coefficients of the primal become constraint right hand side values of the dual
- The objective value coefficients of the dual are the dual values of the primal

In particular, the dual problem is:

$$max \sum_{i \in I} Q_i \lambda_i \tag{3.10}$$

Subjected to the following constraints:

$$\sum M_{p,i} \lambda_i \leq 1 \quad \forall p \in P \tag{3.11}$$

$$\lambda_i \in \mathbb{Z} \tag{3.12}$$

The λ_i is the dual value referred to a specific item constraint. Each dual value gives an indication of how profitable is to add the associate item to a new path. Moreover, to determine the best path to add it is necessary to set up the pricing problem where a new decision variable $y_i \quad \forall i \in I$ which represents how many times a certain item i appears in the new path. More in detail:

$$max \sum_{i \in I} \lambda_i y_i \tag{3.13}$$

Subjected to the following constraints:

$$\sum_{i \in I} L_i y_i \leq L \tag{3.14}$$

$$y_i \in \mathbb{Z} \tag{3.15}$$

Where the (3.14) ensures that the newly added path is feasible and the (3.15) imposes the y_i domain.
Forehead, to decide if a certain path should be added to the RMP it needs to verify the gain obtained by the addition of the new path with the following formula:

$$c - z \leq 0 \tag{3.16}$$

Where c is the original cost from the primal problem and z is the reduced cost computed in the pricing problem.
From the primal, it is possible to get $c = 1$ while from the pricing problem $z =$

$\sum_{i \in I} \lambda_i y_i$. Finally by substituting:

$$1 - \sum_{i \in I} \lambda_i y_i \leq 0 \tag{3.17}$$

Which lastly becomes:

$$\sum_{i \in I} \lambda_i y_i \geq 1 \tag{3.18}$$

Until the previous condition is satisfied by the new generate path this one was added to the primal RMP and the procedure was iterated. To make more clear the overall process the algorithm is shown in the following:

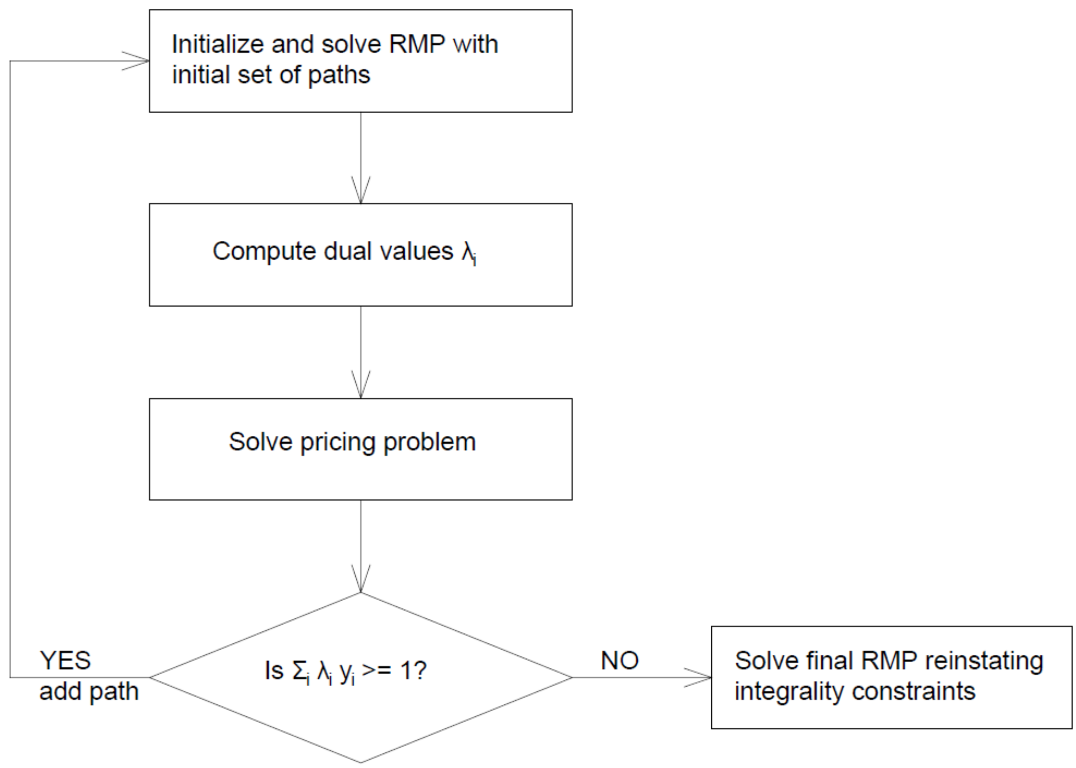

Figure 3.2: Column Generation algorithm for the solution of Cutting Stock Problem

3.3.3 Numerical example

To a deeper understanding of the CSP functioning, in this section, a practical case study will be shown.

The input parameters are provided by the customer which gives the length and the number of items. In this case study it was assumed elements set which contained the items shown in the figure 3.3 (the lengths were assumed as adimensional for simplicity).

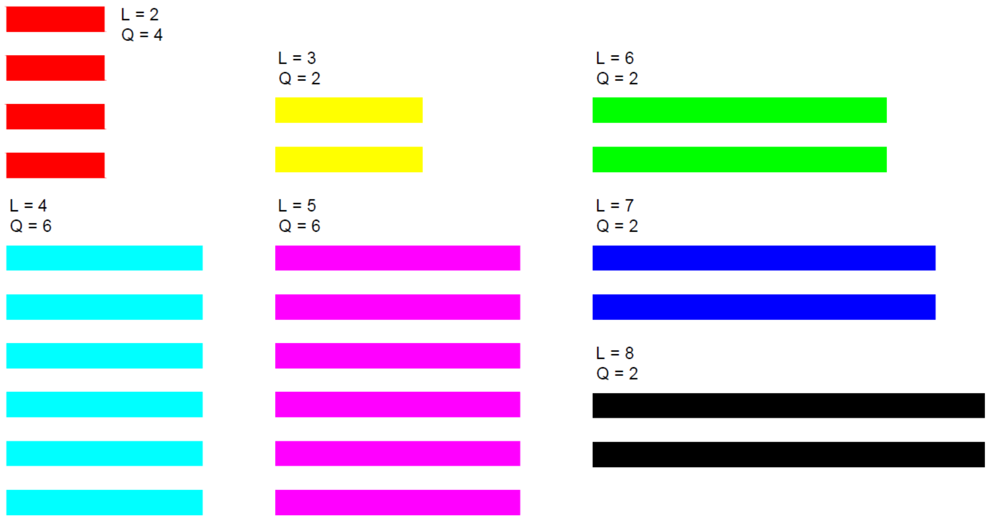

Figure 3.3: Items requested by the customer

This data should be represented in two vectors:

$$I = \{2, 3, 4, 5, 6, 7, 8\} \tag{3.19}$$

$$Q = \{4, 2, 6, 6, 2, 2, 2\} \tag{3.20}$$

Where in the vector (3.19) are collected all the items' lengths and in (3.20) are collected the quantities for each item type.
Meanwhile, the producer provides the length of the bars available in the factory which becomes the bins of the CSP. In this example, the bin length was 9 $L_{bin} = 9$.

First, the matrix of the cutting pattern was initialized as an identity matrix:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.21}$$

The matrix (3.21) simply contains all the pattern which contains only one item as shown in figure 3.4 for clarity: At this point, the initialization phase was over. After-

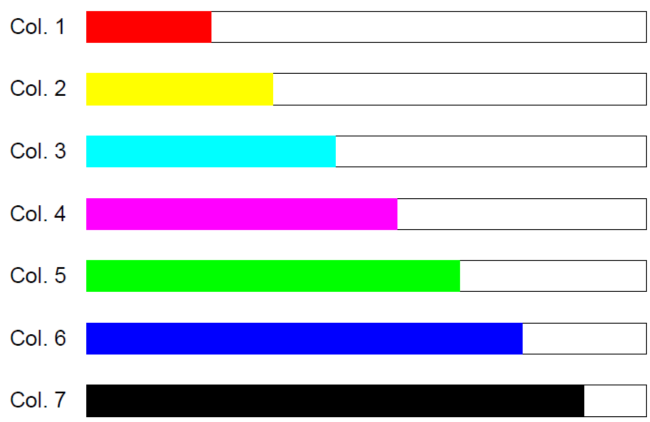

Figure 3.4: Initial pattern at the iteration 0

wards, all the computation step exhibited in the previous subsection was performed. The entire process is iterated until the pricing problem returns a positive reduced cost. At each iteration, a new cutting pattern is added. In the following images, it is possible to see the pattern added for each cycle and the associated reduced cost.

Figure 3.5: Iteration 1 where the reduced cost is -3.0000

Figure 3.6: Iteration 2 where the reduced cost is -1.2500

Figure 3.7: Iteration 3 where the reduced cost is -1.0000

Figure 3.8: Iteration 4 where the reduced cost is -0.5000

Figure 3.9: Iteration 5 where the reduced cost is -0.5000

Figure 3.10: Iteration 6 where the reduced cost is -0.5000

Figure 3.11: Iteration 7 where the reduced cost is -0.5000

Figure 3.12: Iteration 8 where the reduced cost is -0.2500

Figure 3.13: Iteration 9 where the reduced cost is -0.1250

At the end of the algorithm iterations the RMP is recomputed with all the new paths and it is possible to have the final results which are the number of bins ($N_{bins} = 13$) and the final cutting patterns related to each bin (shown in figure 3.14).

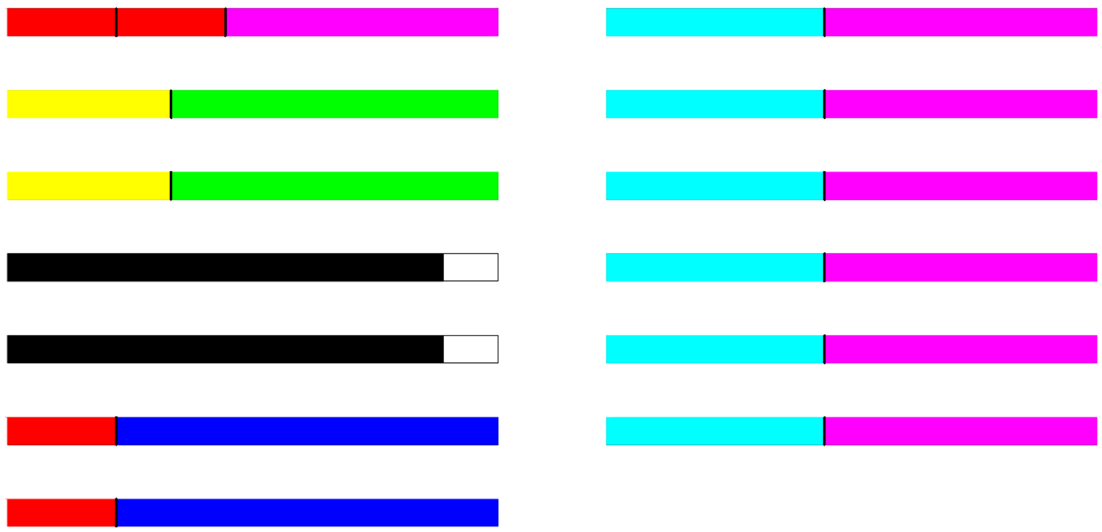

Figure 3.14: Results from the CSP algorithm: bins and relative cutting patterns

Chapter 4

Cutting Stock Problem embedded in Structural Optimization

In the last decades, the Scientific Community challenged on minimizing the structures' price by manipulating material, fabrication and maintenance costs. Specifically, in the structural optimization field, great attention was dedicated to materials cost minimization aiming to achieve a slender structure with optimal resource utilization. In this sense, the traditional approach adopted by researchers and practitioners is to optimize the design cost of structures while simultaneously satisfying safety recommendations provided by specific standard regulations. Undoubtedly, this approach reduces the overall cost of the structure but does not take into account the effective quantity of steel involved in the process which is not only related to the material used in the structure but also to the factory waste resulting from the cutting process (an example of cutting machine in figure 4.1).

As demonstrated in the previous chapter, a significant part of the expenses is also the waste of material from the cutting process. In other words, minimizing the amount of material involved in the construction process without a carefully cutting design, for the minimization of waste, leads to inefficient cost optimization.

Hence, this thesis aims to create a structural optimization which not reduces the structural mass of the structure but the mass of the entire stock of rough bars involved in the construction. This ambitious goal was reached through the implementation of the Cutting Stock Problem (CSP) in the objective function of structural optimization.

Figure 4.1: Beam cutting machine (image from www.asiacnc.com.tw)

In the initial chapters, it is possible to see the overall steps of structural optimization and how these steps work. Moreover, the previous chapters discuss how works the CSP and how this complex combinatorial problem was embedded in some reusing problems.

In the following, by combining the concepts exhibited in the first half of the thesis a new stock mass structural optimization was developed by the author and the respective supervisors of this dissertation.

When considering the cutting stock problem in the context of structural optimization, the goal is to design a structure that minimizes material usage while still meeting the required strength and stiffness criteria. This can be achieved by determining the optimal arrangement in the stock of items that make up the structure.

To solve this problem, mathematical models can be developed that represent the structural design and the material properties, as well as the constraints and objectives of the problem. These models can then be solved using optimization techniques such as linear programming, mixed-integer programming, or metaheuristic algorithms.

In addition to minimizing material waste, the cutting stock problem can also help reduce the overall cost and environmental impact of a structure. By optimizing the use of materials, the structure can be made lighter and more efficient, leading to savings in both production and operation costs, as well as reducing the carbon footprint of the project.

In a nutshell, the method exposed in the following is a weight minimization of the entire stock of factory bars where the truss members were cut. The problem was subjected to the serviceability and strength constraints and solved by a Genetic Algorithm with guided crossover.

The chapter is organized in order to have initially the mathematical formulation of the optimization than the overall algorithm with a detailed explanation of the CSP frow-chart implemented into the process. Afterwards, there is a section dedicated to the metaheuristic algorithm involved in the code and then the case studies and the relative setting parameters.

4.1 Mathematical formulation of the structural optimization

With respect to traditional optimization approaches (i.e [16], [85], [45], [38]) in which the objective function represents the total weight of the structure as a sum of the mass of each element (structural mass), in this study the target function has been evaluated by computing the amount of steel requested during the production phase (stock mass). To achieve this goal, a real-coded guided-Genetic Algorithm (GA) has been developed by the author and the supervisors of the thesis. The design variables are the only discrete cross-sectional areas (taken from the cross-sectional commercial catalogue) of each element in case of a size optimization where is added the members' length in case of a combined size-shape optimization. CSP has been implemented within the optimization process and it has been independently solved for all groups of elements with the same cross-sectional properties. Finally, the solution obtained by the CSP for each group has been adopted for the evaluation of the objective function $W(x)$ expressed as follows:

$$W(x) = \phi_1 \rho \sum_{g=1}^{g=k} n_g A_g L_g \tag{4.1}$$

$W(x)$ is the optimization objective function and represents the total mass, more precisely the sum of the purchased bars' weight for each group g. Specifically, k represents the total number of groups of elements with the same cross-sectional areas. n_g, A_g and L_g are the cardinality, the cross-sectional area and the length of bins belonging to the same group g of elements with the same cross-sectional area A_g, respectively. ρ is material the mass density assumed to be equal for all members composing the structure (in this case the steel density).
The problem is constrained in order to guarantee structural conformity to the technical standards and these constraints are incorporated in the OF as a penalty coefficient ϕ_1. In figure 4.2 it is possible to see the general flowchart of the entire optimization.

STEP 1 Set the input assumption of the optimization problem. First, the model assumption as the features correlated with the geometry of the structure (number of elements and truss topology and eventual symmetry), the loading pattern (position, values and eventual symmetry) and the material properties (Young modulus E and density ρ). Afterwards, set the algorithm's parameters (number of iterations, number of individuals N per population, mutation probability and probability for roulette wheel parents selection)

STEP 2 Generate a random initial population with N individuals (N different trusses having the same topology but different design variables). The design variables are size and eventually shape parameters. The algorithm works with discrete design variables taken from commercial standards.

STEP 3 Perform the structural analysis for the given load condition in order to find the structural behaviour and verify the structural conformity to the Eurocode 3 (EN 1993-12005 and EN 1993-2 2006). The structural analysis is performed by a FEM code which uses the Direct Stiffness Method (DSM).

STEP 4 Computation of the overall penalty ϕ_1 related to the violation of the optimization constraints. The penalties are coefficients that increment the objective function of the individuals which not satisfy the problem constraint in order to penalize the unfeasible solutions. Specifically, the penalties are computed for each violation by penalty functions. The results of the penalty functions are coefficients ϕ_i for each i violation (e.g. strength, deflection, etc...). The final penalty ϕ_1 is the sum of the single penalty ϕ_i.

STEP 5 Verify if at least 1% of the entire population is feasible. If this condition is not satisfied reinitialize the entire population (return to *STEP 2*) with a discrete design variable domain reduce by a certain number of cross-sections. These excluded cross-sections are the smaller ones in order to increase, in the next iteration, the probability to have a population which fits the feasibility request of this decision point.

STEP 6 To obtain the number of standard commercial bars purchased and the relative cutting patterns for each cross-sectional group g. The grouping on the elements is necessary because the items with the same cross-sectional area should be allocated in the same bar typology. The CSP allocates the various structural members into the standard factory bars in an optimal way in order to minimize waste. For more details see figure 4.3 and the relative explanation.

STEP 7 Evaluation of the objective function $W(x)$ which return the total mass of the purchased bars.The OF is simply the sum of the purchased bars' mass resulting from the CSP for each cross-sectional group g multiplied for the overall penalty ϕ_1. The goal of the optimization is to reach the minimum $W(x)$.

STEP 8 Check of the stagnation condition. This step avoids the algorithm stuck in a local optimum while trying to search for a global optimum. Specifically, this condition verifies if the best solution is the same for a predetermined consecutive number of iterations. Whether the response is affirmative the optimization process re-initializes the population (return to *STEP 2*) in order to explore other solution spaces. Otherwise, the optimization process continues with the following steps.

STEP 9 This condition simply check if the imposed number of iteration is reached. Whether the number of the current iteration is lower the process comes to *STEP 3* otherwise continue with the plotting of the output results.

STEP 10 The outputs of the entire optimization process are final member properties, the overall mass of purchased steel, the number of bars for each cross-sectional class and the relative cutting pattern relative to the optimal individual.

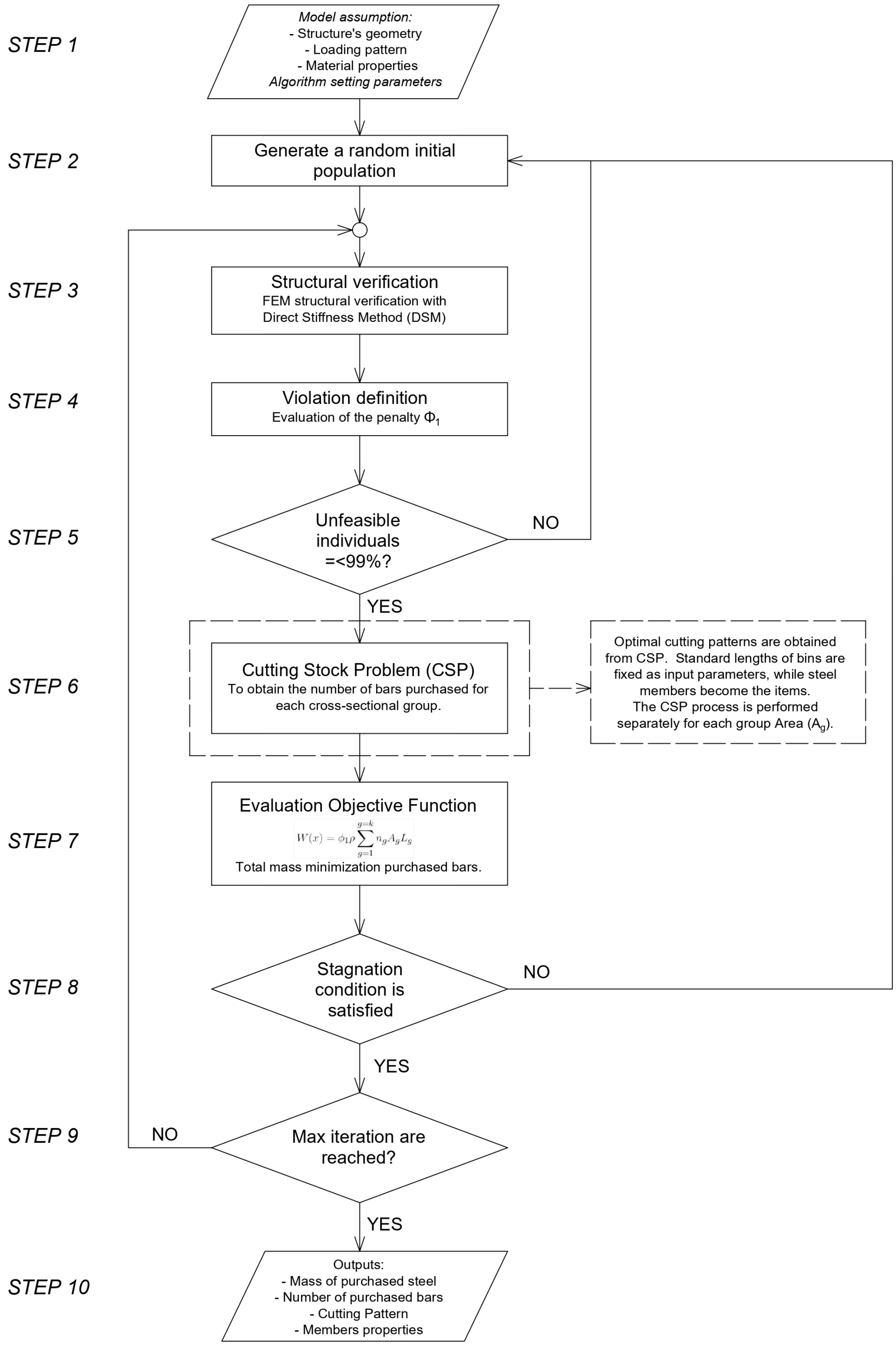

Figure 4.2: Structural Optimization via CSP problem algorithm

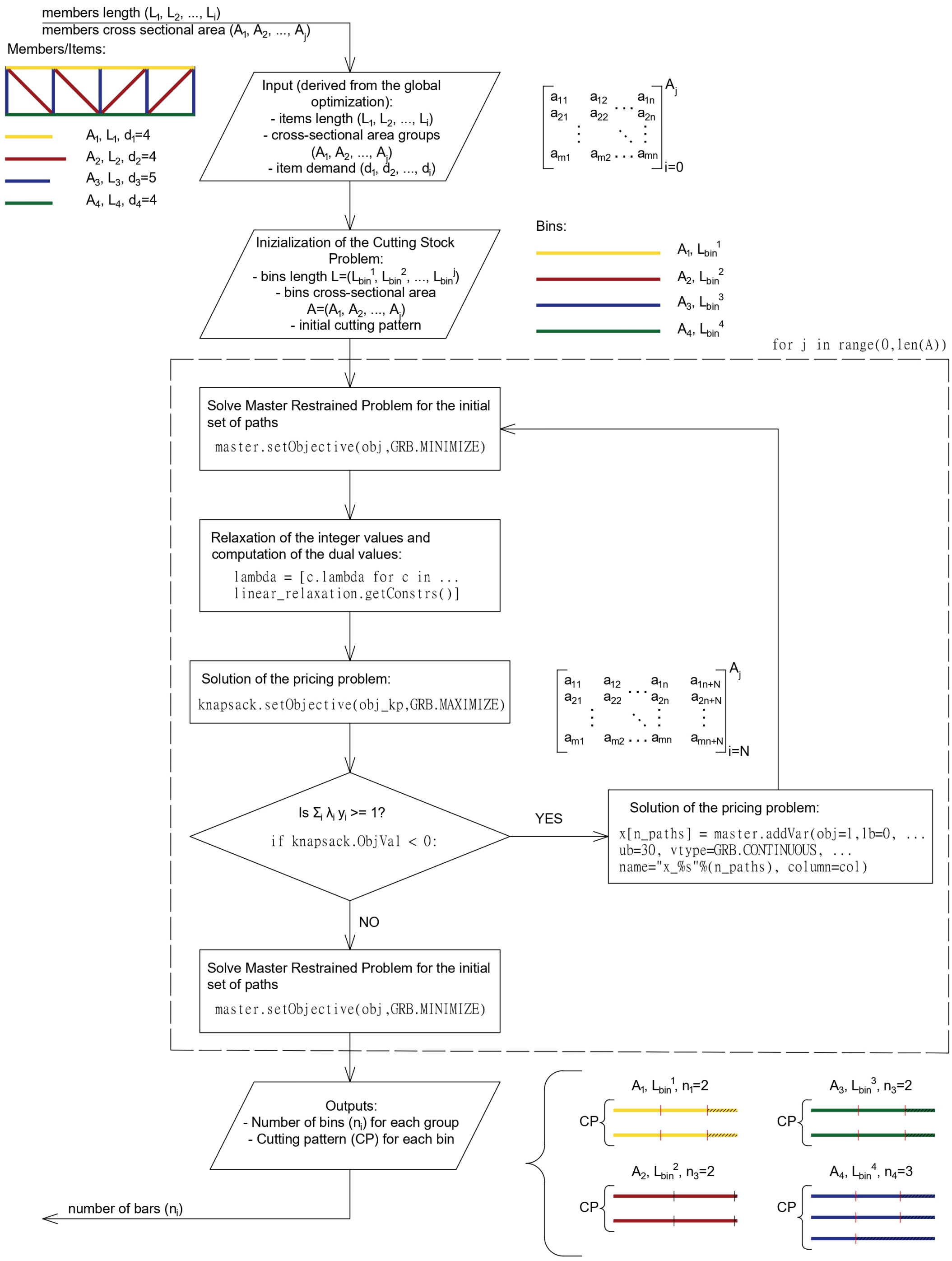

Figure 4.3: Detailed CSP algorithm embedded in the structural optimization

4.2 GA with guided crossover

In order to solve the problem stated in previous section, a real-coded GA is adopted. This is a population-based stochastic optimization technique appropriate for global optimization, which does not require direct evaluation of gradients. Introduced by John Holland [39], it is inspired by Charles Darwin's theory of natural evolution. This algorithm reflects the process of natural selection where the fittest individuals, also called parents, are selected for reproduction in order to produce offspring of the next generation. At the end of the process, the best survival among all the fittest candidates found at each generation is selected as the best globally optimized solution. Although the native GA worked with binary values representing genes, encoded in string structures called chromosomes, in this paper the authors overcome the limits related to the decoding process by using a real-code GA in which genes and chromosomes represent directly the design variables and the solutions of the problem.

At each iteration, traditional GA phases were adopted during the optimization as following:

- **Initial population**: in this phase, individuals with a set of random genes (x_i) composing chromosomes (x) are created by observing lower and upper bounds reported in table. Gene represents, at each generation, the candidate value of a specific design variable involved in the identification procedure. A set of genes (vector form) represent a solution of the problem for the current generation. In this way, the best solution is selected and the optimal set of parameters which govern that specific law is detected.

- **Fitness function**: in this phase, the fitness of the candidate solutions is evaluated by calculating the OF introduced in the previous section.

- **Selection**: During this phase, a Roulette Wheel Selection was implemented in order to guarantee that the two fittest parents are selected for the next steps. Adopting this technique, a probability to each parent is assigned and the parents with higher fitness are more likely to be chosen for the next steps.

- **Crossover**: in this phase, a single point crossover was performed in which recombination of gene pool between parents occurs after a position selected in a random way fro each parent. Lower and upper bounds are imposed at this stage such that if only a gene of the new offspring is not ranged within the imposed interval (higher than the maximum value or lower than the minimum value of that specific parameter), it is forced to assume maximum or minimum value, respectively.

- **Mutation**: aiming to improve the exploration and exploitation ability of the algorithm, a mutation rate of 1% is assigned. In this way, new genes are introduced into the population by modifying gene pools of parents in a random way.

At the end of these stages, a sorted function was implemented aiming to store survivors with the best fitness among the generated offspring, at each generation. The identification procedure can be considered ended when the stopping criteria of the algorithm are satisfied and the optimal set of parameters for each law is found. The entire procedure was run n times in order to check the reliability and robustness of the algorithm. Specifically, the authors observed that stagnation usually occurred for the algebraic models while no differential ones needed to use all the available 200 generations.

4.3 Case study 1: 10-bar truss

Within this section, it will be exhibited a simple application of CSP embedded in structural optimization. The structure under analysis in this section is a ten-bar truss benchmark coming from [79]. The case study is simple and the optimization assumptions are minimal just to check the results.
The section is divided into two subsections, the first show the model definition and the set of algorithm parameters and, afterwards, another subsection exposes and discusses the results.

4.3.1 Model definition and parameters' setting

As reported in Figure 4.4, the structure is a trussed isostatic cantilever composed of 10 steel bars and constrained by two pinned supports at nodes 5 and 6, respectively.

A single-loading condition P_1 has been assumed and, specifically, two equal forces P_1 are applied at nodes 2 and 4.

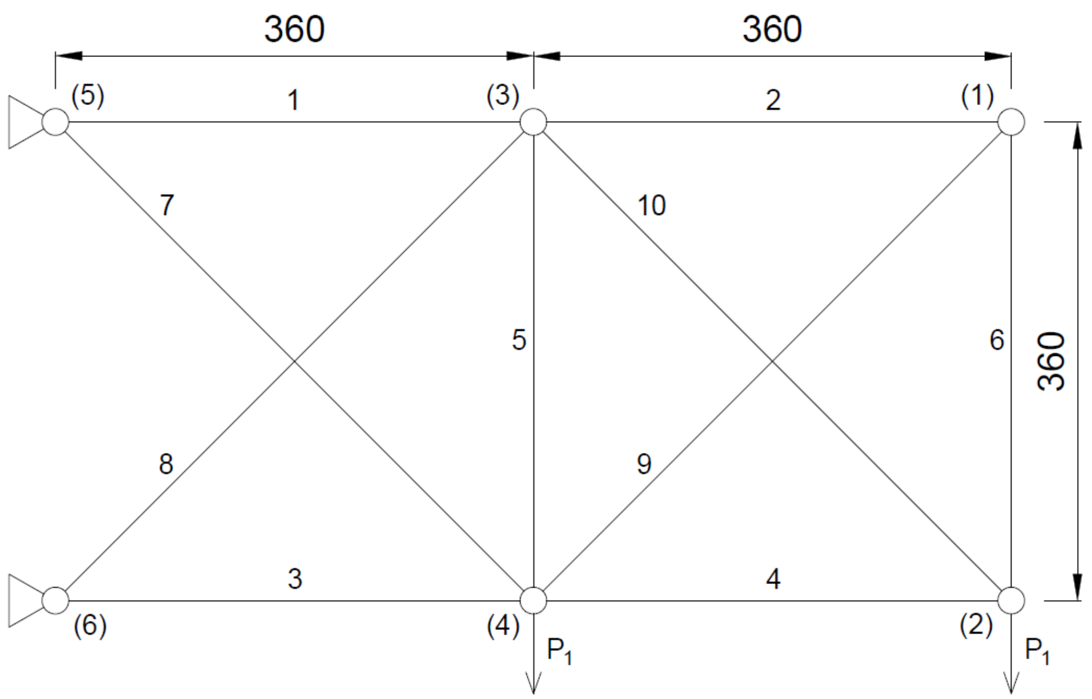

Figure 4.4: Configuration of the in-plane 10-bar truss, measures are expressed in inches (in.)

For having a resume of numerical model assumption on the material and geometry of the truss the table 4.1 was created.

Parameter	**Value**
Modulus of elasticity of steel E	10000 ksi
Steel density ρ	0.10 lb/in^3
Loading P_1	100 $kips$
Length of purchased bars (bins) L_{bin}	1020 in
Number of design variables	10
Bounds of design variables (H_{min}, H_{max})	[1.62, 33.5] in^2

Table 4.1: Model assumption relative to the 10 bar truss.

The design variables of the problem are simply the 10 cross-sectional areas of the truss components. The problem was solved by a GA algorithm which works with discrete values. This procedure allows taking into account only solutions with elements which have cross-sectional areas available in the commercial standards list. Therefore, a set of 42 discrete values (shown in table 4.2) have been used for the possible cross-sectional areas for each member. In this simplified optimization it was not taken into account the elemental buckling and, for this reason, only the cross-sectional areas of the elements are sufficient as sectional properties.

Discrete cross-sectional areas A $[in^2]$						
1.62	1.8	1.99	2.13	2.38	2.62	2.63
2.88	2.93	3.09	3.13	3.38	3.47	3.55
3.63	3.84	3.87	3.88	4.18	4.22	4.49
4.59	4.8	4.97	5.12	5.74	7.22	7.97
11.5	13.5	13.9	14.2	15.5	16	16.9
18.8	19.9	22	22.9	26.5	30	33.5

Table 4.2: Discrete cross-sectional standard area within the design variables are chosen.

Now the assumptions related to the model are imposed. It is possible to proceed with the optimization statement.
The statement of the entire optimization process is the following:

$$\min f(x) = W(x) \tag{4.2}$$

$$\textbf{Subject to} \quad \frac{N_{ED}}{N_{t,RD}} \leq 1 \tag{4.3}$$

$$\frac{N_{ED}}{N_{c,RD}} \leq 1 \tag{4.4}$$

$$u_{max,x} \leq u_{lim,x} \tag{4.5}$$

$$u_{max,y} \leq u_{lim,y} \tag{4.6}$$

The goal of structural optimization is the minimization of the objective function $W(x)$ (see (4.1)) correlated to the stock mass.
Equations from (4.3) to (4.6) represent the structural constraints of the problem.

In detail, strength verifications about tensile stress (without any holes) and compression stress according to Eurocode 3 (EN 1993-12005 and EN 1993-2 2006) are introduced by Equations (4.3) and (4.4) respectively. Other constraints to satisfy is the maximum deflection along x and y directions (represented by Equations (4.5) and (4.6), respectively).
To apply the previously seen constraints to the optimization process a penalty coefficient ϕ_1 is multiplied by the stock mass. In this way, the unfeasible solutions are penalized with respect to the feasible ones.
The penalty applied to the SO is the sum of the single penalty related to the single violation:

$$\phi_1 = \phi_{Nc} + \phi_{Nt} + \phi_{ux} + \phi_{uy} \tag{4.7}$$

In particular, the violation functions are simply equal to the sum of the verification ratios

$$\phi_Q = \sum_{i}^{i=v} \frac{Q^i_{Ed}}{Q_{lim}} \tag{4.8}$$

Where Q is the constraint parameter (stress or deflections) for an element of the truss i which does not satisfy the constraint condition. The overall penalty related to a constraint ϕ_Q is the sum of the violation of all the violated items v. Particularly, the effective solicitation or deflection is at the numerator and the limit value is at the denominator. The displacements of the free nodes in both directions had to be less than ±2 in. and the allowable stress was set to ±25 ksi.
In this specific case study, two attempts were done with the implementation of an additional penalty ϕ_R related to the ripetitivity of the cross-section among the design variables.

$$\phi_R = kN_{single_A} \tag{4.9}$$

Where k is a proportional coefficient which set the impact of the ripetitivity penalty on the objective function. Conversely, N_{single_A} is the number of different areas present within the design variable. This penalty is added to the (4.1) but as it will exposes more in detail in the next paragraph the results are worse compared the original formulation.

The algorithm setting parameters are collected in table 4.3.

Parameter	**Value**
Maximum number of iterations	200
Number of individuals per population	200
Areas excluded if unfeasibility condition isn't satisfied	5 smaller DV
Mutations' probability	1%
Proportional children	1 child for each parent
Stagnation condition	10 iterations

Table 4.3: Optimization algorithm parameters set by the operator.

4.3.2 Results and discussion

In this section, the results of the structural optimization (SO) process with specific regard to the ten-bar case study are pointed out. Specifically, two optimization scenarios have been performed:

- Scenario (a): optimization by considering CSP (minimization of purchased steel bars).
- Scenario (b): optimization via traditional approach by minimizing the total weight of the structure without considering the CSP procedure.

In order to have a comparison between the two mentioned-above approaches, the CSP procedure has been performed at the end of (b) such that the total number of bins requested for the assemblage of the optimal weight structure has been evaluated. Afterwards, a case of the SO via CSP with the implementation of the ripetitivity penalty is exhibited.

Scenario (a) In scenario (a) 10 runs are performed through SO via CSP and the results are summarised in table 4.6 at the end of the section. Afterwards, from the data collected the best result and the mean value and standard deviation are collected in the following table 4.4.

The first observations concern the operation of the CSP in the SO. It can be seen that the results tend to have each bar with a different cross-section, containing 2

	Stock Mass (OF) [lb]	Structural Mass [lb]	Waste Mass [lb]
Best	6825.84	5791.97	1033.87
μ	7320.13	6158.41	1161.73
σ	308.64	257.05	98.37

Table 4.4: Result of the optimization via CSP related to Scenario (a)

elements per bar (corresponding to the maximum number of elements that can be allocated in a bar). These two features allow for obtaining the best results because exploit at the maximum capacity the bars and simultaneously have a good degree of freedom in the size optimization of the elements.
In other words, the OS works on two levels: the correct choice of cutting pattern to maximise the utilisation of the bars and the selection of the minimum cross-sectional area for each bar.
Figure 4.5 gives a more clear view of the bars' exploitation in scenario (a).

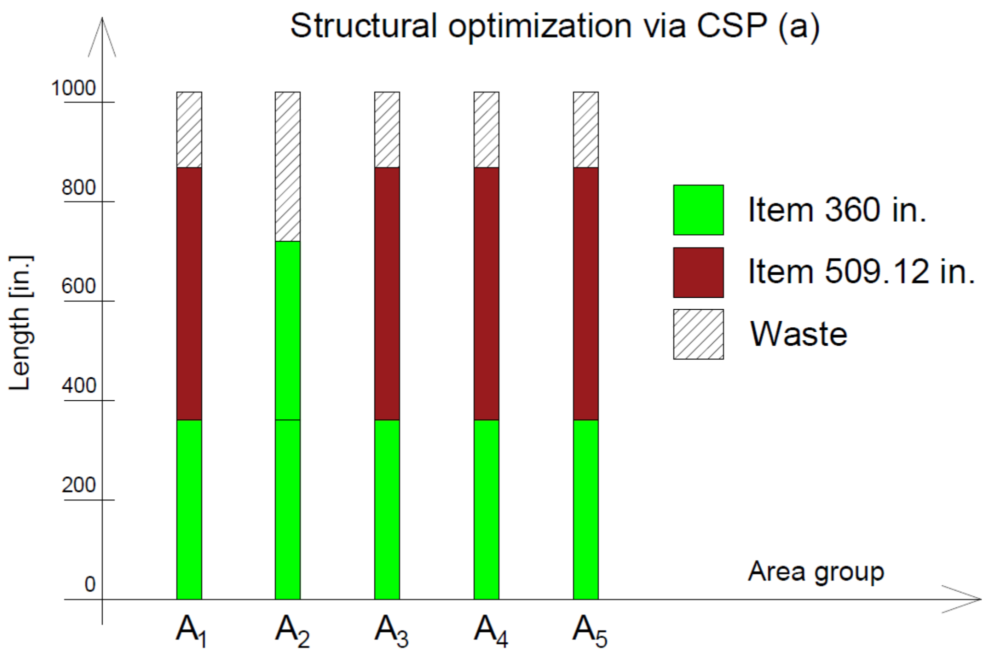

Figure 4.5: Optimal cutting pattern derived by optimization scenario (a)

To show how the algorithm works in more detail, the plots of the best solution (Figure 4.6), the violation (Figure 4.7) and the unfeasibility (Figure 4.8) for each iteration are presented on following.

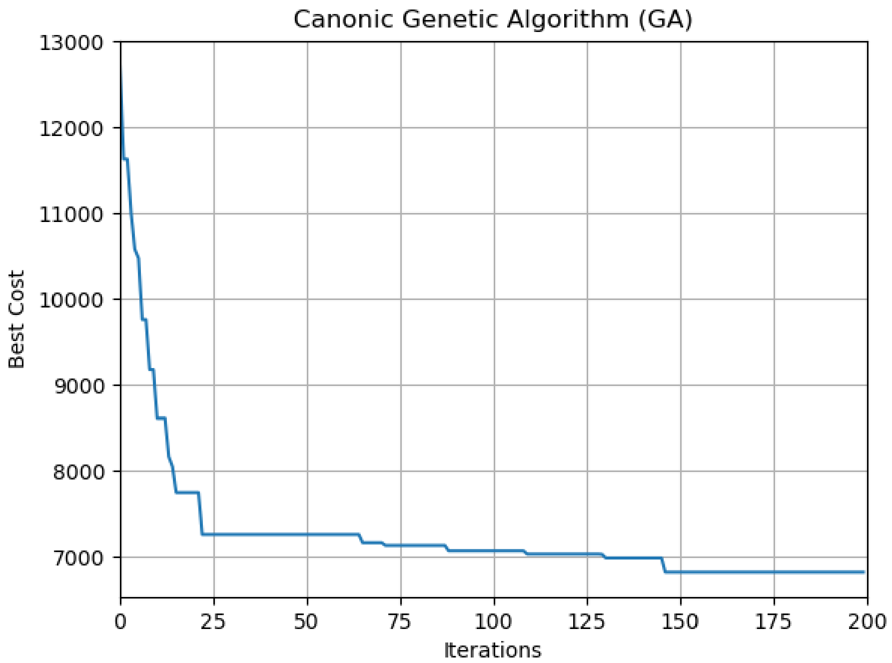

Figure 4.6: OF best solution for each iteration related to Scenario (a)

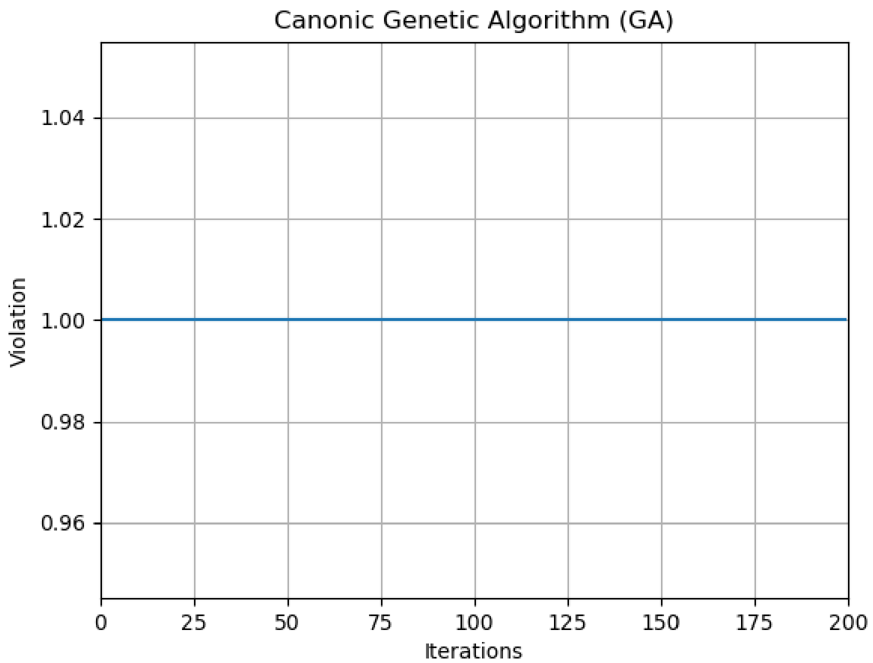

Figure 4.7: OF violation for each iteration related to Scenario (a)

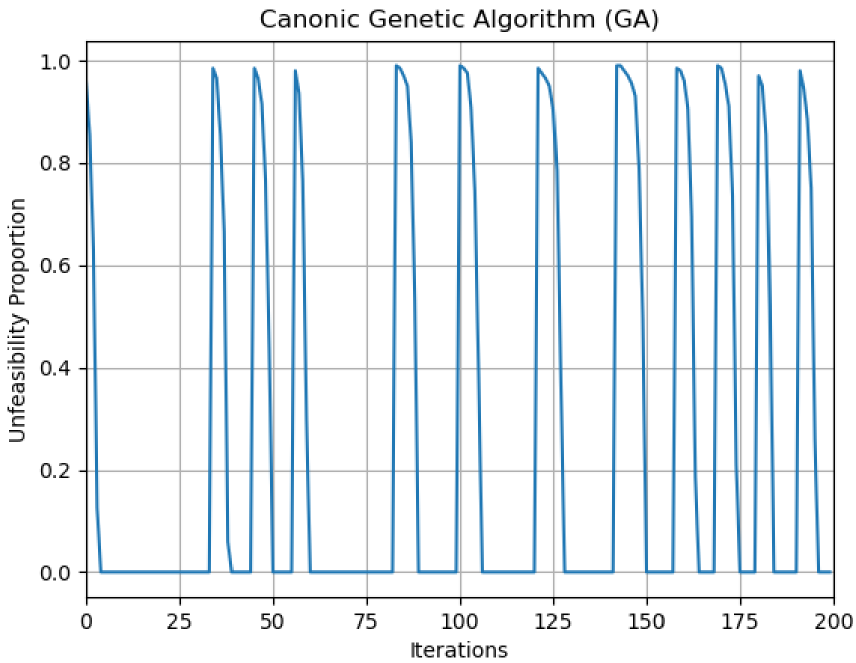

Figure 4.8: OF unfeasibility for each iteration related to Scenario (a)

Scenario (b) Scenario (b) consists again of 10 runs performed with a traditional structural optimization which minimizes the total structural mass only. The mass of the stock was obtained successively by applying the CSP to the cross sections obtained. The results of the 10 runs are summarised at the end of the section in table 4.7. The best solution, the average solution and the standard deviation are collected in the following table.

	Stock Mass (OF) [lb]	Structural Mass [lb]	Mass Waste [lb]
Best	10323.42	5580.42	4743.00
μ	12133.92	5632.88	6501.04
σ	1343.16	64.15	1348.10

Table 4.5: Result of the optimization via traditional approach (b)

On following the stock representation (Figure 4.9) and the plots of the best solution (Figure 4.10), the violation (Figure 4.11) and the unfeasibility (Figure 4.12) for each iteration.

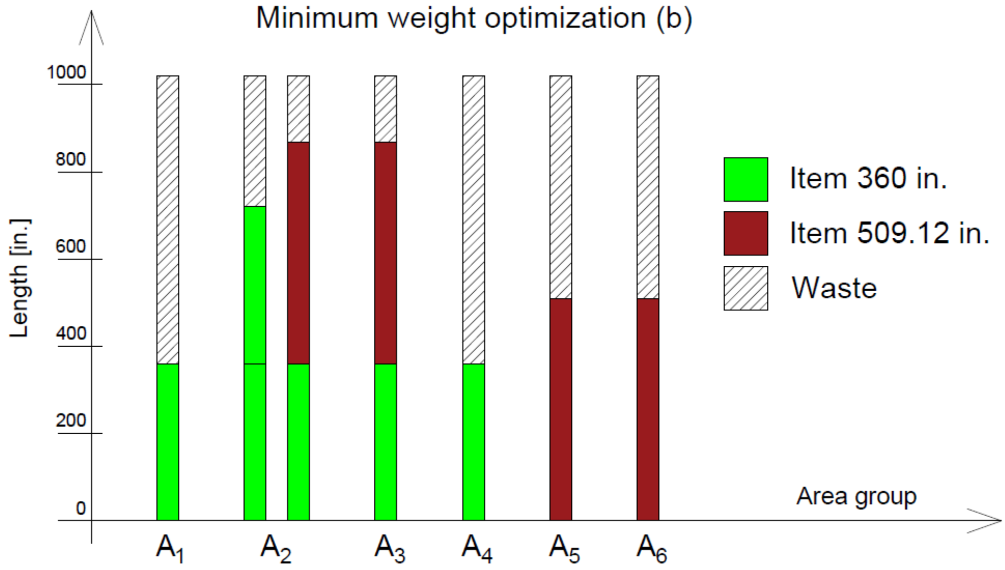

Figure 4.9: Optimal cutting pattern derived by optimization scenario (b)

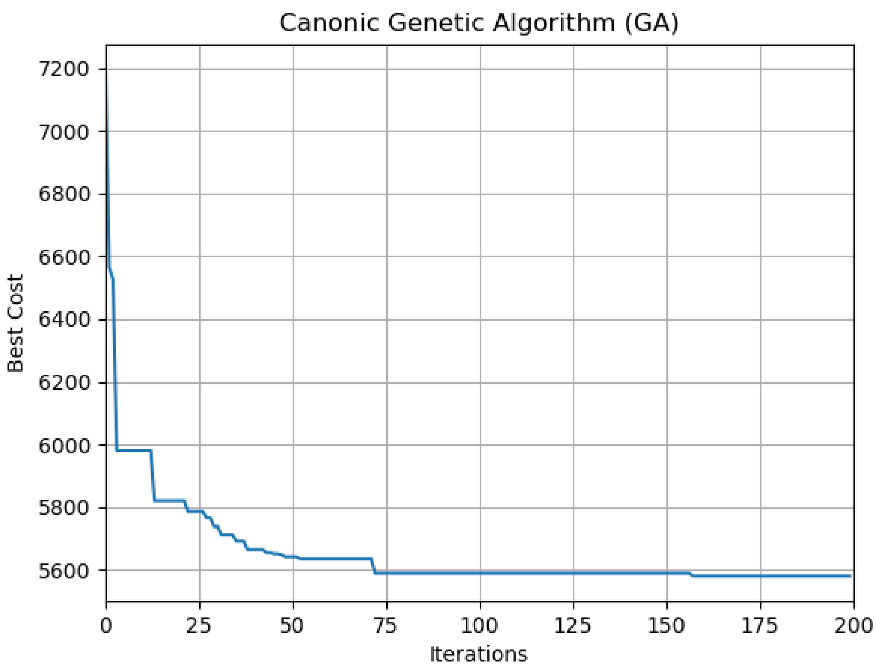

Figure 4.10: OF best solution for each iteration related to Scenario (b)

Now it is possible to compare the results of the two optimizations (Scenario (a) and Scenario (b)). The first important result is that the stock mass is much lower in Scenario (a) versus a small increment of the structural mass which is very

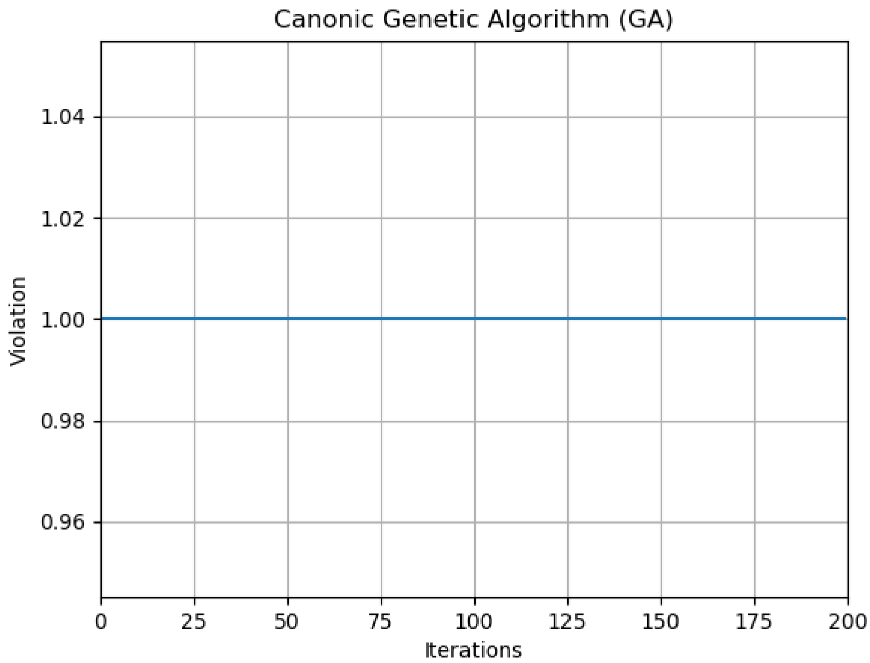

Figure 4.11: OF violation for each iteration related to Scenario (b)

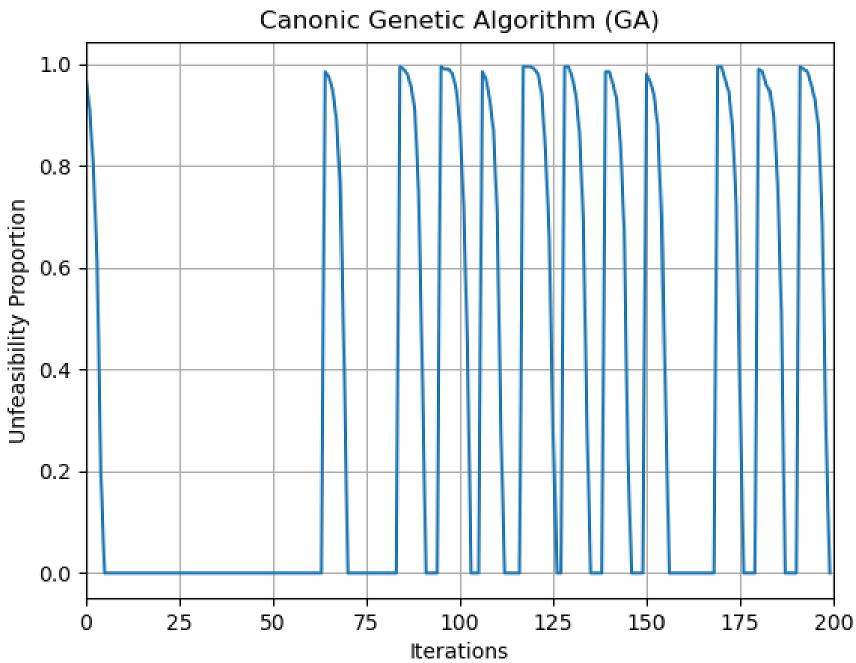

Figure 4.12: OF unfeasibility for each iteration related to Scenario (b)

similar between the two scenarios. From these considerations, it is possible to deduce that the algorithm with embedded the CSP perform well because requests fewer purchased bars with a small increment of the structural weight.
Another consideration is about the variability of the problem which presents better results in terms of a stock mass standard deviation in scenario (a) and regarding the structural mass the standard deviation is lower in scenario (b). Moreover, the standard deviations' values of the relative OF (stock mass for (a) and structural mass for (b)) penalize scenario (a) because the variability of the cutting pattern does not allow to have a strong convergence of the results.

Attempt	Cross-sectional areas [in^2]	Number of bins	Structural Mass [lb]	Stock Mass [lb]	Waste Mass [lb]
1	[30.0 1.99 22.0 22.0 1.99 4.18 13.9 13.9 30.0 4.18]	[1, 1, 1, 1, 1]	6113.27	7351.14	1237.87
2	[18.8 4.22 26.5 18.8 1.8 1.8 26.5 22.0 22.0 4.22]	[1, 1, 1, 1, 1]	6393.24	7478.64	1085.40
3	[30.0 1.8 30.0 13.5 1.8 1.62 13.5 22.0 22.0 1.62]	[1, 1, 1, 1, 1]	5843.82	7029.84	1186.02
4	[22.0 1.62 22.0 22.0 1.99 1.62 22.9 22.0 22.9 1.99]	[2, 1, 1, 1]	6117.41	7192.02	1074.61
5	[30.0 1.99 30.0 13.5 1.99 1.99 13.5 22.0 22.0 1.99]	[1, 2, 1, 1]	5889.66	7086.96	1197.30
6	[30.0 1.62 22.0 11.5 1.62 1.8 11.5 22.0 30.0 1.8]	[1, 1, 1, 1, 1]	5791.97	6825.84	1033.87
7	[30.0 3.13 22.9 15.5 3.13 2.88 15.5 30.0 22.9 2.88]	[1, 1, 1, 1, 1]	6420.43	7589.82	1169.39
8	[26.5 4.49 26.5 13.9 4.49 15.5 15.5 16.9 16.9 13.9]	[1, 1, 1, 1, 1]	6507.30	7883.58	1376.28
9	[26.5 3.87 26.5 13.9 2.88 2.88 26.5 26.5 13.9 3.87]	[2, 1, 1, 1]	6358.10	7512.30	1154.20
10	[26.5 1.99 22.0 18.8 1.99 1.8 26.5 18.8 22.0 1.8]	[1, 1, 1, 1, 1]	6148.88	7251.18	1102.30

Table 4.6: Structural Optimization via CSP results of 10 runs. Scenario (a)

Attempt	Cross-sectional areas [in^2]	Number of bins	Structural Mass [lb]	Stock Mass [lb]	Waste Mass [lb]
1	[30. 1.8 26.5 14.2 1.62 1.62 13.5 18.8 22. 1.62]	[1 1 1 1 2 1 1 1]	5573.62	13264.08	7690.46
2	[30. 1.8 26.5 16. 1.62 1.62 11.5 18.8 22. 1.8]	[1 1 1 1 1 1 1 1]	5545.762334	13078.44	7532.68
3	[22.9 1.62 30. 13.9 1.62 1.62 15.5 22. 22.9 1.8]	[1 2 1 1 1 1 1]	5746.47	11152.68	5406.21
4	[30. 1.62 26.5 11.5 1.62 1.62 7.97 26.5 22. 1.62]	[1 2 1 1 1 1]	5580.42	10323.42	4743.00
5	[30. 3.38 26.5 13.5 1.62 1.8 15.5 16.9 22. 3.47]	[1 1 1 1 1 1 1 1 1 1]	5711.06	13736.34	8025.28
6	[26.5 1.62 22.9 15.5 1.62 1.62 13.5 22.9 22.9 1.62]	[1 2 2 1 1]	5612.90	10663.08	5050.18
7	[30. 1.62 26.5 13.9 1.62 1.62 13.5 22. 18.8 2.38]	[1 2 1 1 1 1 1 1]	5595.03	13292.64	7697.61
8	[26.5 1.8 30. 14.2 1.62 1.62 13.9 22.9 18.8 1.99]	[1 1 1 1 1 1 1 1 1]	5658.64	13434.42	7775.78
9	[30. 1.8 26.5 15.5 1.8 2.13 16. 18.8 18.8 2.88]	[1 1 1 1 1 1 1 1]	5673.77	11588.22	5914.45
10	[26.5 2.13 26.5 16. 1.62 1.8 14.2 18.8 22.9 1.99]	[1 1 1 1 1 1 1 1 1]	5631.08	10805.88	5174.80

Table 4.7: Traditional Structural Optimization (Structural mass only) results of 10 runs. Scenario (b)

Ripetitivity penalty With the idea of helping the algorithm find better results in terms of stock mass, a ripetitivity penalty was introduced that increases the OF value based on the number of different cross-sectional areas between the design variables. Therefore, greater is the number of cross-sectional different areas greater is the penalty.

$$\phi_R = kN_{single_A} \tag{4.10}$$

Where k and N_{single_A} are a coefficient which set the impact of the penalty on the OF and the number of different areas present within the design variable respectively. In the following pages in the tables 4.10 and 4.11 are presented the results of 10 runs (for k=0.1 and k=0.2 respectively). Meanwhile, in the following table the best solution, the average one and the standard deviation are collected (table 4.8 and 4.9)

	Stock Mass (OF) [lb]	Structural Mass [lb]	Mass Waste [lb]
Best	7127.76	6043.71	1084.05
μ	8128.38	6751.39	1376.99
σ	664.19	467.16	248.40

Table 4.8: Result of the optimization via CSP considering ripetitivity ($k = 0.1$)

	Stock Mass (OF) [lb]	Structural Mass [lb]	Mass Waste [lb]
Best	8051.88	6816.97	1234.91
μ	8768.23	7367.95	1400.28
σ	611.95	549.00	209.11

Table 4.9: Result of the optimization via CSP considering ripetitivity ($k = 0.2$)

The algorithms despite the low values of k do not work properly. The OF values are worse and the standard deviation too. These results show that considering the ripetitivity in this specific problem makes the algorithm more rigid because forces purchased bars to have the same cross-sectional areas. Therefore, a reduction of the degree of freedom in the problem has as a consequence less difference in item areas. Moreover, the reduction of the available area produces items with an overestimated cross-sectional area which increase both the structural and stock mass.

Attempt	Cross-sectional areas [in^2]	Number of bins	Structural Mass [lb]	Stock Mass [lb]	Waste Mass [lb]
1	[22.9 13.9 22.9 14.2 7.97 7.97 22. 22. 14.2 13.9]	[1, 1, 1, 1, 1]	6904.97	8258.94	1353.97
2	[26.5 4.97 26.5 13.9 4.97 13.9 13.9 26.5 13.9 26.5]	[2, 1, 2]	7380.30	8748.54	1368.24
3	[30. 4.49 22.9 22.9 1.99 1.99 30. 16. 16. 4.49]	[1, 1, 1, 1, 1]	6418.84	7688.76	1269.92
4	[30. 1.99 22. 13.9 1.99 1.99 13.9 30. 22. 1.99]	[1, 2, 1, 1]	6043.71	7127.76	1084.05
5	[26.5 14.2 26.5 14.2 16.9 3.47 16.9 26.5 26.5 3.47]	[2, 1, 1, 1]	7399.11	8932.14	1533.03
6	[30. 13.9 30. 13.5 13.5 13.5 13.5 13.9 16. 16.]	[1, 1, 2, 1]	7142.55	8863.80	1721.25
7	[22.9 4.8 22.9 15.5 4.8 7.97 15.5 30. 30. 4.8]	[1, 2, 1, 1, 1]	6927.53	8768.94	1841.41
8	[30. 4.18 30. 7.22 4.18 7.22 16. 18.8 16. 18.8]	[1, 1, 1, 1, 1]	6524.25	7772.40	1248.15
9	[26.5 3.87 22. 22. 2.38 2.38 26.5 18.8 18.8 3.87]	[1, 1, 1, 1, 1]	6309.15	7502.10	1192.95
10	[22. 1.99 30. 13.5 7.22 1.99 30. 13.5 22. 7.22]	[1, 1, 1, 1, 1]	6463.50	7620.42	1156.92

Table 4.10: Structural Optimization via CSP including ripetitivity (k=0.1) results of 10 runs

Attempt	Cross-sectional areas [in^2]	Number of bins	Structural Mass [lb]	Stock Mass [lb]	Waste Mass [lb]
1	[26.5 7.22 26.5 7.22 7.22 7.22 26.5 26.5 11.5 11.5]	[2, 2, 1]	6816.97	8051.88	1234.91
2	[30. 4.49 30. 4.59 4.49 4.59 30. 15.5 30. 15.5]	[2, 1, 1, 1]	7446.72	8627.16	1180.44
3	[26.5 22. 26.5 16.9 4.59 4.59 22. 11.5 16.9 11.5]	[1, 1, 1, 1, 1]	6790.31	8311.98	1521.67
4	[30. 4.8 16. 16.9 4.8 16.9 18.8 16. 30. 18.8]	[1, 1, 1, 1, 1]	7474.62	8823.00	1348.38
5	[26.5 4.49 26.5 16. 1.99 16. 16. 16. 16. 16.]	[1, 1, 3, 1]	6551.63	8259.96	1708.33
6	[30. 7.22 30. 7.22 2.38 2.38 30. 30. 13.9 13.9]	[2, 1, 1, 1]	7321.25	8517.00	1195.75
7	[22.9 26.5 26.5 16. 16. 7.97 26.5 7.97 26.5 22.9]	[1, 2, 1, 1]	8441.28	10186.74	1745.46
8	[22.9 26.5 26.5 7.97 7.22 7.22 26.5 22.9 26.5 7.97]	[1, 2, 1, 1]	7809.12	9291.18	1482.06
9	[30. 1.99 30. 7.97 7.97 1.99 30. 15.5 30. 15.5]	[2, 1, 1, 1]	7510.08	8716.92	1206.84
10	[30. 11.5 30. 11.5 4.22 4.22 30. 11.5 30. 11.5]	[2, 2, 1]	7517.51	8896.44	1378.93

Table 4.11: Structural Optimization via CSP including ripetitivity (k=0.2) results of 10 runs

4.4 Case study 2: Symmetric Warren truss

Within this section, a more complex application example is exhibited. The structure under analysis is a Warren truss with 23 members. The structure is subjected to symmetric loading and a symmetric geometry is assumed. This assumption approximately halves the design variables which becomes 12 cross-sectional areas.
The section is divided into two subsections, the first show the model definition and the set of algorithm parameters and, afterwards, another subsection exposes and discusses the results.

4.4.1 Model definition and parameters' setting

This case study examines a single-support Warren truss with 23 elements that is being loaded uniformly in-plane on the lower chord. This decision was made in order to minimize production costs due to the Warren-type truss's reduced joint count as compared to other types (like the Pratt or Vierendeel types). The complete length L of the truss is likewise displayed in Figure 4.13, divided into 6 spans with a spacing of L/6.

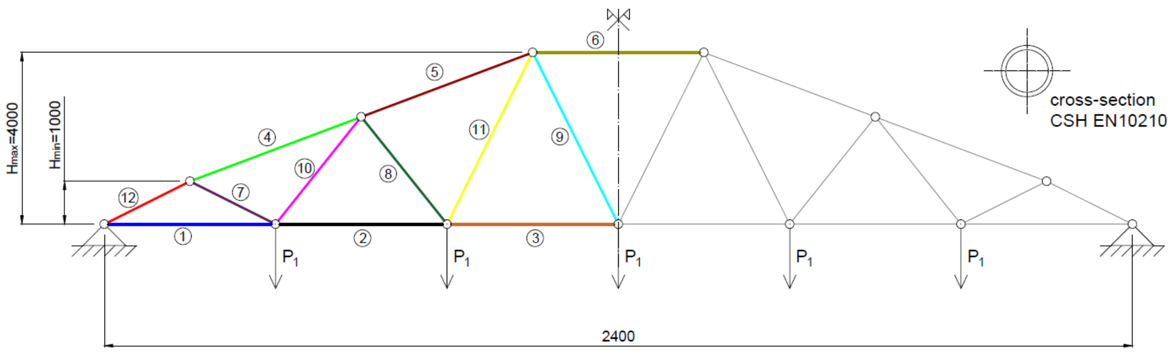

Figure 4.13: Configuration of the Warren truss under analysis, the numbers indicate the design variables which are 12 because the symmetry is considered, measures are expressed in millimetres (mm)

The truss to be optimized is characterized by hollow members in particular in this truss typology Circular Hollow Sections (CHS) are used in particular a standard cross-sectional list from the standard code is used (EN 10210). In fact, hollow sections are particularly effective in compression as previously mentioned because the material is far from the section axis, increasing the resistance to buckling. In some situations, hollow sections can also be more affordable than other profiles, such as when there are lower loads, cheaper steel prices, and higher hourly labour costs [50]. For reducing the computational effort and aiding the algorithm to search for the optimum symmetry of the truss is taken into account. This assumption almost halves the design variables which becomes 12 instead to 23 (the design variables of the problem are indicated in Figure 4.13.

The truss is loading symmetrically by considering a load due to the permanent non-structural load equal to 4 kN/m^2 a snow loading equal to 1.5 kN/m^2 and a maintenance loading of 0.5 kN/m^2. To obtain the in-plane loading of the truss, an influence area of 10 m is considered, which is the distance between two Warren trusses in a roof system. Afterwards, the uniform distributed load along the lower chord of the Warren truss is divided into 5 concentrated forces P_1 concentrated into the lower chord internal nodes. The self-weight of the structure is considered separately in the FEM code used for the structural analysis.

All the specifications regarding the material and the features of the model are reported in table 4.12.

Parameter	**Value**
Modulus of elasticity of steel E	210000 MPa
Steel density ρ	7.85 t/m^3
Loading lower chord nodes P_1	240 kN
Length of purchased bars (bins) L_{bin}	15 m
Number of design variables	12
Bounds of design variables (A_{min}, A_{max})	[182, 24700] mm^2

Table 4.12: Model assumption relative to the symmetric Warren truss.

The statement of the entire optimization process is similar to Case Study 1 and is resumed in the following:

$$\min f(x) = W(x) \tag{4.11}$$

$$\textbf{Subject to} \quad \frac{N_{ED}}{N_{t,RD}} \leq 1 \tag{4.12}$$

$$\frac{N_{ED}}{N_{c,RD}} \leq 1 \tag{4.13}$$

$$\frac{N_{ED}}{N_{b,RD}} \leq 1 \tag{4.14}$$

$$u_{max,x} \leq u_{lim,x} \tag{4.15}$$

$$u_{max,y} \leq u_{lim,y} \tag{4.16}$$

An explanation of the constraints is available in the previous case study. The only noticeable difference is the equation (4.14) which assesses the buckling instability requirement according to Eurocode 3 (EN 1993-12005 and EN 1993-2 2006).
The violation comes from the constraint is computed as in the previous case study, the only difference is that the ϕ_1 used into the $W(x)$ is no more the sum between the various singular violations but the greater of them.

$$\phi_1 = max(\phi_{Nc}, \phi_{Nt}, \phi_{Nb}, \phi_{ux}, \phi_{uy}) \tag{4.17}$$

The settings of the algorithm are shown in the following table 4.13. Considering

Parameter	**Value**
Maximum number of iterations	200
Number of individuals per population	300
Areas excluded if unfeasibility condition isn't satisfied	5 smaller DV
Mutations' probability	5%
Proportional children	1 child for each parent
Stagnation condition	20 iterations

Table 4.13: Optimization algorithm parameters set by the operator.

that the complexity of the problem is grown with respect to the 10-bar-truss case study the number of individuals is increased to 300 and the stagnation is raised to 20 as well. A modification of the stagnation reinitialization is performed. The algorithm does not reinitialize all the individuals but stores the 10 best individuals in order to not lose the genetic pool.

4.4.2 Results and discussion

As in the previous case study, two optimization scenarios are investigated by performing 10 runs per scenario:

- Scenario (a): optimization by considering CSP (minimization of purchased steel bars).
- Scenario (b): optimization via traditional approach by minimizing the total weight of the structure without considering the CSP procedure.

Scenario (a) Regarding the first scenario (a) the results of the optimization are synthesised in the following table and figures referred to the best solution. For more detail about the ten runs see the table 4.16.

	Stock Mass (OF) [kg]	Structural Mass [kg]	Mass Waste [kg]
Best	4341.44	3006.42	1335.03
μ	5249.65	2952.80	2296.85
σ	391.43	292.55	391.10

Table 4.14: Result of the symmetric Warren optimization via CSP approach (a)

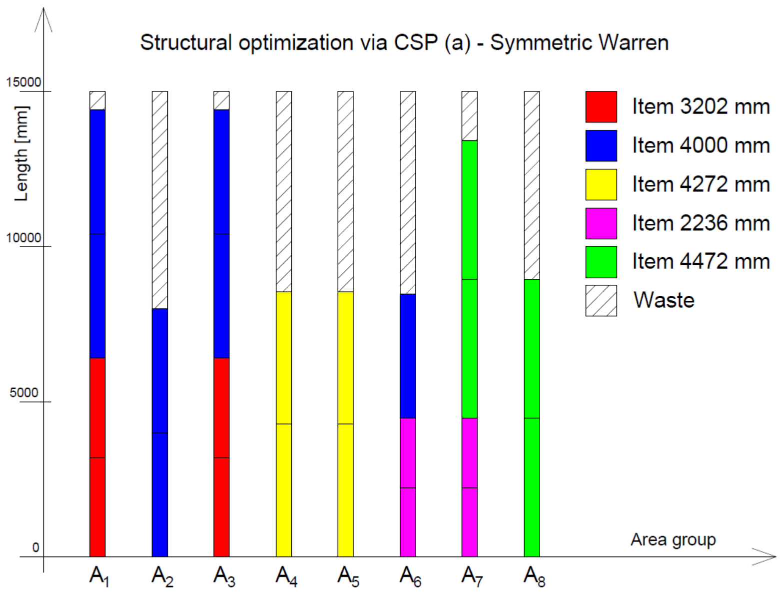

Figure 4.14: Optimal cutting pattern derived by optimization via CSP scenario (a) - Symmetric Warren

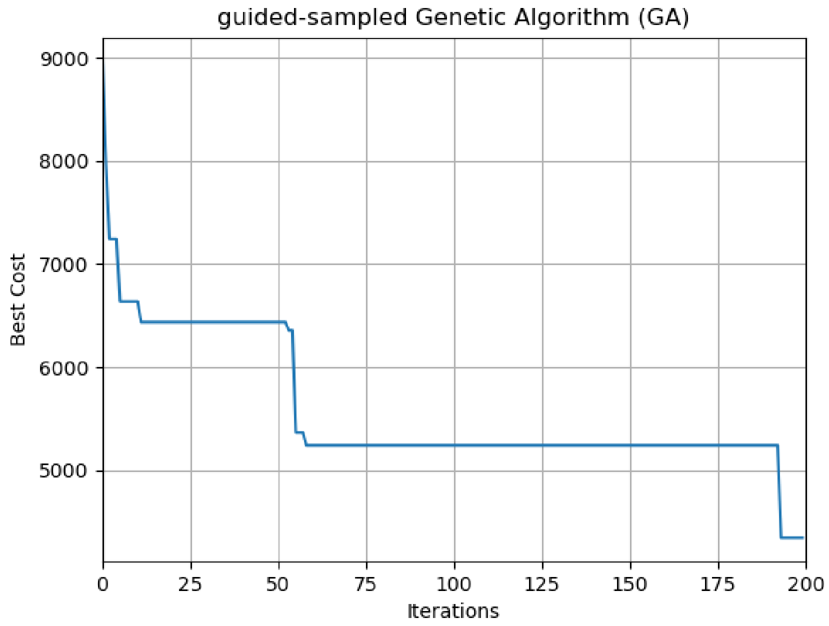

Figure 4.15: OF best solution for each iteration related to Scenario (a) - Symmetric Warren

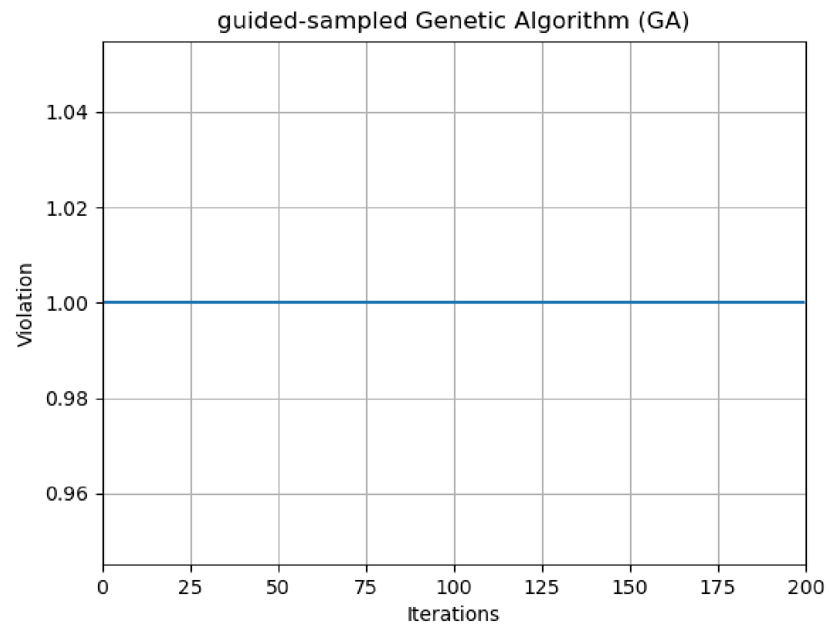

Figure 4.16: OF violation for each iteration related to Scenario (a) - Symmetric Warren

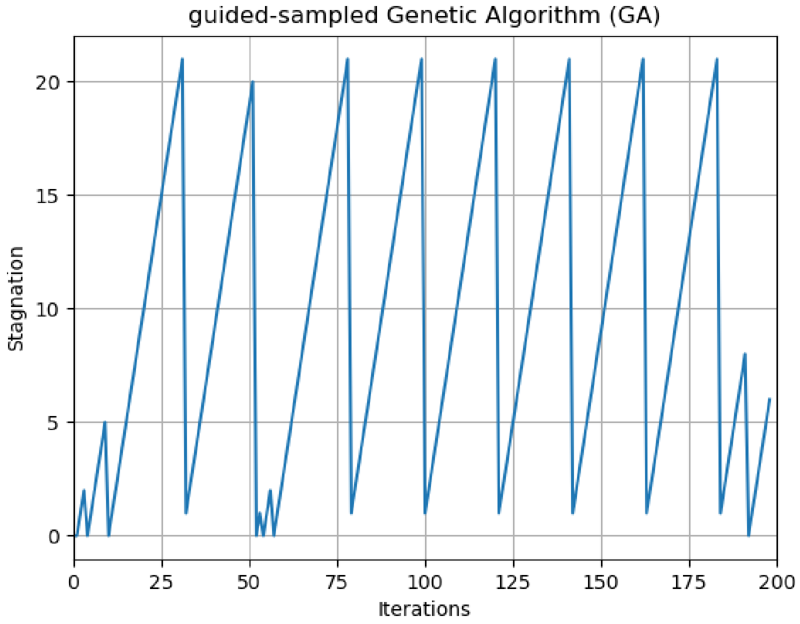

Figure 4.17: OF stagnation for each iteration related to Scenario (a) - Symmetric Warren

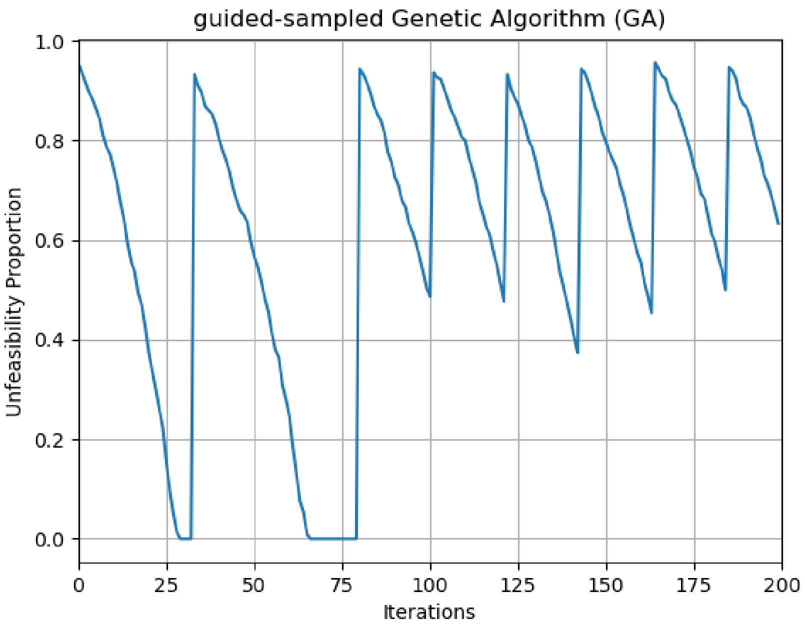

Figure 4.18: OF unfeasibility for each iteration related to Scenario (a) - Symmetric Warren

The results of scenario (a) as it is possible to see in the stock representation (figure 4.14) are not optimal because the grouping of the areas does not reach the minimum in terms of the number of bars. For instance, it is possible to notice that the waste in the bars A_2, A_6, A_8 is greater than the length of each item. As it is possible to see in the best solution evolution through the 200 iterations plot (figure 4.15) achieving the best solution is more difficult and there are long stagnation points. The standard deviation of the stock mass is similar to the 10-bar truss case study and is related to the innate variability of the CSP problem.

Scenario (b) Now on following are exhibited the results and the plots related to scenario (b). More in detail the 10 runs results are shown in table 4.17.

	Stock Mass [kg]	Structural Mass (OF) [kg]	Mass Waste [kg]
Best	4610.62	2395.22	2215.40
μ	5832.42	2717.97	3114.45
σ	531.14	167.85	392.44

Table 4.15: Result of the symmetric Warren optimization via traditional approach (b)

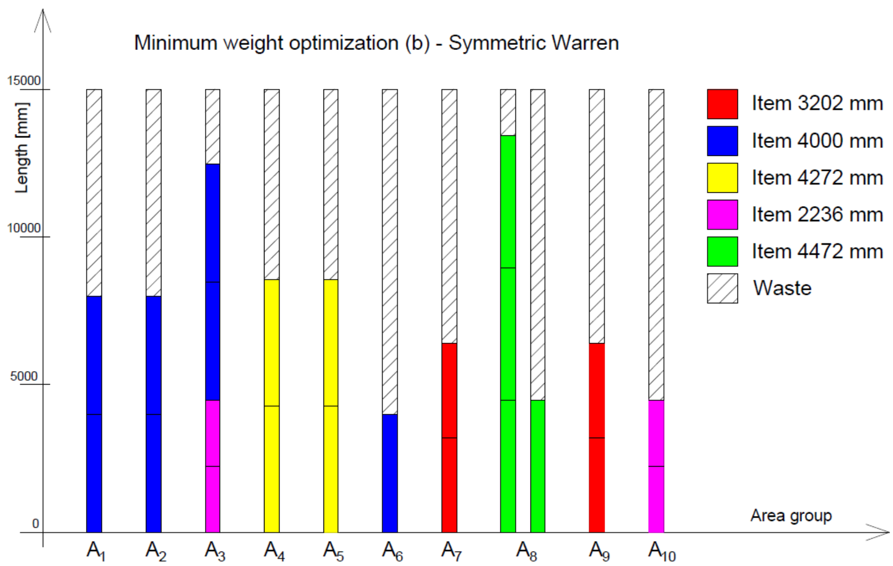

Figure 4.19: Optimal cutting pattern derived by traditional optimization scenario (b) - Symmetric Warren

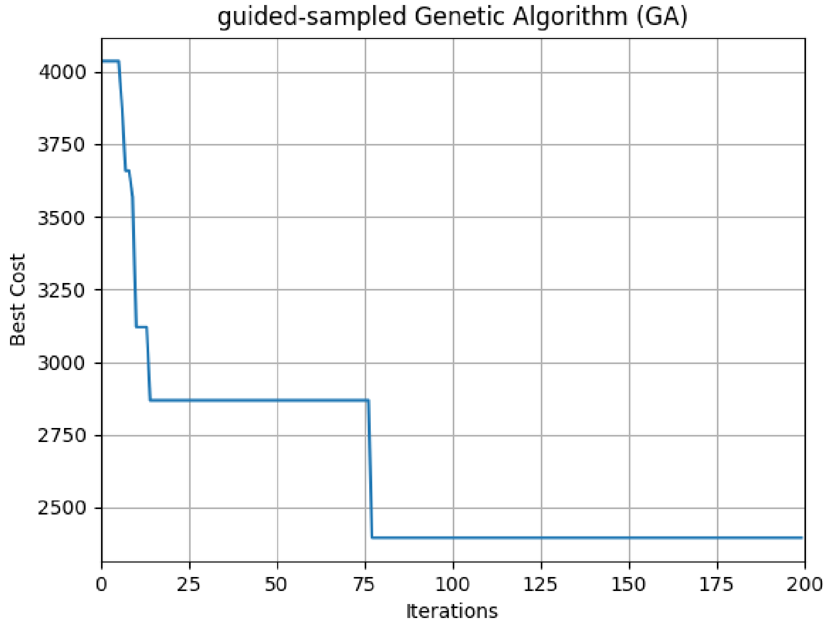

Figure 4.20: OF best solution for each iteration related to Scenario (b) - Symmetric Warren

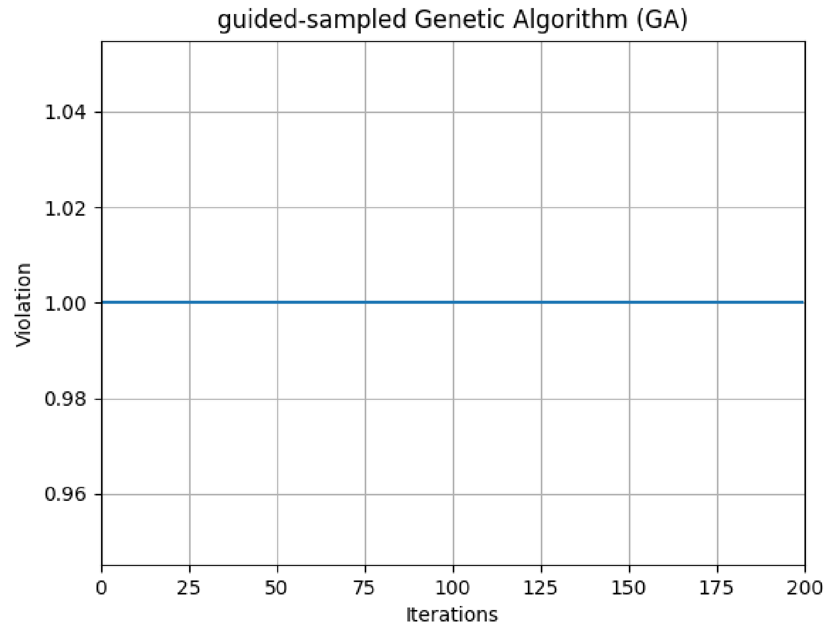

Figure 4.21: OF violation for each iteration related to Scenario (b) - Symmetric Warren

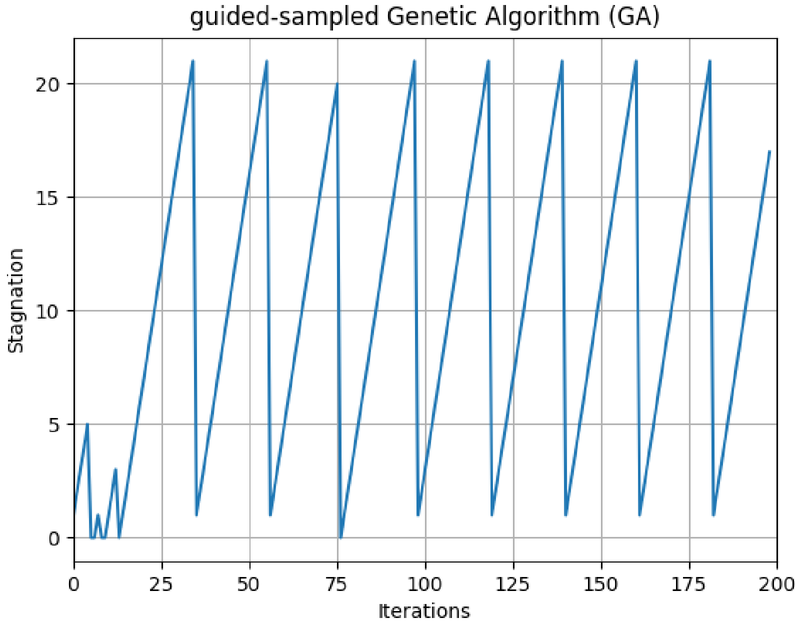

Figure 4.22: OF stagnation for each iteration related to Scenario (b) - Symmetric Warren

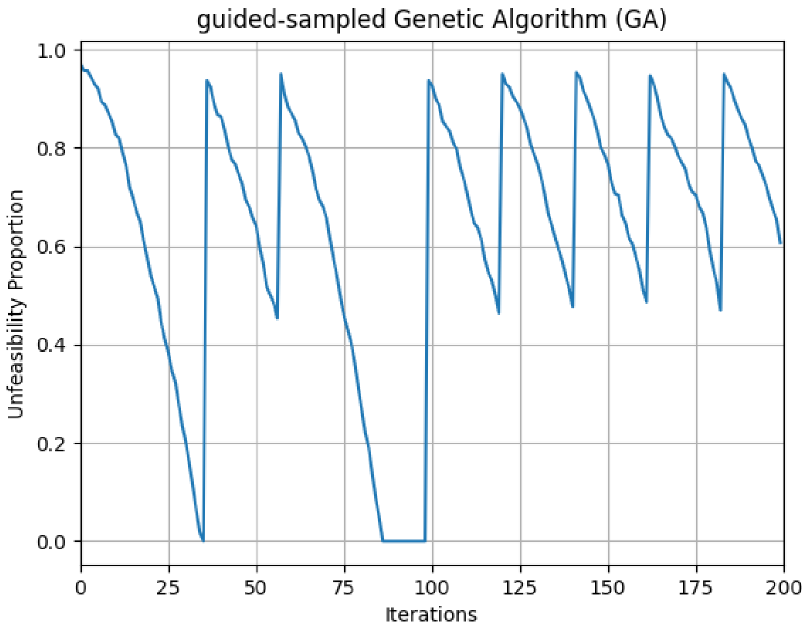

Figure 4.23: OF unfeasibility for each iteration related to Scenario (b) - Symmetric Warren

The results derived from scenario (b) are much worse than the ones obtained from scenario (a). The assignation of the items to the beams in scenario (b) is only dependent on the optimum size for reducing the structural mass. Therefore, taking into account only the structural mass optimization leads to great variability in terms of cross-sectional areas which means a raise in the number of bars. Moreover, a great number of bars results in a low exploitation rate which means a lot of waste material. By comparing the two scenarios in (a) we have the use of 8 bars in the best results meanwhile in (b) the quantity raises to 11 bars. The scenario (a) best solution trend is more rigid (with more numerous and longer stagnation points) but is probably due to the higher complexity of the problem.
In terms of structural mass, the average value of scenario (a) is 234 kg much greater than scenario (b) which is an acceptable compromise because the gain in terms of average stock mass in scenario (a) is about 583 kg. The standard deviations of the results are a little high in both scenarios (less in terms of stock mass in scenario (a) and lower in structural mass in (b)) due to the high variability of the problem.
Unfortunately, the difference between the two scenarios is not high as in the previous case study and, for this reason, an ulterior simplification of the model is performed in the next case study.

Attempt	Cross-sectional areas [mm^2]	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	[6910, 8740, 6570, 10000, 2640, 3760, 1560, 371, 810, 820, 5310, 3760]	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]	2948.01	5592.07	2644.05
2	[4020, 4020, 2960, 5890, 8260, 5940, 689, 5940, 1050, 1120, 373, 9860]	[2, 1, 1, 1, 1, 1, 1, 1, 1, 1]	2666.38	5202.43	2536.05
3	[5890, 10200, 2960, 2640, 8740, 5990, 2040, 454, 1860, 5990, 1120, 4970]	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]	3024.07	5518.24	2494.17
4	[8260, 7940, 4710, 5770, 7120, 5030, 679, 1660, 810, 1250, 454, 5030]	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]	2786.95	5143.67	2356.73
5	[7120, 2520, 3760, 6570, 7370, 6850, 1120, 7120, 1560, 1120, 3760, 6850]	[1, 1, 1, 1, 1, 1, 1, 1]	3006.42	4341.44	1335.03
6	[4710, 3760, 8110, 4670, 8920, 3760, 574, 578, 4670, 5890, 4070, 5890]	[1, 1, 1, 2, 1, 1, 1, 1, 1]	3272.78	5410.85	2138.07
7	[5890, 10900, 9860, 4030, 4970, 8740, 540, 10900, 869, 1320, 540, 1560]	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1]	3354.13	5731.95	2377.82
8	[3360, 3360, 6850, 9110, 3210, 3760, 1720, 2040, 3360, 1390, 182, 7120]	[2, 1, 1, 1, 1, 1, 1, 1, 1, 1]	2552.10	4957.51	2405.41
9	[5310, 6120, 8110, 5990, 6290, 8260, 1710, 289, 1320, 869, 439, 1710]	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]	2620.45	5264.25	2643.80
10	[6290, 11300, 5030, 6910, 9860, 2520, 6910, 1120, 1120, 810, 340, 6290]	[1, 1, 1, 1, 1, 1, 2, 1, 1]	3296.73	5334.08	2037.34

Table 4.16: Structural Optimization via CSP results of 10 runs for symmetric Warren. Scenario (a)

Attempt	Cross-sectional areas [mm^2]	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	[3710, 8110, 4710, 7370, 8770, 12500, 506, 2140, 360, 906, 1070, 1630]	[1 1 1 1 1 1 1 1 1 1 1 1]	2838.33	6097.33	3259.00
2	[4210, 7370, 3310, 6910, 4500, 6120, 1540, 5770, 1120, 1660, 360, 10000]	[1 1 1 1 1 1 1 1 1 1 1 1]	2800.97	6225.44	3424.47
3	[2640, 4030, 7940, 5010, 13000, 8110, 578, 820, 679, 1560, 906, 2960]	[1 1 1 1 1 1 1 1 1 1 1 1]	2748.27	5679.44	2931.17
4	[3310, 7120, 7810, 5010, 4020, 5770, 4500, 733, 707, 3210, 641, 5310]	[1 1 1 1 1 1 1 1 1 1 1 1]	2620.77	5668.60	3047.83
5	[2060, 5990, 4670, 7920, 8740, 15500, 965, 238, 454, 680, 574, 5890]	[1 1 1 1 1 1 1 1 1 1 1 1]	2764.00	6320.94	3556.94
6	[4210, 7370, 4210, 5030, 6290, 8110, 1120, 2040, 1320, 1320, 1540, 11300]	[2 1 1 1 1 1 1 2 1 1]	2806.83	6342.02	3535.18
7	[6660, 3360, 8260, 5010, 2960, 10200, 1710, 707, 1710, 5770, 483, 4500]	[1 1 1 1 1 1 1 1 1 1 1]	2805.83	5842.75	3036.93
8	[5280, 7370, 4020, 6570, 4210, 2060, 4020, 360, 965, 965, 506, 6850]	[1 1 1 1 1 1 1 2 1 1]	2395.22	4610.62	2215.40
9	[12500, 6910, 4500, 5310, 4070, 4030, 1070, 439, 540, 820, 6120, 5770]	[1 1 1 1 1 1 1 1 1 1 1 1]	2922.51	6132.30	3209.79
10	[5770, 7120, 3540, 5990, 3760, 9110, 1370, 1390, 1710, 1120, 810, 4210]	[1 1 1 1 1 1 1 1 1 1 1 1]	2476.97	5404.72	2927.75

Table 4.17: Traditional Structural Optimization results of 10 runs for symmetric Warren. Scenario (b)

4.5 Case study 3: 4 member types Warren truss

Within this section, a further reduction of the design variables regarding the previous model was implemented. The structure is the same as in case study 2 but all the members are grouped into 4 groups which have 4 different cross-sections. In this way, the design variables are only 4 and the computational effort is reduced.
The section is divided into two subsections, the first show the model definition and the set of algorithm parameters and, afterwards, another subsection exposes and discusses the results.

4.5.1 Model definition and parameters' setting

This case study is a simplification of the previous one. As it is possible to see in Figure 4.24 all the members are previously grouped into 4 cross-sectional classes based on the structural behaviour. The four classes are the lower chord, the upper chord the internal web and the external web. The only modification is this radical reduction of the design variable meanwhile the problem statement of the other model and optimization characteristics are equal to the one exposed in case study 2. The final aim of this assumption is to reduce the design variable and to force the grouping of the bars in order to exploit better the stock material.

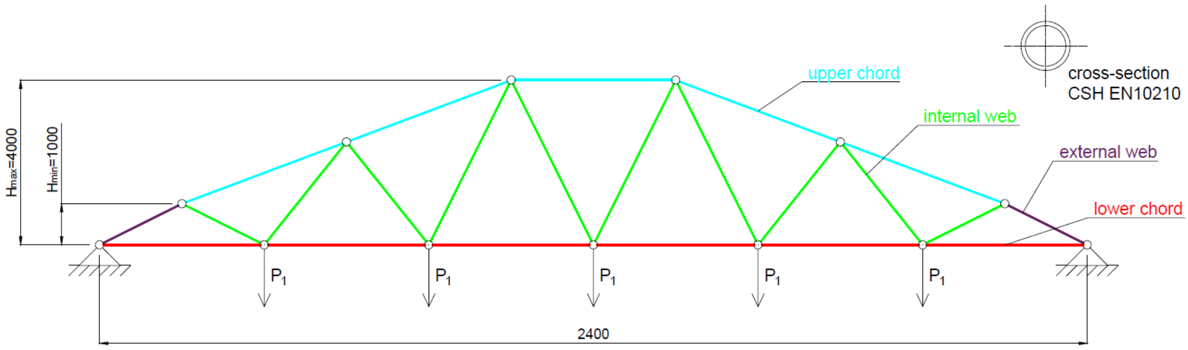

Figure 4.24: Configuration of the Warren truss under analysis, the members are divided into 4 cross-sectional areas which are the 4 design variables, measures are expressed in millimetres (mm)

In the following tables (4.18, 4.19) are summarized the characteristics regarding the model properties in the first one and the setting parameters in the second table.

Parameter	**Value**
Modulus of elasticity of steel E	210000 MPa
Steel density ρ	7.85 t/m^3
Loading lower chord nodes P_1	240 kN
Length of purchased bars (bins) L_{bin}	15 m
Number of design variables	4
Bounds of design variables (A_{min}, A_{max})	[182, 24700] mm^2

Table 4.18: Model assumption relative to the symmetric Warren truss.

Parameter	**Value**
Maximum number of iterations	200
Number of individuals per population	300
Areas excluded if unfeasibility condition isn't satisfied	5 smaller DV
Mutations' probability	5%
Proportional children	1 child for each parent
Stagnation condition	20 iterations

Table 4.19: Optimization algorithm parameters set by the operator.

4.5.2 Results and discussion

As in the previous case study, two optimization scenarios are investigated by performing 10 runs per scenario (for more detail see case study one):

- Scenario (a): optimization by considering CSP (minimization of the stock).
- Scenario (b): optimization via traditional approach (minimization of the structural mass).

Scenario (a) On following the results of scenario (a) with 4 design variables are exposed. For more detail about the 10 runs see table 4.22.

	Stock Mass (OF) [kg]	Structural Mass [kg]	Mass Waste [kg]
Best	2656.44	2189.96	466.48
μ	2770.02	2265.11	504.91
σ	98.30	73.80	37.38

Table 4.20: Result of the 4 DV Warren optimization via CSP approach (a)

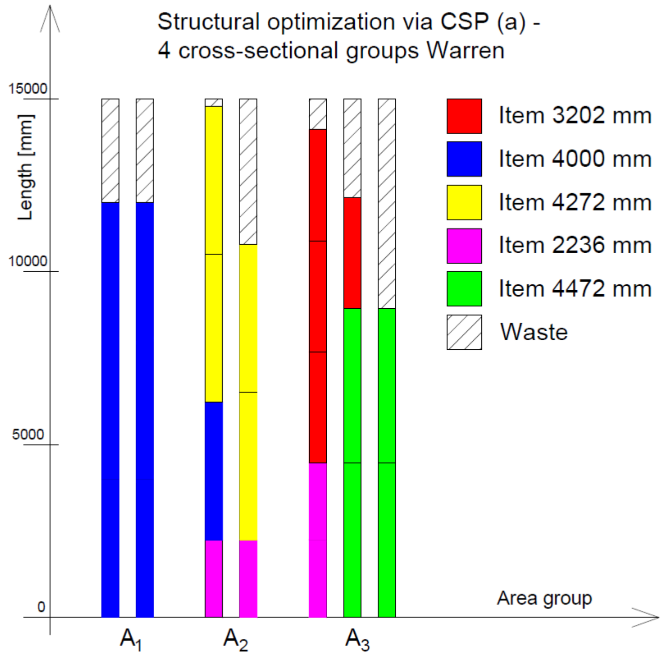

Figure 4.25: Optimal cutting pattern derived by optimization via CSP scenario (a) - 4 DV Warren

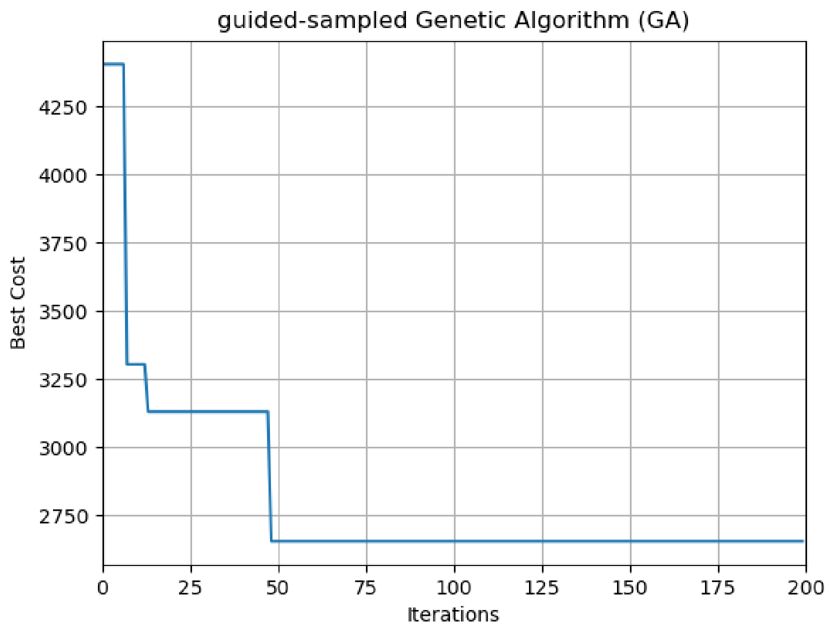

Figure 4.26: OF best solution for each iteration related to Scenario (a) - 4 DV Warren

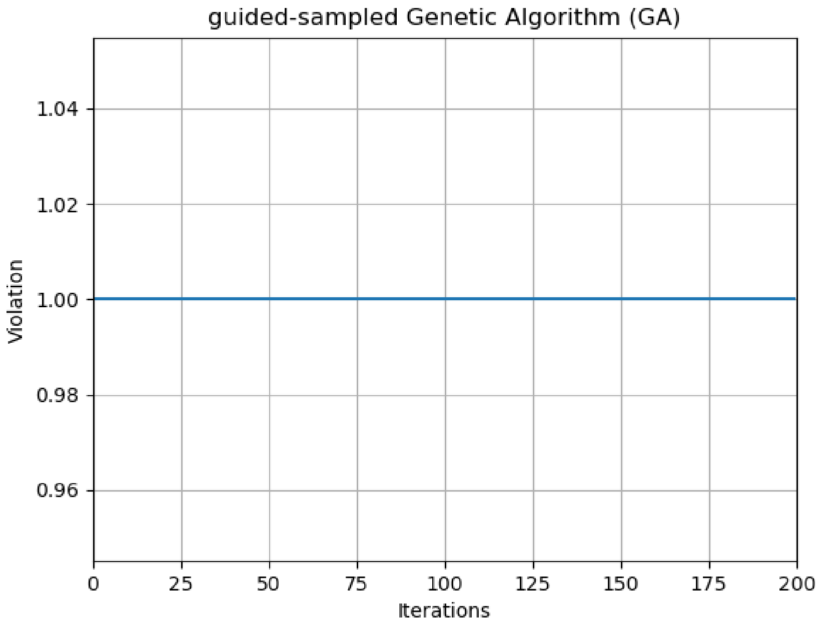

Figure 4.27: OF violation for each iteration related to Scenario (a) - 4 DV Warren

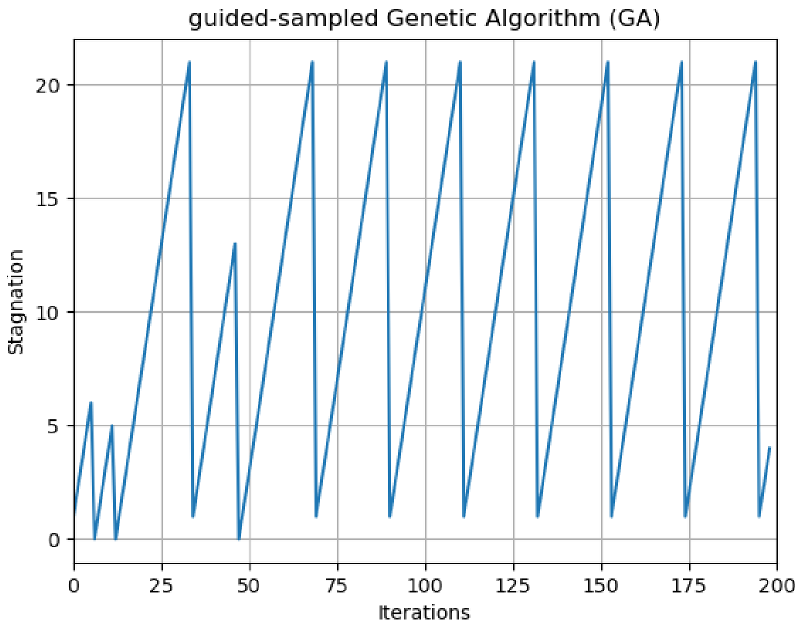

Figure 4.28: OF stagnation for each iteration related to Scenario (a) - 4DV Warren

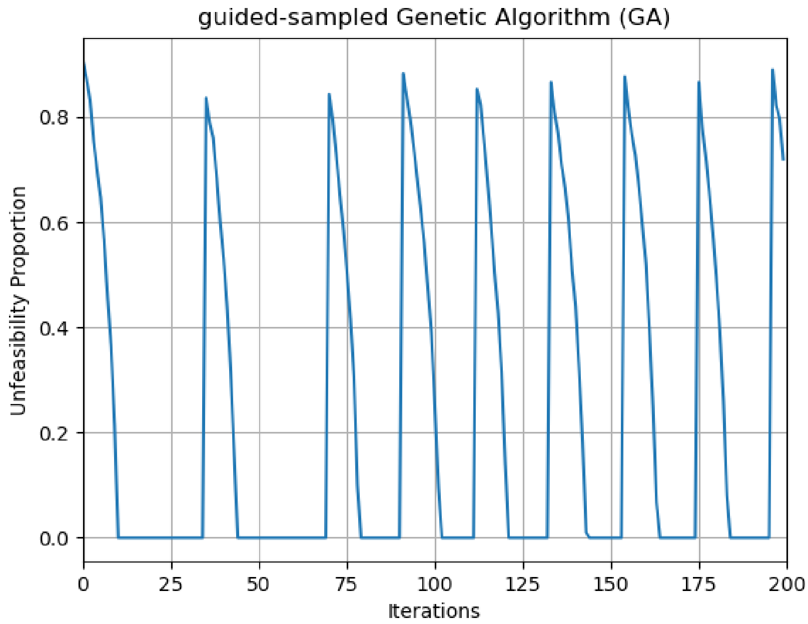

Figure 4.29: OF unfeasibility for each iteration related to Scenario (a) - 4 DV Warren

In this case, the stock representation (Figure 4.25) shows that the rate of exploitation of the bars is the maximum that it is possible to attain. It is interesting to notice that all the 10 runs give as a result the grouping in three classes of areas because the two external webs are simply allocated in one of the other three existing bars. Another indicator of the algorithm's goodness is the decrease of the standard deviations both in stock and structural mass which means that the results tend to converge to similar results. Also, the waste mass is much lower than in the previous case study.

Scenario (b) On following the results of scenario (b) with 4 design variables are exposed. For more detail about the 10 runs see table 4.23.

	Stock Mass [kg]	Structural Mass (OF) [kg]	Mass Waste [kg]
Best	3109.78	2130.44	979.33
μ	3282.09	2146.42	1135.67
σ	222.82	26.38	215.64

Table 4.21: Result of the 4 DV Warren optimization via traditional approach (b)

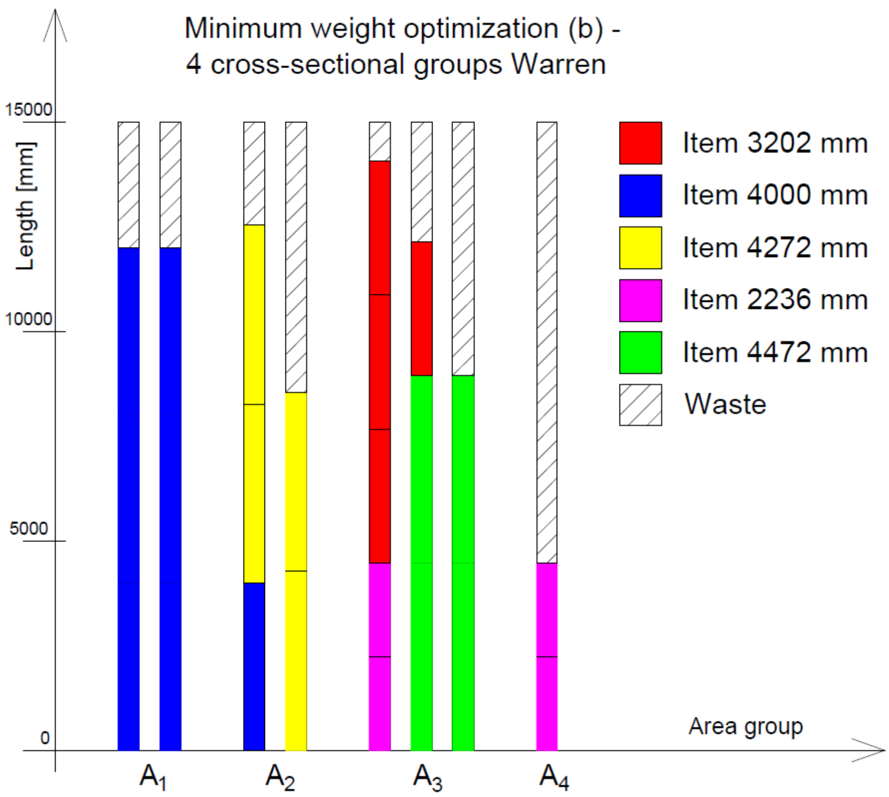

Figure 4.30: Optimal cutting pattern derived by traditional optimization scenario (b) - 4 DV Warren

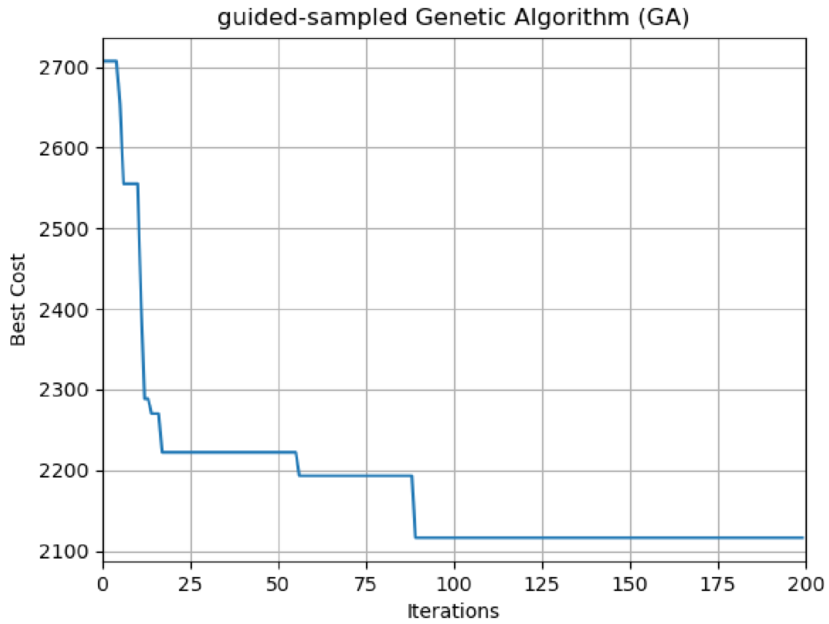

Figure 4.31: OF best solution for each iteration related to Scenario (b) - 4 DV Warren

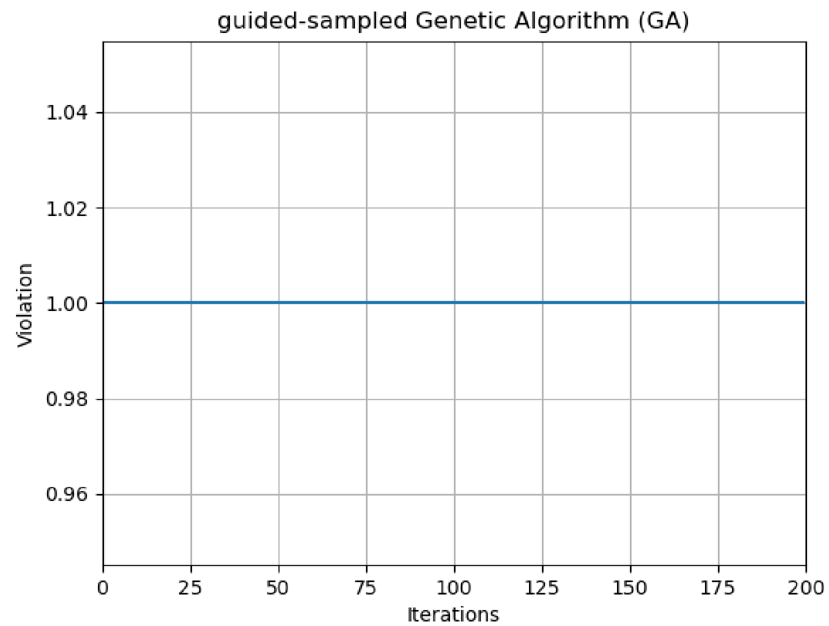

Figure 4.32: OF violation for each iteration related to Scenario (b) - 4 DV Warren

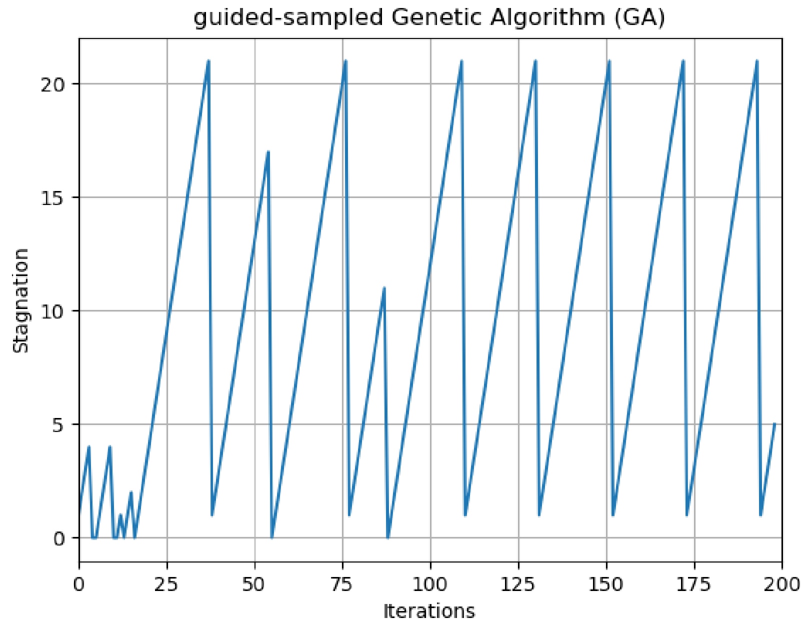

Figure 4.33: OF stagnation for each iteration related to Scenario (b) - 4DV Warren

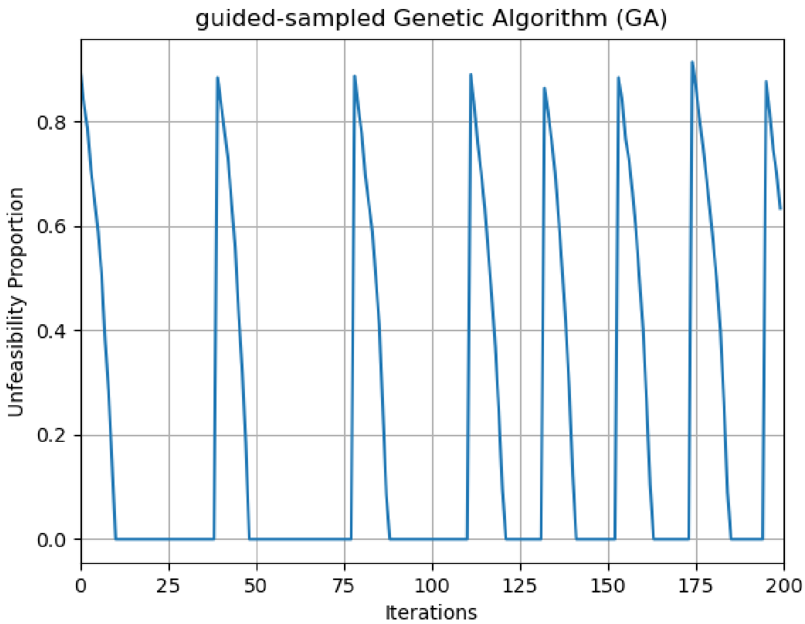

Figure 4.34: OF unfeasibility for each iteration related to Scenario (b) - 4 DV Warren

In scenario (b) the results are a little bit worse than in (a). The increment is almost in all the runs of only one bar. These results are due to the fact that algorithm (b) uses the groups imposed from the operator without the merging of the external webs cross-sectional areal class with another of the three other classes. The principal thought that comes from this outcome is that the traditional structural optimization itself works well and the impact of the CSP optimization, in this case, is minimum.

Therefore, the CSP algorithm in this case has more of a function of finding the best set of cutting patterns instead to group the items' areas which are similar to the one chosen by the traditional structural optimization (in fact the structural mass is very similar between the two scenarios). A noticeable value is the standard deviation which increases in scenario (b).

Attempt	Cross-sectional areas [mm^2]	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	[4210, 5310, 1370, 5310]	[2, 2, 3]	2236.80	2725.91	489.11
2	[4970, 5280, 906, 4970]	[2, 2, 3]	2234.99	2733.92	498.93
3	[4210, 5940, 1120, 5940]	[2, 2, 3]	2294.20	2785.96	491.77
4	[4500, 4710, 2060, 4710]	[2, 2, 3]	2361.53	2896.65	535.12
5	[4030, 6120, 906, 4030]	[2, 2, 3]	2163.95	2710.37	546.42
6	[3710, 5890, 1120, 5890]	[2, 2, 3]	2189.96	2656.44	466.48
7	[4210, 6120, 680, 6120]	[2, 2, 3]	2208.85	2672.93	464.08
8	[4500, 5990, 1390, 4500]	[2, 2, 3]	2381.09	2961.41	580.32
9	[4020, 5890, 1120, 5890]	[2, 2, 3]	2248.37	2729.44	481.08
10	[5280, 4670, 1370, 5280]	[2, 2, 3]	2331.39	2827.18	495.79

Table 4.22: Structural Optimization via CSP results of 10 runs for 4 cross-sectional type Warren. Scenario (a)

Attempt	Cross-sectional areas [mm^2]	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	[4030, 5990, 862, 4210]	[2 2 3 1]	2136.60	3159.94	1023.34
2	[5030, 4710, 965, 4070]	[2 2 3 1]	2136.63	3113.90	977.27
3	[5030, 5280, 869, 3360]	[2 2 3 1]	2179.56	3130.62	951.06
4	[4670, 5280, 641, 5890]	[2 2 3 1]	2137.61	3263.21	1125.59
5	[4500, 5030, 869, 5890]	[2 2 3 1]	2127.14	3244.84	1117.69
6	[4710, 4670, 810, 9110]	[2 2 3 1]	2203.87	3567.83	1363.96
7	[4210, 4970, 1050, 5990]	[2 2 3 1]	2116.05	3238.13	1122.07
8	[4970, 5010, 810, 4020]	[2 2 3 1]	2130.44	3109.78	979.33
9	[5280, 4710, 733, 5010]	[2 2 3 1]	2152.68	3201.50	1048.82
10	[5030, 5030, 679, 5010]	[5 3 1]	2143.65	3791.20	1647.55

Table 4.23: Traditional Structural Optimization results of 10 runs for 4 cross-sectional type Warren. Scenario (b)

Chapter 5

Conclusion

In this thesis, a novel procedure for the optimization of steel truss structures has been introduced to evaluate the minimum number of bins (total mass of purchased steel bars) and the optimal cutting pattern of a stock of elements by solving the Cutting Stock Problem (CSP).
After an introductory part which gives an overview of the scientific literature concerned with Structural Optimization techniques which investigate the minimization of the cost related to the structure, the mathematical formulation of the Cutting Stock Problem in particular the Column Generation methodologies was explored in detail.
Given the necessary prerequisites, the Structural Optimization via CSP developed by the author and supervisors of this thesis is exhibited. The new Structural Optimization aims to minimize the total mass of purchased steel bars by a GA-based size optimization algorithm. The number of bars requested by the objective function is obtained through a CSP column generation code which allocates all the items of the truss under analysis in an optimal way.
Thereafter, some application case studies are investigated for testing the goodness of the algorithm. To have effective feedback the Stock obtained by the Structural Optimization via CSP is compared with the stock obtained by a traditional Structural mass minimization.
In the first case study, which is a simple 10-bar truss used as a benchmark, the algorithm gives satisfactory results. The 10-bar truss under analysis is very simple and the bars have a capacity of a maximum of 2 items per bar. Nevertheless, the ex-

ploration of the domain of solution is very simple due to the standard cross-sectional list which has a small number of cross-sections, a small number of unfeasible solutions and a constant interval between two consecutive areas.
The two next case studies are a 23-bar Warren truss which is modelled in two different ways: as a symmetric truss that almost halves to 12 design variables and as a 4 cross-sectional areas truss (reduce the design variables to the upper chord, lower chord, external and internal web cross-sections). The problem, in this case, is more interesting because of the complexity of the problem that arises (i.e. the number of items per bar, more different items' lengths and a bigger solution domain) and hence the usefulness of the optimization. Due to the innate complexity of the question, the exploration capability of the algorithm is incremented. With respect to the 10-bar truss the solution of the symmetric Warren gives worse results. Probably the principal cause is the solution domain correlated whit the standard cross-sectional list used for the Warren which has a greater number of areas with also a greater percentage of unfeasible solutions and a variable interval between two consecutive solutions. To reduce the variability of the algorithm the 4 cross-sectional areas truss model was created. As expected, forcing the grouping of the area more allows the collection of more items bars and has good results.
The limit of this work is a high variability of the solution due to the innate complexity of the problem, the request for a preventive grouping for more complex structures and the rigidity of the algorithm which have lots of stagnation point across the iterations.

5.1 Future developments

Since the CSP is a problem strictly related to the length of the items and of the bins it would be interesting in the future to add the lengths of the elements to the optimization design variables in order to obtain a size and shape optimization.
Another interesting aspect is to differentiate the length of the purchased bars in order to buy the bar typology which guarantees the minimum waste of material.
Moreover, this thesis treats only the weight as the objective function of the structural optimization, it would be useful to explore more the CSP utilization in cost minimization problem which takes into account more economic factors (i.e. fabrica-

tion cost, connection cost and maintenance).
A further development is to consider the environmental impact of the minimization of the stock bars by a life cycle assessment and/or an objective function which minimize the emission by implementing the optimization of the various environmental indexes.

Chapter 6

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Ringraziamenti

Desidero alla fine di questa tesi, che rappresenta il coronamento del mio percorso di formazione al Politecnico di Torino, spendere due parole per tutti coloro che mi hanno sostenuto in tutti questi ultimi anni e non solo.

Innanzitutto, rivolgo la mia gratitudine ai miei relatori che sono stati protagonisti di questo percorso di tesi. Ringrazio il professor Giuseppe Carlo Marano per aver condiviso con me la sua preziosa esperienza e per avermi dato importanti consigli anche nell'ambito professionale. Rivolgo poi un grande grazie al dottorando Raffaele Cucuzza che mi ha fatto da guida e mi ha insegnato come gestire umanamente e professionalmente problemi complessi, ma soprattutto è stato presente e disponibile durante tutto il periodo di tesi. Ringrazio il professor Nikos Lagaros per l'ospitalità all'NTUA di Atene dove ho approfondito argomenti utili alla redazione di una grossa fetta di questo elaborato e che mi ha permesso di vivere in una vivace città Europea.

Ringrazio tutte le persone che mi hanno sostenuto durante il periodo di tesi all'estero. Ci tengo a ricordare chi mi è stato vicino anche da lontano in particolar modo Flavio, Fabio e Noemi. Voglio anche ricordare con affetto chi era in trasferta da Torino come me e mi riferisco al gruppo di dottorandi presente ad Atene, grazie perchè con voi non mi sono mai sentito solo. Infine, se ripenso ai frutti del soggiorno Ateniese un pensiero speciale va a Gaia, sempre vicina in questi mesi, mi ha aiutato a essere più forte e ha dato colore alla mia quotidianità. Spero in qualche modo di restituirti il bello che hai aggiunto alla mia vita.

Voglio doverosamente ricordare le radici che mi hanno permesso di costruire ciò che sono ovvero la mia famiglia, in particolar modo i miei genitori, mio fratello e mio cugino Alessandro, che con il loro costante sostegno e con la loro presenza sono stati ancora in mezzo al mare e sicurezza contro le tempeste. Senza di loro, non avrei quel bagaglio di risorse che mi ha fatto arrivare fin qui.

Vorrei ringraziare anche coloro con cui ho condiviso quotidianamente le gioie e i dolori della vita universitaria, i miei compagni di corso in particolare Gabriel e Simone che sono stati fondamentali, con voi ho capito cos'è in grado di fare il gioco di squadra. Spero che le nostre strade continuino a incrociarsi e sono convinto che diventerete degli ottimi professionisti, mentre sul fatto che sarete persone straordinarie non c'è da aspettare il domani per averne conferma.

Ringrazio anche tutti i miei amici e le persone che non sono riuscito a citare ma

che nel loro piccolo hanno lasciato il segno sul mio percorso. Tra questi ci tengo a citare tutto il gruppo dei "Vulpian", il gruppo di Malaga, Sergio R. e Francesca C.

Infine, voglio ringraziare me stesso per aver avuto la determinazione e la perseveranza di arrivare fino alla fine di questo percorso. Nonostante gli alti e bassi ho sempre guardato dritto all'obbiettivo ed esplorato tutte le risorse che avevo, malgrado capitasse che avessi paura di non farcela e mi sentissi indugiare. Vorrei questo traguardo sia un piccolo sprono al me stesso di domani per capire che ogni cosa è gestibile e le risorse interne sono maggiori di quelle che crediamo.