POLITECNICO DI TORINO

Master's Degree in ICT for Smart Societies

Master's Degree Thesis

Performance analysis of charging strategies for shared mobility: a queuing network modelling approach

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JULY 2022

Acknowledgements

At the end of this journey I would like to spend few words to acknowledge and give thanks to all the people who supported me during the years of my university experience and in particular during the development of this work.

First of all I would like to express my gratitude to professors Luca Vassio and Danilo Giordano. Thank you for allowing me to take part in this fascinating and challenging project and for guiding me week after week with your precious advice and constant support.

Special thanks to Alessandro Ciociola for your willingness to help me since the very beginning of this work. Thank you for the inspiration and all the suggestions and for being a friend along this important path.

I would like to extend my sincere thanks to professors Marco Mellia and Michela Meo for their important contributions and interest in this thesis work. Thank you for your professional assistance and for the many feedback and thoughts which significantly helped to enrich and improve the final outcome.

Lastly, thank you to my family for being a safe heaven, for always supporting me in my choices and make me feel loved and cared. Thank you to my friends, the old and the new ones, the near and the far. Thank you for bringing happiness in my life and for always walking with me.

"Sal, we gotta go and never stop going 'till we get there.'
'Where we going, man?'
'I don't know but we gotta go."
JACK KEROUAC

Abstract

Sharing transport systems are an established reality in many cities around the world and their role in the urban mobility is set to gain further relevance in the next years as the sharing economy paradigm gains momentum. Free floating, one-way services are the most flexible ones, allowing users to book and pick up a vehicle and leave it anywhere within the operational area boundaries after its usage. This is a great advantage from the users perspective, which has helped establishing this kind of systems as the most successful ones in the urban environment but, at the same time, it makes management operations more difficult. The absence of predefined parking stations and the unpredictability of origin-destination demand patterns, often result in unbalances in the fleet distribution through the area, requiring operator interventions. Relocation of vehicles in shared mobility is one of the central research topic of interest because it allows to increase the overall number of carried out trips, bringing additional revenues to the operator and satisfaction to the customers.

Recently many shared mobility service providers are including electric vehicles in their fleet. This transition has been supported by a general gained interest of the public about environmental issues and by new emerging technologies that make the storage of energy and the batteries charging easier. However a fleet of shared electric vehicles requires additional planning of infrastructures (i.e., charging stations) and new policies to implement charging operations. Although refuelling operations are also needed for internal combustion vehicles, the charging process for electric ones requires indeed more time depending on the available infrastructure's power capacity therefore complicating the operations management. The optimal relocation problem in a shared mobility system is then joined by an optimal charging one.

This thesis research work stems from a wider project on mobility and shared transport systems from the SmartData@Polito research group and it fits into the analysis of free-floating shared mobility services employing an electric vehicles fleet. The main research questions in particular regarded the analysis of network performances studying the planning of charging infrastructures and the impact of different charging policies and relocation techniques.

The used approach is analytical and different models based on queuing theory are proposed. Analytical modelling is used to provide a formal representation of the system of interest with a set of variables describing its working and evolution and using mathematical expressions to obtain performance metrics. Differently from other widespread approaches such as simulation, analytical modelling allows to build a generic model for the studied service with the advantages of requiring low

computational power and time resources. Moreover the convergence of the analytical solution is guaranteed by mathematical proofs and the explicit dependency on the system parameters make the solution easy to interpret with a clear view of how the model variables influence the system. On the other hand some peculiarity and constraints of the system may not be included in the final model because they may increase too much its complexity and may result in a not derivable mathematical formulation. Starting from simple models of single city zones and charging stations, more complex ones are built using network of queues representing the dynamics of a shared mobility fleet through the city. Network of mobility zones only are first proposed to study the general behaviour of the customers' demand and the movement of vehicles through the city. Charging station queues are then included showing the impact of charging operations on the system performances and eventually trips time are modelled through delay queues. Many case studies are presented with model parameters inferred from real data from a *car2go* database of trips for the city of Turin. A spatial characterisation of input data and results is also given through visual maps and graphs. A differentiation in the input data has been considered studying multiple network realisations with distinct input routing and demand rates based on data from particular time intervals during the day and different kind of days.

A particular focus is given to the charging infrastructure planning and recharge operations required by an electric fleet. The placement and aggregation of charging stations in the city network is first studied. Charging policies implementing decisions on when, where to and which vehicles bring to charge are then explored. Relocation is also taken into account jointly with charging operations, with different possibilities considered on where to reposition vehicles when the process is completed. All the performance indicators of interest have been obtained through mathematical expressions and algorithms such as the Mean Value Analysis.

The reference metrics through all the case studies are the general throughput (i.e. total number of trips) of the system and the percentage of unsatisfied user mobility demand as well as their spatial distribution through the city's zones. Different combinations of policies are studied to optimise these values. Moreover the distribution of vehicles in the area, the average utilisation and throughput of the charging station as well as the probability for a vehicle to wait in line before charging are also monitored and discussed. In addition, the impact of the fleet size, the number, positioning and concentration of charging stations and other parameters specific of some policies is shown. Furthermore a brief study on how charging operation affects the power grid is proposed and the effect of varying trip times on the overall operations is observed.

Results showed a promising capacity of the model to represent typical dynamics of a shared mobility system with the advantage that all indicators are obtained with a very small computational power required and in very short negligible time.

With the best combination of policies for charge and relocation it has been observed a decrease in the unsatisfied percentage of mobility demand up to 20.4% for a balanced network, corresponding to an increase of around 48% of the throughput. With an unbalanced routing characterised by higher and more uneven demand instead optimal policies resulted in a 12.2% reduction of fraction of unsatisfied demand corresponding to an increase in throughput of around 38%.

Eventually a simulation tool was considered and compared to the analytical model results. Two of the modelled scenarios were simulated corresponding to a balanced data and an hourly data system to try and reproduce similar dynamics with respect to the one tested with the analytical model. In spite of differences in both the input data and the general network configurations, the analysis and comparison of the simulation results with the previously extracted metrics has shown significant similarities in the general behaviour of the system and it has resulted consistent with the analysis and considerations extracted from the study of the proposed analytical model.

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Acronyms

FFCS
: Free Floating Car Sharing

ICEV
: Internal Combustion Engine Vehicle

EV
: Electric Vehicle

MoD
: Mobility on Demand

FCFS
: First Come First Served

MVA
: Mean Value Analysis

RS-RD
: Repetitive Service with Random Destination

FFBS
: Free Floating Bike Sharing

MINLP
: Mixed Integer Non Linear Programming

MILP
: Mixed Integer Linear Programming

IP

Integer Programming

NLP

Non Linear Programming

DRL

Deep Reinforcement Learning

BLP

Binary Linear Programming

KDE

Kernel Density Estimation

DL

Deep Learning

LPG

Liquefied Petroleum Gas

GHG

GreenHouse Gases

CDF

Cumulative Distribution Function

SoC

State of Charge

WtW

Well-to-Wheel

WtT

Well-to-Tank

TtW

Tank-to-Wheel

Chapter 1

Introduction

Sharing transport systems are already an established reality in many cities around the world and their role in the urban mobility is set to gain further relevance in the next years as the sharing economy gains momentum. Different kind of sharing systems using different means of transport can already be found in many cities; the most common ones employ cars, bikes, scooters and motorbikes.

Limiting the analysis to car-sharing, a further classification following the taxonomy in [1] can be done based on the system specifications and in particular on its mode and on the type of vehicle's engine. The mode defines the operational way of the system that can be station based if shared vehicles are available only in defined city spaces and two-way if each trip must start and end in the same station. One-way models instead allow to end the journey in a different zone with respect to the starting one while free-floating systems lose the constraint of the stations and vehicles can be freely left and picked up in publicly available spots in the whole working area. Free floating car sharing (FFCS) systems are the newest in the market and nowadays are the most common and successful ones. For example, figure 1.1 shows the trend of usage of station based and free floating shared mobility systems in Germany from 2011 to 2021 in terms of number of both vehicles and users. Both types of service show increases in usage and number of customers but free floating ones are clearly more appealing to users.

Most of the available car sharing systems, were originally designed to work with a fleet of internal combustion engine vehicles (ICEV). In the last years however, as the climate crisis awareness has spread, there has been an increasing general concern on the impact of the utilisation of fossil fuels on the environment. The main response in the transport sector and industry was to slowly start to switch towards electric vehicles (EVs). This transition has been supported by new developed technologies which make it easier to store energy and charge batteries and by a general tendency of society to move towards greener forms of energy production and the consequent economic advantages in doing so. Figure 1.2 shows the global

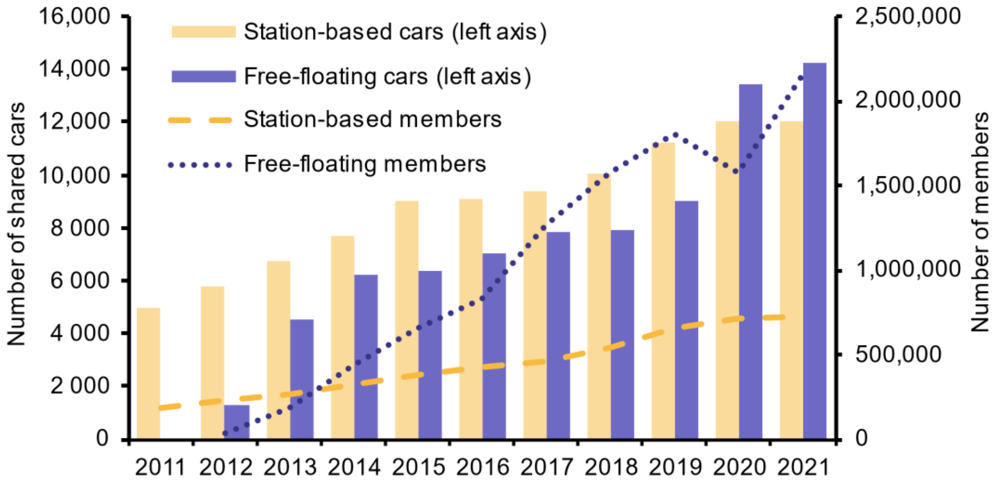


Figure 1.1: Trend in station based and free floating mobility sharing systems in Germany [2]

trend of sales for EV and their market share. It is clear that, although as of 2021 the global market share of EV is only just below 9%, the trend is extremely positive especially in world regions as China and Europe and expected to grow in the next years.

1.1 Operational framework and problem definition

Converting a shared mobility service to use electric vehicles requires additional planning and resources and introduces new organisational and managerial challenges for the system operator that can be summarised as follows:

- Charging stations dimensioning and positioning
- Charging policies and scheduling
- Charging thresholds definition
- Relocation of vehicles after charging
- Workforce management for charging EV
- Management of user contribution to charging operations
- Management of EV lines at the stations

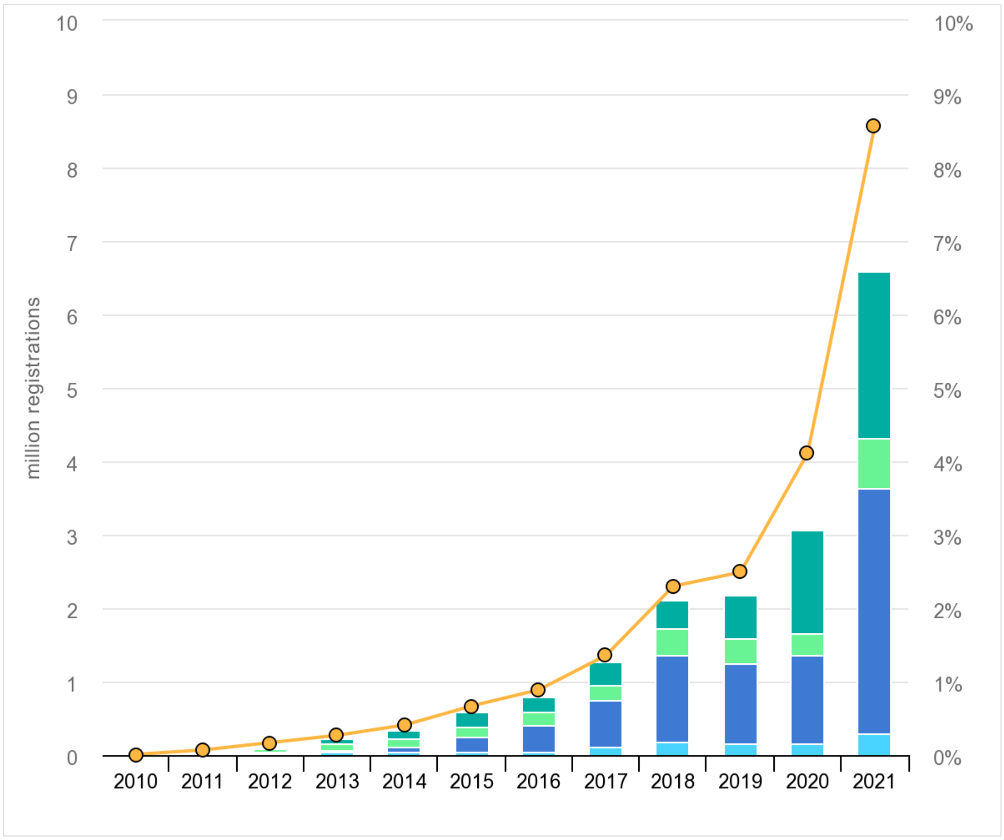


Figure 1.2: Global sales and market share of EV [3]

First a charging infrastructure has to be available and eventually positioned in the most suitable way through the city, then policies are needed to manage the charging operations. Differently from ICEV in fact charging an EV requires much more time which can complicate a lot the management. The main challenge for the system operator is to charge the fleet in an optimal way such that the user demand for mobility is still satisfied and the cost for the charging are limited. The infrastructure planning plays an important role since it may facilitate the operations by reducing the distances between the vehicles in need for power and the nearest charging station. This is particularly true for FFCS systems where vehicles can be parked everywhere while in station-based ones there is an obvious one to one correspondence of mobility and charging stations. Depending on the available power at the charging columns and on the vehicles battery capacities then, a complete recharge may take a non negligible time, making also the scheduling operation critical. Furthermore a set of thresholds has to be defined to trigger the charging operation in a reactive way. Alternatively proactive approaches may be studied which may benefit the overall system operations in spite of further increasing the management complexity. Moreover in most cases the user contribution to the charging operations is not allowed due to difficult or unfamiliar procedure for the average consumer, requiring more workforce and therefore additional costs for the system operator. Eventually the possibility of having queues at the charging stations has to be dealt with, which may add further complexity. These problems introduced with the electrification of the fleet join the classical issues of a generic sharing mobility on demand (MoD) system including the relocation operations needed for the repositioning of vehicles to reduce the customers unsatisfied mobility demand.

The scope of this work is to formulate an analytical model for the representation of a generic electric free-floating MoD system based on queuing theory. Differently from many existing works on the topic of shared mobility, here the approach is purely analytical and supported by real trips data integration. Whilst in a simulation environment it is easier to include many parameters and obtain a quite detailed model, this is often not possible considering analytical representations that would become too complex to be solved mathematically. However analytical modelling can provide useful approximate results to study the possible system behaviours with the important advantage of a much smaller execution time with respect to simulation. Additionally the convergence of the analytical solution is guaranteed by mathematical constraints and the explicit dependency on the system parameters make the solution easy to interpret with a clear view of how the model variables influence the system. An important novel contribution of this work is in the performance analysis of MoD systems regarding the fleet charging. The developed models allowed to perform an in depth study of charging processes and policies for a fleet of EV obtaining important indicators that can help the system operators to optimise the management operations.

The main topics dealt with during the process are the determination of the fluxes inside the network as well as the positioning and dimensioning of the charging infrastructure and the exploration of possible charging policies also including relocation. Moreover the definition of additional nodes in the network is taken into account to associate a time delay to the trips. With the support of real data from a dataset obtained from car2go fleet, different scenarios have been tested and a spatial characterisation of the results has been possible. Eventually results of the model are also compared with the output of a provided simulation tool.

1.2 Methodology recap

The work flow followed for this project can be divided in five steps summarised in the scheme in figure 1.3.

1. **Single queue models definition:** for the definition and construction of

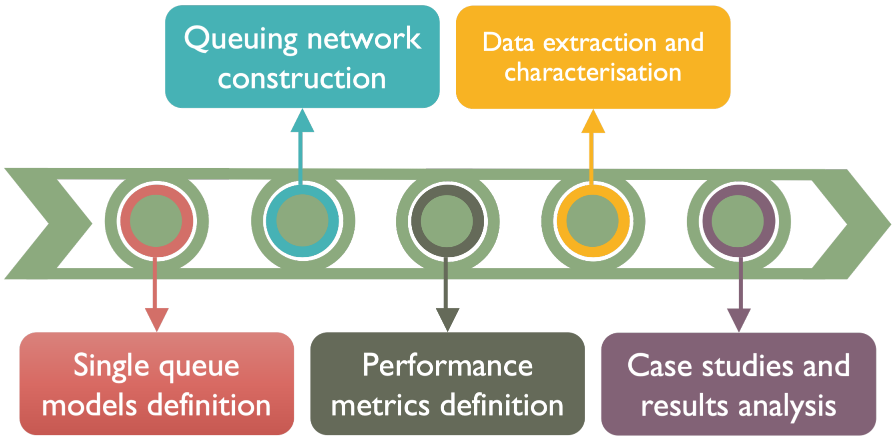


Figure 1.3: Work flow scheme

both single queue and network models, queuing theory has been the main tool exploited. Before constructing the networks, the single queue models for each zone type have been fist studied to associate their variable to the characteristic elements of each represented system. For each queue model in particular a series of parameters has been defined:

- Customers population
- Arrival rate
- Departure rate
- Number of servers
- Capacity
- Discipline

In all the presented cases the customers are the fleet vehicles, both the arrival and departure rates are assumed to be exponentially distributed and the queue discipline is first come first served (FCFS). The number of servers instead is defined differently based on the zone type: mobility queues have a single server, delay ones have infinite servers and charging station queues have a variable number of servers which corresponds to the number of charging outlets. The capacity of mobility and charging queues has been considered in the single queue models while it has been set aside in the network ones. All the considered single queue models are detailed in section 3.1.

2. **Queuing network construction:** looking instead at the network realisations, the starting model is a simple network of mobility zones which is then enriched including charging stations and delay zones. When combining single queues

to build network models additional parameters and network characteristics have to be defined:

- Network type
- Routing matrix

The network type chosen is the closed or Gordon-Newell one which is characterised by a constant and finite population (i.e. fleet size) with no customers that can leave or enter the network. This constraint results in flows at each node that are always dependent on the system state. The network is mashed meaning that all nodes are interconnected with each other and the movement of customers is defined by routing probabilities. Different routing matrices have been inferred from real trips data to establish the movements of vehicles through the mobility zones in different scenarios. The routing towards and from charging stations has been determined instead studying different policies including relocation of vehicles. The formulated network of queues for this work are detailed in section 3.2.

3. **Performance metrics definition:** the performance metrics for the system evaluation have been defined again looking at queuing theory and in particular at known mathematical formulations and algorithms to solve the networks under study. The traffic equations for the network nodes have been written and solved till convergence to determine the relative fluxes of vehicles. The convolution (Buzen's) algorithm has been used to obtain a steady state distribution probability for the network. However the Mean Value Analysis (MVA) was the mainly employed tool, used to define and obtain all the performance metrics for the network. The reference ones were in particular the throughput (i.e. number of trips) per zone and overall for the network and the percentage of unsatisfied requests for mobility by users. All the implemented and used algorithms are explained in detail in chapter 4.

4. **Data extraction and characterisation:** the input data have been obtained from a database of trips for the car2go free-floating car sharing available in the city of Turin. Data have been filtered and grouped to collect spatial and temporal information about user mobility determining the routing probabilities and service rates of the network. Chapter 5 reports all the detail of the work on data extraction and manipulation.

5. **Case studies and results analysis:** eventually different case studies were formulated starting from different configurations of parameters for the network and the relevant performance metrics were obtained through MVA. The obtained results have then been analysed and commented also providing a visual characterisation through graphs and maps. A further comparison has

been made in the end with the output of a simulation tool from which the same metrics of interest have been computed. All the model case studies are reported and detailed in chapter 6 while the comparison with the simulation results is in chapter 7.

Chapter 2

Literature Review

The study of transport systems is a particularly topical issue in research. Different aspect concerning mobility are the focus of many works found in literature. Since the object of this work is to build an analytical model for a car sharing system including charging stations, a particular relevance is given to problems addressing queuing theory applications in the transport sector and modelling of sharing system. Different approaches for the management of charging and relocation operation employed in shared vehicles systems are then studied.

2.1 Queuing models in transport systems

When looking at transport systems in general, one of the most commonly addressed problem is the management of road traffic intensity and in particular the study of traffic behaviour at crossroads mainly using single queue models to describe road intersection with vehicles queuing [4]. Anokye et al. [5] proposed a simple single-queue approach to model vehicular traffic intensity at intersections with traffic lights based on an $M/M/1/\infty$ queue. The system parameters have been inferred from real data and differentiated based on the hour of the day. Assuming steady state behaviour for each interval the average queue metrics were used to predict possible congestion. Another case study using the same approach has been developed by Ekeocha et al.[6] who suggested that increasing traffic light time for vehicles can reduce traffic intensity in particular congested time slots minimising delays. Looking instead at highway systems, a classical example of interrupted flow is represented by toll plazas where vehicles have to stop to pay a fee before continuing the journey and they can also represent a perfect application of a queue model. Duhan et al. [7] applied a simple model of a multi-server queue to the toll plaza congestion problem in which each server represents a station where vehicles stop to pay. Model parameters were inferred form peak hours observations and the computation of the average number of vehicles and the mean waiting time suggested that an increasing in the number of toll booths would help reducing delays. Wang [8] instead presented a more complex model based on M/M/1 queue theory to solve this problem differentiating between three types of toll booths namely human-staffed, automated and with electronic collection of fees. Then a genetic algorithm was also applied to solve the lane number optimisation problem.

Differently from the previous studies analysing interrupted traffic flows, Vandaele et al. [9] described uninterrupted traffic flows modelling road segments as state dependent G/G/1 queues with a general distribution of arrival and service times and making the service rates dependent on the number of vehicles in the system. The performance metrics of the queue were used then to analytically build speed-flow-density diagrams. Another modified approach to traffic flows analysis is proposed by Das and Levinson [10] starting from collected data on a freeway thanks to several loop detector. Queuing theory was used in particular to determine the positions of possible active bottlenecks.

All the cases presented so far rely on single queue models, however a better representation of traffic dynamics would require movement of vehicles and interaction between different nodes (i.e. queues). Queuing networks offer a solution for this new kind of problem. Based on conditions and assumptions made on the system, open, closed or mixed networks may be used. Woensel and Vandaele [11] extended their work on uninterrupted traffic flows [9], considering an open network of consecutive nodes as model of an highway where only feed-forward flows can be considered. They distinguished two cases with an infinite and a finite buffer for the queues including blocking mechanisms in the second one. However exact analytical solutions where not possible and different approximation technique to obtain the system metrics where presented. A Jackson network based solution for single-line uninterrupted traffic flows is presented instead by Raheja [12]. The network is modelled as a closed series of nodes connected in a circle where arrival and service times follow both an exponential distribution allowing a closed form expression for the probability distribution of vehicles.

Queuing network in the transport sector are also used to model sharing systems where queues usually represent city zones or stations from which different types of vehicles can be booked and taken by users. Following the most widespread approach in the study of sharing systems, George and Xia [13] designed a closed network model for a generic station based system with single server queues having Markovian arrival and service rates where vehicles are the customers moving from one station to another following a given routing matrix. The availability of vehicles at stations was the target metric and the network solution was provided through exact MVA and Schweitzer–Bard MVA approximation. An introduction to the MVA and how it can be used to solve queuing networks is presented later in the document in section 4.1.3. Additionally a revenue-based fleet size optimisation problem was formulated starting from the obtained results. A similar network solution was formulated by Fanti et al. [14] focusing on electric cars and differentiating three types of queues at the station for fully charged, partially charged and out of charge vehicles, these last waiting in line to be charged. Other three queues then define trips in the neighbourhood for fully and partially charged vehicles respectively and long trips for fully charged ones only. A routing matrix was eventually constructed to allow short or long trips based on vehicles power levels (i.e. on their origin queue) and dealing with out of charged ones before and after the charging process. The MVA was the chosen tool to solve the network also in this case and an additional fleet size optimisation problem was proposed. Samet et al. [15] focused on a station based bike sharing and studied its performances with a closed queuing network model with finite buffers capacity and three different kind of queues: single server queues to represent the stations and two classes of multiple server queues to manage rejected bikes and to model trips time respectively. The repetitive service with random destination (RS-RD) blocking was chosen to deal with capacities and losses and a general distribution for both arrival and service times characterises the stations. Since no straightforward analytical solutions are possible to solve such a network, an approximate method based on entropy maximisation was employed.

Between sharing systems one of the most popular and convenient solution nowadays is represented by free floating ones where generic vehicles can be left in any publicly available parking spot within the boundaries of the system operational area. From the modelling point of view this can be a complication since the one-to-one correspondence between station and queue is lost and an alternative discretization of space is required. Fricker et al. [16] proposed a division of the service area in many small zones comparable to stations with a total number which tend to infinity. A closed homogeneous Markovian network was developed at first with RS-RD blocking and with parameters inferred from real data. Each zone at steady state is seen as a tandem of an M/M/1 queue for available vehicles and an M/M/∞ for reserved ones. Additionally a more complex model with the possibility for users to book and cancelling their reservation was studied. An alternative approach for free floating sharing systems modelling was proposed by Kim et al. [17] consisting of a network in which vehicles are servers moving between nodes as a consequence of user mobility. Both arrival and service times are considered Markovian processes and losses may happen if no idle servers are available at a node.

2.2 Relocation and charging operation management in shared vehicles systems

All mobility sharing systems and free floating ones in particular, require special attention to re-balance operations which are the focus of many researches within the last years.

Weikl and Bogenberg [18] provided a first categorisation of classical relocation techniques in sharing scenarios employed in literature. The first distinguish is between user-based and operator-based strategies. Entrust users to re-balance the network is certainly cost effective from the operator point of view but some incentives are required for users to modify their behaviours to benefit the overall operations; possible incentives were identified as special-priced rides or free parking. This also requires to prior inform the customer of their possible contribution or to ask them for their intended destination before departure. Operator-based strategies on the other hand require availability of permanent workforce which can be costly and involves additional trips without a revenue. However it can be combined with gas filling or battery charging operations and it is much more reliable than the previous approach. The authors then also provided a two step optimisation model for vehicle repositioning with an offline demand prediction and a online optimisation algorithm to find the best relocation strategies when the vehicle distribution deviate from the predicted positioning. Uesugi et al. [19] studied the optimal assignment of vehicles to users in a one-way car sharing system with user-based relocation in order to optimise the re-balance operations. The optimisation problem aimed at minimising the square residual error sum between the optimum and the real number of parked vehicles in a station, assigning the optimal number of vehicles to each user. Boldrini et al. [20] modelled a single-class closed queuing network applicable to both station-based and free floating shared systems with exponential arrival and service times at the queues and trips time modelled with infinite servers queues. Studying the performances of a free floating bike sharing (FFBS) system using this model, an increase of the fleet size resulted non beneficial in terms of additionally satisfied demand therefore user-based relocation policies were proposed. This study in particular used stackable vehicles such that is simple for a single driver to move more units in a single trip. Two simple heuristic relocation strategies were implemented: a *uniform* one in which users bring an additional vehicle to the planned destination with a certain probability and a *backpressure* one where the same type of relocation happens only if the number of vehicles in the destination zone is less than the number in the origin one. An additional approach is then proposed for an optimal relocation strategy involving a modification of the original network to include load dependent *relocation queues* and defining a parameter which regulates the probability for a user to relocate an additional vehicle when moving to another station. Barth and Todd [21] developed a queuing simulation model with parameters inferred from collected data for a station-based sharing system with electric vehicles. Different algorithms for operator-based relocation were tested in the simulation environment: a *static* one based on real-time needs at particular stations, an *historical predictive* one using prediction of expected demand to determine the best relocation routes and an *exact* one with optimised scheduled operations based on exact knowledge of future demand. The system performances were measured through metrics such as the user average waiting time, the number of waiting customers and the total number of relocation operations required. Additionally a cost analysis was carried out for each scenario based on fixed costs of system deployment and on the trade off between costs for personnel, management and relocation operations and extra earnings from increases in met demand. A similar approach based on a model of a closed queuing network with M/M/1 queues representing city zones and M/G/∞ queues for the interconnections was studied by Bazan et al. [22]. Additionally an optimisation model was developed with constraints derived from the previous one, to minimise the number of empty trips (i.e. relocation trips) while maximising the total revenue. The two models were then combined in a simulation environment to obtained through MVA the optimal fleet size, the total costs of relocation and the average availability of vehicles. Jorge et al. [23] proposed instead an optimisation model for vehicles relocation that aims at maximise the operator revenues from paid trips and taking into account costs of staffed-based operations and maintenance. Also a simulation model was developed to test a real-time policy in which at each minute each station in the network is classified as *supplier* or *demander* based on previous data of departures and arrival at that station for the same time period. The relocation problem is then solved by computing the optimal routes from the *supplier* to the *demander* stations that minimise the operation costs. Some variant of the same policy were tested changing the interval of time to assess if a station is a supplier or introducing different constraints such as a minimum number of vehicles to be left at each station or an initial distribution of vehicles that has to be respected at the beginning of each day. The optimal model applied to a case study provided significant increases in the daily profit of the operator whilst the real time policies resulted in a more limited but still substantial increase. Another case of cost effective relocation is presented by Zhang and Pavone [24] which developed a queuing model for autonomous MoD systems with self-driving vehicles that rebalance themselves according to the system needs. A closed Jackson network was used also in this case comprehending passenger losses. The optimal rebalancing problem was formulated and solved to minimise the number of relocated vehicles guaranteeing their availability through the network together with an additional real-time rebalancing policy. Again performance metrics were obtained through MVA showing the determinant impact of relocation in the management of traffic congestion.

The operation of refuel or recharge of vehicles are another important branch of research in MoD systems. User contributions can be considered providing incentives in a similar way as seen for user-based relocation. However with sharing systems based on EV this may be a further complication. Most of the deployed systems currently rely on operators to connect EV to charging stations whether they are distributed through the operational area or concentrated in a single facility. The management of such operations represent a significant cost both in terms of workforce and of time needed.

Two of the most explored topics regarding the management of sharing systems with charging stations are the stations location and sizing problems. Sadeghi-Barzani et al. [25] proposed an optimal approach for both these problems which minimises the costs for the deployment of a fast charging station infrastructure as a mixed integer non linear programming (MINLP) solved with a genetic algorithm. Costs for the station development and electrification were considered as well as the loss on the electricity grid. Asamer et al, [26] instead developed an optimisation model for an electric taxi system. After inferring the user demand from historical data and identifying the charging demand, the optimal placing regions for the charging infrastructure were obtained as result of a mixed integer linear programming (MILP) based on the maximal covering location problem [27]. Roni et al. [28] formulated an optimisation model for the charging management and planning of the related infrastructure for a FFCS with an EV fleet, as an integer programming (IP) combining the optimal location of charging stations problem together with the optimal assignment of EV to them. Applying it to a case study they found that the dominant component of the total time required for the charging operations is the effective charging time required by the vehicle and that therefore adding more stations in the network is beneficial in reducing the average time to reach them but its contribution is almost negligible looking at the overall performances. Starting from real trips data of a free floating car sharing, He et al. [29] developed a queuing network model with multi-server nodes as charging stations. A particular attention was given to customers' picking behaviours distinguishing vehicles by their level of charge. Two different operator-based approaches were also considered for the charging process: one reactive and threshold activated and one proactive both with the possibility to interrupt the operation before the completing of the battery charge. On top of this network the charging infrastructure planning and fleet sizing optimisation were formulated as a non linear programming (NLP) for which an upper and a lower bound solution were computed. Folkestad et al. [30] proposed instead an optimisation model for both charging and repositioning of vehicles in a FFCS including their assignment to the stations and the routing of staff in charge of the operations. An hybrid genetic search algorithm was proposed as approximate solution and tested on real data bringing to the conclusion that combining repositioning with charging has a positive impact in terms of increased met users' demand. Ma and Xie [31] focused on an online method to assign vehicles to fast chargers and formulated a dynamic MILP to integrate in the optimal charging location problem. The aim of the on-line assignment problem was to minimise travel times and queuing delays resulting from charging operations, based on a given realisation of the system. The charging schedule was optimised following a rolling time-window for 24 hours intervals and results have been obtained from a simulation queuing model. A more complex approach was employed by Liang et al. [32] involving deep reinforcement learning (DRL) combined with binary linear programming (BLP) to solve a charging schedule and EV relocation problem. In particular DRL was used to compute the EV status in the system at specific time instants including their location and level of charge. Then an online scheduling was applied as a BLP to decide optimal repositioning and charging strategies.

2.3 Research group works

The study for this thesis project derives from the wider work on mobility and shared transportation carried out by the research group SmartData@Polito[1]; this center focuses on Big Data technologies, Data Science (from data management, to data modelling, analytics, and engineering), and Machine Learning methodologies applied to several domains of knowledge, finding solutions for both theoretical problems and helping companies toward applications. Some of the group works already addressed the theme of shared mobility in an urban environment.

In [33] a detailed study on the customers' demand prediction in a FFCS is presented starting from real trips data from the car2go database for the city of Vancouver. Several state of the art machine learning algorithms have been tested taking into account their prediction accuracy as well as the complexity of the models for both short and long term predictions. In the same paper the car sharing database was enriched with data from an open dataset from the Municipality census including features describing detailed and diverse socio-demographic characteristics of the city. These feature were then correlated with the demand for mobility in the FFCS and, using machine learning techniques, a tentative of prediction of demand was made based on these data only without any knowledge of past trips to try and highlight relationship between demographics and mobility in the city.

A data-driven model for demand prediction in FFCS systems is instead detailed in [34] with the aim of generalise the probability distribution of observed input data. A time estimation was performed assuming exponentially distributed inter-arrival times between bookings and with different arrival rates for each hour and type of

[1] https://smartdata.polito.it

day. The spatial characteristic instead was obtained fitting a bi-dimensional Kernel Density Estimation (KDE) on spatial data for each one of the previously derived temporal slot. The demand model was then tested in a simulation environment to generate new trips traces. A study on an electric fleet charging management is the proposed with a particular focus on different placement strategies for the charging infrastructure namely a centralised hub and a distributed set of stations.

The same demand model was used in [35] where the scalability problem of an electric FFCS was studied with a simulation approach. The impact of system design choices and infrastructure planning on the performances was initially shown comprehending an economic analysis on costs and revenues. Then the system scalability was investigated increasing the mobility demand and observing how parameters such as the charging infrastructure and the fleet size affected the system capability to cope with it.

A particular focus on the optimisation of charging infrastructure was given in [36, 37]. Charging station placement and car return policies in particular were studied including the possibility for users to return vehicles directly to a charging station if the state of charge is below a certain threshold. Heuristic methods were first explored for the station positioning such as selecting the top zones by average parking time or total number of parkings. Two optimisation algorithms were then proposed aiming at minimising costs and customers' dissatisfaction: a local search and a genetic algorithm. Performances were measured in the developed simulation environment in terms of percentages of infeasible, charges and rerouting trips. The firsts due to discharged batteries, the seconds when users directly plug vehicles to charge them and the latter when customers are forced to change their destination zone in order to charge the vehicle.

The problem of relocation applied to e-scooter sharing was addressed in [38] and treated in a simulation environment. Two prediction models for the demand are proposed: one baseline stationary model simply using averages over hour of day for two day type (i.e. weekday and weekend) of past rentals data to determine future demand and a second one using Deep Learning (DL) to better capture the spatio-temporal dependence of urban mobility. Based on the predicted customers' demand then a relocation schedule is implemented which identifies at each simulated hour pick up and drop off zones according to the expected surplus or lack of scooters respectively. Eventually a greedy strategy selects the pick up and the drop off zones with the highest number of surplus and the highest lack of vehicles respectively and assigns the closest worker to perform relocation of the maximum number of scooters they can move according to the system needs. A similar work on e-scooters is in [39] based on the demand estimation model already described in [34] and focusing on the impact of charging thresholds and heuristic policies on the battery swap operations to charge the fleet. In particular the consequences of an increased number of workers and a reduced time required for the operations are shown as well as the effect of the possible users contribution.

In the end the study in [40] proposes an interesting study of free floating systems comparing different engine types for the fleet namely ICEVs using gasoline, diesel and liquefied petroleum gas (LPG) and EVs. Using simulation different scenarios were studied including the required refuelling or recharging operation required by the fleet and focusing on, besides customers' satisfied demand and system profitability, on the environmental impact of each case by calculating the total greenhouse gases (GHG) emissions.

Chapter 3

Queuing modelling for shared MoD systems

Modelling is in general a powerful tool to provide a formal representation of a system and explain how does it work. The model is represented by means of state variables that describe how the system is working at any time. The relation between the variables and how they change and influence each others, describe the rules according to which the system change its state. Modelling can be very helpful in the design and dimensioning phase to try and forecast future behaviours, or on already developed projects to study peculiar scenarios, solve possible critical situation and obtain performance metrics for their evaluation. In particular analytical modelling provides the mathematical description that relates the elements of the system and derives mathematical expression to obtain performance indicators. A different but complementary approach is given by simulation models which recreate the system behaviour thanks to a simulation environment on a computer and extract performance metrics by simply observe its evolution.

Queuing modelling is a particular branch which uses queues as the studied object. A queuing system is characterised by customers that arrive to a queue in order to receive a service. The main parameters that describe such objects are: the arrival and departure process of customers in the queue, the number of servers which provide the service, the capacity of the waiting line, the customer population size and the queue discipline. The scope of this work is to provide an approximate analytical queuing model to represent a free floating electric car sharing system. To do so two different models are required for starters: the mobility zone queue, which simply represent the zone of the city where car sharing vehicles can be parked, and the charging station queue where electric vehicles are connected to charging outlets in order to increase their battery level. Eventually new queues can be introduced to model, for example, trips time between mobility and charging zones. In the following the first approximated models for the mobility and the charging queue in isolation are illustrated and then combined in the city zone queue. The successive steps is to create a network of zones to relate the fluxes of vehicles between them. A summary of the models presented in the chapter is reported in table 3.1 with single queue models in 3.1a and network ones in 3.1b.

	Queue models	**Assumptions**
Mobility zone	M/M/1	Exponential arrivals and service times
	M/M/1/B_p	Finite capacity
	M/M/1/F	Finite population
	M/M/1/B_p/F	Finite population and capacity
Charging zone	M/M/C	Multiple servers
	M/M/C/B_c/F	Multiple servers, finite population and capacity
City zone	M/M/1/B_p/F + M/M/C/B_c/F	Tandem of finite capacities and finite population queues

(a) Single queue models

	Queue models	**Assumptions**
Mobility zones only network	M/M/1/F	Gordon-Newell network
Charging stations in the network	M/M/1/F + M/M/C/F	Gordon-Newell network with multiple servers queues
Delay zones in the network	M/M/1/F + M/M/C/F + M/M/∞	Single infinite servers queue in the network
	M/M/1/F + M/M/C/F + M/M/∞	Multiple infinite servers queue in the network

(b) Queuing network models

Table 3.1: Single queue and queuing network models summary

A common variable between all the models is the population of customers that is made by the vehicles fleet which is assumed to be a set of homogeneous cars all with the same characteristics. In particular they all present the same battery capacities and average energy consumption.

3.1 Single zone queue models

Study the queue model in isolation is important to give a first characterisation of the its parameters and observe the behaviour of the customers inside each node. The first required step in the construction of the analytical model for the car sharing system, is to define its variables and associate them to the system parameters.

3.1.1 Mobility zone

The mobility zone queue represents a discretized zone of the city where the vehicles are parked. The position of the vehicles inside the zone is not taken into account. The departure process of vehicles from the queue is determined by the request for mobility of a user and it is a parameter that may vary with the city zone and in time. The arrival process, similarly, describes trips that end within the city zone limits. The number of servers for the queue is equal to one because one request for mobility is processed at a time and the service time is considered to be null since a mobility request can be either immediately satisfied if a vehicle is available in the zone, or it can remain unsatisfied if no vehicles are available. The reservation time, which is the time between the user booking request for the car and the actual moment in which the ride begins, has not been considered in this modelling scenario. With this configuration the time spent by vehicles in the queue is given by the single contribution of the waiting time i.e. the one during which idle, parked vehicles wait to be booked. The queue discipline which describe the movement of the customers within the queue is the FIFO; this assumes that the user will book a car within the zone of their choice without distinguishing between the possible available choices. Eventually the state variable which represents the state of the system is the number of vehicles present in the zone.

Users mobility requests are assumed to be random variables of a Poisson distribution and independent from each other such that the mean service time is exponentially distributed with parameter μ_p. Similarly the vehicles arrival process is assumed to be Poissonian with parameter λ_p. These assumption are often made in literature and have been validated for a case of a FFCS by the work in [34]. With this configuration and considering an infinite capacity of the waiting line, the city zone queue model is an M/M/1 queue as in the scheme in Figure 3.1. This is a well known queue for which it is simple to find a close form expression of the probability distribution of its customers and consequently, all the desired indicators can be easily calculated.

Figure 3.1: M/M/1 queue scheme

Mobility zone with finite capacity

Introducing a finite capacity in the queue model can be useful to represent the limited availability of parking spots in a city zone. An accurate dimensioning of this aspect although, would require an extensive knowledge of both the city topography and the state of all vehicles in it at all time which is, of course, impossible to obtain. Nonetheless even an approximate value for the capacity of the waiting line can reflect peculiar characteristics of some city zones for which it is known that the availability of parking spots it is a critical factor. The finite capacity model for an M/M/1 queue is well known and it corresponds to an M/M/1/B_p queue. More details on classic queuing models can be found in the original formulation of Allen [41].

The main difference with the original model is given by the introduction of possible losses and by its finite state space as shown in the Markov chain in figure 3.2. An expression for the steady state distribution of customers π_i in the queue can be obtained with the equations in 3.1 where π_0 is the probability of having an empty queue and ρ_p is its utilisation. In the car sharing scenario the possible queue losses represent cars that can not end their trips in the desired zone of destination and are therefore redirected elsewhere.

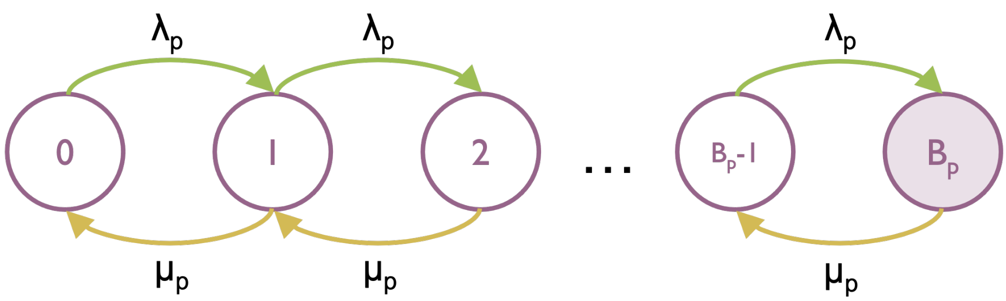


Figure 3.2: Markov chain of M/M/1/B_p queue

$$\pi_i = \pi_0 \cdot \rho_p^i \qquad \rho_p = \frac{\lambda_p}{\mu_p}$$
$$\pi_0 = \frac{1-\rho_p}{1-\rho^{B_p+1}} \tag{3.1}$$

Mobility zone with finite population

Finite population models are useful to describe a system in which the limited number of customers has an impact on the queue indicators. In particular the arrival rate depends on the number of customers that are not already in the queue as shown in its Markov chain in figure 3.3. As for the case of finite capacity queues, the number of possible states for the model is finite but no losses can happen. The equations for the closed form of the probability distribution function at steady state are reported in 3.2.

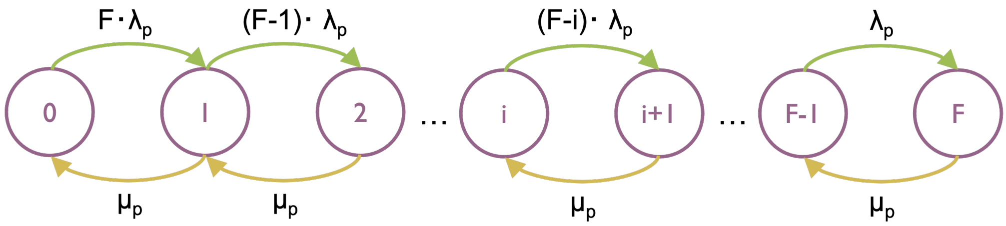


Figure 3.3: Markov chain of M/M/1/F queue

$$\pi_i = \pi_0 \cdot \frac{F!}{(F-i)!} \cdot \rho_p^i \qquad \rho_p = \frac{\lambda_p}{\mu_p}$$
$$\pi_0 = \frac{1}{\sum_{i=0}^{F} \frac{F!}{(F-i)!}\rho_p^i} \tag{3.2}$$

Combining the two described models a new queue of type M/M/1/B_p/F can be obtained which reflects the effects of both the finite population model and the finite capacity of the waiting line. The basic assumption for this case is that not all the the customers can be simultaneously in the same queue, i.e. $B_p < F$. In a shared mobility environment this would mean that non all the vehicles in the fleet can be parked inside the same zone at the same time due for example to a lack of parking spots or even on a general definition of city zones based on smaller areas. Additionally the arrival rate of the cars in each zone depends on how many cars are already inside it; this assumption can result particularly relevant when the fleet size is small and the number of zones is limited. The resulting Markov chain is reported in figure 3.4; this is the same as the one shown in figure 3.3 but truncated in correspondence of the state B_p. Similarly a closed form expression for the probability density function can still be obtained starting from the finite population one, just limiting the space of the summation as shown in the equations in 3.3.

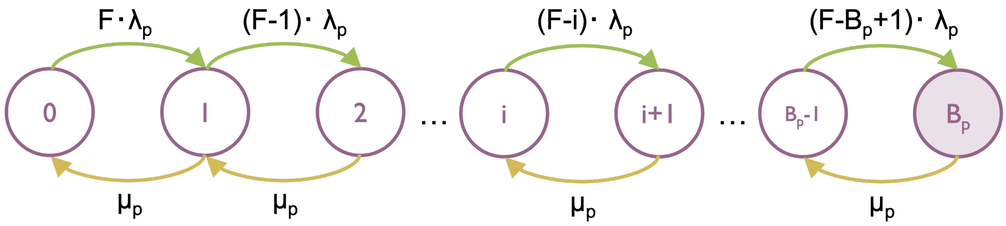


Figure 3.4: Markov chain of M/M/1/B_p/F queue

$$\pi_i = \pi_0 \cdot \frac{F!}{(F-i)!} \cdot \rho_p^i \qquad\qquad \rho_p = \frac{\lambda_p}{\mu_p}$$
$$\pi_0 = \frac{1}{\sum_{i=0}^{B_p} \frac{F!}{(F-i)!} \rho_p^i} \tag{3.3}$$

3.1.2 Charging station

Charging stations are required in an electric car sharing system in order to perform charging operations required by the vehicles. Different approaches can be used allowing the users to connect the cars to the charging outlets or leaving this operation to the car sharing employees which can retrieve vehicles from a city zone and bring them to charge. In all these scenarios the charging station can be modelled as a queue in which the servers represent the charging outlets and consequently the service rate depends on the time needed by a car to complete its charging process. Arrival rates in the zone can be decided according to the need for charging of the fleet and to possible charging policies and strategies.

For this model the state variable is, again, the number of vehicles in the queue and the queue discipline is the FCFS one. In a first approximation both the arrival and departure rates are assumed to be Poisson distributed and independent. The basic model for the charging queue is then the M/M/C where C is the number of servers (i.e. charging outlets) as shown in the scheme in Figure 3.5.

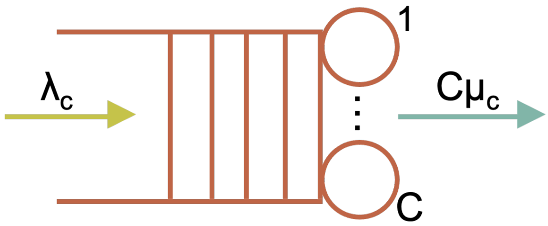


Figure 3.5: M/M/C queue scheme

As shown for the parking zone queue, the model can be improved considering the finite population and the capacity of the waiting line. While the finite population is exactly the same as the one considered for the parking queue (i.e. the vehicles fleet), the limitation on the size of the waiting line is a parameter that can be set a priori by the system operator. Especially when the charging operations are carried out by the car sharing employees in fact, it can be decided to limit the number of vehicles that wait for a plug to become available or even not to allow queuing at the stations. Furthermore the physical meaning of the waiting line in the charging queue can vary whether the cars are brought in its proximity while waiting or if they simply become unavailable to the users and are physically brought to the station only when they can be put on charge.

An additional difference with the model considered in section 3.1.1, is given by the presence of multiple servers. This means that the overall service rate of the queue changes with the occupation of the C servers as it is possible to notice in its Markov chain in figure 3.6. Anyhow, an expression for the steady state distribution of the M/M/C/B_c/F queue can be derived where F is the same fleet size considered for the parking queue and B_c is the chosen capacity and it is shown in equations 3.4.

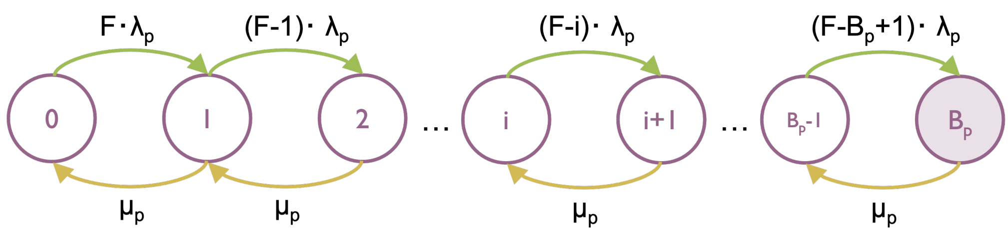


Figure 3.6: Markov chain of M/M/C/B_c/F queue

$$
\begin{aligned}
\pi_i =& \pi_0 \cdot \rho_c^i \cdot \prod_{i=0}^{C-1} \frac{F-i}{i+1} \cdot \prod_{i=C}^{B_c} \frac{F-i}{C} \\
\rho_c =& \frac{\lambda_c}{C \cdot \mu_c} \\
\pi_0 =& \frac{1}{\sum_{i=0}^{B_c} \rho_c^i \cdot \prod_{i=0}^{C-1} \frac{F-i}{i+1} \cdot \prod_{i=C}^{B_c} \frac{F-i}{C}}
\end{aligned}
\tag{3.4}
$$

3.1.3 City zone

A general model for the city zone can be obtained combining the two previously described ones for the mobility and the charging station queues. In particular the final configuration would be a tandem of the charging station and the mobility zone queues where the departing flux of vehicles from the first one is completely directed towards the second one as shown in the scheme in figure 3.7. This is reflected in the real scenario by the cars that, once the charging process is completed, are detached from the charging outlet and become available for users bookings inside the same city zone. The incoming flows of vehicles in the two queues can be instead modelled in different ways depending on the charging policy adopted.

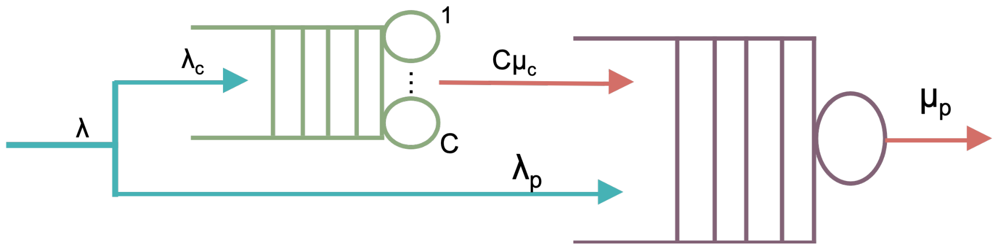


Figure 3.7: City zone queue model scheme

In the first approximation a general arrival process to the city queue, Poisson distributed and with rate λ, is splitted in two parts according to a probability that describes the charging needs of the vehicles in the system. The individual arrival process of the two components queues are therefore branches of a Poisson process and still Poisson distributed with parameters λ_c and λ_p respectively. These parameters can be defined in different ways according to the chosen policy on how to bring vehicles to charge. All the studied alternatives are explained later when considering a network of queues in section 3.3.2. Although the tandem of two queues is a well known model, the introduction of finite capacities in the two sub-components and the consequent possible losses, forbid the formulation of a closed product form expression for its steady state probability. In the following analysis the limitations due to the finite capacities of the queues buffers are set aside and the correspondent simplified model is analytically studied.

3.2 Queuing network modelling

To fully describe a MoD system through an approximate analytical queuing model, it is necessary to consider a network of multiple city zones such as the one described in paragraph 3.1.3. The queuing network allows to relate the flows of vehicles throughout the city by means of routing matrices and traffic equations. Different types of networks can be modelled based on the scenario and on the assumptions made; the most suitable option for the analysed system is a closed one also known as a Gordon-Newell Network. This type of network is defined by a finite and constant population with customers that do not leave the system, exponentially distributed service times and a FCFS discipline. Moreover the sum of the probabilities for a customer to leave a queue i and go to any other queue j in the network p_{ij} must be equal to one and the system must be ergodic (i.e. $\rho_i < 1$ for all queues i). Additionally the possibility to allow cycles has been included with trips departure and arrival zones that can coincide whilst no limitations for the queues capacity are considered in the first attempt. Including limitations on the queues buffer in fact would require to deal with possible losses drastically increasing the complexity of the model.

3.2.1 Closed queuing network of mobility zones

The first network model implemented is composed by a fixed number N of mobility zones queues, each representing an area of the city where car sharing vehicles can be parked. The network is fully meshed meaning that a trip can start and end in whatever city sector with the only constraint that no vehicles can leave the network. A routing matrix can be build to collect the routing probabilities p_{ij} for the entire network as in equation 3.5. Since the system is closed, these routing probabilities have to satisfy equation 3.6, which also implies that the routing matrix R must be stochastic.

$$R = \begin{bmatrix} p_{00} & \cdots & p_{0N} \\ & \cdots & \\ p_{N0} & \cdots & p_{NN} \end{bmatrix} \tag{3.5}$$

$$\sum_{j=1}^{N} p_{ij} = 1, \quad \forall i = 1,2,...,N \tag{3.6}$$

The service times for each queue are still considered independent and exponentially distributed with mean $\frac{1}{\mu_i}$. The parameter μ_i is specific for each zone and reflects the average users demand for mobility in that particular city area.

The scheme for a generic zone i in the network is shown in figure 3.8 where the possible routes are highlighted each with its probability and λ_i^* represent the customers flow through the queue at steady state.

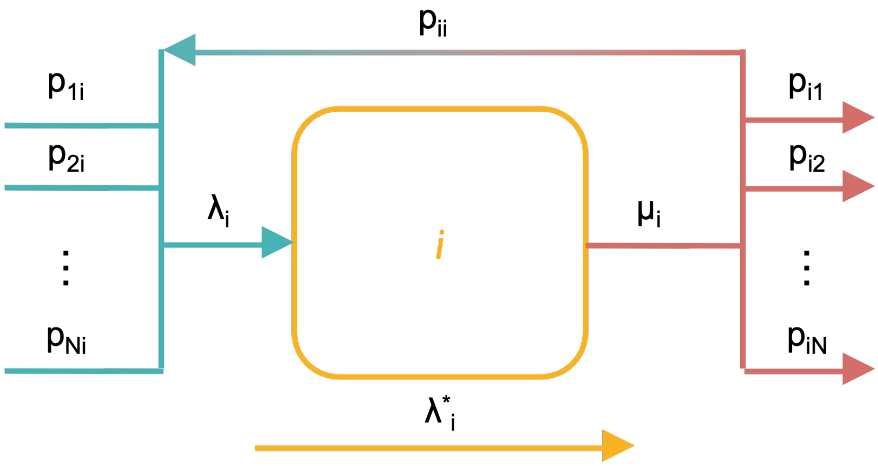


Figure 3.8: Generic zone in the network scheme

3.3 Charging stations in the network

A crucial element in an electric sharing system is the charging infrastructure. Charging stations are usually placed inside one or multiple city zones and consist of one or more charging points each with a socket where the EV can be attached through a plug and a cable. As seen in section 3.1.2, the charging station can be modelled as a multi-server queue where each server represents a charging outlet. This queue representation can be incorporated as a separate node in the network of mobility zones model following the scheme in figure 3.9 for each city zone in which the charging infrastructure has to be included. In a realistic scenario, of course, there is no need to include charging stations in each city zone since it would imply excessive costs and an high unused capacity.

3.3.1 Charging rates and performances

In characterising the charging station queue model, the first parameter to set is the service rate which will quantify the rate of charging of each outlet in the station. This is an intrinsic characteristic of the installed infrastructure, since each

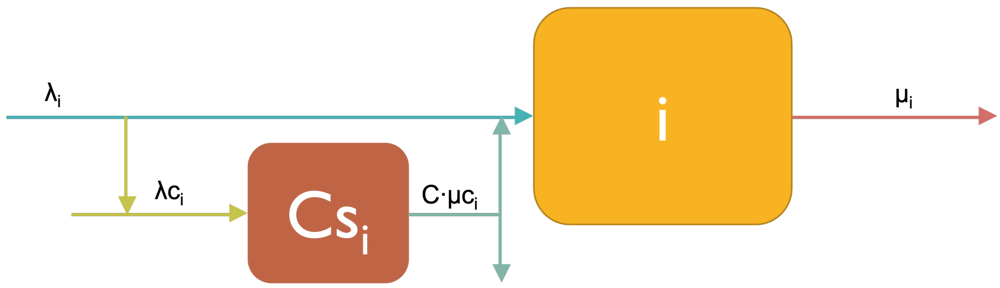


Figure 3.9: Charging station in generic city zone scheme

charging point has its own power. In the model approximation the service rates are exponentially distributed with a mean value μ_{Ci} that can be set according to the system characteristics. An exponential distribution for the charging rates is required by the network model to have a closed form solution and apply the algorithm to solve it as the MVA later explained in section 4.1.3. This assumption may be justified by the fact that, according to the different charging policies explained in the following, vehicles that are brought to charge have a starting battery level that may vary a lot hence requiring a different quantity of time to complete the charge. The overall service rate of the station will be then still exponentially distributed with mean value $C \cdot \mu_{Ci}$ where C is its number of servers (i.e. charging outlets). As of today the majority of the publicly available charging infrastructure has a power around $22kW$. However especially in the last few years, some supercharger alternatives are appearing on the market such as the Tesla supercharger [42] which has an outlet power up to $250kW$. Even so this kind of infrastructure is often still not widespread or even accessible by the general public or by car sharing customers and it is in general more difficult to operate.

Other parameters to be set by the operator that influence the charging performances are the charging thresholds. The minimum threshold is the minimum fraction of residual battery capacity allowed before making the vehicle unavailable for booking and bring it to charge. This is crucial in a sharing system to avoid situations where the EV loses all its battery capacity during a user rent or a vehicle remains available but with a battery status such that no user books it. The maximum threshold instead set the maximum fraction of battery to be charged in a complete charging operation. This is usually set to less than 100% of the total capacity to avoid a fast degradation and preserve the battery and prolong its lifespan.

Table 3.2 shows the impact of charging thresholds and charging stations power on the average time required for a complete charge. The EV took as reference for these scenarios is a Fiat new 500 action [43] in its model with a battery capacity of 23.8kWh an average consumption of 13kWh per 100km that guarantees an autonomy of around 180km.

Charging station power	**EV Battery capacity**	**Charging thresholds**		**Average charging time**
		min	max	
16kW	23.8kWh	20%	90%	63 min
16kW	23.8kWh	10%	1000%	81 min
22kW	23.8kWh	20%	90%	46 min
22kW	23.8kWh	10%	100%	59 min

Table 3.2: Charging infrastructure performances example

3.3.2 Flux in the charging station: charging policies

In order to redefine the routing matrix and the flows in the network considering the newly added charging nodes, different approaches using distinct policies can be employed. The incoming rate in the charging zone in particular can be decided a priori following pre-determined strategies to meet the power needs of the fleet. In the following three policies to determine the entering flux in the charging queue are illustrated namely *opportunistic*, *uniform* and *closest station*, all assuming that the charging operations are carried out by the system operators and that no relocation is employed after.

Opportunistic charging

The first analysed policy is referred as *opportunistic* since it considers only the power necessity of the vehicles entering in the city zones where a charging station is installed. No vehicles are then moved from one zone to another in order to be put in charge. This is a strong assumption especially if no particular criteria are followed to determine the positioning of the charging infrastructure in the city. However if the charging poles are enough and distributed in busy zones and the charging thresholds are set reasonably, it may be safe to assume that the EVs will eventually reach a charging station without run out of power. Even so this is not guaranteed because, depending on the input routing matrix, there may be a possibility, even if small, that repetitive cycles happen in the network which do not include zones with a charging station.

This policy needs a fraction of the flux entering a city node to be redirected to the charging station inside it as depicted in the scheme in figure 3.10.

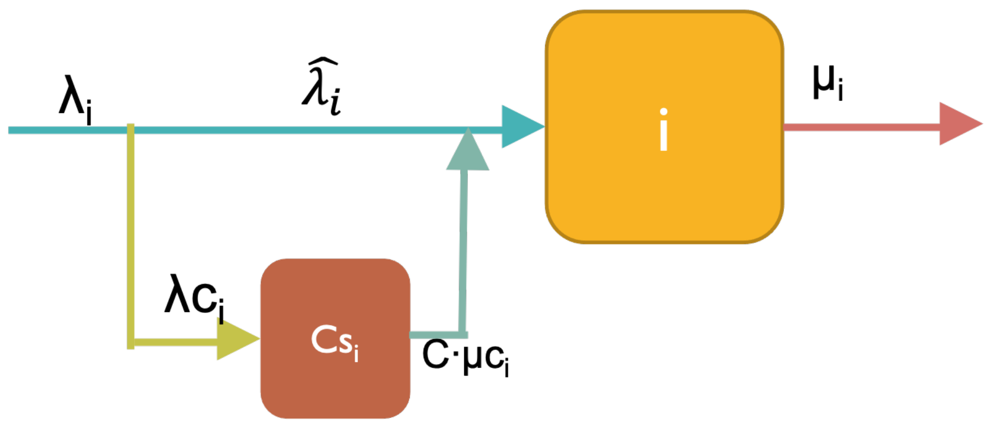


Figure 3.10: Opportunistic charging policy scheme

The flux in the station λ_{C_i} is determined analytically as a function of the EV autonomy and the fraction of the system flux which is directed towards the zones with a charging station as in equations from 3.7 to 3.9. The parameter K is inversely proportional to the average autonomy in terms of trips of the vehicles in the fleet t_a, while the terms CS in the summation indicates the mobility zones with a charging infrastructure within.

$$\lambda_{Ci} = K \cdot \lambda_i \tag{3.7}$$

$$\hat{\lambda}_i = (1 - K)\lambda_i \tag{3.8}$$

$$K = \frac{1}{t_a} \cdot \frac{1}{\frac{\sum_{j \in CS} \lambda_j^*}{\sum_{\forall j} \lambda_j^*}} \tag{3.9}$$

It is important to notice that while the terms K is used as a probability to redirect the flows of vehicles, there are no bounds in its formulation that ensure a value less than one as required by its definition. However in modelling realistic scenarios the eventuality of having a K greater than one is unlikely.

From the point of view of the entire city network, each charging station is considered as a unique node and therefore the original routing matrix is modified consequently to reflect the changes. In particular changing the policy that determines the flux entering in the charging nodes, will result in a modification of the routing probabilities in both the columns corresponding to the charging station and the mobility zone to which it belongs. An example of a modified routing matrix is in 3.10 where a charging station has been included in zone i. Since no relocation after charging is being considered yet, all the flux outside is directly merged into the corresponding mobility zone resulting in the last row of the matrix.

$$R = \begin{bmatrix} p_{00} & \dots & p_{0i} \cdot (1-K) & \dots & p_{0N} & p_{0i} \cdot K \\ & \dots & & \dots & & \dots \\ p_{N0} & \dots & p_{Ni} \cdot (1-K) & \dots & p_{NN} & p_{0i} \cdot K \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{3.10}$$

Uniform relocation charging

More complex approaches for the charging operations can be considered involving the concept of relocation of the fleet. Relocation implies the movement of vehicles from one network's node to another not as a consequence of user mobility but because of system operations. The network operator can use relocation to try and balance the system according to the expected user demand for mobility or, as in this case, to enhance the charging process. A scheme for a generic charging policy including relocation is shown in figure 3.11.

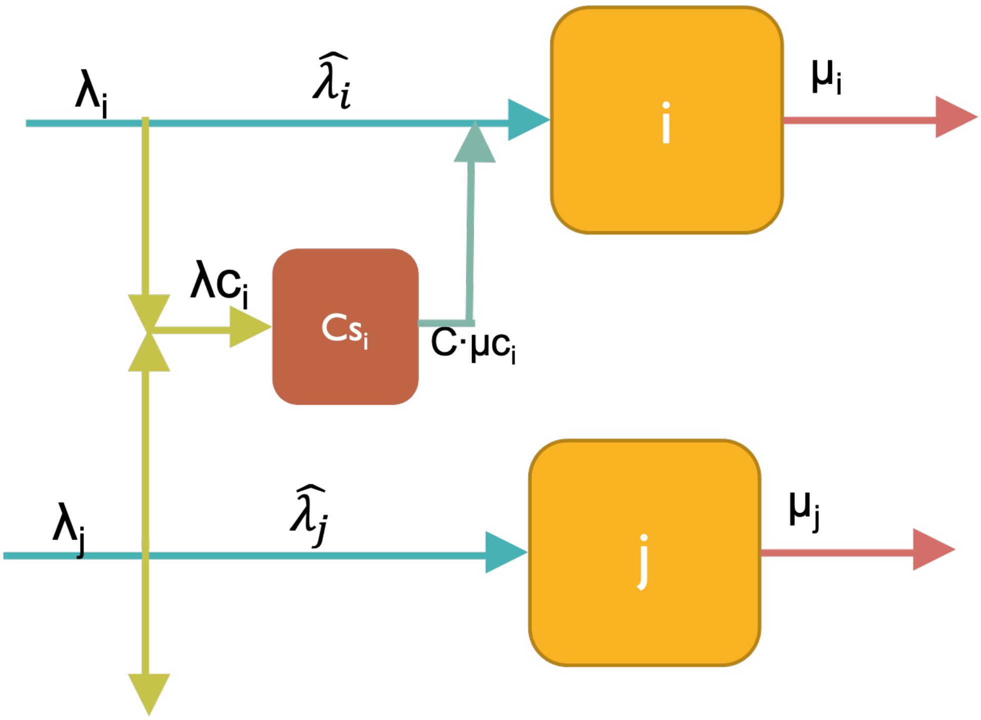


Figure 3.11: Generic charging policy with relocation scheme

The first considered policy exploiting relocation for charging is referred as *uniform* since vehicles are taken from every zone of the city, according to the expected fleet power needs, and are distributed uniformly towards the charging stations in the network. The incoming flux λ_{C_i} in the station under observation is therefore determined by the contributions of multiple fluxes from all over the network as in equation 3.11. The fraction K of the mobility flow to be redirected to the station, in this case, depends on the specific zone to which the flux was originally directed and it can be defined as in 3.13. In fact, given that $\frac{1}{t_a}$ is the fraction of vehicles that has to be taken from each zone and brought to charge, three cases may arise depending on the source zone. If the zone is the one with the charging station within, the entirely fraction is directed into its station since relocation to other charging point would bring no advantages. Consequently if a zone has another charging station within, no vehicles leave that zone to reach an external charging point. In all the other cases (i.e. mobility nodes without a charging infrastructure) the fraction $\frac{1}{t_a}$ of the incoming flux is splitted uniformly and forwarded towards the N_{CS} charging nodes in the network. For all the network nodes the fraction of flux which is not redirected for charging purposes $\hat{\lambda_j}$ is the same and depends only on the autonomy in terms of trips of the fleet as in equation 3.12.

$$\lambda_{Ci} = \sum_{j=1}^{N} K_j \cdot \lambda_j \qquad \forall i \in CS \tag{3.11}$$

$$\hat{\lambda_j} = 1 - \frac{1}{t_a} \cdot \lambda_j \qquad \forall j \tag{3.12}$$

$$K_j = \begin{cases} \frac{1}{t_a} & if \quad j = i \\ \frac{1}{N_{CS} \cdot t_a} & if \quad j \notin CS \\ 0 & if \quad j \in CS, \; j \neq i \end{cases} \tag{3.13}$$

Closest station relocation charging

The last considered policy to determine the incoming flux in the charging nodes, is the *closest station*. This is a possible variant of the *uniform* one previously explained, since it employs relocation for charging following the same scheme as in figure 3.11. The additional aspect introduced by the *closest station* policy is, as the name suggest, the geographical dependency of the system operations. In a real case scenario, in fact, it is safe to assume that the geographical distance between two nodes (i.e. city zones and charging stations) plays an important role in the decision making process, especially considering the operator's costs in terms of both time and fuel. In particular considering the relocation for charging, this policy assumes that vehicles are relocated from zones without charging points to the closest station. The additional spatial element requires to have an explicit knowledge of the network grid over the city in terms of geographical coordinates or, at least, the relative distances between all the zones in the network. Once this information are obtained, it is useful to define for each charging zone in the network the set Z_{CS_i} as the set of mobility zones for which CS_i is the closest charging point. The flux obtained with these hypothesis is in equation 3.14 and in this case the fraction K is equal to the inverse of the vehicles trips autonomy for all the mobility nodes. From the point of view of the system operations, this policy can bring important advantages since all the relocated EV for charging operations from one zone are directed to the same charging node which is also the closest one.

$$\lambda_{Ci} = \sum_{j=1}^{N} K \cdot \lambda_j \qquad \forall j \in Z_{CS_i} \tag{3.14}$$

$$K = \frac{1}{t_a} \tag{3.15}$$

$$\hat{\lambda}_j = 1 - K \cdot \lambda_j \qquad \forall j \tag{3.16}$$

3.3.3 Flux out the charging station: relocation after charging

Once the charging process is completed, the EVs leave the charging nodes servers and become available for the users bookings. In a real case scenario the cars have to be unplugged and moved away in order to make the charging outlet in the station available for new vehicles. Different policies may be actuated to decide where the EVs have to be taken following their charging operations in a similar way as it has been done do determine the fluxes toward the charging station in section 3.3.2. The most simple case implies that cars are simply unplugged and left in the same zone of the charging station while becoming available for booking. More complex policies may involve a further relocation following different criteria to determine the final destination nodes. All these strategies, and in particular the ones involving relocation, require the intervention of the system operator and can be planned beforehand following different optimisation criteria. A general scheme to highlights the flux out of the charging node is presented in figure 3.12. The different policies will provide different definitions for the probability $p_{Cs_i,j}$ from the charging node i to all the mobility zones in the network. Four strategies are proposed in the following: no relocation after charging, *uniform* relocation, *highest demand* relocation and *probabilistic* relocation.

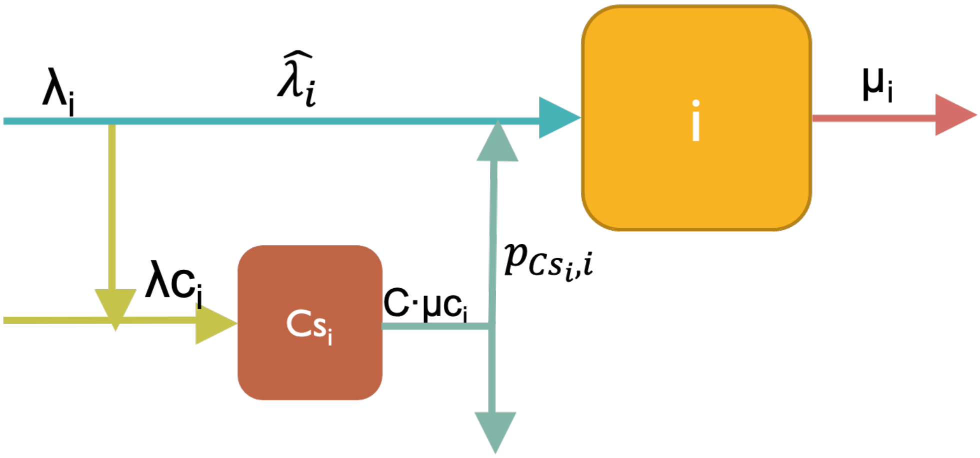


Figure 3.12: Generic scheme for flux outside the charging station

No relocation after charging

Considering the baseline scenario with no relocation after charging implies that all the vehicles leaving the station become available in the corresponding mobility node. Whilst this does not generate movement between city zones, it is still required that vehicles are unplugged to leave the servers available for new charging operations. This also means that cars have to be physically moved to a parking spot available in the same zone. An alternative policy, easier to applied in reality, consists in leaving the cars in the same spot once they have completed the charging. This kind of operation does not require to move the vehicles and it may be therefore more convenient in terms of costs and personnel required. However this would also imply that the charging station stays unavailable until the EVs is booked again and would significantly increase the complexity of the analytical model. For all these reason the first and only considered policy with no relocation after charging, assumes that vehicles leave the station once the charging is completed and become immediately available in the corresponding mobility zone. The simple mathematical formulation for the probability $p_{Cs_i,j}$ following these assumption is in equation 3.17.

$$p_{Cs_i,j} = \begin{cases} 0 \quad if \quad j \neq i \\ \\ 1 \quad if \quad j = i \end{cases} \tag{3.17}$$

Uniform relocation after charging

The first approach involving relocation, is referred as *uniform* and, as the name suggests, implies a uniform distribution of the vehicles towards all the other mobility zones in the network. It is the simplest approach considering relocation to implement, since the probability to move to each of the N mobility node in the network is the same and it is expressed as in equation 3.18. This kind of relocation does not take into account any factor regarding user mobility demand or the spatial distribution of the charging stations.

$$p_{Cs_i,j} = \frac{1}{N} \tag{3.18}$$

Highest demand relocation after charging

A more sophisticated policy using relocation can be formulated taking advantage of the available data on the expected user demand in each mobility node. In particular it is possible to limit the number of destination zones of the relocation operations to avoid to bring vehicles where they will probably remain unused for a long time. This policy requires to select the N_{top} zones with the highest expected user mobility demand where N_{top} is a new parameter that can be set according to the studied scenario. Once the top nodes have been selected a uniform relocation is employed limited to this set. Defining Z_{top} as the set of the N_{top} mobility zones with the highest expected mobility demand, the expression for the probability $p_{Cs_i,j}$ is in equation 3.19.

$$p_{Cs_i,j} = \begin{cases} 0 & if \quad j \notin Z_{top} \\ \\ \frac{1}{N_{top}} & if \quad j \in Z_{top} \end{cases} \tag{3.19}$$

Probabilistic relocation after charging

The final considered policy is a modified version of the *highest demand* one using a probabilistic approach. Starting again from the expected demand for user mobility, the flux out of the charging station is redirected towards each mobility node with a probability proportional to its expected demand rate. This allow for a simpler formulation without the need for an additional parameter such as N_{top}. Given the vector of the service rates for each zone μ its normalised version $\hat{\mu}$ is computed which reflects the out probability as in equation 3.20.

$$p_{Cs_i,j} = \hat{\mu}_j \tag{3.20}$$

3.4 Trips time and delay zones

In the network models seen in sections 3.2 and 3.3, customers movements throughout the city nodes are treated as instantaneous. In fact when a vehicle leaves a mobility zone or a charging station, it will appear immediately in the destination node according to the probabilities in the routing matrix. This assumption may be too simplistic considering a mobility sharing system in which, of course, movement between two points in the city requires a non negligible amount of time.

In order to include the trip time as a parameter in the queuing network model, a possible approach is to consider an additional kind of node whose aim is to associate a certain delay to each trip. The most suitable queue for this job is the M/M/∞ or infinite servers queue shown in the scheme in figure 3.13. Customers entering this kind of node do not queue since there is no buffer and there is always a free server available. The time spent in the queue is therefore given by the only contribution of the service time which in this case is associated to the trip time. It is important to notice that although an exponential distribution may not be the best one to describe the trips times even in an approximate way, this assumption is fundamental for an easy integration of the queue model in the network. Considering other types of distribution of service time such as a constant or a Gaussian one in fact would result in a non Gordon-Newell network, loosing the possibility to obtained a closed form expression from the distribution of customers with a normalisation constant and making impossible to apply the convolution theorem or the mean value analysis to obtain system metrics. On the other hand even if considering an high average for the exponential distribution, there may be very low values for the service time representing trips duration that are likely much longer.

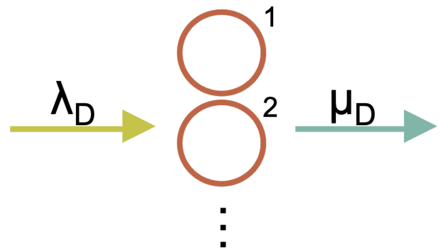


Figure 3.13: M/M/∞ queue scheme

Different approaches may be considered on how to set the network routing to include the trip delay. Ideally a delay node should be placed between each possible couple of origin-destination nodes such that the average service time can be modelled as realistically as possible considering the physical distance between the zones and the average vehicles speed. This although, would exponentially increase the dimension of the network and, consequently, its complexity. The two considering methods instead, use a unique delay zone for the whole network and multiple delay nodes each associate to a origin mobility one respectively.

3.4.1 Single delay zone

In the first approach a single M/M/∞ queue is added to the network and all the routing is diverted to pass through this node before reaching the intended destination. This requires to set a unique mean value for the service time of the delay node that has to reflect as much as possible the general mean trip time for all kind of movements in the network. A generic scheme for this approach is presented in figure 3.14. It can be noticed how all the fluxes are directed to the delay zone except the ones between a mobility zone and the charging station within its space. In fact this kind of movements are associated to a null trip time since the vehicles never leave the city zone.

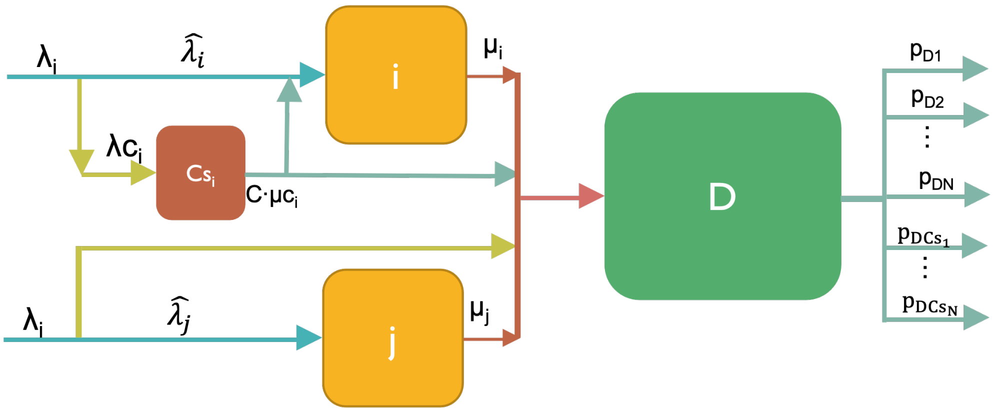


Figure 3.14: Single delay zone scheme

A particularly relevant complication regarding this model is that once all the flows merge into the delay zone, it is impossible to exactly determine their origin nodes and therefore compute the probabilities $p_{D,j}$ and p_{D,Cs_j}. It would be necessary in fact to go back to each individual contribution of the total flow in D and aggregate the ones directed to the same destination node weighting them by the quantity of incoming flux losing the one-to-one correspondence of origin and destination of the routing matrix. An approximation method is implemented in the next chapter to overcome this problem.

3.4.2 Multiple delay zones on departure nodes

In order to have a more punctual estimation of the service times and not to lose the origin-destination coupling of the network routing matrix, the second proposed approach considers a distinct delay zone, modelled still as an M/M/∞ queue, for each origin mobility zone. In this way all the flows of vehicles starting from a node passes through a different delay queue, whose average service time may be set accordingly considering possible peculiarity of the departure zone. In particular all the trips originated from each node are collected and the average distance is computed; the rate for each delay zone is obtained then dividing the average speed of the vehicles in the network by the computed average distance of the corresponding mobility zone. This procedure may result particularly useful for example, for zones that are significantly distant from the rest of the network and therefore generate flows with a greater trip time on average. While the dimension of the network increases by a factor N (i.e. number of mobility zones), the advantages of this method may result worth it especially considering the possibility of retaining the same routing probabilities of the original network. In fact each delay zone would just represent an additional step following the exit from a mobility zone. Starting from the original routing matrix represented in 3.21, the new one can be built with some simple operations and obtaining the result as in 3.22. In particular N new rows and columns are appended to the original matrix, representing the routing probabilities from and to the N delay zones respectively. Then the content of the first N rows in the original matrix is simply moved down to the last N rows since the flows out of each delay zone follow the same probabilities as the flows exiting the mobility zones in the original network. The probabilities out of the charging stations instead do not change. In the end the block in the upper right corner in equation 3.22 is an identity matrix since all the flows from each mobility zone are redirected to the corresponding delay zone. All other possible flows in the network are not allowed and are therefore represented with a zero probability in the routing matrix.

$$R = \begin{array}{cc} \begin{array}{cc} \overbrace{\hphantom{p_{0,0} \quad \cdots \quad \cdots \quad p_{0,N}}}^{N} & \overbrace{\hphantom{p_{0,Cs_0} \quad \cdots \quad p_{0,Cs_{Ncs}}}}^{N_{CS}} \end{array} & \\ \left[\begin{array}{cccc|ccc} p_{0,0} & \cdots & \cdots & p_{0,N} & p_{0,Cs_0} & \cdots & p_{0,Cs_{Ncs}} \\ \vdots & \ddots & p_{j,i} & \vdots & \vdots & & \vdots \\ \vdots & p_{i,j} & \ddots & \vdots & \vdots & \ddots & \vdots \\ p_{N,0} & \cdots & \cdots & p_{N,N} & p_{N,Cs_0} & \cdots & p_{N,Cs_{Ncs}} \\ \hline p_{Cs_0,0} & \cdots & \cdots & p_{Cs_0,N} & & & \\ \vdots & & \ddots & \vdots & & 0 & \\ p_{Cs_{Ncs},0} & \cdots & \cdots & p_{Cs_{Ncs},N} & & & \end{array}\right] & \begin{array}{l} \left.\vphantom{\begin{array}{c} p \\ \vdots \\ \vdots \\ p \end{array}}\right\} N \\ \left.\vphantom{\begin{array}{c} p \\ \vdots \\ p \end{array}}\right\} N_{CS} \end{array} \end{array} \tag{3.21}$$

$$
R = \begin{array}{c}
\begin{array}{ccc} \overbrace{\hphantom{xxxxxxxxxxxxxxxx}}^{N} & \overbrace{\hphantom{xxxxxxxxxxxx}}^{N_{CS}} & \overbrace{\hphantom{xxx}}^{N} \end{array} \\
\left[\begin{array}{cccc|cccc|c}
 & & 0 & & & 0 & & & I \\
\hline
p_{Cs_0,0} & \cdots & \cdots & p_{Cs_0,N} & & & & & \\
\vdots & \ddots & & \vdots & & 0 & & & 0 \\
p_{Cs_{Ncs},0} & \cdots & \cdots & p_{Cs_{Ncs},N} & & & & & \\
\hline
p_{0,0} & \cdots & \cdots & p_{0,N} & p_{0,Cs_0} & \cdots & p_{0,Cs_{Ncs}} & & \\
\vdots & \ddots & p_{j,i} & \vdots & \vdots & \ddots & \vdots & & 0 \\
\vdots & p_{i,j} & \ddots & \vdots & \vdots & & \vdots & & \\
p_{N,0} & \cdots & \cdots & p_{N,N} & p_{N,Cs_0} & \cdots & p_{N,Cs_{Ncs}} & &
\end{array}\right]
\end{array}
\begin{array}{l} \left.\vphantom{\begin{array}{c}0\\0\end{array}}\right\} N \\ \left.\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right\} N_{CS} \\ \left.\vphantom{\begin{array}{c}0\\0\\0\\0\end{array}}\right\} N \end{array}
\tag{3.22}
$$

Chapter 4

Queuing Network performance metrics and indicators

Deriving the system indicators and the performance metrics is much more difficult if a network of queues is considered instead of a simple queue in isolation. The common way is to try and derive the steady state distribution probabilities of the network. In some cases and for certain categories of networks these probabilities can be written in a convenient compact mathematical expression known as product form; the overall expression for the network is therefore the result of the product of the steady state distribution of the single queues as they were considered in isolation. From the obtained expression all the network metrics can be computed, although this may result in particularly computationally complex tasks. For this reason and for the analysis of networks that do not allow a simple product form distribution, different approximation techniques and analysis have been formulated and some of them have been employed in the following steps.

4.1 Mobility zones network

The first network considered is made up of mobility zones only, as previously described in section 3.2; this is a closed, cyclic queuing network whose nodes are all M/M/1 queues. Considering only single server queues is a particularly simplifying condition since it allows to use well known algorithms and procedure to derive the needed indicators.

4.1.1 Customers flows

The peculiar characteristic of the mobility zones network such as the one chosen to represent the car sharing system, is that customer's flows in its nodes are not independent and consequently not Poissonian. In fact both arrivals and departures in each node depend on the current distribution of customers throughout the network. Flows in the network are described through the traffic equations for each node. A system of N traffic equations for the closed network can be written as in equation 4.1 or in matrix form using the routing matrix R as the one in equation 3.5 and the unknown vector of flows λ^* as in 4.2.

$$\begin{cases} \lambda_i^* = \sum_{j=1}^{N} p_{ij}\lambda_j^* \qquad \forall i \in [1, N] \end{cases} \tag{4.1}$$

$$\lambda^* = \lambda^* \cdot R \tag{4.2}$$

Building such a system for the analysed closed queuing network results in a set of N linearly dependent equations since, as stated before, flows in each node are not independent and nor entering flux from outside neither exiting flux from the inside are allowed. The traffic equations system can therefore be solved by fixing the flow inside one of the queue obtaining in this way relative results with respect to the reference node. In a computer environment, starting from a random initial guess of the unknown vector λ^*, an iteration process can start computing the relative fluxes as in equation 4.2, until convergence is reached and the final vector is obtained. It is important to stress the fact that in both cases the solution of the system is a vector of relative flows in the network nodes which does not give a quantitative result on the real throughput.

4.1.2 Steady state distribution probabilities of the network

Whilst for ergodic open networks each queue can be assumed to behave independently at steady state, this assumption is not valid for closed networks. This also means that the steady state probabilities of the network can not be written as a simple product form. Instead, thanks to Gordon-Newell theorem, the joint steady-state probabilities can take a non-trivial product form as in equation 4.3, which takes into account the limited state space including a normalisation constant $G(F, N)$ as in equation 4.4. In fact given the vector $\mathbf{n}$ in equation 4.5 which express the distribution of the customers in the network's nodes, such distribution is valid only when $\mathbf{n}$ belongs to the allowed state space $S(F, N)$ in equation 4.6 which takes into account the constraint on the constant population F.

$$Pr(X(\infty) = x) = \pi_{\mathbf{n}} = \begin{cases} \frac{1}{G(F,N)} \cdot \prod_{i=1}^{N} \beta_i(n_i) & if \sum_i n_i = F \\ 0 & if \sum_i n_i \neq F \end{cases} \tag{4.3}$$

$$G(F,N) = \sum_{\mathbf{n} \in S(F,N)} \prod_{i=1}^{N} \beta_i(n_i) \tag{4.4}$$

$$\mathbf{n} = [n_1, n_2, ..., n_N] \tag{4.5}$$

$$S(F,N) = \{\mathbf{n} \in \mathbb{N}^N : \sum_{i=1}^{N} n_i = F\} \tag{4.6}$$

The factor $\beta_i(n_i)$ express the probability distribution of each single queue i in isolation and depends on its utilisation ρ_i. In particular the formulation of the Gordon-Newell theorem in equation 4.3 is valid for closed network of queues of type $./M/1$ and $./M/C$. Therefore the nodes must have an exponential distribution of their service times but can have different distributions for the arrival ones and in the same network can coexist multiple servers and single server queues. The number of servers C_i in each queue will determine the expression of $\beta_i(n_i)$ as in equation 4.7. The utilisation of each node is expressed by the well known formula in equation 4.8 where λ_i^* and μ_i are the steady state customer flow and the service rate of the queue respectively.

$$\beta_i(n_i) = \begin{cases} \rho_i^{n_i} & if \quad i \text{ is a ./M/1} \\ \begin{cases} \frac{\rho_i^{n_i}}{n_i!} & if \quad n_i < C_i \\ \frac{\rho_i^{n_i}}{C_i! \cdot C_i^{n_i - C_i}} & if \quad n_i \geq C_i \end{cases} & if \quad i \text{ is a ./M/C} \end{cases} \tag{4.7}$$

$$\rho_i = \frac{\lambda_i^*}{\mu_i} \tag{4.8}$$

Normalisation constant computation

As for its definition, the direct computation of the correction factor $G(F,N)$ requires the exploration of all the network state space $S(F,N)$. In particular all the possible combinations of customers in the queues have to be tested. This may lead to a too computationally complex problem when the number of nodes and the population of the network increase. The cardinality of the state space may be computed through equation 4.9 and it is easy to imagine how the complexity of this operation represents a problem even considering relatively small networks and population sizes.

$$|S(F,N)| = \binom{F+N-1}{F} \tag{4.9}$$

A numerical approach can be employed instead, provided by the convolution algorithm or Buzen's algorithm first formulated by the homonymous author [44] whose main step are summarised in Algorithm 1. This recursive method is based on the fact that one can easily compute $G(f,N)$ as function of $G(f-1,N)$. In this way starting from an empty network and increasing at each step the population by one, it is possible to evaluate the correction factor with just $N \cdot F$ multiplications and $N \cdot F$ additions.

Algorithm 1 Convolution (Buzen's) algorithm

procedure CONV(F,N) ▷ for Network of single server queues
    ▷ **Initialisation**
    $g(0,n) = 1 \qquad n = 1,...,N \quad f = 1,...,F$
    $g(f,1) = \rho_1^n \qquad n = 1,...,N \quad f = 1,...,F$
    ▷ **Recursion**
    $g(f,n) = g(f,n-1) + \rho_n \cdot g(f-1,n)$
    ▷ **Termination**
    $g(F,N)$
end procedure

The computation of the normalisation constant, as said, allows to write an expression for the joint steady state probabilities of the network and consequently to extract useful metrics such as the mean number of customers in each queue or their actual utilisation and throughput. A modified version of the algorithm can be employed to include multi-server queues or queues where the service rate depends on the state of the network. However, the dependency on the steady state distribution to exactly compute network indicators can still lead to complexity problems for big networks.

4.1.3 Mean Value Analysis

The MVA is an approximation technique and a powerful tool in analysing closed networks first formulated by Reiser and Lavenberg [45]. It allows to directly compute the network performance metrics, bypassing the need for the steady state distribution calculation and the exploration of its state space. The algorithm is based on a previous result obtained by the same authors [46] stating that the stationary state probabilities when a customer enters a network are equal to the stationary state probabilities at arbitrary times for the same network with one less customer. This allows to find a simple relation between the mean waiting time and the mean queue size of the same system with one less customer. Combining this result with Little's law, a recursive procedure is obtained which increases the population size at each iteration as reported in Algorithm 2.

Algorithm 2 Mean Value Analysis

procedure MVA(F, N) ▷ for Network of single server queues
  ▷ $L_n(f)$: mean number of customers in queue n in a network with f customers
  ▷ $W_n(f)$: mean time spent in queue n in a network with f customers
  ▷ λ_n^*: relative flow of customers in queue n
  ▷ λ_f: real throughput of the network
  ▷ **Initialisation**
  $L_n(0) = 0 \qquad n = 1, ..., N$
  ▷ **Recursion**
  $W_n(f) = \frac{1+L_n(f-1)}{\mu_n}$
  $\lambda_f = \frac{f}{\sum_{n=1}^{N} W_n(f) \cdot \lambda_n^*}$ ▷ Little's Law
  $L_n(f) = \lambda_n^* \cdot \lambda_f \cdot W_n(f) \qquad f = 1, ..., F$
  ▷ **Termination**
  when f=F
end procedure

The results obtained with the MVA algorithm allows to compute the real throughput of the network with equation 4.10 starting from the relative flows obtained through the traffic equations as seen in section 4.1.1 in equation 4.1.

In the car sharing environment, the throughput vector λ represent the total number of trips at steady state per city zone and it is an essential result for computing other metrics such as the occupation vector (i.e. the utilisation of each zone) as in equation 4.11. The occupation vector tells how much of the capacity of a node is used and in the case under study corresponds to the fraction of user demand for mobility that is met by the system. Using this simple result it is then possible to compute a fundamental indicator in the modelling of a sharing system which is the fraction of unsatisfied mobility demand as in equation 4.12. Additionally the actual number of lost mobility requests per unit time can be simply obtained multiplying the unsatisfied fraction by the rate of demand for each zone.

$$\lambda = \lambda_f \cdot \lambda^* \tag{4.10}$$

$$\rho = \frac{\lambda}{\mu} \tag{4.11}$$

$$u_d = 1 - \rho \tag{4.12}$$

Another fundamental outcome of the algorithm is the mean time spent by a customer in each node which, in theory, is the combination of the waiting plus the service time of the queue. In the car sharing system however, the service time is considered to be null since when a mobility request arrives (with rate $\frac{1}{\mu}$), it is either immediately satisfied or rejected if no vehicles are available in the zone. Therefore the mean time in the queue correspond to the waiting time during which a car remains idle and available for booking in a city zone.

Also in this case a modified version of the algorithm can be implemented to deal with multi-server or infinite-server queues.

4.2 Network with charging stations and delay zones

The network model for the electric car sharing system is not complete without considering the charging infrastructure. As introduced before in section 3.3 charging stations are considered as separate nodes and modelled as multi-server queues. Furthermore the introduction of delay zones in the system as shown in section 3.4, modifies again the routing and the fluxes in the network. Similarly to what has been done with the mobility zones in section 4.1, the flux inside the new nodes have to be determined solving the system of traffic equations with the modified version of the routing matrix. Then the convolution algorithm and the MVA in their multi-server and infinite-servers versions can be employed to obtain the overall network indicators and extract the system performances.

4.2.1 Multi-Servers Convolution algorithm and MVA

The introduction of multiple server queues in the network, results in state-dependent service rates. In fact, considering the charging stations, the total rate of charge in the node depends on how many of the available outlets are occupied. Consequently also the utilisation of multi-server queues ρ_i will depend on the number of customers in the node. A new definition of the utilisation is then needed in the Convolution algorithm taking into account the state dependency. The new formulation is shown in Algorithm 3.

Algorithm 3 Multi-server Convolution (Buzen's) algorithm

procedure CONV(F, N) ▷ for Network of multi-server queues
  ▷ $\mu_n(f)$: service rate of queue n when it has f customers
  ▷ s_n: number of servers in queue n
  $\mu_n(f) = min(f \cdot \mu_n, s_n \cdot \mu_n)$
  $\rho_n = \frac{\lambda_n^*}{\mu_n(1)}$ $n = 1, ..., N$
  $A_n(f) = \prod_{j=1}^{f} \frac{\mu_n(j)}{\mu_n(1)}$ $n = 1, ..., N$
  ▷ **Initialisation**
  $g(0, n) = 1$ $n = 1, ..., N$ $f = 1, ..., F$
  $g(f,1) = \frac{\rho_1^n}{A_1(f)}$ $n = 1, ..., N$ $f = 1, ..., F$
  ▷ **Recursion**
  $g(f, n) = \sum_{j=0}^{f} \frac{\rho_n^j}{A_n(j)} \cdot g(f - j, n - 1)$
  ▷ **Termination**
  $g(F, N)$
end procedure

For the same reasons the state-dependency of the service rates of multi-server queues impact also on the MVA Algorithm. In particular a correction factor is required in computing the mean time and the marginal probability $p_n(j, f)$ is introduced and updated in the recursion steps as shown in Algorithm 4.

In dealing with the infinite servers model of the delay zone the modification required on the MVA algorithm are minor and regard only the delay which in these type of queue is given by the only contribution of the service time and therefore can be easily computed as in equation 4.13.

$$W_n(f) = \frac{1}{\mu_n} \tag{4.13}$$

4.2.2 Waiting probability in charging stations

Another useful indicator for the charging queues, is the probability for a vehicle of waiting in line before receiving the service. Following the implemented charging

Algorithm 4 Multi-server Mean Value Analysis

procedure MVA(F, N) ▷ for Network of multi-server queues
 ▷ $L_n(f)$: mean number of customers in queue n in a network with f customers
 ▷ $W_n(f)$: mean time spent in queue n in a network with f customers
 ▷ λ_n^*: relative flow of customers in queue n
 ▷ λ_f: real throughput of the network
 ▷ $p_n(j, f)$: marginal probability of j customers in queue n given a network with f customers
 ▷ s_n: number of servers in queue n
 ▷ **Initialisation**
 $L_n(0) = 0$ $\quad n = 1, ..., N$
 $p_n(0{,}0) = 1$ $\quad n = 1, ..., N$
 $p_n(j{,}0) = 0$ $\quad j = 1, ..., s_n - 1 \qquad n = 1, ..., N$
 ▷ **Recursion**
 $W_n(f) = \frac{1+L_n(f-1)+S_n}{\mu_n}$
 $S_n = \sum_{j=1}^{s_n-1} (s_n - j) \cdot p_n(j-1, f-1)$
 $\lambda_f = \frac{f}{\sum_{n=1}^{N} W_n(f) \cdot \lambda_n^*}$ ▷ Little's Law
 $L_n(f) = \lambda_n^* \cdot \lambda_f \cdot W_n(f) \qquad f = 1, ..., F$
 $p_n(0, f) = 1 - \sum_{i=1}^{N} p_n(i, f)$
 $p_n(j, f) = \frac{\lambda_f \cdot p_n(j-1,f-1)}{\mu_n}$ $\quad j = 1, ..., s_n - 1$ ▷ Update $p_n(j, f)$
 ▷ **Termination**
 when f=F
end procedure

policies, in fact it may happen that an EV is brought to a charging station and finds all the charging outlets already occupied. In this eventuality the vehicle may be kept in queue near the station and plugged whenever one of the outlet is freed. In the case of an heavy utilisation of the infrastructure, the charging queue occupation may increase a lot and it may be difficult to organise the vehicles in line. The waiting probability then become useful to study the impact of this phenomenon and eventually including a capacity for the waiting line. As stated before introducing a limited capacity queue would result in a serious complication of the overall network since possible losses may happen. The waiting probability for multiple server queues can be easily obtained through the Erlang-C formula as in equation 4.14 avoiding the need for the probability distribution of the node. This is a well known results obtained as function of the number of servers s, the flux λ, the service rate μ and the consequent utilisation of the queue ρ.

$$p_w = \frac{\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \cdot \frac{1}{1-\rho}}{\sum_{i=0}^{s-1} \frac{1}{i!}\left(\frac{\lambda}{\mu}\right)^i + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \cdot \frac{1}{1-\rho}} \tag{4.14}$$

4.2.3 Approximate method for single delay zone model

The main concern of introducing a single delay zone in the network, as explained in section 3.4, is that once merged it is not possible to go back to the single contributions of the flow in the node in order to assign the correspondent routing probabilities. The developed approximate method to overcome this problem is based on the assumption that the relative flows in each original network node at steady state remain the same once the delay zone is added. In fact the only scope of including the additional node is to introduce a variable delay associated to the trips but without modify the original routing. Another assumption is that all the flows in the new network have to pass through the delay zone. Therefore the traffic equations are solved considering the original network with the mobility and charging zones only in order to find the relative flows in each node; then the delay zone is introduced and a relative flow equal to the sum of all the flows in the network is assigned to it. Having then the complete vector of flows λ^*, it is possible to directly extract the system metrics with the MVA. However it is important to notice that not all the fluxes in the network imply moving from one city zone to another since there is also an internal routing between some mobility zones and the charging stations inside them. These movements, depending on the assumption made, may just represent vehicles that are unplugged from the charging station and left there or relocated in its proximity therefore requiring very little or not delay at all. Nevertheless the fraction of these trips can be considered negligible with respect to the total amount of flows in the network especially when considering charging policies with relocation of vehicles.

Chapter 5

Real case city scenario applications

Applying the model to real scenarios, requires of course real data to be used as input. In particular the routing matrix has to be extracted from the movement of the vehicles through the city, and the service rates of each zone require an estimation of the user demand which can be spatially and temporarily distributed. Also the identification of the city zone is required as well as the positioning of the charging infrastructure. All these data are obtained in the following starting from real records of bookings of a car sharing system already manipulated to comprehend initial and final position and time of each booking. It is important to notice that in the analysed cases the fleet is made up by internal combustion engine vehicles while the proposed model assumes an homogeneous fleet of EVs.

5.1 Data extraction and manipulation

Real data of a sharing MOD system have been provided by the SmartData@Polito research group which developed an open source tool called UMAP[47]. This is composed by three modules for the acquisition, normalisation and integration of data and their analysis and characterisation. In particular it harvests sharing companies web services to collect snapshots of the system state at periodic time instants [37]. For this work, data for the city of Turin have been considered which contained trips details from the FFCS system of internal combustion vehicles available in the city and for a time period between the years 2016 and 2017. Data showed for each taken trip, the vehicle's plate, initial and final time and position of the trip as well as its total duration and travelled distance and the level of fuel at the beginning and at the end. The input data file is initially filtered to include only valid bookings, therefore trips with a duration less than 3 minutes and greater than 90 minutes are excluded. Moreover a condition on the travelled distance is considered including in the valid dataset only trips with a total distance greater than 500 meters. Then the data are grouped by their start date and start hour of the day and collected according to the desired type of input.

5.1.1 Routing matrix

The routing matrix of the network has to follow the expected movements of vehicles between the city mobility zones. Of course the assumption of having a unique matrix to describe the routing inside the network at any time can not be accurate since in realistic cases it has at least a time dependency on the hour and type of the day. However restricting to much the dataset in order to try to include all the peculiarity of a specific network condition may result in too specific models that may overfit the particular situation and do not provide useful information. Two types of routing matrix have been used in this work to represent different conditions of the network and are explained in the following.

Hourly data routing matrix

The first type of routing matrix is obtained considering only trips that starts within a specific hour of the day and considering weekdays only. The bookings are then mapped to the city grid using their coordinates and the dataset is grouped by the origin and destination id of the city grid zones. The creation of the city grid from the data and the corresponding mapping procedure of the trips are explained in detail in the following section 5.2.1. A simple counting operation is then performed to obtain the number of total departures and total arrivals per zone which are inserted accordingly in the routing matrix. The single probabilities are then obtained enforcing the stochasticity of the matrix normalising it by row such that the sum of the entry of each row is equal to one.

The routing matrices obtained using hourly data result, as expected, unbalanced. In fact considering a single hour interval of the day some specific pattern are easily found with some zones that attract a lot of vehicles and others that generate mobility. A non trivial problem in obtaining this matrices is the lack of data, especially if considering peculiar hours of the night. Some zones in fact may not generate or attract any trip for a particular hour resulting in a complete row or column of zeros in the routing matrix. Having a row with all zeros (i.e. a zone that does not generate any trip) would represent a problem in normalising the matrix when enforcing stochasticity. To obviate to this an artificial flux is created for these zones that starts and end in the zone itself. In this way the network routing is not changed since vehicles have a probability equal to one to "move" to the same zone and the corresponding row of the matrix automatically sums to one.

Balanced daily routing matrix

The second kind of routing matrix employed does not filter data according to a specific hour of the day. In this way the obtained network is much more balanced since all the city zones attract and generate almost the same number of trips during the day. The data are then grouped by starting hour and averaged by day before the same procedure used for the hourly matrix is applied grouping again the trips by zone id and counting the departures and arrivals for each zone. Since much more data are considered in this case the probability to have a zone that does not generate any trip is almost null.

5.1.2 Service rates

The service rates of each mobility node reflects the expected demand for user mobility in that city zone. Although complex methodologies can be developed to try and estimate the demand, in this work the service rates are simply extracted from the available trips data. Since they are hourly rates, they can be obtained by simply summing the rows of the hourly routing matrix before the normalisation procedure. In fact this would directly give the total number of trips generated by each mobility zone in one hour. Also in this case it may happen to have zones that do not generate any trips in the hour interval and therefore having a service rate equal to zero. This would then be a problem in computing different metrics, for example in the MVA, that require to divide by the zones service rates. A small but different from zero artificial rate is then imposed which combined with the modification of the routing matrix performed for these nodes, result in vehicles cycling within this zones. It is important not to set a too high value for this imposed service rate since it may alter the overall performances of the network by creating artificial trips and therefore influencing the system throughput.

If considering instead the case with a balanced routing matrix, the total number of trips generated is still counted for each zone but is then divided by the hours in the interval of time considered (24 if considering all day data) obtaining in this way an average hourly rate of demand.

5.2 Geographical data and spatial characterisation

When analysing mobility systems it may be useful to consider also the spatial dimension for example to take into accounts distances or visualise the results on a map. For these reasons the coordinates of the trips data are considered.

5.2.1 City grid

The first operation concerning the spatial coordinates, is to create a grid of the city. The geopandas library of python has been used for this scope to create a geo dataframe representing a grid of polygons object. Each polygon is a square with a given common side length created within the minimum and maximum latitude and longitude registered in the booking data. An ID is then assigned to each of this grid square and only the objects enclosing the coordinates of the trips are retained. The result is a grid with square zones to which trips data may be mapped. An example obtained with data for the city of Turin is shown in figure 5.1 where each grid cell has a side of 500 meters and the number printed within represents its ID.

5.2.2 Input data characterisation

Just looking at the spatial characteristics of the input data some peculiarity of the dataset can be highlighted. For both routing matrices obtained before in section 5.1.1, the total number of departures and arrivals for each zone is plotted in figure 5.2. In particular figures 5.2a and 5.2b show departures and arrivals per zone respectively with the balanced routing matrix while figures 5.2c and 5.2d show the same metrics obtained with the hourly data routing matrix of trips started between noon and 1 pm during weekdays.

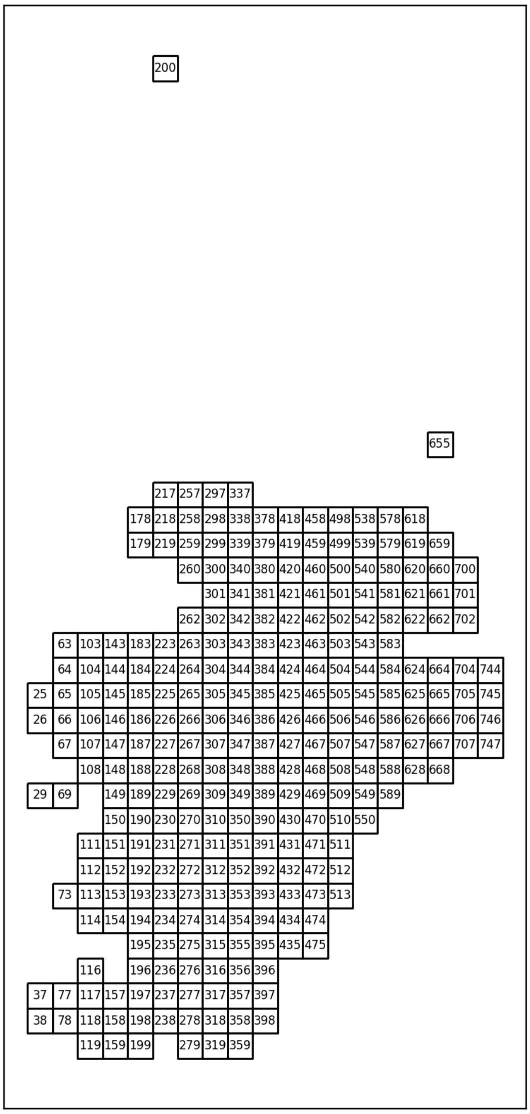
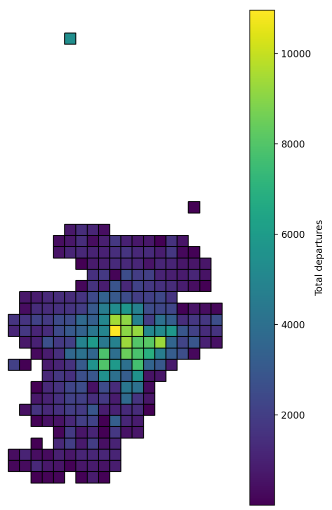
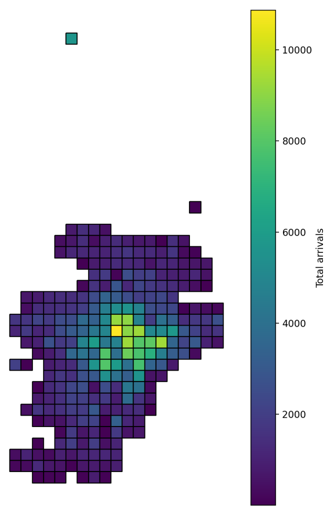


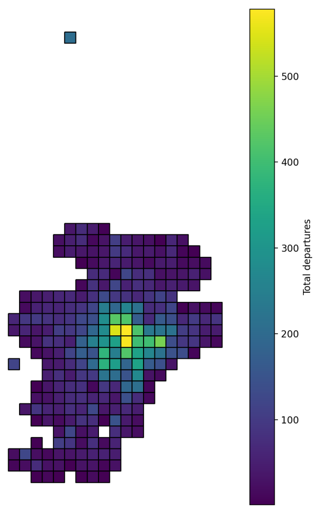
Figure 5.1: Turin city grid example



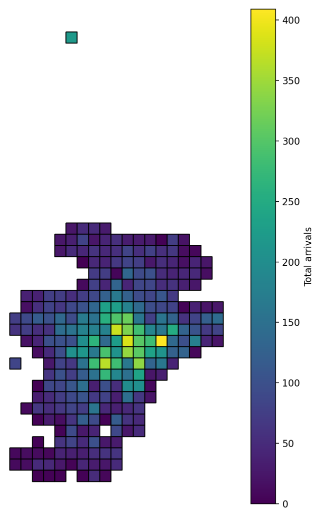
(a) Departures for balanced routing matrix



(b) Arrivals for balanced routing matrix



(c) Departures for hourly data routing matrix (12-1pm)



(d) Arrivals for hourly data routing matrix (12-1pm)

Figure 5.2: Total departures and arrivals per zone with different routing matrices

It is clear from the maps that the great majority of trips both start and end in peculiar zones corresponding to the city center. The only exception is the detached zone in the top of the map corresponding to the airport which also appears to be fairly busy. No particular differences can be spotted between the results distribution on the map obtained with the two different matrices except for the absolute numbers which of course are much bigger in the first case which consider the whole available dataset instead of just a small portion of it used in the second scenario.

Another possible preliminary characterisation of data can be done looking at the average users' demand profile considering hourly intervals during the day. Figure 5.3 shows the system demand obtained as sum of all the mobility zones' service rates in the network for all the possible hourly data routing matrices. Recalling that the service rate of each zone indicates the number of mobility requests arriving on average in the hour, these values give the total system demand for the corresponding interval. The profile shows higher values in what can be considered the peak traffic

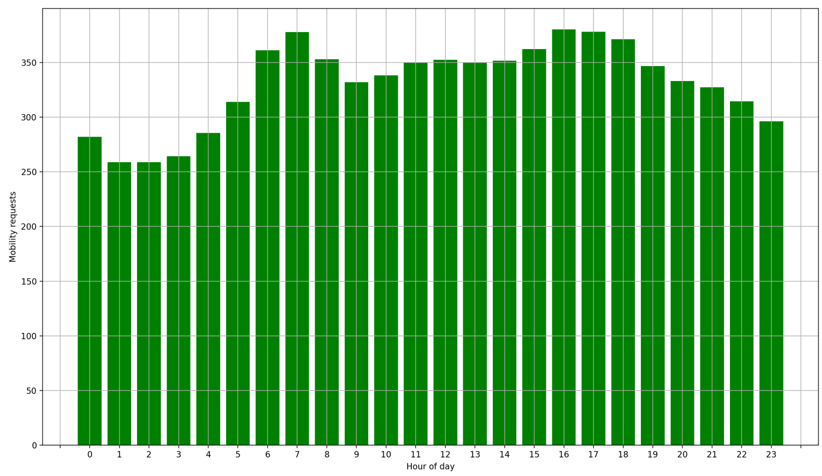


Figure 5.3: System demand for hourly data intervals during the day

hours corresponding to commuting time, i.e. around 8 in the morning and between 4 and 5 in the afternoon. Moreover the lowest values are during the night especially between 1 and 4 in the morning.

5.2.3 Charging stations positioning

Since the available input data refer to a car sharing system which does not employ electric vehicles, the charging infrastructure is not present and therefore has to be placed in city zones of choice. Although the optimisation of the charging station positioning is not the main scope of this work, different criteria are explored. In particular the differences in the network performances following a random positioning and one based on the mobility demand are considered in the following analysis.

Demand based charging stations positioning

Thinking at real sharing scenarios, it is reasonable to assume that positioning the charging infrastructure in the zones which attract a greater amount of vehicles flux would result in a better exploitation of the infrastructure capacity. This is especially true if no relocation is considered in the charging operations such as in the case described by the opportunistic policy in section 3.3.2.

Starting again from the trips data and from the total trips generated and attracted by each zone, it is easy to sort the nodes list by the number of departures or arrivals. Based then on N_{Cs} which is the number of charging nodes to include in the network, the N_{CS} top zones are selected and the charging stations are placed there. Using the available data little differences have often resulted between the top zones ordered by number of departures and by number of arrivals especially when considering the case with a balanced routing matrix. In most real scenarios when using this kind of positioning, the resulting charging zones are often concentrated in a city sector as shown by the 20 red zones in the example in figure 5.4a. It can be useful to avoid this situation and instead try to have a greater distribution of the charging nodes at least to avoid charging stations in neighbouring zones. If for example a neighbours delta of two zones around each charging node is considered for not adding another station, it is obtained a result as in figure 5.4b.

(a) No neighbour delta

(b) Neighbour delta of two zones

Figure 5.4: Charging stations positioning in zones with most departures

Chapter 6

Case studies and results

The FFCS network model formulated in chapter 3 has been tested with real input data taken from the available car2go dataset. All the presented case studies in particular use mobility data for the city of Turin. The city grid has been extracted as explained in section 5.2.1 and data have been filtered depending on the specific analysed scenario. For the first case studies, the simplified version of the network with just the mobility zone as in section 3.2 has been employed while in the following the charging nodes and charging problem have been introduced. Eventually a final analysis including the trip time with the delay zones has been performed. A recap of all tested definitions of system parameters which determined the case studies reported in this chapter, is reported in table 6.1.

6.1 Network of mobility zones

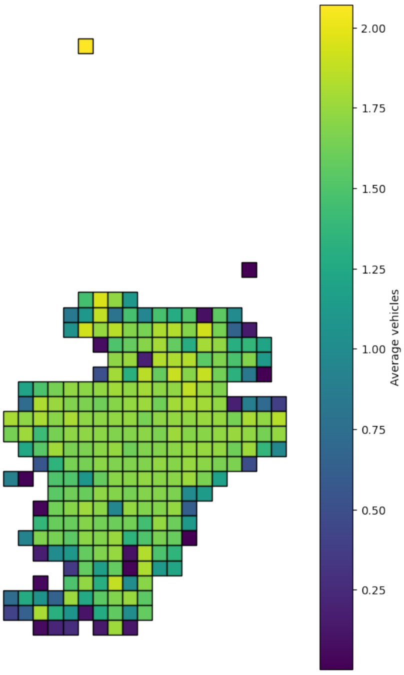
The mobility network without including charging nodes, has been first introduced to study the impact of different routing matrices and service rates on the general fluxes in the network. This analysis does not consider any charging operation of the fleet and therefore it can be associated to a dummy scenario in which every EV has a potentially infinite battery capacity. Moreover no relocation operation of the fleet are considered therefore vehicles move throughout the network just according to the users mobility patterns. Two main cases are studied based on different input routing matrices obtained as in section 5.1.1.

6.1.1 Mobility network with balanced routing matrix

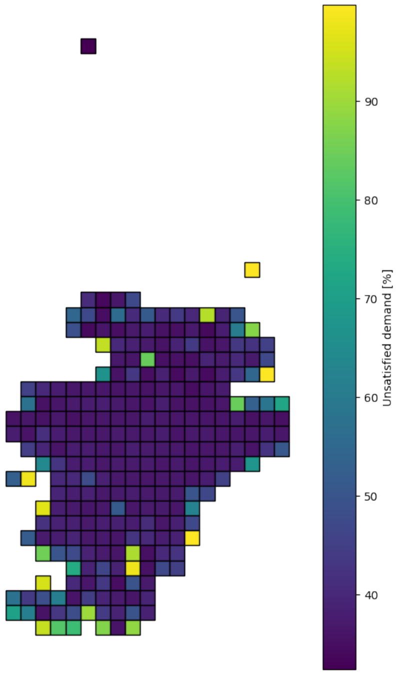
Considering a routing matrix obtained with trips data from the whole day and averaged to get hourly rate, we expect to have a quite balanced network at steady state with an even distribution of vehicles in most city zones.

System parameter	Case studied
Routing matrix	Balanced matrix
	Hourly data matrix
Fleet size	from 400 up to 5500 vehicles
Charging stations positioning	Random
	Demand Based
Charging stations concentration	from 1 to 30 charging stations with 30 to 1 outlets per station
Charging Policy	Opportunistic
	Closest station
	Uniform
Relocation after charging	No relocation
	Highest demand
	Uniform
	Probabilistic
Vehicle consumption	fixed from 0.14 to 0.20 kWh/km
Trip length	fixed, from 0 to 90 minutes
Delay zones	Single zone
	Multiple zones

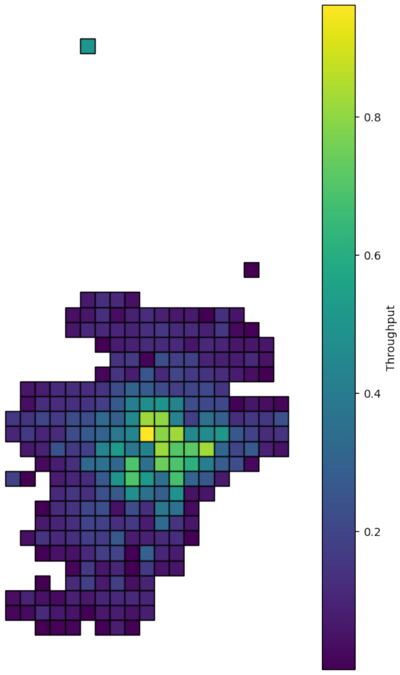
Table 6.1: Parameters definitions for case studies



(a) Average vehicles



(b) Average unsatisfied demand



(c) Throughput

Figure 6.1: Mobility zone network metrics with balanced routing matrix on city grid

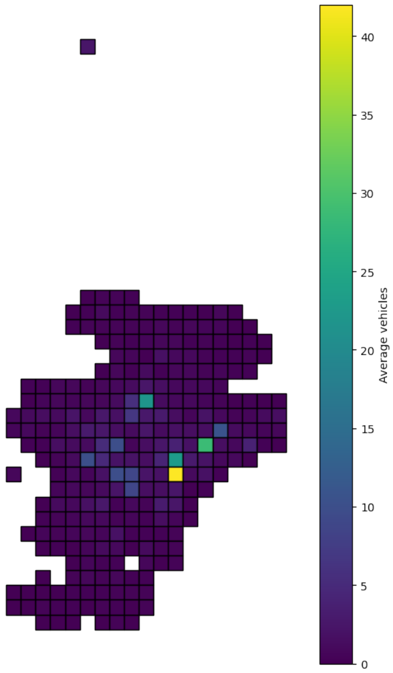
Figure 6.1 shows some of the obtained metrics plotted on the city grid in a scenario with a total of 400 vehicles in the network. In 6.1a in particular it can be seen that the fleet is evenly spread throughout the whole city with the maximum number of accumulated vehicles in a single zone slightly greater than two. Observing figure 6.1b instead it can be seen that the great majority of zones present a similar value for the computed average of unsatisfied demand around 40% of the total mobility requests per zone. Some other zones instead present values much larger (some greater than 90%) and looking at the previous analysed map in 6.1a, it can be seen that they correspond to the city zones where the average number of vehicles is around zero. This can reflect a situation in which there is a certain amount of mobility request from these zones but almost no trips end in them and therefore the number of available vehicles is always less than the needed one.

Lastly, figure 6.1c shows the data of the throughput on the city map. The absolute values appear quite small with less than one trip per zone in the hour in all the cases. This can be explained by the way in which the service rates have been obtained, counting and averaging the number of trips en each hour of each day. In fact there are hour in the nights in which the number of trips is very low (possibly zero) and moreover the ratio of unsatisfied trips, as seen, is not negligible. Looking at the spatial differences in throughput in the map, it is easy to notice that the zones with the highest throughput are the ones in the city center plus the airport (i.e. the detached zone in the upper part of the map). This is an expected result since these zones are the ones that receive most mobility requests and are also very busy destinations.

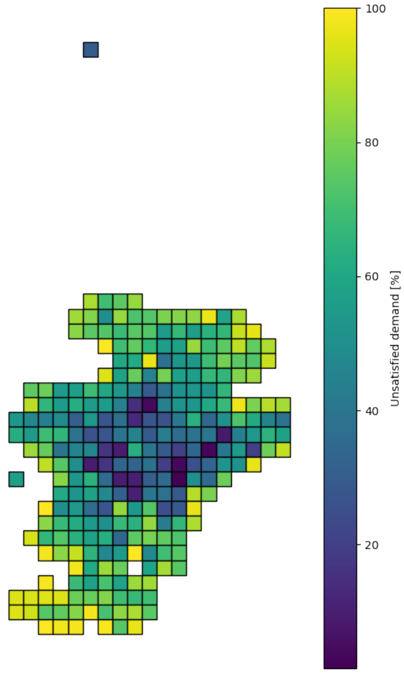
Some global indexes can also be obtained such as the overall system throughput which for this case is equal to 52.12 and indicates the total number of trips that happen in this network each hour. Moreover the average unsatisfied mobility demand has been computed and it is equal to 38.4%. This last value has been obtained by averaging the unsatisfied demand for each zone weighted by the corresponding number of mobility requests.

6.1.2 Mobility network with hourly data routing matrix

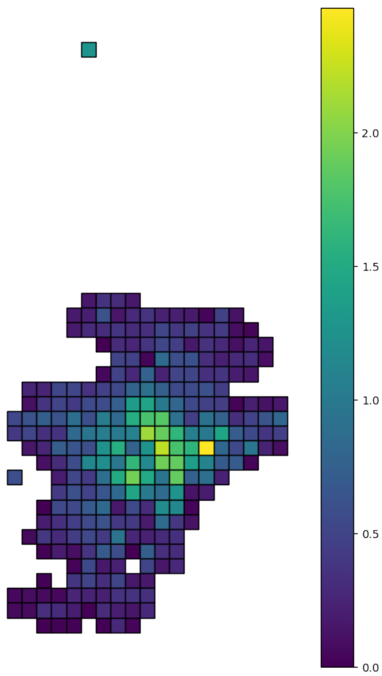
The strong assumption with considering a balanced routing matrix as in section 6.1.1, is that the vehicles' routing and the mobility requests stay constant through the day. To have a more accurate analysis regarding a particular hour or type of day, data can be filtered to obtain hourly routing matrix and service rates as shown in section 5.1.1. Considering only week days or weekends and a particular hour of the day in fact some recurrent pattern can emerge. Figure 6.2 shows the average vehicles, unsatisfied demand and throughput per zone plotted on the city grid when considering input data of trips started between midday and 1pm and only during week days. Differently for what it has been seen for the case with a



(a) Average vehicles



(b) Average unsatisfied demand



(c) Throughput

Figure 6.2: Mobility zone network metrics with hourly data (12-1pm) on city grid

balanced routing matrix, here the distribution of vehicles in the network is much more unbalanced with few zones that tend to accumulate vehicles. Consequently also the average unsatisfied mobility demand has a more stretched range of values since the city zones in which there is an abundance of vehicles will meet all the users' requests while many others will not have enough vehicles to do so. This shows that for the considered hour, some zones generally attract more vehicles.

For what concerns the throughput, also in this case it can be seen that zones in the city center and the airport, have a general greater number of completed trips with respect to other city sectors. Moreover the absolute value of the throughput are also greater than the case studied in section 6.1.1 since this interval hour is usually one of the busiest of the day. The overall system throughput in fact adds up to 150.86 while the weighted average unsatisfied demand for the network is 57.21%.

Average hourly profile of a day

A complete profile of the system throughput, average unsatisfied demand and maximum number of vehicles per zone with hourly data for the whole day and considering weekdays only, can be seen in figure 6.3. It is important to recall that each interval of time is treated separately from the others supposing that the system reaches its steady state conditions with the fixed input routing matrix. This is a strong assumption considering the relatively small time interval with respect to the average trips time. It is in fact very unlikely that the system can reach its steady state in just one hour. However this kind of analysis can be useful to highlight general mobility trends through the day and study peculiar characteristic of the network in each hour.

Looking at the general picture in figure 6.3, to greater values of throughput correspond smaller percentages of unsatisfied demand as expected, and the obtained profile appears to follow a trend of usage with relative peaks in the early morning, at noon and around five in the afternoon supposedly following users' increasing demand at commuting hours. In particular these hour intervals present a much greater value of system throughput with respect to all the others. This results from less unbalanced systems, as confirmed by their maximum number of vehicles in a single zone, where also at steady state many trips manage to be completed. In fact vehicles will still tend to rack up in particularly attractive zones but, due to a concurrent high demand for mobility in them, this effect is mitigated and the total throughput can increase. Consequently the system unsatisfied demand is heavily affected by the low values in these zones since their high demand rates make their weighted contribution much more relevant with respect to other network nodes. Moreover, looking at the maximum number of vehicles in a zone for each hour, it is clear that for most time intervals the vehicles tend to accumulate in a single

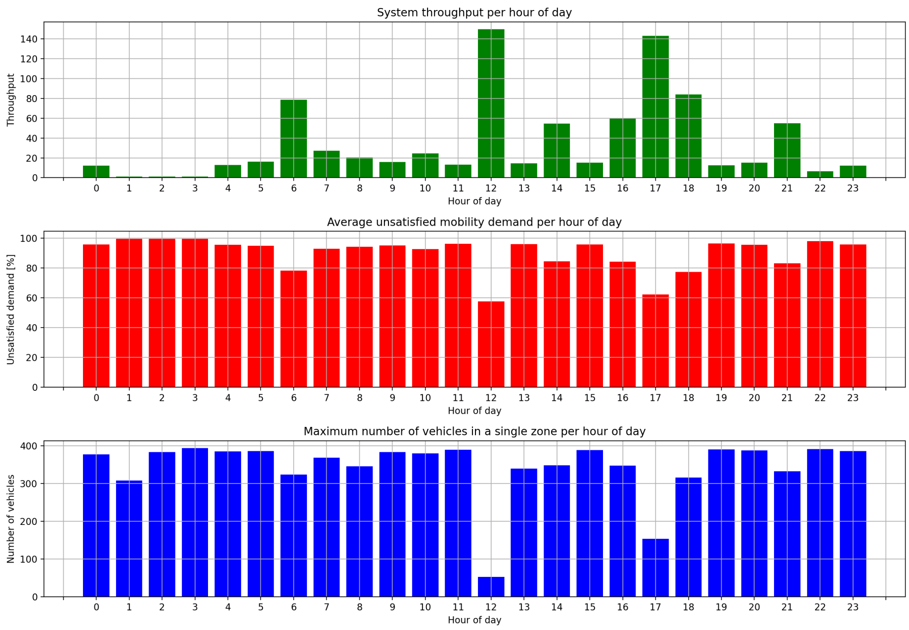


Figure 6.3: System indicators for different hour of day input matrix

zone. This is due to highly unbalanced routing matrices that, in a steady state analysis, brings to an heavily uneven distribution of vehicles in the network. In the end in all these scenarios, there will be a quite big and variable number of dead zones defined for this analysis as nodes for which the average throughput at steady state is less than 0.01. The number of dead zones for each hour interval are reported in 6.4 and are in general coherent with the trends observed in figure 6.3. A net difference is shown considering the three intervals between 1 and 3 in the night which correspond to the lowest values of throughput and exhibit more than half of the mobility zones as dead. For all the other time intervals, except for the one starting at 10 in the evening, the obtained values are not so distant and oscillating not always according to the same profile seen for the throughput. For example looking at the interval starting at 5 in the afternoon, this was the second best one in absolute throughput value, however it presents a number of dead zones (25) greater than 14 others intervals ones. The dependency of these two factors in fact is not direct because the throughput can be increased completing many trips between few zones even if many others are not reached and therefore do not generate any mobility.

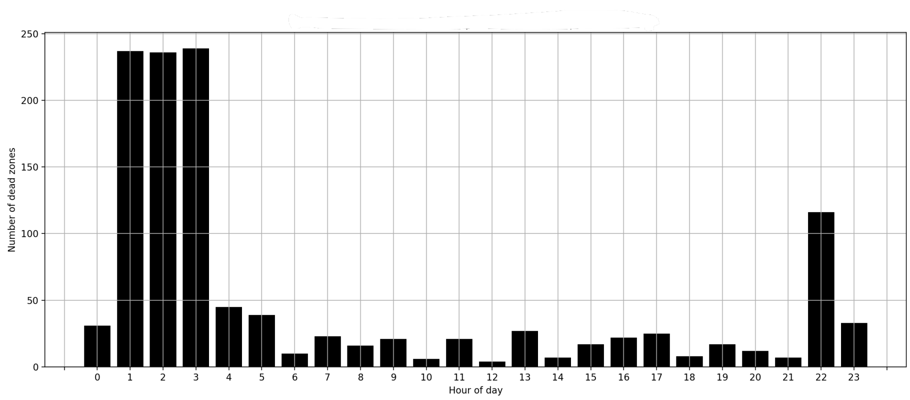
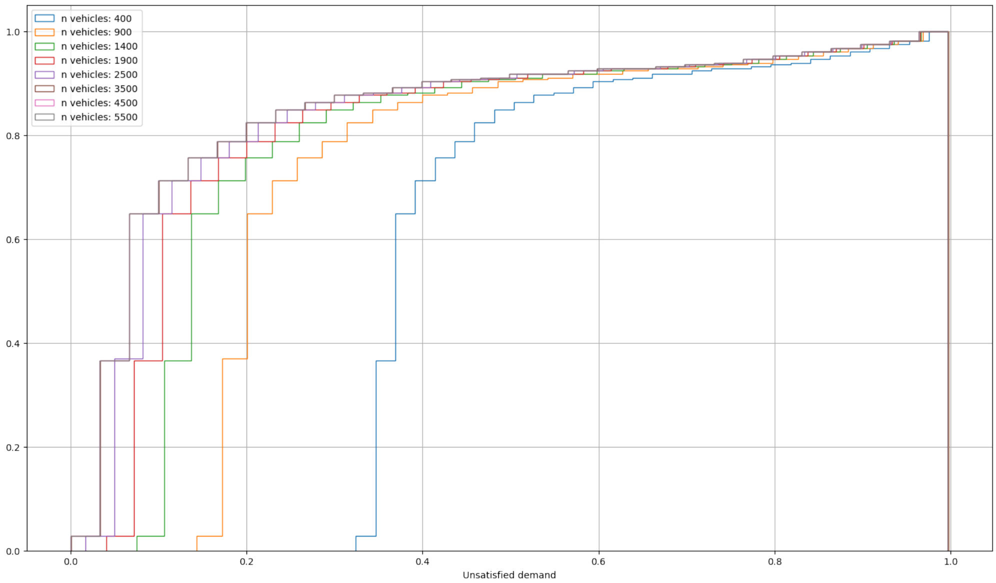


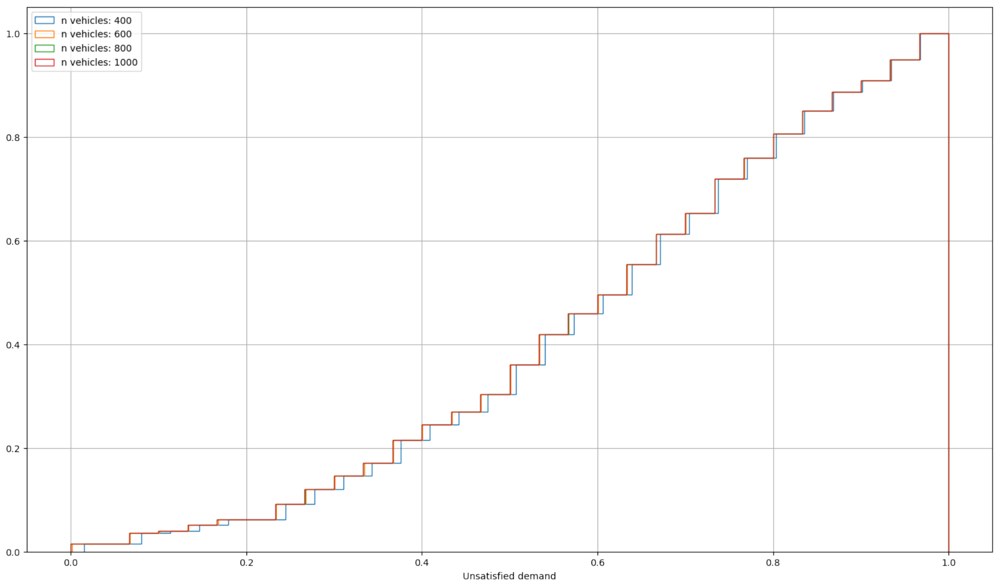
Figure 6.4: Dead zones for different hour of day input matrix

6.1.3 Impact of number of vehicles on user demand

An interesting analysis can be carried out looking at the impact of the total number of vehicles on the system performances. In particular it can be useful for the system operator to evaluate the increase in met mobility requests as function of the fleet size. The obtained results are shown in the form of the cumulative distribution function (CDF) of unsatisfied mobility demand per zone with an increasing number of vehicles in the network and considering, as before, different input routing matrices; figure 6.5a shows the result with a balanced routing matrix, while figure 6.5b with hourly data. The two graphs shows very different behaviours



(a) Balanced routing matrix



(b) Hourly data routing matrix (12-1pm)

Figure 6.5: CDF of unsatisfied mobility demand per zone with increasing fleet size

especially if comparing the relative differences between the curves representing networks with a different fleet size. As observed before for the first case as in figure 6.5a, with 400 vehicles in the network most zones present similar percentage of unsatisfied mobility demand with around 80% of them having between 40% and 50% of lost requests. Figure 6.5b shows on the other hand, a much more regular increase in the CDF which means that there are zones where the user demand is almost always satisfied, while in many others the percentage of dissatisfaction is quite high. To have a direct comparison with the previous case it can be seen that only around the 30% of the zones have a percentage of unsatisfied requests less than 50% with a fleet size of 400 vehicles.

Increasing the fleet size would mean in principle that more mobility requests can be satisfied. However in the steady state analysis of the network which has a fixed routing matrix, this effect is bounded by the unbalances in the repetitive origin-destination patterns. For this reason in figure 6.5a, increasing the fleet size results in a translation of the curve towards the left which means that the average unsatisfied demand in the network decreases with it. However this decrease is not directly proportional to the increase in the number of vehicles but the improvements diminish at each step and reach a plateau after which further increase will not improve the system performances. This means that, although the system is balanced, the fixed routing and no vehicles relocation operations will always cause some unsatisfied mobility requests. A numerical representation of this phenomenon can be seen in table 6.2 where the values of the average weighted unsatisfied demand for the network with an increasing number of vehicles are reported. Of course from the point of view of the system operator, a trade off is necessarily considered between the costs to acquire, maintain and operate a big fleet and the potential increases in revenue due to a greater number of satisfied users' trips.

Number of vehicles	**Average unsatisfied demand**	**Number of vehicles**	**Average unsatisfied demand**
400	30.40%	2500	10.40%
900	21.94%	3500	8.91%
1400	15.73%	4500	8.85%
1900	12.55%	5500	8.85%

Table 6.2: Average unsatisfied mobility demand of the network with balanced routing matrix and increasing fleet size

The situation is very different looking at the results in figure 6.5b; here the system is based on an unbalanced routing matrix and therefore vehicles tend to rack up in few zones leaving many others almost empty. Here increasing the number of vehicles results in a small improvement only if considering small fleet sizes. Further increases in fact do not provide any additional benefits since most vehicles would stay idle and unused. The numerical values of the average unsatisfied mobility demand for the systems confirm these considerations extracted from the graph and are reported in table 6.3.

Number of vehicles	Average unsatisfied demand	Number of vehicles	Average unsatisfied demand
400	57.21%	800	56.56%
600	56.61%	1000	56.55%

Table 6.3: Average unsatisfied mobility demand of the network with hourly data routing matrix and increasing fleet size

6.2 Network with charging stations

The first problem faced when considering charging nodes in the network, is the determination of the number of such stations and their positioning in the city grid. Different approaches for the positioning have already been discussed in section 5.2.3 and their performances are compared in the next analysed case studies. However to study the impact of the number and the concentration of the charging poles is necessary to first define the policies following which vehicles are brought to and taken from them. Therefore for the first problem a single charging policy is chosen while in the next analysis all the others are compared. For all the following analysis some system parameters regarding the charging infrastructure and the vehicles are kept constant and are shown in table 6.4.

Charging outlet power	EV Battery capacity	Charging thresholds		Vehicles consumption	Trip length
		min	max		
20kW	24kWh	20%	90%	0.17 kWh/km	4 km

Table 6.4: Constant charging parameters for case studies

6.2.1 Charging stations number dimensioning

For the dimensioning problem the opportunistic charging policy has been chosen since it is the simplest implemented model as explained in section 3.3.2. Moreover no relocation is took into account after the charging operations which means that vehicles leaving the station become available for booking in the corresponding city zone. It is important to recall that the opportunistic charging policy computes the fraction of vehicles that has to be sent to charge when arriving to a mobility zone with a station inside, based on the fraction of the total flux entering the node. For the algorithm to properly function this fraction of flux must always be less or equal than one therefore in the implementation, the value is clipped to one if it resulted greater than one in the computation. When this happens it means that the charging operations with that specific configuration can not fully satisfy the energy needs of the fleet. Eventually, from a probabilistic perspective, the computed value represents the probability that a vehicle finishing its trip in that mobility zone is redirected to the charging station within, therefore it must always be less or equal than one.

Balanced routing matrix

In the first analysed case the system has been studied with a balanced routing matrix and station positioning following the demand based policy explained in section 5.2.3 such that for each realisation they are placed in the zones with the greatest number of departures in the input data and avoiding, when possible, to put more than one station in a neighbourhood of two zones. The number of charging nodes has been increased at each step considering always just one charging outlet for each. From the computation it resulted that for up to four charging stations in the network the fraction of flux to be redirected to the charging nodes resulted greater than one thus the value has been clipped. This means that at least five charging nodes are required to satisfy in theory the needs of the fleet. Figure 6.6 shows some indicators regarding the stations as their number increases. It is clear from the graphs that increasing the number of charging nodes, the average throughput and utilisation of the single station decrease. The decreasing slope is particularly evident in the first part of the considered range while, after a certain amount of stations are already present, more additions result in less and less changes in performances. Almost the same profile can be seen also in the third graph representing the average number of vehicles in service during the hour. Looking at the absolute numbers it can be seen than even when redirecting the whole flux of vehicles to the charging nodes, as in the first four cases, the average utilisation of the stations is always less than 80% and diminishes rapidly. Consequently the average number of vehicles in charge is always less than one.

When considering the graph in figure 6.7, the total average time spent by a vehicle in the charging stations is plotted together with its average waiting time. The total average time decreases rapidly at the beginning and then stays almost constant starting already from low values on the x-axis. This is due to the fact that the total time is given by the contributions of the waiting time and the service

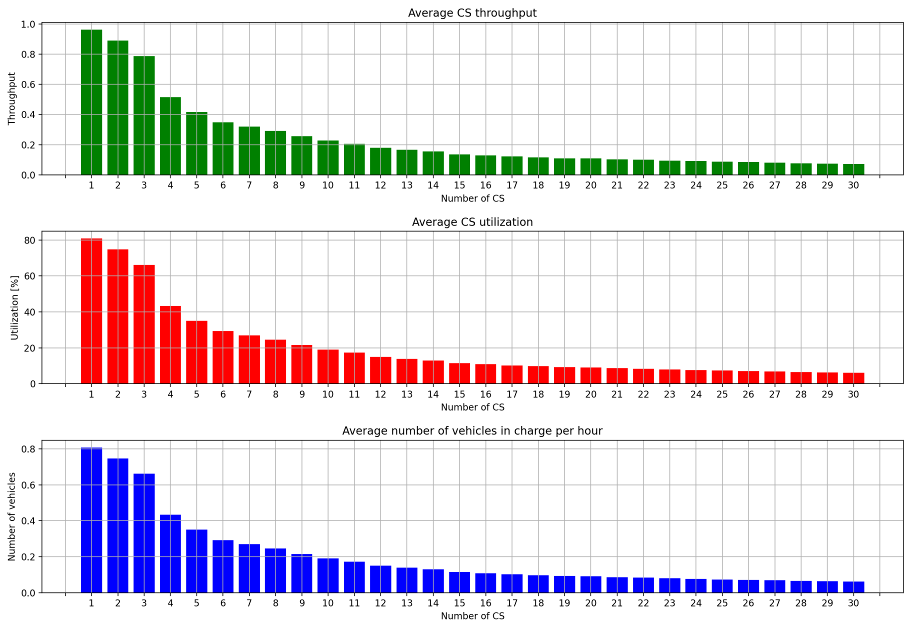


Figure 6.6: Charging stations indicators with balanced routing matrix

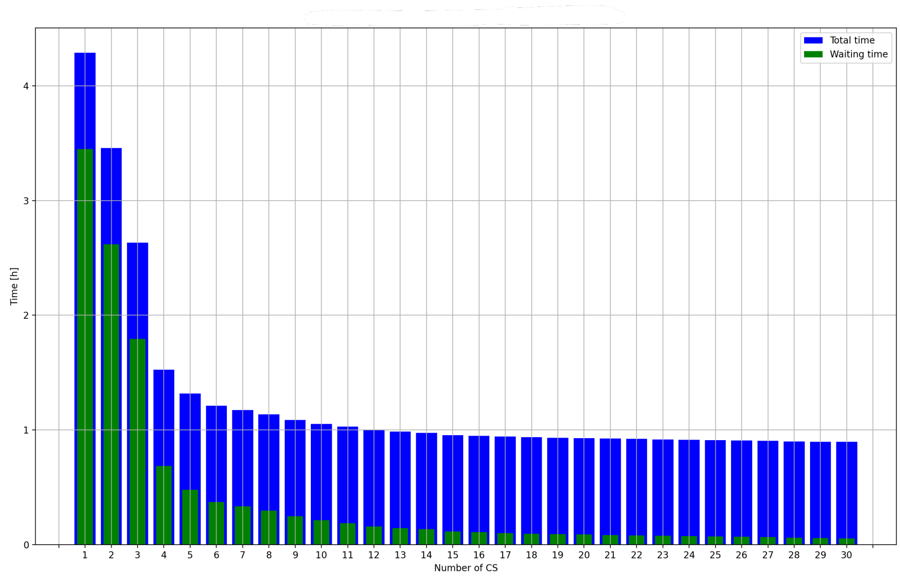


Figure 6.7: Time analysis in charging stations

time. The service time is an exponential with a mean value that is kept constant and depends on the system parameters as set in table 6.4 and in this case it is 1.19 vehicles charged per hour. The waiting time comes into play only when vehicles arrive at the station and find the outlet already occupied. From the figure it can be seen that this contribution is relevant only with few charging nodes in the network while the total time that stays almost constant for greater number of stations it is the result of the only main contribution of the service time. Increasing the number of charging points in the network in fact it can be expected that the probability for a vehicle to wait for an outlet to become available is very low.

Hourly data routing matrix

The same analysis on the number of charging station has been done with an input matrix considering trips data for weekdays between 12 and 1 pm. All the other configurations are kept the same including the positioning strategy of the charging nodes in the zones which follows the greatest number of departed trips criteria.

Figure 6.8 shows similar profiles as in the previous example in figure 6.6. Also in this case for the networks with up to four charging nodes, the value of the fraction of flux to be brought to charge was clipped to one thus more stations are needed to satisfied the fleet energy demands. The main difference with respect to

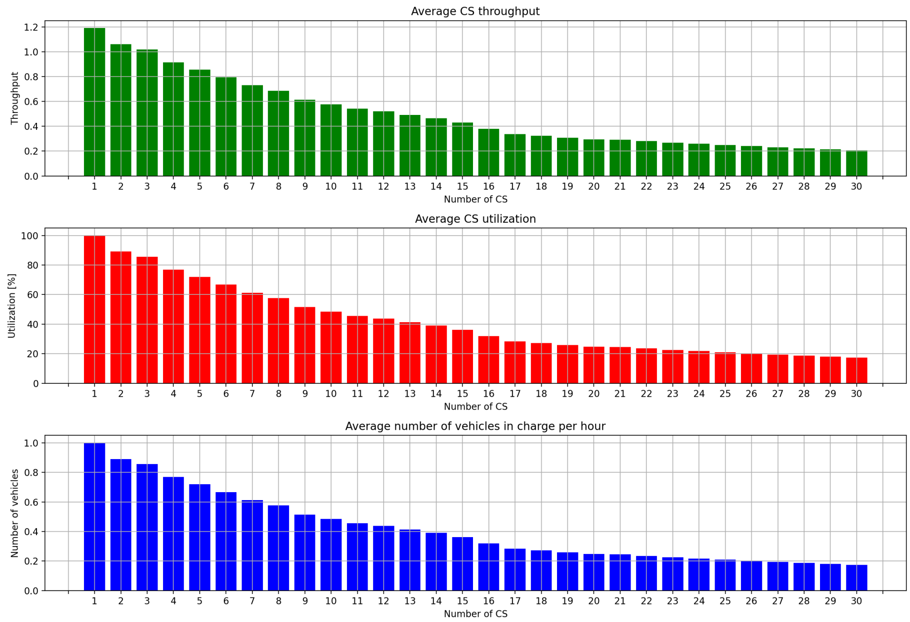
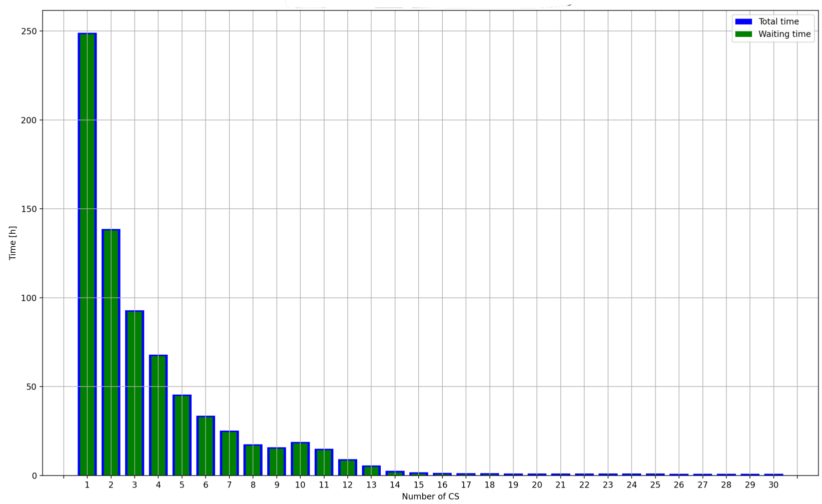
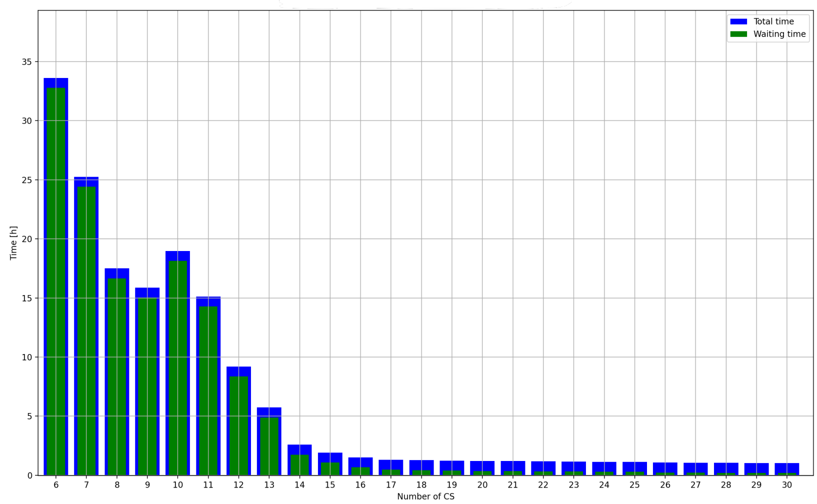


Figure 6.8: Charging stations indicators with hourly data routing matrix (12-1pm)

the case with the balanced routing matrix is in the absolute numbers which are proportionally higher due to an higher general throughput for the mobility zones.



(a) Time analysis



(b) Time analysis (zoomed)

Figure 6.9: Time analysis in charging stations with hourly data routing matrix (12-1pm)

Interesting results can be seen looking at the study of the average time spent by a vehicle in the charging zone as in figure 6.9. In particular huge numbers can be seen in the first bars in figure 6.9a. As seen for this unbalanced network in fact, vehicles tend to rack up in few zones which, clearly, correspond to the ones in which the few charging nodes are placed. Moreover all the flow in these zone is redirected to charge causing many vehicles to accumulate there thus increasing greatly the waiting time at the stations. This effect is of course mitigated including more charging nodes until, as it can be seen in the zoomed graph in figure 6.9b, the waiting contribution on the total time becomes less impacting and the overall values stay almost constant with the increasing of the charging stations number.

6.2.2 Charging stations concentration

Another factor concerning the placement of charging stations in the network, is whether or not including more outlets per each station or spread them as much as possible through the city. For this study the charging policy plays an important role and in particular considering relocation of vehicles for charging purposes has a strong impact on the results. The first case studied is a system with a balanced routing matrix with a total of 30 charging outlets aggregated in different ways through the network and with the charging nodes placed in the zones with most departed trips. Figure 6.10 shows the system total throughput and the corresponding average unsatisfied mobility demand as function of the different aggregation of charging outlets and with all the implemented charging policies. For this case relocation after the charging operations is still not considered. The two graphs are, as expected, mirrored since an increasing in throughput correspond to a proportional decrease in unsatisfied mobility. The most interesting result is that for the opportunistic charging policy the values of throughput stays almost constant with just a slight increase when charging nodes are more spread. For both the closest and the uniform charging policy instead the throughput tend to increase a lot when charging stations are disaggregated. In terms of absolute values, the opportunistic charging policy presents in this case a throughput always greater than the other two which is almost reached in the case when just one outlet is considered for each station. The differences between the least and the most aggregated cases are particularly relevant for the two policies considering relocation with a delta of unsatisfied mobility demand around 30%. This result is due to the fact that no relocation is employed after charging therefore, for these two policies, vehicles are brought from all the network to the zones with a station and are left there when the operations are completed. In this way there is an abundance of vehicles in that zones that unbalances the system causing high values of unsatisfied mobility demand in the other network sectors. Another result that has to be noted is that for up to six charging stations in the network, the opportunistic policy required to

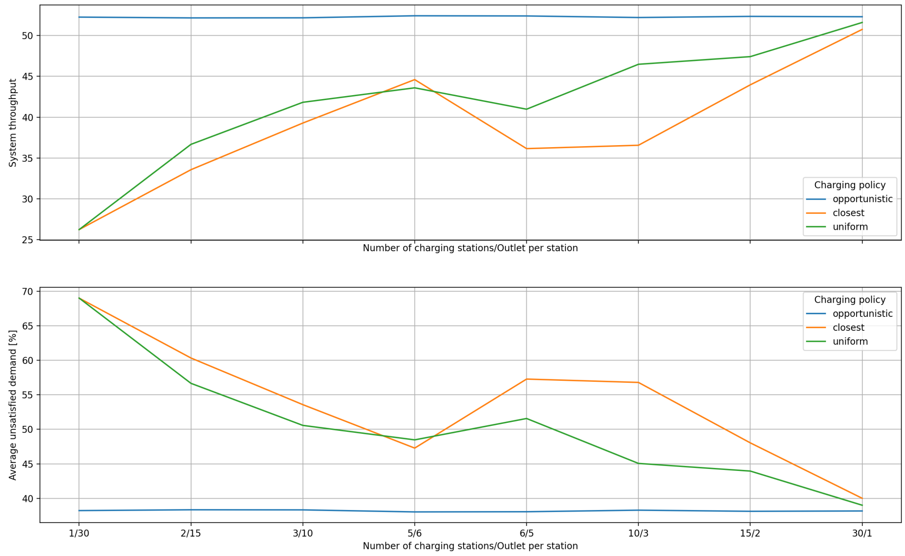


Figure 6.10: System global indices for different aggregation of charging outlets and charging policies with balanced routing matrix

clip the value of the fraction of flux to be brought to charge. This as explained before means that the energy needs of the fleet could not be completely satisfied with this configuration and the whole flux directed to the mobility zones with a station inside is directly queued to charge.

An additional analysis is performed considering a probabilistic relocation after charging and the obtained profiles are shown in figure 6.11. The three curves show

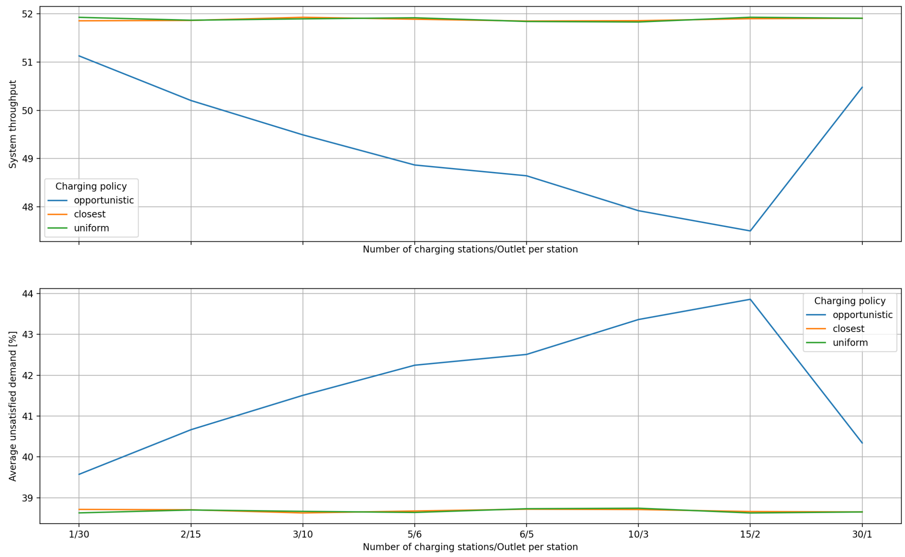


Figure 6.11: System global indices for different aggregation of charging outlets and charging policies with balanced routing matrix and relocation after charging

an almost opposite situation with respect to the one depicted in figure 6.10 with the closest and uniform policies showing almost constant and slightly increasing throughput values as the number of outlet per station decreases. In absolute values the unsatisfied mobility demand for these two cases is comparable with the one obtained with the opportunistic policy and no relocation after charging. On the other hand, the opportunistic charging policy with relocation after charging shows relative poor results, with a decreasing trend as the number of outlet per station decreases. The only anomaly is represented by the value of throughput obtained with just one outlet in all the charging nodes. This can be explained by the fact that this is the only point of the curve obtained without clipping the value of the fraction of flux to be redirected to charge. In fact for up to 15 charging nodes in the network with this configuration the system is not capable of satisfy the energy needs of the fleet. This may be due to a less relative flux entering the nodes with a station within, because of the relocation after charging which tend to re-balance the system by moving vehicles towards all the other zones in the network.

Repeating the same analysis using only data from a one hour time interval during the day (i.e. 12-1 pm) to build the routing matrix, the obtained results are reported in figures 6.12 and 6.13. It can be easily noticed that the curves trends in both figures are very similar to the ones of the previous example with the balanced routing matrix in figures 6.10 and 6.11. Moreover also in this case the fraction of flux to the charging stations determined with the opportunistic policy, has been clipped to one when considering a network with up to 5 charging nodes and with up to 15 charging nodes when considering also relocation after charging. The main difference considering the hourly data routing matrix is in the absolute values of the throughput which are greater due to higher fluxes of vehicles and mobility requests and a greater average unsatisfied demand due to the more unbalanced nature of the system.

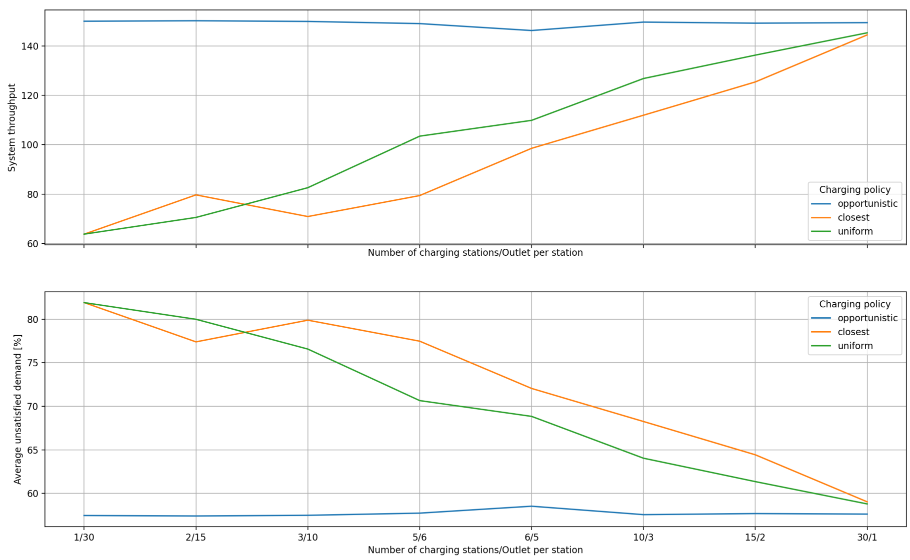


Figure 6.12: Performance indices for different aggregation of charging outlets and charging policies with hourly data routing matrix

Overall the two analysed cases show that for the same number of total outlet in the network, it is better to spread them in different city zones. However the improvements are not so relevant when considering either relocation policies before and after the charge or no relocation at all which appear to be the best possible combination. A detailed analysis of the charging policies and relocation after

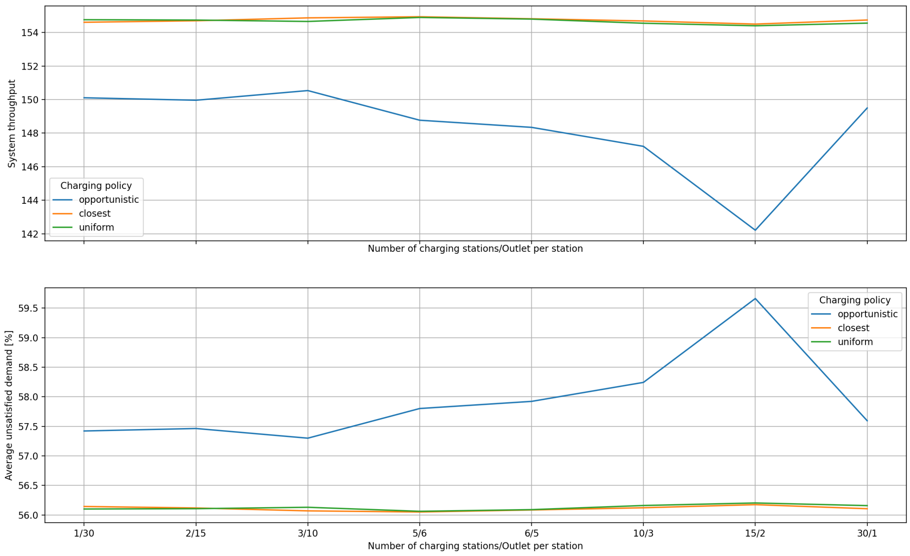


Figure 6.13: System global indices for different aggregation of charging outlets and charging policies with hourly data routing matrix and relocation after charging

charging is carried out in sections 6.2.3, 6.2.4 and 6.2.5.

6.2.3 Charging policies

Charging policies determine how often and where EVs have to be charged, therefore they play a fundamental role in the system operations. The policies implemented to bring vehicles to the charging nodes have been explained in section 3.3.2 and are now analysed in detailed. For the first case study, a balanced routing matrix has been used and a total of 20 charging outlets have been placed, two per station, in the ten zones with most departures but excluding the once in the immediate neighbourhood of an already placed charging node. The resulting positioning on the city map is shown in figure 6.14.

The system charging infrastructure parameters as defined in table 3.2 are kept also for these case studies.

Mobility zones indicators

From the steady state study of the model, some indicators have been extracted and are plotted on the city grid to have also a spatial characterisation of the results. Figure 6.15 shows the average number of vehicles in each city mobility zone with the three different charging policies. Each zone marked with a red dot corresponds to a mobility zone with a charging station within. This analysis allow to see how the vehicles are distributed at steady state and in particular how this distribution is influenced by the charging operations mode. The results shows important differences according to the policy employed and in particular between the opportunistic and the other two which use relocation of vehicles. In the first case, as in figure 6.15a the system appear quite balanced with an even distribution of vehicles in almost all the zones in the network. This result is quite similar to the one shown in figure 6.1a which considered the same network but without charging infrastructure thus taking into account only flows between mobility zones. This means that charging operations using this configuration and the opportunistic policy do not have a strong impact on the mobility of users.

Considering instead the results in figure 6.15b, it can be seen that around half of the fleet is concentrated in a single zone at steady state. This charging policy requires to move vehicles from all the network and bring them into the closest charging station according to the system average energy needs. Since no relocation after charging is considered in this scenario, vehicles are left in the corresponding mobility zones when the charge is completed. The accumulation of vehicles in that specific mobility zone can then be explained by the presence of the charging station in it. Moreover the position of that zone with respect to the other charging nodes and to the city boundaries indicates that the charging station within is the closest

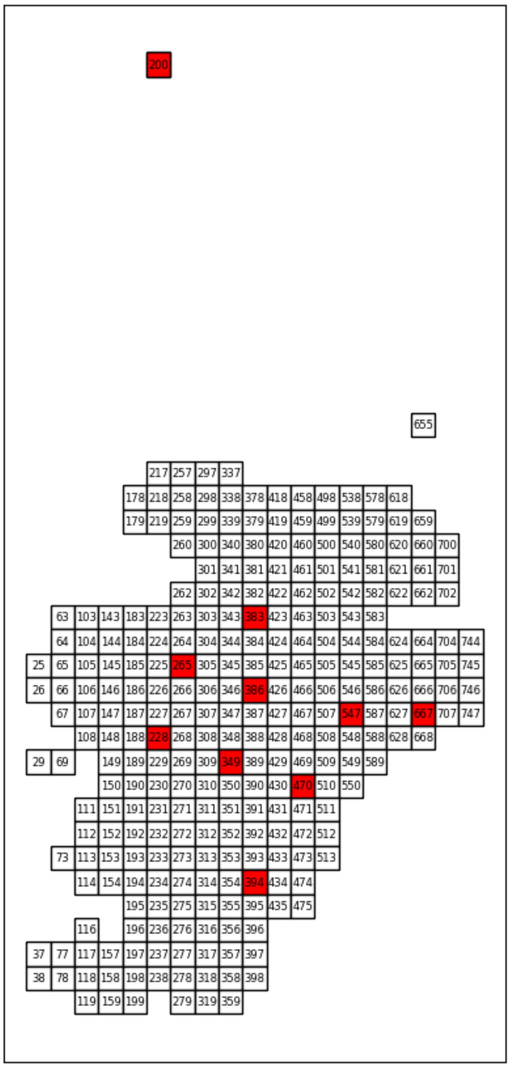
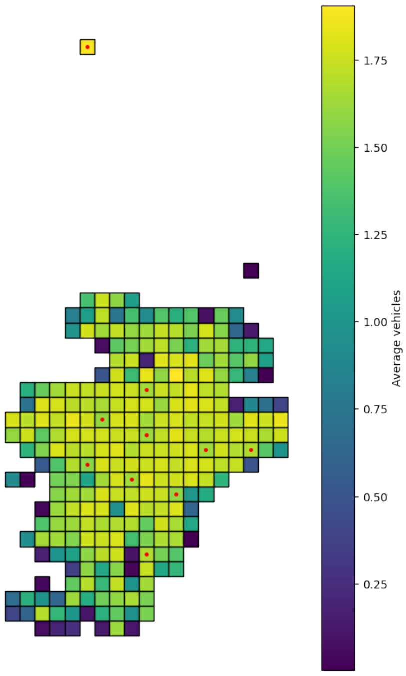
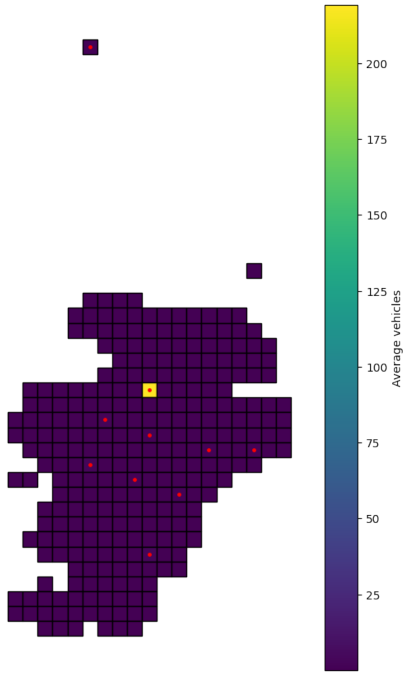


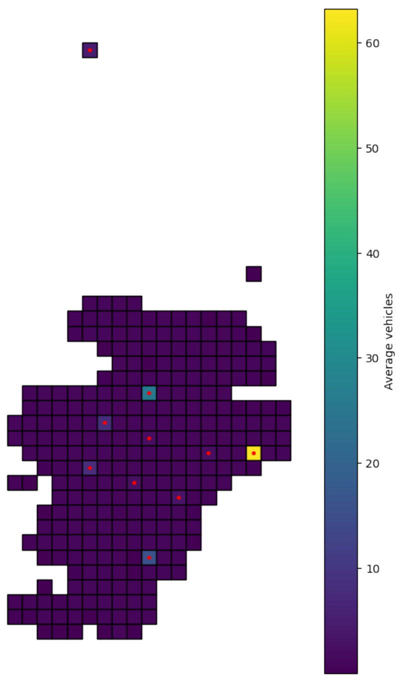
Figure 6.14: Charging station positioning on city map



(a) Opportunistic policy



(b) Closest CS policy



(c) Uniform policy

Figure 6.15: Average vehicles per city zone on map with different charging policies

one for many zones in the north sector of the city. For all these reason it is safe to assume that many EVs are relocated to that specific charging nodes resulting in heavy concentration in the corresponding mobility zone.

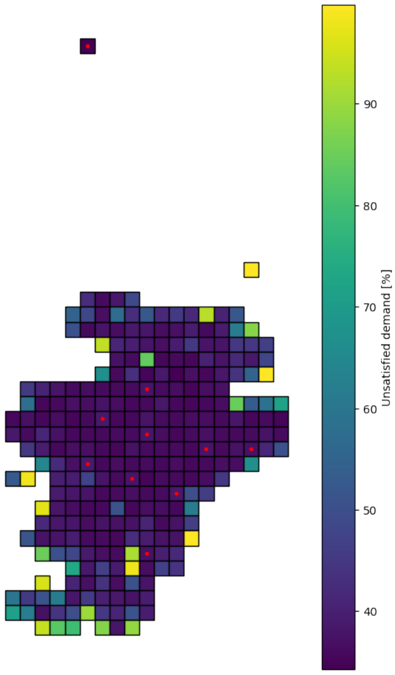
The third graph in figure 6.15c obtained with the uniform charging policy, shows a middle ground situation between the two previously analysed ones. In fact also this policy requires to move vehicles from the whole network but they are distributed uniformly towards all the placed stations. Of course in principle this is a strongest assumption with respect to the closest charging policy since it would require for example to bring vehicles to charge in a faraway zone even if there is a much closer one available. However it is interesting to analyse this results because it can be seen how vehicles still tend to rack up in the mobility zones with a charging station within but, in this case, they are more evenly distributed between them. In fact the biggest average number of vehicle in a single zone is around 60 while in the closest policy was greater than 200.

Another result is plotted in figure 6.16 and shows the average unsatisfied mobility demand for each city zone, still considering the three different policies and no relocation after charging. Again with the opportunistic charging policy the results are quite similar to the network of just mobility zones and can be seen is figure 6.16a. As expected from the even distribution of vehicles in the network in fact, the average unsatisfied mobility demand is similar and around 40% in most city nodes included the ones with a charging station within. Few other zones than present greater values and they correspond to the ones with a smaller average number of vehicles as seen in figure 6.15a.

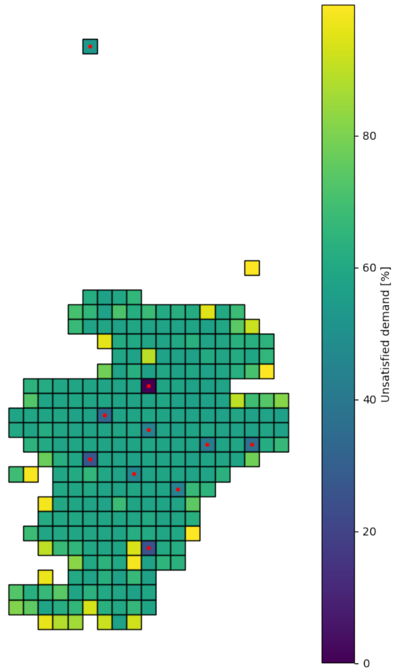
For what concerns the two other policies, the range of unsatisfied demand is almost the same but, again, the spatial characterisation is different. In fact considering the closest station policy in figure 6.16b, it can be seen how the zone where there was a great accumulation of vehicles has a null unsatisfied demand. Intermediate results are then visible in the other zones with a charging station within but they are very similar to the values in all the other mobility zones. Also in this case few zones present higher values probably due to a low incoming rate of vehicles.

Results obtained with the uniform charging policy are then showed in figure 6.16c. Here it can be seen a clear difference between the zone with a charging station that present a lower value of unsatisfied demand and all the other mobility nodes in the network. This is explained as before by vehicles that are uniformly brought to charge and then left in the corresponding mobility zone.

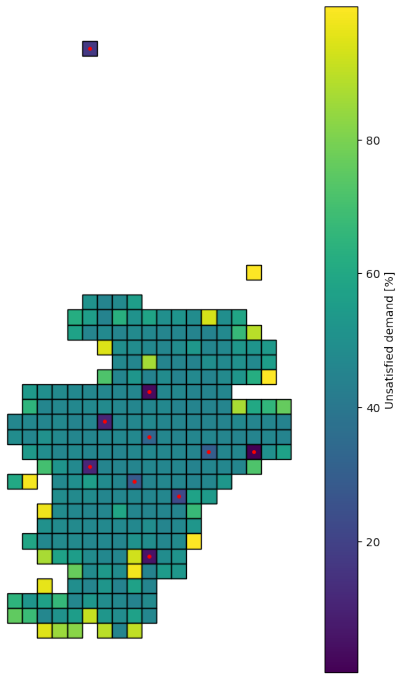
The last map visualisation in figure 6.17 shows instead the values of the throughput for each zone in the network. Here the results are very similar considering all the policies at least in the relative differences between the zones. This is due to the same input matrix and the same demand model employed in all three cases which is indeed the main factor in the determination of the throughput. The only difference



(a) Opportunistic policy

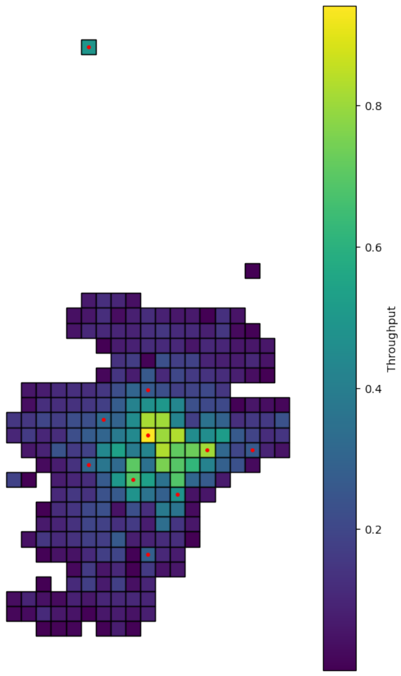


(b) Closest CS policy

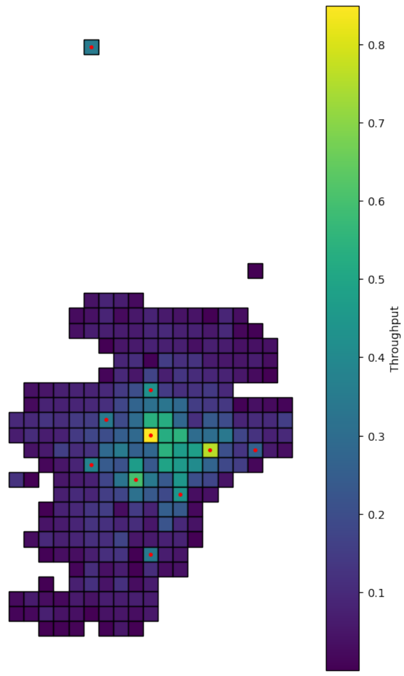


(c) Uniform policy

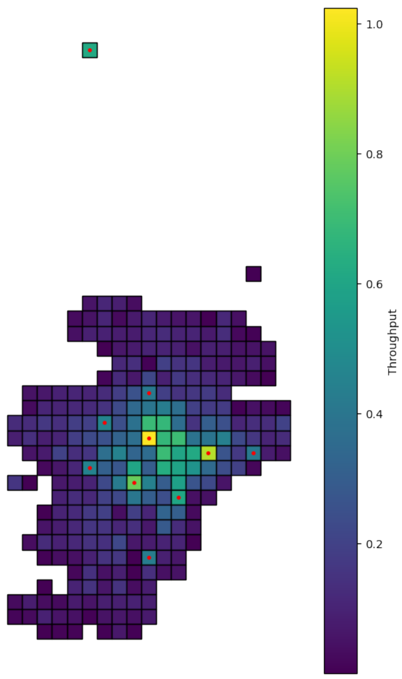
Figure 6.16: Average unsatisfied mobility demand per city zone on map with different charging policies



(a) Opportunistic policy



(b) Closest CS policy



(c) Uniform policy

Figure 6.17: Throughput per city zone on map with different charging policies

is in the absolute range of values; in fact the closest charging policy present the smallest ones suggesting a distribution of vehicles which does not exactly follow the user demand. Recalling the zone in which vehicles accumulate with the closest station policy, this is not the same as the one with the greatest value of throughput probably due to a greater demand for mobility in this last one. The uniform policy, on the contrary, has the biggest throughput values and this can be explained again by the distribution of vehicles obtained with this policy. In this case in fact one of the zone in which vehicles rack up is the one with the greatest mobility demand, increasing in this way the number of satisfied trips from that zone and thus its throughput.

Overall this case study suggests that employing relocation operations to bring vehicles to charge but leaving them there afterwards, causes an uneven distribution of vehicles in the network which, if does not follow the users mobility demand results in a decrease of the general throughput. Moreover unbalanced systems when brought at steady state increase the risk to have dead zones in the network where vehicles do not arrive thus excluding them from the system operations. For the same reasons performances can not be improved in such systems for example increasing the fleet size. An example of this phenomenon is provide in figure 6.18 where the CDF of unsatisfied mobility demand per zone is plotted for the three different policies. For the opportunistic charging policy in figure 6.18a it is clear that increasing the number of vehicles in the network, more user demand can be met moving the curve towards the left. Of course the decrease in unsatisfied demand is not directly proportional to the increase in fleet size since even with a huge number of vehicles the fixed routing and no relocation of vehicles do not allow to satisfy all the users requests. For what concerns the two other policies, unsatisfied demand basically do not decreases adding more than 400 vehicles. The uniform policy in figure 6.18c shows a net improvement when passing from 200 to 400 vehicles but further increases do not benefit the system. The closest station policy as in figure 6.18b instead shows essentially the same results for all the cases from up to 200 vehicles. As stated before this policy with no relocation after charging causes an heavily unbalanced distribution of vehicles which make increasing the fleet size useless.

In the end a summary of the principal network indicators for the three implemented policy is provided in table 6.5. The total number of requests lost per hour has been obtained by multiplying the fraction of unsatisfied demand for each zone by their demand rate and then summing them up for the whole network.

Charging stations indicators

In order to better study the impact of the charging policies on the network, it is useful to also look at indicators regarding the single charging stations to see

Charging policy	**System throughput**	**System average unsatisfied demand**	**Maximum number of vehicles in a zone**
Opportunistic	52.21	38.29%	1.91
Closest CS	35.89	57.58%	219.05
Uniform	46.24	45.34%	63.23

Charging policy	**Number of zones with average unsatisfied demand >90%**	**Total number of lost requests per hour**
Opportunistic	11	32.40
Closest CS	15	48.72
Uniform	13	38.37

Table 6.5: Network indicators recap

how their capacity is exploited and have a time analysis of the overall operations. Figure 6.19 shows for each policy the average throughput in each charging station in red and their average utilisation in green. In table 6.6 instead are printed the average throughput and utilisation for the whole set of stations in the network.

Charging policy	**Average station throughput**	**Average station utilisation**
Opportunistic	0.23	9.60%
Closest CS	0.15	6.10%
Uniform	0.19	7.86%

Table 6.6: Average throughput and utilisation of charging stations

It is useful to recall that the station placement policy is such that the nodes with most departed trips in the dataset are chosen and in the bar plot they are ordered following this criteria. For all three graph in figure 6.19 throughput and utilisation of the stations follow the same path since they are strictly correlated. Figure 6.19a shows a decreasing trend in both throughput and utilisation following the ordered list of stations. An higher departure rate correspond with this configuration to a general higher throughput for the mobility zone and therefore for the station within. With the opportunistic charging policy in fact greater is the flux in the zone, greater would be the number of vehicles that are sent to charge there.

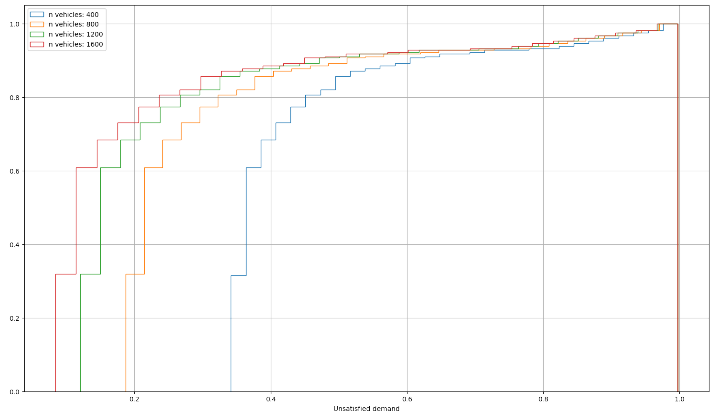
The closest station policy instead takes into account the geographical position of the charging nodes, therefore throughput and utilisation are not directly proportional to the flux in the mobility zone. Figure 6.19b shows a particular low value for the station in the node with zone ID 200. Looking at the city map in figure 6.14 it can be seen that this is the zone detached from the other and therefore far away from the rest of the network (i.e. the airport). For this reason this zone does not receive an incoming flux of vehicles relocated to be charged. Only EVs arriving in that mobility zone are charged in there according to the probability expressed as function of the average energy needs of the fleet. On the other hand the most used station is the one in the zone with ID 383 which looking at the previous examples as in figure 6.15b was the one where vehicles rack up due also to its proximity to all the zones in the city north sector.

Lastly figure 6.19c shows the result obtained with the uniform relocation charging policy. As expected this are the most balanced with respect to the other ones. EVs in fact are relocated from the whole network and sent to all the stations with a uniform probability. For this reason the throughput and utilisation of the single stations are very similar to each other.

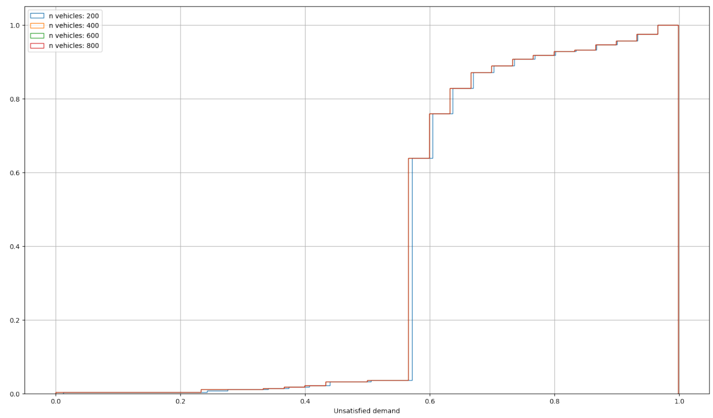
The numerical data in table 6.6 shows again that, considering no relocation after charging, the opportunistic policy ensure a greater utilisation of the charging infrastructure. Even if the uniform policy guarantees a better distribution of the power load, the average station throughput is still lower than the one obtained with the opportunistic. Considering the absolute values however this configuration of the network shows a very low average utilisation of the charging infrastructure which is less than 10% in the worst case. Another important parameter connected to the station utilisation is the probability to wait when arriving at a charging node. As explained in section 4.2.2, waiting at a charging station because the outlet is already occupied can be critical in the organisation of the system operations. The probability that when an EV is brought to charge has to wait for a server to become available can be easily computed with the Erlang-C formula. The obtained results with the three charging policies are plotted in figure 6.20 together with the average number of vehicles in each station. Again the x-axis shows the stations ordered by number of departed trips in the corresponding mobility zone and the general trend of all graphs in figure 6.20 follow the ones in figure 6.19. Again the opportunistic has a proportionality between throughput in the zone and number of vehicles and probability to wait at the charging station. The closest station policy shows an almost zero probability to wait in the station at the airport while the uniform one has very similar values between all the stations. Looking at the numbers, the probability to wait are very low in particular with the two policies using relocation because of the low utilisation of stations for the closest policy and also due to the balanced workload for the uniform one which is the one with the lowest peak value. Figure 6.20a is the only one presenting slightly grater probabilities but still low enough since the most used station has a probability to wait for vehicles arriving around 6%.

6.2.4 Charging policies with hourly data routing matrix

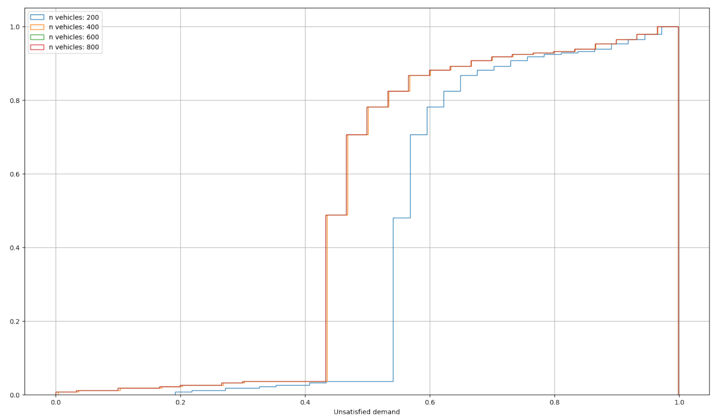
A similar analysis on the effects of the different charging policies as in the whole section 6.2.3 can be carried out using hourly data to build routing matrix and service rates for the network. As done for previous case studies, data of trips in the time interval 12-1 pm and only for weekdays have been considered. The charging related parameters in table 6.4 are still valid for this example. The charging station positioning follows again the zones with most departures which for the input matrix resulted in the distribution in figure 6.21.



(a) Opportunistic policy

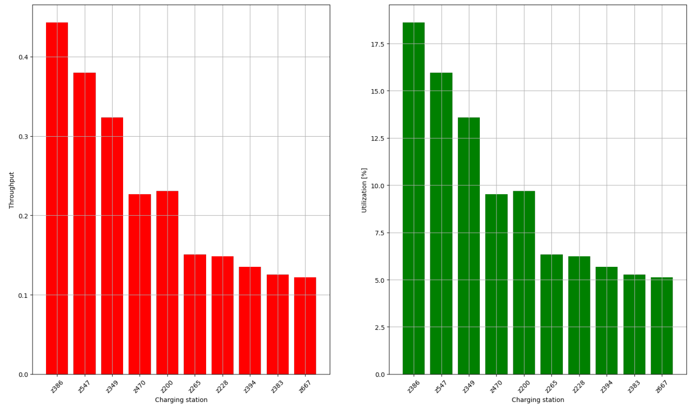


(b) Closest CS policy

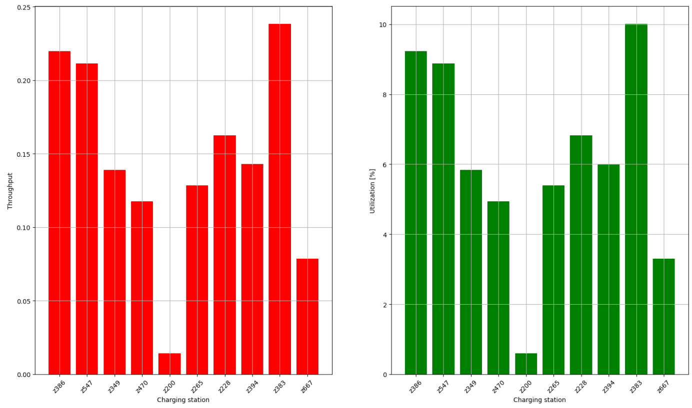


(c) Uniform policy

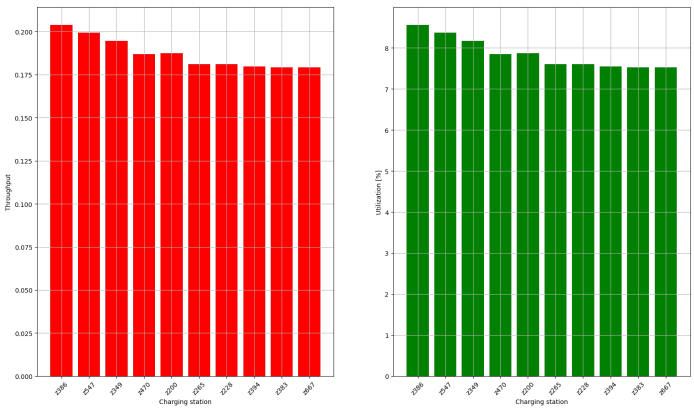
Figure 6.18: CDF of unsatisfied mobility demand per zone with increasing fleet size and different charging policies



(a) Opportunistic policy

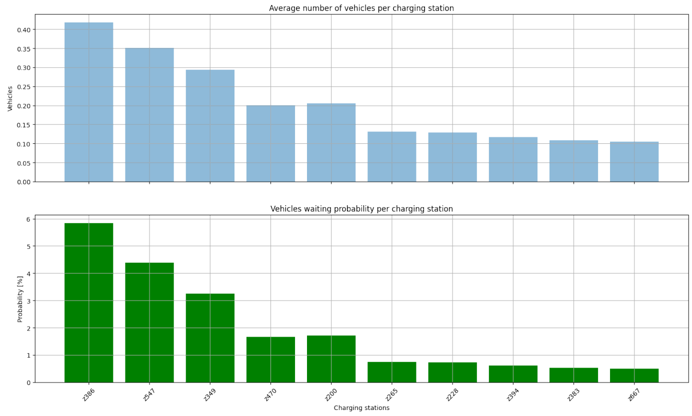


(b) Closest CS policy

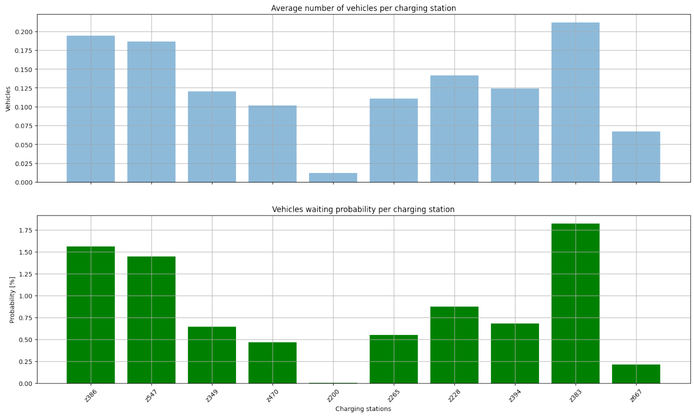


(c) Uniform policy

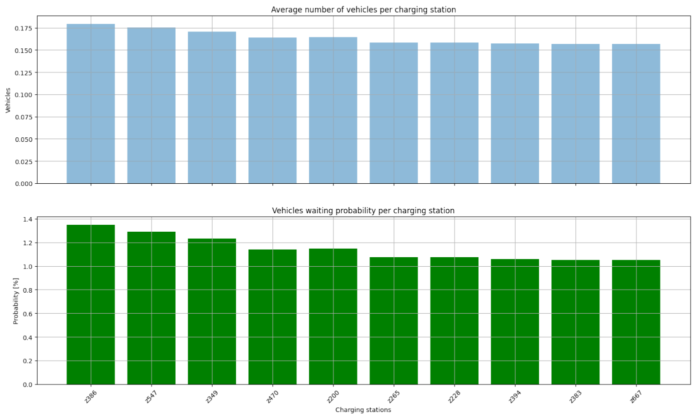
Figure 6.19: Throughput and utilisation of charging stations



(a) Opportunistic policy



(b) Closest CS policy



(c) Uniform policy

Figure 6.20: Average number of vehicles and probability to wait in each charging station

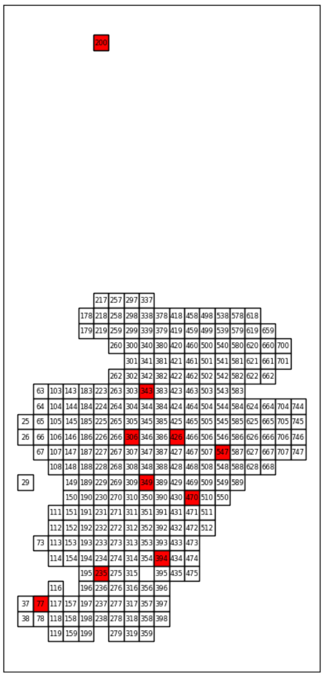
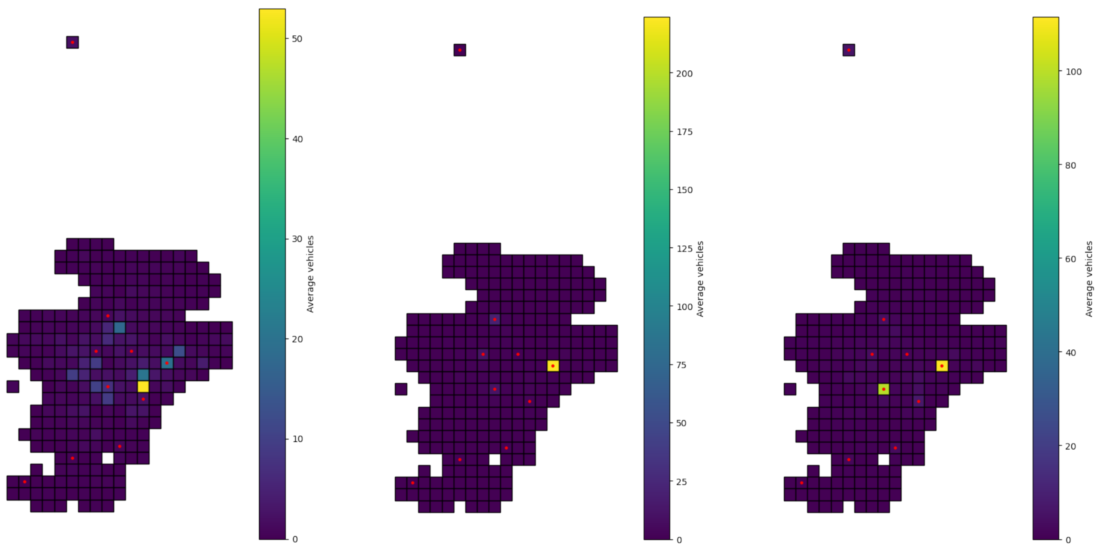


Figure 6.21: Charging station positioning on city map

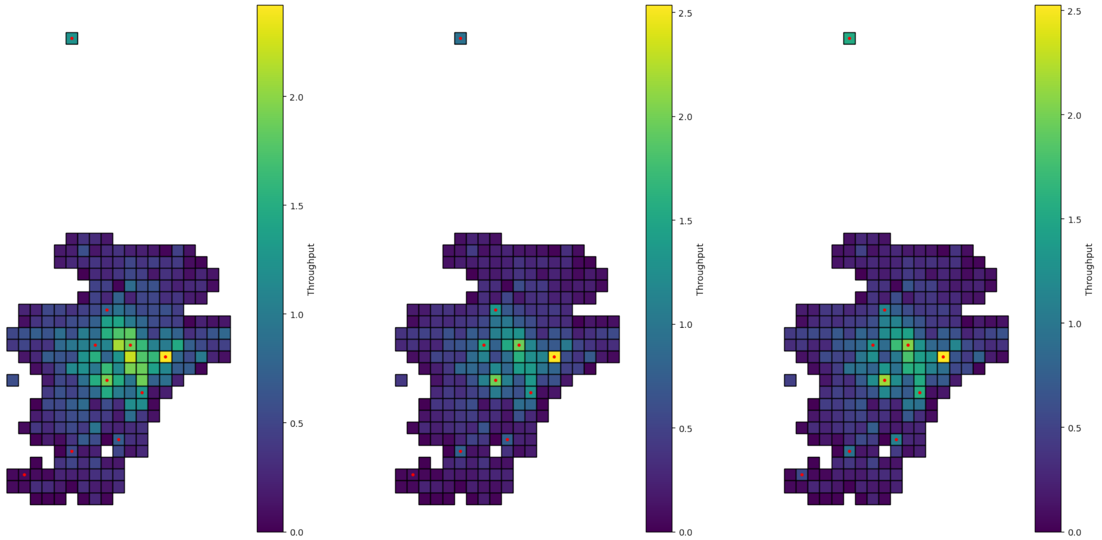
Mobility zones indicators

At first indicators regarding the mobility network have been extracted and plotted over the city grid map. Figure 6.22b shows the average vehicles in each city zone at steady state. The relative differences in the results between the three policy are similar to the one already observed for the balanced network. The closest station policy in fact is the one where vehicles tend to accumulate more in a single zone, which has also a station within as it can be seen in figure 6.22b. Using the uniform policy instead the vehicles distribution is still uneven but there is more than one zone with a charging point in which the number of vehicles is much greater than in the others. In figure 6.22c two zones in particular appear different from the others. The more balanced distribution of vehicles is obtained as expected with the opportunistic policy. However also in this case there are particular zones in which the concentration of vehicles is greater; this result is due as seen before, to the unbalanced routing matrix used as input for the system.



(a) Opportunistic policy **(b)** Closest CS policy **(c)** Uniform policy

Figure 6.22: Average number of vehicles per zone on map with different charging policies (12-1pm)

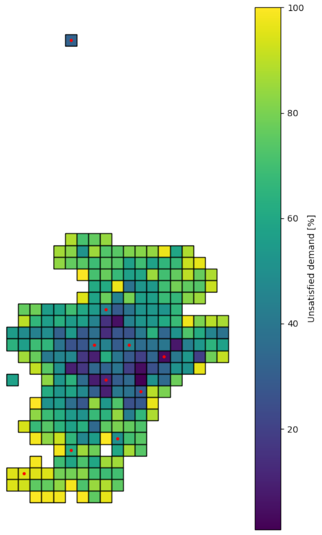


(a) Opportunistic policy **(b)** Closest CS policy **(c)** Uniform policy

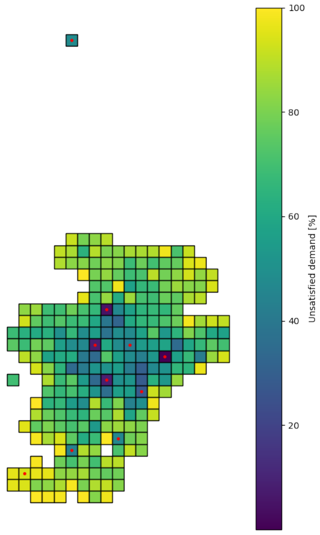
Figure 6.23: Throughput per city zone on map with different charging policies (12-1pm)

Figure 6.23 shows the zones throughput over the city map. The three maps are all similar in the differences between the zones values but the opportunistic in figure 6.23a shows a slightly smaller range of values suggesting a more even distribution of the throughput through the network. Lastly figure 6.24 shows the percentages of average unsatisfied user mobility demand in the network. For all the policies the results show an inverse relation with respect to the throughput as expected. Also considering the absolute values they all present similar numbers, only in figure 6.24a there appear to be more zones in the city center with a blueish colour thus with a smaller percentage of not satisfied requests.

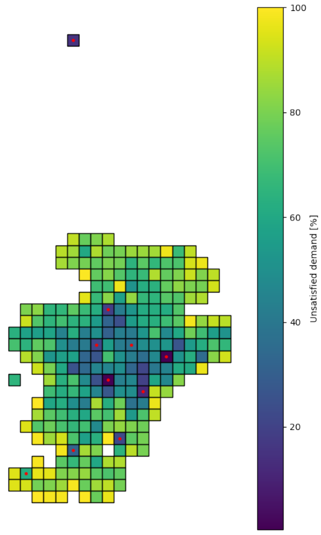
To better mark the differences between the observed cases it is useful to look at the system metrics summarised in table 6.7. From these data is clearer that the opportunistic charging policy shows the best performances as already observed for the previously analysed case study with the balanced routing matrix. The main difference in this case is that the system is already quite unbalanced due to the input routing matrix therefore the effects of the single charging policies are less evident. Moreover the requests rates in this hour interval are much higher resulting in greater absolute values for the throughput. A side effect to this increase in demand which is amplified by the uneven distribution of vehicles in the network is that the lost requests per hour are much more than in the previously analysed case. In the end this configuration creates many "dead zones" in which vehicles rarely arrive and the user mobility demand is almost never satisfied.



(a) Opportunistic policy



(b) Closest CS policy



(c) Uniform policy

Figure 6.24: Average unsatisfied mobility demand per city zone on map with different charging policies (12-1pm)

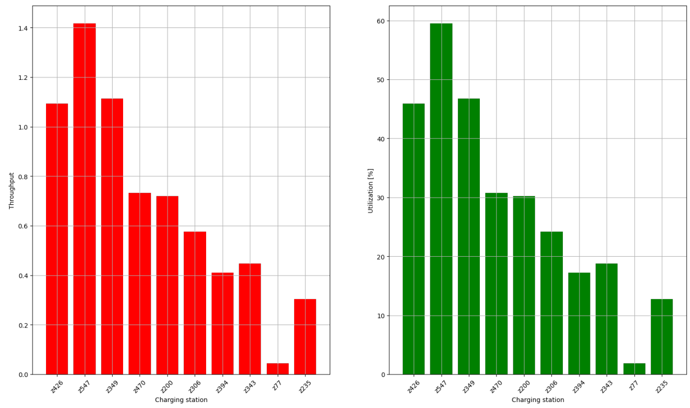
Charging policy	System throughput	System average unsatisfied demand	Maximum number of vehicles in a zone
Opportunistic	149.63	57.56%	52.98
Closest CS	112.35	68.13%	223.91
Uniform	126.60	64.09%	111.50

Charging policy	Number of zones with average unsatisfied demand >90%	Total number of lost requests per hour
Opportunistic	33	202.91
Closest CS	44	240.19
Uniform	37	225.94

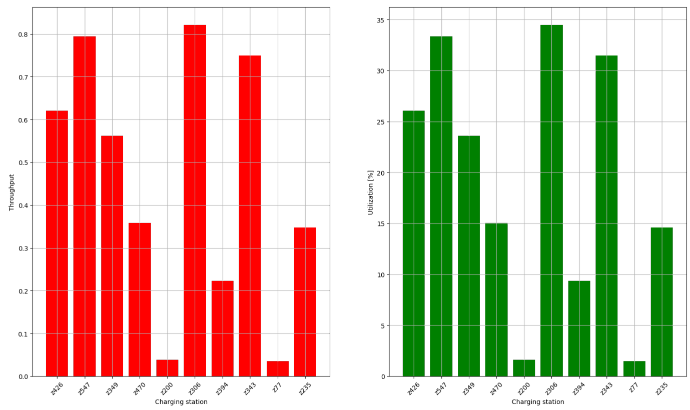
Table 6.7: Network indicators recap (12-1pm)

Charging stations indicators

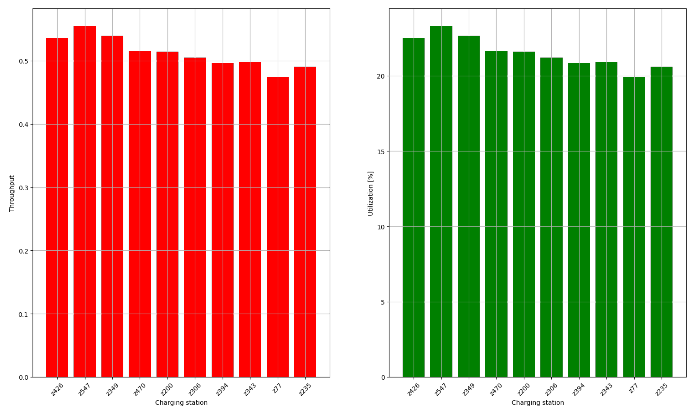
A last study with the same network configuration has been performed focusing on the single charging stations performances.



(a) Opportunistic policy



(b) Closest CS policy



(c) Uniform policy

Figure 6.25: Average throughput and utilisation of charging stations (12-1pm)

Figure 6.25 shows the average throughput and utilisation of all the stations in the network. In all the graphs the order of the stations on the axis follows the decreasing number of departed trips from the corresponding mobility zone. The profile in figure 6.25a is not regular as the one obtained in the previous case study and shown in figure 6.19a. This suggests that the zones with the highest rate of demand do not always correspond to the ones with the greatest flow of vehicles, due again to the unbalanced nature of this network which leads to high values of unsatisfied demand.

Figure 6.25b shows the throughput and utilisation of charging nodes with the closest station policy. The geographical position of the stations is determinant in this case to establish the routing of vehicles towards them. Two nodes in particular appear to have much lower throughput values than the other and they correspond to zones with ID 200 and 77 respectively. Looking at their positioning in the city map in figure 6.21 it is clear that the first corresponds to the airport which is detached from all the rest of the city area while the latter is in a peripheral zone of the city such that it is the closest station to very few mobility nodes. Additionally, as seen before, the nodes in this particular city sector do not have an high throughput resulting in a very low utilisation of the charging station.

The results obtained with the uniform relocation charging policy are displayed in figure 6.25c and shows, as expected, a more uniform distribution of the charging loads between the available nodes. This policy in fact is less affected by the unbalance of the network since whatever is the flux of vehicles in each zone, the relocation operations to bring them to charge are always uniform towards all the stations in the city.

Charging policy	**Average station throughput**	**Average station utilisation**
Opportunistic	0.69	28.81%
Closest CS	0.45	19.10%
Uniform	0.51	21.52%

Table 6.8: Average throughput and utilisation of charging stations (12-1pm)

In table 6.8 are printed the average numerical values for the three policies. As seen in the previous case with the balanced routing matrix in table 6.6, the greatest average utilisation of the stations is obtained with the opportunistic policy while the closest station one leads to the smallest results. The main difference between the two cases is in the absolute values of the throughput which are much higher with the analysed unbalanced network. The average percentage of utilisation consequently is greater for all the policies but still under the 30% of the maximum capacity. However for the opportunistic and the closest station policies the variance is also high therefore it is important to have a further look at the distribution of vehicles in the stations and the consequent waiting probability. Figure 6.26 shows the obtained results with the average number of vehicles in blue and the probability to wait in green for each charging station and charging policy. As expected figure 6.26a shows the most critical results due to the general higher throughput of the stations in this configuration. In particular the station in the mobility zone with ID 547, has an average number of vehicle which is greater than two. Recalling that each charging nodes has two charging outlets, this means that on average there will be a number of EV in the stations such that both the outlets are occupied and a further vehicle arriving would have to wait. The computed value for the vehicles probability to wait is in this case greater than 40%. While this percentage may seem not critical this may open further discussions on the necessity to include a limited capacity of the waiting line in the charging stations. In fact, depending on the physical configuration of the node, it may be difficult to organise a waiting line of vehicles at the charging outlets which requires additional parking space and labor. However, as stated before, no capacities of the waiting line are considered in the developed model since they would introduce potential losses which are not compatible with the network analysis technique employed so far.

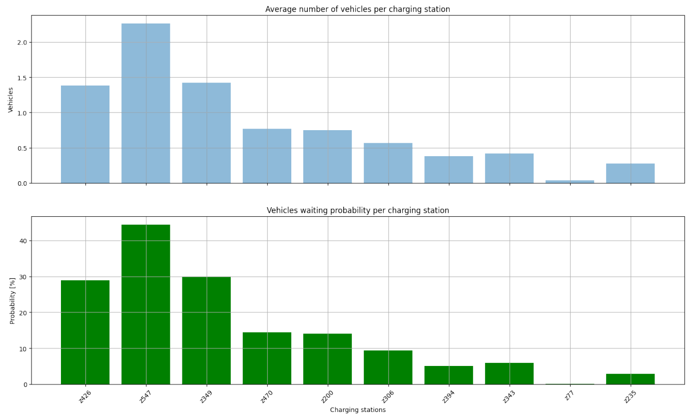
The two other policies in figures 6.26b and 6.26c shows less critical values for the waiting probabilities at the stations and the latter in particular is the one that better mitigate this risk since the load is well balanced through the available charging nodes.

6.2.5 Relocation after charging

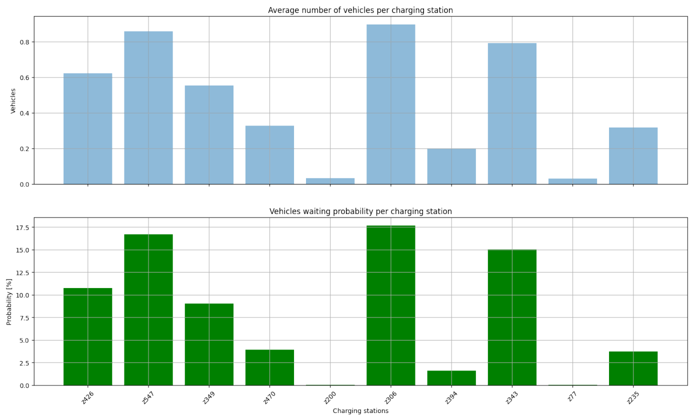
A further level of complexity of the modelled network is introduced considering relocation operations after the EVs charging process is completed. As seen for the charging policy in fact some movement of vehicles not depending on users' mobility can be introduced in order to re-balance the system. In this section are considered specifically operator based vehicles repositioning from the moment an EV is unplugged from the charging outlet. As explained in section 3.3.3 three different relocation after charging policies have been considered namely uniform, highest demand and probabilistic relocation. The results obtained with these different approaches and considering no relocation after charging are compared and combined with the different charging policies illustrated before. A first overview of global system metrics is provided and then the best possible combinations are analysed in detail in the next section.

Charging policies and relocation after charging combinations with balanced routing matrix

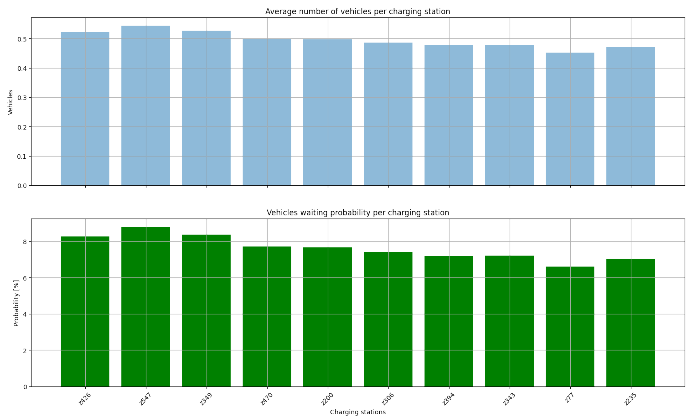
Figure 6.27 shows through heat maps the system throughput and the average unsatisfied demand with all the possible combinations of charging policies and relocation after charging. In particular results in figure 6.27a are obtained with a demand based station positioning in the network while metrics in figure 6.27b with a random placement. In both figure the system throughput is inversely proportional to the unsatisfied mobility demand as for its definition.



(a) Opportunistic policy

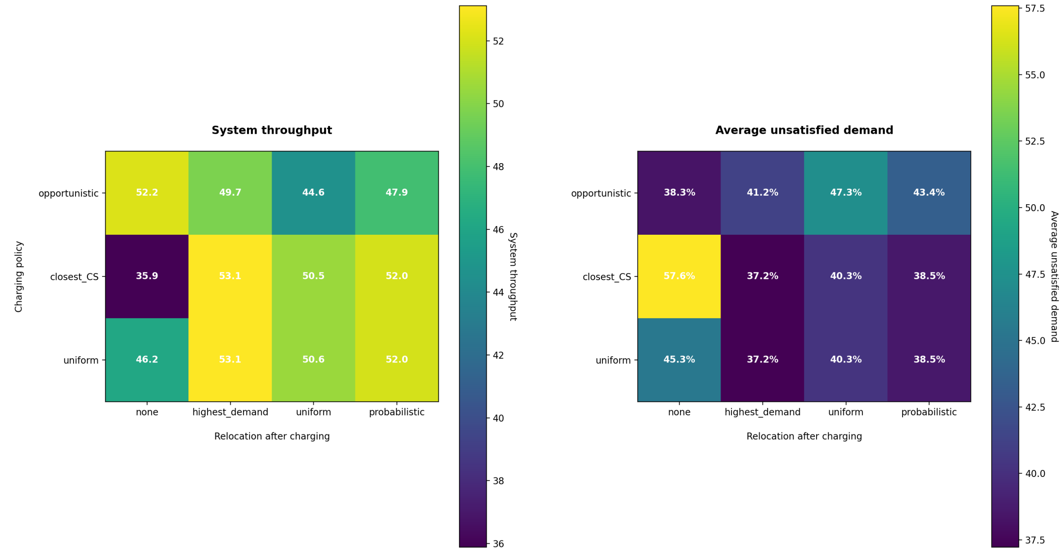


(b) Closest CS policy

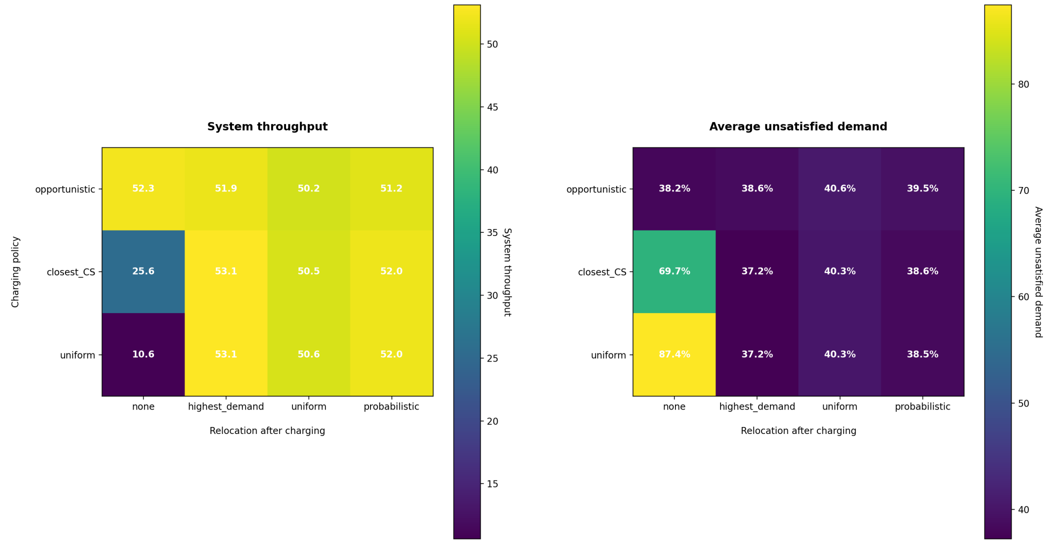


(c) Uniform policy

Figure 6.26: Average number of vehicles and probability to wait in each charging station (12-1pm)



(a) Demand based station positioning



(b) Random station positioning

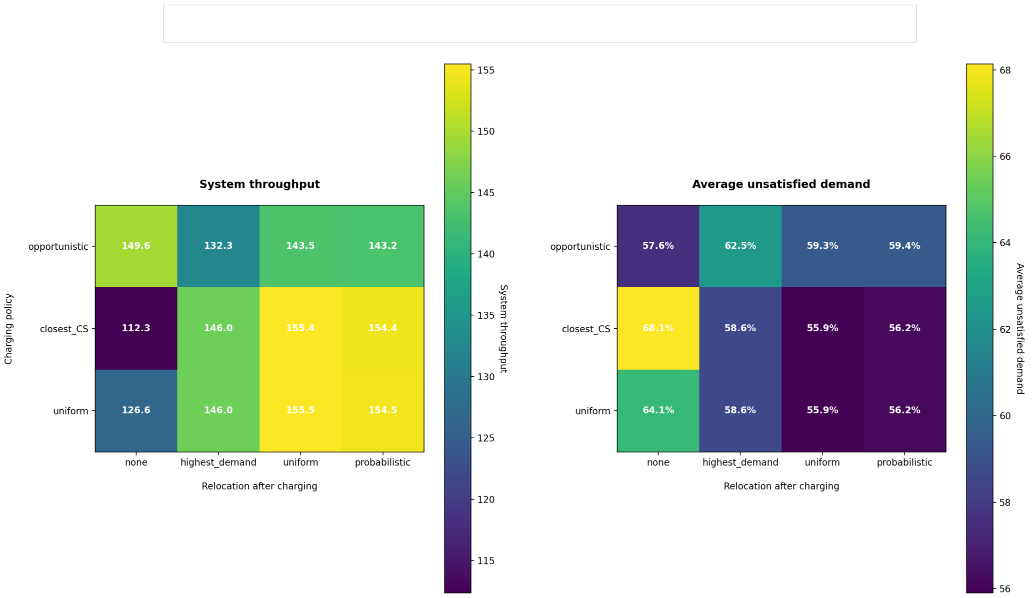
Figure 6.27: Heat maps of possible combinations of charging policies and relocation after charging with different station positioning strategies

Starting from the demand based positioning of charging nodes in figure 6.27a it is evident that the worst result is obtained combining a closest charging station policy with no relocation after charging. The results in the first column of each map (i.e. without relocation after charging) are the same obtained and commented in section 6.2.3 from which it was clear that the best policy was the opportunistic one. However looking at all the other metrics in the map the opportunistic policy with no relocation after charging is just the third best result whilst the greatest system throughput is reached combining either the closest station or the uniform charging policy with the highest demand relocation after charging. Other good but slightly worst values of the throughput are reached considering again either the closest station or the uniform charging policy with the probabilistic relocation after charging. In general is evident that combining no relocation before with relocation after charging policies or vice versa, the obtained throughput is much less than in all the other cases. This results suggest that the best possible performances are obtained by taking vehicles to charge from allover the network, and then by relocating them in the zones in which it is more probable that they will be booked. The difference between the highest demand and the probabilistic relocation after charging is in fact that employing the first one EVs are only relocated in the top N mobility zones of the network by expected demand rates, while in the second case they are brought in all the network according to a probability which follows the users' demand. For the studied case the top 50 zones are considered for the highest demand relocation. Moreover the two best combination produces the same throughput which suggests that for this case there is little difference between bringing vehicles to charge uniformly in all the stations or to the nearest one. Of course these system metrics do not take into account other factor such as the cost for the operator to move vehicles which increases with the travelled distance. Eventually this further consideration suggests that the best combination of policies with this network configuration is considering the closest station policy with the highest demand relocation after charging. The heat map on the right in figure 6.27a shows the average values of unsatisfied mobility demand for the system each obtained weighting the single contributions of each mobility zone. The identified best combination of policies leads to a percentage of unsatisfied mobility demand of 37.2% while in the worst case scenario this reaches 57.6%.

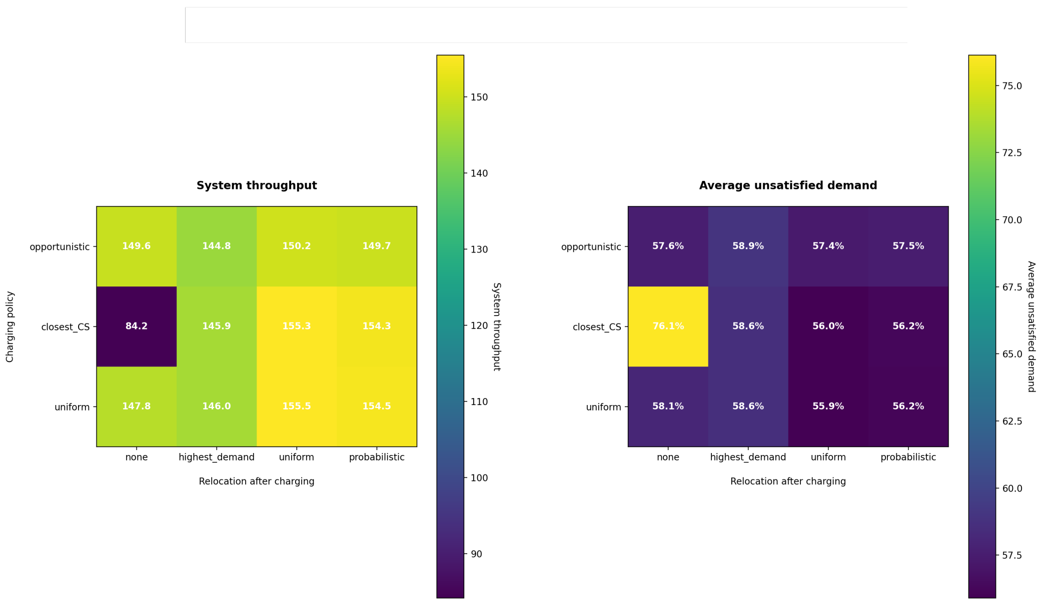
Figure 6.27b shows the same results in terms of throughput and percentage of unsatisfied demand with all the possible combination of charging policies but considering a random placement of the stations in the network. In this way the charging nodes do not necessarily coincide with the highest users' demand mobility zones. The results are not so different with respect to the previous case. In particular considering relocation operations before and after the charging process the station positioning does not have any impact on the overall system throughput unless this would result in an heavy utilisation of a particular station with vehicles queuing and accumulating. Since this is not the case considering a balanced routing matrix and the charging station utilisation is always well below the maximum capacity, the throughput is not affected at all. A similar comment can be made regarding the baseline scenario with opportunistic charging policy and no relocation after charging. Here the station positioning only affects the computation of the fraction of vehicles to be redirected to charge when arriving in the mobility zone with a station within. In this particular case the flux of vehicles in these mobility zones is not enough to guarantee the needed requests for power of the fleet. In fact the values has been clipped by the algorithm meaning that all vehicles are brought to the station but this is still not enough. The resulting throughput of the system is therefore slightly higher than the previous case but it has to be taken into account that more charging operations would be needed. The same phenomenon characterises also all the other combinations including the opportunistic charging policy. The increase in throughput is even more evident when considering relocation after strategies since the random positioning of stations probably does not follow the user mobility demand. For the remaining two combinations of policies, namely the closest station or the uniform charging policy with no relocation after charging, the throughput is even smaller when considering the random placement of stations. This is straightforward since vehicles are brought from the whole network to these nodes and then they are left there where they will tend to pile up since users' demand in these nodes is possibly not high.

Charging policies and relocation after charging combinations with hourly data matrix

The same analysis of possible combinations of charging policies and relocation strategies related to the charging process can be done considering hourly bookings data to describe the routing and the users' demand in the network. The system throughput and average unsatisfied mobility demand for all the cases are reported in the heat maps in figure 6.28. The results for a network following a demand based stations positioning are in figure 6.28a while in figure 6.28b the charging nodes are placed randomly through the city. Again the percentage of unsatisfied mobility demand is inversely proportional to the total throughput of the system.



(a) Demand based station positioning



(b) Random station positioning

Figure 6.28: Heat maps of possible combinations of charging policies and relocation after charging with different station positioning strategies and hourly data matrix (12-1pm)

The first thing that can be notice is that the highest throughput is obtained using uniform relocation before and after the charging operation. Very similar results are also obtained considering the closest station policy with a uniform relocation after charging. This relocation strategies can be in fact more suitable for this kind of network since, as seen in section 6.2.4, the distribution of vehicles through the city tend to be very uneven. A uniform relocation then, act as a re-balance for the system bringing some vehicles in all the mobility zones. For this same reason the throughput results lower if considering a probabilistic approach in the relocation after charging and even lower when using the highest demand strategies. Following the first approach in fact more vehicles are brought in zones where they already tend to rack up and this is even more pronounced considering the second policy which limits the number of destination zones for the relocation. The re-balance effect obtained with both these strategies will eventually be less impacting than with a uniform relocation.

Moreover the worst results are obtained as before including relocation before and not after the charging process as expected. A valid alternative would still be to consider the baseline scenario with opportunistic charging and no relocation but the throughput in this case can be quite lower since as stated before, the re-balancing obtained with the relocation can benefits the system dynamics. In the end it can be seen that if no relocation is considered before charging then including it after would worsen the situation since it would cause a deficit of vehicles in the mobility zones with a station (i.e. the one with the highest mobility demand).

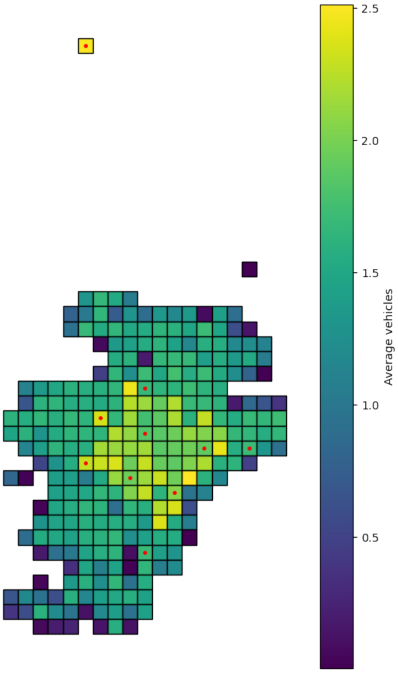
Looking at figure 6.28b, it can be seen the effects of a random placement of stations in the same network analysed before. This impact is not so relevant in the previously identified best configurations of policies since a wider relocation is always preferable wherever the stations are placed. Better results are achieved in this case considering relocation after charging even when employing an opportunistic policy. This is motivated by the fact that stations are possibly placed in zones where the demand is not high therefore moving vehicles from there would benefit the overall throughput.

6.2.6 Best policies for charging case studies

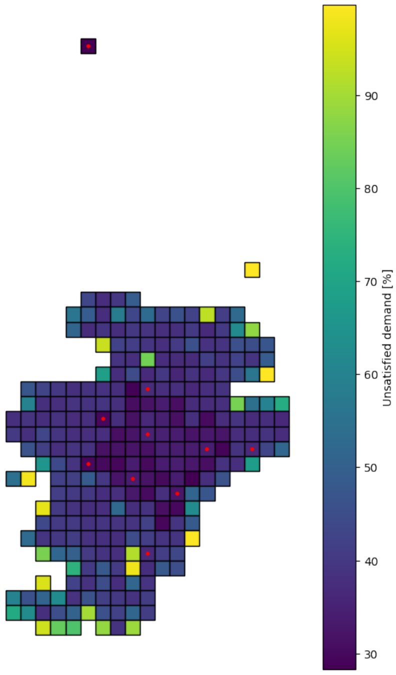
Relocation operations related to the charging process can represent an important tool enhancing the system performances but they also imply a cost for the system operator. Moreover it has been proven that combining different relocation strategies do not always improve the situation therefore, based on the network configuration, general routing and expected users' demand, particular policies schemes are more suitable than others. In the following the best combination of policies in terms of throughput and unsatisfied mobility demand found in section 6.2.5 for the two kind of networks are analysed in detail.

Closest station charging policy and highest demand relocation for balanced routing matrix

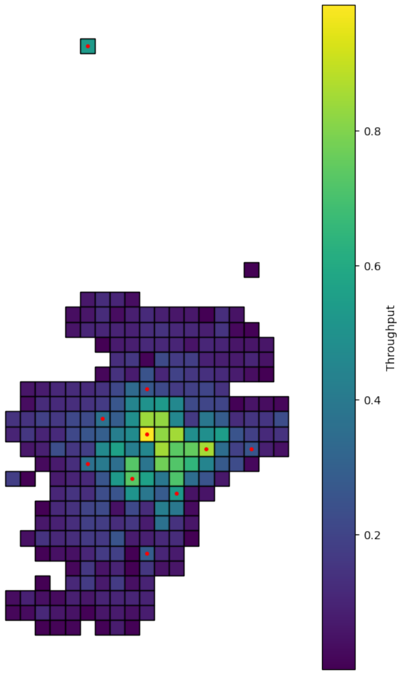
With a balanced network, such as the one obtained with a routing matrix considering all trips in the dataset, the best solution appears to involve a closest station charging policy and a highest demand relocation. Figure 6.29 reports the average vehicles, average unsatisfied demand and throughput for each zone on the city grid.



(a) Average vehicles



(b) Average unsatisfied demand



(c) Throughput

Figure 6.29: Best policies for charging network with balanced matrix on city grid

From figure 6.29a it can be seen that the vehicles distribution is quite even through all the network. This guarantees low values of unsatisfied mobility demand in almost all the zones as shown in figure 6.29b. Very few squares in the map are associated instead to a percentage of unsatisfied users' requests with high values ranging approximately between 70% and 100%. These also corresponds to nodes in which the average number of vehicles is around zero. The zones with a charging station within (represented with a red dot on the map) appear to have a low values of unsatisfied demand. However vehicles do not pile up as seen in figure 6.15b as effect of the closest station policy because in this case a redistribution is guaranteed by the relocation operations after charging. The throughput has still a geographical distribution similar to all the cases studied before with highest values in the city center and at the airport which represent the zones with the highest mobility demand.

Table 6.9 reports some numerical performance metrics obtained for this network. If comparing these results with the one reported in table 6.5 for the opportunistic policy which were the best if not considering relocation after charging, it can be seen that besides worst values of throughput and unsatisfied mobility demand, the vehicles distribution was more uniform through the network in that case and less zones had an unsatisfied demand greater than 90%. However the total number of lost requests per hour is less using the combination of the two relocation policies suggesting that the overall users' demand is better met with this approach.

System throughput	**System average unsatisfied demand**	**Maximum number of vehicles in a zone**
53.11	37.23%	2.51

Number of zones with average unsatisfied demand >90%	**Total number of lost requests per hour**
12	31.50

Table 6.9: Best charging policies network indicators recap

Looking at the charging stations in the network, figure 6.30 shows throughput and utilisation for each while the average values over the network are reported in table 6.10. The peculiar characteristic of the closest station policy are still well observable in the graph with the zones with ID 200 (i.e. the airport) that has the lower utilisation due to its distance from the rest of the network. On the contrary zone 383 has the highest utilisation due to its proximity to most part of the north sector of the city, in spite of the demand rate of the mobility zone which is the second to last with respect to all the other nodes with a station. The overall average values of throughput and utilisation suggest that the charging infrastructure is not heavily used as already seen when considering these input data. Consequently the number of vehicles in the stations is such that the probability to queue there is very low.

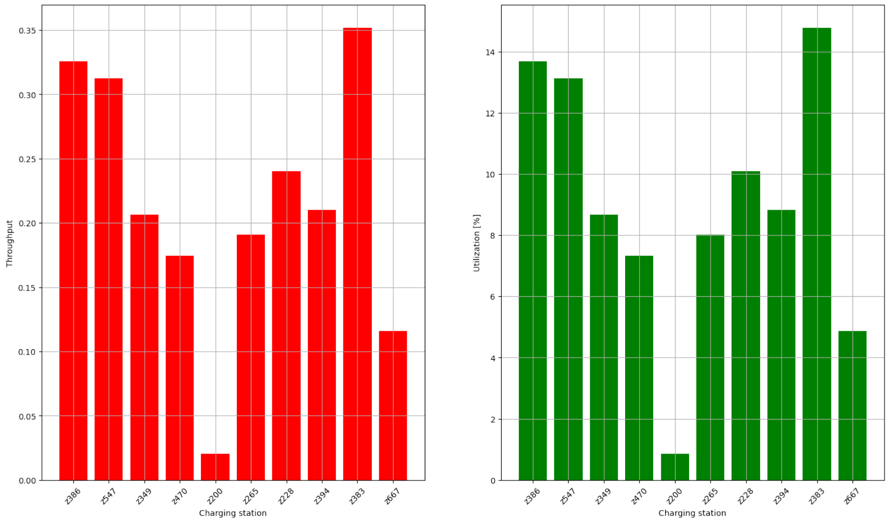


Figure 6.30: Throughput and utilisation of charging stations for best policies network

Average station throughput	Average station utilisation
0.21	9.03%

Table 6.10: Average throughput and utilisation of charging stations for best policies network

The highest demand relocation strategy requires the definition of an additional parameter which is the number of top zones ordered by their expected demand to which limit the relocation of vehicles. The previously illustrated results are obtained with a number of zones equal to 50. However a study of this parameter may be useful to see its impact on the network performances. Figure 6.31 shows the system throughput and the number of lost requests per hour as the number of considered top zones for relocation is increased. The trends appear to be consistent and opposite with respect to each other. After an initial big improvement, increasing the number of zones over 15 appear to worsen both the throughput and the lost requests. This suggests the idea that limiting the relocation after charging to few zones is the best option since the network is already well balanced and only nodes with an high demand can generate more trips with additional vehicles. Moreover the range of absolute values on the y-axis in both plot is narrow, which means that the impact of this factor is not so relevant in the overall performances of the network.

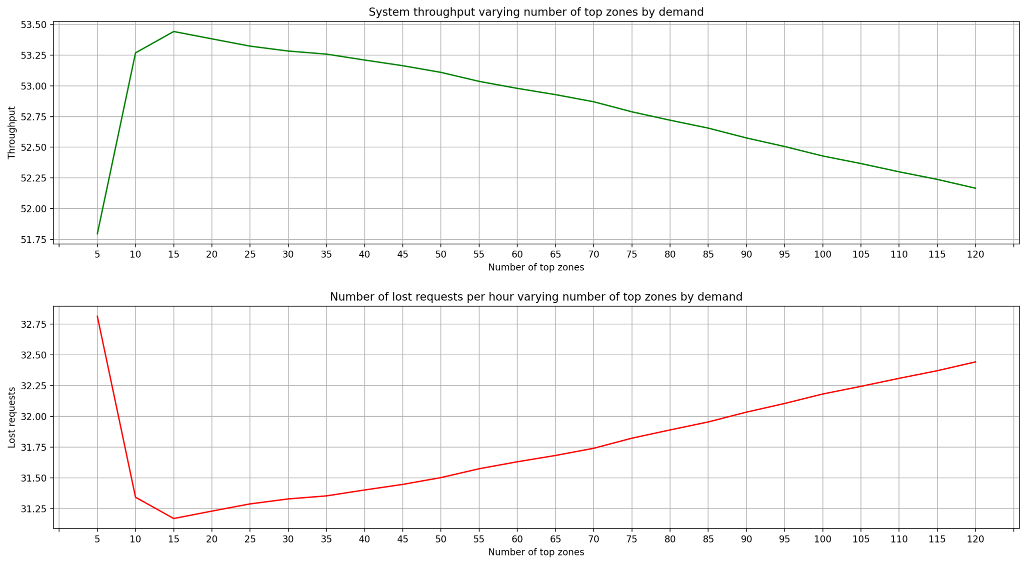
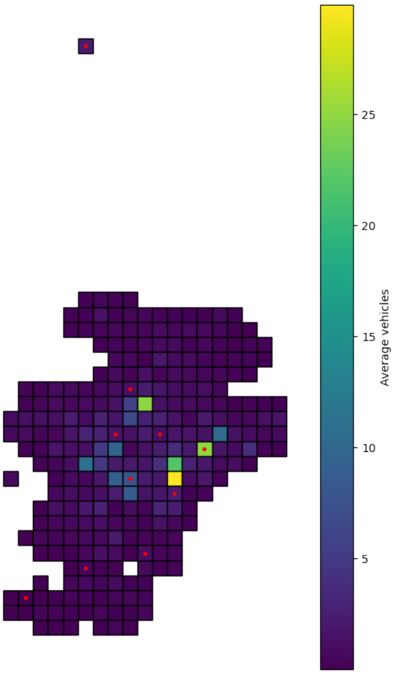


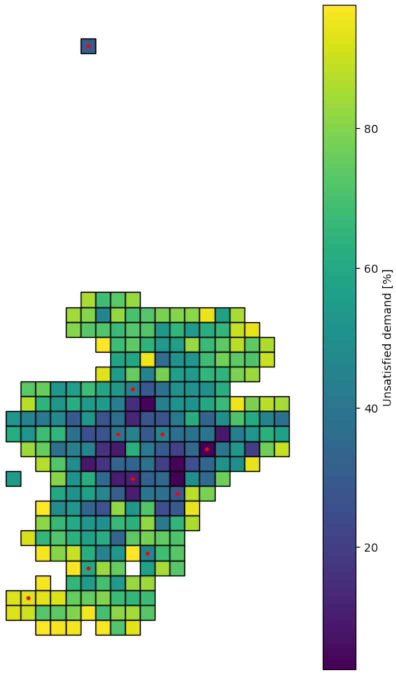
Figure 6.31: System throughput and lost requests increasing number of top zones by highest demand

Uniform charging policy and relocation for hourly data routing matrix

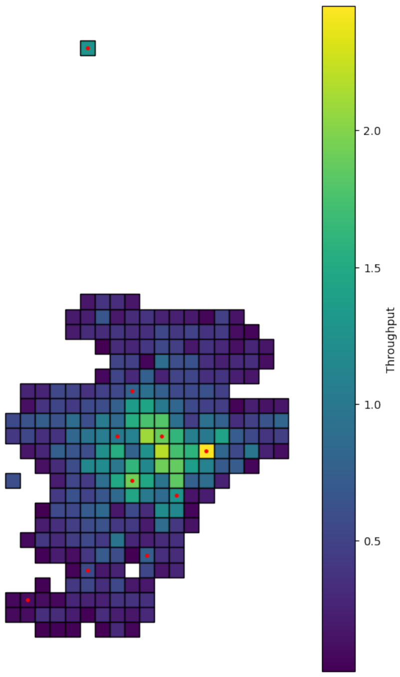
The second network studied follows a routing matrix obtained with hourly trips data from noon to 1 pm, and the best combination of policies for it is using a uniform relocation before and after the charging operations. Figure 6.32 shows the main system metrics on the city map. Observing figure 6.32a, it can be seen that vehicles still tend to accumulate at steady state in particular zones. However the maximum number of vehicles in a zone, as reported in table 6.11 is around 30 and it can be seen on the map that there are few other zones with a relative high number. Especially if comparing this result with the ones obtained with the same network but without relocation after charging as in figure 6.22, this case shows the best distribution of vehicles through the network. Additionally vehicles do not necessarily pile up in zones with a charging station inside but in very attractive nodes according to the users' routing. The range of unsatisfied demand in figure



(a) Average vehicles



(b) Average unsatisfied demand



(c) Throughput

Figure 6.32: Best policies for charging network with balanced matrix on city grid

6.32b is still wide but in addition to the average of the system, both the number of zones with a percentage of lost request greater than 90% and the total number of lost request per hour are significantly improved with respect to all the cases without relocation after charging as reported in table 6.11. All these results highlight the need for relocation operations in an heavily unbalanced network. Eventually the distribution of the throughput on the map in 6.32c follows the usual pattern of users demand.

System throughput	**System average unsatisfied demand**	**Maximum number of vehicles in a zone**
155.45	55.9%	29.94

Number of zones with average unsatisfied demand >90%	**Total number of lost requests per hour**
26	197.07

Table 6.11: Best charging policies network indicators recap (12-1pm)

Moving on the impact of the policies on the charging stations, figure 6.33 shows the throughput and utilisation of each charging node in the network. Both graphs are characterised by similar values for all the stations which is a peculiar characteristic of the uniform charging policy. This kind of network as seen in the previous analysis in section 6.2.4, can cause criticality at the charging station level because the high demand rates and throughput require an heavy utilisation of the infrastructure. However using the uniform approach to put vehicles in charge the load is distributed in the best way. The resulting average utilisation and throughput for the whole set of stations in the network are reported in table 6.12 and do not seem to indicate a particularly critical situation.

Average station throughput	**Average station utilisation**
0.63	26.43%

Table 6.12: Average throughput and utilisation of charging stations for best policies network (12-1pm)

A further study can be done computing the average number of vehicles and the probability to wait at each station. Results are displayed in figure 6.34. Again the values are all similar due to the uniform policy and the number of vehicles is less

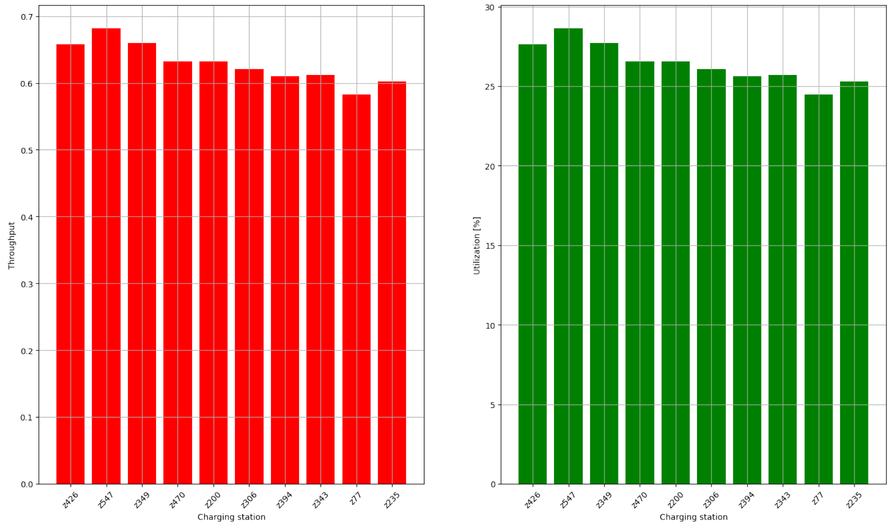


Figure 6.33: Throughput and utilisation of charging stations for best policies network (12-1pm)

than 0.7 for each station. Recalling that there are two charging outlets in each station this suggest that rarely both will be occupied at the same time. In fact the probabilities for a vehicle arriving at the station to wait are quite small and all between 9% and 13%. This result further confirms that a uniform relocation is the best option when dealing with heavy loads on the charging infrastructure.

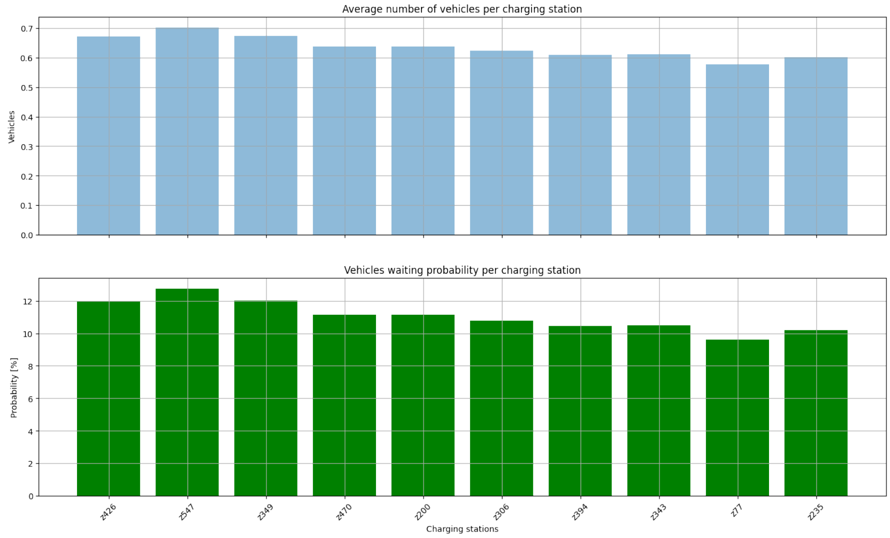


Figure 6.34: Average number of vehicles and probability yo wait in each charging station in best policies network (12-1pm)

6.2.7 Impact of charging operations on the power grid

An interest aspect concerning charging mechanism in the network is the study of the power loads required for the needed operations. With particular configurations of network and at peak hours, the stations heavy utilisation can be critical also in terms of the energy required to the city power grid. A quantitative analysis of the amount of power needed considering hourly data matrix for the whole day is done in the following with the network configurations seen in the paragraphs before. Figure 6.35 shows the power needed by the system fixing the average trip length first and the average EV consumption after. Superimposed in both graph is the bar plot of the system throughput for the same network. For all this network simulations it has been used the opportunistic charging policy and no relocation after charging. The most noticeable aspect in figure 6.35 is that the total average power consumption follows the trend of the system throughput. In general more trips means more energy consumption and therefore more charging operations increasing the power consumed. Looking at the differences between the three curves in both graphs it is easy to see that increasing the average EV energy consumption per kilometre, or the average trip length, the needed power increases. This follows the same consideration made before because more consumption per kilometre means necessity to charge the vehicle more often and a greater trip length implies charges needed every fewer completed trips.

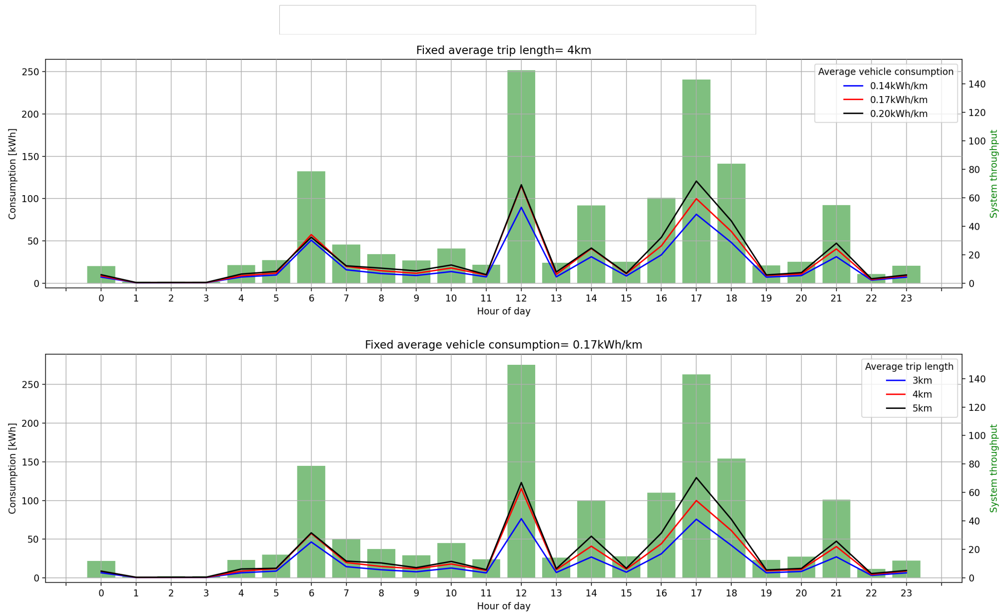


Figure 6.35: System power consumption varying average trip length and average EV consumption

6.3 Network with delay zones

The simplifying assumption made in all the networks studied so far is that once a vehicle leaves a departure zone it appears instantaneously in the destination one. The time component in the mobility flows can although be determinant if the trips time increases for example considering inter-city sharing scenarios. Delay zones have been introduced in the network to study this effect as explained in section 3.4. Table 6.13 reports the usual system metrics for the two networks with different routing matrices, employing the best charging policies and considering the different approaches to include trips delay. For both network configurations and with both methods to add delays, the throughput of the system decreases and consequently the average unsatisfied mobility demand increases. This is expected since including the trip delay diminishes the relative flows in all the mobility zones. In fact part of these flows is now at steady state in the delay queues. No particular differences are visible between the single delay queue and the multiple delay queues approaches. However the throughput is lower using the second one for both studied networks. The service times of the delay queues for this approach have been evaluated using the mean physical distance between each zone and the rest of the network. In the single delay queue the service rate is instead the average of all the service rates computed for the other method.

Balanced matrix		
Delay	**System throughput**	**Average unsatisfied demand**
None	53.11	37.23%
Single queue	52.15	38.37%
Multiple queues	51.22	39.46%

Hourly data matrix		
Delay	**System throughput**	**Average unsatisfied demand**
None	155.47	55.9%
Single queue	152.88	56.63%
Multiple queues	149.24	57.67%

Table 6.13: System performances with best charging policies and different delay strategies

Table 6.14 shows the average number of vehicles in the delay zones and the average time spent there by each. The first metric indicates how many vehicles are moving from one zone to another at steady state and therefore are not available for booking. The values for the multiple queues approach are slightly smaller which also justifies the lower throughput obtained. The main difference however is between the results obtained with the two network configurations. With the second one in fact the number of moving vehicles at steady state is much greater and more than $\frac{1}{8}$ of the total fleet size. The average time in the queue for the single zone approach coincide with the average trip time computed while for the multiple queues one depends also on the routing inside the network. However this values are not so different for both the analysed networks and around 20 minutes which is a reasonable results for the dataset under study.

As explained before in section 4.2.3 an approximate method is used to establish the flux in the single delay zone therefore the results may be affected by a small error. Moreover the multiple delay queues approach can use a more precise estimation of the trips time from each zone. For these reason in the following only this method is considered.

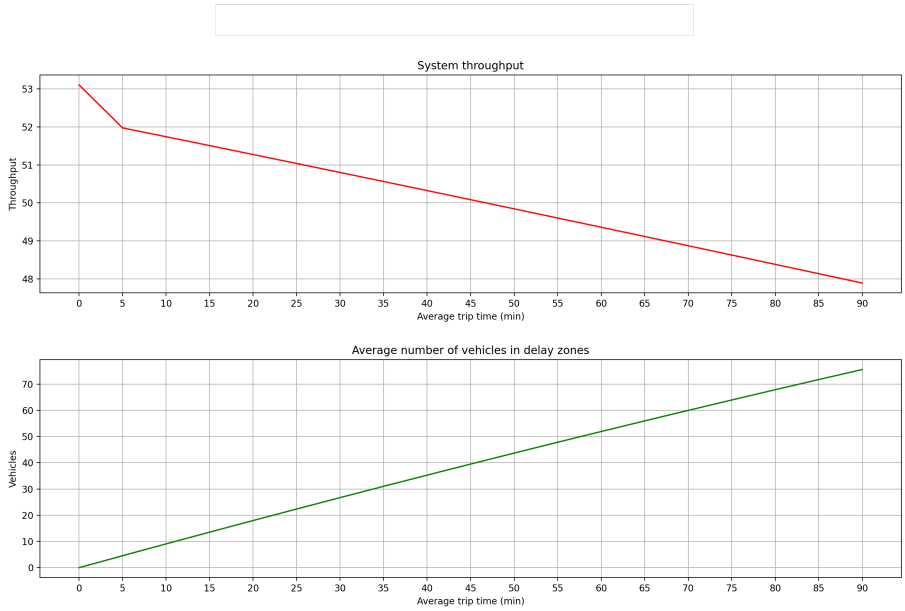
Instead of computing the service rates as function of the estimated time to reach a destination from each zone, arbitrary values of the trip time can be assigned to study the impact of the delay zones in possible different scenarios. Figure 6.36 shows the system throughput and the average number of vehicles in the delay zones as the average trip time is increased. For both networks the trends of the two curves are very similar. The throughput in fact has a net decrease when introducing a

Balanced matrix		
Delay	**Average number of vehicles in delay zones**	**Average time spent in delay zones [min]**
Single queue	18.48	21.26
Multiple queues	18.95	21.04

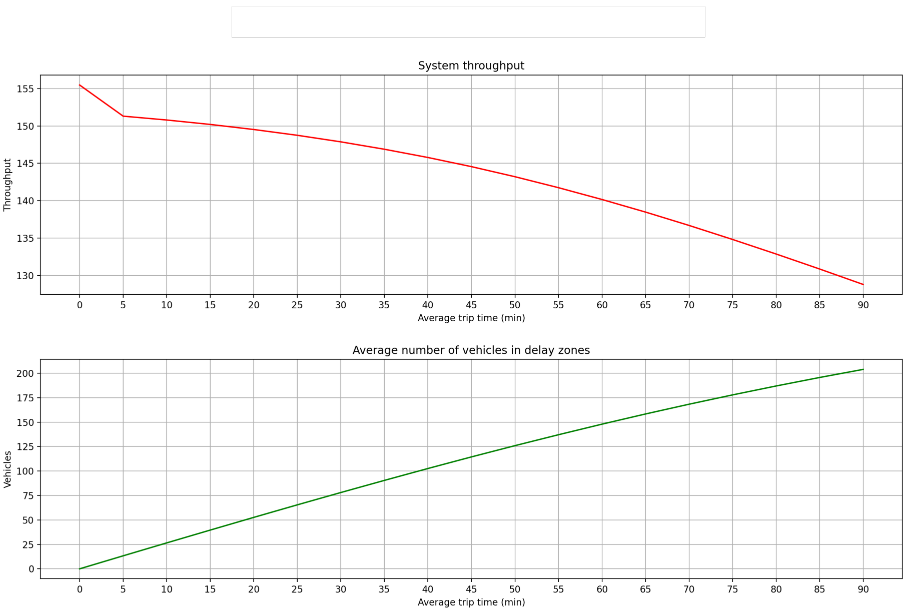
Hourly data matrix		
Delay	**Average number of vehicles in delay zones**	**Average time spent in delay zones [min]**
Single queue	52.87	20.75
Multiple queues	57.45	21.8

Table 6.14: Delay queues metrics with best charging policies

minimum delay and then decreases almost linearly but smoothly for the balanced network in figure 6.36a. This decrease appears even more gradual in the other network in figure 6.36b. On the contrary the number of vehicles in the delay zones, which are the ones moving, increases in a nearly linear way as expected as the trip time increases. In the end with a fixed average trip time on the order of the ones computed in table 6.14 (i.e. around 20 minutes) the loss in throughput are small and the number of vehicles moving at steady state is around 20 for the balanced network and around 50 for the other.



(a) Balanced matrix



(b) Hourly data matrix (12-1pm)

Figure 6.36: System throughput and number of vehicles in the delay zones with increasing average trips time

Chapter 7

Simulation results comparison

All the obtained results showed and analysed in chapter 6, have been obtained through the resolution of analytical models based on queuing theory. As said before these models assume a steady state behaviour for the system with a fixed routing matrix and demand rates to describe the vehicles movement through the city network. To further validate these results a comparison with results from a simulation tool are proposed. The simulation tool used is Odysseus[1] which stands for Origin-Destination Simulator of Shared E-mobility in Urban Scenarios. It was developed inside the SmartData@Polito research group and is a management and simulation software for mobility data with a particular focus on shared electric fleet in an urban environment. The tool simulates vehicles' trips in the city network tracing the origin and destination zones and the corresponding SoC before and after the booking. The path followed by the vehicle is not simulated in detail and the actual trips distance is estimated from the Euclidean distance between the centroids of the starting and ending zone multiplied by a correction factor which is specific for each city [36]. Fleet charging processes are also simulated imposing thresholds for the battery level and strategies for the operations' management and the infrastructure placement.

7.1 Input data adaptation

To have a fair comparison with the results of the model some modifications of the input data to feed to the simulator were required. In particular the input of

[1] `https://odysseus-simulator.readthedocs.io/en/latest/index.html`

the model is a static stochastic routing matrix inferred from real data while the simulator requires directly a trace of trips with an origin and destination time and space in the form of latitude and longitude.

7.1.1 Balanced data

The balanced matrix obtained for the model as in section 5.1.1, includes all trips in the time period for all hours of day and for both weekdays and weekends. In the simulator a new scenario has been created using the complete trips dataset as input. The important difference between the two is that the input routing matrix of the analytical model is extracted from the input trace considering it a stationary process and studying the system behaviour at steady state. The simulation model instead considers this time variability resulting in different routing and demand rates for each day.

7.1.2 Hourly data input

The hourly data input matrix for the analytical model has been obtained filtering input trips with a starting time between noon and one in the afternoon as explained in section 5.1.1. To create an input trace for the simulator which mimics similar dynamics, the same dataset was taken and the trips within the 12-1 pm interval for each day were replicated for all the other hour intervals for the day. Additionally a quite long simulation time is required to overcome the impact of the system initial conditions on the general results. Therefore a six month trace has been generated in which each day presents the same hourly trips pattern repeated 24 times. With this method however an additional variability is present in the input data with respect to what obtained with the fixed routing matrix of the analytical model since each day is different from the other. Nevertheless this day by day variance allows to include in the simulation all trips generated in the considered time interval which were also taken into account in the computation of the analytical model's routing matrix.

7.2 Simulation configurations

The simulator is made by different modules to manage the input data, create the city scenario, generate the demand, provide the supply configuration and complete the simulation. Different parameters and strategies can be set to be used in the modules. In the following the used demand model, the possible configurations for the supply and the followed simulation technique are briefly explained.

7.2.1 City scenario and Demand model

From the given input trips trace, a first data manipulation provides the origin-destination pattern through the network, mapping latitude and longitude on the city grid and creating its visual map. A set of valid zone is extracted and various parameter are computed such as the average speed and average driving distance of vehicles, the set of neighbouring zones through the distance matrix of the network and the actual fleet size.

The demand module would allow to estimate the customers mobility requests in time using prediction models such as the Poisson-KDE proposed in [34]. However for the simple studied simulations, the demand is computed from the input data in a similar way as done for the analytical model, counting for each zone and at each time interval the number of generated trips towards the rest of the network.

7.2.2 Supply model

The system configurations for each simulation are defined in the supply model. Table 7.1 shows a list of the main parameters that can be set for the supply model definition.

Category	Parameter
Vehicles fleet	Vehicles factor
	Engine type
	Profile type
	Vehicle model
Charging infrastructure	CPS placement policy
	Distributed CPS
	System CPS
	Num charging poles
	Num charging zones
Charging policies	Charging strategy
	Charging relocation strategy
	Charging thresholds
	Charging queuing
	Number of workers

Table 7.1: Supply model main configuration parameters list

For what concerns the vehicles, the fleet size is already inferred by the input trips data which also contain the unique plate number of each vehicle. A vehicle factor can be set for the simulation to easily modify this number. Moreover the engine type, the model and the profile type, which determines the voltage and current output, of the EVs are set. All data related to the vehicles can be extracted in this way; for EVs these are the battery capacity, the average consumption and the charging performances with different type of charging stations. The charging stations parameters instead include the possibility to distribute or concentrate the stations in a single zone, to decide the policy for the placement and to set the number of stations and poles to include in the network. For the charging operations it can be choose to adopt a reactive or proactive strategy and how to choose the station to which bring the EV. The thresholds for the charging can be set and the possibility to queue at stations can be enabled. Eventually the number of available workers has to be set together with the average time they took to reach a vehicle to bring it to charge.

For what concern stations positioning in particular, the possibility to place them in the top zones by number of parking was already implemented. This is similar to the demand based approach used in the analytical model and explained in section 5.2.3, however a slight modification to the code was applied to avoid the positioning of stations in neighbouring zones.

In the end other parameters, not included in table 7.1 because they are not of interest for this work and mainly regarding relocation strategies can be set through the supply model.

7.2.3 Simulation technique

The simulation module allows to perform simulation with two different strategies: a model driven and a trace driven one. The model driven approach uses the inferred demand from the Poisson-KDE estimation model while the trace based one follows the list of bookings created by the demand module and it is the chosen approach for the studied cases. The list of booking requests is simply scrolled through and a further check on the validity of the origin and destination zones is performed before a new simulation event is generated. After the simulation is carried out different statistics are generated on the general performances of the system and some graphs are produced.

7.3 Simulation case studies

Two main case studies have been tested with the simulator tool corresponding to different input data as explained before in section 7.1. The balanced data case uses the whole dataset of traces to create a scenario and a demand trace while the hourly data case considers only trips in the 12-1 pm time interval repeated through the whole day.

Excluding the different input data, the other simulation parameter are kept constant in the two simulation. A list of the chosen configuration is in table 7.2.

Vehicles fleet	Vehicles factor	1
	Engine type	Electric
	Profile type	Three-phase
	Vehicle model	Fiat 500e 2020
Charging infrastructure	CPS placement policy	Number of parking
	Distributed CPS	True
	System CPS	True
	Num charging poles	20
	Num charging zones	10
Charging policy	Charging strategy	Reactive
	Charging relocation strategy	Closest free
	Charging thresholds	–% to 90%
	Charging queuing	True
	Number of workers	12

Table 7.2: Simulation main configuration parameters

With respect to the tested analytical models some differences are present in parameters that could not be changed. In particular looking at the charging policy, reactive charging is implemented which means that EV are brought to charge when the battery level is below a security threshold. This threshold is computed based on the vehicles average consumption and the maximum driving distance in the city to have a safe margin and avoid to have EV's battery dying mid trip therefore its value depend on the single case. The maximum threshold instead has been manually set to 90% of the battery capacity. Additionally EV are brought to charge in the nearest station available which is the same strategy implemented for the analytical model and referred as *closest station*. The positioning of the stations then follows the zone with the greatest number of parking avoiding to place two stations in neighbouring zones. This resulted in an almost equal placement with respect to the analytical model which followed the expected user demand. In the end the charging outlet was defined as a three-phase charger set to provide an output voltage of 400V and an output current of 32A which results in a total power of 22.17kW. The maximum charging power allowed by the vehicle with this charging profile was also set to 22kW. This is the most similar solution to the one adopted in the analytical model which considered an output power of 20kW for the charging outlets.

7.3.1 Balanced data simulation

The first simulation case proposed is based on the whole trace of input trips from July to December 2017 including both weekdays and weekends. The computed number of vehicles for this simulation is 439 and the computed minimum threshold to trigger the reactive charging of EV is 19.49% which are both similar values to the one considered in the analytical model. It is important to stress that this simulation simply follows the day-by-day input trace of data while the network in the analytical model resulted from an input fixed routing matrix therefore a direct comparison of the two approaches may be not so informative. However it can be useful to extract overall average metrics for the network to compare the general behaviours of the systems with the two models.

The charging station positioning on the map follows the maximum number of parkings approach explained before and resulted in the grid in figure 7.1. The placement is very similar to the one obtained with the balanced matrix in the analytical model with all the stations placed in and around the city center and an additional one at the airport as shown in figure 6.14.

Mobility network indicators

A first characterisation of the predicted demand can be seen in figure 7.2 where the total number of requests (figure 7.2a) and the hourly demand rate (figure 7.2b) per each zone are plotted. As expected from the input trace an higher mobility demand is generated in the city center zones. Moreover the airport generates a consistent number of requests while in general going towards the city outskirts results in a decreasing of the demand. In terms of rates values these exceed one requests per hour on average only in very few zones in the center of the map.

The actual number of generated and attracted trips by each zone in the network are instead shown in the maps in figure 7.3. The general distribution through the city is very similar to the demand one. Looking at the absolute values of the departures in figure 7.3a, it can be seen that also the range of values is almost the same as the demand one suggesting a good satisfaction of customers' mobility requests.

Each generated event can be tracked through the simulator which gives the possibility to study in detail the trend of users' demand and satisfied bookings not only in the form of aggregated averaged metrics. Figure 7.4a for example shows the profile of events in the simulation time frame in terms of requests, bookings and unsatisfied requests. The data for the plot have been resampled in one hour intervals. The gaps in the curves correspond to time periods for which the input data were not available probably due to a system malfunctioning. The graph suggests that the percentage of unsatisfied demand is always below 50% since the bookings (green) curve is always above the unsatisfied (red) one. Additionally

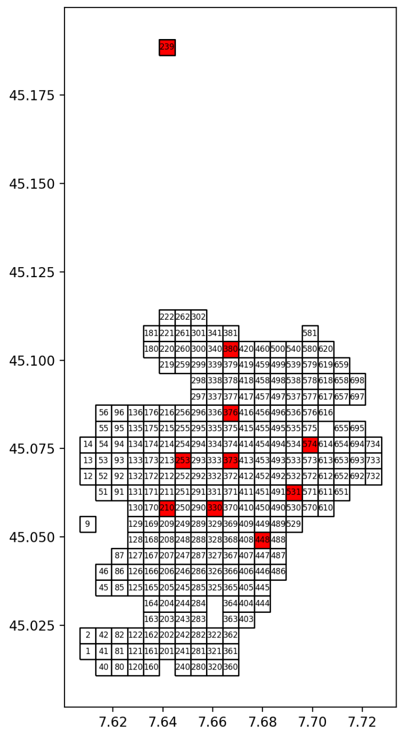
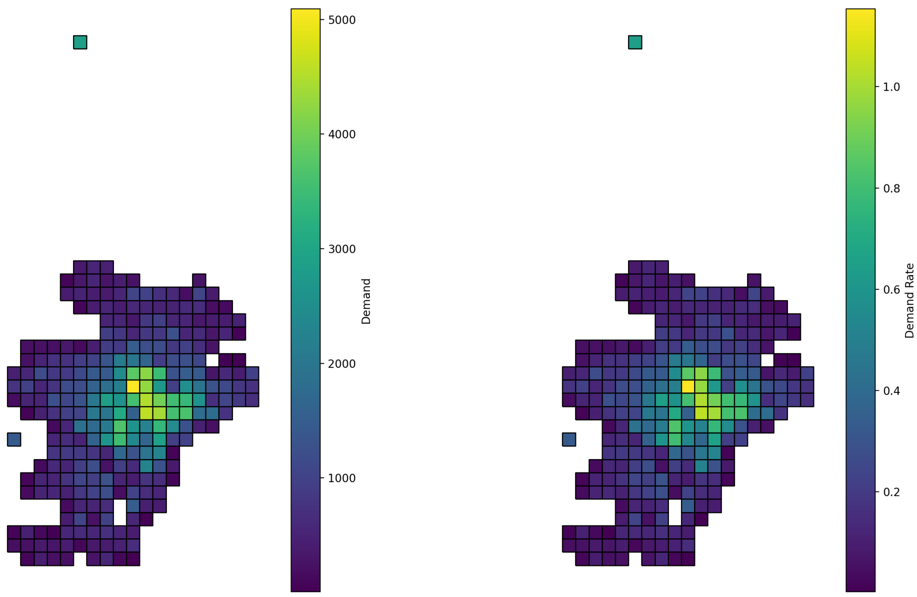


Figure 7.1: Simulation city grid

figure 7.4b shows a zoom on the events generated at the beginning of the simulation. It can be seen that at the very beginning the curve of unsatisfied requests is almost superimposed to the bookings one while it decreases with the simulation time and stays always below the other one after. This suggest the existence of an initial, even if short, transient period. The starting distribution of vehicles probably was not ideal to fully satisfy the customers demand. Going further with the simulation instead that system balances itself reaching an almost stationary condition which can be therefore compared to the steady state condition of the analytical model.

A first comparison with the proposed model can be done observing the system target metrics as in figure 7.5. The most similar case to this simulated one studied with the analytical model is the one obtained with a balanced routing matrix, a *closest station* charging policy and no relocation after charging. The throughput in figure 7.5a appears almost equal in both distribution and values range to the one obtained with the model and showed in figure 6.17b. The total hourly throughput

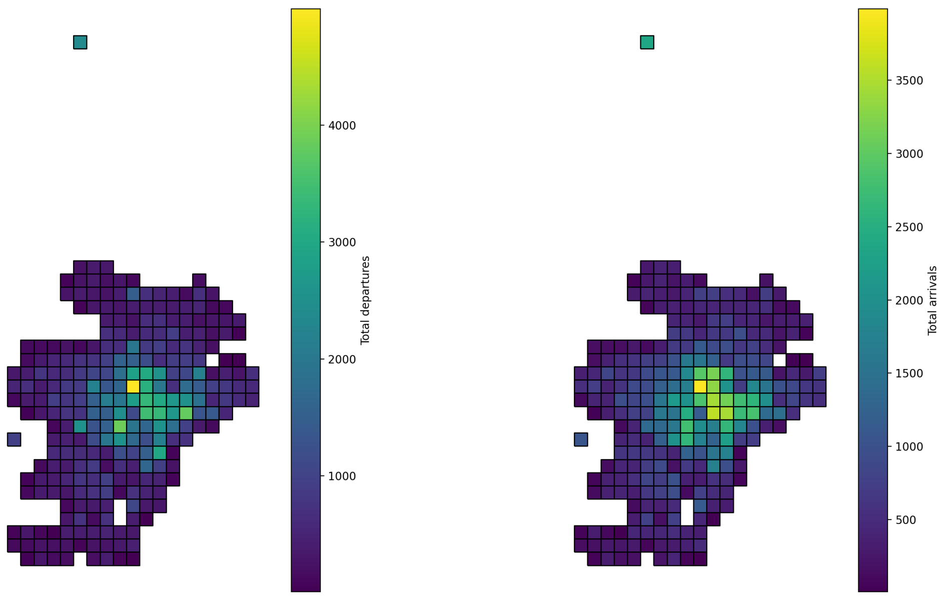


(a) Demand **(b)** Demand rate

Figure 7.2: Simulation demand and demand rate

for the network is 49.13 compared to the total number of mobility request per area which was 64.17. The percentage of unsatisfied mobility request is instead plotted in figure 7.5b. Here it is evident that most of the value are in range between 20% and 40% with few zones with value that approaches 50% and just one which exceed it. The smallest percentages of unsatisfied demand are at the zones with charging stations within and in their immediate neighbourhoods. The used charging policy in fact brings EV to charge in the nearest station and do not relocate them after the process is complete. Therefore it is safe to assume that some EV will accumulate in these zones resulting in most of the demand being met there. An important factor in the configuration of the simulator is that if vehicles are not present in the intended origin zone, a customer can take it from an immediate neighbouring one resulting in zones near the charging station to have similar smaller values of unsatisfied demand.

A further detail on how much this configuration affect the system performances can be seen in figure 7.6 in which the percentage of satisfied trips and between them the percentage of satisfied trips in the intended zone are shown. The total



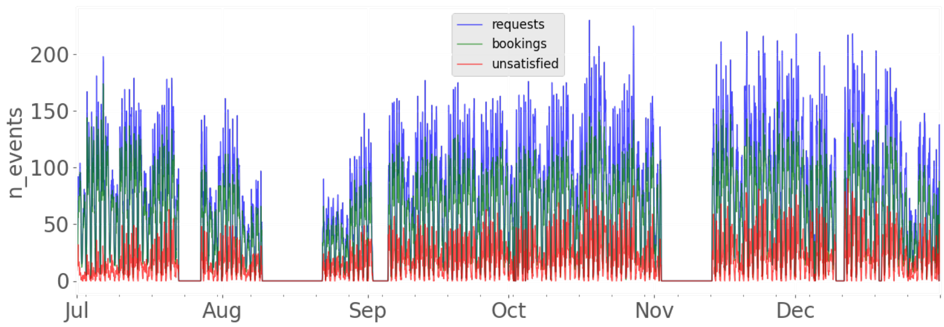
(a) Departures

(b) Arrivals

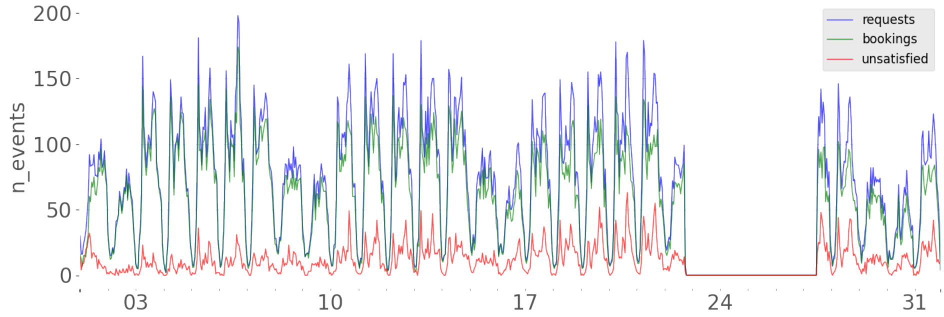
Figure 7.3: Total simulated departures and arrivals per zone

percentage of satisfied trips is 76.46% and the 72.04% of them were from vehicles booked in the intended origin zone which is the 55.08% of all the requests. In a direct comparison with the most similar scenario studied with the analytical model, the overall percentage of unsatisfied mobility demand for that case was 57.58%. The two results confirm that the analytical approximation is very precise for this case in spite of some differences in the initial configurations of the two models. Moreover the initial transient period was not cut off for the computation of these metrics therefore, even if very short, may have contributed in increasing the percentage of unsatisfied requests in the simulation.

In the end a simple analysis of the time complexity can be perform. The simulation for this case took 194.73 seconds which confirms the higher need of time and computational resources for the simulation model with respect to the analytical one. The analytical model in fact took on average around 9.70 seconds including the time to load the data and plot some graphs of which just around 0.68 seconds are used to build the network and extract the system metrics.



(a) Whole period (July-December 2017



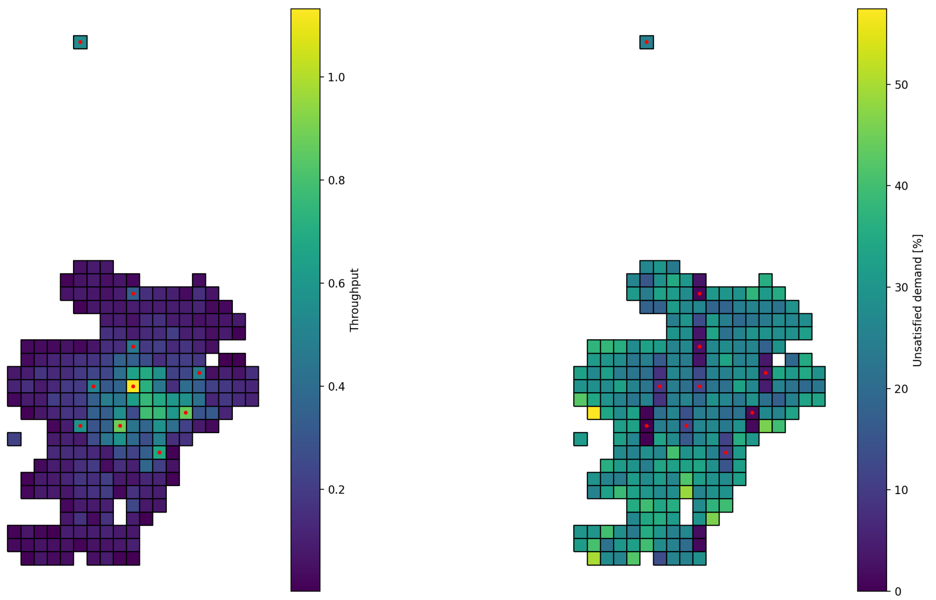
(b) Starting period (1-30 July 2017)

Figure 7.4: Simulation event profile

Charging stations indicators

A further analysis can be done looking at the charging stations in the simulated network scenario. As already displayed in table 7.2, there is a total of 10 stations in the network each with 2 charging outlets which is the same configuration of the analytical model.

Figure 7.7 shows the throughput and the utilisation of each station in the network. The pattern of the stations' throughput does not exactly follow the utilisation one. This happens because, differently from the analytical model, EVs reach the stations with different level of residual battery therefore some charging operation take more time than others. Moreover the only station which present values significantly smaller than the others is the one in the zone with id 239 which corresponds to the airport. The charging policy in fact requires that EVs are brought to the closest station and the airport is far from the rest of the network. It is safe to assume that all the charging events happening at the airport are therefore of vehicles that were already arriving there with no further relocation involved.



(a) Throughput **(b)** Unsatisfied mobility demand

Figure 7.5: System target metrics per zone

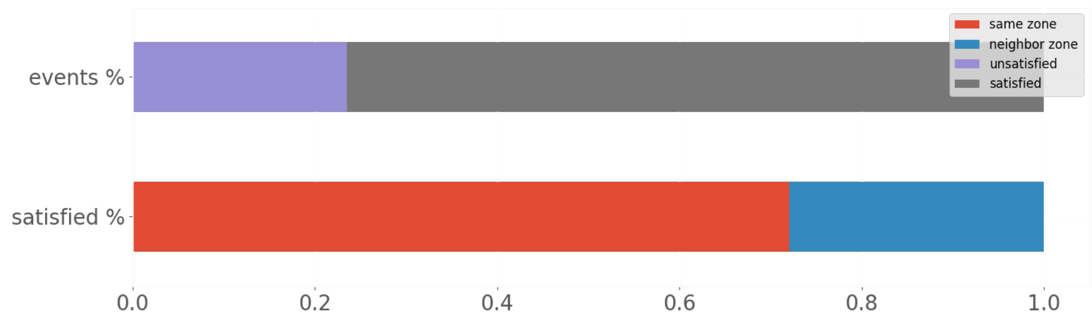


Figure 7.6: Satisfied trips details

Similarly the second station by smallest throughput and utilisation is in the north sector of the city near to few other zones for which the throughput, as seen in 7.5a, is relatively small. Overall the throughput of the most used station is smaller than 0.35 charged EVs per hour and the maximum utilisation is around slightly above 20%. Computing the mean values for all the stations in the network it results an average throughput of 0.24 and an average utilisation of 14.13%. All these values suggest that the infrastructure is not particularly stressed.

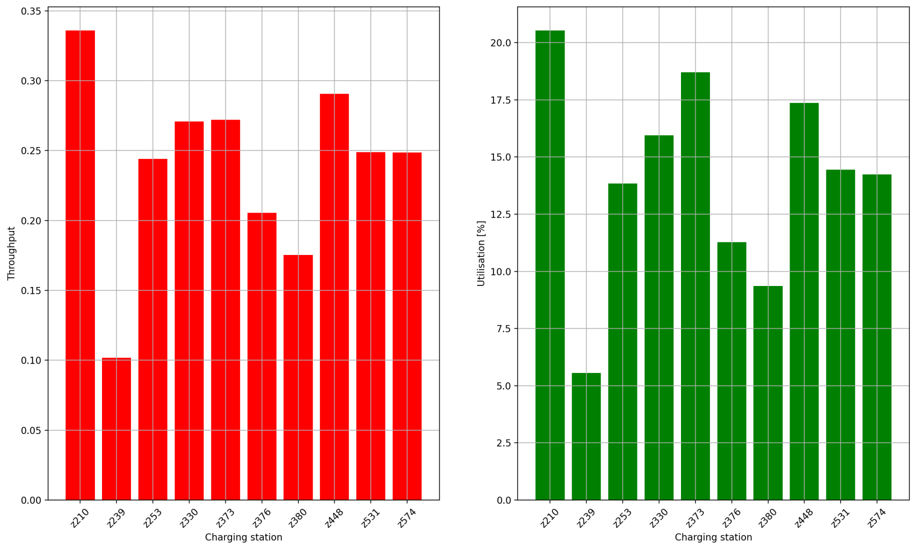


Figure 7.7: Throughput and utilisation of charging stations

Other interesting statistics on the energy consumption and its components and on the general charging operations can be obtained directly through the simulator and are shown in table 7.3.

Parameter	Value
Total charges number	10488
Total WtW energy	377827.17 kWh
Total WtT energy	225816.53 kWh
Total TtW energy	152010.64 kWh
Total charges energy	185641.08 kWh
Total energy cost	13902.63 €
Total WtW CO_2 emissions	87289.11 kg
Average charging duration	3620.52 sec
Average EV SoC	60.12%
Charges by EV	24.50
Average SoC at charging	17.70%

Table 7.3: Simulation charging processes statistics

A particularly useful measurement to compute the total energy consumption of a generic mobility system is given by the well-to-wheel (WtW) which takes into account all the processes required from the extraction of the energy source (well) to its consumption by the vehicles (wheel). This value can then be divided in its two components namely well-to-tank (WtT) and tank-to-wheel (TtW). From the collected data it can be seen that with the given configuration the major component of the total energy consumption is in the WtT phase so from the extraction to the charging of the vehicle. The TtW instead is the actual measure of the energy employed for moving the EVs. The same classification is used for CO_2 emissions but since in this scenario EVs are employed, the TtW emissions component is zero and the total value is entirely due to the extraction and production of the energy required. The total energy quantity provide to the EVs through charging processes is also quantified and it is, as expected, greater than the one consumed (TtW component) at the end of the simulation. With respect to the analytical model, the utilisation and throughput of charging stations are higher in the simulated scenario which is consistent with the higher overall throughput of the mobility zones. Dividing the total amount of energy used in the charging process by the total simulation duration it results an average of 49.58kW of power injected per hour while in the analytical model this same value is 24.40kW.

In the end some measures more specifically concerning the simulated charging processes are provided. The average duration of a charge is around one hour and the average state of charge (SoC) of EV going into the station is 17.70% so slightly less than the set minimum threshold. Moreover the mean SoC of EVs in the system is 60.12% which is above the mid point between the two charging thresholds and an average of 24.5 charges per vehicle are carried out in the six month of simulation time span.

7.3.2 Hourly data simulation

As explained in section 7.1.2 input data for this case study have been filtered to include only trips happened between noon and one in the afternoon and on weekdays. This pattern of trips has then been artificially replicated for all hours of the day. The considered time period of data for the simulation is between July and December 2017. The number of vehicles available for this simulation is 409 while the computed minimum threshold for charging is 18.69% also in this case very similar to the configuration of the analytical model.

Mobility network indicators

Figure 7.8 shows the city map obtained by the simulation with the positioning of the charging stations highlighted in red. The placement is very similar to the one obtained with the analytical model for the hourly data routing matrix with most of the station placed around the city center and one in the airport.

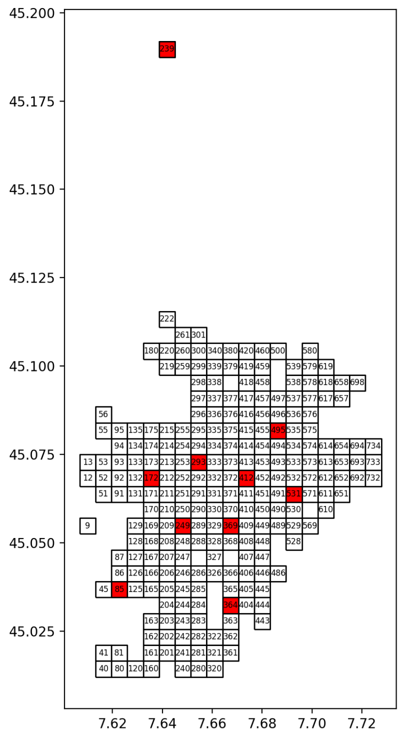
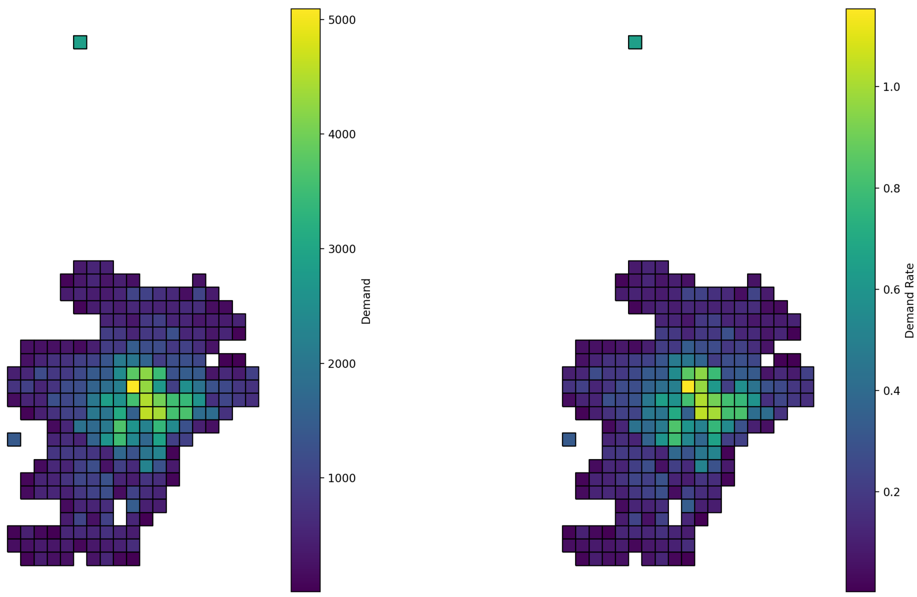


Figure 7.8: Simulation city grid (12-1pm)

The demand computed from the input trace for this scenario has been plotted in figure 7.9. The pattern through the city shows again a greater request for mobility in the city center zone and in part at the airport and the average rates of requests are all below one trips per zone.

An overview of the actual trips generated in the simulation can be seen in figure 7.10 where the total number of departed and arrived trips per zone is plotted on the map. As seen in all the previously studied cases, the zones in the city center are both the most attractive and the ones that generate the greatest number of trips. Some additional gaps are presented in this maps due to zones that have not originated or attracted any trip during the simulation.

The list of generated requests, bookings and the consequent unsatisfied trips are plotted on the simulation time axis in figure 7.11. The first thing that can be



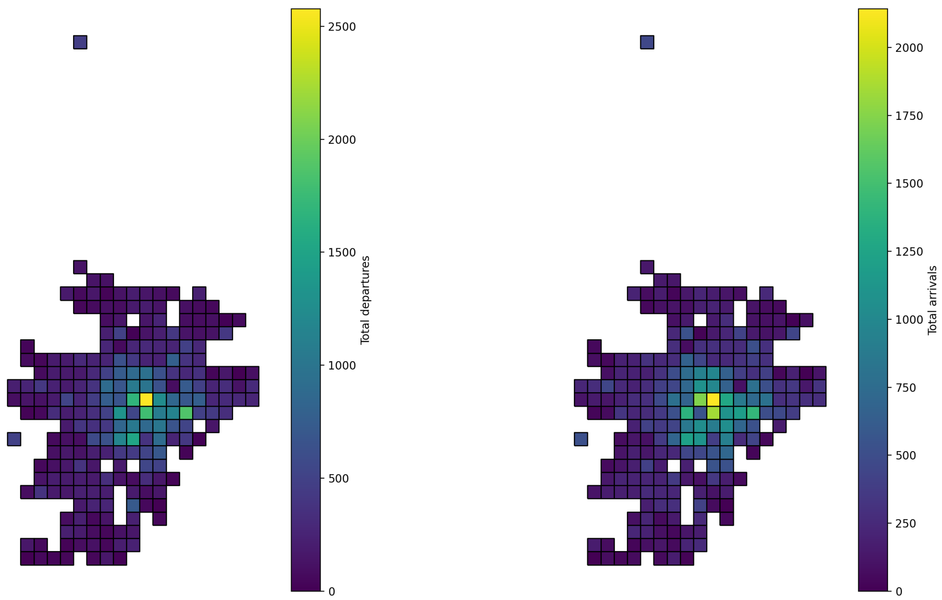
(a) Demand

(b) Demand rate

Figure 7.9: Simulation demand and demand rate (12-1pm)

noticed in figure 7.11a is that most of the time the curve of the bookings is below the one of the unsatisfied requests suggesting that the percentage of unsatisfied mobility demand is almost always above 50%. The gaps in the graphs instead correspond not only to lack of input data but also to weekends for which data were filtered out and the demand was not computed. Figure 7.11b shows a zoom on the first part of the simulation profile. It can be noticed that at the very beginning the number of unsatisfied requests is low and it increases in the following days to remain above the satisfied bookings for the rest of the simulation. This transit effect is due to the initial condition of the system for which in this case vehicles are well distributed through the city. Then due to the heavy unbalances in the demand and trips pattern, the system tend to be unbalance reaching a condition that can be compared with the steady state of the analytical model with the hourly data routing matrix.

Looking at the target system metrics, figure 7.12 shows the hourly throughput and the fraction of unsatisfied mobility demand for each zone. The throughput in 7.12a as seen for the total originated trips in figure 7.10a has higher values in the

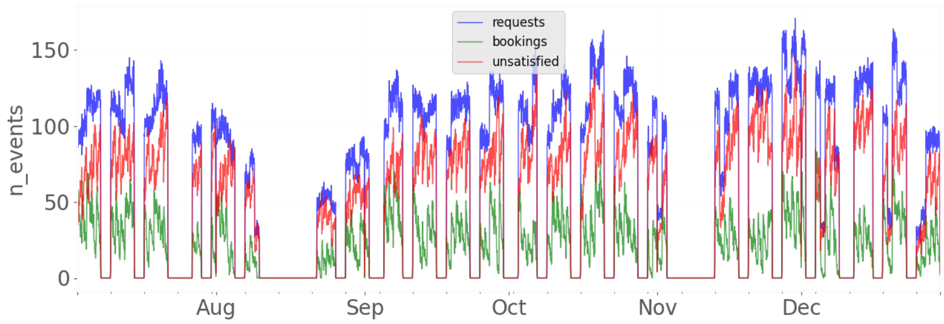


(a) Departures **(b)** Arrivals

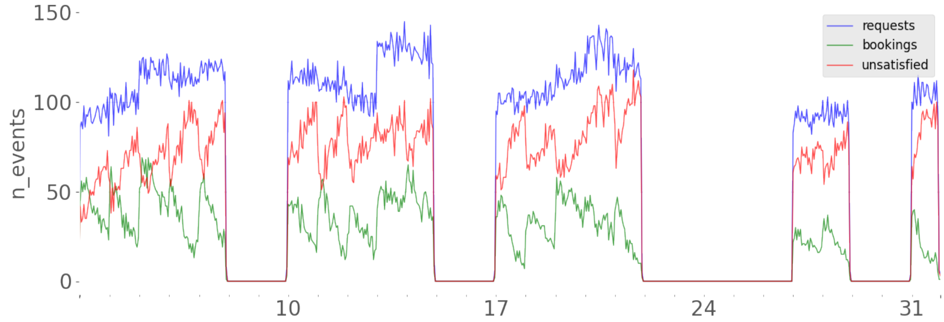
Figure 7.10: Total simulated departures and arrivals per zone (12-1pm)

city centre which are however relatively small compared to the ones seen for the correspondent analytical model. Computing the overall hourly system throughput the obtained value is only 18.60 with a total number of requests per hour which instead reaches 65.39. With the hourly data matrix in the analytical model and considering a closest station charging policy with no relocation, which is the most similar scenario tested to the simulated one, the overall throughput value was 112.35. This difference is the result of much lower values for the simulated model demand and in a general of a greater fraction of unsatisfied mobility. Absolute numbers aside, the distribution of the throughput in the city is similar in the two cases. The unsatisfied demand appears quite evenly distributed through the network with most zones with a percentage of unsatisfied requests around 70%. Few zones have smaller values and are placed where there is a charging station or in a neighbouring zone.

Figure 7.13 gives a detail on the satisfied trips which are just 28.09% of the total demand. Moreover 51.35% of them are satisfied by EV present in a neighbouring zone with respect to the originally indented one. With the similar case in the



(a) Whole period



(b) Starting period

Figure 7.11: Simulation event profile (12-1pm)

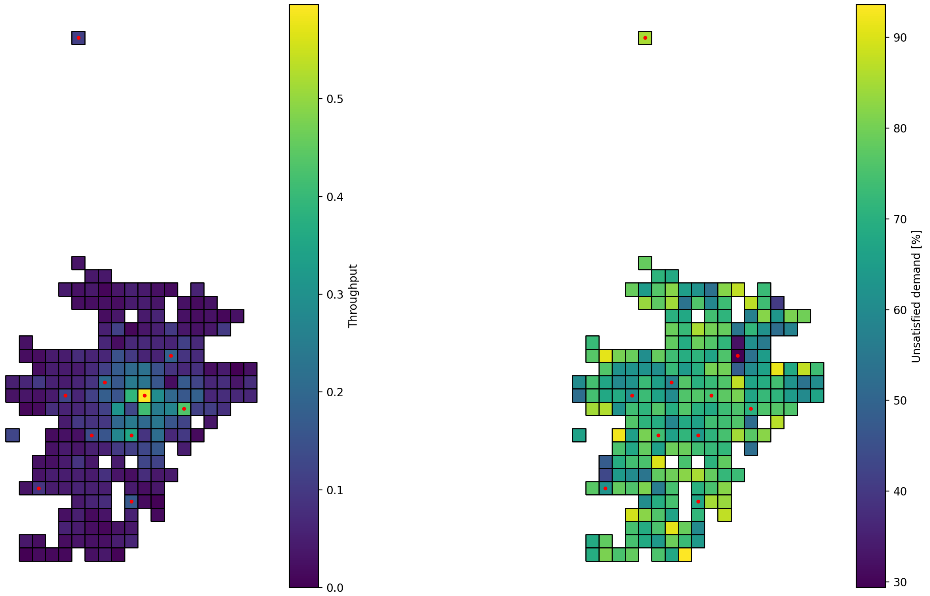
analytical model the fraction of unsatisfied demand was 68.13% which is comparable to the 71.91% obtained here. If instead considering the number of trips originated in the exact same zone as the intended one, this percentage would increase to 86.33% of all the requests.

The time needed to complete this simulation was 112.94 seconds which is smaller than the one required by the previous case study in section 7.3.1 because of the smaller dataset. However this remain much larger than the time required to build the analytical model for the similar scenario which was on average 7.1 seconds including the input data loading and the plots generation and 0.67 seconds for the network building and the performance metrics extraction.

Charging stations indicators

In the end a simple analysis of the charging stations performances for this scenario can be made.

Figure 7.14 shows throughput and utilisation of each single station in the city.



(a) Throughput **(b)** Unsatisfied mobility demand

Figure 7.12: System target metrics per zone (12-1pm)

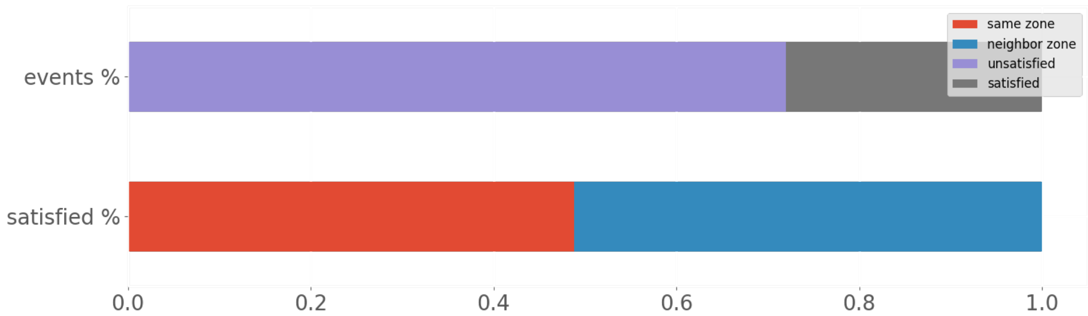


Figure 7.13: Satisfied trips details

Again the station with the lowest throughput and utilisation is the one placed at the airport. On the contrary the most used and with the greatest throughput is the one placed in the central zone which has a throughput much greater than all the other zones. Overall in this scenario even the most used stations have a throughput less than 0.12 charged EVs per hour and a maximum utilisation around 9%. On average the throughput in all the stations is 0.04 and the mean utilisation is 4.41% This is the result of a lower throughput and an high percentage of unsatisfied mobility requests as resulted from the previous analysis.

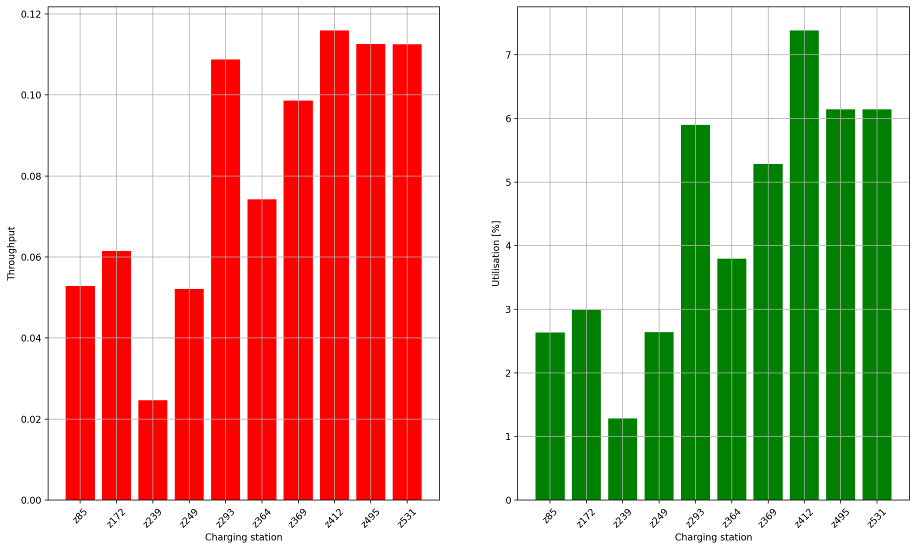


Figure 7.14: Throughput and utilisation of charging stations

The parameters regarding energy consumption and charging process statistics in general for this scenario are reported in table 7.4.

Parameter	**Value**
Total charges number	3485
Total WtW energy	122551.87 kWh
Total WtT energy	73245.77 kWh
Total TtW energy	49306.11 kWh
Total charges energy	62018.00 kWh
Total energy cost	4509.46 €
Total WtW CO_2 emissions	28313.06 Kg
Average charging duration	3640.03 sec
Average EV SoC	60.23%
Charges by EV	8.58
Average SoC at charging	17.79%

Table 7.4: Simulation charging processes statistics

With respect to the previous simulated case here all the absolute numbers relative to total quantities are smaller due to the general lower throughput obtained with this configuration. Looking at the relative results it can be seen that also in this case the major component of energy consumption is in the WtT phase which is again the only period in which CO_2 emissions are produced. In this case the simulated scenario presents much lower values of throughput and utilisation of the charging infrastructure with respect to the analytical one. This result is consistent with the much lower throughput of the mobility system and the higher unsatisfied demand which inevitably lead to a lower need for energy to power the fleet. Computing an hourly average of the used energy for charging the simulation model requires 19.43kW each hour while in the analytical model scenario this value is 76.40kW. For what concern the fleet instead, similar percentages of average SoC in the system and when brought to charge are obtained with respect to the balanced scenario. Also the average charging duration is around one hour while the average number of charges per EV is much smaller since less trips are carried out therefore consuming less energy.

Chapter 8

Conclusions

In this thesis work the problem of modelling analytically a free-floating electric MoD system was addressed. Queuing theory was the theoretical base on which network models were constructed to describe the dynamics of this kind of systems. Parameters for all models have been inferred from real data allowing to spatially and temporally characterise the customers' demand for mobility and the routing of vehicles through the city. Many case studies are presented to study different configurations of input parameters all based on trips data for the city of Turin. A final comparison with results from a simulation model is then provided.

8.1 Contributions and key results

The novel approach implemented considered a closed network of M/M/1 queues representing the city zones of the service's operational area. Moreover charging stations were modelled as multi-server M/M/C queues and additional M/M/∞ nodes were included to take into account delays associated to trips time. Movement of vehicles through mobility zones were dictated by routing matrices extracted from past trips data; similarly the nodes' service rates were modelled according to historic data of users demand. The networks were solved analytically using the MVA to obtain the intended performance metrics.

A particular focus of this work was given to the definition of charging operations and the exploration of possible policies for charging including relocation. A first study on the number of charging stations in the network showed that system performances can be improved up to a certain point from which including additional stations does not provide any further benefit in terms of their throughput or utilisation. Looking at the concentration of stations instead it has been seen that in general it is convenient to spread charging outlets as much as possible through the operational area compared to more aggregated approaches such as the definition of a big single charging hub. Moreover the positioning of the stations in the city has shown to produce better results when they are placed in the busiest zones with different level of improvement according to the selected charging policy.

The exploration of possible charging policies and relocation after charging has resulted in different results based on the configuration of the network. However a general conclusion can be drawn showing that when relocation is considered it is better to relocate vehicles both before and after the charging operations and this in general can improve the system performances (up to 20.4% and 12.2% reduction in unsatisfied demand for balanced and hourly data network respectively). On the other hand considering relocation only before (i.e. to bring EV to the stations) or after (i.e. to take away EV from the station) the charging, in general brings to unbalances in the network that do not enhance the overall throughput. Moreover in all the studied cases, the utilisation of the charging stations did not reach critical values (close to 100%) and the corresponding probabilities for an EV to wait in line to charge were always relatively small (1.75% and 11.06% on average for balanced and hourly data network with the best policies configurations) justifying the assumption of not considering a queue capacity for the stations.

In the end it has been seen that introducing delays associated to the trips resulted in a reduction of the system throughput, as expected, but with results that do not differ much from the original network ones (2.23% and 1.77% reduction in satisfied demand with balanced and hourly data network). Moreover in spite of providing an accurate approach to derive the average trip times for each zone based on the average distance from the rest of the network, the actual trip times would obviously depend on the destination zone for each trip. To have an even more accurate approximation it would be therefore required to define a delay zone between each couple of possible origin-destination zones increasing the complexity of the model by a N^2 factor. However the strongest assumption made for the delay zones is that trip times are exponentially distributed which can be seen as a stretch.

In general the proposed modelling approach seems to provide results that are coherent with all the tested configuration. The steady state assumption required by the analytical model to converge to a solution does not allow to obtain targeted outcomes for specific and transitional network configurations. However this kind of model is much convenient from a computational and time requirements point of view and still can provide very useful insights on the general behaviour of the system. To support these thesis the comparison of obtained results with the simulator ones presented in chapter 7, has shown that the proposed model manages to provide a fine approximation of the system dynamics with a much smaller time required.

8.2 Future work

The proposed model provides an approximation for a generic free-floating electric MOD system and can be further enriched including additional constraint or strategies to better model possible different dynamics. A list of possible improvements and future studies based on this work is proposed in the following.

- **Capacity of mobility queues:** a more accurate model can provide for the possibility of including limited capacity mobility zones to describe the limited availability of parking spots. This may be particularly useful if a different definition based on smaller areas of city zones is used for example to highlight mobility related phenomenon in smaller specific areas. Introducing capacities as said before, can lead to possible losses and consequently to non product form networks for which the MVA is not applicable. Different strategies may be investigated to take into account capacities with blocking mechanisms and rerouting of customers and without loosing the no losses assumption.

- **Capacity of charging stations:** similarly to what said for mobility zones, capacities may be useful in charging queues especially in scenarios with an high average utilisation of the stations which result in frequent lines forming. Different charging strategies may be studied to limit the accumulation of vehicles at stations including again blocking and rerouting.

- **Multi-hour intervals for routing:** considering trips data from multiple hours intervals may allow to extract routing matrices and demand patterns which still follow typical dynamics of a part of the day. The losses in precision considering a bigger interval of time can be negligible in particular scenarios but the steady state assumption would be more justifiable.

- **Non-exponential distribution of service times:** the exponential distribution assumption for service times may be a stretch in particular for delay zones. Different distributions such as Gaussian or even a constant service time based on average trip times for each zone may be more accurate but would result in a different network type for which MVA can not be applicable. Different network solving strategies may be explored to include queues with non-exponential distributions of service times.

- **Additional charging and relocation policies:** in this work three different charging policies and four relocation after charging strategies have been studied in all their possible combination. However other alternatives may be investigated for example including the possibility for users to end their trip directly into charging stations and possibly connect the EV to the outlet.

- **In depth study of peak-hour impact of charging operations on power grid:** in section 6.2.7 a brief study of the generated energy consumption by the fleet charging operations has been presented. This aspect can be crucial in a scenario in which the number of EVs or their high utilisation result in an high quantity of consumed power. The study of power grid energy profiles is an extremely relevant topic in research nowadays and the impact of an entire EV fleet is significant. New implemented charging policies can therefore taking into account common energy profiles to optimise the energy balance on the grid.

- **Simulation integration:** a more accurate comparison of results would require the integration in the simulation model of policies considered in the analytical one and, on the other hand, the definition of additional parameters in the analytical model to better mimics the reality of a MOD system. New artificial traces for the simulation model can be created starting from fixed routing matrix to also have a consistency in the input data.

- **Environmental, sociological and economical impact:** an important aspect which has only been partially treated in the simulation environment regards the environmental, economical and social dimension of the studied system. A study of total emissions due to vehicles mobility or energy production as well as other metrics related to the general psycho-physical wellbeing such as the noise pollution can be carried out imposing additional input model parameters. The economic aspect can be more easily integrated in the model including bookings tariffs and costs for the installation and management of the system.

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