Politecnico di Torino

Department of Mechanical and Aerospace Engineering

Master's Degree Thesis

Dynamic Behaviour of 3D Printed Periodically Structured Mechanical Metamaterials: Experimental Investigations and Computational Simulations

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And with my special thanks to my family, who supported me in every moment of my life, no matter what the situation might be.

Summary

This thesis project is carried out at the dynamic acceleration and vibration laboratory of INRiM (Istituto Nazionale di Ricerca Metrologica) which is a public research centre and is Italy's national metrology institute (NMI) located at Turin. The research represents an overview of mechanical periodically structured metamaterials as well as a more comprehensive investigation on 2D metamaterials capable of reducing transition of mechanical wave. While most of the previous research projects were focused on designing periodic structures by topology optimisation tools and computational simulations, The objective of this research is to experiment, and Validation of the designed metamaterials and the ones extracted from related research works by the experimental setup which is realized in INRiM research centre.

In the first chapter, a literature review is conducted, a general overview of metamaterials is given, the definition and etymology of the word is discussed, and the main typologies are described by giving a more attention to mechanical metamaterials, especially those used to mitigate mechanical vibrations. To better understand the vibrational behaviour of metamaterials, the definitions related to mechanical vibrations of a material and structures are presented. In this section, Bloch's theorem and Brillouin's zone are described, which form the basis for studies on periodic structures.

In the second part, the instrumentation and the method used for the experiments are presented, the necessary information about the experimental equipment, the fabrication method and the material properties are elaborated, and the designed and experimented samples are presented.

In the third chapter which is the main part of the project, the results of the experiments are presented in detail. Effects of various geometrical shapes in the structure is analysed. Moreover, results of the COMSOL Multiphysics simulation are discussed and a comparison between the metamaterials is carried out.

Finally, the conclusion is elaborated, and some possible applications are proposed.

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Introduction

The scientific world has given metamaterials a lot of attention in recent years because of the promising applications and produced devices with never-before-seen properties, starting with electro-magnetic field, and moving on to acoustic and elastic counterparts more recently. moreover, phononic crystals (PnCs) and auxetic metamaterials are attracting a lot of attention, owing to their abilities to control elastic wave propagation and to have a negative Poisson’s ratio.

Metamaterials have a wide range of applications in physics and engineering Because of their unique properties that go beyond those of natural materials such as properties in wave guiding and filtering. consequently, creation of new metamaterials is drawing increasing interest in the scientific world [1]. Additionally, Periodic structures which are a subset of a larger category of metamaterials becoming more popular owing to the Bloch-Floquet theorem, by which a very complicated issue may be reduced to the “simple” study of a single element, the so-called unit cell [1].

While Photonic crystals are an excellent example of periodic structures in the electromagnetic field, their counterparts for acoustic and elastic waves which are the called phononic crystals recently attracting the attention of the scientists [2][3]. Because of its customizable dynamical features and vast variety of possible applications, Phononic Crystals (PnCs) are increasingly being used as smart materials in structures and microstructures.

In the past years, although so much research has been done in the field of metamaterials, a few of them has focused on dynamic mechanical ones and even if there are some considerable papers published, still so many question marks remain about their behaviour and effectiveness. This clearly reveals the importance of experimental research that can cover a specific type of dynamic mechanical metamaterials.

the start point of this project can be traced back to the papers written by L. D'Alessandro et al. in which they have investigated and designed 3D phononic crystals which was able to result an ultra-bandgap in the experiments [4] to mitigate mechanical vibrations. They have also commercialised their PnCs by exploring some interesting applications such as railway systems or industrial plants mechanical isolation. Considering that INRiM research centre have been involved before in research related to acoustic metamaterials[5] and the results were satisfying, further investigations in metamaterials specially on their dynamic behaviour started focusing on 2D designed structures and is summarised in this thesis paper.

Chapter 1

Literature Review

1.1 Definitions

1.1.1 Metamaterials

The term “metamaterial” comes from the Greek word “μετά” half, meaning “beyond”, combined with the Latin word “materia”, meaning “material”. Rodger M. Walser proposed this word in 1999 to express the desire to create artificial materials whose behaviour, in the presence of external stresses, went beyond the natural limits of composite materials [6]. He described them as “a macroscopic substance with a periodic and synthetic cellular architecture designed to provide an optimum mix of two or more responses to a specific solicitation that is not accessible in nature.” Since then, the notion of metamaterial has continued to grow at an extremely high pace, and the fields of application have expanded in continuously by discovering new features of them.

It is impossible to define a general and unambiguous definition to the idea of metamaterial because it has been reinvented so many times [5]. Ari Sihvola’s publications in University of Helsinki [7] were devoted to collecting and debating these initiatives. He remarked that the distinction between metamaterial and other types of structures is still hazy, with definitions that are sometimes imprecise, ambiguous, discordant, or excessively broad. Not to mention the fact that Sihvola addressed the issue solely in terms of electromagnetic metamaterials, entirely ignoring acoustic and mechanical metamaterials and their applications.

According to Sihvola’s publications, following sentences are present in all definitions of metamaterial and can therefore be considered to characterize them [5]:

1- Metamaterials have properties that are not observable in the materials that constitute them. This is not completely correct, although Their geometry and their arrangement in space in a macroscopic level cause different properties, most of their properties (in this sense in the root of the word we go “beyond” the materials); still are related to the constitute materials.

2- Metamaterials exhibit properties not observable in nature. This statement is much more controversial than the first, because, depending on the point of view from which it is read, it could lead to two objections. First problem with the description is that one might think that metamaterials are somehow unnatural, when in reality, they are materials engineered to respond to a specific function, and engineering is a process carried out (inevitably) by exploiting the laws of nature. second problem is that any artificial material (plastics, alloys, etc.) and many man-made objects have characteristics not observable in nature, and therefore this cannot be a phrase that can be used to define a material

3- The characteristics of the metamaterials are given by the inclusion of small artificial inhomogeneities. It is often specified that the dimensions of the inhomogeneities are small compared to the wavelength of the stress on which they must act. It is therefore placed under the focus how the global response function is directly dependent on the size and shape of the inclusions (which can be, for example cavities), and how a certain homogeneity is necessary, in relation to the length d wave of solicitation, so that metamaterials can be called “material”.

4- Metamaterials have a periodic or reticular structure. Metamaterials therefore have a “molecule” or cell: a fundamental unit that repeats itself in the plane or in space. It has not yet been established whether periodicity is a necessary characteristic, or if it is only an engineering trick: a necessity of order linked to the homogeneity. Sometimes it is not even specified whether, or how much the periodicity has an influence on the desired characteristics of the metamaterial. According to some sources, this feature is not necessary for the definition and explicitly says that it may or may not be [8].

Sihvola, in his analysis, does not express a definition but by his experiments he gives a good idea of what we will talk about and emphasises the fact that the term “metamaterial” refers to a broader idea.

1.1.2 Periodic structures

Periodic structures are only a subclass of more general metamaterials that are made of an infinite or finite repetition of a unit cell in one, two or three dimensions. The Bloch-Floquet theorem's ability to simplify a very complicated issue to the "simple" study of a single element, the so-called "unit cell," is one of the factors contributing to the popularity of metamaterials based on periodic structures. So, it follows that when we discuss metamaterials, we primarily take periodic structures into account [1]. In their applications for mechanical waves the properties are mostly obtained by having hollow shapes inside the base material rather than having a composition of materials. In this case the metamaterial is a periodic structure of one type of material used in a particularly designed unit cell and repeated throughout the structure.

1.2 Typology of Metamaterials

In this section, a brief description and overview of different types of metamaterials will be given and a more comprehensive information regarding the mechanical metamaterials which are the topic of this project will be provided.

1.2.1 Electromagnetic and optical metamaterials

The very first kind of metamaterials appeared to manipulate electromagnetic waves and they are used in so many applications so far. Beside from Resonant metamaterials in the field of electromagnetic metamaterials there are other interesting properties are obtained by inducing bi-isotropies in the structure (as in Pasteur and Telleger media) or strong anisotropies such as, in hyperbolic metamaterials. Another class that is worth mentioning is that of metamaterials that present an artificial chirality, alternating inside them mirror cells. In fact, interesting phenomena such as optical activity and circular dichroism can be obtained from chirality, or devices can be created which, by contrasting the attractive forces of Casimir, could in the future allow the construction of nanomachines.

1.3.3.1 Photonic Metamaterials and PCs:

A photonic metamaterial (PM), also known as an optical metamaterial, is a type of electromagnetic metamaterial, that interacts with light, covering terahertz (THz), infrared (IR) or visible wavelengths.[9] The materials employ a periodic, cellular structure.[10]

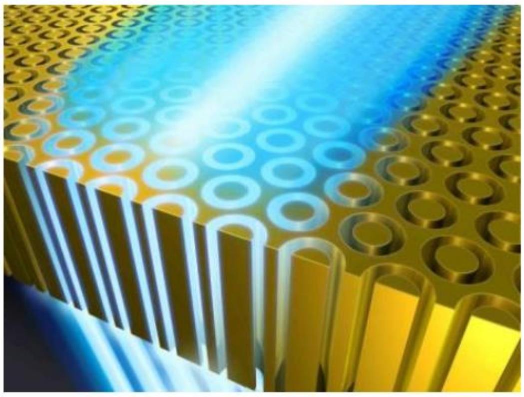


Figure 1 A photonic metamaterial. [11]

Photonic crystals (PCs) are macroporous materials which can demonstrate very interesting properties, especially in the field of optics[12]. They are regular arrangements of matter on a scale roughly equal to the visible light wavelength, or on the order of several hundred nanometers. Photonic crystals affect the propagation of light because they are periodic on the same length scale as light. [13]–[15].

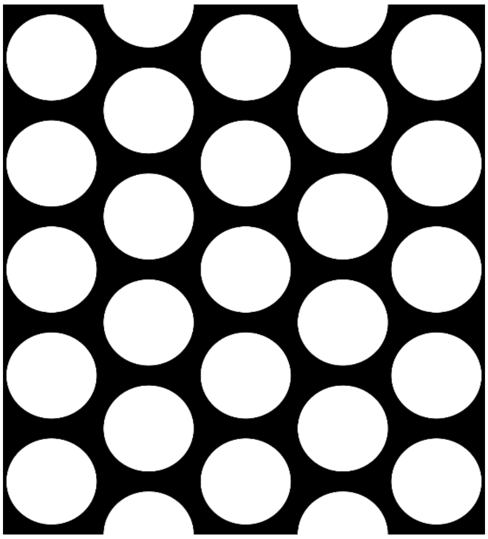


Figure 2 top view of a Photonic crystal. [16]

One-, two-, or three-dimensional photonic crystals can be created and used in the proper application. Layers of thin films can be placed on top of one another to create one-dimensional photonic crystals. While Photolithography or drilling holes in an appropriate substrate can be used to create two-dimensional ones [17]. For three-dimensional ones, fabrication techniques include drilling at various angles, stacking numerous 2-D layers on top of one another, direct laser writing, or, for instance, causing spheres to self-assemble in a matrix and then dissolving the spheres [17].

The difference between photonic crystals and metamaterials is that to have the photonic bandgap the atoms and the lattice constant in PCs must be comparable in size with the wavelength, a≈λ, because the effect of the bandgap arises from diffraction [18].

1.2.2 Mechanical metamaterials:

In recent years, research in this field has been expanding and changing to uncover more intriguing applications. This subcategory of metamaterials is intended to respond to mechanical waves like acoustic or elastic waves or to acquire particular mechanical properties like auxetic properties.

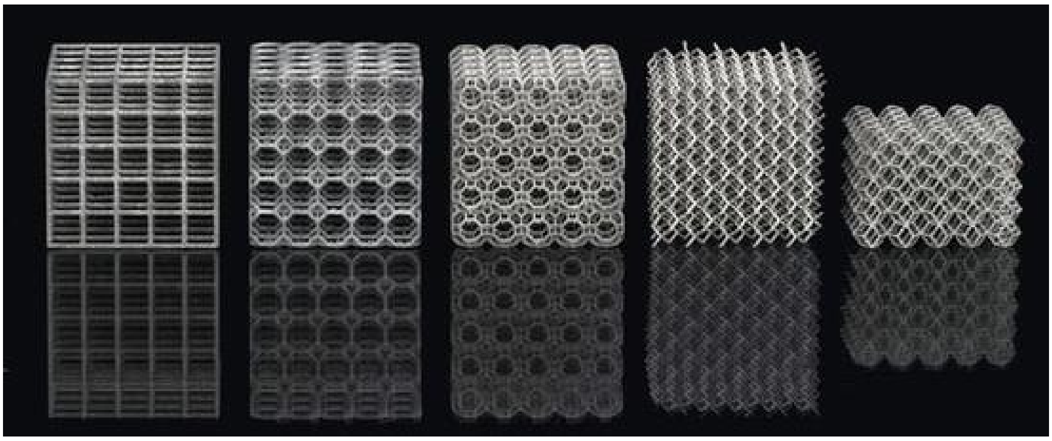


Figure 3 Non Auxetic mechanical metamaterials analysed by de Jonge et al. to compare their mechanical properties.[19]

1.3.3.1 Acoustic metamaterials

Acoustic metamaterials are the mechanical counterparts to the electromagnetic metamaterials. They have grown over the last two decades by using the same concepts and ideas that have previously been observed in the field of acoustics.

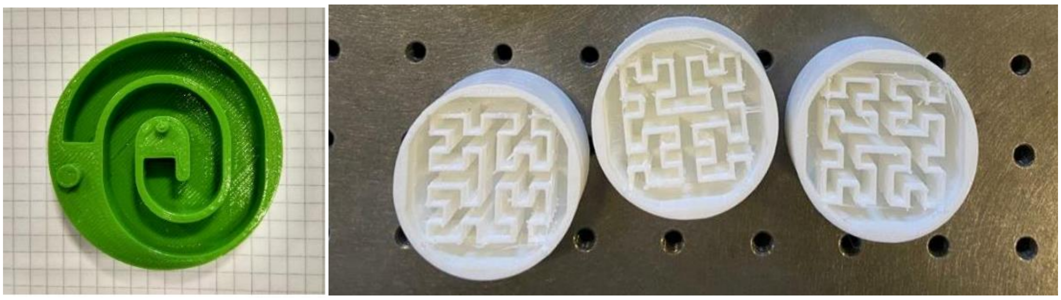


Figure 4 Acoustic metamaterials analysed int the thesis paper of Tomosso Pandolfi conducted in the INRiM research centre [5]

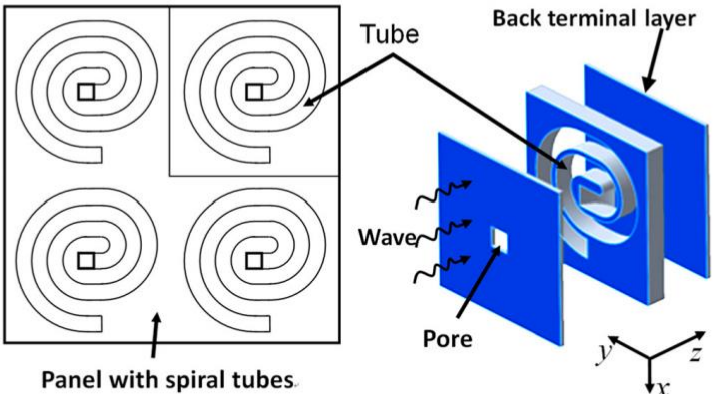


Figure 5 Sound-absorbing metamaterial panel proposed by Cai et al.[20]

1.3.3.2 Phononic Crystals (PnCs):

Phononic crystals (PnCs) are artificial periodic structures with the ability to effectively change the propagation of sound, acoustic waves, or elastic waves [3]. While metamaterials are devoted to sub-wavelength structures, the material behaves more like a phononic crystal when the size of the design units rises, consequently macro arrangement becomes more important in defining the properties [21] There is currently no agreed-upon differentiation between PnCs and metamaterials. Metamaterials, according to some researchers, are artificially structured media that exhibit unique acousto/elastic properties over wavelengths that are significantly longer than their periodicities. PnCs are included in some extensions of the notion of metamaterials [22].

PnCs were introduced about twenty years ago and have gained increasing interest since then, both because of their amazing physical properties and because of their potential applications [3].

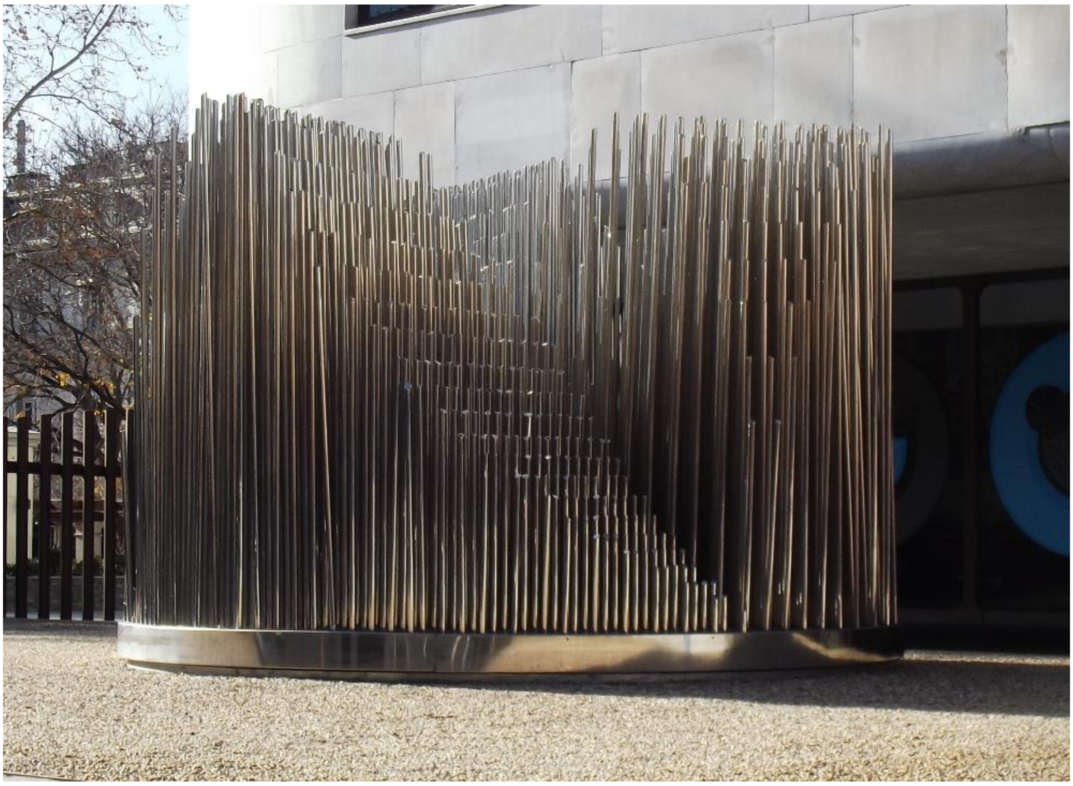


Figure 6 The artwork "Órgano" by sculptor Eusebio Sempere is large-scale example of a phononic crystal: it consists of a periodic array of cylinders in air (the 'metamaterial' or 'crystal structure') and its dimensions and pattern is designed such that sound wav s of specific frequencies are strongly attenuated. It became the first evidence for the existence of phononic band gaps in periodic structures.[23]

In phononic crystals the transmission of acoustic and elastic waves is inhibited by set of crystals, which are periodic structures with frequency ranges called bandgaps [24]. The geometry, topology, and constitutive material properties of a PnC's unit cell determine its bandgap width, frequency level, modal location, and effective direction or isotropy [25]. To attain certain bandgap factors such as bandgap at low frequency range, or maximum relative bandgap width, or the basic features of the unit cell of a PnC can be tuned by means of shape, material, and mechanical characteristics [25].

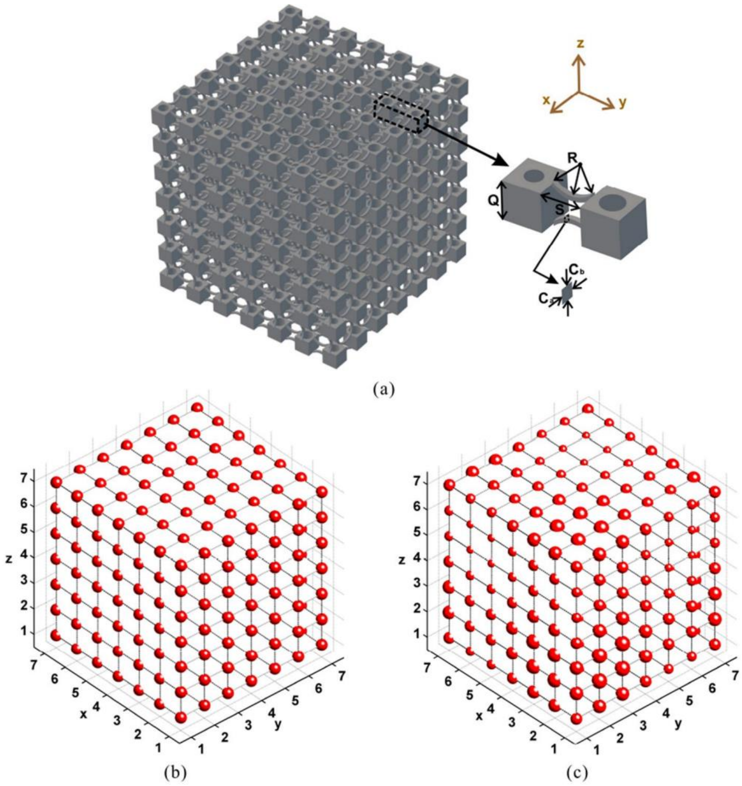


Figure 7 Design of the investigated 3D PnC by H.Meng et al.[22]

In another words, Phononic crystals (PnCs) and metamaterials can be considered as engineered materials with designed macroscopic periodic or non-periodic structures that exhibit acousto/elastic responses not achievable in natural materials [24][22] .

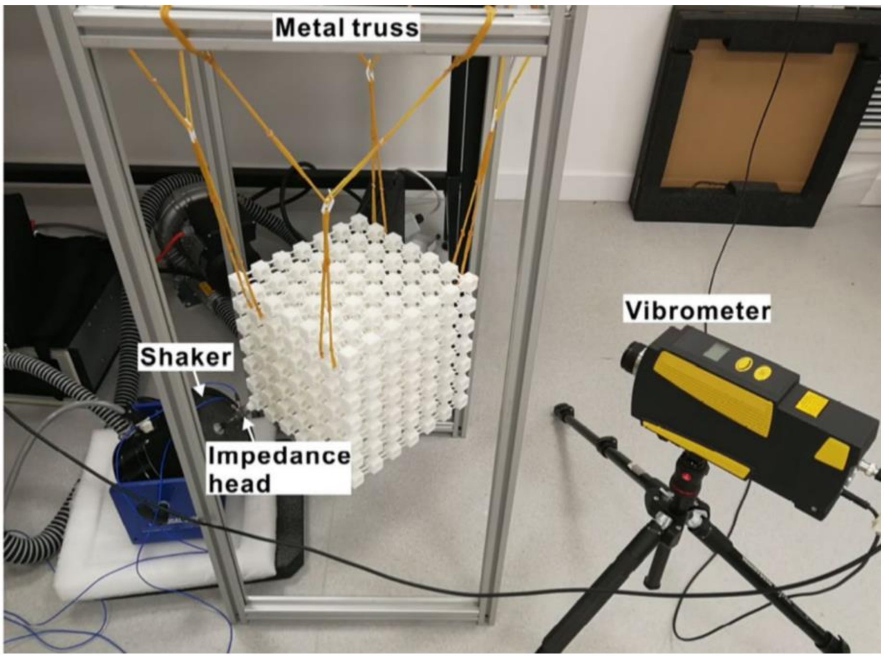


Figure 8 experimental setup for PnCs that H.Meng et al used in their research. [22]

PnCs with periodic structures can be considered as mechanical analogue of the previously mentioned photonic crystals in optics and have been studied since the pioneering work of Lord Rayleigh in 1872. However, PnCs have only recently attracted a lot of attention due to the fact that their design can be supported by advanced optimizers with fast computing performance and cutting-edge additive manufacturing (3D-printing) techniques, which offer significant manufacturing freedom for the creation of complex parts and geometries. [22]. From a structural viewpoint, PnCs are an arrangement of many elementary cells, each containing an inclusion or a cavity inducing non-conventional dispersion properties [26].

1.3.3.3 Metamaterials and Phononic crystals (PnCs) for energy harvesting

In recent years, the applications of metamaterials and PnCs have been extended into the field of energy harvesting. to mitigate undesired vibrations/noise and convert them into electrical energy a direct integration design strategy can be used which yields a multifunctional system as a power converter by means of widely distributed micro-electromechanical systems [27].

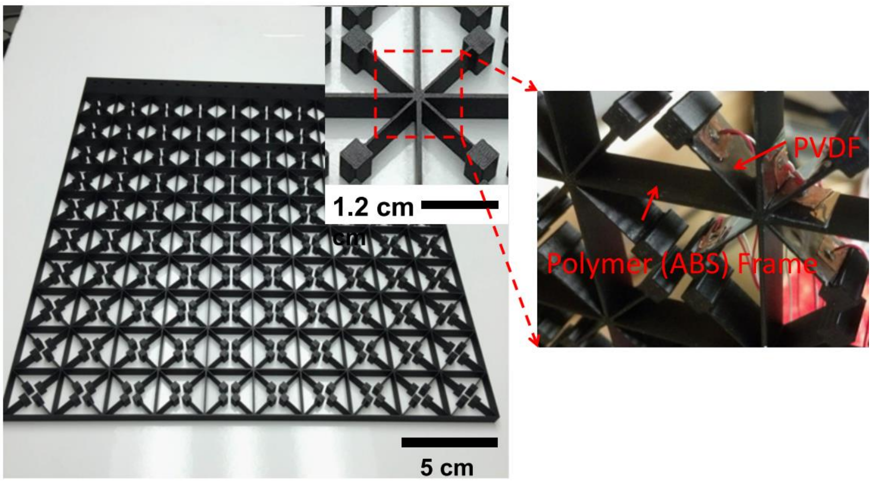


Figure 9 Mechanical metamaterial designed by Ying Li, Evan Baker, Timothy Reissman, et al. for vibration isolation and energy harvesting the surface of the cantilevers are coated with PVDF films to convert kinetic energy into electric energy.[28]*. this design in also experimented in our research only for mechanical isolation.*

Moreover, the defect state mode of metamaterials/PnCs can localize the energy with an amplification effect, which has a great potential for improving energy harvesting efficiency. In addition, through tailoring the refractive index profile of metamaterials/PnCs, the wave focusing phenomenon can be realized to boost the energy harvesting efficiency over a wide frequency range.[29]

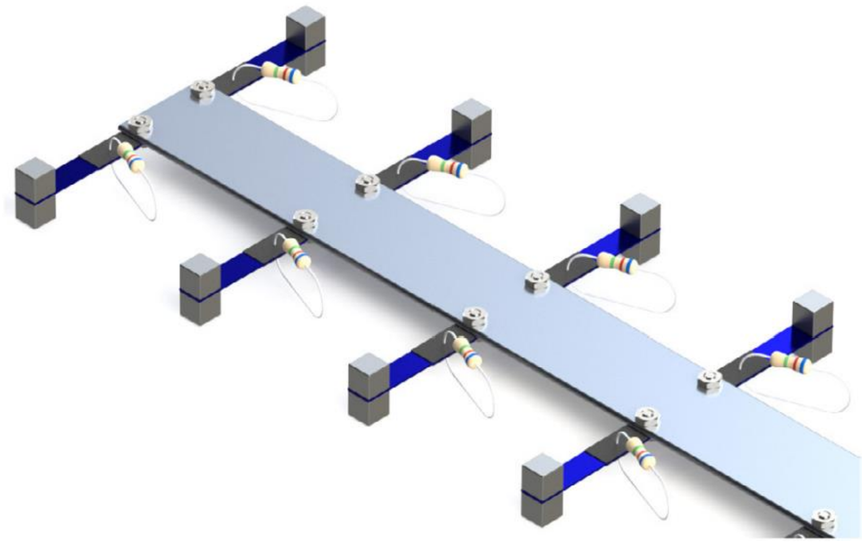


Figure 10 Schematics of the mechanical locally resonant energy harvesting metastructure. small cantilever beams with tip masses act as mechanical resonators attached to the primary beam structure, and piezoelectric elements with a resistive load are bonded to the mechanical resonators to serve as energy harvesters. [30]

A generalized energy harvesting process is to generate electrical energy from its surroundings using special conversion mechanism, environmental energy sources may include kinetic energy in the form of vibrations and noises, electromagnetic radiation, thermal energy, and so on [31].

1.3.3.4 Auxetic metamaterials

Due to their unique microstructures, auxetics are appealing structures with a negative Poisson's ratio. Poisson's ratio is defined as the negative ratio of transverse strain to axial strain from a mechanical standpoint. When auxetic structures are pushed (compressed), they grow (shrink) in the transverse direction[32],[33].

Figure 11 Auxetic metamaterial designed and experimented by Hamzehei et al. [34]

Auxetic structures have grown in popularity over the last three decades as a result of their unique qualities such as high fracture toughness, high strength, improved energy absorption and dissipation, indentation resistance, rising shear modulus, and correct acoustic behavior [34].

1.3.3.5 Dynamic mechanical metamaterials

Mechanical metamaterials or artificial materials that have been intricately engineered to generate anomalous apparent mechanical performance, have been intensively studied in recent decades [35], [36].

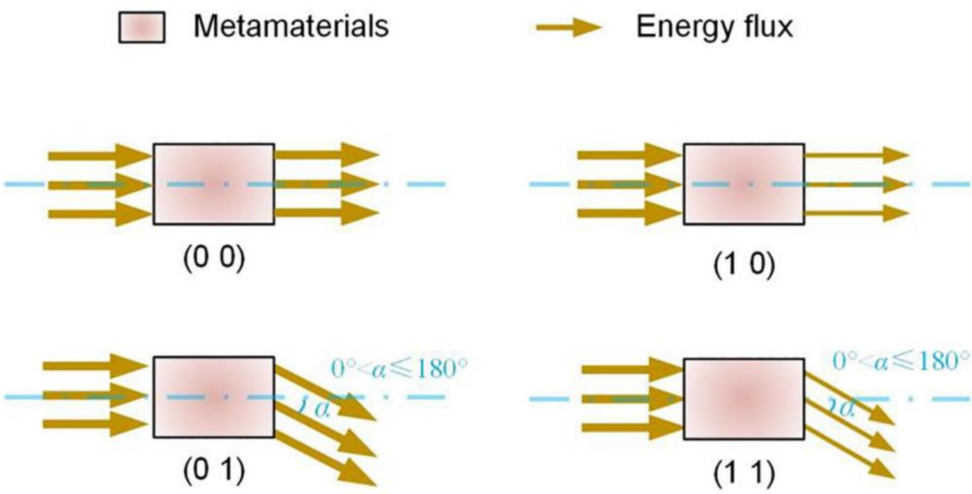


Figure 12 Four categories of dynamic mechanical metamaterials according to the manipulation of magnitude and direction of energy flow.[37]

Unusual static behaviours (e.g., negative thermal expansion, negative Poisson's ratio, and so on) are combined with extraordinary dynamic behaviours (e.g., acoustic wave transmission with expected bandgaps or propagation paths [38]–[41], a unique combination of stiffness, strength, and energy absorption performance [42], full-band mechanical vibration isolation [43], and other behaviours).

The apparent energy flow has two key aspects as a vector field: magnitude and flow direction. It is feasible to characterize dynamic mechanical metamaterials based on whether the apparent energy flow amount and/or direction are modified.[37]

1.3 Review of quantum mechanics

Consider the case of an elastic and isotropic material [44]. By elastic material, we mean a material such that free energy ψ and the first Piola tensor P they can be constitutively written as a function of the strain gradient alone F =∇χ, or:

$$\psi = \hat{\psi}(F)$$

$$P = \hat{P}(F)$$

A constitutive law (and its representation as a constitutive equation), by definition, must satisfy the principle of objectivity and some strong thermodynamic restrictions (Colemann-Noll procedure [45][46]). Thermodynamic restrictions, in the wake of the second law of thermodynamics, together with reasoning regarding polar decomposition, lead to define:

$$P = 2F\frac{\partial\psi(C)}{\partial(C)}$$

Where is it C.is the right Cauchy-Green tensor which, in Cartesian coordinates and not using the covariant formalism, is expressed by $C = F^T F$ The definition of isotropic material, on the other hand, is linked to the concept of material symmetry [44] for an isotropic material each (proper) rotation is a symmetry transformation, i.e. the rotations of the material are orthogonal matrices:

$$\varsigma = Orth+= \{all\, rotations\}$$

This is equivalent to saying that isotropic material is a material whose properties are the same in every direction. By definition of Piola tensors, we can define the Cauchy stress tensor (more widely used in structural mechanics):

$$\sigma = 2\frac{1}{J}F\frac{\partial\widehat{\Psi}(C)}{\partial C}F^T, \qquad J = detF$$

Finally, in the case of non-homogeneous material, they are defined at each point "r" a set of parameters that describe its specific characteristics.

1.3.1 Bloch's theorem

Consider a self-added linear operator in real spacer-space:

$$\mathcal{L}A_\lambda(\gamma) = \lambda A_\lambda(\gamma)$$

With $\mathcal{L} = \mathcal{L}^+, \lambda$ eigenvalues, and $A_\lambda(r)$ eigenvectors. We define the translation operatorT.R.as:

$$T_R f_{(r)} = f(r + R)$$

where $R = n_i a_i$, $i \in \{1,2,3\}$, with n_i integers and a_i non coplanar. The set of discrete vectors R it defines itself as "lattice sites". Suppose the two operators switch:

$$[\mathcal{L}, T_R] = 0$$

These two operators share the same eigenfunctions, but a priori we cannot say anything about the eigenvalues. For this we introduce:

$$b_1 = \frac{2\pi}{V} a_2 \times a_3 b_2 = \frac{2\pi}{V} a_3 \times a_1\ b_3 = \frac{2\pi}{V} a_1 \times a_1,$$

$$V = a_1 \cdot a_2 \times a_3$$

We define $K = m_i b_i$, $i \in \{1,2,3\}$, with m_i integers and b_i not coplanar, called "reciprocal lattice vector" (k-space) and it appears that:

$$e^{jK\cdot R} = 1$$

The Bloch-Floquet theorem states that the eigenfunctions of $\mathcal{L}$ such that:

$$A(r+R;k) = e^{-jR_2k}A(r;k)$$

where the k is defined as the Bloch wave vector. Therefore $A(r;k)$ is pseudo periodic in r-space.

Having assumed the existence of a latex in r-space automatically implies the existence of a lattice in k-space (having correctly defined the functional spaces involved). This analysis can be applied directly to displacement $u(r,t)$ the displacement must satisfy Bloch's theorem, in the sense that:

$$u(r,t) = e^{j(ker-\omega t)}\sum u_k(k)e^{jk\cdot r}$$

The study of waves in 2D non-homogeneous isotropic elastic media thus defined is generally based on the study of dispersive waves: the waves separate into their components as they pass through the medium and the phase/group velocity depends on the frequency.

1.3.2 Brillouin zone:

Having introduced K And "k-space ", we define the first Brillouin zone as a primitive cell uniquely identified in a reciprocal-space. In the case of a square lattice (or more generally rectangular), the first Brillouin zone is represented in the figure 13.

In the specific case 2D, the Bloch wave vector is written in components $k = (k_x, k_y)$ like in the Figure 14. These primitive units contain all the crystallographic characteristics of the structure and therefore the characteristics that are periodically conserved throughout the lattice. specifically, by symmetry, the region identified by $|k_i| \leq \frac{\pi}{a}$ is the first zone of Brillouin. In turn, we define the Brillouin irreducible zone [2] as the region between the points Γ-X-M, which are respectively the centre, the midpoint, and the vertex of a side. Ultimately, data kx, ky from the first Brillouin zone, it all boils down to studying the new dispersion relation:

$$f(\omega, k_x, k_y) = 0$$

starting from the components x, y of the wave vector referred to the “reciprocal lattice”. By choosing a parameter k in function of which the components of the wave vector are defined, we consider the case in which $k \in [0,3]$: the explanation of the choice of this interval derives precisely from the definition of the Brillouin irreducible zone, since:

$$k_i = [\varkappa_{i-1}, x_i] \Leftrightarrow [\Gamma_{i-1}, \Gamma_i] \; i \in \{1,2,3\}, \; \Gamma_i \in \{\Gamma, X, M\}$$

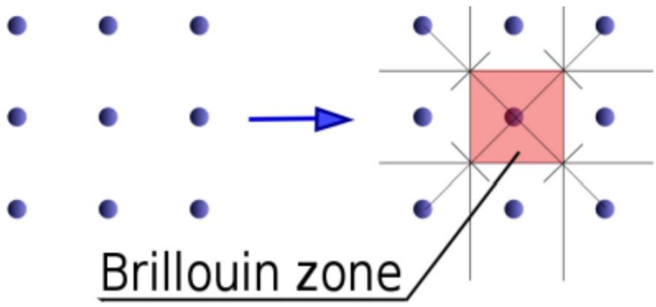


Figure 13 representation of the first Brillouin zone.[47]

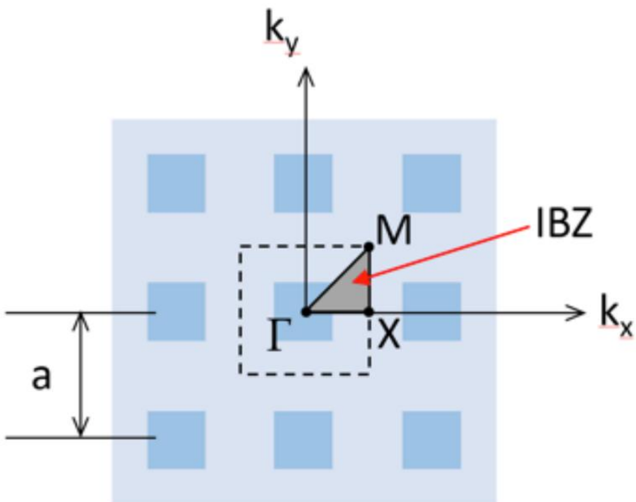


Figure 14 Representation of the wave vector K.[47]

with periodicity for Γ. All the wave vectors of the Brillouin zone can be obtained from those of the irreducible zone. So, each unit interval is associated with a particular path (i.e., $M - \Gamma,\; \Gamma - X,\; X - M$).

1.4 Review of mechanical vibrations:

In this section it is helpful to mention some formulas and definitions in mechanical vibration to better calculate the behavior of a metamaterial or structure.

1.4.1 Frequency response functions:

A Frequency Response Function (FRF) is a function used to quantify the response of a system to an excitation, normalized by the magnitude of this excitation, in the frequency domain.[49]

Bellow some of the most used FRFs are presented by their formula. [50]

Accelerance= Acceleration/ Force	**Apparent Mass= Force/ Acceleration**
Mobility= Velocity/ Force	**Impedance= Force/ Velocity**
Receptance= Displacement/ Force	**Dynamic Stiffness= Force/ Displacement**

Table 1 formulas of most common FRFs.

1.3.3.1 Mechanical mobility and impedance:

The response of a structure to a harmonic force can be expressed in terms of its mobility or impedance.

Mobility is a tensor (or tensor component) which operationally describes the effects upon the resultant velocity (or several velocities) of the application of a force or an array of forces. The concept of mobility can be represented by the matrix equation: V=MF.

where V is a column vector of resultant velocities, F is a column vector of applied forces, and M is a symmetric tensor of mobilities. [48]

$$\text{Mobility}=\frac{V(j\omega)}{F(j\omega)}$$

Impedance is a tensor (or tensor component) which operationally describes the effects upon the resultant force (or several forces) of the application of a velocity or an array of velocities. The concept of impedance can be represented by the matrix equation F=ZV.[48]

$$\text{impedance}=\frac{F(j\omega)}{V(j\omega)}$$

1.4.2 Transmissibility:

Transmissibility in the field of mechanical vibrations cab be defined as the ratio of output to input. More specifically, it is the ratio of force transmitted to the force applied [51]. The force that is being transmitted to the framework or the body of a certain system is referred to as the transmitted force. The external force, also known as the applied force, is what first generates and transmits the force. [51], [52].

1.3.3.1 Acceleration transmissibility and transmission loss:

Transmissibility or Transmission loss (TL) in general "represents the accumulated drop in intensity of a waveform energy as it propagates outwards from a source, via a specific region, or through a certain type of structure" [53].

It's a phrase that comes up a lot in optics and acoustics. In the manufacturing of acoustic equipment such as mufflers and sonars, TL measurements are critical. Measurement of transmission loss can be in terms of decibels.

Mathematically, transmission loss is measured in dB scale and in general it can be defined using the following formula [53]:

$$TL = 10\, log_{10} \left|\frac{W}{W_t}\right|\ dB$$

where:

$|W_i|$ is the power of incident wave coming towards a defined area (or structure)

$|W_t|$is the power of transmitted wave going away from the defined area (or structure).

Chapter 2 :

Instrumentations and Methods

2.1 Experimental setup:

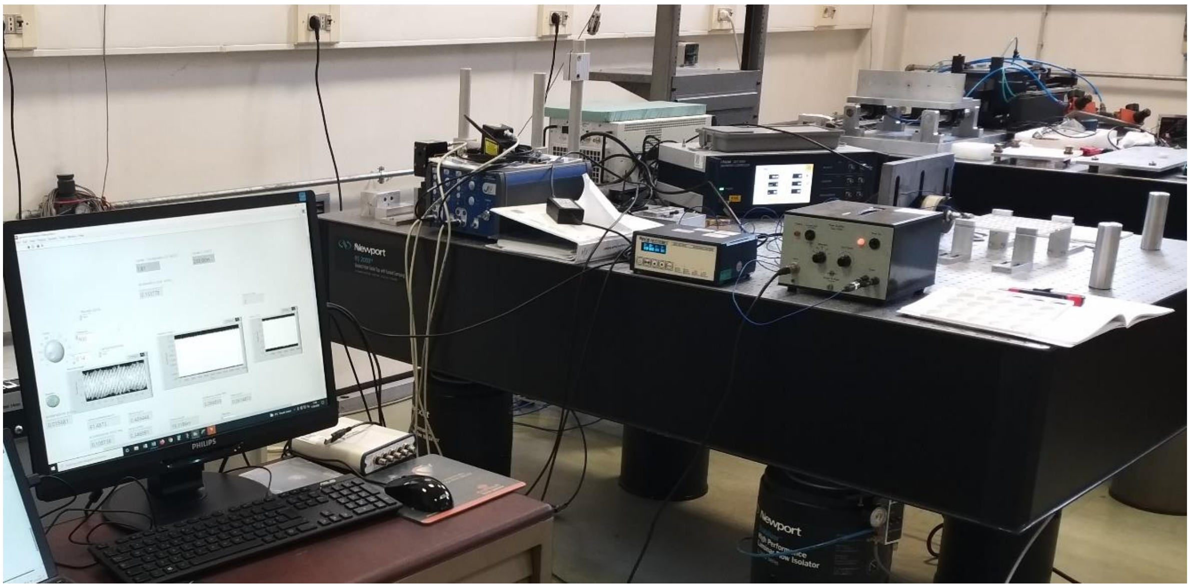


Figure 15 Experimental Setup and Equipment.

In this experimental setup the shaker, which is connected to power amplifier and signal conditioner, exerts the vibrational force and frequency in a point on the horizontal side of the piecework that is hold by the fixtures. From the opposite side the sensor laser is directed to the specimen to measure the output of the signal. The sensor and the controller which are connected to each other along with signal conditioner transfer the data to “data acquisition system” and then the computer to be demonstrated and evaluated by the NI software.

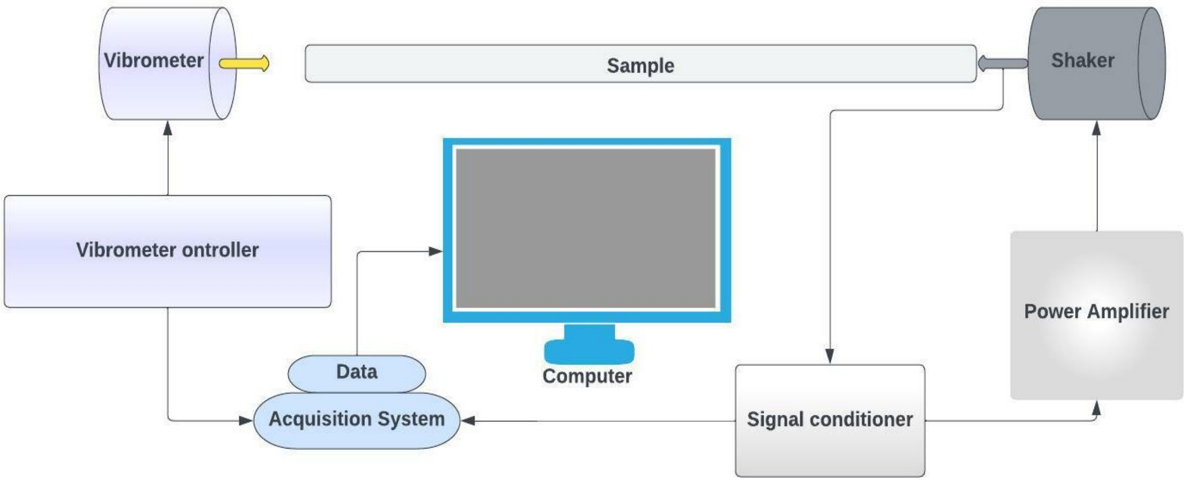


Figure 16 Schematics of the Experimental Setup

2.1.1 Experimental devices:

In the Following a brief description of the devices and their specifications are presented. The information is taken from the products' official catalogues.

1.3.3.1 Vibrometer controller and sensor

In this project the OFV-5000 Controller which is capable of giving a frequency range from near DC to 24 MHz, with velocities up to ±10 m/s and displacements from the picometer to meter range in combination with an interferometric sensor head OFV-505 which is an ultra-precise workhorse for non-contact vibration measurement is used.

Figure 17 Vibrometer controller used in the experiments

OFV-5000 controller contains four slots: two for velocity decoders, one for the displacement decoder and one additional auxiliary slot for a velocity or a displacement decoder.

Figure 18 Laser vibration sensor used in the experiments

1.3.3.2 Power amplifier (Bruel & Kjaer 2706):

The Bruel & Kjaer 2706 Power Amplifier (10 Hz - 20 kHz, 75 VA) has been designed to drive small vibration exciters. In this project it is used to drive the Mini-Shaker Type 4810. For this application, the maximum output current should be limited to 1.8 A. The power amplifier has a flat frequency response from 10 Hz to 20 kHz (± 0.5 dB). The power output capability is 75 VA into a 3 Ω exciter or resistive load and the maximum voltage gain is 40 dB. This enables the power amplifier to be used in acoustical measurement set-ups, even when third-octave narrow band noise is employed.

Figure 19 Bruel & Kjaer 2706 power amplifier

1.3.3.3 Shaker (Bruel&Kjar 4810):

Mini-shaker Type 4810 is of the electrodynamic type with a permanent field magnet. It is well-suited as the force generator in general vibration tests, mechanical impedance and mobility measurements, and experimental modal analysis where only smaller force levels are required. It can also be used in the calibration of vibration transducers, both to determine their sensitivity, by comparison with a standard accelerometer, as well as their frequency response, up to 18 kHz.

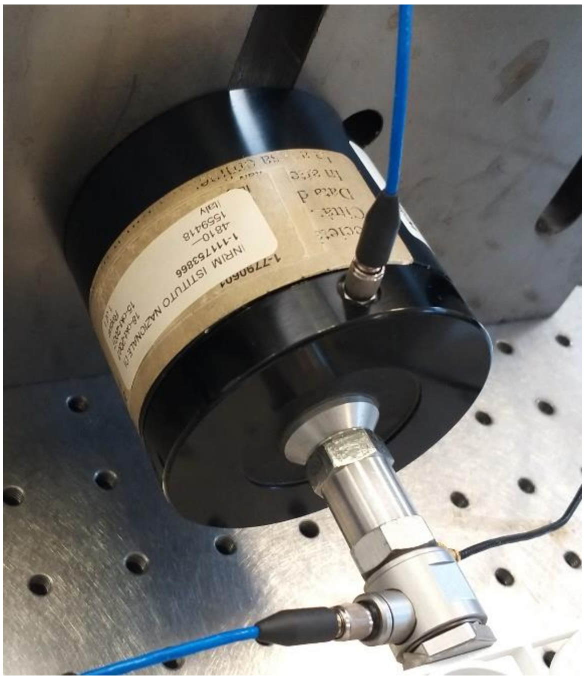


Figure 20 Bruel&Kjar 4810 shaker

1.3.3.4 Signal conditioner (PCB piezotronics 482C):

Signal conditioner which include amplification, filtering, converting, range matching, isolation and any other processes required to make sensor output suitable for processing after conditioning. In this thesis project is the 482C series with 4-channel benchtop signal conditioners that range from units with simple stand-alone operation to more complex units with front panel keypad / display, RS-232, or Ethernet control.

Figure 21 signal conditioner used in the experiments

1.3.3.5 Data acquisition system

Acquisition system consisting of a signal conditioner and a National Instruments high-precision USB-4431 analog I / O module at 16 Bit.

The NI USB-4431 is a five-channel dynamic signal acquisition module for making high-accuracy measurements from IEPE sensors. The USB-4431 delivers 100 dB of dynamic range and incorporates software-selectable IEPE (2.1 mA constant current) signal conditioning for accelerometers and microphones. The module consists of four analogue input channels for reading from IEPE sensors with a single analogue output. The four analogue input channels simultaneously acquire at rates from 2 to 102.4 kS/s. In addition, each channel includes built-in antialiasing filters that automatically adjust to your sampling rate. The USB-4431 is ideal for a wide variety of portable test applications such as frequency response audio tests or suspension shaker tests.

Figure 22 data acquisition system used in the experiments

1.3.3.6 3D printer:

The instrumentation used for the realization of the specimens and the measurements at the acoustic laboratories of INRiM will now be discussed.

Figure 23 MakerBot Replicator 2: samples printed by this model of 3D printer

The 3-D printer used is a MakerBot Replicator 2, a professional fused deposition modeling machine of medium to high end quality. The declared resolution of the machine is 0.1 mm, with a positioning accuracy of the nozzle of 11 microns in the XY plane and 2.5 microns along the z axis. The machine is fed with filaments of thermoplastic material having a diameter of 1.75 mm which is melted and extruded through a 0.4 mm nozzle. The polymer used to make the specimens is polylactic acid or PLA (density: 1.210- 1.430 g cm^{-3}, fusing temperature: 150-160 ° C) which is sold by the printer manufacturer in rolls like the one in the picture.

1.3.3.7 Fixture

In order to prepare this setup, first a fixture for the metamaterial was required to keep it in the proper position and be adjustable for different dimensions (even though we maintained same dimensions for all the structures considering the tolerances and probable change of dimensions is essential). The metallic fixture which is manufactured by machining processes at INRiM machining center and it consists of:

- two side components which are attached from one side to the specimen and from bottom side to the isolator table. They can provide a fully constrained boundary condition in case it is needed.
- one component to hold rigidly the shaker in the proper position and height of the shaker is adjustable.

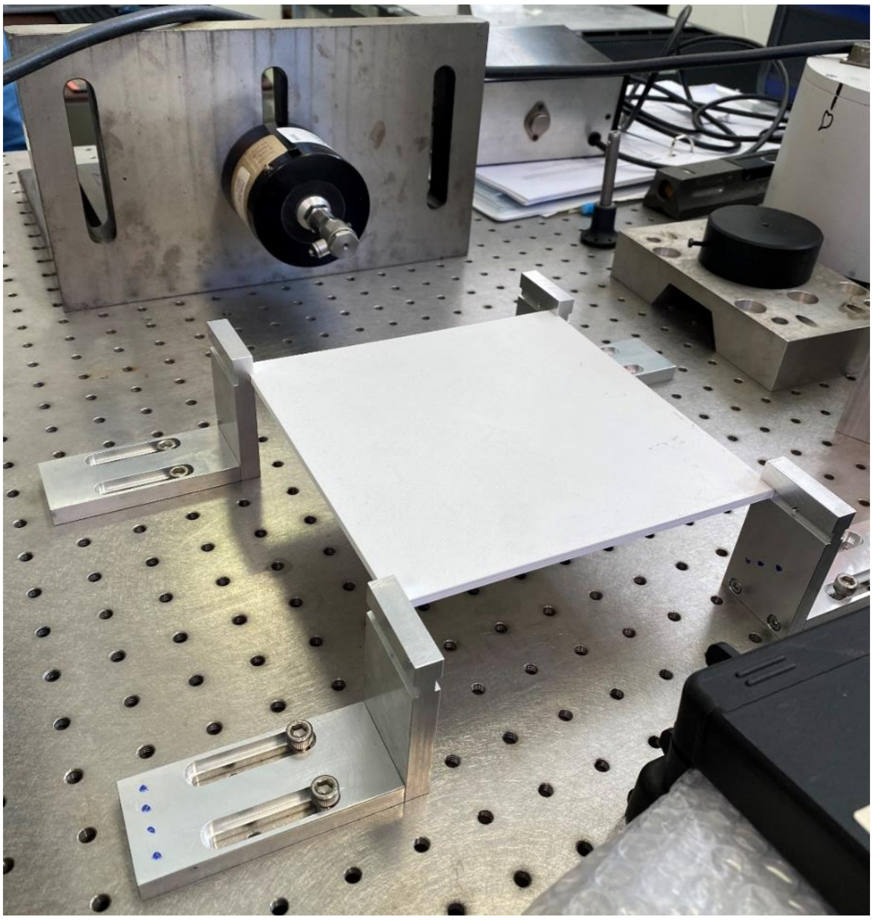


Figure 24 the simple fixture used to hold the specimens

2.2 CAD models:

Set of metamaterials (more specifically can be referred as 2D Phononic crystals) consist of:

- A model representing the metamaterial designed by Ying Li, Evan Baker, Timothy Reissman, et al. [28] but in a replication of five in each direction
- One metamaterial with circles of diameter 32.1 mm inside a PLA plate.
- Two sets of metamaterials which both are part of a specimen with same dimensions but number of periodicities is changing, here also the material consist of a higher free space and around the circles with a thickness of 1 mm are hollow, it is to investigate the effect of void to material ratio.
- By a different categorization we can say, we also designed 2 sets of metamaterials that have same circular shape but in one of them theses circles are inside a full plate

and in the other one also the space around the circles is free. This test will give a clearer view of the effect of making a metamaterial lighter in weight.

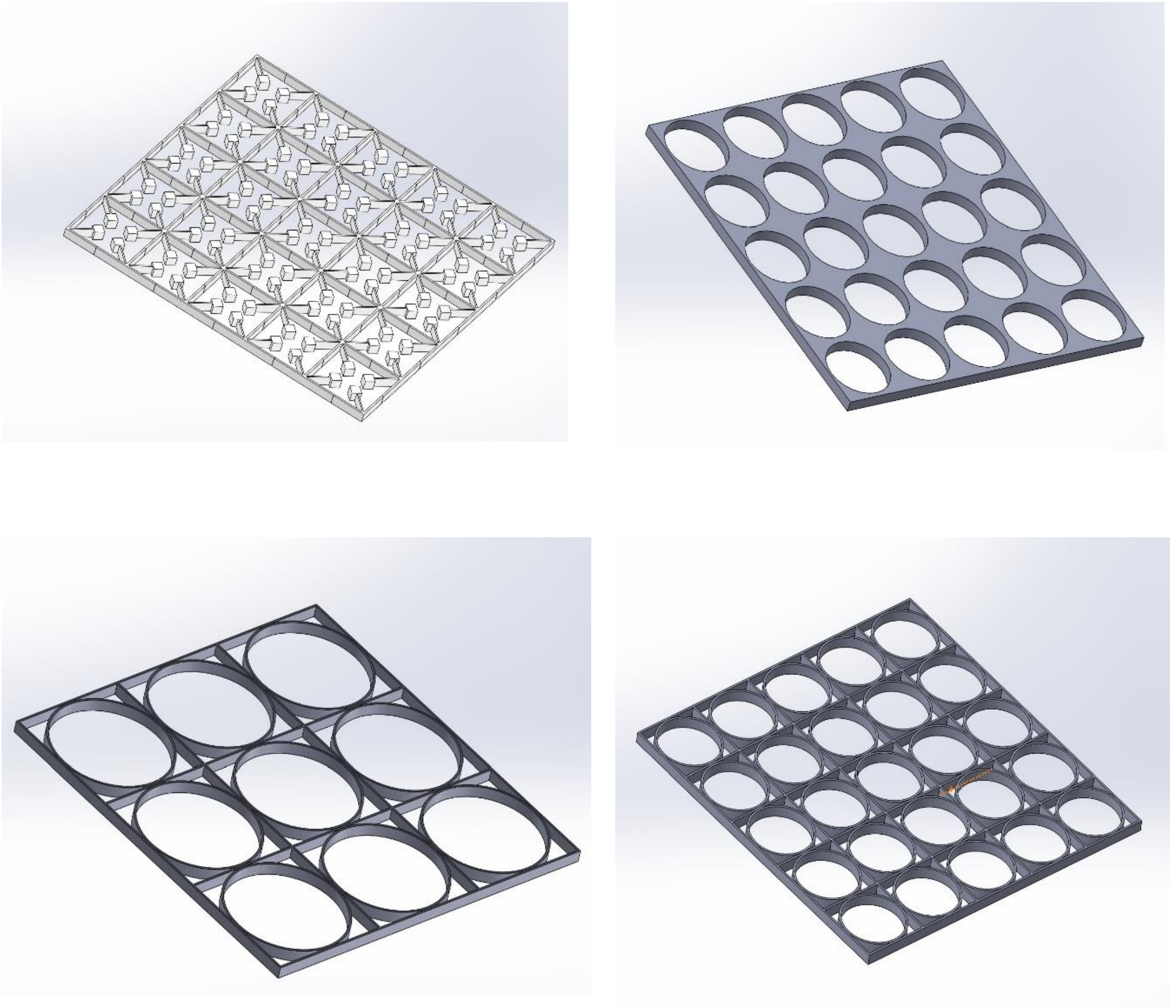


Figure 25 analysed structures during the research, metamaterial 1 is on the top left side, metamaterial 2 is on the top right, metamaterial 3 bottom left and metamaterial 4 bottom right side.

- Dimensions of all the structures are 176.6*176.5 mm with a thickness of 5 mm.
- Totally four sets of metamaterials in different; two of the metamaterials are designed during this research.
- To easily reference the metamaterials, it is decided to call them by the following table with just naming them by numbers.

Metamaterial 1	Metamaterial 2	Metamaterial 3	Metamaterial 4
From the paper of Ying Li, Evan Baker, Timothy Reissman, et al.[28]	Full Plate with 5 holes in each direction	Plate with 5 holes in each direction and hollow space around them	Plate with 3 holes in each direction and hollow space around them

Table 2 specimens represented by their assigned names for ease of referring.

- Ratio of the density between metamaterial 3 and 4 is 0.64. it means that metamaterial 3 is 26 percent lighter in weight with the same volume of base material.

Chapter 3 :

Results & Discussion

3.1 Experimental Results

The experiments are carried out in different conditions and repeatedly if needed (exactly the same experiment).

For all the materials the experiment is repeated in y direction free (only supported) boundary condition in which the material is free in horizontal movements in y direction and in clamped boundary condition in which the material is constrained both in vertical and horizontal movements in all the directions except vibrating axis.

The plots bellow summarizes the experiments performed and demonstrate the key performance indicators of the metamaterials for each metamaterial in both boundary conditions three tests with different amplifying is performed (0.1,0.2,0.3) and in the plots below two of the most reasonable results are depicted in transmissibility plots.

All the data are extracted from LabView NI software to Excel where other related parameters such as displacements and velocity are calculated then the data are imported in MATLAB software for further calculations and execution of the plots.

3.1.1 Plane plate:

After assembling the experimental setup, first a full flat plate of PLA is experimented. Consequently, as expected for a simple plate without any periodic structure the result demonstrates a pick transmissibility at resonant frequency. This frequency is almost same as the result obtained by COMSOL natural frequency analysis. this gives us a better idea of the base behaviour of our PLA material in the specific condition.

According to the results, as expected it is obvious that the transmissibility in the best case is like a simple spring mass system. The material provides some damping and also the effect of clamping prevents waves transmission up to a certain point, however, looking at the mobility plots reveals that for frequencies about less than 1500 Hz even the output mobility is higher than input.

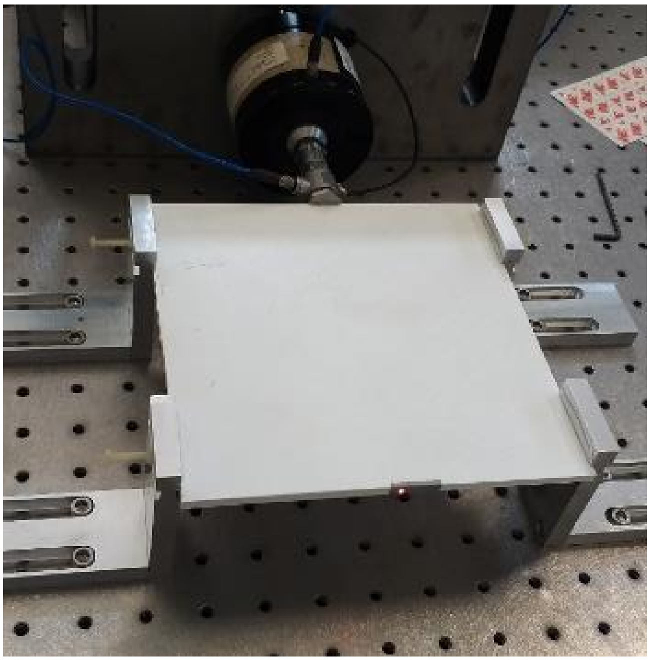


Figure 26 PLA plate clamped in the fixture during the experiments

Furthermore, at the first resonant frequency a higher transmissibility than three is seen that will be compared later with the results obtained in metamaterials.

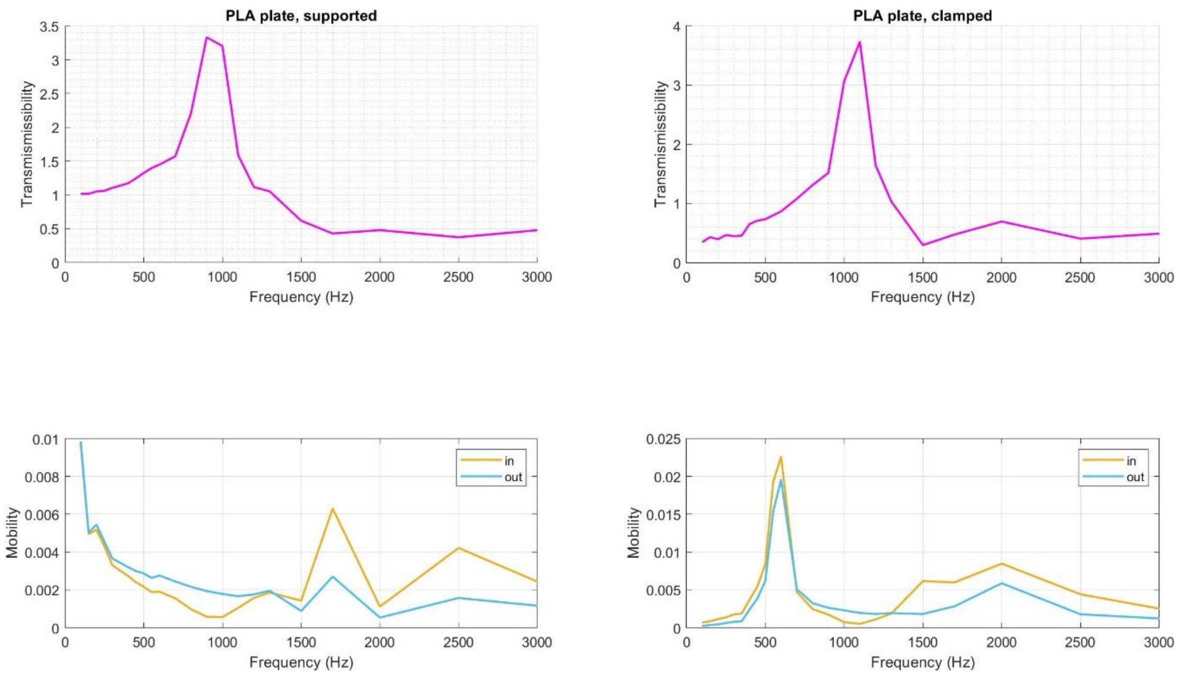


Figure 27 Experimental results of a PLA plate without any macroscopic voids

3.1.2 Metamaterial 1:

This metamaterial that Ying Li, Evan Baker, Timothy Reissman, et al.[28] are experimented to obtain it's energy absorption behviour in our experiments demonstrated a good performance and almost similar to their results. The result is not exclty same because of different material (in their experiments ABS is ued) and printing condition.

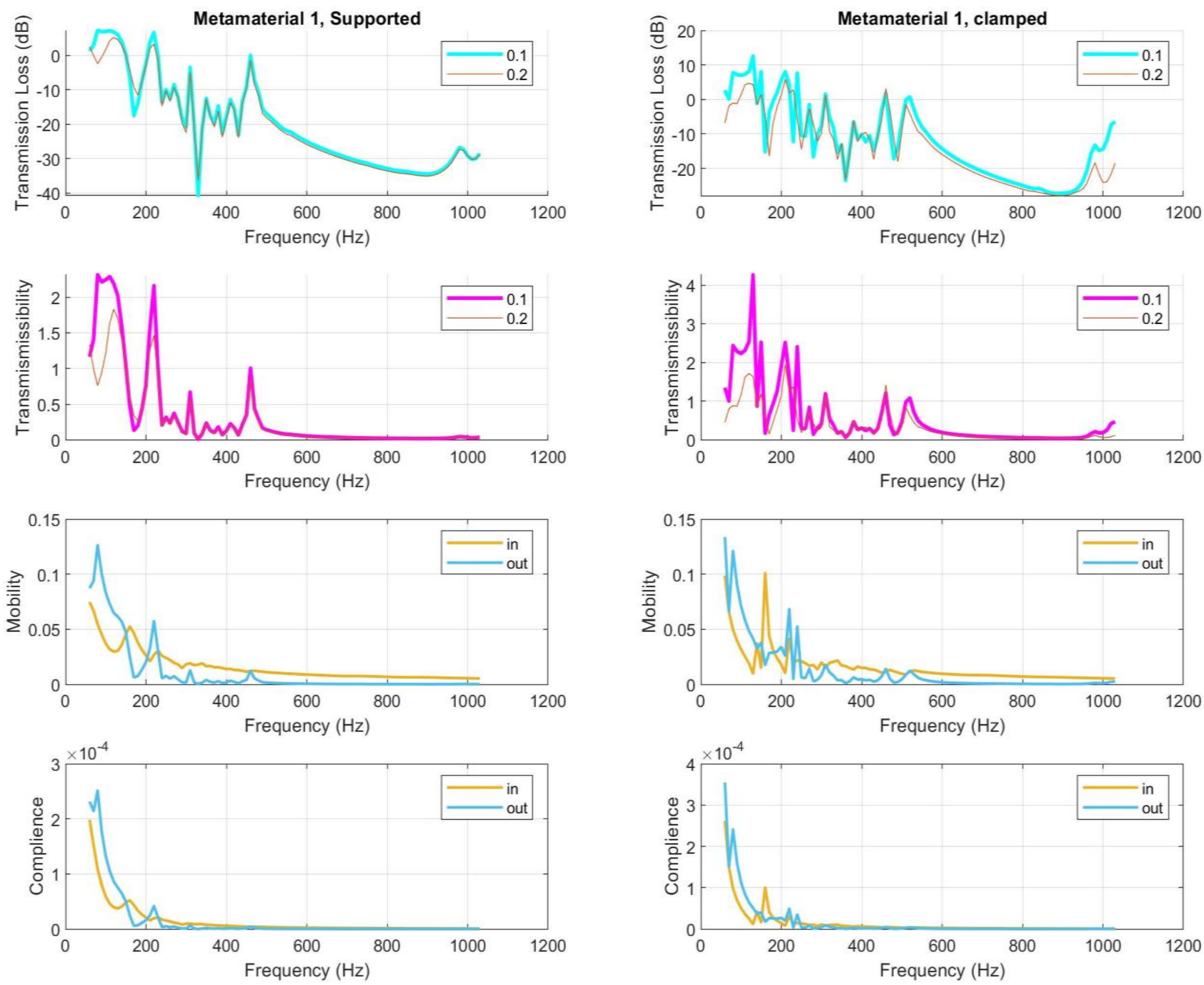


Figure 26 Experimental results for metamaterial 1 represented as Transmissibility, TL, Mobility, and Compliance

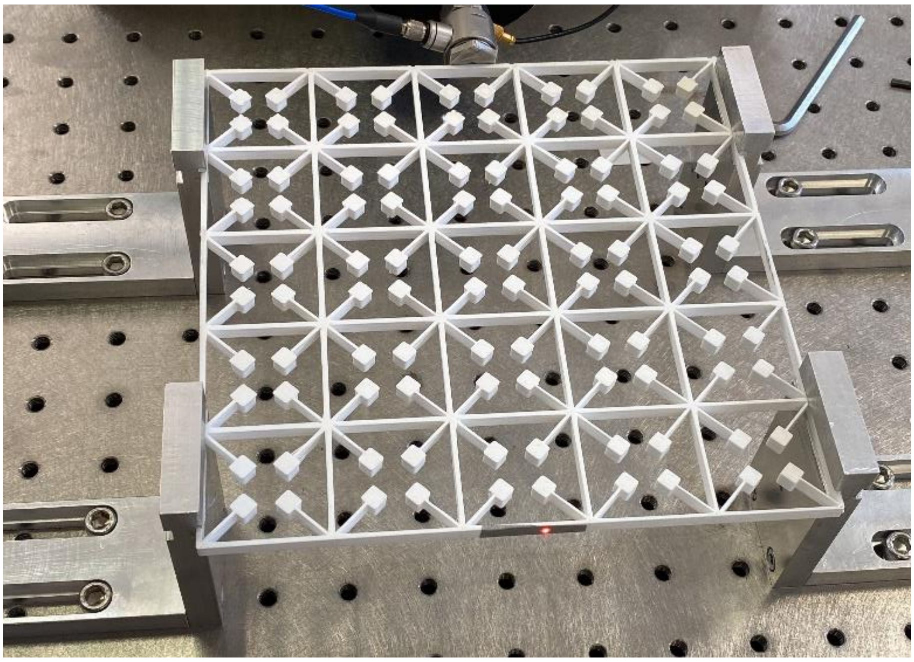


Figure 28 metamaterial 1 during the experiments

3.1.3 Second set of designed metamaterials:

As mentioned before these sets of metamaterials are with almost same structure but hollow circles are inside a full plate. This structure will be also used to validate if periodicity is really not a significant factor in designing a metamaterial as it is expected. In fact, here the size of the plate is the same only number of periodic structures inside is decreased from five to 3 and 3 sets of metamaterials obtained and experimented.

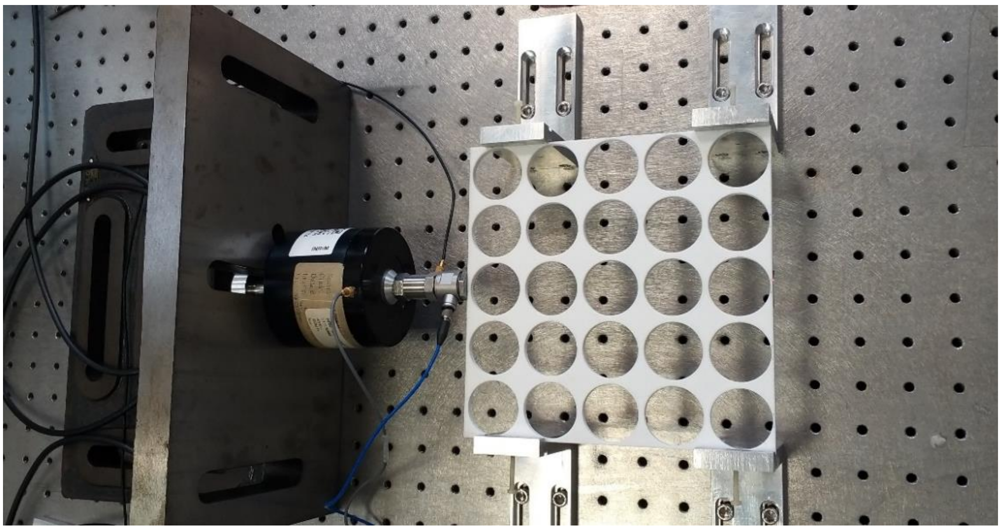


Figure 29 metamaterial 2 during the experiments.

Results are quite unexpected and interesting. According to these results it can be concluded that ratio of hollow space and filled space with material has a significant impact on the isolating behavior of the material even though the overall design of the periodic structure is the same.

1.3.3.1 Metamaterial 2:

Here the voids PLA plate is only in regular circle shape. As the results are demonstrating, theses regular circles cause a very smooth attenuation as the frequency changes. The specimen shows a better attenuation for higher frequencies and even in resonant signal the transmission does not exceed 1.5 times, but it still exists and not mitigated completely.

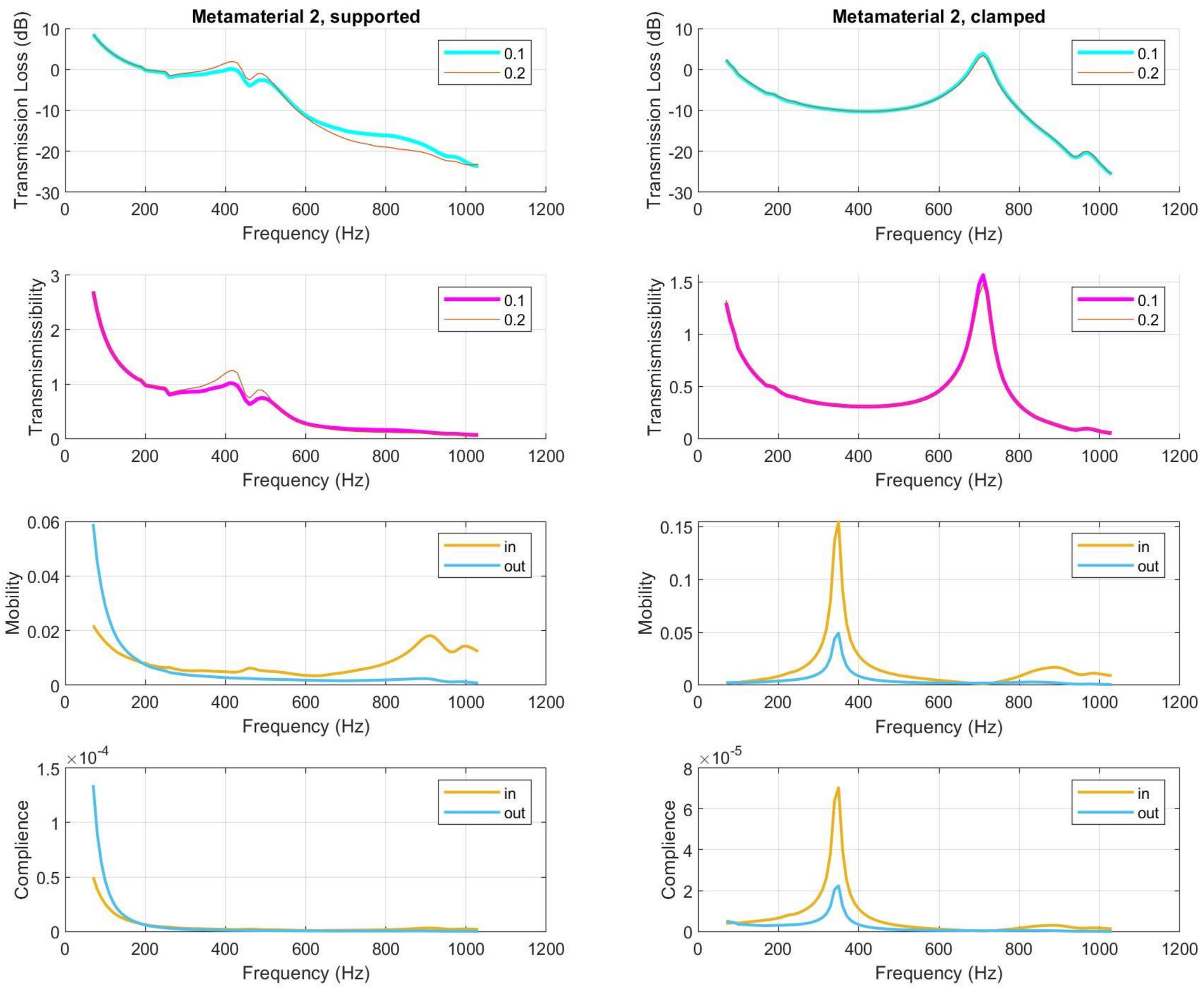


Figure 30 Experimental results for metamaterial 2 represented as Transmissibility, TL, Mobility, and Compliance

1.3.3.2 Metamaterial 3:

Here the change of behaviour of material with respect to previous specimen is due to having more complex voids inside the material, in the way that circular voids and the viouds around the circle both change its attenuation range for different frequencies. Results are considered much better than metamaterial 1 because here almost in both cases of clamped and supported B.Cs transmissibility is lower than 1. In supported condition only in frequencies lower than 200 Hz there is an amplification of input is recorded which can be mostly due to the "non perfect" boundary conditions in reality and its correspondence with resonant frequency of material.

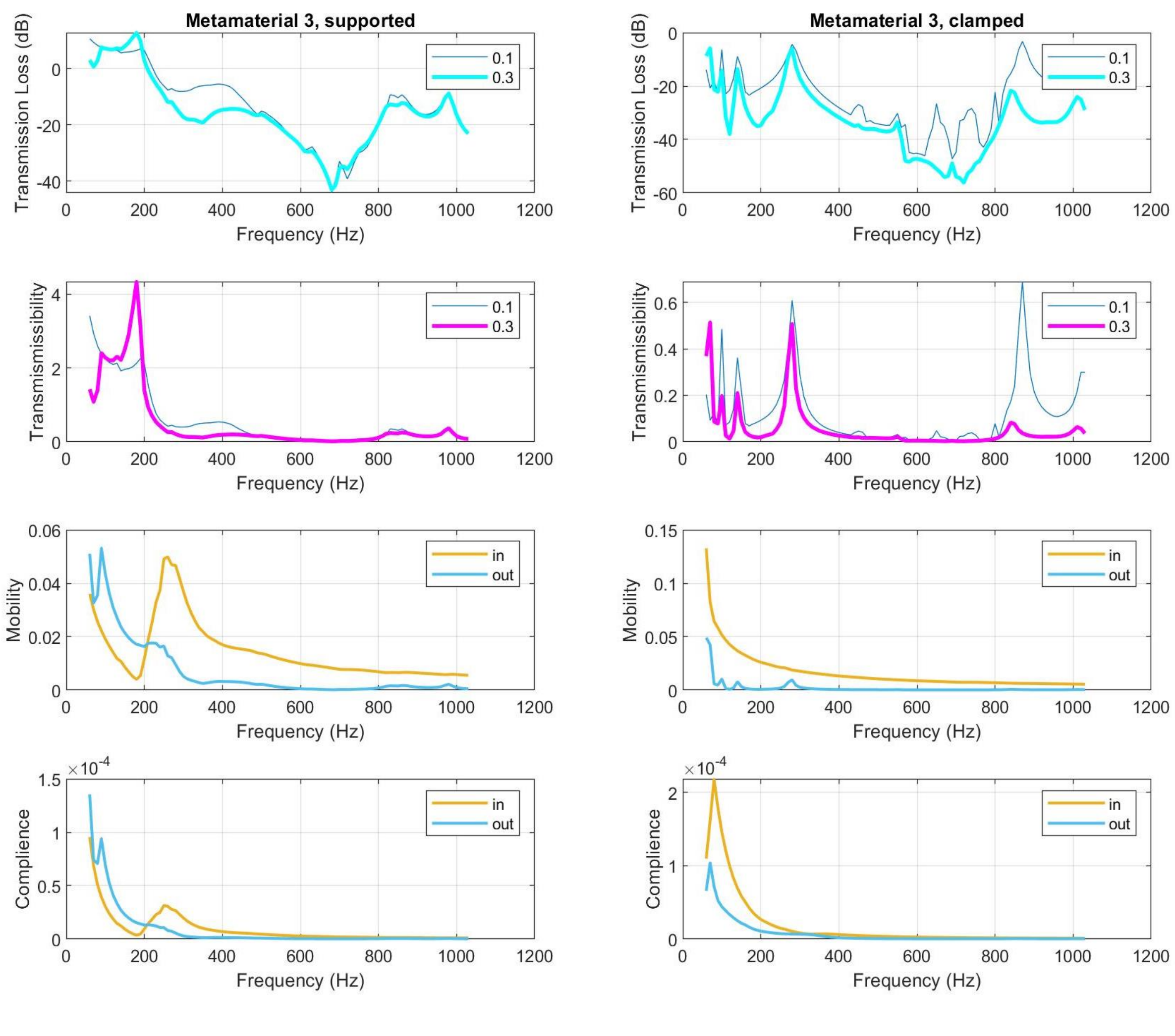


Figure 31 Experimental results for metamaterial 3 represented as Transmissibility, TL, Mobility, and Compliance

1.3.3.3 Metamaterial 4:

Here the voids ratio to the whole strcture is lower than metamaterial 3. This lighter material shows a very good attenuation in frequencies higher than about 500 Hz while the results in lower frequencies is also accepltable. In very low frequency n overloading issue cause the strange pick at the biggining of the transmissibility plot but later this isuue solved by automatical decrease of input force.

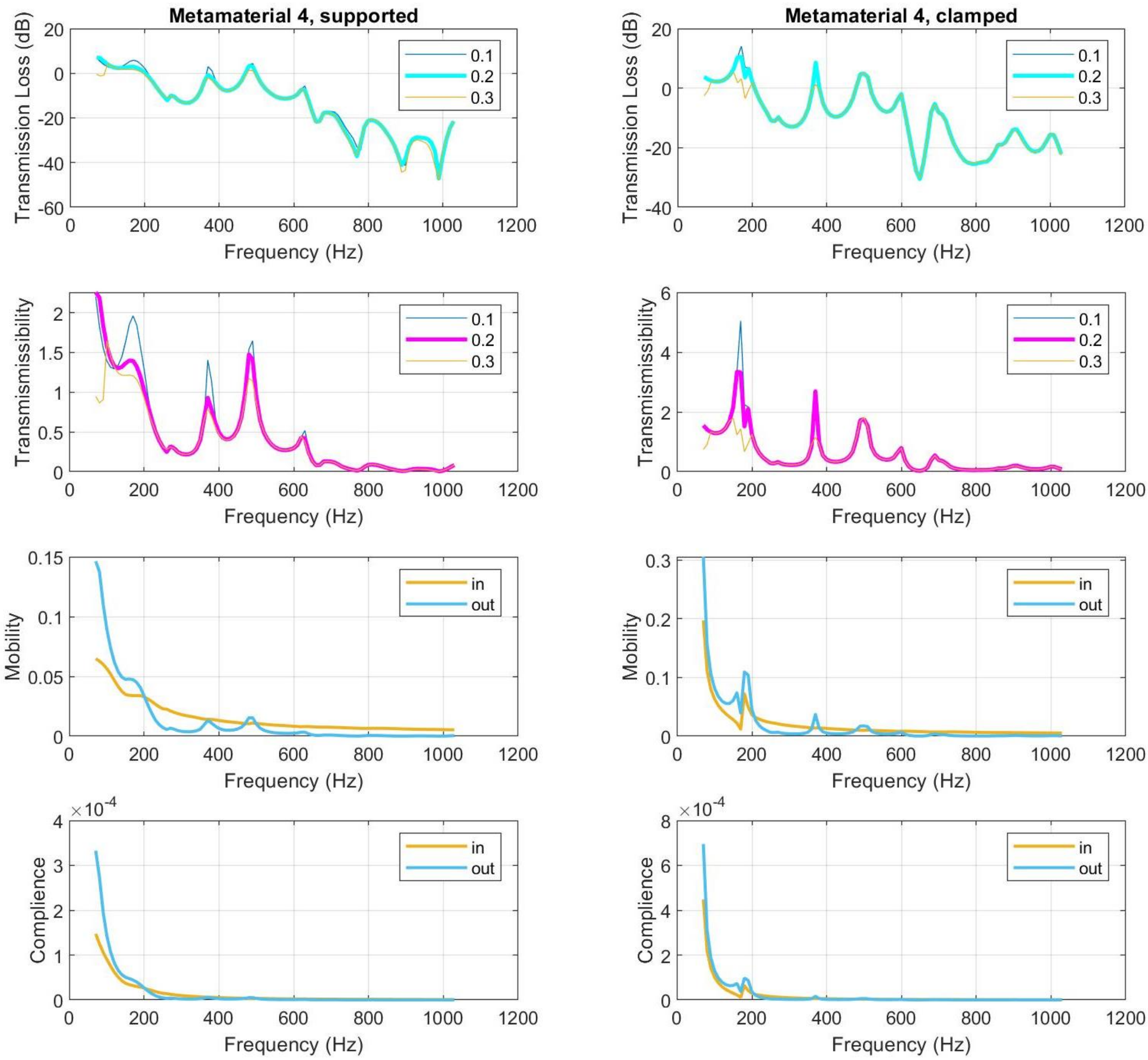


Figure 32 Experimental results for metamaterial 4 represented as Transmissibility, TL, Mobility, and Compliance

3.1.4 Comparison of the results for Metamaterial 3 and 4:

As mentioned previously metamaterial 3 has 26 percent higher void volume inside of its structure. Here the comparison demonstrates how same structure but with different dimensions which means different ratio of voids to the overall material (mass density) can affect the properties of the whole periodic structure.

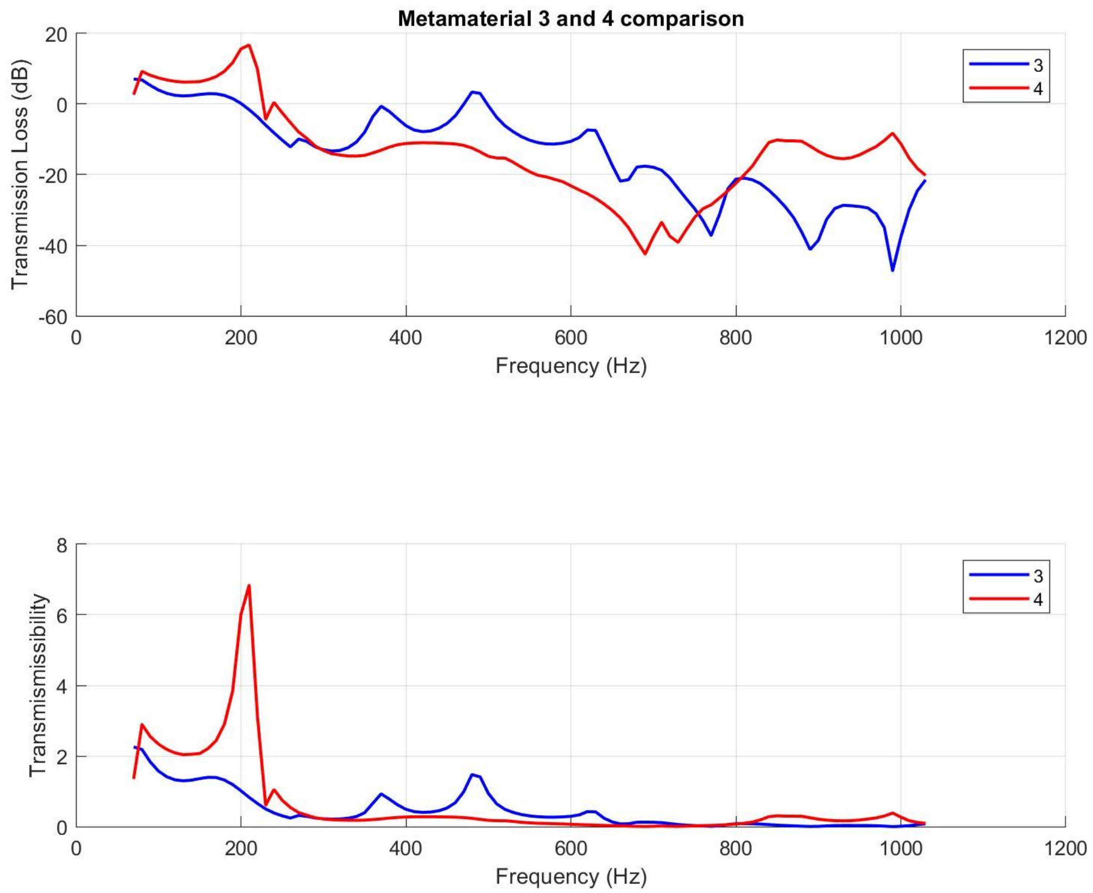


Figure 33 Comparison of Transmissibility and TL of metamaterial 3 and 4

Since the size of the 2D crystals are different in each of the metamaterials their response varies for frequency ranges. While metamaterial 3 has better attenuation for frequencies from 300-750 Hz, metamaterial 4 can better mitigate vibrations with frequencies lower or higher than this range.

Obviously, we cannot conclude that lower or higher density of overall structure can result a better attenuation for all the frequency ranges or even a wide range of frequencies.

3.2 Results of Computer simulations

Having defined the necessary theoretical concept, we introduce the implementation of the specific problem in finite element software COMSOL. The package used to run the simulations is Solid mechanics and the studies carried out are the search for the eigenfrequencies and the representation of the diagram of dispersion.

From the dispersion diagram it is possible to observe the possible presence of band gap if present: band gap is intervals of frequencies in which the propagation of elastic waves is prevented; it is caused by the destructive interference of reflected waves of a certain frequency during their propagation in a periodic medium. In particular, for the research of the band gap a periodic condition of type is set Bloch Floquet and the components of the wave vector are explained in section 4.1. The parameters used in the simulations are described below (section 4.1), the geometry and materials used (section 4.2) and the various cases of cobweb studied, in which the band gap as the size of the structure changes.

3.2.1 Definition of the components of the wave vector k:

For k_x and k_y one can write:

$$k_x = \begin{cases} (1-k)\dfrac{\pi}{A} & k \in [0,1], \;\; M \Leftrightarrow \Gamma \\ (k-1)\dfrac{\pi}{A} & k \in (1,2], \;\; \Gamma \Leftrightarrow X \\ \dfrac{\pi}{A} & k \in (2,3], \;\; X \Leftrightarrow M \end{cases}$$

$$k_y = \begin{cases} (1-k)\dfrac{\pi}{A} & k \in [0,1], \;\; M \Leftrightarrow \Gamma \\ \dfrac{\pi}{A} & k \in (1,2], \;\; \Gamma \Leftrightarrow X \\ (k-2)\dfrac{\pi}{A} & k \in (2,3], \;\; X \Leftrightarrow M \end{cases}$$

While:

$$k_i(0) = k_i(3), i \in \{x, y\}$$

3.2.2 Definition of geometry and materials:

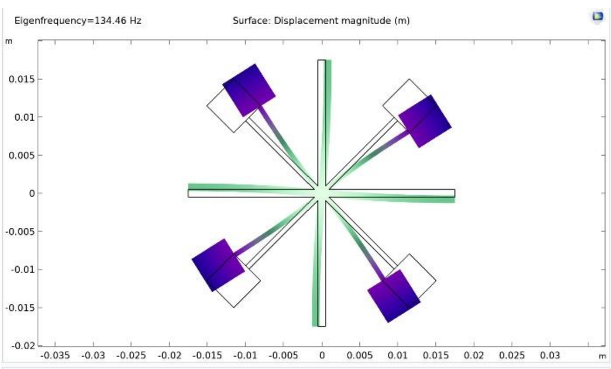
The geometries of each metamaterial are defined only as one of the periodic structures and in the boundary conditions a periodic boundary condition is imposed while the thisckness is chosen 5 mm for all the simulations. In study 1 Regarding the material for all the specimens we used PLA which has following mechanical properties:

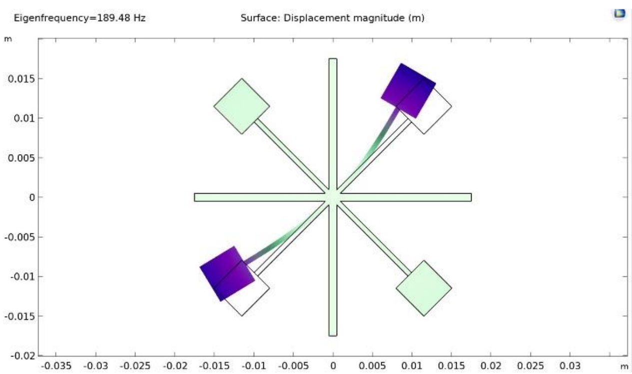
$$\rho = 1240\,\frac{kg}{m^3}\,,\ E = 4.107 \cdot 10^9\,\frac{N}{m^2}\,, \nu = 0.3$$

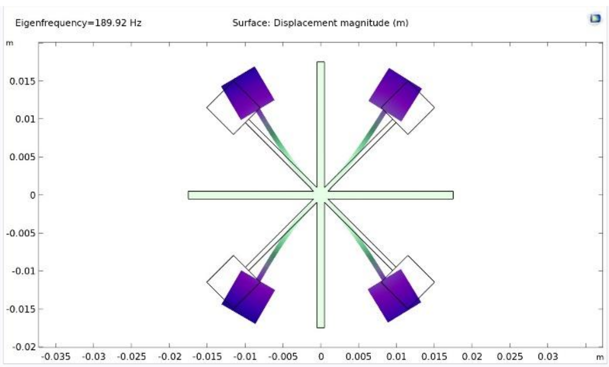
3.2.3 Modes shapes and Dispersion diagrams:

In the following for each of the metamaterials it is shown first mode shapes and eventually dispersion diagrams. Descriptions and analysis are present under each plot.

1.3.3.1 Metamaterial 1







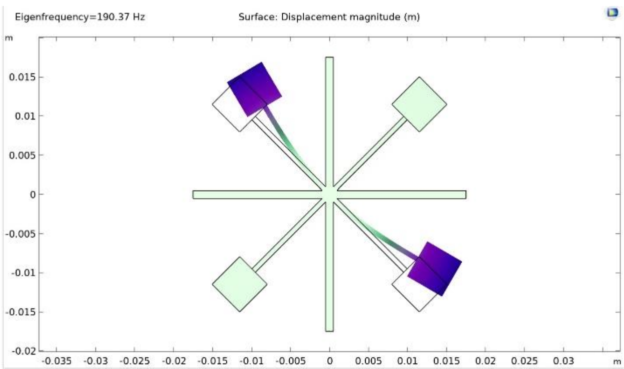


Figure 34 Mode shapes of metamaterial 1 with a deformation scale of 57491 on COMSOL Multiphysics.

It is possible to observe that the eigenfrequencies are: 134.5, 189.5, 189.9, and 190 respectively which is with a good approximation in accordance with the first three picks of experiments plot in clamped mode (Figure 26).

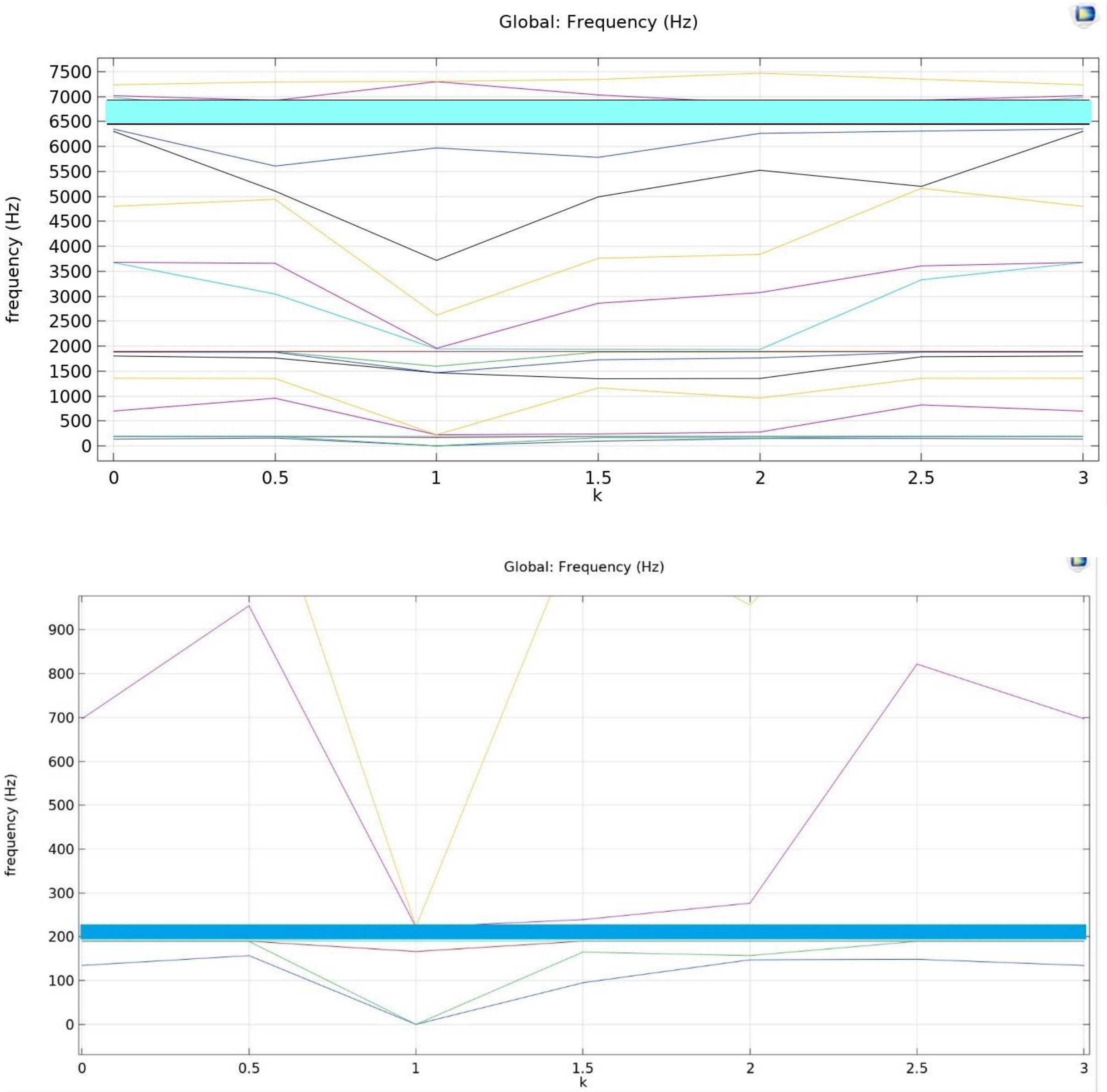


Figure 35 magnified view of the dispersion diagram of the first metamaterial 1, the lower plot is a zoomed in version to show the smaller bandgap.

According to the plots only in some small frequency range the material is able to disperse vibrations. Band gaps are at the frequencies around 200 Hz and also in high frequencies such as 6300-6550 Hz.

1.3.3.2 Metamaterial 2

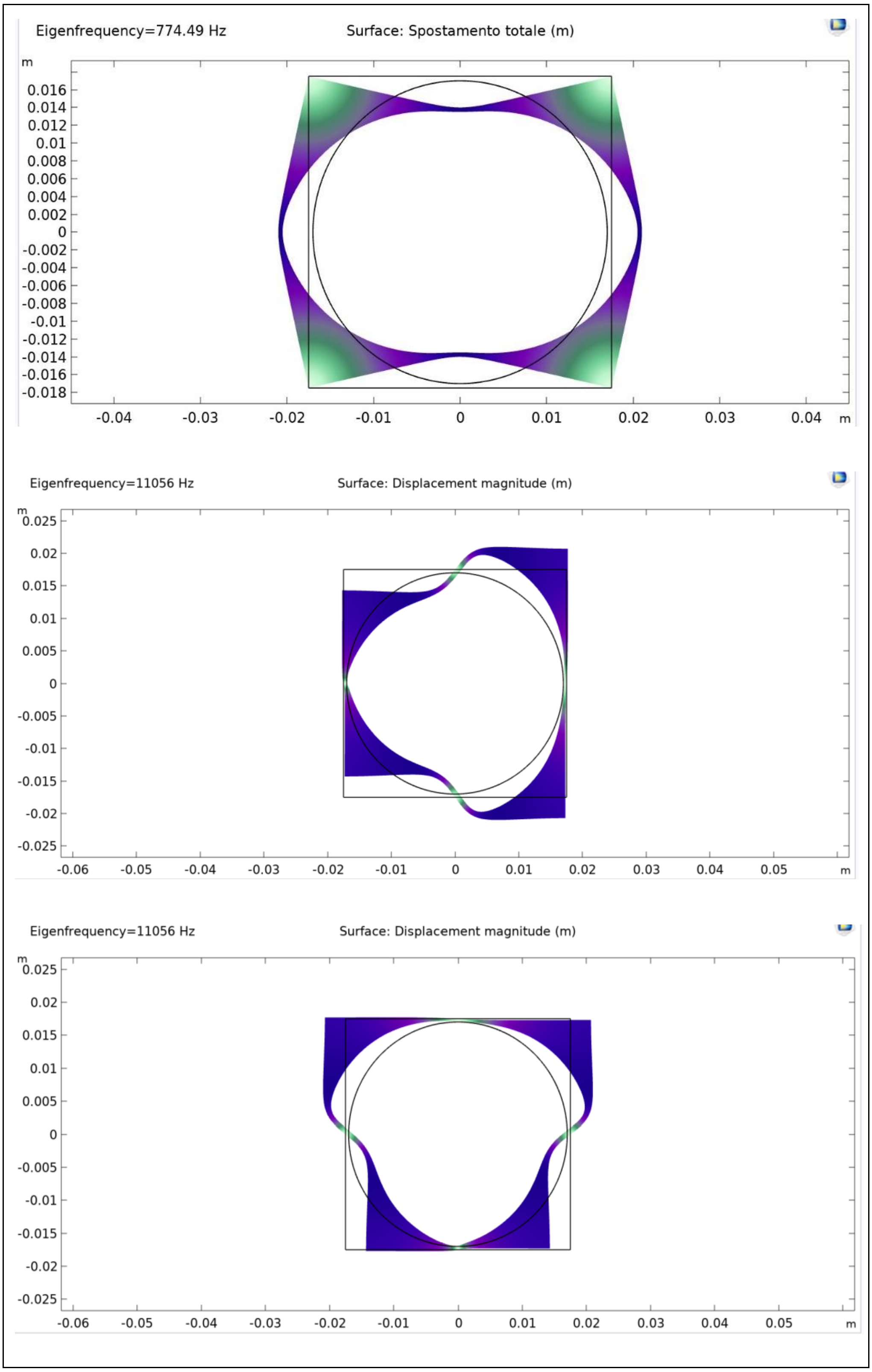


Figure 36 Mode shapes of metamaterial 2 with a deformation scale of 57491 on COMSOL Multiphysics.

It is possible to observe that the first three eigenfrequencies are: 774.5, 11056, and 11056 (with different imaginary part) respectively. First eigenfrequency is exactly accordance with experiments in clamped mode (Figure 28).

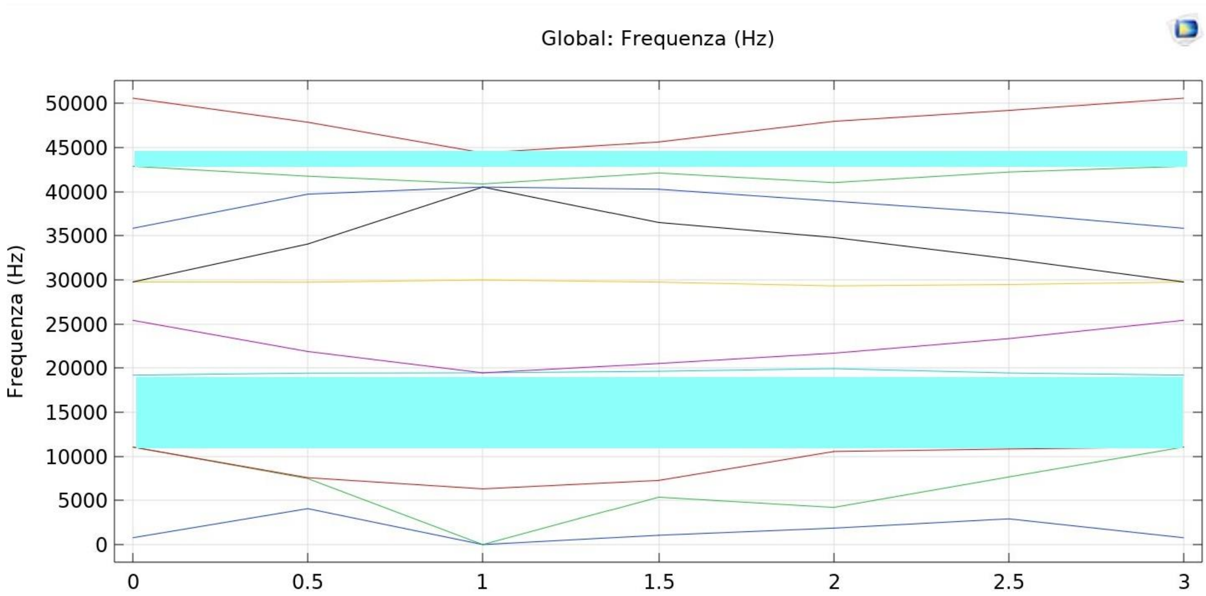


Figure 37 representation of bandgaps for Metamaterial 2.

Regarding the dispersion diagram two band gaps are visible which both of them are at the high frequencies such as 1200-1800 and 2600-3000 Hz as a rough expression.

1.3.3.3 Metamaterial 3:

According to the Figure 36, It is possible to observe that the first three eigenfrequencies are: 466, 873 and 1272.8 respectively. Comparing them with the experiments (Figure 29) demonstrate that either the simulator did not recognize lower eigenfrequencies than 466 Hz or it is due to imprecisions in the experiments. However, second and third eigenfrequency in clamped mode is almost coincident with computer simulations.

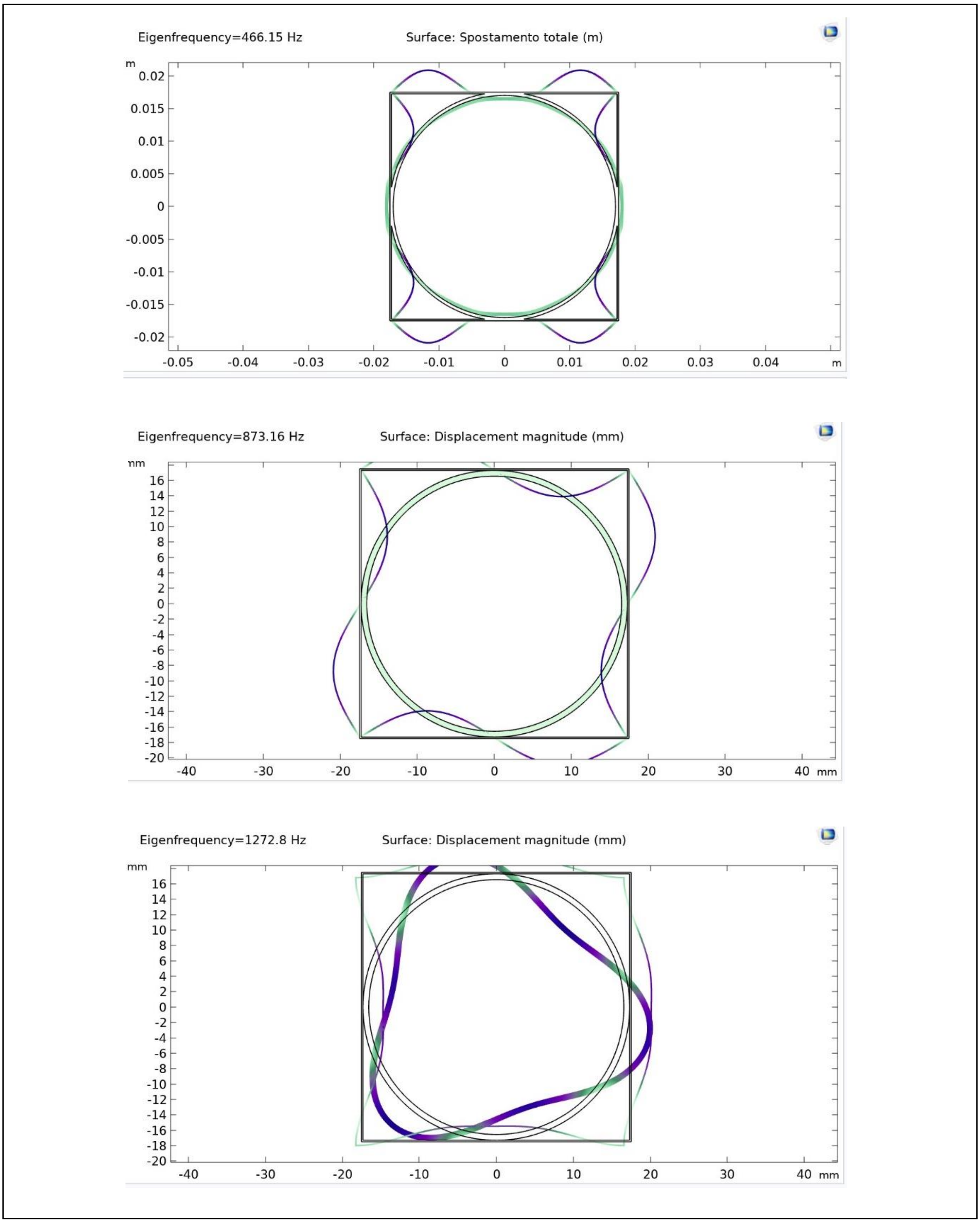


Figure 38 Mode shapes of metamaterial 3 with a deformation scale of 57491 on COMSOL Multiphysics.

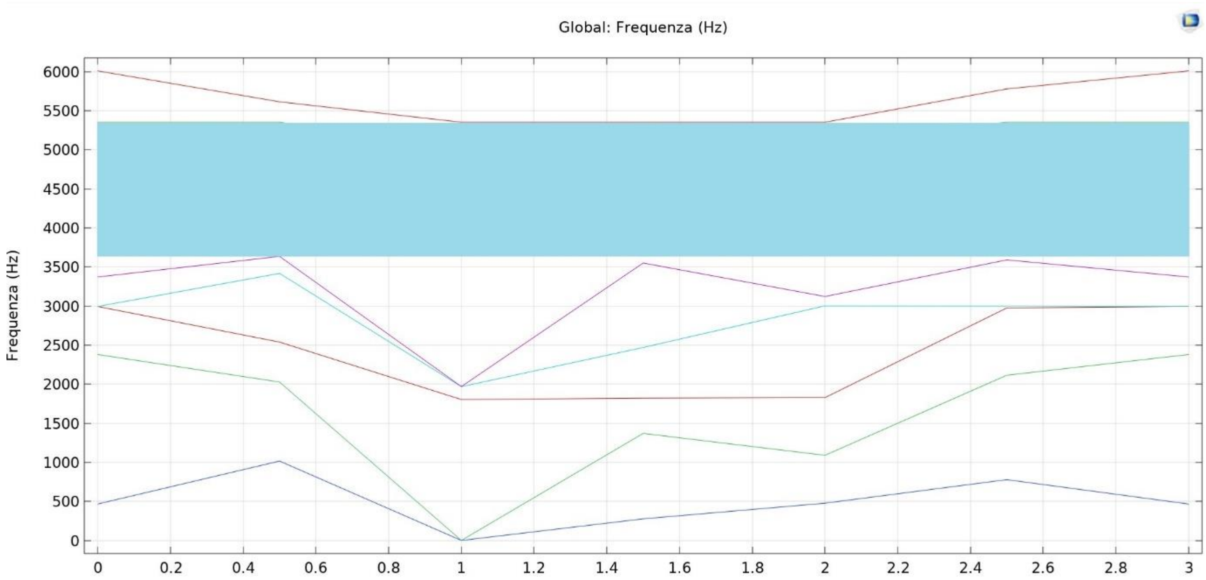


Figure 39 Representation of bandgap for Metamaterial 3.

According to the plot the band gap is relatively wide, and it is around the frequencies from 3700-5300 Hz. However, in the lower frequencies there is no bandgap observable. moreover, in our experiments which was up to 1200 Hz of frequency the transmission Loss of the material was acceptable and better than metamaterial 2 for frequency range of 500-600 Hz. It means that although the waves in lower frequencies are not completely dissipated but still the metamaterial is considerably effective to prevent a high transmissibility.

Conclusion:

In this thesis paper, a comprehensive definition of metamaterials and periodic structures is elaborated, and the most interesting characteristics of mechanical metamaterials are discussed to clarify the aim of the experiments, which is to represent a better analysis of 2D periodic structures designed to attenuate mechanical vibrations and demonstrate their effectiveness by experiments. The latter was missing in most of the research in this area, so filling this gap in the futuristic world of metamaterials is a great advantage for this thesis work, which was fulfilled by the INRiM research centre and the guidance of its researchers.

Starting from the structure proposed by Ying Lu et al [28] in his paper, Design of mechanical metamaterials (or it can be considered as 2D phononic crystal) for simultaneous vibration isolation and energy harvesting**,** more than six metamaterials are design and experimented during this project and at the end the results of the four most interesting specimens are discussed and analysed. Designed structures followed a simple and similar shape to give a better idea of the effect of void to material ratio and vibration attenuation properties. All the specimens are fabricated by fused deposition modelling technology using PLA filament.

Results obtained were very promising. All the specimens were successful in attenuating a wide range of frequencies, and for some smaller ranges their performance was considerably satisfying. These results are obtained by repeating the experiment with different input frequency amplifications and are validated by computer simulations with an approximate estimation. Furthermore, the results obtained from computer simulations also helped to estimate the behaviour of these periodic structures, especially at very high frequencies where it was not feasible to experiment. As it was expected, the computer simulation is not so reliable since it considers a simplified and ideal condition, but it still helps to further realise the frequency range in which the metamaterial is more effective.

Finally, a significant reduction in the transmissibility of the metamaterials can open a new horizon in the applications of 2D metamaterials along with the more proven effectiveness of bandgap 3D structures. Although 3D phononic crystals seems still more useable in the real life, there are some possible application 2D metamaterials such as in industrial facilities where precise machines are working close to each other or in the railway systems where vibrations can be problematic in the vicinity of the rails. on the other hand, the drawback can be in the range of frequencies where the highest effectiveness of the metamaterials is obtainable, and it leaves the field open for future research.

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