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Length measurement of road race courses: experimental comparison of GPS trackers

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Abstract

The world today is on constant development, adjusting technology to all possible requirements to improve people's living standards. The GPS, being one of the most important advances of the last decades, has given us the possibility of knowing where we are located on Earth with better precision than the radio-navigation systems. Most electronic devices have a GPS chip undoubtedly very useful for navigation and tracking applications, encouraging the design of more intelligent devices to respond to new user's demands. Sports wearable devices have also included this technology, however, for professional competitive sports purposes, most of these devices do not have enough accuracy, thus not yet being approved particularly in athletics route tracking.

The following thesis will analyse and assess the accuracy of the GPS data acquired by different tracking devices with the aim of testing their reliability in a competitive sports approach.

This thesis begins with an introduction of GPS devices as well as the most important considerations in the official distance measurements proposed by World Athletics, the one in charge of measuring long distances in marathon competitions.

The work starts with the definition of an outdoor course track and describes a method for the the measurement of the track length. The method is based on an Odometer, which is a standards approach commonly employed also in Athletic courses. Specific calibration and tests were done to characterize this device.

After the odometer calibration, the considered course track is measured thus obtaining the track reference length. In the same way, the track was recorded several times with GPS trackers in order to infer a possible estimated value in terms of average value and relative error. The results showed an overestimation of measured distance with a relative error percentage between 0.04% and 0.28%. In some cases the GPS measurements reached the minimum uncertainty of 0.1% requested by the World Athletics Federation regulations.

Errors associated to each type of GPS device were statistically analysed and discussed.

Eventually, as a further test, the GPSs were compared in static conditions. This test concerned the measurement of a long straight path. As done before, the path length was measured by the odometer as well as by the GPS devices and the results were compared. In this test several GPS measurements were taken at the beginning and at the end of the path, The results were thus analysed in term of Circular Probable Error (CEP) , 95% Radius(R95) and Position Random Walk (PRW).

The present thesis also reports some considerations and limitations to be taken into account for further studies about the accuracy assessment of GPS trackers.

Acknowledgements

The path that has brought me here, up to this moment, has most definitely not been easy. It has been full of thousand of impediments and difficulties, but the human being has been made for standing up time and time again, restarting from scratch if necessary. I can not describe the dimensions of the gratitude I have towards everyone that along my academic career has shared with me a little bit of their strength and knowledge to motivate me to become the best person I can be. To all of them, I'm eternally grateful.

I dedicate this thesis first and foremost to God, for enabling me to come this far, for motivating me to do everything with love, from love, with the objective of becoming more in order to serve the world better.

I would also like to dedicate all the work I have done with this thesis to my parents, to whom I give infinite thanks for the patience they have had with me, you both have always been there for me and without their support I would not have been able to finish this journey. Thank you for all the love you have given me, because of you both I have learned how to get up after each fall, and have learned that even if it takes long, at the end the rewards would come.

I also thank Professor Alberto Vallan for believing in me, for his support and his good attitude with this thesis, for his availability and his recommendations in its elaboration, as well as to all the people who were involved in its realization and participated with their selfless support, I will be eternally grateful.

Finally, I dedicate this work to those loved ones who are not alive today to show them what I have achieved, this is also yours, I hope you'll get to see this, wherever you may be.

To all those who have supported me and believed in me, to all those who have encouraged me to not give up, who gave me hope when I needed it most, and who despite the years remain by my side up to this day, Gracias totales!

Leonardo Rosero Realpe, Torino, March 2022

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Chapter 1

INTRODUCTION

The world today is on constant development and tries to adjust technology to all possible requirements in order to improve people's living standards. The GPS, being one of the most important advances of the last decades, has given us the possibility of knowing where we are located on Earth with better precision than its predecessors, the previous radio-navigation systems. From its origins in the 1960's it has developed and improved along the years, migrating from its initial military use, passing through aeronautic use, finally becoming able to be used in civilian contexts.

In our society the use of GPS has become essential to know where we want to go which explains why most of the population has at least a GPS chip in their electronic devices, proving to be undoubtedly very useful for navigation and tracking applications, its greatest exponent being the "Google Maps" app. On the basis of all the possibilities that the use of these devices could bring, GPS users have opted to apply them on a sports approach, thus increasing production of sports devices with localization capabilities. Nevertheless, there are notable differences between civilian and professional use. Because most of these devices do not have enough accuracy, they have not been approved for use in the competitive sports area, particularly in athletics route tracking.

The following thesis will analyse and assess the accuracy of the GPS data acquired by different devices with the aim of testing their validity in the competitive sports approach, as well as perform a data processing method to be used for assessing the aforementioned accuracy in future occasions.

The static tests made with GPS devices have the goal of evaluating up to what point each device is able to predict accurately and precisely a static location. All the receivers were put under the same conditions, recording a huge number of points at the same time, and in this way, acquiring enough points to infer the mean position and its standard deviation in meters (in Latitude and Longitude coordinates).

Considering the absence, in our laboratory, of a trustworthy long distance measurement method, this thesis proposal raises the construction of an electronic odometer that counts a portion of a bicycle's wheel perimeter in order to find out the real distance or at least a more confident measurement of a particular road race course independently of the traced route.

The next test aims to calculate the total distance between two straight fixed points. In order to achieve this, a static test was made in a static point, after that a long distance (700 m) was travelled using the odometer until locating the second point, the static test was done in this finish point once again. This test should be preferably done in the selected road race course. Finally, at the end of this test the distances were calculated from the mean location coordinates for every device, then compared with the previous odometer measurement.

Once the odometer accuracy was verified, the calibration tests were done following the official regulations of the Athletics Federation, comparing the measurements collected in the sportive area. For this reason, different calibration courses were travelled to calculate the number of counts per kilometer, as odometer or Jones Counter does.

Taking the number of counts per kilometer inferred from the calibration course evaluation, the measurement if the road race course was repeated to establish a more accurate value of the real total distance.

Once a more accurate length of the road race course was known, different GPS handheld devices were used to measure the same route registering all the involved points into the trace. Each GPS device works separately and records its own track of the route. At the same time the starting and finishing times were recorded with the aim of getting these tracks cropped and ensure a fixed travelled distance in this time interval.

This method was repeated many times to assess its repeatability and perform an analysis of results in terms of mean and standard deviation of the measurements. In the end the GPS uncertainty results were compared with uncertainty requirements mentioned in athletics measurement rules.

1.1 Motivation

Many years ago the GPS started to be available to civil population in order to support the needs and location issues of a society in continuous development. Currently, it is one of the most easily accessible tools but few have questioned its quality in terms of location position which may represents an issue for the derived applications as length measurements.

As a result some official organizations as World Athletics federation doubt its credibility and reject the use of these type of devices where the accuracy measurement is required. Due to the above mentioned, this thesis tries to assess the GPS based length measurement method. Through different tests with an experimental approach, the GPS devices are assumed as a black box to the acquisition, consequently no modifications to GPS hardware will be done.

Additionally, the limited literature on this topic makes it a subject with different points of view that will be mentioned and questioned over the thesis.

1.2 Organization

This section explains the content of each chapter and the organization proposed for the thesis.

Chapter 1 - Introduction

Presents the thesis goal and describes its motivation as well as what can be found in each chapter.

Chapter 2 - State of Art

Considering the accuracy requirements for an official sport approach, this chapter explains how the World Athletics federation measures the long distance race course for their competitions around the world. The used methodology and their considerations are explained.

Chapter 3 - GPS approach

This chapter has the aim of describing how the Global Positioning System (GPS) operates, from the signals used up to the significant source of error that can influence the results of length measurements.

Chapter 4 - Test

In order to evaluate the quality and reliability of GPS measurements regarding the level of accuracy required in sports presented in chapter 2, three different tests are proposed to measure the race course properly, compare the results with length estimated by the GPS receivers and assess the validity of these devices.

Chapter 5 - Data Analysis

The results of the tests proposed in Chapter 4 are discussed and analysed. In some parts additional information about the processing of data and literature considerations are explained with the aim of achieving the desired results. In the same way the experimental obtained data are presented by tables and figures.

Chapter 6 - Results

Finally, the resulting data from every test is compared to determine if the devices under consideration are accurate as well as a thesis conclusion and improvements for further studies.

Chapter 2

STATE OF ART

Over the years the International Amateur Athletic Federation and the International Association of Athletics Federations, both abbreviated as IAAF, now jointly called World Athletics since 2009, have tried to evaluate running race routes using many devices, calibrating them in order to warrant accuracy and precision in every competition. With this goal in mind, this federation has aimed to establish rules that have been compiled in the special Calibration and Testing Manual [11], where one of the most important topics is the measurement of distance, although as specified in this document it is not possible nor necessary to measure in athletics with the same accuracy needed in scientific field. As mentioned, the principal goal of this Manual is to indicate how measurement equipment is to be calibrated and the accuracy required for the best resulting measurements. Additionally, it provides suggestions of how federations might more cheaply check the accuracy of their measurement instruments.

The same document states that one of the most important problems is the fact that many manufacturers may be using different measurement techniques which could lead to a lack of consistency between them. For this reason it has been stated that the calibration reports for each device provided by the manufacturers should supply enough information to make certain the measures are adequate for use in World Athletics competitions. Each measuring device to be used in IAAF competitions should be tested and certified by capable organisations, in other words, those that have an ISO 17025 accreditation (which states a high level of competence, impartiality and consistence operation for calibration laboratory activities according with the ISO web page) .

As mentioned in [11], some of the information required for a measurement device in its calibration report broadly regards aspects such as:

- The identification of the method used.
- The description, condition and identification of the item tested or calibrated.

- Reference to the sampling plan and procedures.
- Results and its corresponding units of measurement.
- A statement on estimated uncertainty, its applications and other statement in the case of appropriate results from tested device.
- Deviations, additions or exclusions from the test method, test conditions, evidence that the measurements are traceable for a good interpretation of the test results.

2.1 Measurement Methods

Usually the devices used for measuring distance include Steel Measuring Tapes, Steel measuring Bars, Vernier Callipers and Micrometres and Electronic Distance Measuring devices (EDM) such as infrared, microwave and visible light instruments [13]. In the case of EDM, these are calibrated on a baseline with permanent monuments at known elevations and distances. Their industry maximum standard deviation reaches a value of ± (3 mm + 2ppm) but with the aim of convert the standard deviation to a 95% confidence interval it is multiplied by a factor of 2.0 thus requiring a uncertainty of a single measurement of ± (6mm + 4ppm).

Taking into account the previous information, it is possible to understand that devices such as GPS are not considered, in fact they are not even mentioned as measuring devices that would be apt for the sport field. The scientific accuracy and sportive accuracy required for this field are not satisfied by the GPS performance.

In addition, the World Athletics in cooperation with the Association of International Marathons and Distance Races (AIMS) released in 2008 a book called *"The Measurement of Road Race Courses"* [12] that explains the method used to measure road race courses and its procedures to ensure accuracy on every race. Some principal considerations that must be taken to measure adequately the race route are based on the rules 240 and 260 of the IAAF Competition Rules (2008) and are literally described as follows :

- The course, duly marked, may be on a bicycle path or footpath alongside the road, not on soft ground such as grass verges.
- The start and finish points, measured along a theoretical straight line between them, should not be further apart than 50% of the race distance. The start and the finish of a race shall be denoted by a white line at least 5cm wide.
- In events on roads the course will be measured along the shortest possible route that an athlete could follow within the section of the road permitted for use in the race.

- The length of the course shall not be less than the official distance for the event.
- In competition the uncertainty in the measurement shall not exceed 0.1% (i.e. 42m for the Marathon(1 Marathon = 42.195km)) and the length of the course should have been certified in advance by IAAF approved course measurer.
- For measurement, the calibrated Bicycle Method shall be used. To prevent a course from being found to be short on future re-measurement, it is recommended that a "short course prevention factor" be built in when laying out the course. For bicycle measurements this factor should be 0.1% which means that each km on the course will have an actual "measured length" of 1001m.
- It is recommended that for Road Races staged over standard distances, the overall decrease in elevation between the start and finish should not exceed 1:1000, i.e. 1m per km.
- For the Road Relay, the race shall be run in stages of 5km, 10km, 5km, 10km, 5km, 7.195km. The stages must have been measured and marked during the course measurement with a tolerance of ±1% of the stage distance and must have been verified in accordance with Rule 260.28(e).
- Either the course measurer who measured the course or another "A" or "B" grade measurer in possession of the complete measurement data and maps must validate that the course measured was the course covered.
- The circuit shall be no shorter than 1km and no longer than 2.5km with a possible start and finish in a stadium.

This document is very important to know how competitive athletics around the world work, however, the book describes the relevance of the road course measurement using the calibrated Bicycle Method which is basically a bicycle with a Jones Counter, making it the only method for accurate measurements approved by the IAAF. The Jones Counter was invented in 1970 by Alan Jones and currently its the only approved method for measuring road race courses. It is a mechanical device placed at the hub of the bicycle's front wheel. Some representations of bike parts are shown in Figure 2.1 and Figure 2.2 with the aim of having a better understanding of this device.

The Jones Counter measures the revolutions every time the wheel rotates. This device does not measure the distance directly, instead it measures the revolutions and part-revolutions of the bicycle front wheel. It is made of a gear system in which there must be a clearance between the large gear and the fork and between small gear and spokes. The large gear has a little anchor in one of its teeth that touches one spoke and the movement generated produces a momentum on the entire large gear while the small one is touching it recording the revolutions.

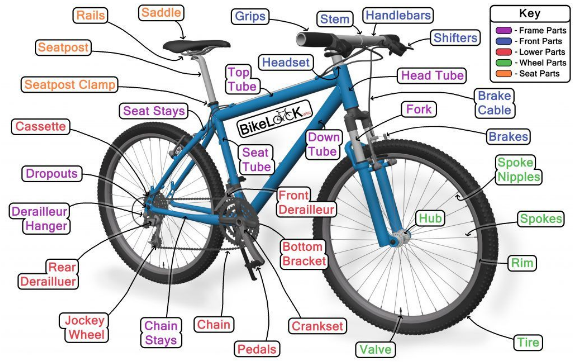


Figure 2.1: Description of the parts of a Mountain bike [3].

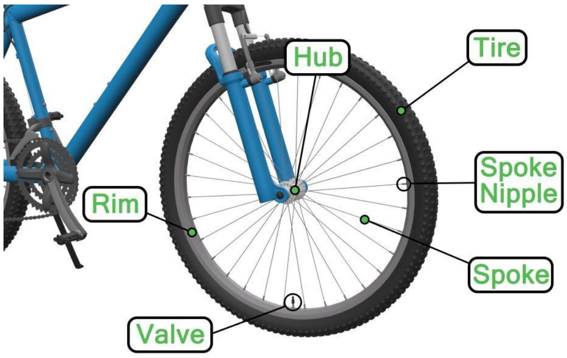


Figure 2.2: General parts of front wheel on a bike [3]

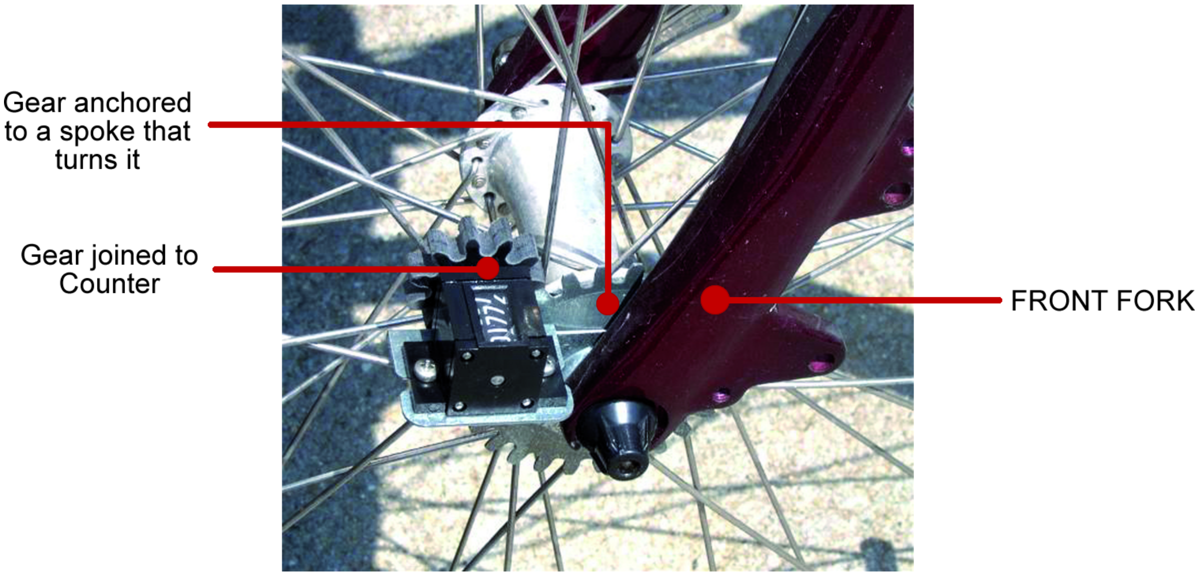


Figure 2.3: Jones Counter [2].

This gear system registers 260/11 counts (23.6363) for each revolution of the wheel due to the standard circumference of the bicycle wheels (2.1 m). Therefore, each count represents approximately 9 cm on the ground. This method compares the number of revolutions of the wheel (in other words the number of counts) along all the race course with the number of revolutions in a calibration course made with a known distance. It could be considered a simple measure but there are also some considerations to be made in order to obtain an acceptable result.

2.2 Methodology and Restrictions

Taking this into account, eight required steps will be mentioned on how the Jones counter must acquire a proper measurement of the race course.

- **Define the road race course:** it is the most important step because the measurement becomes irrelevant if many routes are considered. It is necessary to know about the spaces and permissions required for the organization of the contest and at least a simple map must be designed to mark the entire race route.

- **Select and measure a calibration course:** it is used to calibrate the bicycle and it should be at least 300m in length, although a length of 500m is recommended. The manual recommends a shorter calibration course near to, or on, the race course and explains that its effectiveness depends on a good calibration procedure, suggesting that the calibration course should be completed eight times (four times before and four times after the measurement), ideally in both directions. The road features should be kept as similar as possible to those that will be used during the competition, the surface travelled should be comparable to the entire road race course measured. The steel tape is the standard method of measure of the calibration course, in addition, it would be better for it to have been made by a well-known manufacturer with temperature (20 Celsius degrees) and tension (50 Newtons) specifications and its length should be at least 30m. A spring balance could be used for checking the tension, though a strong pull on the tape is sufficient, similarly a thermometer for checking roadway and tape temperature is recommended. The manual advises to tape the course twice, with the second measurement done in reverse direction from the first one. The number of counts on the calibration course should be similar to other counts on other calibrations courses of the same length.

- **Calibrate the bicycle on the calibration course:** the aim of this step is to calculate the number of counts registered on the Jones/Oerth counter for every kilometre ridden on the bike. It is an important process to deduce the called *working constant*. To calibrate the bicycle it is necessary to complete the following steps:

  - Check the condition of the bicycle tires (firmly inflated) previously ridden before the calibration with the intention of reducing the variation of the counts recorded.
  - When arriving to one endpoint, slowly roll the front wheel forward up to the count at which you will begin the calibration ride for the purpose of keeping the anchor touching the spoke maintaining the same position and thus the same number of counts. To do this it is necessary to lock the

front wheel with the brake placing the axle directly over the endpoint, promptly the number of counts is recorded, repeating this every time a reading is completed, taking care of not changing the number of counts in this process. An example is presented on Figure 2.4

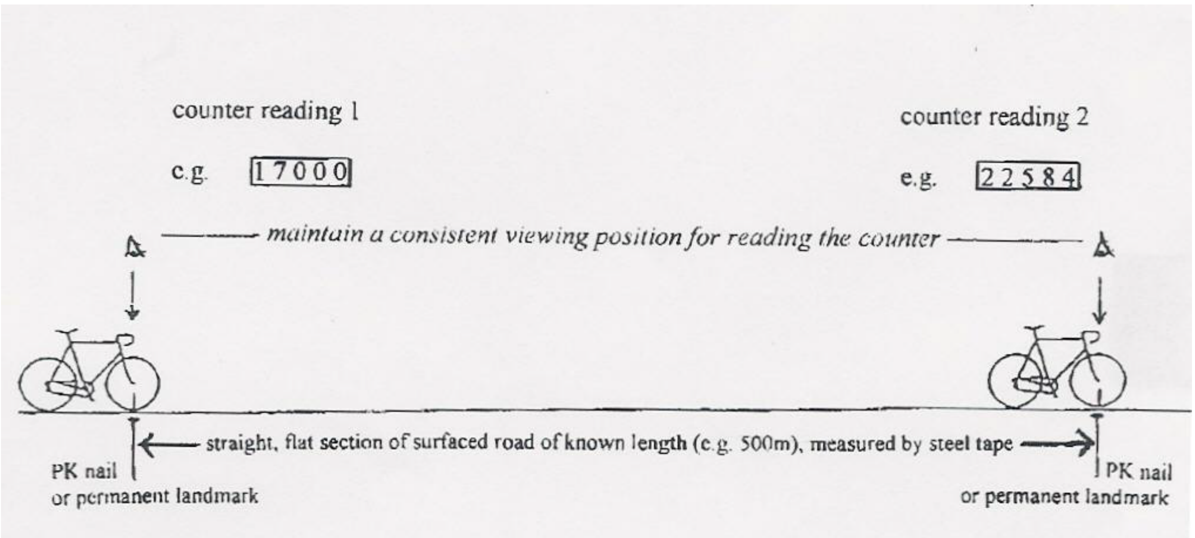


Figure 2.4: Example of how to measure the counts for one ride [12].

– A calibration ride should be one non-stop ride thus it is suggested to ride through the calibration course in straight a line with the same conditions (weight, equipment, position) as will be present when measuring the race course.

– The previous steps have to be repeated until four rides are completed, remembering to complete two ride for each direction.

– Making use of the counter reading, a subtraction between the counts at endpoint and the counts at the previous start point has to be made. In the case in which the number of counts is different from other rides, the entire ride is excluded and an additional ride must be done.

– The number of counts per tour are averaged, obtaining the number of counts for one calibration ride.

– The obtained averaged value is divided by the length of calibration course in kilometers so that the number of counts per kilometre is obtained.

– The preceding value is multiplied by 1.001 to obtain the mentioned Working Constant, which uses the "short course prevention factor" (SCPF) of 1.001 to highlight the error using the calibrated bicycle method, thus specifying that the measure is at least the distance stated. Once the Working Constant is obtained, you will be ready to measure the entire road race course, remembering to return to the calibration course when finished.

- **Measure the road race course:** the measurement should be done taking into account a fixed point that could be the start line or the finish line. In this manner, if the planner has a fixed start line the measure should begin there, otherwise if a fixed finish line is defined, the measure in reverse is done from this point. It is convenient to start the counting from a fixed figure that is easy to start from (around a thousand, could be) and keep it as starting number of counts at the same time the front wheel is locked by the brake. Additionally it is important to take note of the counts through the race road course at a certain fixed distance (every kilometer, mile or every 5km), as well as marking those distances. The manual emphasizes on the shortest possible route defining the road race course, meaning that the measure should be done by the shortest possible route (SPR) a runner could take physically without being disqualified shown in Figure 2.5. This ensures that the runners run at least the announced race distance within the course boundaries, in other words, the runners should run hugging the inside edges of bends, in a way in which the bicycle measures the entire race road course taking a distance lower than 30 cm from kerb and other boundaries, trying to maintain this distance especially on turns and corners.

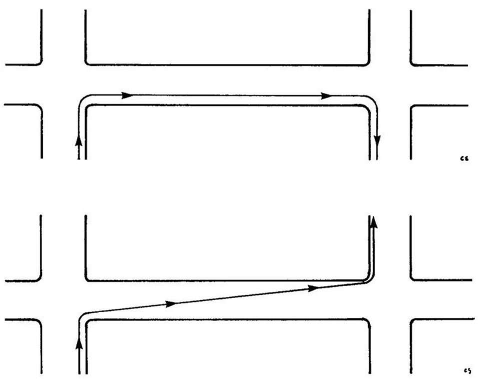


Figure 2.5: Example of Short Possible Route during the measuring over the streets corners [12]

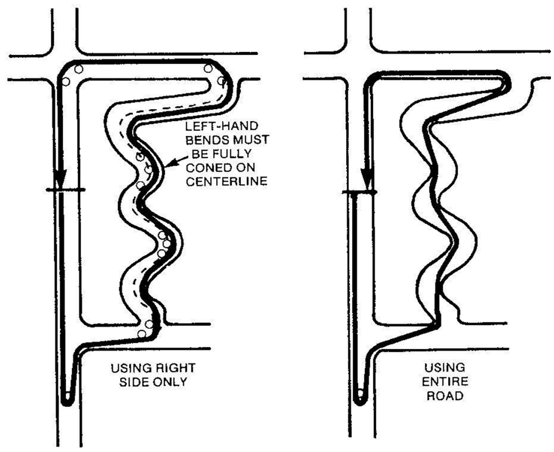


Figure 2.6: Winding roads and theirs disposal [12]

In some cases the contests are made as circuits, in which the runners must complete a defined number of turns in the same race road course. In these situations there are two possible cases, where both allow the winding roads, but the first one uses only half of the road, thus avoid crossing the center line, while the second case allows the use of the entire road, as appreciated in Figure 2.6. The called ultra distance races are usually run on multi-lapcourses, where few runners complete many laps (for example: 50 runners complete 20 x 5km laps in a 100km race). In this case it is imperative to measure accurately the length of the lap because the error will propagate at every turn, hence many measurements must be done considering the shortest registered measurement as the official lap distance (at least 3 measurements).

A non obvious contemplation are the turnaround points, particularly during the measurement. The simplest way to measure them is to set on the turn, then lock the front wheel, record the number of counts and turn carefully the bicycle without modifying the counter to continue to measure. If into the course the limits depend on barriers or cones, the correct position must be indicated on the course map and all the specifications and restrictions must be specified on the map, even where runners will turn the corners because it will have a considerable effect measuring the course. There should be no doubts about the measured path, as indicated on Figure 2.7 and Figure 2.8.

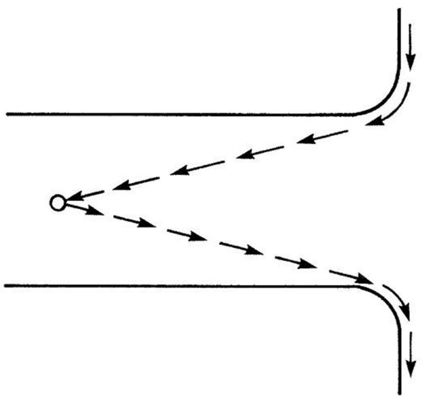


Figure 2.7: Turn around a specific point [12]

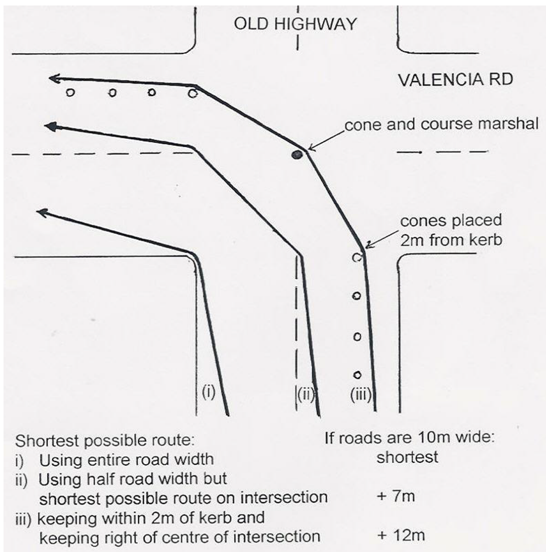


Figure 2.8: Course using different regulations and cones [12]

- **Re-calibrate the bicycle on the calibration course:** At this point it is necessary to check that the conditions have not changed too much. Some conditions as temperature, humidity and air volume on the wheel can change the number of revolutions and part-revolutions of the bicycle and thus, the number of counts defined for the road race course. Seeking a wide sense stationary measure, the organization proposes to perform a post-measure calibration immediately after the complete measurement of the course, before the time conditions change, repeating the pre-calibration steps and doing four rides in both directions. In the end the average values of post-measurement counts should be averaged again and divided by the length of the calibration course in kilometers, then multiplied by 1.001 to obtain the called "finish constant".

- **Calculate the length of the road race course :** when intending to calculate the total course distance it is mandatory to calculate the *"Constant of the day"*. This value is obtained averaging the "Working Constant" (calculated in the first calibration course) and the "Finish Constant" (calculated at the final calibration course). After calculating the total number of counts recorded in the shortest possible route, it is indispensable to divide this figure by the *Constant of the Day* obtaining the length of the road race course. For example, if the Jones counter registered 110526 counts and the Constant of the day is 11059, the distance of the road race course will be 9.9942km.

- **Make final adjustments to the road race course:** this step is only done after the calculation of the road race course distance. Probably in this case it will be necessary to add or subtract some distance to fit it into a desired length using simply a steel tape. Also the intermediate marked points (split points) should be re-positioned if an adjust on the start point is required. If the adjustments are made in the finish line re-positioning of the split points is not needed, otherwise it will be necessary.

- **Document the measurement:** the last step aims to make a documentation sufficiently detailed to check the course and future re-measurements (as mandated, for example, after a world best performance has been set). A clear map of the road race course that will be used in case the race director has to make some changes on some routes must be included. The shortest possible route must be indicated by an unbroken line and the arrowhead will be used to indicate the direction. The indicated trip must show how the bends were rounded, how each turn was taken and which of those turnaround points were restricted and which ones were set up. The road widths on the map should be overemphasized to show all the information clearly.

Chapter 3

GPS APPROACH

3.1 General Notions

The Global Positioning System GPS, originally called NAVSTAR GPS (Navigation Satellite Timing and Ranging Global Positioning System) was developed by the U.S. Department of Defence only for Military purposes and has its origins at the beginning of the Space Race in the mid 1960's. It started with tracking US submarines carrying nuclear missiles using the Doppler effect to document changes in position while having six satellites orbiting. The Department of Defense of the United States (DoD) wanted a robust stable satellite navigation and motivated by the ideas of a navy scientist, the DoD launched the NAVSTAR. Today the GPS is owned and maintained by US Government having many fields of application in which are highlighted the defense, security, civil, commercial and scientific needs. This technology has been available for civil population since 1995. [14]

According to the information found in the official GPS website [9], managed by Official U.S. government, the GPS has its own constellation made of 24 satellites surrounding the earth in geostationary orbits and it is composed by 3 different segments:

- **Space Segment:** describes the GPS space vehicles, in other worlds, the satellites and its constellations. Every satellite has an orbit of 12 hours and travels the same route every time. There are 6 equally-spaced orbital planes with 4 satellites on each Plane, ensuring at least four satellites from virtually any point on the planet, it is possible to get a representation on Figure 3.2. Even if the idea is maintained, the 24 satellites the Space forces flies are more than expected to be able to replace a satellite when it is in maintenance or when it is to be removed. However in 2011 a constellation expansion took place and currently there are 27 satellites covering the orbital planes. These planes are spaced 60° and inclined 55° from equator. They circle the Earth in the Medium Earth Orbit (MEO) at 20.200 Km of altitude with 5-8 space vehicles visible everywhere.[18].

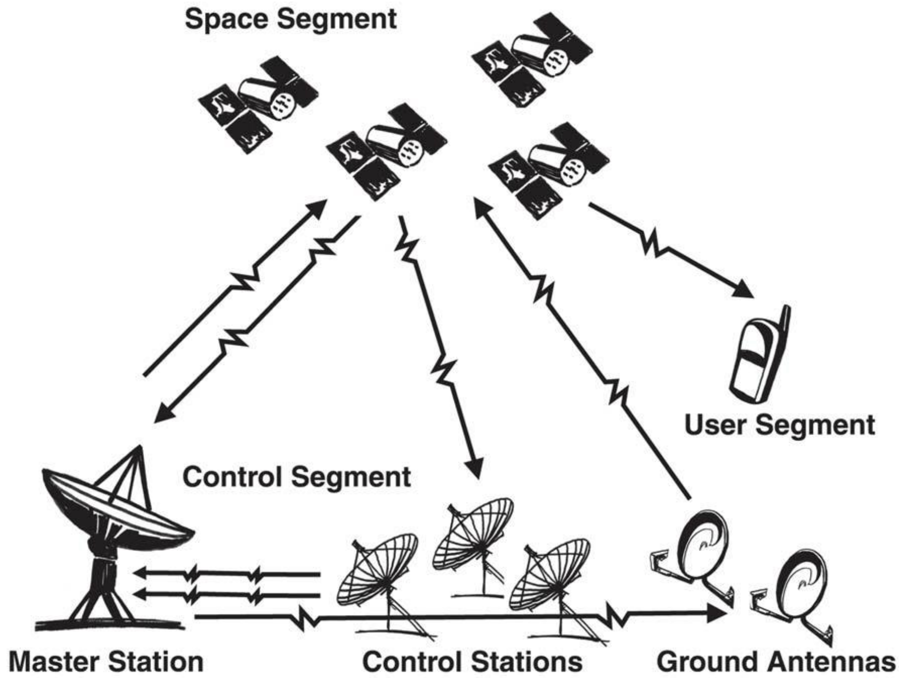


Figure 3.1: GPS Segments and their communications [1] .

- **Control segment:** this segment is located on Earth and these are represented by the 5 Master control stations and few support stations all over the world which monitor, perform analyse, send data and track the GPS satellites. These stations control the satellite signals and send the time corrections to get the correct ephemeris, in other words, the trajectory of astronomical objects in spherical polar coordinates system. The control segment is also divided in 3 elements and each of them performs a specific task :
  - Monitor Station: tracks the GPS satellites, collects navigation signals, range/carrier measurements and atmospheric data using specialized GPS receivers. Collects the information to feed the Master Control Stations.
  - Master Control Station: it is responsible for providing command and control of the GPS satellites and uses a global monitor station to establish the precise position of satellites. It is monitoring permanently the constellation health and its accuracy. Performs maintenance and anomaly resolution, including re positioning. The control station updates the ephemeris and clock data in every satellite.
  - Ground Antennas: they have the duty of sending the commands, navigation data uploads and processor program loads to the satellites. They also collect telemetry. Through S-band they communicate for anomaly resolution and early orbit support.

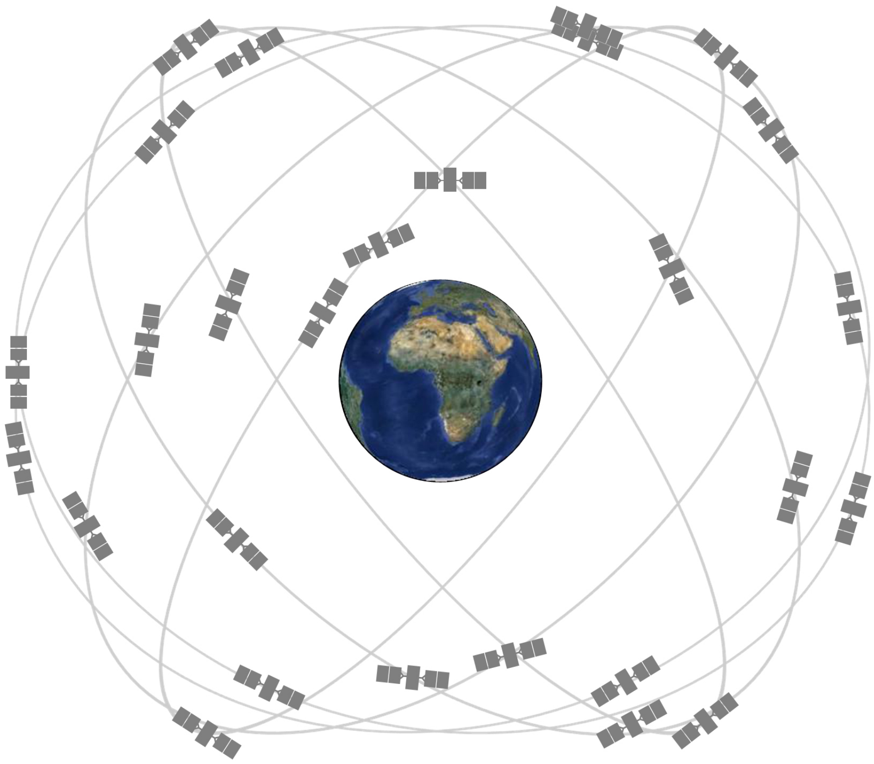


Figure 3.2: GPS Constellation around the earth [9].

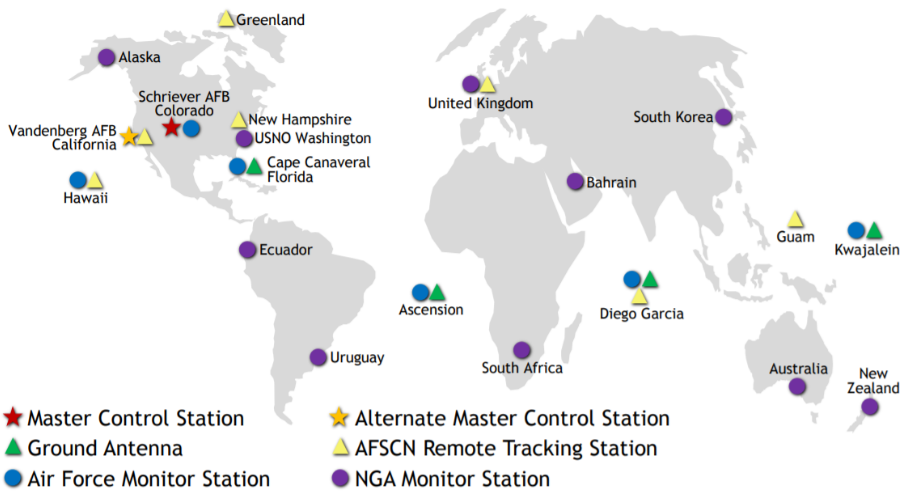
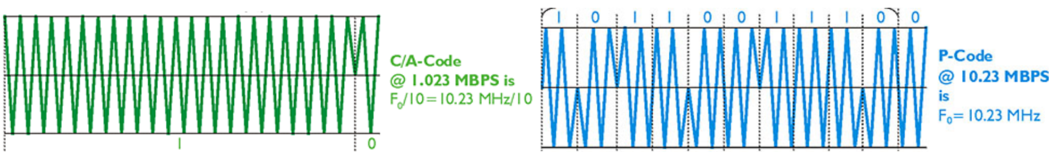


Figure 3.3: Main stations in the Control Segment [9].

- **User segment:** consists of the GPS receiver equipment, which receives the signals from the satellites and uses the information received to calculate the user's 3D position and time. Some sectors as major communication networks, wireless services, banking systems, financial markets and power grids depend on GPS due to its synchronization. The GPS was enabled to enhance the civilians lifestyle in many aspects but currently there are some advantages held for army purposes, for this reason they provide two levels of service. While the civil population uses the SPS (Standard Positioning Service), the military uses the PPS (Precise Positioning Service), They're modulated onto carrier waves and some of their differences are the frequencies and the type of code used[14].

- **Standard Positioning Service (SPS):** uses the Coarse/Acquisition (C/A) code (also known as Civilian Access code) only (Rate=1.023 Million bits per second). This service uses single frequency L1 (centered on 1575.42 MHz). This service is available to all users, free of any direct user charges.
- **Precise Positioning Service (PPS):** uses C/A code and P-code (Precise Code), the last one transmits series of ones and zeros at a rate of 10.23 Million of bits per second and represents the new military signal M-code[10].When P-code is encrypted it is called P(Y)-code Additionally it uses two frequencies L1 and L2 (centered on 1227.60 MHz) and in contrast to SPS, this service is restricted to military and security use

The Penn State Department of Geography [10],explains the features of these two main codes. The signals shown in Figure 3.4 are actually sine waves with sharp peaks and below or above the figures a sequence of zeros and ones is found, indicating the corresponding binary code of the chips. Both types of signals (C/A and P code) are modulated into carried waves using the Phase Modulation, which uses the variation on phase with the aim of carrying the modulation doing a degrees shifting instantly, visually it is perceived as a reversed direction. This technique is used due to its far-reaching distance quality.



(a) (b)

Figure 3.4: (a) C/A Code (b) P Code [10]

As noted, the GPS is a worldwide service supported by many segments, each one contributing to good results in geolocation. At present time, the commercial GPS receivers manufacturers are not authorized to use P(Y) code, thus techniques for carrier wave and a pseudorange indirect measurement on L2 were developed. As a result the manufacturers have started to generate a modernisation on GPS devices proposing new GPS signals [7][10], which will be shown in Table 3.1. However the users must upgrade the GPS receivers to the most recent version to exploit the service of the newest signals.

Signal Use	Name of signal	Description
Civil	C/A code	Represents the main civil use signal for GPS systems and describes a series of ones and zeros with a rate 10 times slower than P(Y) code. The C/A is on L1 radio frequency.
	L2C: Second Civil Signal	This signal is only for commercial demands and it is referred to the radio frequency L2. It allows ionospheric correction to improve accuracy, faster signal acquisition, better reliability and operating range.
	L5: Third Civil Signal	This signal is focused exclusively on aviation safety services and other high-performance applications. Named because of the radio frequency used by the signal (centered on 1176.45 MHz). Used with the previously mentioned signal, it will provide high robust signal through a method called trilaning reaching sub-meter accuracy.
	L1C: Fourth Civil Signal	It enables interoperability between GPS and navigation satellites. The radio frequency of this signal is 1575 MHz, in other words, the L1 frequency, and it is different from L1 C/A. The L1C signal uses a Multiplexed Binary Offset Carrier (MBOC) modulation used for international cooperation without neglecting the U.S security. USA and Europe developed this signal to be used in GPS and GALILEO constellations.

Table 3.1: Types of GPS civilian signals and its use

The military signals are not described in the previous table because they are not of interest because of an inability to access this type of signals.

3.2 Signal Features

Once knowing how the signals are sent by the satellites, it is also relevant to understand how these are processed to determine an specific position. As was been mentioned in [8],the C/A and P Code are both a sequence of ones and zeros carried by L1, L2 or L5 through phase modulation, which means that the encoded signal will be represented by shifting phases of the carrier waves. This modulation takes place multiplying the carrier wave (sine) with the code states wave (square wave containing the C/A and P code) and it is preferred because of its wide band, in other words it has a spread Power Spectral Density (PSD) due to the increment of bandwidth, thus the overall bandwidth of GPS signal is broader than the bandwidth of the transmitted information, nevertheless this increase in bandwidth also decreases its power and therefore it is an important issue when the user wants to determine its position in a covered location. The width of these signals takes more frequencies than needed which improves the signal to noise ratio, the accuracy ranging and the interference artefacts.

The rate of GPS components are multiples of the standard rate of oscillators, from this fundamental clock the frequencies L1, L2 and L5 are derived. While P(Y) code has a rate of 10.23 million bits per second (Mbps), thus the same as the fundamental clock, the C/A has 1.023 Mbps, in other words, it is one tenth of the fundamental clock. So 10 P(Y) codes are generated at the same time in which one C/A code is generated and as a consequence, the P(Y) is more precise.

The amplitude or frequency modulation are not considered, even so the phase modulation that allows a wider spread is the binary phase shift keying (BPSK), which is used to create the NAV Message, C/A and P code, which shifts between 0 (normal state) and 1 (mirror image state), inversely implementing an instantaneous 180° shift in phase in the carrier wave always at zero-crossing [8]. This modulation is rarely for other modulations used in telecommunications and the spectrum of this modulation is the result of the convolution of carrier spectrum and the symbol spectrum as has been mentioned in [5].

According with [4], the frequencies of the carrier signals are between 1 and 2 GHz because current researches determined that in this frequency range the effect of time delays produced by ionosphere is more mitigated and they are less affected by weather. Additionally, the L1 and L2 signals are transmitted at the same time without any type of interference.

The NAV message discriminates the useful code of C/A code and P-Code through the modulo 2-adder port which works as an exclusive OR gate (XOR), thus when both signals have opposite logic levels the output signal will have a high logical value, otherwise the output will have a low logical value. Once both signals pass through this process, they are mixed with the correspondent carry signal (L1 or L2) where the presence of a high logical value, as mentioned previously, correspond to a shift in phase of carrier signal. This could be represented in Figure 3.5.

All the satellites and the GPS devices generate the same signals called Pseudo

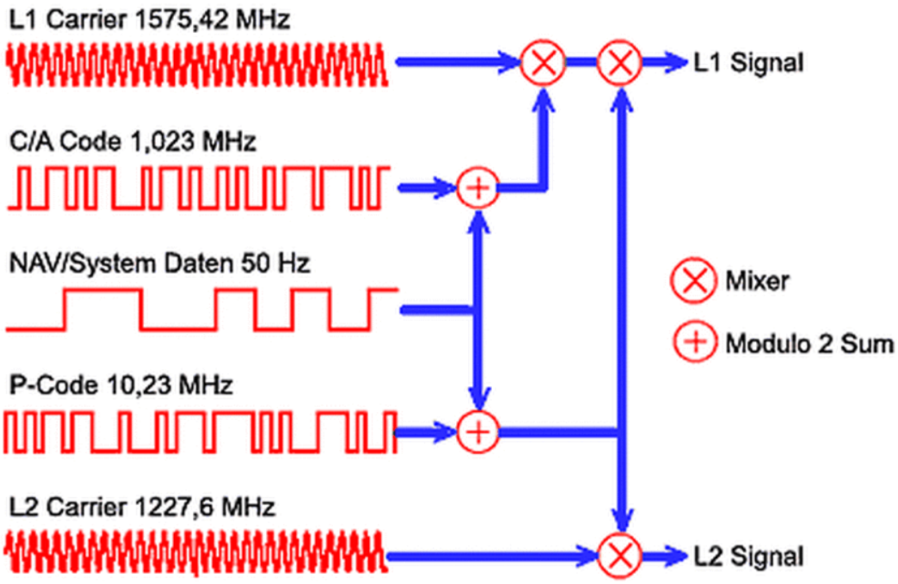
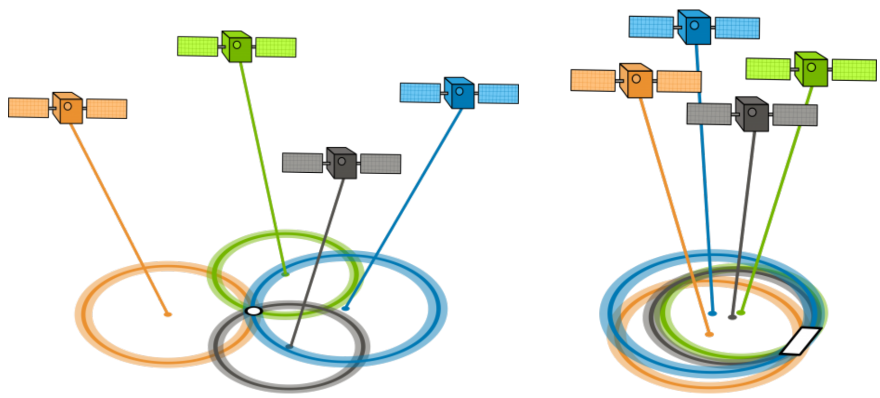


Figure 3.5: Phase modulation on L1 and L2 frequencies. [4].

random Noise (or PRN Codes) which includes the C/A code and the P(Y) and are represented as a noise but actually represent the sequence of zeros and ones code and using a simple equation between distance, velocity and time it's possible to get the distance from one satellite to a receiver. Where the radio waves travel at the speed of light, time is the measured delay (called also time of flight) between the two signals. In this way it's possible to get one distance, thus it's necessary to do the triangulation using at least 4 satellites, obtaining an accurate spatial position in 3D plane. The receiver solves 4 equations, 4 unknown systems and finds its position and local time. More satellites means more accurate localization.

As explained in [15], the Dilution of Precision (DOP) is able to determine a characterization of the geometry of signal between the user antenna and the satellites. Depending on how the satellites are located regarding the user, every satellite sends a signal into a specific range represented with a volume (or an area for 2 dimension purposes) with error bounds, the intersection of this error represents the DOP. When the involved satellites are over the user with low distance between them the DOP increases because the errors of estimated position of every involved satellite affect the measurements. In the other case, when satellites are dispersed (wide angles with each other) in regard to the user the error of every involved satellite is reduced, for that reason smaller DOPs are associated with a better position accuracy, however, this does not mean a low position error because the quality is estimated depending on the geometry measurement (DOP value) and the random source of error.

Nevertheless the measure of DOP can be separated describing the error vertically, horizontally, geometrically and temporarily. The most useful accuracy information



(a) (b)

Figure 3.6: (a) Example of a location with good DOP (b) Example of a location with poor DOP [6]

is quantified through PDOP (Position Dilution of Precision), HDOP (Horizontal Dilution of Precision) and VDOP (Vertical dilution of precision) parameters. Nevertheless these type of values are not easy to calculate, because the identification of the position of all involved satellites estimating one single position is required and most commercial GPS devices receive internal raw signals but their output is made of processed signals on terms of latitude and longitude. As users, it is not possible to access the raw information, seeing as the GPS device itself only outputs processed signals.

3.3 Mathematics Calculation

As explained by Richard Thompson in *The Mathematics of GPS Receivers* [22], each satellite sends its position through the two frequencies (L1 and L2) and the exact times of signal transmission. In this way the receiver device collects these signals to set and calibrate its own internal clock. This is a key step because subsequently the difference in time (Δt) is measured between transmission-reception time. Supposing that there was no atmosphere amid the transmitter and receiver, the distance would be easily calculated knowing the satellite position by $d = c \cdot \Delta t$ in which the light speed represents the speed of this electromagnetic (Radio) wave.

However, the layer of gases around the Earth modify the measurement and factors such as poor satellite signal and limited measured times and distances are solved using the GPS approach. The center of the Earth is considered the reference frame of the coordinate system and = at least four satellites are necessary to position a three dimensional point. In the most simple case, it is necessary to suppose that every satellite S_i with i = 1,2,3,4 is located in a point with coordinates (X_i, Y_i, Z_i) at just the same time as it is transmitting a signal at time T_i and when these signals are received at receiver time $T_i^{'}$ it is possible to calculate the difference of time as $t_i = T_i^{'}$ - T_i. With these considerations in mind, ε would be any error present in terms of time perceived by the receiver.

The receivers calculate the distance of every satellite $d(\Delta \mathrm{t}_i, \varepsilon)$ with only one possible value of ε to detect a point in common, after that the distances are included into a system of equations to determine the position of the receiver denoted by (x_0, y_0, z_0).

$$\left\{ \begin{matrix} (x_0 - X_1)^2 + (y_0 - Y_1)^2 + (z_0 - Z_1)^2 = d(\Delta t_1, \varepsilon)^2 \\ ((x_0 - X_2)^2 + (y_0 - Y_2)^2 + (z_0 - Z_2)^2 = d(\Delta t_2, \varepsilon)^2 \\ (x_0 - X_3)^2 + (y_0 - Y_3)^2 + (z_0 - Z_3)^2 = d(\Delta t_3, \varepsilon)^2 \\ (x_0 - X_4)^2 + (y_0 - Y_4)^2 + (z_0 - Z_4)^2 = d(\Delta t_4, \varepsilon)^2) \end{matrix} \right\} \tag{3.1}$$

When the solution is found, the rectangular position coordinates are transformed into spherical coordinates to deduce the latitude, longitude and altitude.

To summarize, the GPS receiver only needs to receive the time in which the signal was sent and the current position of satellites, then find an approximate solution for the previous system of equations and transform the coordinates.

3.4 Noise Sources

Some of the factors that deteriorate the accuracy on GPS, as mentioned in [22] and [16] are divided in systematic errors and random errors.

The systematic errors, also called bias error. It is the error that comes from persistent problems and can be determined consistently on every measurement product of problems of calibration or a faulty measurement device, thus it is an error than can be replicated easily and can be eliminated. On the other hand, the random errors, as suggested by the name, are unpredictable because of their random nature, additionally those can not be replicated and can not be eliminated.

The GPS can be to some extend calibrated to prevent the systematic error but most sources of error that affect the measurements are random. Some of most known sources of random error are:

- Delay propagation: referred to the delay of speed of GPS signals due to the changes of density of free electrons in the ionosphere and changes of humidity, pressure and temperature in the troposphere.

- Changes in the GPS clocks: cause wrong measurements because of the deviation of the clocks in the different satellites.

- Multipath propagation: the surrounding objects close to the GPS receiver (Buildings, trees, people, etc) that reflect the emitted signal generating a signal with a longer time of travel.

- Ephemeris error: due to error calculating the current orbital position of a GPS satellite at a given time.

- Satellite geometry dilution of precision: refers to the geometry of constellations and their relative position with the user (DOP).

It is important to avoid the systematic error as well as random errors and minimize them as much as possible to estimate more accurate results. Knowing the features of GPS devices and on the basis of a sportive accuracy required approach the next chapters will be dedicated to evaluate their accuracy.

Chapter 4

TEST

The proposed test includes the set-up of a method for measuring an unknown long distance, prove its accuracy and use GPS devices in order to register the tracked course and compare their resulting distances.

4.1 Test 1: Calibration for the Main measurement

Due to the difficulty of acquiring a Jones Counter dedicated to measurement purposes, the idea of building a similar device but based on a magnetic counter was proposed. Accordingly, an Odometer was built using a Reed magnetic sensor to assimilate the Jones Counter Method used in the official distance measuring of the World Athletics.

The Odometer is a device that measures the distance travelled by a vehicle or a bike. It may be constructed mechanically or electronically. Following the previous definition, the Jones Counter could be considered a mechanical Odometer but in the absence of this, a digital odometer made with a reed sensor has been opted for.

The Reed Magnetic sensor closes making use of a magnetic field. When there are no magnetic fields the switch stays open, closing when a magnetic field is nearby. The Reed switch was placed in the front fork of the bicycle, facing the spokes and connected through wires to an Adafruit feather M0 platform based on a Microchip ATSAMD microcontroller. At the same time a 16x2 display, the power bank, and the debounce components were connected and placed together on top of the bicycle stem. Additionally, 4 magnets were placed non-equidistantly in the front wheel spokes.

The system use is simple. Firstly, the perimeter of the front wheel is measured as a distance obtained by the difference between a line marked on the tire and the ground exactly below it, and a new line marked on the ground after a whole turn of the wheel is completed on a straight line course. The aforementioned distance is measured with a steel tape (in this case the mean distance was 2.126 m) and divided by the number of magnets on the wheel, obtaining the distance to be added by each event (where an event is counted every time one of the magnets on the wheel passes by the sensor placed on the bike),thus this system registers 4 counts for each revolution and every count represents 53.15 cm on the ground as shown in Figure 4.1. In this way it would be possible to measure a specific road race course and establish a reference measurement.

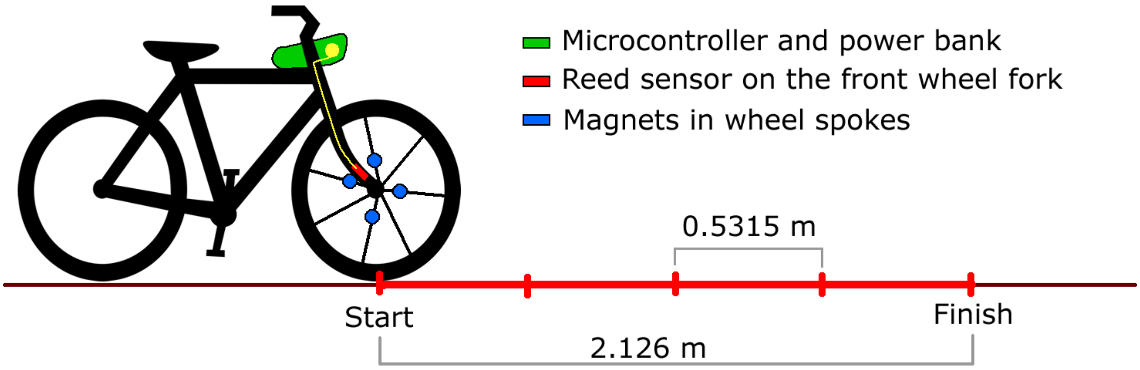


Figure 4.1: Digital Odometer installed in bike.

In steady state the opened switch generates low digital level read by the microcontroller and every time a magnet comes close enough to the Reed magnet Switch, the signal changes to a high logical level. Basically, the microcontroller counts the number of pulses obtained in the pulse wave in order to calculate the total distance of the route.

Even though the magnets have not been placed equidistantly, after a complete wheel turn, the total distance measured is equivalent to the real perimeter of the bike. Each event represents the distance of a part of the perimeter that comes together after a complete wheel turn. It is worth mentioning that the distance travelled using the odometer should not be done at high speed in order to ensure that the micro controller is able to detect the signal level changes.

To measure the bike's wheel an 8 meters steel tape was used. The specifications of steel tape are shown below :

- Nominal distance: 8 meters
- EC class accuracy: Class II (error of ± 2.3mm over a 10m length)
- Year of Manufacture: 2020
- EU Type examination: 0126 that is the designation for the National Weights Measures Laboratory in Middlesex

Due to the previous steel tape information, knowing the class of the instrument and the expected wheel distance, it is possible to have a measurement of 2.126 m $\pm$ 0.7mm of uncertainty. Eventually, the proposed calibration courses will be measured with the same steel tape.

4.2 Test 2: Measurement of road course

With the aim of comparing how similar are the distance measurements of GPS regarding the odometer method, the GPS devices are presented as well as the road course proposed. For its implementation, the entire road course is measured using the odometer many times to infer a reliable measurement in terms of mean value and standard deviation. After that, the four GPS devices are used at the same time to travel the same course numerous times and determine a mean measured distance.

In particular the recording of GPS points through the course is acquired starting and finishing approximately 20 meters before and after a the same reference point, likewise, when all the GPS pass over the reference point, the time stamps (HH:MM:SS) are recorded. The time registered will be used successively to crop the data points recorded with GPS in order to know the exact distance.

4.2.1 GPS devices

Following the previous studies and aiming to compare the performance and the acquired information of the different sources, the devices used for the proves are listed in the Table 4.1.

Device Name	**Producer**	**Type**	**Device Name / Model**
GPS 1	Garmin®	Handheld GPS	eTrex® 30x
GPS 2	Samsung®	Smartphone	Galaxy A52
GPS 3	TomTom®	Sport Watch	Spark3 /Runner 3
GPS 4	Arduino ® /Adafruit	Homemade GPS	Ultimate GPS Breakout V3

Table 4.1: Tested GPS devices

These devices which are considered low-cost receivers have been proposed thinking about ensuring the athlete's capability of acquisition, in order to allow for the reproducibility of the future proves. While the three first GPS devices were purchased considering the availability and the cost, the last one was created using the instruments given by the supervisor in charge and the Department of Electronics and Telecommunication (DET) of the Politecnico di Torino.

Due to the accessibility of the microcontroller board, a simple portable data logger was made to allow data acquirement in the same way as the GPS receivers made by companies do. For the last one an SD card module was also used to save all the track points, a power bank to ensure portability and a Adafruit Ultimate Breakout V3 GPS module. It is necessary to mention that while the GPS module is known by the last device, it was not possible to know exactly the GPS module used in the other devices.

While the collected data from the first three devices generates a *.gpx* file the GPS module used by Arduino can only receive data in NMEA sentences which are then saved in text documents (*.txt*), which has its own considerations. These will be explained in the next chapter.

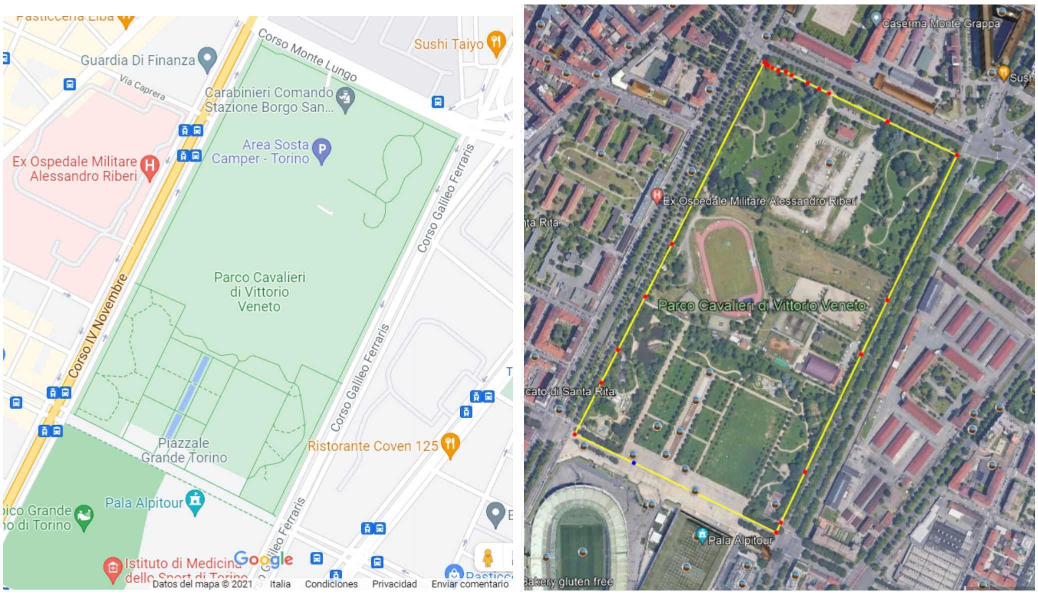
In the following chapters and sections, the figures and tables will be presented in terms of numbered name device, however the brands will be mentioned only as much as necessary.

4.2.2 Road Course proposal

In order to set a road course to evaluate the GPS devices, the route in Cavalieri di Vittorio Veneto Park in Torino (Italy) was selected due to its straight line path and reduced number of turns. This park is located between Corso VI Novembre, Corso Monte Lungo, Corso Galileo Ferraris and the Olympic Stadium. It was also necessary to propitiate all the possible beneficial conditions of the real measurement of the road race. For this reason, the tests were taken in different days during autumn season of 2021, in similar conditions of temperature and humidity, between the 23:00 and 5:00 with the aim of find less people walking by, enabling us to reduce as many obstacles and turns as possible.

According to the tool "Ruler" in Google Earth, the distance of the evaluated running race circuit is around 2.22517 km. With this measurement it is possible to obtain an approximation, however it is not exactly accurate seeing as part of the route traced is covered by trees, skewing its measurement. Another distance value for the complete road race course was found marked with paint on the ground which indicates the starting value 0 km and a full turn distance of 2.228 km but neither one represents a trustworthy value of the real road race course. The marked point in 0 km was used as a reference point for the next measurements made with the GPS.

Considering the lack of assurance of both prior methods of measurement, the previous tested Odometer method was employed to measure the real total distance



(a) (b)

Figure 4.2: Selected Route circuit: a. Top view of the Cavalieri di Vittorio Veneto Park extracted from Google Maps. b. Marked race route circuit

found in one full turn. Once the real distance was established, the GPS recordings were done at least 10 times for every GPS receiver. Nonetheless, the tracks were recorded in a particular way due to the uncertainty of the starting point. As mentioned in the beginning of this chapter, to limit the unpredictable starting and finishing point, the recording started 20 meters before the defined reference point and finished 20 meters after the reference point, with the purpose of cropping the data points.

Because of the time response of the magnetic sensor interfaced to the microcontroller, it was necessary to ride the entire path with a velocity of around 7 km per hour, considering always the shortest possible route delimited by the border of the floor, thus less than 30 cm away from the border.

4.3 Test 3: Static Accuracy

The GPS accuracy in static conditions, was assessed defining a reference "start" point and measuring a long straight path where a "finish" point was marked. All GPS receivers were positioned in both points and left acquiring for a long time (hours). The type of measures and positioning accuracy studied in this test is the Repeatable accuracy, which as it is was explained in [19], allows to return to a predetermined location informing about statistical distribution of the positions regarding a true value considered an average position from measurement series.

Specifically, a 700 m straight path has been proposed due to the road course conditions and an acquisition of at least 13.000 points (more than 3 hours and 36 minutes) was done following the recommendations of minimum significant number of location points made by Mariusz Specht in *"Statistical Distribution Analysis of Navigation Positioning System Errors—Issue of the Empirical Sample Size"* [20]. In this article Specht explains that the determination of mean coordinates ($\overline{\varphi}$ and $\overline{\lambda}$, predicted real mean latitude and predicted real mean longitude, respectively) and the statistical distribution of both values has a significant impact when the population is unrepresentative, which will lead to discrepancies between the coordinates and the actual values.

In this study the author tries to question the assumption of normal distribution of calculating position evaluating with empirical measurements and concluding that the position errors could only achieve a stabilization and be considered normally distributed when a large number of samples are used in the calculations. Some important concepts such as Root Mean Square (RMS) and Distance Root Mean Square (2DRMS) are used to evaluate the accuracy in 1 dimension (1D) and 2 dimensions (2D), respectively, but these measurements can not be applied if the acquired data deviates from the normal distribution and the only possible way to determine these measurements correctly is taking into account a representative number of location points. The effect of sample size on RMS or DRMS is represented in Figure 4.3.

As mentioned in [20], this problem of lack of representativeness of position accuracy measures is due to the influence of the Position Random Walk (PRW) presented on the recorded location points. While a random behavior of points around a mean is expected, the truth is that a PRW behavior is presented in positioning systems, in which a point appears in neighboring sites in regards to its forerunner point.

In contrast to 1 dimension (1D) accuracy measurements, the 2 dimension (2D) accuracy measurements come from the military science of ballistics in which the use of circular areas referred to a precise point to determine the number of points into this region of probability dependent radius is proposed. As mentioned in [15],[21] and [19], most concepts are used to assess the 2D accuracy in regards to the Circular Probable Error (CEP), 95% Radius(R95) and twice the Distance Root Mean Squared (2DRMS).

After the acquisition, the probable density of position errors were calculated, in order to demonstrate the results in [20]. Once the stability of data is demonstrated using the normal distribution as reference, it is necessary to introduce some important concepts used in ballistics in order to measure the static accuracy of each device proposed. The 2D accuracy measurements are presented in Table 4.2 where

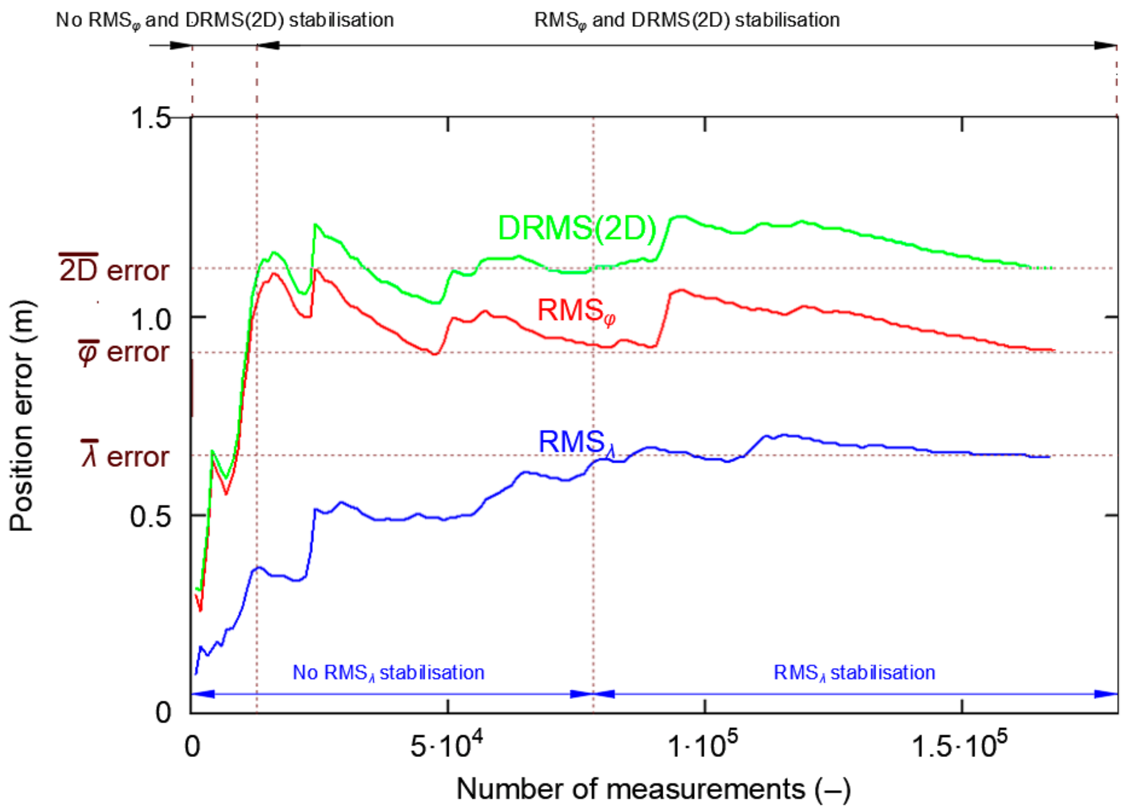


Figure 4.3: Evolution of RMS in latitude ($\overline{\varphi}$) and longitude ($\overline{\lambda}$) as well as 2DRMS depending on the sample size [20].

it is possible to find a description of the measurement, the confidence percentage and the formulas that were extracted in [19] .

As it had been done with the Test 2, more points than required were registered and also the staring and finishing time were recorded with the goal of extracting all the points from the same time window in all GPS devices.

Type of Measurement	Confidence Percentage	Description
CEP	50%	defined as the area in which 50% of estimated measurements are likely to be and represents an area of confidence concerning the GPS collected point. The radius of this circular area has the aim of evaluating the probability that the collected points will fall in with a particular accuracy. $CEP = 0.59 * (\sigma_\varphi + \sigma_\lambda)$
DRMS	63-68%	Represents the Distance Root Mean Square and it is calculated by the square root of average of squared horizontal position errors. $DRMS = \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2}$
R95	95%	Indicates the distance of the radius of a circular area in which the 95% of the collected data is presented. $R95 = 2.08 * CEP$
2DRMS	95-98%	Represents twice the DRMS distance calculated with the horizontal position error. This is the most convenient measurement. $2DRMS = 2 * DRMS = 2\sqrt{\sigma_\varphi^2 + \sigma_\lambda^2}$

Where:
σ_φ represents the standard deviation of latitude geographical component
σ_λ represents the standard deviation of longitude geographical component

Table 4.2: Static Accuracy Measurements

Chapter 5

DATA ANALYSIS

5.1 Analysis of Test 1: Calibration test

As mentioned on Chapter 4, faced with the impossibility of procuring a Jones counter as the official measurement in the World Athletics federation, the idea of using a calibrated odometer was proposed to have a baseline measurement of distance. To verify the accuracy of the device the calibration test was made using 4 different methods:

- Using **100 meters** as the calibration course and counting the number of counts in 4 rides around it.
- Using **40 meters** as the calibration course and counting the number of counts in 4 rides around it.
- Using a **complete wheel rotation** (2,126 m is the wheel's perimeter) and counting the number of counts in 4 consecutive wheel rotations.
- Measuring the **diameter of the wheel** and calculating the perimeter, measured 4 times.

Once the number of counts was acquired in each method 4 times, the mean of the number of counts was calculated for each method. These results were then divided by the known distance expressed in kilometres (for instance in a calibration course for 40 m, it is necessary to divide by 0.04). In this way the counts per 1 kilometre were estimated. The method of the 100 meters calibration course was chosen as the most accurate because there were less unwanted counts per meter. However it would be necessary to consider a longer calibration course to get a more precise result in terms of counts.
The results of the previous calculation were called "Day constants", as explained on Chapter 2. In the next tables (5.1, 5.2, 5.3 and 5.4) the results of the previous calculations are shown.

	Rides	Total Counts	Counts/ Ride	Distance (Complete Turns)[m]
100 Meters Calibration Course	1st	190	190	100.99
	2nd	379	189	201.46
	3rd	569	190	302.46
	4th	760	191	403.99
Average			190	100.99
Constant Day (counts/km)			**1900**	

Table 5.1: Results of the 100 Meters Calibration Course and its Day Constant

	Rides	Total Counts	Counts/ Ride	Distance (Complete Turns)[m]
40 Meters Calibration Course	1st	76	76	40.39
	2nd	152	76	80.79
	3rd	229	77	121.72
	4th	305	76	162.29
Average			76.25	40.53
Constant Day (counts/km)			**1906.25**	

Table 5.2: Results of the 40 Meters Calibration Course and its Day Constant

	Rides	Total Counts	Counts/ Ride	Distance (Complete Turns)[m]
Calibration by Perimeter	1st	4	4	2.126
	2nd	8	4	4.253
	3rd	12	5	6.379
	4th	17	4	8.505
Average			4	2.1263
Constant Day (counts/km)			**1881.46**	

Table 5.3: Results using the front wheel perimeter and its Day Constant

	Attempt	Diameter	Perimeter	Distance/ Attempt	Counts/ km
Calibration by Diameter	1st	68	2.1363	0.5341	1872.4111
	2nd	67.8	2.1300	0.5325	1877.9344
	3rd	67.9	2.1331	0.5333	1875.1687
	4th	68	2.1363	0.5341	1872.4111
Average			2.1339		1874.48
Constant Day (counts/km)			**1874.47**		

Table 5.4: Results using wheel diameter and its Day Constant

5.2 Analysis of Test 2: Measurement of road course

5.2.1 Reference length Measurement

Once the calibration course was completed, the next step was to measure the proposed road course (of an unknown distance) using the total number of counts perceived by the wheel in 5 attempts, aiming to complete the aforementioned attempts under the same conditions of humidity, velocity and space. The results are shown in Figure 5.5.The average number of counts per ride will be the main value and it will represent the total number of counts that the odometer detects for the entire road course, while the standard deviation of measured distance in every attempt represents the variation of the final reference measure.

	Rides	**Counts/ Ride**	**Total Distance [m]**
Rides on the total course	1st	4211	2238.447
	2nd	4216	2241.105
	3rd	4212	2238.978
	4th	4203	2234.1944
	5th	4209	2237.384
Average		**4210.2**	2238.022
Standard Deviation:		4.7644517	**2.532**

Table 5.5: Total counts and distance of complete road course

Type of Calibration	**Constant of the day**	**Calculated Distance (m)**
100 Meters	1900	**2215.894**
40 Meters	1906.25	2208.629
Perimeter	1881.46	2237.721
Diameter	1874.47	2246.064

Table 5.6: Odometer based distances

After obtaining the true number of counts of the entire road course, the next step regards the calculus of the measurement from counts to meters. For this purpose it is necessary to divide the total number of counts by the constant of the day of the respective method, which are portrayed in Figure 5.6. The results determine that the real length of the path is 2215.894 m $\pm$ 2.532 m.

5.2.2 Data Reading

The first step to extract features from the GPS files is to read them in the proper way. The company that manufactured GPS devices uses commonly the *GPX format* that is also called *GPS Exchange Format* which is an easily exchangeable format among GPS receivers and with computers. These types of files are commonly open format focused on software utilization and information such as track points, route points and waypoints. They are saved on XML format which allows an easy importation/exportation of the relevant information such as latitude, longitude (both in Decimal degrees [DD]) and time in UTC (Universal Time Coordinated) time scale. This is the format used by the Garmin GPS, the smartphone and TOM TOM sport watch.

On the other hand, the NMEA files are a special case because the track points are acquired as a text in which each feature of a single point is separated by comas from the rest and they are all collected in one single row. The NMEA data standard has many types of sentences in which the first word of every sentence would determine the information contained on it, that is why it is important to get the correct information and the correct NMEA sentence. During the data acquisition the GPRMC sentences were registered, which represents the recommended minimum data for GPS and has relevant information such as time in UTC, latitude and longitude in DD° MM.MMM'(Degrees and Decimal Minutes format) with their respective direction, speed over ground and current date, among others. The GPS made with Arduino collects the information through NMEA sentences.

In order to analyse and depict the test described on the Chapter 4, the collected data were processed on MATLAB® using the Geographic plots functions which allow the use of the GPX files as input. Moreover, the recorded NMEA are not easy sentences to process, that is why they should be imported to Excel splitting the different kinds of parts that were firstly delimited by comas. After that, it was necessary to save this spreadsheet file and then import the necessary data (Location data and time) by columns through MATLAB® *"Import Data"* option to be stored as a numeric matrix.

Even though the time information was in the same format, the location data instead needed to be processed to have them in the same format. Due to a single NMEA file source, the location data from Arduino GPS should have been transformed into a Decimal degree to standardize and compare all the devices. The Decimal degree format represents the variation of spaces of latitude and longitude by 1° that is expressed in decimals, e.g. the decimal degree coordinate (45.043762, 7.650030) means that the position is 45.043762 degrees north from equator and 7.650030 degrees east from Prime Meridian, in the case of negative latitude and longitude coordinates these represents south and west, respectively.

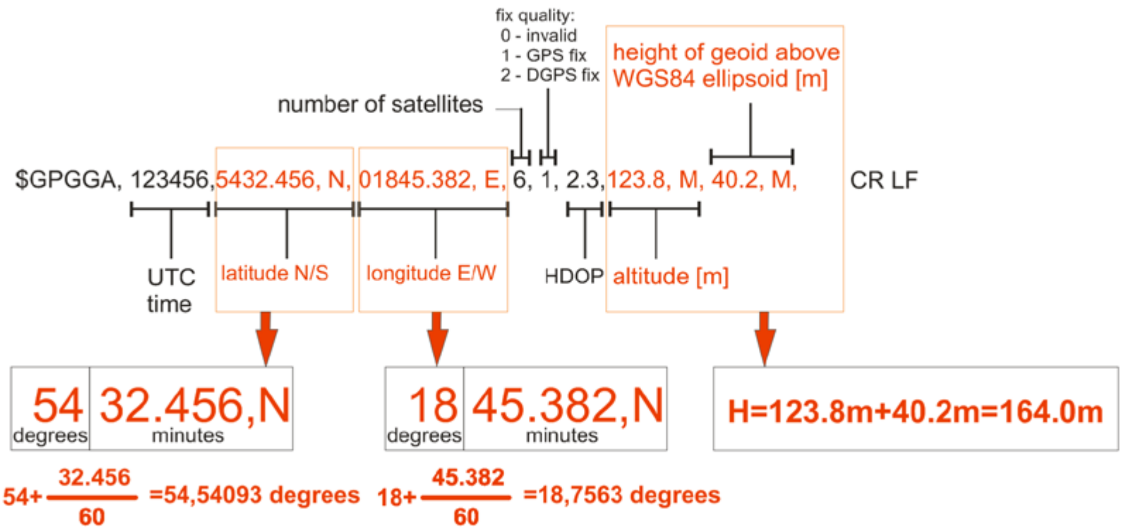


Figure 5.1: Convert Degrees and Decimal Minutes to Decimal Degrees format.[19]

To convert Degrees and Decimal Minutes to Decimal Degrees format it is only necessary to divide the part of decimal minutes by 60 and then add this to the degrees part as shown in Figure 5.1. After this, it is possible to analyse every registered point in the same reference frame.

5.2.3 GPS Distance

Obtaining the real distance of the road race course using the odometer measurements (Test 1), the GPS tracks were registered at least 10 times for every device to restrict the value within an uncertainty range and estimate the distance correctly. It is important to clarify that all the recorded tracks were collected in different days trying to maintain similar conditions. The distance collected by every GPS are presented in Table 5.7 .

Calculated Distances [m]			
GPS1	**GPS2**	**GPS3**	**GPS4**
2229.428134	2215.868367	2200.209848	2231.139055
2224.878055	2225.062551	2212.631292	2238.292947
2215.783862	2229.895731	2234.360000	2212.294709
2224.529813	2225.743597	2222.302483	2219.011146
2216.859174	2225.872222	2216.777285	2218.695248
2203.463649	2215.492352	2219.673982	2211.305338
2216.513766	2213.254244	2216.495082	2228.750652
2223.982144	2209.677751	2217.720011	2226.817201
2212.110209	2216.497325	2207.721932	2216.795289
2214.79605	2223.03913	2219.066194	2217.366852

Table 5.7: GPS measured distances

Value	**GPS1**	**GPS2**	**GPS3**	**GPS4**
Mean value [m]	2218.234486	2220.040327	2216.695811	2222.046843
Std.Deviation [m]	7.60457917	6.682971041	8.993085361	8.78581817

Table 5.8: Mean distance and standard deviation by device

As shown in the Table 5.8 the uncertainty in GPS measured distance varies between 6 and 9 meters. The precedent results will be compared in the Chapter 6.

5.3 Analysis of Test 3 : Static Accuracy Assessment

The static distance points were registered on two different days due to battery limitations of some devices. The first recorded data were collected between 8:20h and 12:00h while the second ones were collected between 9:46h and 13:26h, both at Central European Time (CET), therefore starting from different surrounding environments and different meteorological conditions (humidity, precipitation and presence of clouds).

As was previously explained in Chapter 4 the study requires the assessment of the GPS devices used for distance measuring with the aim of knowing how reliable are the recorded points. The 2D position error values were calculated first finding the standard error distance in latitude as well as in longitude, then the values were calculated using the formulas of Table 4.2 using MATLAB®.

With the aim of proving the absence of enough GPS points presented in the article *"Statistical Distribution Analysis of Navigation Positioning System Errors—Issue of the Empirical Sample Size"*[20], more than 3 hours and 40 minutes of data were registered in order to measure the mean position and infer its probability density function of Latitude and Longitude error. Following the indications made in Chapter 4, the total data were cropped to obtain at least 13.000 GPS data points for each in the beginning and in the end of the straight long distance path using all devices. The Figure 5.2 describe the Probability Density Function (PDF) of Latitude(φ) and Longitude (λ) error of the starting point, in which it is possible to observe that the GPS 1 and GPS 4 (Garmin and Arduino GPS) have an histogram similar to a normal distribution due to the representative number of points recorded, even so, the GPS 2 and GPS 3 (Smartphone and Sportwatch) do not have a precise distribution, but the number of measurements near to the real mean value are higher in this two last cases, additionally the variation of the error distance is small in GPS 2 and 3 which represents a better accuracy, instead the GPS 1 and 4 have a greater dispersion of data.

Moreover the Figure 5.3 describes the Probability density function of Latitude(φ) and Longitude (λ) error of the finishing point, in which its possible to observe that in the same way as starting point the GPS 1 and 4 have a higher dispersion of values while the GPS 2 and 3 are less dispersed and values near to the real mean position have a higher probability.

Nevertheless, it is possible to appreciate a special case in which in GPS 3 seems to be constant for a large number of values. This phenomena was seen in previous static tests and it is thought to happen because of the internal operation of the TomTom GPS, which does not acquire data at least when the internal accelerometer changes its dynamic state, thus it could be precise but it is difficult to know whether it is effectively accurate. It seems that it enables the recording of a different location when it senses a velocity and hence, a change in the position or when the received signals have many variations.

It is feasible to determine normal distribution in GPS 1 and 2 in Latitude and Longitude error with the number of points collected, nonetheless, the GPS 2 and 3 do not present this shape due to internal operation features. Following the results the 2D accuracy measurements were done because using this representative number of samples, the effect of the PRW is mitigated allowing the stability of the values estimation.

As mentioned in [21], there are so many position accuracy measures because the errors of position using GPS devices are not constant and they can fluctuate statistically, which can be correctly observed using the Scatter plot. Initially to assess the system accuracy it is required to know the true coordinates determined

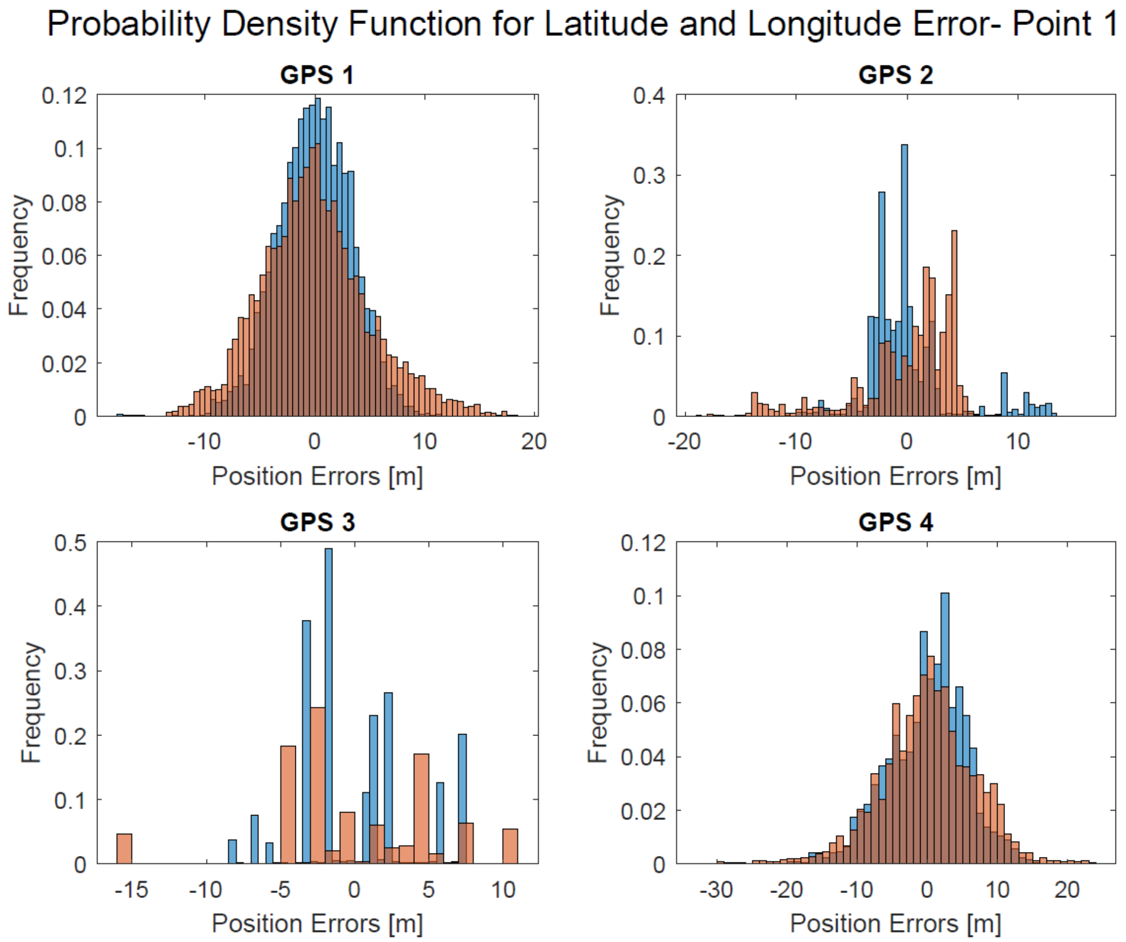


Figure 5.2: Probability Density Function for Latitude (φ in orange bars) and Longitude (λ in blue bars) Error of starting point

with a higher accuracy system but because of its lack in this type of devices the mean coordinates have been considered the true estimated point, which will provide repeatability to the test, although the measurements will undoubtedly contain bias, affecting the results.

After knowing the PDF of both points, the first step to approach the calculation of the 2D accuracy measurements is to find out the values of standard error of all measured points regarding the mean position. This is done calculating the distance between all points and the mean value in Latitude axis as well as in Longitude axis. A brief description of the main position and its respective standard errors are presented in Table 5.9.

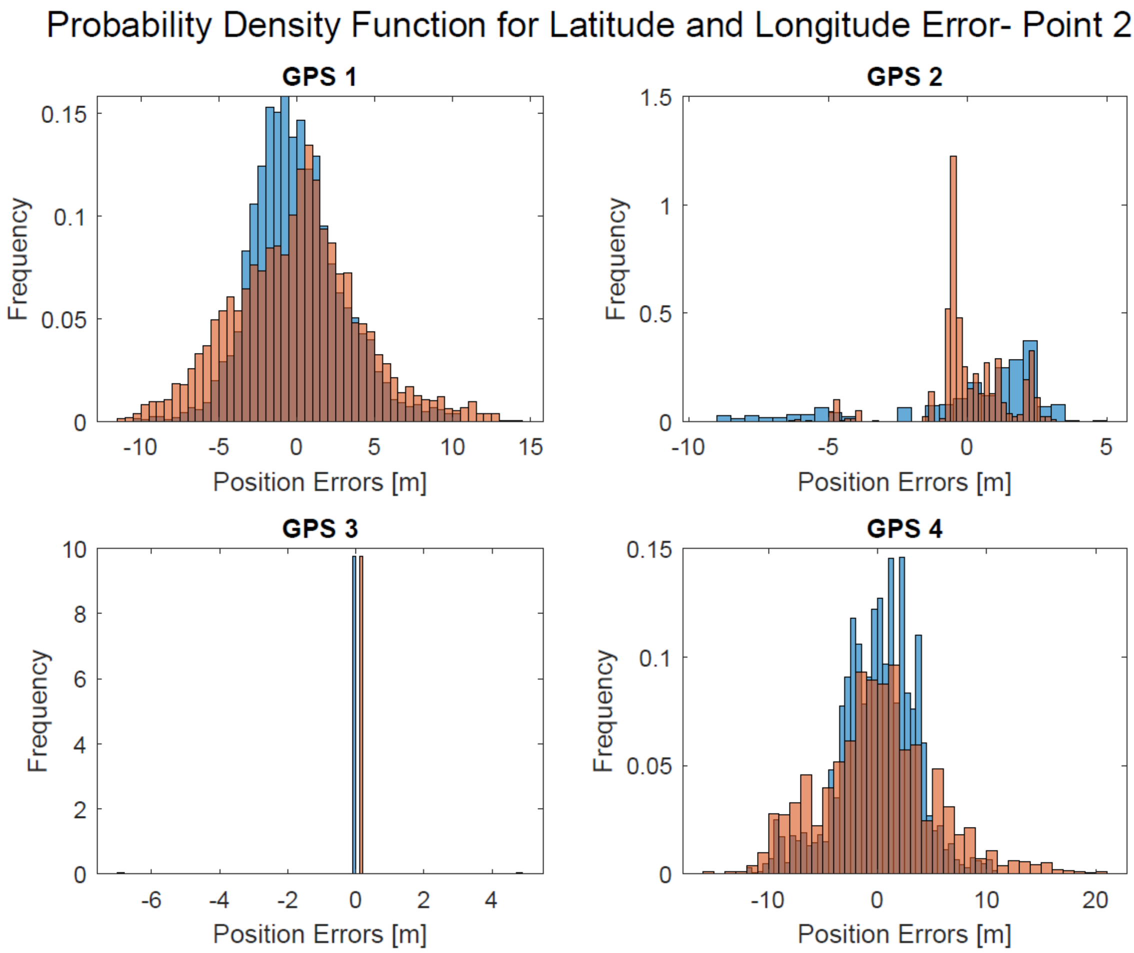


Figure 5.3: Probability Density Function for Latitude (φ in orange bars) and Longitude (λ in blue bars) Error of finishing point

	Device	Mean Latitude [DD-North]	Mean Longitude [DD-East]	Std. Error Latitude(σ_φ) [m]	Std. Error Longitude(σ_λ) [m]
Starting Point	**GPS 1**	45.0438	7.6500	4.940	3.370
	GPS 2	45.0438	7.6500	4.477	3.891
	GPS 3	45.0438	7.6500	5.562	3.864
	GPS 4	45.0438	7.6499	6.783	5.535
Finishing Point	**GPS 1**	45.0494	7.6540	4.105	2.912
	GPS 2	45.0494	7.6540	1.538	2.967
	GPS 3	45.0494	7.6540	0.810	0.555
	GPS 4	45.0495	7.6540	5.364	3.581

Table 5.9: Mean coordinates and its standard errors

It is possible to appreciate that the mean position coordinates converge to a single point which is a good indicator of accuracy and reproducibility, resulting in feasible location at least for the true stationary point. By contrast, it is also evidenced that the standard deviations of the error in meters are not similar but they are found under the real approximated values and the devices with less standard errors are the GPS 2 and 3 at least for the information in the finishing point.

After using the formulas described in Chapter 4,the accuracy measurements values of the starting and the finishing point are shown in Table 5.10, likewise, the representation of the most important values (CEP, DRMS and 2DRMS) are exposed in Figure 5.4 for starting point and Figure 5.5 for starting point.

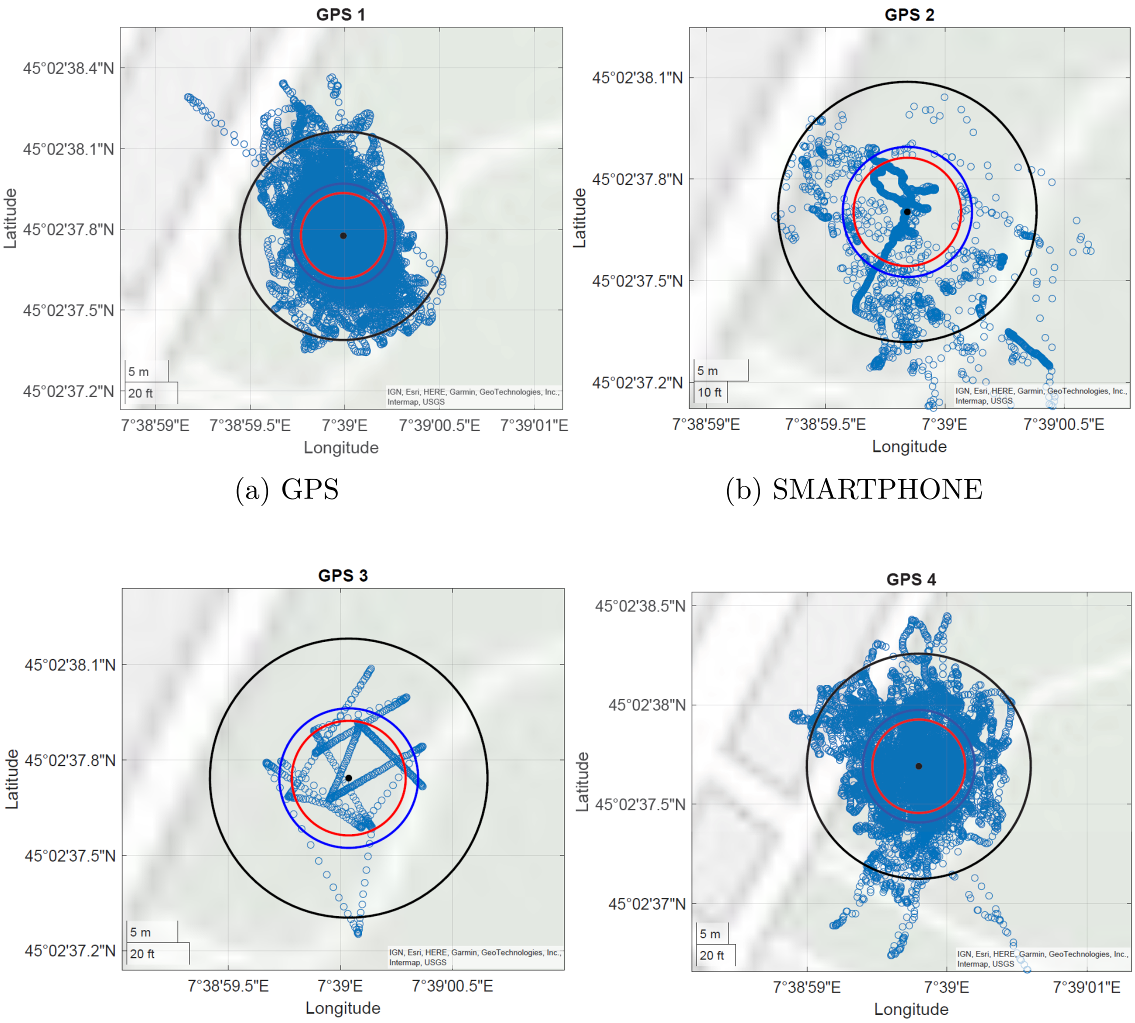
	Device	**CEP (50%) [m]**	**DRMS (63.2%) [m]**	**R95 (95%) [m]**	**2DRMS) (98%) [m]**
Starting Point	**GPS 1**	4.90	5.98	10.19	11.9620
	GPS 2	4.93	5.93	10.27	11.86
	GPS 3	5.56	6.77	11.56	13.54
	GPS 4	7.26	8.75	15.11	17.51
Finishing Point	**GPS 1**	4.14	5.03	8.61	10.06
	GPS 2	2.65	3.34	5.52	6.68
	GPS 3	0.80	0.98	1.67	1.96
	GPS 4	5.27	6.45	10.97	12.90

Table 5.10: Static Accuracy measurements results for each device in starting and Finishing point. 13000 location points

Through the figures mentioned previously, it is possible to appreciate many fluctuations due to the larger number of processed points.

As mentioned previously and then visualized in Figure 5.5 (c), an unusual case is the GPS 3 because of its lack of variance on the samples. GPS3 is a sport watch that automatically stops the acquisition when it is not moving. In the absence of enough different data, the static accuracy of the sport watch can hardly be measured, since its values of CEP, DRMS and 2DRMS are very small and not meaningful.

In the other sub figures (a), (b) and (d) of Figure 5.4 and 5.5, it is possible to identify the trace of all static points received from all the assessed devices. In some cases it is appreciated that points still have a PRW behavior that can be diminished with more larger acquisitions. As seeing, the CEP, DRMS and 2DRMS can state a level of accuracy depending on the radius circumference, the smaller the circumference, the better accuracy the devices have, indicating that a greater number of



(a) GPS

(b) SMARTPHONE

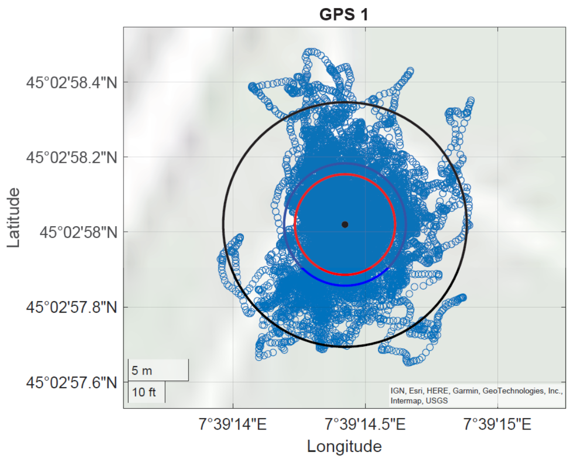
(c) TOM TOM

(d) ARDUINO

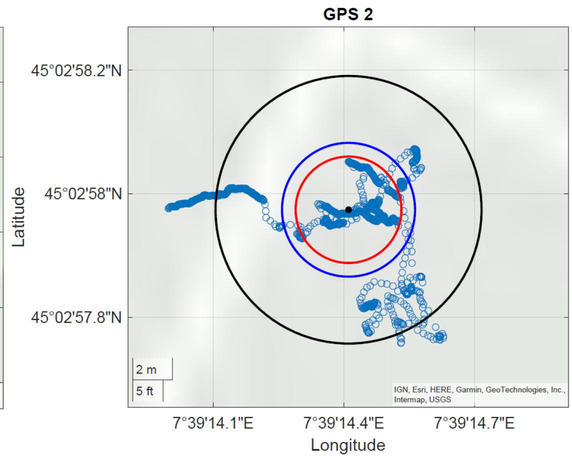
Figure 5.4: Static points measurement for all GPS devices in starting point. The CEP area is limited by the red circumferences, while the DRMS and 2DRMS are limited by blue and black circumference, respectively.

points are less distanced from the mean location coordinates.

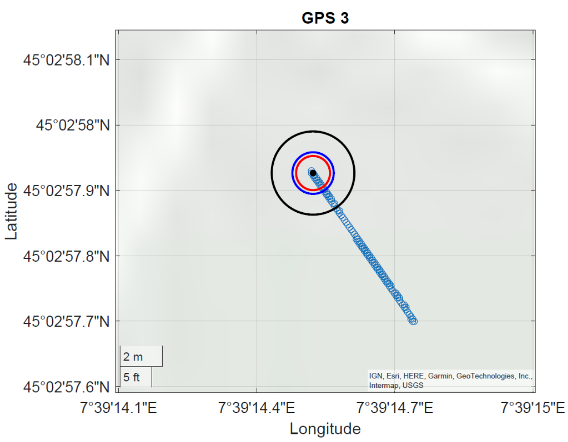
The *GPS WORLD PAGE* [21] expresses that accuracy measurements as CEP or R95 are directly defined from the position error distribution through the statistic empirical rule (in which a percentage of presented data will lie inside a defined number of standard deviation range from the mean value), thus it is feasible to associate them with error probabilities. The article also supports the possibility of deriving the probabilities associated with DRMS and 2DRMS when the distribution along north and east axis is assumed to be normal or Gaussian.



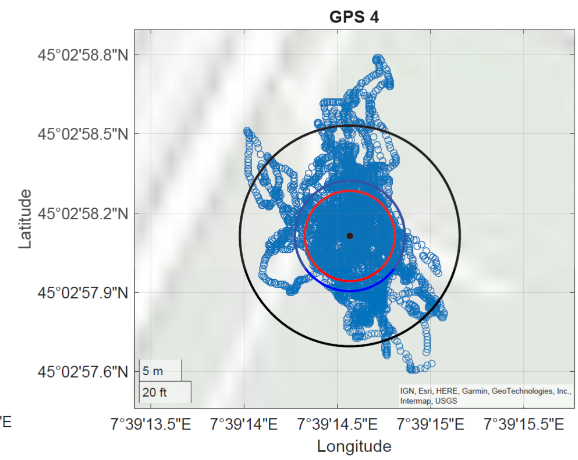
(a) GPS



(b) SMARTPHONE



(c) TOM TOM



(d) ARDUINO

Figure 5.5: Static points measurement for all GPS devices in finishing point. The CEP area is limited by the red circumferences, while the DRMS and 2DRMS are limited by blue and black circumference, respectively.

Considering the precedent part and comparing the results on Table 5.10 the most accurate device is the GPS 2 (Smartphone) because of their lower CEP (4.93m and 2.65m), DRMS (5.93m and 3.34m) and 2DRMS (11.86m and 6.68m) in starting point as well as in finishing point. Moreover, the simple device made with Arduino, obtained the worst results obtaining a distance 47% longer in starting point and 98% longer in finishing point regard to the best device (Smartphone).

It is very important to remember that the acquisition of points was made in different days in different conditions, while the first one was made in a place surrounded by a greater number of trees and walking people the second one was done with few trees surrounding and few walking people near the devices. It is a critical consideration because the effect of scattering and diffraction of GPS signals is demonstrated in the results of both points.

5.3.1 Long distance comparison

The distance between the two points defined in previous Section, was measured using the odometer and it was 700.517m (1318 counts).

Through the figures showed previously, many fluctuations were detected due to the larger number of processed points. The GPS mean position coordinates was considered to assess the length and the error with respect to the odometer was calculated for every device. The results are shown in Table 5.11.

Device	**Measured Distance**	**Percentage Error [%]**
GPS 1	700.21	0.04
GPS 2	702.3	0.25
GPS 3	699.09	0.20
GPS 4	708.46	1.13

Table 5.11: Calculated distance and its percentage error regard to distance measured with odometer

The previous results show that when an acquisition of many number of points is done, more accurate will be the length measurements, at least for straight paths. Additionally, the relative percentage error are well below 1% except for GPS 4 that has an error of about than the 1.2%.

Chapter 6

RESULTS

6.1 Discussion

As shown in Table 5.8 the higher distance error is presented in GPS 3 and GPS 4, while the lower error is in GPS 1 and GPS 2. It is necessary to mention that some of collected values were considered as outliers because they represented more than the 1% error of the mean value, therefore, new measurements were recorded to do the analysis of entire sets.

In the Figure 6.1, the error plot of distances were shown as well as the mean value and the error range of odometer measurement (2215.89m $\pm$ 2.532 m). The mentioned figure shows mean values of GPS are above the real value, in the same way the figure evidences a larger variation of GPS length estimation. Comparing the results of the GPS distance with the main measurement, it is possible to conclude that most GPS receivers overestimate the distance of the real track.

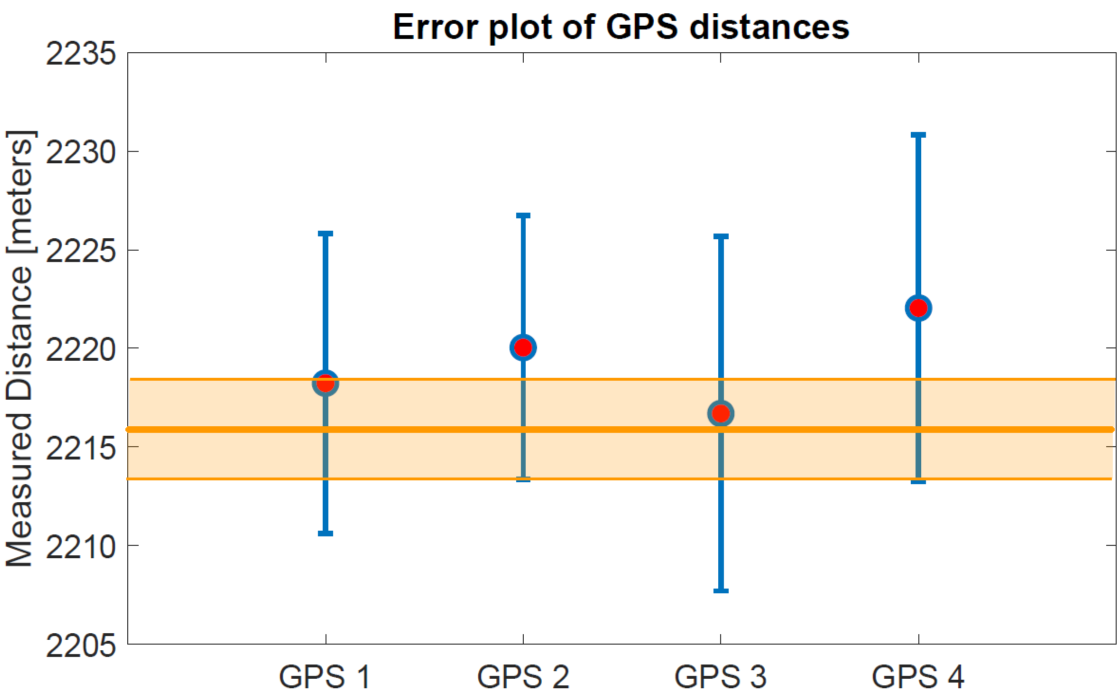


Figure 6.1: Error plot of GPS distance (mean value and standard deviation, see Tab. 5.8, obtained with ten repeated measurements). The orange bar represents the Odometer Measurement (mean value and standard deviation with five repeated measurements).

Following the acquisition of these results, the total distance of the assessed track was calculated using MATLAB® and the raw data. Then, the obtained results were placed in the table and compared with the reference values, in this way calculating the percentage error.

Following the comparisons, the Table 6.1 shows the error distances obtained as the difference between the mean distance measured by the odometer and the mean GPS distance .

	GPS 1	**GPS 2**	**GPS 3**	**GPS 4**
Error Distance [m]	-2.33	-4.14	-0.80	-6.15

Table 6.1: Error Distance with respect to the Odometer measure.

The error is negative and thus the GPS results tend to overestimate the distance measurements as mentioned in [16]. The previous study states that the measurement errors cause a systematic overestimation of measured distance if the interpolation error can be neglected. The authors mention that whenever the errors in latitude and longitude are unbounded (in the case of normal distribution), the expected distance is strictly higher which can be also shown in Table 5.11 with the last test. They also talk about the Overestimation Distance (OED) factor which indicates the residual distance from the estimated and the real value, stating that the OED increases as the spread of GPS measurements error increases assuming a constant distance. In the test 2 the interpolation errors were not be taken into account because most of the total race course was travelled in a straight line and there were few sudden changes in movement such as sharp turns.

Percentage Error (%)

	GPS 1	GPS 2	GPS 3	GPS 4
100 Meters	**0.11**	**0.19**	**0.04**	**0.28**
40 Meters	0.43	0.52	0.37	0.61
Wheel Distance	0.87	0.79	0.94	0.70
Diameter	1.239	1.159	1.308	1.069

Table 6.2: GPS percentage error, with respect to the odometer calibrated in different ways.

According to the manual of measurement used by World Athletics, the uncertainty of the measurement shall not exceed 0.1%. As seen in the table 6.2, only the measurement made by the GPS 3 reached a close value of 0.04%, while the GPS 1 although it is at the limit of the uncertainty, in the same way as GPS 2 and GPS 4 did not comply with the requisite. The calibration performed over a long distance provides the best results, as expected.

6.2 Limitations

Some of previous studies dedicated to assess the GPS watches in measuring distance [17] obtained the data in massive marathon competition with GPS sources from more than 250 participants, which provide a better and higher number of samples with the aim of comparing their accuracy by time and by brand. However in this thesis it was not possible to access one of these events, for this reason all tests were made by the candidate performing repeated tests. For further studies a larger number of GPS total distance recordings would be recommended, although, the bigger the source of data are, the more difficult to control all data sources is, that is why a correct acquisition of measurements by the participant is an important factor for future research.

Due to the lack of number of tested GPS devices and number of tests done, it was not possible to do a non parametric statistical analysis such as Kruskal Wallis test as made by Rebecca E Johansson et al [17].

In most of the cases it is better to maintain a fixed relative position of the GPS over the recording, because carrying the GPS device while running adds another degree of freedom to the measurement that could difficult the acquisition of data due to the increase of scatter probability.

The random errors as mentioned in [16] are solvable up to a point, because the user is not able to control variables such as humidity, reflection due to ionosphere, the effect of scattering of the radio signals.

Moreover, the considered course path only is very simple since it is composed of four linear part with four 90 ° curves. Real courses have a more complex shape and thus errors related to curves could play a more significant role.

6.3 Improvements

- The use of GPS devices with a higher sample rate (more than 2 Hz) may improve the performance of the test obtaining a higher number of points per second and processing them. Interpolation algorithm could also improve the accuracy but weren't employed here for lack of time. An alternative study using GPS, GNSS (Global Navigation Satellite System), and IMU (Inertial measurement unit) could motivate further researches about accuracy. Furthermore, it is recommended to use modern GPS devices because as seen in the data analysis, the values of CEP, DRMS and R95 are smaller in most recent devices such as the Samsung smartphone (2021) and TomTom smartwatch (2016), in contrast the Garmin handheld GPS (2015) and the Adafruit GPS Breakout (2014) had the most distanced values. Thus, the newer the device is, the better the processing included, in this way it is possible to ensure a better performance with regards to the GPS used in this thesis.

- In order to enhance the accuracy of estimated value of the total distance path, a larger number of distance sample is proposed with the goal of knowing a most accurate number of counts made by the odometer reducing the uncertainty of measurement and mitigating the random errors. In this way more samples from a specific population allow to infer a more accurate result.

- For more than 15 years the GPS devices produced by specialized companies implement a extended Kalman Filter (used to predict unknown states exploiting the previous measurements over time) or at least a type of filter into their receivers, which improved considerably the position location obtaining more robust results in terms of accuracy. Regarding the used GPS devices, it is necessary to clarify that the GPS 4 made with Arduino was made only to receive the location data, thus filtering methods as Kalman filter were not used which could have represented a source of error seen in the results. It could be a subject of interest for further studies.

6.4 Conclusion

The used measurement method based on an odometer can be an accessible method to estimate the real length of race courses, however it will not secure a high accuracy as the mechanic Jones counter does.

In general, the previous test proves that the accuracy of GPS devices does not have as badly predicted results, due to the limited percentage error which in most cases does not exceed the 0.5% for the most accurate reference measure.

Nevertheless, only the smartwatch with a percentage error of 0.04% (which represents an error of 0.886m from the reference length) could fit the requirement of a minimum distance uncertainty below the 0.1% of the total road course. The other type of devices are not as accurate as required for official competitions but they still have similar values of percentage error with regards to reference length. Moreover, most of the measurements done could suffer an overestimation in regards to the real distance measured which represents a problem at least in the official competitions of the World Athletics Federation. Thus, although the GPS trackers generate optimal results for users' needs, these devices can not be considered appropriate to measure the length of an official road course.

Taking up the effect of the Position Random Walk (PRW) on the data acquisition, a large number of samples are proposed in order to be close to the real accuracy features of the assessed system. In addition, as mentioned in [20], the accuracy of positioning systems is constantly changing and improvements in technology will increase the accuracy of the GPS systems reaching a better estimation of DRMS and 2DRMS values when representative samples are recorded.

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