POLITECNICO DI TORINO

Corso di Laurea Magistrale
in Ingegneria Meccanica

Tesi di Laurea Magistrale

Techno-Economic Optimization Under Uncertainty of a Wave Energy Converter

Relatori

prof. Giovanni Bracco
prof. Giuliana Mattiazzo
prof Antonello Sergej Sirigu

Candidato

Filippo Giorcelli

Anno Accademico 2020/2021
Sessione di Laurea Dicembre 2021

CONTENTS

ABSTRACT

In the field of renewable technologies, the possibility to obtain energy exploiting seas and oceans' wave motion has been known for a long time. Devices called Wave Energy Converters (WEC) have been developed with this purpose, thanks to which it is possible to transform wave energy into electric energy. Following the design studies carried out in recent years, the research now proceeds towards the development of useful processes for the optimization of these devices.

The purpose of this thesis work is to study, analyze and then apply a robust optimization process to the WEC system, in order to increase its reliability and robustness.

Robust optimization is a probabilistic solving method for real-life optimization problems, in which there are uncertain data that, due to their stochastic nature or uncertainty (linked to design condition changes or to wear), can float around their project value. This method studies these parameters and finds suitable solutions with the aim to avoid unsatisfactory system performances and designs which can compromise their functioning, studying and highlighting at the same time the influence that specific parameters can have. Robust optimization considers uncertain data belong to a specific set called "uncertainty set" (with specific constraints), that must be defined by the designer. Therefore, the robust optimization process final purpose is to obtain a set of solutions able to limit, below specific thresholds, the variation of such uncertain data (and consequently their own uncertainty), finding a robust optimum instead of a global optimum in order to be able to increase the reliability and robustness of the system.

In this work a bibliographic search is at first carried out to describe the state of the art for this field, bringing also examples of engineering design applications in which this optimization technique is employed and analyzing the most used algorithms, with greater attention to evolutionary algorithms, family that also includes genetic algorithms, which are used in this thesis during for the optimization process. Then the case of study is described, in particular this thesis will examinate the PeWEC's robust optimization design problem. The device is considered to be located on the Island of Pantelleria.

Then we proceed with the application of a chosen framework for the WEC robust optimization problem, configuring it with two chosen objective functions and uncertain parameters, which are all defined by a specific uncertainties' probability distribution model, suitable for the relative parameter. The framework's results are then analyzed via post-process analysis. This is done exploiting all the information given by a selected robustness index, employed to study how the system response changes with the variation of some input data, in order to discriminate between influencing and non-influencing factors and understand what their degree of influence is. In this way, an evaluation of the model robustness that allows to compare the different optimization processes is obtained.

INTRODUCTION

Every real-life engineering application, system or design is subject to variations and uncertainties that might be outcomes of the manufacturing process' quality of components or environmental conditions' changes, which in most of the situations are uncontrolled and uncontrollable.
When variations and uncertainties are ignored during an optimization process, a non-robust design is obtained and due to this, unsatisfactory system performances can occur and high system sensitivity to the changes of input parameters can be involved.
Therefore, it is necessary to extend classic optimization problems with the expectation that a design/system insensitive to noises, tolerances or uncertainties can be obtained, in order to increase the probability to reach a certain target of performances and maintain it despite the problem described above. In this way, robust optimization have been developed.

Robust optimization is a probabilistic solving method for real-life optimization problems, in which there are uncertain data that, due to their stochastic nature or uncertainty (linked to design condition changes or to wear), can float around their project value. This method studies these parameters and finds suitable solutions with the aim to avoid unsatisfactory system performances and compromising their functioning, studying and highlighting at the same time the influence that specific parameters can have. Robust optimization considers uncertain data to belong to a specific set called "uncertainty set" (with specific constraints), that must be defined by the designer. Therefore, the robust optimization process' final purpose is to obtain a set of solutions able to limit the variation of output parameters (and consequently their own uncertainty), finding a robust optimum instead of a global optimum in order to be able to increase the reliability and robustness of the system.

Commonly, robustness is attained choosing solution's parameters in order to decrease the influence of negative effects of the uncertain parameters' variations on the solutions' performance. This robustness may be viewed as a passive robustness. In this way, techniques broadly used aim to minimize the sensitivity of the performance without changing or controlling the causes and sources of variations, considering the mean performance and its variation or worst case [2].

There are other techniques studied to solve robust optimization problems that explore the influence of some adjustable variables and parameters which changes may alter (increase, if adjusted in the right way) the performances of the system. This kind of robust optimization processes can be called Active Robust Optimization Processes (AROP). They often have higher performances than classic robust optimization processes but their cost increases, so it is easy to highlight a trade-off between cost and performances in these techniques.

In this thesis work is studied a robust optimization problem for a Wave Energy Converter (WEC). These devices are used to transform wave energy into electric energy. The thesis' case of study is a Pendulum Wave Energy Converter (PeWEC), a "passive" device studied to be located in the Mediterranean sea (precisely, in Pantelleria) which uses the torque produced by the pendulum swinging without the necessity to be powered to produce inertial effect and harvest electrical energy. The main problems for these systems are: the cost of energy, not yet competitive if compared with other sources of renewable energy, and the difficulty for a WEC to deal with conditions in marine environment, e.g. corrosion due to salt water and high loads due to extreme meteorological events (in particular in ocean's environment). In earlier optimization works, as [42], results obtained by the MOP highlighted some noise in hull parameters: pitch turner radius, pitch hull viscous damping, distance between pendulum's hinge and the device's CoG (Center of Gravity), hull cost, pendulum cost and PTO cost. As a result, it is necessary to carry on a robust optimization process in order to increase PeWEC's reliability. To deal with these problems, it is chosen a robust optimization framework approach which will be fully described in next chapters and the results will be analyzed via a post-process based on a selected robustness index. In this way, the purpose is to learn more about the problems with these devices and take another step to make this sustainable and green energy production method more competitive than before.

CHAPTER 1 – STATE OF THE ART

Before starting to describe robust optimization in detail, in this section an overview about optimization is given, with the aim to introduce some principal concepts about optimization's field. Most of what is reported here is taken from [5] which in turn refers to Vangelis'work [4].

1.1 Optimization

The optimization process consists in the research of a solution, defined optimum solution, between all the possible ones which can solve the studied problem and which can be suitable for the design requirements.
Elements involved into an optimization problem are:

- *design parameters:* selected by the designer relatively to the case of study, e.g. mechanical, geometric, dynamical, techno-economic;

- *design constraints:* thresholds that the designer imposes and does not want to overcome or physical limits, e.g. acceptable stresses, strains, displacements, forces, amount of energy produced, etc;

- *objective function:* that has to be optimized as costs, weight, *etcetera*.

A general division for optimization problems is between:

- *single-objective optimization:* there is only one objective function that has to be optimized and it is just one component of an optimization problem. Other components are inputs (parameters, variables) and constraints. In this kind of optimization problem it is possible to pool all the design parameters in a design vector and the objective function converges to a global optimum;

- *multi-objective optimization:* there is more than one objective function that have to be optimized at the same time, which diverge to different optimum. In this other kind of optimization problem, a Pareto n-dimensional set is defined in order to compare the solutions and to highlight a trade-off between them.

Other classifications which can divide optimization processes might be done relatively to the approach that they use to solve the problem. A first division is between:

- *enumerative methods:* these methods consider all the possible solutions to the problem and then choose the suitable one;

- *heuristic methods:* they do not search for an optimum solution, but they search for a solution which is significant and rapid to be found.

1.1.1 Single-Objective Optimization

In [5] is reported a synthetic way to define the single-objective optimization problem:

$$\min[f(\vec{x})], \qquad \vec{x} = [x_1, x_2, \dots, x_n] \in \mathbb{R}^n \quad f \in \mathbb{R}$$

$subjected\ to$:

$$\vec{g}(\vec{x}) \leq 0 \ \ \vec{g} \in \mathbb{R}^p$$

$$\vec{h}(\vec{x}) = 0 \ \ \vec{h} \in \mathbb{R}^q$$

$$x_i \leq X_i \ \ for\ i = 1, \dots, n$$

$bounded\ by$:

$$\vec{x}_L \leq \vec{x} \leq \vec{x}_U$$

It means that in the deterministic field the problem is:

- find the *design vector* $\vec{x} = [x_1, x_2, \dots, x_n] \in \mathbb{R}^n$ bounded in a domain $\vec{x}_L \leq \vec{x} \leq \vec{x}_U$, where $\vec{x}_L$ and $\vec{x}_U$ are the lower and the upper bound of the design variables;

- in X_i, the set of x_i (the whole design space for the n design variables can be denoted as $\vec{X}$;

- that minimizes an objective function $f(\vec{x})$;

- subject to inequality constraints $g_i(\vec{x}) \leq 0$, $\vec{g} \in \mathbb{R}^p$ where $i = 1, \dots p$ are number of inequality constraints;

- subject to equality constraints $h_j(\vec{x}) = 0$, $\vec{h} \in \mathbb{R}^q$ where $i = 1, \dots q$ are number of equality constraints.

It is also described that is possible to define a *feasible set* as:

$$\vec{F} = \{ \vec{x} \in \vec{X} \mid \vec{g}(\vec{x}) \leq 0 , \vec{h}(\vec{x}) = 0 \}, \qquad \vec{F} \in \vec{X}$$

where all equality constraints are considered active at all points of the feasible set $\vec{F}$. The image of $\vec{F}$ is called *criterion space* $V_F = f(\vec{F})$ and the design vector $\vec{x}^* \in \vec{F}$, to wich corresponds the minimum objective function $f(\vec{x}^*) \leq f(\vec{x}) \ \forall \ \vec{x} \in \vec{F}$, is called *global minimizer* (or *maximizer* if the objective function has to be maximized), and the corresponding value of the objective function is called *global minimum* (or *global maximum).*

Then, the following definition is given. It is called *local minimizer* a design vector $\vec{x}' \in \vec{F}$ for which exists a neighborhood $\vec{\chi}$ such that $f(\vec{x}') \leq f(\vec{x}) \ \forall \ \vec{x} \in \vec{\chi}$. The corresponding value $f(\vec{x}')$ is called *local minimum*.

One of the main problems in an optimization process is the difficulty to reach the convergence, and the principal sources of this issue are local optima, in which the algorithm may be trapped during the process. If the algorithm remains trapped or does not depend on the shape of optimization domain, it is possible to distinguish between two sorts of domain: *convex* and *non-convex*. The first is an optimization domain where there is not presence of local optima, instead in the latter is possible to find more than one optimum. That is, a non-convex optimization domain leads to the presence of local optima which can influence the optimization process and compromise the algorithm's convergence.

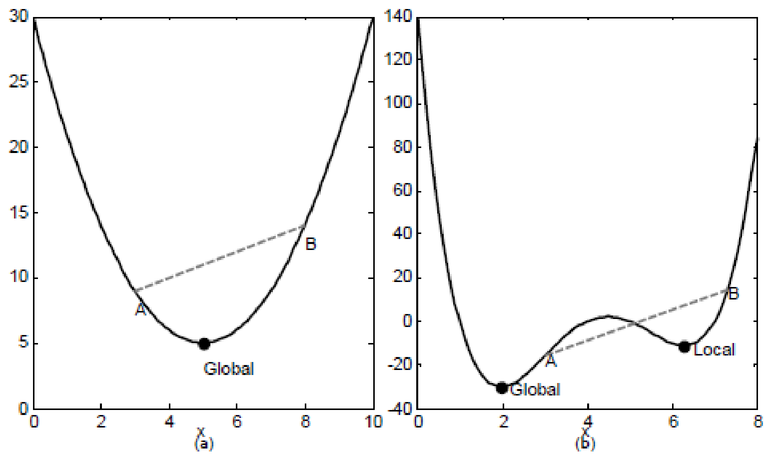

figure 1. 1

To solve the Single Objective Optimization Problem (SOP) there is not only one technique. In Wolpert and Macready's *No Free Lunch Theorem* [9], they demonstrate that it does not exist an algorithm, better than others, that performs the best in every optimization problem. In every study case it is possible to find a specific algorithm that uses a better approach to solve the problem and find suitable solutions, depending on its characteristics.

Hence, during the years, several methods have been developed for SOP. A first raw classification can be between *deterministic* or *probabilistic* methods. Probabilistic methods involve higher complexity, but they also are the best way to handle uncertainties and find safe and suitable solutions. For this reason, during the last decades, with the increase of computational possibilities that make it easier, lots of studies have been conducted in this field. Examples for both cases are reported below.

1.1.1.1 Mathematical Programming (MP)

MP is a deterministic method widely used until computational resources became enough powerful and common that heuristic algorithms were developed and replaced MP. These methods, in particular the gradient's method, adopt an analytical approach to the problem and are considered local methods because they use local curvature information of the objective function together with a gradient evaluation. They find relatively fast the convergence but, on the other hand, they are very sensitive to local optima in non-convex domain.

The most popular method adopted for mathematical programming is the *Sequential Quadratic Programming* (SQP), commonly used to solve Non-Linear Programming problems. Generally, the non-linear constraints are difficult to be treated directly in a gradient search and indirect methods to consider them need to be introduced. A penalty function is used to transform the problem and remove constraints, obtaining an unbounded optimization problem [5].

1.1.1.2 Evloutionary Algorithms (EA)

EA are heuristic methods, not based on rigorous analytical demonstrations, developed after MP showed its difficulty with local optima and its limits in handling complex problems with many design variables. Their heuristic nature makes them less sensitive to local optima than deterministic methods and lets them deal with many types of optimization problems.

The most used EA are:

- *Genetic Algorithms (GA):* population-based algorithms in which, set an initial population (an ensemble of design parameter vectors), natural selection principle is applied by using genetic operators like mutation and crossover operator. The crossover operator creates new off-springs and the mutation maintains the diversity of specimen, while the fitness function determines the survived individuals. Cross-over components combine solutions during optimization and are the main mechanism to explore the search-space (global search-space). However, the mutation operators change some of the solutions slightly, which emphasize exploitation (local search) [5] [7];

- *Particle Swarm methods:* population and swarm-based algorithms similar to GA that, instead of natural selection principle, are based on social contexts. A group of particles are selected and their experience is built by tracking and memorizing the best position encountered by the particles' "flight" along the design space. The process has memory and the global optimum is obtained by keeping into account the previous velocity together with the best ever position of the single particle and of the global swarm. The movement's step indicates direction and speed of solutions. Therefore, such techniques move solutions with the highest speed along different directions in the search space to perform exploration. The magnitude of movement reduces proportionally to the number of iterations to exploit the best position(s) obtained in the exploration phase [5] [7].

Heuristic algorithms consider the optimization problem as a black box, they do not need derivation and they are not based on some gradient formulation. Due to this, we can say that they are problem independent. Furthermore, their inspirations are easy to understand and they handle difficulties of real-world optimization problem better, compared to conventional optimization techniques. In this thesis, it will be used a GA algorithm for the optimization process. Below, a brief introduction about GA is given.

1.1.1.2.1 Genetic Algorithms (GA)

Genetic Algorithms (they take their name from Darwin's evolution theory) are a group of population-based heuristic algorithms, popular thanks to their flexibility, computational economy and ability to deal with high complexity problems.

The idea is that an initial population must generate a new offspring generation. For each iteration step, the best individuals of the population are selected comparing their characteristics. If these do not suit the fitness function the individuals are eliminated, while the best ones remain.

The optimization procedure was first set by Goldberg [10] and it can be summarized in the following steps:

1. *Initialization step:* the first generation can be built randomly or not, possible solutions of the optimization problem (individuals-chromosomes) are part of it, each one characterized by specific design parameters (elements-genes);

2. *Fitness evaluation step:* each member of the population is evaluated by computing the representative and corresponding fitness function, using an appropriate method (some examples: penalty function method, sample methods etc..). The fitness function is an indicator associated to each individual that measures its suitability, compared to the other individuals, to be one of the solutions for the optimization problem. Therefore, the fitness function for an individual also describes its reproduction probability in next generations;

3. *Selection step:* all the individuals of the current population are ranked relatively to their fitness function and the best ones are selected to be the parents that will

be used to create the next generation's offspring. Among the several ways to perform the selection of the strings, the most popular are *tournament selection*, *roulette wheel* and *ranking selection*. Another popular but slightly different technique is the elitism: the best chromosome is selected and directly copied to the next generation without the application of genetic operators. The big advantage of elitism is the increasing performance of the algorithm avoiding the risk of losing the best solution in the genetic operations; on the other hand, the sensitivity to local optima increases (in [11] Maruyama and Igarashi describe an elite reservation technique for the robust GA and applied it to a robust GA for electromagnetic problems);

4. *Generation step:* in order to create the next generation, crossover and mutation operators are applied to the current population. In this step, genes (design parameters) are exchanged between selected parents to create the next generation of the population (crossover). In this same step, random mutation are introduced in specific genes to maintain the diversity in the population and avoid premature convergences to local optima;

5. *Reiteration and Convergence step:* the process is repeated until thresholds or the maximum number of iterations is exceeded.

Each operator adopted in the process (mutation, crossover and fitness) can be defined by different functions, either analytic or heuristic depending on the problem. According to the choice, different evolving algorithms have been developed [5].

1.1.2 Multi-Objective Optimization

Several times optimization problems are different from the straightforward single-objective optimization problems and become multi-objective optimization problems (MOP), in which many objective functions are involved.
In MOP, objective functions involved in the process are often in contrast, thus, as a single solution does not exist and a function cannot be pooled in a single vector that converges to a unique optimum, a trade-off between the possible designs can be highlighted and a *Pareto Set* has to be generated to compare and select the optimal solutions. Therefore, the designer must make a choice in order to select one of the multiple optimal solutions.

1.1.2.1 Pareto Set

It is important to understand the difference between the concept of optimum in SOP and MOP. In SOP the feasible set is completely ranked according to the single value of the objective function f and can be easily compared in order to get the searched optimum [5]. For instance, if figure 1.2 is the solution for a minimization SOP, it is easy to say that x_A is the global optimum solution.

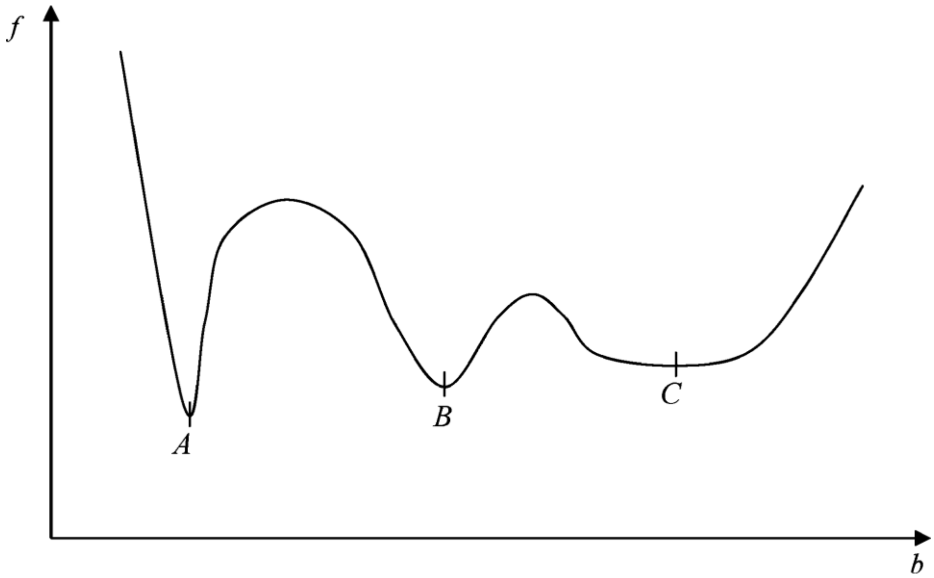

figure 1. 2

In MOP, instead, solutions are only partially ranked, more than one solution can be the optimum solution for the optimization problem and a Pareto Set is generated. Solving the optimization problem achieves a set of Pareto optimal solutions defined in the decision space, after which an image of the objective functions, along with the Pareto front, is calculated over the set of optimal solutions.
A solution is said to be Pareto optimal if there is no alternative to improving one objective without worsening at least another. That is, the feasible point is Pareto optimal when there is no other feasible point with strict inequality in at least one condition. In general, solving a multi-objective optimization problem is not as simple as solving any scalar problem. According to Schaffer (1985), Goldberg (1989) and Deb (2001), evolutionary algorithms are usually best suited to determining the Pareto front [12].

In order to better describe the Pareto optimal concept and multi-objective optimality, some definitions are given, from [5]:

- *Pareto dominance*: an objective vector $\vec{u}$ is said to dominate the objective vector $\vec{v}$ $(\vec{u} \succ \vec{v})$ if and only if:

$$u_i \leq v_i \quad \forall\, i = 1,2,\dots,n;$$

  and $u_i < v_i$ for at least one value of $i = 1,2,\dots,n$;

- *incomparability:* two objective vectors $\vec{u}$ and $\vec{v}$ are incomparable $(\vec{u} \prec\succ \vec{v})$ if neither $(\vec{u} \prec \vec{v})$ nor $(\vec{u} \succ \vec{v})$;

- *rank*: the rank of an individual indicates the order of dominance respect to the others. A rank 1 individual is not dominated by any other, a rank 2 is dominated by the rank 1 individuals, a rank 3 by the 1 and 2 and so on;
- *local Pareto optimality:* a design vector $\vec{x}' \in \vec{F}$ is said to be a local Pareto optimal design vector if, and only if, there is a neighborhood $\vec{\chi}$ of $\vec{x}'$ in which there exists no other $\vec{x} \in \vec{\chi}$ such that $\vec{f}(\vec{x}) \succ \vec{f}(\vec{x}')$;

- *global Pareto optimality*: a design vector $\vec{x}^* \in \vec{F}$ is said to be a global Pareto optimal design vector if, and only if, there exists no other $\vec{x} \in \vec{F}$ such that $\vec{f}(\vec{x}) \succ \vec{f}(\vec{x}^*)$. Or also, there is no other $\vec{x} \in \vec{F}$ such that:

$$\vec{f_i}(\vec{x}) \succ \vec{f_i}(\vec{x}^*) \quad \forall\, i = 1,2, \dots, n;$$

  with $\vec{f_i}(\vec{x}) \succ \vec{f_i}(\vec{x}^*)$ for at least one objective function i. Of course, a global Pareto optimum is also a local Pareto optimum while the reverse is not necessarily true;

- *Pareto set P^**: is the set of all the Pareto optimal design vector$\vec{x}^* \in \vec{F}$. That is, the group of all the non-dominated objective vectors that satisfy:

$$P^* = \left\{ \vec{x} \in \vec{F} \;\middle|\; there\ is\ no\ \vec{x}\ such\ that\ \vec{f}(\vec{x}) \succ \vec{f}(\vec{x}^*) \right\}$$

- *Pareto front*: is the image of a Pareto set P^* in the objective function space.

Figure 1.3 describes the concept of Pareto dominance for an optimization problem with objective function vector $\vec{f} = [\, f_1, f_2\,]$ where both functions have to be minimized. Instead, figure 1.4 shows Pareto front for the same optimization problem.

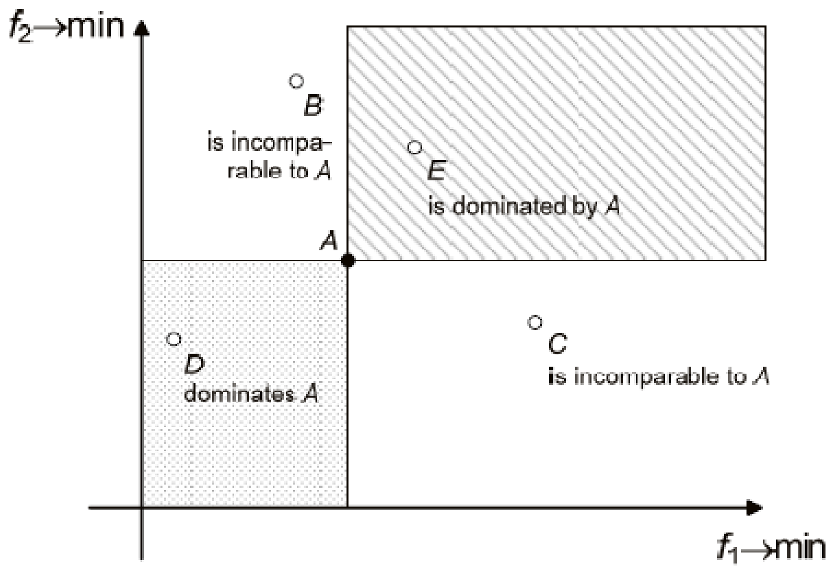

figure 1. 3

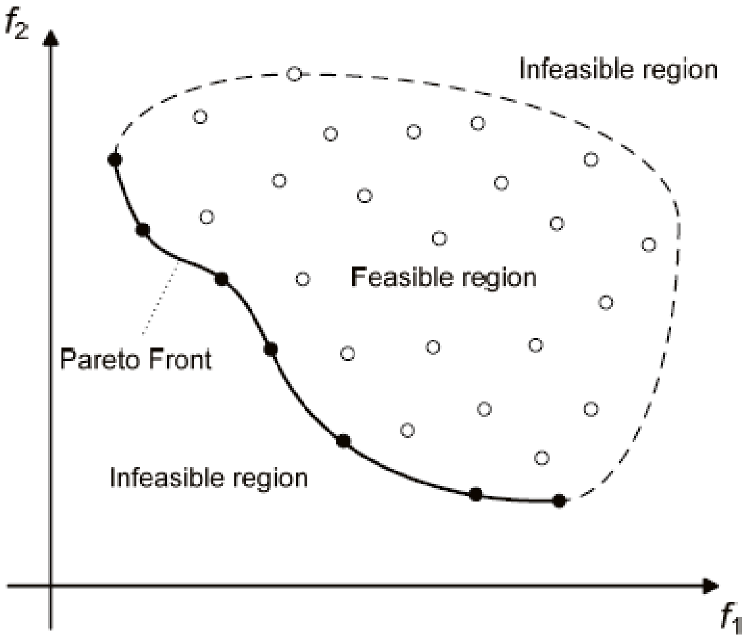

figure 1. 4

According to Miettinen (1998), the solution for the Pareto optimality must satisfy the Karush-Kuhn-Tucker condition [12]:

$$\sum_{i=1}^{k} \omega_i \nabla f_i(x^*) + \sum_{j=1}^{m} \lambda_j \nabla g_j(x^*) = 0$$

$$\lambda_j g_j(x^*) = 0$$

$$\lambda_j \geq 0$$

$$\omega_i \geq 0; \ \sum_{i=1}^{k} \omega_i = 1$$

where x^* is a Pareto optimal solution for the MOP, ω_i is the weighting factor (positive) for the gradient of the i-th objective function, calculated at point x^* and λ_j represents the weighting factor for the gradient of the j-th inequality constraint function. λ_j is called Lagrange multiplier, it is zero when the associated constraint function is not active and represents the sensitivity of the objective function to the j_{th} constraint. The same conditions are applicated in the case of SOP, with $k = 1$ [12].

1.1.2.2 Formulation and methods to solve MOP

Slightly differently form SOP, in [5] is given the MOP formulation and it can be written as:

$$\min[\vec{f}(\vec{x})], \qquad \vec{x} = [x_1, x_2, \dots, x_n] \in \mathbb{R}^n \quad \vec{f} \in \mathbb{R}^m$$

$subjected\ to$:

$$\vec{g}(\vec{x}) \leq 0 \ \ \vec{g} \in \mathbb{R}^p$$

$$\vec{h}(\vec{x}) = 0 \ \ \vec{h} \in \mathbb{R}^q$$

$$x_i \leq X_i \ \ for\ i = 1, \dots, n$$

$bounded\ by$:

$$\vec{x}_L \leq \vec{x} \leq \vec{x}_U$$

It means that in the deterministic field the problem is:

- find the *design vector* $\vec{x} = [x_1, x_2, \dots, x_n] \in \mathbb{R}^n$ bounded in a domain $\vec{x}_L \leq \vec{x} \leq \vec{x}_U$, where $\vec{x}_L$ and $\vec{x}_U$ are the lower and the upper bound of the design variables;
- in X_i, the set of x_i (the whole design space for the n design variables can be denoted as $\vec{X}$;
- that minimizes a vector of objective functions $\vec{f}(\vec{x})$;
- subject to inequality constraints $g_i(\vec{x}) \leq 0$, $\vec{g} \in \mathbb{R}^p$ where $i = 1, \dots p$ are the number of inequality constraints;
- subject to equality constraints $h_j(\vec{x}) = 0$, $\vec{h} \in \mathbb{R}^q$ where $i = 1, \dots q$ are the number of equality constraints.

It is possible to define a *feasible set* as:

$$\vec{F} = \{ \vec{x} \in \vec{X} \mid \vec{g}(\vec{x}) \leq 0, \vec{h}(\vec{x}) = 0 \}, \qquad \vec{F} \in \vec{X}$$

Where all equality constraints are considered active at all points of the feasible set $\vec{F}$. The image of $\vec{F}$ is called *criterion space* $V_F = f(\vec{F})$ and the design vector $\vec{x} \in \vec{F}$ is called *feasible design.*

There are several mehods to solve the MOP. Taking references from descriptions given in [5] and [12], the most broadly used are:

- *perturbation method:* this approach to multi-objective optimization problem is strictly connected to Sensitivity Analysis (SA). Some instances for this kind of solution are described in [12], and others will be discussed in the section about SA. Basically, for each parameter a Pareto set is generated with one objective minimized while the others are constant;

- *min-max method:* connected to the worst case scenario approach, the Pareto set is generated by minimizing the distance between the values of the objective function and its possible maximum or the ensemble of objective values;

- *weighted sum method:* a weight factor ω_i is assigned to every objective function (this is the case of Linear Weighting Method, LWM), depending on the designer sensibility. The weights have to be normalized and will give different Pareto curves from the unweighted case; their sum has to respect $\sum_{i=1}^{k} \omega_i = 1$. It is like transforming a multi-objective optimization problem into a single-objective optimization problem;

- *space investigation parameters (PSI)*: an ensemble of design solutions that cover the entire design space is generated and, for each design, the objective functions are analyzed generating restrictions. In this region an acceptable Pareto set is generated;

- *Evolutionary Multi-Objective Optimization Algorithms (EMO)*: it is a class of population-based Evolutionary Algorithms (EA) which are used to solve MOP. They became popular because they are useful for application and research, working simultaneously on a population of design points, instead of a single one. EAs have a great potential in finding the full Pareto front and are less computationally onerous. Examples of this approach could be found in Deb's works, in particular he built a widely used EMO called NSGA-II.

Another important aspect that has to be considered in order to find the optimal solution in MOP are selection criteria, which are helpful in situations where a trade-off (and therefore a conflict) exists between the problem's solutions.
It is easy to understand that between all the points in Pareto front a trade-off and a conflict exist, thus, when a Pareto front is generated, a selection criteria has to be applied by the decision maker with the aim to define the optimal solution of the problem.

The MOP's solution can be dived in two phases: *search* and *decision making*. In the first one a Pareto set (a Pareto front) is found, in the latter selection criteria are applied, and this (as stated in [5]) can be done in different ways. The author gives the following formulations:

- *decision making before search (a priori method):* the objectives of the MOP are aggregated and the multi-objective problem is transformed in a single-objective one. This option requires a profound knowledge of the solution domain because each choice in this sense influences significantly final results;

- *decision making after search (a posteriori method):* the analysis is performed without any preference given. After that, the designer chooses a trade-off and a final design among the possible ones. For unexplored domain it is the best approach although it implies an increase of complexity;

- *decision making during search:* after each optimization step a number of alternative trade-offs is presented and the decision maker specifies further preferences information, guiding the search process. The final results are heavily influenced by the adopted choices that sometimes may be obscure to the designer being hidden in the optimization process.

The increasing of the problem size leads to the increase of the number of variables and due to this, the non *a priori* methods become inapplicable.

1.2. Robust Optimization

A brief introduction about robust optimization is here reported, in the following chapters others information will be given and more specific robust optimization methods will be presented in order to introduce, thus, the chosen framework for this thesis work.

In every real-world optimization problem, the designer must deal with uncertainties in order to reach an optimal design. Due to this, he cannot leave apart the randomness of the main factors that characterize the problem and uncertain data that, due to their stochastic nature or uncertainty (linked to design condition changes or to wear), can float around their project value. In general, it is possible to distinguish between two classes of uncertainties: *epistemic*, due to lack of knowledge about the system, and *aleatory,* due to the intrinsic randomness of the physical phenomena [5]. Other classifications for uncertainties are given afterwards.

In order to handle this problem, thanks to the studies about probability, combined with the computational capacity reached by computers, some accurate approach is not hard to reach anymore. For instance, the aim of the studies developed in the branches of Computational Stochastic Mechanics and Structural Reliability is to express the reliability of a structure, not in terms of coefficients, but as a failure probability to be kept under a fixed threshold set by the codes [5].
When the number of uncertainties data (as design uncertain parameters of uncertain data set) increases, the computational cost of a direct integration of a joint PDF (probability density function) becomes high. For this reason, mathematical methods have been developed and Pellizari [5], in his work, describes them in the following way:

- *First and Second Order Reliability Method (FORM and SORM):* these methods are based on the approximation of the performance function in the standard Gaussian space by using a polynomial series. Both methods require to know the mean and variance of each random variable and to have a differentiable function that describes the probability to exceed the threshold. The main problem of the SORM and FORM methods is the presence of various local optima, because both methods are quite sensible to them and can stuck in a local optimum rather than in the design point if the dominion is not convex. Moreover, the approximation in the design point neglects the contribute of its neighborhoods that can be important;

- *Response Surface Method (RSM):* this method defines a meta-model to determine the behavior of the system in a defined domain. Generally, the vector set of random variables and the performance function are not given in closed form and it is necessary to perform more experiments to define a response surface with a sufficient level of accuracy. Each experiment is a point in the design space of the random variables for which one value of the function is calculated. With a polynomial interpolation the response surface is approximated between the calculated points in the region of interest. The main advantages of using the RSM consist in the reduction of computational effort for the determination of the limit threshold surface (for a moderate number of random variables) and in the possibility to couple the reliability and optimization algorithms together to reach high efficiency. It could be also used to study approximatively the response of the system before applying other forms of optimization. Unfortunately, for a higher number of random variables, the computational cost for the multiples analysis necessary to determine the polynomial approximation becomes too high and the method is not convenient anymore;

- *Monte Carlo Simulation (MCS):* the MCS methods deal with high complexity problems characterized by a large number of degrees of freedom, significant uncertainty on the input and complex boundary conditions. They are often used to test new models and study uncertainty propagation, where the goal is to determine how random variation, lack of knowledge or errors affect the sensitivity, performance or reliability of the system. The principle is quite simple: many experiments are simulated and the outcomes are used to define in detail the limit state surface. Using statistical sampling, the discrete probabilistic characteristics of the system can be extracted, as mean and variance. The main advantage of the MCS is the capability of handling any type of problem regardless of its complexity by simply repeating the same mechanical analysis several times, avoiding the necessity of modifications of the solution. On the other hand, it is a very expensive computational method, subjected to noise during random sampling that may lead to problems in the response gradient analysis. To reduce the computational effort connected to the MCS, many improved sampling techniques have been developed, for instance: *Importance Sampling (IS), Latin Hypercube Sampling (LHS), Descriptive Sampling (DS)*.

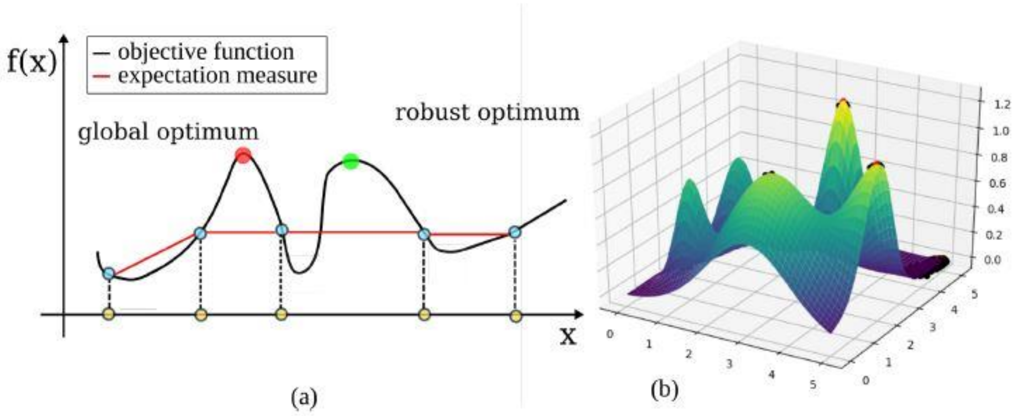

figure 1. 5

Considering the uncertainties problem described above, the deterministic optimization approach shows all its limits. Two forms of probabilistic optimization approach have been developed: *Reliability Based Optimization (RBO)* and *Robust Optimization (RO)*.
The first one aims to keep the probability of exceeding a fixed threshold low, the latter considers uncertain data to belong to a specific set called "uncertainty set" and its final

purpose is to obtain a set of solutions able to limit the variation of uncertain output data (and consequently their own uncertainty), called objective functions, finding a robust optimum instead of a global optimum (difference between global and local optima is highlighted in figure 1.5, for SOP and MOP) in order to be able to increase the reliability and robustness of the system. In this thesis work we will focus on a robust optimization problem applied to a WEC system. We will proceed with the application of a chosen framework for the WEC robust optimization problem, configuring it with two chosen objective functions and uncertain parameters, which are all defined by a specific uncertainties' probability distribution model, suitable for the relative parameter. The framework's results are then analyzed via post-process analysis. This is done exploiting all the information given by a selected robustness index, employed to study how the system response changes with the variation of some input data, in order to discriminate between influencing and non-influencing factors and understand what their degree of influence is. In this way, an evaluation of the model robustness that allows to compare the different optimization processes is obtained.

The first who introduced the concept of robust optimization was G. Taguchi in 1989. He defines robust optimization as "the state where the technology, product, or process performance is minimally sensitive to factors causing variability (either in the manufacturing or user's environment) and aging at the lowest unit manufacturing cost". According to Park et al. [3], the Taguchi method has greatly contributed to quality improvement of various designs. In early cases of study, the Taguchi method was applied to the process design rather than the product design. That is, it was regarded as a method for design of experiments rather than a design methodology. The Taguchi method can be used to determine the settings of control factors for robust design. As described in [13] by Otto and Antonsson, the Taguchi method of product design is an experimental approximation to minimizing the expected value of target variance for certain classes of problems. The method is also extended to solve design problems with constraints, involving the methods of constrained optimization. Finally, the Taguchi method's disadvantages are its difficulty to handle continuous parameters and a large number of constraints, which are present in most real-life situations. Starting from this definition of robustness, researchers gave several and different definitions of it. According with Deb [17] robustness for MOP can be defined as described in figure 1.6.

The four possible positions of the robust front compared to the Pareto-front of the multi-objective solution are: (a) complete Pareto front is robust, (b) complete Pareto front is not robust, (c) a part of Pareto front is not robust and (d) a part of Pareto front is robust [14].

Referring to Wang et al. in [8], a Multi-Objective Robust Optimization Problem (MOROP) aims to find a solution that is feasible, optimal and robust. To come up with a feasible, optimal and robust design, the three following scenarios have been identified: optimal design and robust design are equally important, optimal design is primary and robust design is secondary, robust design is primary and optimal design is secondary.

Mathematically, the RO can be defined as a multi-objective optimization involving the mean and standard deviation of the objective function (OF).

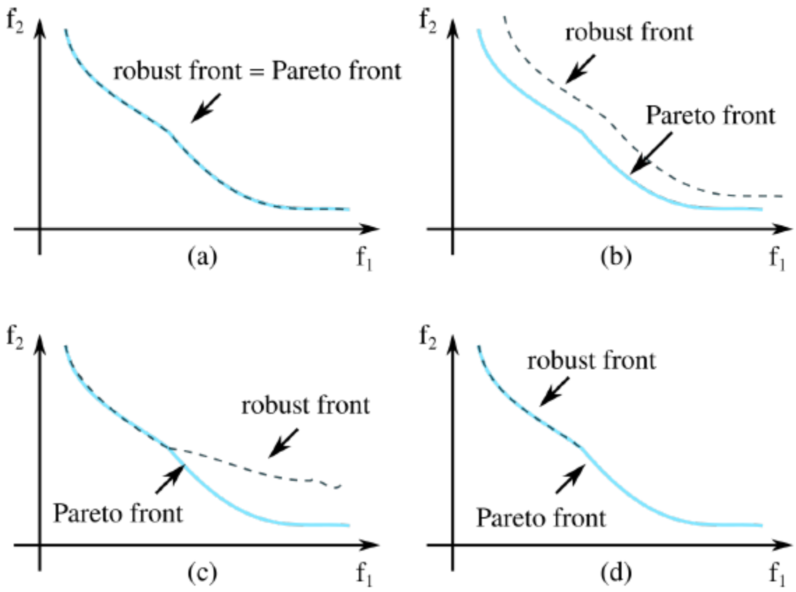

figure 1. 6

In [5], mathematical formulation is given as:

$$\min\left[f(\vec{x}), \sigma_u\left(\vec{x}, \vec{R}\right)\right], \qquad \vec{x} = [x_1, x_2, \dots, x_n] \in \mathbb{R}^n \quad f \in \mathbb{R}, \sigma_u \in \mathbb{R}$$

subjected to:

$$\vec{g}(\vec{x}) \leq 0 \;\; \vec{g} \in \mathbb{R}^p$$

$$\vec{h}(\vec{x}) = 0 \;\; \vec{h} \in \mathbb{R}^q$$

$$x_i \leq X_i \;\; for\; i = 1, \dots, n$$

bounded by:

$$\vec{x}_L \leq \vec{x} \leq \vec{x}_U$$

where:

- $\vec{x} = [x_1, x_2, \dots, x_n] \in \mathbb{R}^n$ *n-design parameters* (or *control parameters* on which the designer has control of the nominal value and have to be optimized in order to minimize the sensitivity to the noise);

- $\vec{R} = [R_1, R_2, \dots, R_n] \in \mathbb{R}^m$ *m-random variables* (or *noise parameters* on which the designer has not control of the nominal value, they are imposed either by the environment or by the unacceptable costs for their control);

- in X_i, the set of x_i (the whole design space for the n design variables can be denoted as $\vec{X}$;
- $f(\vec{x})$ is the scalar function to be minimized;
- subject to inequality constraints $g_i(\vec{x}) \leq 0$, $\vec{g} \in \mathbb{R}^p$ where $i = 1, \dots p$ are the number of inequality constraints;
- subject to equality constraints $h_j(\vec{x}) = 0$, $\vec{h} \in \mathbb{R}^q$ where $i = 1, \dots q$ are the number of equality constraints;
- $\sigma_u(\vec{x}, \vec{R})$ is the standard deviation of the response measure to be minimized.

With the increase of computational possibilities, several techniques were developed during the years for robust optimization and robust design optimization.
In [14] Orosz et al. describe the widely used optimization techniques in electrical machinery and summarize the challenges and open problems in the applications of the robust design optimization and the prospects in the case of the newly emerging technologies.

1.2.1 Sensitivity Analysis (SA) Approach

Robustness can also be described as a product's ability to maintain its performances under conditions of parameters' variations. Due to this, Sensitivity Analysis approaches were developed and discussed.

Taking as a reference [18-20], SA of a mathematical model is defined as the process through which is studied how system's outcomes change with the variation of some input data, in order to discriminate between influencing and non-influencing factors and understand what their degree of influence is and, in this way, obtain an evaluation of the model robustness. Thanks to SA, what-if analysis can be carried or used as an instrument to find criteria and solve decision making problems (for example SA is used with multi-criteria analysis techniques as TOPSIS and SAW).
Generally, a first distinction can be done between the two methods below:

- *derivative:* inputs values are changed in order to evaluate output variations and estimate the robustness of the system and its sensitivity due to this variations; One-At-Time analysis for scenarios' analysis are part of this group of techniques;
- *input value's weights variation and shuffling:* comparison between initial rankings obtained through multi-criteria analysis methods and those obtained after shuffling and appropriate exchanges between sequences of weights; this method

is used in [19].

Sensitivity analysis, therefore, provides useful information on the riskiness of a project. Consequently, the determination of the range of variability of each variable is critical.
For this type of analysis, the ways in which the results are reported are significant. The main methods for graphic representation are tornado diagram and spider plot (figure 1.7).

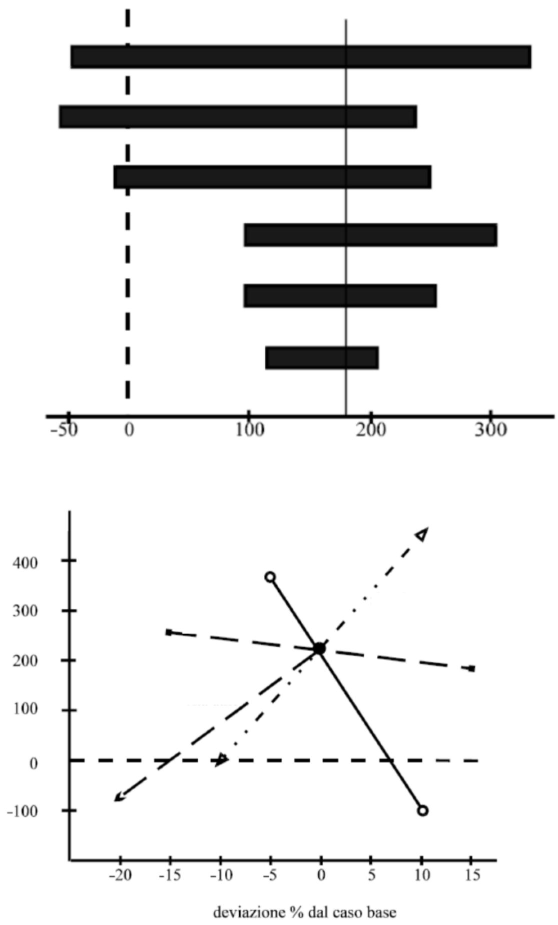

figure 1. 7: tornado diagram up and spider plot down

In [21] and [22] Weck and Willcox well express some SA's concepts.
They describe why sensitivity analysis is an important component of post-processing, after the solution of an optimization problem. They describe SA as the key to understand which design variables, constraints, and parameters are important drivers for the optimum solution: how sensitive is an optimal solution to changes or perturbations in design variables and how sensitive is it to changes in the constraints and fixed parameters. They also highlight the importance of normalization, in order to compare different effects end sensitivities from different design variables.
If the objective function is known in closed form, it is often possible to compute the

gradient vector(s) in closed form and, in this way, calculate the analytical sensitivities.

During SA, it is required that KKT conditions remain valid for a small change in one of the design fixed parameters p:

$$\frac{d(KKT\ conditions)}{dp} = 0$$

It is also described the need to assess when an active constraint will become inactive and *vice versa*. In particular, an active constraint will become inactive when its Lagrange multiplier goes to zero and an inactive constraint will become active when $g_j(x)$ goes to zero.

- $\Delta p = -\frac{\lambda_j}{\delta\lambda_j}$ is the amount by which we can change p before the j_{th} constraint becomes inactive (to a first order approximation);

- $\Delta p = -\frac{g_j(x^*)}{\nabla g_j(x^*)^T \delta x}$ is the amount by which we can change p before the j_{th} constraint becomes active (to a first order approximation).

If during the optimization problem solution p is changed by a great amount, in [21-22] is highlighted that the problem must be solved, again, including the new constraint.

In the paper it is already explained that the Sensitivity Analysis approach is gradient-based, and like every gradient-based method it can have trouble converging to the global optimum, and sometimes fail to find even a local optimum. Due to this, it is very important to interrogate the optimum solution, and features like stopping criteria have to be observed.
Gradient-based algorithm is terminated when a feasible solution is found or algorithm terminates unsuccessfully. It is therefore necessary to decide when a feasible solution is found and when to stop the algorithm without finding an optimal solution, because some thresholds have been exceeded (when progress is unreasonably slow, when a specified amount of resources have been used like time or number of iterations, when an acceptable solution does not exist, when the iterative process is cycling…). The problem is that convergence to a non-optimal solution or a slowdown in iteration can look the same as the convergence to the correct solution, so it is easy to imagine that no set of termination criteria is suitable for all optimization problems and all methods.

Another important aspect which is highlighted by Weck and Willcox is the scaling factor, that can have a large effect on the solution. It is also necessary to think about scale constraints: in a well scaled set of constraints, each constraint is well conditioned with respect to perturbations in the design variables in order to increase robustness of the solution.

Referring again to Wang et al in [8], they propose in their paper a robust optimization problem solution using post-optimality sensitivity analysis technique and they applicate it to a wind turbine design.

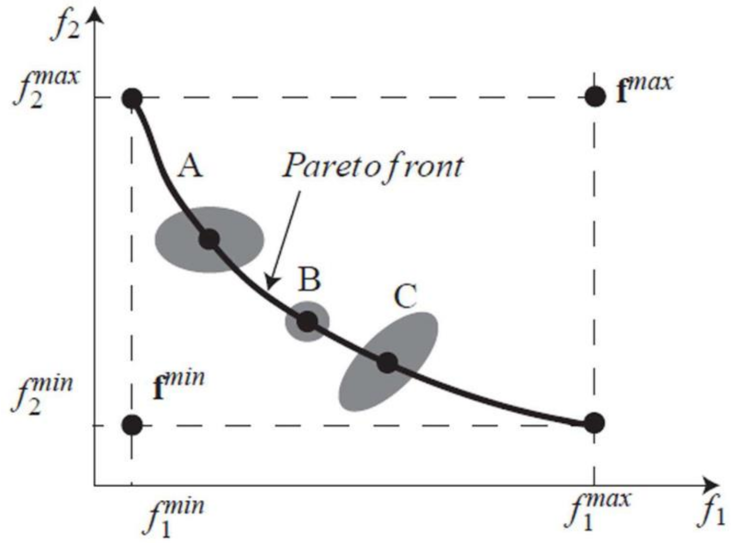

figure 1. 8

In figure 1.8, PF-Space (performance function space) is shown. Points A, B and C are solutions of a multi-objective optimization problem and the size of the grey areas associated with them represent their variation around their nominal values in the PF-Space. After the multi-objective robust problem definition (MOROP), two robustness indices (RI) are introduced in the paper. The first one, I_{RS}, is a RI with regard to the Small Variations in DVs (design variables/parameters) and DEPs (design environment parameters p, which are the uncontrollable parameters) and is defined as a scalar.
In this particular index, the standard deviation σ of the actual performances is used as a measure for the robustness: the smaller the standard deviation, the more robust the design. The absolute value of the difference between the expected value (μ) and the nominal value (f_0) is also defined as a robustness measure: the smaller the absolute difference, the more robust the design. The robustness of each performance function is also normalized, and to do this, they say to divide the sum of standard deviation σ_{f_i} and $\left|\mu_{f_i} - f_{i,0}\right|$ for the i-th performance function by the difference between the two extreme values, already of the i-th performance function, namely, $f_{i,max} - f_{i,min}$. As a result, I_{RS} is defined as follows:

$$I_{RS} = \sqrt{\sum_{i=1}^{m} \left(\frac{\sigma_{f_i} + \left|\mu_{f_i} - f_{i,0}\right|}{f_{i,max} - f_{i,min}} \right)^2}$$

The smaller the I_{RS}, the more robust the design. The second RI described, $I_{RL}(x)$, regards the Large Variations (which affect the environments) in DEPs. They start to define that the initial DEPs are equal to the ones having the maximum PDF amongst the N discrete values and put in evidence that feasible and Pareto optimal solutions may not be the same in the new environments. In order to compare the solution's robustness against large variations in DEPs, the traditional methods are not applicable. Hence, Wang e al. give the following definition: toward a multi-objective optimization problem against large variations in DEPs, solution's robustness against large variations in DEPs is a measure of its ability to be optimal in different design environments. Therefore, the method which they propose aims to compare the solution's relative positions in the PF-Space associated with the new environment. $I_{RL}(x)$ is the second RI ad it is defined as:

$$I_{RL}(x) = 1 - I_F(x) \sum_{1}^{N} I_P(x, p_j) h(p_j)$$

with:

$$I_F(x) = \{1, \forall\, j = 1,2\,, \ldots N, x \in F_j\}$$

$$I_F(x) = \{0, \exists j = 1,2\,, \ldots N, x \notin F_j\}$$

$$I_P(x, p_j) = \frac{1}{I_{rank(x,p_j)}}$$

where Wang et al. define: F_j as the feasible sets; I_F as the Feasibility Index of the solution; I_P as the Pareto optimality Index of the solution; $h(p)$ as the PDF of p; $I_{rank(x,p_j)}$ as the individual's ranking in the new environment where $p = p_j$ and amounts to the number of individuals by which it is dominated amongst the alternative solutions, plus one.
They conclude that in a new environment, if a solution is still non dominated by any other solution, then the I_P value will be equal to one for that solution. Otherwise, the I_P value will be lower than one, but greater than zero. If a solution is feasible in all environments and cannot be dominated by any other solution in all possible environments, then $I_{RL} = 0$. On the other hand, if a solution is non-feasible in some new environments, then $I_{RL} = 1$. If each solution belongs to the set of Pareto optimal solutions, then it is feasible in all environments and its I_{RL} value will be greater than or equal to zero and smaller than one. In case there is continuous probability distributions of the DEPs, N becomes infinite and it is difficult to assess the robustness index I_{RL} for a solution x. However, since the domain of the DEPs can be partitioned into many small parts, we can simplify it as a problem by using a discrete probability distribution of the DEPs. Finally, the smaller the I_{RL}, the more robust the design according to large variations in DEPs.

They chose to represent the RI of a Pareto-optimal solution as a vector. Thanks to this approach, the designer can analyze the robustness of Pareto optimal solutions in the Robustness Function Space (RF-Space, shown in fig 1.9), which is the one with I_{RS} as one dimension and I_{RL} as the other one. Thanks to the proposed method, each Pareto optimal solution has a corresponding position in the RF-Space. If I_{RS} and I_{RL} are not conflicting, then the designer will be able to select the most robust solution, because it is the solution that minimizes both I_{RS} and I_{RL}. If I_{RS} and I_{RL} are two conflicting objectives, then a new Pareto front in the RF-Space will appear.

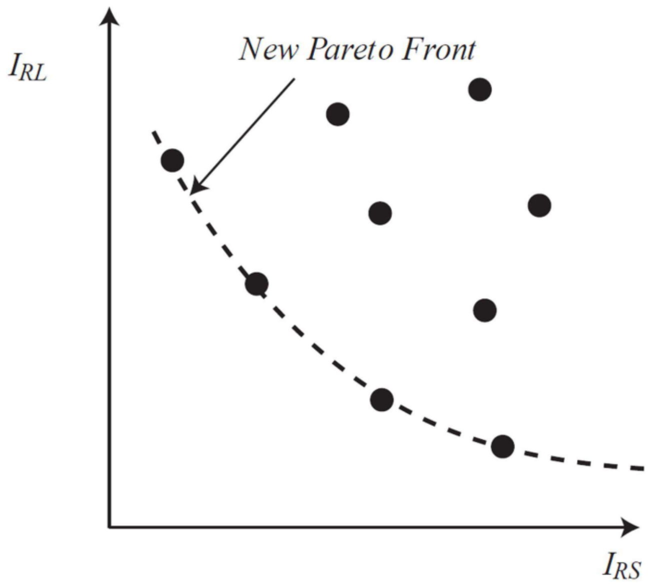

figure 1. 9

A flow chart of the method proposed in [8] is present in figure 1.10.

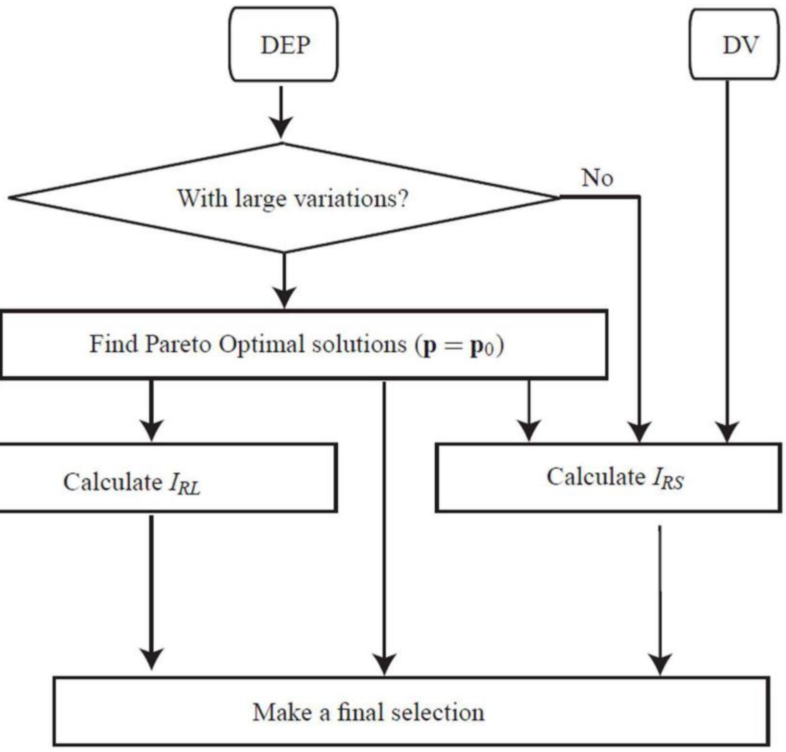

figure 1. 10

A different sensitivity analysis approach to multi-objective engineering problems is given in [12] by Augusto, Bennis and Caro. Their paper proposes two new approaches for problems which performance functions are highly susceptible to small variations in the design variables and/or design environment parameters. In the first method, the designer has to pick up the design variable and/or parameter that causes uncertainties and then he associates a robustness index with each design alternative and adds each index as an objective function in the optimization problem. For the second approach, an *a priori* decision about the interval of variation in the design variables or in the design environment parameters is needed.

For the first approach, it is assumed that the vector of the objective functions is of class C^2 in vector joining the design variables and the parameters v. S can be defined as the ratio of the Euclidean norm of variations in the objective functions and the Euclidean norm of variations in v. S is bounded by the smallest non-zero singular value σ_{min} and the largest singular value σ_{max} of its global sensitivity Jacobian matrix, $J_s = [J_x J_p]$, where $J_x = \frac{\partial \mathrm{f}}{\partial \mathrm{x}}$ and $J_p = \frac{\partial \mathrm{f}}{\partial \mathrm{p}}$. That is, the maximum singular value can be used as a relevant robustness index:

$$R(v) = \sigma_{max}$$

In the paper is highlighted how $R(v)$ makes sense if and only if the terms of J_s are normalized; that is, if they have the same unit. Indeed, the singular values of J_s cannot be compared if their units are different.

In the second approach instead, the designer should know the bounds of variations in the design variables and in the design parameters (maybe some percent of the initial value), with the consequence that he has to accept a tolerance for the variations in the objective functions (already here for some percent in expected value) and in order to maintain valid the constraints which have been defined before; also, variations inside constraints' initial bounds have to be accepted. Due to these approximations, the robust Pareto front is less performing than the nominal one.

In [14] Orosz et al. overview, an introduction about sensitivity analysis approach for electrical machines is given.
The minimization of the sensitivity information to obtain directly a robust solution, which utilizes the second-order sensitivity to find the optimum, is the simplest method used. However, these methods cannot be used for large-scale problems as electrical machines are. Due to this, to avoid this problem are proposed some methods which use response surface methodology or similar meta-modeling techniques, like propagating model-based uncertainty, but both of these approaches risk to converge to a sensitive design. Moreover, the computational complexity increases if a multi-objective optimization problem is considered, but in order to deal with this problem are used advanced sampling methods. Others stochastic gradient methodologies are reported in the paper, in particular Robbins-Monro and Polyak–Ruppert are mentioned because their techniques work with the

problem of how is possible to approximate the shape of an arbitrary, non-convex, non-linear function.

At last, in the paper concludes that electrical machine optimization during a sensitivity analysis task is often considered as a single objective optimization problem. The advantage of this constrained single-objective formalism is that the mathematical programming-based approaches can be used for the solution of these problems and in the robust optimization problem scenario, sensitivity analysis generally considers the role of the uncertainties as a single variable problem. As said above, the Sensitivity Analysis approach is a gradient-based approach and like every gradient-based method can have trouble converging to the global optimum, and sometimes fail to find even a local optimum. In order to avoid this problem, heuristic methods have been developed. The best solutions from a heuristic technique should be checked with KKT conditions or used as an initial conditions for a gradient-based algorithm. For these reasons, various heuristics and artificial intelligence-based methods were introduced to handle this kind of analysis, where the selected design variables are perturbed with the required tolerances to calculate the model sensitivities.

1.2.2 Heuristic Algorithms/Genetic Algorithms (GA) Approach

Heuristic algorithms are a group of computational techniques studied to solve robust optimization problems which become widely used thanks to the increase of computational possibilities. Into this family, a first raw classification can be done between stochastic and deterministic approaches. As reported in [14], in electrical machines field stochastic optimization algorithms are broadly used because they allow a gradient-free search of the solutions, and they do not need an initial solution to converge to the global optimum. In the same paper a more accurate and schematic classification is given and here reported in figure 1.11.

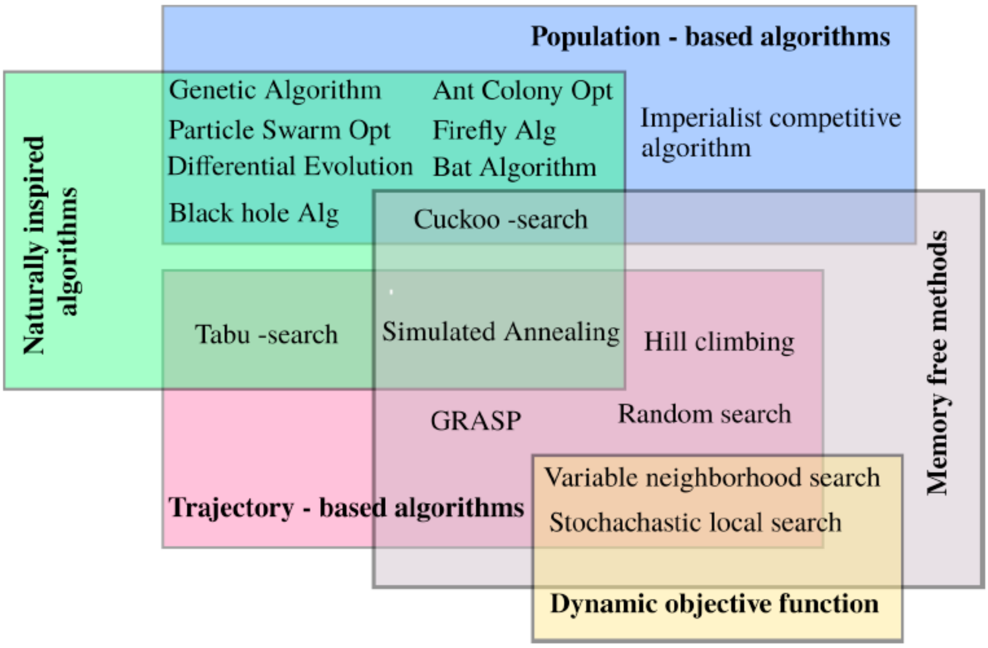

figure 1. 11

In this categorization, it is important to put in evidence the population-based algorithms. In [7] Mirjalili divides in two phases the mechanism by which population-based algorithms improve the set of initial solutions over the course of iterations: exploration (diversification) and exploitation (intensification).
The main objective for the first phase (exploration) is to discover the promising areas of the search landscape; in this way, a rough estimation of the global optimum is found and the search can be conducted with frequent or large changes in the solution. In order to provide exploratory behavior, different operators for different algorithms are developed. For instance, the author describes how in PSO the inertia weight maintains the tendency of particles toward their previous directions and emphasizes exploration. Then, in GA, a high probability of crossover causes more combination of individuals as the main mechanism for exploration.
The second phase (exploitation phase) main objective is defined in the paper as a local search around the best solutions found in the exploration phase. Exploitation is carried on in order to find the best possible solution, in a more accurate way than the first global search, around the global best optima reached before. Exploitation phase mechanisms are different for each algorithm. Taking as examples PSO and GA, the author describes the two different exploitation mechanisms: in the first case, a low inertia rate causes low exploration and a higher tendency toward the best personal/global solutions is obtained, in the second case the mechanism that brings exploitation is mutation.

The problem highlighted in the paper about exploration phase and exploitation phase is that they are in conflict. Exploration does not give the possibility to improve the accuracy of the solution found, meanwhile the exploitation can increase accuracy of the solution but results sensitive to local optima and can lead the algorithm to premature convergence. A good balance between these two phases is difficult to find because of their different search space but it can lead to a robust solution for the algorithm.
In the paper is suggested a broadly used method, where exploration and exploitation are tuned proportional to the number of iterations, in order to let the algorithm smoothly transit from exploration to exploitation as the iteration counter increases. Another popular technique proposed by the author is to promote exploration at any stage of optimization if there is no improvement in the best solution obtained.

According to Wolpert and Macready No Free Lunch Theorem [9], as said before, it does not exist an algorithm better than others, that performs to the best in every optimization problem. In every case of study, instead, it is possible to find a specific algorithm that uses a better approach, depending on its characteristics, to solve the problem and find suitable solutions. Due to this, many different algorithms have been developed, also among population-based ones which are already the most widely used in electrical machines field, as described by Orosz et al. in [14].
In the above mentioned paper it is put in evidence that the main difference between these algorithms is that they use different operators and methodologies to share the information between the selected parents and offspring. For instance, the PSO algorithm needs the best individual's information to generate the new position of the particle, GA instead does not care about it. However, there are many modifications and improvements that have been introduced in the last two decades. In order to give a brief overview on these methods, the paper reports the following table (table 1.1) about the widely used nature-

inspired multi-objective optimization techniques.

table 1. 1

Technique	Name	Description
NSGA-II	Non-dominated Sorting Genetic Algorithm II	It is a widely used optimization method. It uses the elitist strategy with the crowding distance operator to preserve diversity and the efficient non-dominated sorting operator to select the Pareto-dominant solutions
NSGA-III	Reference point based non-dominated sorting genetic algorithm	This algorithm is designed for many-objective problems (more than two). It uses similar operators, like NSGA-II with reference points and the niche preservation operator, where the reference point can be associated with some solutions, and it keeps the solutions that are close to the reference point
ϵ-MOEA	ϵ-dominance based Multi-objective Evolutionary Algorithm	It uses the epsilon-dominance concept with an epsilon archiving strategy limiting and sorting the Pareto-dominant solutions, which can be faster than the non-dominated sorting, and it can be advantageous in many cases. However, the sorting time is not relevant in most cases of numerically expensive electrical machine design problems
MOEA/D	Multi-objective Evolutionary Algorithm with Decomposition	This algorithm explicitly decomposes the problem into scalar optimization subproblems. It solves these subproblems simultaneously. At each generation, the population is composed of the best solution found so far for each subproblem. This algorithm can significantly reduce the computational complexity compared to NSGA-II [
GDE3	Generalized Differential Evolution	GDE3 uses the weak-dominance concept to select the Pareto-dominant solutions and an improved crowding distance operator and non-dominated sorting for the results
PAES	Pareto Archived Evolution Strategy	It uses a non-dominated bounded archive to maintain the Pareto-optimal solutions
PESA2	Pareto Envelope based Selection Algorithm	It uses a region-based selection operator, instead of individual based ones, like NSGA-II
SPEA2	Strength based Evolutionary Algorithm	SPEA-2 uses the strength-based diversity operator, the calculation and sorting time of which are more expensive than the case of NSGA-II; however, the diversity and convergence of the results can be better, which can be advantageous in the case of expensive optimization problems
IBEA	Indicator Based Evolutionary Algorithm	It uses a flexible integration of preference information. Therefore, an arbitrary performance indicator can be used for the search. It does not need any diversity preservation techniques; moreover, the population size can be arbitrary
MO -CMA-ES	Covariance Matrix Adaption Evolution Strategy	The adaptive grid archiving strategy, which was presented in PAES, is merged with the covariance matrix adaption evolutionary strategy. In contrast to most other evolutionary algorithms, the CMA-ES is quasi-parameter-free
OMOPSO	Optimized Multi-objective Particle Swarm Optimization	Uses crowding distance to filter out leader solutions and the combination of two mutation operators to accelerate the convergence of the swarm and e-dominance based archiving strategy
SMPSO	Speed-constrained Multi-objective Particle Swarm Optimization	It is very similar to the OMOPSO algorithm, and there are only three differences: the usage of the speed constriction factor and the polynomial mutation operator and velocity handling at the borders of the search space
NSPSO	Non-dominated Sorting based	It uses the main mechanisms of NSGA-II (crowding distance, non-dominated sorting), and the global leader is selected

	multi-objective Particle Swarm Optimization	randomly from the leaders' archive. OMOPSO and SMPSO clearly outperform this variant
AMOPSO	Another Multi-objective Particle Swarm Optimization	It selects leaders from a non-dominated external archive. Three different selection are techniques used: Roundsby to preserve diversity, calledRandom to promote convergence, and called Prob, a weighted probability method. OMOPSO and SMPSO outperform this method
MOFA	Multi-objective Firefly Algorithm	Uses random weights to select the best from the Pareto-optimal solutions. Very quickly converges to the solution; however, it contains usually more than one function evaluation in every iteration step because the selected firefly makes one step with a new evaluation of a dominating one. This is a disadvantage in the case of expensive optimization problems

Authors divide Multi-objective Evolutionary Algorithms (MOEA) into three main paradigms: the Pareto-dominance based MOEAs (like NSGA- II), the Indicator Based Evolutionary Algorithms (IBEAs), and the decomposition based MOEAs. The first algorithm's paradigm works on the principle of non-dominated sorting where to each solution a rank based on its Pareto-dominance is assigned. In this case, elitist selection criteria are applied. The second, directly contains performance indicators to select the most appropriate offspring. In the third one, decomposition is a procedure that breaks down the given problem into smaller pieces and then optimizes them sequentially or in parallel. In this last approach, it is suggested in the paper to incorporate this paradigm with metaheuristic algorithms, e.g., Tabu-search, PSO, *etcetera.*

In the paper it is also highlighted the problem of comparing different algorithms and, at the same time, handling uncertainties in order to solve the robust optimization problem with these techniques. For this reason, mathematical benchmarks have been produced to test and compare the different optimization algorithms. For instance, the difficulties that benchmarks mimic are: slow convergence, a large number of local optima, a large number of variables, the dependency of variables, constraints, deceptive search spaces, flat search spaces and uncertainties. Due to this, there are many techniques that have been introduced which can increase the performance of the evolutionary algorithm-based calculations without increasing the number of samples. Paper [14] report some examples.
Parmee's [23-24] method proposes an evolutionary algorithm-based space decomposition method. It divides the search region into high and low-performance regions as a function of the sensitivity of the examined parameter. Another possibility reported in the paper is explicit averaging over time, where 'explicit averaging' means a form of resampling. Increasing the sample size is equivalent to reducing the variance of the estimated fitness function. Aizawa and Wah [25-26] were the first ones to propose to adapt the sample size during the algorithm. They proposed to start the calculation with a relatively small sample size and increase the number of individuals with the number of generations.
Other methods suggest modifying parts of the algorithm. For instance, in the paper some researchers' ideas, which propose to modify the selection operator and to use deterministic selection schemes in genetic and evolutionary algorithms to better handle the different types of uncertainties, are reported. Markon et al. [27] propose a threshold during the selection process in an evolutionary strategy, Branke and Schmidt [28-29] proposed a de-randomization for a better handling of uncertainties in the selection

process. The paper already reports some literature suggestions for selecting an evolutionary algorithm in a noisy environment, thanks to Rakshit and Conar's research [30-32]. They introduce four principles. Firstly, adapt in time the sample size during the optimization, increased exponentially with the generation number. Secondly, they propose to use deterministic selection schemes. Thirdly, they propose to use a clustering approach. Finally, they develop a robust crowding distance scheme that can work better in noisy environments.
At last, Orosz et al., in the paper mentioned above, suggest using meta-modeling techniques (as RSM, MCS, etc..) in order to evaluate the fitness function and avoid complex numerical simulations. Aspects of electrical machines robust optimization design are reported in paper [14], with the applications in some other fields of new technologies.

In [6] and [15] different features are shown, which can help to handle robust evolutionary algorithms.

In [15] Zielinski et al. show that, due to different characteristics of the algorithms, different stopping criteria are required to achieve a good convergence behavior. They examinate eleven stopping criteria for two different algorithms.

Reference [6] (Goldman's thesis work) presents two methods to increase the robustness of EAs. The author idea is that EAs can be made more robust by designing methods that automatically configure parts of an EA or by changing how EAs work to avoid configurable aspects, allowing them to reach better performance on a wider variety of problems with less requirements on the user. In this way, the author emphasizes the need to develop techniques that avoid or reduce the impact of configuration.
The first method is a novel algorithm designed to automatically configure and dynamically update the recombination method which is used by the EA to exploit known information to create new solutions. In particular, the following operators are dynamically tuned: population size, offspring size, n in n-point crossover, Gaussian mutation's step size, bit flip mutation's mutation rate, parent selection tournament size, and survivor selection tournament size. It is found out that a single dynamic parameter is more effective than using strictly static parameters.
The second method introduces the Self-Configuring Crossover operator encoded with linear genetic programming, which addresses these shortcomings while relieving the user from the burden of crossover configuration. It is described in the work that when performing crossover, each individual uses its own operator to create a child. This child receives a version of the parent's operator as part of its genome. Quality crossover operators tend to create quality children, therefore increasing their ability to propagate. If the operator is ineffective, the offspring will tend to be poor, resulting in a lower likelihood of survival and propagation.
Achieving automated configuration of an EA is one of the great unsolved challenges in the field of Evolutionary Computation and Goldman's thesis work is a good reference for an introduction in this field. In our case of study we will not try to implement dynamic configuration, but this problem can be the next step for further work in WEC's robust optimization field.

Solomon et al. [1] present the difference between active and passive robust optimization. Is defined passive robustness when it is attained by choosing the solution's parameters such that the solution's performance is less influenced by negative effects of the uncertain parameters' variations. Active robust optimization considers products able to adapt to environmental changes. Thus, by adaptation, losses in performance due to environmental changes and uncertainties are reduced. Evolutionary Algorithm for Solving AROP (Active Robust Optimization Problem) are given in the research.
AROP is a multistage problem, in which, during the design phase, the possible configurations vector $\vec{y}$ of the design vector $\vec{x}$ of the uncertainties involved are considered, and the optimal configuration for a sampled set of scenarios is searched. Authors conclude that an adaptive solution which possesses active robustness is able to achieve better performance than an equivalent non-adaptive solution.

1.2.3 Application Fields

Most of real-world optimization problems can be considered as robust optimization problems due to the presence of uncertainties, trade-off between objective-functions that have to be optimized and constraints.
García and Peña [33] present an overview on ROP common application fields and give definitions for some typology of robustness. The robustness models that they present are:

- *strict robustness:* the essence of strict robustness is that all scenarios can occur and all of them have an important criticality. In real problems, this type of robustness is necessary in critical systems where a failure is not tolerable;

- *cardinality constrained robustness:* in cardinality constrained robustness, it is considered unlikely that all the uncertainty parameters change at the same time when analyzing the worst case. Then, we can restrict the cardinality of the uncertainty space by varying only some parameters; the others are modeled with their representative values. It is a way to relax the strict robustness;

- *adjustable robustness:* it is another way to relax the space of uncertainty for strict robustness. It corresponds to divide the space into groups of variables. A first group is evaluated before that the scenario is determined, another group after that the scenario is known;

- *light robustness*: it is another approach used to relax strict robustness, relaxing the constraints in favor of the quality of the solution;

- *regret robustness*: relax the problem trough the objective function;

- *recoverable robustness:* recoverable robustness uses the concept of recovery algorithm and, like adjustable robustness, it obtains the solution in two stages relatively to the scenario.

Robust optimization is widely used in the following application fields (as reported in [33]): energy management [34], water management [35], machine learning, logistics, public goods, and a lot of engineering systems, as mechanical and electrical, both for structural problems and design problems [2], [36-37]. For WEC systems some examples are reported in literature, some of them for WEC's control system [38-43].
Thanks to the flexibility of this optimization technique, RO is also used in society problems, like in Phebe Vayanos' research concerning data driven decision making under uncertainties [44] about health care.

This last section concludes this first chapter and the overview about optimization process and robust optimization. This thesis work will continue in Chapter 2, which will describe the specific case of study (the PeWEC system) for this work, the chosen robust optimization framework (with a previous survey about different robust optimization framework and uncertainties' model) and the robustness indices used to perform it.

CHAPTER 2 – CASE OF STUDY: PEWEC

In the near future, the world will be forced to cope with climate change and its consequences. The energy demand is an important side of this challenge and in order to deal with it, several technologies have been developed and studied in the renewable energy field during the last decades. Solar, wind and geothermal energy technology have reached their commercial maturity. Instead, an already unexploited source for renewable energy with high potential is wave's motion energy: Wave Energy Converters (WEC) devices have been developed with the purpose of harvest this energy. The main problems for these systems are: cost of energy not yet competitive if compared with other sources of renewable energy, and the difficulty to deal with hostile conditions of the marine environment, e.g. corrosion due to salt water and high loads due to extreme meteorological events (in particular in ocean's environment).

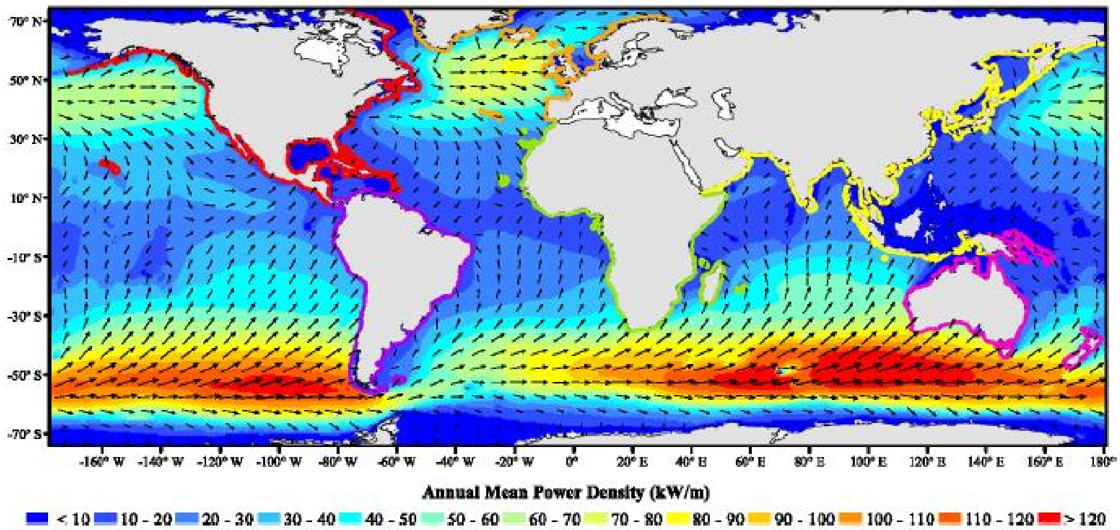

figure 2. 1

According to [45] it is possible to classify WEC in different ways. A first classification can be done depending on their installation location and their distance from the shore and bathymetry [45].

- *shoreline devices:* usually based on oscillating water column technology (OWC, in figure 2.2). They are typically embedded with harbor infrastructures, where waves force air to remain trapped in a chamber and induced it to flow through a Wells turbine in order to generate electric energy. Advantages for this kind of WEC are the proximity to the shore, that implies proximity to end-users and the reduction of maintenance costs, because shore marine environment is less wearing than open sea or ocean environment;

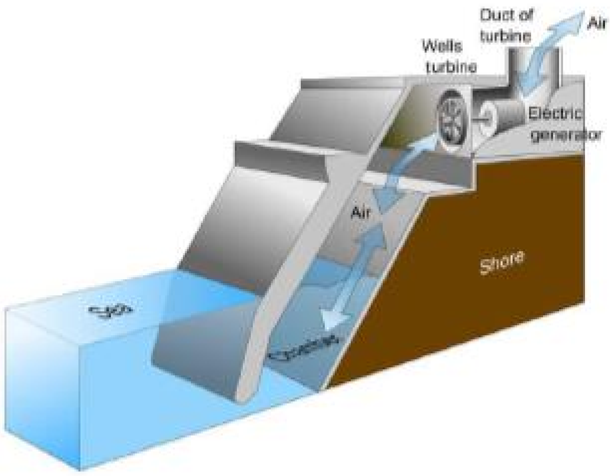

figure 2. 2

- *nearshore devices:* they are devices located relatively near the shore and with a bathymetry range between $10 - 25\ m$. Typically anchored to the seabed in order to trying to avoid mooring systems. The advantages for these devices are mainly the higher production of energy and greater closeness to the shore than offshore devices. This last can also be a disadvantage due to the social impact that the presence of WECs can have close to the shore, in particular if they are installed in arrays;

- *offshore devices:* they are located far from the shore and with a bathymetry higher than $40\ m$. For this kind of WECs structures, the technological solutions widely adopted are submerged structures moored to seabed or floating devices. Offshore devices can lead to a grater harvest of wave energy than all other typologies of WECs described above, already with a minor social impact. The distance from the shore, sharpens the difficulties caused by salty water and high loads, present in open ocean and sea environments.

The second classification described in [45] is respect WEC's size and working direction, with reference to the dominant wave.

- *Attenuators:* they are multi-body structures formed by several floating sections moored to the seabed and aligned with the direction of the dominant wave. They are linked by joints which relative motion is damped by hydraulic power take off

(PTO) to harvest the wave energy;

- *Terminators:* their main dimension is along the perpendicular to the incoming dominant wave direction [45]. They are WECs systems which exploit wave energy damping the pitching motion of the structures;

- *Point Absorbers:* these group of devices are characterized by a small dimension compared with the incident wavelength. They are floating or submerged structures moored at the sea-bed and they harvest wave energy damping the motion of the structure [45]. Examples for this WEC devices are ISWEC (Inertial Sea Wave Energy Converter) and PeWEC, both developed by Politecnico di Torino during last decades.

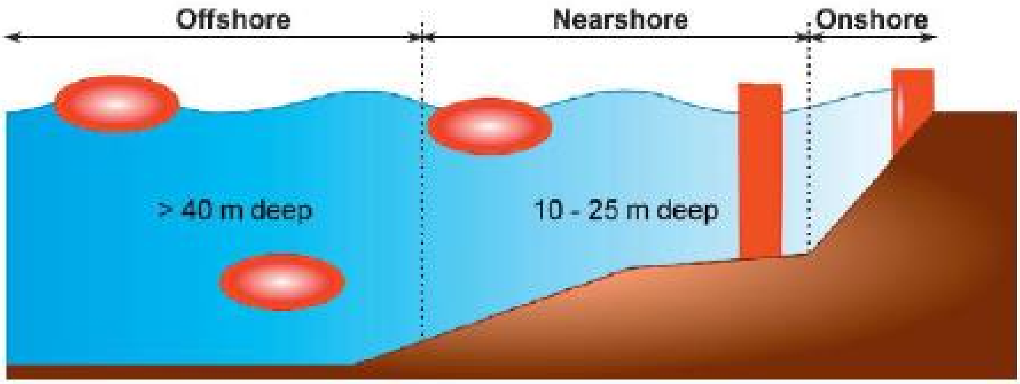

figure 2. 3

In order to deal with the disadvantages caused by the difficulties described above, which a WEC device has to cope with, several solutions were designed in particular to avoid relative motion between mechanical parts of the structure that, in marine environment, can quickly lead to corrosion and biofouling, thus, to a loss of reliability and efficiency. One of the most broadly used solution is to add a mass which has the function of inertial damper, reducing the structure's dynamic motion and, in this way, harvest wave energy damping the floater motion through rotating or translating masses inside the hull [45]. In this category it is possible to distinguish between pendulum and gyroscopic systems: the first ones exploit wave energy damping the pitching motion of the floater and therefore they need a mooring system that allows the device to align with the dominant sea-state [45], the latter reduce the rocking motion of a floater and harvest wave energy [45]. These devices can harvest the major possible quantity of energy when their floating body's resonance frequency matches with the incoming sea-state. Therefore, the device should be operating as near as possible to the resonance condition to maximize the extraction power. For this reason, floating structures of wave energy devices are characterized by their Response Amplitude Operator (RAO) which defines the behavior of the floater in frequency domain. A technological solution to this problem is an oscillating floating WEC embedded with a sloshing water ballast tank, which dynamic characteristics can be modified during operation through a proper control logic, in order to achieve the resonance-matching of the WEC for several sea state conditions. [45]. This is where the WEC's optimization problem arises: due to the sea-state high variability (and mechanical uncertainties) it is hard to tune design parameters and guarantee high performances in each environment that the device will deal with during its life. One of the robust optimization process objectives is to find the best possible set which can lead to increase reliability coping with environment's variability that brings uncertainties and, thus, make the device adaptable with more than one site, in order to make a step forward in the pre-commercialization phase.

In this thesis work the focus will be paid to the floating point absorber devices, in particular on PeWEC (Pendulum Wave Energy Converter). In the next paragraphs the description of the system will be resumed, taking as references [82], [49] and [42], which are papers written by researchers and professors form Politecnico di Torino who work at the MORElab. Since 2006, at Politecnico di Torino, Bracco et. al. started the research in wave energy converter field with a particular focus on the Mediterranean Sea, which is characterized by a high frequency of waves and, therefore, it is suitable for the use of inertial devices where the forces used to generate power are proportional to the incoming excitation frequency [49]. Consequently, these studies lead to the development of two devices: ISWEC and PeWEC. The first one is an inertial "active" device, since the inertial response can be adapted by varying the gyro speed. Therefore, the device needs to drain a small amount of the produced power to keep the gyroscope in rotation [Sirigu et al., 2016] [49]. The latter is a "passive" device which uses the force produced by a pendulum, without the necessity to be powered to produce inertial effect (for example to maintain in rotation mechanical parts) and studied to be located in Mediterranean site. In particular, PeWEC is developed for the Island of Pantelleria, one of the most powerful Mediterranean sites [Liberti et al., 2013] [49].

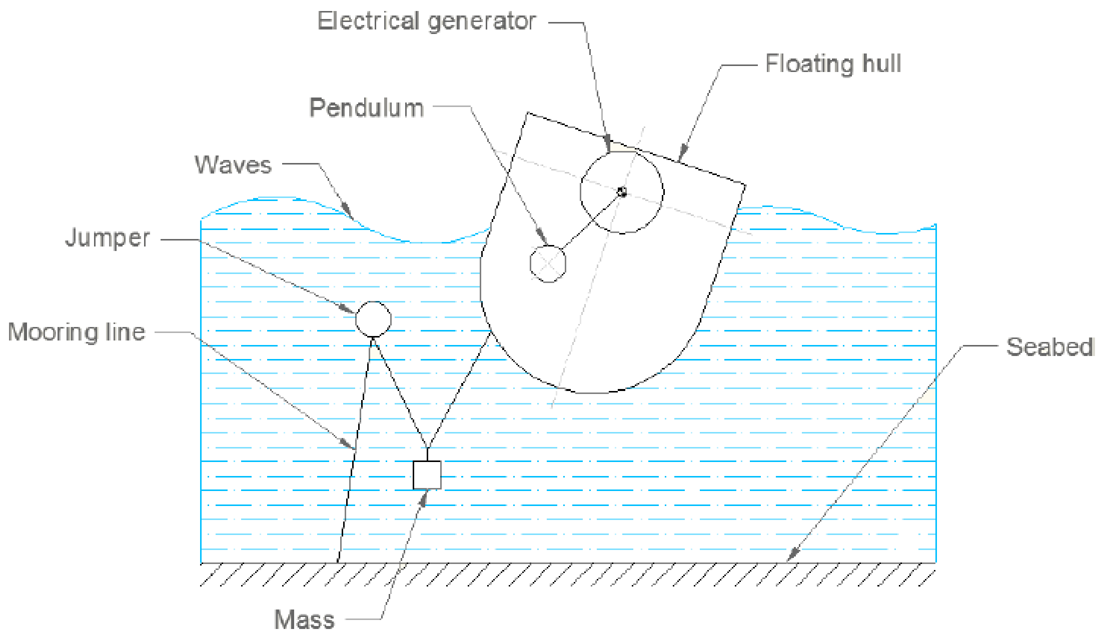

figure 2. 4

PeWEC layout is described in figure 2.4. It is composed of a floating hull anchored to the seabed and a pendulum connected to the shaft of an electrical generator, which is integral with the hull structure [49]. By protecting the pendulum, electrical generator and all the other parts of the system which are needed for the WEC's functioning from the direct contact with salt marine water, the device's durability increases.
The mooring system that anchors the device to the seabed can be divided in a submerged jumper, a clump weight and three chain sections. This arrangement is designed in order to not significantly interfere with the functioning of the WEC, paying particular attention to the weight per unit of length of chains, the mass of the clump weight and the length of chain sections. In ref. [49] Pozzi, Bracco et. al. explained the working principle of PeWEC starting from a qualitative description with a bi-dimensional point of view (given in figure 2.5). At first the system is considered to be at rest (2.5.1). When waves tilt the hull, it begins its motion along three directions: surge, heave and pitch [49]. In 2.5.2 is shown that pendulum hinge moves with the hull structure and consequently when hull structure start to tilt, the pendulum oscillations are induced. The pendulum rotation respect to the hull motion is used to drive the electrical generator shaft [49] and, thus, harvest electrical energy by damping these oscillations through a PTO (that has to be correctly sized), which is controlled as a rotary damper coupled to the pendulum. A control law is implemented in the PTO driver to adjust the pendulum dynamics to the instantaneous sea conditions to maximise the output energy [82].
A brief description of the PeWEC's mechanical device in all its components is given in the following paragraphs.

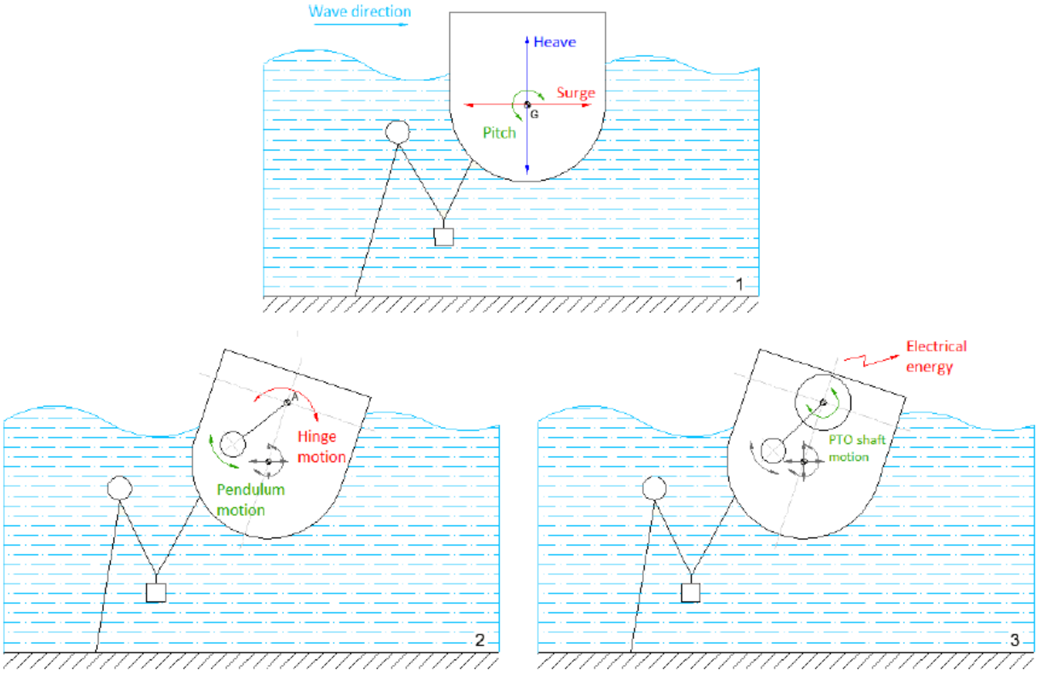

figure 2. 5

The device's hull is a steel structure, which has been sealed in order to separate the components within it from the hostile marine environment. It is made up of a curved keel, two side walls and a flat coping. Inside the hull, three internal sand weights (on the keel, stern and bow) ensure the distribution of the masses necessary to guarantee the required inertial properties [82]. The connection between the pendulum shaft and the PTO system described above takes place through a gearbox, which (as defined in the previous works) ensures a suitable coupling between the pendulum swing speed and the nominal speed of the PTO [82].

Before talking about the optimization problem, an introduction to the mathematical model used to evaluate the objective function and on which the optimization software relies is given, continuing to take as a reference the discussion made in [42] and [48]. Because of computational convenience reasons, the model is built as a fully linear model based on linear potential flow and it is implemented in frequency domain. Then, in order to avoid unrealistic or/and problematic motion, various constraints on displacements and loads are included. Potential flow-based models are usually divided in two phases A first phase in which the excitation coefficients $\boldsymbol{F_w(\omega)}$, the added mass $\boldsymbol{A(\omega)}$ and the radiation damping $\boldsymbol{B(\omega)}$ are evaluated for a representative set of frequencies by means of a boundary element method (BEM) software called NEMOH (this proceeding needs to be implemented and automatised in the genetic algorithm). A second phase in which, relatively to the designed installation site and for a comprehensive set of sea states, the dynamic response is evaluated taking care of appropriate energy-maximising PTO control parameters, tuneable with respect to the incoming sea state using a multi-variate simplex algorithm for each device. Therefore, the optimal control is independent of the design optimization (performed by the genetic algorithm modifying geometrical and technical characteristics of the device).

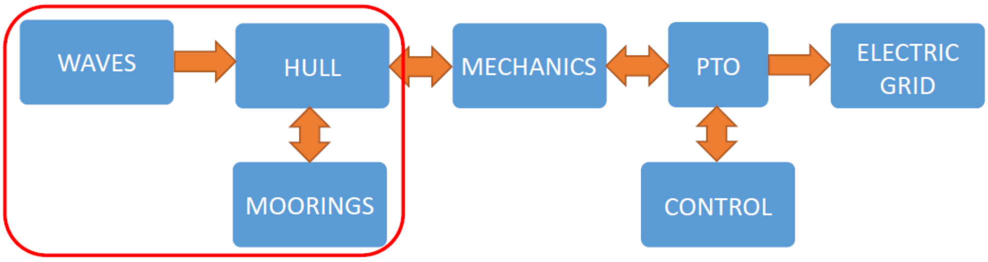

figure 2. 6

Therefore, the following equation describe the device's dynamic behaviour and motion:

$$\left(\boldsymbol{M} + \boldsymbol{A}(\boldsymbol{\omega})\right)\ddot{\boldsymbol{X}} + \boldsymbol{B}(\boldsymbol{\omega})\dot{\boldsymbol{X}} + (\boldsymbol{K_h} + \boldsymbol{K_p})\boldsymbol{X} = A_w(\omega)\boldsymbol{F_W}(\boldsymbol{\omega}) + \boldsymbol{T_{PTO}}$$

where: $\boldsymbol{M}$ is the mass matrix (and consequently the inertial reaction forces), $\boldsymbol{K_h}$ and $\boldsymbol{K_p}$ are respectively the hydrostatic stiffness and the restoring force of the pendulum, $A_w(\omega)$ is the wave amplitude, $\boldsymbol{T_{PTO}}$ is the PTO action and $\boldsymbol{X}$ the state vector. $\boldsymbol{K_p}$ and $\boldsymbol{M}$ represent the coupling between the pendulum and the hull. Looking at the last one under the assumption of mono-directional waves aligned with the longitudinal axis of the hull and also considering the pendulum oscillation, the state has four dimensions:

$$\boldsymbol{X} = \begin{bmatrix} x \\ z \\ \delta \\ \varepsilon \end{bmatrix}$$

with x for surge motion, z for heave motion, δ for pitch motion and ε for pendulum oscillation. Considering the PTO acting on the rotational degree of freedom of the pendulum:

$$\boldsymbol{T_{PTO}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -k_{PTO}\varepsilon - c_{PTO}\dot{\varepsilon} \end{bmatrix}$$

In order to evaluate the coupling between the pendulum and the hull, $\boldsymbol{M_P}$ and $\boldsymbol{K_P}$ matrices can be defined after linearization in the following way, also considering the total inertia of the device with respect the pitch axis like $I_{yy} = I_b + I_{p,A} + I_f$.

$$\boldsymbol{K_P} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -gm_p(d-l) & gm_p l \\ 0 & 0 & gm_p l & gm_p l \end{bmatrix}$$

$$\boldsymbol{M_P} = \begin{bmatrix} m_T & 0 & m_p(d-l) & -m_p dl \\ 0 & m_T & 0 & 0 \\ m_p(d-l) & 0 & I_{yy} + m_p d(d-2l) & I_{p,A} - m_p dl \\ -m_p l & 0 & I_{p,A} - m_p dl & I_{p,A} \end{bmatrix}$$

where m_T is the total mass of the system; I_f is thefloater moment of inertia; g is the acceleration of gravity; d is the distance between the device centre of gravity (CoG) and the pendulum fulcrum; l is the pendulum length; m_p and m_b are the masses of the pendulum and the bar holding the pendulum, respectively, and $I_{p,A}$ and I_b are their moments of inertia, where the first is the inertia computed with respect the rotation axis

[82]. Taking as references the works done before, for example in [82], with the purpose to maintain valid the model's assumptions, mooring and viscous effects are neglected because the first are mean drift forces and second order effects and the seconds would require specific tuning for each device. However, despite all these simplifications, the model has been validated trough an experimental campaign and ensure appropriate accuracy and representativeness for global optimisation problems [82]. Always with the aim of keeping the computational cost as low as possible, assuming the system excited only by harmonic loads, the response will also be harmonic, and the system can therefore be easily solved. In particular, the steady-state response of the device can generally be determined based on the system's transfer function as follow:

$$H(\omega) = \left[-\omega^2[\boldsymbol{M} + \boldsymbol{A}(\boldsymbol{\omega})] + j\omega\boldsymbol{B}(\boldsymbol{\omega}) + \boldsymbol{K_h} + \boldsymbol{K_p}\right]^{-1}$$

Therefore, this last equation provides the frequency representation of the system described. Then, this equation can be used in order to describe the spectral-domain model probabilistic representation of the waves and of the PeWEC response through the power spectral density (PSD) matrix.

$$S_q(\omega) = H(\omega)S_f(\omega)H^{\mathcal{H}}(\omega)$$

Where: $F_{ext}(t)$ is the excitation force, $S_f(\omega)$ is the power spectral density matrix of the excitation force, $S_q(\omega)$ is the response spectrum matrix and $H^{\mathcal{H}}(\omega)$ denotes the Hermitian transpose of $H(\omega)$ [82]. In the following sections a more detailed description of the robust optimization PeWEC's problem and its chosen solution method are reported. The general optimization problem is given according to Sirigu et al. in [42].

2.1 Optimization's Problem Definition

What the optimization problem tries to optimize is the Levelized Cost of Energy (or Cost of Energy CoE), defined as the ratio between capital cost and Annual Energy Production (AEP) for the whole life of the installation. The optimization software requires, for its implementation, the definition and parametrization of the PeWEC device, in order to fully and univocally define the system so as to completely identify it by its shape, dimensions, mass, inertia, ballast's distribution, pendulum and PTO characteristics.

In this thesis work we refer to the same 13 constructive parameters used in previous works as paper [42] or [82] and they are: six for the hull, five for the pendulum and two for the PTO [42]. Referring to earlier research, the shape of the hull is assumed to be composed of a bottom circumference, tangential to two circumferences in the bow/stern sections, as shown in figure 2.7, while the transversal section is assumed to be constant

[42]. This, for both manufacturability and hydrodynamic performance. The design parameters reported by Sirigu et. al. are:

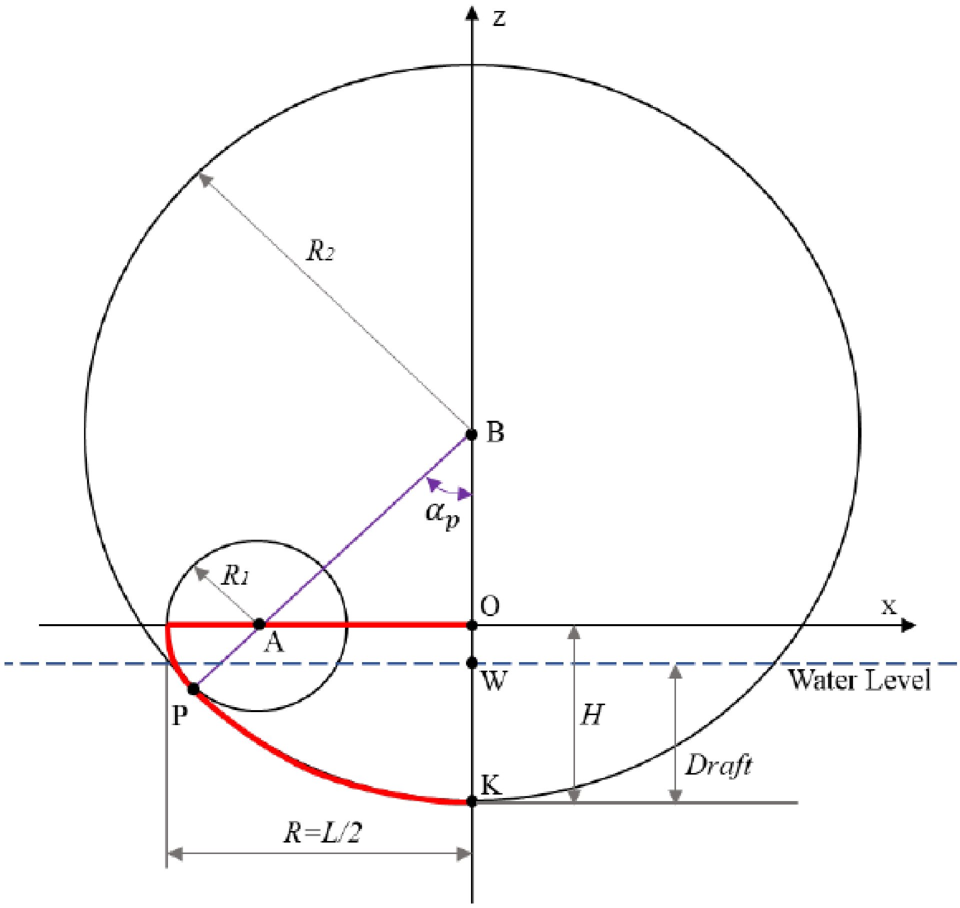

figure 2. 7

- R: semi length of the floater;
- H: overall height of the hull—i.e., the distance between the keel and the deck;
- D: draft of the hull;
- R_1: radius of circumference C_1;
- (x_A, z_A): x- and z- coordinates of the centre C_1, respectively, with $z_A = 0$;
- R_2: radius of circumference C_2;
- (x_B, z_B): x- and z- coordinates of the centre C_2, respectively, with $x_B = 0$;
- $\alpha_p = P\hat{B}O$;
- L: length of the floater;
- W: width of the floater;
- $h = \frac{x_A}{h}$ bow/stern circumference ratio;
- $k = \frac{H}{R}$ height ratio;
- $j = \frac{D}{H}$ draft ratio;
- BRF: ballast filling ratio, defined as the ratio of ballast located in aft/fore ballast tanks over the total ballast ($BFR = 1$: all the ballast is stored in aft/fore ballast tanks; $BFR = 0$: all the ballast is stored in bottom ballast tank).

An important design aspect is the mass distribution and the calculation of the ballast mass. Three different masses are defined by designers in [42]: ballast (sand) required (M_{bal}), mass of the displaced volume of water (M_{tot}), mass of the hull structure (M_h) and the mass of PTO-units. M_{bal} is univocally defined by the others. For more detailed aspects of the mechanical point of view the reader is suggested to look and referring to [42] or [82]. The design of the structure is expressly determined to ensure that the allocation of M_{bal} settle the total moment of inertia of the system. The mass of each unit is defined as follow:

$$M_{unit} = \frac{\beta_U (M_{tot} - M_h)}{N_p}$$

where β_U is a design parameter, of which complementary fraction represent the ballast mass. In order to resume the design parameters used by the genetic algorithm to univocally define an individual [42], the authors give the following table.

Design Parameter	Symbol	Units
Hull Length	L	m
Hull Width	W	m
Bow/stern circumference ratio	h	$-$
Height ratio	k	$-$
Draft ratio	j	$-$
Ballast filling ratio	BRF	$-$
Number of pendulum/PTO	N_p	$-$
Unit mass ratio	β_U	$-$
Pendulum shape factor	σ_p	$-$
Pendulum arm factor	γ_p	$-$
Pendulum fulcrum factor	λ_p	$-$
Gearbox ratio	r_g	$-$
PTO ID	ID_{PTO}	$-$

table 2. 1

where the shape factor σ_p defines volume, radius, height and decides if the pendulum is large and short ($\sigma_p = 0$) or small and long ($\sigma_p = 10$) [42], the length of the swinging arm is defined by γ_p and the height of the fulcrum by λ_p. Note that not all possible combinations of geometric and pendulum property parameters are feasible due to volume and weight physical restrictions [42]. The energy conversion stage is carried out by a one permanent magnet synchronous motor (PMSM) for each pendulum with the help of a planetary gearbox defined by the design parameter r_g, sized to be suitable for high torque and low speed application. Finally, different PTO systems are used (ID_{PTO}), with different combinations of nominal speed (n_{PTO}) and nominal torques (T_{PTO}) [42].

As said before, regarding the techno-economical side of the optimization problem, the aim is to minimize the ratio (CoE) between the overall capital expenditure ($CapEx$) over productivity, which is the energy produced by the plant during its lifetime N_y (assumed by Sirigu et. al. as 25 years for PeWEC):

$$CoE = \frac{CapEx}{N_y AEP}$$

where AEP is the annual energy production. In this thesis work, like in [42], operational expenditure $OpEx$ is neglected and for the same reason presented in the paper: because its impact is about 2% of the overall expenses. A second techno-economical parameter is CWR (Capture Width Ratio) which measure the ability of the device to convert energy and is formulated as:

$$CWR = \frac{AEP}{E_w}$$

where E_w is the annual available energy. CWR has to be maximized. CWR and CoE are in contrast, thus for instance, a multi-objective optimization can be performed and a trade-off between these parameters will be highlighted in order to obtain a set of design variable which can maximize the production of energy and minimize costs.

A drawback for WEC technology, which is underlined in these studies, is the strong dependence of device's design from the wave climate. For example, a pitching floating device's design dimension increases with the increment of wave's length or, indeed, longer is the wave period longer should be the hull length in order to be in resonance condition with the most probable sea states of the deployment site [45]. Therefore, enormous devices can lead to significant techno-economic, manufacturing and installation issues [45]. For this reason, the productivity is calculated thanks to a scatter diagram for the specific installation site target, which is near the Island of Pantelleria, as written before. In particular, for each sea state the incoming wave power of the device is defined as [42]:

$$P_w = \frac{\rho g^2}{64\pi} H_s^2 T_e W$$

where ρ is the water density, H_s the significant wave height, T_e the energy period and W the hull width, which is shown to be proportional to the incoming wave power by the equation described above. In figure 2.8 scatter diagrams for occurrences and waves energy are reported. In this figure red crosses represent a specific wave characterized by

a wave period with corresponding height, energy and occurrences during a specific amount of time. In this way the energy resources are predictable and can be used by the control logic in order to set the WEC's dynamic damping device in order to achieve the resonance-matching and maximize the quantity of harvested energy.
Thus, the incoming sea state influence the mean absorbed power, in particular the expected value can be described with the following equation:

$$E(P_{abs,avg}) = -c_{PTO} \sum_{k=1}^{N} S_{\varepsilon\dot{\varepsilon}}(\omega_k)\Delta\omega$$

where $S_{\varepsilon\dot{\varepsilon}}$ refers to the power spectral density function of the degree of freedom related to power extraction [82].

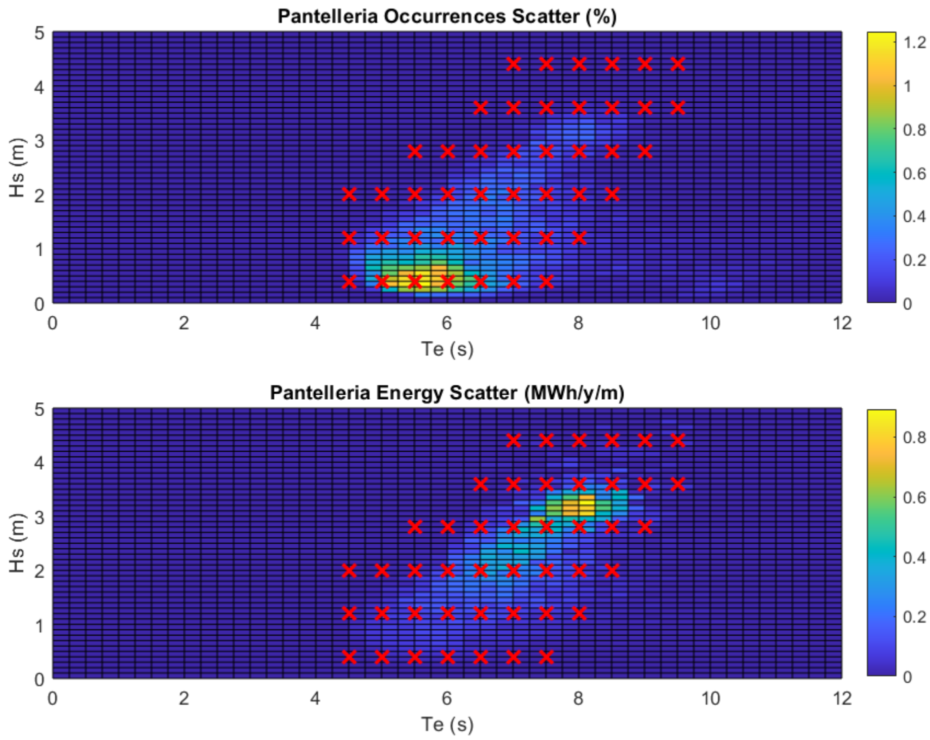

figure 2. 8

In figure 2.9 a digital representation of the 1:45 PeWEC prototype and its subcomponents is reported

2.2 Robust Optimization Design: Uncertainties' Models and Treatments

In this thesis work, a robust optimization will be performed in order to increase the device's reliability and performances at the same time. In earlier optimization works, as [42], the results obtained by the MOP highlighted some noise in hull parameters: pitch turner radius, pitch hull viscous damping, distance between pendulum's hinge and device's CoG (Center of Gravity), hull cost, pendulum cost and PTO cost. For this study, uncertainties effects are considered only for the first three parameters. This choice has been made due to the economic factors' randomness. The cost's variations are not caused only by market fluctuations in the prices of materials, but also by the possibility of choosing different suppliers, who sell their products at different prices. This makes defining a range in which these three cost parameters can fluctuate very difficult.
Referring to paper [68], which in turn refers to [69], four main categories of uncertainties can be found in real-world problems:

- operating conditions, which affect environmental conditions when a system is operating;
- parameters, which come up from manufacturing imprecisions and parameters which inadvertently change during system operation;
- outputs, carried on by simulations and meta-model's inaccuracies and error;
- constraints and feasibility uncertainties, meaning the situation in which boundaries of search space are varied but the system does not change proportionately.

In this thesis work, the attention will be placed on the first and second type of uncertainties.

Referring to Venigella master thesis [51], the uncertainty can be defined as the deviation of a value, estimated after a calculation or an observation, from the nominal/true value. A more general classification can be made distinguishing between aleatory uncertainties and epistemic uncertainties.
Aleatory uncertainties describe, when sufficient data are available, the fluctuation for a specific parameter, and due to their stochastic (aleatory indeed) nature, they are usually modeled by probability theory. Due to the specific parameter's nature, different variables may follow different probability distributions [51]. In the paper, the authors suggest that measurement errors, dimensions, costs and prices can be modeled with a probability distribution. In particular, they mention the normal distribution as suitable for those cases.
Epistemic uncertainties, instead, usually arise when lack of knowledge or incomplete information about the parameter in exam and the system studied are present. This kind of uncertainties can be described using both probability and non-probability theories. Thus, when available data are not sufficient, interval approach can be used. In this model,

uncertainty is denoted using a simple range [51], which must be defined and limited by the decision maker. The smaller the range, the more robust will be the parameter.
In the paper, three methods are given to deal with these two different uncertainties: one uses only the random-probabilistic model in order to handle aleatory uncertainties, one uses only the interval model in order to handle epistemic uncertainties and the last one considers both the model and the uncertainties type in the system (this last approach will be, of course, more accurate than the others).
Therefore, looking to Castrup work in [73], the importance to have sufficient available data and estimates can be highlighted. These data and estimates are obtained computing a standard deviation from a sample of measurements (type A estimates) or by forming an estimate based on experience (type B estimates) [73] in order to estimate the most possible accurate and suitable uncertainties probability distribution (data-based experimental distribution) and choose the right one. Another assumption made in this thesis work is that each uncertainty is considered to be independent.

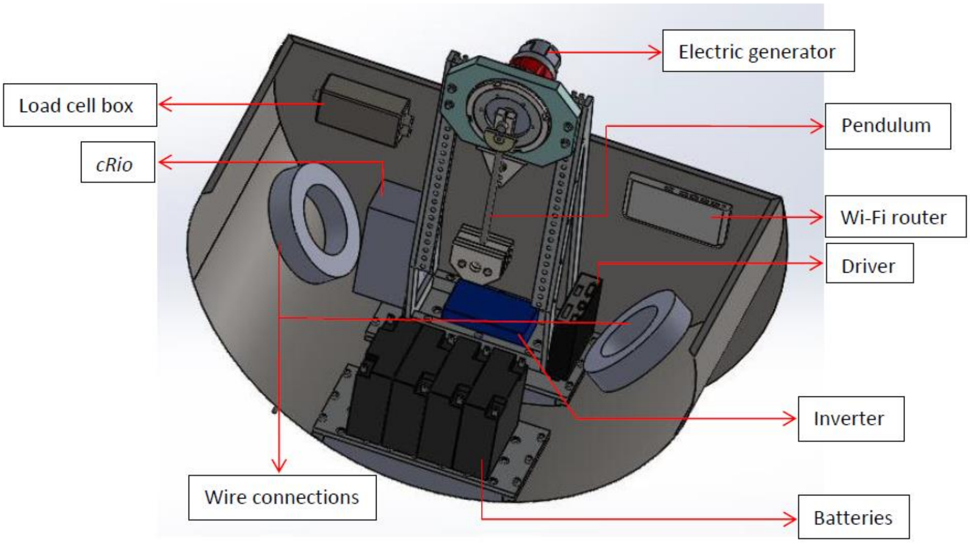

figure 2. 9

Therefore, the necessity to settle an uncertainties probability distribution model in order to choose and define our robust design optimization process can be emphasized.
As previously stated, it can be highlighted how important is to have sufficient available data and estimates in order to reach the most possible accurate and suitable uncertainties probability distribution (data-based experimental distribution) and choose the right one.

Castrup, still in [73], give us a summary and some guidelines upon the most common different probability distribution and their characteristics. Resuming:

- *Uniform distribution:* the probability for a single event of lying between upper and lower limits is constant and the probability of lying outside is zero. This distribution is used when the decision maker does not know for sure how events are distributed and prefer a conservative approach. Due to these reasons, it is one of the distribution most commonly utilized for uncertainties analysis;

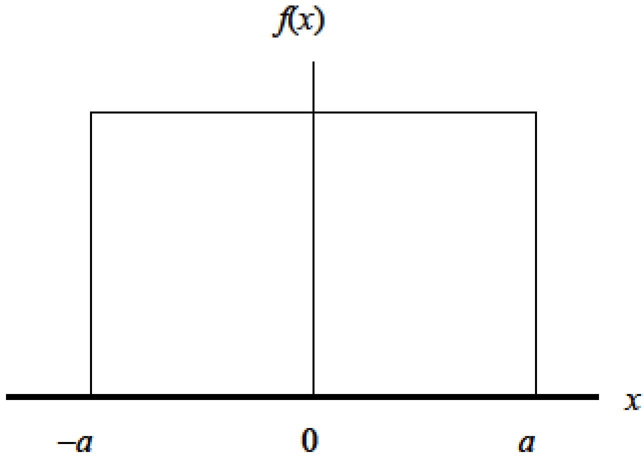

figure 2. 10

- *Normal (or Gaussian) Distribution:* defined by mean and standard deviation. It is usually obtained for a sample of repeated measurements. It is also widely used to represent physical phenomena or manufacturing process parameters and instrument parameters subjected to usage;

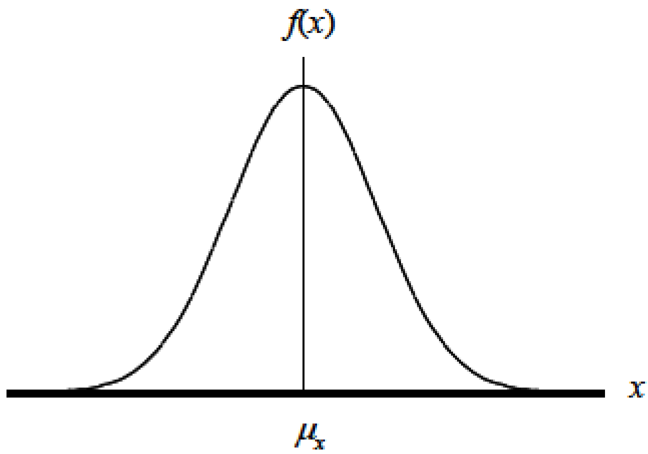

figure 2. 11

- *Logonormal Distribution:* useful for parameters subjected to physical limit and constraints or with asymmetric tolerances;

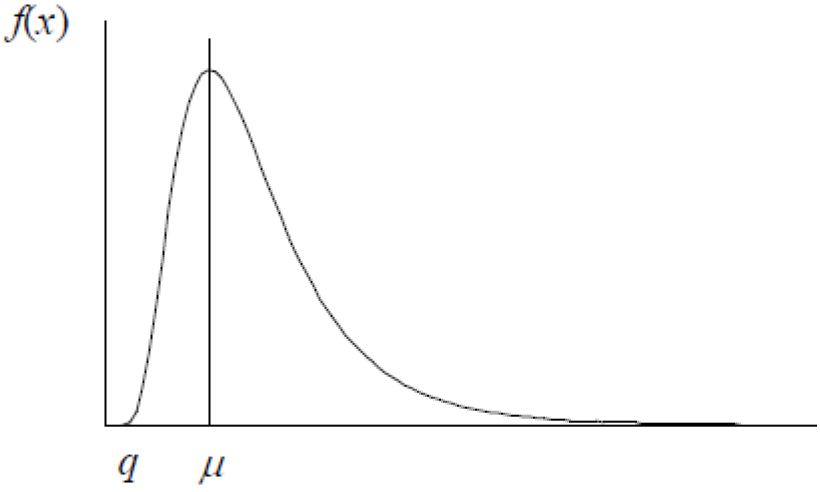

figure 2. 12

- *Triangular Distribution:* used when containment is 100% like uniform distribution but with central tendency;

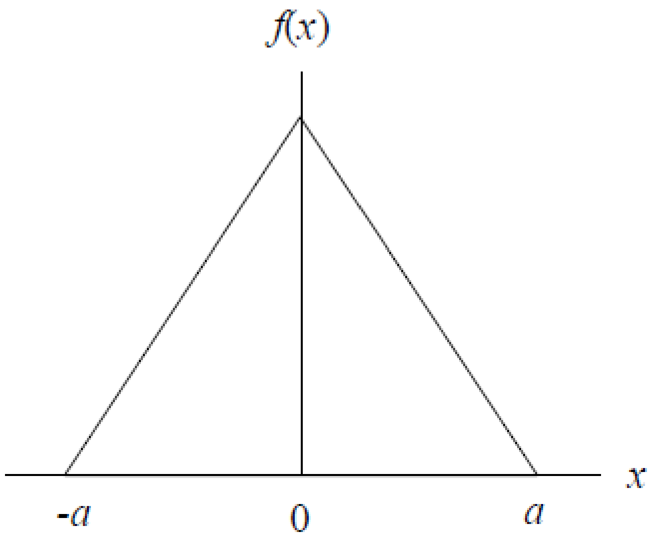

figure 2. 13

- *Quadratic Distribution:* represent a central tendency but eliminates the abrupt change at the zero-point respect to the triangular distribution without discontinuities and linear behaviour;

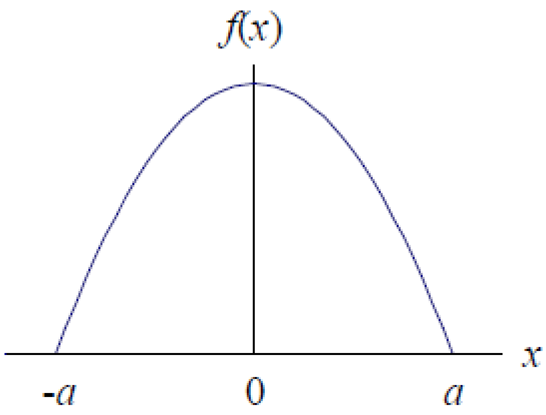

figure 2. 14

- *Cosine Distribution:* a 100% containment distribution with a central tendency and lacking discontinuities [73] used when the parameters present a central tendency and it is subjected to random usage and handling stress.

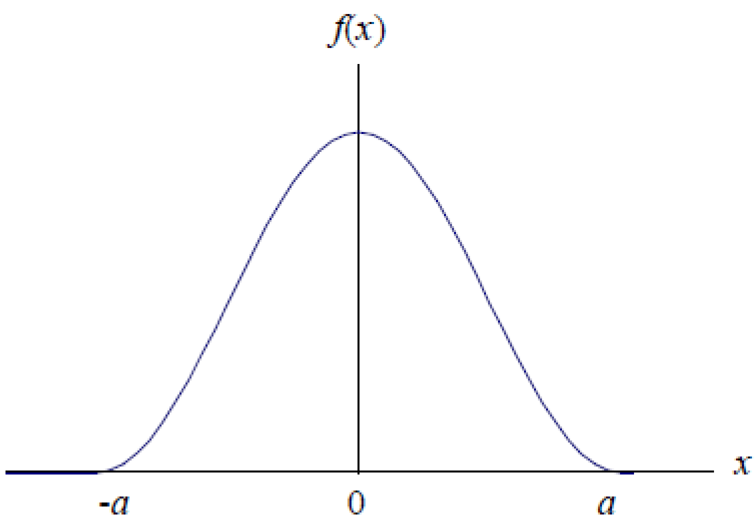

figure 2. 15

For more accurate notions upon these distributions or different distributions (half-cosine, U and Student) the reader is suggested to look to the paper cited above.

If sufficient data are available, an approach used to investigate which distribution model can be the suitable one for the parameter under examination is the Kolomogorov-Smirnov test. This test can be used in order to compare samples among them or with a reference distribution chosen. For example: perform a Kolomogorov-Smirnov test in order to verify if an experimental/empirical obtained distribution or CDF follows a normal distribution behavior.

Some studies and papers which explores WEC's uncertainties probability distribution have been found in literature.
Hiles, Beatty and de Andres in [74] explore WEC's outputs uncertainties because of their influence on economic viability of a wave energy project, especially for MAEP (Mean Annual Energy Production) which is a standardization metric for the quantification of annual electricity production. Authors describe their research made in two phases. First, they use high fidelity time-domain numerical models to simulate several long-term WEC's deployments in order to obtain sufficient WEC's performance data. Second, they use those data to evaluate the associated MAEP and then study the relative uncertainty. With obtained results they established that the population of MAEP follow a normal distribution and authors decide to use the standard deviation σ_{MAEP} to describe the population variance. Due to the dependence of the results on WEC's typology, location and climate condition, authors conclude it seems reasonable to set a target level for σ_{MAEP}.
Orphin, Penesis and Nader use Monte-Carlo method in [75] in order to investigate uncertainties propagation and validate experimental data obtained with physical-scale model and their reliability.
In [76], uncertainty analysis is performed for the energy conversion chain in a WEC. The authors' purpose with this paper is to provide guidelines for International Towing Tank Conference (ITTC) members in order to perform an uncertainty analysis for WEC's model tests experiments and collect high quality data minimizing uncertainty sources and effects. A specific uncertainties classification is given and then also a list of possible uncertainty sources is discussed, for example: inaccuracy for WEC model characteristics, undesired facility related to hydrodynamic effects, errors in PTO equipment parameters, disturbance for test arrangement of the model, measurement inaccuracies. At last, a test procedure in phases (resumed in a flow chart) is described and provided with an application example.

Studies about uncertainties quantification (UQ) are carried on by UTOPIAE (Uncertainty Treatment and Optimization in Aerospace Engineering) project. It is a European research project with the purpose to develop mathematical methods and algorithm in order to bridge the gap between UQ and optimization, referring to Probability Theory and Imprecise Probability Theory, in aerospace field. Example of papers published thanks to this work are from [77] to [81].
In particular in [77] Filippi, Vasile, Riccardi and Absil present a novel solution for a Resilience Optimization Problem in preliminary design optimization process under uncertainty. This kind of problem is considered as the union of Robust (related to the impact of uncertainties on the prediction of the objective function value) and Reliability-based (related to the ability of a system to maintain its operational capacity under uncertainties) Optimization Problem, seen as two aspect of the resilience one. In the paper

authors divide uncertainties for this early design phase in two large categories. Epistemic, associated to lack of knowledge or imprecise modelling and poorness of data background. Aleatory, related to the occurrence of random-aleatory events. The methodology presented to the designer in order to cope with these issues is based on Evidence Theory (a part of Imprecise Probability Theory), which operates on deductions from the available evidence instead of assuming complete knowledge of the probability distribution [77]. Thus, the framework proposed creates an evidence-theoretic model evaluating Belief (which collects the probability masses associated to possibilities satisfying a specific statement [77]) and Plausibility (which collects the masses of possibilities not contradicting a specific statement [77]) of a given proposition and that makes possible to handle both epistemic and aleatory uncertainty while using the same approach. Two other application examples for Imprecise Probability Theory and Evidence Theory to deal with optimization under uncertainties are presented in papers [78] and [79].
In papers [80] and [81] instead, Sabater, Bekemeyer and Görtz (which work in UTOPIAE project too) present another way to assess uncertainties and perform it for two different applications. Specifically, the proposed framework and a surrogate-based optimization algorithm and an also surrogate-based approach for uncertainty quantification. The first one utilizes a quantile method with the purpose to balance exploration and exploitation phase during the process, the latter is designed with an active infill criterion for the quantification of the quantile.

2.3 Robust Optimization Design: Robust Optimization Frameworks

Referring to paper [69] by Beyer and Sendhoff, a comprehensive introduction to different framework typologies can be set up. Especially, a survey about approaches where uncertainties are involved in the process using any kind of simulation technique, in order to obtain an objective value evaluation, which may be regarded as robust optimization direct approaches, is given. Authors state in the paper that functionally to the way chosen to manage raw data obtained from simulations, three different direct approaches (direct optimization algorithms are methods which use only objective function values to be optimized ad inputs and not gradient or higher-order derivative information) categories can be listed:

1. *Monte Carlo (MC) strategies*, which use statistical parameters, calculated given a fixed design point, as inputs for direct and gradient/derivative free numerical optimization algorithms;

2. *Meta-model approach,* constructed as an estimate of the real robust optimizer model given a set of design points carefully chosen;

3. *Perturbated system;* in which the perturbated observed output performance is used directly as inputs of optimization algorithm (this approach especially suits for noisy uncertainties).

A possible categorization for the main methods attributable to case 1, which can be implement in algorithms in order to handle with uncertainties (therefore, handle a robust optimization problem) is given by Marquez-Calvo and Solomatine in [60]. They divide these techniques in four categories:

A. *Methods minimizing the mean of the objective function.* Which minimizes a smoothed version of an objective function, which uses the average of the objective function values in the proximity of a given point [60];

B. *Methods minimizing the mean-variance of the objective function.* In this method also the standard deviation is estimated beside the smoothed mean-objective function and their combination is optimized instead the of the nominal objective function;

C. *Methods using an additional objective function related to robustness.* This method adds to the original objective-functions' vector $\vec{f}$ another objective function f_i which represents a selected measure of robustness;

D. *Methods using additional constraints related to robustness.* In this method a vector of constraints is added to the original inequality constraints of the problem, which sets a boundary to a given measure of robustness [60];

E. *Method based on comparing the cumulative distribution functions.* Using Monte Carlo simulation, a number of CFD are generated and the purpose of this approach is to reduce during the optimization the difference between the CFD characterizing uncertainty of a real solution and that related to an 'ideal' design.

According to this, can be concluded in agree with paper [69] that if the research interest is in expectancy measure, the minimizing of the mean (or mean variance) method over a fixed number of samples represent a simple possibility. Drawbacks for this method are computational costs (which can be very expensive) and the connection between degree of accuracy and number of samples (high number of samples is required to ensure a certain quality and reliability of the robustness measure).
In order to handle with issues described before, the second approaches' category can be observed. The meta-model is usually a simplified function depending on a set of model parameters tuned with the purpose of predict the observed data. Observing this simplified model, optima can be easily calculated and used as approximations of the real robust optima. Broadly used meta-model techniques are: response surface methodology, neural networks, Kriging models. For this second approach the main problem is that it is not suitable for large-scale robust optimization problems with large number of design variables due to three principal issues: model complexity, degree of outputs' approximation (optimum obtained will be a first approximation and the metamodel need to be repeatedly applied in order to get closer to valid results) and the data uncertainties which can lead to uncertainties in model parameters.

The third approach, indeed, represents the most direct usage of noisy information [69] and four different types of search methods which handle with noise can be identified:

- Gradient estimation techniques, usually referred to stochastic approximation methods. The idea is to combine gradient-based search strategy with iterative update, assuming the exitance of the first moment of the function to be optimized;

- Pattern search methods, which create search points and accept only points in which appear an improvement referring to prior search points scheme. They do this without local gradient approximation or Hessian of the function to be optimized. Their appeal is represented by their characteristic of inspecting pattern points on the basis of necessity [69];

- Optimization techniques based on RSM. They are very similar to the second type approach before described. They iteratively generate sequences of response surfaces and predicted improvement directions or local optima are used as predictors for improved design points. Due to the computational expensiveness, simply response surfaces must be used;

- Tabu search algorithm.

The first three techniques reported are widely used also in first case approach, in order to optimize expectancy robustness measures.

A special mention has to be done for Evolutionary algorithms, using them it is possible to deal with most of the methods mentioned before.
They have been successfully applied in robust optimization, in particular for noisy environments because they can be robustness optimizer per sé. For insance, considering the mutation/recombination operator as a robustness tester by directly adding noise in the process: indeed, in biology, mutations are noise for the inheritance (and cell division) processes against which the system has to be sufficiently robust to succeed [69]. In order to emulate mutations' effects and noise effects simply with another mutation, this last one and parameter uncertainties must enter the system at the same level [69].
EA are also widely used in order to achieve case 1 approach results and cope with necessities of that method.

As paper [61] states, the broadest spread method used in order to explore the influence of uncertain parameters on system's outputs, is analyze them through a sensitivity analysis approach after performing a deterministic optimization process. The same paper also highlights how in literature most of the works consider the stochastic nature of renewable energy as source of uncertainty. Then, authors (Roberts et al.) presents a method which modifies a GA using a worst-case scenario approach.

In paper [64] Zojionvic et al. present an urban drainage robust optimization process using two different methods, both based on minimizing the mean of the objective function. In this work, authors consider the uncertain variables to have normal probability distribution, with supposed mean value and standard deviation (the last one considered as

a percentage of the first one). The optimization is processed with NSGA II.
The first approach (called multiple realization method) incorporates uncertainties in the evaluation of objective function through multiple realizations of uncertainties in each generation [64], this is carried on running the case of study's meta-model for a specific number of samples (sampled using LHS technique) for each chromosome on the population.
The second approach refers to previous studies cited in the paper and it is practically a modification of the NGSA II algorithm which incorporates uncertainties during the optimization process with a single realization of the uncertainty sets in each generation (a meta-model is elapsed in each generation). It is done averaging present and past values over the chromosome's age in order to calculate the objective value.
Two similar methods are presented in [66]. In his thesis work, Kebede performs at first a sensitivity analysis to select sensitive parameters which most affects the performance of the system. Then, due to time and computational power required to include uncertainties in the optimization framework [66], he chooses to represent those uncertainties' effect to the system with the most effective certain parameters mentioned before. For example: in his work he wants to minimize flooding, therefore all parameters found with the sensitivity analysis which are most sensitive to flooding are used to represent uncertainties. The next step in Kebede's work is to perform a multi-objective robust optimization which includes uncertainties during the process. This is tried out with two different methods as mentioned before, both working by integrating sampling techniques (LHS), samplings, metamodel simulation (in particular an hydraulic metamodel) and NSGA II optimizer. The first method is very similar to the first method presented in paper [64], for each chromosome a metamodel simulation is performed for all sets of the uncertain parameters sampled and average of the objective functions over the entire sample realizations are used to evaluate the chromosome fitness in a generation [66]. In the second one NSGA II is modified with the purpose of obtain a more efficient optimum solution search, therefore the metamodel simulation was performed for one sample realization for the evaluation of the fitness of the chromosome [66], one time for each generation, and outputs are used for objective value calculation [66].

Jin and Sendhoff in [67] suggest a method with the purpose to exploit all information available in the current population with a perturbation of the design variables during the fitness evaluation in order to obtain a robustness measure trying to avoid additional fitness evaluation, which can be computational expensive. Then the robustness measure is treated as a new objective function (case C) in order to find a trade-off between optimality and robustness of the system.

Another kind of approach is described by Mirjalili, Lewis and Dong in [68]. Their method uses a novel dominate operator in the optimization process to find the Pareto-front based on both robustness level and confidence level and so-called confidence-based Pareto dominance and Confidence-based Robust Multi-Objective Particle Swarm Optimization. This method allows to design different confidence-based robust optimization variants of meta-heuristics based on different methods [68]. The paper distinguish between two main techniques used to achieve a robustness measure during an optimization method which can fall within case A or B. The first obtains the objective function value referring to an expectation measure (mean and/or standard deviation) calculated by averaging a representative set of neighboring solutions around the nominal point (computational expensive due to their need to evaluate the objective function several times for the sampled neighborhood); the second obtains the objective function value referring to an archive-based technique (with meta-heuristic search's agents) or a surrogate meta-model technique, which stores the previous sampled points and uses them to achieve a robustness measure. Principal differences between these two approaches are the computational costs (bigger for the first minimize average approach than the second one) and the introduction of another level of uncertainty with the surrogate technique (the second one). The last one's main advantage is that it does not approximate the objective function evaluation because the sample points are generated at some time during the optimization process. However, the drawback of this technique is the unreliability in finding enough number of points with high distribution within the neighborhood of a solution due to the stochastic nature of stochastic optimization algorithms [68]. That is, if in a meta-heuristic archived-based method the number of archive members and true function evaluations are reduced, then the reliability of the robustness measure decrease. In addition, archive-based methods that only use previously sampled points are very unstable in the initial steps of optimization due to the fewer number of sampled points [68]. The paper's confidence-based method states a confidence measure based on number of sampled points in the neighborhood, radius of the neighborhood, distribution of the available points in the neighborhood and it has the purpose to address these issues without additional samples.

In [70], Korolev and Toropov present a Multipoint approximation Method (MAM) that consists in perform iteratively a meta-model built in trust regions (sub-space of the design space where the meta-model is applied in order to perform response and they change size and translate as optimization progress), adapted for large-scale optimization problem with uncertainty. In their work, authors define a typical response for optimization under uncertainty:

$$F(\vec{x} + \vec{u}, \vec{y})$$

where $\vec{x}$ is the vector of deterministic design variables, $\vec{y}$ is the vector of environmental uncertainties, $\vec{u}$ is a vector of additive noise uncertainties with known probability distribution. In order to mapping and thus convert this response into a deterministic function that can be optimized a risk measure or robustness measure is applied to the response:

$$\tilde{F}(\vec{x}) = R(F(\vec{x} + \vec{u}, \vec{y}))$$

The risk/robustness measure can be provided in different ways:

- $R(F) = \mu(F)$ is the mean of F and in this way the average performance of the system is optimized;

- $R(F) = sup(F)$ is the worst-case scenario choice;

- $R(F) = \mu(F) + k\sigma(F)$ where $\sigma(F)$ is the standard deviation of the response and $k > 0$ is a constant. This risk measure is sometimes referred to as the safety margin. Optimization using this risk measure is referred to as robust design optimization and reflects the paradigms of a '3σ-design', a '6σ-design' [70];

- $R(F) = q_F(\alpha)$ and this choice represents the definition of reliability-based design optimization, where $q_F(\alpha)$ is the value of the quantile function corresponding to a given probability. $1 - \alpha$ corresponds to the probability of failure and the purpose is to maintain it below a chosen or given threshold;

- $R(F) = \overline{q_F}(\alpha)$ is the super-quantile choice.

The same robustness measure can be used not only in meta-model techniques but also in MC-approaches if computational costs are sustainable. Also in MAM presented method, the chosen risk measures have been two different: super-quantile and robust design optimization. Especially for the last one, two others example can be cited.
In [71] Knoch, Yang and Gu present a design for six sigma approach, which is defined as a robust optimization formulation that incorporates approaches from structural reliability and robust design with the concepts and philosophy of six sigma [71]. Six sigma philosophy measures the system's quality referring to probability of constraints' satisfaction and performances' sensitivity. σ stands for standard deviation and due to its definition, it is used to describe the variability of system parameters and performances. The method directly refers the probability of failure in a particular range with the specific σ-level (each one characterized by a specific area under the normal probability distribution bell). Metrics proposed in the paper to express and measure the quality of the system are: percent variation, defects per million in a short-term and defect per million in a long-term (long and short term describe the maintenance of mean performance value over time). That is, in design for six sigma the objective is to maintain the six-standard deviation ($\mu + 6\sigma$) of performance variation and mean under chosen thresholds (lower specification limit and upper specification limit).
In the second paper [72] Jin, Chen and Sudjianto provide an abridgment about global sensitivity analysis and uncertainty propagation in order to facilitate a robust optimization process using meta-model technique. Then, they develop one approach based on the minimizing mean and variance approach, applying it to a robust design optimization process of engine piston.

2.4 ROBUST OPTIMIZATION DESIGN APPROACH CHOSEN

In this thesis work a Monte-Carlo robust design optimization approach is chosen in order to cope with uncertainties for the PeWEC's case described before. The new optimization approach arises from the modifications of previous studies, which used an

NSGA-II genetic algorithm for techno-economic optimization purpose (read Carapellese et al. ‘s work in [82] for more specific information). The algorithm is changed adding a Monte-Carlo inner loop for each device during each generation-step, in which outputs functions (AEP, $COST$, CoE) are evaluated for different input design variables, affected by uncertainty, deriving from an a priori univocal sampling scheme previously calculated according to a chosen number of samples and applied for each individual during the optimization process.

The optimizer approach flow-chart is given in figure 2.16. The NSGA II optimizer works as a single box, which in input receives the fitness values under uncertainties from the Monte-Carlo simulation and returns as output the individual’s characteristics. Between the Monte-Carlo simulation and the nominal fitness values evaluation, uncertainties’ effects are included. Due to the computational cost (as will be highlighted in the following pages) the Monte-Carlo simulation is run with a parallelization technique using a cluster solution. The simulation is carried out for each individual in each generation of the genetic algorithm, in order to derive an experimental distribution for each output function we want to examinate. Using the specific output’s distribution, it is also possible to obtain some statistical parameters evaluation, which can be used to calculate the robustness index for the specific device/individual.

The sampling model technique used to implement the uncertainties’ effects in the three parameters defined before is a Latin Hypercube Sampling technique (LHS). This technique, unlike the classic Monte-Carlo sampling approach, does not draw random numbers in a selected range but first divides the sample space by creating a n-by-m table and only after that, it pulls out a single sample for each cell created with the application of this table. This proceeding allows the simulation to avoid drawing very close numbers and also allows it to cover the entire sample space more evenly. Therefore, LHS is performed in order to better exploit the few number of samples that we can use without excessively increasing the computational cost of the process, which, due to the complexity of the system and in particular due to the time required for computational hydro-dynamic simulations, takes about 7 seconds to evaluate the objective functions of a single point/sample of the Monte-Carlo simulation.

About uncertainties’ model, in this work it was chosen to apply distinct distribution models for each one of the specific parameters described before on which uncertainties effects are considered to be relevant. It is possible to consider the pitch turner radius variable and the distance between pendulum’s hinge and device’s CoG (Center of Gravity) variable as two mechanical uncertainties parameters. Hence, a gaussian probability distribution model can be applied for them. Thanks to this consideration, it is possible to modify the LHS sampling model for those parameters, creating a non-uniform n-by-m table and increasing the quantity of cells in a specific central zone of the sample space (it means to create a higher number of cells but of smaller dimensions) following the gaussian probability distribution. The observations made above cannot be done for the pitch hull viscous damping variable, due to its non-mechanical nature. This uncertainty’s parameter describes a very complex phenomenon, of which we have many areas of missing knowledge. Due to all these considerations, in this thesis work it was chosen to use a uniform probability distribution model as the uncertainty’s probability distribution model applied to the damping parameter. Consequently, the sampling technique chosen is a classic LHS.

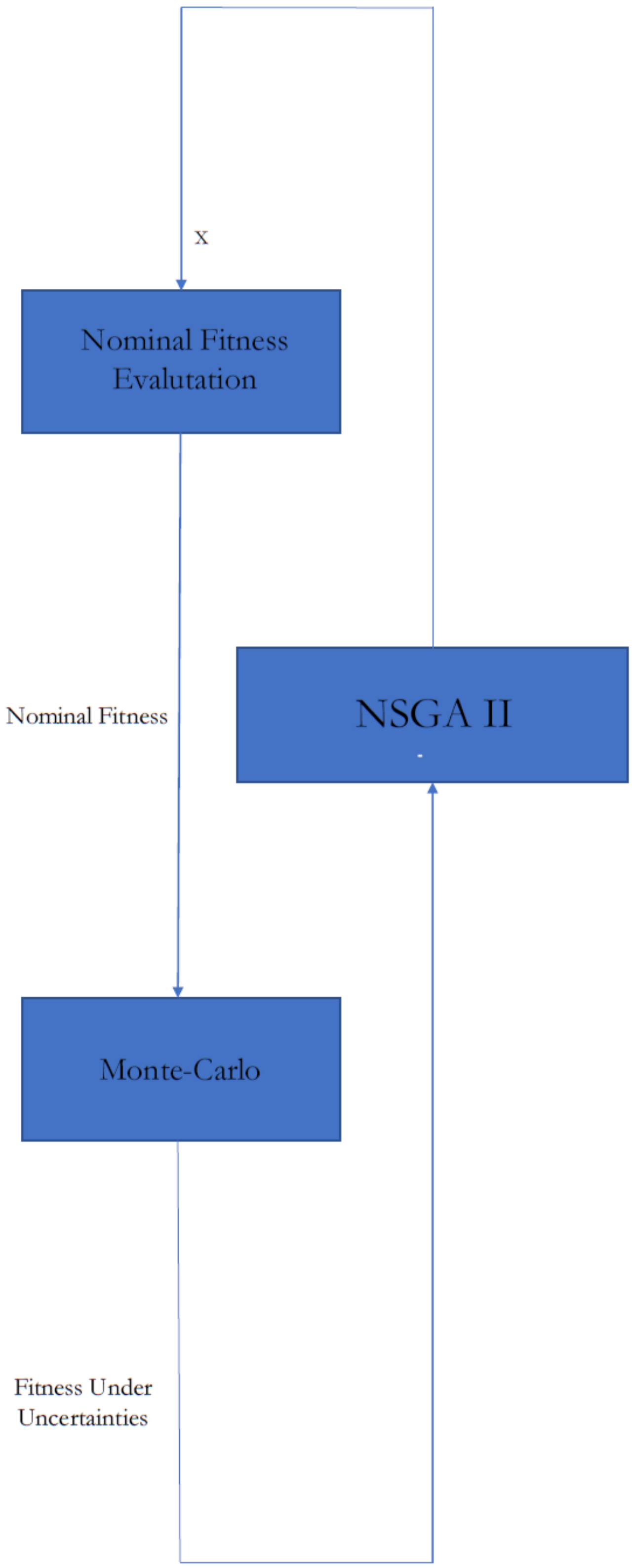

figure 2. 16

As mentioned before, using the specific objective functions output's distributions obtained after the Monte-Carlo LHS simulations for a single device/individual, it is possible to make some statistical considerations. In particular, these distributions can be used in order to define specific robustness indcies, by which a specific device can be ranked.

In previous pages of this chapter, different types of robustness have been highlighted, also in literature it is possible to find a lot of robustness indices used by as many authors. Among those indices which exploit statistical information given by the obtained distributions with the purpose of evaluate a measure of the device's reliability and robustness, it is possible to distinguish between two general approaches: assuming the index as a symmetric risk measure or assuming it as an asymmetric risk measure.

The first one is defined as a symmetric risk measure because it considers in a negative way and as something to be penalized any variation from the nominal value, indiscriminately this is an improvement or a pejorative one for the specific objective function under consideration. Referring to this approach a robustness index can be formulated, taking inspiration from Wang et al. in [8], as follow:

$$R = \sqrt{\sum_{i=1}^{m} \left(\frac{s_{f_i} + |\mu_{f_i} - f_{i,0}|}{f_{i,0}} \right)^2}$$

Where m is the number of objective functions, s_{f_i} and $|\mu_{f_i} - f_{i,0}|$ are the dispersion around the nominal value and the difference between the mean of the distribution obtained after the simulation (μ) and the nominal value (f_0) for the i-th objective function, which can be seen as an offset between the nominal value and the system response. They both represent a measure for the device robustness. The nominal value $f_{i,0}$ is used in the index to normalize the results for different devices, in order to compare them. Using Monte Carlo, the mean and dispersion around the nominal value are calculated as follow:

$$\mu_{f(\vec{x})} = \frac{1}{N} \sum_{i=1}^{N} f(\vec{x_i})$$

$$s_{f(\vec{x})} = \sqrt{\frac{\sum_{i=1}^{N} \left[f(\vec{x_i}) - f_{i,0} \right]^2}{N-1}}$$

The smaller R is, the more robust the system under consideration is.

The index which follows the second approach, instead, is defined as an asymmetric risk measure because it considers in a negative way and as something to be penalized only the variations from the nominal value which lead to worse performances and consequently to worse values of the objective function in question. Therefore, the risk measure for this approach is evaluated as follow (m still is the number of objective functions):

$$Q = \sqrt{\sum_{i=1}^{m} \left(\frac{Q_{i,0}}{f_{i,o}}\right)^2}$$

Where $Q_{i,0}$:

- if the i-th performance must be minimized, it is the objective function distribution’s value under which the q % (usually q is bigger than 90 %) of the occurrences appear. In this case, the smaller Q, the better is the risk measure;
- if the i-th performance must be maximized, it is the objective function distribution’s value under which the $(100 - q)$ % (usually q is bigger than 90 %) of the occurrences appear. In this case, the bigger Q, the better is the risk measure.

Also for this approach the nominal value $f_{i,0}$ is used in the index to normalize the results for different devices, in order to compare them. The information that the Q_i index gives us back is not only an asymmetric risk measure with respect to a fixed percentage threshold q, the Q_i’s value also provides us some data regarding the quality of the device's functioning with respect to the i-th objective function. In particular, for a minimization problem:

- if $Q_i < 1$, the device’s functioning for the i-th objective function is (relatively to the percentage threshold set q) better than what we expected;
- if $Q_i = 1$, the device’s functioning for the i-th objective function is (relatively to the percentage threshold set q) equal than what we expected;
- if $Q_i > 1$, the device’s functioning for the i-th objective function is (relatively to the percentage threshold set q) worse than what we expected.

A graphic representation for both Q (in a minimization case) and R index, is reported in figure 2.17. In the figure, a plot of the simulation's outcome distribution for the objective function f is presented with the three vertical lines used to describe the two indices Q and R. The lines depict Q_0, the nominal value f_0 and the mean value $\mu(f)$.

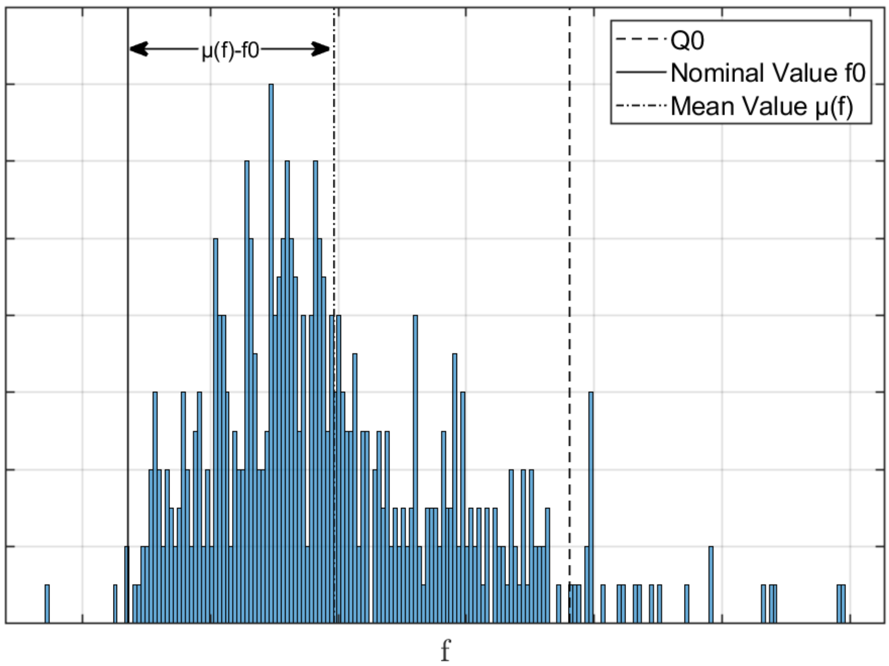

figure 2. 17

In the graph, thence, the trend of the objective function's behavior for a chosen device is highlighted. In particular, in the chart provided as example, it is possible to read qualitatively some details. Considering the studied one as a minimization problem, so long as $Q_0 > f_0$, it is possible to state that (always keeping in mind that all these considerations are relatively to the percentage threshold set q) statistically the device works with worse performances than the expected design performances. Despite, if it had been a maximization problem and the one described by the graph was always the Monte-Carlo simulation's statistical distribution results, it would have been possible to say that the device would have worked statistically with better performances than the expected design ones. The distance between $\mu(f)$ and f_0 is also highlighted, the smaller this distance, the more the device will behave statistically similarly to the design expectations.

Therefore, the difference in the approaches dictated by these two indices lies in the two different visions with which they approach the measurement of risk and robustness. They are both valid, but they stand for two dissimilar applications and purposes of the robust optimization process.
The aim of the first one (R) is to emphasize and favor devices that respect a strict robustness formulation, which is usually requested in case of study where any variation from the nominal value can generate issues and deteriorates the quality of the product. For instance, the manufacturing process of mechanical parts and components on which geometric tolerances represent a very hard constraint in order to guarantee the correct mounting and functioning of the mechanical assembly. In general, cases of study for which an approach with asymmetrical risk measurement is required are those in which the final product of an industrial production process must comply with specific constraints (geometric, temperature, weight, color shade or any other characteristic).

The Q index purpose instead, is to improve the devices' performances and it is not to limit the variations on both sides around the nominal value of the specific objective function under consideration. In this way, using an asymmetric risk measurement, a specific device is penalized during the optimization process just in the case, relatively to the percentage threshold set q, its function is worse than the nominal one. Thus, relaxing the definition of robustness, this index allows the designer to exploit all the system's performances and improve them during the robust optimization process. For these reasons, this second kind of approach is appropriate for the design phase of an entire system, from which the optimization process wants to get the best it can offer. Keeping in mind all the considerations which has been said above, it is possible to conclude that for the case study in question for this thesis work the second index Q is the suitable one.

Therefore, after these observations, the complete robust optimization framework chosen will be summarized and described in Chapter 3, reassuming the way in which the optimization parameters have been set up. In the next chapter, also all the attempts carried out during this thesis work will be presented with their results. Finally, all possible considerations about the obtained results will be discussed in the final chapter of this thesis work.

CHAPTER 3 – OPTIMIZATION SETUP AND RESULTS

As anticipated in the previous chapter's conclusion, the next pages will summarize the complete robust optimization framework chosen and the way in which the optimization parameters have been set up will be reassumed. Then, the results obtained will be presented in detail along with the first observations, even finally giving the possibility to choose which are the most interesting devices to observe and on which to focus more attention and therefore to be studied more in detail.

3.1 Optimization Parameters Configuration

In order to fully define the optimizer, it becomes necessary to define all the parameters that will totally characterize the optimization process. With this purpose, a brief recap of the framework used is given in next lines.

As stated in the general description of the robust optimization framework given in the previous chapter, in this thesis work a genetic algorithm is utilized in the optimizer, in order to cope with our problem. Specifically, as already anticipated, a NSGA-II GA is carried out, which, after some attempts, has been set up with the following parameters. The number of individuals for each generation has been configured to 75 and all along the process, 41 generations have been generated before an unforeseen error interrupted the trial. To cope with the problem of the optimization under uncertainty in a robust way, the approach chosen for this work is a Monte-Carlo approach which use an additional objective function related to robustness. Due to this, we are faced with a Multi-Objective Optimization Problem (MOP), in particular a Bi-Objective Optimization Problem. The first of the two objective functions chosen for this work is the nominal value of the Cost of Energy, which is defined as the ratio of all the device's overall capital expenditure ($CapEx$) over productivity, which is the energy produced by the plant during its lifetime N_y (as mentioned before, assumed by Sirigu et. al. as 25 years for PeWEC):

$$CoE = \frac{CapEx}{N_y AEP}$$

The second objective function is the asymmetric robustness index Q, described in Chapter 2, referred only to the CoE. In order to perform the optimization using this index, another parameter which has to be set up is the percentage threshold q. For the definitive optimization, q has been configured to a value of 95%.

$$Q = \frac{Q_{CoE,0}}{CoE_0}$$

The next step required in order to continue the optimization process configuration is the uncertainties' model calibration.
The parameters involved during this step are: the variables subject to uncertainties, on which the selected uncertainties' probability distribution models are applied, the just mentioned uncertainties' probability distribution models, the sampling technique utilized and the number of samples performed. Quickly recalling the three variables on which the uncertainties' effects are considered, they are:

1. *Pitch Turning Radius*, which is a mechanical parameter but at the same time it is representative for the device's inertia;

2. *Pitch Hull Viscous Damping*, which is an uncertainty's parameter that describes a very complex phenomenon of non-mechanical nature, about which there are many areas of missing knowledge;

3. *Distance Between Pendulum's Hinge and Device's Center of Gravity*, which is a mechanical parameter too.

In 2.4 have been already described the sampling technique selected and the probability uncertainties distributions. The first one is the Latin Hypercube Sampling Technique, the last three are two gaussian distribution for the two mechanical parameters and a uniform distribution for the damping one. Specifically, for the first two parameters (pitch turning radius and viscous damping) the uncertainties' effects are considered as a percentage applied to the nominal value while for the last (distance between CoG and pendulum hinge) the uncertainties' effects have been modelled as a value obtained from the probability distribution that is directly added (or subtracted) to the specific device nominal value of the parameter.
The calibration phase of the uncertainties' effects mainly unfolds in two directions: the direct customization of the uncertainty models for the relative parameter on which they are applied and the exploration space setting, where the optimization algorithm will operate and will search the optima individuals with respect to the variables that are involved in the process. From this last one direction considered an iterative procedure arises, in which the previously mentioned exploration phase alternates with an exploitation one. The purpose of this operation is to increase the optimizer's performances, helping the algorithm with convergence. The exploration phase is performed with a statistical analysis of the occurrences relating to a specific system's variable, which is the object of the optimization. The exploitation phase, instead, is the one in which the choices are made about restriction and modification of the search space for the algorithm, so that it searches for optimal individuals in the areas of the aforementioned optimization space where it is more likely to find them. This argument

can also help and be extended to the uncertainties' model calibration, regarding the customization of the uncertainty models applied to the variables which are subject to those. This occurs due to the improvement of the algorithm research process quality, that consequently improve the quality of the sampling made during the Monte-Carlo simulation and thus also the quality of the uncertainty modelling improves. Briefly summarizing the logical flow of the reasoning previously reported. Given a limited and fixed number of possible samples due to the computational costs required by the system under examination, it can logically be stated that the more the search space of the optimization algorithm is limited and the smaller are the steps in which, for a uniform probability distribution, is divided the range of uncertainties or in which, for a Gaussian probability distribution, is divided the width of the dispersion zone around the mean value, the denser our sampling will be and therefore the better the quality of the analysis. Metaphorically it is as if the optimization designer had a limited and fixed number of arrows to shoot in order to hit a target whose position is not known and to cope with this situation he did not choose to shoot them randomly but in the areas that, thanks to previous analysis, turn out to be those in which our target will most likely be positioned. Following this procedure, it is therefore possible to state that fixed the same range of variation for a uniform probability distribution and fixed a standard deviation for a Gaussian probability distribution, the smaller the search space of the optimizer, the more qualitative the uncertainty model will be.

Therefore, following this reasoning, a first step of this iteration procedure has been carried out and a statistical analysis has been performed for some selected parameters and the most significant results are reported below.

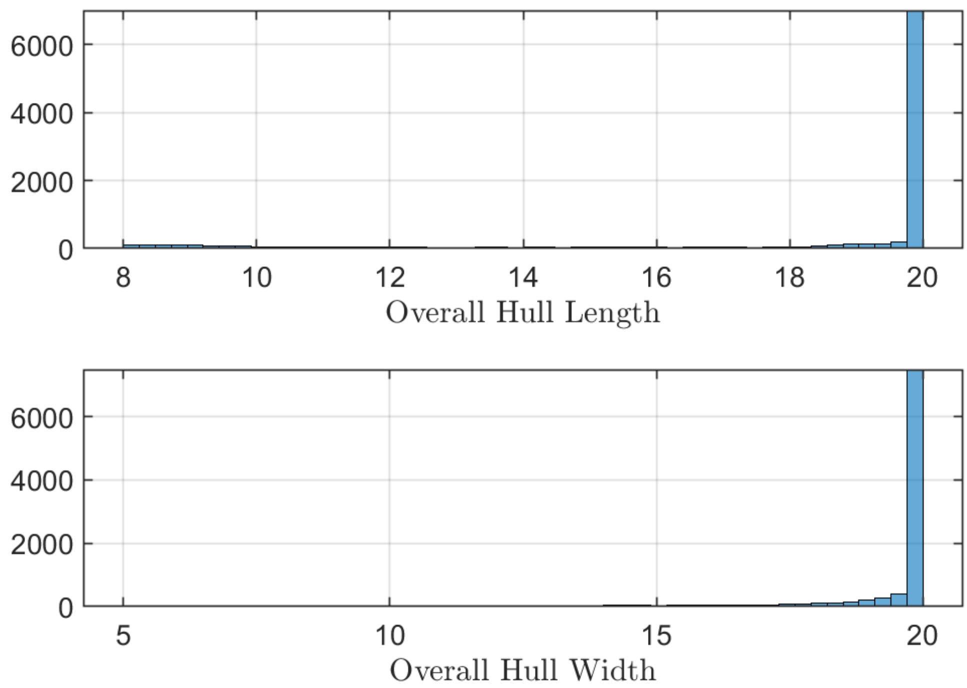

figure 3. 1

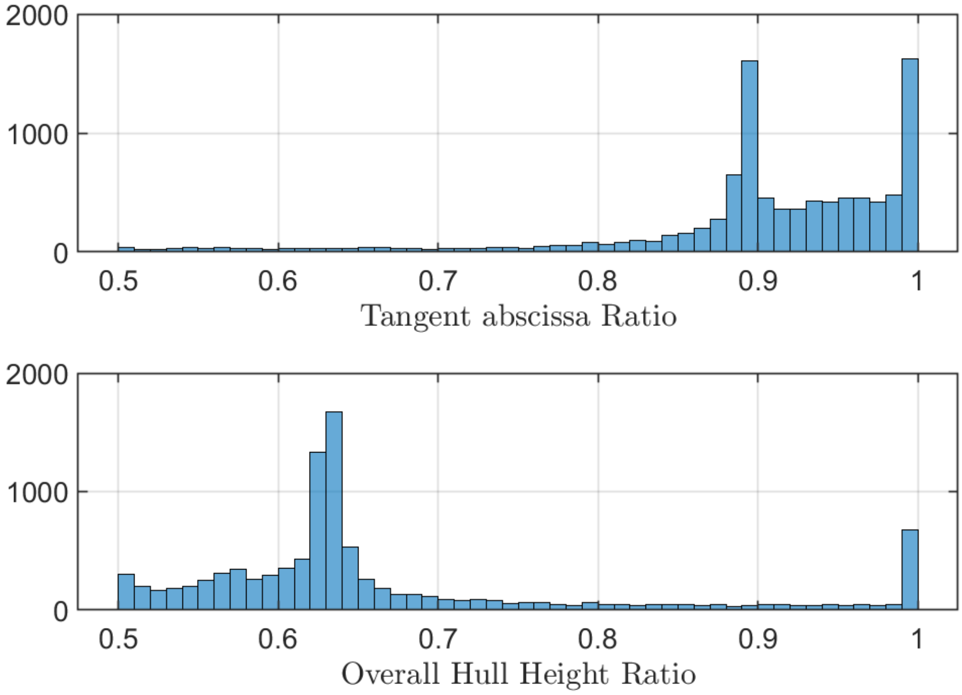

figure 3. 2

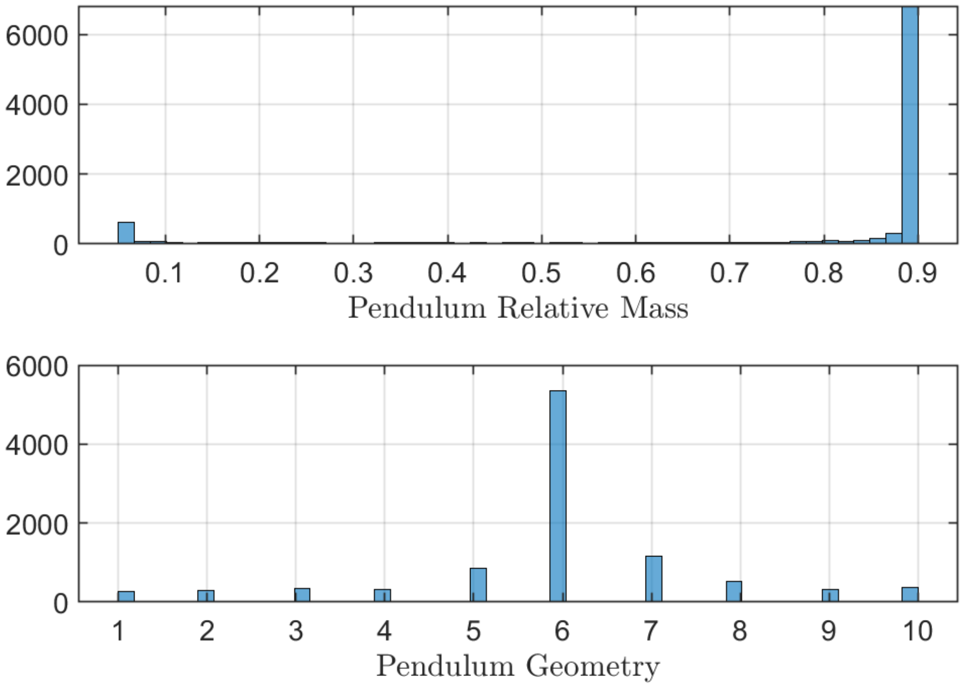

figure 3. 3

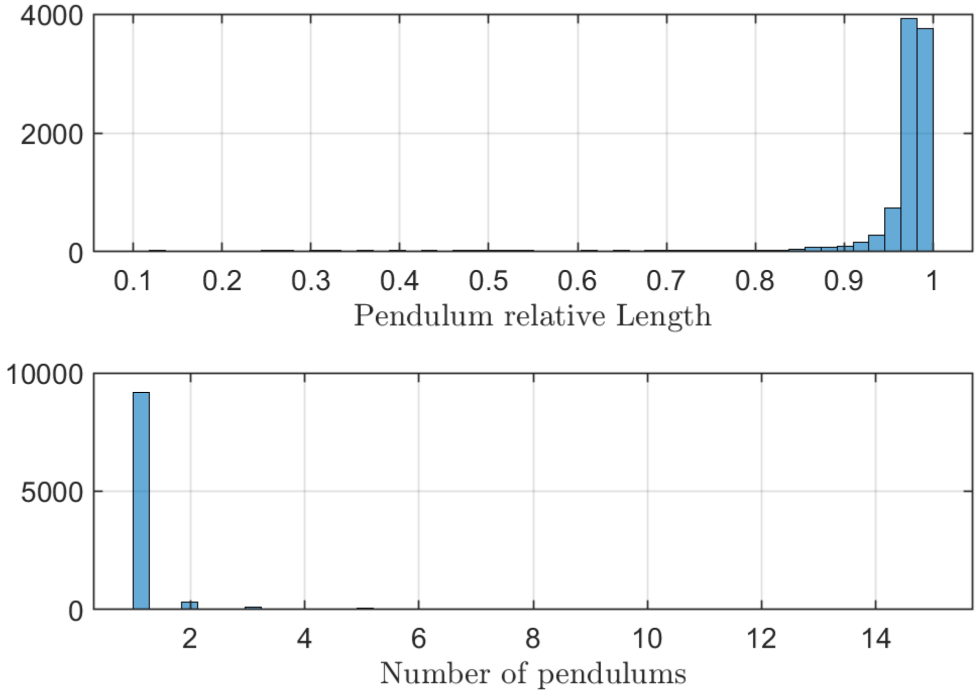

figure 3. 4

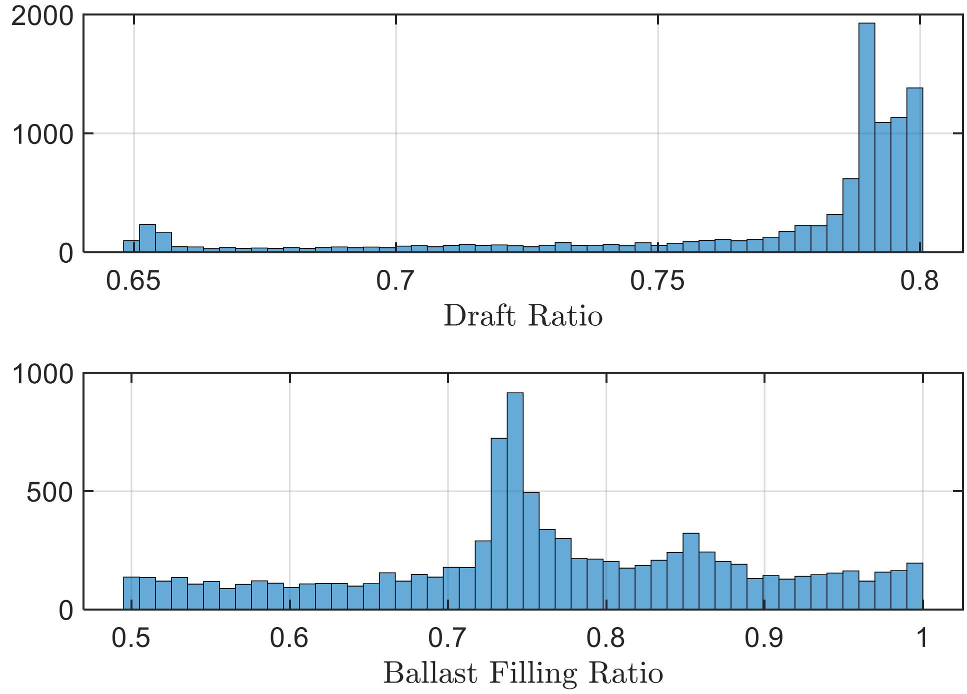

figure 3. 5

Observing the plots, it is easy to understand that especially for some variables it is possible to substantially reduce the search area of the optimization algorithm, for example as regards the hull length and width. The main drawback for this process, however, is the time required by each iteration, which increases as the number of samples increases. In particular, in the case of study examined in this thesis work, a long time is required for the evaluation of a single point / individual (the evaluation of a single sample during the Monte-Carlo simulation takes about 7 seconds, as previously mentioned, mainly due to time requested by the computational hydro-dynamic simulation). Therefore, by limiting the number of possible samples, the number of iterations required necessarily increases if the designer want to be more precise and detailed. For this initial work, it was chosen to proceed with a single iteration, which using 50 samples took about 3 days, and then launch the final optimization. A process which uses 250 samples was configured for the definitive optimization. These first configuration parameters are resumed in table 3.1

Parameter	**Value**
Genetic Algorithm	NSGA-II
Population	75
Generations	41
Objective Function 01	$CoE\ \left[\frac{€}{MWh}\right]$
Objective Function 02	Q [\]
Sampling Techniques	Latin Hypercube Sampling
q	95 %
Pitch Turner Radius Distribution	Gaussian Probability Distribution $\mu_{PTR} = 1\ [m]\quad \sigma_{PTR} = 0.001\ [m]$
Pitch Hull Viscous Damping	Uniform Probability Distribution [0%; 10%]
Distance Between Pendulum's Hinge and Device's Center of Gravity	Gaussian Probability Distribution $\mu_{CoG} = 0\ [m]\quad \sigma_{CoG} = 0.003\ [m]$
Samples	250

table 3. 1

Taking a closer look at the NSGA-II optimizer setup in more detail, it is possible to highlight the following aspects. NSGA-II stands for Non-Dominated Sorting Genetic Algorithm, it is a multi-objective optimization algorithm based on the standard operators of genetic algorithms selection, reproduction, crossover and mutation. In this work, the same optimization setup for the NSGA-II used in [82] is configured. The optimization is performed in a MATLAB environment and utilize the variant *(gamultiobj* function) of the original NSGA-II to perform it. That function introduces controlled elitism of the solutions, which increases the diversity of the population and avoids premature convergence in local minima. The others operator used are not the default crossover and mutation functions but a modified version of them, called bounded exponential (BEX) for the crossover phase and power mutation (PM) for the mutation one. Instead, the truncation procedure is based on the Shopova method [83]. The constraints handling is based on the penalty function as well. The tuning factors of the algorithm are set in table 3.2 and 3.3 [82].

Name	**Symbol**
Selection Function	Tournament [85]
Crossover Function	BEX-Bounded Exponential
Mutation Function	PM-Power Mutation
Truncation Procedure	Shopova Method [83]
Constraints Handling	Penalty Function

table 3. 2

Name	**Symbol**	**Value**
Population Size	N	75
Maximum Generation Count	M	41
Convergence Threshold	Δ	$1.00 * 10^{-5}$
Tournament Size	k	4

table 3. 3

At last, the control and design parameter search spaces for the definitive optimization set up, configurated referring to the previous analysis, are given in table 3.4.

Design Parameter	**Symbol**	**Units**	**Lower Bound**	**Upper Bound**	**Typology**
Hull Length	L	m	12	20	Continue
Hull Width	W	m	12	20	Continue
Bow/stern circumference ratio	h	$-$	0.5	1	Continue
Height ratio	k	$-$	0.5	1	Continue
Draft ratio	j	$-$	0.55	0.75	Continue
Ballast filling ratio	BRF	$-$	0.7	1	Continue
Number of pendulum/PTO	N_p	$-$	1	3	Discrete
Unit mass ratio	β_U	$-$	0.05	0.9	Continue
Pendulum shape factor	σ_p	$-$	1	10	Discrete
Pendulum arm factor	γ_p	$-$	0.5	1	Continue
Pendulum fulcrum factor	λ_p	$-$	1	10	Continue
Gearbox ratio	r_g	$-$	10	30	Discrete
PTO ID	ID_{PTO}	$-$	2	12	Discrete

table 3. 4

3.2 RESULTS

Referring to the previously described optimization setup, the results of this process are reported below. In this section, these results will be divided and presented in different sub-chapters, in order to differentiate the analysed aspect. In the firsts subsections, the attention will be paid to the process in general, by looking at the set of results obtained, in order to evaluate the statistics and the general behaviour of the optimization. Subsequently, the analysis will move to specific devices belonging to the Pareto frontier, giving rise to possible comparisons and to the search for possible correlations and trends between devices which differ according to specific parameters and performances in question. For example, by comparing dimensions and characteristics of hulls with different robustness index Q and / or R.

3.2.1 Optimization Process Statistics

Focusing initially on the data relating to the optimization process per sé, the time requested by the algorithm to reach the predetermined number of generations and the aspects relating to the mortality rate of the optimization process (for each generation and the overall one) have been examined with the statistics relating to the causes for which a device is considered unfeasible, i.e. the violation of specific constraints.

In figure 3.6 the time requested for the convergence to a specific generation is reported and a linear behaviour it is highlighted.

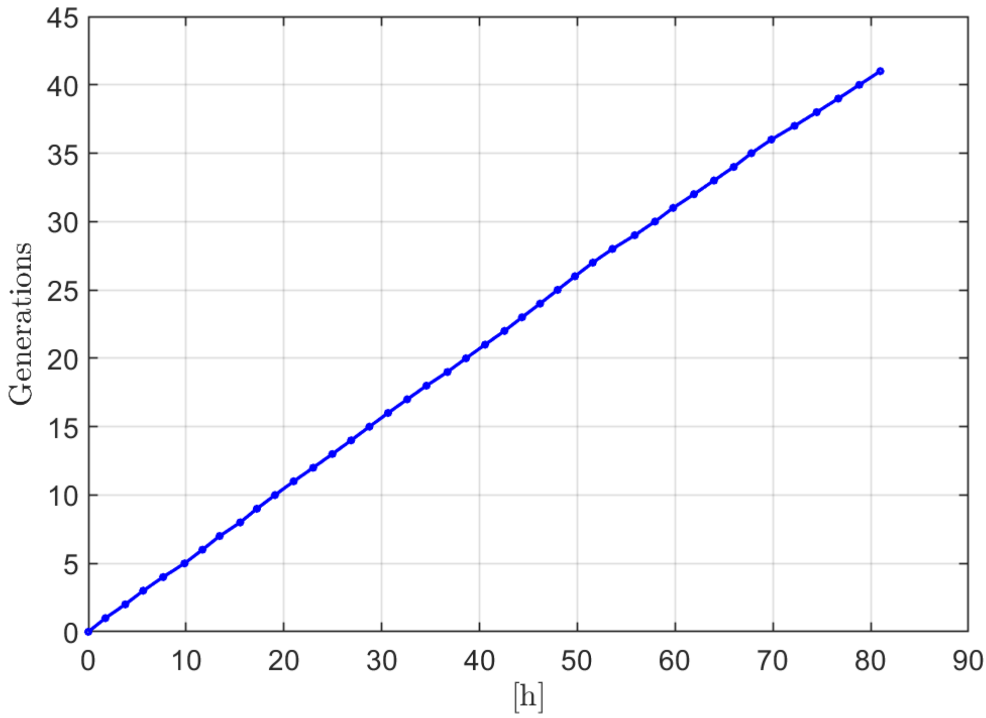

figure 3. 6

Then, figures 3.7 and 3.8 describe the mortality rate and the violated boundaries statistics for those unfeasible devices.

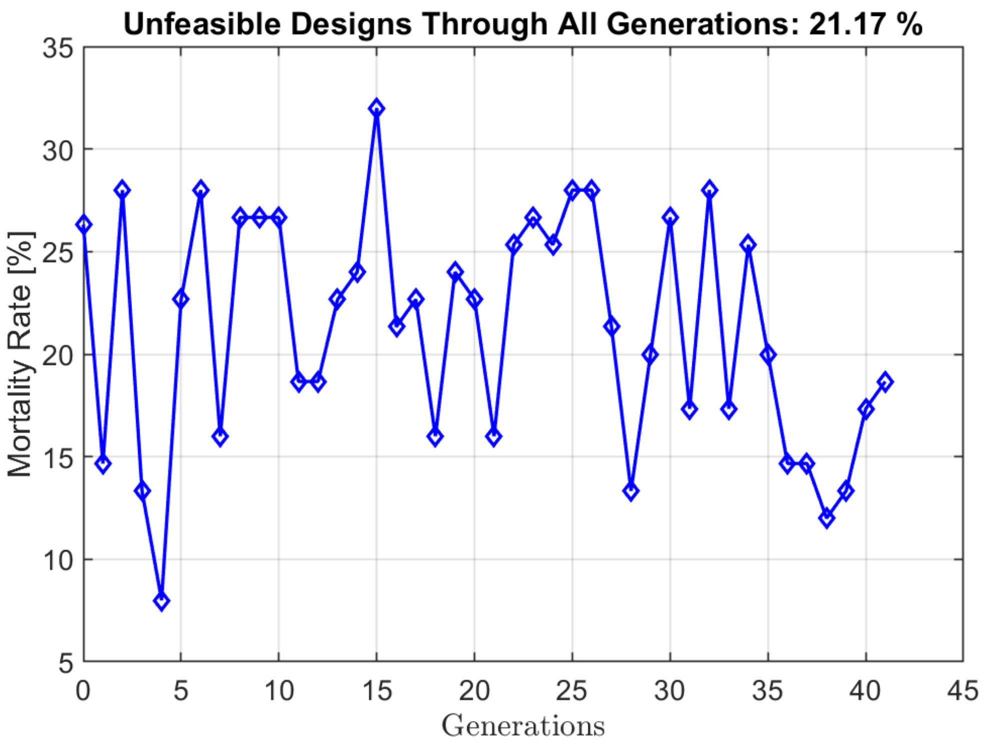

figure 3. 7

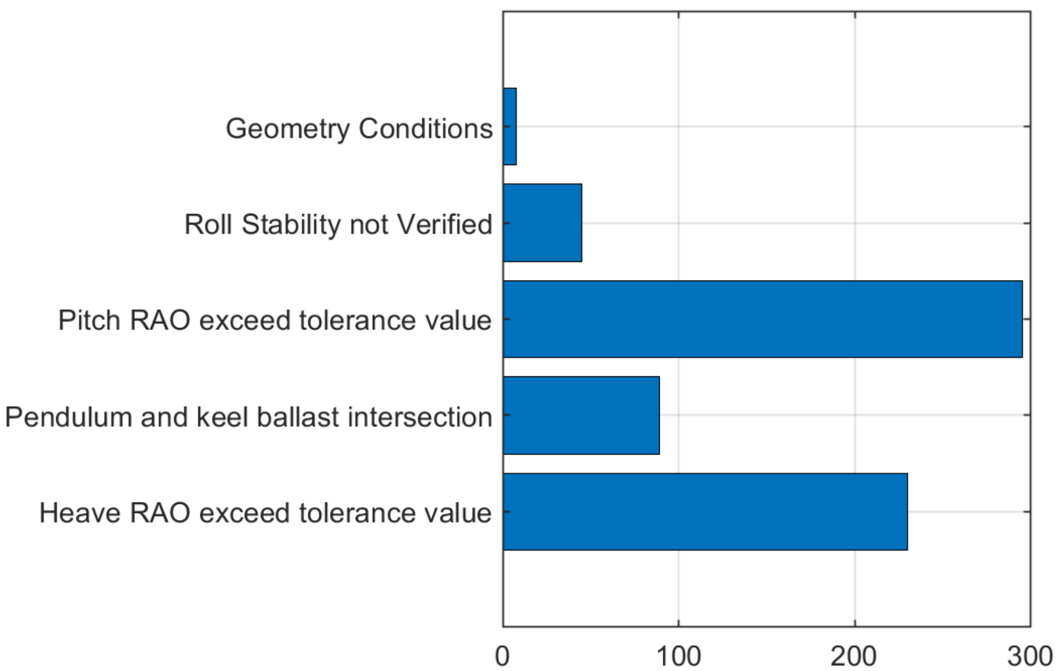

figure 3. 8

The mortality rate is defined as:

$$Mortality\ Rate = \frac{unfeasible_{GEN_i}}{population\ size}$$

And the general mortality rate trough all generations:

$$Mortality\ Rate\ Through\ All\ Generatios = \frac{unfeasible_{TOT}}{total\ individuals}$$

The unfeasible devices statistics, instead, put in evidence that the most occurring violated constraint is the pitch RAO that exceed the tolerance value settled.

3.2.2 Pareto and Generations

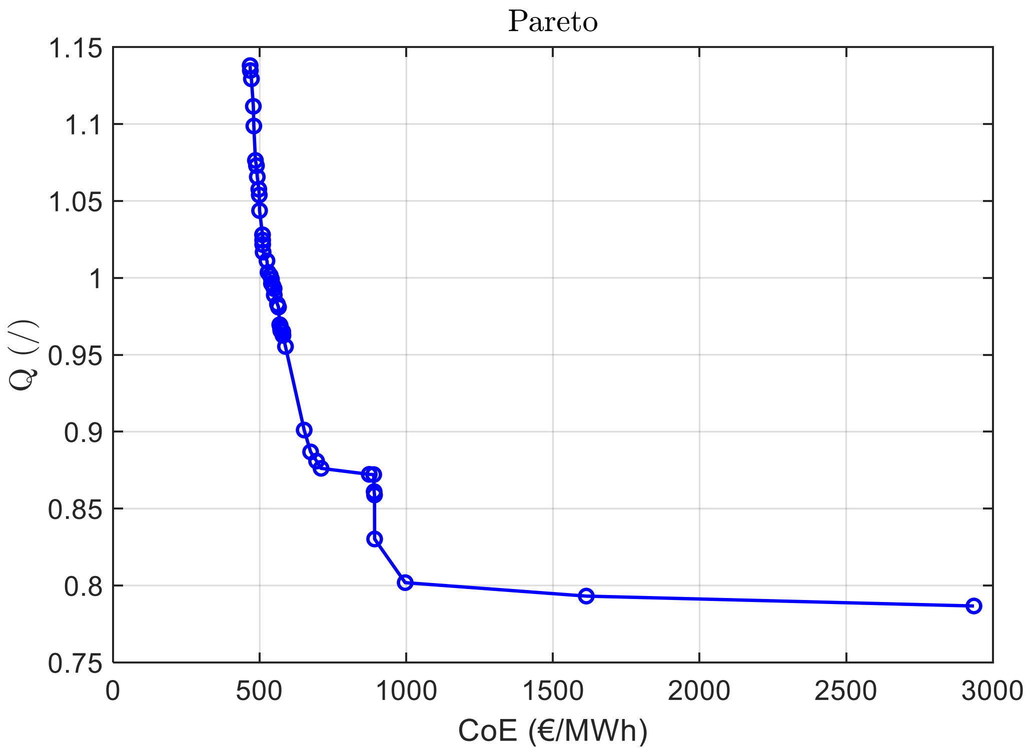

figure 3. 9

This second subsection reports the results of the optimization process inherent in the search for the Pareto front in the objective space, which is formed by the two objective functions in question. The first objective function, the Cost of Energy CoE, is plotted on the abscissas, while on the ordinates we find the second objective function, i.e. the asymmetric strength index Q. As it was imagined, the devices resulting from the robust multi-objective optimization process create a Pareto front that highlights the trade-off between the two performances examined. In particular, it is possible to note that the cost of energy describes values that are in a range between $460\frac{€}{MWh}$ and $3000\frac{€}{MWh}$, with a particular concentration of devices in the range of the objective function space delimited by the intervals $\Delta_{CoE} = [460; 590]\frac{€}{MWh}$ and $\Delta_Q = [1.5; 0.95]$.

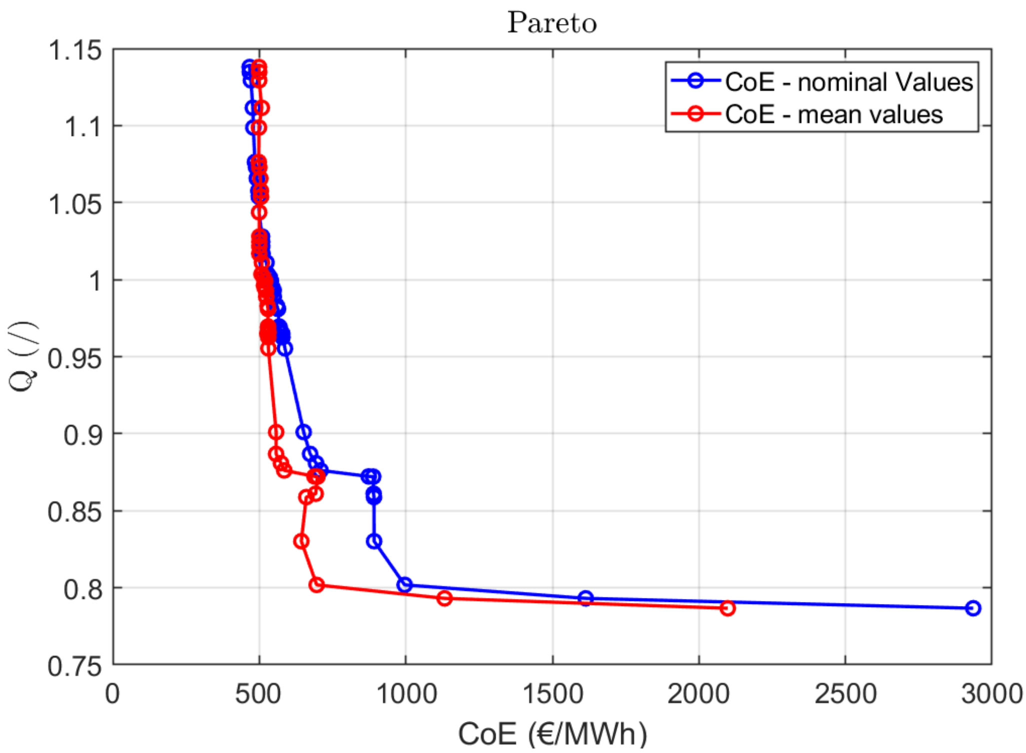

figure 3. 10

Given the general will of the robust optimization process to consider the variation of the performances due to the effects of the uncertainties on the specific devices, in figure 3.10 a comparison is represented between the pareto front obtained thanks to the robust optimization (where each point is equivalent to the representation of a specific device) and the representation of the same devices but using, instead of their nominal Cost of Energy value CoE_0, their mean value, $\mu(CoE)$. It can be noted that the mean values, for $Q \lesssim 1$, are lower than the nominal values CoE_0, while when $Q \gtrsim 1$, the mean CoE values are higher than the nominal values. This trend denotes the behaviour required to the optimizer, that is to move the 95% of the occurrences simulated during the Monte-Carlo loop for the specific individual as far as possible to the left of the nominal value CoE_0. This last consideration, together with the observations made regarding the areas with higher population density of the Pareto front in the objective functions space, allows us to hypothesise that if the unexpected error had not occurred, the optimisation could perhaps have converged to a Pareto front in which the devices described by $Q < 1$ would have been characterised by somewhat lower CoE values than those resulting now. This, however, could not have modified the trade-off between the two optimization parameters, which clearly tells us that in order to obtain devices that in 95% (value of the parameter q set previously) of the cases in which they suffer the effects of uncertainties work with a CoE lower than the design one, it is necessary to plan devices with this nominal value higher than those which instead work in 95% of the cases at CoE values similar to the nominal ones or a little higher.The optimization process evolution, generation by generation and looking at the pareto front convergence, is represented in figure 3.11 and 3.12.

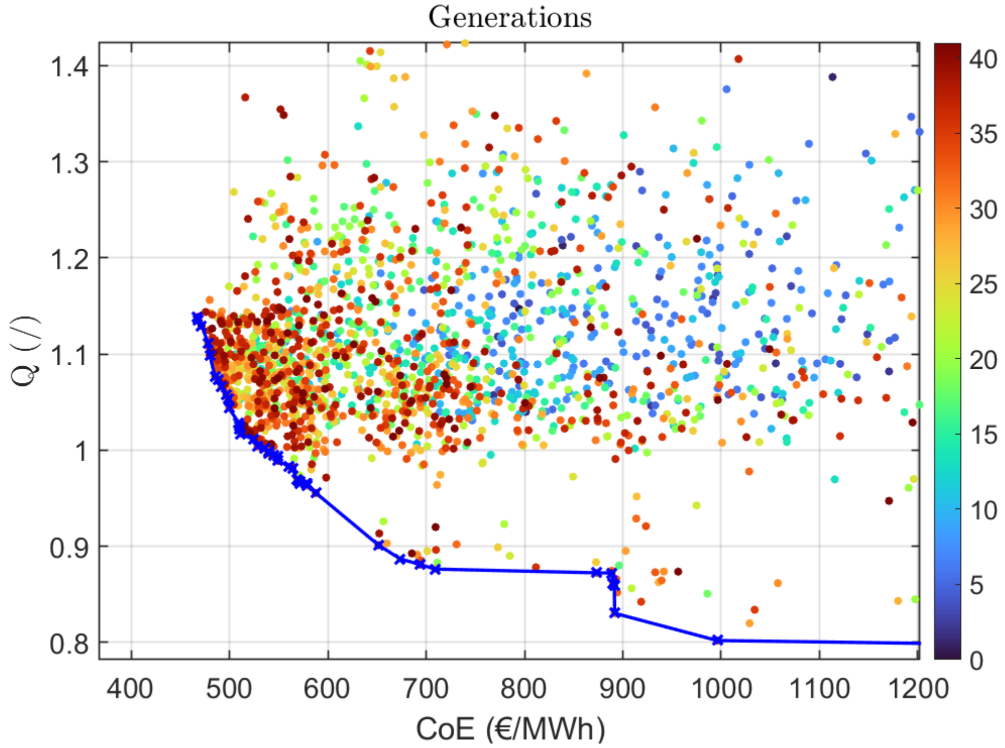

figure 3. 11

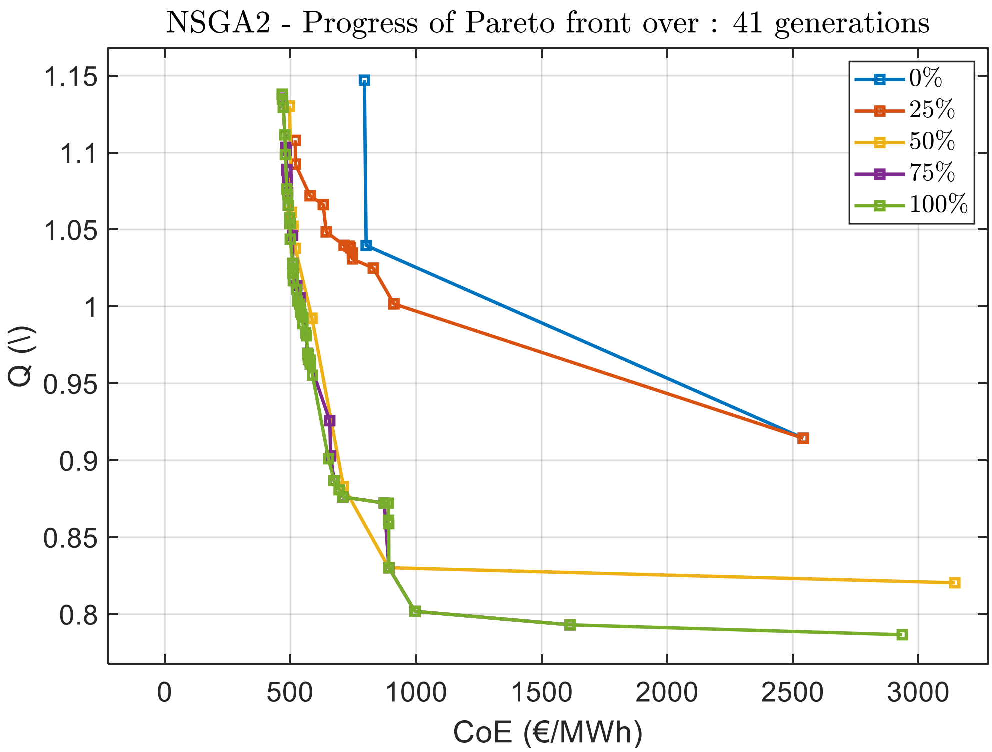

figure 3. 12

3.2.3 Pareto Variables and Statistics

The devices belonging to the pareto front are then further analysed in this subsection, in compliance with further and specific parameters and variables of the system.

Figure 3.13 shows the values of the two robustness indices. From these graphs it is easy to see that the area of greatest interest in our results, i.e. the one mentioned above and delimited by the interval on the x-axis of$\Delta_{CoE} = [460; 590]\frac{€}{MWh}$, corresponds to the area in which the symmetrical robustness index R is lowest. This shows a very evident behaviour of the obtained results: the largest number of devices belonging to the Pareto front is at minimum CoE_0 and at Q values around 1, for these rages the symmetric robustness index R is at its minimum values (which are sufficiently low), this means that for those devices not only the offset of the system response (distance between CoE_0 and $\mu(CoE)$) is very low, but also the dispersion of the results around the nominal value. Those units are therefore very robust, considering a definition of robustness in a strict way, and consequently very reliable. This trend can also be read as the ability of those systems to exploit their potential very well, especially in terms of annual production AEP, since in this thesis work the nominal value of the individual device costs was considered fixed and not subject to the effects of uncertainties.

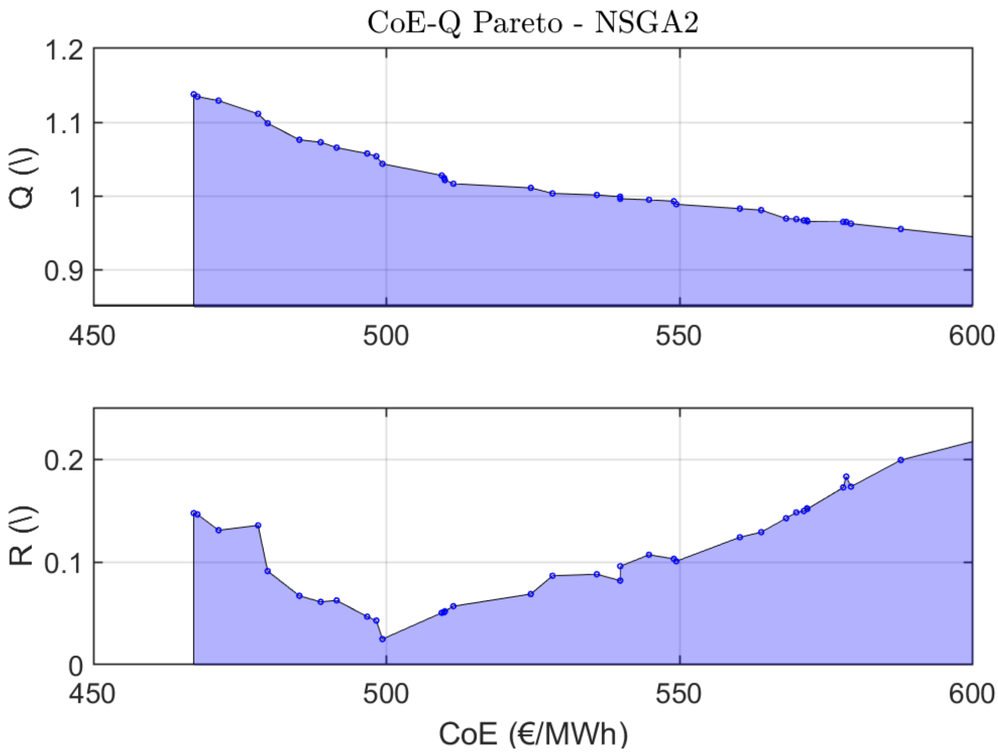

figure 3. 13

Turning now to an analysis of these performances, we observe Figures 3.14 and 3.15. These graphs show the nominal (3.14) and average (3.1) AEP and $COST$ values, again for devices belonging to the Pareto front. First of all, it can be noticed that there are no differences between $COST$ and $\mu(COST)$, this because for the assumptions made the cost parameters are not considered affected by uncertainties. Then, it can be observed that the highest values of AEP ($> 60\,\frac{MWh}{y}$) always correspond to the devices belonging to the area of greatest interest highlighted above and that for those same devices the lowest costs are also described. From the plots it is well evidenced that what influences the Q and R values is therefore the annually energy production and that the difference between AEP and $\mu(AEP)$ increases if you leave the area of interest in the Pareto front to which we have referred up to now, with a trend conforming to that required by the optimization process, i.e. for values of $Q < 1$ corresponds to values of $\mu(AEP) > AEP_0$ in 95% of occurrences.

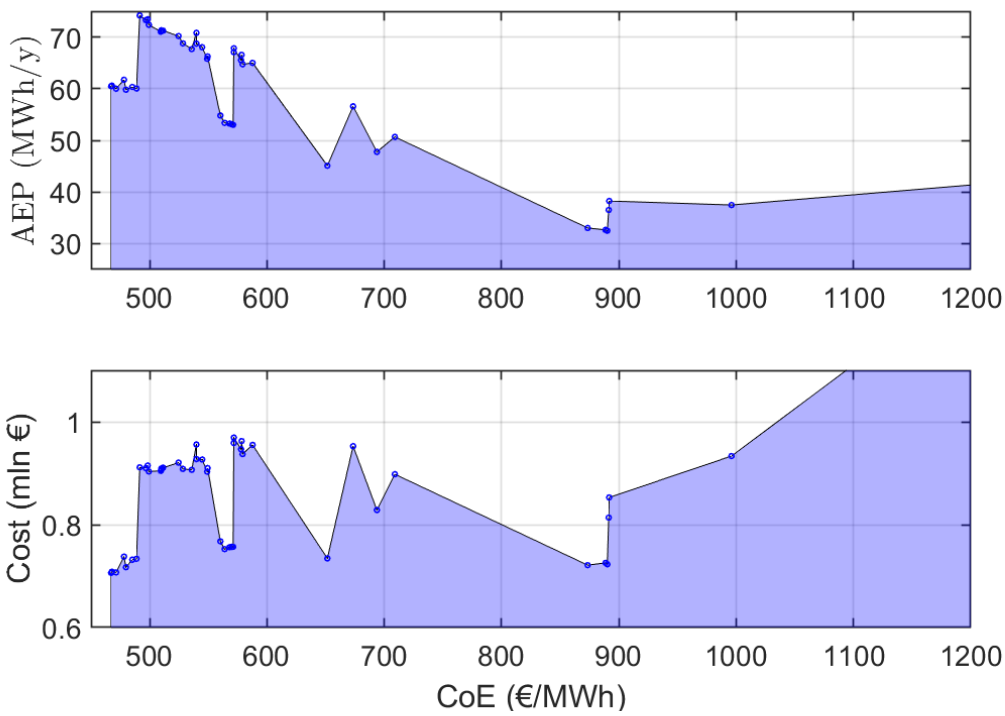

figure 3. 14

In order to reduce the Cost of Energy, we can therefore think in three directions (which can also be practicable at the same time): increase annual productivity AEP, decrease plant's costs (and in this sense perform a cost analysis) and increase life profit of the plant N_y.

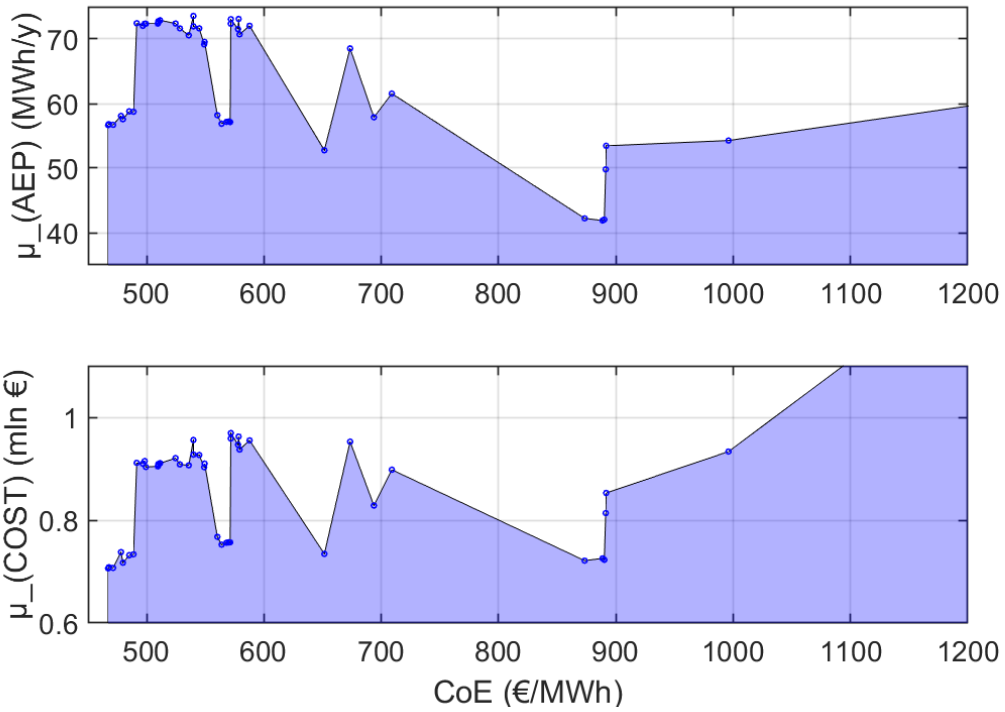

figure 3. 15

Further project parameters of the devices belonging to the Pareto front are shown in the figure 3.16 to 3.18.

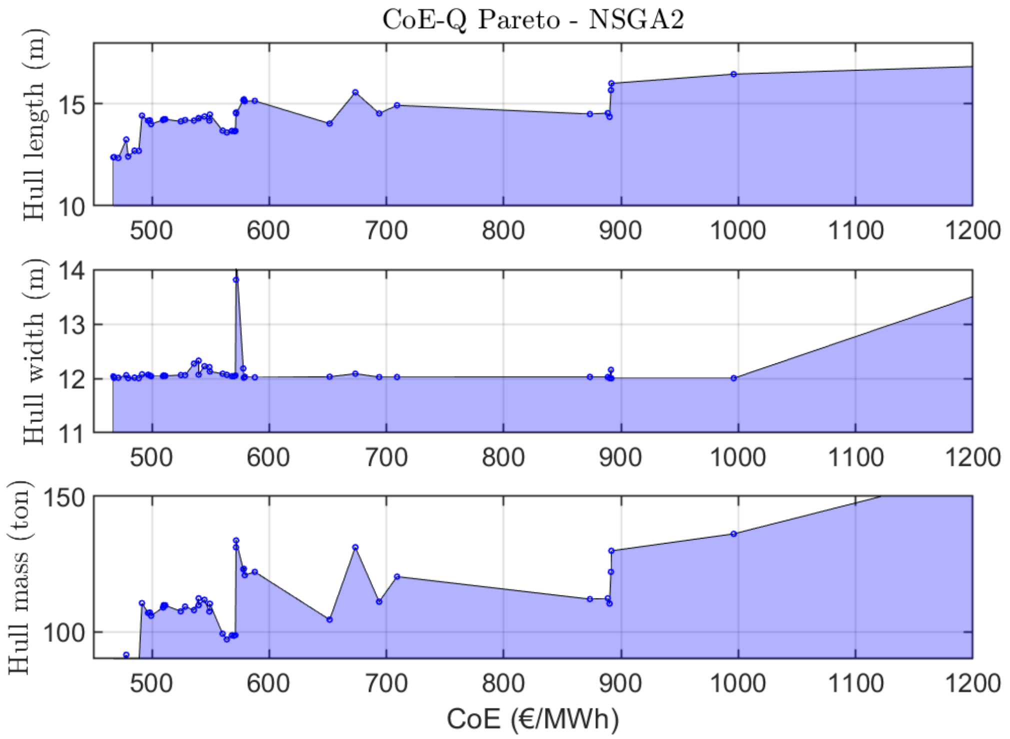

figure 3. 16

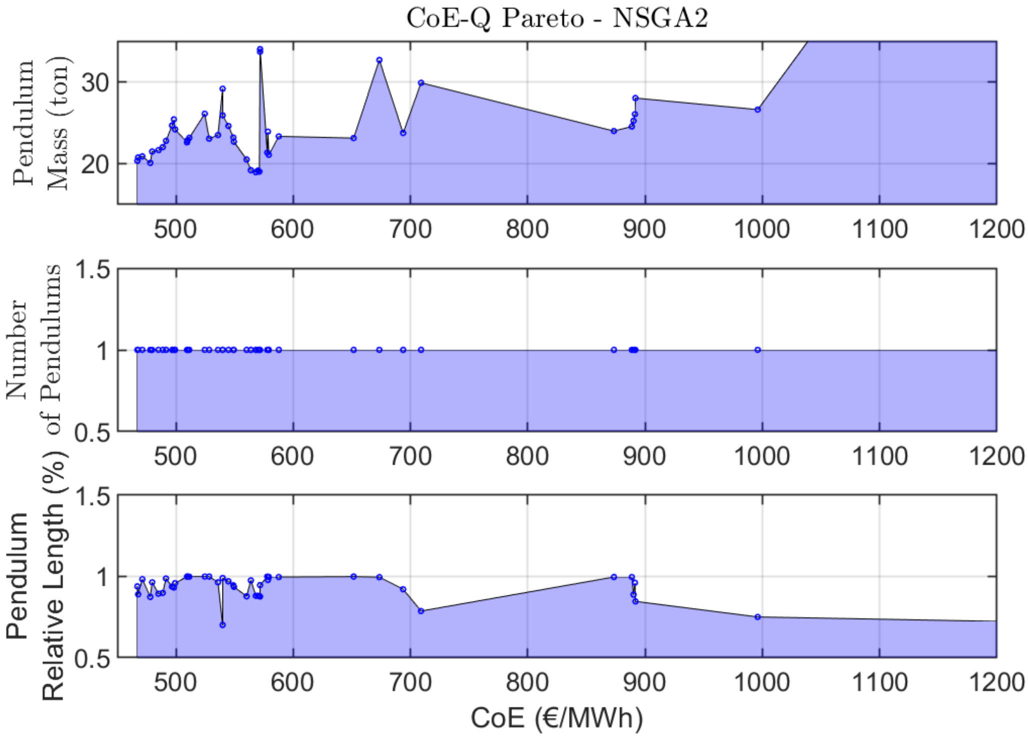

figure 3. 17

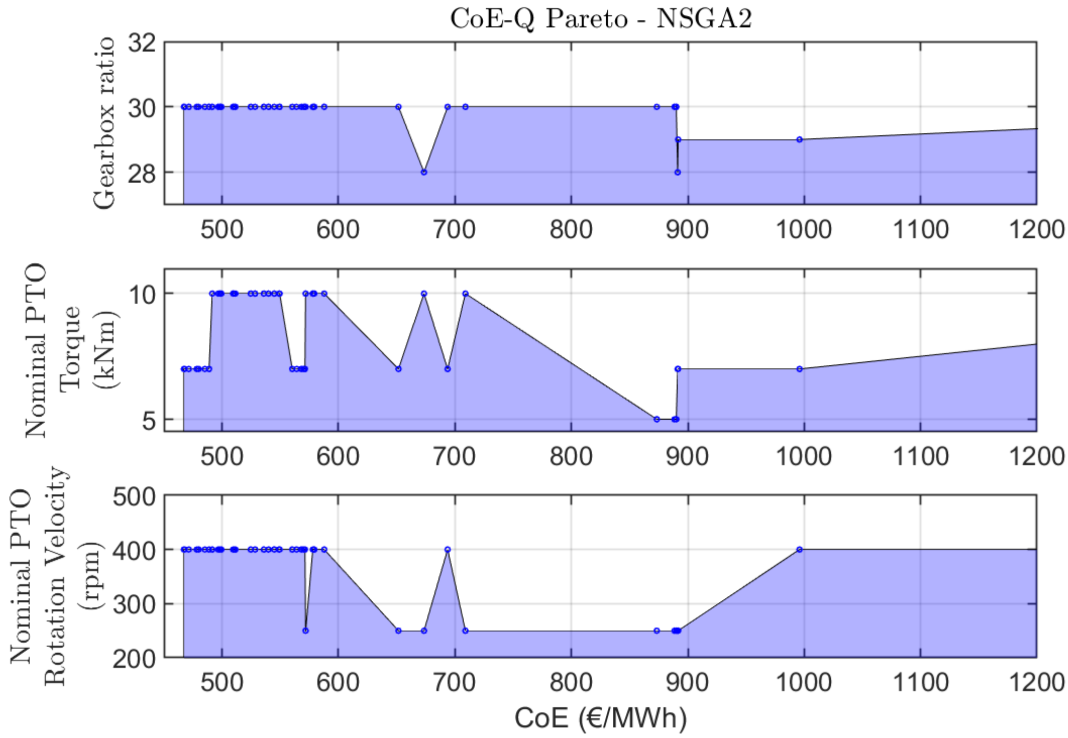

figure 3. 18

An attempt made in order to try to stress the robustness of the systems was to perform an optimization settled with the same set-up parameters but doubling the uncertainties on the pitch hull viscous damping parameter and on the distance between the pendulum's hinge and the device's Centre of Gravity. In the following considerations, this attempt will be referred to as Case 02. Therefore, comparing the results obtained in the previous case with the latter we obtain the Pareto fronts in figures 3.19 and 3.20.

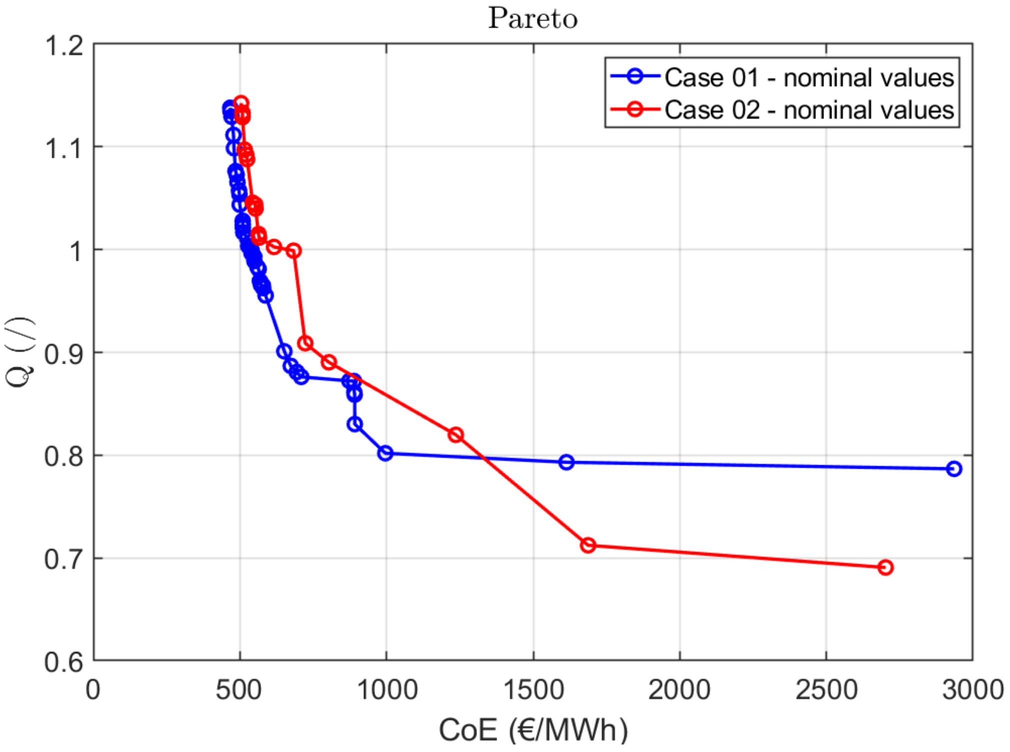

figure 3. 19

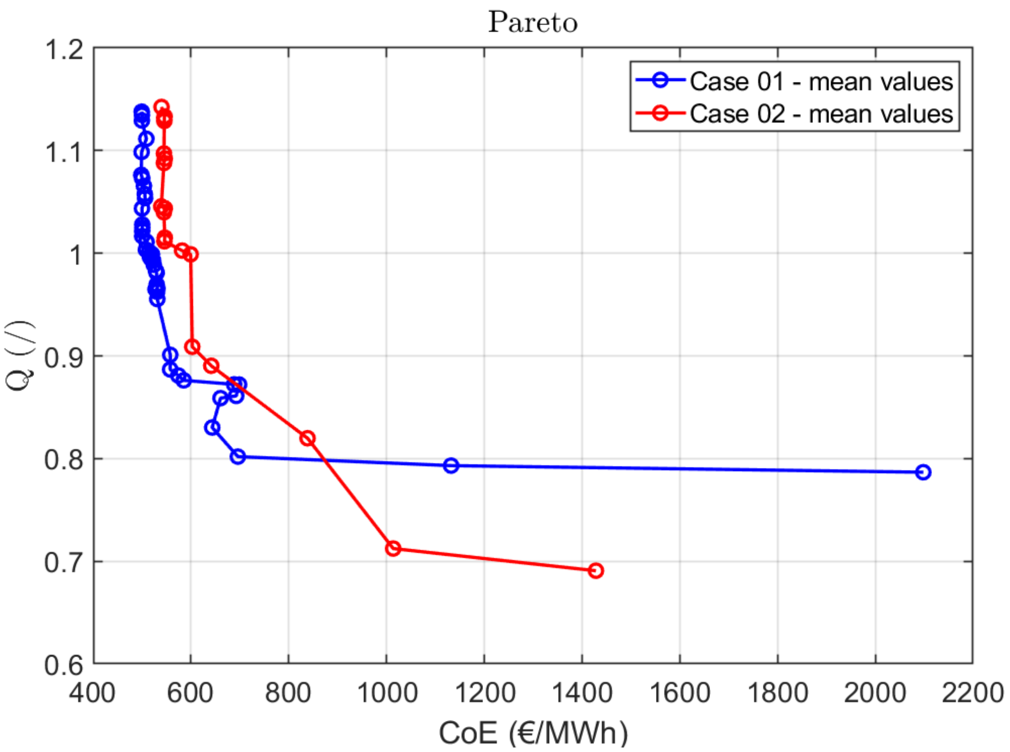

figure 3. 20

In the first graph, the results of the two cases under examination are likened by plotting the CoE nominal values CoE_0, instead in the second the mean values are reported. The main difference between Case 01 and Case 02 turns out to be the CoE_0 values achieved. Specifically, the devices obtained downstream of the second attempt have little higher nominal cost of energy. This difference can also be caused by the three fewer generations that have been bred in the case 2 optimization. Instead, the robustness index Q evaluations are found to be very similar.

Also observing the other variables relating to the Pareto front devices obtained in Case 02, it can be stated that the difference from the results obtained following the first attempt is small and tends to be described as a slight translation towards higher CoE values.

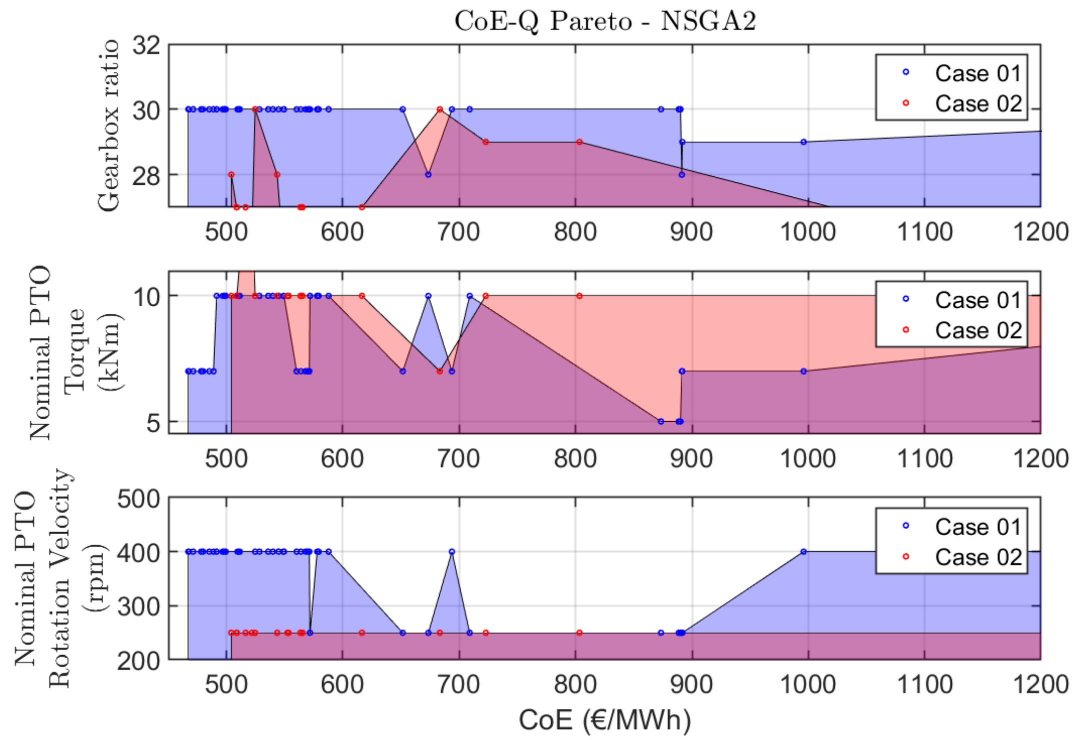

figure 3. 21

The main odds are found in the nominal PTO rpm and pendulum mass (figure 3.21 and 3.23). Instead, focusing the attention in the most interesting range $\Delta_{CoE} = [460; 590]\frac{€}{MWh}$, the behaviour looks very similar for both the trials.

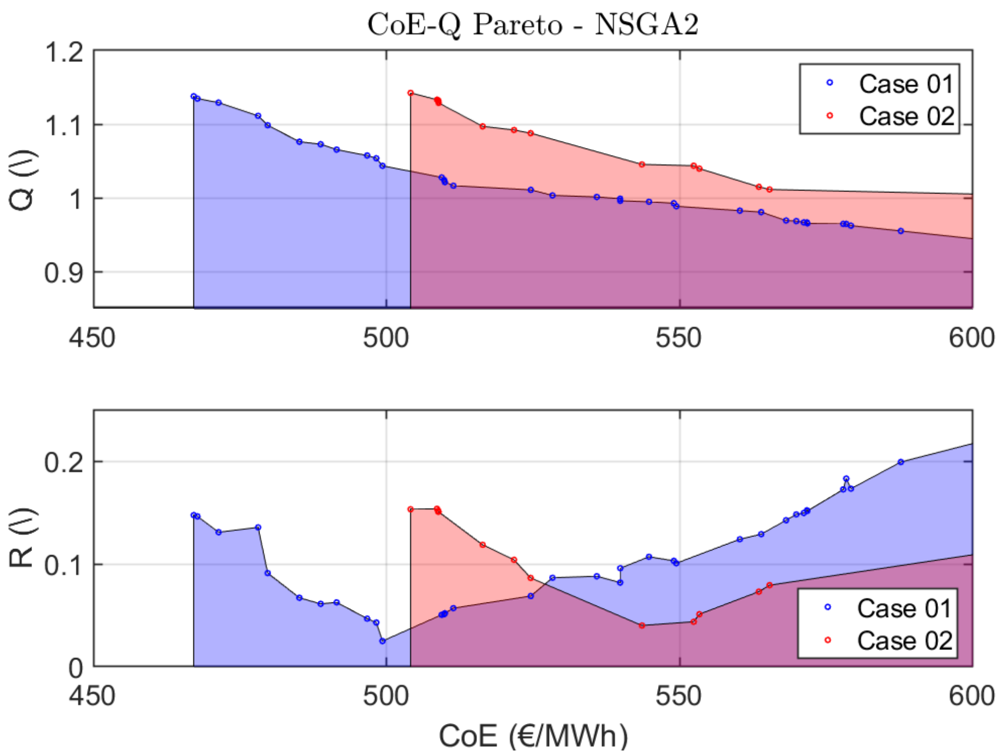

figure 3. 22

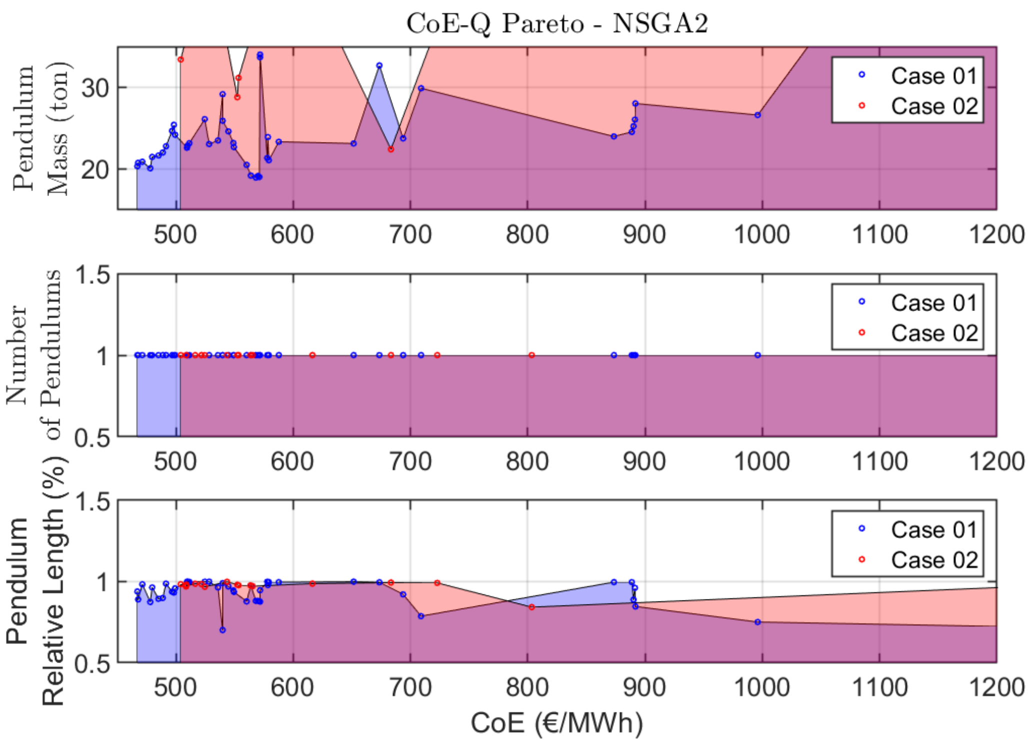

figure 3. 23

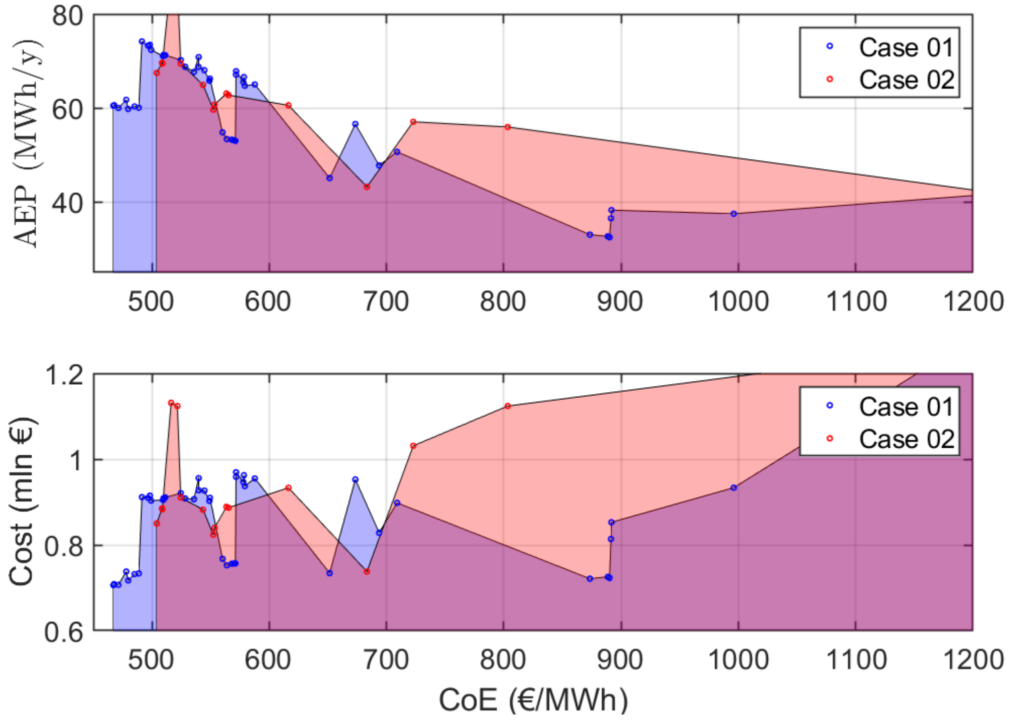

figure 3. 24

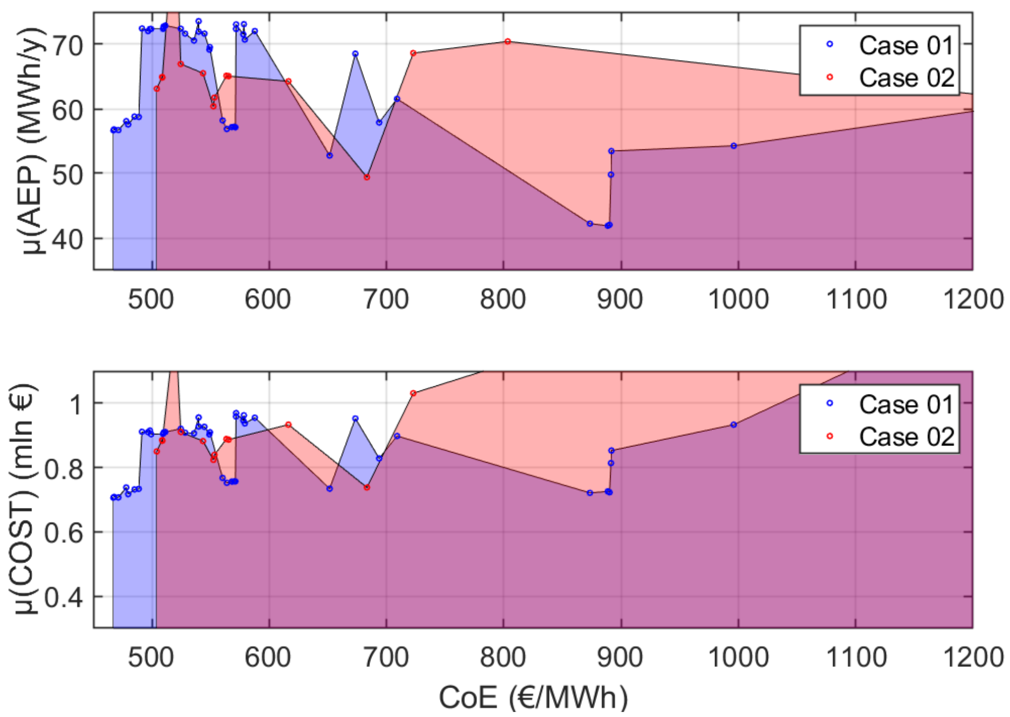

figure 3. 25

3.2.4 Device Analysis

Finally, in this last subsection, specific devices belonging to the Pareto front are analysed. In particular, the aim is to analyse the most representative cases of the results obtained, which will be defined as follow:

- *Device_A*: the device with lower CoE and higher Q;
- *Device_B*: the device where there is a large step in the CoE values for approximately equal Q values;
- *Device_C*: the device with higher CoE and lower Q.

Figures 3.26 show 2-D representations of the hulls at points A-B-C of the Pareto front listed above. From top to bottom the order of the devices is: A, B, C. Moving from hull A to hull C, one moves from a device with lower CoE and R to one with higher CoE and R as well as from a device with higher Q to one with lower Q. Looking at the hulls, however, what is seen to increase are the dimensions of the hull, including: hull width, hull length and pendulum arm, which consequently leads to a different positioning, relative to the hull, of the pendulum's hinge.

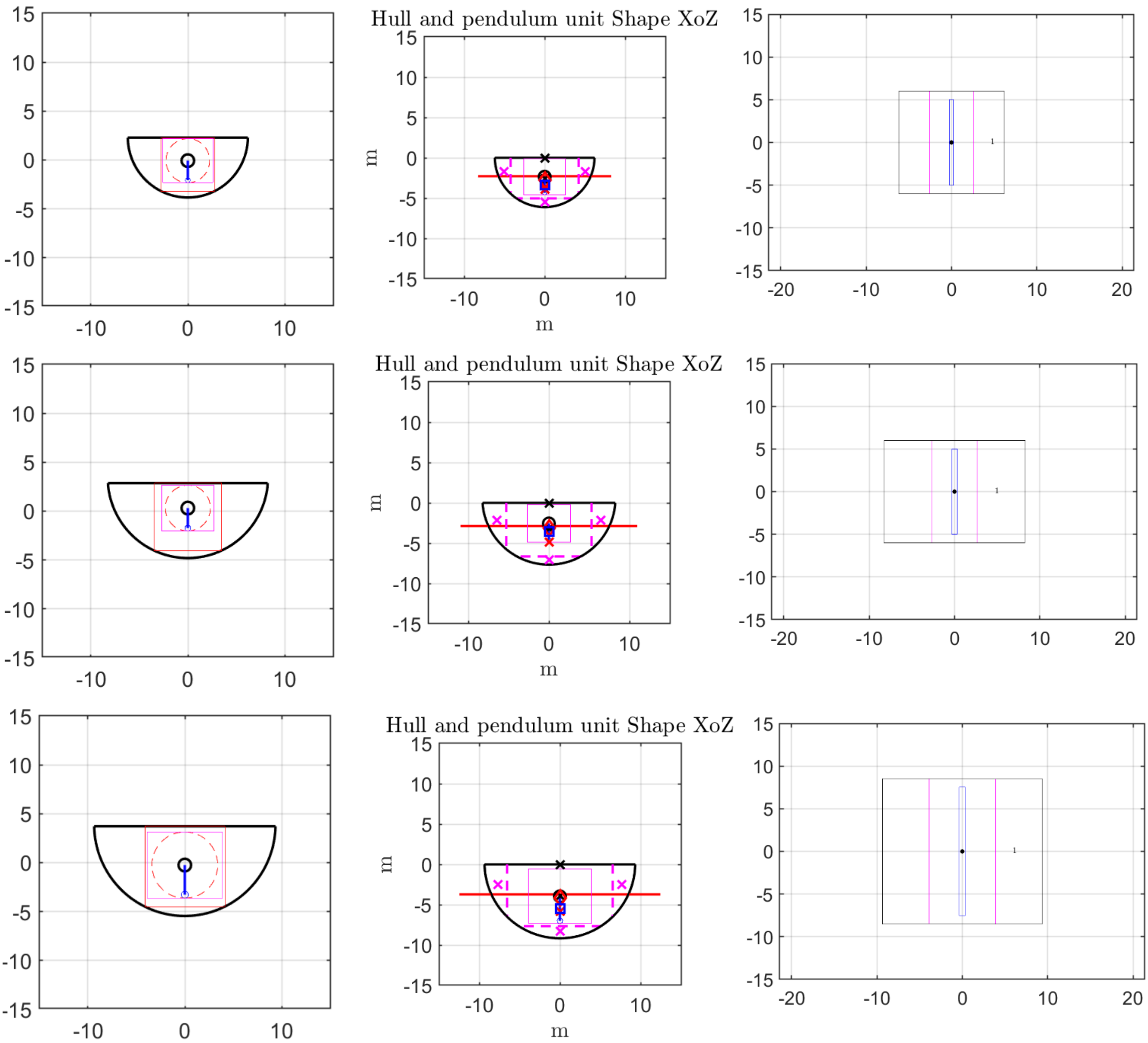

figure 3. 26

Between the three devices, substantial differences can also be seen when examining the period of the pitch oscillation peak. Specifically, moving from device A to device C, we can see that T_{res} increases (figure 3.27), while the height of this peak decreases slightly.

The frequency responses with respect to pitch motion, heave motion and surge motion for devices A, B and C are depicted in figures 3.28-29-30.

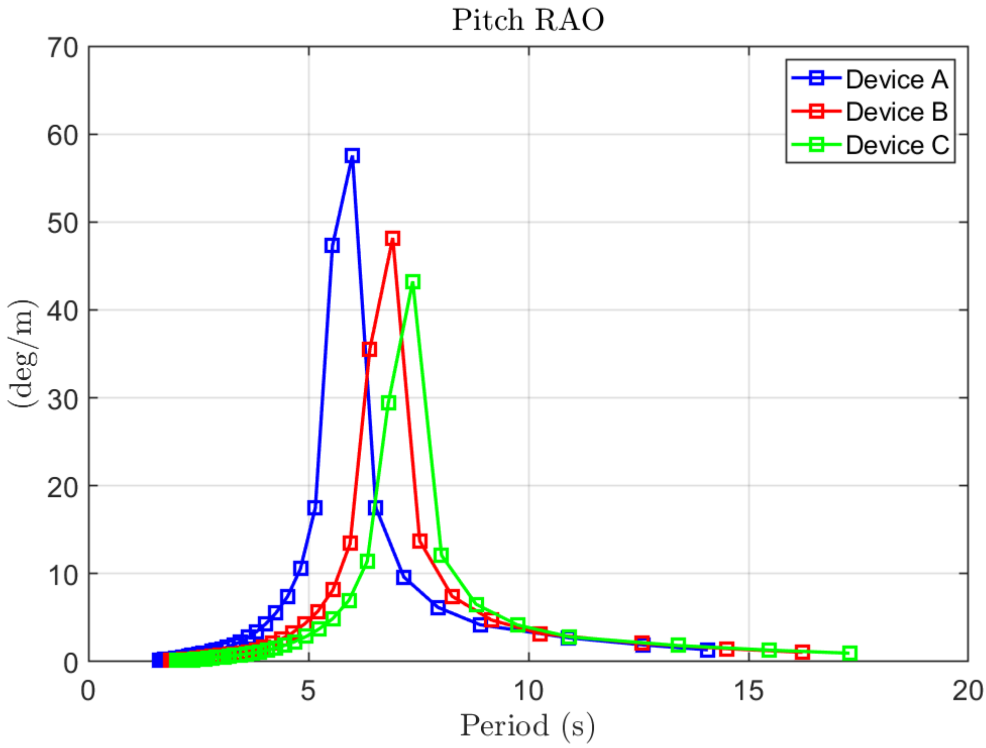

figure 3. 27

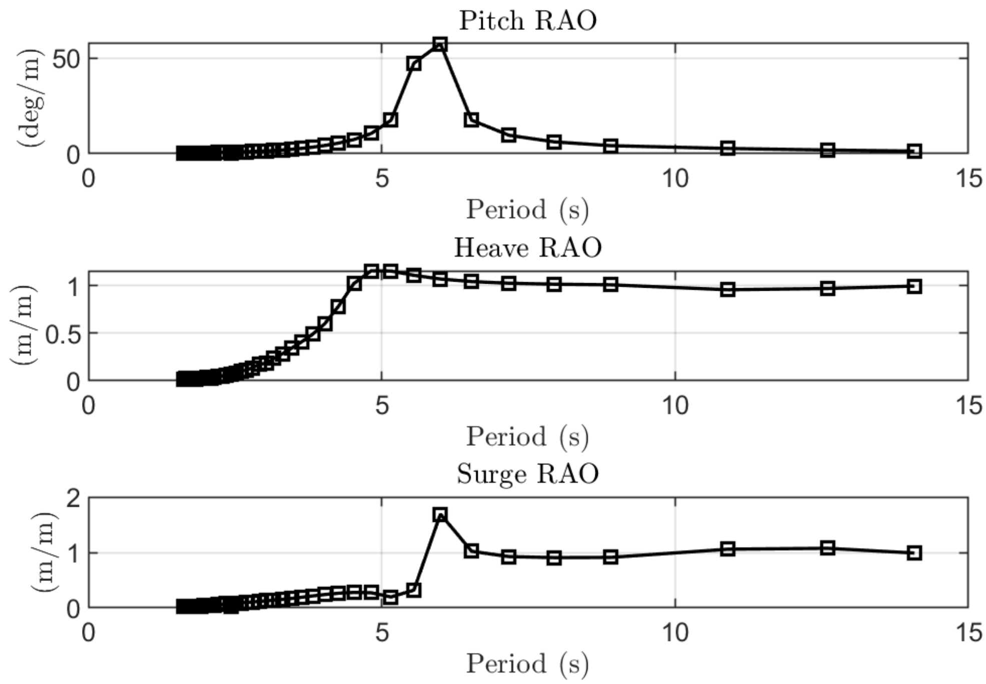

figure 3. 28

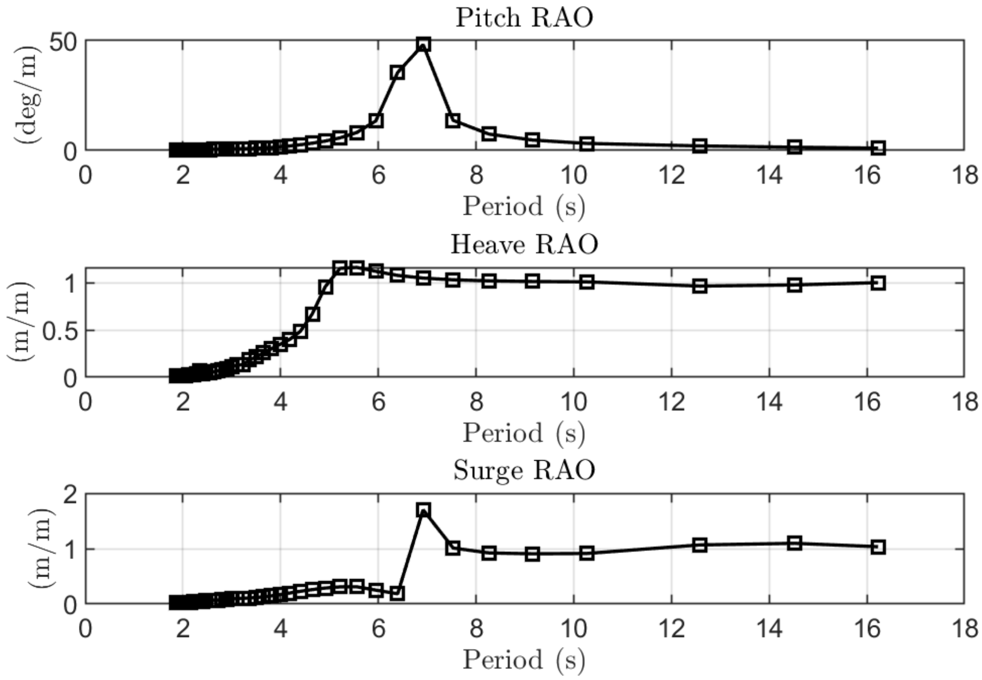

figure 3. 29

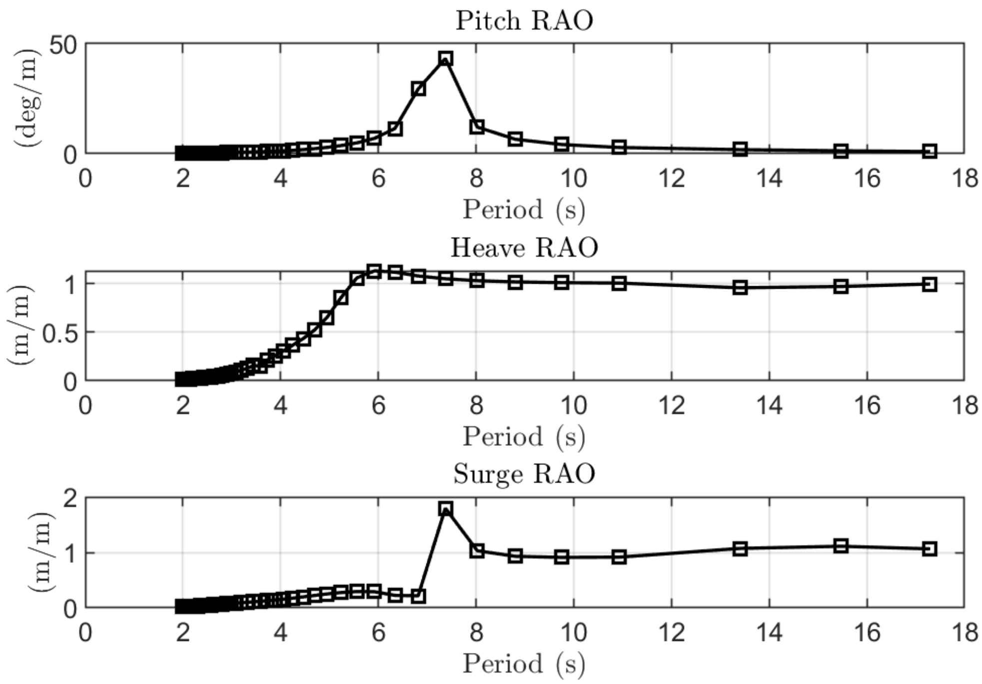

figure 3. 30

In the second part of this subsection, the same hulls listed above are examined via sensitivity analysis. Specifically, the procedure is carried out as follows: after assessing the characteristic parameters for the specific device under examination, the performances produced by this device is calculated by applying a sensitivity indicator to a single parameter subject to the effects of uncertainties at a time, in order to assess the influence of that specific variable on the performance under examination. In our case, the examined performance is always the cost of the energy CoE and the variables of which we want to evaluate the influence are always the three parameters previously indicated and on which we know to act the effect of the uncertainties. The sensitivity values applied are $\pm 0.1\%$ with respect to the design value for the pitch turning radius and for the pitch hull viscous damping (for the latter, the negative sensitivity value has not been applied, as it has no mean for the phenomenon that the parameter describes), a sensitivity of $\pm 20\ cm$ is applied instead to the input parameter that defines the distance between CoG of the device and the hinge of the pendulum.
The tornado plots in figure 3.31-32-33 describe the results of this analysis. In particular, the blue colour identifies the influence at a positive sensitivity while the orange colour identifies the influence at a negative sensitivity.

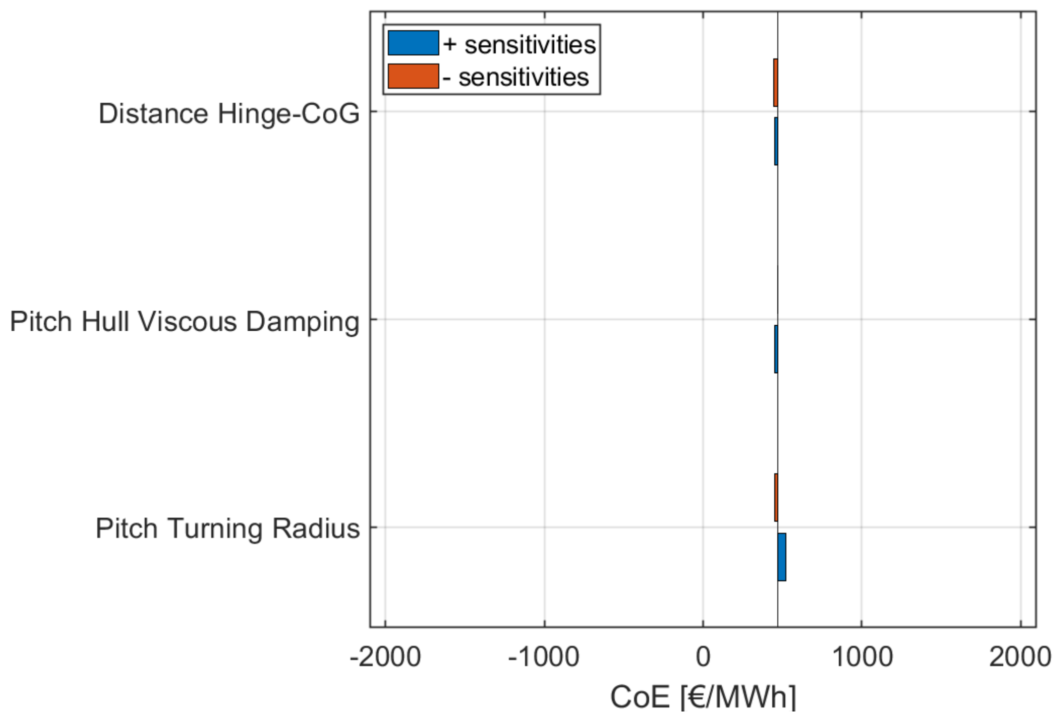

figure 3. 31, Device A

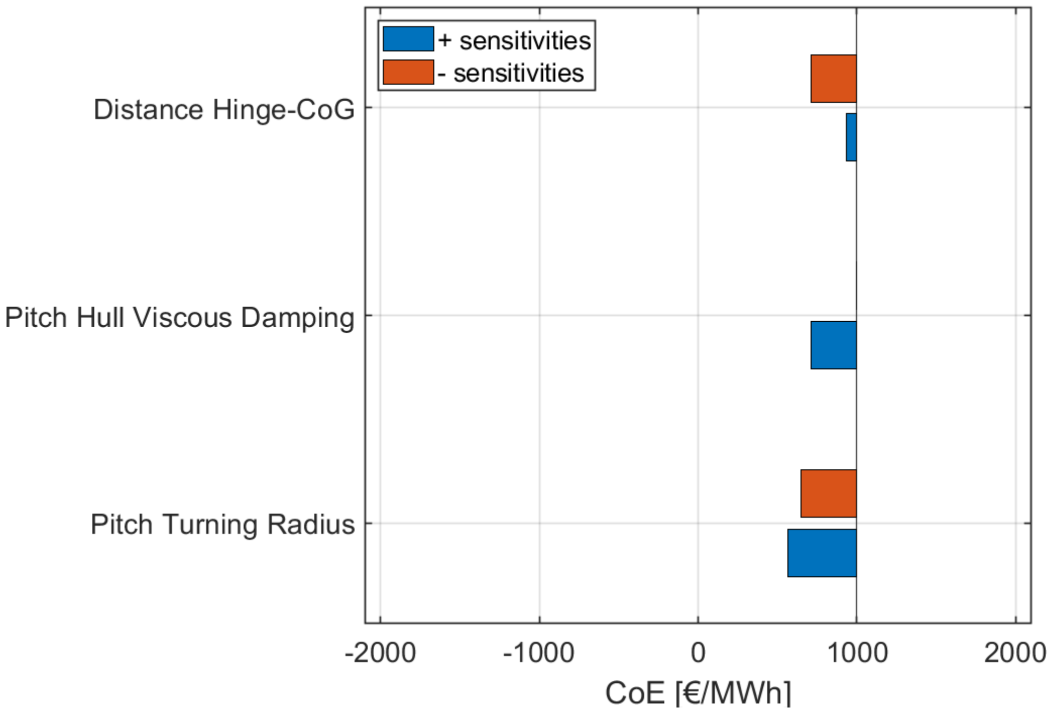

figure 3. 32, Device B

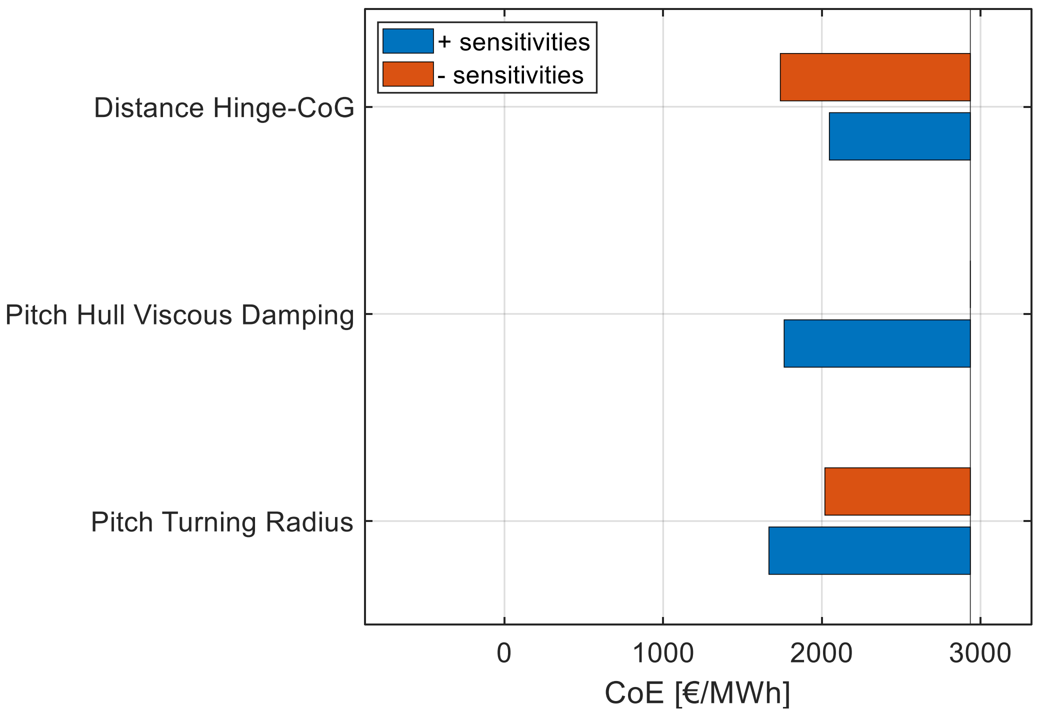

figure 3. 33, Device C

This latter analysis shows that the units examined tend to have robust behaviour and thus verifies the reliability of the hull. The analysis also shows which are the most influential parameters on the evaluation of the CoE, that is: the increase of the hull pitch viscous damping and of the pitch turning radius and a variation in negative direction of the distance between the hinge of the pendulum and the CoG of the device. In particular, Device C is, among the three examinated, the device that is the least robust (lowest R among the three) and that suffers most the effect of the uncertainties.

The same analysis was then conducted on two further devices: *Device D* and *Device E.* These, together with *Device A*, will form a new triplet of devices which is representative of three different values of the symmetrical robustness index R. In particular, *Device D* represents the device with the lowest index R $(0.0250\,[/])$, this device is also defined by $CoE = 499\left[\frac{€}{MWh}\right]$ and $Q = 1.04\,[/]$. Device E, instead, is defined by the following values: $CoE = 587\left[\frac{€}{MWh}\right]$, $Q = 0.9554\,[/]$, $Q = 0.1991\,[/]$.

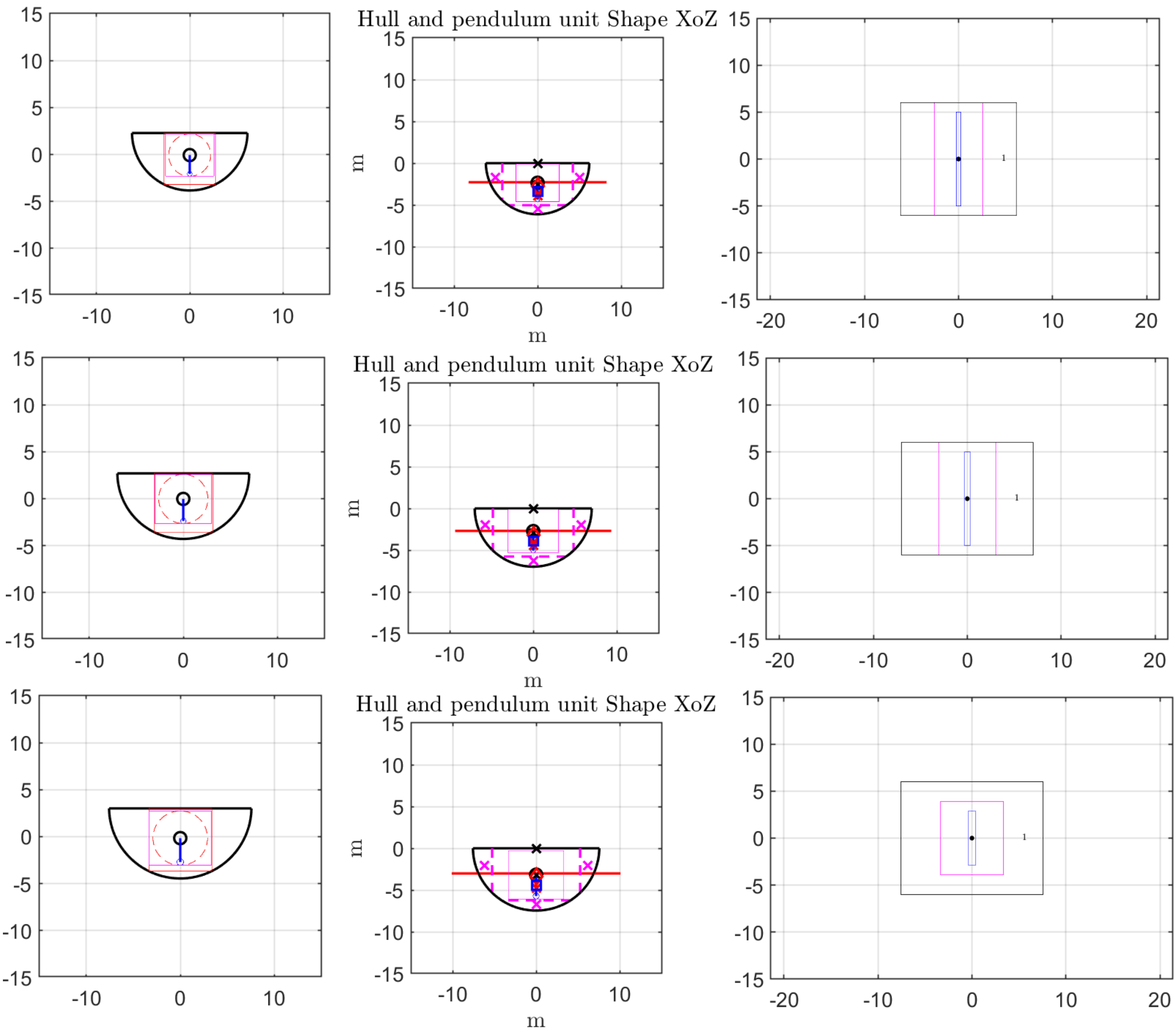

figure 3. 34

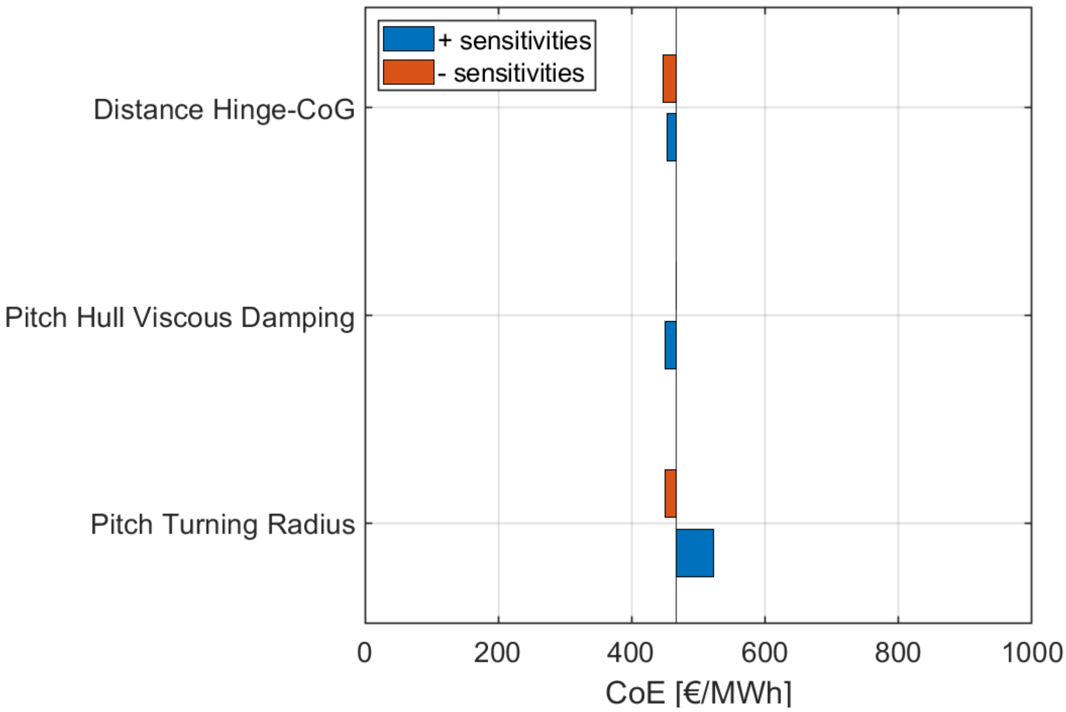

figure 3. 35, Device A

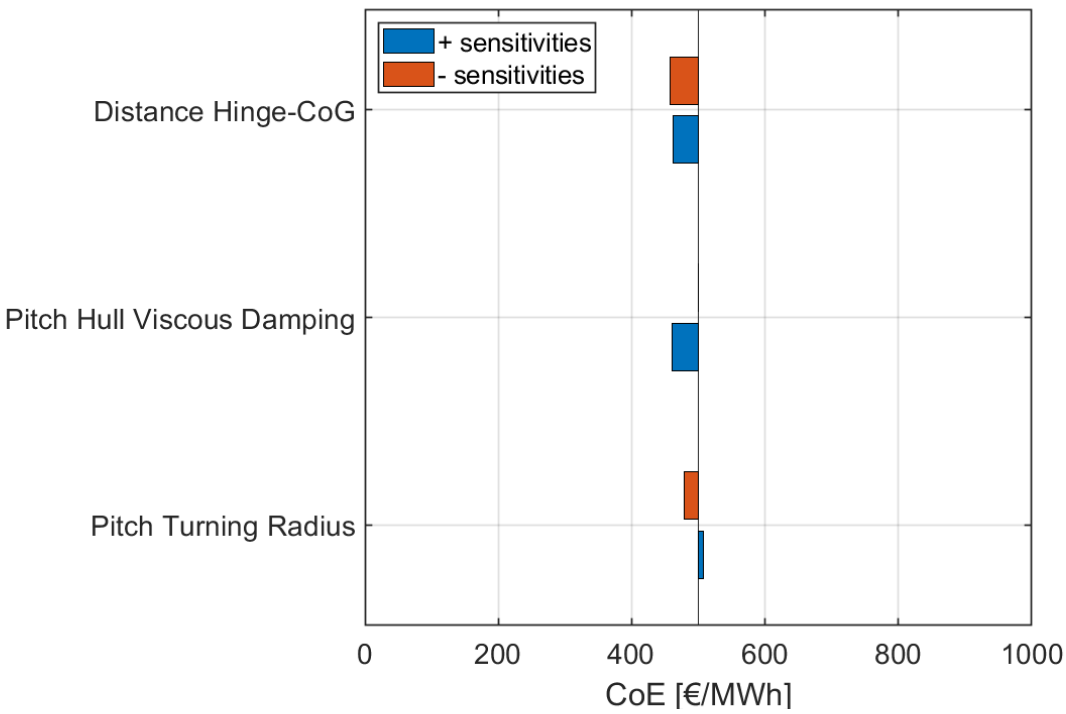

figure 3. 36, Device D

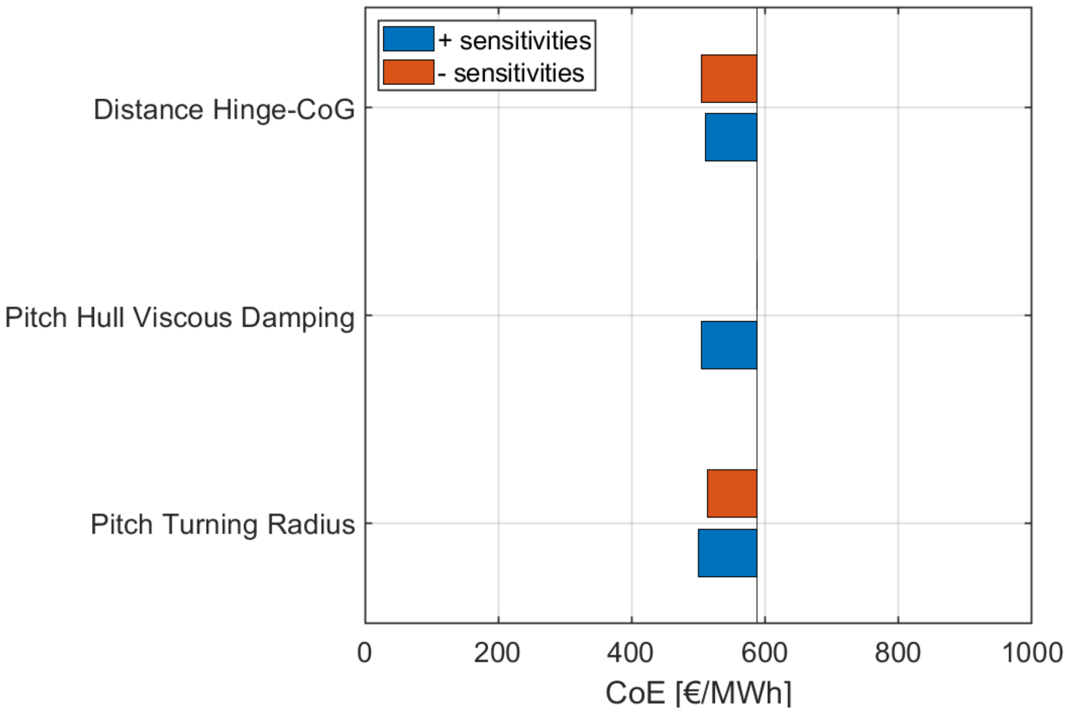

figure 3. 37, Device E

The results of these analyses show that, in some cases, given a lower CoE value, it may be preferable to choose a device with an asymmetric risk index Q value around unity but also characterised by a very low R value.

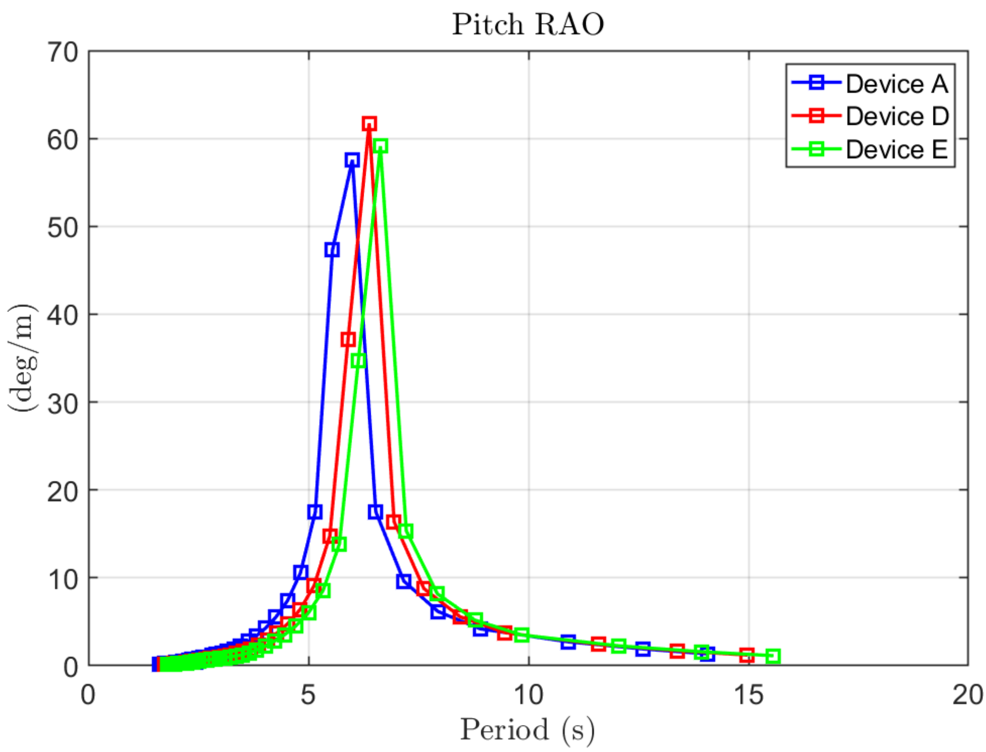

figure 3. 38

Device	A	D	E	B	C
Overall Hull Length [m]	12.38	14.01	15.14	16.45	18.68
Overall Hull Width [m]	12.04	12.04	12.02	12.00	17.06
Tangent Abscissa Ratio [/]	0.68	0.97	0.86	0.53	0.51
Overall Hull Height Ratio [/]	0.99	0.10	0.99	0.93	0.98
Draft Ratio [/]	0.63	0.61	0.60	0.63	0.59
Ballast Filling Ratio [/]	0.700	0.70	0.71	0.81	0.70
Pendulum Relative Mass [/]	0.07	0.06	0.06	0.05	0.07
Pendulum Geometry [/]	10	10	5	10	10
Pendulum Relative Length [/]	0.94	0.96	0.10	0.71	0.94
Number of Pendulums [/]	1	1	1	1	1
Pendulum Position [/]	10	10	8	10	7
Power Take Off ID [/]	9	10	10	9	11
Gearbox Ratio [/]	30	30	30	29	10

table 3. 5

Tables 3.5 and 3.6 provide a concise summary of the data, performances value and parameters for the hulls analysed in these surveys.

Device	A	D	E	B	C
Power Take Off ID [/]	9	10	10	9	11
COST [mln €]	0.70	0.90	1.0	0.93	1.7
AEP [MWh/y]	60	72	65	37	24
CoE [€/MWh]	467	499	588	956	2936
L [m]	12.38	14.01	15.14	16.45	18.68
W [m]	12.04	12.04	12.02	12.00	17.06
Q [/]	1.14	1.04	0.96	0.80	0.79
R [/]	0.15	0.03	0.20	0.61	0.58
Total Device Mass [ton]	391.4	493	544	636	1162

CHAPTER 4 - CONCLUSIONS

The purpose of this last chapter is to draw conclusions from the results presented in Chapter 3, to think accordingly on the possible developments of the track begun with this thesis work and finally to suggest the possible directions in which to conduct eventual future researches and works.

In this thesis work, a robust optimization method was designed and applied for the analysis of a specific wave energy converter (PeWEC). In order to cope with this problem an approach based on a genetic algorithm has been chosen, that involve an inner loop during which a Monte-Carlo simulation is performed for each individual, so as to be able to evaluate, from the distributions obtained, two robustness indices (Q and R) previously defined. From the final results obtained with this procedure, it is possible to highlight a Pareto front in the objective functions space $CoE - Q$. The devices belonging to this set show that, net of an annual energy production (AEP) $> 60\frac{MWh}{y}$, Q and CoE values result to be positive and consistent with the researches previously carried out. These results can be considered positive not only because they are characterized by a low asymmetrical risk index Q, but also by low symmetrical robustness index R. In fact, sometimes a device defined by a Q index tending to 1 and a small R is suitable rather than a device with index $Q < 1$ and big R. What is certainly always preferable would be to have, with the same amount of annual energy production AEP, costs as low as possible.
The most interesting results obtained stand around the ranges $\Delta_{CoE} = [460; 590]\frac{€}{MWh}$ and $\Delta_Q = [1.5; 0.95]$. In this range it is possible to find the devices characterized by the lowest R and CoE in the Pareto front and with asymmetric robustness index values around $Q = 1$. These results were also found to be valid following an attempt to stress the robustness of the devices by doubling the values of the uncertainties for the viscous hull damping parameter and for the parameter describing the distance between the CoG of the unit and the hinge of the pendulum. In this way, the reliability of devices in this Pareto front range was assessed. This was then verified via a sensitivity analysis. The latter analysis also showed which parameters are the most influential on the evaluation of the CoE, namely: an increase in the viscous damping of the pitch hull and of the pitch turning radius, or the variation in a negative direction of the distance between the pendulum hinge and the CoG of the device.

Now, known these results, it is necessary to contextualize them according to the purpose for which the wave energy converters were designed. The aspects that must be compared with those of technologies suitable for recovering energy from different renewable sources is the levelized cost of energy, which can be described like the parameter CoE analyzed in this thesis work.
For instance, it is possible to take as a reference the report given by Lazard in its website [87] and drawn up in 2021, where an extensive analysis is presented concerning the levelized cost of energy at different energy sources. In particular, as shown in one the graph present in the report mentioned above, which describes the mean levelized cost of energy over the years as the energy source varies, it can be seen that the cost of energy

extracted using PeWEC still has a fairly high CoE compared to those of other forms of technologies which recover energy from renewable sources for current generation. It is therefore necessary to think of a way to try and further improve the performance of the wave energy converter analyzed. This can be achieved by working in different directions: increasing the lifetime N_y of the plant while keeping costs and annual energy produced constant, studying the costs of the plant in order to optimize them as much as possible and decrease them, or finding ways to increase annual productivity while keeping costs more or less constant. With regard to the latter, one idea might be to use PeWEC array, evaluating the performance of the overall system and how the individual WEC that make it up interact with each other in the set. For this hypothesis, the costs must also be assessed. In addition, what should always be kept in mind is the dependence of the performance of the wave energy converter technology on the site where it is planned to establish the plant. Therefore, it is also necessary to pay particular attention to the study of siting and develop possible future work in this direction.

One of the assumptions made during this thesis work was to exclude from the parameters subject to the effects of uncertainties those relating to costs, in particular the PTO cost, hull cost and pendulum cost. This choice has been motivated by the difficulty to estimate a probability model distribution suitable for these parameters, inasmuch it is very influenced by market fluctuations of materials prices and by additional factors such as the prices with which each specific retailer chooses to sell the materials and products. One way in which it is possible to try to deal with this aspect of the case study examined is suggested by the chance offered by some software, which are able to generate an estimate of the production cost of a given component as output, known: the 3D model of the latter, the materials and the production process in its totality. Therefore, a possibility would be to use these outputs to estimate statistics in order to study the probability distribution model that best suits the cost of the component under consideration.

Another aspect that has not been examined in depth during this preliminary work is the degradation of the device's performances and how this decay affects its robustness. The only assumption made in respect of this topic is the device's lifetime under consideration, which has been set at 25 years.

Focusing again on the improvement of the PeWEC device, together with possible revisions of the system’s numerical model, eventual developments and modifications inherent of the optimization framework can be considered and present several possible paths ahead. The first, preserving the same framework design used in this thesis work, consists in making different choices regarding the objective functions and modifying the constraints configured for the optimization algorithm (like type D approach among those listed in Chapter 2). For example, a chance could be to set the Annual Energy Production (AEP) and the total device cost as objective functions and then set up the maximum value Q_{max} as a constraint, beyond which the value of the asymmetric risk index Q (which could in any case be calculated with respect to the CoE) cannot rise. For example, if $Q_{max} = 1.1$ was enforced, this would mean that the number of occurrences defined by the chosen percentage q would be below a value of 10% greater than the nominal value. Therefore, by setting this type of constraint, an optimization algorithm could be obtained which in output returns devices that respect a sort of safety coefficient relating to the device’s performances. Another option would be to set, for example, together with the previous established constraint, a lower limit that define the minimum AEP required for a single device.

A second way forward, on the other hand, is based on the radical change of the optimization framework structure and consequently of the optimization under uncertainty problem solution approach. What has already been highlighted as one of the major difficulties encountered during the analysis performed with an approach based on a Monte-Carlo simulation method is the onerous computational power required and hence the great processing time required in order to elapse the whole optimization process. In particular, what weighs more on these factors is the computational fluid-dynamics calculations for a complex and full of non-linearities system like the one in question. Therefore, these reasons necessarily involve constraints from which it is not possible to escape if the chose made by the designer is to apply an approach based on a Monte-Carlo simulation, such as the one chosen for the thesis work carried out. Between the time required for the convergence of the algorithm and the number of samples used in the Monte-Carlo, there is, thence, a trade-off, net of the parallelization capacity of the hypothetical cluster on which the optimization takes place. A way to avoid this problem can be to study a new approach, this time based on the application of one or more metamodels, for one or more of the examined performances or for the whole PeWEC system in general. This different typology approach, help to streamline and speed up the whole optimization process by their nature.

LIST OF TABLES

LIST OF FIGURES

REFERENCES

[1] S.Salomon, G. Avigad, P. J. Fleming, R. C. Purshouse, *Active Robust Optimization: Enhancing Robustness to Uncertain Environments* in IEEE Transactions on Cybernetics, November 2014 https://www.researchgate.net/publication/267101763

[2] Stéphane Caro, Fouad Bennis, Philippe Wenger. *Tolerance Synthesis of Mechanisms: a Robust Design Approach*. Journal of Mechanical Design, American Society of Mechanical Engineers, 2005, 127, pp.86-94. 10.1115/1.1825047. hal-00463707 https://hal.archives-ouvertes.fr/hal-00463707

[3] G. J. Park, T. H. Lee, K. H. Lee, K. H. Hwang, *Robust Design: an Overview*, in AIAA Journal, January 2006 https://www.researchgate.net/publication/245426180

[4] Vagelis Plevris, *Innovative Computational Techniques for the Optimum Structural Design Considering Uncertainties,* PhD Thesis, National Technical University of Athens School of Civil Engineering Institute of Structural Analysis and Seismic Research, June 2009 https://www.researchgate.net/publication/287997537

[5] F.Pelizzari, *Robust optimization of MTMD systems for the control of vibrations A robust optimal design based on genetic algorithms of MTMD systems excited by random vibrations***,** Master's Degree Thesis in Civil Engineering, Politecnico di Torino, March 2020

[6] Goldman, Brian Wesley, "*Robust evolutionary algorithms*" (2012). Masters Theses. 5148 https://scholarsmine.mst.edu/masters_theses/5148

[7] S. Mirjalili, *Introduction to Evolutionary Single-objective Optimisation*

[8] Wejun Wang, Stéphane Caro, Fouad Bennis, Ricardo Soto, Broderick Crawford, *Multi-objective Robust Optimization using a Post-optimality Sensitivity Analysis Technique: Application to a Wind Turbine Design,* Journal of Mechanical Design, Ameican Society of Mechanical Engineers, 2015, Journal of Mechanical Design, 137, pp.011403- 1-011403-11. 101115/1.4028755. hal-01084715 https://hal.archives-ouvertes.fr/hal-01084715

[9] D. H. Wolpert, W. G. Macready, *No Free Lunch Theorems for Optimization***,** IEEE Transactions On Evolutionary Computation, Vol. 1, no. 1, April 1997

[10] D.E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning,* Addison-Wesley Publishing Company Inc, January 1989

[11] Maruyama , Takayuki ; Igarashi , Hajime , *An Effective Robust Optimization Based on Genetic Algorithm* , Hokkaido University, IEEE Transactions on Magnetics, 44(6), 990-993, https://doi.org/10.1109/TMAG.2007.916696 ;

[12] O.B. Augusto, F. Bennis, S. Caro, *Multi-objective Engineering Design Optimization Problems: A Sensitivity Analysis Approach,* in Pesquisa Operacional, December 2012, https://www.researchgate.net/publication/259496309

[13] K.N. Otto, E.K. Antonsson *Extensions to the Taguchi Method of Product Design* in the ASME Journal of Mechanical Design, January 1991

[14] T. Orosz, A. Rassõlkin, A. Kallaste, P. Arsénio, D. Pánek, J. Kaska, P. Karban *Robust Design Optimization and Emerging Technologies for Electrical Machines: Challenges and Open Problems*, MDPI Appl. Sci. 2020, 10, 6653; doi:10.3390/app10196653, September 2020

[15] K. Zielinski, D. Peters, R. Laur, *Stopping Criteria for Single-Objective Optimization,* Institute for Electromagnetic Theory and Microelectronics (ITEM), Bremen

[16] R.Jin. X. Du, W. Chen, *The Use of Metamodeling Techniques for Optimization Under Uncertainty,* https://www.researchgate.net/publication/226317334_The_Use_of_Metamodeling_Techniques_for_Optimization_Under_Uncertainty

[17] K. Deb, H. Gupta, *Introducing Robustness in Multi-Objective Optimization,* February 2006, https://www.researchgate.net/publication/6687491_Introducing_Robustness_in_Multi-Objective_Optimization

[18] Simone Poli , *La Sensitivity Analysis*, Università Poitecnica delle Marche, 2018

[19] Maurizio Rosina, *L'analisi di sensitività basata sulla perturbazione dell'ordine dei pesi applicata ai metodi SAW e TOPSIS*, Marzo 2017

[20] *Analisi di Sensitività, Università* di Padova;

[21] O.De Weck, K. Willcox, *Multidisciplinary System Design Optimization (MSDO), Gradient Calculation and Sensitivity Analysis, Lecture 9*, Massachusetts Institute of Technology

[22] O.De Weck, K. Willcox, *Multidisciplinary System Design Optimization (MSDO), Post-Optimality Analysis, Lecture 16*, Massachusetts Institute of Technology

[23] Parmee, I.; Johnson, M.; Burt, S. *Techniquesto aid global search in engineering design*. In Proceedings of the 7th International Conference on Industrial and Engineering Applications of Artificial Intelligence and Expert Systems; CRC Press: Boca Raton, FL, USA, 1994; p. 377.

[24] Parmee, I.C.; Bonham, C. *The maintenance of search diversity for effective design space decomposition using cluster oriented genetic algorithms (COGAs) and multi-agent strategies (GAANT).* In Proceedings of the ACEDC; University of Plymouth: Plymouth, UK, 1996

[25] Aizawa, A.N.; Wah, B.W. *Dynamic control of genetic algorithms in a noisy environment*. rn 1993, 2, 1.

[26] Aizawa, A.N.; Wah, B.W. *Scheduling of genetic algorithms in a noisy environment*. Evol. Comput. **1994**, 2, 97–122

[27] Markon, S.; Arnold, D.V.; Back, T.; Beielstein, T.; Beyer, H.G. *Thresholding-a selection operator for noisy ES*. In Proceedings of the 2001 Congress on Evolutionary Computation (IEEE Cat. No. 01TH8546), Seoul, Korea, 27–30 May 2001; pp. 465–472.

[28] Neri, F.; Caponio, A. *A differential evolution for optimisation in noisy environment*. Int. J. Bio-Inspired Comput. **2010**, 2, 152–168.

[29] Branke, J.; Schmidt, C. *Selection in the presence of noise.* In Genetic and Evolutionary Computation Conference; Springer: Berlin/Heidelberg, Germany, 2003; pp. 766–777.

[30] Rakshit, P.; Konar, A. *Noisy Multi-objective Optimization for Multi-robot Box-Pushing Application*. In Principles in Noisy Optimization; Springer: Berlin/Heidelberg, Germany, 2018; pp. 243–305.

[31] Hughes, E.J. *Evolutionary multiobjective ranking with uncertainty and noise*. In International Conference on Evolutionary Multi-Criterion Optimization; Springer: Berlin/Heidelberg, Germany, 2001; pp. 329–343.

[32] Teich, J. *Pareto-front exploration with uncertain objectives.* In International Conference on Evolutionary Multi-Criterion Optimization; Springer: Berlin/Heidelberg, Germany, 2001; pp. 314–328.

[33] J. García, A. Peña, *Robust Optimization: Concepts and Applications,* http://dx.doi.org/10.5772/intechopen.75381

[34] Zugno, M., & Conejo, A. J. (2013). *A Robust Optimization Approach to Energy and Reserve Dispatch in Electricity Markets*. Technical University of Denmark. Technical Report-2013 No. 05, https://orbit.dtu.dk/en/publications/a-robust-optimization-approach-to-energy-and-reserve-dispatch-in--2

[35] I. Vertommen, K. van Laarhoven, M. da Conceição Cunha, *Robust Design of a Real-Life Water Distribution Network under Different Demand Scenarios,* Water 2021, 13, 753, https://doi.org/10.3390/w13060753

[36] Z. Bontinck, O. Lass, S. Schöps, H. De Gersem, S. Ulbrich, O. Rain, *Robust Optimization Formulations for the Design of an Electric Machine,* IET Research Journals, August 2018

[37] X. Chen, J. Fan, X. Bien, *Structural robust optimization design based on convex model,* https://www.sciencedirect.com/science/article/pii/S2211379717313049?via%3Dihub

[38] G.Li, G. Weiss, M. Mueller, S. Townley, M.R. Belmont, *Wave energy converter control by wave prediction and dynamic programming,* http://www.elsevier.com/copyright

[39] M. P. Shoen, J. Hals, T. Moan, *Wave Prediction and Robust Control of Heaving Wave Energy Devices for Irregular Waves* in IEEE Transactions on Energy Conversion, July 2011, https://www.researchgate.net/publication/224216344

[40] H.W. Fang, Y. Z. Feng, G. P. Li, *Optimization ofWave Energy Converter Arrays by an Improved Differential Evolution Algorithm*, Energies 2018, 11, 3522; doi:10.3390/en11123522, www.mdpi.com/journal/energies , December 2018

[41] R. Alamain, R. Shafaghat, M. Reza Safaei, *Multi-Objective Optimization of a Pitch Point Absorber Wave Energy Converter*, Water 2019, 11, 969; doi:10.3390/w11050969, www.mdpi.com/journal/water , May 2019

[42] S. A. Sirigu, L. Foglietta, G. Giorgi, M. Bonfanti, G. Cervelli, G. Bracco, G. Mattiazzo *Techno Economic Optimisation for Wave Energy Converter via Genetic Algorithm*, J. Mar. Sci. Eng. 2020, 8, 482; doi:10.3390/jmse8070482, www.mdpi.com/journal/jmse , June 2020

[43] L.V. Snyder, M.M. Moarefdoost, *Optimizing Wave Farm Layouts Under Uncertainty*, Proceedings of the 3rd Marine Energy Technology Symposium, METS2015, April 27-29, 2015, Washington, D.C.

[44] P. Vayanos, A. Georghiou, H. Yu, *Robust Optimization with Decision-Dependent Information Discovery,* University of Southern California, April 2020

[45] S.A. Sirigu, *Development of a Resonance-Tunable Wave Energy Converter,* Doctoral Programm in Mechanical Engineering ScuDo, Politecnico di Torino, 2019

[46] D. Song, Y. Shiqiang, H. Duanfeng, M.Qingwei, *Overview on Hybrid Wind-Wave Energy Systems,* International Conference on Applied Science and Engineering Innovation (ASEI 2015), College of Shipbuilding Engineering, Harbin Engineering University, Harbin 150001, China School of Mathematics, Computer Science and Engineering, City University London, London EC1V 0HB, UK

[47] Q. Dongsheng, H. Rizwan, Y. Jun, N. Dezhi, L. Binbin, *Review of Wave Energy Converter and Design of Mooring System,* Sustainability 2020, 12, 8251; doi:10.3390/su12198251, www.mdpi.com/journal/sustainability , October 2020

[48] N. Pozzi, *Numerical Modeling and Experimental Testing of a Pendulum Wave Energy Converter (PeWEC),* Corso di Dottorato in Ingegneria Meccanica ScuDo, Politecnico di Torino, Ottobre 2017

[49] N. Pozzi, G. Bracco, B. Passione, S.A. Sirigu, *Wave Tank Testing of a PEndulum Wave Emnergy Converter 1:12 Scale Model,* International Journal of Applied

Mechanics, March 2017, https://www.researchgate.net/publication/313827379

[50] L. Mian, A. Shapour, A. Vikrant, *A Multi-Objective Genetic Algorithm for Robust Design Optimization,* University of Maryland, June 2005

[51] Venigella, Pavan Kumar, "Robust Mechanism synthesis with random and interval variables" (2007) Masters Theses. 6829 https://scholarsmine.mst.edu/masters_theses/6829

[52] C. Barrico, C. Henggeler Antunes, *A New Approach to Robustness Analysis in Multi-Objective Optimization*, MOPGP'06: 7th Int. Conf. on Multi-Objective Programming and Goal Programming, Tours, France, June 12-14, 2006

[53] K. Deb, H. Gupta, *Searching for Robust Pareto-Optimal Solutions in Multi-Objective Optimization*, Lecture Notes in Computer Science, March 2005, https://www.researchgate.net/publication/221228385

[54] K. Deb, H. Gupta, *Introducing Multi-Objective Optimization,* Evolutionary Computation, February 2006, https://www.researchgate.net/publication/668749

[55] T. Jin, B. Sendhoff, *Trade-Off between Performance and Robustness: An Evolutionary Multiobjective Approach,* Lecture Notes in Computer Science, April 2003, https://www.researchgate.net/publication/233885579

[56] S. A. Starks, V. Kreinovich, L. Longprè, M. Ceberio, G. Xiang, R. Araiza, J. Beck, R. Kandathi, A. Nayak and R. Torres, *Towards Combining Probabilistic and Interval Uncertainty in Engineering Calculations,* NASA Pan-American Center for Earth and Environmental Studies (PACES), University of Texas, El Paso, TX 79968, USA, Reliable Computing, January 2004, https://www.researchgate.net/publication/248133543

[57] Evan. J. Huges, *Evolutionary Multi-Objective Ranking with Uncertainty and Noise,* Conference Paper, March 2001, https://www.researchgate.net/publication/221228598

[58] S. Tsutsui, A.Ghosh, *Genetic Algorithms with a Robust Solution Searching Scheme,* IEEE Transaction on Evolutionary Computation, Vol. 1, No. 3, pp. 201-208, September 1997, https://www.researchgate.net/publication/3418520

[59] I. Paenke, J. Branke, Y. Jin, *Efficient Search for Robust Solutions by Means of Evolutionary Algorithms and Fitness Approximation*, IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 10, NO. 4, AUGUST 2006, https://www.researchgate.net/publication/216301204

[60] O. O. Marquez-Calvo, D. P. Solomatine, *Approach to robust multi-objective optimization and probabilistic analysis: the ROPAR algorithm,* Journal of Hydroinformatics, March 2019, http://creativecommons.org/licenses/by-nc-nd/4.0/

[61] J. J. Roberts, A. Marotta Cassula, J. L. Silveira, E. da Costa Bortoni, A. Z. Mendiburu, *Robust Multi-Objective Optimization of a Renewable Based Hybrid Power System,* April 2018, https://doi.org/10.1016/j.apenergy.2018.04.032

[62] S. Gunawan, *Parameter Sesnsitivity Measures For Single Objective, Multi-Objective, And Feasibility Robust Design Optimization,* University of Maryland, 2004

[63] P. Limbourg, *Multi-Objective Optimization of Problems with Epistemic Uncertainty,* Institute of Information Technology, Department of Engineering, University of Duisburg-Essen

[64] Z. Vojinovic, S. Sahlu, A. S. Torres, S. D. Seyoum, F. Anvarifar, H. Matungulu, W. Barreto, D. Savic, Z. Kapelan, *Multi-objective rehabilitation of urban drainage systems under uncertainties,* Journal of Hydroinformatics May 2014

[65] Kapelan, Z. S., D. A. Savic, and G. A. Walters (2005), *Multi-Objective Design of Water Distribution Systems Under Uncertainty*, Water Resour. Res., 41, W11407, doi:10.1029/2004WR003787

[66] Seneshaw A. Kebede, *Optimal Design of Urban Stormwater Drainage System Under Uncertainty,* UNESCO-IHE Institute for Water Education, MSc Thesis MWI SE 2014-08, April 2014

[67] Y. Jin, B. Sendhoff, *Trade-off Between Performance and Robustness: An Evolutionary Multiobjective Approach*, Lecture Notes in Computer Science, April 2003, https://www.researchgate.net/publication/233885579

[68] S. Mirjalili, A. Lewis, J.S. Dong, *Confidence-based robust optimisation using multi-objective meta-heuristics*, Swarm and Evolutionary Computation BASE DATA (2018), doi: 10.1016/ j.swevo.2018.04.002

[69] H.G. Beyer, B, Sendhoff, *Robust Optimization – A Comprehensive Survey,* March 2007

[70] Y. M. Korolev, V. V. Toropov, Queen Mary University of London, London, E1 4NS, UK S. Shahpar, Rolls-Royce plc, Derby, DE24 8BJ, UK, *Design Optimization Under Uncertainty Using the Multipoint Approximation Method,* Copyright © 2017 by Rolls-Royce plc. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission, https://www.researchgate.net/publication/313458374

[71] P.N. Koch, R.-J. Yang, L. Gu, *Design for six sigma through robust optimization*, Structural and Multidisciplinary Optimization, February 2004, https://www.researchgate.net/publication/225392469

[72] R. Jin PTO Engineering & IT Solutions, Ford Motor Company, W. Chen Integrated DEsign Automation Laboratory (IDEAL), Northwestern University, A.

Sudjianto V-Engine Engineering Subsystem Engineering, Ford Motor Company, Analytical Metamodel-Based Global Sensitivity Analysis and Uncertainty Propagation for Robust Design, Downloaded from SAE International by Imperial College London, Thursday, August 09, 2018, SAE TECHNICAL PAPER SERIES

[73] H. Castrup, *Distributions for Uncertainty Analysis,* presented at the 2001 IDW Conference, Knoxville, TN. Revised 27 May 2004

[74] C. E. Hiles, S. J. Beatty, A. de Andres, *Wave Energy Converter Annual Energy Production Uncertainty Using Simulations*, J. Mar. Sci. Eng. 2016, 4, 53, doi:10.3390/jmse4030053 www.mdpi.com/journal/jmse, Journal of Marine Science and Engineering, September 2016, https://www.researchgate.net/publication/307615610

[75] J. Orphin, I. Penesis, J-R.Nader, *Uncertainty Analysis for a Wave Energy Converter: the Monte Carlo Method,* National Centre for Maritime Engineering & Hydrodynamics, Australian Maritime College, University of Tasmania Locked bag 1395, Launceston, Tasmania, 7250, Australia, September 2018, https://www.researchgate.net/publication/327687023

[76] International Towing Tank Conference, *Recommended Procedures and Guidelines – Uncertainty Analysis for a Wave Energy Converter,* 2017

[77] C. Ortega Absil, G. Filippi, A. Riccardi, M. Vasile, *A Variance-Based Estimation of the Resilience Indices in the Preliminary Design Optimisation of Engineering Systems Under Epistemic Uncertainty*, EUROGEN 2017, September 13-15, 2017, Madrid, Spain, https://www.researchgate.net/publication/323242901

[78] G. Filippi, M. Vasile, P. Z. Korondi, M. Marchi, C. Poloni, *Robust Design Optimisation Of Dynamical Space Systems,* September 2018, https://www.researchgate.net/publication/328496884

[79] G. Filippi, M. Vasile, M. Marchi, P. Vercesi, *Evidence-Based Robust Optimisation of Space Systems with Evidence Network Models,* July 2018, https://www.researchgate.net/publication/328455814

[80] C. Sabater, S. Görtz, German Aerospace Center (DLR), Institute of Aerodynamics and Flow Technology, 38108 Braunschweig, Germany, *An Efficient Bi-Level Surrogate Approach for Optimizing Shock Control Bumps under Uncertainty*

[81] C. Sabater, P. Bekemeyer, S. Görtz, German Aerospace Center (DLR), Institute of Aerodynamics and Flow Technology, 38108 Braunschweig, Germany, *Robust Design of Transonic Natural Laminar Flow Wings under Environmental and Operational Uncertainties*

[82] Fabio Carapellese*, Sergej Antonello Sirigu, Giuseppe Giorgi, Mauro Bonfanti, Alberto Ghigo and Giuliana Mattiazzo, *Multiobjective optimisation approaches* *applied to a Wave Energy Converter design*

[83] E. G. Shopova and N. G. Vaklieva-Bancheva, *Basic—a genetic algorithm for engineering problems solution* Computers Chemical Engineering, vol. 30, no. 8, pp. 1293–1309,2006.[Online].Available: https://www.sciencedirect.com/science/article/pii/S009813540600055X

[84] H. Jalota and M. Thakur, *Genetic algorithm designed for solving linear or nonlinear mixed-integer constrained optimization problems,* in International Proceedings on Advances in Soft Computing, Intelligent Systems and Applications, M. S. Reddy, K. Viswanath, and S. P. K.M., Eds. Singapore: Springer Singapore, 2018, pp. 277–290.

[85] M. Thakur, S. S. Meghwani, and H. Jalota, *A modified real coded genetic algorithm for constrained optimization,* Applied Mathematics and Computation, vol. 235, pp. 292–317, 2014. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0096300314003415

[86] X. Ma, *A new hybrid evolution genetic algorithm with laplace crossover and power mutation*, in 2013 Ninth International Conference on Computational Intelligence and Security, vol. 2. Los Alamitos, CA, USA: IEEE Computer Society, dec 2009. [Online]. Available: https://doi.ieeecomputersociety.org/10.1109/CIS.2009.64

[87] Lazard Levelized Cost of Energy Analysis – Version 15.0, https://www.lazard.com/perspective/levelized-cost-of-energy-levelized-cost-of-storage-and-levelized-cost-of-hydrogen/