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Theoretical analysis and comparison of CAPM and APT and their roles in risk management

Relatore:
Prof.ssa Elisa Ughetto

Candidata:
Gunel Taghizade

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Abstract

Financial markets are constantly changing and becoming more complex, and everyone involved tries to understand the causes of unpredictable fluctuations. With the volume and variety of financial products on the market today, it is more difficult than ever to predict the outcome of an investment. Investors are risk-averse. That is, they are willing accept higher risk only if you are expecting a higher return. Unfortunately, it is not possible to actually observe the risks and expected returns associated with financial instruments. This is because financial instruments are constantly affected by the individual institutions that issue financial instruments and the unlimited number of events that affect their sectors. To the economy or the whole world. However, while it is possible to estimate them using statistical methods and historical records of securities performance, these estimates are not always accurate and it is certain that new and unexpected events will occur at some point.

Throughout the years, many scholars have tried to come up with theories that could reflect the market fluctuations. The two most focused models of asset pricing are Capital asset pricing model and the Arbitrage pricing theory. These models try to demonstrate the relationship between risk and return. Many empirical studies have been conducted on these two models to check if they were truly functional. In different years, and in different parts of the world, scholars have obtained various results.

At some points, certain methodologies performed in the testing of these models, have lead to biased data, and they have been evolved and become more effective. The purpose of this study is to discuss the methodologies used to conduct empirical studies on CAPM and APT and their differences have been examined.

Theoretical analysis and comparison of CAPM and APT and their roles in risk management

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1. Literature overview

1.1. Modern portfolio theory

Modern portfolio theory (MPT), also known as mean-variance analysis, is the theory that helps create a portfolio of assets that give the maximum possible expected return is at a certain level of risk. It is a mainly idea is that diversification - owning different kinds of financial assets is less risky than owning just one type. The theory states that the effect of portfolio's overall risk and return should also be considered when an asset's risk and return is assessed. The variance of asset prices is used to represent the risk.

Modern portfolio theory was developed by Economist Harry Markowitz in 1952 as "Portfolio Selection" in the Journal of Finance . For his work he was awarded Nobel Memorial Prize in Economic Sciences.

One of the assumptions of the modern portfolio theory is that investors are risk averse, that is, investors will prefer the less risky portfolio when they have to choose between two portfolios having the same expected return. Which means an investor will accept to take higher risk if they think they are likely to obtain higher expected returns. Or vice versa, they should be ready to take higher risk in order to get increased expected return.

Expected Return

The expected return is the amount that an investor expects to earn or lose on an investment. It is calculated by multiplying the possible returns with the probability of each consequence and summing these together:

$$\text{Expected Return} = \Sigma\ (\text{Return}_i \times \text{Probability}_i)$$

Where i denotes each of the containing assets.

The calculation of expected return is usually done by considering historical data and which means it may not be true in all cases. But it is usually helpful to form an idea on expectations.

The expected return of the portfolio is found by computing the the weighted average of the expected returns of the containing assets of the portfolio. Expected return on portfolio of two securities is calculated as:

$$\text{Expected return} = \overline{R}_A X_A + \overline{R}_B X_B = \overline{R}_P$$

Where:

- $\overline{R}_A$, $\overline{R}_B$ are the expected returns of each security
- X_A, X_B are the corresponding weight of the security

Variance. Standard deviation.

One of the most common ways to measure the volatility of a security's return is using variance. Variance is calculated by finding the sum of squared deviations from the expected return of return.

The formula of variance is written as:

$$\sigma^2 = \frac{\Sigma(\mathrm{R} - \overline{R})}{N}$$

Where:

- R is the actual return
- $\overline{R}$ is the expected return
- N is the

Standard deviation could be found by calculating the square root of the variance:

$$\mathrm{SD}=\sqrt{\sigma^2}$$

Covariance and Correlation

In statistics these terms measure how the expected returns of two different stocks are related. Covariance shows the relationship between the returns on two assets in terms of direction. If the value of covariance is positive, this indicates a positive dependency, which means both of the return of the two securities are above or below its average at the same time. If the return of the one security is above its average value, and the other one's value is below, this shows negative dependency, and will result in a negative value of covariance. The value of the covariance will be zero if there is no relationship between the two returns.

The formula of covariance is written as:

$$\sigma_{A,B} = \text{Cov}\ (R_A, R_B) = \text{Expected value of } [(R_A - \overline{R}_A) \times (R_B - \overline{R}_B)]$$

Where:

- σ_{AB} is the covariance of the two securities A and B
- $\overline{R}_A$, $\overline{R}_B$ are the expected returns of each security
- R_A, R_B are the actual returns of each security

Correlation is an important concept in finance and investment industries and shows how two securities move in with respect to one another. It also indicates how weak or strong the relationship is between the returns. Correlation is calculated as the correlation coefficient, and has a value in the interval of -1.0 and +1.0.

$$\rho_{AB} = \text{Corr}\ (R_A, R_B) = \frac{\text{Cov}\ (R_A, R_B)}{\sigma_A \times \sigma_B}$$

Where:

- Cov (R_A, R_B) is the covariance of the two securities A and B
- σ_A and σ_B are the variances of each security

The variance of a portfolio.

The formula for the variance of a portfolio that contains two securities can be expressed as:

$$\sigma^2_p = X^2_A\ \sigma^2_A + 2\ X_A\ X_B\ \sigma_{A,B} + X^2_B\ \sigma^2_B$$

Where:

- σ^2_p is the variance of the portfolio containing the securities A and B
- X_A, X_B are the corresponding weight of the security
- σ^2_A and σ^2_B are the variances of the two securities
- σ_{AB} is the covariance of the two securities

The formula shows that the variance of a portfolio is depends on the variances of each security in the portfolio and the covariance of them. The variance of a security indicates the spread of the return of that particular security, and the covariance will show the relationship between them. If the value of the covariance turns out positive, then this will increase the value of the variance of a portfolio. If the covariance is negative, then the variance of a portfolio will be decreased. This means that if the securities move in the opposite directions, they are compensating each other, which in financial terms is called hedging. Otherwise if the securities are moving in the same direction, then there is no hedging, and the risk is higher.

The standard deviation of the portfolio's return

The standard deviation of a could be found by calculating the square root of the variance:

$$SD=\sqrt{\sigma^2{}_p}$$

Diversification effect

Diversification occurs when the correlation of securities falls below 1, which means securities are not perfectly correlated, then the risk is reduced.

Efficient frontier

The graph of efficient frontier represents the relationship between the return and variance of a portfolio. The curve above the point "Minimum variance portfolio" plots the maximum possible return for a certain risk level. The minimum variance portfolio point is where the variance is lowest, thus the risk is the least.

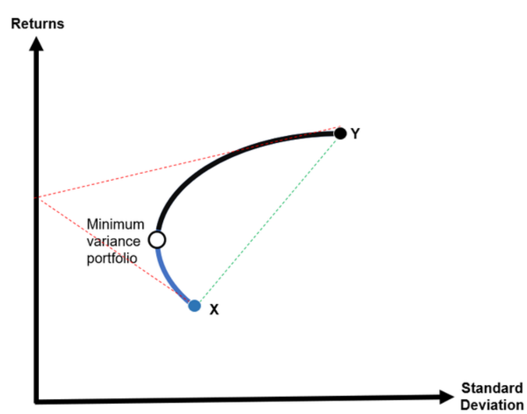

Figure 1.1. The graph of efficient frontier

Diversification

Diversification is the selection of a portfolio containing different classes of assets in order to reduce risk. Also, securities from the foreign market could be selected, as they are less likely to have much correlation with domestic securities.

Systematic and unsystematic risk

Systematic risk, also called undiversifiable risk, volatility, market risk or aggregate risk, affects the whole market or the market segment. It is not possible to foresee or totally avoid this type of risk. Systematic risk depends on important changes in politics, inflation, recession, change of interest rates, catastrophes, etc.

The beta of a security can give information about the relationship between that security and the systematic risk of market. If the value of beta is less than 1, this suggests that the systematic risk of the security is less than the that of the market. If the value of beta is more than 1, this shows that that the systematic risk of the security is less than systematic risk of the market.

The formula of beta is expressed as:

$$\beta_M = \frac{Cov\ (R_i, R_M)}{\sigma^2(R_M)}$$

Where:

- $Cov\ (R_i, R_M)$ is the covariance of between the return on Asset i and the return on the market portfolio
- $\sigma^2(R_M)$ is the variance of the market

Unsystematic risk, however, corresponds to a particular security and it is possible to mitigate with diversification. It is associated with internal factors in the activities of enterprises issuing these securities. Non-systematic risks of two different enterprises are not related to each other and can occur independently of each other.

1.2. The Capital Assets Pricing Model

The capital assets pricing model (CAPM) is a model that describes the relationship between the expected return of a portfolio of securities and the degree of its risk. It was proposed by the financial economist William Sharpe, in his book "*Portfolio Theory and Capital Markets"*(1970). The formula to demonstrate the relationship between risk and expected return can be expressed as:

$$\overline{R} = R_F + \beta\ (\overline{R}_m - R_F)$$

Where:

- $\overline{R}$ is the expected return
- R_F is the risk-free rate
- β is Beta of the security
- $\overline{R}_m - R_F$ is the risk premium

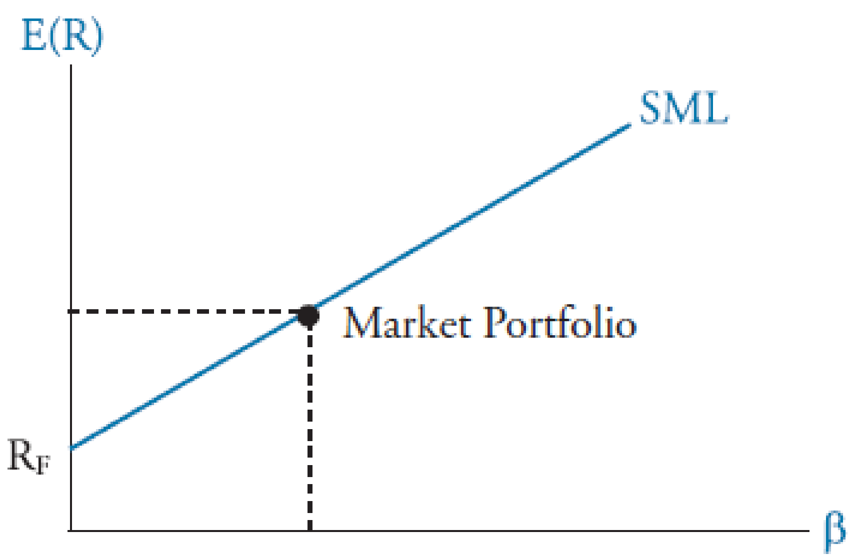

Figure 1.2. The security market line

The security market line is the graphical expression the above equation. The intercept is the point where beta is zero, the point where there is no systematic risk. It represents the asset that is the only one which is not correlated to the market fluctuations, and promises a certain return.

The CAPM is based on the following assumptions:

- all investors are risk averse, with the same risk level, they choose securities with higher returns and with the same level of return, they choose securities with lower risk.
- all assets in the capital market can be completely subdivided. All assets are fully liquid, marketable and decentralized.
- Market participants can borrow and lend unlimited amounts at risk free rate
- all investors have the same thoughts on probability distribution of the returns on assets, there is only one efficiency boundary in the market. The same expected value for the expected rate of return, the standard deviation, the covariance between securities are the same for all
- all investors can get information on market in time and free of charge.
- there is no inflation, discount rate stays same, there is no tax and transaction costs when buying and selling securities.
- Individual investors cannot affect market prices through buying and selling, they are all price takers

1.3. **The arbitrage pricing theory**

The arbitrage pricing theory was proposed by Ross S. in 1976, in the "Journal of Economic Theory."

The APT model is based on the fact that the investor seeks to increase the return on his portfolio without increasing risk whenever the opportunity arises. Investors use the principle of arbitrage, that is, they strive to increase the profitability of their portfolio without increasing risk every time there is an opportunity to obtain a risk-free profit by using different prices for the same securities or other assets.

The arbitrage pricing model, unlike the CAPM, makes it possible to include any number of risk factors. So the required rate of return can be a function of three or more factors.

The main assumptions of the APT model are that the profitability of each share depends, on the one hand, on general macroeconomic factors, and on the other, on internal factors (hindrances) in the company's activities.

The profitability of an individual share in the theory of arbitrage pricing is calculated using the formula:

$$\overline{R}_i = R_F + \beta_n \, (\overline{R}_i - R_F)$$

Where:

- $\overline{R}_i$ is the expected return of asset i
- R_F is the risk-free rate
- β_n is sensitivity of asset to the macroeconomic factor n
- $\overline{R}_i - R_F$ is the risk premium

To use the APT model, it is necessary to determine the list of macroeconomic factors, then estimate the expected risk premium for each of the factors and determine the sensitivity of each stock to these factors. If the actual value of profitability does not coincide with the calculated one, the more profitable asset will be bought, and its price will rise; the less profitable asset will be sold and its price will fall.

Arbitrage pricing theory is also based on a number of assumptions that critically affect its practical applicability and the possibility of a qualitative forecast of market movements. Here are the main ones:

- Since the main goal of an investor is to maximize profit, he owns such a number of shares that the unsystematic risk becomes negligible.
- Markets are frictionless (no barriers due to transaction costs taxes, or lack of access to short selling).
- There are no arbitrage opportunities and if they are uncovered, they will be quickly exploited by investors

2. Analysis of Capital Asset Pricing Model

Empirical tests of the model revealed a large number of weaknesses, such as inaccurate estimates of the beta coefficient. Therefore, some economists began to deviate from the basic concept. Blume (Blume 1970), Black, Jensen and Scholes (Black, Jensen, Scholes 1972) applied the model not to individual securities but to a portfolio. This approach turned out to be very successful.

Jensen (1968) presented a time series test that rejected the classic Sharp-Lintner-Mossin version of the CAPM. Jensen modified the classical model to include the so-called Jensen's alpha, a constant. Jensen's alpha also called "Jensen's return index" or "Jensen's coefficient" is used to evaluate the abnormal return of a security or portfolio of assets in relation to the expected market returns. The latter is calculated based on the capital asset pricing model (CAPM).

The higher the alpha, the greater the portfolio's return relative to the expected return.

This indicator was first used by Michael Jensen in 1968. It was originally used to assess the performance of mutual fund managers, to assess how well managers can obtain above market returns over time. It turned out that the constant is significant and even more than the risk-free rate. This direction of testing the CAPM model is one of the main and most popular. Studies by Miller and Scholes (Miller, Scholes 1972), Blume and Friend (Blume, Friend 1973) and Fama and French (Fama, French 1992) arrive at similar results as Jensen.

Another area of testing the model was beta testing. It was tested whether this ratio can explain one or another level of expected returns (Fama, MacBeth 1973).

However, Roll (Roll 1977) argued in his work that all tests of the model are not reliable and never will be, because the proxies used in the tests are not complete, they cannot be designed to include absolutely everything. assets. Continuing the criticism of Roll, Fama and French (Fama, French 2004) consider the impossibility of testing the model and overly simplistic assumptions as a possible reason for this empirical "failure". Another problem, according to the authors, is that some definitions are misinterpreted, such as the market portfolio

However, this did not stop the researchers from continuing to test the CAPM modifications. For example, Gibbons (1982) rejects both Black's CAPM and the classical version of the model. However, in the same year, Stambaugh (1982) comes up with opposite conclusions.

Subsequently, a large number of studies of conditional CAPM appeared. For the first time, the ability of conditional CAPM to explain differences in average returns on a large number of portfolios was tested in (Jagannathan, Wang 1996). They concluded that the conditional CAPM is quite capable of dealing with market inefficiencies. The same conclusions were reached (Lettau, Ludvigson 2001; Gomes et al. 2003; Petkova, Zhang 2005). On the other hand, in (Lewellen, Nagel 2006), the authors conclude that conditional CAPM is unable to explain specific market anomalies, but more importantly, conditional constants are large and significant, which rejects the applicability of conditional CAPM.

The Estrada model also received widespread publicity among researchers. So in the articles (Post, Vliet 2004; Estrada 2006; Estrada 2007) shows that the one-sided approach to estimating the beta coefficient is more preferable than the classical one. Likewise, (Abbas et al 2011) argues that DCAPM is the solution to the problem of asset pricing.

Over time, models with higher moments were tested as a measure of risk. In particular, a model with skewness (Harvey, Siddique 2000) and kurtosis (Dittmar 2002) was tested in the American stock market. The authors conclude that such models are more suitable for the American market than the classic one.

Conditional CAPM tests using multivariate GARCH models allow us to take into account the fact that conditional covariance varies greatly over time, as a result, and the beta coefficient also changes over time.

The first work devoted to testing CAPMs using the multivariate GARCH model is considered to be (Bollerslev, Engle, Wooldridge 1988). It is the first to use the multidimensional diagonal VEC GARCH model. The authors test the CAPM for quarterly yields of 6-month US Treasury bills, 20-year US Treasury bonds, and stocks for the period from Q1 1959 to Q2 1989. US Treasury bills. They conclude that the conditional covariance model performs better than the unconditional covariance model. The authors conclude that CAPM is rejected because the lagged dependent variable added to the model is significant, i.e. the classic CAPM is unable to fully predict excess returns.

A slightly different methodology was used in (Ng 1991). It tested CAPM not on individual securities, but on a portfolio. Various portfolio construction techniques were applied. The portfolios consisted of all securities traded on the New York Stock Exchange from January 1926 to December 1987. Monthly returns were used. Both the classic Sharpe-Lintner-Mossin CAPM and Black's CAPM without a risk-free asset were tested. The author concludes that the testing results depend on the portfolio construction methodology. Thus, when testing the CAPM on portfolios sorted by size, it is concluded that the models are rejected. When testing the model on beta sorted portfolios, the CAPM is not rejected.

Similar studies have become very popular and have been carried out in other developed markets as well. In particular, the article (Hansson, Hordahl 1998) conducted tests of conditional CAPM in the Swedish stock market for the period from 1977 to 1990. The authors used a variety of portfolio construction techniques and monthly returns. They conclude that the conditional CAPM cannot be rejected in favor of other modifications (for example, the authors used zero-beta CAPM as an alternative model).

The next round of CAPM testing using multivariate GARCH models was the transition from developed to developing markets. For example, (Godeiro 2013) tests a conditional CAPM in the Brazilian stock market using the multivariate VCC GARCH. The daily returns of stocks included in the Brazilian market index for the period from January 1, 1995 to March 20, 2012 are used. The author does not use a portfolio, but tests the CAPM for each stock separately. He concludes that the conditional CAPM is being rejected.

2.1. **Methodologies for empirical analysis in CAPM**

Most of the empirical test conducted on the capital asset pricing model were by the use of time series test. After estimating the betas this way, using the cross-sectional regression, the hypothesis was tested. Below are listed different studies with different methodologies applied to test the model, and the discussion of the methodology.

Lintner and Douglas studies

In 1965 John Lintner carried out the first empirical test of CAPM. Later Douglas in 1967 replicated the study with results similar to Linter's original test. Lintner applied a two-pass procedure for individual stocks. The data he used corresponds the decade covering 1954 and 1963 with one measurement for each year. He examined the returns of 301 companies listed on the New York Stock Exchange and used the predecessor of the S&P 500 Index as proxy for the market portfolio. To estimate the beta of each security, Lintner used the following regression. In this regression, the annual returns of each security are regressed against the returns of the market index.

$$R_{it} = \alpha_i + \beta_i * R_{mt} + \varepsilon_{it} \qquad (1)$$

Where:

β_i is the beta of the asset, R_{it} is the return on the asset at time t, R_{mt} is the return on the market at time t, ε_{it} is the error term and α_i is the intercept.

After the estimation of the betas, the second pass cross sectional regression is carried out.

$$R_i = y_0 + y_1 \beta_1 + y_2 \sigma^2(\varepsilon_i) + \varepsilon_i \qquad (2)$$

Where y_1 is the slope of the beta of the asset, R_i is the return on the asset i, y_2 is the slope of the residual variance and ε_i is the error term and y_0 is the intercept.

If the results were in compliance with CAPM, the coefficients have to be: $y_0 = 0$, $y_1 = R_i - R_i$ and $y_2 = 0$. But the results do not comply with the CAPM, the coefficient for the intercept, y_0, was larger than risk-free rate, the coefficient for the residual risk, y_2, was significantly different from zero and slope of the SML, y_1 was much different from the market risk premium.

Miller & Scholes study

In 1972 Miller and Scholes also performed a study on CAPM using the two-pass method also coming up with very similar results. It turns out that, the slope was too flat and crucially different from the market risk premium. The intercept and residual variance were too much different from zero.

To investigate the statistical problems of the two-pass method, Miller and Scholes arranged numbers and ran a simulation test. According to the CAPM model, they generated numbers that exactly corresponded to the expected beta ratio of the rate of return. That is, the simulated returns are created to exactly match the CAPM. However, the CAPM using the two-pass method was discarded because the results were similar to those generated with the actual data. This clearly proves that there are some statistical problems with the two-pass methodology. They have shown that the model may be valid even if the CAPM is rejected using this method. They further argue that the methodology used to test the model continues to result in model rejection and is the result of errors in the beta measurement of the second cross-sectional regression.

Due to statistical problems, it is clear why Miller and Scholes conclude that there is a problem with the two-pass method. Regression equation (1) estimates the coefficients at the same time, and these estimates are interdependent. When estimating the intercept of a single variable, the regression relies on the estimation of the slope coefficient.

In summary, if the beta estimates are distorted, so are the section estimates. Miller and Scholes conducted a CAPM study using the procedure twice and obtained very similar results. However, the slope is too flat and deviates significantly from the market risk premium. The intercept and residual variance are not statistically significant.

To test the statistical problems of the two-pass procedure, Miller and Scholes performed a simulation test by generating random numbers. Following the CAPM model, they identified numbers that exactly matched the expected return beta relationship. That is, the simulated rate of return has been set to fully agree with the CAPM. However, the results were almost identical to those produced with real data, so the CAPM was rejected using the two-pass technique. Obviously, this demonstrates that there are some statistical problems with the two-pass method. They demonstrated that even if CAPM is rejected using this approach, the model can still be valid. They further assert that the methodology used to test the model and result in the model being rejected is a consequence of the error in the beta measurement for the second cross-sectional regression.

Due to statistical problems, Miller and Scholes apparently concluded that the two-pass procedure was problematic. The regression equation estimates the coefficients simultaneously, and the estimates depend on each other. When estimating the intersection of a single variable, the regression depends on the estimate of the slope. As a conclusion, if the beta estimate is biased, so is the intercept estimate.

They show that the statistical problems caused by taking into account the beta from the first pass regression estimated with significant sampling error must be addressed. Therefore, the estimated beta is not a proper input for the second pass regression, but provides misleading results and ultimately does not test the validity of the model. By using the beta estimated by the measurement error, the regression in the second run causes the slope y_1 to be the downward factor and the y-intercept y_0 to be the upward factor. Empirical testing of the above model shows that this is exactly the case. That is, the intercept is higher than predicted by the CAPM results and the gradient is much flatter than predicted by the model.

Fluctuations in beta estimates are also caused by the fact that other major but unmeasured causes of common stock volatility are at work. Bias in the model specifications misrepresents beta estimates because the market model does not take into account other significant systematic impacts on stock market volatility.

Black, Jensen & Scholes study

The rejection of the CAPM in the initial verification tests above does not necessarily mean that the model is defective. It may be due to a statistical error in the beta measurement. To avoid measurement errors that effectively distort SML estimates, Black, Jensen, and Scholes provided additional insights into the nature and structure of security returns. Instead of testing individual stocks, they suggested ways to improve the accuracy of the estimated beta by grouping the stocks into a portfolio.

In their article, in previous tests of the model, the structure of the process, where the data appears to be generated, leads to a cross-section test of misleading importance and does not provide a direct test of the effectiveness of the CAPM. Aggregation makes portfolio beta estimates less susceptible to measurement errors than individual securities beta estimates. This reduces the statistical error that can occur in beta factor estimates. They examined all New York Stock Exchange securities from 1926 to 1965 using an evenly weighted portfolio of all New York Stock Exchange stocks as a substitute for the market index.

The outline of their study began with estimating the beta of each securities by using equation (1) to regress monthly returns against the market index for the first 60 months of the period. Then, by sorting the securities according to the calculated beta, 10 portfolios were formed. Portfolio 1 consisted of the highest beta stocks and Portfolio 2 consisted of the next highest beta stocks. By doing this, they found that the portfolio had a huge expansion in beta.

However, the beta used to select the portfolio is still subject to measurement error, so building a portfolio of securities based on the estimated beta gives an unbiased estimate of the portfolio beta. To solve this problem, they built a portfolio for the following year using betas of individual securities estimated in the previous period. In this way, many of the sample fluctuations in the estimated beta of individual securities can be eliminated.

The next step is to calculate the monthly returns for each portfolio for the following year. This step is then repeated once a year for the entire sampling period. They then calculated the average monthly return and estimated beta factor for each of the 10 portfolios. Finally, they used equation (2) to regress the average portfolio return against the portfolio beta for the entire sample period and the various sub-periods. Evidence from their empirical analysis led to the rejection of traditional forms of CAPM because the duration of interception was higher than the risk-free rate. However, the SML gradient term was significant and was positively linear as predicted by the model, so the results appeared to be consistent with zero-beta CAPM. In general, the results were similar for the entire and partial test period.

Sharpe and Cooper study

In 1972 Sharpe and Cooper performed a simple test of CAPM in the form of a simulated portfolio strategy. From 1931 to 1967, stocks were investigated and tested on the New York Stock Exchange. The original purpose was to determine if the higher the beta stock, the higher the return. Portfolios constructed in different betas by first regressing individual asset returns with market returns based on the last 5 years using equation (i). Once a year, they evaluated all strains according to beta and divided them into 10 categories. They formed portfolios that were evenly weighted by category, stocks with the highest beta in one portfolio, stocks with the second highest in another portfolio, and so on. The investment strategy followed was to hold only one category of securities for the entire period. The results of this study show that high beta stocks produce higher returns. They found that 95% of the fluctuations in expected return could be explained by the difference in beta, so there was a linear relationship between realized average return and that beta. However, the intercept, or risk-free rate, is 5.54%, which is significantly higher than the 2% for that period, so it does not match the Sharpe Lintner version of the model. However, these results support zero beta CAPM.

Fama & MacBeth

Fama and MacBeth have followed a similar methodology to Black to test the CAPM, but add another explanatory variable, the square of the beta coefficient, to the equation to test that there is no non-linearity in the relationship between risk and reward. bottom. The importance of linear conditions was largely overlooked in early empirical testing of the model. Another difference from the BJS study is that Fama and MacBeth use one period beta to predict returns for later periods, while the BJS method calculates beta and average returns for the same period. is.

To test the effectiveness of CAPM the following model was used:

$$R_{pt}=y_{0t}+y_{1t}B_p+y_{2t}B_i^2+y_{3t}\sigma^2+\eta_{pt} \qquad (3)$$

Index p is related to the constructed portfolio, but not to individual stocks. Using equation (3), Fama and MacBeth tested some hypotheses about CAPM. They tested the expected value of the risk premium y1t, that is, whether the slope of SML is positive, that is, E (y_{1t}) = E (R_{mt}) - rf)> 0. To test the linearity, the beta-square variable of the equation, the hypothesis tested under this condition is E (y_{2t}) = 0. Hypotheses are tested in connection with testing non-beta risks, E (y_{3t})=0. We also test that the intercept is equal to the risk-free rate, that is, E (y_{0t}) = rf. The mean of the disturbance term η is zero and is assumed to be independent of all other variables in the equation. Like Jensen, Black & Scholes, Fama and MacBeth find that the intercept is larger than the risk-free rate and reject the Sharpe Lintner CAPM. However, the results of their research confirmed the important testable implications of zero-beta CAPM. The SML slope is positive because the coefficient y_1 is significantly different from zero, which indicates a positive risk-return trade-off. The coefficients y_2 and y_3 were not significantly different from zero. There is no evidence of non-linearity or residual variance affecting the return. Finally, the observed fair game characteristics of risk-return regression coefficients and residuals are consistent with an efficient capital market where prices perfectly reflect all the information available.

Modigliani study

Empirical testing and research focuses primarily on US-listed stocks. In essence, the literature is not very extensive when it comes to the European market. The reason for this could be the availability of data and the increased efficiency of the US stock market. In 1972 Franco Modigliani conducted the first test of the CAPM on the European stock exchange. Prior to his research, CAPM was not represented in capital markets other than the US market.

The tests were conducted in eight major European markets and reproduced previous tests in the US market (Black, Jensen & Scholes (1972), Friend & Blume (1970), Jacob (1971)). Modigliani found that the European market is generally considered less efficient than the US market. If so, that would mean that the risk assessment of European securities is less rational than that of American securities.

For the test, the data to be used included the daily prices and dividend data for 234 common shares corresponding to eight major European countries covering the period of March 1966 to March 1971. Data have been revised for all capital adjustments. Since the data are from eight countries, the stocks were analyzed individually so that the regression results are displayed individually for each country. The results of the US market were used for similar periods for comparison.

The results of the study supported the hypothesis by showing that systematic risk is an important factor in the pricing of European securities. In addition, there was a positive correlation between realized returns and risk for 7 of the 8 markets tested. Moreover, there were no signs that the European stock market was unreasonable or inefficient. Nevertheless, the test period was as short as 5 years and the samples were limited.

Roll´s critique

Richard Roll criticized previous work on the capital asset pricing model. He suggested that the methodology was not accurate because the tests were only performed using market portfolio proxies, not the actual market portfolio.

Roll stated that the only verifiable hypothesis related to CAPM is that the market portfolio has mean-variance efficiency, and all other suggestions of the model, such as the linear relationship between returns and beta, are due to the efficiency of the market portfolio. He pointed out that this is the reason why it cannot be tested independently.

Roll argued that the real estate market portfolio contained all assets, including human capital and real estate. He also points out that using only one proxy in the market portfolio is not enough to state or reject whether CAPM works. He further argued that it was impossible to define a real market portfolio and therefore could not seriously test the CAPM.

Roll & Ross and Kandel & Stambaugh study

Roll, Ross, Kandel, and Stambaugh performed further study on Roll's critique. They say that the refusal of the risk-return relationship specified by CAPM does not refute the model relationship between average returns and beta, but because the market portfolio proxies are mean-variance

inefficient. Insisted that there was a possibility. They show that market portfolio proxies, which are not significantly inefficient and should produce a positive linear risk-return relationship in large samples, cannot establish a significant relationship. Kandel and Stabaugh explain how many people don't see the meaning of CAPM separately because one implies the other.
The CAPM suggestions could be seen in two forecasts:

- the market portfolio is efficient
- the security market line (the expected relationship of return and beta) is capable of demonstrating the risk-return trade-off in an accurate way, in other words, alpha values equal to zero.

Their article further shows that one implication is close to perfection and the other implication can fail dramatically. They tested Schwarz's Zero Beta CAPM using a two-pass method with a market portfolio proxy, as an efficient portfolio is only theoretical.
After performing the first-pass time series regression (1), the following generalized least-squares the second-pass regression was performed. In this way, they took into account the correlation between the residuals.

$$r_i - r_z = y_0 + y_1 * (Estimated\ \beta i)$$

Where r_i is return on stock i, r_z is, y_0 is the intercept, y_1 is the beta and βi is the estimated beta. Their conclusion is that terms proportional to the relative efficiency of the selected proxy that represents the true market portfolio is the reason why the intercept and slope coefficients are biased. If the market index used in the regression is perfectly efficient, the test is well specified. However, if the proxy is inefficient for the market portfolio, the second path regression provides a bad test for the CAPM. Therefore, without a reasonably efficient market proxy, the model cannot be meaningfully tested.

David W. Marines

In 1982, David W. Mullins made a complete and detailed evaluation on the effectiveness of CAPM. He found the model incomplete, but said it was an important tool for investors in determining the return on total assets needed. In addition, he suggests that even if the model assumptions are theoretical and unrealistic, it is important to simplify the reality in order to build a comprehensive model for determining asset prices.

Clare, Smith & Thomas study

In 1997, Clare, Smith, and Thomas tested both conditional and unconditional versions of CAPM on the UK Stock Exchange. Their dissertation creates an important link between formal asset pricing and evidence of predictability of excess returns. They use both market value and dividend yield ranked portfolios to find the right risk return spread. The results show that beta plays an important role in explaining expected return.

Pettengill, Sundaram & Mathur study

In 1995 Pettengill, Sundaram & Mathur found a consistent and very important relationship between beta and portfolio returns. They make a significant distinction between their tests and previous empirical tests, and as predicted by the Sharpe-Lintner-Black model, and they realize that the positive relationship between returns and beta is based on expected returns rather than realized returns. They consider the impact of using the realized market revenue as a substitute for the expected market revenue. If the excess market return is negative, there should be an inverse relationship between beta and portfolio returns. Balancing expectations for negative excess market returns finds a consistent and very important relationship between beta and portfolio returns across sample and sub-periods. There is also a positive risk-return trade-off.

Fama & French study (1992)

Fama & French wanted to investigate, in addition to beta, factors that could affect stock returns, such as book-to-market capital and size. The data they used are from all non-financial companies in NYSE, AMEX, NASDAQ, and annual industry files containing income statements and balance sheet data from 1962 to 1989. They had values for required accounting variables, so they were able to estimate the beta of the portfolio and appropriate that beta to each stock in the portfolio. This way, its possible to use individual stocks in Fama-MacBeth regression. Prior to 1969, it's recognized that there was a positive relationship between beta and average returns (Black, Jensen & Scholes, 1972; Fama & MacBeth, 1973). However, their results from 1969 to 1990 suggest that the relationship between average returns and beta does not exist. In addition, they found a weak beta-return relationship for the 50 years from 1941 to 1990. In conclusion, their tests did not support the prediction of the basic Sharpe-Lintner-Black model.

Fama & French study suggests that poor beta results may be due to other explanatory variables that correlate with the true beta. This obscures the relationship between average return and estimated beta. However, the beta alone does not explain why it seems to have little explanatory power in each period.

The results of Fama & French also show that the book-to-market equity-to-revenue-to-price ratio is an inadequate alternative to beta. However, their key findings conclude that the easily measurable variables size and book-to-market fairness appear to explain the cross-section of average returns.

The summary of the results are as follows:

- If there is variation in beta that is not related to the size, the relation between beta and average return is not reliable.
- The opposing roles of market leverage and book leverage in average returns are precisely described by book-to-market equity.
- The relation between E/P and average return is balanced with the help of the connection of size and book-to-market equity.

Fama & French study (2004)

Additional research on CAPM includes another article by Fama and French covering the latest related research on CAPM. They state that the CAPM presented by Sharp and Lintner has never had empirical success, but the Black version has produced some positive results.

However, as the survey began to include variables such as size, various pricing ratios, and dynamics, it also helps explain the average revenue that the beta provides. The model results faced a problem that Fama & French said was sufficient to explain most invalidations. CAPM application.

The three-factor model presented by Fama & French adds market characteristics and size to the official book and forms the three factors together with the beta. With this model, you can achieve up to 90% of diversified portfolio returns. These results are supported since Fama & French. In this study the focus will be on Pharma & Macbeth's previous approach, which does not take into account non-beta factors.

2.2. **Analysis of data and explication of the model through decades**

The Capital Asset Pricing Model (CAPM) test does not directly test the CAPM assumptions, but the properties of the stock market line are tested. Which means, it tests if there is a positive correlation between the security or portfolio beta factor and the expected return.
Since the capital asset pricing model briefly describes the relationship between the returns and risks of different assets in the capital market, the capital asset pricing model is naturally very important if it can describe the actual situation of the capital market well. For this reason, many empirical tests have been conducted on asset pricing models.
Below are some empirical tests conducted on the capital asset pricing model and the analysis of data and results.

The Black, Jensen, and Scholes study
To conduct the study, all stocks on the NYSE covering the interval 1926 – 1965 were examined and as proxy for the market index, equally weighted portfolio of all stocks were taken.
The study was carried out based on the following steps:

1. Monthly returns of the first 60 month period, 1926-1930 is used for the estimation of Beta of each stock.
2. 10 portfolios were created by ranking the betas: top 10 portfolios named portfolio no. 1, the successive portfolios named by increasing order and the last and no. 10 containing the stocks with the smallest betas.
3. Each portfolios returns for 12 months in 1931 were calculated.
4. Steps 1 to 3 were applied to all 10 portfolios for throughout the interval 1926-1965, each time taking 5 year interval and estimating the beta and the return of the stock in the successive year.
5. The mean portfolio returns were regressed against the portfolio betas throughout the period 1926-1965, to estimate the ex post Security Market Line.

The results of this study rejects the traditional version of the CAPM, because the intercept turns out to be greater than risk free rate. But it is consistent with zero beta CAPM.

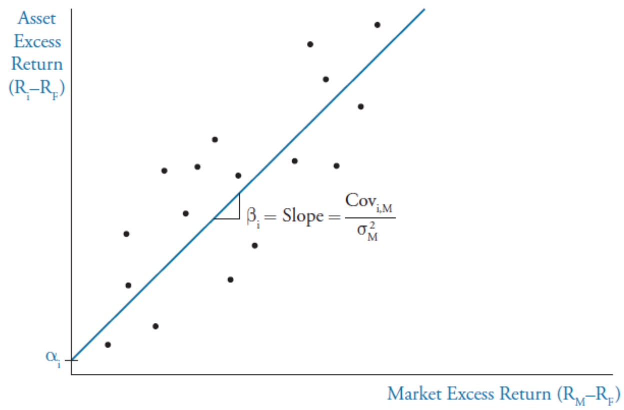

Figure 2.1. Estimating beta with regression

Equilibrium models for the pricing of capital assets has always been focused on by many scholars throughout the years. The most prominent equilibrium model is the mean variance model of Sharpe initiated in 1964. This model was in the following years advanced and explicated by the scholars Lintner, Mossin, Fama, Long. Later, Treynor, Sharpe, Jensen came up with models which would be used for also portfolio evaluation. These asset pricing models were built on or extended from the mean variance model of Sharpe. The idea behind the model demonstrates the connection between the expected value of risk premium and the systematic risk. This relation could be expressed such:

$$E\,(\tilde{R}_j) = \gamma_1 \beta_j \qquad (1)$$

Where:

$E\,(\tilde{R}_j) = \frac{E\,(\tilde{P}t) - Pt-1 - E\,(\tilde{D}t)}{Pt-1}$ - r_{Ft} is expected excess returns on the *j*th asset

$\gamma_1 = E\,(\tilde{R}_M)$

$\tilde{D}t$ is dividends paid on the *j*th security at time *t*

r_{Ft} is riskless rate of interest

$E\,(\tilde{R}_M)$ is expected excess returns on a "market portfolio" consisting of an investment in every asset outstanding in proportion to its value

β_j is systematic risk of the *j*th asset

It can be seen from relation (1) that the expected return of an asset is directly proportionate with its beta. Expressing α_j in a way that:

$$\alpha_j = E(\tilde{R}_j) - E(\tilde{R}_M)\,\beta_j$$

leads to the result that α of every asset is equal to zero according to relation (1).

If relation (1) proves to be true in practice, it can eventually lead to the simplification of analysis in many economic matters including portfolio selection, capital budgeting, cost benefit analysis or many other issues that require the connection of risk and return. The study carried out by Jensen (1968; 1969) about the connection of expected return and risk return of mutual funds demonstrates that there could be a rational explanation of the relation between risk and return of assets. But the studies carried out by Douglas (1969), Lintner (1965), later by Miller and Scholes (1972) states otherwise, that the model is unable to explain this connection. The study conducted by Miller and Scholes describes that the α of a security is dependent on the β of that security in a systematically. In other words securities with higher systematic risk are inclined to show negative alpha, and on the other hand, securities with lower systematic risk are inclined to show negative alpha.

In the study of Jensen, Miller, Scholes (1972), the main objective is to perform further tests of the capital asset pricing model and eliminating the errors in the earlier tests carried out. The tests that were performed before were based on the cross-sectional procedure, which is the regression of the average return of the group of securities over a specific time period on the approximation of the systematic risk of each security. The equation is expressed as:

$$\overline{R}_j = \gamma_0 + \gamma_1\hat{\beta}_j + \tilde{u}_j$$

Where:

$\overline{R}_j$ is the mean excess return

$\hat{\beta}_j$ is approximation of the systematic risk

This lead to the results showing γ_0 is sufficiently different from 0, γ_1 is sufficiently different from $\overline{R}_M$, which is the slope demonstrated in the model.

In the study certain aspects will be discussed including: it is shown that the cross-sectional tests may result with errors because of the organization of the procedure that creates the data, the reliability of the model is tested using methods that have been able to eliminate the complications related to cross-sectional tests. The results obtained demonstrate that the (1) version of the asset pricing model does not lead to accurate security returns. It is shown that expected excess returns on assets with higher beta prove to be lower than that proposed by (1) and expected excess returns on assets with lower beta prove to be higher than that proposed by (1).

The alternative hypothesis of capital assets pricing arises from the relaxation of one of the traditional forms of assumptions in the capital goods pricing model. Relaxing the assumption that risk-free lending and borrowing opportunities exist leads to the formulation of a two-factor model. In equilibrium, the expected return E ($\tilde{r}_j$) of an asset is given by:

$$E\ (\tilde{R}_j) = E\ (\tilde{r}_z) + [E\ (\tilde{r}_M) - E\ (\tilde{r}_Z)\ ]\ \beta_j \quad (2)$$

Where:

r's indicate total returns

E ($\tilde{r}_z$) is the expected return on a portfolio with zero covariance (β_z=0)

$\tilde{r}_M$ is the return on the market portfolio.

In this model, the expected return on a 30-day Treasury bill of return (used as a substitute for a "risk-free" rate) only represents the return on a particular asset in the system. Therefore, by subtracting $\tilde{r}_F$ from both sides of (2), (2) can be rewritten as follows in terms of "excess" returns:

$$E\ (\tilde{R}_j)= \gamma_0 + \gamma_1 \beta_j \quad (3)$$

Where:

$\gamma_0 = E\ (\tilde{R}_z)$

$\gamma_1 = E\ (\tilde{R}_M) - E\ (\tilde{R}_z)$

Whereas in the traditional model, $\gamma_0 = 0$ and $\gamma_1 = E\ (\tilde{R}_M)$, the two factor model demonstrates that $\gamma_0 = E\ (\tilde{R}_z)$, and is not equal to zero in all cases, and $\gamma_1 = E\ (\tilde{R}_M) - E\ (\tilde{R}_z)$.

Additionally, several other models have emerged as a result of relaxing some assumptions of traditional asset pricing models, implying $\gamma_0 \neq 0$ and $\gamma_1 \neq E\,(\tilde{R}_M)$,

These models include and briefly account for the problem of R_M measurement, the existence of non-marketable assets, and the existence of differential taxes on capital gains and dividends. Our main focus has been to test the rigorous traditional forms of asset pricing models. That is, $\gamma_0 \neq 0$. It has not been attempted to provide a direct test for these other alternative hypotheses.

To test the traditional model, all securities listed on the New York Stock Exchange between 1926 and 1966 were used. The problem faced was getting an efficient estimate of the mean beta and its variance. An alternative hypothesis can be tested by randomly selecting one security, estimating a beta coefficient based on a time series, and finding out whether the average return differs significantly from that predicted by traditional forms of capital asset pricing models. However, this would not be an efficient testing procedure.

For efficiency reasons, securities were grouped into 10 portfolios so that the portfolio had large spread of β′s. It was known that grouping securities based on estimated β′s would not provide an unbiased estimate of the "beta" portfolio, because the β's used to select the portfolio contains measurement errors. This procedure would lead to selection bias in the tests.

To remove this bias, the previous period's estimated beta was used an instrumental variable, to select a stock portfolio group for the following year. Using these procedures, ten portfolios whose β′s estimate is an unbiased estimate of the "Beta" portfolio was constructed.

It was found that much of the sampling variation of the estimated β′s for individual securities is eliminated using portfolio groups. So the β's of the built-up portfolio varies from 0.49 to 1.5, and the β's portfolio's estimates for the sub-periods shows significant stationarity.

Summary of Statistics for Time Series Tests, Entire Period (January, 1931-December, 1965)
(Sample Size for Each Regression =420)

Item*	1	2	3	4	5	6	7	8	9	10	$\bar{R}_M$
	Portfolio Number										
$\hat{\beta}$	1.5614	1.3838	1.2483	1.1625	1.0572	0.9229	0.8531	0.7534	0.6291	0.4992	1.0000
$\hat{\alpha} \bullet 10^2$	-0.0829	-0.1938	-0.0649	-0.0167	-0.0543	0.0593	0.0462	0.0812	0.1968	0.2012	
$t(\hat{\alpha})$	-0.4274	-1.9935	-0.7597	-0.2468	-0.8869	0.7878	0.7050	1.1837	2.3126	1.8684	
$r(\tilde{R}, \tilde{R}_M)$	0.9625	0.9875	0.9882	0.9914	0.9915	0.9833	0.9851	0.9793	0.9560	0.8981	
$r(\tilde{e}_t, \tilde{e}_{t-1})$	0.0549	-0.0638	0.0366	0.0073	-0.0708	-0.1248	0.1294	0.1041	0.0444	0.0992	
$\sigma(\tilde{e})$	0.0393	0.0197	0.0173	0.0137	0.0124	0.0152	0.0133	0.0139	0.0172	0.0218	
$\bar{R}$	0.0213	0.0177	0.0171	0.0163	0.0145	0.0137	0.0126	0.0115	0.0109	0.0091	0.0142
σ	0.1445	0.1248	0.1126	0.1045	0.0950	0.0836	0.0772	0.0685	0.0586	0.0495	0.0891

* $\bar{R}_M$= average monthly excess returns, σ = standard deviation of the monthly excess returns, r = correlation coefficient.

Table 2.1

Portfolio number 1 includes the highest-risk securities and portfolio number 10 contains the lowest-risk securities. The critical intercepts α are shown in the second row of table 2.1. Below that the student "t" values are placed. The table also includes the correlation of the market and portfolio returns, r($\tilde{R}_M$, $\tilde{R}_K$) and the autocorrelation of the residuals.

The autocorrelation turns out to be small, but the correlation of the market and portfolio returns are high as anticipated. In the table for each portfolio, is demonstrated the standard deviation of the residuals, the average monthly return, the standard deviation of the monthly excess return. Time series regression of portfolio excess returns against market portfolio excess returns has a significantly negative intercepts for high beta securities and a significantly positive intercepts for low beta securities, as opposed to the traditional form of forecasting in the model.

Over the 35 year interval, the predicted amount to be earned by high risk securities was higher than the actual one, and the low risk securities happened to earn more than predicted. There was also important evidence that this effect became stronger over time and was most pronounced around 1947-65.

To check the stationarity of the empirical relations, the 35 year period was divided into 4 equal subperiods each consisting of 105 months.

Summary of Coefficients for the Subperiods

*Item**	*Subperiod†*	*Portfolio Number* 1	2	3	4	5	6	7	8	9	10	M_M
$\hat{\beta}$	1	1.5416	1.3993	1.2620	1.1813	1.0750	0.9197	0.8569	0.7510	0.6222	0.4843	1.0000
	2	1.7157	1.3196	1.1938	1.0861	0.9697	0.9254	0.8114	0.7675	0.6647	0.5626	1.0000
	3	1.5427	1.3598	1.1822	1.1216	1.0474	0.9851	0.9180	0.7714	0.6547	0.4868	1.0000
	4	1.4423	1.2764	1.1818	1.0655	0.9957	0.9248	0.8601	0.7800	0.6614	0.6226	1.0000
$\hat{\alpha} \bullet 10^2$	1	0.7366	0.1902	0.3978	0.1314	-0.0650	-0.0501	-0.2190	-0.3786	-0.2128	-0.0710	
	2	-0.2197	-0.1300	-0.1224	0.0653	-0.0805	0.0914	0.1306	0.0760	0.2685	0.1478	
	3	-0.4614	-0.3994	-0.1189	0.0052	0.0002	-0.0070	0.1266	0.2428	0.3032	0.2035	
	4	-0.4475	-0.2536	-0.2329	-0.0654	0.0840	0.1356	0.1218	0.3257	0.3338	0.3685	
$t(\hat{\alpha})$	1	1.3881	0.6121	1.4037	0.6484	-0.3687	-0.1882	-1.0341	-1.7601	-0.7882	-0.1978	
	2	-0.4256	-0.7605	-0.8719	0.5019	-0.6288	0.8988	1.1377	0.6178	1.7853	0.8377	
	3	-2.9030	-3.6760	-1.5160	0.0742	0.0029	-0.1010	1.8261	3.3768	3.3939	1.9879	
	4	-2.8761	-2.4603	-2.7886	-0.7722	1.1016	1.7937	1.6769	3.8772	3.0651	3.2439	
$\overline{R}$	1	0.0412	0.0326	0.0317	0.0272	0.0230	0.0197	0.0166	0.0127	0.0115	0.0099	0.0220
	2	0.0233	0.0183	0.0165	0.0168	0.0136	0.0147	0.0134	0.0122	0.0126	0.0098	0.0149
	3	0.0126	0.0112	0.0120	0.0126	0.0117	0.0109	0.0115	0.0110	0.0103	0.0075	0.0112
	4	0.0082	0.0082	0.0081	0.0087	0.0096	0.0095	0.0088	0.0101	0.0092	0.0092	0.0088
σ	1	0.2504	0.2243	0.2023	0.1886	0.1715	0.1484	0.1377	0.1211	0.1024	0.0850	0.1587
	2	0.1187	0.0841	0.0758	0.0690	0.0618	0.0586	0.0519	0.0494	0.0441	0.0392	0.0624
	3	0.0581	0.0505	0.0436	0.0413	0.0385	0.0364	0.0340	0.0289	0.0253	0.0203	0.0363
	4	0.0577	0.0503	0.0463	0.0420	0.0391	0.0365	0.0340	0.0312	0.0277	0.0265	0.0386

* $\overline{R}_M$ = average monthly excess returns, σ = standard deviation of the monthly excess returns.

†Subperiod 1 = January, 1931-September, 1939; 2 = October, 1939-June, 1948; 3 =July, 1948-March, 1957; 4 = April, 1957-December, 1965.

Table 2.2.

The table 2.2 has demonstrated the regression statistics with the data relating to each of ten portfolios. It can be seen that apart from portfolio 1 and 10, the risk coefficients β were quite stationary.

On the other hand, the section of critical intercepts were not stationary throughout the period.

The positive indication of α for high risk portfolios in the subperiod 1 (January, 1931-September, 1939) shows that these securities had earned more than the forecast of the model. The negative α of the low risk portfolios show that they earned less than the prediction of the model.

The following 3 subperiods have shown opposite results and have deviated from the prediction of the model even further.

In the cross-sectional test of the model, securities were grouped and then with the time series method the risk measure was estimated for each portfolio. Then the cross-sectional parameters were estimated by portfolio mean return over risk coefficients.

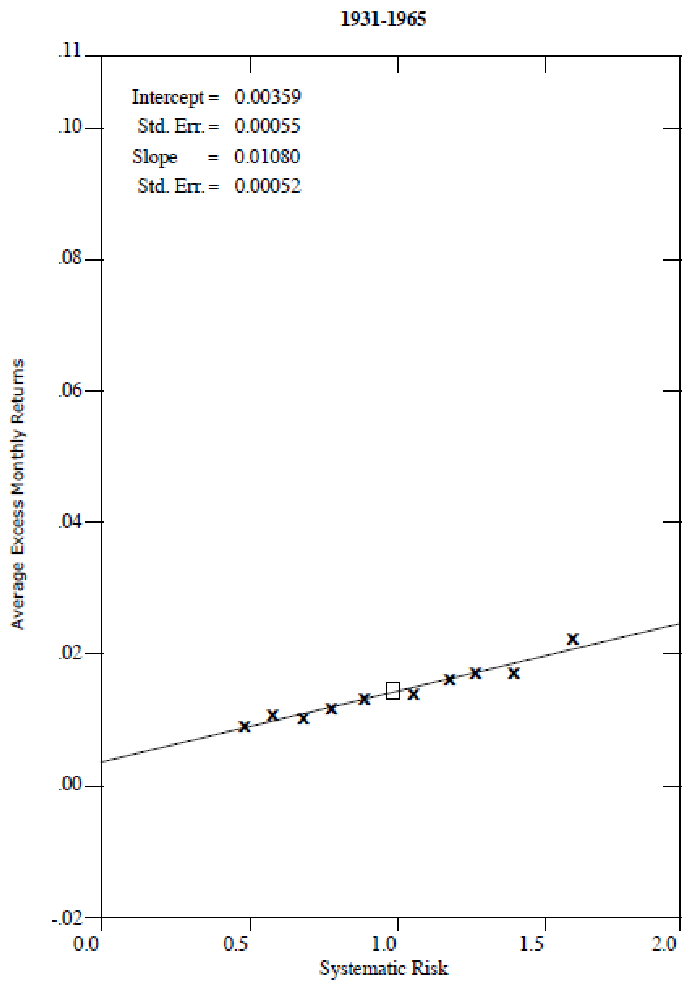

Figure 2.1.

Figure 2.1. plots the mean monthly returns against systematic risk for 35 year interval 1931-1965 for each of 10 portfolios and the market portfolio. The points expressed with the sign x denote average excess return and risk of all the portfolios, whereas the point □ is average excess return and risk of the market portfolio.

The traditional version of the capital asset pricing model demonstrates that intercept should be zero, and the slope is the mean excess return on the market portfolio. Over this interval, theoretical values of the mean excess return on the market portfolio was calculated to be 0.0142. The 35-year interval was divided into four subperiods. The plots of average excess return versus risk of all the portfolios over these subperiods are shown in figure 2.2.

Summary of Cross-sectional Regression Coefficients and Their t Values

	Time Period				
	Total Period	*Subperiods*			
	1/31-12/65	1/31-9/39	10/39-6/48	7/48-3/57	4/57-12/65
$\hat{\gamma}_0$	0.00359	-0.00801	0.00439	0.00777	0.01020
$\hat{\gamma}_1$	0.0108	0.0304	0.0107	0.0033	-0.0012
$\gamma_1 = \overline{R}_M$	0.0142	0.0220	0.0149	0.0112	0.0088
$t(\hat{\gamma}_0)$	6.52	-4.45	3.20	7.40	18.89
$t(\gamma_1 - \hat{\gamma}_1)$	6.53	-4.91	3.23	7.98	19.61

Table 2.3.

A cross-section plot of the mean excess return and the estimated β's of the portfolio shows that the relationship between the mean excess return and β is linear. However, the slope of the intersection and the cross-sectional relationship differs depending on the subperiods, which does not match the traditional form of the capital asset pricing model.

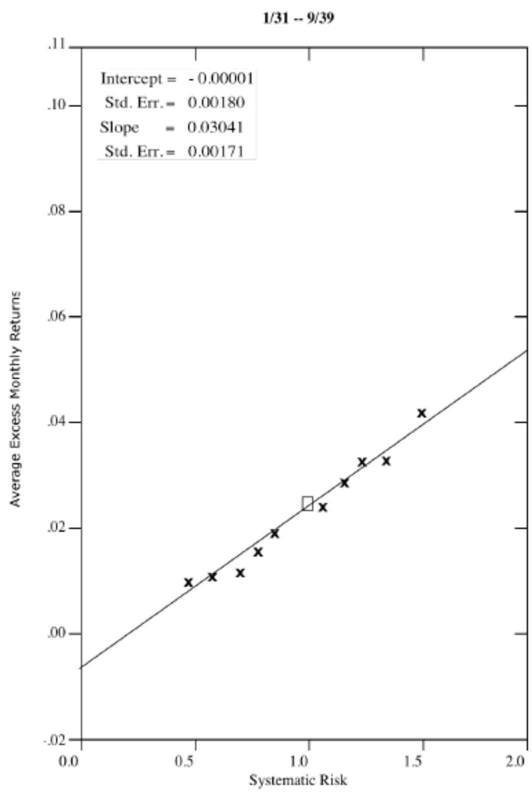

Figure 2.2.

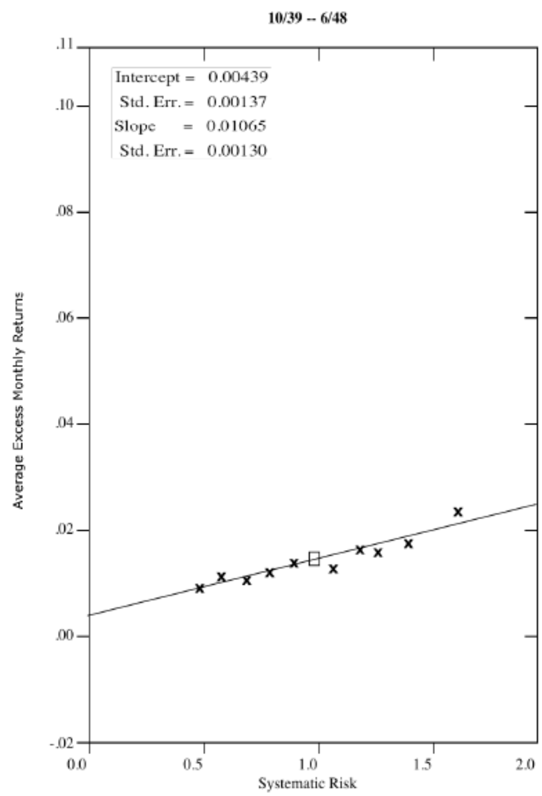

Figure 2.3.

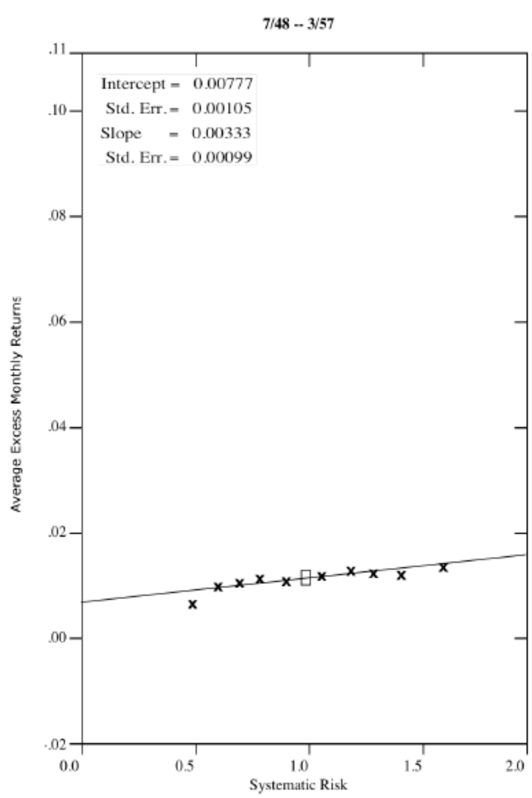

Figure 2.4.

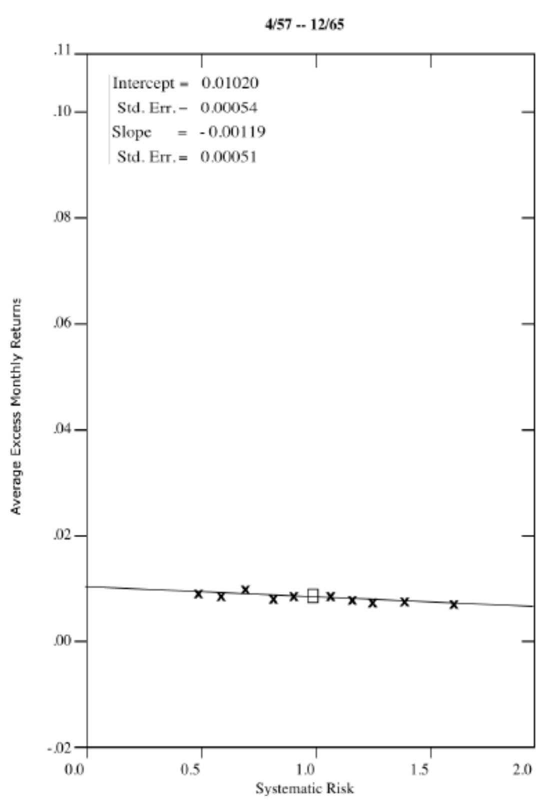

Figure 2.5.

In the two sub-periods of 105 months before the war investigated, the slope in the first period was steeper than predicted in the traditional form of the model and flatter in the second period. In each of the two periods of 105 months after the war, it was significantly flatter than expected. Based on evidence from both time series and cross-section execution, the hypothesis that γ_0 in (3) is zero was rejected.

As a result, it was concluded that the traditional form of asset pricing model is inconsistent with the data.

Explicit time series estimate of beta returns were made to get more efficient estimates of mean and variance, thereby allowing to directly test that the mean excess return of beta is zero. From the return data of the portfolio, a minimum-variance, unbiased linear estimator of the returns of the β factor was derived. It was demonstrated that, when independence of the residuals is given, the optimum estimator requires that of each portfolio's unobservable residual variances is known, but that this difficulty could be avoided if they were equal. The time series of returns on the beta factor was calculated using this assumption of equal residual variances.

Extensions of the CAPM.

The capital asset pricing model has been extended in many ways. The most well-known extensions include:

- allowing heterogeneous beliefs (Lintner, Merton),
- Elimination of the possibility of risk-free lending and borrowing (Black),
- setting some assets not marketable (Mayers)
- examination of multiple periods and investment options that change from one period to the next (Merton, Breeden);
- Expansion of international investment (Solnik; Stulz; Adler and Dumas);
- Use of weaker assumptions considering arbitrage pricing (Ross).

For most CAPM extensions, a single portfolio of risky assets is not optimal for all. Rather, investors distribute their assets differently to several risky portfolios that are aggregated to all investors to form a market portfolio.

3. Analysis of Arbitrage Pricing Theory

An alternative to CAPM is the Arbitrage Pricing Theory (APT), developed in 1976 Stephen Ross. APT , unlike CAPM, does not assume that shareholders evaluate decisions using the mathematical expectation and variance of returns. It assumes that the performance of a stock depends partly on macroeconomic factors, and partly on events related directly to the company itself. Instead of defining a stock's return as a function of one factor (the market portfolio return), it presents that return as a function of several macroeconomic factors on which the market portfolio depends.

Diversification allows to eliminate the specific risk of a particular security, leaving only macroeconomic risk as the main determinant of the required return.

The APT model does not specify which factors are the main ones: they can be the stock market index, the value of the gross national product, oil prices, interest rates, and other indicators. Typically, some companies are more sensitive to some factors than others.

In theory, it is possible to create a zero-risk portfolio (i.e. a portfolio with zero "beta"), which will provide returns equal to the interest rate in the absence of risk. If such a portfolio provided a higher return, then investors could make a risk-free return by borrowing to buy such a portfolio at a risk-free rate. This process, called arbitrage (i.e., making a profit with zero risk), would continue until the expected risk premium for a given portfolio was equal to zero.

APT avoids the inherent problem with CAPM of identifying a market portfolio. However, it poses even more difficult challenges. First, the use of APT requires the identification of macroeconomic variables. Secondly, it is necessary to solve the problem of assessing risk premiums for all factors and to determine the sensitivity of the values of stock returns to each of them.

APT proponents point out that it is not really necessary to isolate the relevant factors. To determine APT parameters you can use the mathematical apparatus of factor analysis. Initially, data on hundreds and even thousands of shares are taken into account, then several different portfolios are formed that do not closely correlate with each other in terms of profitability. Thus, in this set of portfolios, each of them is more strongly influenced by one of the known factors. Then the required return of each portfolio is considered as λ for this factor, and the sensitivity of the return of each individual stock to the return of this portfolio becomes the sensitivity of the factor (β). Unfortunately, the results of factor analysis are not easy to interpret, since it does not allow deep penetration into the essence of the main economic components of risk.

Earlier research also suggests that the multivariate APT model explains expected rates of return better than the univariate CAPM does. On the one hand, its variables are not specified. In addition, there is no consensus as to which variables should have the greatest impact. On the other hand, APT is more complicated in that for each company and for each specific period of time, coefficients must be generated for several factors, and not just for one factor, when they are going to be used. Finally, the CAPM also takes into account internal risk factors, in contrast to the APT.

The following factors have been identified and proposed to be used in the APT model by Chen, Roll and Ross:

- The spread between short-term and long-term interest rates, the yield curve
- Expected vs unexpected inflation
- Industrial production
- The spread between low risk and high risk corporate bond yields

The arbitrage pricing model has a number of significant drawbacks:

- the model does not take into account systemic factors affecting risk and profitability;
- there is no guarantee that all relevant factors were identified and taken into account;
- the factors taken into account can change over time, that is, undergo a temporary transformation;
- the calculation of the model is quite complicated and requires a more thorough preparation of information, the collection of more statistical data and their more detailed analysis.

In addition, this approach is retrospective, since it is based on the tacit assumption that in the future exchange rates will behave in the same way as in the past.

3.1. **Methodologies for empirical analysis in APT**

To implement a test on APT one of the three approaches could be chosen for the estimation of factors:

The first approaches includes the algorithmic analysis of the estimated covariance matrix corresponding to asset returns. Roll and Ross (1980), Chen (1983) and Lehman and Modest (1985a, 1985b) use factor analysis. While Chamberlain and Rothschild (1983) and Connor and Korajczyk (1985, 1986) use principal component analysis.

The second approach is for the researcher to start with an estimated covariance matrix of asset returns, use his judgment to select factors, and then estimate the matrix.

The third approach is purely judgmental. Researchers primarily use intuition to select factors and estimate the load of the factors to see if they explain the estimated cross-sectional variation in expected return. Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986) choose financial and macroeconomic variables as factors. These include variables such as stock index returns, the difference between short-term and long-term interest rates, private sector default premium indicators, inflation rate, industrial production growth rate, and total consumption. Due to business intuition, researchers continue to add many new elements.

The first two approaches are implemented to match the underlying factor structure of APT. The first approach is based on algorithmic design, and the second approach is to ensure that the factor they are using actually leaves the unexplained portion of the asset's earnings almost uncorrelated. The third approach is implemented without considering the factor structure.

Gehr

In 1975, Gehr collected monthly securities return data from CRSP tapes for 30 years. He uses the Principal Component Analysis (PCA) method to extract common factors, which rotate obliquely. The first step is to regress each industry index of factor values to extract potential factors and their corresponding loads. In the next step, he performed a cross-sectional OLS regression to get an estimate of the coefficients of the APT model. His results show that over the 30-year study period, only one factor seemed to have a significant price. Gehr's empirical evidence has not been tested against the specified alternatives, so it can be seen that there is no empirical power to separate CAPM from APT. In addition, Gehr's work doesn't make sense for several other reasons. On the other hand, factor solutions are arbitrarily rotated into one of many acceptable solutions. This creates strong multicollinearity between the explanatory variables in the second phase of the experiment, ignoring the difficulty of interpreting the statistics into the explanatory variables. Second, the variable declared in the design of experiments is the average (mean) return of the industry index, not the return of individual stocks. This implicitly limits the experimental design of the APT test. Finally, it does not try to deal with the variable error in the section OLS regression used. Therefore, the estimated risk premium is highly biased.

Roll and Ross

Roll and Ross in 1980 performed the most detailed APT test to date. Although they can overcome the first and second drawbacks of Gehr's tests, their cross-section evaluation tests are still at the expense of variable errors. The Roll and Ross study selected approximately 42 groups of 30 stocks listed on the New York Stock Exchange in alphabetical order to cover the period from July 3, 1962 to December 31, 1972.

They perform the following procedure for each group to adjust the daily arithmetic return for all capital changes and dividend payments:

- Calculate the covariance matrix of the time series sample for each of the groups of 30 stocks that cover the analysis period.
- The maximum likelihood factor analysis (MLFA) method is used to extract the number of factors and the corresponding factor load.
- At the end, regresses the factor loading matrix of average securities returns to provide an estimate of the factor risk premium whose significance is assessed.

To test the validity of the systematic risk hypothesis, the standard deviation of returns were imported into the APT model and the cross-sectional regression was repeated. Therefore, the explanatory variables are the standard deviations of the factor load estimates and returns. Their results show that of the four factors evaluated in cross section, three factors have significant prices. Therefore, their results support APT and empirically distinguish it from CAPM based on pricing equations.

Brown and Weinstein

In 1981 Brown and Weinstein try to replicate some of the empirical results of Roll and Ross (1980) based on another method. It could be seen that none of the grouped securities has an obvious five-factor return generation process. In addition, cross-section pricing results show that the average revenue of five or less factors is inconsistent with the APT framework. Eventually, they found that the results responded slightly to normality because of the surprising responsiveness of APT evidence to the test methodology.

3.2. Analysis of data and explication of the model through decades

To have a better understanding of the testing of APT and methodology, it would be a best to have a detailed look on the study of Roll and Ross.

The empirical test includes two major steps:

In the first step, the expected return and factor factors are estimated from the time series data of the individual returns of the asset.

The second step uses these data to test the basic conclusions about APT pricing. This procedure is similar to the known empirical CAPM task of using time series analysis to capture the market beta and perform a cross-sectional regression of the expected return on the estimated beta over different time periods. Although this has flaws in several respects, the two-step approach does not pose a major conceptual problem it shows in CAPM testing. In particular, APT applies to a subsets of whole of assets.

Data are described in Table 3.1. In selecting them, some arbitrary choices were needed. Although daily data were available through 1977, the calculations used data only through 1972. The objective was to make sure to have a calibration sample without losing the benefit of a large estimation sample, large enough for some statistical reliability even after aggregating the basic daily returns into monthly returns. The calibration sample is also used for later application and investigating the problems like non-stationarity.

Data Description

Source:	Center for Research in Security Prices Graduate School of Business University of Chicago Daily Returns File
Selection Criterion:	By alphabetical order into groups of 30 individual securities from those listed on the New York or American Exchanges on *both* 3 July 1962 and 31 December 1972. The (alphabetically) last 24 such securities were not used since complete groups of 30 were required.
Basic Data Unit:	Return adjusted for all capital changes and including dividends, if any, between adjacent trading days; i.e., $[(p_{j,t} + d_{j,t})/p_{j,t-1}] - 1$, where p = price, d = dividend, j = security index, t = trading day index.
Maximum Sample Size per Security:	2619 daily returns
Number of Selected Securities	1260, (42 groups of 30 each)

Table 3.1

In the empirical analysis, an estimated covariance matrix of returns was calculated for each group of assets. Since the calculation of covariance requires simultaneous observations, the start and end dates are set to exclude very short-lived securities. This resulted in a significant increase in the time series samples for each group, but there were still some differences in the number of observations between the groups.

Among the 42 groups, none included all the data for the 2619 trading day. However, the minimum sample size was still 1445 days, and only 3 groups had less than 2000 days. There were at least 2400 observations in 36 groups (86%). The size of 30 individual securities had to be a compromise. For purposes likes estimating the number of return factors present in the economy, the optimal group size includes all individual assets. But, this would mean a covariance matrix too large to process.

Estimating factor model

The model analysis consists of the following phases: 1. For individual asset groups, an exemplary product-moment covariance matrix is calculated from time series returns. 2. Maximum likelihood factor analysis is conducted on the obtained covariance matrix. This estimates the number of factors and the matrix loading. 3. The individual asset factor load estimates found are used to account for the cross-sectional fluctuations in the individual estimated expected returns. The procedure here is similar to the generalized least squares regression of a cross section. 4) Estimates from the cross-section model are used to measure the size and statistical significance of the risk premium associated with the estimated factors. This procedure is similar to estimating the size and importance of the factor score. 5. Steps (1) through (4) are repeated for all groups and the results are set in table.

Cross-sectional generalized least squares regressions of arithmetic mean sample returns on factor loadings, (42 groups of 30 individual securities per group, 1962–72 daily returns, standard errors of risk premia (λ) computed from time series)

1 FACTOR	2 FACTORS	3 FACTORS	4 FACTORS	5 FACTORS
I. $\bar{R}_j - 6\% = \hat{\lambda}_1 \hat{b}_{j1} + \cdots + \hat{\lambda}_5 \hat{b}_{j5}$ (λ_0 assumed at 6%)				
Percentage of groups with at least this many factor risk premia significant at the 95% level				
88.1	57.1	33.3	16.7	4.8
Expected Percentage of groups with at least this many risk premia significant at the 95% level given no true risk premia ($\underline{\lambda} = \underline{0}$)				
22.6	2.26	.115	.003	.00003
Percentage of groups with factor's risk premium significant at the 95% level in natural order from factor analysis				
76.2	50.0	28.6	23.8	21.4
II. $\bar{R}_j = \hat{\lambda}_0 + \hat{\lambda}_1 \hat{b}_{j1} + \cdots + \hat{\lambda}_5 \hat{b}_{j5}$ (λ_0 estimated)				
Percentage of groups with at least this many factor risk premia significant at the 95% level				
69.0	47.6	7.1	4.8	0
Percentage of groups with this factor's risk premium significant at the 95% level in natural order from factor analysis				
35.7	31.0	23.8	21.4	16.7

Table 3.2

The first panel demonstrates 6% λ per year during the sample period from July 1962 to December 1972. The first results in Table 3 show the percentage of groups in which more than a certain number of factors are associated with a statistically significant risk premium. In daily data, 88.1% of the group had at least one important risk premium factor, 57.1% had two or more important factors, and one-third of the group had at least three risk premiums. These percentages are well above what is expected to be ineffective under the null hypothesis by chance alone. The next row in Table 3 shows the expected relevant percentages under this null hypothesis. For λ=0, the probability of significantly observing at least the specified number of s at the 95% level is the top of the binomial distribution with a success probability of $p = .05$.

In fact, if the four factors are really important, the percentage of importance of 4.8 observed in the five factors is about the same as expected at the 95% level. If three are really important, then 16.7% of the groups where at least four are found to be important exceed 9.75%, which occurs by chance alone. If there are less than three important factors, the disparity will be much larger. From this it can be concluded that at least three factors are important in determining pricing, but it is unlikely that it will exceed four. The second result set still has λ at 6%, but shows the percentage of groups in which the first, second, and remaining factors generated by factor analysis have a significant risk premium. As can be seen, all factors are well above the probability level (5%), with the first two being particularly heavily weighted. The remaining three are important, and can be due to the order of the elements in the group being mixed.

The second part of the table also shows similar consequences but λ is estimated and not assumed nevertheless the results comply with the previous data. It can be concluded that three factors are surely included in expected returns.

Historical development of arbitrage pricing theory

The basic work of the asset pricing model begins with the development of the Capital Asset Pricing Model (CAPM). CAPM is a one-factor model and its empirical validity depends on the market portfolio, but the argument that the true market portfolio cannot be inspected severely limits the acceptance of the model. Ross argues that CAPM has never been tested and will not be tested because it cannot be identified the market portfolio. Roll expanded his criticism until he completely rejected the CAPM and became a supporter of Ross's arbitrage pricing theory (APT). The APT model includes two pricing identifications.

One is called the factor likelihood APT (FLAPT) and the other one is related to the pre-specified macroeconomic variable APT (PMVAPT). PMVAPT shows the relationship between the expected rate of return and the number of randomly selected macroeconomic variables. Conversely, FLAPT provides an intuitive linear relationship between expected return and asymptotically large potential factors.

It is assumed that the asset return rates obtained in APT are determined by "n" independent factors. These risk factors have different effects on financial assets in different situations and times. However, it is not possible to give any explanation about the exact number and nature of these factors. It is assumed that there is a linear relationship between asset return rates and these risk factors.

It is possible to examine the Arbitrage Pricing Models in the literature under three headings. These are: Single Risk Factor Arbitrage Pricing Model, Two Risk Factor Arbitrage Pricing Model and Multi Risk Factor Arbitrage Pricing Model.

Single Factor Arbitrage Pricing Model and Arbitrage Pricing Line.

The simplest case of Arbitrage Pricing Model is accepted as Single Factor Arbitrage Pricing Model. In this model, it is assumed that only a single systematic risk affects the return on assets. The relationship between a risk factor and the expected rate of return is represented by a linear function.

$$r_i = E(r_i) + \beta_{1,i} F_1 + e_i$$

Graphically showing this equation, which reveals the relationship between the expected rate of return and a single risk factor, is called the Arbitrage Pricing Line (APL).

Two Factor Arbitrage Pricing Model and Arbitrage Pricing Plane.

According to the Arbitrage Pricing Model with two risk factors, it is assumed that two different systematic risk factors will be affected by the return rates of their assets. The relationship between these two risk factors and the expected rate of return is shown in the following equation with the help of a linear function.

$$r_i = E(r_i) + \beta_{1,i} F_1 + \beta_{2,i} F_2 + e_i$$

The graphical expression of the relationship between the expected return on assets and the two risk factors seen in the equation is called the Arbitrage Pricing Plane. Arbitrage Pricing Plane is called an extended form of APL. As with APL, all assets in equilibrium must have a point on the

Arbitrage Pricing Plane. According to the Arbitrage Pricing Plane, expected return is on the vertical axis.

Multi-Factor Arbitrage Pricing Model

According to Arbitrage Pricing Theory, it is assumed that asset return rates are determined by "k" independent factors. As a result of these risk factors, they have different effects on financial assets in different situations and time periods.

$$r_i = E(r_i) + \beta_{1,i} F_1 + \beta_{2,i} F_2 + \ldots + \beta_{n,I} F_n + e_i$$

However, it is not possible to make an explanation about the exact number and nature of these factors. It is assumed that there is a linear relationship between asset return rates and these risk factors.

4. Comparison of Arbitrage Pricing Theory (APT) and the Capital Asset Pricing Model (CAPM). Their impact on risk management

When making a comparison between the capital asset pricing model and arbitrage pricing theory, it could be seen that the capital asset pricing model is based on and Markowitz's optimal portfolio selection procedure. The arbitrage pricing theory is based on factor models.

As we know, CAPM is a factor model that describes the expected return on a portfolio as a function of the risk-free rate and excess return on the market portfolio as one factor. APT also uses factor models, but includes more than one factor. Although the CAPM model states that the relative risk factor is the market portfolio, the APT model does not say exactly what the risk factor is.

While the CAPM in practical application uses the market index as the only factor, a simplification of the original market portfolio model is obtained. The index can best represent the market portfolio. It is possible that many factors in the APT model give only an approximate value of the market, while one market index can represent it better in the CAPM model. The CAPM model requires all investors to act according to the quadratic utility function and have the same expectations. That is, we can look at CAPM as a special case of APT, with certain additions and restrictions.

Also, the APT model can be viewed as a sampling model: it can include economic factors that are best sensitive to a security or portfolio. Sharp fluctuations of one factor, therefore, causes structural changes in the indicator of the expected return of a security.

The capital asset pricing model and arbitrage pricing theory play a significant role in risk management. When mentioning risk, it could be understood as both a positive or negative result. Obviously when taking opportunities with high risk, higher return would be expected, and vice versa, when taking opportunities with low risk, one would expect lower return. Risk does not always represent the amount of the loss or gain. In many cases potential loss is big but it could be forecasted and somehow be given attention to, using risk management techniques. The main issue is determining the variability of the potential loss. Especially if the loss has the probability of rising too much.

Risk management contains certain order of procedures, which have the ability to decrease the size of or completely get rid of the potential loss. In terms of both expected and unexpected losses, the duty of risk management is to be able to predict the and consider how much risk could be taken in exchange for the potential returns.

The activities of the risk management procedure are formulated such that, as a result they determine if the expected return is worth the expected risk.

The main steps are the following:

1. Identifying risks
2. Measuring and managing risks
3. Distinguishing the type of risk: expected or unexpected
4. Consider the connection between the risks
5. Design a risk mitigation plan
6. Monitor the mitigation strategy and regulate when necessary

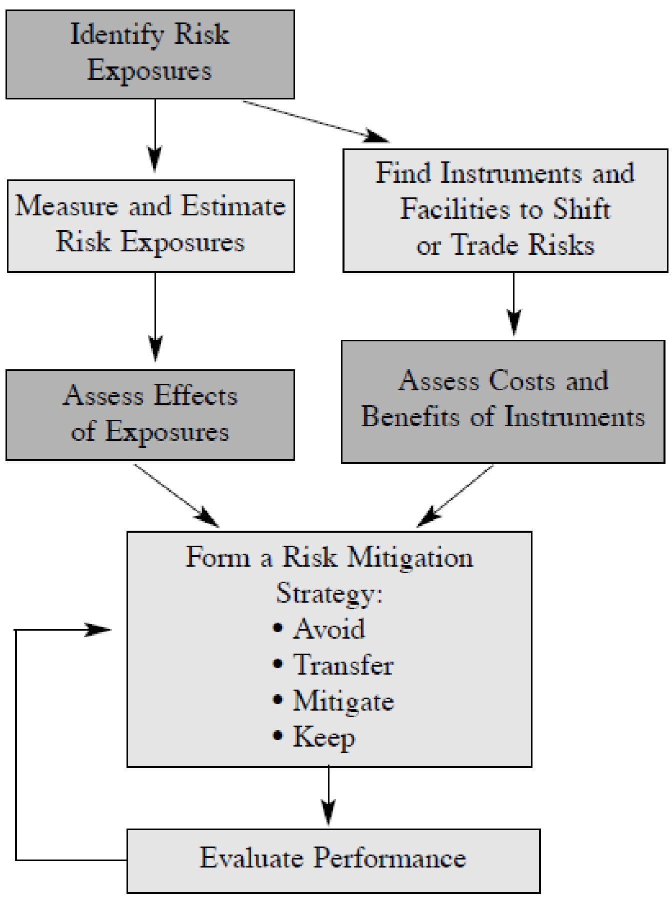

Figure 4.1. The risk management process

The identification of the risk can be carried out by brainstorming. The participants are the key business leaders, who could also involve their subordinates. Industry-level resources could be used for this purpose such as regulatory standards, etc. One of the most common methods is scenario analysis, which is analyzing actual loss to determine the amount and rate of different losses.

Risk management involves taking important decisions including:

- Avoiding the risk
- Retaining the risk, considering the expected return taking into account the frequency and amount of expected loss
- Mitigating risk, trying to reduce the impact of the risk factor
- Transferring the risk, with the help of derivatives, or purchasing insurance

To understand risk management and its techniques better, firm theoretical foundation is necessary, including some key theories and models that have an important connection with the approaches to risk management. These certain key theories and models propose specific simplifying assumptions. Implementing these theories and models in the real world is not so simple in all cases, and many factors in real life is difficult to express in models. The duty of models is to clarify and make less complicated and concentrate on the most significant factors. As stated by Milton Friedman, models must be assessed according to their ability to forecast. The assumptions made should not be considered when evaluating. At the same time, a model does not have to contain all the factors and details in the real world. In other words, the degree to which it is complicated is not the main requirement from a model.

The main indication that a model is useful is its power of prediction, and the degree to which it aids in making decisions.

Markowitz's study in 1952 on the principles of portfolio selection is one of the fundamentals in modern risk analysis. In this study, Markowitz has demonstrated that an investor acts in a way such that, when considering different portfolios, the main factors to focus on are the mean, the

variance of the rate of return. The assumptions made by Markowitz in this study are the following: capital markets are perfect, rate of returns are normally distributed.
According to certain theories based on the interest of shareholders, executives of corporations should not be involved in managing the risk of the corporations.
Franco Modigliani and Merton Miller set forth the idea that the value of a firm does not change with transactions. The assumption made by Modigliani and Miller is that the capital markets are perfect: they are highly competitive and the participants are not exposed to transaction costs, information costs or taxes.
In 1964 William Sharpe established the capital asset pricing model, and in this study Sharpe states the idea that firms should not focus on risks concerning the firm itself, but rather concentrate on mutual risks with other firms, because the specific risk can be eliminated by diversification, without costs, under the assumption of perfect capital markets. As a result firms do not have to manage risk that individual investor can manage on their own.
The role of theoretical models in this risk management could be expressed such that, the models assist in determining the risk, and deciding which measures are most suitable in certain cases. The models also reveal the significance of the difference between specific and systematic risk, the connection between financial modeling and key parameters.

Conclusion

Certain models such as CAPM and APT have been devised to predict the outcome of an investment in the financial markets that are constantly changing. The capital assets pricing model (CAPM) is a model that describes the relationship between the expected return of a portfolio of securities and the degree of its risk. It was proposed by the financial economist William Sharpe, in his book “Portfolio Theory and Capital Markets”. An alternative to CAPM is the Arbitrage Pricing Theory (APT), developed in 1976 Stephen Ross. APT , unlike CAPM, does not assume that shareholders evaluate decisions using the mathematical expectation and variance of returns. It assumes that the performance of a stock depends partly on macroeconomic factors, and partly on events related directly to the company itself. Instead of defining a stock's return as a function of one factor (the market portfolio return), it presents that return as a function of several macroeconomic factors on which the market portfolio depends.

The studies on the models goes further, in order to check the validity of them. To test the CAPM, most of the empirical test conducted on the capital asset pricing model were by the use of time series test. After estimating the betas this way, using the cross-sectional regression, the hypothesis was tested. In 1965 John Lintner carried out the first empirical test of CAPM. Later Douglas conducted the same test with similar results. Lintner applied a two-pass procedure for individual stocks. But the results do not comply with the CAPM. Miller and Scholes also performed a study on CAPM using the two-pass method also coming up with very similar results. To investigate the statistical problems of the two-pass method, Miller and Scholes arranged numbers and ran a simulation test, coming to a result that the method had statistical problems. The rejection of the CAPM in the initial verification tests above does not necessarily mean that the model is defective. To avoid measurement errors that effectively distort SML estimates, Black, Jensen, and Scholes provided additional insights into the nature and structure of security returns. Instead of testing individual stocks, they suggested ways to improve the accuracy of the estimated beta by grouping the stocks into a portfolio. the SML gradient term was significant and was positively linear as predicted by the model, so the results appeared to be consistent with zero-beta CAPM. In general, the results were similar for the entire and partial test period.

As for the tests implemented on the APT, In 1975, Gehr collected monthly securities return data from CRSP tapes for 30 years. He uses the Principal Component Analysis (PCA) method to extract common factors, performed a cross-sectional OLS regression to get an estimate of the coefficients of the APT model His results show that over the 30-year study period, only one factor seemed to be significantly priced, the estimated risk premium was highly biased.
Roll and Ross in 1980 performed the most detailed APT test to date. Their results show that of the four factors evaluated in cross section, three factors have significant prices. Therefore, their results support APT. In 1981 Brown and Weinstein try to replicate some of the empirical results of Roll and Ross (1980) based on another method. It could be seen that none of the grouped securities has an obvious five-factor return generation process.
When making a comparison between the capital asset pricing model and arbitrage pricing theory, it could be seen that the capital asset pricing model is based on and Markowitz's optimal portfolio selection procedure. The arbitrage pricing theory is based on factor models.
As we know, CAPM is a factor model that describes the expected return on a portfolio as a function of the risk-free rate and excess return on the market portfolio as one factor. APT also uses factor models, but includes more than one factor. Although the CAPM model states that the relative risk factor is the market portfolio, the APT model does not say exactly what the risk factor is.

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