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M-346 Simulator Model: development of the mathematical model of the flight dynamics

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Abstract

Flight simulators are nowadays used as powerful professional training tools and also as entertainment tools, and this has led to a strong rise in the development of high-fidelity simulators with a large aircraft collection. The simulator allows to reproduce the flight of a plane by noting the aerodynamic parameters characteristic for that aircraft in different flight conditions through an appropriate mathematical model of flight dynamics. In literature it is possible to find mathematical models for the aircrafts that result to be the most long-lived and the most studied, while such models are not available, for example, for military aircrafts of new generations. To this last category, the advanced military trainer M-346 aircraft belongs object of study of this thesis.

The purpose of this thesis is to develop the mathematical model of the flight control system useful to define the flight dynamics of the M-346 for an entertainment flight simulator. In order to reach such scope, it has been started from the realization of the CAD of the aircraft in scale 1: 1 and submitting the obtained element to fluid dynamic simulations with the purpose to obtain forces and moments applied to the aircraft. Several fluid dynamic simulations have been conducted in order to consider the different flight regimes and thus allowed to cover the whole flight envelope. From the results all the needed parameters are obtained for the mathematical model that tries to reproduce the behaviour of the aircraft in flight as close as possible to the real case.

The resulting mathematical model can be used to evaluate the responses of the aircraft to the given commands.

Chapter I – Overview

Recently, technological progress has led to the development of high-performance calculators capable of performing complex simulation activities. This has led to the spread of software capable of simulating virtual environments that require a high level of computational calculation in a very short time.

Reality emulation programmes cover several sectors, including the aerospace industry, where the use of simulators is not a recent introduction, but a technology that has been used for many years in flight crew training processes. Obviously, these training simulators were very bulky and expensive (in terms of design and execution), and above all they were only available for a very specific environment that was exclusively for flight crews. This limited field of application of aerospace simulators has been overcome thanks to the technological development of very low-cost, high-performance computers. The performance gap has led to the spread of simulation programmes, not only in aerospace but in all fields, among enthusiasts who want to try out these virtual experiences. It should be noted that simulators dedicated to the training of flight crews are still very complex systems (such as the dynamic simulators) and very expensive, while those developed for the public are still complex but very cheap and easy to use.

Aerospace simulators for the public are distributed as video game programmes where the concept of a video game is no longer limited to entertainment but goes beyond that and becomes a tool for evaluation, study and, of course, recreation. There are several flight simulators on the market that offer different experiences to the user. In fact, it is possible to customise the mission in every aspect: from the choice of aircraft to the environmental conditions. As imaginable, this vast ability to customise the simulation in all its parts is made possible by the detailed design of every single aspect concerned, starting with the flight mechanics of the used aircraft.

The study of the flight mechanics of an aircraft is the foundation of any simulator because it is related to the ability to faithfully reproduce the in-flight behaviour of the aircraft and so the mathematical model of the flight dynamics is the centrepiece of the entire flight simulator. Simulating the dynamics of an aircraft requires the knowledge of numerous parameters that must be known in advance in order to create the code that reproduces the response of the aircraft according to the input conditions: these parameters can be obtained from flight tests, tunnel tests or tests with CFD (Computational Fluid Dynamics) programmes. Once the parameters characterising the motion of the aircraft are known, it is possible to start designing one of the many parts of a flight simulator.

The considerations made in the previous paragraph must be carried out for each aircraft model (since each one is different from the others) and for each accuracy level desired in the simulation. The work of this thesis is therefore the implementation of the mathematical model of the M-346 aircraft for a flight simulator.

The M-346 is a state-of-the-art military aircraft that has recently entered into service, so there are currently simulators dedicated exclusively to the training of flight crews, while there are no simulators for the amateur public. The development of a programme for the emulation of the behaviour of the M-346 for the public represents an excellent opportunity, despite the criticality of the confidentiality aspect of the aircraft itself. One of the critical issues in the development of the mathematical model is the lack of specific data that are essential for the simulation of the aircraft behaviour, so it is necessary to find them.

The determination of the required aerodynamic derivatives and aerodynamic coefficients will be the result of a CFD analysis of the aircraft model, followed by the process of setting up the mathematical model and evaluating the results it will be able to generate: all this work will be done and reported in detail in the coming chapters.

Chapter II - M-346 aircraft characteristics

Introduction

The M-346 aircraft is a jet trainer and light combat aircraft developed by Leonardo Aircraft Division. This aircraft took off for the first time in 2004, and now is used by several air forces (such as the one from Italy, Israel, Republic of Singapore and Poland).
This aircraft is characterised by a long-term reliability, a cost-effective operation and a wide range of operations. The last feature is due to the easiness of the aircraft to change its own configurations and, so, to perform several roles: from training capabilities to combat operations. [1][2]

Design

The design of the M-346 was based on the operations performed by the aircraft itself. This has resulted in a twin-engine aircraft, tandem-seat airframe, sensors, weapons and an easy integration of the mission system suite. Thanks to the possibility of performing different roles, the aircraft is available in two versions, one suitable for training and one dedicated to light combat: the first is called the Advanced Jet Trainer Variant (M-346AJT), while the second is called Fighter Attack Variant (M-346FA). [3]
Both versions of the M-346 aircraft have approximately the same technical characteristics, with the exception of some details related to the tactical aspect such as all armaments and dedicated software for fight management (aspects that will be discussed below).

The M-346 geometrical & technical characteristics are reported in $Table$ 1. [3][4][5][6]

Table 1 M-346 Geometrical and technical characteristics

Length	$11.49\ m$
Height	$4.76\ m$
Wingspan	$9.72\ m$
Wing Area	$23.52\ m^2$
Wing Mean Aerodynamic Chord	$2.426\ m$
Wing Aspect Ratio	4.02
Wing Taper Ratio	0.83
Wing Sweep Angle	28 °
Horizontal Tail Area	$5.4\ m^2$
Horizontal Tail Span	$4.91\ m$
Horizontal Tail Aspect Ratio	4.46
Vertical Tail Area	$5.43\ m^2$
Vertical Tail Span	$2.76\ m$
Take Off Weight	$7600\ kg$
Ramp Weight (Max)	$9600\ kg$
Limit Speed	$1060\ km/h$
Service Ceiling	$13715\ m$
Rate of Climb	$6705\ m/min$
Sustained Turn Rate	$12.5\ deg/s$

The plane presents a classic control surface configuration: wing, horizontal tail and vertical tail. The wing is a hybrid delta wing, in which there is a substantial variation in the length of the chord of the profile near the middle of the wingspan. The specific geometry of the wing and the whole plane are reported in $Figure$ 1 [3]. In this figure are represented the three projections of the aircraft and the top and frontal view show the difference in the external equipment between the two versions of the aircraft.

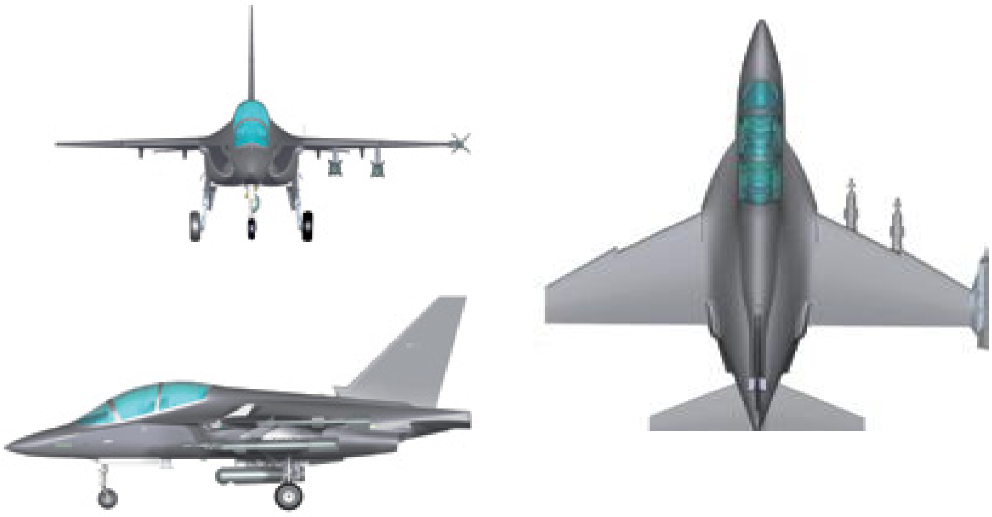


Figure 1 M-346 projections [3]

The horizontal tail is an all-moving tail, so the steady stabiliser and the elevators are one moving body with a wide range of rotation. The vertical tail has a significant surface to guarantee a well-balanced flight and it's compound by the fixed vertical tail and the rudder. The respective ultimate angles of rotation of the moving surfaces are shown in $Table$ 2.

Table 2 Ultimate angles of rotation of Control Surfaces

Min Elevator Angle Limit	$-15°$
Max Elevator Angle Limit	$+30°$
Trim Limit Elevator Angle	$19.5°$
Rudder Angle Limit	$\pm 30°$
Aileron Angle Limit	$\pm 30°$
LEF Angle Limit	$-20°$

Additional moving surface introduced in the M-346 are the air brake, the leading-edge flap and the trailing-edge flap. Flaps are used as a function of the speed (in particular CAS-Calibrated Air Speed), the flight mode (take off, cruise or land) and the position on the lending gear.

One of the common elements of both models, M-346AJT and M-346FA, is the same engine: they are powered by two Honeywell F124 turbofan engines that permit to the plane to be capable of transonic flight without using an afterburner. A single example of this type of propeller can generate 2,850 kg of thrust, which allows the plane to have high performance on different missions. Both thrusters are located at the rear of the plane with the intakes placed between the leading edge of the wing and the central part of the body. The aircraft is equipped with APU, to provide an autonomous engine starting, and FADEC (Full Authority Digital Engine Control) to optimise engine performance.

Other common features to both models concern the communication system and the human-machine interaction. As for the first one, the aircraft include two independent VHF/UHF transceivers, IFF transponder, radio-aided navigation based on TACAN and VOR/ILS/MB, autonomous navigation based on Embedded GPS/INS Radar-altimeter (EGIR), Mid-air Collision Avoidance System and Ground Proximity Warning System. On the second aspect, however, the M-346 is provided with the newest generation of the Human-Machine Interface (HMI): this one is compounded by six Multi-Function Displays (MFD), two Head-Up Displays (HUD), an up-Front Control Panel (UFCP), a digital moving map, a hands-On Throttle And Stick (HOTAS) controls, an integrated Helmet Mounted Display (HMD) system, a Night Vision Goggles (NVG) and a Get Home Display (GHD).

For a better synergy between the pilot and the plane during the training, the Embedded Tactical Training System (ETTS) enriches the training experience by the emulation and interaction of all the carried equipment. The highlight of the ETTS is the generation, evaluation and analysis of mission data, recorded during all the flights: all these roles offer a wide training simulation function in flight. This training system is matched with the Integrated Training System (ITS) and the Ground-Based Training System (GBTS) providing in this way a better training experience to all the new pilots. The ITS simplify the training process providing more knowledge, skills and practice, and for these reasons the teaching effectiveness is maximized. The practice is enhanced through the support of a digital map, which emulate threats, targets and Computer-Generated Forces. Other on-board simulation regard on-board sensors as targeting pod and active/passive electronic countermeasures, armaments and a LVC (Live-Virtual-Constructive) environment. The ETTS can support Stand Alone, a single plane mission, or multi-Ship networked operations through the adoption of a specific Training Data Link, the ETTS exchange the tactical scenario with further pilots/planes.

All these details are common to both models, as long as the M-346FA is directly derived from the M-346AJT, but as mentioned above, the differences between the two versions are essentially related to the presence of armaments. The additional equipment added to the basic

model aims to make the aircraft suitable for combat mode and to preserves all the qualities and high performances. More precisely the aircraft is suited with precision-guided munitions (such as IRIS-T or AIM-9 Sidewinder air-to-air missiles, air-to-surface missiles, anti-ship missiles, free-fall and laser-guided bombs and rockets), tactical reconnaissance elements, a multi-mode fire-control radar and seven hardpoints. The Fighter Attack is also provided with a low-observability radar and self-protection system like DASS (Defensive-Aids Support System). To increase survivability and functionality in case of hazardous situation, all primary systems are duplicated and reconfigurable. [1][2][3][4][5][6]
Despite the additional payload (up to 3000 kg), the M-346FA is a highly cost-effective multirole plane: it maintains its high thrust-to-weight ratio and all these elements have a slight impact on performance, in fact, it can be observed that the addition of these components increases the maximum take-off and ramp weight, accompanied by a decrease in the autonomy, the load factor and the turn rate. In $Table$ 3 it's possible to compare the difference between the M-346AJT and M-346FA. [3]

Table 3 Difference between the M-346AJT and M-346FA

	M-346AJT	M-346FA
Wingspan	$9.72\ m$	$10.14\ m$
Take Off Weight (Clean)	$7600\ kg$	$8100\ kg$
Max Level Speed, Low Altitude	$1090\ km/h$	$1065\ km/h$
Sustained Load Factor, SL	$8g$	$7.3g$
Sustained Turn Rate	$12.5\ deg/s$	$11.3\ deg/s$
Ferry Range, 10% reserve, Int. Fuel	$1925\ km$	$1665\ km$
Ferry Range, 10% reserve, 3 Ext. Tanks	$2550\ km$	$2220\ km$

As noticeable, the training version and the Fighter Attack have a different wingspan since the second one has to carry armaments, in particular there are some missiles located on the tip of each wing (in Figure 1 this difference is remarkable). Supplementary weights mean a greater consumption, and so the training aircraft has a bigger autonomy that the one for attacks. Relating to kilometric range, the three external fuel tanks ($630\ l$ each) increase greatly it, and it can be rides further via in-flight by a single point pressure refuelling system with a removable refuelling probe.

Despite the performance variation, all the supplementary elements allow the M-346FA to be capable to perform ground attack, homeland defence and air policing missions and reconnaissance: all these elements permit to compare and to overtake the last modern military aircrafts.

With these specifications, the M-346 family allow pilots to switch from training mode to combat mission mode without the necessity to change aircraft between the two phases. All of this is enabled by the use of the Integrated Training System that make the M-346 one of the most modern trainers nowadays.

Note the differences between the trainer and the Fighter Attack, the M-346 aircraft has a different designation according to its role or the country where it’s used: the designation for the basic type is M-346; for the Italian Military is T-346A; for the Polish Military is M-346LCA (Light Combat Aircraft); M-346FT (Fighter Trainer), a multirole variant capable of switching between training and combat operations; the classic M-346FA (Fighter Attack), the Grifo-M346 radar which implements countermeasures and stealth features; and, at the end, for the USA Military the name is T-100.

Flight Control System

FCS Functions

The Flight Control System (FCS) of the M-346 is responsible for manoeuvrability and controllability of the aircraft for a large set of angles of attack. The FCS is a Full-Authority/Full-Time digital fly-by-wire system, redundant four time in order to increase the reliability and safety of the aircraft in case of damage. This system has several functions, such as:

- Air data sensing and computation: the system needs air data for the control law processing, navigation and aircraft state.

  Air data are obtained by a dedicated system called ADS (Air Data System) which consists of Integrated Multifunction Probe (IMFP) and Total Air Temperature (TAT).

  From the Air Data Computation, Static and Dynamic Pressure-AOA-AOS are calculated.
- Aircraft trimming.
- Stability and control augmentation system.
- Aerodynamic configuration control: this permit to optimise performances, stability and manoeuvrability; plane's configuration can be changed by moving the LEFs and TEFs.
- Manoeuvre limitation for carefree handling: reduce pilot's work and increase safety (e.g. limiting the angle of attack and the load factor).
- Mass and store management function (MAMS).
- Autopilot and flight director functions.

All these digital supports lead to an improved three-axis control (pitch, roll and yaw) of the vehicle.

The FCS is able to guarantee the essential functions for specific flight phase automatically afterwards a single or multiple failure by the continuous monitoring of the flight, by the failure detection and by the build-in test routines that are performed and highlighted to the pilots.

FCS structure

As said before, for an increased safety of the aircraft the FCS hardware components are redundant several times, in fact all system is made by four FCC (Fight Control Computers), four IMFP, four ATU (Aircraft transducer Units), one TAT, one MSU (Magnetic Strap-down Unit) and various aerodynamics surfaces actuators and position feed-back transducers. The general structure of the M-346's FCS is represented in $Figure\ 2$.

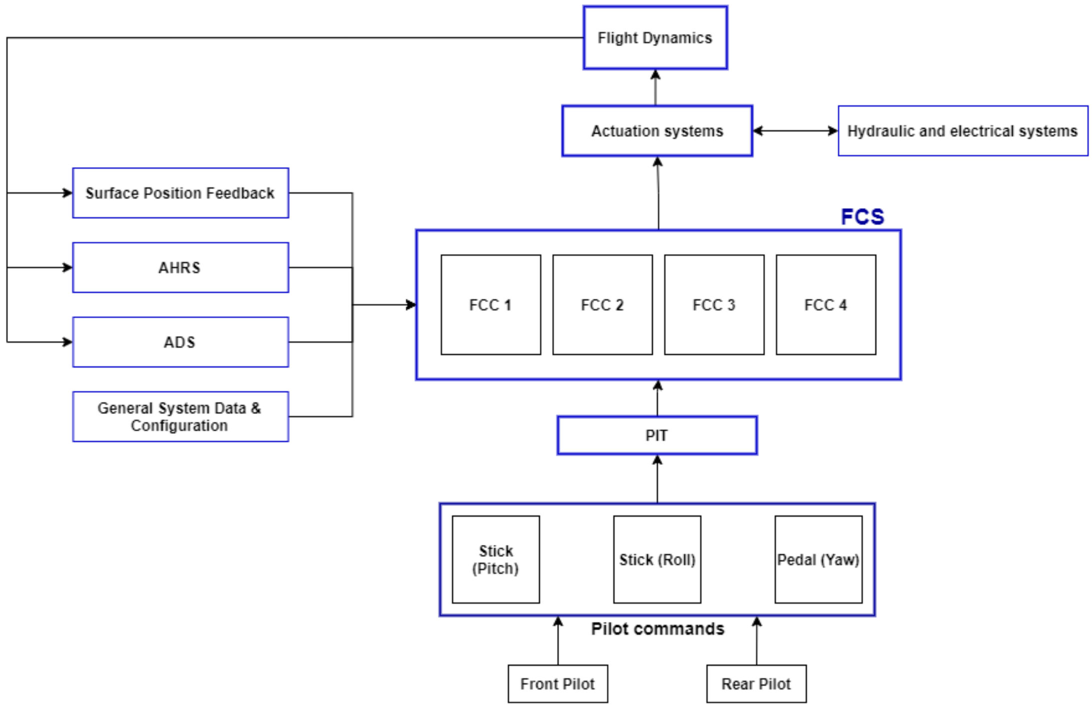


Figure 2 General structure of the M-346's FCS

Every single FCC implements the aircraft control laws works given the analogue-digital-discrete inputs, and then produces in output the desirable actuator commands for the control surfaces. Not only these computers compute the necessary action for the actuators, but they implement an actuator servo loop control and monitoring logic, a warning-caution logic, a BIT capability, an input monitoring logic, and an aircraft state sensor interface processing. All the operations made by a single computer is subject to a cross check, through the CCDLs (Cross Channel Data Links), with the equivalent value obtained from the other FCC: this check improves reliability of the aircraft and the mission. All the communications of the FCCs and the aircraft are based on MIL-ST-1553B bus, but they also have an ARINC 429

and RS 422 interface. For each flight computer, there are two buses (input and output) and two 28 $V\ DC$ batteries, which are redundant for greater safety.
The ARINC 429 and RS 422 interface is dedicated for the Attitude and Heading Reference System (AHRS): this system gives to the flight control computers the aircraft heading and inertial information, like magnetic heading, body rates and linear accelerations.
Other FCS's inputs are pilots' commands: these are converted into electrical signals proportionally to the movement of the stick by the PITs (Pilot Input Transducers), because commands are provided by a mechanical link assembly. In both directions, electrical signals and pilot commands are filtered by the AFU (Artificial Fell Units) in order to report the effort of the aircraft's response to the pilot's command.
The FCCs' outputs are the position commands of the control surfaces, in particular the computer outputs are the control signals of the actuators that move the individual surfaces. The required surface motion is achieved by servo-actuators, so two actuation systems can be identified: primary actuation systems and secondary actuation systems. The first one controls the primary control surfaces and it's made up of five actuators: one for the rudder, one for each aileron and one for each horizontal stabilizer. The secondary actuation system includes three sub-system: LEFAS - Leading Edge Flap Actuation System (a dual-hydraulic and dual-duplex electrical system), TEFAS - Trailing Edge Actuation System (a single hydraulic and dual electrical system), and Airbrake System (a single hydraulic and dual electrical system).

Control and Stability Augmentation

The Control and Stability Augmentation System provide a full-time augmentation function in the three-dimensional space to ensure and maintain aircraft controllability and stability in all configurations and in all flight envelope.
The aim of the three-axis feedbacks of AHRS-AOA-AOS data is to augment the natural stability of the aircraft according to the pilot's commands, that are such as to adapt to the different phases of the flight, to support the handling features variations and to limit the carefree handling for full stick commands. The Control and Stability Augmentation System can be separated for the Longitudinal and Lateral-Directional plane.
The Longitudinal Control and Stability Augmentation System involves different operating modes according to velocity, load factor (N_z), angle of attack and stick position.

The Lateral-Directional Control and Stability Augmentation System is based on velocity, Mach number, normal acceleration, angle of attack and stick position.
Each mode of the CAS-SAS depends on the type and quality of input and feedback parameters, so in case of a missing data or an error, for example, the system will change the operating mode with a view to maintaining the manoeuvrability and controllability of the M-346. [3][7][8]

Chapter III - SolidWorks & M-346 CAD

Introduction to SolidWorks

SolidWorks is a software designed to create models and assemblies and today is also used for the development of mechatronics systems. It's a solid modelling computer-aided design (CAD) and computer-aided engineering (CAE) program used for design and analysis of mechanical, electrical, and software elements. This program is realised by Dassault Systèmes, but it was developed in 1997 by J. Hirschtick, a MIT engineer.

The ability to operate in different areas (like mechanical, electrical or electronics) and to respond to design needs, makes this software one of the most important CAD software in the engineering field worldwide.

SolidWorks is a multifunctional program for the development and study of an element/system. In the first stage of the design process, the software application is related for planning, modelling, feasibility assessment, and project management. After the element's definition, it can be used for management and analytics of the system throughout its life cycle.

SolidWorks is a solid modeler based on a parametric feature approach and the geometry of the model or assembly is determined by the values of the constraints and the parameters. The former can be numerical or geometrical, and to define and maintain the design of the complex they can be associated with the other parameters using relations. The advantage of using parametric design is the simplicity with which the bodies and their relationships can be modified.

The design process to obtain the final component begins by drawing the outline of the relative base of the component. After that, the three-dimensional component or part of it can be generated by some specific SolidWorks' functions, such as extrusion. Once the extruded component has been obtained, it is possible to continue to work on any part of it in order to obtain the desired three-dimensional geometry. To do this, the software allows geometries to be drawn on any plane in the workspace and then extruded or to carry out extruded cutting. Therefore, the component design process consists of plotting the orthogonal projections of the component on the program and then obtaining the 3D shape from these projections. Following this simple design logic, highly complex objects can be obtained, and by joining several parts together, an assembly can be formed, all using a single application.

SolidWorks is not only an important design tool, but it also provides the ability to evaluate the behaviour of the generated part due to the availability of specific applications that are included in the program itself. The program allows countless types of simulation to be carried out, depending on the field of application of the component. In the case of simulations of mechanical objects, these are carried out using the finite element theory (Finite Element Analysis - FEA): this permits to predict the CAD models' real-world physical behaviour.
Each simulation study is set up by the user and the degree of precision of the results to be obtained depends entirely on the user's choice, the model complexity and the computer's computational capabilities, since as the precision increases, the computational costs increase. [9][10]

M-346 CAD

SolidWorks is the software used for the design of the M-346 solid body because it is the right tool for modelling and simulations (necessary to obtain forces and moments). Given the two versions of the M-346, it has been decided to consider only the Advanced Jet Trainer version for the CAD and all subsequent work.

The process of obtaining a solid body suitable for the simulations required a special design method because it started from an existing aircraft drawing. The CAD available was not suitable for carrying out the simulations as it was simply a set of surfaces tracing the shape of the aircraft, as can be seen from $Figure$ 3. Despite this shortcoming of the CAD provided, an important advantage of it is that it reproduces the aircraft in its actual size, on a $1:1$ scale.

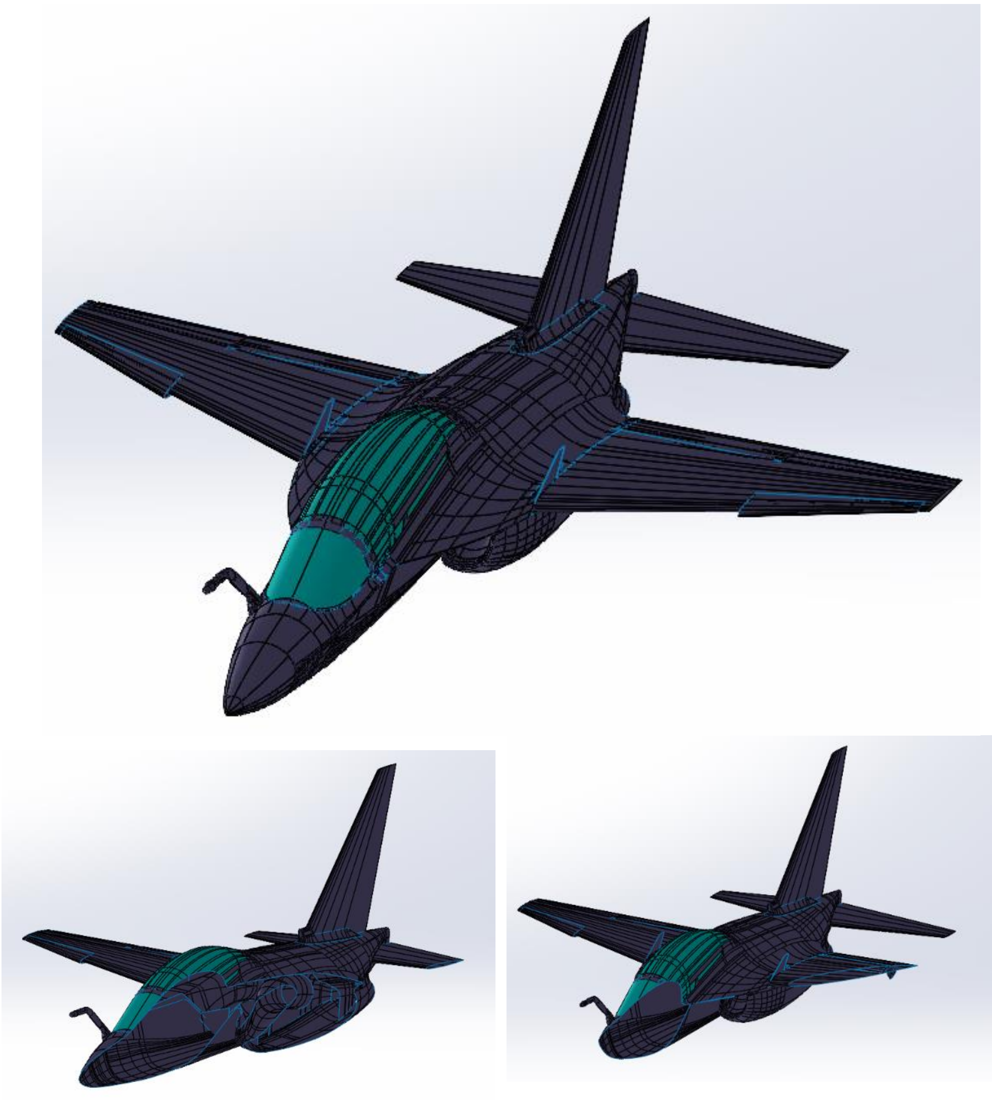


Figure 3 Initial three-dimensional drawing of the M-346

As can be seen, this file only shows the shape of the aircraft without highlighting the different moving surfaces, which are essential elements for studying the dynamics of the aircraft. Given the geometry of this empty 'body', it is clear that, in order to run the simulations, it is necessary to redesign the aircraft as a solid body in which the various moving surfaces are also visible. The logical and operational approach to obtaining the CAD from the external surface is shown in $Figure$ 4: the basic concept for deriving the aircraft is to generate intersection planes with the basic drawing, from the intersection highlight the aircraft's edges and then extrude three-dimensional bodies from the numerous sketches and guidelines.

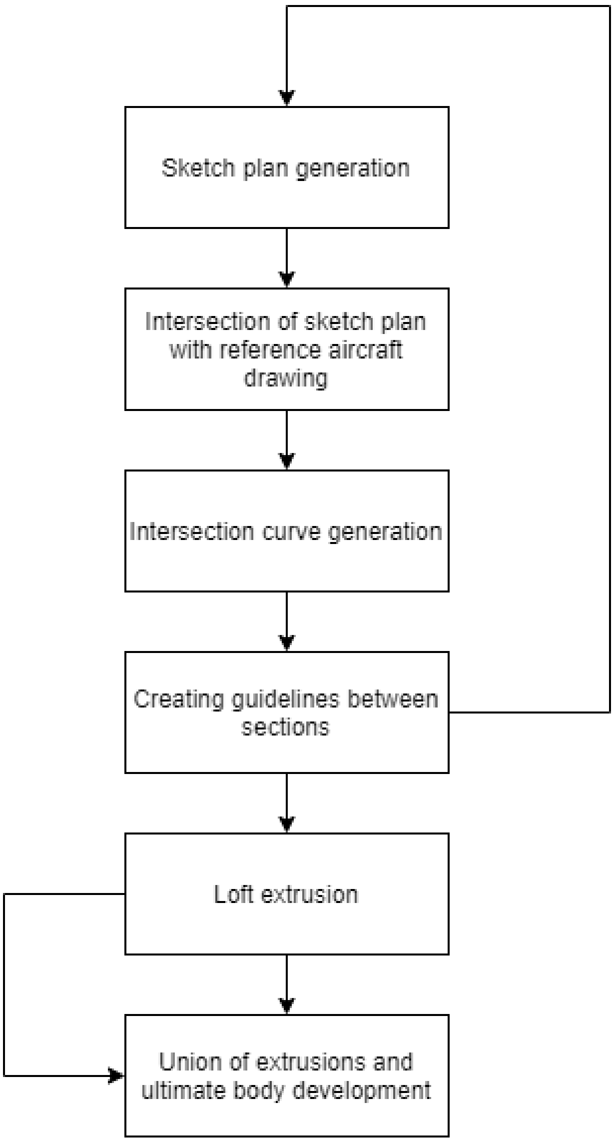


Figure 4 The logical and operational approach of the CAD design

The first step in the realisation is the creation of a large number of sketch planes which intersect the entire basic aircraft. For the generation of the planes, the reference system is the $x - y - z$ coordinate triplet of the aircraft, and consequently plans have been created along each direction:

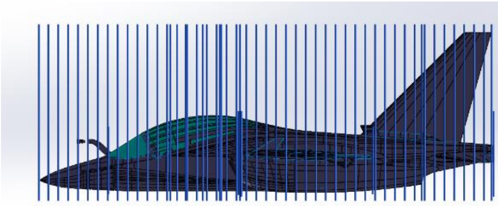
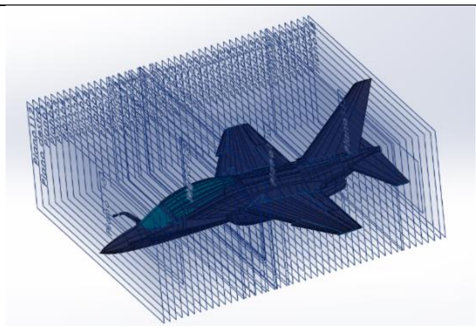
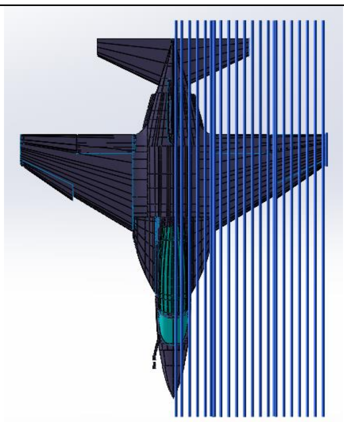
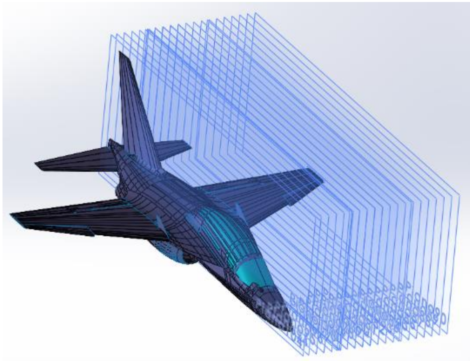
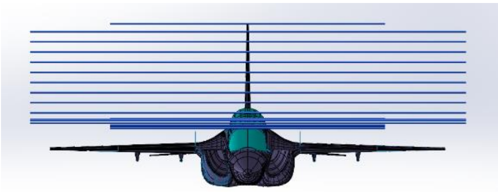
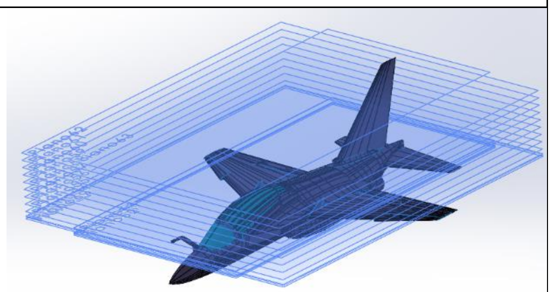
- x-axis: A total of 60 planes were generated along this axis; most of these are equidistant ($0.24\ m$), while others have been added at strategic points where there is a strong geometric variation that could not be followed simply with only two sketches of the equidistant planes. The planes along this direction are dedicated to the creation of the intersections of the fuselage, or rather the entire central body of the aircraft.

- y-axis: in the lateral direction a total of 68 planes have been added, of which 44 are for the wing (22 per half-wing) and 24 for the horizontal stabiliser (12 per side in the symmetry). The division of the planes for the wing and stabiliser is merely a technical choice to associate each sketch with its own drawing plane. The planes are all equidistant by $0.25\ m$, but not all of them since the planes of the extremities do not have the y-axis as normal but instead the normal to the extremity surface: this is a peculiarity of the extremities only.

- z-axis: The rudder was created from a total of 19 planes, most of which were equally spaced ($0.25\ m$), while others have been inserted at more complex geometric points such as the base of the rudder itself.

$Table$ 4 shows the images of the constructed planes along the three directions: a very complex matrix is obtained.

Table 4 Constructed drawing planes along the three directions

x-axis		
y-axis		
z-axis		

The purpose of these intersection planes is to generate the intersection curves of the source aircraft with the individual planes, and, by doing so, to obtain the sections of the aircraft with which to reconstruct it as a solid.

The generation of the intersection curves is done thanks to a specific function of SolidWorks, called "Intersection curve", which traces the intersection between the selected plane and the surface being considered: the result is a curve representing the generic section of the aircraft. By repeating this function on each previously created plane, the searched cross-sections are produced and as can be seen from $Figure$ 5, as together they compose the skeleton of the aircraft.

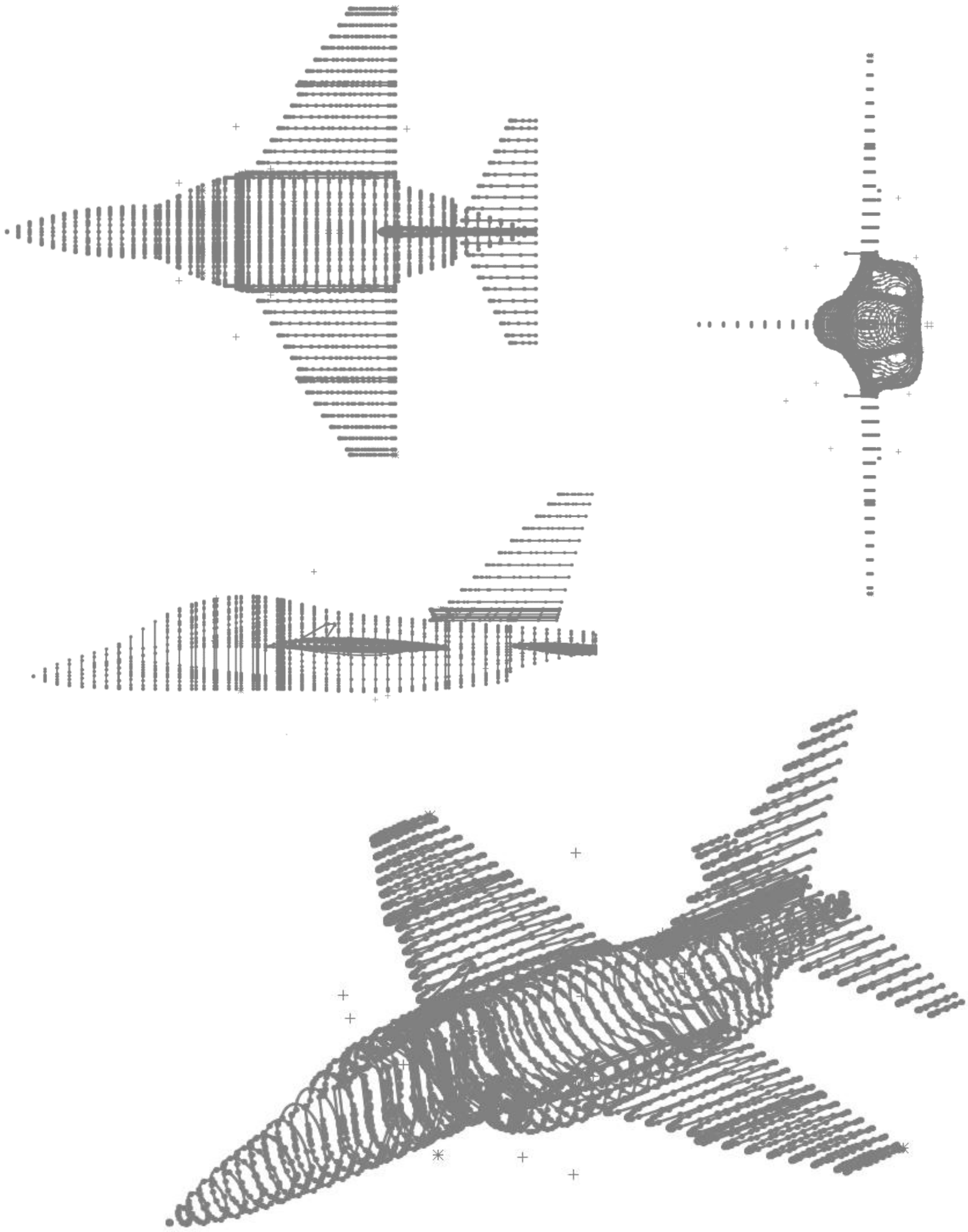


Figure 5 Searched cross-sections of the skeleton of the M-346

Once the various sections of all the elements of the aircraft (like fuselage, wings, rudder and horizontal stabiliser) have been obtained, guidelines are added to help the user and the program in the generation of 3D solids that reflect the curves of the aircraft. The addition of guidelines for each pair of sketches is essential in achieving the most accurate replication of the aircraft's volume pattern. In particular, the guide liners have been essential in the development of the central body of the aircraft, while they have been less essential for the wing and horizontal stabiliser, as they have very similar shaped profiles but different dimensions.

An example of these particular guidelines can be found in $Figure$ 6 where it is possible to see the frame of a half-wing and a fuselage leg.

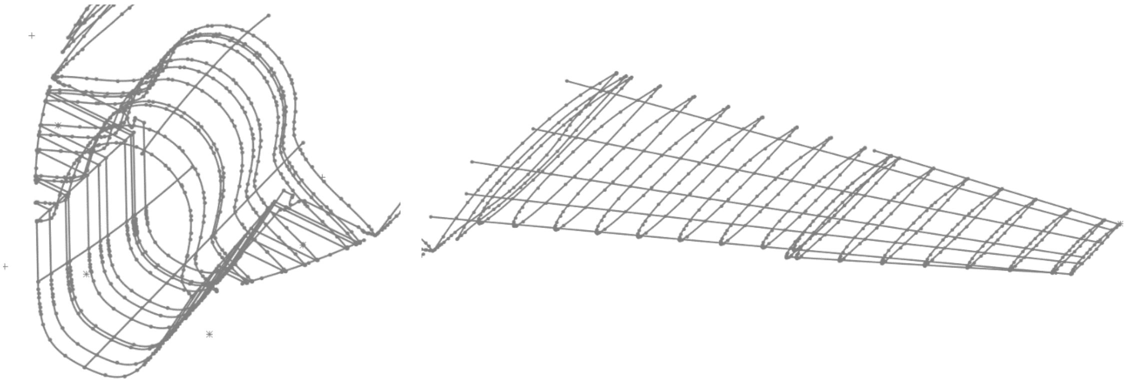


Figure 6 An example of the sketch guidelines

The following step has been the production of the three-dimensional body through the specific function called "extrusion loft", which allows to extrude a solid from two sketches following the guidelines: this extrusion process can be illustrated as follows in $Figure$ 7.

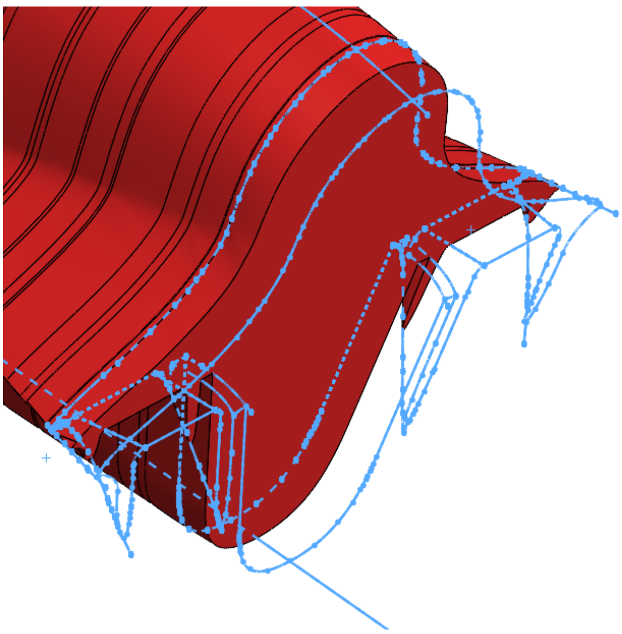


Figure 7 Detail of the solid body extrusion process

By carrying out this process, always taking two consecutive sections, it is possible to obtain the complete CAD of the M-346 ($Figure$ 8), which is a single solid body and so suitable for flight simulations.

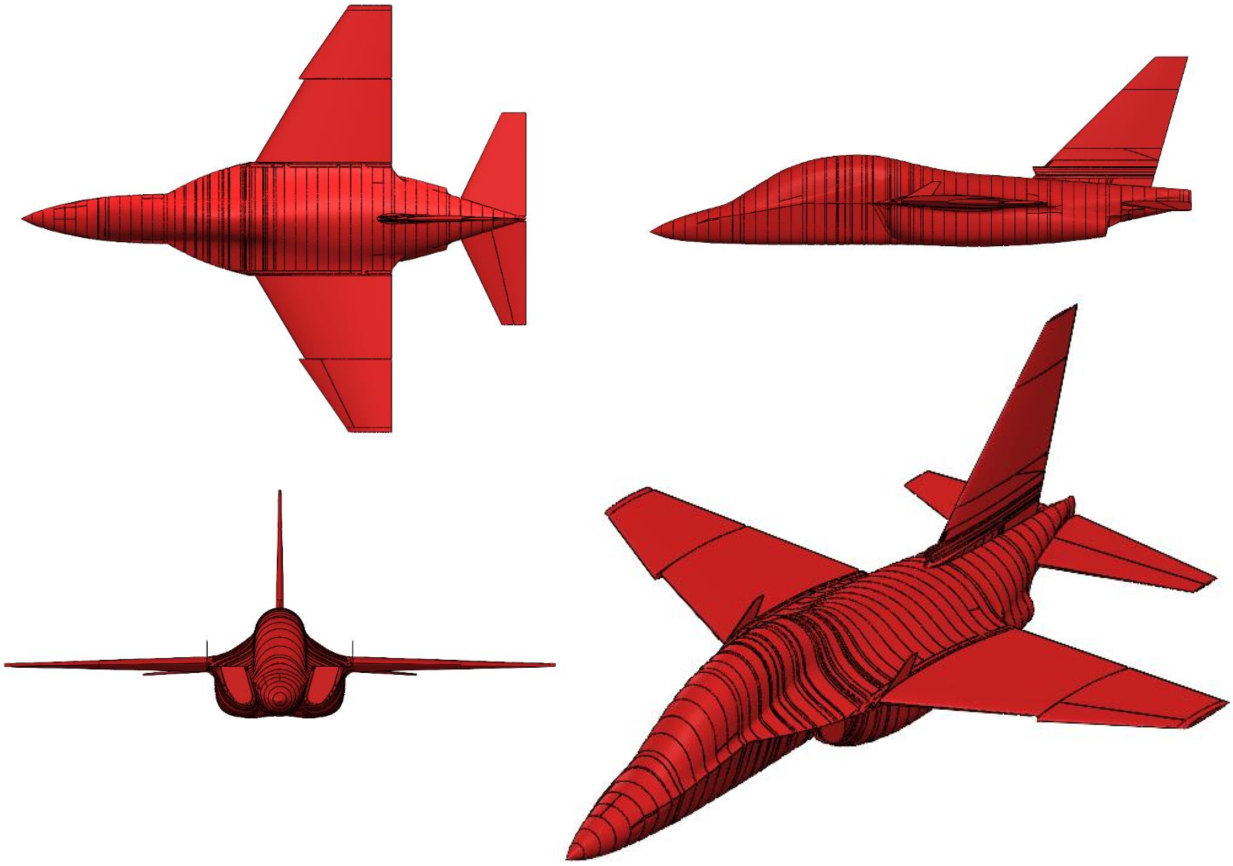
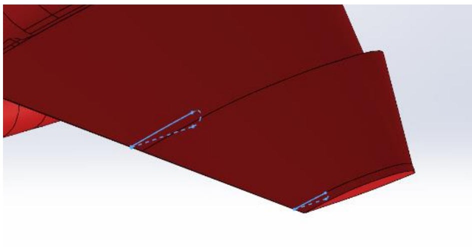
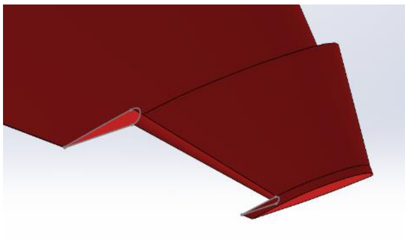
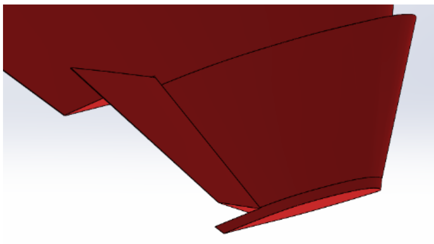


Figure 8 CAD of the M-346

What is obtained by the logical process, is a single three-dimensional body in which the control surfaces have not yet been delimited. For the exact delimitation of the control surfaces, no geometric measurements were available (only the dimensions of the individual areas were at disposal), so the geometric data have been obtained proportionally from images of the real aircraft, projections of the same and some surfaces/guidelines in the source CAD file. In this way, measurements of the ailerons, LEFs, flaps and rudder have been gotten.
The creation of the individual control surfaces has been carried out in a three-stage process ($Table$ 5): firstly, the sections delimiting the individual surface have been drawn co-regulating the volume development, afterwards the space corresponding to the control surface was cut out and finally the 'cut out' part was rebuilt in the same position but as an independent object from the rest of the aircraft.

Table 5 The three-step process for the construction of the moving surfaces

1^{st} stage	
2^{nd} stage	
3^{rd} stage	

In this way, the control surfaces on the two wings and the tail rudder were obtained, while the horizontal stabiliser was not affected by this process, since it is not made up of a fixed part and a mobile part (as explained above). The control surfaces are thus completely rotatable and allow the development of simulations of the dynamics of the aeroplane in a complete form. The admissible range of rotation of each control element has already been expressed in the previous chapter in $Table$ 2, and now in Figure 9 these angle excursions can be seen.

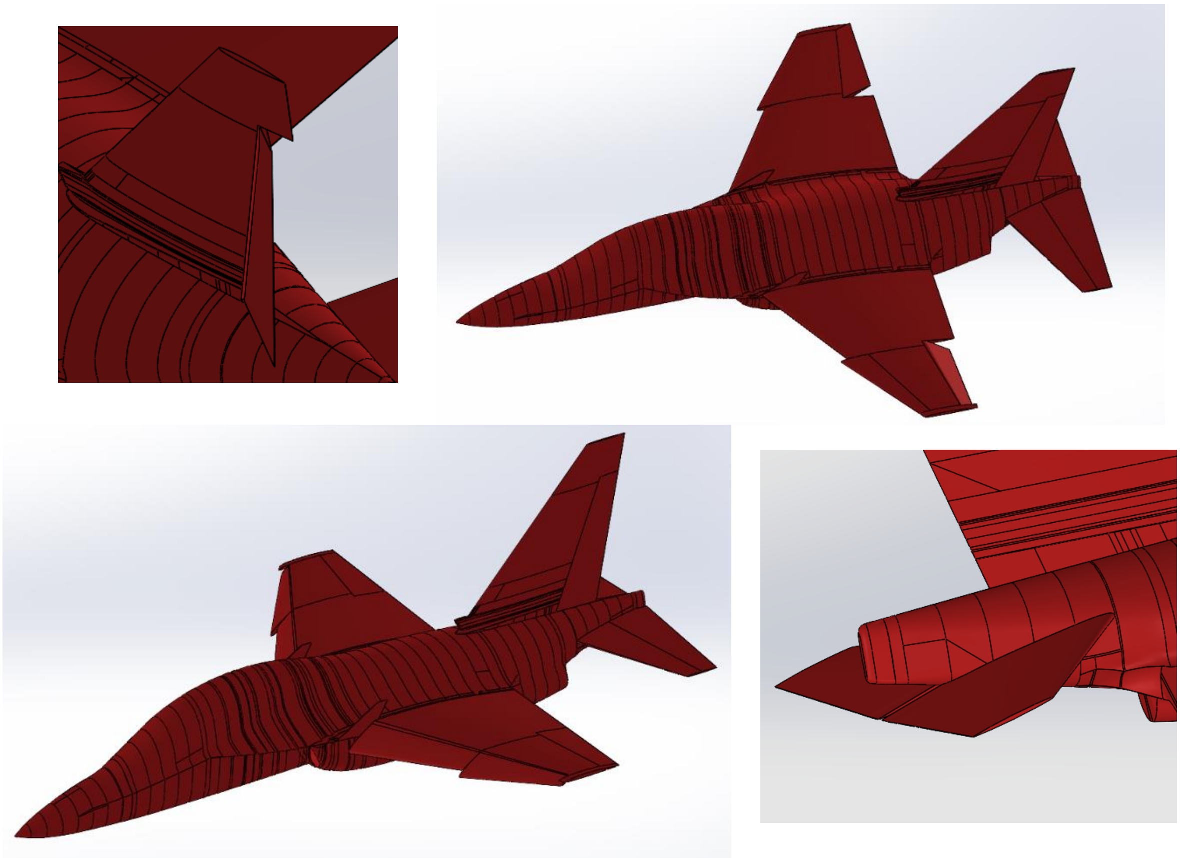


Figure 9 Representation of the rotation of individual control surfaces

The CAD of the M-346 is suitable for aerodynamic simulations, but for greater fidelity between the real flight and the simulated one, a weight of $9000\ kg$ has been assigned to the CAD thanks to the software's ability to assign a density to the designed objects. The allocated mass is an interpolation value derived from the take-off mass data in clean configuration and with the maximum ramp for both M-346AJT and M-346FA: for the trainer aircraft the two masses are $7600\ kg$ and $9600\ kg$ respectively, while for the attack aircraft they are $8100\ kg$ and $10500\ kg$.

Having chosen the mass to be assigned, it was possible to use SolidWorks to read the occupied volume ($19.81\ m^3$) of the aircraft and then calculate a fictitious density to be assigned, which turns out to be equal to $454.31\ kg/m^3$.

Given the mass and geometry of the aircraft, SolidWorks also evaluates the moments of inertia, which for the M-346 are shown in $Table$ 6:

Table 6 Moments of inertia of M-346 CAD

I_{xx}	$9441.8\ kg \cdot m^2$
I_{yy}	$45151.7\ kg \cdot m^2$
I_{zz}	$51222.8\ kg \cdot m^2$
I_{xy}	$-0.72\ kg \cdot m^2$
I_{xz}	$663.6\ kg \cdot m^2$
I_{yz}	$-0.19\ kg \cdot m^2$

These moments of inertia are evaluated from the centre of gravity of the CAD of the aeroplane, which is $0.28078\ m$ forward of the focus of the aerodynamic mean chord.

Chapter IV - Calculation of aerodynamic derivates and coefficients

Introduction

The aerodynamic coefficients, crucial to the study of aircraft flight dynamics, are calculated and extrapolated from the forces and moments applied to the aircraft. In this case, due to unavailability of flight tests of the M-346, it was necessary to calculate them.
As already mentioned, SolidWorks is a powerful program not only for drawing but also for simulation, in fact, through the "Flow Simulation" application, it is possible to exploit this characteristic to simulate different flight conditions obtaining forces and moments acting of aircraft 3D model. Specifically, the designed M-346 CAD is used to obtain all the required data through various simulations.
"Flow Simulation" is an application dedicated to the fluid-dynamic study of a body immersed in a stream where it is possible to define all the input parameters according to the needs of the user. For simplification of calculations of different coefficients by using a dedicated Excel spreadsheet and to avoid the possible propagation of errors, the setting values of the flow simulation have been considered as constants for all the simulations carried out. The inputs considered to be constant for different simulations are shown in $Table$ 7: obviously these are the basic parameters that are maintained fixed, while other inputs such as angular rotation speed or sideslip angle will be suitably modified for those simulations that require it.

Table 7 Input parameters of the simulation environment

Analysis Type	External (Exclude cavities and internal space)
Gravity	$-9.81\ m/s$ (z-axis local coordinate system)
Fluid	Air (Standard conditions)
Flow Type	Laminar and Turbulent
Wall Conditions	Adiabatic wall without roughness
Pressure	$101325\ Pa$
Temperature	$288.15\ K$

Velocity Parameter:	
Defined by	Aerodynamic angles
Velocity	$154\ m/s$
Longitudinal Plane	XZ (Local coordinate system)
Longitudinal Axis	X (Local coordinate system)

As the parameters are almost exclusively environmental (related to the stream around the body), the value that determines the greatest variation in the results is the flow velocity, which has been set at $154\ m/s$ (equals $300\ knots$). This value has been arbitrarily chosen considering the speed range at which the aircraft can fly (maximum speed $302.8\ m/s$).
Another very important factor in simulations is the quality of the results, and this is represented by the parameter "Mesh", i.e. the density of the integration grid for each iteration of the calculation. The greater the mesh density, the better the results obtained: the drawback is the number of iterations to be carried out which increases considerably. For this reason, SolidWorks allows this parameter to be chosen in a range from 1 to 7, where 7 represents the maximum density, but in the carried-out simulations the values 3 (aircraft configuration with rotation of control surfaces) or 4 (simple aircraft configurations, i.e. with the control

surfaces in a neutral position) have been used in order to limit the number of iterations maintaining accurate results.
All the simulations performed aim to calculate the forces and moments acting on the aircraft along the three $x - y - z$ axes of the reference system. The resulting calculations produce three forces (F_X, F_Y, F_Z) and three moments (M_X, M_Y, M_Z), from which it is possible to obtain the different aerodynamic coefficients of the M-346 for both the wind and the body axis systems.
For both reference systems, the simulations to evaluate the dynamic behaviour of the aircraft have been performed by varying only the following parameters in a defined range or to precise values:

- α - Angle of Attack (AOA).
- β - Angle of Sideslip (AOS).
- p – Roll speed.
- q – Pitch speed.
- r – Yaw speed.
- Aileron rotation.
- Elevator rotation.
- Rudder rotation.
- LEF rotation.

Despite the variation of numerous input parameters, the outputs will always be the same (three forces and three moments) from which the relevant coefficients can be derived. The forces and moments that the program provides are oriented according to the CAD design reference system, which differs from the classic reference system used in aeronautics (x-axis oriented from the tail towards the nose, y-axis oriented to the right of the pilot, z-axis oriented towards the ground). To solve this problem, a new reference system has been generated, centred on the centre of gravity and with the axes oriented according to the aeronautical reference system. Forces and moments are oriented in relation to this new tern. Taking this correction into account, dividing the outputs by the appropriate denominators gives the six coefficients expressing the forces and moments along the axes. The formulas used for the calculation are as follows:

$$c_X = \frac{F_X}{\frac{1}{2} * \rho * S * V^2}$$

$$c_Y = \frac{F_Y}{\frac{1}{2} * \rho * S * V^2}$$

$$c_Z = \frac{F_Z}{\frac{1}{2} * \rho * S * V^2}$$

$$C_l = \frac{M_X}{\frac{1}{2} * \rho * S * V^2 * b}$$

$$C_m = \frac{M_Y}{\frac{1}{2} * \rho * S * V^2 * c}$$

$$C_n = \frac{M_Z}{\frac{1}{2} * \rho * S * V^2 * b}$$

The denominators of the previous equations contain these terms: air density ρ, wing area S, flight speed V, mean aerodynamic chord c and wingspan b.

The calculation of aerodynamic coefficients and derivatives is necessary to reproduce the behaviour of the aircraft and to evaluate its dynamics. It is therefore essential to obtain these data accurately and according to the appropriate reference systems. The realization of a mathematical model for the evaluation of the aircraft dynamics is very complex, therefore some simulation systems have been evaluated as a conceptual framework.
Among the numerous references found in the literature on flight simulators and mathematical models, the one that is most similar to the aircraft under study is the F-16 simulator developed by Prof. G. Balas and his collaborators & R. Russell [11]. This simulator develops two mathematical models of different fidelity: the low-fidelity model simplifies and linearizes the problem and is based on the equations of "Aircraft Control and Simulation", by B. Steven & F. Lewis [12], while the high-fidelity model is based on "NASA Technical Paper 1538" [13 and it does not make simplifications (e.g. it does not neglect the

presence of LEF). The latter model is more similar to the desired characteristics of the M-346 model, so it will be considered as a simple guideline.

Axis reference system

The dynamics of the aircraft can be studied according to different reference systems that can be fixed with respect to the motion or vary with it. In this case, two triples of axes have been considered: the wind axis system and the body axis system.

In the wind axis reference system (indicated by the subscript W), the origin of the Cartesian tern is located at the aircraft's centre of gravity and the $x-$axis is directed according to the relative speed vector in relation to the atmosphere (the $y-$axis is directed to the right and the $z-$axis is normal to the xy plane and directed downwards). A clearer arrangement of the axes can be seen in the following figure, in fact $Figure$ 10 [14] shows the differences between the different reference systems, in particular between the body axis (B), wind axis (W) and vertical-axis (V) reference systems.

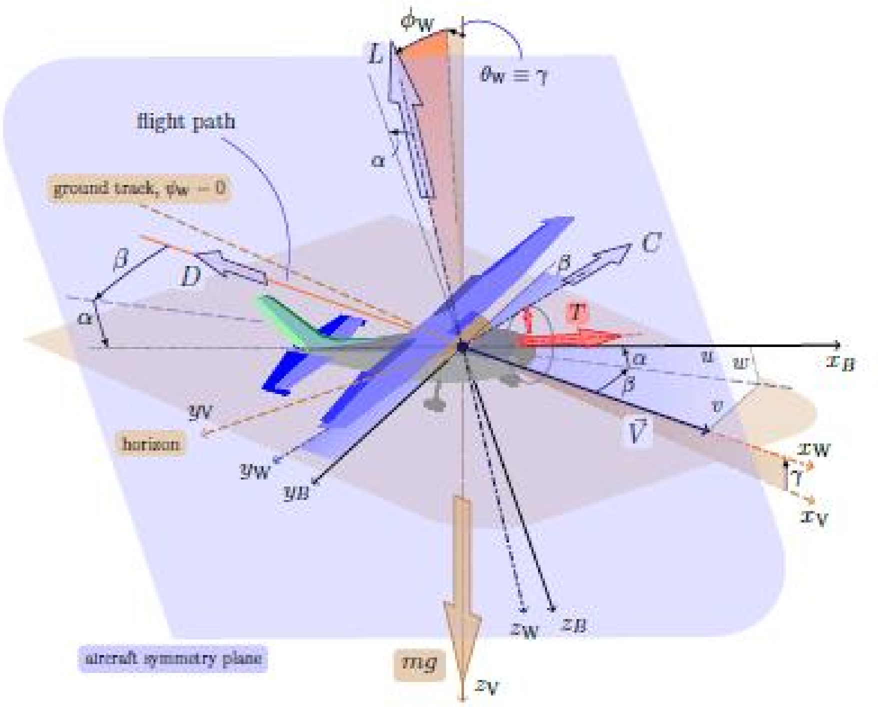


Figure 10 Overview of the reference axes of an aircraft [14]

Another reference system centred on the centre of gravity is the body axis system. The primary characteristic of this system is that it remains integral to the body during motion and the x-axis coincides with the longitudinal axis of the aircraft and has a positive direction according to the direction of motion. The y-axis is oriented to make a left-hand coordinate system, as the z-axis lies in the plane of longitudinal symmetry of the aircraft pointing downwards. $Figure$ 10 clearly indicates the differences between this reference system and the previous one, from which it is clear that the only way to move from one reference system to another is to rotate the axes.

Defining the reference systems is necessary as the whole development of the mathematical model of the simulator is a function of the axis system, so it has been chosen to use the body axis reference system as, the simulator on which the future mathematical model will be implemented uses this reference system.

Stability Derivates

The study of the aircraft dynamics is based on a set of derivatives, called aerodynamic derivatives, which can be obtained by running precise simulations or by performing calculations that include the whole configuration of the aircraft. The set of stability derivatives to be extracted contains the following parameters: C_{D_0}, C_{L_α}, $C_{L\dot{\alpha}}$, C_{L_q}, $C_{L_{delta_e}}$, C_{m_0}, C_{m_α}, $C_{m\dot{\alpha}}$, C_{m_q}, $C_{m_{delta_e}}$ for the longitudinal plane, and C_{Y_β}, $C_{Y_{delta_r}}$, C_{Y_p}, C_{Y_r}, C_{l_β}, $C_{l_{delta_a}}$, $C_{l_{delta_r}}$, C_{l_p}, C_{l_r}, C_{n_β}, $C_{n_{delta_r}}$, $C_{n_{delta_a}}$, C_{n_r} e C_{n_p} for the lateral directional plane. These derivatives, and specifically those involving lift and drag, are calculated in wind axis reference system.

The first simulation carried out concerned the study of the behaviour of the Lift, Drag and Pitch Moment of the M-346 as the angle of attack varies, since in this way it is possible to have a first interpretation of the quality of the CAD and the simulator. The AOA has been varied in unit steps from $-180°$ to $+180°$ and $Figure$ 11 shows the trend of the three coefficients.

To obtain the values of C_L and C_D, firstly lift L and drag D have been calculated by forces along the x and z axis as

$$L = F_X * \cos(\alpha) - F_Z * \sin(\alpha)$$

$$D = F_X * \sin(\alpha) + F_Z * cos(\alpha)$$

$$C_L = \frac{L}{\frac{1}{2} * \rho * S * V^2}$$

$$C_D = \frac{D}{\frac{1}{2} * \rho * S * V^2}$$

With α the angle of attack, ρ the air density, S the wing area and V the aircraft speed.

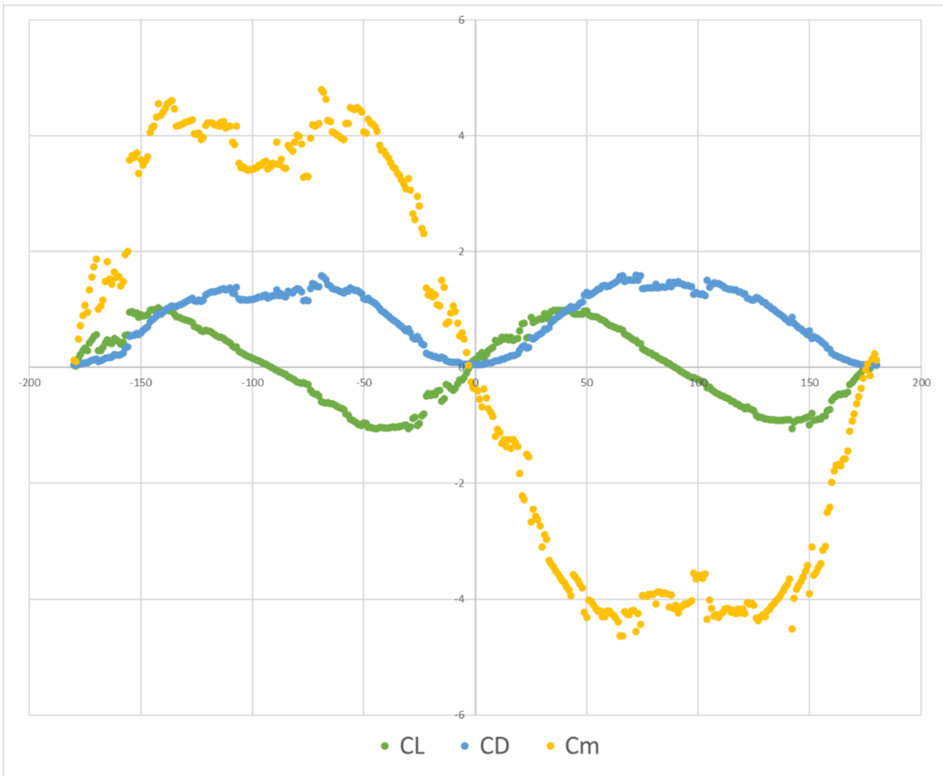


Figure 11 Chart of M-346 lift, drag and moment coefficient

Neglecting fluctuations and observing the symmetry between positive and negative angles, it can be deduced from the macroscopic trends of the three coefficients that the CAD is suitable for simulations and that the results can be considered reliable.

The aircraft has an operational range of the angle of attack between $-10°$ and $+25°$, so considering an additional safety margin of 5°, a simulation was carried out with a step of 0.5°, obtaining that the drag coefficient had a parabolic trend while the lift coefficient had an almost linear behaviour (with discrepancies from linearity towards greater angles), meaning that at low angles of incidence, the two coefficients follow the theory. The graph in $Figure$ 12 shows the C_L and C_D in the limited AOA range, it is simply a localised magnification of $Figure$ 11 with a higher node density.

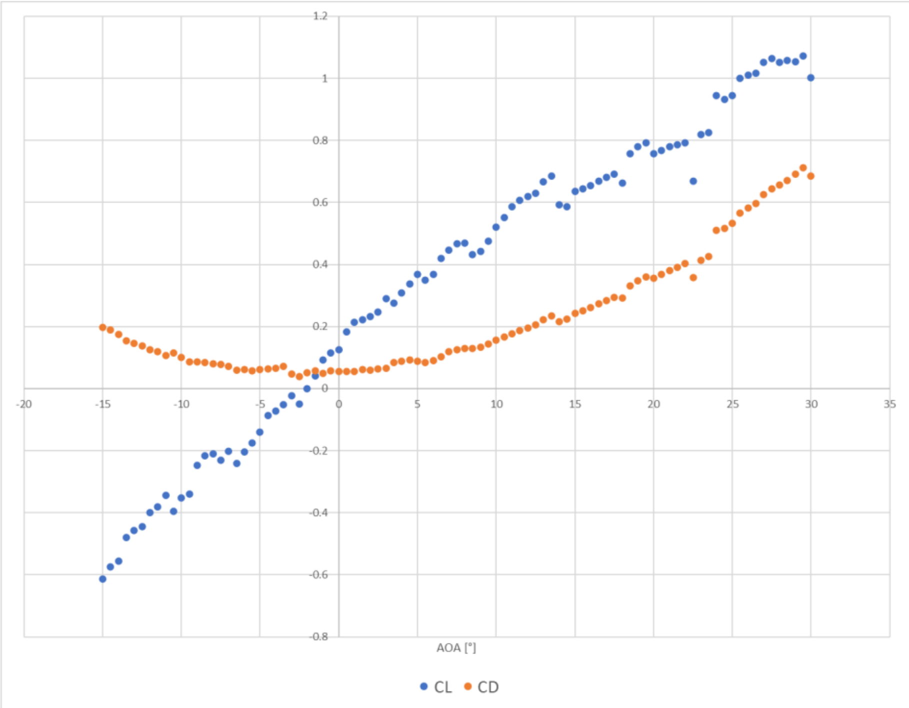


Figure 12 Lift and drag coefficient evaluated in the operating AOA range

Plotting the lift coefficient as a function of the drag coefficient, we can obtain the polar plot ($Figure$ 13) of the aircraft from which to derive, according to what is written in "Flight mechanics" by C. Casarosa at page 130 [15], the drag coefficient at zero lift C_{D0}, which turns out to be 0.0264.

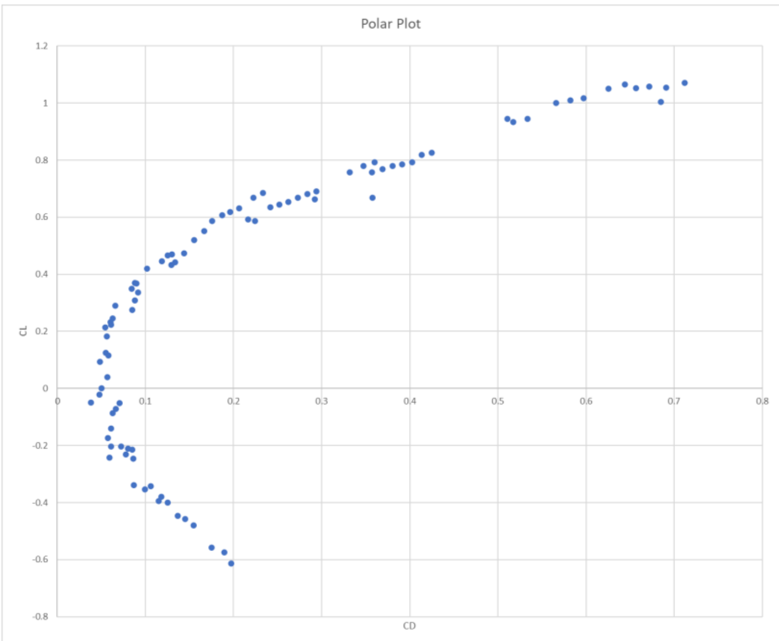


Figure 13 M-346 polar plot

From $Figure$ 13 (above) it is possible to extrapolate the data to evaluate the slope of the lift curve in order to obtain the aerodynamic derivative $C_{L\alpha}$, but given the fluctuation of the data, for greater precision this element has been calculated according to the method given in "Airplane Design" by J. Roskam [16], which allows the value of 3.2650 to be obtained. The latter book has been considered as a guideline for the calculation and extrapolation of all remaining aerodynamic derivatives.

Once the C_{L_α} is known, the C_{m_α} can be calculated: this new term is negative, thus stating that the aircraft is stable. Similarly, the $C_{m\dot{\alpha}}$ is derived from the $C_{L\dot{\alpha}}$ by following again the formulas of the reference texts. All the obtained results are shown in $Table$ 8, in particular the derivatives calculated so far refer to the longitudinal plane.

The calculation of the remaining stability derivatives is more complicated as it should be evaluated for each individual AOA. In the first instance this calculation has been made for a specific angle of attack, while in a second instance it has been done for each α. This first step of research has been based only on the variation of the angle of attack, while in addition to the main variation of the latter angle, the aircraft is subjected to other variations of angles and velocities.

The simulations carried out subsequently concerned the evaluation of the variation of the lift coefficient and pitching moment regarding the position of the elevator. In particular, the derivative of the moment coefficient has been calculated from the derivative of the lift coefficient, which has been obtained by extrapolation from the relevant graph. The simulations run in this case also considered the same range of AOA, but this time the angular position of the elevator has been varied, placing it in addition to the neutral position of 0°, at the minimum and maximum deflection (-30° and +15°).

The rotations of the other two control surfaces, ailerons and rudder, and their effects are also evaluated through the derivatives of the roll and yaw moment and the derivative of the force along y. The evaluation of these five derivatives is carried out by running two simulations, one for each surface, and keeping the angle of attack between $-15°$ and $+30°$. Specifically, the ailerons have been rotated to $-30°$, 0° and $+30°$, and the rudder has been placed in the same positions because they have the same deflections).

The first change concerns the addition of the sideslip angle of the relative velocity vector: this allows the calculation of the derivatives of C_Y, C_l and C_n with respect to β. To extrapolate these three derivatives, three simulations have been executed maintaining the same range of variation of the angle of attack ($-15°/+30°$) but setting each time a different sideslip angle equal to $0°$, $+5°$ and $+10°$. Given the symmetry of the aircraft and the flight conditions for the extrapolation of the data, the simulations with a sideslip angle of $-5°$ and $-10°$ have not been performed. This allowed to calculate the variation of the force coefficient along the y-axis, the roll and yaw coefficients respect to β.

The derivatives of the angular velocities p, q and r (C_{l_q}, C_{m_q}, C_{y_p}, C_{y_r}, C_{l_p}, C_{l_r}, C_{n_r} e C_{n_p}) still have to be evaluated: these can be derived from the results of the simulations, but they can also be analytically calculated following the guidelines of J. Roskam's book. In the first instance the second method has been followed, and subsequently they have been derived from the data obtained from the simulations.

The calculation of derivatives according to the literature formulas requires many parameters (many geometric and performance based) which must be known a priori or obtained from other simulations. One of the parameters required to calculate these derivatives is, for example, a_t (slope of the tail lift curve) for which a specific simulation was carried out to obtain this value. Some of the parameters used for the calculation of the latter aerodynamic derivatives are given in $Appendix\ A$.

$Table$ 8 reports the values obtained for the considered stability derivatives. Specifically, the listed data of the derivatives of the angular velocities have been calculated only with respect to the equilibrium angle of attack α_{eq}. On the other hand, the angle of attack considered for the derivatives of the sideslip angle and the rotation of the primary control surfaces is different as the tabulated values refer to horizontal rectilinear flight with $\alpha = 0°$. The choice of reporting only specific values is linked to the fact that there are numerical values to interpret the stability of the aircraft, so these values need to be considered only as example data.

Table 8 Numerical values of stability derivatives

C_{D0}	0.0264
$C_{L\alpha}$	3.2650
$C_{L\dot{\alpha}}$	1.5081
CL_q	3.3356
CL_{δ_e}	0.0905
Cm_0	0.3638
Cm_α	-1.2961
$Cm_{\dot{\alpha}}$	-2.6172
Cm_q	-5.7885
Cm_{δ_e}	-0.1595
CY_β	-0.3043
$CY_{\delta_{rudder}}$	2.3287
CY_p	-0.1227
CY_r	0.1957
Cl_β	-0.1012
$Cl_{\delta_{aileron}}$	-0.0351
$Cl_{\delta_{rudder}}$	0.0177
Cl_p	-0.2237
Cl_r	0.1147
Cn_β	0.5550
$Cn_{\delta_{rudder}}$	-0.0021
$Cn_{\delta_{aileron}}$	-0.0068
Cn_r	-0.0542
Cn_p	-0.0239

These derivatives assume plausible values for the category of aircraft under consideration and the signs that the individual derivatives assume would appear to be correct, so this indicates that the aircraft seems to be stable.

The values just obtained are useful and essential for the evaluation of the stability of the aircraft, but since the mathematical model on which the simulator is based is set on the body axis reference system, it is necessary to carry out further simulations to obtain the additional data required.

Body axes reference system coefficients

For the body axes reference system, it has been necessary to conduct specific simulations because in implementing the model for evaluating the dynamic stability of the aircraft, it is necessary to have a set of aerodynamic coefficients from which only those necessary and specific for the flight condition can be extracted.
For the subsequent simulations, a new angle-of-attack range has been set to consider the high incidence (up to 90° of AOA). The new vector covers a wide range of angles of attack without increasing the number of iterations in each simulation because the angle increment is no longer 0.5°, but 5° and 10°. This led to the setting up of a new vector (called "Alpha1") of twenty AOA:

$$Alpha1 = \{-20, -15, -10, -5, 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 70, 80, 90\} \quad [deg]$$

The first set of simulations carried out concerned the aircraft in a clean configuration, with all its moving surfaces in a neutral position (or rather with a rotation angle of 0°), subjected only to the angular velocities p, q and r. These simulations have made it possible to derive the aerodynamic derivatives C_{x_q}, C_{y_r}, C_{y_p}, C_{z_q}, C_{l_r}, C_{l_p}, C_{m_q}, C_{n_r} *and* C_{n_p} which express the variation of forces and moments with angular velocities. These derivatives have been extrapolated graphically from the relationships linking the relative coefficients with the dimensionless rotation speed. From these simulations, nine vectors of twenty elements each have been constructed, from which the desired data can be extrapolated with the aircraft in a clean configuration and in absence of sideslip angle.
The next step consisted of introducing the LEFs into the simulations, in particular the leading-edge flaps were extended to the maximum aperture in order to evaluate their influence on the performance of the aircraft: the force and moment coefficients are then calculated, adding the influence of the sideslip angle.
By looking at the presence of β, all the involved simulations have been done considering that this angle assumes certain values contained in a vector of 19 elements like:

$$\beta = \{-30, -25, -20, -15, -10, -8, -6, -4, -2,0,2,4,6,8,10,15,20,25,30\} \quad [deg]$$

The vector of AOAs has also been shortened to reduce the number of iterations to 14 elements ("Alpha2" vector):

$$Alpha2 = \{-20, -15, -10, -5, 0, 5, 10, 15, 20, 25, 30, 35, 40, 45\} \quad [deg]$$

Conducting these calculations with the variation of α and β, the vector of each coefficient is made up of 266 items. The arrangement of the latter has a specific structure in which the nineteen values dependent on β are shown one after the other for each α angle. With this structure, the vectors of the six coefficients, called $C_{x_{lef}}$, $C_{y_{lef}}$, $C_{z_{lef}}$, $C_{l_{lef}}$, $C_{m_{lef}}$ and $C_{n_{lef}}$, are constructed.
In addition to evaluating the force and moment coefficients with LEFs, it is also necessary to evaluate how these data vary in the presence of angular velocities, hence the need to carry out some simulations to have $C_{x_{q_{lef}}}$, $C_{y_{r_{lef}}}$, $C_{y_{p_{lef}}}$, $C_{z_{q_{lef}}}$, $C_{l_{r_{lef}}}$, $C_{l_{p_{lef}}}$, $C_{m_{q_{lef}}}$, $C_{n_{r_{lef}}}$ and $C_{n_{p_{lef}}}$. These nine unknown values have been obtained for angles of attack defined by the vector "Alpha2" and their values have been extrapolated from the coefficient - dimensionless angular velocity graphs.

In addition to LEFs, the aircraft's primary control surfaces have an important impact on the coefficients/derivatives as the angle of attack and sideslip change. For this work the single surfaces have been rotated individually and then simulations have been run.
The influence of the elevator is the first element of analysis: it is placed at five different positions (vector $DH1 = \{-30, -15, 0, 5, 15\} \quad [deg]$) and for each position α and then β are varied. The result contains a large amount of data for each of the force coefficients C_x C_y C_z and moment coefficients C_l C_m C_n. These values are no longer extrapolated but are simply calculated according to the formulas previously given.
The coefficients related to the longitudinal plane C_x C_z C_m are vectors composed of 1900 elements as they are the result of the variation of the angle of attack "Alpha1", the angle of sideslip "Beta1" and finally the elevator angle "DH1".
For the lateral directional plane, the number of coefficients is lower: for the force coefficient C_y the vector is 380 elements long because the elevator has no significant effect on the lateral force, so the simulation only takes into account the variation of AOA ("Alpha1") and β ("Beta1"); the roll moment coefficients C_l and yaw coefficients C_n are influenced by the

elevator in a way that their linear extrapolation can be less accurate, so the simulations have been made considering only three elevator positions (vector $DH2 = \{-30, 0, +15\} \quad [deg]$), while the other two angular variables remained the same. With this alteration, the two vectors constructed in this way contain 1140 elements each.

Following the evaluation of the elevator, the ailerons and rudder have also been examined: individually, they have been rotated until they reach their maximum opening, which is $30°$ for both. With this position of the surfaces, the simulations have been conducted by varying only the angle of attack in the range $-20° - +90°$ ("Alpha1") and that of sideslip between $-30° - +30°$ ("Beta1"): it has been calculated the 380 elements for each single vector of coefficients $C_{y_{r30}}$, $C_{n_{r30}}$, $C_{l_{r30}}$, $C_{y_{a30}}$, $C_{n_{a30}}$ and $C_{l_{a30}}$.

A further simulation that has been done concerned the simultaneous presence of ailerons (rotated to their maximum, $+30°$) and leading edge flaps (extended to their maximum) to assess the influence of both on the aircraft by use of the coefficients $C_{y_{a30_{lef}}}$, $C_{n_{a30_{lef}}}$ and $C_{l_{a30_{lef}}}$. Due to the presence of LEFs, the aircraft has been evaluated during the simulation as a function of the variation of the sideslip angle - "Beta1" and the angle of attack, which is represented by the dimensionally reduced vector "Alpha2": thus, producing only 266 values for each coefficient.

The latest simulations have involved the construction of three vectors $deltaC_{n_\beta}$, $deltaC_{l_\beta}$ and $deltaC_m$ as a function of only the variable angle of attack (since high incidences had to be covered, "Alpha2" has been used as an operating range). The derivatives $deltaC_{n_\beta}$ and $deltaC_{l_\beta}$ evaluate the variation of the moment coefficients with respect to β for each α; while $deltaC_m$ is only the pitch moment coefficient evaluated for each AOA.

The purpose of carrying out so many simulations is to obtain as many as possible of the aerodynamic coefficients and derivatives that can describe in the most accurate way the flight behaviour of the M-346. The organisation of the data in the form of vectors is intended to facilitate the simulation code when extrapolating values according to flight conditions. The process of extrapolation is very simple because, given α, β, position of LEFs and movement of the primary control surfaces, it is possible to go to each vector and take (directly or by interpolation between two values) the data required for the simulation and evaluation of flight dynamics.

Chapter V - Coefficient arrangements

The coefficients and derivatives calculated for the development of this mathematical model have been obtained simply by applying the mathematical relations linking the forces and moments to their corresponding coefficients, while any work has been done on filtering their behaviour.

Studying the evolution of the behaviour of the coefficients and derivatives as the angle of attack varies is necessary because of the method used to obtain the forces and moments. These have been extracted by performing CFD simulations in which a certain density of the analysis grid has been imposed, like the "Mesh" factor on Solidworks: the choice of this parameter consequently implies the presence of a certain error around the calculated point. The presence of some level of uncertainty at each data set means that the values of the individual points are not regular and/or deviate from a continuous behaviour. This, together with the mathematical nature of the forces and moments developed on the incidence base, are the reasons for a review and re-evaluation of each individual vector.

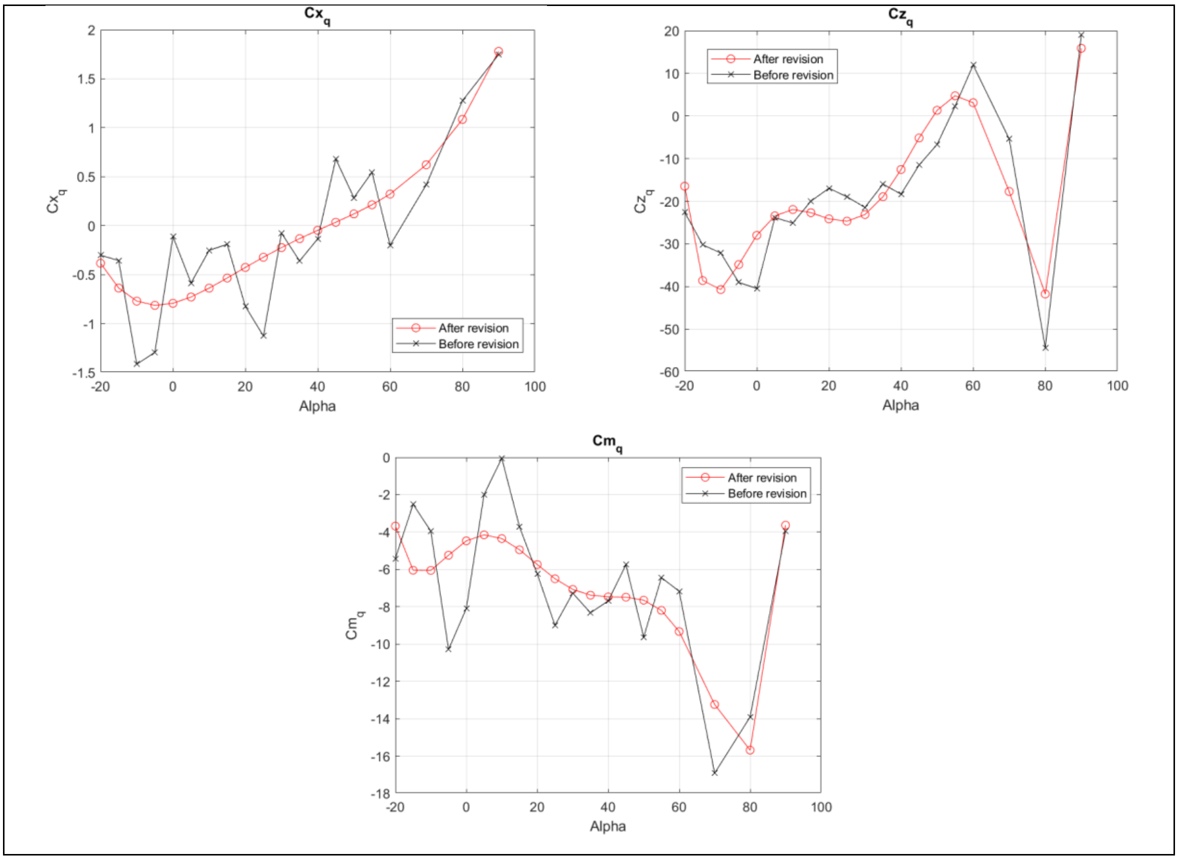
The revision process involved all coefficient vectors and followed a precise path: in fact, in the first phase the coefficients have been graphically plotted as a function of the angle of attack; then the polynomial function that best represents the behaviour of that specific data set has been calculated by a dedicated Matlab code; subsequently, from the interpolating polynomial, the "new" data have been calculated for each angle of attack; finally, the new vector has been plotted over the previous one in order to evaluate the goodness of the new values and whether to adjust the interpolating polynomial. With this intermediate step, the possible propagation of iteration and integration errors of the CFD simulation program has been countered locally. In addition, the optimisation of the data behaviour also makes it possible to reduce interpolation errors between two successive values in the event that there is a change in the curve slope between the two points compared to the general trend.

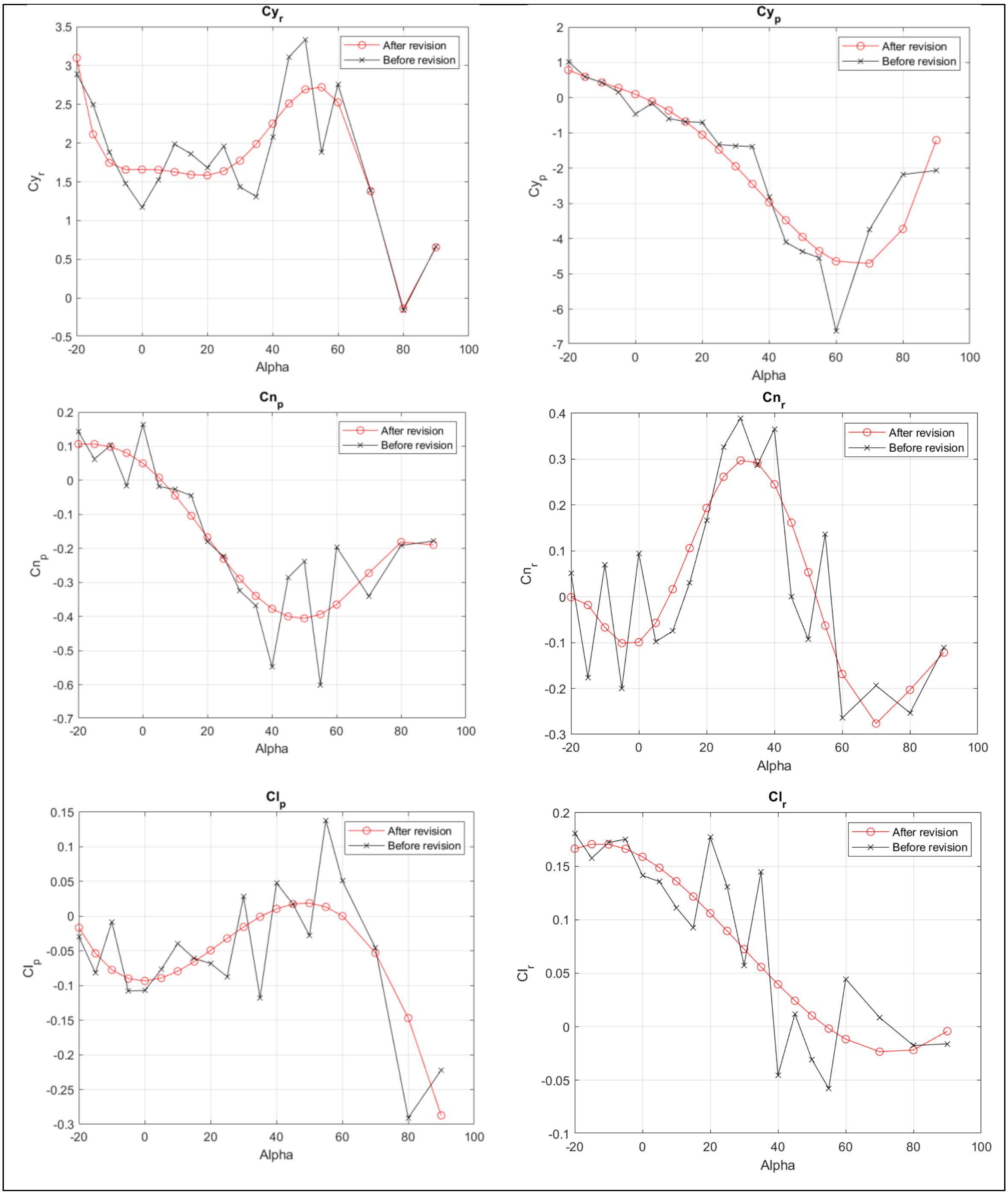
The selection criteria for the degree of the interpolation polynomial has been based exclusively on a graphical perspective: observing the distribution of the points, it starts with a polynomial of degree one and in consecutive steps this order is increased until the curve optimally represents the starting values. The search for the interpolating polynomial has been conducted using predefined Matlab commands that implement the method of least squares in order to obtain the best possible fitting. A Matlab code dedicated exclusively to data interpolation has optimised the calculation process and also the degree of accuracy of the calculated data. In general, to ensure greater accuracy, interpolating polynomials have an order of 4 or 6, as these values are the most commonly used depending on the complexity of the points.

The results of the logical procedure just described are shown below where each figure represents the same data before (experimental data obtained from CFD simulations and calculation) and after the review. On the following pages, not all sets of coefficients are listed, but only some examples of the most significant ones.

The first values to be revised are those of the aerodynamic derivatives in relation to angular velocities along the three axes of the reference system. As can be seen in Table 9 the individual values for each derivative have a high dispersion resulting in considerable local variation of the trends.

Table 9 Review of aerodynamic derivatives in relation to angular velocities

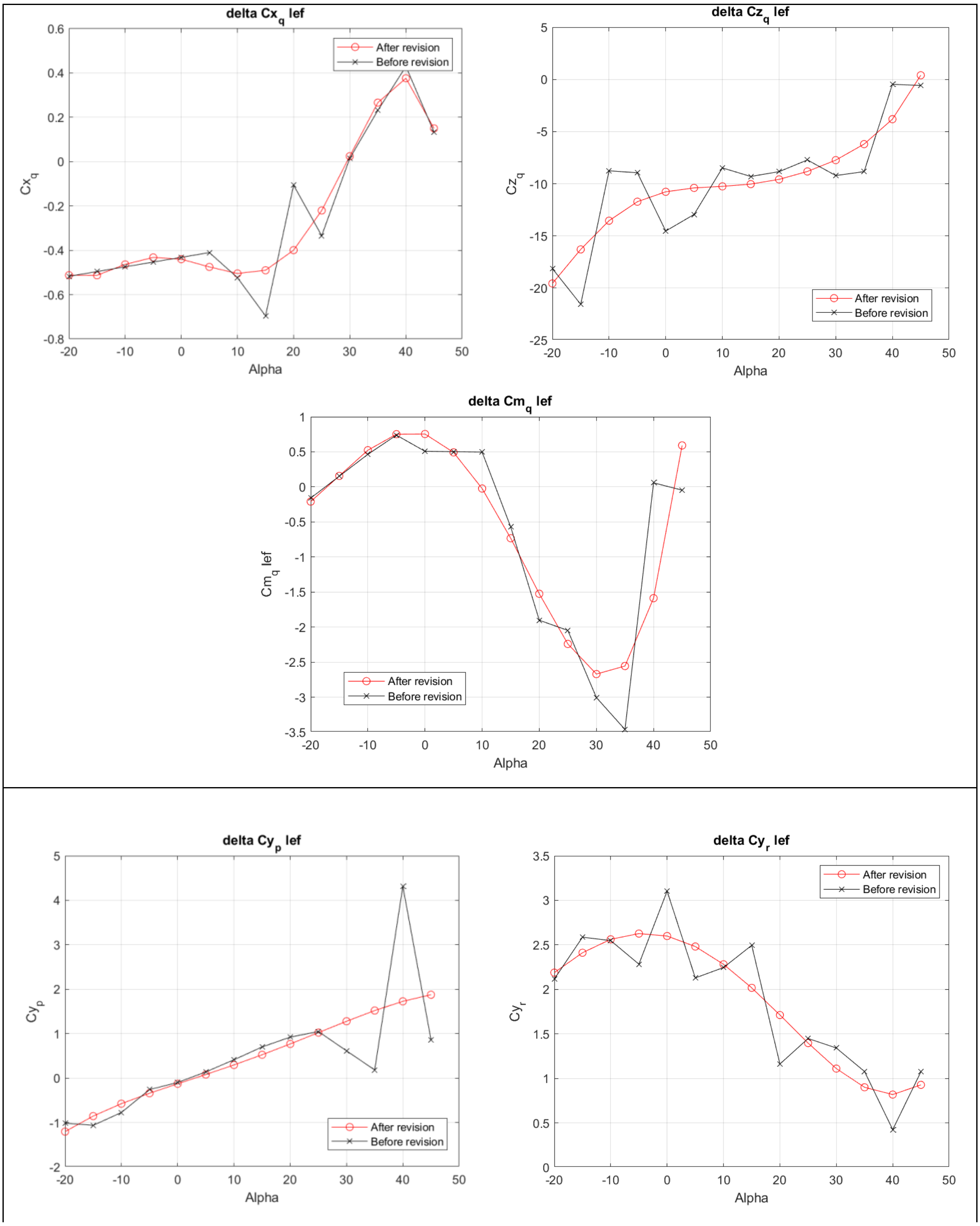


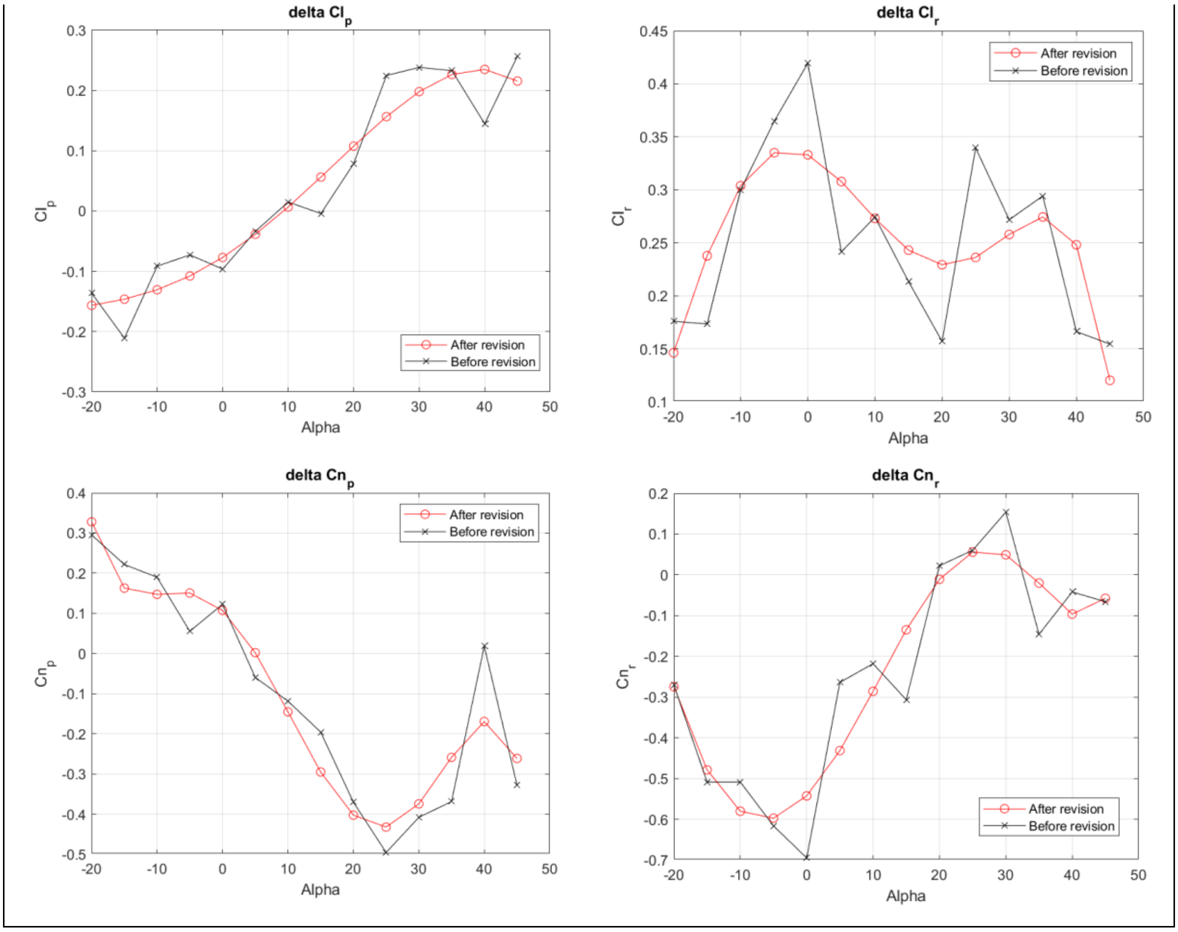


From these images it can be deduced that at high incidences there is greater non-uniformity of data values and curve trends, and this is linked to the presence of a greater number of calculation errors by the simulation program at high AOA values.

Similarly, the same coefficients can also be compared when leading edge flaps ("LEF") are used on the aircraft (Table 10). By comparing the following curves with the corresponding previous ones, the effect of the presence of LEFs can be noticed and how the individual curves change and translate into the space.

Table 10 Review of aerodynamic derivatives in relation to angular velocities in the presence of LEFs



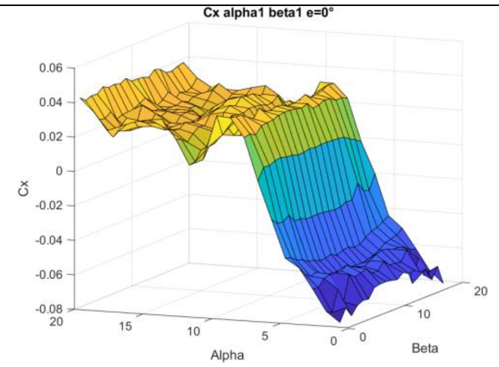
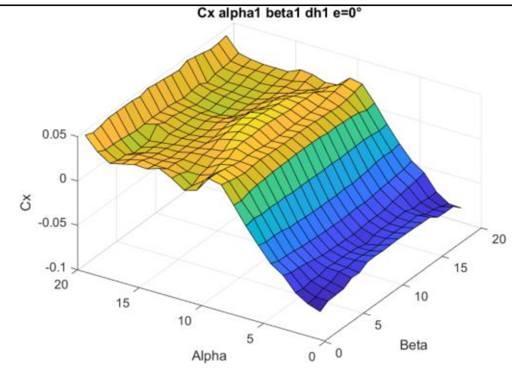


Revision is obviously a process to improve the overall behaviour of the individual coefficient as the angle of attack changes, but it is not without error. The most important negative consequence that can also be seen visually is occasionally a marked localised discrepancy.

The previous coefficients are a function of the angle of attack only, so their representation is two-dimensional, while other coefficients have been expressed as a function of two parameters: α and β. The presence of the sideslip angle is a further parameter of variation which, if considered together with the variation of alpha, makes it possible to obtain three-dimensional charts where the coefficients are represented like a surface. The review of the coefficients that are represented as a function of the two parameters has only been made in relation to the angle of attack, while no assessment of the trend in relation to the sideslip angle has been made.

This category of two-parameter coefficients includes all force and moment coefficients evaluated along the three axes of the aircraft like C_x, C_y, C_z, C_l, C_m and C_n. These six coefficients have also been assessed against the position of the elevator, but in Table 11 only those relative to the elevator at 0° are represented, therefore with the aircraft in clean configuration.

Table 11 Review of aerodynamic coefficients against alpha and beta with null rotation of the elevator

<table>
<tr><th>Before revision</th><th>After revision</th></tr>
<tr><td>
</td><td>
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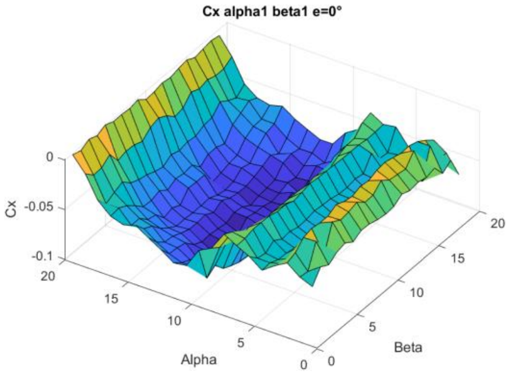
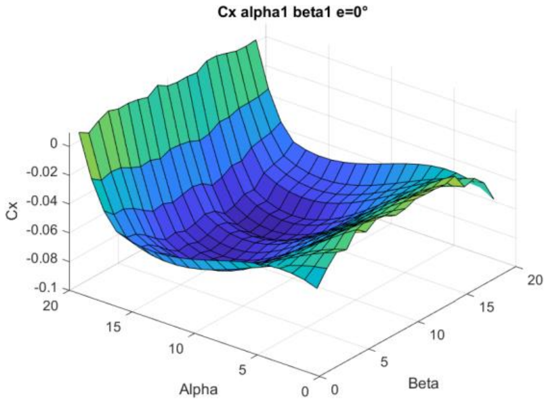
<tr><td>
</td><td>
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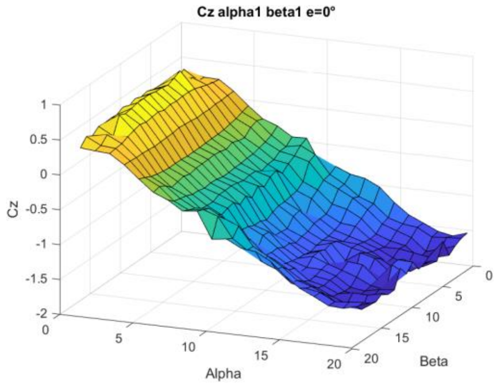
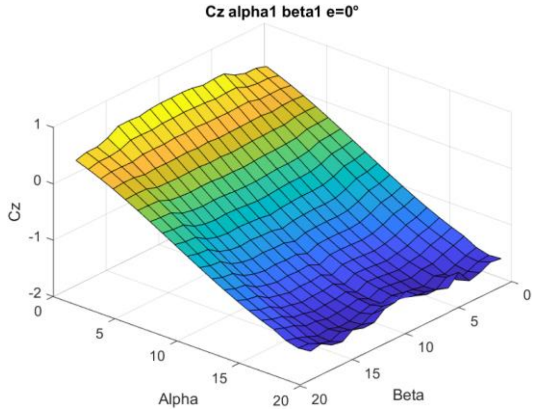
<tr><td>
</td><td>
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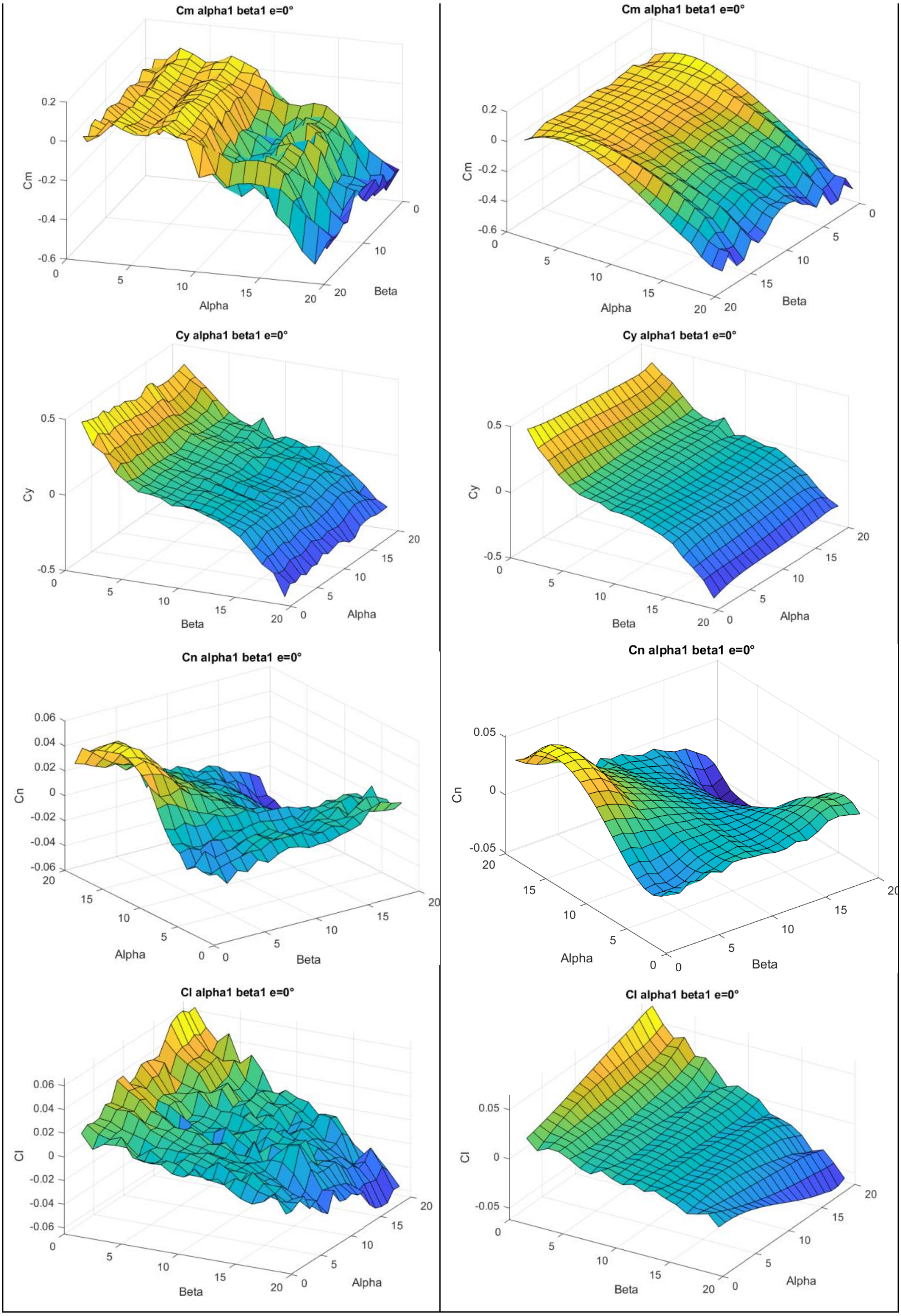
Cm alpha1 beta1 e=0°
Cm
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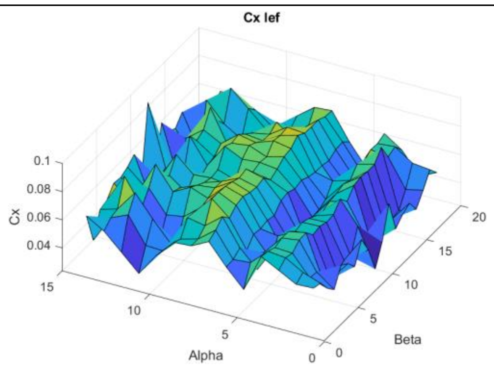
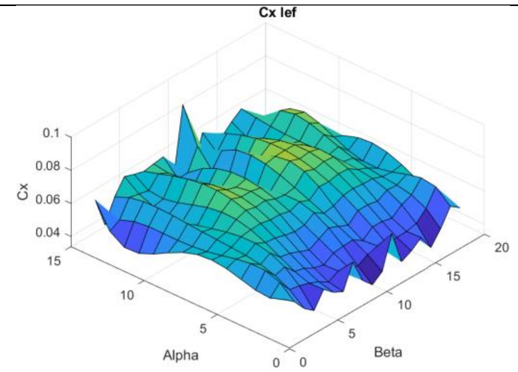
From the observation of the surfaces, it is evident that working on just one parameter allows a considerable improvement in the development of surfaces with a smooth progression.
The previous data do not take into account the possible presence of the leading-edge flaps, consequently the corresponding values considering these surfaces have also been manipulated. As expressed in the previous chapter, these six coefficients were evaluated only as a function of alpha and beta, while the elevator was in the null position. The trends obtained for these data sets are shown in Table 12.

Table 12 Review of aerodynamic coefficients against alpha and beta with null rotation of the elevator with extended LEFs

<table>
<tr><th>Before revision</th><th>After revision</th></tr>
<tr><td>
</td><td>
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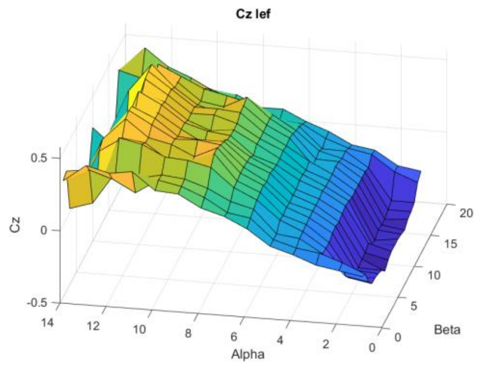
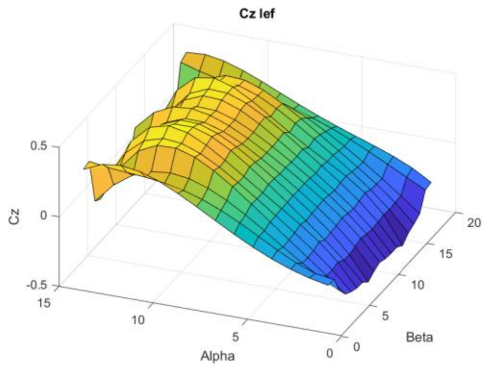
<tr><td>
</td><td>
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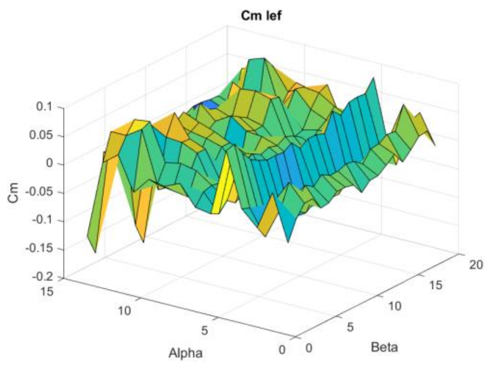
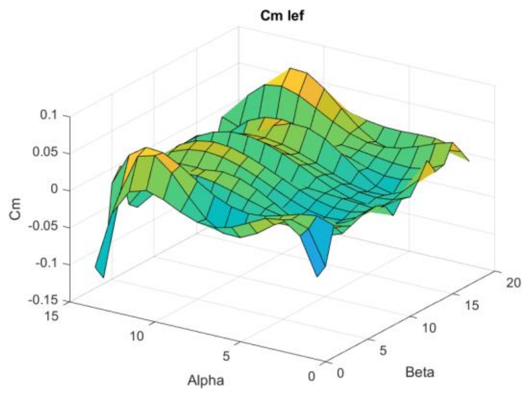
<tr><td>
</td><td>
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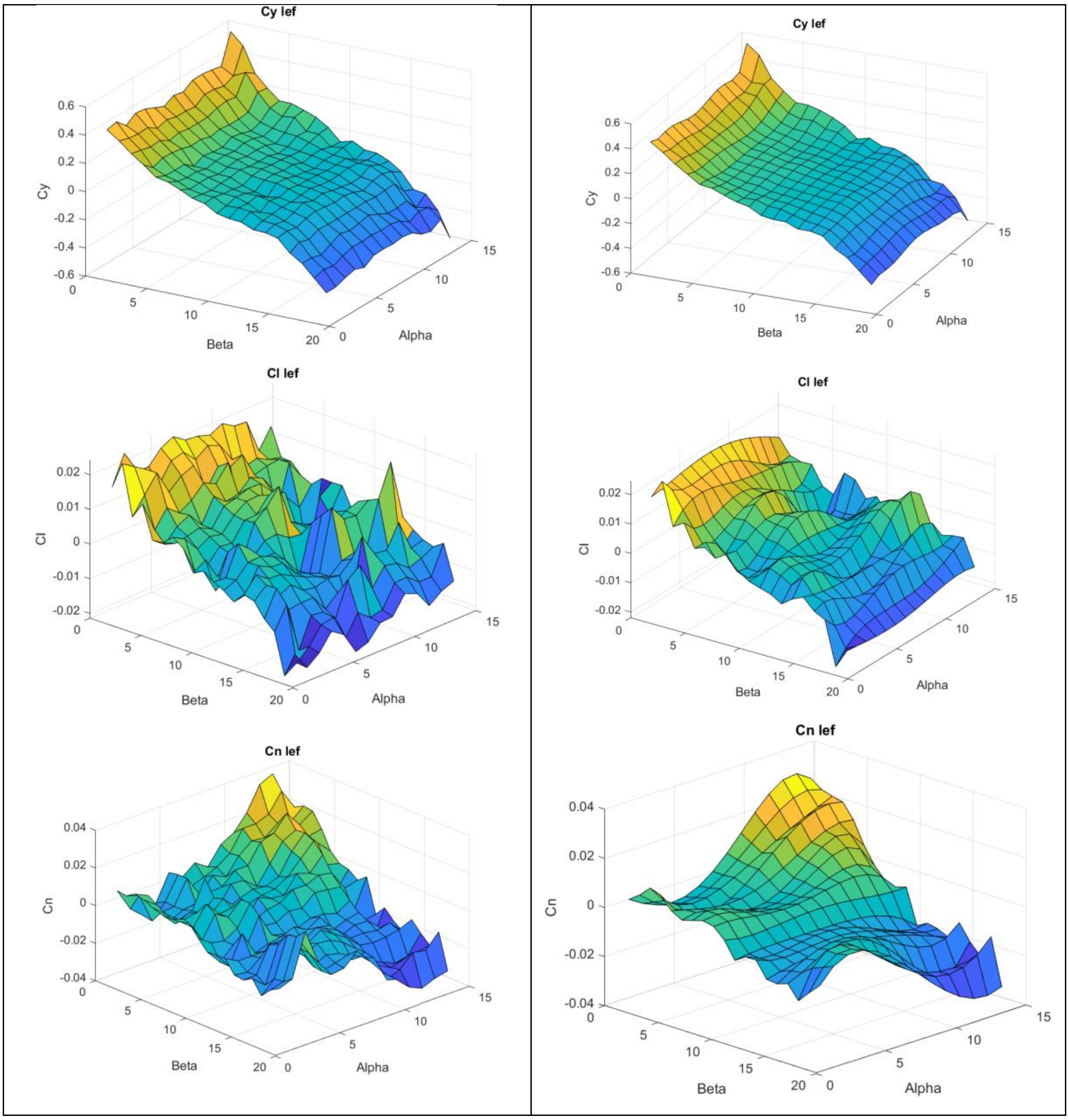
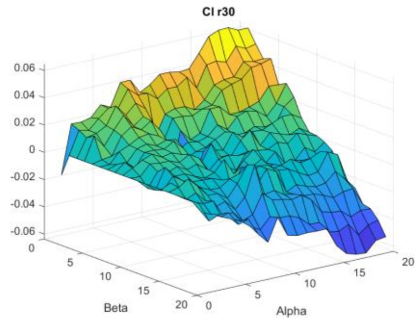
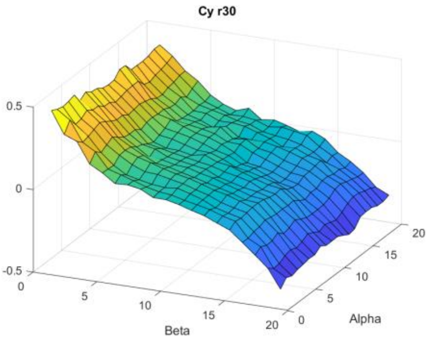
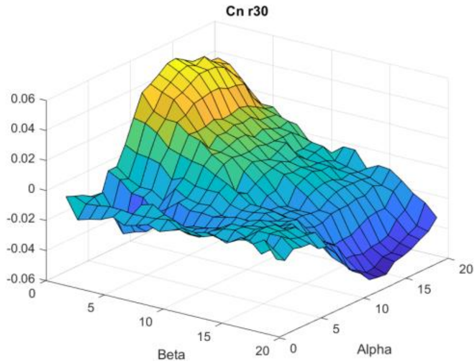
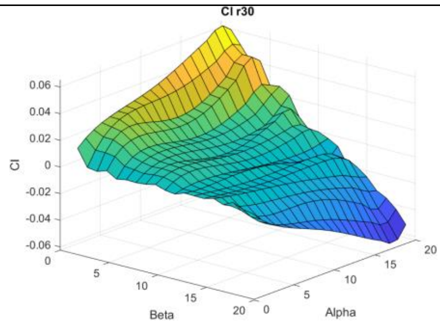
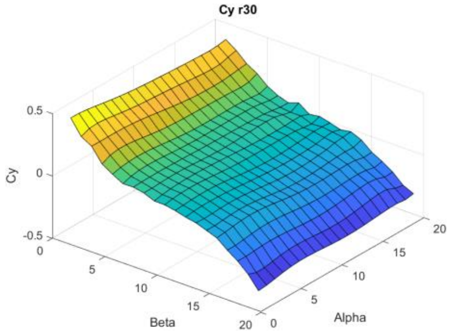
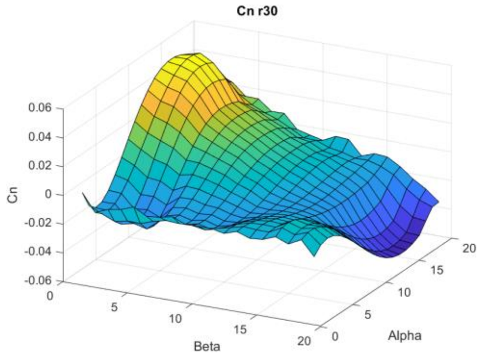
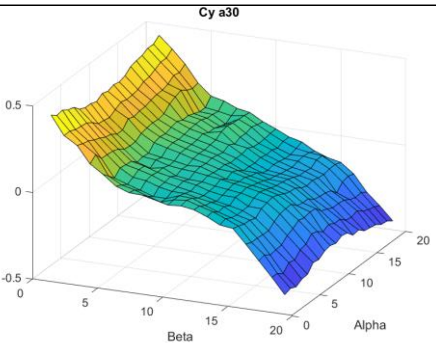
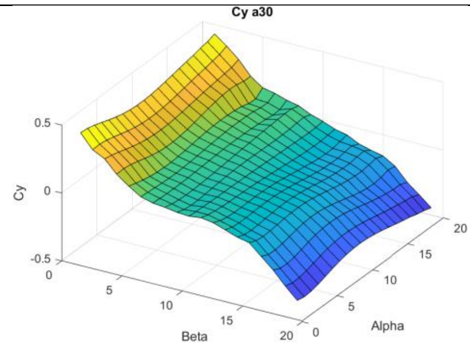
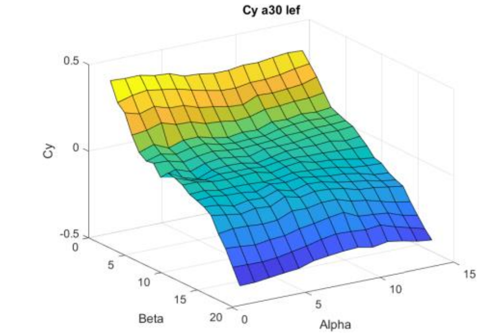


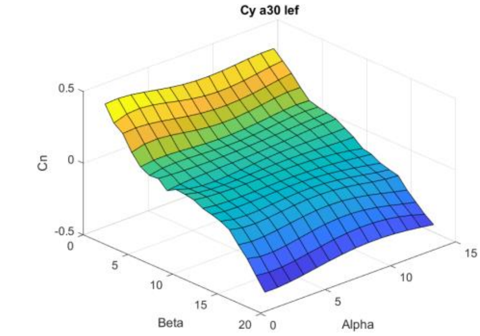
By comparing the above values with the previous ones, the presence of the leading-edge flaps results in coefficients with quite comparable trends to the previous case but with a greater dispersion of the data. This is partly corrected by the revision, but some residual scatter remains.

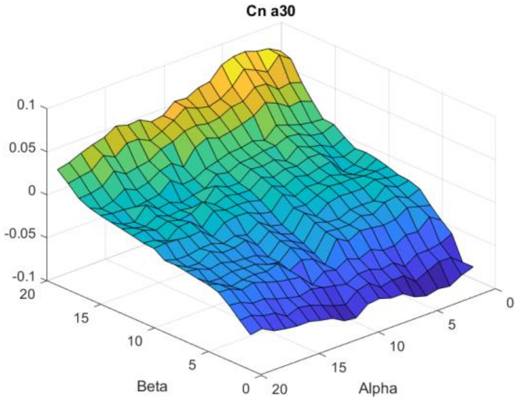
Among the many coefficient evaluations, there are also those where the coefficients are evaluated as a function of rudder and aileron position. The actions of these primary control surfaces are again a function of AOA and sideslip angle, so even in this case the coefficients, both original and modified, can be represented as surfaces. Comparison of the data before and after revision can be done using the images in Table 13.

Table 13 Revision of aerodynamic coefficients when ailerons, rudder and LEFs are actuated

Before revision	After revision
  	  
	


Cy a30 lef
Cy
Beta
Alpha


Cy a30 lef
Cn
Beta
Alpha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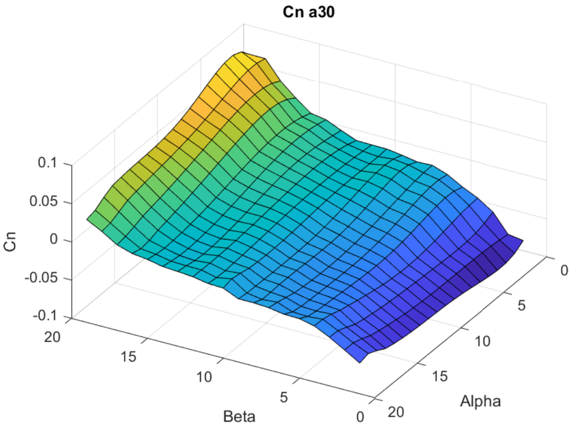
Cn a30
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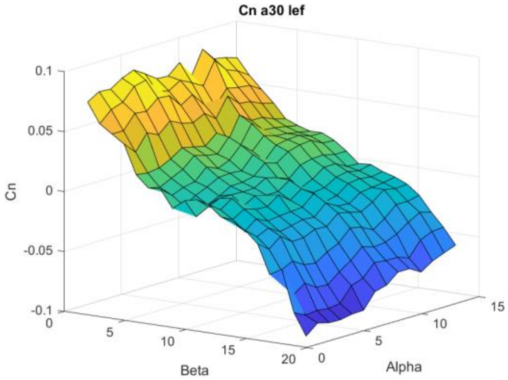
Cn a30 lef
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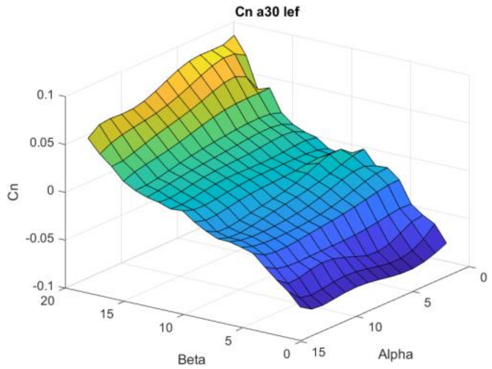
Alpha


Cn a30 lef
Cn
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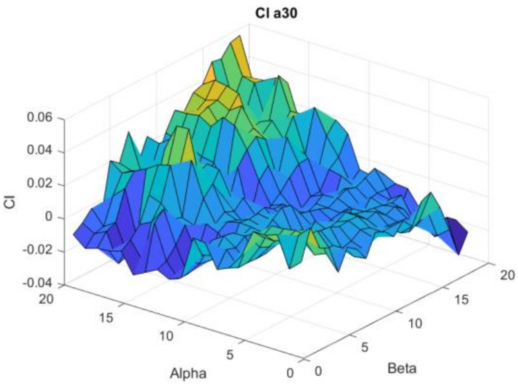
Alpha


Cl a30
Cl
Alpha
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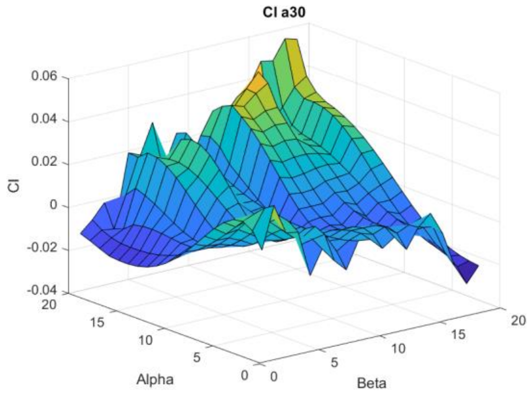
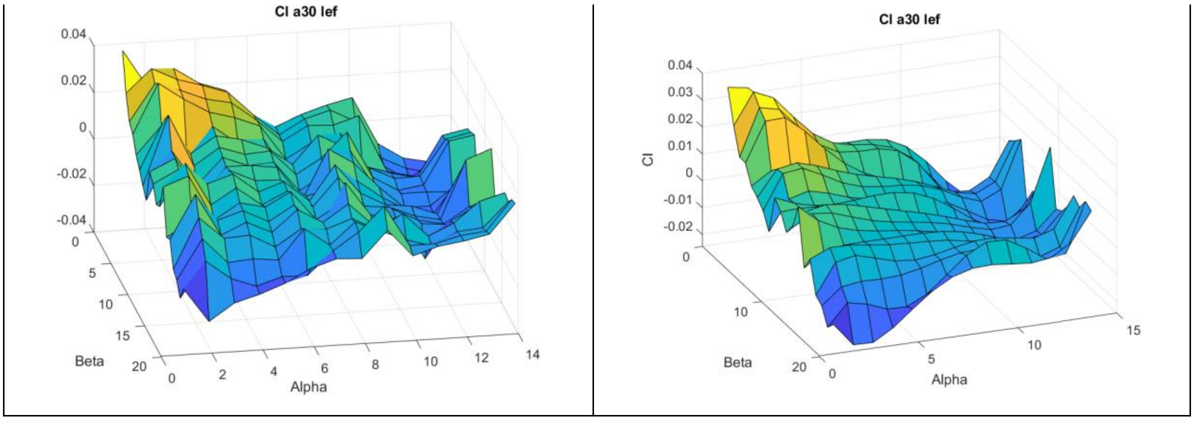

Cl a30
Cl
Alpha
Beta



The process of revisions to the coefficients involving the rotation of the control surfaces has not significantly altered the data; in fact, as can be seen, the corrections have been minimal.

This revision process can be considered as a filter that allows the calculated coefficients to be modelled so that errors due to the fluid dynamic simulation process are reduced. The reprocessing of the data has therefore made possible the reconstruction of the same vectors of coefficients as in the previous chapter and the application of these to the mathematical model simulating the behaviour of the M-346.

Chapter VI –Flight Model

Mathematical Flight Model Structure

The aim of the work carried out so far is to obtain a set of data under specific conditions from which to interpolate the values necessary to represent the flight conditions that the user wishes to analyse. The study of the trim condition and flight dynamics of an aircraft, of the M-346 in this case, is carried out by a code capable of reproducing the behaviour of the aircraft after the definition of certain input parameters. This code represents the mathematical model, which is essential for transforming the previously calculated coefficients into aircraft responses.

The operation of the mathematical model developed for the M-346 can be schematised as in Figure 14. This block diagram summarises the functioning of the entire model.

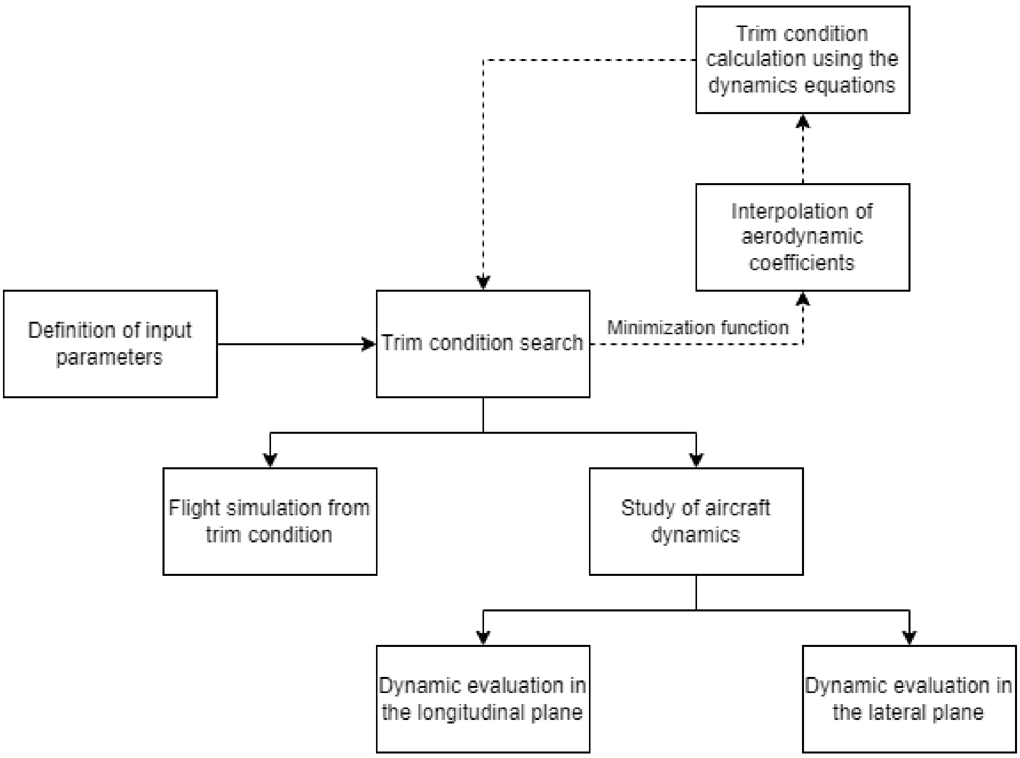


Figure 14 Functional diagram of the developed mathematical model

In order to develop the mathematical model, it is necessary to define the input parameters, which are all the values that characterise the aircraft's dynamics, so in first place the code request the flight altitude and the speed. To follow up, the angular rotation speeds and any disturbances on the control surfaces can be included as input parameters, but in the following result these values have been fixed to zero to evaluate the trim condition in a steady level flight. These are the main inputs that the user has to submit to the code, even if inside the model there are many other values that can be considered as incoming values, with the peculiarity they are not directly chosen by the user, since they are function of the speed and the altitude.

These secondary input values are the angle of attack, the thrust and the positions of the primary control surfaces (hence the angular position of ailerons, rudder and elevators). For the first iteration loop only, these five data are expressed as a function of the altitude, specifically the angle of attack and the thrust which have the main variation with altitude. In fact, once the tangency altitude is known, it is divided into steps of $2000\ ft$ and at each step the conditions change, which have been obtained by applying the lift and resistance formulas given by the respective coefficients, speed and density at that specific altitude. The five parameters thus interpolated are used for the first cycle of finding the minimum trim condition, which, according to the number of cycles that the user may decide, calculates for each loop the minimum trim condition taking each time as initial reference values the minimum conditions found in the previous one. The number of cycles is at the user's discretion and serves to achieve greater accuracy, at the expense of higher computational consumption.

Each search cycle allows trimming conditions to be derived and to do this it uses a minimisation function, which tries to find, for the given input, the trim values by an iterative method.

The minimisation procedure needs an optimisation condition which defines the aspects that determine the achievement of the minimum condition: this is represented by a vector containing four elements which are the tolerance of the function, the tolerance of the variable of the function itself, the maximum number of iterations and the maximum number of function evaluations allowed. The dimensions of these quantities are shown in Table 14:

Table 14 Numerical values of characteristic parameters for the minimisation function

Tolerances on the function	10^{-30}
Tolerances on the variable of the function	10^{-30}
Maximum number of iterations	$5 * 10^{5}$
Maximum number of function evaluations	10^{5}

When one of the values of the previous table is reached, the code considers this value as the minimum condition researched. These values make it possible to achieve high accuracy in the calculated data, resulting in an even more reliable trim condition calculation scheme.
The maximum number of iterations and evaluation of the function are essential because this minimisation technique is based on a cost function: this means that each time every single parameter is perturbed until all the state derivates are minimized. Given the importance of the parameters of the cost function and the fact that any variation in these parameters affects the results obtained, these data have been set using as a reference those applied in the development of the model of an aircraft of a similar category.
The minimisation function for finding the trim condition needs to minimise the variables of the state vector, which is provided by a further function that calculates the forces and moments acting on the aircraft in those determining flight conditions. This function represents the heart of the mathematical model as it implements all the equations of the aircraft dynamics. This code is flanked by another one that only takes care of interpolating the different coefficients known as angle of attack, angle of sideslip and control surfaces position.

Once the search for the trim condition has been completed, the programme allows two separate paths to be followed. The first one allows to simulate for a certain period of time (defined in the code) the flight of the M-346 starting from the trim condition and then to evaluate if actually this condition represents an equilibrium condition or not. The simulation of the aircraft starting from the trim condition uses a Simulink model that reproduces the dynamics of the aircraft even when using the state vector calculation function (this is the same function used to calculate the forces and moments for the trim condition).
The second path, on the other hand, is used to research and evaluate the dynamics of the aircraft and this study is carried out independently for the longitudinal plane and the latero-directional plane. The separation of the two characteristic planes is useful for a simpler study

of the dynamics; in fact, by separating the entire dynamics into two, it is possible to work with smaller matrices and the evaluation of the aircraft's responses is facilitated, as are the implementation of any compensations system.
All the input and output values of each function and of the system in general are continuously checked to ensure that they do not exceed the permissible limits, and if they do, they are automatically set to the limit values. This multi-point control check aims to make the study of aircraft dynamics as close as possible to the real case where the performance of the various components, both hardware and software, have operational limits.

Trim status

The search for the trim condition is a very important part of a mathematical model as it allows to evaluate if what has been done before (calculation of coefficients and aerodynamic derivatives and implementation of the functions of the model itself) is correct for the calculated quantities.

The study of the trim condition can be done through the analysis of the data obtained from the minimisation function, so through the angle of attack α, thrust, elevator angle δ_e, aileron angle δ_a and rudder angle δ_r. To extract these five data, the trim search is performed by providing the code with the altitude and speed at which the aircraft must be trimmed, and these two values alone are therefore sufficient to determine whether the aircraft is trimmable. The variation range of the altitude and the speed must be designed to take into account the entire flight envelope of the M-346. It is essential that the aircraft is trimmable throughout its entire operational range. The flight envelope considered for the study does not take into account flight conditions where the speed is below $150\ kts$ because at speeds below that the pilot directly controls the aircraft by deactivating the aircraft control system. With this approach, the flight envelope considered for the study of the trim condition is reduced compared to the original, guaranteeing in any case a vast field of application: the resulting diagram is shown in Figure 15.

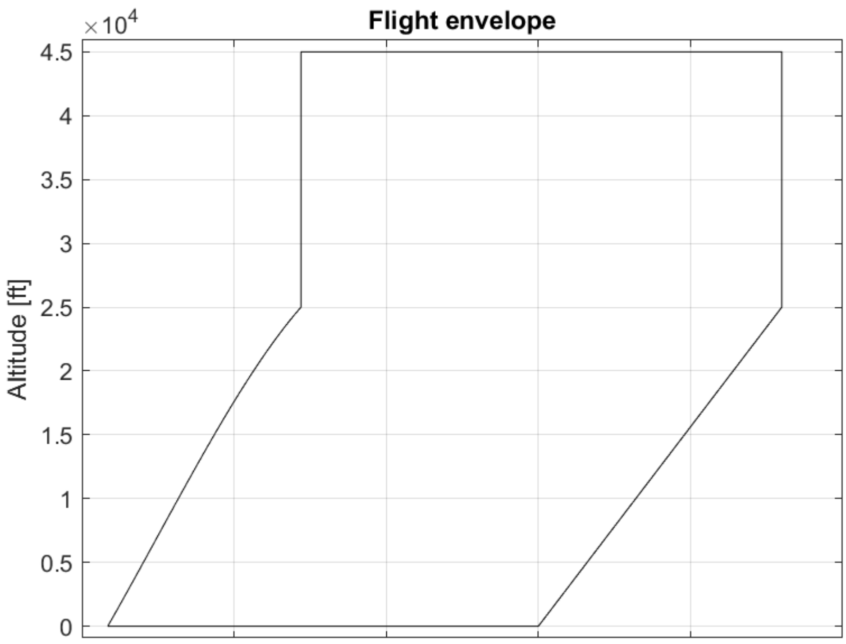


Figure 15 M-346 simplified Envelope Diagram

The envelope diagram is useful as it defines the speed and altitude limits at which the model should ensure that the trim condition is reached. The operational range of the aircraft is represented by the data shown in Table 15.

Table 15 M-346 altitude and speed operational limits

	Min	Max
Altitude	$0\ ft$ (On ground)	$45000\ ft$
Speed	$150\ kt$	$570\ kt$

Providing only an altitude and a speed, the system generates a trim condition only for these data and this does not permit to evaluate the global behaviour of the aircraft as it is only a local data, so for a general evaluation it is necessary to vary both data simultaneously to see how the five characteristic parameters evolve. The study of this evolution occurs through a code in which two vectors are defined, speed and altitude respectively, containing a set of equally spaced data. These are then inserted into a double iteration loop in which the velocity is varied in the innermost cycle and the altitude is varied in the outermost cycle. In this way it is possible to diagram the curves of a magnitude as a function of speed for different altitudes.

These types of representation provide a way of evaluating the effects of the two input variables simultaneously. In fact, considering a single variable, it will have a certain continuous behaviour with the variation of the speed (increasing and/or decreasing) at the same altitude, while the general effect of the altitude change is to translate in the space this curve. Obviously, the altitude has also an influence on the curve's trend, but the effect is minor compared to that of the translation.

The curves obtained for the five variables that describe the trim condition are shown specifically on the following pages. The relative graphs show, for explanatory purposes only, the values at specific altitudes so as not to generate too much confusion in the reading of the data. The altitudes that will be reported allow you to cover the entire flight envelope, from ground to tangency altitude, with an equidistant variation of $5000\ ft$.

Angle of Attack - α

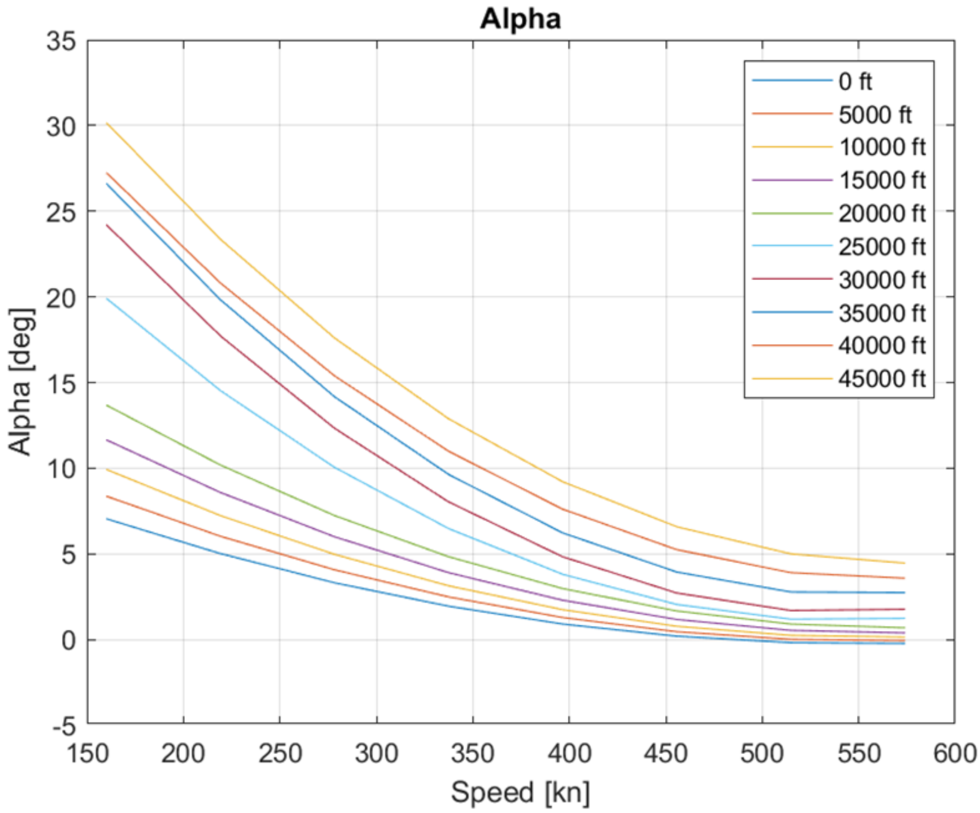


Figure 16 Angle of attack at trim conditions

The angle of attack is undoubtedly one of the fundamental variables for studying the flight mechanics of an aircraft, because it measures the inclination of the longitudinal axis of the aircraft in relation to the direction of the flow. Figure 16 shows just this angle, which represents the incidence of the aircraft in the trim condition, as all values it assumes at the predefined altitudes are the incidence at which the aircraft is in equilibrium condition.

The first effect that can be seen is that the angle of attack has a decreasing tendency with speed in a hyperbolic manner, reaching at very high speeds a situation of tangency, a symptom of the fact that the speed term within the force and moment calculation formulae assumes a significant weight, so for the same quantity assessed, the relative aerodynamic coefficient can be smaller (with a consequent reduction in the angle of attack).

The translational movement due to altitude is clearly visible: as the distance from the ground increases, the curves of the angle of attack move to higher incidences. The difference in angle between one altitude and the next one tends to decrease as the speed increases. The reason for this behaviour is related to the physical nature of the equations of forces and moments: speed is a term with a second degree, the angle of attack (expressed through the coefficient) is of the first, and finally the air density changes as it depends on the altitude. It

can be deduced that as the speed increases, the relative term is always greater, while the air density term decreases exponentially, so the compensation is given by the rise of the aerodynamic coefficients.
It should also be pointed out that at low speeds and high altitudes a slight error is introduced due to the mathematical model, specifically the minimisation function, which has a higher cost when compared to the same cases at low altitudes. Being a minimisation error, it can be assumed that it is related to the decrease in density or even to the local values assumed by the coefficients.
The trends of the angle of attack as a function of speed and altitude reflect the predictions of the theory of flight mechanics, and as far as the range of variation of the same angle is concerned, it remains within the operational limits of the aircraft, so it can be deduced that the values obtained seem plausible.

Thrust

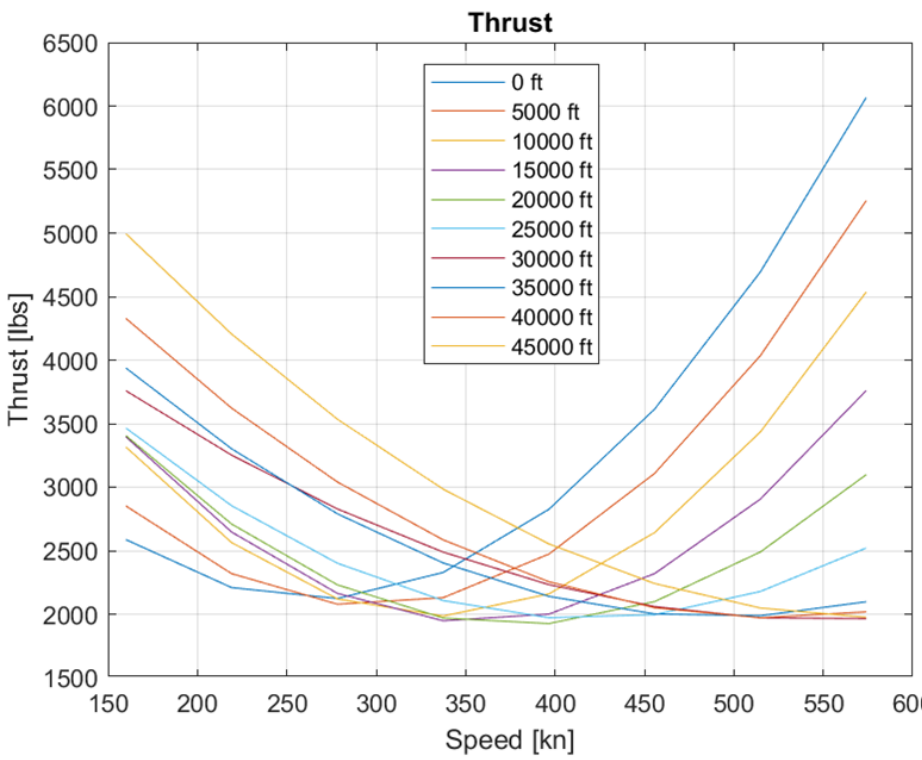


Figure 17 Thrust at trim conditions

Thrust is a required force to ensure the equilibrium in a nominal flight condition as it serves to balance the drag to which the aircraft is subjected and thus allow the longitudinal motion of the aircraft. As mentioned, the thrust must balance the aircraft's resistance component, so it should have a parabolic behaviour, if evaluated as a function of speed, with a minimum within the operating range.

The behaviour of the trim thrust calculated in the case of M-346 can be seen in Figure 17. In this image, it is worth noting the inverse behaviour of the curve as altitude increases, in fact, at low speeds the required thrust for the trim condition increases as altitude increases, while at high speeds the required thrust decreases as altitude increases. This general behaviour is always due to the physical nature of the equations that govern the flight mechanics, in particular in this case the variables that have a greater impact are speed and density, which vary considerably and in different ways.

Further evidence is that the minimum conditions are obtained for intermediate velocities, in specific, the point of minimum moves at progressively greater velocities as the altitude increases.

Elevator - δ_e

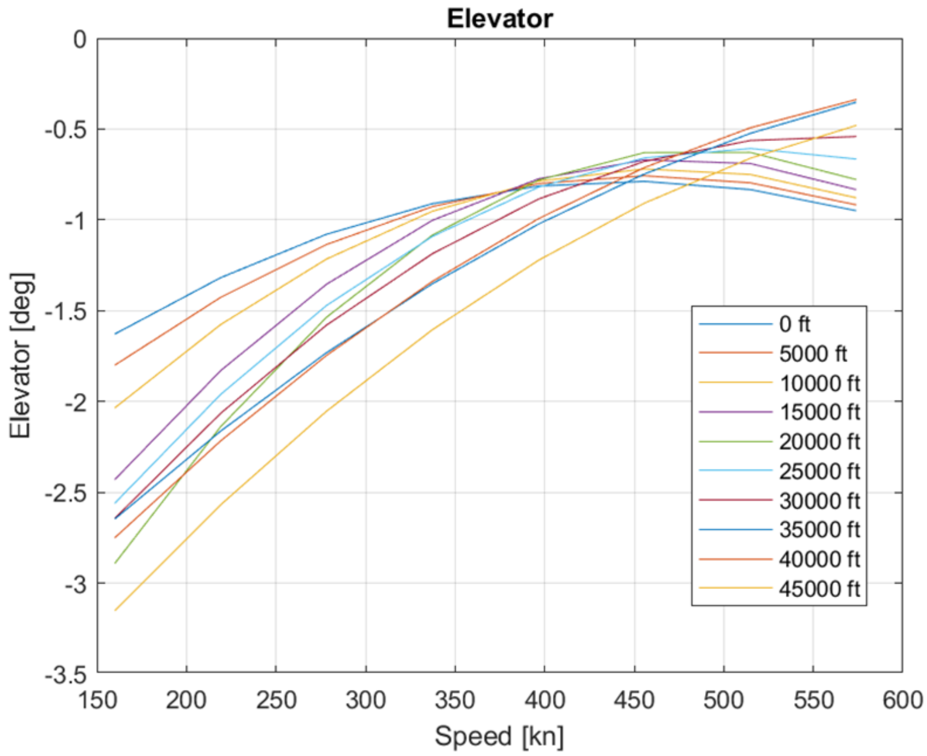


Figure 18 Elevator at trim conditions

The elevator is the control surface that ensures the longitudinal control of the aircraft, so the angular positions it assumes at different speeds affect the controllability and stability of the aircraft. On standard aircraft this surface is an assembly called a horizontal stabiliser consisting of a fixed and a moving surface, but in the case of the M-346 it is an all-moving tail, meaning there is only one moving surface that rotates. Since it is necessary to ensure longitudinal control, this surface takes on different angular positions depending on the speed, as can be seen in Figure 18.

The elevator angle tends to decrease with increasing speed at the same altitude and this behaviour is also present with increasing heights. In this case there is not a simple translation of the curves due to the effect of altitude but there is also a change in the slope of the curve which increases as well. The behaviour of every single curve obviously is subject to the errors present in the mathematical model that is correlated to the coefficients or to the function of minimization, in fact it is for this motive that the curves introduce of the "erroneous" points. Despite any local errors, the general trend seems to respect the flight mechanics by providing a downforce tail surface.

Aileron - δ_a

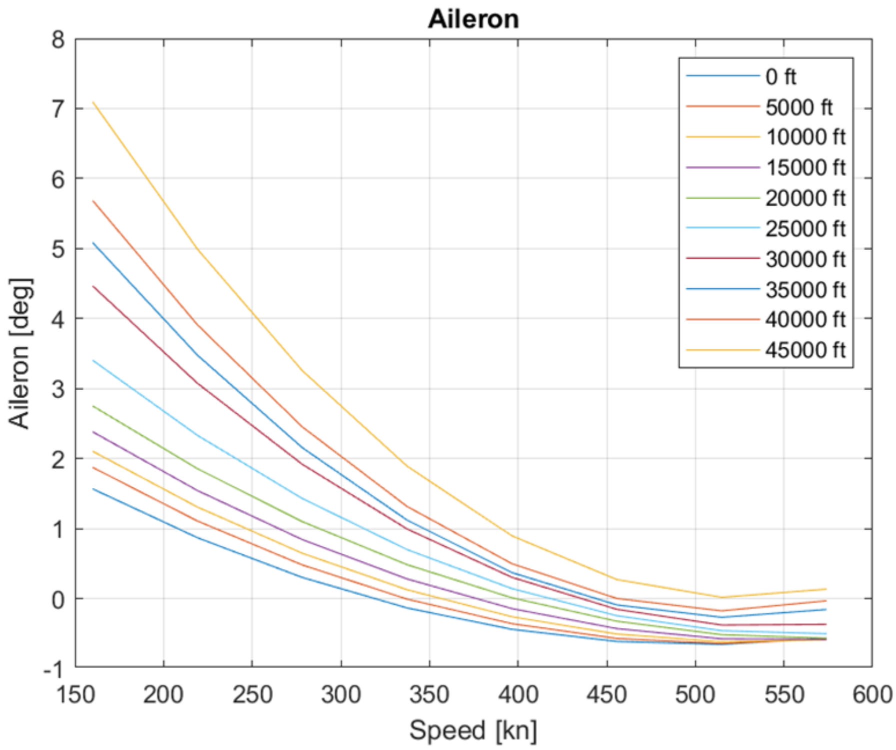


Figure 19 Aileron at trim conditions

Ailerons are responsible for the control of the roll moment, consequently in trim conditions these surfaces should not present particularly large rotations as the aircraft should remain as much as possible with levelled wings and consequently, they should only balance the coupling of the dynamics of the other control surfaces and the aircraft itself. The roll moment compensation for the trim condition is more affected at low speeds, while at high speeds this effect is minor, and this can be seen from Figure 19 where it is observed that at the same altitude the ailerons are positioned at lower angles as the speed is increased.

The reductive attitude of the aileron angle with speed is maintained in all the different curves representing the several altitudes, as the distance from the ground increases, the curves translate vertically at greater angles, with a behaviour similar to that of the angle of attack. Indeed, also in this case at the same speed, the angle gap between two different altitudes is greater at low speeds, while it progressively decreases for higher speeds.

Although the curves represent equally spaced dimensions, graphically this is not observable between the curves and the cause is always to be traced back to the minimization function and interpolation errors of the various aerodynamic coefficients and derivatives. Since these are interpolated values from tables and vectors it is possible that inaccurate data may be spread leading to these results. Although this aspect is important, the diagrammed values faithfully reflect what is expected from these control surfaces.

Rudder - δ_r

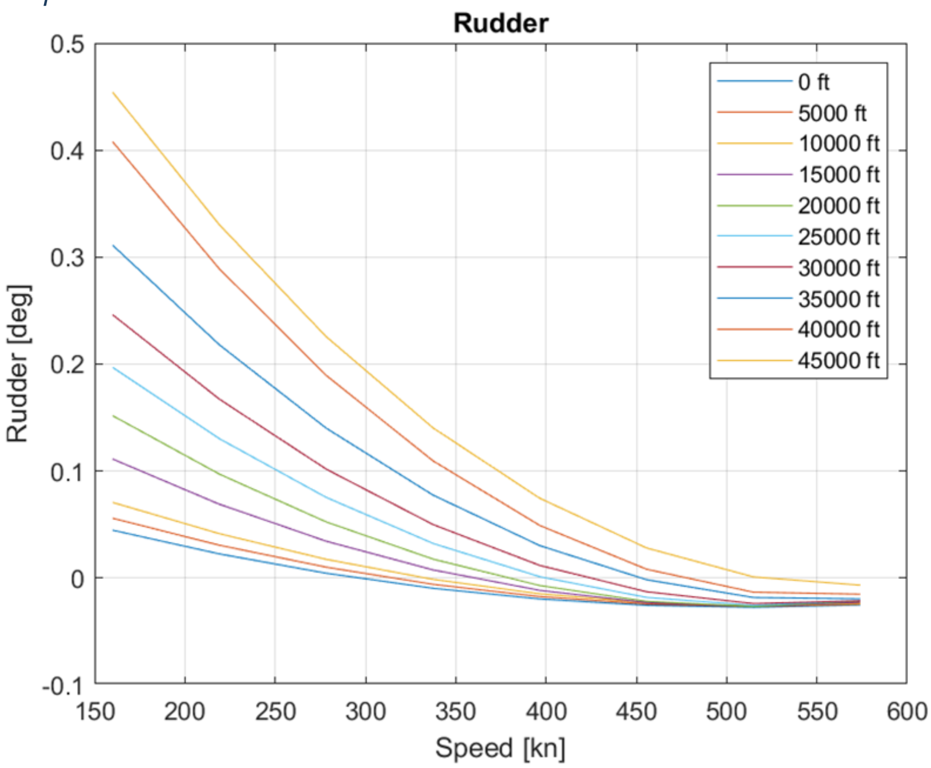


Figure 20 Rudder at trim conditions

The control of the aircraft along the z-axis is provided by the rudder, which is used to generate and compensate the yawing moment. As in the case of the ailerons, the rudder should be in or around the neutral position as much as possible to compensate for yawing due to aircraft dynamics. Unlike the previous controls, the rudder has a smaller rotation, even by orders of magnitude, so it macroscopically remains around the neutrality. In detailed analysis, it varies with speed and also with altitude: this angle decreases continuously as the aircraft flies faster and this trend is found at all altitudes whose relative curves shift and increase in slope as the height above ground increases. The δ_r trends are shown in Figure 20 where the angle of each altitude trends asymptotically towards values very close to each other at maximum speed.

In the view of what is also shown in the chart, the values and trends representing the trim conditions are quite plausible and reliable.

Once the search function for the trim condition and the values obtained have been verified, the mathematical model of the M-346 can be extended with additional parts such as the code dedicated to flight simulation (given the trim conditions), and the code dedicated to studying the aircraft dynamics.

Maintaining trim status

Trim represents the equilibrium condition of the aircraft, so if it maintained these conditions over time, it would have to perform a horizontal linear motion. The previously obtained conditions from a theoretical point of view should be trim conditions indeed, but to verify that they really are, it is possible to simulate the flight behaviour of the aircraft from the flight conditions obtained from the trim search function. Simulation can be carried out through the implementation of a Simulink scheme which emulate the behaviour of the aircraft over time. This scheme is a continuous loop in which, starting from the conditions at zero time of the angle of attack, thrust and the positions of the control surfaces, it reiterates for the simulation time the values obtained from the integration of the state vector (calculated from the dynamics equations). With this simple interaction over time, it is possible to simulate the flight of an aircraft and assess its behaviour.

Several sample tests have been carried out on the M-346, but in the following pages only two simple examples are reported, evaluated at the same speed but at different altitudes. The choice of reporting these two cases is also intended to highlight the influence of altitude on the trimming condition. The speed at which the two examples will be assessed has been chosen arbitrarily, considering the entire flight envelope, in fact it has been selected the intermediate speed which corresponds to $360\ kt$ ($\cong\ 600\ ft/s$). An arbitrary choice has also been made on the altitudes, taking into account a relatively low altitude of $10{,}000\ ft$ and a plausible cruising altitude of $35{,}000\ ft$. The selected values are purely for example, as the same behaviour is observed for the entire flight envelope.

The first example case is for low altitude and the corresponding results of the simulation are shown in Figure 21. This figure shows the most characteristic data, including the three-dimensional movement in the air and the altitude trend: carefully, for the entire simulation time the aircraft loses $0.8\ ft$ of altitude, which is an imperceptible amount. This loss is not related to the Simulink scheme, but to the values that are provided to the scheme, therefore to the calculated trim values. As already expressed above, the computed trim conditions are subject to an error due to the calculation method and this fault can be preserved in this case. Despite this consideration, the loss of elevation found is negligible compared to the overall complex.

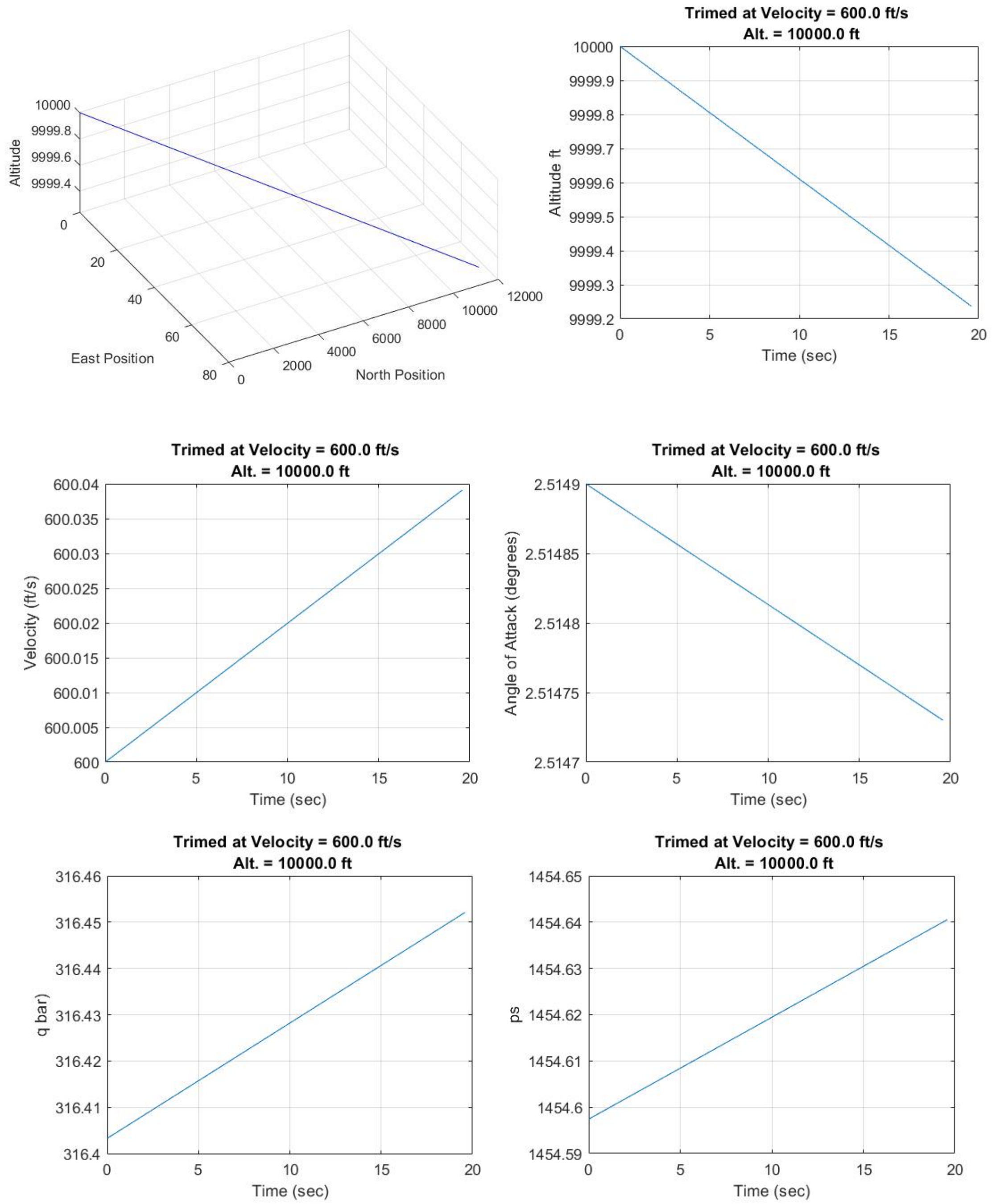


Figure 21 Maintaining trim condition at 10000 ft altitude

The slight loss of altitude is also observed in the Θ angle and α angle, which decrease by a very small percentage, while there is a very small increase in speed and pressure (lower altitude means greater air density), and therefore also in dynamic pressure. All this is a symptom of what has already been expressed above.

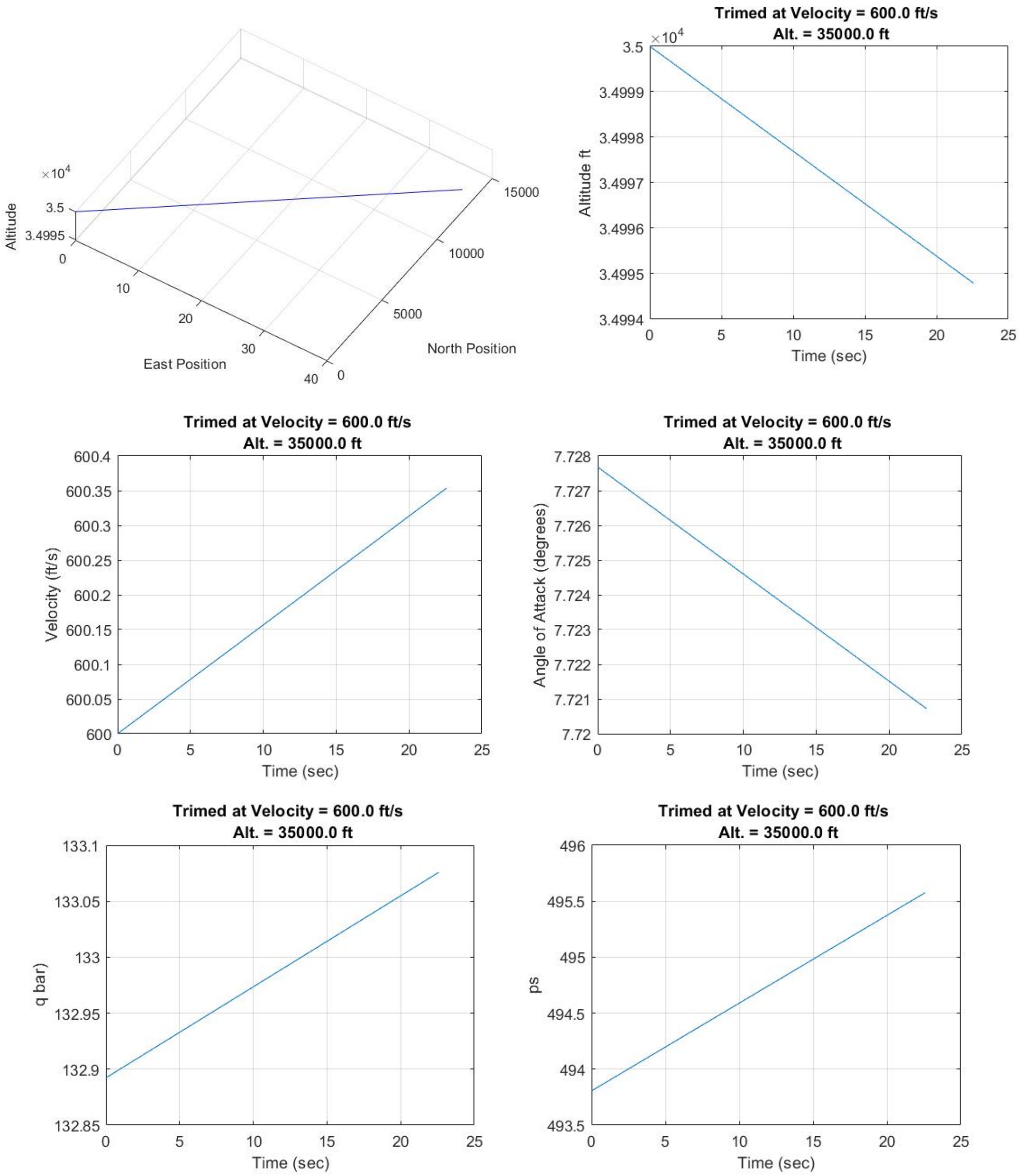


Figure 22 Maintaining trim condition at 35000 ft altitude

Maintaining the same speed and increasing the altitude up to cruising level, the achieved data does not differ from what has been noted for the results obtained at low altitude, which is confirmed by the results in Figure 22.

The loss of height in this example is greater than in the previous case, with the same simulation time the loss of altitude is $4.5\ ft$, which is still a very low percentage compared to the initial altitude. Also in this scenario, the descent of the aircraft is associated with an increase in speed and dynamic pressure, with a reduction in the angle of attack: these variations are always very small and negligible on the whole. These fluctuations are always attributable to the propagation of errors from the method used for the fluid dynamic analysis of the aircraft.

These two simplified examples are a further aid in checking the validity, even if not in a rigorous manner, of the previously determined trim conditions for different speeds and altitudes.

Study of Aircraft Dynamics

Aircraft dynamics is an integral part of a mathematical model for a flight simulator and consists of a system of equations of multiple variables. The derivation of these equations is correlated to the considered reference system of the aircraft, which is, for this model, the aircraft-based body system.

The study of the dynamics of the M-346 is carried out using a dedicated Matlab code, which creates a time invariant model by applying linearisation of the equations. This dynamics evaluation model was based on the dynamics model of an F-16 flight simulator. The advantage of using such an approach is that the user has only to pick the altitude and speed at which to investigate the dynamics, while the code takes care of generating the matrices for the dynamics study.

The code used for the study of dynamics has a block structure, in which each block performs a different function. The first step is the determination of the trim conditions for the known altitude and speed: this step serves for the next block, since the five parameters obtained will be used for the integration process. The latter is used to obtain the four matrices that define the study of the dynamics through a linearisation process.

The calculation of the individual matrices uses a Matlab function (the "linmod" function) to extrapolate a continuous-time linear state-space model around an operating point represented by the trim status. This Matlab function requires a Simulink model which is able to interpolate the coefficients and aerodynamic derivatives given the trim conditions, apply these data to the equations of dynamics in order to obtain the derivative of the state vector, integrate this vector and iterate the whole process. The vector of state variables in the model consists of 12 elements that define the aircraft's dynamics:

$$x = [\, n_{pos}\; e_{pos}\; altitude \quad \Phi\;\Theta\;\Psi \quad v\;\alpha\;\beta \quad p\;q\;r]$$

In addition to the state vector, the Matlab function for matrix extraction is also provided with the structure of the command vector, which is made up as follows:

$$U = [\, Thrust \quad \delta_e \quad \delta_a \quad \delta_r \,]$$

By providing the extraction command with the Simulink model, the command vector and the initial trim conditions, it generates the four fundamental matrices for studying the dynamics: the state matrix A, the control matrix B, the observation matrix C and the D matrix.
These matrices have a very high dimension because they incorporate all the dynamics of the aircraft, as the longitudinal plane is coupled to the lateral plane. If the analysis of the dynamics is carried out with this approach in mind, the analysis is very complex. For this reason, the two planes are separated and the four dynamics matrices for the longitudinal and lateral planes are created by extracting only the terms of interest from the previous ones. The possibility of separating the planes derives from the assumptions made upstream of the calculation of the equation of the dynamics, including the theory of small perturbations, the aircraft considered as a rigid body with constant mass and constant mass distribution. The assumptions adopted make it possible to neglect coupling effects and to analyse the two planes separately.
The extracted elements for the formulation of the state matrix, the control matrix, the observation matrix and the D matrix for the longitudinal plane refer to the following data:

$$x_{longitudinal} = [\, altitude \quad \Theta \quad v \quad \alpha \quad q \,]$$

$$U = [\, Thrust \quad \delta_e \,]$$

On the other hand, the elements selected for the latero-directional plan are those referring to the following variables:

$$x_{lateral} = [\, \Phi \quad \Psi \quad v \quad \beta \quad p \quad r \,]$$

$$U_{lateral} = [Thrust \quad \delta_a \quad \delta_r \,]$$

The separation of the planes is yet another block of the code of dynamics that is implemented, after which the transfer function for both pairs is formulated. Once these are obtained, we can proceed to the next step, which is to study the system's eigenvalues, thus the response of the aircraft in its two planes.
The code for the evaluation of dynamics implements one more block: if the aircraft is not stable, action is taken to stabilise it using a stabilisation and control system. The M-346 is

supposed to be naturally stable, but despite this it has a stabilisation and control system in the event of special conditions that could destabilise it.

The file dedicated to the study of aircraft dynamics allows therefore to study the response of the aircraft in its entire flight envelope, but some examples are given below of the responses that are obtained at different altitudes and different speeds trying to cover different flight conditions.

The data contained in Table 16 – 17 – 18 – 19 – 20 refer to the study of the aircraft dynamics at different altitudes at the maximum speed of $570\ kt$:

Table 16 Aircraft dynamics evaluation at maximum speed at an altitude of 40000 ft

$$Altitude = 40000\ ft$$

Longitudinal plane

$\lambda_{1-2} = -0.00244 \pm 0.04701$	$\lambda_{3-4} = -2.0529 \pm 4.5309$
$\omega_{1-2} = 0.047073\ \frac{1}{s}$	$\omega_{3-4} = 4.9743\ \frac{1}{s}$
$\zeta_{1-2} = 0.051863$	$\zeta_{3-4} = 0.4127$
$T_{1-2} = 133.47\ s$	$T_{3-4} = 1.26\ s$
$t_{1/2} = 284.08\ s$	$t_{1/2} = 0.33\ s$

Lateral plane

$\lambda_1 = -\ 0.049828$	$\lambda_2 = -0.42489$	$\lambda_{3-4} = -0.080532 \pm 3.5344$
$\omega_1 = 0.049828\ \frac{1}{s}$	$\omega_2 = 0.42489\ \frac{1}{s}$	$\omega_{3-4} = 3.5353\ \frac{1}{s}$
$\zeta_1 = 1$	$\zeta_2 = 1$	$\zeta_{3-4} = 0.02779$
$T_1 = -$	$T_2 = -$	$T_{3-4} = 1.78\ s$
$t_{1/2} = 13.91\ s$	$t_{1/2} = 1.63\ s$	$t_{1/2} = 8.61\ s$

Table 17 Aircraft dynamics evaluation at maximum speed at an altitude of 30000 ft

$$Altitude = 30000\ ft$$

Longitudinal plane

$$\lambda_{1-2} = -0.00567 \pm 0.0460 \qquad \lambda_{3-4} = -3.1827 \pm 6.3521$$

$$\omega_{1-2} = 0.046394\ \frac{1}{s} \qquad \omega_{3-4} = 7.1048\ \frac{1}{s}$$

$$\zeta_{1-2} = 0.12228 \qquad \zeta_{3-4} = 0.44796$$

$$T_{1-2} = 153.43\ s \qquad T_{3-4} = 0.88\ s$$

$$t_{1/2} = 288.81\ s \qquad t_{1/2} = 0.22\ s$$

Lateral plane

$$\lambda_1 = -\ 0.04997 \qquad \lambda_2 = -0.6577 \qquad \lambda_{3-4} = -0.13627 \pm 4.0607$$

$$\omega_1 = 0.04997\ \frac{1}{s} \qquad \omega_2 = 0.65771\ \frac{1}{s} \qquad \omega_{3-4} = 4.0630\ \frac{1}{s}$$

$$\zeta_1 = 1 \qquad \zeta_2 = 1 \qquad \zeta_{3-4} = 0.03354$$

$$T_1 = - \qquad T_2 = - \qquad T_{3-4} = 1.5464\ s$$

$$t_{1/2} = 13.87\ s \qquad t_{1/2} = 1.05\ s \qquad t_{1/2} = 5.09\ s$$

Table 18 Aircraft dynamics evaluation at maximum speed at an altitude of 20000 ft

$$Altitude = 20000\ ft$$

Longitudinal plane

$$\lambda_{1-2} = -0.010407 \pm 0.044968 \qquad \lambda_{3-4} = -4.6101 \pm 7.2394$$

$$\omega_{1-2} = 0.046156\ \frac{1}{s} \qquad \omega_{3-4} = 8.5826\ \frac{1}{s}$$

$$\zeta_{1-2} = 0.22547 \qquad \zeta_{3-4} = 0.53714$$

$$T_{1-2} = 136.13\ s \qquad T_{3-4} = 0.73\ s$$

$$t_{1/2} = 66.6\ s \qquad t_{1/2} = 0.15\ s$$

Lateral plane

$$\lambda_1 = -0.051562 \qquad \lambda_2 = -0.95539 \qquad \lambda_{3-4} = -0.19077 \pm 4.7349$$

$$\omega_1 = 0.051562\ \frac{1}{s} \qquad \omega_2 = 0.95539\ \frac{1}{s} \qquad \omega_{3-4} = 4.7387\ \frac{1}{s}$$

$$\zeta_1 = 1 \qquad \zeta_2 = 1 \qquad \zeta_{3-4} = 0.040259$$

$$T_1 = - \qquad T_2 = - \qquad T_{3-4} = 1.33\ s$$

$$t_{1/2} = 13.44\ s \qquad t_{1/2} = 0.73\ s \qquad t_{1/2} = 3.63\ s$$

Table 19 Aircraft dynamics evaluation at maximum speed at an altitude of 10000 ft

$$Altitude = 10000\ ft$$

Longitudinal plane

$$\lambda_{1-2} = -0.01709 \pm 0.042615 \qquad \lambda_{3-4} = -6.4552 \pm 7.9154$$

$$\omega_{1-2} = 0.045914\ \frac{1}{s} \qquad \omega_{3-4} = 10.214\ \frac{1}{s}$$

$$\zeta_{1-2} = 0.37222 \qquad \zeta_{3-4} = 0.632$$

$$T_{1-2} = 136.85\ s \qquad T_{3-4} = 0.62\ s$$

$$t_{1/2} = 40.56\ s \qquad t_{1/2} = 0.11\ s$$

Lateral plane

$$\lambda_1 = -\ 0.052482 \qquad \lambda_2 = -1.3506 \qquad \lambda_{3-4} = -0.25551 \pm 5.5235$$

$$\omega_1 = 0.052482\ \frac{1}{s} \qquad \omega_2 = 1.3506\ \frac{1}{s} \qquad \omega_{3-4} = 5.5294\ \frac{1}{s}$$

$$\zeta_1 = 1 \qquad \zeta_2 = 1 \qquad \zeta_{3-4} = 0.04621$$

$$T_1 = - \qquad T_2 = - \qquad T_{3-4} = 1.14\ s$$

$$t_{1/2} = 13.2\ s \qquad t_{1/2} = 0.51\ s \qquad t_{1/2} = 2.71\ s$$

Table 20 Aircraft dynamics evaluation at maximum speed at an altitude of 0 ft

$$Altitude = 0\ ft$$

Longitudinal plane

$$\lambda_{1-2} = -0.026043 \pm 0.037199 \qquad \lambda_{3-4} = -8.8050 \pm 8.1886$$

$$\omega_{1-2} = 0.045409\ \frac{1}{s} \qquad \omega_{3-4} = 12.024\ \frac{1}{s}$$

$$\zeta_{1-2} = 0.57352 \qquad \zeta_{3-4} = 0.73228$$

$$T_{1-2} = 138.37\ s \qquad T_{3-4} = 0.52\ s$$

$$t_{1/2} = 26.62\ s \qquad t_{1/2} = 0.08\ s$$

Lateral plane

$$\lambda_1 = -\ 0.053002 \qquad \lambda_2 = -1.8635 \qquad \lambda_{3-4} = -0.33932 \pm 6.3994$$

$$\omega_1 = 0.053002\ \frac{1}{s} \qquad \omega_2 = 1.8635\ \frac{1}{s} \qquad \omega_{3-4} = 6.4084\ \frac{1}{s}$$

$$\zeta_1 = 1 \qquad \zeta_2 = 1 \qquad \zeta_{3-4} = 0.05295$$

$$T_1 = - \qquad T_2 = - \qquad T_{3-4} = 0.98\ s$$

$$t_{1/2} = 13.07\ s \qquad t_{1/2} = 0.37\ s \qquad t_{1/2} = 2.04\ s$$

Analysing the tabulated data, it can be seen that at high speed the aircraft's dynamics are stable as it has the characteristic solutions for the longitudinal and lateral directional planes. In particular, the characteristic frequency, damping, period and half-time characteristics present the Phugoid mode and the Short Period mode for the longitudinal plane, while the Spiral mode, the Roll mode and the Dutch roll mode for the latero-directional plane.

Since the stability at high speeds has been validated, the solutions of the dynamics at intermediate points to the envelope diagram are shown in Table 21 – 22 instead. Again, as an arbitrary choice, solutions are shown at speeds of $360\ kt$ and at altitudes of $10000\ ft$ and $30000\ ft$.

Table 21 Aircraft dynamics evaluation at 360 kt speed and 10000 ft altitude

$$Altitude = 10000\ ft$$

Longitudinal plane

$\lambda_{1-2} = -0.059618 \pm 0.073097$	$\lambda_{3-4} = -3.8241 \pm 4.7038$
$\omega_{1-2} = 0.07334\ \frac{1}{s}$	$\omega_{3-4} = 6.0621\ \frac{1}{s}$
$\zeta_{1-2} = 0.08129$	$\zeta_{3-4} = 0.63082$
$T_{1-2} = 86.67\ s$	$T_{3-4} = 1.04\ s$
$t_{1/2} = 11.63\ s$	$t_{1/2} = 0.18\ s$

Lateral plane

$\lambda_1 = -0.078587$	$\lambda_2 = -0.7942$	$\lambda_{3-4} = -0.16123 \pm 3.6741$
$\omega_1 = 0.078587\ \frac{1}{s}$	$\omega_2 = 0.7942\ \frac{1}{s}$	$\omega_{3-4} = 3.6777\ \frac{1}{s}$
$\zeta_1 = 1$	$\zeta_2 = 1$	$\zeta_{3-4} = 0.04384$
$T_1 = -$	$T_2 = -$	$T_{3-4} = 1.71\ s$
$t_{1/2} = 8.82\ s$	$t_{1/2} = 0.87\ s$	$t_{1/2} = 4.3\ s$

Table 22 Aircraft dynamics evaluation at 360 kt speed and 30000 ft altitude

$$Altitude = 30000\ ft$$

Longitudinal plane

$\lambda_{1-2} = -0.002928 \pm 0.064607$	$\lambda_{3-4} = -1.9392 \pm 3.7828$
$\omega_{1-2} = 0.064673\ \frac{1}{s}$	$\omega_{3-4} = 4.2509\ \frac{1}{s}$
$\zeta_{1-2} = 0.045273$	$\zeta_{3-4} = 0.4562$
$T_{1-2} = 97.15\ s$	$T_{3-4} = 1.48\ s$
$t_{1/2} = 236.73\ s$	$t_{1/2} = 0.36\ s$

Lateral plane

$\lambda_1 = -0.047139$	$\lambda_2 = -0.46768$	$\lambda_{3-4} = -0.062779 \pm 2.6301$
$\omega_1 = 0.0047139\ \frac{1}{s}$	$\omega_2 = 0.46768\ \frac{1}{s}$	$\omega_{3-4} = 2.6309\ \frac{1}{s}$
$\zeta_1 = 1$	$\zeta_2 = 1$	$\zeta_{3-4} = 0.023862$
$T_1 = -$	$T_2 = -$	$T_{3-4} = 2.39\ s$
$t_{1/2} = 14.7\ s$	$t_{1/2} = 1.48\ s$	$t_{1/2} = 11.04\ s$

From the tabular values above, it is clear that even when sampled at intermediate speeds, the system has stable solutions for both planes.

Evaluating the dynamics at even lower speeds, regardless of the altitude, the solutions obtained at very low speeds and in the vicinity of the minimum speed are no longer stable, at least in the longitudinal plane where the Phugoid-Short Period pair is no longer present as it is replaced by the Third Mode. The absence of stabilisation even though the aircraft is naturally stable may lead to the conclusion that there are errors in the mathematical model, which is probably correct as in the trim conditions there were greater fluctuations in the data. Despite this, the destabilisation is due to several factors including the influence of speed and altitude, as well as the loss of reliability of the data necessary to evaluate the aircraft response: this last aspect is already considered in the aircraft FCS which, in case of loss of some data, activates a fixed gain stabilisation and control system.

A stability-increasing system has been generated with fixed gains and applied whenever there is reduced stability, or the Third Mode occurs. Since in general the latero-directional plane is stable, the following is an example of how the longitudinal plane stabilisation system acts in the case of the occurrence of Short Period and Phugoid cohesion in the Third Mode.

The example shown in Table 23 is for a velocity of about 200 kt at 3000 ft altitude, which is arbitrarily chosen so that a low-altitude example can also be shown. The emergence of the

Third mode in this case can be expressed by the fact that the trim values used for the study of the dynamics are not entirely correct since, the cost of the trim function is relatively high (of some order) compared to the situations in which a stable response occurs, therefore this is one of the cases in which the destabilisation is correlated to the interpolation function of the aerodynamic coefficients and the trim conditions. Although this is the case, the implemented stabilisation system is capable of returning the aircraft to a safe condition. The stabilisation process can be observed in Figure 23 – 24 – 25 where the whole process is represented.

Table 23 Aircraft dynamics evaluation at 200 kt speed and 3000 ft altitude

Longitudinal plane

$\lambda_1 = 1.0231$	$\lambda_2 = -6.4683$	$\lambda_{3-4} = -0.004759 \pm 0.10059$
$\omega_1 = 1.0231\ \frac{1}{s}$	$\omega_2 = 6.4683\ \frac{1}{s}$	$\omega_{3-4} = 0.1007\ \frac{1}{s}$
$\zeta_1 = -1$	$\zeta_2 = 1$	$\zeta_{3-4} = 0.047257$
$T_1 = -$	$T_2 = -$	$T_{3-4} = 62.4\ s$
$t_{1/2} = -$	$t_{1/2} = 0.11\ s$	$t_{1/2} = 145.65\ s$

Lateral plane

$\lambda_1 = -0.19494$	$\lambda_2 = -1.879$	$\lambda_{3-4} = -0.25402 \pm 2.2568$
$\omega_1 = 0.19494\ \frac{1}{s}$	$\omega_2 = 1.879\ \frac{1}{s}$	$\omega_{3-4} = 2.2711\ \frac{1}{s}$
$\zeta_1 = 1$	$\zeta_2 = 1$	$\zeta_{3-4} = 0.11185$
$T_1 = -$	$T_2 = -$	$T_{3-4} = 2.78\ s$
$t_{1/2} = 3.36\ s$	$t_{1/2} = 0.37\ s$	$t_{1/2} = 2.73\ s$

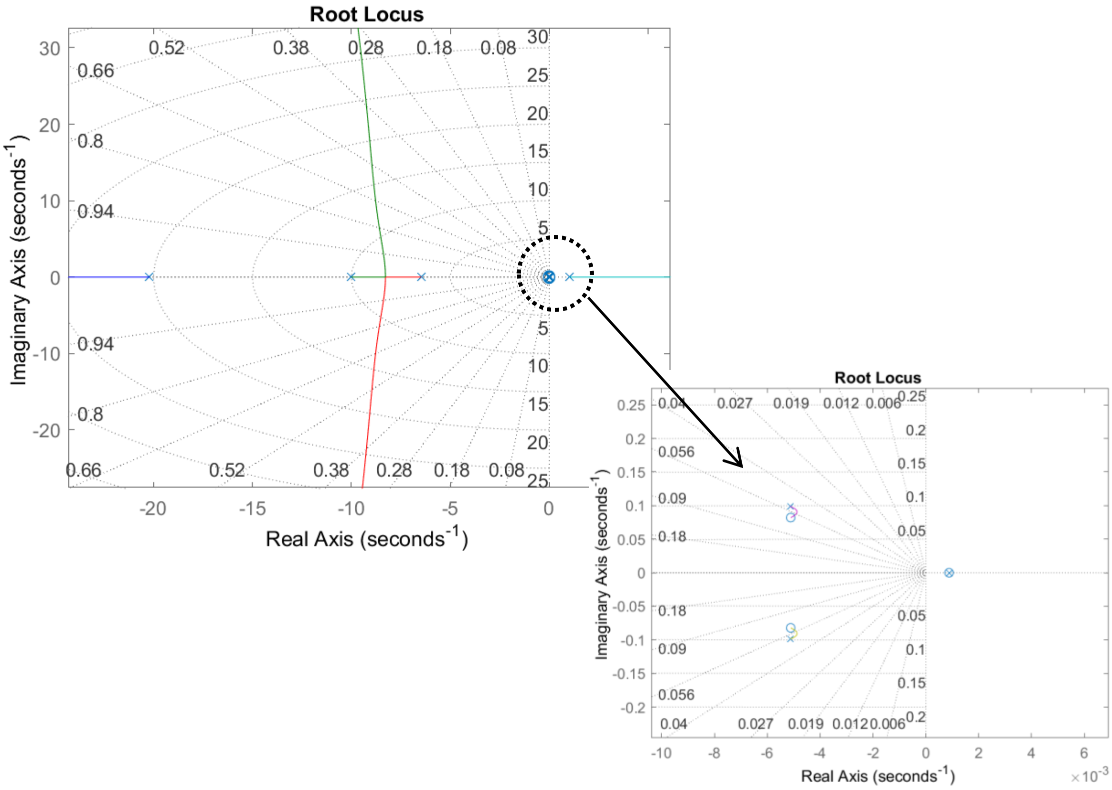


Figure 23 Rlocus longitudinal dynamics diagram at 200 kt speed at 3000 ft altitude

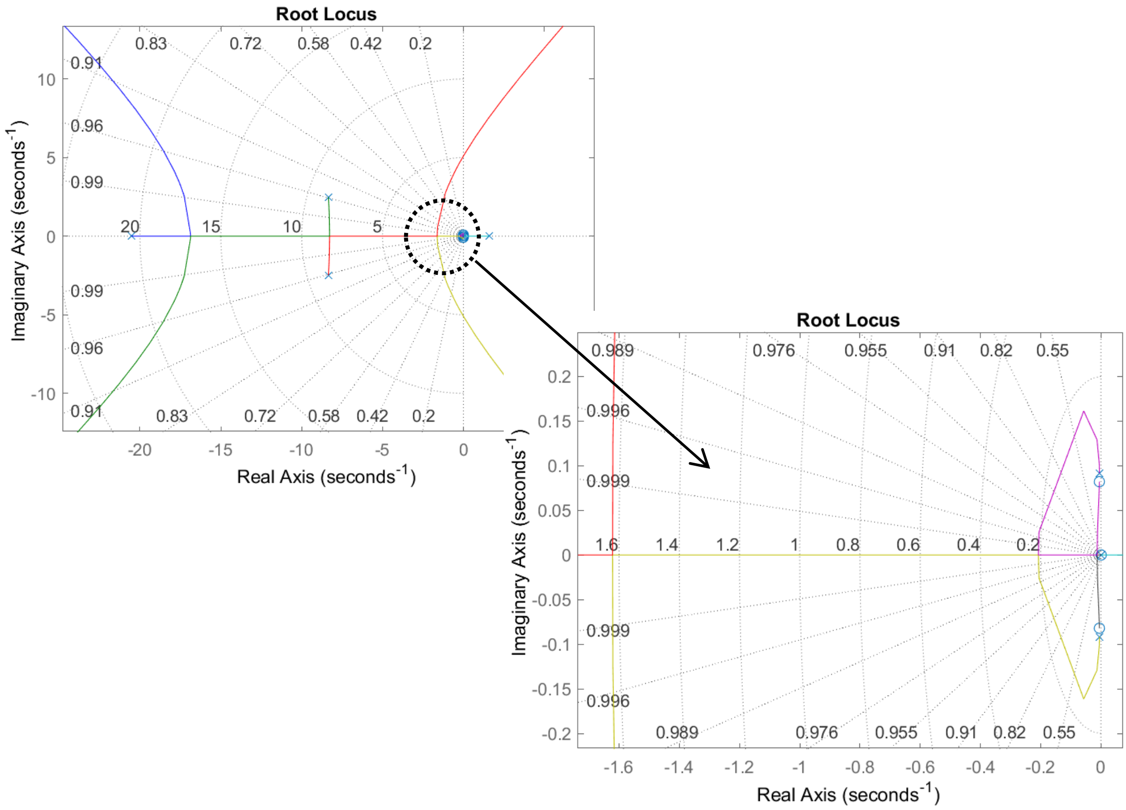


Figure 24 Rlocus diagram of the longitudinal dynamics applying the gain K_α: short period and phugoid mode are obtained

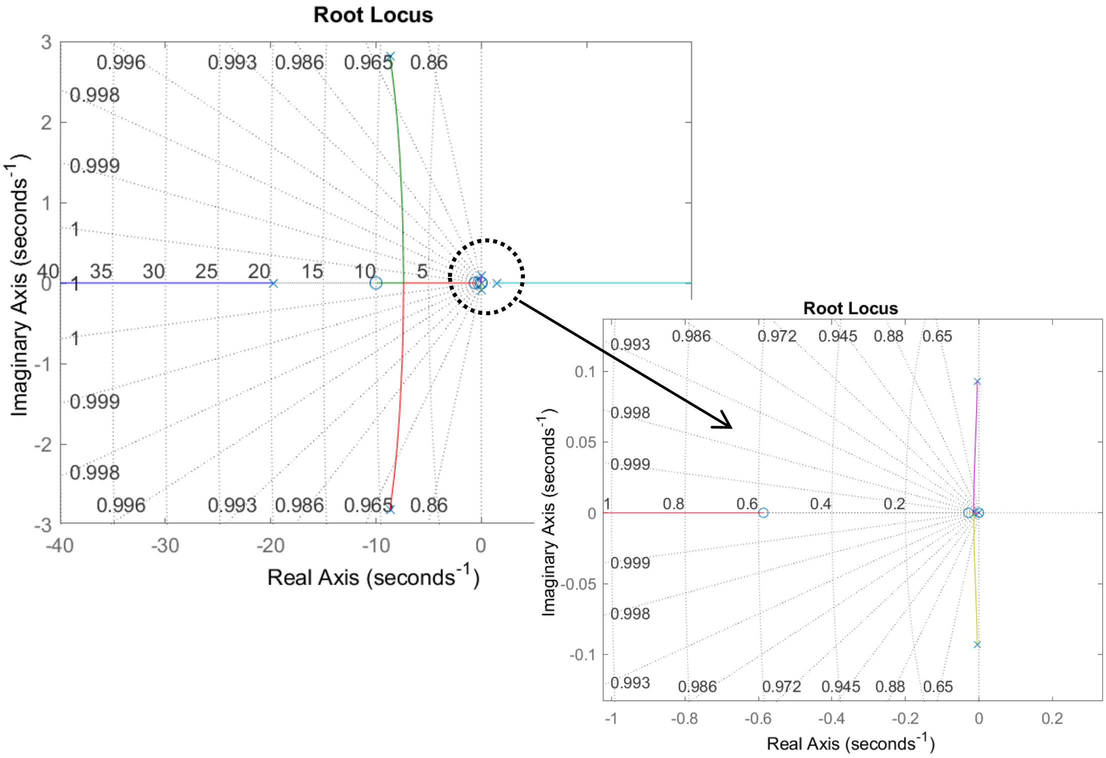


Figure 25 Rlocus diagram of longitudinal dynamics applying K_q gain: short-period and phugoid modes are stabilised

As the aircraft is naturally stable, cases such as this one should not occur, and if they do, it is necessary to investigate the reason for their occurrence. In this case the cause of the destabilising behaviour is known, while in other cases it would be appropriate to increase the number of cycles for the calculation of the trim condition or the weights of the cost function in order to obtain a figure that reflects the real dynamics of the aircraft.

Conclusion

From a simple CAD of the M-346 aircraft, it has been possible to derive the various aerodynamic coefficients and derivatives to be used for the development of a mathematical model through which it has been possible to evaluate the trim condition and dynamics. The results obtained for the various steps reported in this project are at the same time plausible and promising for future projects. In fact, in the previous chapters, a number of critical points have already been pointed out, which should be subjected to further study that also implements other methods of analysis.

An initial approach to validate the aerodynamic coefficients and the aerodynamic derivatives may consist in comparing the calculated data with the respective values obtained from flight tests or tunnel tests. This comparison has not been made in this thesis due to the impossibility of having such data available. Due to the impossibility of a confrontation with real values, it may be possible to try to carry out a comparison with data obtained from a different fluid dynamic simulation program or to increase the computational capacity and the density of the analysis nodes in the program used in this work. These measures would make it possible to considerably reduce the presence and propagation of errors.

Getting coefficients and derivatives with a higher degree of accuracy also means obtaining more precise interpolation results and consequently trim conditions with a very low minimisation cost. This last aspect also plays a fundamental role for the model: being able to reduce the cost function as much as possible means increasing the level of detail of the model results. As seen in some of the previous charts, at certain positions, especially at low speeds and high altitudes, the solutions obtained were not entirely accurate and lead to a lack of trimming and dynamics. These problems can therefore be mitigated if detailed data are already available at the start of program modelling. The weaknesses of the mathematical model may be subject to refinements with the ultimate aim of representing the behaviour of the study aircraft as faithfully as possible.

The mentioned points represent the most critical aspects that have been identified in this work, but with the undertaken path an attempt to mitigate their effect overall has been made, obtaining in the end the expected results:

- The CAD model is highly accurate.
- Aerodynamic coefficients and aerodynamic derivatives show comparable trends and values to those of an aircraft of similar category.
- The trim conditions present global trends that reflect the predictions of flight mechanics theory and the values obtained are plausible for the aircraft in question.
- The study of dynamics confirms the general natural stability of the aircraft.

The achieved outcomes are a good point of arrival but can also be the starting point for future projects. One of the many future tasks would be studying an autopilot that controls the aircraft in a fully automatic manner. Of the numerous autopilot models, it would be interesting to develop an autopilot for altitude holding, attitude holding and thrust holding. These three examples, if implemented, would form the basis for the development of other types of autopilots.

In addition to the development of autopilots, whose programs would only allow us to observe how the characteristic quantities behave, being able to integrate what has been done so far and what is supposed to be further work, into a flight simulator would be the final step in validating all the project, in addition to the entertaining and creative aspect of seeing the behaviour of the aircraft.

The final milestone of the next activity would be to compare the work done by the flight simulator implementing this mathematical model with the work done by the real aircraft in flight: if this were to happen, it would be the final step in a process that began with a simple CAD drawing of the M-346.

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Appendix A

The parameters used to calculate the aerodynamic derivatives are listed below.

S wing	23.52	[m^2]
b	9.72	[m]
Mean Aerodynamic Chord	2.426	[m]
Root Chord	2.65	[m]
Aspect ratio	4.02	[-]
Taper ratio	0.83	[-]
Wing Sweep Angle	28	[deg]
S horizontal tail	5.4	[m^2]
L_t	4.21	[m]
Horizontal stab span	4.91	[m]
Mean Chord Horizontal tail	1.099796	[m]
AR_h	4.46	
S vertical tail	5.43	[m^2]
l'_r	3.34	[m]
Vertical stab span	2.76	[m]
z_r	1.78	[m]
Elevator Area	1.11	[m^2]
Max Elevator Angle	-15	[deg]
Min Elevator Angle	30	[deg]
Trim Limit Elevator	19.5	[deg]
Rudder Area	0.92	[m^2]
Rudder Angle Limit	30	[deg]
Density (rho)	1.225	[kg/m^3]
V	154	[m/s]
V_H	0.398426	
V_H	0.382716	
d_epsilon/d_alpha	0.470688	
a_tail	4.186	
a_F	1.2595	
V_V	0.079331	
l_F	3.34	

A conclusione di questo elaborato, desidero menzionare tutte le persone, senza le quali questo lavoro di tesi non sarebbe stato concluso.
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