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# Advanced control strategies for the management of energy storage systems in a non-residential building

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# Abstract


## Italiano-Strategie di controllo avanzato per la gestione di sistemi di accumulo energetico in un edificio non residenziale

La crescente disponibilità e penetrazione delle risorse energetiche rinnovabili insieme alla progressiva elettrificazione degli usi finali di energia rende necessario porre particolare attenzione all'integrazione di tali fonti con la rete elettrica e alla stabilità della stessa. Le fonti di energia rinnovabili come il solare e l'eolico sono infatti tipicamente non programmabili; dunque, la loro disponibilità non segue in generale l'andamento della domanda. La flessibilità richiesta per l'adattamento alle esigenze della rete (Demand Response) trae beneficio dai sistemi di accumulo energetico la cui gestione, però, richiede sistemi di controllo adeguati. Le strategie di controllo tradizionali come l'On-Off e il PID sono essenzialmente orientate al mantenimento delle condizioni operative dei sistemi controllati. Le strategie di controllo avanzate permettono invece di includere obiettivi di efficientamento e flessibilità energetica grazie alle loro capacità predittive e alla possibilità di individuare soluzioni ottimali. Esigenti dal punto di vista computazionale, queste strategie stanno attraendo una crescente attenzione da parte del mondo della ricerca vista la sempre maggior disponibilità di potenza di calcolo. Il presente lavoro formula diverse strategie di controllo e propone un framework per la loro verifica in sede simulativa. Le strategie sono applicate a un caso studio che consiste in un edificio servito da un impianto di riscaldamento, una pompa di calore per la generazione di calore, un serbatoio di accumulo termico, una batteria elettrica e dei pannelli fotovoltaici per la produzione di energia elettrica. È inoltre possibile lo scambio di energia con la rete. La prima parte del lavoro dimostra come strategie di controllo Rule Based opportunamente formulate possano raggiungere obiettivi di efficienza, flessibilità e risparmio economico pur in assenza di capacità predittiva o di ottimizzazione. Successivamente, il problema viene affrontato servendosi di un controllo predittivo, ovvero il Model Predictive Control (MPC), che permette di minimizzare l'acquisto di energia dalla rete attraverso l'ottimizzazione di una funzione di costo opportunamente formulata. I risultati del controllo predittivo sono confrontati con quelli un RBC adottato come Baseline e mostrano come il primo sia più efficace nel conseguimento degli obiettivi di risparmio energetico prefissati. In particolare, Self Sufficiency, Self Consumption e spesa per l'acquisto dell'energia, adottati come indici per il confronto tra diverse strategie, mostrano significativi miglioramenti rispetto al caso Rule-based adottato come baseline.

## English- Advanced control strategies for the management of energy storage systems in a non-residential building

The increasing availability and penetration of renewable energy sources, along with the progressive electrification of final energy uses makes it necessary to manage the integration of such sources with the power grid with care with respect to the stability of the grid itself. Indeed, renewable energy sources such as wind

and solar are typically non-programmable, therefore their availability generally does not follow the demand profile. The flexibility required for the adaptation to the power grid needs (Demand Response) takes advantage from energy storage systems; however, their management calls for adequate control strategies. Traditional control strategies such as On-Off and PID are essentially oriented towards maintaining operating conditions of controlled systems. Advanced control strategies on the other hand allow to include energy efficiency and flexibility objectives thanks to their prediction capabilities and the ability to find optimal solutions. These strategies are computationally demanding; however, they are attracting increasingly large attention from the research community given the continuously growing availability of computational power. The present work formulates different control strategies and proposes a framework for their testing in a simulation fashion. Strategies are applied to a case-study consisting in a building served by a heating system, a heat pump for heat generation, a thermal storage tank, an electric battery and PV panels for electricity production. Power exchange with the grid is also allowed. The first part of the work proves how Rule Based control strategies can achieve efficiency, flexibility and economic savings goals despite lacking predictive and optimizing capabilities. The problem is then dealt with by employing a predictive control strategy, namely the Model Predictive Control (MPC), that allows to minimize the energy acquired from the grid by optimizing a suitable cost function. Results of the MPC control are compared to that of an RBC adopted as baseline and show how the former is more efficient in reaching the set energy savings goals. In particular, Self Sufficiency, Self Consumption and monetary expense, adopted as indexes for the comparison between different strategies, display a significant improvement with respect to the RBC baseline.

# Index

# 1 Chapter 1

## 1.1 Building automation and Smart Buildings

Definitions regarding building automation and its main features are abundant in research and regulatory literature. Of particular relevance are the definitions of Building Automation and Control (BAC), Building Automation and Control Systems (BACS) and Technical Building Management (TBM). BAC refers to the set of products, pieces of software and engineering services for automatic control, monitoring and optimization, human intervention and management that have the purpose to achieve economic, safe and energy wise efficient operation of building services equipment. BACS are all the products and services related to BAC; an alternative term often found in literature is Building Management System (BMS). TBM comprises control systems as well as their effective operation and integration within the whole building system.

These definitions often interlap with each other and *"clarify the multidisciplinary role of automation and control, involving hardware and software"* (Aste, Manfren, and Marenzi 2017). More in general, building automation is aimed at optimizing the following functions:

1. Heating, Ventilation and Air conditioning Systems (HVAC);
2. domestic hot water (DHW);
3. lighting system control;
4. shading system control;
5. energy conversion and storage;
6. locally produced energy management;
7. monitoring and data acquisition;
8. communication and security management.

Historically, the evolution of building automation has gone through the following phases, as reported in (Aste, Manfren, and Marenzi 2017):

1. *Dedicated Systems* (1980-1985): subsystems have individual functions and are managed independently;
2. *Integrated Multifunctioning Systems* (1985-1990): individual subsystems were grouped into functional areas;
3. *Building Level Integrated Systems* (1990-1995): a first attempt at integrating subsystems at the building level of automation;
4. *Computer Integrated Building* (1995-2002): the integration of subsystems at the building level in reached by exploiting the network technology capabilities;
5. *Enterprise Network Integrated System (ENIS)* (after 2002): integration is carried out on a higher level and connects more than one building together.

The automation of buildings is not restricted to the sole single facility domain. In fact, as point 5. in the forementioned historical phases suggests, the concept of smart buildings is becoming increasingly related to that of the smart city, seen as a set of actors interacting with each other. Particularly important is the role of energy exchange between buildings and the electric power grid.

Dealing with the grid interaction with different actors for the purpose of including a significant share of energy production from renewable sources (RES) to substitute fossil fuel based primary resources has posed a serious challenge to the management of the power grid. Indeed, the main drawback of RES is that they are mostly non-controllable, and their productivity is hard to foresee, so that a significant share of electricity production must be provided by traditional combustion based thermal plants.

An all-renewable energy production scenario was investigated by (Johnson, Rhodes, and Webber 2020). The authors noted how, due to the non-synchronous nature of most renewable resources such as wind and solar, *"the transition to a grid dominated by non-synchronous energy generation should be handled with care"*. Moreover, RES tend to be fragmented into smaller production sites or even into local production nodes where consumers produce part of their needs with, say, roof mounted photovoltaic panels. These are often referred to as *distributed energy resources* or DER for short. A comprehensive literature review work on the topic of RES and prosumers integration into Smart Grids can be found in (Espe, Potdar, and Chang 2018).

Electrification of final uses is increasing in most domains, including the building sector, as electric energy is much easier to manage and solar and wind sources produce indeed energy in this form. Heating and hot water demand accounts for 79% of the overall final energy consumed by residential buildings only (Psimopoulos et al. 2020). As electrification takes place, heating needs are covered by heat pumps (HP) which will, accordingly, have a serious impact on the grid stability. (Freier et al. 2020) observed how building electrification, particularly through the use of HPs, poses a challenge for the grid management. Similarly, (Zappa, Junginger, and van den Broek 2019) estimated that in order to have an 100% renewable scenario for the European power scenario, one of the conditions would be *"the well managed integration of heat pumps [...] into the power system to reduce demand peaks"*.

All of the forementioned criticalities lead to the necessity of best managing energy by means of time-shifting between production and use, and that can be achieved with storage systems. The integration of thermal and electrical storage systems with heat pumps in building is an increasingly studies subject matter. Rule based control is the state of the art control technique for the operation of such systems, as it can achieve significant energy savings if properly implemented, despite its solution being suboptimal (Mařík et al. 2011). Other, more advanced techniques will be reviewed in the present chapter, while Chapter 2 will focus on MPC formulation and applications. The interaction between storage systems, heat pumps and the grid will be dealt with four rule based control strategies in Chapter 4, to be then translated into an MPC strategy in Chapter 5.

## 1.2 Generalities on Control for Smart building applications

### 1.2.1 State space representation of dynamic systems

A dynamical system is a system whose behaviour changes with time, in general as a result of external actions. The discipline of control has the goal of ensuring that such behaviour is as close as possible to a set desired profile.

The study of dynamical systems and therefore of control theory comes from the cultural heritage of mechanics and that of electrical/electronic engineering. In particular, from mechanics comes the idea of the *state space* model, while from the study of electrical circuits and components come the *input-output* approach.

In mechanics, the dynamic evolution of a system is described by the time dependency of some specific quantities that describe the system itself; such quantities are called *state variables* and will be hereafter indicated with the letter x, following a common convention. More in general, a finite set of $n \in \mathbb{N}$ state variables collected in a more convenient vectorial notation:

$$x(t) = [x_1(t), x_i(t), \dots, x_n(t)]^T \tag{1.1}$$

in which the time dependence has been highlighted.

A common example is that of the point mass whose motion is completely described once its position $x(t)$ and its velocity $\dot{x}(t)$ are known; by denoting $x = x_1$ and $\dot{x} = x_1$, it is possible to describe the system as the evolution of vector $x(t) = [x_1(t), x_2(t)]^T$. From the definition of state variable naturally comes the definition of a *state space*, that is the set of the possible states in which a system can be found.

A different view of a dynamic system emerged from the study of circuits and their components: according to this approach, attention is paid to the system response to external actions, that is the relationship between *inputs* and *outputs*. Inputs and outputs are in general time dependent functions and can be indicated as u(t) and y(t) respectively. Being *m* the number of the input variables and p the number of the output variables, vectors for the inputs and for the outputs can be defined:

$$\begin{gathered} u(t) = [u_1(t), u_i(t), \dots, u_m(t)]^T, u \in \mathbb{R}^m \\ y(t) = \left[y_1(t), y_i(t), \dots, y_p(t)\right]^T, y \in \mathbb{R}^p \end{gathered} \tag{1.2}$$

Following the forementioned notation, the evolution of a dynamic system can be expressed through the *state equations*, which are the equations that express the variation of the states as a function of the current states and the inputs acting upon the system. Such equations, when in standard form, make up a set of *n* coupled first order ordinary differential equations in the following form:

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_i, .., x_n, u_1, u_j, \dots, u_m, t) \\ \dot{x}_i = f_i(x_1, x_i, .., x_n, u_1, u_j, \dots, u_m, t) \\ \vdots = \vdots \\ \dot{x}_n = f_n(x_1, x_i, .., x_n, u_1, u_j, \dots, u_m, t) \end{cases} \tag{1.3}$$

where each $f_i(x_1, x_i, .., x_n, u_1, u_j, \dots, u_m, t)$ is in general nonlinear and time varying. A more convenient vectorial notation can be used to represent such a system:

$$\dot{x} = f(x, u, t) \tag{1.4}$$

#### 1.2.1.1 Observability

In general, the states of a system might not be directly measurable; moreover, their choice is often such that they do not even represent an actual physical quantity; this is the case, for instance, of lumped parameters. *Observability* is a concept introduced by Rudolf E. Kalman to measure how precisely states values can be inferred from the values of the outputs. Indeed, outputs are in principle known since they are the results of measurements. Since only inputs and outputs can in general be considered as known, the sole state equations are not sufficient to describe the evolution of a dynamic system for the purpose of control: another set of equations, called *output equations*, are defined as follows:

$$\begin{cases} y_1 = g_1(x_1, x_i, .., x_n, u_1, u_j, \dots, u_m, t) \\ y_i = g_i(x_1, x_i, .., x_n, u_1, u_j, \dots, u_m, t) \\ \vdots = \vdots \\ y_p = g_n(x_1, x_i, .., x_n, u_1, u_j, \dots, u_m, t) \end{cases} \tag{1.5}$$

again, these equations can be written in the more compact vectorial form:

$$y = g(x, u, t) \tag{1.6}$$

#### 1.2.1.2 Linear time-invariant systems

A system is called *time invariant* when there is no explicit time dependency, so that the equations can be written as:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= g(x, u) \end{aligned} \tag{1.7}$$

In case functions f and g are linear and time invariant, the system takes the following form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1.8}$$

where:

- $A$ is a constant $n \times n$ matrix, called *dynamics matrix*;
- $B$ is a constant nxn matrix, called *control matrix*;
- $C$ is a constant nxn matrix, called *sensor matrix*;
- $D$ is a constant nxn matrix, called *direct matrix*;

### 1.2.2 Classification of control strategies in buildings

A large and varied number of control techniques have been employed in building applications throughout the years and more strategies are currently research material, so that a classification framework is necessary to both the researcher and the designer in order to make the proper design choices.

(Gholamzadehmir et al. 2020) proposes a classification of control strategies that will be followed in the present work; an illustration of such classification can be seen in Figure(1).

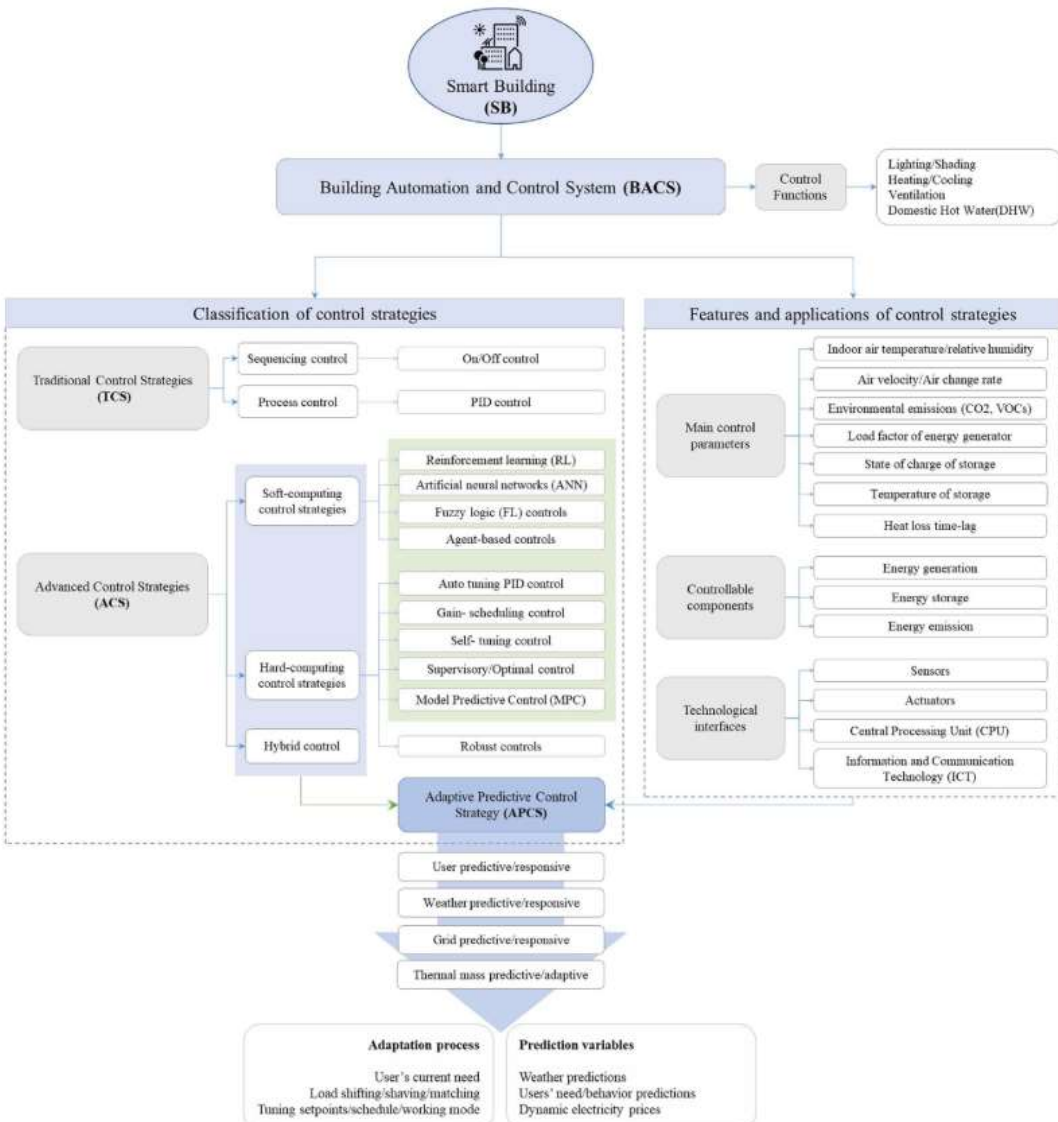


*Figure 1-Control methods for building classification (Gholamzadehmir et al. 2020)*

Control strategies are in the first place divided into two main groups: *traditional control strategies* (TCS) and *advanced control strategies* (ACS). Traditional control strategies are further classified as *sequencing control* strategies and *process control* strategies. Advanced control strategies are categorized as follows:

- *soft computing control strategies*: these strategies generally do not employ and analytical model, but instead make use of approximate or statistical computations. Among those strategies, the most used are *reinforcement learning* (RL), deep learning based on *artificial neural network* (ANN), *fuzzy logic* (FL) controls and agent-based controls.
- *Hard computing control strategies*, such as auto tuning PID control, gain scheduling control, self-tuning control, supervisory optimal control, model predictive control (MPC) and robust control. These strategies employ mathematical models for the systems to be controlled, and therefore require precise input data for a correct computation.
- *Hybrid control strategies*: these strategies are a combination of hard and soft control methods.

A different distinction can be drawn between adaptive and robust control methods. A control strategy is *adaptive* when it has the capability of changing its parameters during the operation: such controllers will therefore include mechanisms such as system identification tools in order to automatically re-tune and re-calibrate the parameters. On the other hand, control is defined as *robust* when parameters are set at design time and assumed to be able to handle uncertainties without the need of any re-tuning during operation.

## 1.3 Traditional control strategies

Traditional control strategies are the most consolidated control techniques in the field of HVAC applications for buildings, as well as in most other applications, such as process industry. Their main advantage lays in their simplicity, in that they employ a minimal computational capability, if any at all. Indeed, they can be at times reached by means of passive actuators, such as regulation valves.

### 1.3.1 ON OFF control

On/Off control belongs to the set of control strategies known as *discontinuous controllers*. Such controllers are able to yield values of the control signal with a set of few discontinuous values, whereas *continuous controllers* can produce outputs that continuously change in dependence of the input the controller receives. Due to this simplifying characteristic, there will be a certain range in which the control signal is not affected by the fluctuations of the system output, resulting in a less accurate control of the system. In practice, most discontinuous controllers are *two-position controllers*, which means that their output has only two possible states, being a maximum and a minimum value or an On and an Off status of a particular actuator. The analytical representation of a two-position control strategy requires a discontinuous formulation, as shown in the following expression:

$$u(t) = \begin{cases} u_{max}, & e < 0 \\ u_{min}, & e \geq 0 \end{cases} \tag{1.9}$$

where $u(t)$ represents the control signal, while $e$ is the control error.

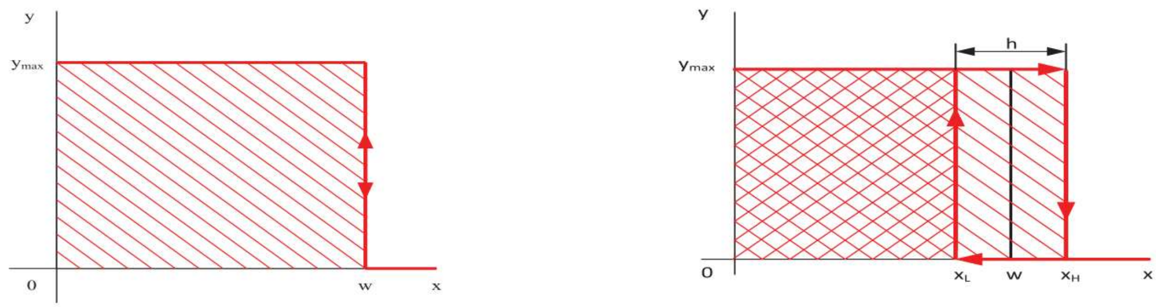


*Figure 2-On-off control (Uriča and Simonová 2017)*

Figure(2) (Uriča and Simonová 2017) shows the working principle of a simple two-position controller. Here, $y$ is the action variable while $x$ is the process variable. A particular desired value of the latter is set as a *setpoint* $w$, so that any deviation from that value results in a nonzero value of the control error $e$. The action variable responds to a negative control error, which occurs when the process variable is above the setpoint value, with an On status, that is the value $y_{max}$, whereas when the process variable is below the setpoint value, the action variable is on its minimum value (Off status here). Figure(2) highlights the pat followed by the process in the $xy$ plane.

In the proximity of the setpoint value, this type of control might yield a high frequency intermittent control signal, which can damage the controlled components. To avoid this phenomenon, *hysteresis* is introduced. Given a hysteresis value h, the output of the controller changes its state from value $y_{max}$ to 0 when the process variable reaches $x_H = w + h/2$ and switches back to value 0 when the process value reaches $x_L = w - h/2$. The resulting path in the $xy$ plane can be seen in Figure(2); such path is closed, hence the hysteresis denomination.

The choice of an On/Off strategy does not only depend on the characteristics of the controller, but is often due to the fact that the controlled object itself is able to work in two modes only; examples are fixed-speed pumps, non-modulating heat generators and two positions valves.

The main advantages and disadvantages of On/Off control are highlighted in the following table, from (Behrooz et al. 2018):

| Control Method | Advantage | Disadvantage |
|---|---|---|
| On/Off control | (1) It is the most easiest and intuitive method to implement.<br>(2) It is a basic method and very simple.<br>(3) Low initial cost.<br>(4) Plain structure.<br>(5) Quick response [9].<br>(6) Feedback controller [9]. | (1) It is unable to control moving process with time delays [3].<br>(2) It is not accurate enough.<br>(3) It does not have enough quality.<br>(4) PID controllers should be used for reducing the fluctuation produced by on/off controller. But parameter's setting for the PID controllers is difficult [13,14].<br>(5) Accepts only binary inputs [9].<br>(6) Due to inability to set point tracking accurately, this method is inefficient [9].<br>(7) Not useful and effective in the long run [9]. |

*Figure 3-Main pros and cons of On-Off control (Behrooz et al. 2018)*

### 1.3.2 PID control

Two position control is a strategy adopted for system whose control variables or actuators cannot modulate in their value. Most technological components nowadays can be defined as *continuous* as opposed to *discontinuous* in that their outputs change continuously as a function of the input it receives from the control unit. Common examples in HVAC applications include modulating heat generation systems, regulating valves and variable-speed pumps.

In a feedback control system, the controller is thus enabled to react in response to the output it receives from the plant. The simplest and most intuitive feedback mechanism is that of *proportional control.* In proportional control, the feedback control input $u(t)$ is computed from the error $e(t)$ as:

$$u(t) = K_p e(t) \tag{1.10}$$

where constant $K_p$ is called *proportional gain*. In terms of transfer functions, signal $U(s)$ is computed as $K_p E(s)$ where $E(s)$ is the error transfer function. The process scheme is represented in Figure(4) from (Wang 2020)

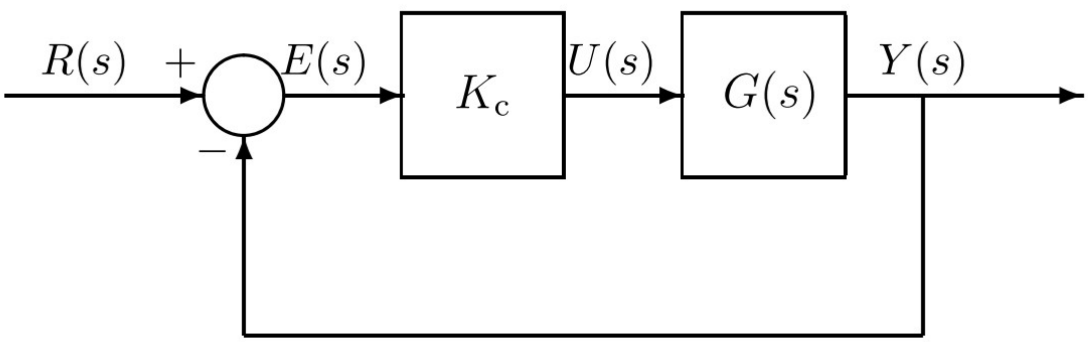


*Figure 4-Closed loop proportional control (Wang 2020)*

The main drawback of proportional control is that it does not eliminate the steady state error of the controlled system. Instead, in response to a disturbance, the plat will settle to a new steady state configuration that will have an error with respect to the reference value. To overcome this limitation, an *integral* action is introduced in the so called proportional-integral control (PI for short).

The integral term is not proportional to the error at instant *t* but rather on the integral in time of the error itself. The control signal of a PI controller is computed as:

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau) d\tau \tag{1.11}$$

The constant value $K_I$ is called *integral gain* and is often represented as:

$$K_I = \frac{K_I}{\tau_I} \tag{1.12}$$

where $\tau_I$ is the *integral time constant*.

The resulting Laplace transform of the PI controller is:

$$U(s) = K_p E(s) + K_I E(s) \tag{1.13}$$

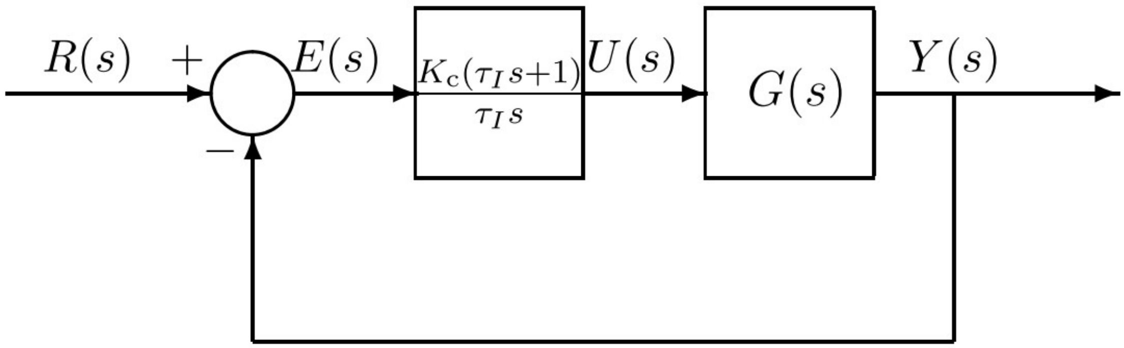


*Figure 5-Closed loop PI control (Wang 2020)*

Another limitation of both proportional and integral control is that the control signal profile can be at times very steep, which poses a threat to the technological elements of the controlled plant.

This limitation is overcome by the introduction of a *derivative* term, which is proportional to the derivative of the feedback error. A resulting PD controller follows Eq(1.14) in computing the control signal:

$$u(t) = K_p e(t) + K_D \frac{de(t)}{dt} \tag{1.14}$$

Constant $K_D$ is called *derivative gain* and is often represented as:

$$K_D = K_C \tau_D \tag{1.15}$$

where $\tau_D$ is called *derivative time constant*.

Again, Laplace transform of the PD control system shall be:

$$U(s) = K_p E(s) + K_D s E(s) \tag{1.16}$$

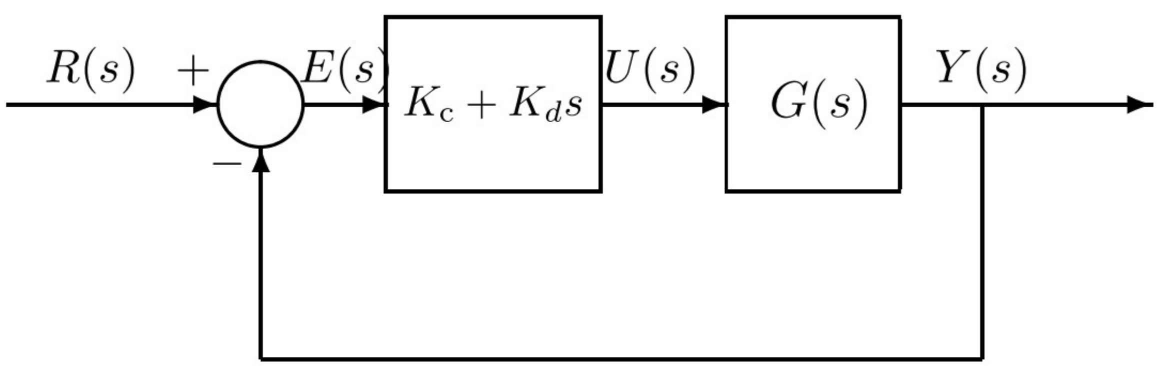


*Figure 6-Closed loop PD control (Wang 2020)*

All three terms can be summed up to obtain the *proportional-integral-derivative* control, or PID for short. The main advantage of classical PID control is that, being purely reactive in nature, does not require a detailed knowledge of the controlled systems, that is, a plant model is not necessary. However the tuning process, which is the process of choosing the most appropriate values of $K_p$, $K_I$ and $K_D$ is a delicate stage of the design phase of the controller.

Classical tuning methods are the Cohen-Coon method and the Ziegler-Nichols methods. More advanced tuning methods for HVAC applications can be found in (Almabrok, Psarakis, and Dounis 2018) and in (Fütterer, Stinner, and Müller 2016).

## 1.4 Building energy modelling for control

A key component of an advanced control strategy lies in its prediction capability. Predicting the behaviour of both the building in terms of its envelope and of its technological components allows these control strategies to compute actions adapted to the forecasted boundary conditions. Moreover, a reliable building model is crucial in the design phase as it allows a simulated testing of the intended strategies. For the same reason, simulation is a key in research.

Despite its importance, "*developing a high-fidelity forecasting model for building systems is not an easy task*" (Li and Wen 2014). For the most part building systems are complex system which display a nonlinear behaviour and are influenced by a vastly diverse range of hard-to-foresee disturbances such as weather conditions, building operating modes, hydraulic circuits and mixing tools, storage systems and occupant schedules. Moreover, economic and practical reasons make it hard to monitor and accurately measure building systems. Sensors are less common and less precise than those typically employed in industrial applications.

Regarding HVAC systems modelling, (Afroz et al. 2018) pointed out how "*considering all of these discrete, non-linear and highly constrained characteristics and parameters of HVAC systems, it is a challenging task to develop an accurate and effective model for these systems that accurately represents reality. For the development of modelling research it is necessary that the building research and management community become informed about the application, role, strengths and weaknesses of the various modelling techniques associated with research studies and how the developed models perform in real world situations*".

### 1.4.1 Classification of modelling techniques

A key factor in building and HVAC systems modelling is that these systems are dynamic in nature and therefore require transient models to be described, and of course the same applies to forecasting models for weather and other disturbances. A first distinction therefore shall be drawn on the basis of the time scale needed for the prediction horizon of the model; (Li and Wen 2014) point out how literature has focused on three categories:

1. long-term load forecasts for system planning;

2. medium-term forecasts for system maintenance;
3. short-term modelling for daily operation, scheduling and load shifting plans.

Short-term models are the ones used for day-to-day control of the building system can be further categorized in three main classes which follow three different approaches:

1. physics based, known as white box/mathematical/forward models;
2. data-driven, known as black box/empirical/inverse models;
3. hybrid models which combine the first two approaches, known as grey box models.

These modelling paradigms can yield models either linear or nonlinear, static or dynamic, explicit or implicit, discrete or continuous, deterministic or probabilistic, deductive, inductive or floating.

The distinction between static (steady state) models and dynamic (unsteady state/transient) models lies in the fact that in models of the first type parameters are constant while in the second type parameters change with time. Dynamic modelling is particularly relevant for system start-up, shutdown and reaction to disturbances. Following this categorization, generally physics-based techniques are deductive and data-driven models are inductive, while grey box modelling can fall under both the inductive and the deductive class. All modelling paradigms can result in linear/nonlinear, static/dynamic, and explicit or implicit models. Usually physics-based techniques yield continuous and deterministic models whereas data-driven techniques commonly result in discrete and deterministic or stochastic models.

### 1.4.2 White box models

White box models are developed on the basis of the physical equations of energy, mass and momentum conservation which yield a set of equations that can be derived and solved. When writing these equations, the designer must determine the structure of the equations, meaning their order, number of dimensions and number of parameters, while setting the exact values of such adopted parameters. Indeed, according to (Drgoňa et al. 2020a) *"the main challenge in white-box modelling is the significant effort required to describe the building properties".* The values adopted are based on the physical characteristics of the modelled objects and need to be obtained by design plans, manufacture catalogues or on-site measurements. Models for building energy use often include hundreds if not thousands of parameters, which increases the chances of having potential sources of inaccuracy, other than making the modelling process highly demanding. However, when the parameters choices are accurate, their results are extremely reliable.

White box models can be either distributed or lumped parameter type; however, lumped parameter models have proven to be more efficient because of their ease of use.

The main drawback of white box models lies in their highly demanding computational needs, which makes them hard to employ in most control applications that require simulations and optimization processes to be conducted within the span of a sampling timestep. On the other hand, their high accuracy makes them a viable choice for simulation and testing.

Physics-based dynamic HVAC system models are usually employed for slower processes such as zone temperature dynamics, zone humidity dynamics, heating/cooling coil dynamics etc. Static models are often developed for faster dynamics systems, like mixed air temperature and carbon dioxide concentration in a mixing box or fluid flowrate through a valve, or to compute fans and pumps energy consumption.
In the following paragraphs, a chiller modelling will be presented as an example of white box model.

#### *1.4.2.1 Chiller model*

Chillers are reversed thermal machines that reject heat from a fluid through a vapor compression cycle or an absorption cooling cycle. Chillers are made up of four main components, namely the evaporator, the condenser, the compressor and the expansion valve.
The chiller power consumption is a function of the fluid flowrate, the heat capacity of the fluid, the difference in temperature between the incoming and the outgoing fluid in the chilled fluid loop and the COP of the chiller. The COP varies according to the load on the chiller.
Due to their widespread diffusion in households and non-residential buildings they account for a significant share in final energy consumption. According to (Browne and Bansal 2002) "*it is a well-known fact in the HVAC industry that for the majority of the time these machines operate under part-load conditions (away from design conditions) and in an unsteady manner. This generally results in a decrease in the coefficient of performance (COP) and hence electrical costs are greater than necessary*". From this premise follows that engineers might benefit from chiller models that can help in both the design and the control phase.
The model proposed by (Browne and Bansal 2002) will be here shortly presented and reviewed. The authors proposed a dynamic model, noticing that while steady-state models can be useful under many conditions, under common strongly dynamic conditions these models become "*unacceptably inaccurate*". The purpose of the work was to model the dynamic performance of vapor-compression liquid chillers under various operating conditions. Although their approach was mostly physic-based, in one of the chillers they had to employ a regression model for the compressor and apply some degree of empiricism for the evaporator tube wall mass to predict the start-up process for that particular chiller adequately. Four simplifying assumptions were made:

- The mass flow rate of the refrigerant is assumed to be the same throughout the system and equal to the mass flow rate through the compressor.
- The temperature of the walls does not vary through the cross-section or along the length of the tubes.
- The refrigerant properties within each component are assumed to be homogeneous with pressure drops being neglected.
- The expansion process through the EEV/orifice plate was assumed to be isenthalpic.

The evaporator load and its energy balance equations used are:

$$\dot{Q}_e = \alpha_{ei} A_{ei} (T_{chw} - T_{we}) \tag{1.17}$$

$$\dot{m}_r (\Delta h_{ev}) = \alpha_{eo} A_{eo} (T_{we} - T_e) \tag{1.18}$$

similarly, the same equations across the condenser are:

$$\dot{Q}_c = \alpha_{ci} A_{ci} (T_{wc} - T_{cw}) \tag{1.19}$$

$$\dot{m}_r (\Delta h_{ev}) = \alpha_{co} A_{co} (T_c - T_{wc}) \tag{1.20}$$

Here, $\dot{Q}_e$ and $\dot{Q}_c$ represent the heat power exchanged at the evaporator and the condenser respectively, $\alpha_{ei}$ and $\alpha_{co}$ represent the evaporator and the condenser heat transfer coefficients, $A_{ei}$ and $A_{ci}$ represent the heat exchangers areas, $\Delta h_{ev}$ and $\Delta h_{ev}$ stand for the enthalpy difference occurring across the components for the refrigerant of flowrate $\dot{m}_r$.
The prediction result of this model was found to be within ±10% as a result of a number of simplifying assumptions.

### 1.4.3 Black box models

Black box modelling is a paradigm that captures the correlation between the inputs and the outputs of a dynamic system without any assumption regarding the physics involved. Such an approach requires a sufficient amount of input-output data for the training phase, which establish the relationship between the two sets. After the training phase, the system is able to infer outputs on the basis of new input.
The main advantage of data-driven modelling is its low development cost; on the other hand, huge amounts of data are required for the training and the model is not reliable outside the training range.
The data-driven nature of black box models suggests applications on existing systems for which measurements are available, thus excluding its employment in design of new system and in research. However, the recent trend in research has been to couple black box models to white box models which replace the real system in providing the required training data, allowing researchers and designers to explore new solutions and applications of black box models.
(Afroz et al. 2018) propose a classification of data-driven models into nine types.

#### *1.4.3.1 Frequency domain model*

Slow dynamic processes, such as the dynamics of zone temperature and humidity, can be modelled with first and second order models with dead time. Second order models are in this case over-damped.

As to first order models, their general transfer function shall be:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} e^{-Ls} \tag{1.21}$$

while for second order models the transfer function shall be:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{as^2 + bs + c} e^{-Ls} \tag{1.22}$$

where $K$ is the gain, $\omega_c$ is the natural angular frequency and $L$ is the dead time of the process; therefore:

$$a = \frac{\left[c - Re\left(\frac{e^{-j\omega_c Ls}}{G(j\omega_c)}\right)\right]}{{\omega_c}^2} \tag{1.24}$$

$$b = Im\left[\frac{\left(\frac{e^{-j\omega_c L}}{G(j\omega_c)}\right)}{\omega_c}\right] \tag{1.25}$$

$$c = \frac{1}{K} \tag{1.26}$$

$$K = \frac{\Delta y(\infty)}{\Delta u(\infty)} \tag{1.27}$$

Applications of the first order model can be found in (Huang, Wang, and Xu 2009) and (Huang, Wang, and Xu 2010); the authors used the model in both works to describe the dynamics of AHUs to design a robust control strategy of an air conditioning system. In their examples, the output $Y$ represents the outlet air temperature after the cooling coil, the input $U$ represents the openness of the valve which controls the chilled water flow rate through the cooling coil. A similar approach was followed by (Bi et al. 1999) who proposed a robust identification of first-order plus dead-time model from step response applied to an AHU. The authors here formulated an identification method based on a step-response test through linear regression equations outperforms the existing estimation methods that use step-test responses, while being robust in the presence of large amounts of measurement noise. The effectiveness of the identification method was demonstrated through several simulation examples and a real-time test.

The second order model finds an application in the design of an optimal PID control system conducted by (Kurokawa et al. 2020).

Both the first and the second order model can be applied for SISO as well as for MIMO systems. Their main advantages lie in the simple structure and the small number of parameters to be determined from measurement data; moreover, it is a well-established method supported by abundant literature. On the other hand, collecting data is an intrusive process as the step response test requires to pause the ordinary operation of the system. Furthermore, these models can only be applied to linear, time-invariant systems.

#### *1.4.3.2 Data mining algorithms*

Data mining algorithms are designed to find patterns in input-output datasets provided by the designer. The algorithms are capable of finding a relationship between the inputs and the outputs of the provided set so that future outputs will be inferred from new inputs. This process of training requires sufficient amount of suitable data, but allows to model complex and non-linear dynamics.

The most widely used data mining algorithms for HVAC systems and building dynamics are Artificial Neural Network (ANN) and Support Vector Machine (SVM).

#### *1.4.3.3 Fuzzy logic models*

Fuzzy logic was introduced by Lotfi Aliasker Zadeh with the intention of creating a computer logic able to mimic the human mind decision making process. In classical theory, sets are called crisp as any given element can either belong or not belong to a certain set; in fuzzy logic theory, elements can be said to belong to a set "to some degree". This approach follows closely human reasoning and language. People describe and define reality through several categories that cannot possibly be translated in "crisp" terms: adjectives like "reliable", "safe", "convenient" or "expensive" can hardly be attributed to elements of a discourse in a definite way. Fuzzy logic comes in handy in providing a mathematical formalization of such concepts to allow ultimately a computer-based formulation, offering interesting prospects to control systems applications since these are based on a decision-making algorithm. This approach naturally follows under the domain of the black box modelling rationale, since it allows experience-based knowledge to be translated into logical rules.

In formal terms, given a generic element $u$ belonging to a universe of discourse $U$ (see Usó-Doménech, Nescolarde-Selva, and Gash 2018 for a more detailed definition of universe of discourse and existance), a fuzzy set $A$ can be defined by a membership function $\mu_A: U \rightarrow [0,1]$ or rather, function $\mu_A(u)$ gives $u$ a measure of its degree of membership to set $A$. By contrast, a crisp set membership function could only assign either 0 or 1 value to element $u$. It is important to note that the uncertainty expressed by fuzzy logic should not be interpreted as a form of probability distribution as it is not based on randomness but rather on the vagueness of the definition of a fuzzy set.

Fuzzy logic is applied to decision making processes through the so-called fuzzy inference, that is the process of mapping inputs to outputs on the basis of previous knowledge. A Fuzzy Inference System (FIS) has a general architecture that can be summed up in three stages:

1. Fuzzification: crisp inputs are evaluated in their degree of membership to the fuzzy sets to obtain the fuzzy variables. For example, a temperature value, rather than being either cold or hot, might be

assigned a value of membership to the two categories, on the condition that the sum of the two values equals one.

2. Inference: fuzzy variables go now through the decision-making process described by the fuzzy rules to obtain fuzzy outputs.
3. Defuzzification: fuzzy outputs are translated back into crisp values.

The two most common Fuzzy Inference Systems are the Mamdani and the Sugeno methods and find many applications to control systems and fuzzy modelling.

Fuzzy logic is a popular choice for modelling as the resulting models are "*generally very simple and easy to understand*" (Afroz et al. 2018); moreover, FL allows designers to incorporate previous experience and, due to the very nature of fuzzy logic, human-like reasoning in the design and modelling phases. Several examples of FL applied to the modelling and control of HVAC components and building energy systems are found in literature. (Calvino et al. 2004) developed a fuzzy PID regulator for the PMV value in an office room subject to different outdoor climatic conditions. The experimental results proved the control system to be stable under diversified boundary conditions and able to provide "*an effective and fast control of the indoor microclimate conditions*". Thermal comfort is also the modelling technique developed in (Chen, Jiao, and Lee 2006). The authors pointed out how the very notion of comfort is fuzzy in nature and therefore prone to be translated in terms of FL; the study is carried out employing the architecture of the Fuzzy Adaptive Network (FAN).

### 1.4.4 Grey box models

Grey box modelling is often referred to as *hybrid modelling* for it combines white and black box paradigms together. The rationale behind this approach is that of writing the equations describing the system, as physics-based modelling suggests, while leaving the parameters to be determined by a data-driven process. This gives grey box modelling two main advantages:

- a reduced amount of data required for the training process compared to a purely data-driven model;
- a possibly simplified structure of the equations compared to a solely physics-based model.

Indeed, "*usually, the physics in grey-box models is simplified by means of state space dimensionality reduction or linearization*" (Drgoňa et al. 2020a), which means that the designer can choose to linearize the equations describing the system or to convert them to lumped-parameter differential equations on the basis of their intended employment. Grey box modelling is particularly apt to be used in MPC applications as it allows to adapt to the needs of an optimization solver.

Another advantage of grey box modelling is its portability. (Reynders, Diriken, and Saelens 2014) argued that "*only few model types are needed to represent the majority of buildings*".

### 1.4.5 RC equivalent circuits

As previously illustrated, one of the main advantages of grey box modelling lies in the option given to the designer to reduce the complexity of the chosen model. Simplification is directly linked to the application for which the model will be used. When it comes to modelling buildings from the point of view of their thermal behaviour, the simplest and therefore most desirable solution is that of a lumped-parameter, possibly linear model, that could provide a set of linear ordinary differential equations. The thermal-electrical analogy applied to heat transfer comes in handy in formulating such equations with the optional graphical aid of a circuit representation. More in detail, it is possible to assign to each thermophysical quantity an equivalent electrical quantity, as summarized in Table(1) :

| **Thermophysical quantity** | **Electrical quantity** |
|---|---|
| Thermal resistance [R]=K/W | Electrical resistance [R]= Ω |
| Heat flux [$\dot{\Phi}$]=W | Current [I]=A |
| Temperature [T]=K | Voltage [V]=V |
| Thermal capacity [C]=J/K | Capacitance [C]=F |

*Table 1-Thermal/electrical analogy*

Following this analogy, heat transfer through a component in one dimension shall be modelled with the Eq.(1.29):

$$\dot{\Phi} = \frac{\Delta T}{R} \tag{1.29}$$

so that the heat flux $\dot{\Phi}$ will be driven by the difference in temperature $\Delta T$ between the two boundaries of the element interested by the heat conduction, just as current flow is driven by a difference in voltage between two nodes of a circuit. Similarly, resistance will oppose such flow in both cases.

In order to model heat transfer in a non-steady state regime, capacity must be taken into account, yielding the following equation:

$$\dot{\Phi} = C\frac{dT}{dt} \tag{1.30}$$

that is, the variation in temperature is proportional to the heat flux interesting the object. Again, the analogy with the model for an electric capacitance holds perfectly.

A simple and effective application of this analogy to building models has been proposed by Henrik Madsen in (Bacher and Madsen 2011). This methodology is applied to single zone buildings and can be summarized in three main steps:

1. Model choice, that is the choice of a circuit and therefore of the set of equations to adopt;
2. Parameters estimation, which means assigning a value to the equations parameters on the basis of considerations of the physical properties of the system components;
3. Parameters tuning and model validation by means of an input-output data set coming from field measurements or an accurate white box simulation.

This procedure, referred to as *Forward selection,* is recursive; the authors suggest starting from the simplest possible model and then increase its complexity until no extension to the model yields and improvement below a given p-value, usually set to 5%.
The most complete model is showed in Figure(7):

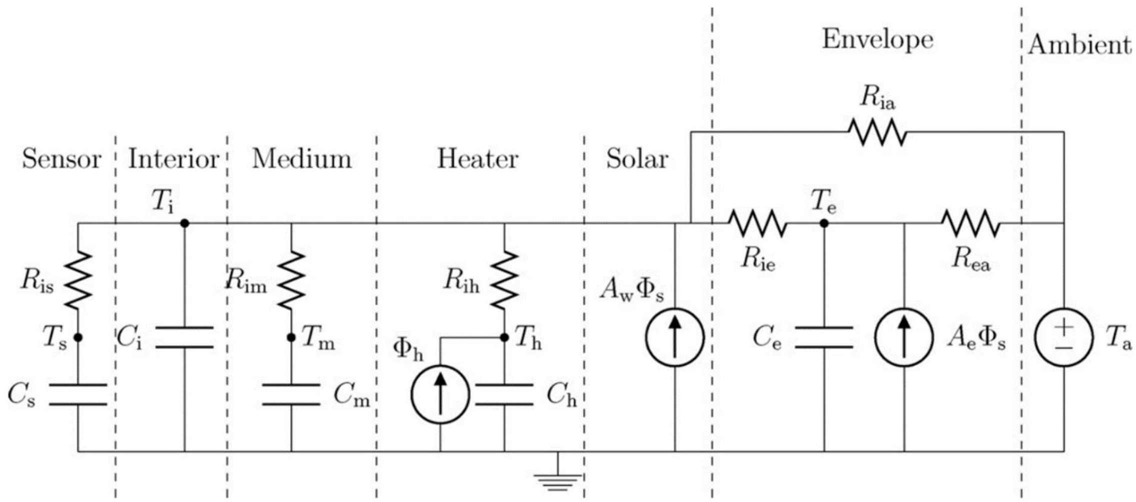


*Figure 7-Complete RC equivalent model (Bacher and Madsen 2011)*

The model is made of the following elements:

- Five significant circuit nodes, $T_s$, $T_i$, $T_m$, $T_h$, and $T_e$ representing five states of the systems whose equations will determine their values in time. Treated as voltages in the circuit, they represent temperatures of different elements of the building, as it will be detailed later.
- Five capacitances, $C_s$, $C_i$, $C_m$, $C_h$, and $C_e$ representing the thermal capacity of the forementioned elements.
- Six resistors, $R_{is}$, $R_{im}$, $R_{ih}$, $R_{ie}$, $R_{ea}$, and $R_{ia}$ representing thermal resistances.
- Three current generators, $\Phi_h$, $A_w\Phi_s$ and $A_e\Phi_h$ representing heat fluxes.
- One voltage generator $T_a$, representing the external temperature.

These circuit elements are employed to model the dynamics of the different components of the building and their interaction, to allow the formulation of the state space equations. These components are here analysed in detail.

### 1.4.5.1 Sensor

An accurate modelling of the dynamic behaviour of any system should consider the non-steady state nature of sensors, particularly when dealing with on field measurements. Here, along with the sensing element temperature $T_s$, whose reading indicates the internal ambient temperature, a resistance ($R_{is}$) separates the sensor from the internal air. The thermal capacity ($C_s$) is responsible for the lag between the temperature variations of the internal ambient air and the respective reading on the measurement instrument.

### 1.4.5.2 Interior

The portion of the circuit called interior models the dynamics of the internal air mass. This air mass is separated from the other elements by several thermal resistances and includes a usually significant thermal mass ($C_i$).

### 1.4.5.3 Medium

The medium portion models all the masses inside the building envelope other than the internal air. Indeed, such masses have usually thermal proprieties significantly different from air, so that the internal energy stored or released might need a dynamic modelling of its own. Internal masses can include furniture or internal walls between spaces of the same thermal zone. The medium temperature, capacity and its resistance with respect to the internal air are here called $T_m$, $C_m$and $R_{im}$ respectively.

### 1.4.5.4 Heater

The heater section accounts for the contribution of a heating system, in terms of sensible heat. The most accurate model separates a lumped parameter "heater" node temperature ($T_h$) from the internal air node with a resistance between the two ($R_{ih}$). The heating terminal might have a non-neglectable thermal dynamic that requires a capacity to be included ($C_h$). This is particularly relevant for radiant systems whose time constant is comparable to that of the internal thermal mass.

### 1.4.5.5 Solar

The solar portion consists of a current generator representing the heat flux entering the building through the transparent surfaces. Indeed this contribution is estimated as the product of the incident solar radiation $\Phi_s$ as power per unit of surface and the area of the transparent surface $A_w$. This parameter accounts for the g-value of the employed glass as well.

### 1.4.5.6 Envelope

The envelope is modelled as a temperature node exchanging heat with the internal temperature node and the external temperature node through two resistances $R_{ie}$ and $R_{ea}$. As the envelope mass is often non-neglectable, a capacity is associated with it ($C_e$). The solar radiation on the envelope surface is an additional heat flux entering the envelope node and it is modelled as $A_e\Phi_s$ in analogy with the solar contribution through the transparent components; clearly, $A_e$ stands for the opaque surface.

An additional branch connects directly the external ambient air and the internal air nodes with resistance $R_{ia}$. This quantity accounts for the infiltration losses of building, which can be computed as:

$$\dot{\Phi}_{inf} = \dot{m}_{air} c_p (T_i - T_a) \tag{1.31}$$

so that:

$$\dot{\Phi}_{inf} = \frac{(T_i - T_a)}{R_{ia}} \; with \; R_{ia} = \frac{1}{\dot{m}_{air} c_p} \tag{1.32}$$

#### *1.4.5.7 Ambient*

Ambient air temperature is imposed to the system from data (predicted or measured), and therefore an ideal voltage generator, so that the value of the temperature in node $T_e$ is always known.

#### *1.4.5.8 State space representation*

This model yields the following set of equations by applying Kirchoff's nodal law to the five temperature nodes:

$$\begin{cases} \dfrac{dT_s}{dt} = \dfrac{1}{R_{is}C_s}(T_i - T_s) \\ \dfrac{dT_i}{dt} = \dfrac{1}{R_{is}C_i}(T_s - T_i) + \dfrac{1}{R_{im}C_i}(T_m - T_i) + \dfrac{1}{R_{ih}C_i}(T_h - T_i) + \dfrac{1}{R_{ie}C_i}(T_e - T_i) + \dfrac{1}{R_{ia}C_i}(T_a - T_i) + \dfrac{A_w\, \Phi_s}{C_i} \\ \dfrac{dT_m}{dt} = \dfrac{1}{R_{im}C_m}(T_i - T_m) \\ \dfrac{dT_h}{dt} = \dfrac{1}{R_{ih}C_h}(T_i - T_h) + \dfrac{\Phi_h}{C_h} \\ \dfrac{dT_e}{dt} = \dfrac{1}{R_{ie}C_e}(T_i - T_e) + \dfrac{1}{R_{ea}C_e}(T_a - T_e) + \dfrac{A_e\, \Phi_s}{C_e} \end{cases} \tag{1.33}$$

These equations can conveniently be written in matrix form according to the standard state-space representation notation; since the states are the five temperatures $\underline{T} = [T_s, T_i, T_m, T_h, T_e]^T$, the inputs are $\underline{u} = [T_a, \Phi_s, \Phi_h\ ]^T$, the equations take the following form:

$$\frac{d\underline{T}}{dt} = A\underline{T} + B\underline{u} \tag{1.34}$$

where matrixes A and B contain the sole parameters.

The only output of this system is the internal temperature as read by the sensor. The output equation is:

$$Y_s = T_s + e \tag{1.35}$$

since the output $Y_s$ is affected by an error $e$.

### 1.4.6 Thesis objectives and structure

The original contribution of the present thesis-work is to formulate and test different control strategies for the management of the energy systems serving a non-residential building. In particular, the strategies control the storage systems serving the building, namely a hot water tank for thermal storage and a battery for electric energy storage, plus their interaction with the public power grid. The suggested strategies are oriented towards different objectives regarding energy efficiency, economic savings and optimal operation of technological components. Moreover, a methodology of modelling, designing, testing and analysis is suggested as it could be employed in future applications.

Chapter 2 is dedicated to the Model Predictive Control and illustrates the main features of such strategy. Theoretical fundamentals are provided, including some notes on mathematical optimization. The chapter, albeit relatively general in many respects, focuses primarily on building and HVAC applications and provides an overview of previous model predictive control strategies applied to this field.

Chapter 3 presents the case study, which is a non-residential building equipped with an energy system made of thermal energy and an electrical energy storage system, photovoltaic arrays for local renewable energy production, a heat pump for heat generation and a connection to the public power grid. The chapter presents the building and the plant main features, along with their modelling process.

Chapter 4 is dedicated to the formulation and testing of four Rule-Based Control strategies, two of which are oriented to the enhancement of energy related and economic goals respectively. The two strategies are compared to each other and to two simpler control logics that shall serve as baselines. The modelling and simulation process is described in detail.

Chapter 5 presents a Model Predictive Control strategy applied to the same case-study. Again, simulation and modelling are detailed throughout the chapter. The MPC strategy is then compared to a Rule-Based control strategy that serves as baseline. The baseline case is properly designed to provide an equal comparison benchmark.

Finally, observations and future research suggestions are brought forward in Chapter 6.

# 2 Chapter 2: Model Predictive Control

Model predictive control (MPC) is a term used to designate a class of control methods first developed in the Seventies within the field of process industry, particularly chemical industry (Hechavarría, Rodney; López 2013) to be later adopted by other industrial fields, robotics, clinical anaesthesia and autonomous driving systems.

As a well-established control method in industrial applications, MPC has attracted increasing attention by research in the field of building automation, energy management and HVAC control systems. Particularly interesting are the review works of (Serale et al. 2018) and (Drgoňa et al. 2020) which prove how the research community has dedicated increasing focus on the implementation of MPC strategies for building applications.

## 2.1 Working Principles

The authors of (Hechavarría, Rodney; López 2013) indicate three main elements of an MPC controller:

- the presence of a system model with the purpose of predicting its outputs in future time instants over a given time horizon ahead in the future;
- the calculation of a control sequence that minimizes a given objective function;
- the so-called *receding strategy*, that is the shifting at any timestep of the time horizon and the actual application of the sole first control signal of the control sequence.

Therefore, an MPC controller does not only choose its control action to be valid at the present time instant, but plans their evolution over a determined time horizon in the future. This is possible thanks to an explicit model of the controlled system that allows to make a prevision of the time profile of the controlled variables.

Multiple reasons justify the adoption of the MPC strategy; among the many, the following make MPC a much more effective competitor against classical control methods.

1) Multi-input-multi output systems (MIMO): Model predictive control is a much more suitable choice for MIMO systems compared to a traditional control strategy. For instance, let's consider a system with two output $y_1$ and $y_2$ and two control actions $u_1$ and $u_2$, controlled by a PID controller. Action $u_1$ will depend solely on the value of $y_1$, while $u_2$ will depend on $y_2$. However, $u_1$ will influence $y_2$ as well as $u_2$ shall influence $y_1$, thus greatly increasing the complexity of the control system design. MPC on the other hand offers the possibility of formulating a single strategy able to manage multiple inputs and multiple outputs at once.
2) Optimization: in traditional control strategies the choice of the control actions is exclusively oriented towards keeping the controlled systems outputs as closely as possible to a desired profile; however, control actions can have the most varied consequences upon a dynamical system and the reaching of a specific objective might be conflicting with the reaching of another. In general terms, in order to

consider multiple contrasting objectives, it is necessary to find a compromise or, in more rigorous terms, to solve an *optimization problem*. More formal aspects concerning mathematical optimization will be dealt with later in the present work; some examples might nonetheless help grasp the concept intuitively. A heat exchanger employed to recover waste heat will allow greater savings as its surface increases; at the same time, with a larger surface comes a higher monetary investment: it is therefore desirable to find some criteria to help reaching a compromise. Another example, more suited to the purpose of this thesis, is that of keeping a comfortable air temperature inside of a building: this, in general, goes against energy and monetary savings. Optimization problems come in handy when dealing with such matters.

In model predictive control an optimizer works alongside the system model, driving the controller towards a choice of actions that minimize a function known as *cost function*. The cost function sums up the different quantities to be minimized, possibly assigning to each one a weight representing its relative priority. Following the previous example, we could define a parameter that accounts for the difference between the actual internal air temperature and its desired value and a parameter accounting for energy consumption: reducing both shall be desirable, but a relative importance could be assigned to one of the two elements.

3) Disturbances: traditional control strategies merely react to the disturbances acting upon the controlled plant. Disturbances are, by nature, non-controllable as it is their very definition. A control system with the capability to react directly disturbances is therefore most desirable. In order to include disturbances in the control logic, a previous knowledge of their profile is required, or, in other words, their prediction is necessary. Model Predictive Control is able to include predictions of disturbances in its structure so that the controller can to some extent "anticipate" them.

### 2.1.1 Receding horizon strategy

Neither the predictive model nor the disturbances previsions are sufficient to guarantee the MPC controller to operate as a purely open-loop, feedforward control strategy: indeed, the prediction model embedded in the controller shall never be perfect and often requires a great deal of simplification given the computational burden of the optimization process. At the same time, disturbances will never be exactly predicted and the system is in general affected by unpredictable disturbances and measurement noises. A feedback mechanism of some kind is therefore necessary in the MPC algorithm. Such mechanism is known as the *receding horizon strategy*. A description of this strategy follows along with the adopted notational conventions.

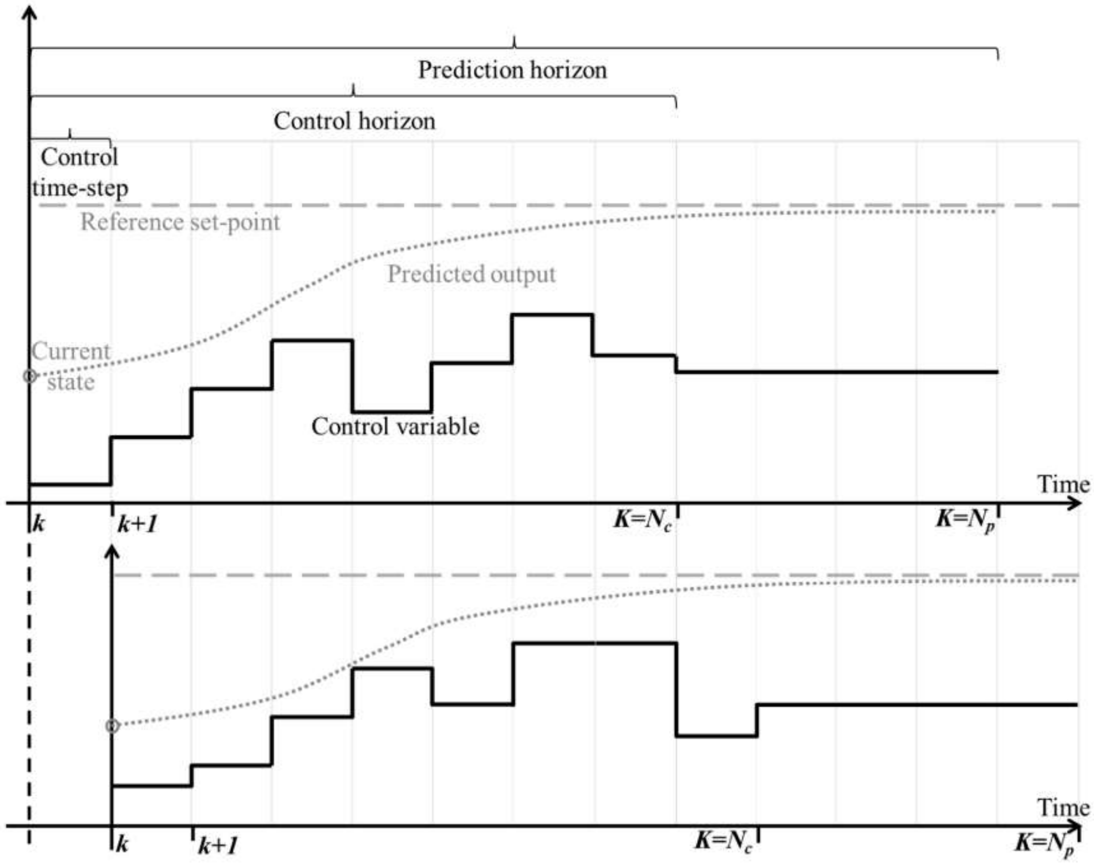


*Figure 8-MPC receiding horizon strategy (Serale et al. 2018)*

The receding horizon strategy is based on the following quantities:

- Current instant $k$: the present instant in the discretization of the time variable.
- Control timestep $T_s$: the time interval used in the discretization of time. It is usually also the sampling time and is for most applications considered to be constant.
- Prediction horizon $N_p$: also called *planning horizon*, it is the span ahead in time for which the MPC will predict the profile of the cost function under the given constraints. It is measured in number of timesteps, therefore it is given as a natural number.
- Control horizon $N_c$: also called *execution horizon* or *manipulated input horizon*, it is the span ahead in time for which the MPC will predict the profile of the manipulated variables. It is always set that $N_c \leq N_p$, which means that the MPC algorithm might predict the values of the outputs and the cost function over a longer horizon than that on which it chooses the values of the manipulated variables. In case $N_c < N_p$, for the remaining timesteps between $K = N_c + 1$ and $K = N_p$the control variable will be evaluated as constant.

The algorithm's working principle follows this general structure:

1) at time instant $k$, assumed as the present instant, the controller evaluates the state of the plant and reads the values of the disturbances' predictions (if available);
2) an optimal sequence of control actions is computed over the control horizon, that is, the size of the sequence will be equal to the control horizon $N_c$;
3) of all the Nc steps computed by the optimizer, only the first action will be actually implemented until the following sample instant $k + 1 = k + T_s$; all the other steps of the computed sequence will be discarded;
4) as time advances to the next sampling instant, the whole time window is shifted by one position, that is, time $k + 1$ becomes the new time instant $k$, and the procedure will start all over from point 1), hence the name *receding horizon strategy*.

The fact that the plant states and outputs are measured or at least estimated at each sampling time, and that the structure of the above described algorithm is intrinsically recursive guarantees that the MPC strategy has a *feedback* nature.

## 2.2 Optimization

A *mathematical optimization problem*, often referred to as just *optimization problem*, is formalized as follows:

$$\begin{cases} \quad\quad\quad minimize\ f_0(x) \\ subject\ to\ \ f_i(x) \leq 0, \quad i = 1,2, \ldots, m \\ \quad\quad\quad\ h_j(x) = 0, \quad i = 1,2, \ldots, p \end{cases} \tag{2.1}$$

Aim of optimization is to find a set of *n* real values that minimize a given cost function (or objective function), here indicated as $f_0(x)$. This is a function in real scalar values defined on a domain included in $\mathbb{R}^n$. The solution to the problem therefore shall be a vector in $x^* \in \mathbb{R}^n$ so that $f_0(x^*)$ has the minimum value; the $x$ variable is called *optimization variable*. Not all points $\mathbb{R}^n$ are eligible as solutions: the choice of all possible candidate $x$ for the solution to the problem is to be found within the set of the so-called *feasible points*; points not included in such set are defined as *unfeasible*.

Feasibility of points is determined by inequalities $f_i(x) \leq 0,\ i = 1,2, \ldots, m$ and equalities $h_j(x) = 0,\ i = 1,2, \ldots, p$ called *inequality constraints* and *equality constraints* respectively, while the corresponding functions $f_i\colon \mathbb{R}^n \to \mathbb{R}^n$ and $h_j\colon \mathbb{R}^n \to \mathbb{R}^n$ are called *inequality functions* and *equality functions*. In case no constraints are present ($p = m = 0$), the problem is said to be *unconstrained*.

The set of points for which the objective function and all the constraint functions are defined is called *domain* of the optimization problem:

$$D = \bigcap_{i=0}^{m} dom(f_i) \cap \bigcap_{j=0}^{p} dom(h_j)$$

A point $x$ is feasible if it belongs to domain $D$ and satisfies all the constrains. If a problem has at least one feasible point, the problem itself is defined as *feasible*; likewise, it is defined as *unfeasible* when it has no feasible points. The set of all feasible points of a problem is called *feasible set* or *constraints set*.
We can define an *optimal value of the problem* $p^*$ a point such that:

$$p^* = inf\ \{f_0(x) | f_i(x) \leq 0, i = 1, \ldots, m, h_j(x) = 0, j = 1, \ldots, p\}$$

We say that $x^* \in \mathbb{R}^n$ solves the problem (or is an *optimal point*) if $x^*$ is feasible and $f_0(x^*) = p^*$. Defining an optimal point leads to the definition of an *optimal set*, that is the set of all optimal points. We can describe such a set as:

$$X_{opt} = \{x \mid f_i(x) \leq 0, i = 1, \ldots, m, h_j(x) = 0, j = 1, \ldots, p, f_0(x^*) = p^* \}$$

The definition of optimality for feasible points gives us a criterium for assessing the solvability of an optimization problem; indeed, a problem is *solvable* if $X_{opt}$ has at least one element, and we therefore say that the problem is *attained* or *achieved*. In case $X_{opt}$ is an empty set, the optimal value is not attained or not achieved.

Finding the optimal set is evidently the most desirable goal for anyone dealing with an optimization problem: optimal points are in fact the "best choices" for problem at hand in that they minimize the cost function in absolute terms compared to all the other feasible points. However, practical limitations might make it necessary to find a compromise with respect to the optimal solution, and to identify as solutions points that are not optimal, but nonetheless must respect some kind of requirement for us to accept them as reasonable solutions. Such alternatives are offered by *sub-optimal* solutions and *locally optimal* solutions.

Given a value of $\varepsilon > 0$, a feasible point $x$ is called *ε-suboptimal* if $f_0(x^*) \leq p^* + \varepsilon$ and the set of all ε-suboptimal points is called *ε-suboptimal set* for the problem.

A feasible point is *locally optimal* if there exists a real value $R > 0$ such that:

$$f_0(x) = inf\ \{f_0(z) | f_i(z) \leq 0, i = 1, \ldots, m, h_j(z) = 0, j = 1, \ldots, p, \|z - x\|_2 \leq R\}$$

which is equivalent to saying that point x solves the following problem:

$$\begin{cases} \text{minimize } f_0(z) \\ \text{subject to } f_i(z) \leq 0, \quad i = 1,2,\dots,m \\ h_j(z) = 0, \quad i = 1,2,\dots,p \\ \|z - x\|_2 \leq R \end{cases} \tag{2.2}$$

The meaning of local optimality is that the point $x$ does actually minimize the cost function $f_0$, but solely in within the neighbourhood of a point $z$. Indeed, many authors specifically refer to the ordinary optimal solution as the *global solution* to avoid confusing it with the *local solution*.

If a point $x$ is feasible and $f_i(x) = 0$, the *i-th* inequality constraint $f_i(x) \leq 0$ is defined as *active at x*. In case $f_i(x) \leq 0$, the *i-th* inequality constraint is called *inactive*. From this definition, it is obvious that all equality constraints are active for all feasible points. In case a constraint does change the feasible set, such constraint *redundant*.

It might be useful, before solving an optimization problem, to determine whether the given set of constraints are consistent, and in such case find a point that satisfies those constraints. This is achieved by solving a *feasibility problem*, which can be written as:

$$\begin{cases} \text{find } x \\ \text{subject to } f_i(x) \leq 0, \quad i = 1,2,\dots,m \\ h_j(x) = 0, \quad i = 1,2,\dots,p \end{cases} \tag{2.3}$$

### 2.2.1 Equivalent problems: slack variables

A non-rigorous definition of equivalence for optimization problems can be stated as: "two problems are equivalent if the solution of one can easily be obtained from the solution of the other". In other words, it is possible to manipulate the problem to obtain a new one whose solution is the same as that of the original problem, or at most is closely related to it. Such manipulations are of different nature and their application is justified by computational convenience.

A commonly used transformation that is particularly important in MPC applications is the introduction of the so-called *slack variables*. Any constraint inequality can undergo the following transformation:

$$f_i(x) \leq 0 \iff \exists s_i \geq 0: f_i(x) + s_i = 0$$

which states that inequalities can be restated as equalities with the addition of a value $s_i$ indeed called slack variable. Using slack variables, the optimization problem can be turned into an equivalent one:

$$\begin{cases} \text{minimize } f_0(x) \\ \text{subject to} \\ f_i(x) + s_i = 0, \quad i = 1,2,\dots,m \\ s_i \geq 0 \quad i = 1,2,\dots,m \\ h_j(x) = 0, \quad i = 1,2,\dots,p \end{cases} \tag{2.4}$$

where the problem variables are $x \in \mathbb{R}^n$ and $s \in \mathbb{R}^m$. In such a case, the solution of the problem shall be a pair $(x, s)$.

## 2.3 Model Predictive Control for buildings

The following section aims at sketching an overview of Model Predictive Control by presenting its main features. Particular attention will be paid to building applications and specifically to the thermal control achieved through the employment of HVAC systems; however, since the adoption of MPC strategies by this sector is a fairly recent phenomenon, many examples will be drawn from the existing literature concerning various other engineering fields in which MPC has found applications so far.

Literature consulted in the making of this thesis showed that a general agreement in MPC notation, formulation and nomenclature has yet to find consensus among the scientific community, therefore different examples will display variations in the notation.

### 2.3.1 Problem formulation

The MPC problem formulation can be adapted to the control of the building thermal behaviour as indicated by a general standard form proposed by (Drgoňa et al. 2020):

$$\begin{gathered}
\min_{u_0,\dots,u_{N-1}} l_N(x_N) + \textstyle\sum_{k=0}^{N-1} l_k(x_k, y_k, r_k, u_k, s_k) \\
subject\ to \\
x_{k+1} = f(x_k, u_k, d_k),\ k \in \mathbb{N}_0^{N-1} \\
y_k = g(x_k, u_k, d_k),\ k \in \mathbb{N}_0^{N-1} \\
u_k = f_{HVAC}(x_k, a_k, m_k),\ k \in \mathbb{N}_0^{N-1} \\
s_k = h(x_k, y_k, u_k, r_k),\ k \in \mathbb{N}_0^{N-1} \\
x_k \in X,\ u_k \in U,\ a_k \in A,\ s_k \in S, k \in \mathbb{N}_0^{N-1} \\
d_k = d(t + kT_s), k \in \mathbb{N}_0^{N-1} \\
r_k = r(t + kT_s), k \in \mathbb{N}_0^{N-1} \\
x_0 = \hat{x}(t)
\end{gathered} \tag{2.5}$$

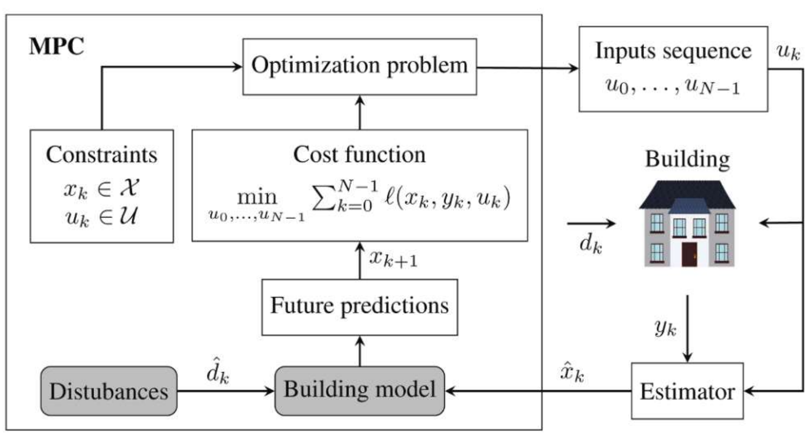


*Figure 9-MPC actors and interaction with the controlled system (Drgoňa et al. 2020)*

The building with its behaviour and its dynamics is affected by the disturbances. Disturbances are all those actions that act upon the system that cannot be controlled by the controller, i.e. people presence, internal heat gains such as lighting systems and electrical appliances and, of course, weather conditions. Disturbances, however, can be quantified and above all predicted to be fed to the MPC control unit: this justifies their presence in the first two constraint equations above stated.

The $n_y$ outputs of the system, collected in a vector $y_k \in \mathbb{R}^{n_y}$, are the quantities that can actually be measured and therefore will constitute the feedback element of the MPC logic. The object indicated as *estimator* in the picture estimates the value of the $n_x$ state variables that will take the form of vector $x_k \in \mathbb{R}^{n_x}$. Building envelope inputs are here represented with the vector $u_k \in \mathbb{R}^{n_u}$, while HVAC actuators are considered separately in a vector $a_k \in \mathbb{R}^{n_a}$, and their relationship is represented by the third constraint equation. Vector $m_k \in \mathbb{R}^{n_m}$ collects additional measured variables, $d_k \in \mathbb{R}^{n_d}$ denotes the disturbances, $r_k \in \mathbb{R}^{n_r}$ the reference signals and $s_k \in \mathbb{R}^{n_s}$ is the vector of the slack variables.

The objective function is given as the sum of two terms: $l_N(x_N)$ represents the *terminal penalty term*, used to ensure stability and convergence; however, most building applications omit this term. The second term $l_k(x_k, y_k, u_k, r_k, s_k)$ is *stage cost* because for its stage *k* this function assigns a cost to the choice of values $x_k, y_k, u_k, r_k, s_k$.

Slack variables represent a violation of algebraic constraints, usually a deviation from the set values of comfort. The conditions $x_k \in X,\ u_k \in U,\ a_k \in A,\ s_k \in S, k \in \mathbb{N}_0^{N-1}$ are bounding constraints on the possible values of the state variables, building envelope inputs, actuator inputs and slack variables. Forecasts of disturbances and reference signals are given by the equations $d_k = d(t + kT_s)$ and $r_k = r(t + kT_s)$. Finally, initial conditions of the states are given at each sampling instant as $x_0 = \hat{x}(t)$, where $\hat{x}$ denotes an *estimation* of the states, since they often cannot be directly measured.

### 2.3.2 MPC problem classes

#### *2.3.2.1 Linear MPC*

An MPC problem is said to be linear when both the plant model and the constraints are linear and the cost function is either linear or quadratic. In such a situation, the relative optimization problem is guaranteed to be convex, being in particular a Linear Program (LP) or a Quadratic Program (QP) for the linear and the quadratic cost function respectively. In such situations, a large number of algorithms available guarantee a solution time shorter than the sampling time for most applications.

(Serale et al. 2018) propose a typical MPC formulation for a Linear Time Invariant (LTI) plant model:

$$\begin{cases} x(k+1) = Ax(k) + B_u u(k) + B_v v(k) + Gw(k) \\ y(k) = Cx(k) + D_u u(k) + D_v v(k) + d(k) \end{cases} \tag{2.6}$$

where $x(k+1)$ is the vector of the predicted state for the time instant $k+1$, $x(k)$ is the vector of the states at the present instant $k$, $u(k)$ is the vector of the controlling inputs, $v(k)$ is the vector of the measured disturbances that affect the system (e.g. the weather), $w(k)$ is the unmeasured random noise on the measurement of the states and $d(k)$ is the unmeasured random noise on the measurement of the outputs $y(k)$. The terms $A$, $B_u$, $B_v$, $C$, $D_u$, $D_v$ and $G$ are state matrix, manipulated input matrix, measured disturbances matrix, output matrix, direct transmission matrix for manipulated inputs, direct transmission matrix for measured disturbances, and the matrix of the unmeasured random noise on the states respectively.

Linear systems can be integrated in a very simple way by recursive substitutions of the state variables moving forward in time. Such a formulation of the problem is referred to as *dense formulation* and the computational complexity of this algorithm is of order $O(N^3 n_u^3)$, where N is the number of steps of the control horizon and $n_u$ is the number of inputs. An alternative to the dense formulation is the *sparse formulation*, whose complexity increases to $O(N^3(n_x + n_u)^3)$ where $n_x$ indicates the number of states of the problem. However, if the solver makes proper use of the sparsity of the problem, the algorithm complexity drops significantly to order $O(N(n_x + n_u)^3)$. Clearly, dense formulation is an appropriate choice when the number of states is large, which is a situation typically found in building control applications.

### 2.3.3

#### *2.3.3.1 Nonlinear MPC*

The vast majority of real processes from the most diverse applications and fields are nonlinear in nature. However, most MPC applications have so far used linear models. Linearity for MPC has been often preferred for its computational convenience, since employing a linear plant model along with a quadratic objective function yields a convex optimization problem (Quadratic Programme). As (Camacho-Bordons) noted, there are two other important reasons that suggest the use of a linear model: firstly, system identification for linear models from process data is much easier; secondly, linearity well approximates the behaviour of a system in the neighbourhood of the operating point, assumed to be in steady-state.

Not all applications, however, support a linear model; in case of building physics applications, the building model is often assumed linear but coupled with a nonlinear modelling of the HVAC system and the effects of the disturbances.

#### *2.3.3.2 Hybrid MPC*

Many systems contain variables that cannot be described as continuous but require a discrete description. Such quantities can be related to a wide range of physical and technical objects, such as switches, two-state valves, fixed speed pumps and ON-OFF commands in general. For many years, the control of discrete quantities was considered as completely set apart from the control of continuous variables, for which mathematical tools like differential equations and transfer functions apply. As to discrete variables, common modelling tools have been state transition graphs, finite states automatas and Petri nets, supported by theoretical fields such as switching theory and graph theory. Traditionally, discrete systems dynamics have been fields of interests for computer scientists.

A change started to appear under the horizon in the beginning of the 1990s, since when there has been a growing interest in in processes that incorporated both discrete and continuous time-dependent quantities, giving birth to the study of the so-called *hybrid systems*.

Notable research in the field of hybrid systems, particularly in the perspective of MPC applications, have been carried on by Alberto Bemporad among others. Important results can be found in (Bemporad, Heemels, and De Schutter 2002), in which five main classes of hybrid systems, namely linear complementarity, extended linear complementarity, mixed logical dynamical, piecewise affine and min-max-plus-scaling systems were firstly proven to be equivalent, albeit under specific assumptions related to well-posedness and boundedness of some variables. Starting from this result, the paper moves on to prove that "for linear or hybrid plants in closed-loop with a model predictive control (MPC) controller based on a linear model, fulfilling linear constraints on input and state variables, and utilizing a quadratic cost criterion [...] the closed-loop system is a subclass of any of the former five classes of hybrid systems". As mentioned by the authors, this result is important for the development of hybrid MPC controllers with increased robust stability.

The most commonly used approach for hybrid MPC is to translate the control model into a *piecewise affine system* (PWA). A PWA system can be described by the following state space representation:

$$\begin{cases} x(k+1) = A^i x(k) + B^i u(k) + f^i \\ y(k) = C^i x(k) + g^i \end{cases} \tag{2.7}$$

Apex *i* refers to the choice of a given "configuration" depending on the particular combination of the values taken by the discrete variables. In other words, for each choice of the discrete variables, different indexes *i* will change form $1, .., s$ yielding a different description of the system dynamics. This dynamics, however, is nonetheless linear, albeit different under different conditions, hence the name "piecewise affine". More formally, the space of the states and input space is partitioned by hyperplanes (polyhedral partitioning) so that:

$$\begin{pmatrix} x(k) \\ y(k) \end{pmatrix} \in X_i \tag{2.8}$$

where $\{X_i\}_{i=1}^{s}$ is the polyhedral partition.

If the hybrid dynamic model is indeed piecewise linear, two scenarios might arise:

1) if the objective function is linear, the corresponding optimization problem shall be a Mixed Integer Linear Programming (MILP);
2) if the objective function is quadratic, the corresponding optimization problem shall be a Mixed Integer Quadratic Programming (MIQP);

In case the dynamical model itself contains nonlinearities, the corresponding problem will be a Mixed Integer Nonlinear Programme (MINLP), a problem of particularly difficult solution.

### 2.3.4 MPC solution

This section aims at presenting the main strategies adopted for the solution of MPC problems, as found in the existing literature. Along with the already illustrated *receding horizon* strategy, MPC solvers require some other steps, the first being *state estimation*. State estimation is the process of inferring the values of state variables that cannot be directly measured as outputs.

#### 2.3.4.1 Optimal control solution method

The problem that stems out of Model Predictive Control requires the solution of an optimal control problem (OCP). Such solution is traditionally achieved through three different approaches:

1) *Direct methods*: this class of methods is the most widely used for MPC applications to the present day; as their name suggests, they consist in the translation of the OCP into the corresponding optimization problem, which is then solved via the most appropriate optimization algorithm.
2) *Indirect methods*: these methods reformulate the OCP as a boundary value problem. The stage cost and the costate equations are incorporated into the control Hamiltonian, which is later minimized (or maximized).
3) *Dynamic programming*: dynamic programming employs the Hamilton-Jacobi-Bellman equations solved recursively. The main disadvantage of this methods lies in the fact that it cannot be applied to systems with too many state variables ((Drgoňa et al. 2020b) refer to this limitation as *curse of dimensionality*). This drawback can be overcome by applying the so called approximate dynamic programming, which takes advantage of reinforcement learning techniques.

The following paragraphs will focus on direct methods, since they are the most implemented choice for the solution of optimal control problems.

One possible method takes the name of *single shooting method*, *dense formulation* or *state condensing* method. The basic working principle of this method is to try out different candidate trajectories for the control actions in different directions until it finds the one that satisfies the boundary conditions. This method is suited to the solution of system whose computational burden is low, such as with linear systems.

A second approach is the *multiple shooting* method, also known as *sparse formulation*.

#### 2.3.4.2 Implicit MPC

Implicit MPC refers to a paradigm for MPC solution based on direct methods. This method finds the optimal choice of the control actions sequence at the present time instant $k$ by solving online the optimization problem. Online solution evidently requires more computational time; however, buildings have slow dynamics, so typically implicit MPC is considered a viable choice for building applications.

#### 2.3.4.3 Explicit MPC

Solving online optimization problems for each timestep can be computationally demanding. In some cases, the computational time required for such a solution exceeds the sampling time itself, calling for a more capable processor for the MPC application. Explicit MPC tackles this problem by pre-computing the optimal solution for a range of initial values of the state variables. These precomputed solutions are then stored and made available offline in the form of lookup tables or simple algebraic equations in the following form:

$$u(x) = f\big(x(k)\big) \tag{2.9}$$

By doing this, the computational time is drastically reduced as the optimizer merely evaluates a simple function rather than solving a whole optimization problem at each sampling instant, allowing for the employment of cheaper hardware. As noted by (Alessio and Bemporad 2009) the functions to which the optimization is translated offline is often piecewise affine, so that "the MPC controller maps into a lookup table of linear gains".

The main drawback of explicit MPC is that the precomputed solutions, while relieving the processor from the computational burden, increase the need for memory storage. Therefore, explicit MPC is recommended for small scale systems with no more than 10 parameters.

#### 2.3.4.4 Approximate MPC

An alternative to the solution of the optimization problem comes from the Machine Learning (ML) world. With *approximate MPC*, a ML model is trained to mimic the behaviour of MPC, which is used to generate training data for the approximate controller. These training data are generated in closed loop simulations using implicit MPC. The resulting machine learning model shall be an approximation of the MPC control law called *control policy*.

Control policy is basically a parametric solution to the MPC problem whose representation is a function that goes from the space of the parameters ξ to the space of the control variables, in the following mathematical form:

$$f_{MPC}: \mathbb{R}^{n_\mathrm{\xi}} \rightarrow \mathbb{R}^{n_\mathrm{u}}$$

The explicit MPC used as an expert teacher of the ML model generates a set of *m* training data $\{(\xi^{(1)}, \mathrm{u}^{(1)}), \ldots, (\xi^{(m)}, \mathrm{u}^{(m)})\}$ linking a set of control actions to each set of states, so that $\xi^{(i)} \in \mathbb{R}^{n_\xi}$ and $\mathrm{u}^{(i)} \in \mathbb{R}^{n_\mathrm{u}}$. The training algorithm will result in a function $f_\Theta: \mathbb{R}^{n_\xi} \rightarrow \mathbb{R}^{n_\mathrm{u}}$ called *response* or *target variable* that predicts the values of the control actions (*future vector*) given a state ξ, so that $u = f_\Theta(\xi)$. The main drawbacks of approximate MPC are:

- an accurate but nonetheless suboptimal solution of the MPC problem
- the need for a large training data set
- no guarantees on stability and constraints handling.

### 2.3.5 Models and architecture

One of the main challenges in formulating an MPC controller is that of choosing an appropriate model for the system at hand. In case of buildings, we are dealing with systems whose dynamics is, albeit slower, in many ways more complex than that of, say, industrial processes. Indeed, disturbances in an industrial setting are much easier to control and predict. At the same time, building control is not solely related to the envelope, but encompasses the control of the HVAC systems a well, whose behaviour is conversely much faster in its dynamics and much harder to model with simple, linear equations.

The challenge in the MPC formulation thus becomes that of finding a trade-off between model accuracy and computational feasibility.

An MPC controller for a building and its HVAC system requires two models in the first place:

- A *control oriented* model, that is a model embedded into the MPC controller unit, capable of offering predictions upon the behaviour of the building and its HVAC system thermodynamics wise. This model cannot be accurate at will but must be simple enough to allow the optimizer to find a feasible solution, whether it be online or offline.
- A *disturbance* model, whose aim is to provide a forecast of the profile of the uncontrolled variables affecting the system, e.g. the weather conditions, occupants behaviour and energy pricing.

These two models are always necessary for the implementation of an MPC controller. They provide a prediction of the behaviour of the system and a choice for the actions to be taken by the actuators so that, according to the standard MPC scheme, the effects of such actions are measured on the actual physical plant, that is the building and/or its HVAC system, to allow the controller to act as in a closed-loop, feedback fashion. However, in the design phase or in research contexts, a building with its mechanical and piping systems are not available, which calls for a third type of model:

- A *surrogate* model, whose job is to simulate the plant with the best possible accuracy, since its computational burden will not affect the controller itself, and it will not appear in the final implementation as it is a mere substitute for the real system.

#### *2.3.5.1 HVAC systems*

Objects usually constituting an HVAC system, such as boilers, heat pumps, combustion-based heat generators, chillers, filters, pumps, heat exchangers and pipes, have often complex and non-linear characteristics, making it challenging to find a suitable compromise between complexity and computational demand.

For instance, pumps and fans have nonlinear characteristics and are coupled with nonlinear relations of mass flow rates and pressure differences along the piping and duct systems. Clearly Computational Fluid Dynamics (CFD) codes are far too demanding for the computational simplicity required by the MPC controller; in literature, different approaches can be found to tackle this problem.

- A first approach separates the HVAC system from the building including solely the latter into the MPC formulation, thus solving a higher level optimization problem. The solution of this problem might return, for instance, the temperature setpoints to be reached by the HVAC system. Controllers on a lower hierarchical level of the control architecture will work towards those setpoints by means of more traditional ON-OFF or PID logics.
- A second approach takes the problem of modelling the HVAC system head on, choosing an appropriate mathematical description. This method contemplates two cases: integrating this model within the MPC formulation, thus solving one single optimization problem, or separate it from the building, so that the building demand will be treated as a predicted disturbance by the MPC controller. The first method requires a higher computational effort, since the states considered for the description of the system increases to take into account the states relative to the HVAC system objects. The second method yields a nonlinear problem in case one of the states multiplies a control variable, like it might happen if a temperature value, assumed as a state, multiplies a controllable mass flow rate value.

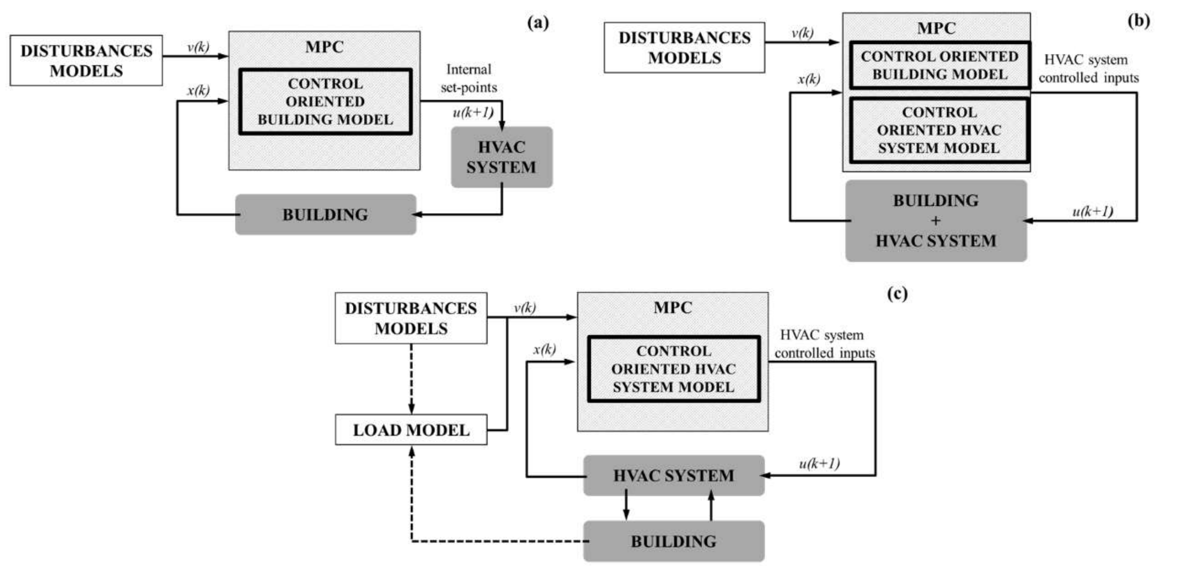


*Figure 10-MPC approaches for building applications (Serale et al. 2018)*

### *2.3.5.2 Disturbances prediction models*

Disturbances are those inputs that influence the system but cannot be controlled. The capability of handling disturbances is indeed one of the main advantages of MPC with respect to traditional control methods; however, disturbances must be predicted with sufficient precision in order to be correctly evaluated by the controller. A classification proposed by (Serale et al. 2018) is reported in the following paragraph.

Disturbances can be either *measured* or *unmeasured*. Measured disturbances can take part into the feedback mechanism of the receding horizon, therefore are generally included in the system dynamic model. Unmeasured disturbances on the other hand cannot be included in the embedded model regardless of the significance of their impact upon the system, so they affect the uncertainty and the accuracy of the model response.

Measured disturbances can be assumed as *ideal* in case no noise (white, stochastic etc.) is significantly affecting their measurement, otherwise thy are considered as *affected by uncertainty* and their signal will require to undergo some suitable processing.

Disturbances affecting buildings and their HVAC systems usually follow under one of three categories:

1. *Climatic disturbances*, such as external temperature, humidity ratio, relative humidity, solar radiation, wind velocity, ground temperature.
2. *Occupant behaviour* related disturbances, such as the number of occupants in a given thermal zone or the gain dew to the use of electrical appliances.
3. *Grid and energy distributor* related disturbances, such as real-time energy prices.

In some cases, disturbances can be lumped together in one single factor. Regarding indoor occupancy, models can provide either the heat gain profile, the occupant count or the count along with a behaviour prediction, since people behaviour influences window opening, appliances usage and heat emission. According to (Drgoňa et al. 2020b), "*the state-of-the-art occupancy behaviour models […] are computationally too expensive to be included in MPC. Therefore, less computationally demanding approaches are typically adopted in the context of MPC, for example models based on heuristics (e.g. anticipated schedules) or machine learning*"

Predictions of disturbances can be obtained in a simple way from commonly available sources, such as Representative Meteorological Year, Typical Meteorological Year, International Weather for Energy Calculations files or building demand patterns as proposed by ASHRAE. This approach only provides general trends but would be hardly suited for precise, short-term predictions. They are therefore used in the design stage rather than in operation.

A more precise solution for design purposes is to analyse historical data collected by the BAS, which provides a more accurate description of the case at hand.

For real applications the MPC controller requires accurate short-term predictions that can be obtained by two main methods:

- *Online predictions*: these are internet based predictions that take advantage of computations made by a third party, relieving the processor *in loco* from the computational burden.
- *Offline predictions*: these methods do not require am internet connection but a forecast model in necessary for their employment. They are compulsory for the prevision of occupancy related disturbances, since these are specific to the single building.

### 2.3.6 Constraints and objective functions

#### *2.3.6.1 Cost function formulation*

The MPC algorithm aims at minimizing a cost function; as previously mentioned, a control strategy might have multiple control targets to be addressed. These targets can often conflict with each other so that a trade-off between them needs to be found. (Drgoňa et al. 2020a) indicate as the most widely used approaches for multi-objective optimization the strategies of *goal attainment*, *minmax* and *Pareto front*.

**Goal attainment**. Goal attainment formulation aims at finding a balance between different goals by assigning each a weight, that is in practice a coefficient that multiplies each term to be minimized. For instance, a combination of goals could be that of minimizing discomfort while minimizing energy consumption as well. As these are clearly contrasting objectives, Eq.(2.5) could be reformulated as follows Eq.(2.10):

$$\min_{u_0,\dots,u_{N-1}} \sum_{k=0}^{N-1} (\|Q_s s_k\|_2^2 + Q_u \kappa_k u_k) \tag{2.10}$$

Here the term $\|Q_s s_k\|_2^2$ represents discomfort as a weighted squared 2-norm of the slack variables, as these represent the distance from the desired behaviour of the system, and therefore are to be minimized to guarantee comfort for the occupants. Since these deviations can assume both signs, the adoption of a squared norm is justified. In the second term, $\kappa_k$ represents the time varying weights associated with emission or price profiles. The matrix $Q_u$ collects the weighting factors.

**Minmax**. Minmax formulation is employed when the optimization goal is to minimize the worst element of a cost function. Indeed, norm 1 is often used for minmax problems.

**Pareto front**. The Pareto front technique is used to find trade off solutions between contrasting objectives.

#### *2.3.6.2 Comfort satisfaction*

Internal comfort for the occupants is the main purpose of HVAC systems in nonindustrial applications. Usually, thermal comfort is the main constituent of indoor environmental quality (IEQ) (Drgoňa et al. 2020b). In order to achieve thermal comfort, it is necessary to maintain internal air temperature close to a set value, as indicated by technical standards.

The MPC algorithm requires a quantification of thermal comfort. A simple solution is that of minimizing the displacement of the internal air temperature, assumed as a system output $y$, from the desired setpoint. However, more advanced indexes can be found across the literature. A well established metric for the quantification of indoor thermal comfort is the PMV, introduced by Fanger in 1973 and later adopted by most international standards, such as ISO7730, ASHRAE55 and EN15251. The main drawback of the PMV model is its nonlinear nature, which makes it particularly demanding for MPC computations.

#### 2.3.6.3 Minimization of cost

Minimizing the energy consumption of a building might not always go along with the purpose of minimizing operational costs. Indeed, storage systems allow load shifting, which could make it more economically convenient to store energy when energy prices are lower, to then use stored energy when prices are higher.
The dynamics of fuel prices such as gas, oil and wood, is neglectable when compared to the prediction horizon of an MPC controller for a building, the former being much slower. Subsequently, fuel prices factor could be updated offline in the general formulation when major variations in tariffs occur.

#### 2.3.6.4 Minimization of green-house gasses emission

An important objective for control is that of minimizing greenhouse gasses (GHG) emission in order to reduce the carbon footprint of the building energy needs. A formulation for such objective function could be that of replacing the cost factor of an economic MPC with by an emission factor that multiplies the amount of used energy.
Minimizing energy consumption will match the aim of minimizing GHG emission solely when conventional fossil energy sources are employed. On the other hand, electricity might come from a mixture of sources so that a decrease in consumption is not necessarily proportional to a decrease in GHG emission. An extreme case is that in which the electricity distributor guarantees that the provided electricity comes from renewable sources only: in this case, GHG emissions related to the direct use of electric energy are zero, leaving no room for optimization.

#### 2.3.6.5 Maximization of the share of renewable sources

Maximization of the usage of renewable resources is one of the most relevant objectives for MPC energy applications. If the building has a local production of renewable energy, this quantity can be added to the cost function associated to a negative cost factor, thus pushing the MPC to maximize their exploitation. (Drgoňa et al. 2020b) suggest the introduction of an abstract factor $\kappa_k = 1 - R_k$ where $R_k$ stands for the fraction of renewable energy employed by the load at timestep $k$.

### 2.3.7 Design and tuning

Designing an MPC controller requires to choose a suitable model for the building and, if included, its HVAC system. Along with the choice of the model, factors in the problem formulation must be properly set in the *tuning* phase. Tuning requires experience on the part of the MPC designer, and the design stage, with its use of a surrogate model, is essential in that it allows a trial-and-error procedure.
Literature and past experience suggest however some general rules of common practice that serve as a rule of thumb basis for further refinements. Parameters related to time should be based on the dynamics of the controller system. A general rule prescribes a choice of a sampling time $T_s$ large enough to allow computing, communicating and implementing the next control signal, according to the complexity of the optimization problem that the controller must be solve. At the same time, it must be small enough to control the system in

a stable way. Practice suggests a choice of $T_s$ that allows to have at least 10 to 20 samples in the rise time $T_{90}$ of the process step response. Building dynamics is slower than that of many industrial systems, so that a common choice for $T_s$ is between 15min and 180min. Therefore, the prediction horizon spans from 5h to 48h. As previously mentioned, the prediction horizon is larger or equal to the control horizon: the reason behind this choice is that the effects of computed actions decreases with each step in the future, so that only the first few computed control actions would have a significant impact on the system behaviour. Choosing a shorter control horizon thus allows to reduce the computational demand of the optimization problem. Indeed if $N_c = N_p$, the number of optimized variables will be equal to $n_u N_p$, whereas with a reduced control horizon, this number drops to $n_u N_c$. It is recommended to set $N_c$ at about 20% of the prediction horizon, with $N_c \geq 2$.

# 3 Chapter 3: Case study

## 3.1 The PVZEN building

The PVZEN building is a project developed by the Politecnico di Torino with the interaction of the Energy department (DENERG), the Architecture and Design department (DAD) and the Department of Electronic and Telecommunications (DET) and consists in the design of an experimental NZEB building.
The PVZEN building is bound to be built within the area of the Politecnico di Torino campus, in the city of Turin, Italy. The exact coordinates are the following:

- latitude: 45.06557°N;
- longitude: 7.6584 °E.

The location of the facility is shown in Figure(11):

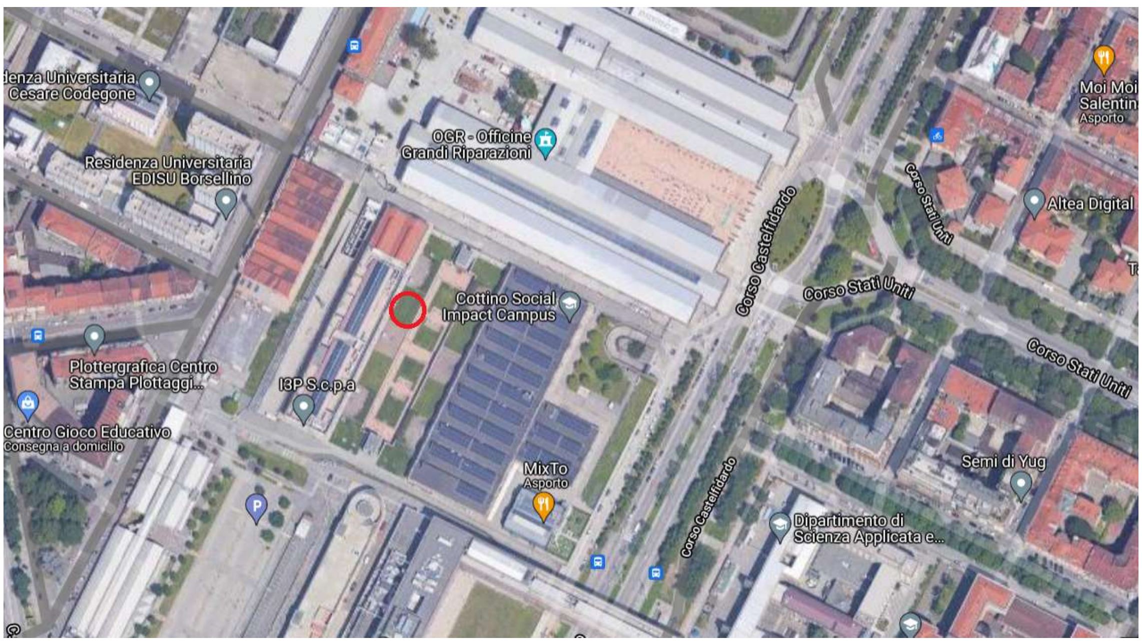


*Figure 11-PVZEN building future location (Amato 2013)*

where the red circle indicates the position that the building will occupy. A 3D rendering obtained from Google Earth (from Amato 2013)) shows how the structure will be influenced by the nearby pre-existing buildings in terms of shadowing and wind speed:


*Figure 12-PVZEN building location inside the PoliTo campus (Amato 2013)*

The building destination of use is that of a study room at the service of the University students. In detail, the facility is made of four rooms overall, namely two study rooms, one control room and a technical room. The two study rooms are intended to host a maximum number of ten people each, while the control room is supposed to be occupied by at most one technician at a time. As to the technical room, it will host the energy system technical equipment and it is therefore not intended to have a fixed occupation.

The two study rooms are equipped by computers, lights, projectors at the disposal of the students. These pieces of equipment will each have a schedule of activity based on the most likely behaviour of the occupants in the different simulations presented in the present work; consequently, they will represent both a source of internal heat gains and an electrical load for the system. The control room is as well equipped with a computer and lighting systems at need, whereas the technical room shall host the energy conversion systems such as the inverters, which are once again a source of heat gains.

### 3.1.1 Envelope properties and load profiles

Table(2) summarizes the main geometric and thermo-physical features of the building.

| Feature | Value |
|---|---|
| Conditioned floor area | 96.8m$^2$ |
| Conditioned volume | 501m$^3$ |
| Envelope surface/conditioned volume ratio | 0.85 m$^{-1}$ |

| | |
|---|---|
| Transparent/opaque envelope surface ratio | 6.6% |
| Opaque envelope surface | 400 $m^2$ |
| Mean U value | 0.184 $W/m^2K$ |

*Table 2-Building main thermophysical and geometric properties*

## 3.2 Thermal and electrical system

Purpose of the work is to study the energy interactions of a working building with the public power grid. The building is intended to be all-electric energy wise, thus including the thermal needs that well therefore be covered by an inverse cycle thermal machine, namely, a heat pump for the winter season. The preferred electricity source are the roof-mounted photovoltaic panels; however, a grid connection allows for energy to be exchanged in both directions with the public power grid.

The key elements of the energy system, however, are the two storage systems, namely a battery for electricity storage and a water tank for heat storage in the form of hot water. The role of energy storage systems is indeed that of allowing energy to be stored when conditions are convenient, such as when the price is low or when the photovoltaic production is available. Evidently, for the storage systems to be most effective, suitable control strategies must be employed.

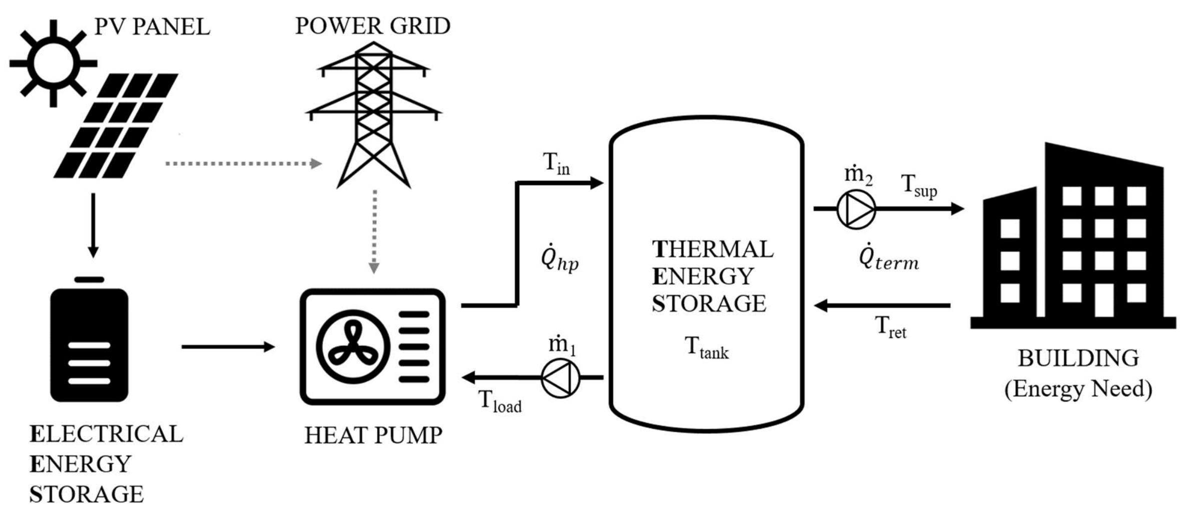


*Figure 13-Energy plant layout*

Figure(13) shows the general layout of the energy system plant serving the facility. Key element of the system is the water tank, which covers the function of storing the heat produced by the heat pump. The tank separates the plant in two main hydraulic circuits. A primary circuit allows the water to be heated up by the heat pump, while a secondary circuit makes the water circulate from the tank to the HVAC system where heat is exchanged through the terminals.

The primary circuit pumps the water from the bottom of the tank where it is at its lowest temperature into the heat pump inlet; from there, water is heated up to the set temperature $T_{in}$ and sent to the tank inlet at the top of the TES, where hot water tends to stratify. Reversely, the secondary circuit pumps the water from the top

of the TES where it is at its highest temperature and sends it to the HVAC system, where, by exchanging heat with the building, its temperature decreases to the value $T_{ret}$ to be then sent to the bottom of the storage.
With such a configuration, the primary and the secondary circuit are independent in their operation as long as a minimum availability of hot water is guaranteed inside the tank. This independent behaviour is key in allowing the system to operate the heat pump when it is most convenient without affecting the satisfaction of the heating needs, or, in other words, without compromising the thermal comfort of the occupants.
An analogy holds between thermal and electrical storage. Indeed, the battery serves the purpose allowing the time shifting of the renewable energy locally produced, by storing it for the heat pump to use. Electric loads are indeed the sum of the internal appliances needs and the power required by the heat pump. The former must be guaranteed for the needs of the occupants to be satisfied and are not object of control, but rather disturbances to be predicted. On the other hand, the latter can be time-shifted thanks to the presence of the TES as already explained. Control strategies shall therefore operate choices in terms of the electrical balance between the power yielded or acquired from the grid, the charge or the discharge of the battery, and the operation of the heat pump, all for the purpose of maximizing set goals and within the boundaries of guaranteeing a comfortable and correct operation of the building and the storage systems themselves, which are constrained by their maximum and minimum state of charge.
According to previous works (Amato 2013) 24 photovoltaic modules are installed on the rooftop of the building for a total installed power of 8.64kW.

## 3.3 EnergyPlus model

The building being still in design phase, an accurate white-box model was needed for the simulations of the different strategies presented in this work. The platform of choice for this modelling has been the software EnergyPlus, which allows an accurate description of a building geometry and thermophysical properties, occupancy and equipment usage schedules, along with precise estimations of heating demand, local renewable energy production and many more data regarding the behaviour of the building. Moreover, boundary conditions such as weather conditions are included in the model through suitable IGDG files.

As to the geometry and orientation of the building, the software SketchUp allows to get 3D renderings as shown in Figure(14):

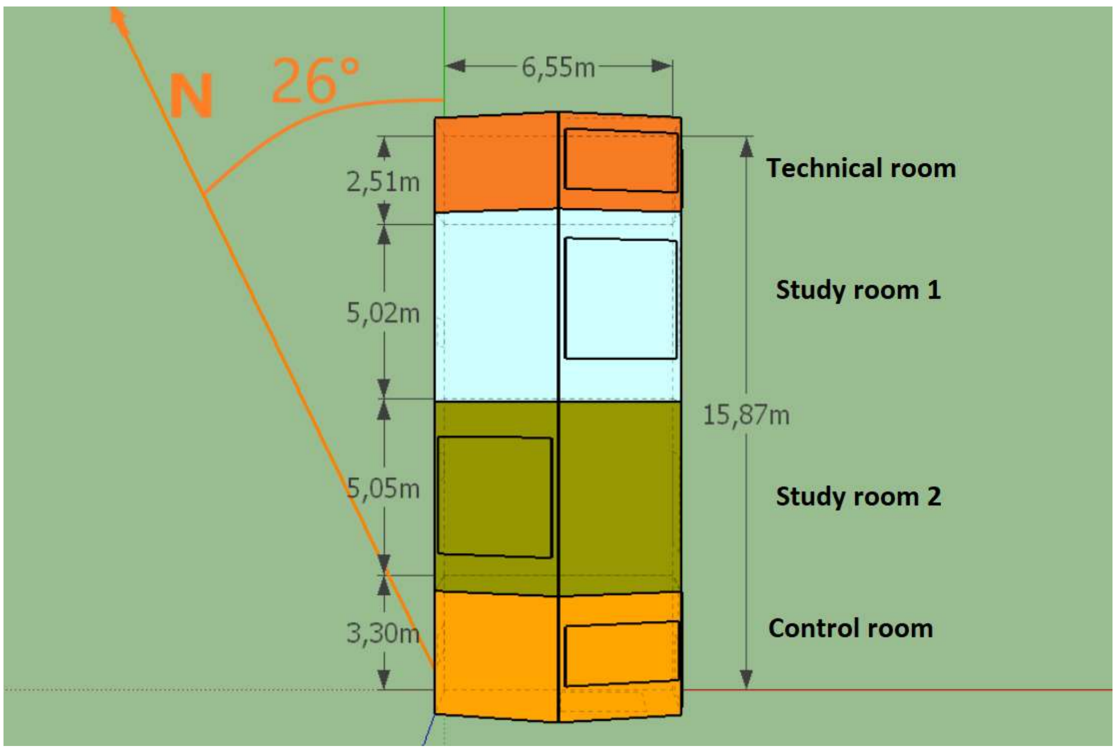


*Figure 14-SketchUp geometrical modelling of building geometry (Amato 2013)*

The facility layout is represented with its rooms and main dimensions. The orange line represents the north-axis orientation of the building.

The model includes the presence of nearby buildings that will project their shadow on the structure, as shown in Figure(15):

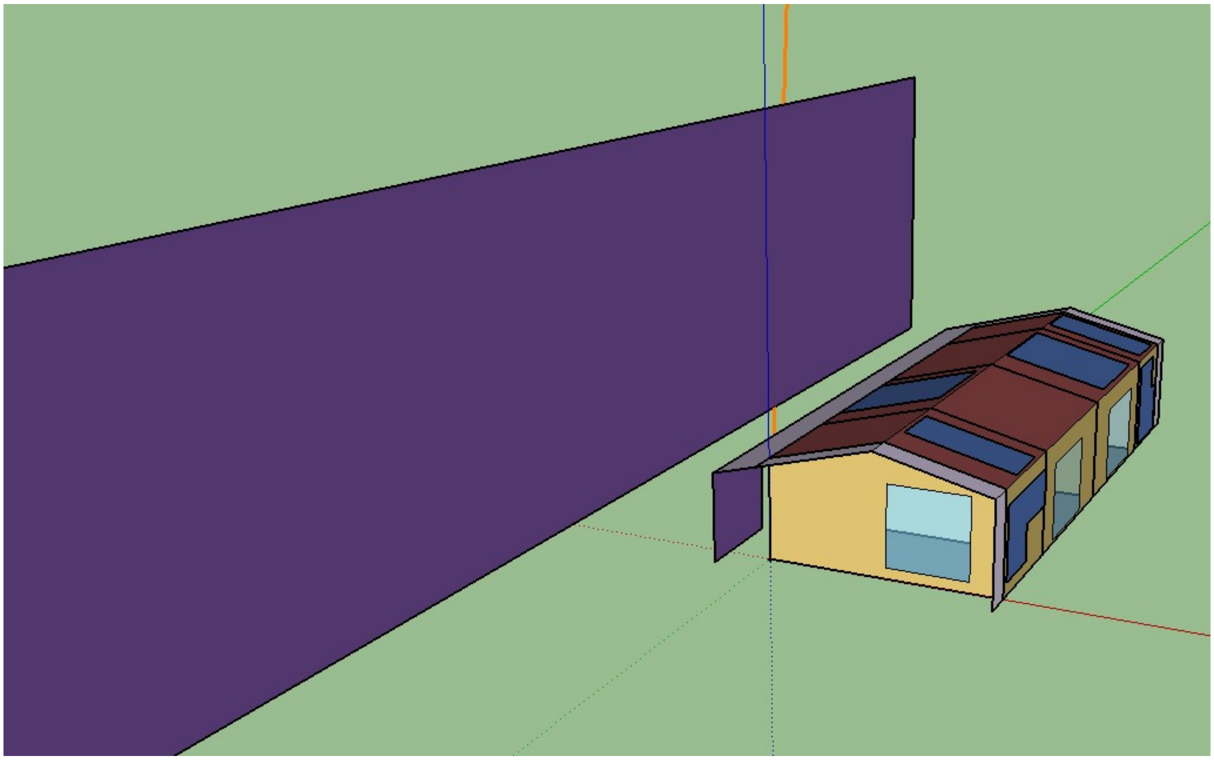

*Figure 15-Nearby shadowing building (Amato 2013)*

Materials properties are described in the *Material* object, and include thickness, thermal conductivity, mass density and specific heat of opaque surfaces, and U-factor, solar heat gain coefficient and visible transmittance of transparent components. Object *Construction* packs together the materials of all the layers making up a given surface, to be then assigned to its specific geometric surface.

Photovoltaic production is modelled first by placing the solar panels in their right position. For the positioning, the work (Amato 2013) was taken as a reference. Their productivity depends on the solar radiation whose value is provided by the weather file and is modelled through the single-diode model, available in EnergyPlus as the *PhotovoltaicPerformance:EquivalentOne-Diode* object. The positioning of the photovoltaic arrays is shown in Figure(16) from (Amato 2013):

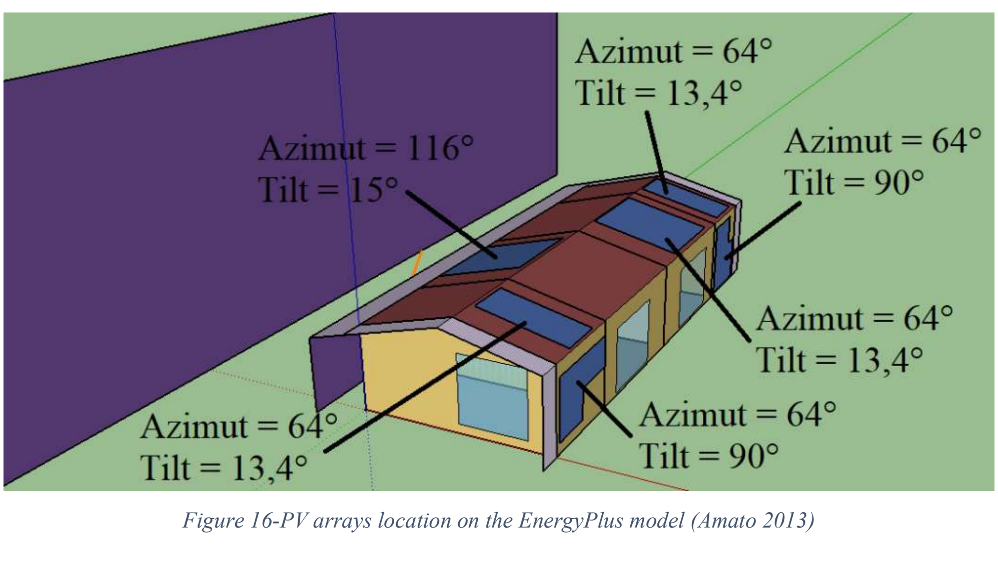


*Figure 16-PV arrays location on the EnergyPlus model (Amato 2013)*

## 3.4 Gray box modelling

In the following section, the procedure adopted for the modelling of the building thermal dynamics is described. The methodology follows that presented in (Bacher and Madsen 2011) as already introduced in Chapter 1. The forementioned article suggests a number of different RC equivalent circuits of increasing complexity; the specific choice of one of them is thereinafter justified.

The model therein identified will play the role of the reduced model embedded into the MPC controller to capture the building dynamics and it will be recalled in Chapter 5 when the formulation of the MPC strategy is presented.

The building has been assumed to be described with a single thermal zone for all the four rooms with the purpose of reducing the number of states thus facilitating the computation of the MPC optimizer.

The circuit of choice is that of Figure(17):

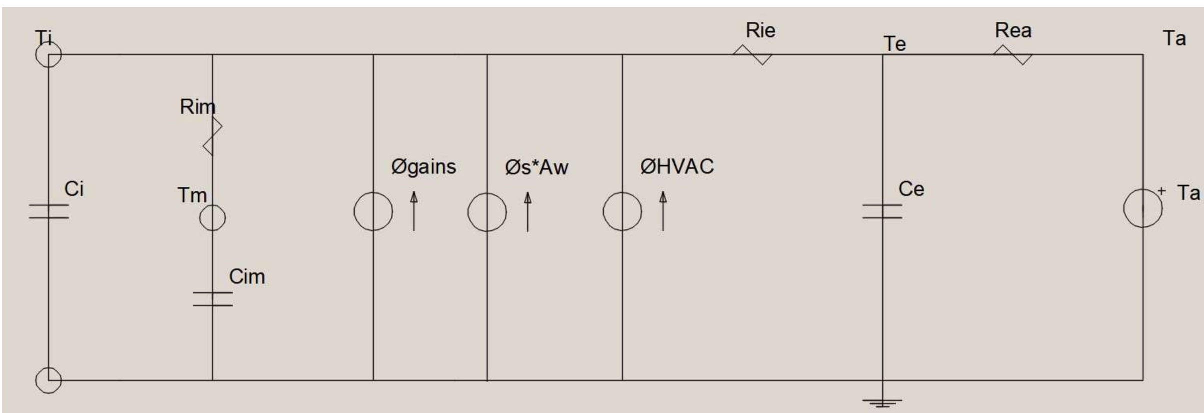


*Figure 17- Adopted RC equivalent model*

The states selected for the state space equations are:

- Internal temperature $T_i$, representing temperature of the whole internal air mass.
- Medium temperature $T_m$, accounting for the internal walls. As the model considers one single zone for the whole building, the three internal walls separating the four rooms belonging to the same zone are treated as internal masses. These wall masses have considerable heat capacity and thermal resistance, therefore a medium temperature node gives much more precise results in terms of dynamic description of the building.
- Envelope temperature $T_e$: this parameters lumps in the temperature at approximately the centre of the building envelope.

As to the heat gains, the following heat fluxes have been embedded into the circuit:

- Internal gains: this quantity incorporates all the internal heat gains including lighting, electric equipment and people dissipated sensible heating rates.
- Solar heat gains: this gain accounts for the heat entering the building through the transparent components.

- HVAC heating system: this heat flowrate is the one provided by the heating system only to cover the sensible heat losses through the envelope. As to the ventilation power demand, it will be treated separately, as detailed further in the thesis.

Since all the heat gains contribute equally in adding heat to the internal air control volume, they have been summed up in the system of equations as heat flux $\Phi_{int}$. The resulting model is:

$$\begin{cases} \dfrac{dT_i}{dt} = T_i\left(-\dfrac{1}{C_iR_{im}} - \dfrac{1}{C_iR_{ie}}\right) + T_e\left(\dfrac{1}{C_iR_{ie}}\right) + T_m\left(\dfrac{1}{C_iR_{im}}\right) + T_a(0) + \Phi_{int}\left(\dfrac{1}{C_i}\right) + \Phi_{sol}\left(\dfrac{A_w}{C_i}\right) \\ \dfrac{dT_e}{dt} = T_i\left(\dfrac{1}{C_eR_{ie}}\right) + T_e\left(-\dfrac{1}{C_eR_{ie}} - \dfrac{1}{C_eR_{ea}}\right) + T_m(0) + T_a\left(\dfrac{1}{C_eR_{ea}}\right) + \Phi_{int}(0) + \Phi_{sol}(0) \\ \dfrac{dT_m}{dt} = T_i\left(\dfrac{1}{C_mR_{im}}\right) + T_e(0) + T_m\left(-\dfrac{1}{C_mR_{im}}\right) + T_a(0) + \Phi_{int}(0) + \Phi_{sol}(0) \end{cases} \tag{3.1}$$

This system of equations can be written in a more compact form that separates the states from the input values:

$$\dot{x_b} = A_b x_b + B_b u_b \tag{3.2}$$

where dynamics matrix $A_b$ and control matrix $B_b$ are respectively:

$$A_b = \begin{pmatrix} \left(-\dfrac{1}{C_iR_{im}} - \dfrac{1}{C_iR_{ie}}\right) & \left(\dfrac{1}{C_iR_{ie}}\right) & \left(\dfrac{1}{C_iR_{im}}\right) \\ \left(\dfrac{1}{C_eR_{ie}}\right) & \left(-\dfrac{1}{C_eR_{ie}} - \dfrac{1}{C_eR_{ea}}\right) & (0) \\ \left(\dfrac{1}{C_mR_{im}}\right) & (0) & \left(-\dfrac{1}{C_mR_{im}}\right) \end{pmatrix} \tag{3.3}$$

$$B_b = \begin{pmatrix} (0) & \left(\dfrac{1}{C_i}\right) & \left(\dfrac{A_w}{C_i}\right) \\ \left(\dfrac{1}{C_eR_{ea}}\right) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix} \tag{3.4}$$

### 3.4.1 Parameters estimation

Parameters listed in the previous section require a first guess for the identification process be started. This first guess is based on physical considerations, such as the thermophysical properties of the materials making up the envelope stratigraphy and the transparent components. Nonetheless, these parameters, as it is usual with grey box type models, should not be interpreted under a rigorous physical prospective. In fact, as we are dealing with a lumped parameter model, very few terms of a simple system of equations substitute a rather complex thermophysical system, with different parts interacting with each other albeit being considered as uniform under the assumption of a homogeneous thermodynamical system. Moreover, since more than one reference circuit can provide a working model for the description of the same building, it is obvious that the same parameter must represent different physical quantities depending on the overall model of choice in which it finds itself.  Bearing this concept in mind, parameters are here listed along with the computation of their first guess.

#### 3.4.1.1 Envelope thermal resistance

The thermal resistance for a one-dimensional multi-layered surface can be computed as follows:

$$\tilde{R} = \frac{1}{h_i} + \sum_j \frac{s_j}{\lambda_j} + \sum_k R_k + \frac{1}{h_e} \tag{3.5}$$

where $h_i$ and $h_e$ stand for the convective heat transfer coefficient of the internal and the external layers respectively, $s_j$ and $\lambda_j$ are the thickness and the thermal conductivity of the j-th wall layer and $R_k$ is the resistance of layer $k$.

This thermal resistance is then divided by the area $A$ of the conduction surface so that, in steady state conditions, heat transfer is governed by equation:

$$\dot{\Phi} = A \frac{1}{\tilde{R}} (\Delta T) \tag{3.6}$$

where heat flux $\dot{\Phi}$ (W) is driven by the temperature difference between the internal and the external air $\Delta T$.

Heat conduction between the internal and the external ambient shall take place through all the envelope surfaces with the exception of the floor, which has been assumed as adiabatic. Since this building is made of different walls with differentiated properties, the overall resistance value has been computed by assuming heat conduction happening between an internal temperature node and an external temperature node. Consequently, the various wall resistances are summed up following the rule of parallel resistances in circuit theory, that is:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i} \tag{3.7}$$

Finally, the adopted model separates the envelope resistance into two series resistances, so that their sum equals the overall value. Since the walls making up the envelope of the studied building differ greatly stratigraphy-wise, it was chosen to divide them equally and let the tuning process choose the relative weight of the two resistances.

#### 3.4.1.2 Envelope capacity

The thermal capacity of a surface $j$ of unitary area made of $k$ layers is computed as:

$$c_j = \sum_k \rho_k c_{p,k} s_k \tag{3.8}$$

where $\rho_k$, $c_{p,k}$, and $s_k$ are the mass density ($kg/m^3$), the specific thermal capacity ($\frac{J}{kgK}$) and the layer thickness ($m$). All the surfaces making up the envelope can be seen as parallel capacitances between an internal temperature node and an external temperature node and were therefore multiplied by their areas and then summed up:

$$C_e = \sum_j A_j c_j \tag{3.9}$$

#### 3.4.1.3 Internal capacitance

The internal capacity accounts for the air thermal capacity and is therefore computed as:

$$C_i = V_i \rho_{air} c_{p,air} \tag{3.10}$$

where $V_i$is the internal air volume, $\rho_{air}$is the air density and $c_{p,air}$ its thermal capacity.

#### 3.4.1.4 Medium capacity and resistance

Medium elements in the building include the internal walls and the floor surface. Their capacity and resistance are computed as that of the other opaque surfaces. For the floor, the entire mass contributes to the overall medium capacity, while only half of the resistance shall be included into the value of the equivalent resistance.

### 3.4.2 Collecting data for tuning and testing

The grey box modelling process requires a suitable amount of input and output data so that the model can be trained to correlate the two sets at best. Generally, an additional amount of data is required for a testing phase, in which the quality of the obtained model is indeed tested on real data to assess the prediction capability of the model itself.

The selection of input and output data is strictly correlated to the model of choice as well as to the availability and the technical practicality of the measurements. On field measurements are often expensive, particularly for building applications where the investment in sensor is not easily covered by productivity as it is often the case in industrial applications. Indeed, industry usually relies on more precise measurements compared to the building sector. This work, being entirely simulative, did not allow any on field collected data, so that they must come necessarily from simulations. While this opens up to the chance of obtaining a much broader amount of data from the white box model provided by EnergyPlus. However, in order for the present work to be of greater significance for future applications, limits of real-life measurements were born in mind even during the simulation.

#### 3.4.2.1 Simulation setup

The required input-output dataset for the tuning process was obtained by a simulation of the building behaviour carried out on the EnergyPlus software, as it has been done with any other simulation involving the building dynamics on this thesis work. On the software, thermophysical properties of the building components are modelled so that the simulation can be thought as an acceptable substitute for the real building. Clearly, the process therein described can be replicated once the real building is in place with virtually identical steps.

Along with the properties of the building materials and its geometry, EnergyPlus is able to model the building occupation, equipment usage profiles and the heating power delivered by the HVAC system. Since all these elements are collected under the “internal gain” heat flux into the model, it has been chosen to consider the building as empty during the data collection phase, that is, no internal gains come from people occupation or lights and equipment usage, because what the system sees mathematically is a power profile that sums up all the forementioned profiles along with the HVAC heating profile. So, it was convenient to activate the heating system as the sole internal heating source in order to have a wide range of possible profiles, including free running intervals in which no internal gains of any kind are present.

As to the boundary conditions to the problem, external temperature and solar radiation were read from a climatic file (Torino-Caselle 1605901 (IGDG)).

The only output of the system was in this case the internal air temperature. As to the other two temperature nodes, medium temperature and envelope temperature, they were inferred from the internal air temperature in a simple way for the initial conditions:

$$T_m = T_i + 1.5°C$$
$$T_e = T_i - 1.5°C$$

The simulation setup was thus made of the following components.

1. EnergyPlus, containing a description of materials and geometrical properties.
2. Schedules for heating rate.
3. A Climatic file, containing the boundary conditions

#### 3.4.2.2 *Data collection and testing periods*

The grey box modelling procedure requires a period of time for the collection of input-output data which has to be suitable for the tuning phase. Following the tuning phase, the identified model must be put to the test; this requires an additional period to compare real data to the results provided by the identified model. For the present work, two weeks were adopted for the tuning phase, and the following week was chosen for testing. Boundary climatic conditions were referred to year 2019 starting from the 11$^{th}$ of February.

#### 3.4.2.3 *Input schedules and boundary conditions*

As to the heating power, a maximum rate value was assumed as equal to 3500W of sensible heat through the *OtherEquipment* object of EnergyPlus. This object receives a fractional value from the schedule, e.g. between 0 and 1, and models the introduction of sensible, all convective heat into the thermal zone equal to the received fraction times the maximum set power. This allowed the building to be subject to an internal heating rate that could range from 0 to 3500W. The heating rate profile for the training phase was chosen as follows:

- Five days of random values between the minimum and the maximum heating rate, changing every 60 minutes;
- Three days of free running;
- Six days of random values changing every 120 minutes

For the testing week, the following schedule was created:

- Three days of random values, changing every 60 minutes;
- One day of free running;
- Three days of random values changing every 120 minutes.

The choice of changing heating rate inputs with a minimum frequency of 1 hour came from the fact that the MPC controller will work with that timestep.

Figure(18) shows the resulting heating profile:

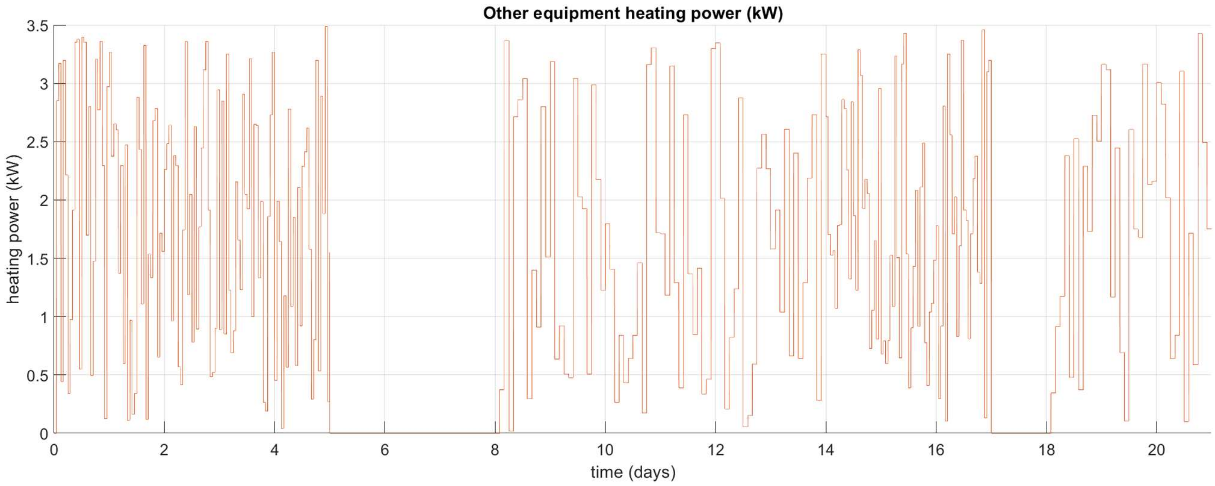


*Figure 18-Heating schedule for training and testing*

Boundary conditions include the external air temperature and the site direct solar radiation.

The external air temperature for the selected three weeks follows the profile of Fig(19):

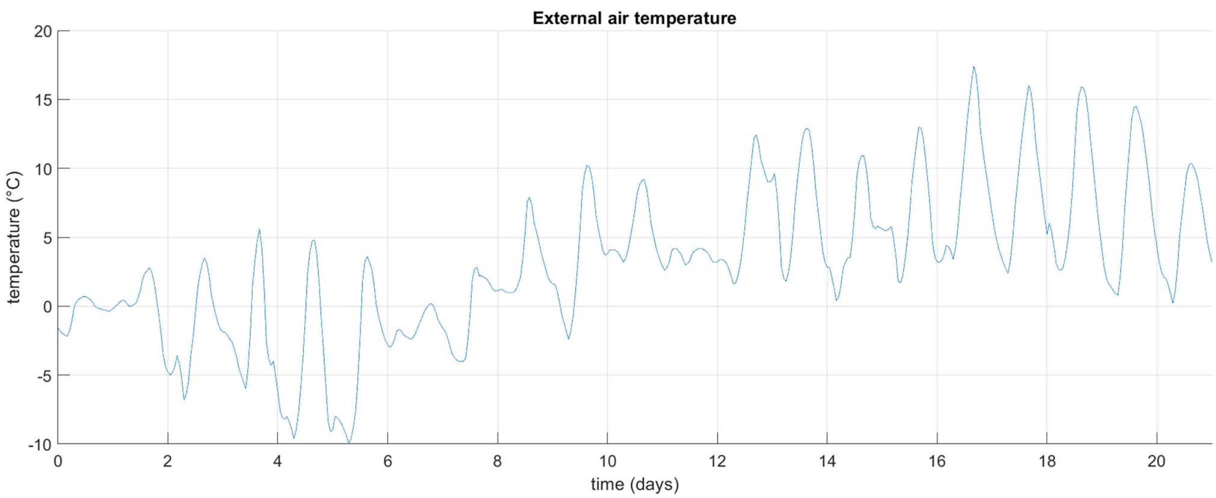


*Figure 19-External air temperature during the data collection phase*

while the solar radiation profile is shown in Figure(20):

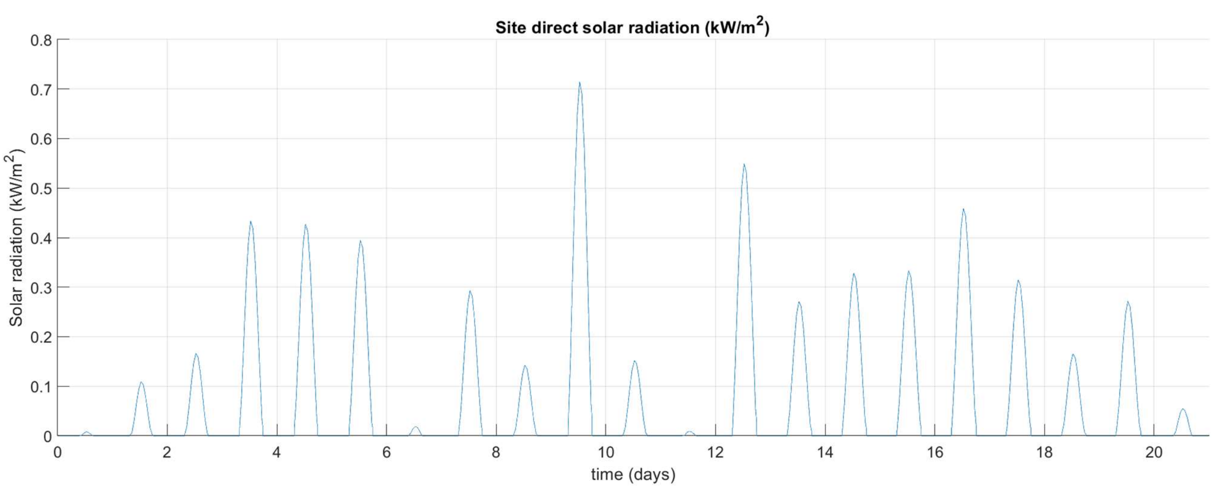


*Figure 20-Site direct solar radiation*

### 3.4.3 Results

Table(3) shows the results of the system identification process. In the first column are reported the guess values, that is the values of the parameters inferred from physical considerations. Since the identification process needs to change those values, minimum and maximum acceptable values were provided to the identification toolbox. The resulting identified parameters can be read in column 5.

| Parameter | Guess value | Minimum value | Maximum value | Adjusted value |
|---|---|---|---|---|
| $R_{ie}$ (K/kW) | 3.560 | 0.3560 | 35.60 | 1.8113 |
| $R_{ea}$ (K/kW) | 14.240 | 1.4240 | 142.40 | 15.6469 |
| $R_{im}$ (K/kW) | 887.90 | 88.790 | 8879.0 | 88.7900 |
| $C_i$ (kJ/K) | 604.206 | 60.4206 | 6042.06 | 631.237 |
| $C_e$ (kJ/K) | 3.4979e+4 | 3.4979e+3 | 3.4979e+5 | 3.1663e+3 |
| $C_m$ (kJ/K) | 4.0425e+4 | 4.0425e+3 | 4.0425e+5 | 8.4758e+3 |
| $A_w$ (kJ/K) | 7.3503 | 0.73503 | 73.503 | 2.0165 |

*Table 3-System identification results*

Noticeably, the transparent component area was reduced by the identification process; indeed, while the guess value considered the actual area of the windows, solar radiation is measured in terms of its direct, vertical component, therefore a reduced identified value for the window area was expected.

The last week served as a testing dataset to compare the behaviour predicted by the identified system to the real one provided by the measured (in this case, simulated) dataset. This comparison is made on the output of the system, which is the internal air temperature. Figure(21) compares the predicted behaviour (blue line) to the actual one (grey line).

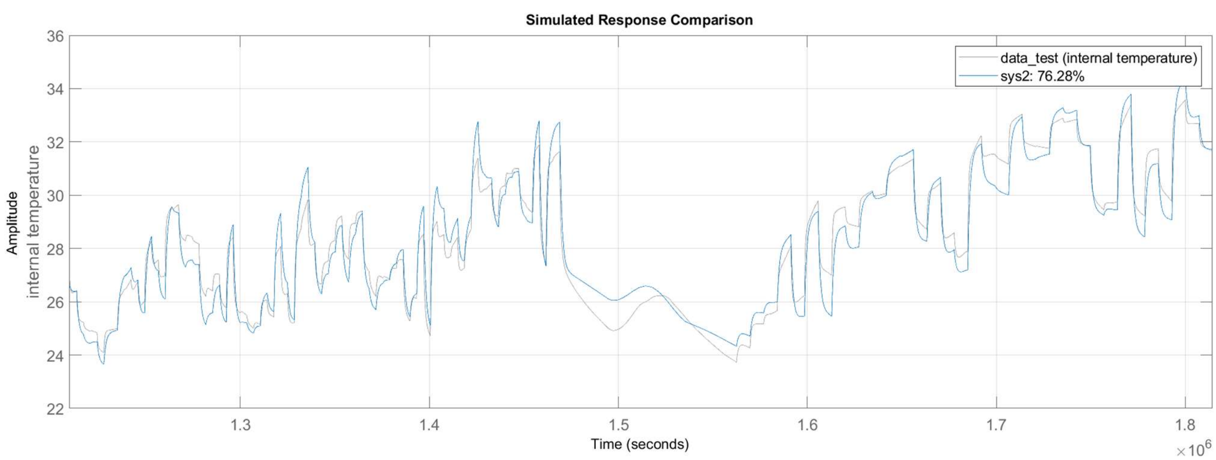


*Figure 21-Simulated response of the identified system*

The identification toolbox provides the Normalized Root Mean Square Error (NRMSE) between the predicted and the actual temperature across the entire week as a metric to quantify the quality of the predicted system. In the continuous domain, the simulated response comparison resulted in a NRMSE equal to 76.28%.

The last step is the discretization of the identified system. In fact, the RC model will be employed in the MPC control unit which operates by timesteps of one hour each.

Moreover, the control signal coming from the MPC controller are staircase inputs, that is inputs that are constant throughout the whole timestep. Therefore, the discretization method of choice was the Zero-Order Hold method (ZOH) that indeed considers uniform inputs at each timestep:

$$u(t) = u[k]$$

where $t$ represents the continuous time coordinate, while $k$ is the discrete timestep.

The discretization was obtained with the "c2d" Matlab function, and its comparison with the collected data is shown in Figure(22):

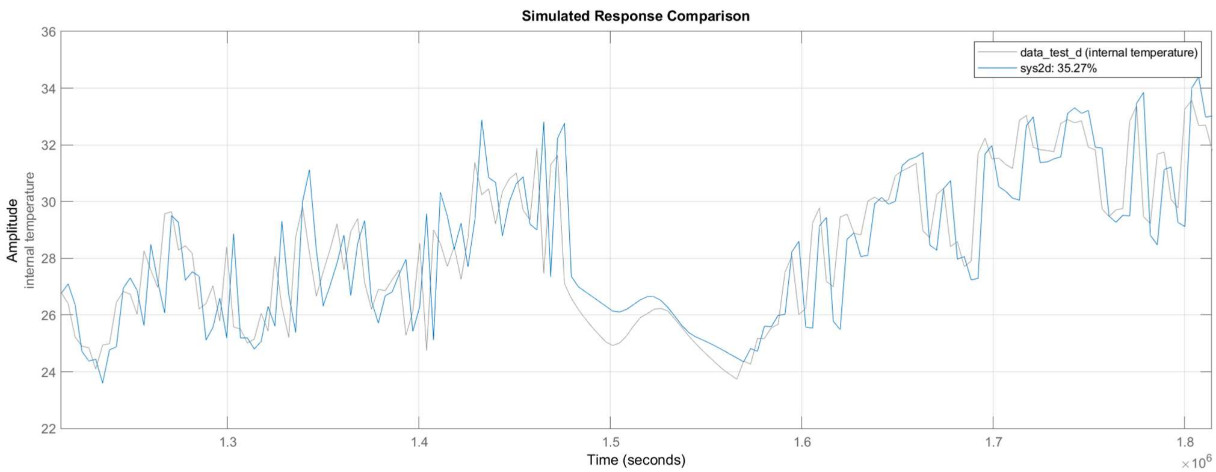


*Figure 22-Simulated response of the Identified system in the discrete domain*

The NRMSE decreases in the discrete domain to 35.27%.

In discrete domain matrixes $A_b$ and $B_b$ become:

$$A_{b,d} = \begin{pmatrix} 0.0559 & 0.9196 & 0.0197 \\ 0.0183 & 0.9737 & 8.6493e-4 \\ 0.0015 & 0.0032 & 0.9953 \end{pmatrix}$$

$$B_{b,d} = \begin{pmatrix} 0.0049 & 1.7514 & 3.5318 \\ 0.0071 & 0.0769 & 0.1552 \\ 9.4691e-6 & 0.006 & 0.0121 \end{pmatrix}$$

## 3.5 Control objectives and KPIs

The different strategies studied in the present work are oriented towards the attainment of various, often contrasting goals. The actual attainment of these objective requires quantitative indexes to be assessed. The following goals have been considered in the different proposed strategies.

- **Self Sufficiency**. Self Sufficiency (SS) is a measure of how "independent" the system is energy wise, that is, how much the system relies on external input of energy for its operation. In detail, SS is defined as follows:

$$SS(\%) = \frac{E_{lgc}}{E_{load}} \times 100 \tag{3.11}$$

where $E_{lgc}$ is the locally generated and consumed energy and $E_{load}$ is the energy required by the facility, indeed in terms of its loads.

- **Self Consumption**. Self Consumption (SC) is a measure of how well a system exploits locally generated energy over a period of time. Indeed, its definition is:

$$SC(\%) = \frac{E_{lgc}}{E_{pv}} \times 100 \tag{3.12}$$

where at the denominator stands the locally produced energy $E_{pv}$.

- **Monetary cost**. The monetary cost considered in this work is the sole expense related to the energy acquired from the grid, and does not consider other factors, such as maintenance costs. As a consequence, reducing the cost will depend on both the amount of energy acquired per se and the pricing of energy itself. As will be shown in Chapter 4, when pricing varies significantly from high to low tariffs, the choice of the most convenient price might lead to greater economic savings, even if the overall energy acquired is higher. Therefore, monetary cost and energy related indicators can at times be contrasting goals.

# 4 Chapter 4: Rule Based Control Strategies

In this chapter Rule Based control (RBC) is investigated. Rule based control is a control technique based on a set of *if-then-else* rules that operate decision on the control action to be sent to the plant. Traditional rule based control is a simple, reactive technique that does not include optimizing or prediction capabilities. However, literature shows how it can be very effective in reaching specific goals. For instance, (Mařík et al. 2011) observed that RBC can achieve significant energy savings when applied to heat pump operation. Two simple rule based control algorithms were tested and compared to two predictive RBC strategies for the management of an HP coupled with photovoltaic local production to improve Self consumption and minimize the final energy needs. Self consumption was enhanced in (Baggio, Bee, and Prada 2018) with a RBC strategy based on instantaneous PV production.

Four RBC strategies are formulated for the control of the energy system serving the PVZEN building, whose details are specified in Chapter 3. Two of these strategies are assumed as baseline and do not aim at attaining any specific goal. Two enhanced, objective oriented RBC strategies were then compared to the baselines by means of the KPIs defined in Chapter 3.

## 4.1 Simulation environment

The strategies formulated were tested in a simulative fashion by means of different pieces of software of different nature that operated in synergy with each other. These are:

- EnergyPlus. This piece of software provides the building white box model and was used to return the energy thermal and electrical needs, which were assumed to be fixed, or rather non-controllable by the proposed strategies.
- Matlab. Simpler, lumped-parameter equations were written on Matlab codes for the modelling of the energy system serving the building, namely the hot water tank, the heat pump, the electric battery and the electric power exchange between the system, the battery, the photovoltaic panels and the power grid.
- BCVTB. This piece of software, called *Building Control Virtual Test Bed* or BCVTB for short, enables different pieces of software to be connected to create a *co-simulation* environment. BCVTB acts at each given timestep of a set simulation period by allowing an exchange of data between the different actors of a system. Being intended for control, the present co-simulation linked the response of a system, modelled with both EnergyPlus and Matlab scripts to a controller, again written in Matlab. In detail, at each timestep the building response, quantified through the suitable output parameters, is sent to the Matlab programme that reads these values and computes a consequent control action. this establishes a proper closed loop or feedback mechanism that is required for a control system.

## 4.2 Components modelling and methodology

The methodology suggested in this work uses models of different nature. As already mentioned, the building dynamic behaviour is predicted by means of EnergyPlus, which can be assumed to simulate the exact building response. The other components are described through easier equations as better detailed below.

The simulation environment and the modelling choice, along with the suitable KPIs to compare the different results, make up a paradigm that could easily be reproduced in other works to test control strategies in a similar setting. Moreover, this methodology is perfectly suitable for testing on field with a real system.

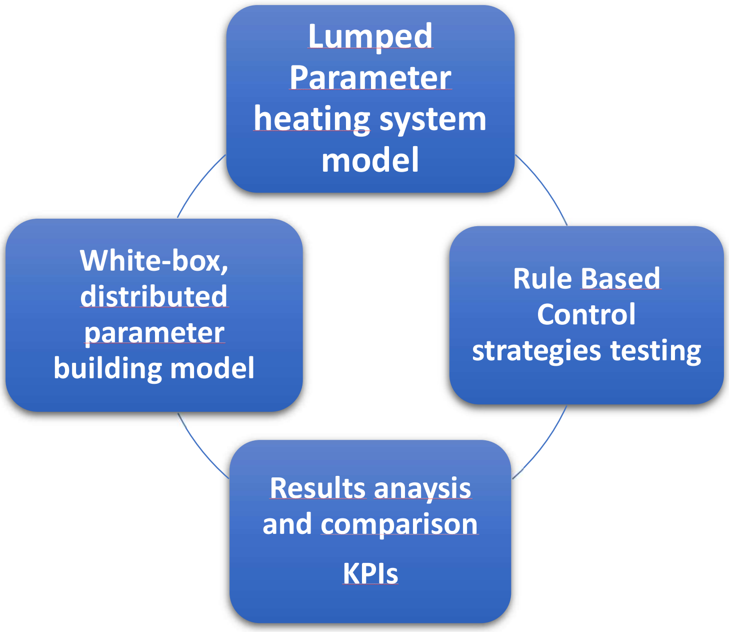


*Figure 23-Simulation and testing methodology*

The system layout is recalled in Figure(24):

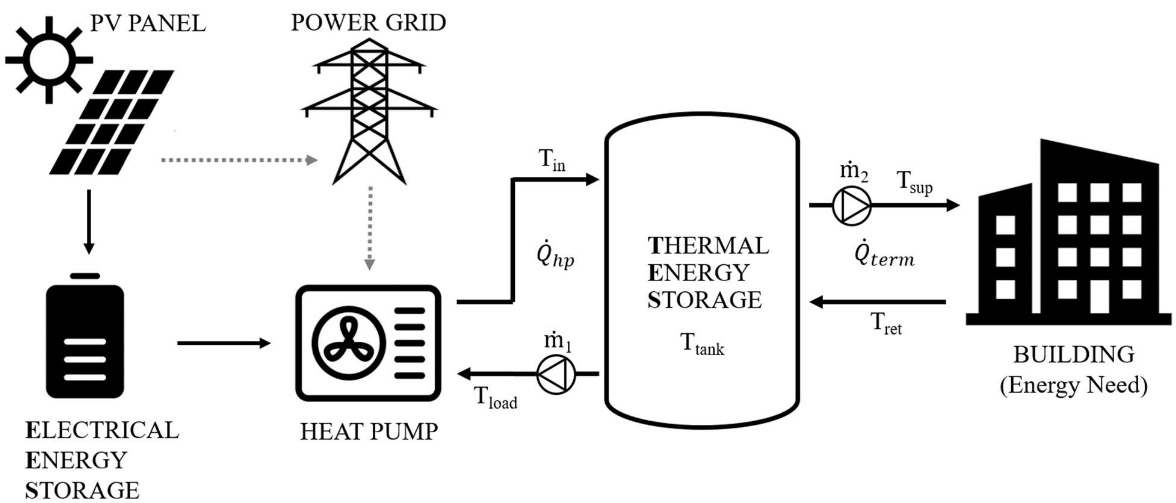


*Figure 24-Plant layout*

### 4.2.1 Thermal energy storage

The thermal energy storage system is a water tank that stores the hot water produced by the heat pump. The tank has a total of two inlets and two outlets, and it is connected to the rest of the system by two hydraulic circuits.

Phenomena happening inside water tanks include mixing, natural and forced convection, and heat conduction through the different layers of the fluid. These phenomena require detailed models which are highly demanding in the modelling phase as well as during simulation, as their computational needs can be substantial. As a result, a simpler, lumped parameter model was chosen for the water tank, that describes the TES dynamics by means of a single time dependent variable, this variable being its internal temperature $T_{tank}$. Clearly this is possible on the assumption that perfect mixing takes place inside the tank: while this is not the most accurate choice, two reasons motivate it. Firstly, convection and conduction inside the tank will make the temperature field of the inside volume more uniform with time; secondly, the model preserves the first principle of thermodynamics as the overall internal energy inside the TES varies according to the heat delivered by the heat pump or subtracted by the building HVAC system in accordance with the energy conservation equation. In detail, heat exchange takes place as follows:

- Heat provided by the HP as:

$$\dot{Q}_{\mathrm{hp}} = \rho_w c_{pw} \dot{m}_1 (T_{in} - T_{load}) \tag{4.1}$$

- Heat subtracted by the HVAC system:

$$\dot{Q}_{\mathrm{term}} = \rho_w c_{pw} \dot{m}_2 \left(T_{sup} - T_{ret}\right) \tag{4.2}$$

- Internal temperature variation

$$V \rho_w c_{pw} \frac{dT_{tank}}{dt} \tag{4.3}$$

- $T_{load} = T_{ret} = T_{tank}$ since parameters are lumped

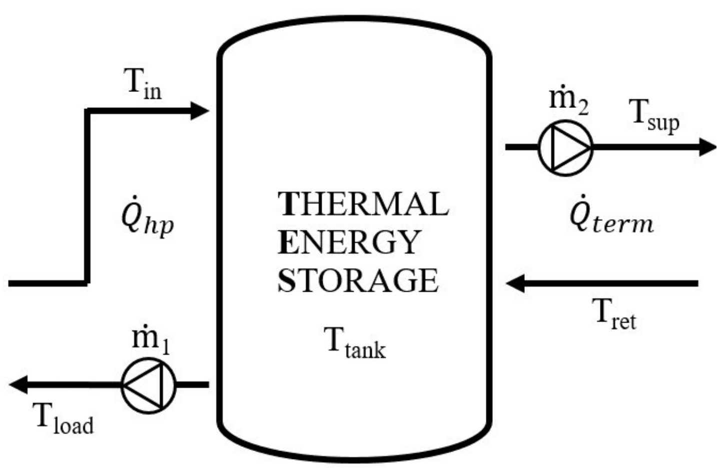


*Figure 25-TES working scheme*

The overall energy balance equation shall be:

$$V\rho_w c_{pw}\frac{dT_{tank}}{dt} = \rho_w c_{pw}\dot{m}_1(T_{in} - T_{tank}) + \rho_w c_{pw}\dot{m}_2(T_{ret} - T_{tank}) \tag{4.4}$$

Here, $c_{pw}$ represents the water thermal capacity and $\rho_w$ is the water density. This equation can be easily solved in discrete form with the Forward Euler Method.

The main parameters of the tank are summarized in Table(4):

| **Quantity** | **Value** |
|---|---|
| Minimum water temperature | 35°C |
| Maximum water temperature | 45°C |
| Volume | 1,335 $m^3$ |
| Water specific heat capacity | 4186 J/kg/°C |
| Water density | 1000 kg/$m^3$ |
| Primary pump flowrate (nominal) | 0.4 kg/s |

*Table 4-Tank sizing data*

### 4.2.2 Heat pump

The heat pump was assumed to have a constant COP equal to 3, a mass flowrate of 0.4 kg/s and a fixed temperature $T_{in}$ for water production. Outlet temperature and pump flowrate being set, the thermal power delivered by the heat pump shall be computed at each instant as:

$$\dot{Q}_{\text{hp}} = \rho_w c_{pw}\dot{m}_1(T_{in} - T_{load}) \tag{4.5}$$

where $T_{load}$ is the temperature of the cold water returning from the tank. This simulates a realistic operation logic for a heat pump, as the settings can be given in terms of pump flowrate and water production regulations, while the power delivered is unknown and computed *a posteriori*.

As a maximum thermal power was set to 9.2kWth, during operation the following situation might occur: if temperature $T_{load}$ is too low, fixed $T_{in}$ and pump flowrate would result in a thermal power higher than the maximum capability of the HP. To avoid this, either the flowrate or the water production temperature must be decreased. From a technological perspective, it is easier for the system to reduce the pump motor rotational speed rather than the temperature at the outlet, as this might affect the water tank as well. Therefore, mass flowrate shall be recomputed as follows:

$$\dot{m}_1 = \frac{\dot{Q}_{\text{hp,max}}}{\rho c_{pw}(T_{in} - T_{load})} \tag{4.6}$$

### 4.2.3 HVAC system

The HVAC system and its distribution piping network where not modelled in detail. The heat delivered by the terminals was computed by EnergyPlus through the object *IdealLoads* which returns the heat necessary to balance the heat losses of the building, given its heat gains. However, an assumption was made on the nature of the terminals to work at low temperature, namely 35°C. This means that water in the tank can be stored at any temperature equal or greater than 35°C; if greater, a mixing valve could easily regulate the temperature to be suited for the optimal operation of the terminal. Moreover, this assumption is conservative, since a lumped parameter model considers the average temperature inside the whole control volume; if the average temperature is equal or above 35°C, water on the upper outlet of the tank will be equal to that temperature in the worst case.

### 4.2.4 Battery and electric power exchange

The battery State of Charge (SOC) evolves in time through the following equations, valid for charging and discharging mode respectively:

$$SOC(t) = SOC(t-1) - \eta_{charge}\frac{P_{bat}(t)\Delta t}{C_{bat}} \tag{4.7}$$

$$SOC(t) = SOC(t-1) - \frac{P_{bat}(t)\Delta t}{C_{bat}} \tag{4.8}$$

where $SOC(t)$ is the State of Charge at instant $t$, $SOC(t-1)$ is the SOC at the previous time instant, $\eta_{charge}$ represents the charging efficiency, $C_{bat}$ is the battery storing capacity and $\Delta t$ is the timestep of the adopted discretization. $P_{bat}$ is the power exchanged by the battery, with the convention that a negative value accounts for an increase in SOC of the battery.

The battery of choice was a commercially available model, the US2000 by Pylontech®, which was used as a reference for the sizing parameters, summed up in Table(5):

| | |
|---|---|
| **Battery capacity** | 2400 Wh |
| **Minimum State of Charge** | 4% |
| **Max. charging power** | 1325 W |
| **Max. discharging power** | 1231 W |
| **Charging efficiency** | 96% |

*Table 5-Battery data*

The electric power fluxes of the system are regulated by a Rule Based logic that involves the following quantities, other than $P_{bat}$:

- The power produced by the PV arrays $P_{pv}$, as provided by EnergyPlus;
- The building demand power load $P_{load}$, that accounts for all the non-HVAC power demand (computers, lights etc…) and for the HP electric demand;

- A quantity called $P_{grid}$, that makes up for the energy balance with an exchange with the power grid, according to the equation:

$$P_{grid} = P_{load} - P_{pv} - P_{bat} \tag{4.9}$$

The logic adopted for the management of this power exchanges is rule-based and favours the usage of locally produced energy over the acquisition of power from the grid.

### 4.2.5 Occupation and internal loads

The people occupancy profile was modelled according to the most likely behaviour for a study room, as summarized in the top plot of Figure 1; accordingly, the electrical consumption of the non-HVAC equipment (computers, projectors and lighting) follows the profile shown in the bottom plot of Figure(21).

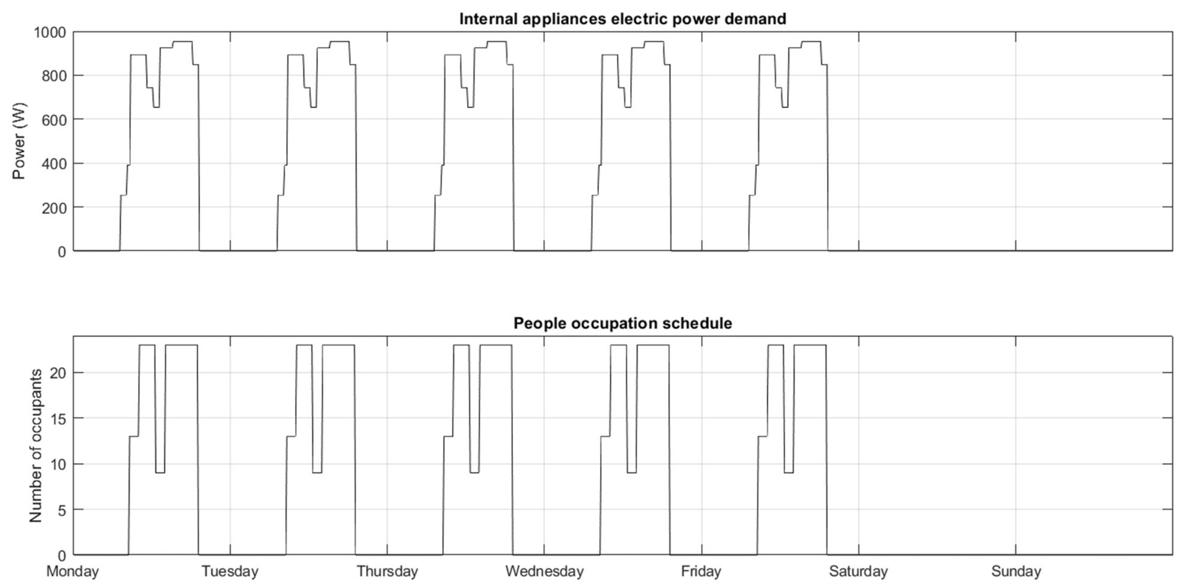


*Figure 26-Non-HVAC power demand and occupancy profile*

## 4.3 Strategies formulation

The strategies formulated in this section are Rule-Based and aim at improving the operation of the system with respect to set goals. All the different strategies are tested with the methodology illustrated in the previous paragraphs under the same boundary conditions of occupancy, heating demand, time of the year, internal gains and internal loads. Essentially, the overall power demand of the building will remain the same for all simulations, while the various control strategies will operate their choices for the best management of the storge systems.

All the implemented Rule Based strategies operate a choice between two working modes for the heat pump:

- Mode 1. This working mode solely guarantees a minimum temperature inside the water tank of 35°C. This means that for any temperature above 35°C the heat pump will not operate; the temperature of 35°C is maintained through an ON-OFF controller.
- Mode 2. In this mode, the heat pump operates by producing water at 45°C until the storing capacity of the tank reaches its limit. Again, this translates into an ON-OFF control for the tank temperature around 45°C.

The formulated strategies differ from each other in the way they choose between mode 1 and mode 2. In detail:

- RBC1: the heat pump only operates in mode 1;
- RBC2: the heat pump only operates in mode 2;
- RBC3: mode 2 is the mode of choice when the battery SOC is above 90% and mode 1 is the choice otherwise;
- RBC4: mode 2 is the mode of choice when the electricity price is at its lowest tariff level and mode 1 is the choice otherwise;

Of the two working modes for the heat pump, mode 2 is the one that favours heat storage, at the expense of instantaneous consumption on behalf of the HP. The criterium adopted for the choice of such a mode will therefore push the system towards the enhancement of a given objective. RBC3 operates in mode 2 when the battery SOC is above 90%, that is when locally produced energy is available. In fact, the rule based logic governing the battery exchanges does not charge the battery with energy coming from the grid. RBC3 is therefore expected to perform in terms of Self Sufficiency and Self Consumption, possibly at the expense of an increased monetary cost. Conversely, RBC4 favours the operation of the HP for the storage of heat when electricity prices are at their lowest, with no regard to energy related criteria such as battery state of charge or photovoltaic production. As a consequence, this strategy is intended to reduce the economic expense for the energy acquired.

RBC1 and RBC2 do not take any decision on the basis of either economic or energy related factors: they are, indeed, assumed as baselines and are expected to perform poorer on both fronts.

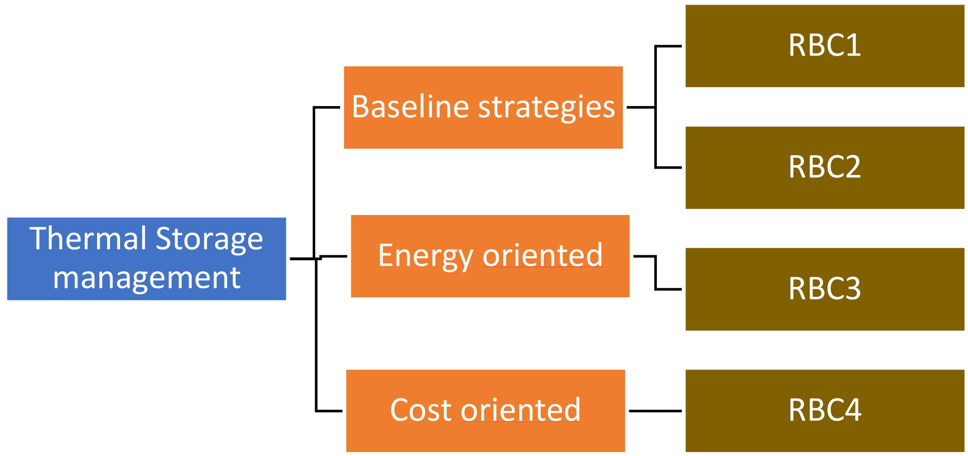

*Figure 27-RBC strategies objectives*

The pricing adopted for the simulation of non-constant electricity tariffs follows this schedule:

- higher price (0.3€/kWh): from Monday to Friday between 8am and 7pm;
- medium price (0.165€/kWh): from Monday to Friday between 7am and 8am and between 7pm and 11pm; on Saturdays from 7am to 11pm;
- lower price (0.03€/kWh): all of the remaining time.

## 4.4 Results

The simulation results are shown in Figure(28). The resulting profiles have been reported for the sole first week (February 14th-20th) of simulation for better graphical clarity (quantitative results will be computed on the entire simulation period).

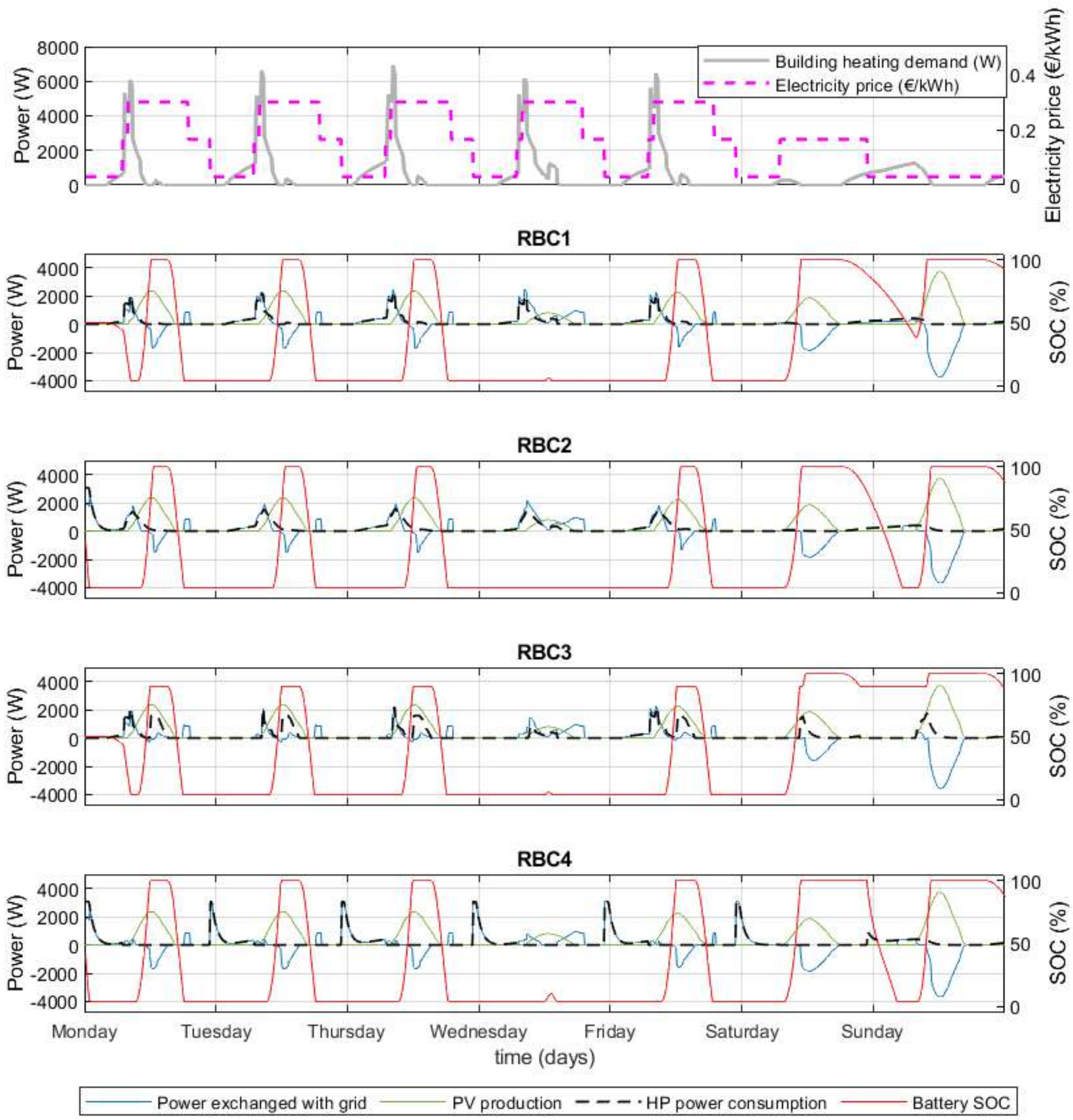


*Figure 28-RBC strategies simulation results*

The top plot shows the time profile of the electricity prices (y-axis on the left) and of the thermal power delivered to the building.

Focusing on RBC1 and RBC2, it is easy to notice how the HP power profile closely follows the heating power demand of the building in qualitative terms. Indeed, with these two strategies the TES merely functions as a hydraulic separator, with hardly any storing capability being employed and no criteria of convenience being followed. As expected, RBC2 works with a TES at its highest working temperature and therefore allows a smoother profile of heat pump usage, thus proving to be a preferable choice.

RBC3 and RBC4 display much more varied profiles. Chiefly, they notably allow a time shifting between the heat pump usage and the building power demand profile. This proves how these strategies are able to decouple the operation of the heat pump from the instantaneous thermal needs of the facility. More in detail, RBC3 has the heat pump work during the central hours of the day, when photovoltaic production tends to be at its highest. Indeed, the purpose of this strategy is to better employ the locally produced renewable energy when available. Moreover, with this control system, the battery SOC tends to stay away from saturation and that limits the amount of energy yielded to the grid, thus increasing Self Consumption.

RBC4, on the other hand, shifts most of the operating time of the heat pump to the night hours, when prices are at their lowest. However, these hours of the day are the ones with no photovoltaic production, which conversely peaks during daytime when tariffs are high.

All of the strategies were tested over the same period of time, and then compared qualitatively and in terms of their KPIs. Since the building energy demand is the same in terms of both thermal and electrical energy, different performances are due to the control of the storage systems, particularly that of the TES, whose function is to decouple heat production form the heating energy demand. Indeed, the overall energy consumed by the heat pump is the same for all cases, with the exception of the difference in possible residual energy stored in the tank. This proves how control systems can noticeably affect the performance of energy system by simply time shifting the load profile of the heat pump.

At the same time, power production from the photovoltaic arrays and the electric power demand of internal loads are the same regardless of the strategy adopted, therefore the control system must manage the electricity exchanges and the heat pump operation according to the intended goal. In this instance, two were the goals: reducing the expense and maximizing the exploitation of locally generated electric power. The former goal has been attained by storing thermal energy when electricity prices were at their lowest. Achieving the latter is possible depending on the per se amount of energy acquired from the grid, with no regard to its price. A simple energy balance computed on the simulation period shows that the sum of the energy acquired from the grid and the energy produced must equal the sum of the energy consumed by the facility and the energy yielded to the grid.

Numerical results, quantified by the previously defined KPIs, confirm the expectations regarding the outcome of the compared strategies. Indeed, both RBC1 and RBC2 result in a poorer performance with respect to energy management and economic savings. As expected, RBC3 displays a remarkable improvement with respect to Self Sufficiency and Self Consumption, albeit at the expense on a poorer economic performance. Conversely, RBC4 decreases its energy related KPIs when compared to both RBC3 and the two baseline strategies, while significantly decreasing the economic expense.

| Control strategy | Self-sufficiency | Self-consumption | Gross expense |
|---|---|---|---|
| RBC1 | 59.10 % | 52.75 % | 13.95 € |
| RBC2 | 62.29 % | 57.59 % | 13.67 € |
| RBC3 | 72.89 % | 67.39 % | 10.84 € |
| RBC4 | 55.85 % | 51.63 % | 6.53 € |

*Table 6-RBC simulation results*

## 4.5 Conclusions

The RBC strategies presented in this chapter proved to be effective albeit simple. Indeed, RBC control strengths lie in the following characteristics:

- Simple design and implementation. These strategies are based on simple rules that can easily transferred to different buildings and different system layouts. Being entirely reactive, RBC does not require any prediction model, and few, easy-to-obtain measurements such as water tank temperature and battery SOC provide all the input data for the controller. Moreover, the simple nature of the controller poses no concerns regarding computational capabilities of the control logic units.
- Goal attainment. The simple structure of the RBC strategies does not prevent them from obtaining significant improvements in attaining the set purposes. Indeed, both RBC3 and RBC4, being the two enhanced strategies, proved to be effective in improving the desired KPI.
- Consolidate experience. Rule based control has been in use for decades by control engineers in the building energy field and has been extensively investigated by researchers. Clearly, future advancement in control technologies may take advantage of this sound, extensive pre-existing experience.

On the other hand, this work has highlighted some drawbacks of Rule-Based control techniques. Chiefly, the following flaws are mentioned.

- Non optimal solutions. Control actions computed by classical Rule-Based controllers are not specifically tailored for the exact boundary conditions that influence the plant and for the states in which the plant finds itself at a given time. In other words, while improvements can be achieved by these controllers, these improvements are not in general optimal, in that a better choice for the control action is in general possible.
- No prediction capabilities. RBC is reactive in nature, which means that the control action is not based on any predictions as how the plant might evolve as a consequence of that action. Moreover, predictions might involve disturbances too. Classical RBC cannot include such models, and that excludes potentially improvements in the selection of the most appropriate control action.
- Single objective control. This work compared the results of strategies (RBC3 and RBC4) that were oriented towards one specific goals. In fact, when improving on Self Sufficiency and Self Consumption, RBC3 resulted in a higher expense when compared to RBC4, which, conversely, greatly reduced the gross monetary cost at the expense of decreased SS and SC

The forementioned flaws of Rule-Based control call for a different control strategy that would be able to compute optimal actions by exploiting a prediction capability, all the while reaching improvements under more than one specific goal. All these requirements are met by Model Predictive Control, which will be investigated in Chapter 5 by formulating a control strategy on the same case study of Chapter 4 and comparing it to a baseline RBC logic.

# 5 Chapter 5: MPC strategies

## 5.1 Simulation environment

Model Predictive Control involves different actors in its operation. In order to test an MPC strategy in a simulative fashion, a proper simulative environment must be set to enable the interaction between the elements that make up the controller and the control system.

Firstly, the controller must include a reduced model of the controlled plant and an optimizer for the cost function to be minimized. The reduced model and the problem constraints were written in the HYSDEL language (Torrisi and Bemporad 2004) and translated into an optimization problem by the MPT Toolbox. MPT (Multi-Parametric Toolbox) Toolbox is a free toolbox for Matlab that allows the design, the analysis, and the deployment of optimal controllers for the solution of hybrid, linear and nonlinear systems.

The optimization problem is formulated by MPT, then Matlab calls the Gurobi optimizer for the solution of the problem.

The surrogate model of the system is provided by EnergyPlus for the building response, while Matlab models substitute the pieces of storage equipment.

## 5.2 Ventilation load estimation

For the MPC simulation, an explicit estimation of the ventilation load in the building was conducted. In fact, for previous RBC strategies this load was computed by EnergyPlus and simply read by the control system, then the heat subtracted from the water tank included this load. Such an approach is justified for a reactive control system simulation. Indeed, reactive control does not require a knowledge of the system as a whole, but can focus on a single portion of it, the single portion being the energy system serving the building rather then the building itself. RBC strategies of Chapter 4 would work both in a simulation and in a real application with any thermal and electrical power profile used by the building as no prediction of that portion of the system is a requirement for the control logic to operate its choices.

MPC control, on the other hand, must know all the quantities involved in the controlled process, at least at the level at which control operates. Any quantities involved in the MPC optimization must be either a prediction or a control choice of the controller. Heat delivered to the building to make up for the envelope thermal losses will be in this formulation computed with the aid of the RC model as control variables. Ventilation consumption, on the other hand, will be estimated as therein explained and passed to the MPC as a prediction.

Ventilation thermal power can be computed as:

$$Q_{vent} = \dot{V}_{vent}\rho_{air}c_{p,air}(T_i - T_a) \tag{5.1}$$

where $\dot{V}_{vent}$ is the air volume flow rate while $\rho_{air}$ and $c_{p,air}$ are the air density and heat capacity respectively. Ventilation is proportional to the difference between the internal air and the external air temperature $(T_i - T_a)$.

Ventilation volume flowrate was computed according to the UNI10339 as $6 \cdot 10^{-3} m^3/s$ per occupant and the ventilation system was supposed to work with a constant flowrate for an occupancy of ten people during the opening hours of the facility.

While flowrate, density and heat capacity are known values (density and capacity were assumed to be constant), temperature difference $(T_i - T_a)$ is a source of non-linearity for the system. Indeed, while external air temperature is a predicted quantity, internal air temperature is a state whose value in time along the prediction horizon changes depending on the actions taken by the controller. Consequently, $Q_{vent}$ could not be considered a prediction, being a function of state $T_i$ itself. To overcome this problem, internal temperature $T_i$ was assumed as constant and equal to 20°C.

## 5.3 Model Predictive Control formulation

The MPC strategy is intended to manage the optimal operation of the energy system serving the PVZEN building as described in Chapter 3. The system layout is reminded in Figure(29):

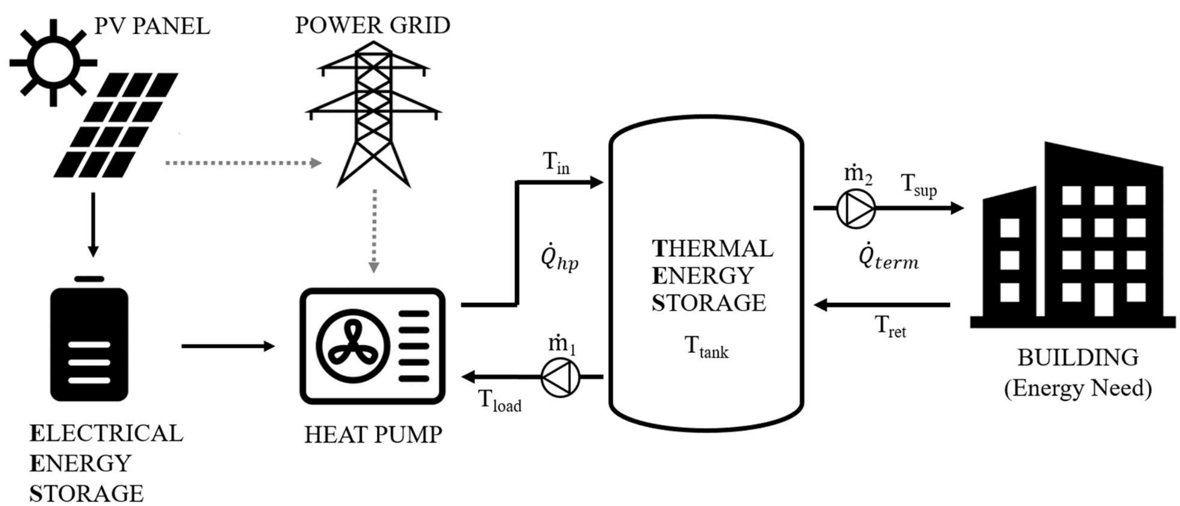


*Figure 29-Plant layout*

MPC controllers work by controlling the system within the desired operating conditions while optimizing given criteria. Therefore, when formulating the MPC problem, priority is given to the maintenance of the states of the system within the boundaries indicated by the constraint.

The system dynamics is described by state space equations in the reduced model which will be embedded into the MPC control unit. The control unit will then compute the control action on the basis of the reduced model and the optimizer.

### 5.3.1 Reduced model of the system

The first step of the design process was the formulation of a state space set of equations that would make up the reduced model to be embedded into the control unit. Firstly, it is necessary to define what states will be employed for an accurate dynamic description of the entire system.

For each system component, one or more states have been pinned out as the most representative physical quantity for its description. This paragraph will describe all these reduced model equations that allowed a single system to be written.

For the building, the equivalent RC circuit was chosen as the reduced model. The identification process of this grey box model is detailed in Chapter 3. As explained in that chapter, three states were considered for a proper dynamical description of the temperature evolution of the system. The equations are now recalled:

$$\begin{cases} \dot{x}_b(t) = A_b x_b(t) + B_b u_b(t) \\ y_b(t) = C_b x_b(t) + D_b u_b(t) \end{cases} \tag{5.2}$$

Vector $x_b$ collects the three temperature nodes of the RC equivalent model that were selected as states; in detail, $x_b = [T_i, T_e, T_m]^T$, where $T_i$, $T_e$ and $T_m$ are the internal air temperature, the envelope temperature and the medium temperature respectively. Inputs of the systems, including both control signals and disturbances, are collected in vector $u_b = \left[T_a, \Phi_{gains} + Q_{hvac}, \Phi_{sol}\right]^T$, where $T_a$ is the external air temperature, $\Phi_{gains}$ is the sum of the internal heat gains (people occupancy, lighting and appliances heating power), $Q_{hvac}$ is the power provided by the HVAC heating system and $\Phi_{sol}$ is the direct solar radiation on the facility site. All these quantities have been considered as inputs for the system. However, $Q_{hvac}$ is of a different nature in the MPC perspective. In fact, external air temperature, internal gains and the solar radiation will not be computed by the control system as control variable, in that they are boundary conditions that will influence the system as disturbances. $Q_{hvac}$, on the other hand, is a control variable whose value is not predicted by is computed as an output of the control unit.

The tank model is a simplified one that uses only one state for the description of the tank dynamics. The equation is:

$$\dot{T}_{tank} = \frac{1}{\rho c_p V}\left[Q_{HP} - (Q_{hvac} + Q_{vent})\right] \tag{5.3}$$

where $T_{tank}$ is the water temperature inside the tank (see Chapter 3 for details of the model), $\rho$ is the water density, $c_p$ is the water thermal capacity and $V$ is the thermal storage volume. $Q_{HP}$ is the heat produced by the heat pump which enters the TES as a heat flux, increasing its temperature. $Q_{vent}$ is the ventilation heat load that will be seen by the tank as a heat loss, along with $Q_{hva}$ which accounts for the heat transferred from the thermal storage to the building. Again, while $Q_{hvac}$ and $Q_{HP}$ are control input computed by the controller, $Q_{vent}$ is here treated as a prediction.

This equation is discretized with the selected timestep $\Delta t$ as:

$$T_{tank,k+1} = T_{tank,k} + \frac{1}{\rho c_p V}\left[Q_{HP,k} - \left(Q_{hvac,k} + Q_{vent,k}\right)\right]\Delta t \tag{5.4}$$

A similar model is employed for the battery, whose state variable is the State of Charge. The SOC evolution follows Eq(…):

$$S\dot{O}C(t) = -\eta_{charg}\ \frac{P_{bat}(t)}{C_{bat}} \tag{5.5}$$

where $SOC$ is the battery state of charge, $\eta_{charge}$ is the charging efficiency, assumed in this case as equal to the discharging efficiency, $C_{bat}$ is the battery capacity and $P_{bat}$ is the power exchanged by the battery. $P_{bat}$ is a control variable.

In order for the reduced model to provide predictions of the state's evolution, these state space equations must be collected into a single, solvable system. This was made possible by the presence of $Q_{hvac}$ and $Q_{HP}$ which both belong to two different equations. The resulting system, in continuous form, becomes:

$$\begin{cases} \dot{x}_b(t) = A_b x_b(t) + B_b u_b(t) \\ \dot{T}_{tank} = \dfrac{1}{\rho c_p V}\left[Q_{HP} - (Q_{hvac} + Q_{vent})\right] \\ S\dot{O}C(t) = -\eta_{char}\ \dfrac{P_{bat}(t)}{C_{bat}} \\ P_{grid} = P_{loads} + \dfrac{Q_{HP}}{3} - P_{pv} - P_{bat} \end{cases} \tag{5.6}$$

### 5.3.2 Constraints

Constraints are requirements on the values that both states and control variables can have during the time evolution of the system. The rationale behind constraints is that some quantities in the system cannot go below or above a set threshold, for instance to satisfy technological or comfort constraints.

Constraint can be formulated in two forms:

- **Hard constraints**. Hard constraints are expressed as inequalities in the MPC formulation, and they cannot be violated. For instance, for a given state $x$ to be constrained between a minimum value $x_{min}$ and $x_{max}$, a hard constraint shall be formulated as:

$$x_{min} \le x \le x_{max}$$

- **Soft constraints**. Hard constraints might be too strict for the system to be satisfied; in fact, a single violation of a hard constraint yields a non-feasible solution. Often a soft constraint formulation is a preferrable choice when setting constraints. Soft constrained are introduced in the MPC formulation by means of slack variables. The previous example of a hard constraint would become:

$$x_{min} - s \le x \le x_{max} + s$$

where $s$ is the slack variable. The slack variable $s$ is then treated as a control variable, whose value will be chosen by the MPC. The slack variable itself will then be constrained:

$$\begin{cases} s \geq 0 \\ s \leq s_{max} \end{cases}$$

where $s_{max}$ is the maximum value allowed for the slack variable.

The constraints related to the building states is on the internal air temperature. Clearly, this will be motivated by comfort requirements; in this work, the simulation being conducted in winter season, a minimum temperature will be set at $T_{i,min} = 19°C$. In order to allow violation to this condition, a slack variable $s_{Ti}$ is introduced, whose maximum acceptable value is 2°C. The soft constraint becomes:

$$T_i \geq T_{i,min} + s_{Ti} \tag{5.7}$$

The water tank must operate between two temperatures, set as 35°C ($T_{tank,min}$) in the lower boundary and as 55°C ($T_{tank,max}$) as the upper boundary. A slack variable $s_t$ allows the constraint to be of the soft kind. The inequalities become:

$$T_{tank,min} \leq T_{tank} \leq T_{tank,max} \tag{5.8}$$

The limitations on the battery are formulated as hard constraints. Indeed, while temperature limitations in a component such as the water tank or in the building might be violated with possible repercussion on the component optimal operation or on the occupants' comfort, these "extreme" temperatures are physically possible. On the other hand, a battery State of Charge cannot possibly be violated under any circumstances, as a SOC below zero or above the maximum capacity would be physically meaningless. The constraint becomes:

$$SOC_{min} \leq SOC \leq SOC_{max} \tag{5.9}$$

where $SOC_{min} = 4\%$ and $SOC_{max} = 96\%$.

Another technological limitation affecting the battery is related to its maximum charging and discharging power. This translates into the following condition, again a hard constraint being once more a technological limit:

$$P_{bat,min} \leq P_{bat} \leq P_{bat,max} \tag{5.10}$$

where $P_{bat,min} = -1.3kW$ and $P_{bat,max} = +1.3kW$. The sign is due to the fact that the power can be exchanged by the battery in both directions.

The heat pump maximum power was set to $10kW_{th}$, and the maximum power that can be delivered by the heating system is equal to $8kW_{th}$. These constraints are of the hard kind:

$$
\begin{aligned}
0 \le Q_{HP} \le 10kW \\
0 \le Q_{hva} \;\; \le 8kW
\end{aligned} \tag{5.11}
$$

An additional condition has been added to smoothen the time profile of the heat delivered as $Q_{hvac}$. Indeed, as no limitations are given on the steepness of that profile, the control input coming from the MPC unit might be to steep or even bang-bang like. To overcome this, a fictitious state called $Q_{hva\ ,last}$ recorded the last value of the $Q_{hvac}$ input sent to the plant and actually implemented. The equation related to this state is such that the value remains the same for the whole prediction horizon, so:

$$
Q_{hvac,last} = Q_{hvac,last} \tag{5.12}
$$

while a hard constraint is added to make sure that any new value of $Q_{hvac}$ during that control horizon is never higher or lower that value of more than 1.5kW:

$$
Q_{hvac,last} - 1.5 \le Q_{hv} \quad \le Q_{hvac,last} + 1.5 \tag{5.13}
$$

### 5.3.3 Boolean condition

The main goal of the here proposed MPC system is to minimize the energy acquired from the grid. In fact, since the HYSDEL language does not allow a specific destination for predictions, they must be included in the model as states by means of a techniques defined as *rolling sequence* by (Torrisi and Bemporad 2004). As a consequence, only one tariff has been attributed to the purchase of electricity from the grid. Still, in order to compute the expense at each timestep, the controller needs to know the direction in which the exchange with the grid takes place. In fact, by defining a variable *cost* as equal to the tariff times the power exchanged with the grid ($P_{grid}$), price would be equal for sold and bought energy. This problem calls for a Boolean condition to be introduced to discern between selling and purchasing conditions.

Boolean conditions in an MPC formulation yield a Hybrid Model Predictive Control and require the introduction of a discontinuous variable, that is a variable whose values can either be 0 or 1 depending on a given logical condition. A variable *selling* was therefore defined as:

$$
selling = \left(P_{loads} + \frac{Q_{HP}}{3} - P_{pv} - P_{bat}\right) \le 0 \tag{5.14}
$$

so that when the electric power balance is negative, power is yielded to the grid and the whole system is in the “selling scenario”.

An auxiliary variable *cost* is therefore computed according to an *if-then-else* condition on the Boolean variable *selling*:

$$cost = \begin{cases} \left(P_{loads} + \frac{Q_{HP}}{3} - P_{pv} - P_{bat}\right) R_1 \; if \; selling = 1 \\ \left(P_{loads} + \frac{Q_{HP}}{3} - P_{pv} - P_{bat}\right) R_2 \; if \; selling = 0 \end{cases} \tag{5.15}$$

This formulation allows to attribute two different tariffs for the sale and the purchase of energy ($R_1$ and $R_2$ respectively). In particular, $R_1$ was chosen to be zero as no remuneration was assumed for energy yielded to the grid, while $R_2$ was set to 30c€/kWh.

### 5.3.4 Cost function

The cost function is the core of the MPC control strategy: it formalizes and quantifies the objectives of the designed control by assigning a weighting factor to the quantities involved in the problem. The sum of all those quantities multiplied by their respective weighting factors yields the cost, which the optimizer will try and minimize under the conditions defined as constraints.

The mpt toolbox allows the following general formulation of the cost function:

$$\min_{u_0,\ldots,u_{N-1}} \|P_N x_N\|_p + \sum_{k=0}^{N-1} \left[\|Q x_k\|_p + \|R u_k\|_p\right] \tag{5.16}$$

Here, $x$ is the vector of the states while $u$ is the vector of the inputs. Since these vectors evolve with time, their value at a given timestep $k$ between the first instant $k = 0$ and the last $k = N$ is indicated as $x_k$ and $u_k$. The first term of the cost function $\|P_N x_N\|_p$ is a cost on the terminal state, that is, a cost on the system state at the last timestep of the control horizon; $P_N$ is the weight on the terminal state. $Q$ is a diagonal matrix that contains the weights to attribute to the states, while $R$ contains the weights for the inputs. Norms in the cost function vary according to the value of $p$: for tracking problem, norm2 ($p = 2$) is advised. In the present work, the interest in formulating an economic MPC, therefore norm1 was the norm of choice.

The choice of weights is crucial in designing an MPC controller. Choices adopted in this work are therein reported and justified.

Inputs variables were given the following weights:

| Quantity | Dimensions | Weight |
|---|---|---|
| HVAC delivered energy $Q_{hvac}$ | kW | 0 |
| HP delivered energy $Q_{HP}$ | kW | 0 |
| Battery energy exchange $P_{bat}$ | kW | 0.05 |
| Internal temperature slack $s_{Ti}$ | °C | 300 |
| Tank temperature slack $s_{Ttank}$ | °C | 200 |

*Table 7-weights on input variables*

No weight has been assigned to the heat delivered by the heat pump and the HVAC system. Indeed, the system should be able to regulate the temperature profile of the building internal air and the water tank with no limitations provided that constraints are respected. This allows in particular the water tank to be filled when necessary and with no associated cost, so that its storing capability is best exploited.

While the same rationale might hold for the battery, a small weight was given to $P_{bat}$ that is justified by technological reasons. In fact, batteries lifetime is a relevant element in their deployment and limiting the number of charging/discharging cycles but giving a little price to the charging and discharging power was considered to be a preferrable choice. Weights on temperature slack variables $s_{Ti}$ and $s_{Ttank}$ are much higher as their values are in order of magnitude of the unit (°C) and violation of soft constraints, albeit tolerated, are preferably to be avoided.

State variables were given the following inputs:

| Quantity | Dimensions | Weight |
|---|---|---|
| Cost | c€/kWh | 1 |
| Deviation from $T_i$ setpoint $T_{i,dev}$ | °C | 0.01 |

*Table 8-weights on state variables*

Finally, no terminal cost was defined.

## 5.4 Baseline strategy

In order to assess the effectiveness of the MPC implemented, an RBC was adopted as baseline. The RBC of choice was RBC3; this strategy is in fact oriented towards the best exploitation of locally produced energy to minimize the amount of energy acquired from the grid. As electricity price is assumed as constant in the MPC, RBC4 would not be suitable for a fair comparison with the former, since RBC4 bases its choices on the variation of energy tariffs.

RBC3 was adjusted in all sizing parameters to provide a comparable and consistent benchmark for the evaluation of the MPC performance. However, as mentioned in section 5.2, the MPC based controller must provide the heating power for the building as a control action, whereas RBC strategies of Chapter 4 act by reading the power delivered by the HVAC system as it is determined by other control systems that do not interfere with the proper operation of the storage system RBC. For a fair comparison between RBC and MPC, an ON-OFF control was included in the control system to determine the power delivered to the building by the HVAC system.

## 5.5 Results

The simulations were conducted for nine days between Monday 11th and Tuesday 19th of February of 2019, year of the weather file.

External air temperature, internal heat gains and site direct solar radiation are reported in Figure(30):

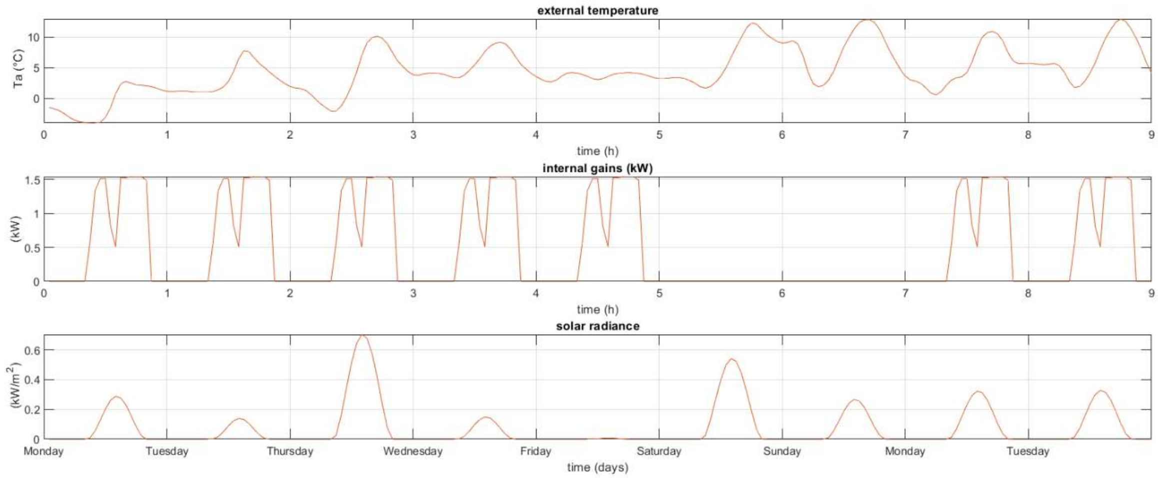


*Figure 30-Internal air temperature, internal gains and Site Solar radiation*

Photovoltaic production and internal electric power demand are displayed in the plots of Figure(31):

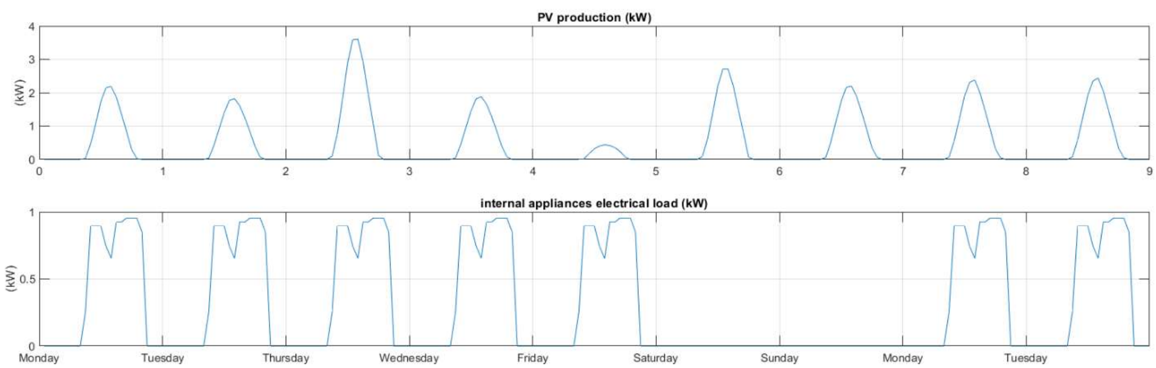


*Figure 31-Photovoltaic production and non-HVAC predicted electricity demand*

Internal electric power demand accounts for the sum of computers, projectors and lights.

### 5.5.1 MPC simulation

Internal air temperature control depends on the $Q_{hvac}$ power profile. $Q_{hvac}$ is a control variable for both the MPC and the RBC strategy. Figure(32) shows the temperature profile obtained by the MPC controller as it is influenced by the HVAC power:

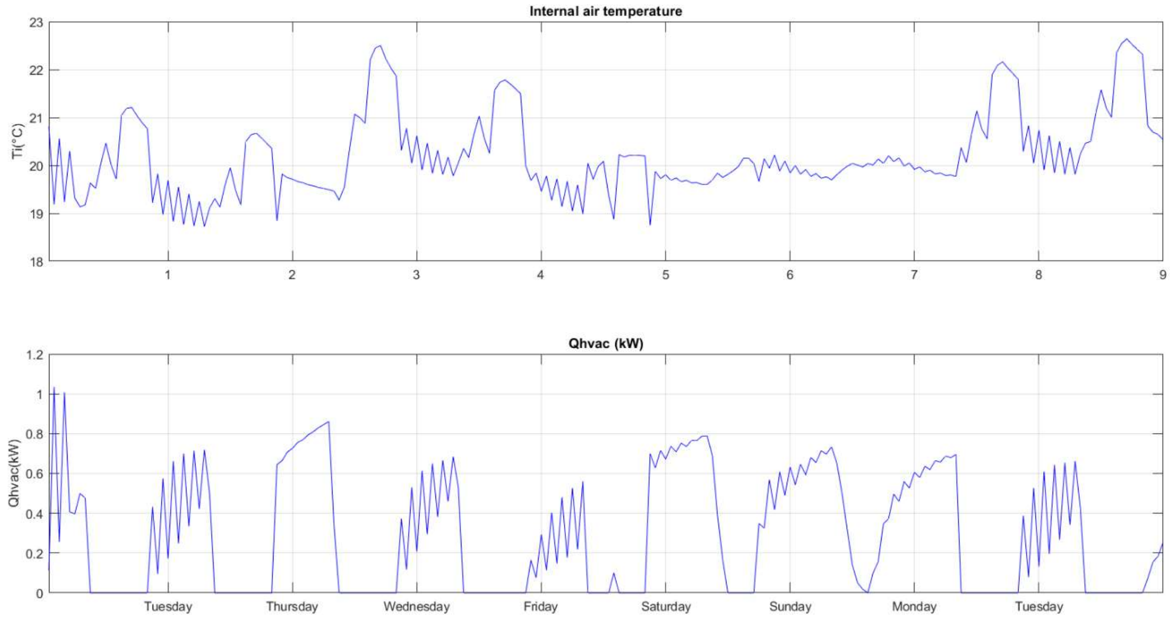


*Figure 32-Internal air temperature and HVAC delivered power*

Temperature time profile, displayed in the top plot, shows some overshoots above 21°C due to the high contribution of internal gains on a small, well insulated building, particularly during daytime. These overshoots are not controllable by the controller since the HVAC system works in heating mode only during this time of the year. However, as to the lower boundary set to 19°C, the MPC manages to control temperature much more precisely, so much that it rarely drops below the lower boundary setpoint. Indeed, the relatively high cost on the internal temperature slack variable guarantees that the constraint, albeit soft, is for the most time respected. As to the $Q_{hv}$ profile, the constraint that limits its variation between two consecutive timesteps to 1.5kW is never violated.

The second component controlled by the MPC is thermal energy storage. Inside, water temperature was restricted between 35°C and 55°C, and the null cost on heat pump operation allows the temperature to increase and increase to best employ the storing capability of the tank. The water tank state of charge is influenced by the ventilation thermal power as well. Water tank temperature is compared to the heat subtracted by the heating and the ventilation powers in Figure(33).

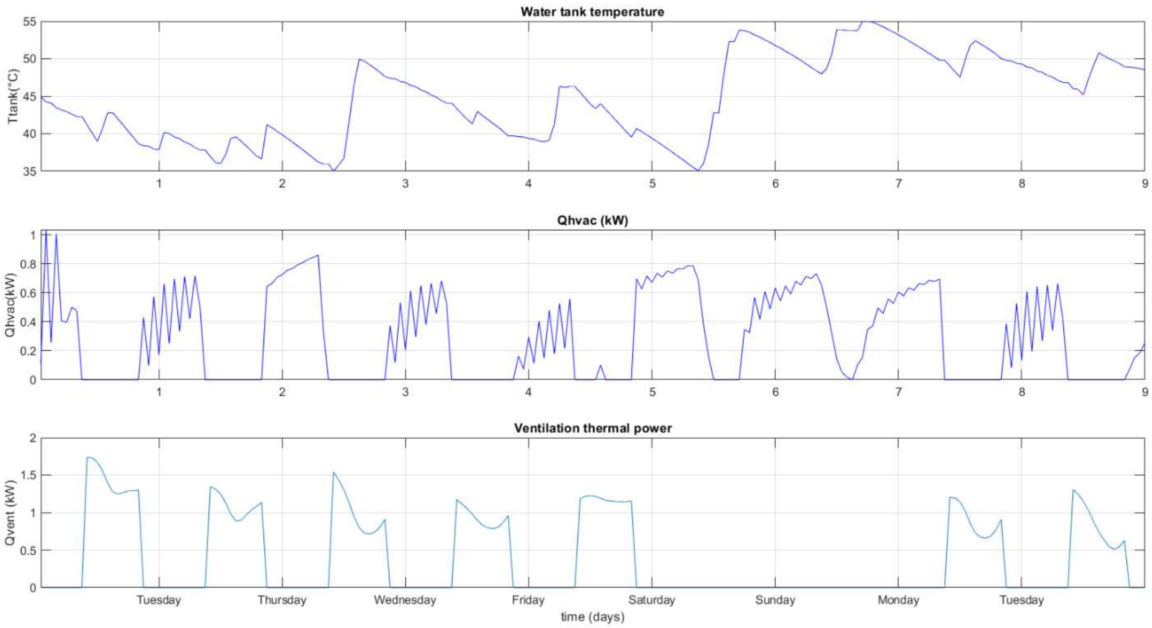


*Figure 33-Water tank control (MPC)*

Thirdly, the controller manages the electric power exchanges between the battery, the grid, and the electricity demand. The latter is made of the internal loads contribution, which is a predicted quantity, and the heat pump electric consumption. Since the aim of the controller is to minimize the amount of purchased power from the grid, the heat pump operation is expected to be shifted to the hours of the day in which PV production is at its highest values. Figure(34) shows that the MPC controller effectively shifts HP operation during daytime when the PV production peaks.

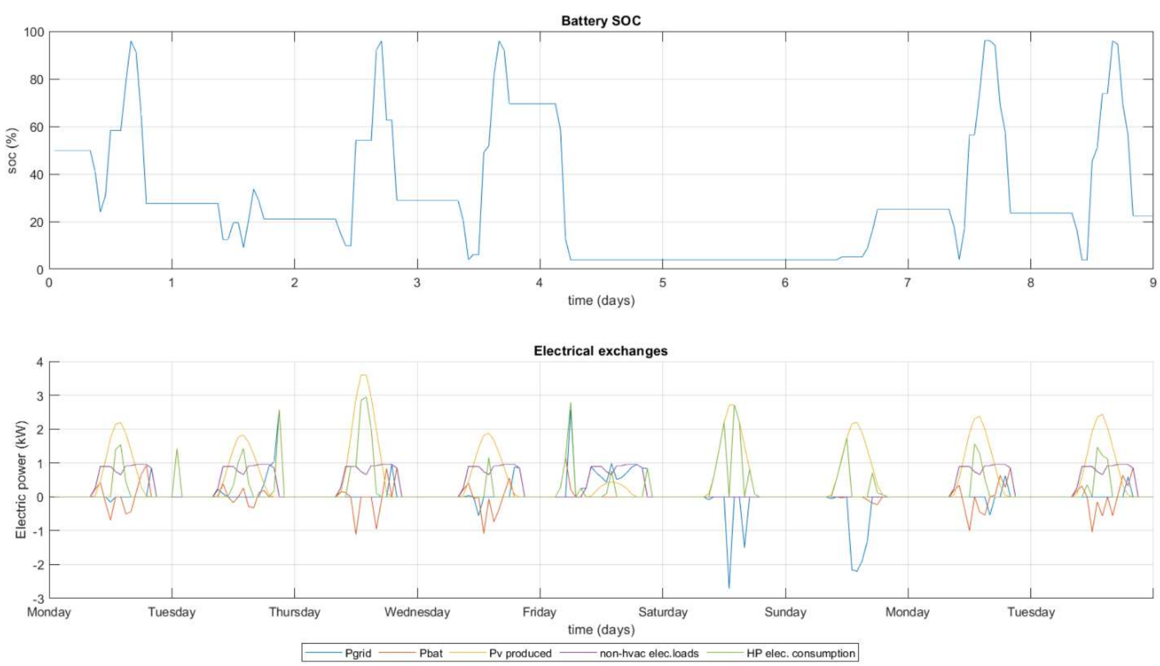


*Figure 34-Electric power exchanges (MPC)*

### 5.5.2 Baseline

The Rule Based strategy adopted as baseline controls the internal air temperature through an ON-OFF logic with a hysteresis of 2°C around a setpoint of 20°C. The maximum value of the power delivered by the HVAC system was set to 1.5kW and is delivered with a bang-bang type profile. This results into a much less regular internal temperature profile, with very steep variations in short time intervals. Air temperature variations as a reaction of the delivered thermal power are shown in Figure(28):

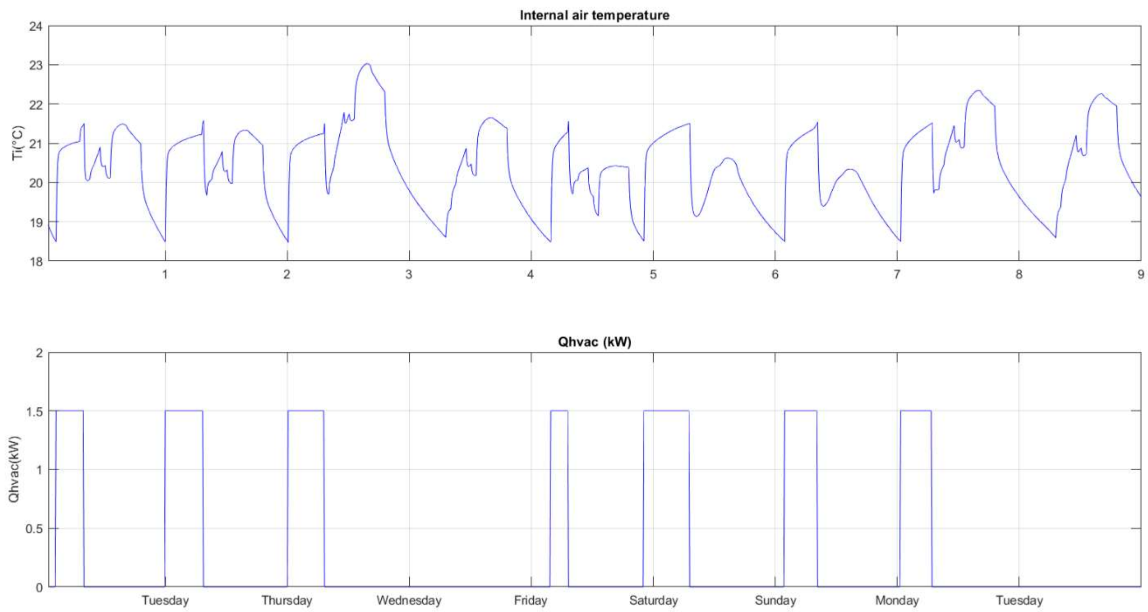


*Figure 35-Internal air temperature control (RBC)*

The control of the tank water temperature is shown in Figure(36):

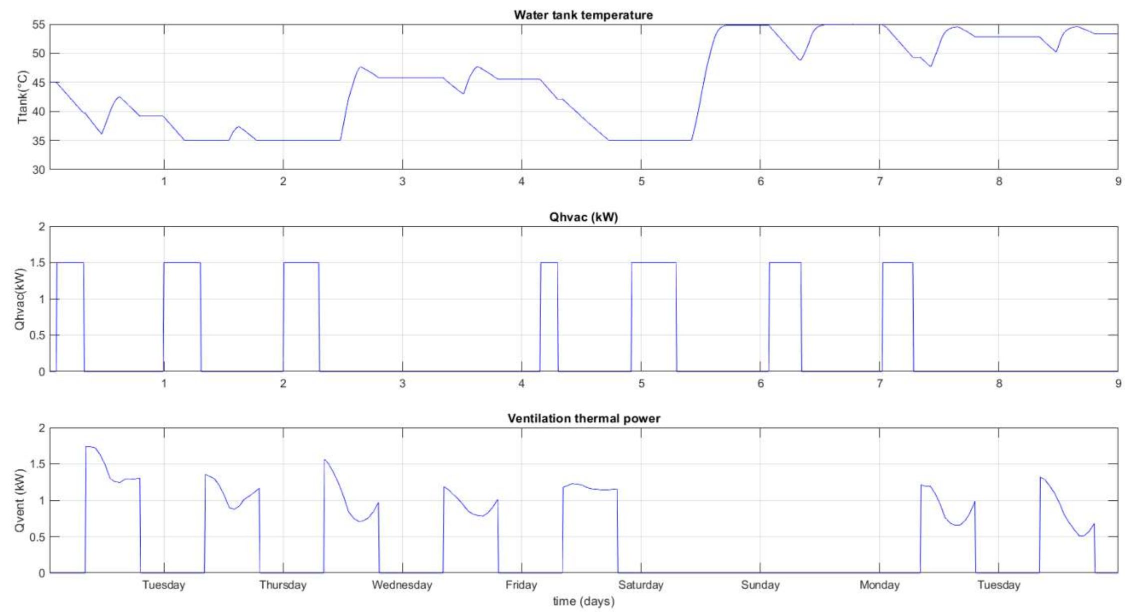


*Figure 36-Water tank control (RBC)*

It can be noticed that is kept in many instances at either 35°C or 55°C, which are the set lower and upper boundary values, as the RBC logic is to choose between these two setpoint at any time during operation. Since the external temperature is the same for both the MPC and the RBC simulations, ventilation power follows very similar profiles in both cases. This confirms that the assumption of an internal air temperature of 20°C as a constant value when estimating the ventilation thermal power proves to be solid.
Finally, electrical exchanges between the system and the power grid are shown in Figure(37).

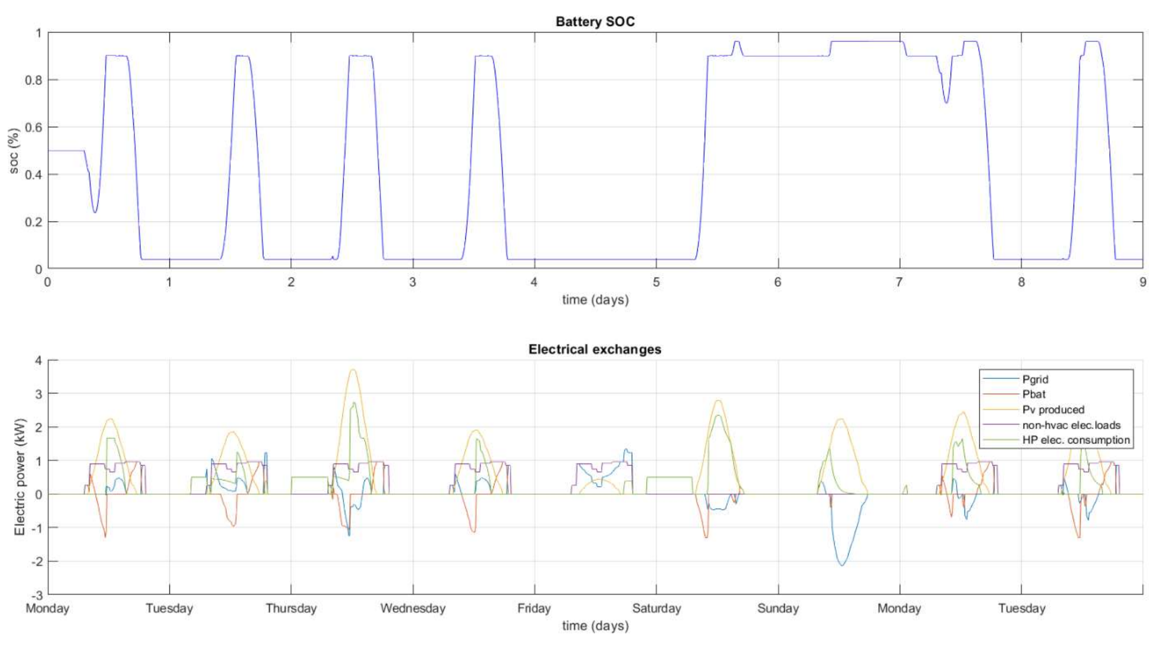


*Figure 37-Electric power exchanges (RBC)*

Qualitatively, as already observed when analysing the results of RBC3 in Chapter 4, the rule based strategy manages shifting most of the heat pump operating time during daytime, when PV production peaks, for a better exploitation of the locally produced energy. However, as quantitative results will prove, this improvement is still not as good as that achieved by the MPC, as its control is of optimal nature.
Quantitatively, simulation results in terms of the indexes defined in Chapter 3 are reported in Table(9):

| | MPC | RBC |
|---|---|---|
| Self Sufficiency | 80.83% | 75.51% |
| Self Consumption | 89.2619% | 86.2306% |
| Total purchased energy | 22.8398kWh | 30.1705kWh |
| Total monetary expense | 6.81€ | 9.05€ |

*Table 9-Quantitative results of the MPC simulation*

All KPIs are substantially improved by the MPC strategy. Clearly, expense and purchased energy are in fact the same indicator as the tariff was considered constant.

# 6 Conclusions

The present thesis-work chiefly aimed at formulating and testing a Model Predictive Control strategy for a building energy application. The main challenge of this formulation was the integration of many components whose interaction led to a complex MPC formulation. The resulting formulation was that of a Hybrid Model Predictive Control system, as it included a discrete variable whose value was determined by a Boolean condition. The strategy was tested in a simulative fashion on a case-study consisting in an experimental building served with electrical and thermal storage systems and equipped with rooftop-mounted photovoltaic panels for the local generation of energy. Finally, the connection to the public power grid allowed a bidirectional flux of electric power. The MPC strategy aimed at minimizing the amount of energy acquired from the grid, increasing Self Sufficiency and Self Consumption while ensuring comfort conditions for the occupants and working conditions for the system equipment operation. Results showed a significant improvement with respect to these objectives when compared to a Rule Based strategy adopted as baseline and tested under the same boundary conditions.

MPC showed, however, some limitations that call for further research to be conducted on the subject. The main obstacle that prevents a wider employment of MPC strategies lies in the optimization capabilities of software and hardware involved in the control process. Indeed, any MPC problem is translated into an optimization problem, whose solution becomes increasingly demanding as the complexity of the controlled system increases. Moreover, many building applications require hybrid formulations that make the computational burden even greater.

Another aspect to be investigated is that of robustness of the control system with respect to non-perfect predictions. In fact, predictions in this work were assumed to be perfect, which was allowed by the simulation. Real applications, particularly in the building sector, might suffer from non-perfect predictions such as inexact weather forecasts or predicted occupants' behaviours that are not met.

Nonetheless this work and more significantly previous literature examples proved that MPC is a potent, promising technology that, if properly implemented and deployed, could lead to significant results in terms of energy savings, building integration in Smart Grids and attainment of comfort conditions for occupants.

As to Rule Based Control, the enhanced strategies formulated in Chapter 4 proved that this control paradigm, albeit simple, is capable of achieving interesting results towards different goals. The main limitations of Rule Based Control lies in its non-optimal solutions and its difficulty in working towards contrasting goals. However, their simplicity, robustness and their consolidated knowledge make them a solution still to be considered in both research and applications.

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