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MASTER'S DEGREE THESIS

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Department of Environment, Land and Infrastructure Engineering Master's Degree in Petroleum and Mining Engineering

Master's Degree Thesis

Implicit Resources Modelling with vector indicator kriging



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Abstract

The purpose of this project is to use Geostatistical estimation tools to define an ore deposit by using geologic variables or structural information. The misclassification of the estimation domains has a significant effect on the resource estimate such as dilution of gold grades. The objectives of this project were to carry out a geostatistical resource estimation for a structural control gold deposit, increase the industry's understanding of geostatistical methods, numerical algorithm and computer applications to overcome huge computational time and processes.

Currently, computer aided system and software serve for geological modeling. The accuracy and speed of computer enable evaluation of various scenarios within reasonably short times. Computer application is very essential for geological resources estimation studies. Variogram analysis was conducted on composite string file to quantify the continuity of the Gold mineralization. Semi-variograms were generated for this project which were validated to allows us compare estimated and true values using only the information available in the sample data set.

In this project, SURPAC, LEAPFROG, SGEMS and MATLAB software have been used for ore estimation and geostatistical studies of the sample data obtained from exploration activities. Understanding the assayed AU percentage of the deposit, conducting basic statistics on composites string file and creation of the block model was achieved by using Surpac statistical tools. The estimation of AU percentage was accomplished by two different estimation methods i.e. inverse distance method and ordinary kriging method.

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Nomenclature

JORC	Joint Ore Reserve Committee			
RC	Reverse Circulation			
DD	Diamond Drilling			
SMU	Selective Mining Unit			
CV	Coefficient of Variation			
OK	Ordinary Kriging			
IDW	Inverse Distance Weighting			
KNA	Kriging Neighborhood Analysis			
KE	Kriging Efficiency			
BV	Block Variance			
KV	Kriging Variance			
R	Regression			
DTM	Digital Terrain Model			
P-value	Probability value			
Q_Q	Quantile-quantile			
Q_Q °	Quantile-quantile Degrees (angle)			
-	· · ·			
0	Degrees (angle)			
° +/-	Degrees (angle) plus or minus			
。 +/- g (h)	Degrees (angle) plus or minus Variance at a lag h			
° +/- g (h) N	Degrees (angle) plus or minus Variance at a lag h Number of samples			
° +/- g (h) Ν μ	Degrees (angle) plus or minus Variance at a lag h Number of samples Lagrange multiplier			
ο +/- g (h) N μ s	Degrees (angle) plus or minus Variance at a lag h Number of samples Lagrange multiplier Standard deviation			
 ° +/- g (h) N μ s s2 	Degrees (angle) plus or minus Variance at a lag h Number of samples Lagrange multiplier Standard deviation Variance			
 +/- g (h) N μ s s2 a 	Degrees (angle) plus or minus Variance at a lag h Number of samples Lagrange multiplier Standard deviation Variance Range (metres)			
 +/- g (h) N μ s s2 a A 	Degrees (angle) plus or minus Variance at a lag h Number of samples Lagrange multiplier Standard deviation Variance Range (metres) Area			
 +/- g (h) N μ s s2 a A Au 	Degrees (angle) plus or minus Variance at a lag h Number of samples Lagrange multiplier Standard deviation Variance Range (metres) Area Gold			

Co	Nugget variance		
$E\{Z\}$	Expected value of Z		
g/t	Grams of gold per tonne of rock		
h	lag (metres)		
km	kilometre		
m	Metre		
tonnes	One thousand kilograms		
V	Volume		
Z*	Estimator		

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CHAPITER 1

1.1. Introduction

This chapter describes the overview of the thesis, the definition of the problem statement, the precise goals of this project, the approach used to accomplish the purpose of the study and how the thesis is structured.

1.2. Statement of the problem

Estimating a shear zone hosted deposit without defining the mineralized zones into uniform domains is usually a common practice because the gold grades have little variation after estimation with a good correlation. However, this seems to be quite inaccurate as the domaining method is crucial and must be embraced as a major technique when dealing with shear zone deposit. Tectonic driven factors (faults) have displaced the ore body that characterizes the project region. The fault-induced displacement has divided the orebody into three separate domains. The key driving tools to assess the feasibility of a successful mining project are accurate recoverable resource estimation and grade control practices. Extra effort is put in place to achieve robust local estimates in an industry characterized by data abundance. In addition, the quality of an estimate is fully dependent on the quality of the data and any patterns or correlations between them will be disclosed by a detailed statistical analysis of the drillhole data. Statistical analysis helps to determine the quality of the data for a precise resource or reserve calculation that will be fitted into the model. In mineral resource estimation, domaining is a common practice that consists of partitioning the orebody into several zones (Anderson, E. M. 1951). The statistical parameters, kriging variance and spatial dependence between geological variables are analyzed within each domain. The discrepancy of geological boundaries, such as dilution, over or underestimate, tonnage, ore loss or population mixture, have been found to have ramifications for resource estimation. This poses a big threat to recoverable resources if a stepwise approach is not adopted to evaluate the consequences of defining inadequate estimation domains.

1.3. Objective of the project work

The objectives of this thesis are:

Resource Modelling and running a statistical analysis on gold data from a sample drillhole.

Emphasize and understand the significance of statistical analysis prior to the form of grade interpolation using Indicator Kriging

Investigate and analyze the effects of geological domains on estimates of mineral resources.

Provide necessary recommendations

1.4. Method used

The methods that were used to achieve the objective includes:

The collection and processing of geological data.

Extraction of drillhole cross sections and digitizing of ore zones within the deposit to create wireframes and solid model.

Analysis of drillhole data using GEOVIA Surpac 6.7.4 modelling software.

Geostatistical Modeling with MATLAB and Stanford Geostatistical Modeling Software (SGeMS).

Definition of domains.

Estimate resource using IDW and OK.

1.5. Thesis organization

The thesis is organized into several chapters.

Chapter one discusses the statement of the problem, goals, methodology, scope of the work and order of the presentation.

Chapter two gives details explanation of the location of the deposit, regional geology and geology of the concession area.

Chapter three elaborates on the review of literature associated to geostatistical theory of ore estimation.

Chapter four talks about the methodology used and the processes involved in sample evaluation using the research data. This includes data processing, data validation and digitization of ore zones, ore modeling, definition of domains, wireframing, statistical analysis and variogram modeling.

Chapter five concentrates on the results obtained from the Block modelling and resource estimation.

Chapter six talks about the conclusion and recommendation of the project.

CHAPITER TWO: GEOLOGY OF THE DEPOSIT

2.1. Location of the deposit

The Lega Dembi gold mineis located in the Sidamo region of southern Ethiopia. It is the largest gold producer in the country. The deposit is situated 500 km south of Addis Ababa, within the late Proterozoic meta-volcanosedimentary Megado belt of the Adola granite-greenstone terrane, which forms the southernmost extension of the Neoproterozoic Arabian-Nubian Shield (Astrup J, 1948) (Figure 2.1).



Figure 2.1:The location of Lega Dembi gold mine in the Sidamo region of southern Ethiopia (Google Earth, 2020)

It is in the north-south trend volcano-sedimentary Megado belt, which forms part of the late Proterozoic Adola granite-greenstone terrain in southern Ethiopia in the late-Precambrian metamorphosed sediments. It is trending north-south, steep northerly dipping quartz-vein system. The lode-gold mineralization occurs following the structural contact between the underlying feldspathic gneisses and the Megado belt volcano-sedimentary sequence (Bell T. H, 1981 and Astrup J, 1950).

2.2. Regional Geology

Gneissic terrain is intruded by large tonalite bodies. Gneissose granites are concerned with the gneissic terrain. The greenstone belts and the post tectonic granites occur marginally in Gneissic Terrane. The tectonic contact between the Megado belt and the gneissic terrane is eastern and is regionally referred to as the shear zone of Lega Dembiata (Chater A. M, 1971). However, the western edge of the Megado belt is distinguished by the growth of gneissose tonalite showing primary intrusive contacts with the supracrustal assembla (Figure 2.2).

Two main metamorphic events (M1-M2) have been identified as having Granite Adola-Greenstone Terrane. Data of an early M1 event is only described by relict cordierite-containing mineral parageneses, which have been largely overprinted by the subsequent M2 event that has omnipresent impacted the region's rocks. Mineral assemblages usually include quartz, plagioclase, biotite, muscovite and accessory rutile, chlorite and epidote in the sedimentary rocks of the Megado belt (Chater A. M, 1971).



Figure 2.2: Location and regional geological setting of the Adola granite-greenstone terrane in southern Ethiopia (Chater A. M, 1971).

Actinolite, hornblende, plagioclase (albite-oligoclase), epidote and chlorite are prevalent in the basic rocks. However, in places where kyanite-bearing rocks found within the series of greenschist-facies point to metamorphic conditions of a locally higher grade. The rocks of the terrain of the gneiss and the belt of Kenticha have the metamorphism of the staurolite-almandine and kyanite-almandine-muscovite sub facies which was influenced by amphibolite±facies during the M2 metamorphism (Figure 2.2). The metamorphic grade rises in the SE of the gneissic terrane from lower and mid amphibolite facies in the NW to upper-amphibolite and lower-granulite facies. (Boulter C. A; Fotlos M. and Phillips G. N, 1987).

There are three different versions that are proposed for the disposition of the linear greenstone N-S trend gneissic terrane overlying belts. The first model is proposing the origin of the sequences of greenstone as ophiolites, which were thrust onto the Terrane of the Gneiss. Later, the ophiolites were refolded by significant N-S trending folds and then adjusted by strike-slip shaving shearing. Intra-continental rifting, accompanied by thick-skinned tectonics, indicates the second model, whereby both the greenstones and the gneisses were driven in an eastward direction (Boulter C. A; Fotlos M. and Phillips G. N, 1987).

There are scarce age indicators on the rocks in the Adola area. The supracrustal sequence of the Megado belt was associated with simplified geological map of the Lega Dembi region showing the position of the

gold deposit along the contact between the Megado belt greenstone sequence (in the west) and the gneissic basement (in the east) similar upper Proterozoic Arabian-Nubian Shield assemblages providing the volcano-sedimentary sequence with a minimum age constraint. For the gneissic basement which is associated with gneisses of the Mozambique belt and the upper Proterzoic ages are also inferred. Mafic units form a N-S trending ridge parallel to the Megado belt's structural grain. Stringer and pod-like tonalite bodies that display penetrative planar and linear fabrics are locally intruded into the mafic units (Kazmin V, 1972).

2.3. Geology of the deposit

The Lega Dembi deposit is divided into four open pits interconnected operations locally referred to as Southern, Central, Northern and Upper Lega Dembi (SLD, CLD, NLD) (from south to north and ULD, respectively). The mining areas and some resistant lithologies are mainly confined to outcrop. The geomorphology of the heavily weathered and densely forested region dominates the N-S trending ridges (Guha J. Dube; P. Chawn; E. H. Archambault and G. Bouchard, 1988).

At the Lega Dembi, the steep westerly-dipping lithostratigraphic sequence can be subdivided into: a collection of quartz-feldspathic, biotite gneisses and amphibolites belonging to the eastern gneissic terrane, and the western Megado belt volcano-sedimentary sequence (Kazmin V; Shiferaw A and Balcha T, 1978) (Figure 2.3).



Figure 2.3: Simplified geological map illustrating the area of Lega Dembi the gold deposit location along the contact between the contact of the gold deposit. The Megado belt (in the west) and the gneissic basement (in the east) series of greenstones (Kazmin V; Shiferaw A and Balcha T, 1978).

Along the contact between the gneissic terrain and the supracrustal series, the ultramafic talc schists are commonly formed. The moderate-to-steep $(40\pm70^{\circ})$ westerly dipping contact is characterized by mylonitic fabric formation. It is clearly tectonic and generally sharp, although local intercalations of talc schist bands are observed within the gneisses. The width of the talc schists at NLD is less than 5 m, pinching out locally, but eventually increasing to the south and reaching a maximum thickness of approximately 180 m south of Reji (Chowaka. S and de Wit M. J, 1981). The succession of meta-sedimentary rocks can be subdivided into a lower leucocratic muscovite-quartz plagioclase schist, which is overlain by laminated darkgreyish, graphite-rich, locally graded feldspathic arenites and quartz wackes. The mineral assemblage of the sediments consists of quartz, biotite, muscovite, plagioclase, together with accessory rutile, epidote, graphite, tournaline and chlorite. The latter overgrows the main foliation. Kyanite is present locally in proximity to quartz veins (Gilboy C. F, 1970) (Figure 2.4).



Figure 2.4: The mineral assemblage of the sediments that consists of Lega Dembi the gold deposit (Gilboy C. F, 1970).

The succession of sedimentary reaches at CLD, the overall thickness is approximately 280 m and steadily decreases to less than 20 m at ULD, where it is trapped between the massive meta-gabbro in the west and the quartz-feldspathic gneisses in the east (Kazmin V, 1972). Together with minor amphibolites, the huge meta-gabbro forms the western margin of the deposit of Lega Dembi (Figure 2.4). The mafic units parallel to the Megado belt's structural grain form a N-S trending ridge. Stringer and pod-like tonalite bodies that represent penetrative planar and linear fabrics are locally intruded into the mafic units and are folded on a meter scale along with the amphibolites (Ghebreab W, 1989).

CHAPITER 3: LITERATURE REVIEW

3.1. Introduction

The literature on geostatistical ore estimation is discussed in this chapter, with domaining being the underlining process in the estimation process.

3.2. Geostatistical Theory

Geostatistics is "the application of the formalism of random functions to the recognition and estimation of natural phenomena." For example, sample grade is a regionalized variable because it is distributed throughout a space. A method called the 'variogram' defines the variability of the grade in that space (Matheron G, 1962). Samples and the variogram is used to measure the mean grade of a point, area, or volume using a method known as 'kriging.' Kriging gives the best estimate of a regionalized variable's mean value.

During kriging, each sample is assigned a sample weight. The weighted samples are then linearly combined to give the best estimate. It is the 'best' estimate because the procedure minimizes the expected error between the estimated grade and the true grade. Sample weights are calculated such that the variance of the estimate is a minimum. The variance can be calculated using the sample positions and the variogram function. Having the estimation variance is extremely useful because it allows the user to explore the risk of the estimate (Snowden, D.V, 2001).

3.2.1. Regionalized Variables and Random Functions

Variables are distributed throughout a space are regionalized variables. Several variables are regionalized in earth science. There are two characteristics that are obvious in a regionalized variable for mining application. The first characteristic is a local random that gives the appearance of a random variable. A general, structural pattern that can be represented by a function is the second. Using random functions, the two can be interpreted probabilistically. A sample's grade can be considered a particular realization of a random variable. A random function is all possible realizations of that random variable. At a point, a grade is considered a random variable. However, a pair of points that are separated by a distance are independent, but also correlated through a certain spatial structure known as the variogram (Ortiz, J. M. and Emery X, 2006).

3.2.2. Stationarity

That indicates that the random variables expected average must be constant in any location. Quasistationarity is believed to occur in practice. Under that theory, for a limited distance known as range, the spatial structure is invariant. In a deposit, the range can be rationalized as the limits of a homogeneous region (Rossi, M.E. and Deutsch, C.V, 2014).

3.2.3. The Covariance Function or Variogram

The covariance function is one that describes a covariance as a function of variable separation. At zero separation distance, two variables have the maximum covariance (Co) as their separation increases to the range, covariance decreases (Figure 3.1). At a certain distance, the covariance will approximately equal zero. One type of covariance function common in geostatistics is known as a transition structure (Rossi M.E. and Deutsch C.V, 2014).



Figure 3.1: Relationship between the covariance function and semi-variogram (Rossi M.E. and Deutsch C.V, 2014).

The variogram function is another way of expressing the degree of covariance between two variables separated by a distance (Rossi M.E. and Deutsch C.V, 2014). It is closely related to the covariance function through the following relationship:

g(h) = sill + nugget - C(h)

where:

g(h) = variance of two variables separated by h distance

sill = the ceiling value for g(h) - nugget value

nugget = value of g (h) very close to h=0 (described later)

C(h) = covariance of two variables separated by h distance

That relationship is shown graphically in Figure 3.1. The covariance structure can be anisotropic, meaning that there can be many different covariance functions that describe the covariance structure along different orientations. For example, variables spaced in the horizontal direction might follow a certain covariance function, while those spaced in the vertical direction might follow another (Clark I, 1986).

3.2.4. Nugget effect

Two variables have the maximum Covariance at a separation distance of zero. The corresponding variogram value g(0) = 0 implies that the variance is equal to zero for two variables located at the same point. The function variogram is discontinuous, it has a certain value known as the nugget value at a very small separation distance. Measurement errors and microvariability in mineralization are cause by the nugget value (Clark I, 1986).

3.2.5. Sill (S) and Range (R)

The sill is the value of the y-axis at which the model variogram levels out. As the range is called, the distance at which the model begins to flatten out. All samples with separation smaller than the range are autocorrelated, and those with separation larger than the range are not. The larger the selection, the greater the sample correlation (Goovaerts P, 1997).

3.3. Basic statistics

There are two main groups of statistics that we need to understand for geostatistics:

- The measures of central tendency:
- The measures of spread:

The statistics used to describe measures of central tendency are the mean, mode and median.

• The mean is the sum of all the sample values divided by the number of samples. The mean is the same as the average value.

Sum of the samples

The median is the middle value. It is determined by sorting the data into ascending order and selecting the middle value. The median is the same as the 50th percentile where half of the data lies below this sample value and half the data lies above this sample value (Snowden, D.V, 2001).

median = 50th percentile

The mode is the most frequently occurring sample value.

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mode = highest frequency value
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The statistics used to describe measures of spread are the inter-quartile range, variance, standard deviation and coefficient of variation.

The simplest spread statistic is the range. The range is the difference between the highest and lowest sample value.

range = maximum value – minimum value

The inter-quartile range tells us something about the range of the sample values which represent the middle 50% of the samples. It is calculated by sorting the data into ascending order and determining the sample which has 25% of the data below this value (25th percentile) and the sample which has 75% of the data below this value (75th percentile). The inter-quartile range is the difference between the 75th and 25th percentiles (Isaaks E.H. and Srivastava R.M, 1989).

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inter-quartile range = 75th percentile – 25th percentile
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The variance measures the typical difference between the actual sample values and the overall average value. The differences between individual sample values and the average values are each squared (to prevent positive and negative differences cancelling each other out). These squared differences are accumulated and divided by one less than the number of samples. A value of one is subtracted from the

number of samples because the sample values are being compared to a statistic that is based on the sample values themselves rather than the population (Royle A.G, 1992). Ignoring this would tend to underestimate the true variance, hence n-1 is used to account for the bias. This is called "losing a degree of freedom".

Variance =
$$\frac{\text{sum of (sample value - mean value)}^2}{\text{number of samples}}$$

standard deviation = square root of variance

The coefficient of variation (CV or COV) is used to compare the variability of datasets. It is often called the relative standard deviation as it is the ratio of the standard deviation to the mean value. The standard deviations of datasets can only be compared directly when the datasets have the same means (Journel A.G. and Huijgregts C.J, 1978).

$$CV = \frac{\text{standard deviation}}{Mean}$$

Population distribution is also an important aspect of statistical analysis. The distribution can be described in terms of the skewness sign which is an indication of the difference between the mean and the median (mean – median). When the data distribution is plotted as a histogram (number of samples within a series of grade bins plotted against that grade), a distribution is not skewed if it shows a symmetrical, bell shaped curve. In this instance the mean, median and mode are approximately equal (Journel A.G. and Huijgregts C.J, 1978).

If the distribution tails to the right then the mode is less than the median, which is less than the mean and the distribution is positively skewed (Figure 3.2). Examples of positively skewed data include gold, copper, sulphide nickel, platinum and many contaminants in iron ore. If the distribution tails to the left then the mode is greater than the median, which is greater than the mean and the distribution is negatively skewed. An example of negatively skewed data includes iron in iron ore deposits. Raw mean and variance are sensitive to extreme values as the level of skewness increases. I t becomes harder to produce reasonable estimates that reflect the population characteristics (Journel A.G. and Huijgregts C.J 1978).



Figure 3.2: Population distribution (Journel A.G. and Huijgregts C.J, 1978).

The population distribution can also be used to determine the presence of two or more statistical populations. Single statistical populations will show a smooth curve with a single peak on the histogram while mixed populations will show up as multiple peaks (Figure 3.3) (Journel A.G. and Huijgregts C.J 1978).



Figure 3.3: Single and mixed population shape (Journel A.G. and Huijgregts C.J 1978).

3.3.1. Histograms, cumulative distribution functions and probability plots

There are three main graphs that are used for statistical analysis. Histograms, cumulative distribution functions and probability plots. Histograms provide a graphical summary of the number of samples that have grade within a range of intervals (Figure 3.4). For non-skew, normally distributed data the histogram should show a normal bell-shaped curve (Deutsch C. V, 1996).



Figure 3.4: Histogram (Deutsch C. V, 1996).

A cumulative distribution function (CDF) is an accumulated histogram where the proportion of samples below each grade threshold (cumulative probability) is plotted against that grade (Figure 3.5). CDFs can be generated by sorting the data in ascending order, calculating the percentile values for each sample and plotting the percentiles against the sample grades. The percentile is simply the relative position of the grade, for example the 10th percentile has 10% of the samples being lower grade and 90% being higher grade.

CDFs are "S" shaped when the data is not skewed. CDFs for negatively skewed data are steep at the highgrade end while CDFs for positively skewed data are steep at the low-grade end (Cressie N, 1993).



Figure 3.5: Cumulative distribution function (Cressie N, 1993).

Probability plots are a variation on the CDF where the probability scale is adjusted so that the graph forms a straight 1:1 line if the population is normally distributed (Cressie N, 1993). The graph effectively maps the data distribution against a standard normal (bell-shaped) distribution (Figure 3.6).

Probability plot



Figure 3.6: illustrate histograms, CDF"s and probability plots for normal, negatively skewed and positively skewed distributions, respectively (Cressie N, 1993).

3.4. Resource Estimation

Resource estimation is used to determine and define the ore tonnage and grade of a geological deposit from the developed block model. There are different estimation methods used for different scenarios dependent upon the ore boundaries, geological deposit geometry, grade variability, the amount of time and money available. Estimation methodology has developed over time out of a need for an unbiased estimate of the grade at an unsampled point given the values of known points (Isaaks EH, and RM Srivastava, 1989).

3.4.1. Kriging

Kriging is the method of geostatistical estimation developed to provide the ideal linear and unbiased estimates. In terms of the variogram (or correlogram), it relies on expressing the spatial variance of the property and minimizes the prediction errors that are calculated. Kriging considers covariances between samples in an approximate block model, which would then reduce the weight of a sample cluster, reducing the effect of variable sample spacing. The efficacy of kriging depends on the correct input of parameters describing the model of the variogram since kriging is robust but also fragile, and even a naive set of parameters can provide an approximation comparable to many other methods of grid estimation (Deutsch C.V. and A.G. Journel, 1998). Kriged estimates should not be systematically higher or lower than the true value used, which is why kriging weights are calculated by solving a set of equations shown below that minimize the variance of the estimation error (Figure 3.7).

Kriging uses a set of simultaneous linear equations for each point on the output grid such that all the actual input data is optimally weighted according to distance using the semivariogram. These equations are often written using matrix notation. The correlation matrix on the left-hand side records all the redundancies between the samples and ensures that the kriging weights considers sample clustering (Deutsch C.V. and A.G. Journel, 1998).

$g^* = \sum_{i=1}^{i=n} w_i * g_i$	$\sum_{i=1}^{n}$	$w_i = 1$	$R = g^* - G$
$w_i = weights, g^* = estimates$	ate, R = estimate	ite error	
$\underbrace{ \begin{bmatrix} C_{11} & \cdots & C_{1n} & 1 \\ \vdots & \ddots & \vdots & 1 \\ C_{n1} & \cdots & C_{nn} & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix} }_{}$	$\times \qquad \underbrace{\left[\begin{array}{c} w_1\\ \vdots\\ w_n\\ \mu\end{array}\right]}$	=	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$
Sample redundancy	Kriging wei	ghts Statistica	d distance

Figure 3.7: Kriging weights calculation equation use to minimize the variance of the estimation error (Deutsch C.V. and A.G. Journel, 1998).

Kriged estimates can be represented graphically in geostatistical grids to provide geological knowledge. The key to kriging is not only to provide interpretation but also to provide optimized interpretation, which is why changes can be made to establish variations in terms of anisotropy and trend inclusion (Tulcanaza E, 1972).

3.4.2. Polygonal estimation

In the past, traditional methods have used either a nearest neighbour or an averaging approach to assign grades to points or volumes. The simplest method is to assign the value of the nearest sample to the unknown point; this is called nearest neighbour estimation. However, this approach under-utilized the available data and ignores any correlation between samples (Journel A. G, 1985).



Figure 3.8: Polygonal estimation (Journel A. G, 1985).

The polygonal method is a nearest neighbor estimate which assigns the grade to a volume. A volume or polygon is defined around each sample point and the grade of that sample point is assigned to the polygon (Figure 3.8). This method ignores any correlation between samples and ignores the volume variance effect (larger volumes have lower variance). Hence, the extreme grades estimated by this method cannot be achieved during mining (Dowd P. A, 1992).

3.4.2. Inverse distance

Assuming there is some spatial correlation between the samples. It makes sense that samples closer to the unknown point are more related to it. Inverse distance estimation is a linear three techniques which was developed to attempt to account for this sample to distance relationship (Journal, A. G, 1983).

Inverse distance estimation is like the averaging approach look at previously, except that each sample is assigned a weight according to the inverse of their separation distance from the point of estimation (Figure 3.9). This means that close samples receive a higher weight than samples further away (Journal, A. G, 1983).



inverse of sample to estimation point distance

Figure 3.9: Inverse distance estimation – samples weighted by distance (Journal, A. G, 1983).

To ensure an unbiased estimate, the weights (inverse of the separation distances) are rescaled, so they sum to one to ensure that the estimated grade is unbiased when compared with the sample grades. The equation for inverse distance estimation is:

 $Estimate = \frac{sum of (sample value x inverse distance weight)}{sum of (inverse distance weights)}$

Where:

(sample to estimation point distance) power

1

The inverse distance weights can also be raised to a power. This power is selected by the user in an arbitrary way. One way to think about the power parameter is to consider its effect on the estimate. If the power is high (say about three), then the closer samples receive even more weight than the samples further away. The higher the power, the more weight is assigned to the closer samples. When the power is low (say one), then the closer samples still receive greater weight than the samples further away. (Sinclair A.J. and Blackwell G.H, 2002).

3.4.3. Ordinary Kriging

inverse distance weights =

Ordinary kriging is like inverse distance in that it applies weights to account for similarity in grades according to distance. The difference is that the weights are selected via the variogram model according to the samples distance and direction from the point of estimation (Journal A. G. and Rossi M.E, 1989). This means that the weights consider the spatial correlation between the samples and the point of estimation (Figure 3.10).



Figure 3.10: Ordinary kriging estimation-samples weighted by variogram (Journal A. G. and Rossi M.E, 1989).

The kriging estimation equation is written as: Estimate = sum of sample value x kriging weight

Kriging was created to serve a need; an estimation method was sought which, of all possible linear estimators would provide an estimate that had the least overall error and would be unbiased (Best Linear Unbiased Estimator) (David M, 1977).

Consider these two criteria in detail:

• Least overall error means the overall variance between the estimate and the true values must be a minimum, so the average difference between the estimate and the true values must be smaller for this estimator than for any other linear estimator. This can be written as:

 $(estimate - real value)^2 = minimum$

• The estimate must be unbiased; so, the sum of the weights must equal one. This can be written as:

(sum of weights -1) = 0

These two criteria can be combined into the following equation where the LaGrange multiplier is an adjustment factor applied to ensure that the weights sum to one.

 $(estimate - real value)^{2} + [LaGrange multiplier x (sum of weights - 1)] = minimum$

Given that the estimate is generated as follows: estimate = sum of (sample value x weight)

The expression can be updated to read:

[sum of (sample value x weight) - real value] 2 + [LaGrange multiplier x (sum of weights-1)] = minimum

This equation can be differentiated (using standard calculus methods) to determine what weights are required to minimize the expression. The result is the kriging system of equations that are used to derive the kriging weights (David M, 1977).



Figure 3.11: Kriging system (David M, 1977).

The kriging system that is used to derive the weights can be summarized in three matrices: Matrix A, X and B where:

- Matrix X is the matrix of weights the system is attempting to estimate.
- Matrix B is the matrix of variogram values for the distances (and directions) between the samples and the point to be estimated.
- Matrix A summarizes the variogram values between all the samples used for the estimation. Matrix A takes care of the decluttering in the system by ensuring that lower weights are assigned in areas of clustering.

Since Matrix A and B are essentially derived from the variogram model that is supplied to the kriging system, all that remains is to solve the kriging system to derive the weights.

Consider the equation: ax = b, where a and b are known; x is simply b/a.

This same logic is applied to resolve the equation for the weights. Matrix A and B are known (from the variogram) and we need to work out what X needs to be to satisfy the equation. This is done using standard matrix algebra. The weights are derived are then applied to the sample values to estimate a grade for the unsampled location (Journal A and Huijbregts C, 1978).

Note the last column and last row of matrix A contain ones (except the very last entry which is a zero), the last entry of matrix B is a one and the last entry of matrix X is a m (shorthand for LaGrange multiplier). These entries invoke the constraint that the weights add up to one (Journal A and Huijbregts C, 1978).

3.4.4. Indicator kriging

The indicator kriging process is described below. The process is illustrated at each step using the ninesample example used previously. As described during statistical analysis (Chiles J. and Delfiner P. 1999).

Select indicators to describe the population distribution.

Code the composited sample data for each indicator to 1 if the sample grade is less than or equal to the indicator cut-off otherwise to 0. Calculate mean and median statistics for the intervals between each indicator (Chiles J. and Delfiner P. 1999).



Figure 3.12: Worked example – indicator variogram models (Chiles J. and Delfiner P. 1999).

Carry out ordinary kriging using the coded data (1 and 0) and appropriate indicator variogram for each indicator. The result is an estimated value between 0 and 1 for each indicator which is the probability that the grade will be less than the indicator grade. These probabilities create a CDF at each estimation point which describes the full range of grades and the corresponding likelihood that the grade will be less than any given grade. The probability that the grade will be less than the indicator grades and the corresponding likelihood that the grade will be less than any given grade. The probability that the grade will be less than the indicator grades are converted to the probability that the grade will be in the intervals between indicator grades (Deutsh V. Clayton, 1989). This is carried out by simply subtracting the probability of being less than the lower interval grade from the probability of being less than the higher interval grade. This is repeated for all intervals to create the equivalent of a histogram of likely grades for the estimation location (Figure 3.13).



Figure 3.13: Worked example – CDF and histogram of probabilities from estimation of indicators (Deutsh V. Clayton, 1989).

Grades are assigned to each interval using the mean of the interval. The final indicator kriged estimate of the expected grade at a particular location is then generated by multiplying each interval mean grade by the probability that the location will have a grade in the interval and summing the results (Deutsh V. Clayton, 1989).

		estimated probability			statistics		calculated
indicator	interval	grade being < indicator	grade being within interval		mean grade within interval		grade within interval
0.95	0 to 0.95	0.40	0.40	х	0.70	3115	0.280
1.9	0.95 to 1.9	0.63	0.23	х	1.45	=	0.334
3.2	1.9 to 3.2	0.87	0.24	x	2.80	=	0.672
	> 3.2	1.00	0.13	X	8.10	=	1.053
						1	2.339

Figure 3.14: Worked example for calculating the grade in indicator estimation (Deutsh V. Clayton, 1989).

When there is a positively skewed tail, and the mean grade of the last interval is calculated from a small set of data. The mean grade will be biased by a few extremely high grades. In this situation, the median grade is believed to provide a better representation of the grade conditions. It is used in place of the mean grade for the last interval (Matheron, 1963).

The basic statistics of the domain data are useful as a guide for selecting the most appropriate estimation techniques (Figure 3.15). Generally, ordinary kriging is the best estimation method in all situations except for mixed populations and highly skewed populations where indicator kriging is required (Matheron, 1963).



Figure 3.15: Selecting an estimation method (Matheron, 1963).

The main advantages and disadvantages of the methods discussed are summarized in Table 1:

Technique	Advantages	Disadvantages		
Inverse Distance	Quick and easy to use.	Choice of power is arbitrary.		
	Only a few parameters to set.	Sensitive to data clustering.		
		Weighting is directly related to distance, irrespective of ranges of influence.		
Ordinary kriging	Uses spatial relationship between.	Time and effort to do variography.		
	samples to weight the samples.	Negative weights need to be controlled.		
	Built in delustering.			
Indicator kriging	Adapts weights according to grade ranges.	Time and effort to do full indicator variography.		
	Allows estimation for spatially integrated populations.	Indicators are estimated independent of each other.		
	Allows estimation when continuity rotates according to grade ranges.	Order relation problems need to be controlled.		
	Copes with highly skewed populations.			

Table 1: Advantages and disadvantages of estimation technique

<u>CHAPITER FOUR: DOMAINING, ORE MODELLING AND</u> <u>SAMPLE DATA ANALYSIS</u>

4.1. Domaining

Domain is a very critical step in mineral resource estimation and must be done with special care. A domain, as described earlier, represents an area or volume within which the characteristics of mineralization are more similar than outside the domain. The definition of domains is usually accompanied by a lithological interpretation and delineation of ore bodies with the same structural and geological characteristics. Most cases, geological units are the same as mineral domains, such as, Gold, iron ore deposits, metal sedimentary deposits or metal sulphide syngenetic deposits (O. Kaufmann T. Martin 2008).



Figure 4.1(a): Ore model and domains that characterized our ore body



Figure 4.1(b): Ore domaining in Surpac Modelling Software

4.1.1. Type of geological domains

All deposits that are syngenetic or epigenetic show some difference from ore to non-ore. If the concentration of metals in rock is below the cut-off grade, the rock is considered as waste and cannot be mined for benefit. The two types of domains are hard and soft boundary, and both will be described in detail in the subsequent pages (Ortiz J. M. and Emery X, 2006).

a) Hard Boundary

Since geological domains are representative of a stationary randomized homogeneous variable, the concept of hard boundary is based on five processes. The stationary decision is a five-step process:

Choose a domain number and form

Modeling Domain Boundaries

Determine the nature of the cross-domain transitions

Quantify large-scale patterns in domains

Predict with the Model of Trend

Hard boundaries reveal an abrupt transition across borders. (Figure 4.2(a). Also, the grades assessed on either side of the boundary are distinct, and there is no evidence of spatial association across borders (Ortiz J. M. and Emery X, 2006).



Figure 4.2(a): Contact plot showing abrupt changes in Z across hard boundaries (Ortiz J. M. and Emery X, 2006).

Hard boundaries greatly facilitate resource estimation. The selection of a natural cut does not cause over or under grade smoothing. Grade misclassification problems are rarely occurring and there is no overestimation or underestimation of metal concentrations. The distance between samples and Z is the grade or any element being examined. In vien deposits, hard boundaries are sometimes characterized as a region of non-deposition or unconformity obeying the principle of lateral continuity. In addition, the boundaries identified are clearly distinguished by the deposition and mineralization style (Larrondo P. and Deutsch C.V, 2005). Hard domains do not allow grades to be interpolated and simulated across borders.

b) Soft boundary

A soft boundary is present when the grade indicates a substantial trend in at least one domain, but there is no significant change in the grade level at the boundary. The soft boundary also shows the transition zone between two domains making it difficult to define the exact threshold layout as shown in (Figure 4.2(b). It allows grades from the other side of the boundary to be used for estimating all domains to varying degrees (Larrondo P. and Deutsch C.V, 2005).



Figure 4.2(b): A contact plot showing gradational change in Z (u) across soft boundaries (Larrondo P. and Deutsch C.V, 2005).

Boundaries are often defined by a shift in the mean grade, where there is no spatial dependence across the boundary within the domain.

4.2. Definition of domains

The classification of the domains must be carried out carefully considering the geological knowledge of the deposit and how it was formed (Boulter,C. A; Fotlos M and Phillips G. N, 1987). Other petrophysical properties of rocks assist in the identification and definition of domains. Some of these are:

- > Thickness of reefs or mineralized rocks and its corresponding accumulation.
- > Specific gravity and the strength of the rocks
- > Structural control of grades such as faulting, unconformity etc.

4.2.2. Dependency between grade domains

It is prudent to evaluate the dependence between domains caused by the spatial continuity of the deposit. In geostatistics, the samples collected must display continuity in the direction and the construction of domains may influence this spatial continuity due to the formation of geological or statistical boundaries. The process of estimating grades within domains separately implies that they are independent entities (Boulter,C. A; Fotlos M. A, and Phillips G. N, 1987).
4.2.3. Data distribution analysis

The initial step is to plot and observe the nature of the assay data using statistical techniques. Statistical tools help to enhance the interpretation of data, ensure the accuracy of data and evaluate the confidence of projections.

The first step is the construction of histograms and frequency distribution curves to determine the overall impression of the distribution of the assay. For a normal population, the arithmetic means or the median is known to be a good grasde estimator (Pyrcz, M. J and Deutsch C. V, 2014).

The histogram plot can be used to identify a bimodal distribution and outliers. Bimodal indicates the use of data with a mixed population and should be statistically separated to help produce the best grade and tonnage estimates within a deposit. Outliers are values different from the rest of the sample and raise the suspicion that they may be from a different population. Usually, the effect of outliers on variograms is a very erratic curve which is difficult to interpret. Outliers are mostly treated by capping or cutting of grades and must be accepted as a real member of the complete population (Pyrcz, M. J and Deutsch C. V, 2014).

(i) Normal (Gaussian) distribution

Normal distribution is commonly used to describe a discrete data set. It is defined by a bell-shaped curve, symmetrical around the mean, the mode and the median of the distribution (Figure 4.3). Normal curves can be applied to an unbiased histogram to indicate the probability that the variable in question is normally distributed. Its mean is denoted by μ and its standard deviation by σ . A continuous random variable x that has a normal distribution is called a normal random variable (Daly C. and Verly, G. W, 1994).



Figure 4.3: Gaussian distribution (Daly C. and Verly, G. W, 1994).

Each different set of values of μ and σ gives a different normal distribution. The value of μ determines the center of a normal distribution curve on the horizontal axis, and the value of σ gives the spread of the normal distribution curve (Daly C. and Verly, G. W, 1994).

(ii) Lognormal distribution

The distribution of variable x is said to be lognormal if the transform log [t = ln(x)] of the variable has a normal distribution.

Where ln(x) = logarithm of raw data,k= constant, t = normally distributed random quantity

It has been established that in most geological formations, assay values do not adhere to normal distribution, but rather that their logarithms appear to be normally distributed (Daly C. and Verly, G. W, 1994) (Figure 4.4).



Cumulative frequency (%)

Figure 4.4: Parameter lognormal plot as a log probability plot (Daly C. and Verly, G. W, 1994).

4.3. Ore Modelling

3D geological modeling allows for the creation of complex geological models and geological objects in the 3D environment using geological maps, geological analysis records, structural information, geophysical and geochemical data. Every point in space has X, Y, Z and one or more other attributes referring to this space point. Exploration holes drilled to intersect mineralized rocks are carefully modeled in 3D to define the shape, size, structure and size of the ore body. After comprehensive geological interpretation. The variables of interest are carefully grouped together and working sections with (hand) drawn interpretations allow for a complex understanding of geological controls and better management of future data collection campaigns (Sides E. J, 1997).

A sound geological model is the foundation for robust resource estimation, efficient mine planning and effective near-mine exploration. The main objective of orebody modelling is to estimate and predict the tonnage and grade of the ore body. There are two types of orebody modeling; explicit and implicit geological models that have developed over the last few years (Sides E. J, 1997).

Explicit modeling (conventional method) produces parts that are the goals of explicit modelling workflow. The workflow consists of digitizing the geological features of a segment and Joining these interpretations to generate a 3D model (Figure 4.5). It makes modeling at times a challenging task, as it is difficult to update a model as more data becomes available.



Figure 4.5: Explicit modelling in Surpac

Implicit modeling removes complex tasks by using algorithms to create a 3D model from data (Figure 4.6). A mathematical constraint is developed that can be used to visualize various aspects of 3D data. Most implicit models are dynamic, thus any changes to the geological parameters that gets applied automatically updates the 3D model. It therefore offers greater flexibility, reliability, efficiency and smooth idealized surfaces (Covan E.J; Beatson R.K and Ross H.J, 2003).



Figure 4.6: Implicit modelling in Leapfrog



Figure 4.7: Implicit Modeling vs Explicit modeling

4.3.1. Creating geological database

A geological database is the systemic collection of drillholes data. Drillhole data is the starting point for all mining projects. It constitutes the bases on which feasibility study and all reserve estimations are done. The vital importance of drillhole data is to determine the location of the orebody and to create a block model with constraint with accurate grade and attributes. In creating a geological database once require two tables, the mandatory and the optional table (Turner A.K, 2006). The mandatory table give information about the collar and the survey and the optional table include the assay and the geology.

The drillhole database for this project deposit contains 195 drillholes. The database is organized as a Surpac readable access file complete with logs of lithology, collar information and assay data from all sampled intervals.



Figure 4.8(a): Drillholes collars in Surpac Geomodelling Software

Drillhole samples were used for the estimation of AU content in the block model and geostatistical evaluation and grade estimation. The locations of drill hole used for estimation are shown in Figure 4.8 (a).

The (Figure 4.8 (b)) below shows a section of drillholes highlighting AU percentages and classification with a cutoff grade of =< 0.3 g/t.



Figure 4.8(b): Drillholes display according to assay value

The white portion from =<0.299 g/t are the waste zones. The ore zone is represented by the four other colors based on classification as either low, medium, high or super high grade. Low grade AU zones (0.3 to 0.75 g/t) are displayed in blue, medium grade AU zones (0.75 to 1.5 g/t) are shown in yellow, high grade AU zones (1.5 to 3.5 g/t) are displayed in plum. The super grade AU zones (>3.5 g/t) represents red (Figure 4.8(b).

4.3.2. wireframing

Wireframes are usually considered essential to geological modeling. It offers tools for constructing, handling and analyzing 3D solids and 3D surfaces for advanced exploration, resource estimation, mining and geological modelling. It uses powerful and intuitive tools to visually create 3D models, such as underground mine designs or geological representations, using data such as centerlines, profile shapes or polygons. The objective of interpretation and wireframing is to divide domains and provide each of them with a wireframe of their own from a spatial, geological and statistical point of view. It should be known that a domain can consist of one or more wireframes, these wireframes reflect a geological, statistical and spatial environment (Henley S, 1998).

Wireframe is used to calculate digitize segment grade. It refers to the string or segment representation the ore zone (Figure 4.9). The 3D wireframing method is simply an extension of the 2D drawing process, where all lines are joined manually by adding tie lines that connect nodes from one sectional polyline to another. Each sectional polyline, which are connected in three dimensions, are an ambiguous representation of a solid volume.



Figure 4.9: Wireframing in Surpac

The object is basically a wireframe. It does not have an embedded information on places within or outside the object, without these essential facts regarding the object interface positions in space, the wireframe cannot be used for two tasks which can be required for aid modelling:

- I. Selecting assay data in space (usually expressed as midpoints of assay composite intervals)
- II. Documenting the interpreted shape and volume of the orebody in 3D, which informs the percentage of ore/ lithology within a particular block in a block model.

For computer software to determine the inside and outside of this object, the connected polylines must be transformed into a triangulation surface model; that is, the surface at any set by the triangulation face becomes part of the continuous surface of the solid object to complete the above two tasks. It is important that the wireframe model is closed and that the facets of the triangulation (which are facing the directions) are all consistently aligned.

4.3.3. Building Solid model

The solid model is a three-dimensional data triangulation. It is an object can be shaped by stitching triangles around strings, which often reflect sections through the shape being modelled. A model allows you to use triangulation to create three-dimensional models based on digital terrain models (DTMs) and string files (Glacken I. M and Snowden D.V, 2001).

A solid model is created by creating a set of triangles from the points in a string. These triangles may tend to overlap when you are in the plane view, but they do not overlap or intersect when the third dimension is considered.



Creating a solid model step by step process

Step 1:

Choose Solids > Triangulate > Between segments.

Enter the information as shown and click Apply.

Define	the trisolation to be	e created	×		
Function	TRIANGULATE AUTO	MATIC			
Layer name	ore_sectioning_et_di	gitising.str			
Object	t 8				
Trisolation	1				
0		Apply	X Cancel		

Figure 4.10: Defining the trisolation type in Surpac

You are prompted to Select a point on the first segment to be triangulated.

You are prompted to Select a point on the next segment to be triangulated.

Continue using the Between segments function to conclusion.

Press ESC.

The part of the solid created using triangulate between segments is displayed



Figure 4.11: Triangulate between segments in Surpac

TICK Triangulate end segment Direction to copy segment > forward Copy segment by > perpendicular to segment plane Distance > 50 Slope > 90



Figure 4.12: The ore model unvalidated

4.3.4. Model validation

Solid validation must have a geological context, free the model from defects, acceptable as a solid, grade distributions inside the geological domain and should reveal a precise estimation when displaying a normal distribution (Figure 4.14). If the data distribution is lognormally distributed, more data treatments can be used to normalize it as this avoids the squashing of high or low data values (grades). The best validation methods are the comparison of the estimated data with the production data, high correlation between these sets of data increases the level of confidence in relation to the precision of the resource model (Cowan E.J, 2002).

Validate objects and trisolations					
Layer n	name	ore.str			
Report	file	ore_valid	-		
Object	Range				
Trisolat	ion Range				
Attemp	t to autocorrect proble	ms			
Set vali	id objects to	Solid Ovid			
Colour	s for highlighting invalio	l triangles			
	Problem	Highlight colour			
1	Holes	GREEN			
2	Intersecting triangles	CYAN			
3	Invalid edges only	ORANGE			
4	Open edges	MAGENTA			
5	Reversed triangles	RED			
			-		
		🖌 Apply	Cancel		

Figure 4.13: Validation of an Ore model



Figure 4.14: 3D model modelled in Surpac



Figure 4.15(a): 3D model modelled in Surpac with drill holes intersecting the ore model



Figure 4.15(b): 3D model modelled of the strings inside the ore model

4.4. Sample Data analysis

The drill hole spacing was 25 m x 25 m with a mix of Diamond Drill (DD) and Reverse Circulation Drilling (RC) samples. Diamond drilling was mostly used for exploration. At an angle of 50 to 80 degrees downhole, all the holes were drilled to intersect the orebody at a predetermined spacing of 25 m along strike and dip of the deposit. Figure 4.16 shows the stepwise approach used for this study.



Figure 4.16: Phases in mineral resource Estimation

4.4.1. Sample selection and statistical analysis

The drillhole data in excel format (.csv) was converted to string files (.str) and data values which were used for further statistical analysis. To ensure that the samples for estimation represent equal volume (support), downhole compositing was done to ensure that each sample represents the same length at every 1m. It is evident that more than 90% of the samples were taken every 1m in length. The raw length descriptive statistics of the geological data is shown in table 2. The Assaying was done by using the fire assay standard technique. This technique separates metal concentrates from impurities with the aid of heat and dry

Table 2: Descriptive statistics	of the geological data
---------------------------------	------------------------

Variable	Gold
Number of samples	8599
Minimum value	1
Maximum value	2.287309
Mean	1.06983
Median	1.0306
Geometric Mean	1.064864
Variance	0.013258
Standard Deviation	0.115145
Coefficient of variation	0.107629
First quatile	1.00473
The median quatile	1.0306
The third quatile	1.419129

Alternatively, further investigation was done to observe if the thickness and accumulation will be good variables for domain definition. It was later abandoned since it is evident that the project area is a hard boundaries domain. The composite inside the ore body was used for the geostatistical estimation (Table 3)

Number of	Total	Minimum	Maximum	Mean	Standard deviation	Variance
samples						
19376	29031.73	1.13	1.50	1.50	0.020	0.000

Table 3: Composite sample length statistics

4.4.2. Data distribution analysis

The normality test was performed by plotting the histogram and the probability plot to know the type of distribution that follows the data and the distribution of the data values. The composite samples were used, and the distribution was skewed positively. Therefore, it was necessary to separate data into homogeneous domains. The probability plot test for normality showed that it did not come from a distribution of the same type because the points departed substantially from the line pattern. Data distribution analysis (a) Histogram plots of sample data (Figure 4.17 to 4.20).





Figure 4.17: Histogram plot of samples data (A)



Figure 4.20: Top Cut Analysis for the samples data

The domain distribution of the gold mineralization plotted was characterized by positively skewed distribution as is expected for gold. This implies that the sample population contains many low values to medium grades values and relatively very few high values. The positive skewness is confirmed by the histogram of gold grades for the ore zone.

Many problems arise when dealing with skewed distributions and a more preferred distribution is the normal or the Gaussian distribution. This is because there are many values at the opposite side of the tail for a skewed distribution. This discrepancy causes an imbalance in the frequency of sample values thereby rendering the estimation biased. Thus, the assay values for gold followed a lognormal distribution and the original data values from the respective domains were transformed by finding a logarithm of the gold value (au) (Davis J. C, 1986).

This is expressed mathematically as:

AUnormalise = ln (au) where, In is natural logarithm Au value of gold in grams per tonne (g/t)

4.4.3. Removal of outliers

Treatment of outliers in mineral resource estimations is a perplexing problem for which there is no generally accepted solution. Each deposit may have a unique distribution of outlier values which may require multiple methods of treatment to fully understand the issues and their effect on the mineral resource estimate. If good sampling and QA/QC protocols have been followed (i.e. good results in high grade duplicates) and the geological description of these intervals matches with the high grade value. There is sufficient close spaced sampling to limit the influence of the high grade in the estimation with no theoretical or scientific ground to cap these outliers (Arik A, 1992). Data values that were somewhat distant were removed in the domains to produce a robust variogram for statistical analysis

Some commonly used techniques to identify possible outliers:

Histograms or Log probability Plots - It is easy to detect outliers from a sample grade distribution or histogram: they are separated from the rest of the distribution by gaps. Note gaps on a histogram with a log scale are more significant than gaps on a histogram with an arithmetic scale. However, we did not detect an outlier in our data distribution this indicates that the samples were correctly logged.

4.5 Variogram Analysis

Variography is the analysis of spatial variability of grade within a region. Gold deposits have high spatial variability between samples. Understanding how sample grades relate to each other in space is a vital step in informing resource estimation. The variogram characterizes the spatial continuity or roughness of a data set. Ordinary omi-dimensional variogram statistics for two data sets may be nearly identical, but the spatial continuity may be quite different. A variogram is used to quantify this spatial variability between samples. In estimation, the variogram is used for: selecting appropriate sample weighting in Kriging and RBF estimators to produce the best possible estimate at a given location and calculating the estimators' associated quality and diagnostic statistics (Calder C.A; Cressie N, 2009). Variogram analysis consists of the experimental variogram calculated from the data and the variogram model fitted to the data.

The experimental variogram is calculated by averaging one-half the difference squared of the z-values over all pairs of observations with the specified separation distance and direction. The variogram model is chosen from a set of mathematical functions that describe spatial relationships. The appropriate model is chosen by matching the shape of the curve of the experimental variogram to the shape of the curve of the mathematical function. Mathematical function is defined as the variance of the difference between field values at two locations and across realizations of the field (Cressie, 1993).

$$2\gamma(x,y) = \operatorname{var}(Z(x) - Z(y)) = E\left(|(Z(x) - \mu(x)) - (Z(y) - \mu(y))|^2\right).$$
$$2\gamma(x,y) = E\left(|Z(x) - Z(y)|^2\right),$$

Where $\gamma(x, y)$ itself is called the semi-variogram. In the case of a stationary process, the variogram and semi-variogram can be represented as a function $\gamma_s(h) = \gamma(0, 0 + h)$ of the difference $h = y - x_1$ locations only, by the following relation (Cressie, 1993).

$$\gamma(x,y) = \gamma_s(y-x).$$

If the process is furthermore isotropic, then the variogram and semi-variogram can be represented by a function $\gamma_i(h) := \gamma_s(he_1)$ only (Cressie, 1993).

$$\gamma(x,y) = \gamma_i(h)$$

Where" h" is called lag distance.

4.5.1. Variogram calculation

The maximum lag distance is initially taken as the full span of the boreholes (i.e., 650 meters). Then from the experimental variogram the effect of the lag distance on the number of pairs of boreholes is studied. It is observed that with lag distance greater than 70 meter the number of borehole pairs are either zero or comparatively much less.

The best suited experimental variogram is selected based upon the following criteria:

- It should be smooth.
- The variance should be minimum.
- Range should be maximum for a given sill. •

Now whichever set of lags; bearing, azimuth and dip of search ellipsoid satisfies the above three criteria is selected to construct the experimental variogram.

In this project different lag distance value is 15 was used to construct the variograms. The generated variogram files are presented through the following figures (Figures 4.21 - 4.25), respectively.





Figure 4.21: Horizontal Variogram model.



Figure 4.23: Variogram model for an azimuth 45 degree



Figure 4.24: Variogram model for an azimuth 90 degree



Figure 4.25: Variogram model for an azimuth 135 degree

The semi-variogram was used to measure the spatial trends in the data. The composited geological data was imported into Surpac for geostatistical analysis. Grades were not estimated for the waste domain and therefore no variograms were computed for it.

Number of samples	13239
Mean	2.398275
Variance	47.941463
Standard Deviation	6.923977

Table 4: Statistics summary of the variogram analysis

Azimuth	Dip	Spread	Spread Limit	Nugget (Co)	С	Range
0	0	22.5	200	0.231	0.53	19
0	0	90	200	0.39	0.12	22.6
45	90	22.5	10	0.21	0.91	7.9
0	-90	22.5	8	0.22	0.89	7.8
135	15	22.5	10	0.20	0.48	6.4

Table 5: Statistics summary of the variogram analysis

The semi-variograms for downhole samples were also generated with Co in the range between 0.20 to 0.39 g/t (Table 5). Semi-variograms were generated from the composite string files and the lag sizes were generally kept close to the drill spacing of 25 m to capture sample variance. The resulting semi-variogram is usually erratic if lag distances are not close to sample spacing.

The orientation used for the variogram computation is specified together with a summary of the data values masked for variogram calculation. The longest direction of continuity was generally along the eastings(X) (rotated to match the strike of mineralization) with the intermediate direction of continuity along the Y axis (rotated to match the down dip direction of mineralization). Ranges along strike (direction of maximum continuity) were shorter because of the continuity of the ore body and the less variability between grade values.

4.5.2. Validation of variogram

Cross validation allows us to compare estimated and true values using only the information available in the sample data set. Thus, it gives the statistics of kriging errors. The estimation of unsampled locations depends heavily on the semi-variogram model as the representation of the true spatial structure for that measurement in that area. The kriged estimates were cross validated to justify the precision of the semi variogram model and the estimated block grades were compared to the true grades in this process (Figure 4.26)



Figure 4.26: Distribution of Kriging errors

Table 6: Summary	statistics of	of Kriging	errors
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True Value	Estimated value	Error	Std_dev	Mean	Avg Krig variance
2.146	7.237	5.0912	0.7405	0.0045	0.6089

The true grades (actual sample measurement) in this case were analogous to the composite grades because grade compositing was done every 1 m to reduce spatial variability (Table 6).

CHAPTER FIVE: BLOCK MODELING AND RESOURCE ESTIMATION

5.1. Block modeling

A block model is a simplified representation of an ore and its surroundings, which you can think of as a stack of computer-generated "bricks" that reflect small volumes rock at the deposit (ore and waste). Each "brick" or cell contains the "brick" estimates of data, such as grade element, density, etc, values of geological. Ore body model is later reconstructed to form block models that are spatially georeferenced and divided into fixed size blocks (Abzalov M.Z, 2006).

The block model is filled with geological attributes and serves as a model for grade introduction. The geometry of the block model depends on the characteristics of the deposit, the geographical highlights being modeled, mine planning necessities, such as equipment size and type to be utilized by the operation. Block size and geometry is an important decision in resource modelling (Stephenson P, 2015).

A model framework describes the rectangular area of the space inside which the cells of the model are located. It needs to be an origin distance for each axis. Angle of rotation within this framework are individual blocks with a designated length (X-increment), width (Y-increment), and height (Z-increment) (Figure 5.1). The block position may be defined by a centroid (Xc, Yc, Zc), or a block origin (Xmin, Ymin, Zmin) (Stephenson P, 2015).



Figure 5.1: Block model block definition (Stephenson P, 2015).

In order to enhance model boundaries within the model space, blocks may be subdivided into smaller cuboid sizes (or rectangular prisms), known as sub-blocks or sub-cells, while retaining the storage and computational efficiency of the standard block model. Subshells are typically stored separately from the parent blocks. The flexible approach allows the subdivision to vary based on the angle of intersection of a specific block with a boundary surface that controls the subdivision. The subdivision is infinitely variable, allowing for a better volumetric understanding of the boundary surface and producing less blocks at the same degree of accuracy as the octree process be required.

The model should be broad enough to cover the full spectrum of input data, most of which consist of one or more wireframes and drilling databases. Set the minimum coordinates of the starting blocks-the lowest

angle, the south-west of the model, and the maximum block coordinates-the highest angle to the north-east of the model to calculate the scale of the block model.

The Sub-cells are not only useful for the accurate construction of a 3D grade model for the ore body, but also for the identification of continuous sensitive Au-grade variations in a single ore body.

To select the size of the block model, we should also consider several factors, such as:

- The size of the blocks depends on the characteristics of the source,
- The size of the forms of bodies,
- Type of ore,
- Zonality,
- Length of sample analysis,
- Based on the cross-section view of the geometric shape of the bodies (such as width and height)

5.1.1. Block modelling concepts

The block model is a form of spatially referenced database that provides a means to model a 3D body, such as drillhole sample data, from point and interval data. The block model consists of interpolated values instead of real measurements (Figure 5.2). It provides a method for estimating volume, the tonnage and the average grade of the 3D body are based on the drillhole data.



Figure 5.2: Filled block model

5.1.2. Building the block model

To achieve accurate estimation and eliminate misclassification, choosing an acceptable block size is extremely necessary. The optimal block size for estimation is mainly a function of drillhole spacing. A quarter of the spacing of the drillhole could be appropriate if the deposit has consistent mineralization and a small nugget. Block models tend to be generated using block sizes which are larger than the anticipated mining selectivity unit (SMU). Change of support is the process of adjusting the block model estimates so that the results reflect the expected grade tonnage relationship at the anticipated SMU. Also, to quantify the

effectiveness of the estimation process at a variety of block sizes, statistical optimization can also be performed. This may provide useful support for the selection of block sizes. However, it should always be tempered with reality and practical considerations. Once an appropriate block size has been chosen, constructing the block model is the next step. The block model is commonly referred to at this point as a volume model because it specifies the volumes of the approximate domain but contains no grade information. It is important to set up the volume model to cover the total area of interest. For example, the mine design process may require an expanded area around the mineralization to allow for pit walls and/or dilution. (Figure 5.3(A and B)).



Figure 5.3(a): Display of constrained block model Block Model Sub-celling of a block mode in surpac



Figure 5.3(b): Display of constrained block model Block Model Sub-celling of a block mode in surpac

	Y	X	Z
Minimum Coordinates	4975.1	4998.023	1462.573
Maximum Coordinates	6430.1	5793.023	2162.573
User Block Size	15	15	5
Minimum block size	15	15	5
Total Blocks		55531	
Au	Gold		
Density	2.7		

Table 7: Block model summary

Block modelling is a model for the variation of ore quality in the solid model. For this purpose, solid model is divided into blocks, and the average grade value of each block is calculated by the interpolation method or simulation method. Block sizes and their shapes may be variable as they can be fixed. Block grades can be estimated by the methods like nearest neighborhood, inverse distance weighting and geostatistical methods. This workflow that demonstrates the steps to accurately achieve to a block model (Figure 5.4).



Figure 5.4: Step for building a block model

5.2. Grade interpolation

A density of 2.7 was used for tonnage calculation. This may deviate a little from the actual value but was still maintained for tonnage calculation.



Figure 5.5: Blockmodel of the project area colored by grade range IDW estimation (Black dots show the graphic workspace).



Figure 5.6: Blockmodel of the project area colored by grade range OK estimation (Black dots show the graphic workspace).



Figure 5.7: Constrained block model colored according to AU grade 3D block model of the research work with IDW estimation



Figure 5.8: Constrained block model colored according to AU grade 3D block model of the research work with OK estimation



Figure 5.9: The 3D block model colored according to AU grade and drillholes interesting the Model with inverse distance estimation



Figure 5.10: The 3D block model colored according to AU grade and drillholes interesting the Model with OK estimation

5.3. Cut-off grade

Cut-off grade and financial optimization according to the SAMREC Code of the mineral reserve is that portions of the mineral resource, which is valuable, legally, economically and technically feasible to extract (SAMCODE, 2009). A cut-off grade of 0.3 g/t Au was selected for this project. The commonly accepted method for determining if material forms part of the mineral reserve is to calculate a cut-off grade at the current economic conditions (Table 8). The only material above this grade is then considered to have

economic value and included in the mine plan (Figure 5.11). The material with a grade lower than the cutoff grade remains in the resource; although with rising commodity prices and/or lower mining costs, it may later be included in the mineral reserve.

Grade range	Attribute	Volume	Tonnes	Average	Density
		(m3)		Grade (g/t)	
$0.0 \rightarrow 0.3$ (Waste)	Au	769500	2077650	0.301	2.7
0.3 -> 0.75 (LG)	Au	2908125	7851937	0.648	2.7
0.75 -> 1.5 (MG)	Au	3709125	10014637	1.165	2.7
1.5 -> 3.5 (HG (a))	Au	3426750	9252225	2.328	2.7
3.5 -> 999.0 (HG (b))	Au	2020500	5455350	6.901	2.7
Grand Total	Au	12834000	34651800	2.210	2.7

Table 8: OK estimate of the project



Figure 5.11: Grade-tonnage curve

Every orebody is unique and thus should be considered individually for the determination of cut-off grades. The grade-tonnage curve is then automatically genre steps involved in the generation of the geostatistical resource estimate as a representative example of the deposit.

Risk Assessment

The evaluation of the risk associated with the various processes and steps involved in generating the geostatistical resource estimate. This is in line with the current JORC Code published in 2012

a) Integrity of databases (Low risk)

The company provided the drillhole database for this project and all database validation checks were audited and passed.

b) Geological Interpretation (Low risk to moderate risk)

Except for areas disturbed by faults, geological stability is good. Similar models were seen by an overall exploration (from mapping and literature) to grade control, and this significantly assisted in the description of geological domains prior to geostatistical estimation.

c) Techniques of estimation and modeling (Low risk)

For the deposit, semi-variograms were modelled. For estimation, OK and IDW were used as the number of assay data available was adequate to produce an accurate estimate.

CHAPTER SIX: CONCLUSION AND RECOMMENDATION

6.1. Conclusion

- \checkmark The original data sample values have a high concentration of medium grade values.
- ✓ The orebody's geological understanding aided in the description of geological domains. Domains three and four were partitioned by a normal fault.
- ✓ In mineral resource estimation, statistical analyses are important, and an effective data analysis can result in an unbiased estimate and smoothed grade interpolation.
- ✓ Data gain from the nature of the grade distribution help in the selection of an effective estimation model.
- \checkmark IDW and OK estimation methods were used to test the total block model estimates.
- ✓ The total tonnage for the deposit was 34651800 tonnes, volume is 12834000 with an average grade of 2.210 g/t as estimated using OK.
- ✓ The overall cutoff grade is 0.3 g/t.

6.3. Recommendation

- ✓ Mineral resource estimation should always take place within homogeneous domain boundaries in order to boost estimates.
- ✓ The technique used in this research project should be applied as a standard operating procedure for other areas of the deposit during geostatistical analysis.
- ✓ Mineral resource estimates should be compared to mine output data for reconciliation purposes and to measure the degree of conditional bias.
- ✓ Another evaluation of block size optimization is needed. This will assist in deciding the best block size for estimating mineral resources. To assess the degree of confidence of block estimates, other estimation techniques such as Simple Kriging and Indicator Kriging can be used.

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