DEVELOPMENT AND MANUFACTURE OF A DELIVERING UAV
DEVELOPMENT AND MANUFACTURE OF A DELIVERING UAV

Thesis presented to the Graduate Program in Mechanical Engineering at the Polytechnic School of the University of São Paulo, Brazil and the Polytechnic University of Turin, Italy to obtain the dual degree in Master of Science.

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Mechanical Engineering

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Turin
2021
ACKNOWLEDGMENTS

This thesis is ultimately a result of the belief of multiple people I have had the pleasure to make connections with.

I would like to thank Aloisio Neto, Ana Paula Manfrinatti Neuenschwander, André Marra, André Pimenta Mathias, Célia Moreira de Almeida, Eckart Spielkamp, Eduardo Naoki Akiyoshi Ichige, Fábio Alves Tomaz, Fernanda Cirillo, Fernando Paiva, Flávia Cristina Escobar Santana, Gabriela Soares, Gustavo Rosolen, Jéssica Gomes, Kristiane Silveira Fernandes, Lucas de Oliveira Fracarolli, Luiz Fernando Portella Staub, Marcelo Seraphim, Miyuki Ichige, Nanci Saraiva Moreira, Paulo Macedo, Rafael Herrera, Rafael Nass de Andrade, Rubens Mario Marques de Freitas, Thiago Falcone e Tunico Barros that gave me not just the financial support, but the motivation to cross the ocean pursuing a diploma on Europe.

Thanks to Ana Virginia Kesselring and the Virginia Center that not just did help improving my English skills, but built the confidence on which later made me learn also Italian.

Thanks to Beatriz Lucca and Felipe Sereno that provided me friendship, support, but more than that, made sure everything would run smoothly while I was abroad.

Thanks to Elvis and Felipe for providing me so many laughs and good times and for helping me out in times of crisis.

Thanks to professor Dr. Edilson Hiroshi Tamai that besides teaching me the technicalities of Mechanical Engineering, help me turning ideas into reality.

Thanks to Felipe Labate, that put me on contact with so many people that would help me in so many ways. Thank you for the long phone calls, for passing your experience to me and for motivating me when I was about to departure for the exchange program.

Thanks also to my mentor, friend, sponsor and second mother, Solange Cunha, for the numerous calls, meetings, planning and the patience to endure me. This would not be possible without you! It was not a second easy, but this is not longer a dream!

Thanks to all the friends that I had the pleasure of living with during my masters: Mário, Pedro (PG), Arthur, Thiago, Gilberto, Daniel, Fabrizio, Guilherme, Marcelo, Felipe and Tiago, I am glad to call you brothers.

I am thankfully also to have had the opportunity to be a member of the Association des États Généraux des Étudiants de l’Europe (AEGEE), my big family in Europe, where I’ve learnt so much and had the luck of meeting my partner Dorottya, who I am also thankful for supporting me all the way through this work.

Also, special thanks for my advisors Prof. Carlo Novara and Prof. Dr. Roberto Spinola Barbosa for the numerous reviews and feedback throughout the work.

Last but not least, thank to my family for being my rock and to give me everything any person could ask for. Mom and Dad, you are true heroes!
"Que Deus me guarde, pois eu sei que ele não é neutro
Vigia os rico, mas ama os que vem do gueto"

Racionais MC's
ABSTRACT

This thesis presents the steps for the development of a multicopter of the type quadrotor for delivering usage by means of semi-autonomous techniques. After a brief introduction of the state of the art for the UAV (Unmanned Aerial Vehicles) technology, the equations of motion using Euler’s angles approach are presented. In sequence, after a review of the propulsion components available on the market, a method for choice of the best fit of commercial components, and therefore dimensioning, is presented. Following, the control approach is introduced as the MATLAB simulation and results. The last part of the work focuses, on the manufacture of the airframe using additive manufacturing processes and on the final assembly of the UAV. Lastly, suggestions for next works are summed up and presented.

Keywords – UAV, Quadrotor, Control, Propulsion Optimization, Autonomous Robot.
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1 INTRODUCTION

The use of Unnamed Aerial Vehicles (UAV), popularly known as drones, have been increasing over the years. Their application covers a wide spectrum of possibilities and is helpful where its use is an advantage in terms of safety and productivity. The Amazon Prime Air, which aims to deliver products by using drones [1], is one of many examples of its potential.

Following this line, the purpose of this thesis is the development and manufacture of an UAV for the transport of goods in low scale. On the next pages, the steps for the construction of such a robot are presented from the introduction of the equations of motion to the manufacture of the airframe, passing through the control and choice of best fit of commercial components.

Before stepping into the content of the work itself, let’s take a quick look in the general concept of flight of a multicopter. For example, for the quadrotor, a multicopter with 4 rotors, the movement is achieved through the rotation of propellers connected to rotors. Each rotor generates moment and thrust. Considering figure 1, in order to have hover, the moment in relation to the geometric center generated by the thrust on the rotor #1 should cancel the moment generated by the thrust on the rotor #2. Similarly, the moment caused by the thrust on #4 should cancel the moment caused by the thrust on #3. Also, the sum of all reaction torques pointing outwards the paper on the figure 1 should be null.

Any unbalance of thrust or reaction moment causes the quadrotor to move. For example, if an yaw movement is desired, the sum of the reaction torques pointing outwards the paper need to be different than zero, making the quadrotor yaw. If, for example, the quadrotor is desired to move to the right on the figure 1, the thrust on #3 overcomes #4 and similarly, the thrust on #2 overcomes #1 by the same amount. This unbalance, makes the quadrotor to tilt and move to the right. Any other movement, can be similarly achieved by the actuation on the rotors.

As it will be present later, the thrust and the reaction moments change with the
angular speed of the rotors.

1.1 Objectives

The main goal of this work is the construction of a functional prototype that simulates the transportation of goods. In order to achieve such goal, the work will be divided into 5 main parts all interconnected. Those parts are:

- **Development of Equations of Motion**: In this step the mathematical model for the UAV will be defined. This first part will be fundamental when implementing the control method.

- **Components Optimization**: In this part several commercial components for the propulsion system will be further analyzed and a method for the definition of the best set of commercial components will be presented. The main dimensions of the UAV are therefore determined in function of the best fit of components chosen.

- **Control and Simulation**: with the mathematical model, the size of the quadcopter and its inertial properties, the control model can be defined and simulations can be performed before the real flight take place. The first tuning of PID parameters realized on this stage might be modified according to the flight performance obtained during eventual tests.

- **Airframe Manufacture**: With the dimensions of the UAV, the airframe can be designed and simulated through FEM analyses. Then, the manufacture process itself can be started. Considerations about the manufacturing method in order to increase mechanical properties and keep vibrations responses below safe flight conditions will be presented as well.
2 DYNAMIC EQUATIONS OF MOTION

2.1 Attitude Representation

2.1.1 Coordinate frames

Firstly, the two coordinate frames to be used along the development of the dynamic equations of motion are defined. As shown on figure 2, the coordinate frame $E_{e_1e_2e_3}$ is called Earth-fixed Coordinate frame (Earth) and is the inertial frame of reference. The frame described by $B_{b_1b_2b_3}$ is called the Aircraft Body Coordinate frame (Body) and is fixed to the multicopter [3]. The following versors are used for those frames:

- Earth: $\{ e_1 \ e_2 \ e_3 \}$
- Body: $\{ b_1 \ b_2 \ b_3 \}$

![Figure 2: Coordinate frames](image)

Adapted from:[3]

As one can easily notice from figure 2, both coordinate frames are hand-side oriented.
2.2 Euler’s Angles

In order to obtain the equations of motion, the multicopter will be considered as a quadrotor (4 rotors) moving in X-configuration as in figure 3. The origin of the right-hand oriented Body is coincident with the the center of mass of the quadrotor (figure 3).

Figure 3: Aircraft Body Coordinate frame (Body)

The angles $\theta$, $\phi$ and $\psi$ are called pitch angle, roll angle and yaw angle, respectively. The positive direction of such angles are shown on the figure 3 and coincide with the positive direction of the Body axis. The Euler’s Angles representation used is the ZXY, obtained by a sequence of rotations on the following order: rotation $\psi$ around $z$, rotation $\phi$ around the new $x$ axis after the previous rotation; and rotation $\theta$ around the new $y$ axis after the previous rotation. The rotation matrices are described as follow:

$$R(y, \theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$R(x, \phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}$$

$$R(z, \psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
The resultant matrix $R$ is defined as

$$R = R(z, \psi)R(x, \phi)R(y, \theta)$$

(2.1)

In order to simplify the notation, the following convention will be adopted: $\sin x := sx$ and $\cos x := cx$. Therefore, the matrix $R$ results:

$$R = \begin{bmatrix}
c\psi c\theta - s\psi s\phi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\
s\psi c\theta + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\theta s\phi c\psi \\
-c\phi s\theta & s\phi & c\phi c\theta
\end{bmatrix}$$

(2.2)

By pre-multiplying a vector described in the Body for the rotation matrix $R$ (2.2), the vector described in the fixed frame (Earth) is obtained.

Such results show that from the Euler Angles $\theta$, $\phi$ and $\psi$ the rotation matrix can easily be obtained. Now, the inverse problem would be further studied: how to determine the Euler’s Angles from the rotation matrix $R$. In order to solve this, let’s assume that the rotation matrix $R$ is known and is represent by the following notation:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33} \end{bmatrix}$$

(2.3)

As mentioned before, the positive direction of the Euler’s angles are shown in figure 3.:

1. **Pitch angle**: the value range for $\theta$ is $[-\pi, \pi]$;
2. **Roll angle**: the value range for $\phi$ is $[-\pi/2, \pi/2]$;
3. **Yaw angle**: the value range for $\psi$ is $[-\pi, \pi]$.

from (2.2) and (2.3) the following system of equations can be easily obtained:

$$\begin{align*}
\phi &= \arcsin(r_{32}) \\
\theta &= \text{atg2}(\frac{r_{12}}{\cos \phi}, \frac{r_{13}}{\cos \phi}) \\
\psi &= \text{atg2}(\frac{r_{22}}{\cos \phi}, \frac{r_{32}}{\cos \phi})
\end{align*}$$

(2.4)

Here, the $\text{atan2}$ function is used over the $\text{atan}$ since the sign of the angle matters.
from the system of equations (2.4) \( \phi \) is well defined on the interval \([-\pi/2, \pi/2]\). However, when \( \phi \) is equal to \(-\pi/2\) or \(\pi/2\) the cosine is null and both \(\theta\) and \(\psi\) are undefined. This is know as the singularity problem and is the main drawback of the Euler representation.

2.3 Newton’s Second Law

The Newton’s Second Law can be written as

\[
f = \sum_{i=1}^{N} f_i = m \frac{d^E v^e}{dt}
\]  

(2.5)

where \(v^e\) denotes the linear velocity of the referential of the Body with respect to the Earth expressed in the Earth referential, \(f_i\) is the force applied on the \(i\) -th rotor, and \(m\) is the total mass of the quadrotor. Furthermore, the overscript \(E\) indicates that the derivative is taken in relation to the Earth. The \(N\) on the sum symbol indicates the number of rotors and, as discussed before, in the quadrotor case is equal to 4.

Alternatively, the Second Law can be expressed as

\[
f = \frac{d^E L}{dt}
\]  

(2.6)

where \(L\) is the Linear Momentum. The expression (2.6) states that the sum of the forces acting on the body equals the rate of change of the linear momentum.

The figure 4 shows the free body diagram of the quadrotor. The gravity field (not shown on the figure) points on the negative direction of \(e_3\). The position vector of the Body in relation to the Earth described in the Earth is the following

\[
r(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}
\]  

(2.7)

Then, by considering the equation (2.5) and the free body diagram on figure 4, one have

\[
m\ddot{r}(t) = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ -(f_1 + f_2 + f_3 + f_4) \end{bmatrix} + f_{aero}
\]  

(2.8)

where \(f_{aero}\) represents further aerodynamic forces that will be introduce on section 2.5.
2.4 Euler’s Equations

The Euler’s Equations describe the rotational dynamics and can be written as

\[ \tau_C = \frac{d^E H_C}{dt} \]  \hspace{1cm} (2.9)

where \( \tau_C \) is the vector of moments about \( C \), \( H_C \) is the Angular momentum of the center of mass. The expression of the angular momentum is

\[ H_C = J_C \cdot \omega^B \]  \hspace{1cm} (2.10)

where \( \omega^B \) is the angular rotation of the Body with respect to the Earth expressed in the Body and \( J_C \) is the inertia matrix of the quadrotor expressed on the Body and described as

\[
J_C = \begin{bmatrix}
J_{xx} & J_{zx} & J_{zz} \\
J_{zx} & J_{yy} & J_{zy} \\
J_{zz} & J_{zy} & J_{zz}
\end{bmatrix}
\]  \hspace{1cm} (2.11)

At this point, the following assumptions are adopted: (1) \( e_1, e_2 \) and \( e_3 \) are axis of symmetry of the quadrotor (figure 5); and (2) the origin of the Body coincides with the Center of Gravity (CoG) of the quadrotor. from those assumptions equations (2.9) and (2.11) can be further simplified into:

\[ \tau_C = \frac{d^B H_C}{dt} + \omega^B \times H_C \]  \hspace{1cm} (2.12)
and

\[ J_C = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \]  \hspace{1cm} (2.13) \]

The angular velocity can be written in a more convenient notation as

\[ \omega^B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}^T \]  \hspace{1cm} (2.14) \]

from figure 4, one can easily obtain

\[ \tau_C = \begin{bmatrix} \frac{dv^2}{2} (f_1 - f_2 - f_3 + f_4) \\ \frac{dv^2}{2} (f_1 + f_2 - f_3 - f_4) \\ -\tau_1 + \tau_2 - \tau_3 + \tau_4 \end{bmatrix} \]  \hspace{1cm} (2.15) \]

Then the equation (2.12) can be rewrite as

\[ J_C \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{dv^2}{2} (f_1 - f_2 - f_3 + f_4) \\ \frac{dv^2}{2} (f_1 + f_2 - f_3 - f_4) \\ -\tau_1 + \tau_2 - \tau_3 + \tau_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times J_C \begin{bmatrix} p \\ q \\ r \end{bmatrix} \]  \hspace{1cm} (2.16) \]

Figure 5: Quadrotor X-configuration

Source:[3]

2.5 Quadrotor Aerodynamics: Thrust and Moments Model

The rigid body equations (2.8) and (4.31) of the quadrotor are here summarized:
\[ m\ddot{\mathbf{r}} = mge_3 + R\mathbf{f} + \mathbf{f}_{\text{aero}} \]  
\[ J \cdot \dot{\omega} = \tau - \omega \times J\omega \]  

(2.17)

where \( \mathbf{f} \) and \( \tau \) combine the main nonconservative forces and moments applied to the quadrotor due to the aerodynamics of the rotors [4], while \( \mathbf{f}_{\text{aero}} \) is the contribution due to the blade flapping phenomenon, that can be neglected if comparing with the other forces.

In the hover case, the steady-state thrust generated by the rotor [3] can be modeled as

\[ f_i = C_T \rho A_r i r_i^2 \omega_i^2 \]  

(2.18)

where \( C_T \) is the thrust coefficient dependent on the geometry of the propeller, \( \rho \) is the air density, \( r_i \) is the radius of the \( i-th \) propeller, \( A_r \) is the circumferential area defined by the radius \( r_i \), and \( \omega_i \) is the angular speed of the \( i-th \) propeller (rotor) measured in \( \text{rad/s} \). Similarly, the reaction torque due to the rotor drag can be modeled as

\[ \tau_i = C_r \rho A_r i r_i^3 \omega_i^2 \]  

(2.19)

where \( C_r \) is the thrust coefficient dependent on the geometry of the propeller. Considering the variation of the air density on the altitude range of the quadrotor negligible for the evaluation of the forces and moments, the following constants can be defined

\[ c_T = C_T \rho A_r i r_i^2 \]  

\[ c_r = C_r \rho A_r i r_i^3 \]  

(2.20)

Combining (2.18), (2.19) and (2.20), and from figure 4 and 5 the following matrix system is obtained

\[
\begin{bmatrix}
\mathbf{f} \\
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix} =
\begin{bmatrix}
c_T & c_T & c_T & c_T \\
dc_T \frac{\sqrt{2}}{2} & dc_T \frac{\sqrt{2}}{2} & -dc_T \frac{\sqrt{2}}{2} & -dc_T \frac{\sqrt{2}}{2} \\
-dc_T \frac{\sqrt{2}}{2} & dc_T \frac{\sqrt{2}}{2} & dc_T \frac{\sqrt{2}}{2} & -dc_T \frac{\sqrt{2}}{2} \\
c_M & -c_M & c_M & -c_M
\end{bmatrix}
\begin{bmatrix}
\omega_1^2 \\
\omega_2^2 \\
\omega_3^2 \\
\omega_4^2
\end{bmatrix}
\]  

(2.21)

The matrix system (2.21) obtained by a quadrotor can be easily extended for a general case of more rotors. for further details consult [3].

Given the desired thrust and moments, the matrix form allows to solve for the required rotor speeds.
2.6 Summary

This chapter aimed to present the dynamic equations of motion for a quadrotor. By introducing the Euler’s angles representation, the rotation matrix to transform the vector on the Body space into the Earth (inertial) one is obtained. Moreover, the aerodynamic characteristics of the non-conservative forces and moments are presented and the concept of blade-flapping is briefly introduced.

The Dynamic Equations of Motion obtained are here summarized

\[
\begin{align*}
    m\ddot{\mathbf{r}} &= mg\mathbf{e}_3 + R\mathbf{f} \\
    J \cdot \dot{\omega} &= \tau - \omega \times J\omega
\end{align*}
\]

Such equations are a necessary step toward the control of the UAV and will be further discussed throughout the project.
3 OPTIMIZATION OF PARAMETERS FOR COMPONENTS CHOICE

3.1 Propulsion System Modeling

The modelling of the propulsion system is an important initial step towards the design of the UAV and will follow the modelling methodology presented in [3].

The propulsion system is compounded by a number of propulsors, motors and electronic speed controllers that are equal in number, connected to a centralized source of energy. Each of those components will be modelled in order to evaluate the performance of the propulsion system in some particulars flight conditions.

Additionally, the airframe model will be discussed as well, and first expressions for its mass and its surface area will be derived.

3.1.1 Environmental Parameters

Before proceeding to the model of the propulsion system itself, the environmental parameters are first analyzed. The main atmospheric parameter to be considered is the density of the air \( \rho \) [kg/m\(^3\)], which is function of the local altitude \( h \) [m] and the temperature \( T_t \) [°C] according to the following expression [5]:

\[
\rho = \frac{273 P_a}{101325(273 + T_t)} \rho_0
\]

(3.1)

where \( \rho_0 \) is the standard air density (0 °C) and \( P_a \) is the atmospheric pressure given by

\[
P_a = 101325 \left(1 - 0.0065 \frac{h}{273 + T_t}\right)^{5.2561}
\]

(3.2)

On the height range where the multicopter usually operates, the temperature can be considered constant, and so does the atmospheric pressure \( P_a \) and the air density \( \rho \).
Another important parameter is the gravitational acceleration $g$, also considered constant on the height range operation of the drone.

The environmental parameters are summed up on the table 1.

<table>
<thead>
<tr>
<th>Environmental Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude $h$</td>
<td>200 m</td>
</tr>
<tr>
<td>Temperature $T_t$</td>
<td>40 °C</td>
</tr>
<tr>
<td>Air Density $\rho$</td>
<td>1.103 kg/m$^3$</td>
</tr>
<tr>
<td>Atmospheric Pressure $P_a$</td>
<td>99132.49 Pa</td>
</tr>
<tr>
<td>Gravity Acceleration $g$</td>
<td>9.81 m/s$^2$</td>
</tr>
</tbody>
</table>

Source: Author

3.1.2 Propeller Modeling

When searching for a propeller, 4 main parameters may be used to evaluate which might better match a determined application. Those are: the Propeller Diameter $D_p$, the Propeller Pitch $H_p$, the Number of Blades $B_p$ and the Propeller Mass $m_p$.

Usually propellers are described by a four number digit, as in the figure 6, where the 2 first numbers represents the diameter of the propeller, and the last two represent the pitch of the propeller both in inches. The pitch is defined as the distance a propeller would move into a soft solid after one revolution. The figure 6 shows a 9545 propeller with 2 blades.

![Figure 6: T-Motor 9545 Propeller](source: [6])

The performance of the propeller is based on its thrust $T$ and torque $M$ given by the following expressions

\[ T = \frac{P}{2} \]
\[ M = \frac{T}{2} R \]
\[ T = C_T \rho \left( \frac{N}{60} \right)^2 D_p^4 \]
\[ M = C_M \rho \left( \frac{N}{60} \right)^2 D_p^5 \]  
(3.3)

The dimensionless parameters \( C_T \) and \( C_M \) are the thrust coefficient and the torque coefficient respectively. The can be expressed as [3]

\[ C_T = 0.25 \pi^3 \lambda \xi^2 B_p K_0 \frac{\epsilon \arctan \frac{H_p}{\pi D_p} - \alpha_0}{\pi A + K_0} \]  
(3.4)

\[ C_M = \frac{1}{8A} \pi^2 C_d \xi^2 \lambda B_p^2 \]

where

\[ C_d = C_{fd} + \frac{\pi A K_0^2 \left( \epsilon \arctan \frac{H_p}{\pi D_p} - \alpha_0 \right)^2}{e \left( \pi A + K_0 \right)^2} \]  
(3.5)

The values for \( A, \epsilon, \lambda, \xi, e, C_{fd}, \alpha_0 \) and \( K_0 \) are dependent of the type, material and technology of the propeller. However, [3] indicates that a set of a mean of those parameters match with experimental data. The values of those parameters are summed on the table 25. All the propellers discussed on this text will be considered as with the same values present in table 25.

<table>
<thead>
<tr>
<th>( A )</th>
<th>( \lambda )</th>
<th>( \epsilon )</th>
<th>( \xi )</th>
<th>( C_{fd} )</th>
<th>( \alpha_0 )</th>
<th>( K_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.75</td>
<td>0.83</td>
<td>0.55</td>
<td>0.015</td>
<td>0</td>
<td>6.11</td>
</tr>
</tbody>
</table>

Source: [3]

### 3.1.3 Airframe Modeling

For this initial analysis, the 2 main parameters to be taken into consideration for the modeling of the airframe will be the diameter \( D_p \), of the propeller and the material which the airframe would be manufactured.

The size of the multicopter must match the size of the propellers in order to have a compact configuration without the propellers interfering with each other. In the figure 7 a model of a quadrotor is shown.

In the figure 7, \( R \) denotes the measure of the arm of the quadrotor. The size of the arm \( R \) is geometrical limited by the size of the propeller. In order to avoid aerodynamic interference caused by the rotation of the propeller, a concentric circumference of diameter \( D_{p_{max}} = 1.2 D_p \) is used to set the size of the airframe. For the quadrotor case, the expression for the size of the airframe arm \( R \) as function of the propeller dimension is given by:
\[ R = \frac{r_{\text{max}}}{\sin(45^\circ)} = \frac{0.6D_p}{\sin(45^\circ)} \] (3.6)

Similarly, for a general multicopter with \( n \) rotors, one have the following expression

\[ R = \frac{r_{\text{max}}}{\sin(180^\circ/n)} = \frac{0.6D_p}{\sin(180^\circ/n)} \] (3.7)

### 3.1.3.1 Mass of Airframe

In order to have a first estimation of the mass of the airframe for a quadrotor, a structure such the one of the figure 8 will be used.

All the dimensions of the structure are function of the propeller diameter. Additionally, the thickness \( d \) of the structure will be set equal to 10 mm. For the quadrotor case, the expression for the volume of the structure becomes then

\[ V = (2Rl + (2R - l)t)d = (4Rl - t^2)d \]

\[ R = \frac{0.6D_p}{\sin(45^\circ)} \] (3.8)

For a \( n \)-rotor, with \( n < 2R/l \), a similar structure can be used and one can easily obtained the following expression for the volume
The mass is then given by

\[ m_{af} = 1.25\rho_{af}V \]  

(3.10)

where a 0.25 factor is added to take into consideration additional weight, as the protection case for the electronic components, bolts, etc. The \( \rho_{af} \) is the density of the material to be used. In a vibration point of view, the material has to be rigid enough to prevent the bending and torsion of the arms, being at the same time light in order to improve the endurance time.

### 3.1.3.2 Surface Area of the Airframe

The surface area of the airframe is an important parameter for the aerodynamic of the flight. As bigger the surface of the drone, as higher drag forces will be present which may lead to poor flight performance.

From the figure 7, the maximum surface area of the drone can be considered as, in a general mode, as the total area of the discs delimited by the propellers, plus the area of the internal polygon, minus the area of the intersection between the discs and the internal polygon. For a n-rotor, the following expression can be obtained for the surface area \( S \)

\[
V = (nRl - l^2)d
\]

\[
R = \frac{0.6Dp}{\sin(180°/n)}
\]

(3.9)
\[ S = \frac{nD_{p_{\text{max}}}^2}{4} \left[ \pi + \frac{1}{\ln(n)} - \frac{\pi}{2} \left( 1 - \frac{2}{n} \right) \right] \]  \hfill (3.11)

### 3.1.4 Motor Modeling

The equivalent motor model is shown in figure 9, where \( U_m \) is the equivalent motor voltage [V], \( I_m \) is the equivalent motor current [A], \( R_m \) is the armature resistance [Ω], \( L_m \) is the armature inductance [H], \( I_0 \) is the no-load current required to overcome losses due to friction and electrical effects [A], and \( E_a \) is the back-electromotive force [V].

![Motor Model](source: [3])

Commercial motors can be evaluated by the following 6 parameters: the KV Value \( K_{V0} \); the Maximum Continuous Current \( I_{m_{\text{Max}}} \); the Nominal No-Load Current \( I_{m0} \); the Nominal No-Load Voltage \( U_{m0} \); the Motor Resistance \( R_m \); and the Mass of the Motor \( m_m \). Those parameters are defined as:

- **the KV Value** \( K_{V0} \) measures the rotational speed of the no-load motor in RPM’s by the voltage applied. A KV920 motor, for example, will have an angular velocity of 920 RPM when 1 V is applied on it.

- **the Maximum Continuous Current** \( I_{m_{\text{Max}}} \) is a threshold of continuous current which the motor can operate safely with. A maximum continuous current of 15 A, indicates that the motor can safely operates with a continuous current up to 15 A.

- **the Nominal No-Load Current** \( I_{m0} \) and the **Nominal No-Load Voltage** \( U_{m0} \): The nominal no-load current, is the current passing through the stator in the idle operation with nominal voltage applied. This voltage is the Nominal No-load Voltage.
• **the Motor Resistance** $R_m$ is the internal resistance of the motor, which can overheat it and reduce its efficiency.

Those parameters are found in a typical test report as in figure 10. The motor of the report is the 2212 KV920 of the T-Motor (figure 11). The 2212 indicates that the motor has a stator diameter of 22 mm (two first digits) and a stator height of 12 mm (two last digits).

![Figure 10: Motor Test](source: [7])

Here, some reasonable simplifications are adopted:

- The armature inductance $L_m$ is neglected;
- The transient process caused by the switching elements are ignored;
- The no-load current $I_0$ is considered constant for a given angular speed;
Figure 11: T-Motor 2212 KV920

Source: [7]

- The Nominal No-Load Current $I_{m0}$ can be used as an estimation for the no-load current $I_0$.

The aim of the motor modeling is to obtain the equivalent current $I_m$ and the equivalent voltage of the motor $U_m$ using the angular speed $N$ and the torque $M$ obtained from the propeller model. As shown in [3], $I_m$ and $U_m$ can be obtained from the following expressions:

$$U_m = \frac{MK_{V0}U_{m0}}{9.55(U_{m0} - I_{m0}R_m)} + I_{m0}R_m + \frac{U_{m0} - I_{m0}R_m}{K_{V0}U_{m0}} N$$

$$I_m = \frac{MK_{V0}U_{m0}}{9.55(U_{m0} - I_{m0}R_m)} + I_{m0}$$  \hspace{1cm} (3.12)

3.2 Electronic Speed Controller Modeling

The Electronic Speed Controller, figure 12, is the device in between the battery and the motor. The ESC converts the DC voltage of the battery into the three-phase alternating signal which is synchronized with the rotation of the motor (figure 13).

A model of the ESC is shown in figure 14, where $U_e$ is the ESC input voltage [V], $I_e$ is the ESC input current [A], $U_{e0}$ is the equivalent DC voltage, and $R_e$ is the ESC internal resistance[Ω]. Other 2 important parameters are the Maximum ESC Current $I_{eMAX}$ [A], and the ESC mass $m_e$ [kg], where the first is the parameter that usually names the ESC. As in figure 12, an ESC 15 has a maximum current of 15 A.

The aim of the ESC model is to obtain the ESC input voltage $U_e$, the ESC input current $I_e$ and the throttle command $\sigma$ given the parameters resulting from the motor model.

From figure 14
Figure 12: Electronic Speed Controller

Source: [8]

Figure 13: ESC Wiring

Source: [8]

Figure 14: ESC Model

Source: [3]

\[ U_{e0} = U_m + I_m R_m \]  

(3.13)

The throttle command is then obtained by
\[ \sigma = \frac{U_e}{U_e} \approx \frac{U_e}{U_b} \]  

(3.14)

where \( U_b \) is the battery voltage.

The ESC input current \( I_e \) can be expressed by the following relation:

\[ I_e = \sigma I_m \]  

(3.15)

The equation 3.15 is based on the fact that the input power of the ESC is equal to its output power.

The ESC voltage \( U_e \) is supplied by the battery and can be expressed by

\[ U_e = U_b - I_b R_b \]  

(3.16)

where \( R_b \) is the battery internal resistance [\( \Omega \)] and \( I_b \) is the battery current [\( A \)]. For a multicopter with \( n \) propulsors, the battery current \( I_b \) can be expressed as

\[ I_b = n I_e + I_{other} \]  

(3.17)

where \( I_{other} \) includes the losses and current from other devices, usually accounts around 1 A.

### 3.2.1 Battery Modeling

The parameters taken into consideration for the battery modeling are the capacity \( C_b \), the resistance \( R_b \), the total voltage \( U_b \), the maximum discharge rate \( K_b \) and the battery weight \( m_b \). The capacity \( C_b \) indicates how much electrical charge the battery has and is measured in mAh. The discharge rate is the reason between the current of discharge and the capacity. The maximum discharge rate \( K_b \) establishes a threshold for the current of the battery as it will be seen on the section 3.4.3.

As it can be seen on figure 15 the capacity \( C_b \) and the maximum discharge rate \( K_b \) are the parameters that usually name the battery.

Finally, the battery model mainly aims to obtain the time of endurance \( t_b \) from the previous models.
It is well known that the discharge of the battery is non-linear with its voltage. Here, a simplified method is used where the voltage is considered to remain constant with the discharge process and the capacity of the battery decreases linearly with time \[3\]. Then, the following expression can be used to express the time of endurance \(t_b\):

\[
t_b = 0.06 \frac{C_b - C_{\text{min}}}{I_b}
\]

(3.18)

where \(C_b\) is expressed in mAh, \(C_{\text{min}}\) [mAh] is in the range \(0.15C_b : 0.20C_b\), and \(t_b\) is given in minutes.

### 3.3 Evaluation of Optimal Parameters

In order to evaluate the set of components which best match the multicopter application, a MATLAB simulation was run by combining dozens of propellers, motors, ESC’s and batteries, and three different configurations of multicopter generating more than 10,000,000 combinations. By a sequence of compatibility analyses, incompatible components are then taken out of the analyses. The sets were evaluated by its performance on the following parameters:

- **Maximum time of endurance in hover** \(t_{e\text{MAX}}\)
- **Maximum system efficiency** \(\eta_{\text{MAX}}\)
- **Maximum produced propeller thrust** \(T_{\text{MAX}}\)
- **Maximum payload mass** \(m_{\text{MAXLOAD}}\)
• Maximum pitch angle $\theta_{MAX}$
• Maximum forward speed $V_{MAX}$
• Maximum flight distance $Z_{MAX}$
• Minimum mass of propulsors $m_{MIN}$
• Parameters of minimum cost

For each of these items the set which maximize the particular parameter may be determined, and then the set is evaluated on all the parameters.

A second approach, that is the final goal of this report, involves an optimization process which determines a single set that maximizes a series of key parameters with the maximums values obtained on the first approach.

On the table 3 the parameters of the components were summed up. For reference, the important expressions for the modelling of the propulsion system were summed up on the table 4. The complete list of components and its parameters are present on appendixes A to D and the MATLAB codes on appendixes E and F.

### 3.3.1 Maximum Endurance in Hover

The particular condition of flight when the thrust generated by the propulsor group equals the weight $G = gm$ of the multicopter is called hover. In this condition the resultant force on the gravitational field direction is null and the multicopter stays fixed in the air at a height $h$.

In this section, the expression of the maximum endurance in hover will be determined. Given an $ijkr$ set of components, with $0 < i \leq n_b$, $0 < j \leq n_e$, $0 < k \leq n_m$, and $0 < r \leq n_p$, being $n_b$, $n_e$, $n_m$ and $n_p$ the number of batteries, ESC’s, motors, and propellers being analyzed; the Endurance in Hover $t_e$ is evaluated through the following system of equations

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller $r$</td>
<td>$\Theta_{pr} = (D_{pr}, H_{pr}, B_{pr}, m_{pr})$</td>
</tr>
<tr>
<td>Motor $k$</td>
<td>$\Theta_{mk} = (K_{V0_k}, I_{MAX_k}, I_{0_k}, U_{0_k}, R_{mk}, m_{mk})$</td>
</tr>
<tr>
<td>ESC $j$</td>
<td>$\Theta_{ej} = (I_{MAX_j}, R_{ej}, m_{ej})$</td>
</tr>
<tr>
<td>Battery $i$</td>
<td>$\Theta_{bi} = (C_{bi}, R_{bi}, U_{bi}, K_{bi}, m_{bi})$</td>
</tr>
</tbody>
</table>

Source: Author
Table 4: Propulsion System Modelling

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller model</td>
<td>Thrust Coefficient $C_T = 0.25\pi^3\lambda^2B_pK_0\frac{\tan\Theta_p}{\pi A + K_0} \alpha_0$ eq 3.4</td>
</tr>
<tr>
<td></td>
<td>Torque Coefficient $C_M = \frac{1}{5A}\pi^2C_d\lambda B_p^2$ eq 3.4</td>
</tr>
<tr>
<td></td>
<td>$C_d$ Coefficient $C_d = C_{fd} + \frac{\pi A K_T^2}{e} \frac{(\tan\Theta_p - \alpha_0)^2}{(\pi A + K_0)^2}$ eq 3.5</td>
</tr>
<tr>
<td></td>
<td>Thrust [N] $T = C_T\rho\left(\frac{N}{60}\right)^2D_p^4$ eq 3.3</td>
</tr>
<tr>
<td></td>
<td>Torque [N/m] $M = C_M\rho\left(\frac{N}{60}\right)^2D_p^5$ eq 3.3</td>
</tr>
<tr>
<td>Motor model</td>
<td>Back-electromotive Force Constant $K_E = \frac{U_{m0} - I_{m0}R_m}{K_v0U_{m0}}$</td>
</tr>
<tr>
<td></td>
<td>Torque Constant $K_T = 9.55K_E$</td>
</tr>
<tr>
<td></td>
<td>Equivalent Current [A] $I_m = \frac{M}{K_T} + I_{m0}$ eq 3.12</td>
</tr>
<tr>
<td></td>
<td>Equivalent Voltage [V] $U_m = I_m R_m + K_E N$ eq 3.12</td>
</tr>
<tr>
<td>ESC model</td>
<td>Circuit Principle $U_{e0} = U_m + I_m R_e$ eq 3.13</td>
</tr>
<tr>
<td></td>
<td>Input Throttle $\sigma = \frac{U_{e0}}{U_e} \approx \frac{U_{e0}}{U_b}$ eq 3.14</td>
</tr>
<tr>
<td></td>
<td>Input Current [A] $I_e = \sigma I_m$ eq 3.15</td>
</tr>
<tr>
<td></td>
<td>Input Voltage [V] $U_e = U_b - (nI_e + I_{other})R_b$ eq 3.16</td>
</tr>
<tr>
<td>Battery model</td>
<td>Battery Current [A] $I_b = nI_e + I_{other}$ eq 3.17</td>
</tr>
<tr>
<td></td>
<td>Endurance [min] $t_b = 0.06(C_b - C_{min})/I_b$ eq 3.18</td>
</tr>
</tbody>
</table>

Source: Adapted from [3]
\[ m_{ijkr} = m_{af} + m_b + 1.25n(m_{pr} + m_{mk} + m_{ej}) \]
\[ T_{ijkr} = \frac{g m_{ijkr}}{n} \]
\[ N_{ijkr} = 60 \sqrt{\frac{T_{ijkr}}{\rho D_p^2 C_{T_p}}} \]
\[ M_{ijkr} = C_M \rho \frac{(N_{ijkr})^2}{60} D_p^5 \]
\[ U_{m_{ijkr}} = U_m(\Theta_{mk}, M_{ijkr}, N_{ijkr}) \]
\[ I_{m_{ijkr}} = I_m(\Theta_{mk}, M_{ijkr}, N_{ijkr}) \]
\[ \sigma_{ijkr} = \sigma(\Theta_{ej}, \Theta_b, U_{m_{ijkr}}, I_{m_{ijkr}}) \]
\[ I_{ijkr} = I_{eijkr} + I_{b_{ijkr}} + I_{other} \]
\[ t_{hover_{ijkr}} = t_{hover}(\Theta_b, I_{b_{ijkr}}) \]

where the 0.25 factor on the first equation of the system 3.19 is due to the extra weight of wires and other electrical components, the second equation of 3.19 is due the fact the thrust generated by the propulsor group has to equals the weight, and the third equation of 3.19 is obtained from the expression of the thrust on equation 3.3.

Furthermore, the ESC input voltage is obtained as

\[ U_{e_{ijkr}} = U_{b_{ijkr}} - I_{b_{ijkr}} R_{b_{ijkr}} \] (3.20)

The maximum endurance on hover is obtained from

\[ t_{eMAX} = \max t_{hover_{ijkr}} \] (3.21)

The ESC input voltage for the maximum endurance on hover is the \( U_{e_{ijkr}} \) for the set \( ijkr \) that satisfy equation 3.21.

### 3.3.2 Maximum System Efficiency

The system efficiency is the indicator of how much of the electrical energy generated by the battery is converted into mechanical energy, and therefore thrust. As higher is the efficiency as lower is the dissipative losses.

The efficiency of the propulsor set is defined on the maximum throttle mode (\( \sigma = 1 \)), meaning that the battery voltage \( U_b \) reaches its peak. Using a numerical solver for non-linear equations, \( M, N \) and \( I_m \) are determined through the following system of non-linear
where the first equation of the system 3.22 is the expression of the maximum throttle mode. The system of equations 3.22 is derived from the equations 3.14, 3.3, and 3.12.

From the results of 3.22 one can proceed as

\begin{align}
I_{eijkr} &= I_{mijkr} \\
I_{bijkr} &= n I_{eijkr} + I_{other} \\
U_{eijkr} &= U_{bijkr} - I_{bijkr} R_{bijkr}
\end{align}

The system efficiency of the set \(ijkr\) is then given by the ratio of the propeller output mechanical power and the power generated by the battery as

\[ \eta_{ijkr} = \frac{2\pi n M_{ijkr} N_{ijkr}}{U_b I_{bijkr}} \]  

(3.24)

The maximum system efficiency is represented by the set \(ijkr\) that satisfy the following expression

\[ \eta_{MAX} = \max \eta_{ijkr} \]  

(3.25)

The ESC input voltage for the maximum system efficiency is the \(U_{eijkr}\) for the set \(ijkr\) that satisfy the equation 3.25.

### 3.3.3 Maximums Payload Mass, Produced Propeller Thrust and Pitch Angle

From the results of the system of equations 3.22, the maximum payload mass possible for the \(ijkr\) set is given by

\[ m_{LOAD_{ijkr}} = n T_{ijkr} - m_{ijkr} g \]  

(3.26)

The maximum pitch angle possible for the set \(ijkr\) is given by
\[
\theta_{\text{MAX}ijkr} = \arccos \frac{m_{\text{ijkr}}g}{nT_{ijkr}} \tag{3.27}
\]

From the results above is possible to determine the maximum payload mass it possible be achieved from all sets as

\[
m_{\text{MAXLOAD}} = \max m_{\text{LOAD}ijkr} \tag{3.28}
\]

Similarly, the maximum pitch angle among the sets is given by

\[
\theta_{\text{MAX}} = \max \theta_{\text{MAX}ijkr} \tag{3.29}
\]

Although it may be confusing, each set \(ijkr\) presents a maximum pitch angle denoted as \(\theta_{\text{MAX}ijkr}\). The maximum value of \(\theta_{\text{MAX}ijkr}\) from all the components is the Maximum Pitch Angle \(\theta_{\text{MAX}}\).

### 3.3.4 Maximum Forward Flight Speed

As show in figure 16 on a forward movement (neglecting blade flapping effects) the thrust generated by the action of the propellers is split in a component parallel to the direction of the forward flight and a component parallel to the gravitational field. In order to have the movement and the multicopter in a fixed altitude \(h\), the component parallel to the movement has to at least equals the drag force \(F_{\text{drag}}\), the dissipative force contrary the movement of the UAV; and the component parallel to the gravitational field has to equals the weight \(G\) of the multicopter. Then

\[
F_{\text{drag}} = mg \tan \theta \tag{3.30}
\]

The expression of the drag force is given by [10]

\[
F_{\text{drag}} = \frac{1}{2} C_D \rho V^2 S \tag{3.31}
\]

where \(V \ [\text{m/s}]\) is the forward flight speed, \(S \ [\text{m}^2]\) is the maximum surface area of the multicopter given by equation 3.11 and \(C_D\) is the drag coefficient of the whole multicopter. The drag coefficient \(C_D\) is function of the pitch angle \(\theta\) and can be evaluated experimentally through a CFD analyses. The fitting curve for the CFD results obtained
in [11] gives the following relation to $C_D$

$$C_D = C_{D1}(1 - sin^3 \theta) + C_{D2}(1 - cos^3 \theta)$$

where $C_{D1}$ is equal to 3 and $C_{D2}$ is equal to 1.5. Although the expression 3.32 may not be exactly the same for the quadrotor in study, the expression will be used as a rough approximation in a first effort towards the design.

Isolating the forward speed $V$ in the equation 3.31 and replacing the equation 3.32 on it, the following expression comes

$$V(\theta) = \sqrt{\frac{2mgtan\theta}{\rho S[C_{D1}(1 - sin^3 \theta) + C_{D2}(1 - cos^3 \theta)]}}$$

As it is highlighted in equation 3.33, the forward speed of the multicopter is function of its pitch angle.

For the pitch angle $\theta$ interval $0 \leq \theta \leq \frac{\pi}{2}$ the equation 3.33 represents a rising function in $\theta$. Therefore, the maximum forward flight speed for a generic multicopter can be defined by
\[ V_{\text{MAX}} = V(\theta_{\text{MAX}}) \]  
(3.34)

For a multicopter with the \( ijk \) set of components, the equation 3.33 is written as

\[ V_{ijkr}(\theta) = \sqrt{\frac{2m_{ijkr} g \tan \theta}{\rho S_r [C_{D1}(1 - \sin^3 \theta) + C_{D2}(1 - \cos^3 \theta)]}} \]  
(3.35)

moreover, the maximum forward flight speed for the multicopter with the \( ijk \) set of components is then

\[ V_{\text{MAX}} = \max_{ijkr} V_{ijkr}(\theta_{\text{MAX}}) \]  
(3.36)

Note that the equations 3.34 and 3.36 are different in essence: the equation 3.34 represents the maximum forward flight speed of a given multicopter with its maximum possible pitch angle; while equation 3.36 represents the set with the maximum forward flight speed among all the maximums forward flight speed of all sets. Although the same notation is used here for simplification, the expression ”Maximum Forward Flight Speed” from now on on the text will represent the function 3.36 save if told otherwise.

### 3.3.5 Maximum Flight Distance

In order to calculate the maximum flight distance, the total flight time has to be evaluated. The set of equations to calculate the total flight time is similar to that of the endurance in hover.

From the figure 16 a \( ijk \) set has the following expression for the thrust \( T_{ijkr} \)

\[ T_{ijkr} = \frac{m_{ijkr} g}{n \cos \theta} \]  
(3.37)

Replacing equation 3.37 in the expression for thrust 3.3, the angular speed \( N \) can be isolated and written as

\[ N_{ijkr} = 60 \sqrt{\frac{m_{ijkr} g}{\rho C_T D_p^4 n \cos \theta}} \]  
(3.38)

And the torque \( M_{ijkr} \) is expressed by
\[ M_{ijkr} = \frac{m_{ijkr} g C_M D_p c}{C_T n \cos \theta} \] (3.39)

Hence, the total time of flight of the multicopter compounded by the \( ijk \) set, namely \( t_{fly,ijkr} \) is calculated through

\[
\begin{align*}
U_{m,ijkr} &= U_m(\Theta_{mk}, M_{ijkr}, N_{ijkr}) \\
I_{m,ijkr} &= I_m(\Theta_{mk}, M_{ijkr}, N_{ijkr}) \\
\sigma_{ijkr} &= \sigma(\Theta_{ej}, \Theta_{bi}, U_{m,ijkr}, I_{m,ijkr}) \\
I_{e,ijkr} &= \sigma_{ijkr} I_{m,ijkr} \\
I_{b,ijkr} &= n I_{e,ijkr} + I_{other} \\
t_{fly,ijkr} &= t_{fly}(\Theta_{bi}, I_{b,ijkr})
\end{align*}
\] (3.40)

Note that the set of equations 3.40 follows the similar procedure it has been seen in the section 3.3.1 for the hover endurance and the flight time is given in minutes.

The flight distance for the set \( ijk \), namely \( Z_{ijkr} \) [m], can be then written as a function of \( \theta \) as

\[ Z_{ijkr}(\theta) = 60 V_{ijkr}(\theta) t_{fly,ijkr}(\theta) \] (3.41)

The maximum flight distance \( Z_{MAX} \) is then calculated as

\[ Z_{MAX} = \max Z_{ijkr} \] (3.42)

### 3.3.6 Financial Aspects

Apart from the technical parameters, two financial related parameters will be discussed briefly: the cost and the price.

For now, the costs taken into consideration are exclusively that of the propulsor group. Meanwhile, the pricing analysis was made through a survey with around a hundred people.

#### 3.3.6.1 Costs

The cost of the propulsion system is one of the key parameters for the project, and as it will be presented later on, it has a weight on the decision criterion.
The figure 17 shows how endurance time, efficiency of the system, payload mass and flight distance changes in function of cost for the propulsor sets of a quadrotor case.

**Figure 17: Theoretical Results for a Quadrotor**

Source: Author

### 3.3.6.2 Pricing

For a product be likely profitable its price has to be bigger than its costs. In other words, the market has to be able to pay for a product, more than it costs to develop and manufacture it.

In a survey with around 100 people from 20 countries, an indirect question that could be translated simply into ”how much would you be willing to invest in a drone?” was asked. A scale called willingness from 0 to 5 was then defined and the results obtained were

- the willingness for paying until 50 USD for the drone was of 3.4 out of 5;
- the willingness for paying until 100 USD for the drone was of 3.1 out of 5;
- the willingness for paying until 500 USD for the drone was of 1.8 out of 5;
- the willingness for paying until 1000 USD for the drone was of 0.8 out of 5;
From the results, ideally, the costs of the drone should be at least lower than 100 USD in order to have a product that might be profitable.

It’s important to remind this is a really simplistic analysis that is not adequate in this specific case, since a highly potential market strategy to be used would be the one where the client is not buying the drone itself, but the service offered for it. In fact, as it will be seen it on the next chapters, the margin for revenue by selling the drone itself for a final client would nonexistent, since the costs of the propulsion components themselves are around twice the price determined by the survey.

Although simplistic however the study is important to define a market strategy by cutting off a first approach of selling drones to the final consumer. In order to have a competitive product in this way, the costs would have to be reduced by cutting intermediaries and therefore manufacturing most of components. Moreover, for this, a further study of market would be needed, what will not be covered here, since it is out of the scope of the project.

### 3.4 Compatibility Aspects

All the combinations of the components have the compatibility evaluated in 3 aspects: the *maximum angular Speed*, *Motor/ESC Compatibility*, and *Battery Current Compatibility*. Naturally, if the set does not match any of those criteria, its assembly is physically impossible and the parameters assume the values on table 5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance in Hover $t_e$</td>
<td>0</td>
</tr>
<tr>
<td>System Efficiency $\eta$</td>
<td>0</td>
</tr>
<tr>
<td>Propeller Thrust $T$</td>
<td>0</td>
</tr>
<tr>
<td>Payload Mass $m_{LOAD}$</td>
<td>0</td>
</tr>
<tr>
<td>Pitch Angle $\theta$</td>
<td>0</td>
</tr>
<tr>
<td>Forward Speed $V$</td>
<td>0</td>
</tr>
<tr>
<td>Flight Distance $Z$</td>
<td>0</td>
</tr>
<tr>
<td>Mass of the set $m$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Cost of the set</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Source: Author
3.4.1 Angular Speed Compatibility

The study of the parameters requires the calculation of the angular speed $N$ from the propeller parameters (as in the equations 3.3). However, the angular speed calculated $N_{calc}$ may be so high that a determined motor couldn’t spin and make the flight possible. In this way, a threshold is established for each motor taking into considerations the maximum angular speeds given by its manufacturer. This threshold is the maximum angular speed $N_{MAX}$.

In this way, the angular speed compatibility establishes a condition for the pair propeller-motor where the relation

$$N_{calc} \leq N_{MAX}$$  \hspace{1cm} (3.43)

has to be respected.

For case where $N_{calc}$ is actually bigger than $N_{MAX}$ the parameters are set as in table 5, indicating that the pair motor-k and propeller-r is not feasible and so is not the multicopter flight.

3.4.2 Motor/ESC Compatibility

In order to have the best efficiency of the motor, the current limit of the ESC has to be slightly higher than the maximum current for the motor. Hence, the compatibility of the pair motor k - ESC j is checked through

$$I_{eMAX,j} \geq 1.2I_{mMAX,k}$$  \hspace{1cm} (3.44)

In case the relation is not satisfied, the parameters are set as on the table 5.

3.4.3 Battery Current Compatibility

The battery current for the set $ijkr$ $I_{b_{ijkr}}$ cannot be greater than the maximum current that the battery can withstand. Therefore, the following relation must be respected

$$I_{b_{ijkr}} \leq \frac{C_{b_i}K_{b_i}}{1000}$$  \hspace{1cm} (3.45)

with $C_{b_i}$ in mAh and $K_{b_i}$ in C.
For a $I_{b_{ij}}$ bigger than the threshold, the parameters assume the values of the table 5.

### 3.5 Results

The models approached were implemented in a MATLAB simulation with

- 43 different types of propellers;
- 50 different types of motors;
- 38 different types of ESC’s;
- 41 different types of batteries;
- 3 different multicopter configurations.

A total of 10,049,100 different combinations of sets were evaluated where the incompatible ones were excluded. The sets evaluated were those proposed on chapter 3.3, for what the set of components that optimize each of these parameters were determined.

Figures 18, 19 and 20 present the distribution of the theoretical results for tricopters, quadrotors and hexacopter, respectively.
Figure 18: Theoretical Results for a Tricopter

Figure 19: Theoretical Results for a Quadrotor

Figure 20: Theoretical Results for a Hexacopter

Source: Author
3.5.1 Best Set of Components by Optimal Parameters

This section presents the set of components that optimize some of the parameters introduced in chapter 3.3. Those results will be of great importance when defining the decision criterion, which will lead to the set of components to be used as base of the project.

The data of the components are presented in a first table and the results obtained in a second one. All the data and results present on the table are self-explanatory and extra comments will be used only where is strictly necessary.
3.5.1.1 Set with the Maximum Time of Endurance in Hover \( t_{eMAX} \)

For the maximum time of endurance in hover, the propeller, the motor and the battery models are the same for both the tricopter and quadcopter cases. On the meantime, the ESC model is the same for all cases.

### Table 6: Maximum Time of Endurance in Hover Data

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Propeller</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_p )</td>
<td>0.1270 m</td>
<td>0.0650 m</td>
<td></td>
</tr>
<tr>
<td>( H_p )</td>
<td>0.1143 m</td>
<td>0.1118 m</td>
<td></td>
</tr>
<tr>
<td>( B_p )</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>( m_p )</td>
<td>0.0037 kg</td>
<td>0.0004 kg</td>
<td></td>
</tr>
<tr>
<td>( m_{af} )</td>
<td>0.0886 kg</td>
<td>0.1483 kg</td>
<td>0.0841 kg</td>
</tr>
<tr>
<td>( S )</td>
<td>0.0557 ( m^2 )</td>
<td>0.0779 ( m^2 )</td>
<td>0.0345 ( m^2 )</td>
</tr>
<tr>
<td><strong>Motor</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_{V0} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_{mMAX} )</td>
<td>1400 RPM/V</td>
<td>4500 RPM/V</td>
<td></td>
</tr>
<tr>
<td>( I_m )</td>
<td>12 A</td>
<td>14 A</td>
<td>4 A</td>
</tr>
<tr>
<td>( U_m )</td>
<td>0.2 A</td>
<td>0.76 A</td>
<td></td>
</tr>
<tr>
<td>( R_m )</td>
<td>10 V</td>
<td>7 V</td>
<td></td>
</tr>
<tr>
<td>( m_m )</td>
<td>0.325 ( \Omega )</td>
<td>0.145 ( \Omega )</td>
<td></td>
</tr>
<tr>
<td><strong>ESC</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_{eMAX} )</td>
<td>40 A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_e )</td>
<td>0.01 ( \Omega )</td>
<td>0.0026 kg</td>
<td></td>
</tr>
<tr>
<td>( m_e )</td>
<td>0.0026 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Battery</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C_b )</td>
<td>3800 mAh</td>
<td>3700 mAh</td>
<td></td>
</tr>
<tr>
<td>( R_b )</td>
<td>0.016 ( \Omega )</td>
<td>0.016 ( \Omega )</td>
<td></td>
</tr>
<tr>
<td>( U_b )</td>
<td>7.4 V</td>
<td>14.8 V</td>
<td></td>
</tr>
<tr>
<td>( K_b )</td>
<td>65 C</td>
<td>45 C</td>
<td></td>
</tr>
<tr>
<td>( m_b )</td>
<td>0.14 kg</td>
<td>0.285 kg</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author

### Table 7: Maximum Time of Endurance in Hover Results

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Endurance in Hover</strong></td>
<td>26.8 min</td>
<td>22.6 min</td>
<td>16.54 min</td>
</tr>
<tr>
<td>System Efficiency ( \eta )</td>
<td>72.55%</td>
<td>74.47%</td>
<td>69.68%</td>
</tr>
<tr>
<td>Produced Propeller Thrust ( T )</td>
<td>1.53 N</td>
<td>1.53 N</td>
<td>5.47 N</td>
</tr>
<tr>
<td>Payload mass ( m_{LOAD} )</td>
<td>0.113 kg</td>
<td>0.178 kg</td>
<td>2.819 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle ( \theta_{MAX} )</td>
<td>40.68°</td>
<td>44.69°</td>
<td>80.79°</td>
</tr>
<tr>
<td>Maximum Flight Speed ( V_{MAX} )</td>
<td>5.68 m/s</td>
<td>5.84 m/s</td>
<td>32.38 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance ( Z_{MAX} )</td>
<td>6395.07 m</td>
<td>5071.74 m</td>
<td>6050.28 m</td>
</tr>
<tr>
<td>Mass of Multicopter ( m )</td>
<td>0.3546 kg</td>
<td>0.4447 kg</td>
<td>0.5264 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>138.82 USD</td>
<td>181.08 USD</td>
<td>205.93 USD</td>
</tr>
</tbody>
</table>

Source: Author
3.5.1.2 Set with the Maximum System Efficiency $\eta_{MAX}$

Table 8: Maximum System Efficiency Data

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller</td>
<td>$D_p$</td>
<td>0.2540 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_p$</td>
<td>0.2286 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_p$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_p$</td>
<td>0.0041 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_{af}$</td>
<td>0.3545 kg</td>
<td>0.5932 kg</td>
</tr>
<tr>
<td></td>
<td>$S$</td>
<td>0.223 m²</td>
<td>0.312 m²</td>
</tr>
<tr>
<td>Motor</td>
<td>$K_{V_0}$</td>
<td>450 RPM/V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_{m_{MAX}}$</td>
<td>40 A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_{m_0}$</td>
<td>0.6 A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U_{m_0}$</td>
<td>22 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_m$</td>
<td>0.06 Ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_m$</td>
<td>0.085 kg</td>
<td></td>
</tr>
<tr>
<td>ESC</td>
<td>$I_{e_{MAX}}$</td>
<td>60 A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_e$</td>
<td>0.01 Ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_e$</td>
<td>0.06 kg</td>
<td></td>
</tr>
<tr>
<td>Battery</td>
<td>$C_b$</td>
<td>3700 mAh</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_b$</td>
<td>0.016 Ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U_b$</td>
<td>14.8V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_b$</td>
<td>45 C</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_b$</td>
<td>0.285 kg</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author

Table 9: Maximum System Efficiency Results

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance in Hover $t_{e_{MAX}}$</td>
<td>17.27 min</td>
<td>13.25 min</td>
<td>8.12 min</td>
</tr>
<tr>
<td>System Efficiency $\eta$</td>
<td>87.31 %</td>
<td>87.83 %</td>
<td>88.35 %</td>
</tr>
<tr>
<td>Produced Propeller Thrust T</td>
<td>11.60 N</td>
<td>11.60 N</td>
<td>11.60 N</td>
</tr>
<tr>
<td>Payload mass $m_{LOAD}$</td>
<td>2.277 kg</td>
<td>3.03 kg</td>
<td>4.336 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle $\theta_{MAX}$</td>
<td>68.75 °</td>
<td>68.75 °</td>
<td>67.04 °</td>
</tr>
<tr>
<td>Maximum Flight Speed $V_{MAX}$</td>
<td>11.53 m/s</td>
<td>11.25 m/s</td>
<td>10.27 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance $Z_{MAX}$</td>
<td>3889.50 m</td>
<td>2891.01 m</td>
<td>1707.58 m</td>
</tr>
<tr>
<td>Mass of Multicopter $m$</td>
<td>1.269 kg</td>
<td>1.694 kg</td>
<td>2.757 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>487.69 USD</td>
<td>634.45 USD</td>
<td>927.97 USD</td>
</tr>
</tbody>
</table>

Source: Author
3.5.1.3 Set with the Maximum Produced Propeller Thrust $T_{MAX}$ and the Maximum Payload Mass $m_{MAXLOAD}$

In this case, the payload mass $m_{MAXLOAD}$ is around 13 kg for all the multicopter configurations, what is a really significant result. However, the endurance in hover is around 5 min for the tricopter and quadrotor and less than 3 minutes for the hexacopter. Furthermore, as it will be present on the next pages, the time of flight decreases exponentially with the increase of the payload mass being carried by the UAV. Therefore, those sets would be unviable for the delivering application.

Table 10: Maximum Produced Propeller Thrust and Maximum Payload Mass Data

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller $D_p$</td>
<td>0.5080 m</td>
<td>0.457 m</td>
<td>0.457 m</td>
</tr>
<tr>
<td>$H_p$</td>
<td>0.1524 m</td>
<td>0.254 m</td>
<td>0.1829 m</td>
</tr>
<tr>
<td>$B_p$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$m_p$</td>
<td>0.0470 kg</td>
<td>0.0041 kg</td>
<td>0.0320 kg</td>
</tr>
<tr>
<td>$m_{af}$</td>
<td>1.418 kg</td>
<td>1.922 kg</td>
<td>4.1594 kg</td>
</tr>
<tr>
<td>$S$</td>
<td>0.891 m$^2$</td>
<td>1.01 m$^2$</td>
<td>1.727 m$^2$</td>
</tr>
<tr>
<td>Motor $K_{V0}$</td>
<td>300 RPM/V</td>
<td>240 RPM/V</td>
<td></td>
</tr>
<tr>
<td>$I_{mMAX}$</td>
<td>40 A</td>
<td>40 A</td>
<td></td>
</tr>
<tr>
<td>$I_{m0}$</td>
<td>1.1 A</td>
<td>0.9 A</td>
<td></td>
</tr>
<tr>
<td>$U_{m0}$</td>
<td>15 V</td>
<td>18 V</td>
<td></td>
</tr>
<tr>
<td>$R_m$</td>
<td>0.063 Ω</td>
<td>0.085 Ω</td>
<td></td>
</tr>
<tr>
<td>$m_m$</td>
<td>0.136 kg</td>
<td>0.136 kg</td>
<td></td>
</tr>
<tr>
<td>ESC $I_{eMAX}$</td>
<td>60 A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_e$</td>
<td>0.01 Ω</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_e$</td>
<td>0.06 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Battery $C_b$</td>
<td>1400 mAh</td>
<td>1300 mAh</td>
<td>1400 mAh</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.016 Ω</td>
<td>0.016 Ω</td>
<td>0.016 Ω</td>
</tr>
<tr>
<td>$U_b$</td>
<td>22.2 V</td>
<td>22.2 V</td>
<td>22.2 V</td>
</tr>
<tr>
<td>$K_b$</td>
<td>120 C</td>
<td>120 C</td>
<td>120 C</td>
</tr>
<tr>
<td>$m_b$</td>
<td>0.230 kg</td>
<td>0.212 kg</td>
<td>0.230 kg</td>
</tr>
</tbody>
</table>

Source: Author
3.5.1.4 Set with the Maximum Payload Mass $m_{MAXLOAD}$ - Tricopter

Particularly for the tricopter case, the set with the maximum payload mass differs from the one with maximum produced thrust and its data and results are presented in this dedicated section.

Table 11: Maximum Produced Propeller Thrust and Maximum Payload Mass Results

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance in Hover $t_{eMAX}$</td>
<td>5.15 min</td>
<td>4.75 min</td>
<td>2.30 min</td>
</tr>
<tr>
<td>System Efficiency $\eta$</td>
<td>79.70 %</td>
<td>82.21 %</td>
<td>85.24 %</td>
</tr>
<tr>
<td>Produced Propeller Thrust $T$</td>
<td>48.73 N</td>
<td>40.56 N</td>
<td>32.09 N</td>
</tr>
<tr>
<td>Payload mass $m_{LOAD}$</td>
<td>12.287 kg</td>
<td>13.35 kg</td>
<td>13.47 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle $\theta_{MAX}$</td>
<td>79.64 °</td>
<td>79.07 °</td>
<td>71.62 °</td>
</tr>
<tr>
<td>Maximum Flight Speed $V_{MAX}$</td>
<td>13.41 m/s</td>
<td>13.14 m/s</td>
<td>10.09 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance $Z_{MAX}$</td>
<td>820.38 m</td>
<td>787.62 m</td>
<td>397.80 m</td>
</tr>
<tr>
<td>Mass of Multicopter $m$</td>
<td>2.617 kg</td>
<td>3.187 kg</td>
<td>6.157 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>683.50 USD</td>
<td>730.20 USD</td>
<td>1539.00 USD</td>
</tr>
</tbody>
</table>

Source: Author
Table 12: Maximum Payload Mass Data - 3Rotor

<table>
<thead>
<tr>
<th>Propeller</th>
<th>$D_p$</th>
<th>0.3810 m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_p$</td>
<td>0.254 m</td>
</tr>
<tr>
<td></td>
<td>$B_p$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$m_p$</td>
<td>0.0041 kg</td>
</tr>
<tr>
<td></td>
<td>$m_{af}$</td>
<td>0.7975 kg</td>
</tr>
<tr>
<td></td>
<td>$S$</td>
<td>0.501 $m^2$</td>
</tr>
<tr>
<td>Motor</td>
<td>$K_{V0}$</td>
<td>340 RPM/V</td>
</tr>
<tr>
<td></td>
<td>$I_{mMAX}$</td>
<td>40 A</td>
</tr>
<tr>
<td></td>
<td>$I_{m0}$</td>
<td>0.9 A</td>
</tr>
<tr>
<td></td>
<td>$U_{m0}$</td>
<td>22 V</td>
</tr>
<tr>
<td></td>
<td>$R_m$</td>
<td>0.055 Ω</td>
</tr>
<tr>
<td></td>
<td>$m_m$</td>
<td>0.1080 kg</td>
</tr>
<tr>
<td>ESC</td>
<td>$I_{eMAX}$</td>
<td>60 A</td>
</tr>
<tr>
<td></td>
<td>$R_e$</td>
<td>0.01 Ω</td>
</tr>
<tr>
<td></td>
<td>$m_e$</td>
<td>0.06 kg</td>
</tr>
<tr>
<td>Battery</td>
<td>$C_b$</td>
<td>1300 mAh</td>
</tr>
<tr>
<td></td>
<td>$R_b$</td>
<td>0.016 Ω</td>
</tr>
<tr>
<td></td>
<td>$U_b$</td>
<td>22.2 V</td>
</tr>
<tr>
<td></td>
<td>$K_b$</td>
<td>120 C</td>
</tr>
<tr>
<td></td>
<td>$m_b$</td>
<td>0.2120 kg</td>
</tr>
</tbody>
</table>

Source: Author

3.5.1.5 Set with the Maximum Forward Speed $V_{MAX}$

As it can be seen on the table 15 the relatively high flight speeds make those sets strong candidates for racing drones.

Table 13: Maximum Payload Mass Results - 3Rotor

<table>
<thead>
<tr>
<th>Endurance in Hover $t_{eMAX}$</th>
<th>8.31 min</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Efficiency $\eta$</td>
<td>82.98 %</td>
</tr>
<tr>
<td>Produced Propeller Thrust $T$</td>
<td>46.76 N</td>
</tr>
<tr>
<td><strong>Payload mass</strong> $m_{LOAD}$</td>
<td>12.590 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle $\theta_{MAX}$</td>
<td>83.08 °</td>
</tr>
<tr>
<td>Maximum Flight Speed $V_{MAX}$</td>
<td>17.96 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance $Z_{MAX}$</td>
<td>1470.39 m</td>
</tr>
<tr>
<td>Mass of Multicopter $m$</td>
<td>1.7079 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>518.57 USD</td>
</tr>
</tbody>
</table>

Source: Author
Table 14: Maximum Forward Speed Data

<table>
<thead>
<tr>
<th>Propeller</th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_p)</td>
<td>0.0650 m</td>
<td>0.0406 m</td>
<td>0.0650 m</td>
</tr>
<tr>
<td>(H_p)</td>
<td>0.1118 m</td>
<td>0.0914 m</td>
<td>0.1118 m</td>
</tr>
<tr>
<td>(B_p)</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>(m_p)</td>
<td>0.0004 kg</td>
<td>0.0012 kg</td>
<td>0.0004 kg</td>
</tr>
<tr>
<td>(m_{af})</td>
<td>0.0232 kg</td>
<td>0.0152 kg</td>
<td>0.0841 m</td>
</tr>
<tr>
<td>(S)</td>
<td>0.0145 (m^2)</td>
<td>0.0080 (m^2)</td>
<td>0.0345 (m^2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor</th>
<th>(K_{V0})</th>
<th>4500 RPM/V</th>
<th>6500 RPM/V</th>
<th>4100 RPM/V</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{mMAX})</td>
<td>14.4 A</td>
<td>15.1 A</td>
<td>20.2 A</td>
<td></td>
</tr>
<tr>
<td>(I_{m0})</td>
<td>0.76 A</td>
<td>0.59 A</td>
<td>1.2 A</td>
<td></td>
</tr>
<tr>
<td>(U_{m0})</td>
<td>7 V</td>
<td>7 V</td>
<td>10 V</td>
<td></td>
</tr>
<tr>
<td>(R_{m})</td>
<td>0.145 (\Omega)</td>
<td>0.12 (\Omega)</td>
<td>0.097 (\Omega)</td>
<td></td>
</tr>
<tr>
<td>(m_{m})</td>
<td>0.0085 kg</td>
<td>0.0085 kg</td>
<td>0.0165 kg</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESC</th>
<th>(I_{eMAX})</th>
<th>60 A</th>
<th>40 A</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_{e})</td>
<td>0.01 (\Omega)</td>
<td>0.01 (\Omega)</td>
<td></td>
</tr>
<tr>
<td>(m_{e})</td>
<td>0.0600 kg</td>
<td>0.0026 kg</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Battery</th>
<th>(C_b)</th>
<th>1300 mAh</th>
<th>3700 mAh</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_b)</td>
<td>0.016 (\Omega)</td>
<td>0.016 (\Omega)</td>
<td></td>
</tr>
<tr>
<td>(U_b)</td>
<td>22.2 V</td>
<td>14.8 V</td>
<td></td>
</tr>
<tr>
<td>(K_b)</td>
<td>120 C</td>
<td>45 C</td>
<td></td>
</tr>
<tr>
<td>(m_b)</td>
<td>0.2120 kg</td>
<td>0.285 kg</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author

Table 15: Maximum Forward Speed Results

<table>
<thead>
<tr>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{eMAX})</td>
<td>6.03 min</td>
<td>7.97 min</td>
</tr>
<tr>
<td>System Efficiency (\eta)</td>
<td>64.33 %</td>
<td>65.07 %</td>
</tr>
<tr>
<td>Produced Propeller Thrust (T)</td>
<td>10.23 N</td>
<td>3.64 N</td>
</tr>
<tr>
<td>Payload mass (m_{LOAD})</td>
<td>2.5805 kg</td>
<td>1.05 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle (\theta_{MAX})</td>
<td>80.21 °</td>
<td>72.77 °</td>
</tr>
<tr>
<td>Maximum Flight Speed (V_{MAX})</td>
<td>48.02 m/s</td>
<td>41.46 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance (Z_{MAX})</td>
<td>3392.98 m</td>
<td>5245.33 m</td>
</tr>
<tr>
<td>Mass of Multicopter (m)</td>
<td>0.5466 kg</td>
<td>0.4327 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>267.72 USD</td>
<td>156.15 USD</td>
</tr>
</tbody>
</table>

Source: Author

3.5.1.6 Set with the Maximum Flight Distance \(Z_{MAX}\)

As it can be seen on table 17, the high endurance time and payload mass and the relatively low costs, make those sets strong candidates for the delivering application.
Table 16: Maximum Flight Distance Data

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_p$</td>
<td>0.0650 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_p$</td>
<td>0.1118 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_p$</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_p$</td>
<td>0.0004 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{af}$</td>
<td>0.0232 kg</td>
<td>0.0388 kg</td>
<td>0.0841 kg</td>
</tr>
<tr>
<td>$S$</td>
<td>0.0145 $m^2$</td>
<td>0.0204 $m^2$</td>
<td>0.0349 $m^2$</td>
</tr>
<tr>
<td>Motor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{V_0}$</td>
<td>4500 RPM/V</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{m_{MAX}}$</td>
<td>14.4 A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{m_0}$</td>
<td>0.76 A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_{m_0}$</td>
<td>7 V</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_m$</td>
<td>0.145 Ω</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_m$</td>
<td>0.0085 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ESC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{e_{MAX}}$</td>
<td>40 A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_e$</td>
<td>0.01 Ω</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_e$</td>
<td>0.0026 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Battery</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_b$</td>
<td>3700 mAh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.016 Ω</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_b$</td>
<td>14.8 V</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_b$</td>
<td>45 C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_b$</td>
<td>0.285 kg</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Author

Table 17: Maximum Flight Distance Results

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance in Hover $t_{e_{MAX}}$</td>
<td>16.78 min</td>
<td>17.24 min</td>
<td>16.54 min</td>
</tr>
<tr>
<td>System Efficiency $\eta$</td>
<td>69.24 %</td>
<td>69.46 %</td>
<td>69.68 %</td>
</tr>
<tr>
<td>Produced Propeller Thrust $T$</td>
<td>5.47 N</td>
<td>5.47 N</td>
<td>5.47 N</td>
</tr>
<tr>
<td>Payload mass $m_{LOAD}$</td>
<td>1.250 kg</td>
<td>1.778 kg</td>
<td>2.819 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle $\theta_{MAX}$</td>
<td>75.63 °</td>
<td>78.49 °</td>
<td>80.79 °</td>
</tr>
<tr>
<td>Maximum Flight Speed $V_{MAX}$</td>
<td>33.51 m/s</td>
<td>33.74 m/s</td>
<td>32.38 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance $Z_{MAX}$</td>
<td>8364.99 m</td>
<td>7579.94 m</td>
<td>6050.28 m</td>
</tr>
</tbody>
</table>

Source: Author

3.5.1.7 Set with the Minimum Cost
### Table 18: Minimum Cost Data

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller</td>
<td>$D_p$</td>
<td>0.0310 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_p$</td>
<td>0.0483 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_p$</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_p$</td>
<td>0.0003 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_{af}$</td>
<td>0.0053 kg</td>
<td>0.0088 kg</td>
</tr>
<tr>
<td></td>
<td>$S$</td>
<td>0.0033 $m^2$</td>
<td>0.0046 $m^2$</td>
</tr>
<tr>
<td>Motor</td>
<td>$K_{V0}$</td>
<td>8000 RPM/V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_{m_{MAX}}$</td>
<td>11.5 A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_{m0}$</td>
<td>0.58 A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U_{m0}$</td>
<td>3 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_m$</td>
<td>0.163 Ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_m$</td>
<td>0.0042 kg</td>
<td></td>
</tr>
<tr>
<td>ESC</td>
<td>$I_{e_{MAX}}$</td>
<td>15 A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_e$</td>
<td>0.01 Ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_e$</td>
<td>0.0044 kg</td>
<td></td>
</tr>
<tr>
<td>Battery</td>
<td>$C_b$</td>
<td>250 mAh</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_b$</td>
<td>0.016 Ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U_b$</td>
<td>4.35 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_b$</td>
<td>60 C</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_b$</td>
<td>0.0065 kg</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author

### Table 19: Minimum Cost Results

<table>
<thead>
<tr>
<th></th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance in Hover $t_{e_{MAX}}$</td>
<td>2.43 min</td>
<td>1.95 min</td>
<td>1.35 min</td>
</tr>
<tr>
<td>System Efficiency $\eta$</td>
<td>49.92%</td>
<td>52.14%</td>
<td>54.56%</td>
</tr>
<tr>
<td>Produced Propeller Thrust $T$</td>
<td>0.18 N</td>
<td>0.18 N</td>
<td>0.18 N</td>
</tr>
<tr>
<td>Payload mass $m_{LOAD}$</td>
<td>0.010 kg</td>
<td>0.014 kg</td>
<td>0.019 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle $\theta_{MAX}$</td>
<td>34.38 °</td>
<td>35.52 °</td>
<td>33.80 °</td>
</tr>
<tr>
<td>Maximum Flight Speed $V_{MAX}$</td>
<td>7.38 m/s</td>
<td>7.33 m/s</td>
<td>6.71 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance $Z_{MAX}$</td>
<td>901.85 m</td>
<td>701.38 m</td>
<td>447.24 m</td>
</tr>
<tr>
<td>Mass of Multicopter $m$</td>
<td>0.0468 kg</td>
<td>0.0614 kg</td>
<td>0.0940 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>55.33 USD</td>
<td>72.34 USD</td>
<td>106.43 USD</td>
</tr>
</tbody>
</table>

Source: Author

#### 3.5.1.8 Best Overall Set of Components

On the section 3.5.1 the sets that optimize the parameters introduced in 3.3 were obtained. However, for a determined application, the ideal would be select a set of components that optimizes a series of key parameters simultaneously. Therefore, a decision criterion should be used in order to determine a single set that could match with the application proposed. Hence, the **criterion P** expression is defined as
\[ P_{ijkr} = \frac{\sum_{n} w_n \frac{p_n}{p_{n,MAX}}}{\sum_{n} w_n} \] (3.46)

where \( N \) is the number of parameters in study, \( w_n \) is the weight of the parameter \( p_n \), and \( p_{n,MAX} \) is the maximum value the parameter \( p_n \) in the set of values in study. Ideally, the set \( ijkkr \) holds all parameters as their maximum values and \( P_{ijkr} \) is equal to 1.

For the delivering application, the key parameters are the time of endurance, the payload mass and the flight distance, being the two first parameters the most important ones. Then, the equation 3.46 assumes the values from table 20.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 ) Time of Endurance ( t_e )</td>
<td>( w_1 = 4 )</td>
</tr>
<tr>
<td>( p_2 ) Inverse of Propulsor Cost ( Cost^{-1} )</td>
<td>( w_2 = 1 )</td>
</tr>
<tr>
<td>( p_3 ) System Efficiency ( \eta )</td>
<td>( w_3 = 1 )</td>
</tr>
<tr>
<td>( p_4 ) Payload Mass ( m_{LOAD} )</td>
<td>( w_4 = 4 )</td>
</tr>
<tr>
<td>( p_5 ) Flight Speed ( V )</td>
<td>( w_5 = 1 )</td>
</tr>
<tr>
<td>( p_6 ) Flight Distance ( Z )</td>
<td>( w_6 = 2 )</td>
</tr>
</tbody>
</table>

Source: Author

By evaluating the criterion \( P \) for each of the set of parameters for the 3Rotor, 4Rotor and 6Rotor, the set of components which best optimize the parameters according to 3.46 was determined. The data for the sets chosen are presented in table 21 and the results summed up on table 22.

Naturally, increasing the the mass carried by the UAV, the time of endurance will decrease. The figure 21 shows how the time of endurance decreases exponentially as the payload mass is increased up to the maximum admissible value of payload mass for the three multicopter configurations. It can be seen that even for the best set of components, when the multicopter is carrying the maximum payload, the endurance time is reduced to a few minutes for all curves. Nevertheless, the blue curve (tricopter) has an endurance of around 25 min, and performs better than the hexacopter in terms of endurance and is able to carry more weight on. However, the quadrotor presents a slight better performance for payloads heavier than 1 kg and a higher payload capacity, and despite having a higher propulsor cost and a lower endurance-cost ratio than the tricopter, presents less complications for the controlling. In this way, for a flight where the quadrotor carries the maximum payload, the flight time is around 2 minutes, which makes the UAV suitable for a delivery distance radius of 1.4 km at maximum speed considering the return to the
Therefore, the configuration to be used on the project will be the 4Rotor. The complete data for the components and the results obtained are present in the tables 21 and 22.

**Table 21: Best Overall Set of Components Data**

<table>
<thead>
<tr>
<th>Component</th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller</td>
<td>$D_p$</td>
<td>0.1524 m</td>
<td>0.0650 m</td>
</tr>
<tr>
<td></td>
<td>$H_p$</td>
<td>0.1143 m</td>
<td>0.1118 m</td>
</tr>
<tr>
<td></td>
<td>$B_p$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$m_p$</td>
<td>0.0049 kg</td>
<td>0.0004 kg</td>
</tr>
<tr>
<td></td>
<td>$m_{af}$</td>
<td>0.1276 kg</td>
<td>0.2135 kg</td>
</tr>
<tr>
<td></td>
<td>$S$</td>
<td>0.080 $m^2$</td>
<td>0.112 $m^2$</td>
</tr>
<tr>
<td>Motor</td>
<td>$K_{V0}$</td>
<td>1700 RPM/V</td>
<td>4500 RPM/V</td>
</tr>
<tr>
<td></td>
<td>$I_{m,MAX}$</td>
<td>37.2 A</td>
<td>14.4 A</td>
</tr>
<tr>
<td></td>
<td>$I_{m0}$</td>
<td>1 A</td>
<td>0.76 A</td>
</tr>
<tr>
<td></td>
<td>$U_{m0}$</td>
<td>10 V</td>
<td>7 V</td>
</tr>
<tr>
<td></td>
<td>$R_m$</td>
<td>0.076 Ω</td>
<td>0.145 Ω</td>
</tr>
<tr>
<td></td>
<td>$m_m$</td>
<td>0.0331 kg</td>
<td>0.0085 kg</td>
</tr>
<tr>
<td>ESC</td>
<td>$I_{e,MAX}$</td>
<td>45 A</td>
<td>40 A</td>
</tr>
<tr>
<td></td>
<td>$R_e$</td>
<td>0.01 Ω</td>
<td>0.01 Ω</td>
</tr>
<tr>
<td></td>
<td>$m_e$</td>
<td>0.0042 kg</td>
<td>0.0026 kg</td>
</tr>
<tr>
<td>Battery</td>
<td>$C_b$</td>
<td>3700 mAh</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_b$</td>
<td>0.016 Ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U_b$</td>
<td>14.8 V</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_b$</td>
<td>45 C</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_b$</td>
<td>0.285 kg</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author

**Table 22: Best Overall Set of Components Results**

<table>
<thead>
<tr>
<th>Criterion</th>
<th>3Rotor</th>
<th>4Rotor</th>
<th>6Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance in Hover $t_{e,MAX}$</td>
<td>24.8 min</td>
<td>21.5 min</td>
<td>16.5 min</td>
</tr>
<tr>
<td>System Efficiency $\eta$</td>
<td>77.72 %</td>
<td>77.92 %</td>
<td>69.68 %</td>
</tr>
<tr>
<td>Produced Propeller Thrust $T$</td>
<td>13.68 N</td>
<td>13.68 N</td>
<td>5.47 N</td>
</tr>
<tr>
<td>Payload mass $m_{LOAD}$</td>
<td>3.542 kg</td>
<td>4.798 kg</td>
<td>2.819 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle $\theta_{MAX}$</td>
<td>81.36 °</td>
<td>81.93 °</td>
<td>80.79 °</td>
</tr>
<tr>
<td>Maximum Flight Speed $V_{MAX}$</td>
<td>23.94 m/s</td>
<td>23.51 m/s</td>
<td>32.38 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance $Z_{MAX}$</td>
<td>6802.59 m</td>
<td>5484.84 m</td>
<td>6050.28 m</td>
</tr>
<tr>
<td>Mass of Multicopter $m$</td>
<td>0.6423 kg</td>
<td>0.781 kg</td>
<td>0.526 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>161.46 USD</td>
<td>199.48 USD</td>
<td>205.93 USD</td>
</tr>
</tbody>
</table>

Source: Author
3.6 Summary

The aim of this chapter was to define and study proper models for the components of the propulsion system as a first step towards the design process.

After defining the environmental parameters, the models for propellers, motors, ESC’s and battery are presented, and additionally, the mass of the airframe and its surface are defined as function of the propeller dimensions.

After the modelling process, the optimal parameters to be looking for are defined as:

- Maximum time of endurance in hover $t_{e\text{MAX}}$
- Maximum system efficiency $\eta_{\text{MAX}}$
- Maximum produced propeller thrust $T_{\text{MAX}}$
- Maximum payload mass $m_{\text{MAXLOAD}}$
- Maximum pitch angle $\theta_{\text{MAX}}$
- Maximum forward speed $V_{\text{MAX}}$
- Maximum flight distance $Z_{\text{MAX}}$
- Minimum mass of propulsors $m_{\text{MIN}}$
• Parameters of minimum cost

For each of these items a set of components which maximize the particular parameter is determined and the results for each of those sets are presented for each configuration of multicopter. Such results show that each set might be used for a particular application, but none of them are completely adequate for the application the report focus on: delivering. Nonetheless, those results are important to define the maximum value of each of the parameter analyzed.

The last section defines the criterion decision method, and with the aim of the maximum values defined previously, the set that best fit the delivering application is determined in three configurations: tricopter, quadrotor and hexacopter.

The set chosen was then determined and the data of the components are once more presented on the following table.

<table>
<thead>
<tr>
<th>Table 23: Best Overall Set of Components Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Motor</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ESC</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Battery</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Source: Author

The results are summarized once more as well.
Table 24: Best Overall Set of Components Results

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of Endurance in Hover $t_{eMAX}$</td>
<td>21.5 min</td>
</tr>
<tr>
<td>System Efficiency $\eta$</td>
<td>77.92 %</td>
</tr>
<tr>
<td>Produced Propeller Thrust T</td>
<td>13.68 N</td>
</tr>
<tr>
<td>Payload mass $m_{LOAD}$</td>
<td>4.798 kg</td>
</tr>
<tr>
<td>Maximum Pitch Angle $\theta_{MAX}$</td>
<td>81.93 °</td>
</tr>
<tr>
<td>Maximum Forward Flight Speed $V_{MAX}$</td>
<td>23.51 m/s</td>
</tr>
<tr>
<td>Maximum Flight Distance $Z_{MAX}$</td>
<td>5484.84 m</td>
</tr>
<tr>
<td>Mass of Multicopter $m$</td>
<td>0.781 kg</td>
</tr>
<tr>
<td>Cost of Propulsors</td>
<td>199.48 USD</td>
</tr>
</tbody>
</table>

Source: Author
4 CONTROL SIMULATION

4.1 Overview of Modeling

As it has been presented on a previously, the movement of the quadrotor can be described by two set of equations: the Newton’s Second Law which describes the acceleration of the center of mass of the quadrotor and therefore its linear movement; and the Euler’s Equations which describes the angular movement. On the figure 22 the forces and moments acting on the airframe of the quadrotor are presented, where the forces $F_i$ are the forces generated by the propeller and are responsible for the vertical movement, and the roll and pitch of the quadrotor; while the moments $M_i$ are the drag moments also generated by the action of the propellers and responsible for the yaw movement. The directions $x_B, y_B$ and $z_B (B b_1 b_2 b_3)$ define the systems of coordinates of body frame, meanwhile $x_W, y_W$ and $z_W (E e_1 e_2 e_3)$, on figure 2) define the inertial frame. As the equations for the movement will be described in the body frame, for simplicity of notation the position vector will be described simply as $r = [x, y, z]$. Lastly, the aerodynamical model will be presented due to its importance on the evaluation of the motor dynamics.

Figure 22: Forces and moments acting on the quadrotor frame

\[
\begin{align*}
F_1 & \quad M_1 \\
F_2 & \quad M_2 \\
F_3 & \quad M_3 \\
F_4 & \quad M_4
\end{align*}
\]

Source [12]
4.2 Newton’s Second Law

As previously presented, the Newton’s second Law for the quadrotor can be described as

\[ m\ddot{r} = mg\mathbf{e}_3 + \mathbf{R}f \]  

(4.1)

where \( \mathbf{e}_3 = [0 \ 0 \ 1]^T \) is the versor of the z direction, \( f \) is the total thrust generated by the rotors altogether and \( \mathbf{R} \) is the rotation matrix defined as

\[
\mathbf{R} = \begin{bmatrix}
c\psi c\theta - s\psi s\phi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\
s\psi c\theta + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\theta s\phi c\psi \\
-c\phi s\theta & s\phi & c\phi c\theta
\end{bmatrix}
\]  

(4.2)

Notice that the blade flapping are not be taken into consideration, since its effects can be neglected in comparison with the other forces.

4.3 Euler’s Equations

As presented previously, the Euler’s Equations describe the rotational dynamics and can be written as

\[ J \cdot \dot{\mathbf{\Theta}} = \tau - \mathbf{\Theta} \times J\Theta \]  

(4.3)

where \( \mathbf{\Theta} = [\phi \ \theta \ \psi]^T \).

4.4 Aerodynamics Model

The set of equations for the thrust and moments acting on the airframe in a quadrotor moving in X configuration (figure 23) can be expressed as

\[
\begin{bmatrix}
f \\
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix} =
\begin{bmatrix}
c_T & c_T & c_T & c_T \\
\frac{1}{2}c_T & \frac{1}{2}c_T & -\frac{1}{2}c_T & -\frac{1}{2}c_T \\
-\frac{1}{2}c_T & \frac{1}{2}c_T & \frac{1}{2}c_T & -\frac{1}{2}c_T \\
c_M & -c_M & c_M & -c_M
\end{bmatrix}
\begin{bmatrix}
\omega_1^2 \\
\omega_2^2 \\
\omega_3^2 \\
\omega_4^2
\end{bmatrix}
\]  

(4.4)
4.5 Linearization of the Model

The set of equations 4.1 and 4.3 despite describing properly the dynamics of the quadrotor are non-linear and highly coupled which makes the analysis and control much more complex. Hence, to simplify the control some reasonable assumptions will be taken in order to linearize the equations. Firstly, the equations for the acceleration of the center of mass will be rewritten as

\[
\begin{align*}
\ddot{x} &= \frac{L}{m}(c\psi s\theta + c\theta s\phi s\psi) \\
\ddot{y} &= \frac{L}{m}(s\psi s\theta - c\psi c\theta s\phi) \\
\ddot{z} &= \frac{L}{m}c\phi c\theta - g
\end{align*}
\]

(4.5)

Here two main assumptions are adopted:

- The angles are considered small (which according to [12] the model will be quite robust for angles up to 60°);

- The thrust generated by the motors approximate the weight of the quadrotor (hover condition).

According to the small angles approximation, one has the following relations: \(\sin\phi \approx \phi\), \(\cos\phi \approx 1\), \(\sin\theta \approx \theta\), \(\cos\theta \approx 1\) and moreover \(f \approx mg\). Therefore the set of equations 4.5 can be rewritten as
\begin{equation}
\begin{align*}
\ddot{x} &= g(\phi s \psi + \theta c \psi) \\
\ddot{y} &= g(-\phi c \psi + \theta s \psi) \\
\ddot{z} &= \frac{L}{m} - g
\end{align*}
\end{equation}

where the two first equations are considered to be approximately $mg$, while on the third equation this assumption is not valid, otherwise there wouldn’t be any input on the altitude channel. Notice that 4.6 is now a linear model.

Similarly, the system of equations 4.3 can be rewritten as

\begin{equation}
\begin{align*}
J_{xx} \dot{p} &= \tau_x - qr (J_{zz} - J_{yy}) \\
J_{yy} \dot{q} &= \tau_y - pr (J_{xx} - J_{zz}) \\
J_{zz} \dot{r} &= \tau_z - pq (J_{yy} - J_{xx})
\end{align*}
\end{equation}

It can be assumed the yaw angle remains constant and therefore the component $r$ is small. Moreover, around the hover conditions the simplifications $p \approx \dot{\phi}$, $q \approx \dot{\theta}$ and $r \approx \dot{\psi}$ can be considered, which is translate into

\begin{equation}
\begin{align*}
J_{xx} \ddot{\phi} &= \tau_x \\
J_{yy} \ddot{\theta} &= \tau_y \\
J_{zz} \ddot{\psi} &= \tau_z - \dot{\phi} \dot{\theta} (J_{yy} - J_{xx})
\end{align*}
\end{equation}

Usually due to the simmetry $J_{yy} \approx J_{xx}$ and the subtractive term on the third equation can be neglected. However, this is not true for the quadrotor in question since there are no simmetry around the axis $z_b$ as it can be seen by its airframe in figure 24.
4.6 The Nested Control Loop

After the overview of the model, the Nested Loop can finally be presented. The Nested Loop consists of the loops that form the controlling logic. As in figure 25 the 4 fundamentals blocks for the controlling of the quadrotor are:

- **Position Controller:** or *outer loop* is the block responsible for the translational dynamics. It has as input the desired trajectory and is feedback by the current position and velocity of the quadrotor in order to adjust the error. This block aims to deliver the desired roll and pitch angles and thrust.

- **Attitude Controller:** or *inner loop* receives the desired angles command from the position controller and the desired yaw angle from the trajectory planner and is feedback by the current angular position and angular velocity from the quadrotor and with those data calculates the desired moments.

- **Motor Dynamics:** The motor dynamics block has as inputs the desired thrust from the position controller and the desired moments from the attitude controller. However, the motor dynamics has a delay before achieving the desired thrust and moments. Roughly speaking, this delay is the core of the motor dynamics block.

- **Rigid Body Dynamics:** This is the block which represents the quadrotor itself by means of its mathematical model and has as input the actual thrust and moments generated by the motors. In practice, the actual angular position and velocities as
the linear position and velocities of the quadrotor are estimated by sensors installed on the airframe and those values differ from the actual ones according to the dynamic of the sensor. However, in order to simplify the approach, the dynamic of the sensors will be neglected and the feedback values will be obtained through the mathematical model of the quadrotor.

4.6.1 Position Controller

The aim of the position controller (outer loop) is to generated as output the desired thrust for the motor, and the desired angular position for the attitude controller. In order to achieve such outputs a command acceleration $r_c$ need to be generated by the position controller. Thus, the position error $e_p \in \mathbb{R}^3$ is defined as the difference between the desired trajectory $r_d$ and the actual trajectory $r$, with $r_c,r \in \mathbb{R}^3$; and then $e_p = (r_d - r)$. Ideally the position error $e_p$ should become null exponentially in time. From the Control Theory (as it described in details in [13]), this can be satisfied by means of a PID controller designed as

$$ (\ddot{r}_d - \dot{r}_c) + K_d(\dot{r}_d - \dot{r}) + K_p(r_d - r) + K_i \int (r_d - r) = 0 \quad (4.9) $$

where $K_d,k_p$ and $K_i \in \mathbb{R}^{3x3}$ are the diagonal matrices of derivative, proportional and integrative gains, respectively. Isolating $\dot{r}_c$ in the equation 4.10 one can have

$$ \dot{r}_c = \ddot{r}_d + K_d(\dot{r}_d - \dot{r}) + K_p(r_d - r) + K_i \int (r_d - r) \quad (4.10) $$
By multiplying the first of the equations 4.6 by $s\psi$ and the second by $-c\psi$ and summing both equations the following expression is obtained

$$\phi = g^{-1}(\dot{x}s\psi - \dot{y}c\psi)$$

(4.11)

Similarly, an expression for the pitch angle $\theta$ can be obtained as

$$\theta = g^{-1}(\dot{x}c\psi + \dot{y}s\psi)$$

(4.12)

Those equations can be used together with the command acceleration $r_c$ to obtain the expression for the desired attitude as

$$\phi_d = g^{-1}(\dot{x}_c s\psi_d - \dot{y}_c c\psi_d)$$

$$\theta_d = g^{-1}(\dot{x}_c c\psi_d + \dot{y}_c s\psi_d)$$

(4.13)

or in matrix form

$$\Theta_d = g^{-1}A_\psi r_{c,h}$$

(4.14)

where $r_{c,h} \in \mathbb{R}^2$ is the horizontal commanded acceleration.

To complete the position controller as in figure 25, the only step left is calculating the desired thrust, the input of the motor dynamics block, which is obtained by rewriting the third equation of 4.6 and making $f = f_d$ and $\ddot{z} = \ddot{z}_d$. Then, one has

$$f_d = m(g + \ddot{z}_d)$$

(4.15)

which completes the position controller.

### 4.6.1.1 Direction Guaranteed Saturation Function

As discussed previously, the goal of the controller is to make the error between the desired quantity to the actual one goes exponentially to zero. However, it can happen of this error being too large on practice, which can cause problems. Therefore a threshold should be applied in order to avoid such issues. Before introducing the Direction Guaranteed Saturation Function as in [3], the simple saturation function will be analyzed. The saturation function $\text{sat}(x, a)$ with $x \in \mathbb{R}$ and $a \in \mathbb{R}_+$ is defined as

$$\text{sat}(x, a) = \begin{cases} 
  x_k, & |x_k| \leq a \\
  a \cdot \text{sign}(x_k), & |x_k| > a 
\end{cases}$$

(4.16)
In case $x \in \mathbb{R}^n$, the saturation function becomes

$$sat(x, a) = \begin{bmatrix} sat(x_1, a) \\ \vdots \\ \vdots \\ sat(x_n, a) \end{bmatrix}$$ \hspace{1cm} (4.17)

For the first case where $x$ is a scalar value, the simple saturation function works fine, controlling the error and making it assume a pre-determined value in case it becomes larger than the threshold. The limitations arise when the $x \in \mathbb{R}^n (n \in \mathbb{Z})$ where despite the error been keep limited, the direction of it can stroll from the desired one as shown in figure 26.

To avoid such issue, a Direction-Guaranteed Saturation Function is designed as

$$sat_{gd}(x, a) = \kappa_a(x) \cdot x$$ \hspace{1cm} (4.18)

where

$$\kappa_a(x) = \begin{cases} 1, & ||x||_\infty \leq a \\ \frac{a}{||x||_\infty}, & ||x||_\infty > a \end{cases}$$ \hspace{1cm} (4.19)

and $x \in \mathbb{R}^n$, $a \in \mathbb{Z}^+$ and $||x||_\infty = \text{max}(|x_1|, ..., |x_n|)$. As it can be seen in figure 26 besides keeping the error under a pre-determined value, the Direction Guaranteed Saturation Function has the advantage of maintaining the direction of the vector $x$.

Figure 26: Simple saturation function and direction guaranteed saturation function

![Diagram of Simple and Direction-Guaranteed Saturation Functions](source: [3])

Therefore, using the Direction Guaranteed Saturation Function the equations 4.13
and 4.15 are rewritten as

\[
\begin{align*}
\Theta_d &= \text{sat}_{gd}(g^{-1}A_{\phi}r_{c,h}, \theta_{\text{max}}) \\
f_d &= \text{sat}_{gd}(m(g + \ddot{z}_d) - \frac{f_{\text{min}} + f_{\text{max}}}{2}, \frac{f_{\text{max}} - f_{\text{min}}}{2}) + \frac{f_{\text{min}} + f_{\text{max}}}{2}
\end{align*}
\]  

(4.20)

where \(\theta_{\text{max}}\) has been evaluated from the previous report, \(f_{\text{max}}\) is established by the motor manufacturer and \(f_{\text{min}}\) is considered as 10% of the weight of the quadrotor.

### 4.6.2 Attitude Controller

The aim of the attitude controller is evaluate the desired moments required for keeping with the trajectory. As in [3] the converge ratio of the inner loop should be 4-10 higher than the position controller in order to guarantee that the outputs of the position controller have been already achieved and by consequence the attitude controller will comply with its tasks.

According to reference [12] the desired moment can be evaluated by using PD control laws as

\[
\tau_d = \begin{bmatrix}
k_{p,\phi}(\phi_d - \phi) + k_{d,\phi}(p_d - p) \\
k_{p,\theta}(\theta_d - \theta) + k_{d,\theta}(q_d - q) \\
k_{p,\psi}(\psi_d - \psi) + k_{d,\psi}(r_d - r)
\end{bmatrix}
\]  

(4.21)

On practice saturation values need to be taken into consideration and the vector of desired moments is rewritten as

\[
\tau_d = \text{sat}_{gd}\left(\begin{bmatrix}
k_{p,\phi}(\phi_d - \phi) + k_{d,\phi}(p_d - p) \\
k_{p,\theta}(\theta_d - \theta) + k_{d,\theta}(q_d - q) \\
k_{p,\psi}(\psi_d - \psi) + k_{d,\psi}(r_d - r)
\end{bmatrix}, \tau_{\text{max}}\right)
\]  

(4.22)

with \(\tau_{\text{max}} \in 3x3\). Recollecting the set of equations 4.4 the maximum moments for the axes x, y and z are given by

\[
\begin{align*}
\tau_{\text{max},x} &= \tau_{\text{max},y} = l \cdot f_{\text{max}} \\
\tau_{\text{max},z} &= \frac{2}{n\cdot c_T} f_{\text{max}}
\end{align*}
\]  

(4.23)

where \(c_M = 2.925 \cdot 10^{-9} \text{ [Nm/rpm}^2\text{]}\) and \(c_T = 1.4865 \cdot 10^{-7} \text{ [N/rpm}^2\text{]}\) as from [3].
4.6.3 Motor Dynamics

The motor dynamic is a really complicated function that depends on many factors as range of speed, acceleration or deceleration condition, etc. Moreover, the effective thrust and moments combine the dynamic of the propeller, which is compounded by really complicated relations as well. In order to simplify the analysis without losing generality, the dynamics of the motor is simplified by a first order function described by

\[
\omega_i = \omega_{d,i} \left( 1 - e^{\frac{t}{\tau_m}} \right) \tag{4.24}
\]

where \( \omega_{d,i} \) is the desired speed of the i-th motor and \( \omega_i \) is its actual speed. The \( \omega_d \) is calculated by the inversion of the matrix

\[
\begin{bmatrix}
  f_d \\
  \tau_d
\end{bmatrix} =
\begin{bmatrix}
  c_T & c_T & c_T & c_T \\
  \frac{1}{2}c_T & \frac{1}{2}c_T & -\frac{1}{2}c_T & -\frac{1}{2}c_T \\
  -\frac{1}{2}c_T & \frac{1}{2}c_T & \frac{1}{2}c_T & -\frac{1}{2}c_T \\
  c_M & -c_M & c_M & -c_M
\end{bmatrix}
\begin{bmatrix}
  \omega_{d,1}^2 \\
  \omega_{d,2}^2 \\
  \omega_{d,3}^2 \\
  \omega_{d,4}^2
\end{bmatrix} \tag{4.25}
\]

The actual thrust and moments generated by the motors are obtained by evaluating the equation 4.4 using the values of \( \omega_i \) values obtained thorough equation 4.24.

4.6.3.1 Adjust to PID Parameters

By inverting 4.25, one has

\[
\begin{bmatrix}
  \omega_{d,1}^2 \\
  \omega_{d,2}^2 \\
  \omega_{d,3}^2 \\
  \omega_{d,4}^2
\end{bmatrix} =
\begin{bmatrix}
  \frac{1}{4c_T} & \frac{1}{2c_T} & -\frac{1}{2c_T} & -\frac{1}{2c_T} \\
  \frac{1}{4c_T} & \frac{1}{2c_T} & \frac{1}{2c_T} & -\frac{1}{2c_T} \\
  \frac{1}{4c_T} & -\frac{1}{2c_T} & \frac{1}{2c_T} & -\frac{1}{2c_T} \\
  \frac{1}{4c_T} & -\frac{1}{2c_T} & -\frac{1}{2c_T} & -\frac{1}{2c_T}
\end{bmatrix}
\begin{bmatrix}
  f_d \\
  \tau_d
\end{bmatrix} \tag{4.26}
\]

that can be rewritten into

\[
\begin{bmatrix}
  \omega_{d,1}^2 \\
  \omega_{d,2}^2 \\
  \omega_{d,3}^2 \\
  \omega_{d,4}^2
\end{bmatrix} =
\begin{bmatrix}
  \frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} \\
  \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{4} \\
  \frac{1}{4} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\
  \frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{4}
\end{bmatrix}
\begin{bmatrix}
  f_d \\
  \epsilon_T \\
  \epsilon_T \\
  \epsilon_T
\end{bmatrix} \tag{4.27}
\]
which finally leads to
\[
\begin{bmatrix}
\omega_{d,1}^2 \\
\omega_{d,2}^2 \\
\omega_{d,3}^2 \\
\omega_{d,4}^2
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} \\
\frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{4} \\
\frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{4}
\end{bmatrix}
\begin{bmatrix}
u_{d,1} \\
u_{d,2} \\
u_{d,3} \\
u_{d,4}
\end{bmatrix}
\] (4.28)

or in the short form
\[
\begin{bmatrix}
\omega_{d,i}^2
\end{bmatrix} = M_{4x4} \cdot \mathbf{u}_d.
\] (4.29)

This way the coefficients \(c_T\) and \(c_M\) that usually are unknown can be compensated by the PID parameters.

### 4.6.4 Rigid Body Dynamics

The actual values for position \(\mathbf{r}\) and velocity \(\dot{\mathbf{r}}\), and the angular position \(\Theta\) and angular velocity \(\dot{\Theta}\) can be obtained by the combination of sensors (as accelerometers, GPS’s, ultrasounds, cameras, etc) installed on board of the quadrotor. For the sake of the simulation, such values will be obtained by means of the rigid body movement equations. Those equations are

\[
\ddot{x} = g(\phi s \psi + \theta c \psi)
\]
\[
\ddot{y} = g(-\phi c \psi + \theta s \psi)
\]
\[
\ddot{z} = \frac{L}{m} - g
\] (4.30)

and

\[
J_{xx}\ddot{\phi} = \tau_x
\]
\[
J_{yy}\ddot{\theta} = \tau_y
\]
\[
J_{zz}\ddot{\psi} = \tau_z - \dot{\phi}\dot{\theta}(J_{yy} - J_{xx})
\] (4.31)

whose the values can be integrated in order to obtain the linear and angular velocity. Those can be integrated once more into position and attitude.

### 4.7 Non Dimensional Tunning

After obtaining the controller model, the only task left is to tunning the PID parameters, which can be a really complicated task. In order to simplify this approach, the non dimensional tunning presented on reference [12] will be used as a first step of the tunning. Starting by the attitude controller, the equation of motion for the rotation around the
axis x is used as an example. This equation is expressed by

\[ J_{xx}\ddot{\phi} + k_{d,\phi}\dot{\phi} + k_{p,\phi}\phi = 0 \]  \hspace{1cm} (4.32)

Since this a second order system, it will be compared to the general equation of motion given by

\[ \ddot{\phi} + 2\xi\omega_n\dot{\phi} + \omega_n^2\phi = 0 \]  \hspace{1cm} (4.33)

where \( \xi \) is the damping ratio and \( \omega_n \) is the natural frequency. By comparing equations 4.32 and 4.33, the PD parameters can be obtained as

\[ k_{p,\phi} = J_{xx}\omega_n^2 \]  \hspace{1cm} (4.34)

and

\[ k_{d,\phi} = 2J_{xx}\xi\omega_n \]  \hspace{1cm} (4.35)

According to tests, \( \xi \approx 1 \) and the natural frequency \( \omega_n \approx 9 \text{ rad/s} \). A similar method for position controller with the damping ratio around 1 and a natural frequency equal to 4 rad/s will be adopted as well.

Although this is not a perfect method for tuning, it is useful as a first attempt and with the results obtained by this first simulation, the PID parameters can be optimized until reaching satisfactory values.

### 4.8 Simulations

In order to evaluate the effectiveness of the control model, a simulation on MATLAB (which the code is available on the appendix G) has been carried out. All the physical parameters used on the simulation are present on the table 25, while the PID parameters are shown on the table 26.

The quadrotor is expected to fly at a constant altitude describing a square shape trajectory. The desired trajectory \( r_d \) and \( \psi_d \) are presented on the table 27 and the initial conditions are
Table 25: Quadrotor parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m ) [kg]</td>
<td>1.3420</td>
</tr>
<tr>
<td>( g ) [m/s(^2)]</td>
<td>9.81</td>
</tr>
<tr>
<td>( J_{xx} ) [kg( \cdot )m(^2)]</td>
<td>0.0034</td>
</tr>
<tr>
<td>( J_{yy} ) [kg( \cdot )m(^2)]</td>
<td>0.0054</td>
</tr>
<tr>
<td>( J_{zz} ) [kg( \cdot )m(^2)]</td>
<td>0.0050</td>
</tr>
<tr>
<td>( l ) [mm]</td>
<td>182.9</td>
</tr>
<tr>
<td>( f_{\text{max}} ) [N]</td>
<td>68.9839</td>
</tr>
<tr>
<td>( f_{\text{min}} ) [N]</td>
<td>1.4554</td>
</tr>
<tr>
<td>( \tau_{\text{max,x,y}} ) [Nm]</td>
<td>12.6172</td>
</tr>
<tr>
<td>( \tau_{\text{max,z}} ) [Nm]</td>
<td>0.6787</td>
</tr>
<tr>
<td>( \theta_{\text{max}} ) [rad]</td>
<td>1.4299</td>
</tr>
<tr>
<td>( T_{\text{m}} ) [s]</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Source: Author

Table 26: PID parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{p,x} )</td>
<td>16</td>
</tr>
<tr>
<td>( k_{d,x} )</td>
<td>8</td>
</tr>
<tr>
<td>( k_{i,x} )</td>
<td>0.01</td>
</tr>
<tr>
<td>( k_{p,y} )</td>
<td>16</td>
</tr>
<tr>
<td>( k_{d,y} )</td>
<td>8</td>
</tr>
<tr>
<td>( k_{i,y} )</td>
<td>0.01</td>
</tr>
<tr>
<td>( k_{p,z} )</td>
<td>64</td>
</tr>
<tr>
<td>( k_{d,z} )</td>
<td>160</td>
</tr>
<tr>
<td>( k_{i,z} )</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Source: Author

\[
\begin{align*}
\mathbf{r}_0 &= [0 \ 0 -10]^T \quad [m] \\
\dot{\mathbf{r}}_0 &= [0 \ 0 0]^T \quad [m/s] \\
\mathbf{\Theta}_0 &= [0 \ 0 0]^T \quad [\text{rad}] \\
\dot{\mathbf{\Theta}}_0 &= [0 \ 0 0]^T \quad [\text{rad/s}] \\
\omega_0 &= [4000 \ 4000 \ 4000 \ 4000]^T \quad [\text{rpm}]
\end{align*}
\]

Table 27: Desired trajectory

<table>
<thead>
<tr>
<th>( \Delta t ) [s]</th>
<th>( x_d ) [m]</th>
<th>( y_d ) [m]</th>
<th>( z_d ) [m]</th>
<th>( \psi_d ) [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 5)</td>
<td>2t</td>
<td>0</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td>[5, 10)</td>
<td>10</td>
<td>2(t-5)</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td>[10, 15)</td>
<td>10 - 2(t-10)</td>
<td>10</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td>[15, 20)</td>
<td>0</td>
<td>10 - 2(t-15)</td>
<td>-10</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Author

The results are presented on the figures 27, 28, 29 and 30 where the dashed lines represent the desired values and the continuous line, the actual ones. As it can be seen on the figures, the control method and the PID parameters adopted return satisfactory values. For the z position, for example, even though the values overshoot, the error is limited to less than 3 cm.

Lastly, the figure 31 shows the trajectory for the quadcopter. It can be visualized that the most problematic zones are the corners of the square, where the quadcopter needs to change direction abruptly. Those regions corresponds to the peaks on the graphics of the
figures 29 and 30, which is coherent with the practice, since the quadrotor changes its attitude in order to manoeuvre.
4.9 Summary

This chapter presented the control method for a quadrotor tracking a trajectory. The control was split into 4 blocks: position controller, attitude controller, motor dynamics and rigid body dynamics. Considerations about the saturation has been presented and the direction-guaranteed functions has been introduced. Lastly, the simulation results were presented.

The main equations for the control are here summarized:

- **Position Controller**

  \[
  \ddot{\mathbf{r}}_c = \ddot{\mathbf{r}}_d + \mathbf{K}_d (\ddot{\mathbf{r}}_d - \ddot{\mathbf{r}}) + \mathbf{K}_p (\mathbf{r}_d - \mathbf{r}) + \mathbf{K}_i \int (\mathbf{r}_d - \mathbf{r}) \\
  \dot{\Theta}_d = \text{sat}_{gd} (g^{-1} \mathbf{A}_\psi \mathbf{r}_{c,h} \cdot \dot{\Theta}, \theta_{max}) \\
  f_d = \text{sat}_{gd} \left( m(g + \ddot{z}_d) - \frac{f_{min} + f_{max}}{2}, \frac{f_{max} - f_{min}}{2} \right) + \frac{f_{min} + f_{max}}{2}
  \]

- **Attitude Controller**

  \[
  \tau_d = \text{sat}_{gd} \left( \begin{bmatrix} k_{p,\phi} (\dot{\phi}_d - \dot{\phi}) + k_{d,\phi} (p_d - p) \\ k_{p,\theta} (\dot{\theta}_d - \dot{\theta}) + k_{d,\theta} (q_d - q) \\ k_{p,\psi} (\dot{\psi}_d - \dot{\psi}) + k_{d,\psi} (r_d - r) \end{bmatrix} , \tau_{max} \right)
  \]

- **Motor Dynamics**

  \[
  \begin{bmatrix} \omega_{d,i}^2 \end{bmatrix} = \mathbf{M}_{4x4} \cdot \mathbf{u}_d \\
  \omega_i = \omega_{d,i} \left( 1 - e^{-\frac{t}{\tau_m}} \right) \\
  \mathbf{u} = \mathbf{M}_{4x4}^{-1} \cdot [\omega_i^2]
  \]
• Rigid Body Dynamics

\[
\begin{align*}
\ddot{x} &= g(\phi s \psi + \theta c \psi) \\
\ddot{y} &= g(-\phi c \psi + \theta s \psi) \\
\ddot{z} &= \frac{f}{m} - g \\
J_{xx} \ddot{\phi} &= \tau_x \\
J_{yy} \ddot{\theta} &= \tau_y \\
J_{zz} \ddot{\psi} &= \tau_z - \dot{\phi} \dot{\theta} (J_{yy} - J_{xx})
\end{align*}
\]
5 UAV DESIGNING AND ASSEMBLING

In this part, the design, the placement of the electronics and the assembly of the UAV will be presented. The chapter is divided in 4 parts: Design of the Airframe, Manufacturing the Airframe, Onboard Electronics and Assembling the UAV. The first one presents the design of the airframe from its first sketches on paper until the design on the AutoDesk Inventor software. The second one explains the manufacturing process of the airframe through the relatively new process of additive manufacturing by the FDM molding technology. After that, the onboard electronics, its main components and the placement of the electronics are shown. Lastly, the final assembly of the UAV is presented.

5.1 Design of the Airframe

The initial design of the airframe is presented on the figure 35. As it can be seen on the figure, the design differs from the conventional commercial “drones” nowadays and was at the same time inspired in a F1 race vehicle (observe the supports of the frontal motors/propellers) and a fighter aircraft. It’s important to mention that the airframe has been sized using as a main dimension the size of the propellers as presented on the section 3.1.3.

On figures 33 and 34 its presented the internal part of the CAD model of the airframe. As it can be see by the figures the airframe has a shallow profile with several cavities on it, which contributes to decrease the total weight of the structure and provide a better cooling surface to avoid the overheating of the electronics and the battery. Also, it can be noticed some differences between the initial draft and the CAD model; those were implemented after the FEM analysis of the previous models.

On 36 the details inscribed on the internal part of the airframe (base of the battery). Those inscriptions name the 4 motors of the UAV and shows the orientation of the mobile axis. The “RHO” indication stands for “Right-Hand Orientation” and places the z-axe (pointing outside of the paper).
Lastly, figures 37, 38, 39, 40 and 41 shows the complete CAD model of the airframe with the cover included. The cover and base part of the airframe are assembled together with the aim of 8 M3 screws.
5.2 Manufacturing the Airframe

The prototype of the project has been manufactured making use of the additive manufacture method FDM (Fused Deposition Modelling). The 3D printer used was the Ender-3 from the Creality, which is one of the most suitable machines for beginners since it is easy to learn compared to other 3D printers and is relatively cheap (it costs around USD 300). The characteristics of the printing are presented on the table 28.
The Creality Under-3 comes with an SD-card compartment, where the printable file can be transferred from a computer. The process of creating the printable file from the
Autodesk Inventor is quite simple: the first thing is saving the CAD into a .stl file which can be read by a kind of software called "slicer". This last converts the .stl file into a series of coordinates that the printer should move in order to build up the prototype. This series of coordinates is the well known file ".g" used on CNC machines for example. Besides the coordinates, those file carries the main characteristics of the printing. Those characteristics are presented on the table 29 below.

Mechanical Strength is not the strong suit of the FDM process. Said that, the PETG
material is the best choice when comes to prototyping due its lower prices if compared to other materials, its relatively high strenght, and its one of the materials that are the most easy to work with. It is worth mentioning that an important characteristic when working with FDM process, its the ability of the material to stick to the bed of the printer: if a material does not stick properly, there is a risk of detachment from the bed during the print process ruinning the next layers of the printing; a material that usually has this problem is the PLA. From other side, there are some materials that stick so good to the
bed, that sometimes the bed is ruined after the printing, when taking out the printed part; and this might be the case of the PETG. Of course there are methods to circumvent those negative characteristics of those materials, however it is out of the scope of this work. On this thesis, the material used was the "BQ Easy Go PET-G" which its technical sheet is present at the end of this text on the annex A.

Before starting the printing of the actual airframe, a smaller prototype in scale 1:4 has been printed in order to identify potential printing issues and parts where the design should be improved. This first printing took **2h 40 min** (room temperature: 23 °C), and the result is showed on the picture 42 (without a scale).

**Figure 42: Airframe Scaled Prototype**

![Airframe Scaled Prototype](source: Author)

After the prototype, some configurations of the printing process was changed, as the
placement of the supports, since the central part of the print had no support at all and the filament was dropped "in the air".

The actual airframe has been printed in two parts: first, the cover has been printed in a process that took **1 day 6 hours 46 minutes and 7 seconds** (figure 43) while, the main part of the airframe took **3 days 5 hours 52 minutes and 55 seconds** (figure 44)

![Figure 43: Airframe Cover Printing](image)

Source: Author

On the figures 45, 46, 47 and 48 the final printing after the removal of the supports are presented.

On the figure 47, the details previously presented on the figure 37 is highlighted.
5.3 Onboard Electronics

In this section the onboard electronics of the project will be briefly discussed. The electronics are compounded by: a battery, an ESC, a microcontroller, 4 motors and a MEMS consisting of a 3-axis Accelerometer and a 3-axis Gyroscope. The main characteristics of the electronics elements are presented on the tables 30, 31, 32, 33 and 34. The motors, ESC and battery have been chosen through the method present on the section 3.3.

Table 30: Motor parameters

<table>
<thead>
<tr>
<th>Motor Name</th>
<th>Brand</th>
<th>XING CAMO X2306 2-6S FPV NextGen Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>iFlight</td>
</tr>
<tr>
<td>$K_{V0}$</td>
<td></td>
<td>1700 RPM/V</td>
</tr>
<tr>
<td>$I_{m,MAAX}$</td>
<td></td>
<td>37.2 A</td>
</tr>
<tr>
<td>$I_m$</td>
<td></td>
<td>1 A</td>
</tr>
<tr>
<td>$U_m$</td>
<td></td>
<td>10 V</td>
</tr>
<tr>
<td>$R_m$</td>
<td></td>
<td>0.076 Ω</td>
</tr>
<tr>
<td>$m_m$</td>
<td></td>
<td>0.0331 kg</td>
</tr>
</tbody>
</table>

Source: [15]

Table 31: ESC parameters

<table>
<thead>
<tr>
<th>ESC Name</th>
<th>Brand</th>
<th>F45A 6S 4IN1 V2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>iFlight</td>
</tr>
<tr>
<td>$I_{e,MAAX}$</td>
<td></td>
<td>45 A</td>
</tr>
<tr>
<td>$R_e$</td>
<td></td>
<td>0.01 Ω</td>
</tr>
<tr>
<td>$m_e$</td>
<td></td>
<td>0.0042 kg</td>
</tr>
</tbody>
</table>

Source:[16]
The microcontroller used was an Arduino based one, due to its relatively easy to use softwares and hardwares, besides the possibility of consult plenty of online material. Moreover, the arduino counts with libraries whose make the interface with sensors and actuators easier. For this reason, the MPU6050 was choose as the main "sensor" of the project, since it is arduino compatible and the data provided by it is quite easy to gather.

The figure 49 shows how the electronics are connected. All the pins used are hightled in blue. Also, the ESC has a Battery Eliminatio Circuit (BEC), which means that the battery is connected to the ESC and this one powers up the microcontroller, without the need of an extra battery.
The base arduino code used in combination with the schematic circuit of the figure 49 is presented on the appendix H. As it will be seen on the next section, the codes lacks an effective filter for better results provided by the MPU6050, and the PID parameters must be improved in order to achieve a smooth stabilization and actuation of the motors.

The figure 50 shows how the electronics components have been assembled inside the UAV. As it can be seen on figure 33 the components are placed in two levels and in the figure 50 just the superior level is visible showing the ESC (in black) and the MPU6050 (blue, on the middle). The microcontroller is on the ground level and it cannot be seen on the figure.

5.4 Assembling the UAV

Finally, the UAV completed assembly is shown on figures 51, 52 and 53 below.
Figure 50: Onboard Electronics

Source: Author

Figure 51: Assembly

Source: Author
Figure 52: Assembly

Source: Author

Figure 53: Assembly

Source: Author
6 CONCLUSION

This work has presented the development of a Unhamed-Aerial-Vehicle (UAV) to perform the task of delivering.

The first chapter has introduced the topic with a brief explanation of the working principles of UAVs of the type multicopter, following the presentation of the objectives of this thesis.

The second chapter aimed to present the dynamic equations of motion for a quadrotor. By introducing the Euler’s angles representation, the rotation matrix to transform the vector on the Body space into the Earth (inertial) one is obtained. Moreover, the aerodynamic characteristics of the non-conservative forces and moments are presented and the concept of blade-flapping is briefly introduced. The equations obtained on that section were a necessary step toward the control of the UAV.

The chapter 3 aimed to define and study proper models for the components of the propulsion system as a first step towards the design process. After defining the environmental parameters, the models for propellers, motors, ESC’s and battery were presented, and additionally, the mass of the airframe and its surface were defined as function of the propeller dimensions. After the modelling process, was defined the optimal parameters to be analyzed, where for each of the parameters, a set of components which maximize the particular parameter was determined and the results for each of those sets were presented for each configuration of multicopter (3-rotor, 4-rotor and 6-rotor). Such results showed that each set might be used for a particular application, but none of them are completely adequate for the application of delivering. Nonetheless, those results were important to define the maximum value of each of the parameter analyzed. The chapter also presented the criterion decision method, and with the aim of the maximum values defined previously, the set that best fit the delivering application has been determined in three configurations: tricopter, quadrotor and hexacopter. At the end of the chapter the set of components has been chosen and the parameters evaluated for this choice of component presented.
The fourth chapter presented the control method for a quadrotor tracking a trajectory. The control was split into 4 blocks: position controller, attitude controller, motor dynamics and rigid body dynamics. Considerations about the saturation has been presented and the direction-guaranteed functions has been introduced. Lastly, the simulation results were presented.

The chapter 5, the manufacture process of the airframe has been presented, from its first sketches on paper to the images of its CAD images. Following this first section the 3D printer and its characteristics were presented as the parameters set on the "slicer". Lastly the onboards electronics and its components are presented finishing the thesis with the assembling of the prototype.
7 SUGGESTIONS FOR FUTURE WORKS

As this work focuses from the choice of commercial components to the assembly of the UAV, in this section it will be provided some points to be improved and implemented on next works:

- **Batteries**: the flight endurance of the nowadays multicopters is intrinsically related to the batteries technology that is on use. A bottleneck to the use of drones with electric propulsion as an alternative to the delivery of goods is the few time they can fly without a charging landing. Although there have been a great development of the batteries on the past few years, it will still be a challenge to the industry to provide new battery models at a reasonable price, that are secure to use and with a green discard after its life. The reference [3] gives some examples of technological advances of the batteries and also some alternatives to the electric propulsion as combustion drones and drones with a power supply fixed on the ground and connected to the drone through an electric cable.

- **Control Model**: the *PID Control Method* presented on the section 4 although is a good start point to controlling the UAV under the conditions stated is not the most robust method. In fact, such a method has been used exclusively due to its didactical approach and wouldn’t be suitable to perform the quick acceleration and roll/pitch of competition drones, for example. For proper material regarding the robust control methods, you could start by consulting the references [12], [20], [21] and [22]. For a general reference about control theory, please refer to [13].

- **Manufacturing and Materials**: please notice that the airframe manufactured by the 3d printing method presented is a **functional prototype only** and does not have the mechanical properties required by a true and proper flight, and therefore, the structure might not resist a fall, for example. The limitations of strength are due to the mechanical properties of the material itself and the characteristics of the manufacturing process. The additive manufacturing process has been chosen due
to its flexibility to make changes on the project due to the analysis provided by the printed part. However, as presented on the section 5.2 the complete printing takes around 4.5 days and therefore it would be impossible to manufacture in scale using such method. Optionally, instead of printing your own design you might want to see commercial airframes in carbon fiber, which have a really good mechanical strength. A good website to buy such parts is the HobbyKing.com [23] which besides airframes has pretty much everything else you would need to build an UAV.

- **Prototype Improvements**: although many modifications have been made on the prototype in comparison with the initial project, there are still some improvements that could be made. *The first modification* would be related with the base of the ESC. In the maximum throttle configuration the ESC heats up and such heat is not dissipated properly by the base. A possible solution for such problem would make holes on the base for increase the contact area with cold air which would therefore better dissipated the heat. *A second modification* is related to the battery compartment of the UAV. Such compartment can easily deformates as showed by the FEM figure 54 below which could compress the battery and cause a possible damage. Also, FEM studies could be further exploited for improvements of the prototype.

![Figure 54: Battery compartment](image)

Source: Author

- **Data filtering**: the measurements given by the MPU6050 (the 6 axis sensor utilized) cannot be used directly since it presents a lot of noise and inaccuracies. Therefore, a method should be used in order to provide a better estimation of the measured variables. A useful method that can be used is the Kalman filter that
combines a series of measurements to produce a more precise result. Information regarding the Kalman filter can be consulted on the reference [3]. Another important point to be mentioned is that to have coherent results through the MPU6050, this one should be properly calibrated. For a practical calibration method for such sensor please check [24].

- **Telemetry Data:** as it can be seen on the figure 49 on the F45A ESC there are two pins not in usage: the pin ”CURR” and the pin ”TELE”. The ”CURR” pin provides the current required by the motors, while the pin ”TELE” provides informations as the temperature of the ESC and the rotational speed of the motors. Those data could be stored and displayed in real time if the drone is wireless connected to a computer and it could be useful for improving the control method and predicting the battery status of charge.

- **Autonomous Flight:** to have the UAV fly autonomously, sensors that provide data from which it is possible to place the UAV in space (position sensors) and to perform obstacle avoidance are fundamental. One can think that the position can be integrated from the MPU6050 acceleration data and although this assumption makes sense in theory, it is showed impossible on the practice, since the noise present on the acceleration and therefore accumulated in two integration steps (from acceleration to velocity and to velocity to position) makes the data completely incorrect even if a Kalmann filter is used. Sensors that can be used for the determination of the position more accurately are: GPS, IMU, Cameras and RGB-D (those last two are used to obstacle avoidance as well); which one of those sensors presents its positive points and drawbacks. A reference for autonomous flights in indoor and outdoor environment is [20].

- **Flight Safety:** it cannot be highlighted enough how important is the safety during the flight. Therefore methods and procedures to a safer fly are always welcome. To increase the safety, several measures can be implemented as: redundant turn off bottoms in case of emergency; protection case for the propellers; real time temperature measurement of battery and critical components; and robust obstacle avoidance methods, to name a few. The reference [3] provides a introduction to the safety topic.

Please always remove the propellers when testing the UAV on ground.
Please read through the safety tips on the manual that comes with the battery.
- **Flight Tests**: flight tests should be performed and from the evaluation of those the control method on the section 4 could be validated and improved. As stated previously such method is not advised for competition UAVs and for those more robust methods should be implemented. Also, tests using payload should be performed in order to evaluate the maximum payload capacity of the UAV as it its endurance and those results should be confronted with the theoretical values presented on the table 3.6 for validation.
REFERENCES


5 Mustafa Cavcar. The International Standard Atmosphere (ISA). *Anadolu University*.


APPENDIX A – LIST OF PROPELLERS

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APPENDIX E – SET OF PARAMETERS
MATLAB CODE

Source: Author

% SET OF PARAMETERS
clear
clc

%INITIALIZATION
tic
fprintf('Initializing...\n');
tic

general_data = readtable('set_of_parameters.xlsx','Sheet', ... 'PARAMETERS','Range','E:E');

env_data = readtable('set_of_parameters.xlsx','Sheet', ... 'PARAMETERS','Range','I:I');

rho = env_data{3,1}; %AIR DENSITY

g = env_data{5,1}; %GRAVITY ACCELERATION

num_of_prop = general_data{1,1};

BMCF = general_data{4,1};

Iother = general_data{5,1};

Cd1 = general_data{6,1};

Cd2 = general_data{7,1};

num_prp_data = 43; %max number of propellers being analyzed

num_mot_data = 50; %max number of motors being analyzed

num_esc_data = 38; %max number of esc's being analyzed

num_bat_data = 36; %max number of batteries being analyzed

%PROPELLER

prp_data = readtable('set_of_parameters.xlsx', 'Sheet', 'PROPELLER', ... 'Range','C2:L53', 'ReadVariableNames', false);
Dp_table = prp_data(3:end,1);
Hp_table = prp_data(3:end,2);
Bp_table = prp_data(3:end,3);
mp_table = prp_data(3:end,4);
cost_prp_table = prp_data(3:end,5);
CD_table = prp_data(3:end,6);
CT_table = prp_data(3:end,7);
CM_table = prp_data(3:end,8);
maf_table = prp_data(3:end,9);
S_table = prp_data(3:end,10);

Dp = zeros(num_prp_data,1);
Hp = zeros(num_prp_data,1);
Bp = zeros(num_prp_data,1);
mp = zeros(num_prp_data,1);
cost_prp = zeros(num_prp_data,1);
CD = zeros(num_prp_data,1);
CT = zeros(num_prp_data,1);
CM = zeros(num_prp_data,1);
maf = zeros(num_prp_data,1);
S = zeros(num_prp_data,1);

for r=1:num_prp_data
    Dp(r) = str2double(Dp_table{r,1});
    Hp(r) = str2double(Hp_table{r,1});
    Bp(r) = str2double(Bp_table{r,1});
    mp(r) = str2double(mp_table{r,1});
    cost_prp(r) = str2double(cost_prp_table{r,1});
    CD(r) = str2double(CD_table{r,1});
    CT(r) = str2double(CT_table{r,1});
    CM(r) = str2double(CM_table{r,1});
    maf(r) = str2double(maf_table{r,1});
    S(r) = str2double(S_table{r,1});
end

%---------------------------------------

%MOTOR---------------------------------------

mot_data = readtable('set_of_parameters.xlsx', 'Sheet', 'MOTOR', ...
                      'Range', 'C2:L53', 'ReadVariableNames', false);

KV0_table = mot_data(3:end,1);
ImMAX_table = mot_data(3:end,2);
Im0_table = mot_data(3:end,3);
Um0_table = mot_data(3:end,4);
Rm_table = mot_data(3:end,5);
mm_table = mot_data(3:end,6);
cost_mot_table = mot_data(3:end,7);
KE_table = mot_data(3:end,8);
KT_table = mot_data(3:end,9);
NMAX_table = mot_data(3:end,10);

KV0 = zeros(num_mot_data,1);
ImMAX = zeros(num_mot_data,1);
Im0 = zeros(num_mot_data,1);
Um0 = zeros(num_mot_data,1);
Rm = zeros(num_mot_data,1);
mm = zeros(num_mot_data,1);
cost_mot = zeros(num_mot_data,1);
KE = zeros(num_mot_data,1);
KT = zeros(num_mot_data,1);
NMAX = zeros(num_mot_data,1);

for k=1:num_mot_data
    KV0(k) = str2double(KV0_table{k,1});
    ImMAX(k) = str2double(ImMAX_table{k,1});
    Im0(k) = str2double(Im0_table{k,1});
    Um0(k) = str2double(Um0_table{k,1});
    Rm(k) = str2double(Rm_table{k,1});
    mm(k) = str2double(mm_table{k,1});
    cost_mot(k) = str2double(cost_mot_table{k,1});
    KE(k) = str2double(KE_table{k,1});
    KT(k) = str2double(KT_table{k,1});
    NMAX(k) = str2double(NMAX_table{k,1});
end

esc_data = readtable('set_of_parameters.xlsx', 'Sheet', 'ESC', ...
    'Range', 'C2:F53', 'ReadVariableNames', false);

IeMAX_table = esc_data(3:end,1);
Re_table = esc_data(3:end,2);
me_table = esc_data(3:end,3);
cost_esc_table = esc_data(3:end,4);
IeMAX = zeros(num_esc_data,1);
Re = zeros(num_esc_data,1);
me = zeros(num_esc_data,1);
cost_esc = zeros(num_esc_data,1);

for j=1:num_esc_data
    IeMAX(j) = str2double(IeMAX_table{j,1});
    Re(j) = str2double(Re_table{j,1});
    me(j) = str2double(me_table{j,1});
    cost_esc(j) = str2double(cost_esc_table{j,1});
end

%----------------------------------------

%BATTERY-----------------------------------
bat_data = readtable('set_of_parameters.xlsx', 'Sheet', 'BATTERY', ...
    'Range', 'C2:H53', 'ReadVariableNames', false);

Cb_table = bat_data(3:end,1);
Rb_table = bat_data(3:end,2);
Ub_table = bat_data(3:end,3);
Kb_table = bat_data(3:end,4);
mb_table = bat_data(3:end,5);
cost_bat_table = bat_data(3:end,6);

Cb = zeros(num_bat_data,1);
Rb = zeros(num_bat_data,1);
Ub = zeros(num_bat_data,1);
Kb = zeros(num_bat_data,1);
mb = zeros(num_bat_data,1);
cost_bat = zeros(num_bat_data,1);

for i=1:num_bat_data
    Cb(i) = str2double(Cb_table{i,1});
    Rb(i) = str2double(Rb_table{i,1});
    Ub(i) = str2double(Ub_table{i,1});
    Kb(i) = str2double(Kb_table{i,1});
    mb(i) = str2double(mb_table{i,1});
    cost_bat(i) = str2double(cost_bat_table{i,1});
end

%----------------------------------------

%PRP x MOT x ESC x BAT HOVER-------------------
ntotal = num_prp_data*num_mot_data*num_esc_data*num_bat_data;
Ielogical = 0;
Iblogical = 0;
Imlogical = 0;
Nlogical = 0;

mass_array = zeros(ntotal,1);
cost_array = zeros(ntotal,1);

prpxmotxescxbat = zeros(ntotal,4);
N_hov = zeros(ntotal,1);
M_hov = zeros(ntotal,1);
Im_hov = zeros(ntotal,1);
Um_hov = zeros(ntotal,1);
Ue0_hov = zeros(ntotal,1);
sigma_hov = zeros(ntotal,1);
Ie_hov = zeros(ntotal,1);
Ib_hov = zeros(ntotal,1);
t_hov = zeros(ntotal,1);
t_hov_over_cost = zeros(ntotal,1);
Ue_hov = zeros(ntotal,1);

eff_nl = zeros(ntotal,1);
Um_nl = zeros(ntotal,1);
Im_nl = zeros(ntotal,1);
M_nl = zeros(ntotal,1);
N_nl = zeros(ntotal,1);
Ib_nl = zeros(ntotal,1);
Ue_nl = zeros(ntotal,1);
eff_nl_over_cost = zeros(ntotal,1);
T_nl = zeros(ntotal,1);
mload_nl = zeros(ntotal,1);
mload_nl_over_cost = zeros(ntotal,1);
pitch_nl = zeros(ntotal,1);

V_max = zeros(ntotal,1);
Ie_z = zeros(ntotal,1);
Ue_z = zeros(ntotal,1);
Ib_z = zeros(ntotal,1);
N_z = zeros(ntotal,1);
Z_max = zeros(ntotal,1);
Z_max_over_cost = zeros(ntotal,1);
clc
fprintf('Evaluating...
');
for i = 1:num_bat_data
    for j = 1:num_esc_data
        for k = 1:num_mot_data
            for r = 1:num_prp_data
                mass_array(n) = maf(r) + 1.25*(mb(i) + num_of_prop*me(j) ... + num_of_prop*mm(k) + num_of_prop*mp(r)); %TOTAL MASS ...
                cost_array(n) = num_of_prop*cost_prp(r)+ ... num_of_prop*cost_mot(k)+num_of_prop*cost_esc(j) + ...
                cost_bat(i); %cost

                prpxmotxescxbat(n,1) = i; %NUMBER OF THE BATTERY
                prpxmotxescxbat(n,2) = j; %NUMBER OF THE ESC
                prpxmotxescxbat(n,3) = k; %NUMBER OF THE MOTOR
                prpxmotxescxbat(n,4) = r; %NUMBER OF THE PROPELLER

                N_comp = ... 60*sqrt(g*mass_array(n)/(rho*num_of_prop*Dp(r)^4*CT(r)));
                if N_comp > NMAX(k) %ANGULAR SPEED CHECK
                    Nlogical = 1;
                else
                    N_hov(n) = N_comp; %ANGULAR SPEED N (1)
                    M_hov(n) = rho*Dp(r)^5*CM(r)*(N_hov(n)/60)^2; %TORQUE ...
                    M_hov(n) = M_hov(n)/KT(k) + Im0(k); %Im (3)
                    Um_hov(n) = KE(k)*N_hov(n) + Rm(k)*Im_hov(n); %Um (4)
                    Ue0_hov(n) = Um_hov(n) + Im_hov(n)*Re(j); %Ue0 (5)
                sigma_hov(n) = Ue0_hov(n)/Ub(i); %sigma (6)
            end

            if IeMAX(j) < 1.2*ImMAX(k) %MOTOR CURRENT CHECK
                Imlogical = 1;
            end

            if sigma_hov(n)*Im_hov(n) > IeMAX(j) %ESC CURRENT CHECK
                Ielogical = 1;
            else
                Ie_hov(n) = sigma_hov(n)*Im_hov(n); %Ie (7)
            end
if num_of_prop*Ie_hov(n) + Iother > Cb(i)*Kb(i)/1000 \%THE ... CURRENT REQUIRED BY THE ESC'S OUTSTAND THE MAX ... AVAILABLE FROM THE BATTERY
   Ib_logical = 1;
else
   Ib_hov(n) = num_of_prop*Ie_hov(n)+Iother; \%Ib (8)
end

Ue_hov(n) = Ub(i) - Ib_hov(n)*Rb(i); \%Ue (9)
t_hov(n) = 0.06*(1-BMCF)*Cb(i)/Ib_hov(n); \%Thover (10)
t_hov_over_cost(n) = t_hov(n)/cost_array(n); \%Thover over ...
   cost (11)

\%NON LINEAR SYSTEM SOLVER 1-------------------------------
fun = @(x) sysEff(x, Ub(i), Rb(i), Re(j), Dp(r), CM(r), ...
   KV0(k), Um0(k), Rm(k), Im0(k), rho, num_of_prop, Iother);
x0 = [10, 5, 20, 5000, 10, 10]; \% 6 OUTPUTS
options = optimset('Display','off');
sol = fsolve(fun, x0, options);
   Um_nl(n) = sol(1); \%(12)

if sol(2) > IeMAX(j) \%ESC CURRENT CHECK
   Ie_logical = 1;
else
   Im_nl(n) = sol(2); \%(13)
end

if sol(4) > NMAX(k) \%ANGULAR SPEED CHECK
   N_logical = 1;
else
   M_nl(n) = sol(3); \%(14)
   N_nl(n) = sol(4); \%(15)
end

if sol(5) > Cb(i)*Kb(i)/1000 \%BATTERY CURRENT CHECK
   Ib_logical = 1;
else
   Ib_nl(n) = sol(5); \%Ib (16)
end

if Ie_logical || Ib_logical || Im_logical || N_logical
   \%SKIP
else

Ue_nln(n) = sol(6); \%(17)
over_eff_nln = 2*pi()/60*num_of_prop*M_nln(n)*N_nln(n);
under_eff_nln = Ub(i)*Ib_nln(n);
eff_nln(n) = over_eff_nln/under_eff_nln*100; %SYSTEM ...

EFFICIENCY IN PERCENTAGE (18)
eff_nln_over_cost(n) = eff_nln(n)/cost_array(n); ...
%SYSTEM EFFICIENCY IN PERCENTAGE/TOTAL COST (19)
T_nln(n) = rho*Dp(r)^4*CT(r)*(N_nln(n)/60)^2;%THRUST (20)
if num_of_prop*T_nln(n)/g < mass_array(n)
  else
    mload_nln(n) = num_of_prop*T_nln(n)/g - ...
    mass_array(n);% (21) PAYLOAD MASS(it contains ...
    a 25% factor on the mass of the propulsor to ...
    account for the wires, sensors, ...
    microcontroller, etc)
mload_nln_over_cost(n) = ...
    mload_nln(n)/cost_array(n); %PAYLOAD OVER COST ...
  (22)
pitch_nln(n) = ...
    acos(g*mass_array(n)/num_of_prop/T_nln(n));%MAX ...
    PITCH ANGLE (23)
  end
end

%FORWARD FLIGHT SPEED-----------------------------
over_V_max = sqrt(2*g*mass_array(n)*tan(pitch_nln(n));
CDRAG_V_max = ...
Cd1*(1-(sin(pitch_nln(n)))^3)+Cd2*(1-(cos(pitch_nln(n)))^3));
under_V_max = rho*S(r)*CDRAG_V_max;
V_max(n) = over_V_max/under_V_max; % (24) FORWARD FLIGHT ...
SPEED

%FLIGHT DISTANCE--------------------------------
[Ie_z(n),Ue_z(n), Ib_z(n), N_z(n), Z_max(n)] = dist(g, ...
  mass_array(n), pitch_nln(n), rho, S(r), Cd1, Cd2, ...
  CT(r), Dp(r), num_of_prop, CM(r), KT(k), Im0(k), ...
  KE(k), Rm(k), Ub(i), Iother, BMCF, Cb(i), Re(j)); % ...
FLIGHT DISTANCE

if N_z(n) > NMAX(k) %ANGULAR SPEED CHECK
Nlogical = 1;
end

if Ie_z(n) > IeMAX(j) %ESC CURRENT CHECK
Ielogical = 1;
end

if Ib_z(n) > Cb(i)*Kb(i)/1000 %THE CURRENT REQUIRED BY ... THE ESC'S OUTSTAND THE MAX AVAILABLE FROM THE BATTERY
Iblogical = 1;
end

if Ielogical || Iblogical || Imllogical || Nlogical
cost_array(n) = Inf;
mass_array(n) = Inf;

N_hov(n) = 0;
M_hov(n) = 0;
Im_hov(n) = 0;
Um_hov(n) = 0;
Ue0_hov(n) = 0;
sigma_hov(n) = 0;
Ie_hov(n) = 0;
Ib_hov(n) = 0;
t_hov(n) = 0;
t_hov_over_cost(n) = 0;
Ue_hov(n) = 0;

eff_nl(n) = 0;
Um_nl(n) = 0;
Im_nl(n) = 0;
M_nl(n) = 0;
N_nl(n) = 0;
Ib_nl(n) = 0;
Ue_nl(n) = 0;
eff_nl_over_cost(n) = 0;
T_nl(n) = 0;
mload_nl(n) = 0;
mload_nl_over_cost(n) = 0;
pitch_nl(n) = 0;

V_max(n) = 0;
Ie_z(n) = 0;
Ue_z(n) = 0;
Ib_z(n) = 0;
N_z(n) = 0;
Z_max(n) = 0;

Ielogical = 0;
Iblogical = 0;
Imlogical = 0;
Nlogical = 0;

else
    Z_max_over_cost(n) = Z_max(n)/cost_array(n);
end

n=n+1;
end
end
clc
fprintf('Step %d/%d: %3.1f \% completed\n',i, num_bat_data, ...
n/ntotal*100);
end

%RESULTS 1-----------------------------------------------
clc
w_counter = 1;

w_counter = writing_function(t_hov, prpxmotxescxbat, 'D2', 'F2', ...
    'G2', 'H2', 'I2', 'T2', 'W2', 'Z2', 'AC2', Ie_hov, Ue_hov, ...
    Ib_hov, N_hov, w_counter);

w_counter = writing_function(-cost_array, prpxmotxescxbat, 'D4', ...
    'F4', 'G4', 'H4', 'I4', 'T4', 'W4', 'Z4', 'AC4', Ie_hov, Ue_hov, ...
    Ib_hov, N_hov, w_counter);

w_counter = writing_function(t_hov_over_cost, prpxmotxescxbat, 'D6', ...
    'F6', 'G6', 'H6', 'I6', 'T6', 'W6', 'Z6', 'AC6', Ie_hov, Ue_hov, ...
    Ib_hov, N_hov, w_counter);

w_counter = writing_function(eff_nl, prpxmotxescxbat, 'D8', 'F8', ...
'G8', 'H8', 'I8', 'T8', 'W8', 'Z8', 'AC8', Im_nl, Ue_nl, Ib_nl, ...
N_nl, w_counter);

%MAXIMUM SYSTEM EFFICIENCY OVER COST-----------------------------
w_counter = writing_function(eff_nl_over_cost, prpxmotxescxbat, ...
'D10', 'F10', 'G10', 'H10', 'I10', 'T10', 'W10', 'Z10', 'AC10', ...
Im_nl, Ue_nl, Ib_nl, N_nl, w_counter);

%MAXIMUM PRODUCED PROPELLER THRUST-------------------------------
w_counter = writing_function(T_nl, prpxmotxescxbat, 'D12', 'F12', ...
'G12', 'H12', 'I12', 'T12', 'W12', 'Z12', 'AC12', Im_nl, Ue_nl, ...
Ib_nl, N_nl, w_counter);

%MAXIMUM PAYLOAD MASS--------------------------------------------
w_counter = writing_function(mload_nl, prpxmotxescxbat, 'D14', 'F14', ...
'G14', 'H14', 'I14', 'T14', 'W14', 'Z14', 'AC14', Im_nl, Ue_nl, ...
Ib_nl, N_nl, w_counter);

%MAXIMUM PAYLOAD OVER COST----------------------------------------
w_counter = writing_function(mload_nl_over_cost, prpxmotxescxbat, ...
'D16', 'F16', 'G16', 'H16', 'I16', 'T16', 'W16', 'Z16', 'AC16', ...
Im_nl, Ue_nl, Ib_nl, N_nl, w_counter);

%MAXIMUM PITCH ANGLE---------------------------------------------
w_counter = writing_function(pitch_nl, prpxmotxescxbat, 'D18', 'F18', ...
'G18', 'H18', 'I18', 'T18', 'W18', 'Z18', 'AC18', Im_nl, Ue_nl, ...
Ib_nl, N_nl, w_counter);

%MAXIMUM FORWARD FLIGHT SPEED-----------------------------------
w_counter = writing_function(V_max, prpxmotxescxbat, 'D20', 'F20', ...
'G20', 'H20', 'I20', 'T20', 'W20', 'Z20', 'AC20', Ie_z, Ue_z, ...
Ib_z, N_z, w_counter);

%MAXIMUM FLIGHT DISTANCE------------------------------------------
w_counter = writing_function(Z_max, prpxmotxescxbat, 'D22', 'F22', ...
'G22', 'H22', 'I22', 'T22', 'W22', 'Z22', 'AC22', Ie_z, Ue_z, ...
Ib_z, N_z, w_counter);

%MAXIMUM FLIGHT DISTANCE OVER COST-------------------------------
w_counter = writing_function(Z_max_over_cost, prpxmotxescxbat, 'D24', ...
'F24', 'G24', 'H24', 'I24', 'T24', 'W24', 'Z24', 'AC24', Ie_z, ...
Ue_z, Ib_z, N_z, w_counter);
```matlab
w_counter = writing_function(-mass_array, prpxmotxescxbat, 'D26', ...
    'F26', 'G26', 'H26', 'I26', 'T26', 'W26', 'Z26', 'AC26', Ie_hov, ...
    Ue_hov, Ib_hov, N_hov, w_counter);

fprintf('Finishing...
');
clc
toc

%FUNCTIONS---------------------------------
%NON LINEAR SYSTEM SOLVER
function F = syseff(x, Ub, Rb, Re, Dp, CM, KV0, Um0, Rm, Im0, rho, ...
    num_of_prop, Iother)
    F(1) = -Ub + x(1) + x(2)*Re;
    F(2) = x(3) - rho*Dp^5*CM/60/60*x(4)^2;
    F(3) = x(1) - (x(3)*KV0*Um0/9.55/(Um0-Im0*Rm) + Im0)*Rm - ...  
        (Um0-Im0*Rm)/KV0/Um0*x(4);
    F(4) = x(2) - (x(3)*KV0*Um0/9.55/(Um0-Im0*Rm) + Im0);
    F(5) = x(5) - num_of_prop*x(2) - Iother;
    F(6) = x(6) - Ub + x(5)*Rb;
end

%MAXIMUM FLIGHT DISTANCE
function [Ie_z, Ue_z, Ib_z, N_z, Z_max] = dist(g, mass, teta_max, ...
    rho, S, Cd1, Cd2, CT, Dp, num_of_prop, CM, KT, Im0, KE, Rm, Ub, ...
    Iother, BMCF, Cb, Re)
    teta = 0:0.01:real(teta_max);
    over_V_max = sqrt(2*g*mass*tan(teta);
    CDRAG = (Cd1*(1-sin(teta).^3)+Cd2*(1-cos(teta).^3));
    under_V_max = rho*S*CDRAG;
    V_max = over_V_max./under_V_max; %FORWARD FLIGHT SPEED
    N = 60*sqrt(g*mass/rho/CT/Dp^4/num_of_prop./cos(teta));%ANGULAR ...
    SPEED N
    M = mass*CM*Dp/CT/num_of_prop./cos(teta);%TORQUE M
    Im = M/KT + Im0; %Im
    Um = KE*N + Rm*Im; %Um
    sigma = (Um + Im*Re)/Ub; %sigma
    Ie = sigma.*Im; %Ie
    Ib = num_of_prop*Ie + Iother; %Ib
    T_flight = 0.06*(1-BMCF)*Cb./Ib; %TIME OF FLIGHT
    Z = 60*V_max.*T_flight; %FLIGHT DISTANCE
    [Z_max, Z_max_index] = max(Z);
```
Ie_z = Ie(Z_max_index);
Ib_z = Ib(Z_max_index);
Ue_z = Ub - Ib_z*Re;
N_z = N(Z_max_index);
end

%RESULTS WRITING-----------------------------------------------

function w_counter = writing_function(array, prpxmotxescxbat, ...
    RangeString1, RangeString2, RangeString3, RangeString4, ...
    RangeString5, RangeString6, RangeString7, RangeString8, ...
    RangeString9, Ie_array, Ue_array, Ib_array, N_array, w_counter)
    [opt, opt_index] = max(array(1:end,1));
    PRP_model = prpxmotxescxbat(opt_index,4); %PROPELLER MODEL NUMBER
    MOT_model = prpxmotxescxbat(opt_index,3); %MOTOR MODEL NUMBER
    ESC_model = prpxmotxescxbat(opt_index,2); %ESC MODEL NUMBER
    BAT_model = prpxmotxescxbat(opt_index,1); %BATTERY MODEL NUMBER
    file = 'set_of_parameters.xlsx';
    R = 'RESULTS';
    writetable(array2table(abs(opt)),file,'Sheet',R,'Range',RangeString1);
    writetable(array2table(PRP_model),file,'Sheet',R,'Range',RangeString2);
    writetable(array2table(MOT_model),file,'Sheet',R,'Range',RangeString3);
    writetable(array2table(ESC_model),file,'Sheet',R,'Range',RangeString4);
    writetable(array2table(BAT_model),file,'Sheet',R,'Range',RangeString5);
    Ie = Ie_array(opt_index);%ESC INPUT CURRENT
    Ue = Ue_array(opt_index);%ESC INPUT VOLTAGE
    Ib = Ib_array(opt_index);%BATTERY CURRENT
    N = N_array(opt_index);%MOTOR SPEED N
    writetable(array2table(Ie),file,'Sheet',R,'Range',RangeString6);
    writetable(array2table(Ue),file,'Sheet',R,'Range',RangeString7);
    writetable(array2table(Ib),file,'Sheet',R,'Range',RangeString8);
    writetable(array2table(N),file,'Sheet',R,'Range',RangeString9);
    fprintf('Writing... %d/13\n', w_counter);
    w_counter = w_counter + 1;
end
%---------------------------------------------------------------
% GRAPHICS FOR THE SET OF PARAMETERS
% This code is to be used on the command window by using the results of the
% simulation "set_of_parameters_opt_5th_attempt"

figure(1)
scatter(cost_array, t_hov,10,'filled');
set(gca, 'FontSize', 14)
title('Endurance Time x Cost');
xlabel('Cost [USD]');
ylabel('Endurance in Hover t_e [min]');

figure(2)
scatter(cost_array, eff_nl,10,'filled');
set(gca, 'FontSize', 14)
title('Efficiency of System x Cost');
xlabel('Cost [USD]');
ylabel('System Efficiency $\eta$');

figure(3)
scatter(cost_array, mload_nl,10,'filled');
set(gca, 'FontSize', 14)
title('Maximum Mass of Payload x Cost');
xlabel('Cost [USD]');
ylabel('Maximum Payload Mass m_{LOADMAX} [kg]');

figure(4)
%FLIGHT DISTANCE x COST
scatter(cost_array, Z_max,10,'filled');
set(gca, 'FontSize', 14)
title('Flight Distance x Cost');
xlabel('Cost [USD]');
ylabel('Flight Distance Z [m]');

figure(5)
%TIME OF ENDURANCE IN HOVER x PAYLOAD MASS
scatter(t_hov, mload_nl,10,'filled', 'k');
set(gca, 'FontSize', 14)
title('Maximum Mass of Payload x Endurance Time');
xlabel('Endurance in Hover t_{e} [min]');
ylabel('Maximum Payload Mass m_{LOADMAX} [kg]');

figure(6)
%FLIGHT DISTANCE x PAYLOAD MASS
scatter(Z_max, mload_nl,10,'filled', 'k');
set(gca, 'FontSize', 14)
title('Maximum Mass of Payload x Flight Distance');
xlabel('Flight Distance Z [m]');
ylabel('Maximum Payload Mass m_{LOADMAX} [kg]');

%DECISION CRITERION
w_t = 4;
w_cost = 1;
w_eta = 1;
w_m = 4;
w_V = 1;
w_Z = 2;
W = w_t + w_cost + w_eta + w_m + w_V + w_Z;

tMAX = max(t_hov);
costMIN = min(cost_array);
etaMAX = max(eta_nl);
mMAX = max(mload_nl);
VMAX = max(V_max2);
ZMAX = max(Z_max);
p1 = w.*t.*hov/tMAX;
p2 = (w.*cost.*costMIN/costarray)';
p3 = w.*eta.*eff.nl/etaMAX;
p4 = w.*m.*mload.nl/mMAX;
p5 = w.*V.*V_max2/VMAX;
p6 = w.*Z.*Z_max/ZMAX;
P = (p1+p2+p3+p4+p5+p6)/W;
[P_max, P_max_index] = max(P);
PRP_model = prpxmotxescxbat(P_max_index,4); %PROPELLER MODEL NUMBER
MOT_model = prpxmotxescxbat(P_max_index,3); %MOTOR MODEL NUMBER
ESC_model = prpxmotxescxbat(P_max_index,2); %ESC MODEL NUMBER
BAT_model = prpxmotxescxbat(P_max_index,1); %BATTERY MODEL NUMBER
fprintf('#########################TRICOPTER########################
');
fprintf('PRP: model %d
', PRP_model);
fprintf('MOT: model %d
', MOT_model);
fprintf('ESC: model %d
', ESC_model);
fprintf('BAT: model %d
', BAT_model);
fprintf('P_MAX = %3.4f
', P_max);
fprintf('t_e: %3.1f min
', t_hov(P_max_index));
fprintf('Efficiency: %3.2f %
', eff.nl(P_max_index));
fprintf('T: %3.2f N
', T.nl(P_max_index));
fprintf('m_{LOADMAX}: %2.3f kg
', mload.nl(P_max_index));
fprintf('theta: %1.2f rad
', pitch.nl(P_max_index));
fprintf('V_{MAX}: %2.2f m/s
', V_max(P_max_index));
fprintf('Z_{MAX}: %4.2f m
', Z_max(P_max_index));
fprintf('m: %1.3f kg
', mass_array(P_max_index));
fprintf('cost: %4.2f USD
', cost_array(P_max_index));

%PAYLOAD MASS FOR THE SET x TIME OF ENDURANCE OF THE SET
mass_payload = 0:0.01:mload.nl(P_max_index);
mass = mass_array(P_max_index) + mass_payload;
N = 60*sqrt(g*mass/(rho*num_of_prop*Dp(PRP_model)^4*CT(PRP_model))); ... %ANGULAR SPEED N (1)
M = rho*Dp(PRP_model)^5*CM(PRP_model)*(N/60).^2; %TORQUE M (2)
Im = M/KT(MOT_model) + Im0(MOT_model); %Im (3)
Um = KE(MOT_model)*N + Rm(MOT_model)*Im; %Um (4)
Ue0 = Um + Im*Re(ESC_model); %Ue0 (5)
sigma = Ue0/Ub(BAT_model); %sigma (6)
\[ I_e = \sigma \cdot I_m; \quad \%I_e (7) \]
\[ I_b = \text{num\_of\_prop} \cdot I_e + I_{\text{other}}; \quad \%I_b (8) \]
\[ U_e = U_b(\text{BAT\_model}) - I_b \cdot R_b(\text{BAT\_model}); \quad \%U_e (9) \]
\[ t_e = 0.06 \cdot (1 - \text{BMCF}) \cdot C_b(\text{BAT\_model}) / I_b; \quad \%t_{\text{hover}} (10) \]

\text{hold on}
\text{figure(7)}
\text{plot(mass\_payload, t_e, 'LineWidth', 2);}
\text{set(gca, 'FontSize', 12)}
\text{title('Time of Endurance x Payload Mass');}
\text{xlabel('Payload Mass m\_{LOAD} [kg]');}
\text{ylabel('Time of Endurance t_e [min]');}
\text{legend('Trirotor', 'Quadrotor', 'Hexarotor');}
\text{hold on}
\text{figure(7)}
\text{plot(mass\_payload, t_e, 'LineWidth', 2);}
\text{set(gca, 'FontSize', 12)}
\text{xlabel('Payload Mass m\_{LOAD} [kg]');}
\text{ylabel('Time of Endurance t_e [min]');}
\text{legend('No Cost Filter', 'Cost Filter');}

\%MULTIPLE BATTERIES
\[ n_{\text{bat}} = 1:1:1000; \]
\[ \text{mass1} = \text{maf}(\text{PRP\_model}) + 1.25 \cdot (n_{\text{bat}} \cdot \text{mb}(\text{BAT\_model}) + \ldots \\
\text{num\_of\_prop}\cdot \text{me}(\text{ESC\_model}) + \text{num\_of\_prop}\cdot \text{mm}(\text{MOT\_model}) + \ldots \\
\text{num\_of\_prop}\cdot \text{mp}(\text{PRP\_model})); \quad \%\text{TOTAL MASS m}\ (\text{it contains a 25\% ... \\
\text{factor on the mass of the propulsor to account for the wires, ... \\
sensors, microcontroller, etc})} \]
\[ \text{mass} = \text{mass1} + \text{mload\_nl}(\text{P\_max\_index}); \]
\[ N = 60 \cdot \sqrt{g \cdot \text{mass} / (\rho \cdot \text{num\_of\_prop}\cdot D_p(\text{PRP\_model})^4 \cdot CT(\text{PRP\_model}))}; \ldots \\
\%\text{ANGULAR SPEED N (1)} \]
\[ M = \rho \cdot D_p(\text{PRP\_model})^5 \cdot CM(\text{PRP\_model}) \cdot (N/60)^{0.7}; \quad \%\text{TORQUE M (2)} \]
\[ I_m = M / KT(\text{MOT\_model}) + \text{Im0}(\text{MOT\_model}); \quad \%I_m (3) \]
\[ U_m = KE(\text{MOT\_model}) \cdot N + \text{Rm}(\text{MOT\_model}) \cdot I_m; \quad \%U_m (4) \]
\[ U_e = U_m + I_m \cdot \text{Re}(\text{ESC\_model}); \quad \%U_e (5) \]
\[ \sigma = U_e / U_b(\text{BAT\_model}); \quad \%\sigma (6) \]
\[ I_e = \sigma \cdot I_m; \quad \%I_e (7) \]
\[ I_b = \text{num\_of\_prop} \cdot I_e + I_{\text{other}}; \quad \%I_b (8) \]
\[ U_e = U_b(\text{BAT\_model}) - I_b \cdot R_b(\text{BAT\_model}); \quad \%U_e (9) \]
\[ t_e = 0.06 \cdot (1 - \text{BMCF}) \cdot n_{\text{bat}} \cdot C_b(\text{BAT\_model}) / I_b; \quad \%t_{\text{hover}} (10) \]

\text{plot(n\_bat, t\_e);}
%CONTROL

close all
clear
clc

n = 4;

m_{af} = 0.708;
m_p = 5/1000;
m_m = 36/1000;
m_{e} = 39/1000;
m_b = 478/1000;
m_c = 39/1000;

m = 0.8*m_{af} + n*(m_p + m_m) + 1.1*(m_{e} + m_b + m_c);
g = 9.81;

I_{xx} = 0.002 + 2*(m_m + m_p)*(1.1293)^2;
I_{yy} = 0.004 + 2*(m_m + m_p)*(1.1293)^2;
I_{zz} = 0.005;

c_{T} = 1; %Corrected with PID parameters
c_{M} = 1; %Corrected with PID parameters

l = 182.9/1000;
f_{max} = n*1.758*g;
f_{min} = 0.1*m*g;
T_{max} = f_{max}*1;
tau_{max} = f_{max}*1;
tau_{max_z} = (2/n)*(2.925*10^−9/(1.4865*10^−7))*f_{max};
w_{max} = sqrt(f_{max}/n/c_{T});
\[
\theta_{\text{max}} = 81.93 \times \pi / 180;
\]

% TUNNING OF CONTROL PARAMETERS

\[
\begin{align*}
\xi_{\text{ang}} &= 1; \\
\omega_{\text{ang}} &= 9; \\
\xi_{\text{lin}} &= 1; \\
\omega_{\text{lin}} &= 4; \\
\end{align*}
\]

\[
\begin{align*}
\kappa p_{\phi} &= I_{xx} \omega_{\text{ang}}^2; \\
\kappa p_{\theta} &= I_{yy} \omega_{\text{ang}}^2; \\
\kappa p_{\psi} &= I_{zz} \omega_{\text{ang}}^2; \\
\kappa d_{\phi} &= 2I_{xx} \xi_{\text{ang}} \omega_{\text{ang}}; \\
\kappa d_{\theta} &= 2I_{yy} \xi_{\text{ang}} \omega_{\text{ang}}; \\
\kappa d_{\psi} &= 2I_{zz} \xi_{\text{ang}} \omega_{\text{ang}}; \\
\kappa d_x &= 2\xi_{\text{lin}} \omega_{\text{lin}}; \\
\kappa d_y &= 2\xi_{\text{lin}} \omega_{\text{lin}}; \\
\kappa d_z &= 20 \times 2 \xi_{\text{lin}} \omega_{\text{lin}}; \\
\kappa p_x &= \omega_{\text{lin}}^2; \\
\kappa p_y &= \omega_{\text{lin}}^2; \\
\kappa p_z &= 4 \omega_{\text{lin}}^2; \\
\kappa i_x &= 0.01; \\
\kappa i_y &= 0.01; \\
\kappa i_z &= 40 \times 0.01; \\
T_m &= 0.09; \\
K_d &= \begin{bmatrix}
\kappa d_x & 0 & 0 \\
0 & \kappa d_y & 0 \\
0 & 0 & \kappa d_z
\end{bmatrix}; \\
K_p &= \begin{bmatrix}
\kappa p_x & 0 & 0 \\
0 & \kappa p_y & 0 \\
0 & 0 & \kappa p_z
\end{bmatrix}; \\
K_i &= \begin{bmatrix}
\kappa i_x & 0 & 0 \\
0 & \kappa i_y & 0 \\
0 & 0 & \kappa i_z
\end{bmatrix};
\end{align*}
\]

% TIME ARRAY DEFINITION

\[
\begin{align*}
t_{\text{step}} &= 0.01; \\
t_{\text{init}} &= 0; \\
t_{\text{end}} &= 20; \\
t_{\text{array}} &= t_{\text{init}}:t_{\text{step}}:t_{\text{end}}; \% \text{Time array}
\end{align*}
\]

r0 = [0; 0; -10];
dr0 = [0; 0; 0];
ddr0 = [0; 0; 0];
theta0 = 0;
phi0 = 0;
psi0 = 0;
dphi0 = 0;
dtheta0 = 0;
dpsi0 = 0;
w0 = [4000/60;4000/60;4000/60;4000/60];

%ALLOCATION

% ACTUAL POSITION
r = zeros(3,length(t_array)+1);
dr = zeros(3,length(t_array)+1);
ddr = zeros(3,length(t_array)+1);

e_p = zeros(3,length(t_array)+1);
e_v = zeros(3,length(t_array)+1);
dev = zeros(3,length(t_array)+1);
int_e_p = zeros(3,length(t_array)+1);

% TRAJECTORY TO BE TRACKED
r_T = zeros(3,length(t_array)+1);
dr_T = zeros(3,length(t_array)+1);
ddr_T = zeros(3,length(t_array)+1);
dddr_T = zeros(3,length(t_array)+1);

ddr_c = zeros(3,length(t_array)+1);

% COMMANDED ACCELERATION
phi_c = zeros(1,length(t_array)+1);
theta_c = zeros(1,length(t_array)+1);
psi_c = ones(1,length(t_array)+1)*psi0;
dphi_c = zeros(1,length(t_array)+1);
dtheta_c = zeros(1,length(t_array)+1);
dpsi_c = zeros(1,length(t_array)+1);

phi = zeros(1,length(t_array)+1);
theta = zeros(1,length(t_array)+1);
psi = psi_c;
dphi = zeros(1,length(t_array)+1);
dtheta = zeros(1,length(t_array)+1);
dpsi = zeros(1,length(t_array)+1);
u1_c = zeros(1,length(t_array)+1);
u2_c = zeros(3, length(t_array)+1);

w_des_sq = zeros(4, length(t_array)+1);

w_des = zeros(4, length(t_array)+1);

w = zeros(4, length(t_array)+1);

u = zeros(4, length(t_array)+1);

%INITIAL CONDITIONS

r(:,1) = r0;

dr(:,1) = dr0;

phi(1) = phi0;

theta(1) = theta0;

dphi(1) = dphi0;

dtheta(1) = dtheta0;

w(:,1) = sqrt(m*g/n/c_T)*ones(4,1);

for t_i = 1:length(t_array)

%TRAJECTORY TO BE TRACKED

if t_array(t_i) < 5
    r_T(:,t_i) = [t_array(t_i)/5*10;0;-10];
elseif (5 <= t_array(t_i)) && (t_array(t_i) < 10)
    r_T(:,t_i) = [10;(t_array(t_i)-5)/5*10;-10];
elseif (10 <= t_array(t_i)) && (t_array(t_i) < 15)
    r_T(:,t_i) = [10-(t_array(t_i)-10)/5*10;10;-10];
else
    r_T(:,t_i) = [0;10-(t_array(t_i)-15)/5*10;-10];
end

%FIRST DERIVATIVE OF THE TRACKED POSITION

for t_i = 1:length(t_array)
    dr_T(:,t_i) = (r_T(:,t_i+1)-r_T(:,t_i))/t_step;
end

%SECOND DERIVATIVE OF THE TRACKED POSITION

for t_i = 1:length(t_array)
    ddr_T(:,t_i) = (dr_T(:,t_i+1)-dr_T(:,t_i))/t_step;
end

%THIRD DERIVATIVE OF THE TRACKED POSITION
for t_i = 1:length(t_array)
    dddr_T(:,t_i) = (ddr_T(:,t_i+1)-ddr_T(:,t_i))/t_step;
end

for t_i = 1:length(t_array)

%CONTROLLED ACCELERATIONS

    e_p(:,t_i) = x_T(:,t_i) - r(:,t_i);
    e_v(:,t_i) = dr_T(:,t_i) - dr(:,t_i);
    int_e_p(:,t_i) = ...
        [sum(e_p(1,1:t_i));sum(e_p(2,1:t_i));sum(e_p(3,1:t_i))];

    ddr_c(:,t_i) = ddr_T(:,t_i) + Kd*e_v(:,t_i) + Kp*e_p(:,t_i) + ...
        Ki*int_e_p(:,t_i);

%COMMANDED ANGLES

    phi_c(t_i) = sat_gd(1/g*( ddr_c(1,t_i)*sin(psi0) - ...
        ddr_c(2,t_i)*cos(psi0) ), theta_max);
    theta_c(t_i) = sat_gd(1/g*( ddr_c(1,t_i)*cos(psi0) + ...
        ddr_c(2,t_i)*sin(psi0) ), theta_max);

if t_i ==1
    dphi_c(t_i) = dphi0;
    dtheta_c(t_i) = dtheta0;
else
    dphi_c(t_i) = (phi_c(t_i)-phi_c(t_i-1))/t_step;
    dtheta_c(t_i) = (theta_c(t_i)-theta_c(t_i-1))/t_step;
end

%INPUTS

    u1_c(t_i) = sat_gd(m*(g + ddr_c(3,t_i))-(f_max+f_min)/2, ...
        (f_max-f_min)/2 ) + (f_max+f_min)/2;

    u2_c(1:2,t_i)= sat_gd([ kp_phi* (phi_c(t_i) - phi(t_i)) + ...
        kd_phi*(dphi_c(t_i) - dphi(t_i) );
        kp_theta* (theta_c(t_i) - theta(t_i)) + ...
        kd_theta*(dtheta_c(t_i) - dtheta(t_i) )],tau_max);

    u2_c(3,t_i) = sat_gd(kp_psi* (psi_c(t_i) - psi(t_i)) + ...
        kd_psi*(dpsi_c(t_i) - dpsi(t_i) ),tau_max_z);

%CONTROL ALLOCATION

    M_4 = [ c_T c_T c_T c_T;
            l*c_T/2 l*c_T/2 -l*c_T/2 -l*c_T;
            -l*c_T/2 l*c_T/2 l*c_T/2 -l*c_T;
            c_M -c_M c_M -c_M];
if \( t_i = 1 \)
\[
\text{w}_{\text{des}} \text{sq}(:,t_i) = \text{w(:,t_i)}^2;
\]
else
\[
\text{w}_{\text{des}} \text{sq}(:,t_i) = M_4 \\{ u1_c(t_i); u2_c(:,t_i) \};
\]
end
\[
\text{w}_{\text{des}}(:,t_i) = \sqrt{\text{w}_{\text{des}} \text{sq}(:,t_i)};
\]

%MOTOR DYNAMICS
\[
\text{w}(;t_i) = \text{w}_{\text{des}}(;t_i) \ast (1-\exp(-t_{\text{array}}(t_i)/T_m));
\]
\[
\text{u}(;t_i) = M_4 + \text{w}(;t_i)^2;
\]

%RIGID BODY DYNAMICS
\[
\text{ddr}(1,t_i) = g \ast \left( \phi(t_i) \ast \sin(\psi(t_i)) + \ldots \theta(t_i) \ast \cos(\psi(t_i)) \right);
\]
\[
\text{ddr}(2,t_i) = g \ast \left( -\phi(t_i) \ast \cos(\psi(t_i)) + \ldots \theta(t_i) \ast \sin(\psi(t_i)) \right);
\]
\[
\text{ddr}(3,t_i) = \frac{u(1,t_i)}{m} - g;
\]
\[
\text{dx1dt} = @(t,x1)[x1(2); u(2,t_i)/Ixx]; \ % roll channel
\]
\[
\text{dx2dt} = @(t,x2)[x2(2); u(3,t_i)/Iyy]; \ % pitch channel
\]
\[
\text{dx3dt} = @(t,x3)[x3(2); (u(4,t_i) - \ldots \phi(t_i) \ast \text{dphi}(t_i) \ast (I_{yy} - I_{xx})/I_{zz}); \ % yaw channel
\]
\[
\text{dx4dt} = @(t,x4)[x4(2); g \ast \left( \theta(t_i) \ast \cos(\psi(t_i)) + \ldots \phi(t_i) \ast \sin(\psi(t_i)) \right)]; \ % x channel
\]
\[
\text{dx5dt} = @(t,x5)[x5(2); g \ast \left( \phi(t_i) \ast \cos(\psi(t_i)) - \ldots \theta(t_i) \ast \sin(\psi(t_i)) \right)]; \ % y channel
\]
\[
\text{dx6dt} = @(t,x6)[x6(2); \frac{u(1,t_i)}{m} - g]; \ % z channel
\]
\[
x_{10} = [\phi(t_i), \text{dphi}(t_i)];
\]
\[
x_{20} = [\theta(t_i), \text{dtheta}(t_i)];
\]
\[
x_{30} = [\psi(t_i), \text{dpsi}(t_i)];
\]
\[
x_{40} = [r(1,t_i), \text{dr}(1,t_i)];
\]
\[
x_{50} = [r(2,t_i), \text{dr}(2,t_i)];
\]
\[
x_{60} = [r(3,t_i), \text{dr}(3,t_i)];
\]
\[
\text{tLim} = [t_{\text{array}}(t_i) - \text{t_step}, t_{\text{array}}(t_i) + \text{t_step}];
\]
\[
[t_{1Sol}, x_{1Sol}] = \text{ode45}(\text{dx1dt}, \text{tLim}, x_{10}); \ % roll channel
\]
\[
[t_{2Sol}, x_{2Sol}] = \text{ode45}(\text{dx2dt}, \text{tLim}, x_{20}); \ % pitch channel
\]
\[
[t_{3Sol}, x_{3Sol}] = \text{ode45}(\text{dx3dt}, \text{tLim}, x_{30}); \ % yaw channel
\]
\[
[t_{4Sol}, x_{4Sol}] = \text{ode45}(\text{dx4dt}, \text{tLim}, x_{40}); \ % x channel
\]
[t5Sol, x5Sol] = ode45(dx5dt, tLim, x50); % y channel
[t6Sol, x6Sol] = ode45(dx6dt, tLim, x60); % z channel

% ATTITUDE CONTROLLER FEEDBACK
phi(t_i+1) = sum( x1Sol(:,1) )/length(t1Sol);
dphi(t_i+1) = sum( x1Sol(:,2) )/length(t1Sol);
theta(t_i+1) = sum( x2Sol(:,1) )/length(t2Sol);
dtheta(t_i+1) = sum( x2Sol(:,2) )/length(t2Sol);
psi(t_i+1) = sum( x3Sol(:,1) )/length(t3Sol);
dpsi(t_i+1) = sum( x3Sol(:,2) )/length(t3Sol);

% POSITION CONTROLLER FEEDBACK
r(:,t_i+1) = [sum(x4Sol(:,1) )/length(t4Sol); sum(x5Sol(:,1) ... ]/length(t5Sol); sum(x6Sol(:,1) )/length(t6Sol) ];
dr(:,t_i+1) = [sum(x4Sol(:,2) )/length(t4Sol); sum(x5Sol(:,2) ... ]/length(t5Sol); sum(x6Sol(:,2) )/length(t6Sol) ];
end

% GRAPHICAL ANALYSIS
figure(1)
plot(t_array,r(1,1:end-1),'r','LineWidth', 2 );
xlabel('Time [s]');
ylabel('x(t) [m]');
hold on
plot(t_array,r_T(1,1:end-1),'--k');
legend('Actual','Tracked')
set(gca,'FontSize',22)

figure(2)
plot(t_array,r(2,1:end-1),'g','LineWidth', 2 );
xlabel('Time [s]');
ylabel('y(t) [m]');
hold on
plot(t_array,r_T(2,1:end-1),'--k');
legend('Actual','Tracked')
set(gca,'FontSize',22)

figure(3)
plot(t_array(1:end-1),r(3,1:end-1-1),'b','LineWidth', 2);
xlabel('Time [s]');
ylabel('z(t) [m]');
hold on
plot(t_array,r_T(3,1:end-1-1),'--k');
legend('Actual','Tracked')
set(gca,'FontSize',22)
figure(4)
plot(t_array,dr(1,1:end-1),'r','LineWidth', 2 );
xlabel('Time [s]');
ylabel('$\dot{x}(t)$ [m/s]', 'interpreter', 'latex');
set(gca,'FontSize',22)
hold on
plot(t_array,dr_T(1,1:end-1) );
legend('Actual','Tracked')

figure(5)
plot(t_array,dr(2,1:end-1),'g','LineWidth', 2 );
xlabel('Time [s]');
ylabel('$\dot{y}(t)$ [m/s]', 'interpreter', 'latex');
set(gca,'FontSize',22)
hold on
plot(t_array,dr_T(2,1:end-1) );
legend('Actual','Tracked')

figure(6)
plot(t_array(1:end-1),dr(3,1:end-1-1),'b','LineWidth', 2 );
xlabel('Time [s]');
ylabel('$\dot{z}(t)$ [m/s]', 'interpreter', 'latex');
set(gca,'FontSize',22)
hold on
plot(t_array,dr_T(3,1:end-1) );
legend('Actual','Tracked')

figure(7)
plot(t_array,ddr(1,1:end-1) );
xlabel('time step');
ylabel('$\ddot{x}(t)$', 'interpreter', 'latex');
hold on
plot(t_array,ddr_T(1,1:end-1) );
legend('Actual','Tracked')

figure(8)
plot(t_array,ddr(2,1:end-1) );
xlabel('time step');
ylabel('$\ddot{y}(t)$', 'interpreter', 'latex');
hold on
plot(t_array,ddr.T(2,1:end-1) );
legend('Actual','Tracked')
figure(9)
plot(t_array,ddr(3,1:end-1) );
xlabel('time step');
ylabel('$\ddot{z}(t)$', 'interpreter', 'latex');
hold on
plot(t_array,ddr.T(3,1:end-1) );
legend('Actual','Tracked')
figure(10)
plot(t_array(1:end-1),phi(1:end-1-1) ,'r','LineWidth', 2 );
xlabel('Time [s]');
ylabel('$\phi(t)$ [rad]', 'interpreter', 'latex');
hold on
plot(t_array,phi_c(1:end-1), '--k');
legend('Actual','Commanded')
set(gca,'FontSize',22)
figure(11)
plot(t_array,theta(1:end-1),'g','LineWidth', 2 );
xlabel('Time [s]');
ylabel('$\theta(t)$ [rad]', 'interpreter', 'latex');
hold on
plot(t_array,theta_c(1:end-1),'--k');
legend('Actual','Commanded')
set(gca,'FontSize',22)
figure(12)
plot(t_array,psi(1:end-1) ,'b','LineWidth', 2);
xlabel('Time [s]');
ylabel('$\psi(t)$ [rad]', 'interpreter', 'latex');
hold on
plot(t_array,psi_c(1:end-1),'--k');
legend('Actual','Commanded')
set(gca,'FontSize',22)
figure(13)
plot(t_array(1:end-1),dphi(1:end-1-1),'r','LineWidth', 2 );
xlabel('Time [s]');
ylabel('$\dot{\phi}(t)$ [rad/s]', 'interpreter', 'latex');
set(gca,'FontSize',22)
hold on
plot(t_array,dphi_c(1:end-1));
legend('Actual','Commanded')

figure(14)
plot(t_array,dtheta(1:end-1),'g','LineWidth', 2);
xlabel('Time [s]');
ylabel('\dot{\theta}(t) [rad/s]', 'interpreter', 'latex');
set(gca,'FontSize',22)
hold on
plot(t_array,dtheta_c(1:end-1));
legend('Actual','Commanded')

figure(15)
plot(t_array,dpsi(1:end-1),'b','LineWidth', 2);
xlabel('Time [s]');
ylabel('\dot{\psi}(t) [rad/s]', 'interpreter', 'latex');
set(gca,'FontSize',22)
hold on
plot(t_array,dpsi_c(1:end-1));
legend('Actual','Commanded')

figure(16)
plot3(r(1,1:end-1),r(2,1:end-1),r(3,1:end-1), 'Color',[26 188 156]/255,'LineWidth', 2);
xlabel('x [m]')
ylabel('y [m]')
zlabel('z [m]')
xlim([-2 12])
ylim([-2 12])
zlim([-12 -8])
box on
hold on
plot3(r_T(1,1:end-1),r_T(2,1:end-1),r_T(3,1:end-1),'-k');
legend('Actual','Tracked')
set(gca,'FontSize',22)

figure(17)
plot(r(1,1:end-1),r(2,1:end-1), 'Color',[26 188 156]/255,'LineWidth', 2);
xlabel('x [m]')
ylabel('y [m]')
xlim([-2 12])
ylim([-2 12])
box on
hold on
plot(r_T(1,1:end-1),r_T(2,1:end-1), '--k');
legend('Actual', 'Tracked', 'Location', 'bestoutside')
set(gca,'FontSize',22)

figure(18)
plot(t_array(1:end-9),u1_c(1:end-10))
xlabel('Time step')
ylabel('u1')
hold on
plot(t_array(1:end-9),u(1,1:end-10))
legend('Controlled', 'Actual')

figure(20)
plot(t_array, w_des(1,1:end-1));
hold on
plot(t_array, w(1,1:end-1));
legend('Controlled', 'Actual')

%----------------------------------------
%DIRECTION GUARANTEED SATURATION FUNCTION----------------------------------------
function u_sat = sat_gd(u,a)
    mod_inf = max(abs(u));
    if (mod_inf < a) || (mod_inf==a)
        kappa = 1;
    else
        kappa = a/mod_inf;
    end
    u_sat = kappa*u;
end
%----------------------------------------
APPENDIX H – BASE ARDUINO CODE

Source: Author
```cpp
#include <Wire.h> // Importing the I2C library.
#include <Servo.h>
#include <PID_v1.h>

Servo M1, M2, M3, M4;

const int I2C_address_MPU = 0x68; // I2C Address of the MPU6050;
int16_t acc_x_raw, acc_y_raw, acc_z_raw; // Variables for the Accelerometer sensor
int16_t gyro_x_raw, gyro_y_raw, gyro_z_raw; // Variables for the Gyroscope
int16_t temp_raw; // Variable in which the temperature is saved
float acc_x_offset, acc_y_offset, acc_z_offset; // offsetting values
float gyro_x_offset, gyro_y_offset, gyro_z_offset; // offsetting values
int16_t acc_x, acc_y, acc_z; // Variables for treated data
float gyro_x, gyro_y, gyro_z; // Variables for treated

const int n = 4;
const float m = 1.3420; // [kg]
const float g = 9.81; // [m/s^2]
const float Ixx = 0.0034; // [kg*m^2]
const float Iyy = 0.0054; // [kg*m^2]
const float Izz = 0.0050; // [kg*m^2]
const float l = 0.1829; // [m]
const float c_T = 1; // Corrected with PID parameters
const float c_M = 1; // Corrected with PID parameters
const float f_max = n*1.758*g;
const float f_min = 0.1*m*g;
const float T_max = f_max*1;
const float tau_max = f_max*1;
const float tau_max_z = (2/n)*(2.925*10^-9)/(1.4865*10^-7) * f_max;
const float w_max = sqrt(f_max/n/c_T);
const float theta_max = 81.93*PI/180;
const float M_40[] = {.25, .5, -.5, .25};
const float M_41[] = {.25, .5, .5, -.25};
const float M_42[] = {.25, -.5, .5, .25};
const float M_43[] = {.25, -.5, -.5, -.25};
```

### FIXED QUANTITIES

```cpp
const int xi_ang = 1;
const int omegan_ang = 9;
```
const int xi_lin = 1;
const int omegan_lin = 4;

const float kp_phi = Ixx*omegan_ang*omegan_ang;
const float kp_theta = Iyy*omegan_ang*omegan_ang;
const float kp_psi = Izz*omegan_ang*omegan_ang;
const float kd_phi = 2*Ixx*xi_ang*omegan_ang;
const float kd_theta = 2*Iyy*xi_ang*omegan_ang;
const float kd_psi = 2*Izz*xi_ang*omegan_ang;
const float kd_x = 2*xi_lin*omegan_lin;
const float kd_y = 2*xi_lin*omegan_lin;
const float kd_z = 20*2*xi_lin*omegan_lin;
const float kp_x = omegan_lin*omegan_lin;
const float kp_y = omegan_lin*omegan_lin;
const float kp_z = 4*omegan_lin*omegan_lin;
const float ki_x = 0.01;
const float ki_y = 0.01;
const float ki_z = 40*0.01;
const float T_m = 0.09;

const float Kd[][3] = {
    {kd_x, 0, 0},
    {kd_y, 0, 0},
    {kd_z} 
};
const float Kp[][3] = {
    {kp_x, 0, 0},
    {kp_y, 0, 0},
    {kp_z} 
};
const float Ki[][3] = {
    {ki_x, 0, 0},
    {ki_y, 0, 0},
    {ki_z} 
};

// INITIAL CONDITIONS
const float r0[] = {0, 0, 0};
const float dr0[] = {0, 0, 0};
const float ddr0[] = {0, 0, 0};
const float theta0 = 0;
const float phi0 = 0;
const float psi0 = 0;
const float dphi0 = 0;
const float dtheta0 = 0;
const float dpsi0 = 0;
const float w0[] = {4000/60, 4000/60, 4000/60, 4000/60, 4000/60};

// DEFINING VARIABLES
unsigned long t0, t1;
float t_step;

unsigned long t0_int, t1_int;
float t_step_int;
float r[] = {}; //ACTUAL POSITION
float dr[] = {};
float ddr[] = {};
float dr_int[] = {};
float ddr_int[] = {};

float e_p[] = {};
float e_v[] = {};
float de_v[] = {};
float int_e_p[] = {};

float r_T[] = {}; //TRAJECTORY TO BE TRACKED
float dr_T[] = {};
float ddr_T[] = {};
float dddr_T[] = {};

float ddr_c[] = {}; //COMMANDED ACCELERATION

float phi_c;
float theta_c;
float psi_c = psi0;
float dphi_c;
float dtheta_c;
float dpsi_c;

float phi;
float theta;
float psi = psi_c;
float dphi;
float dtheta;
float dpsi;
float phi_c0;
float theta_c0;
float dphi_c0;
float dtheta_c0;
float dphi_int;
float dtheta_int;
float dpsi_int;

float u1_c;
float u2_c[] = {};
float u_c[] = {};

float w_des_sq[] = {};
float w_des[] = {};
float w[] = {};

float u[] = {};

float a;
float u_sat;
float kappa;
float mod_inf;

bool offset = true;
bool init_high = true;
bool init_low = true;

void setup() {
  Serial.begin(9600);
  Wire.begin();
  Wire.beginTransmission(I2C_address_MPU); //Starting the I2C transmission
  Wire.write(0x6B);
  Wire.write(0);
  Wire.endTransmission(true);

  M1.attach(3,1000,2000); // (pin, min pulse width, max pulse width in microseconds)
  M2.attach(4,1000,2000); // (pin, min pulse width, max pulse width in microseconds)
  M3.attach(5,1000,2000); // (pin, min pulse width, max pulse width in microseconds)
  M4.attach(6,1000,2000); // (pin, min pulse width, max pulse width in microseconds)
}

for(int i = 0; i < 3; i++){
  r[i] = r0[i];
  dr[i] = dr0[i];
  r_T[i] = 0;
  dr_T[i] = 0;
  ddr_T[i] = 0;
}

phi = phi0;
theta = theta0;
dphi = dphi0;
dtheta = dtheta0;
dphi_c0 = dphi0;
dtheta_c0 = dtheta0;

for(int i = 0; i < 4; i++){
  w[i] = sqrt(m*g/n/c_T);
  w_des_sq[i] = w[i]*w[i];
}

void loop() {
  e_p[0] = r_T[0] - r[0];
\[ e_v[0] = dr_T[0] - dr[0]; \]
\[ e_v[1] = dr_T[1] - dr[1]; \]

\[ \text{int}_e_p[0] += e_p[0]; \]
\[ \text{int}_e_p[1] += e_p[1]; \]
\[ \text{int}_e_p[2] += e_p[2]; \]

\[ ddr_c[0] = ddr_T[0] + Kd[0][0] \cdot e_v[0] + Kp[0][0] \cdot e_p[0] + Ki[0][0]\cdot \text{int}_e_p[0]; \]

```
// COMMANDED ANGLES

double t1 = millis();
t_step = (t1-t0)/1000.0;
t0 = millis();

u_sat = 1/g*( ddr_c[0]*sin(psi0) - ddr_c[1]*cos(psi0) );
a = theta_max;
sat_gd();
phi_c = u_sat;

u_sat = 1/g*( ddr_c[0]*cos(psi0) + ddr_c[1]*sin(psi0) );
a = theta_max;
sat_gd();
theta_c = u_sat;

dphi_c = ( phi_c - phi_c0 )/t_step;
dtheta_c = ( theta_c - theta_c0 )/t_step;
Serial.print(phi_c0);
Serial.println();

phi_c0 = phi_c;
theta_c0 = theta_c;
```

```
// INPUTS
```

```
// COMMANDED ANGLES

u_sat = m*(g + ddr_c[2])-(f_max+f_min)/2;
a = (f_max-f_min)/2;
sat_gd();
u1_c = u_sat + (f_max+f_min)/2;

u_sat = kp_phi* (phi_c - phi) + kd_phi*(dphi_c - dphi );
a = tau_max;
```
sat_gd();

u2_c[0] = u_sat;

u_sat = kp_theta*(theta_c - theta) + kd_theta*(dtheta_c - dtheta);
a = tau_max;
sat_gd();

u2_c[1] = u_sat;

u_sat = kp_psi*(psi_c - psi) + kd_psi*(dpsi_c - dpsi);
a = tau_max_z;
sat_gd();

u2_c[2] = u_sat;

/////////////////////////////////////////////////////////////////////////
////////////////////////////////////////
//CONTROL ALLOCATION - MOTOR
////////////////////////////////////////
/////////////////////////////////////////////////////////////////////////

u_c[0] = u1_c;
u_c[1] = u2_c[0];
u_c[2] = u2_c[1];
u_c[3] = u2_c[2];

for (int i = 0; i < 4; i++){
  w_des_sq[0] += M_40[i]*u_c[i];
  w_des_sq[1] += M_41[i]*u_c[i];
  w_des_sq[2] += M_42[i]*u_c[i];
  w_des_sq[3] += M_43[i]*u_c[i];
  //Serial.print(tau_max);
  //Serial.println();
}

w_des[0] = sqrt(w_des_sq[0]);
w_des[1] = sqrt(w_des_sq[1]);
w_des[2] = sqrt(w_des_sq[2]);
w_des[3] = sqrt(w_des_sq[3]);

/////////////////////////////////////////////////////////////////////////
////////////////////////////////////////
//FEEDBACK
////////////////////////////////////////
/////////////////////////////////////////////////////////////////////////

if (offset == true){
  for (int i = 0; i < 2000; i++){
    read_MPU6050();
    acc_x_offset += acc_x_raw;
    acc_y_offset += acc_y_raw;
    acc_z_offset += acc_z_raw;
    //temp += temp_raw;
    gyro_x_offset += gyro_x_raw;
    gyro_y_offset += gyro_y_raw;
    gyro_z_offset += gyro_z_raw;
  }
  acc_x_offset /= 2000.00;
  acc_y_offset = abs(acc_x_offset/16384.00*(-g));
  acc_y_offset /= 2000.00;
```c
acc_y_offset = abs(acc_y_offset/16384.00*(-g));
acc_z_offset /= 2000.00;
acc_z_offset = abs(acc_z_offset/16384.00*(-g));
//temp /= 2000;
gyro_x_offset /= 2000.00;
gyro_x_offset = abs(gyro_x_offset/131.00);
gyro_y_offset /= 2000.00;
gyro_y_offset = abs(gyro_y_offset/131.00);
gyro_z_offset /= 2000.00;
gyro_z_offset = abs(gyro_z_offset/131.00);
offset = false;
}
read_MPU6050();

//ACCELERATION CONTROLLER FEEDBACK
ddr[0] = acc_x_raw/16384.00*(-g);
ddr[0] += acc_x_offset;
ddr[1] = acc_y_raw/16384.00*(-g);
ddr[1] += acc_y_offset;
ddr[2] = acc_z_raw/16384.00*(-g);
ddr[2] += acc_z_offset;

//ATTITUDE CONTROLLER FEEDBACK

t0_int = millis();
t_step_int = (t1_int - t0_int)/1000.0;
dphi = gyro_x_raw/131.00;
dphi += gyro_x_offset;
dphi *= PI/180;
dtheta = gyro_y_raw/131.00;
dtheta += gyro_y_offset;
dtheta *= PI/180;
dpsi = gyro_z_raw/131.00;
dpsi += gyro_z_offset;
dpsi *= PI/180;

phi = (dphi - dphi_int)*t_step_int;
theta = (dtheta - dtheta_int)*t_step_int;
psi = (dpsi - dpsii_int)*t_step_int;

//POSITION CONTROLLER FEEDBACK

dr[0] = (ddr[0] - ddr_int[0])*t_step_int;

r[0] = (dr[0] - dr_int[0])*t_step_int;
r[1] = (dr[1] - dr_int[1])*t_step_int;

t0_int = millis();
dphi_int = dphi;
dtheta_int = dtheta;
dpsi_int = dpsii;
```
ddr_int[0] = ddr[0];
ddr_int[1] = ddr[1];
ddr_int[2] = ddr[2];

// Serial.print(w_des[0]);
// Serial.println();
// Serial.print(w_des[1]);
// Serial.println();
// Serial.print(w_des[2]);
// Serial.println();
// Serial.print(w_des[3]);
// Serial.println();
// Serial.print("------------------------");
// Serial.println();
// delay(1500);

///////////////////////////////////////////////////////////
// MOTOR COMMANDS
///////////////////////////////////////////////////////////

w_des[0] *= 1000;
w_des[1] *= 1000;
w_des[2] *= 1000;
w_des[3] *= 1000;

w_des[0] = (int) w_des[0];
w_des[1] = (int) w_des[1];
w_des[2] = (int) w_des[2];
w_des[3] = (int) w_des[3];

w_des[0] = map(w_des[0], 0, 1000, 0, 180);  // scale it to use it with
the servo library (value between 0 and 180)
w_des[1] = map(w_des[1], 0, 1000, 0, 180);
w_des[2] = map(w_des[2], 0, 1000, 0, 180);
w_des[3] = map(w_des[3], 0, 1000, 0, 180);

// Serial.print(w_des[0]);
// Serial.println();
// Serial.print(w_des[1]);
// Serial.println();
// Serial.print(w_des[2]);
// Serial.println();
// Serial.print(w_des[3]);
// Serial.println();
// Serial.print("------------------------");
// Serial.println();
// delay(1500);

if(init_high == true){
    M1.write(180);  // Send the signal to the ESC
    M2.write(180);
    M3.write(180);
    M4.write(180);
    init_high = false;
    delay(8000);
//Serial.print(1);

if(init_low == true) {
    M1.write(0);    // Send the signal to the ESC
    M2.write(0);
    M3.write(0);
    M4.write(0);
    init_low = false;
    delay(8000);
    //Serial.print(2);
}
else{
    M1.write(w_des[0]);    // Send the signal to the ESC
    M2.write(w_des[1]);
    M3.write(w_des[2]);
    M4.write(w_des[3]);
    delay(15);
    //delay(30);
}

/////////////////////////////////////////////////////////////////////////
////////////////////////////////////////
/////////////////////////////////////////////////////////////////////////
////////////////////////////////////////
// FUNCTIONS
///////////////
//////////////////////////////////////////////////////////
////////////////////////////////////////
/////////////////////////////////////////////////////////////////////////
////////////////////////////////////////
////////////////////////////////////////
////////////////////////////////////////
// MPU6050
/////////////////////////////////////////////////////////////////////////
////////////////////////////////////////
/////////////////////////////////////////////////////////////////////////
void read_MPU6050() {
    Wire.beginTransmission(I2C_address_MPU);    //Starting the I2C transmission
    Wire.write(0x3B);
    Wire.endTransmission(false);
    Wire.requestFrom(I2C_address_MPU, 7*2, true);
    acc_x_raw = Wire.read()<<8 | Wire.read();
    acc_y_raw = Wire.read()<<8 | Wire.read();
    acc_z_raw = Wire.read()<<8 | Wire.read();
    temp_raw = Wire.read()<<8 | Wire.read();
    gyro_x_raw = Wire.read()<<8 | Wire.read();
    gyro_y_raw = Wire.read()<<8 | Wire.read();
    gyro_z_raw = Wire.read()<<8 | Wire.read();
}

/////////////////////////////////////////////////////////////////////////
////////////////////////////////////////
// DIRECTION GUARANTEED SATURATION FUNCTION
///////////////////////////////////////////////////////////////////////
```c
void sat_gd(){
    mod_inf = abs(u_sat);
    if ( (mod_inf < a) || (mod_inf == a) ){
        kappa = 1;
    }
    else{
        kappa = a/mod_inf;
    }
    u_sat = kappa*u_sat;
}
```
ANNEX A – PETG: TECHNICAL SHEET

Source: [28]
PET-G is the best material for advanced users:

- Fast solidification, higher heat resistance (Glass transition temperature: 70ºC - 75ºC), very hard and flexible.
- Does not require Kapton tape.
- Heated bed needed at 60ºC-80ºC.
- Due to the nature of the PET-G, we recommend regular cleaning of the hotend exterior and the nozzle.
- Keep in a dry environment and away from light.

**Important:** do not clean the bed with alcohol.

PET-G is the best alternative to ABS: it is non-toxic, recyclable and releases fewer particles into the air.

**Filament Diameter:** 1.75 mm
**Thickness:** 1.27 g/cm³ (ASTM D792)
**Weight:** 1 kg
**Spool Size:** 175 mm x 77 mm
**Enclosure:** 187 x 187 x 83 mm
**Spool axle diameter:** 44 mm

Compatible with printers with heated bed and 1.75 mm filament.