POLITECNICO DI TORINO

Master Degree in

Automotive Engineering

Equilibrium systems for motorcycles at low velocities

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Abstract

The analysis of the behavior of motorcycles' roll stability at rest and at low velocities is the aim of this project. Motorcycles present a significant difference with the other vehicles because of the presence of only two wheels, meaning that without the action of a driver it is a naturally unstable vehicle which when left alone to its own action would fall to the ground.

After having analyzed the geometry of a motorcycle, the system was considered as an inverted pendulum with two equivalent point masses in order to study its behavior for what concerns the roll stability at low velocities in case of small roll angles. In order to do so the equivalent system was studied as an LTI system which was stabilized by means of a controller.

After having studied the results with different types of the geometric parameters considering the motorcycle at rest, the influence of velocity and gyroscopic effects on the system was studied.

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Of course I can not forget my little sister Benedetta, you have been the greatest sister I could have wished for, thank you for your support and help during all of my life, I love you.

Sommario

Introduction

In the world of motorcycles the stability of the vehicle at low velocity is one of the issue that is more difficult to deal with because of the characteristics of the motorcycle is to be a system which support itself only with two wheels. Stability at medium to high speed is generally high, while at low speed it is very reduced, in fact when the vehicle is standing still it is necessary to support it with the rider's feet on the ground. That is a great difference with respect to cars, in fact they are hyperstatic structure because of the presence of four wheels.

Based on the idea of developing motorcycle with good self-standing performances, the aim of this project is to study the possibility of an effective attitude stabilization method by using the forces generated only through the steering control while the vehicle is standing still or it is moving at low speed, without adding weights or inertia to the vehicle in order to generate restoring roll moments used to shift the tilted motorcycle to an upright position, so that the effects on the maneuverability during medium to high speed travel is not influenced negatively.

This stabilization of the motorcycle can be of great importance from a safety and comfort point of view, in fact many motorcyclists in their experience can testify the casualty of falling at low velocity or even at standstill position. Having a motorcycle able to stand up for itself can be a great help for the less experienced drivers from a safety point of view and furthermore it can be of great comfort when standing in traffic being able to keep the feet on the footrests instead of putting them down on the ground.

This project is focused on the dynamic model of the inverted pendulum, by studying the system an attitude stabilization method for a stationary or slowly moving motorcycle to use only the forces generated by steering control, in developing this method a dynamic model for motorcycles focusing on the conservation of momentum to represent the forces that are generated according to the geometry. During this study the effects of tire slipping, lateral forces, deformation and camber thrust are not considered. The study is   performed modifying the most important geometric parameters for the steering behavior of motorcycles.

Chapter one of this project is focused on the description of motorcycles and their basic parameters involved in their kinematic and dynamic behavior, chapter two is focused on the description of the automatic control systems and how to properly chose a control behavior of a system and chapter three is the study of the roll stability of motorcycles at low velocities.

1. Kinematics of motorcycles

The kinematic study of motorcycle is important because it effects the dynamic behavior of motorcycles, in this chapter the kinematic study is used to show its influence on the directional stability and maneuverability of motorcycles.

1.1 Definition of motorcycles

Altough motorcycles are composed of a great variety of mechanical parts, from a kinematic point of view and by considering the suspensions to be rigid bodies, a motorcycle can be defined as a spatial mechanism composed of four rigid bodies:

- The rear assembly(frame, saddle, tank and motor-transmission drivetrain group)
- The front assembly (fork, steering head and the handlebar)
- The front wheel
- The rear wheel

These rigid bodies are connected by three revolute joints (the steering axis and the two wheel axles) and are in contact with the ground at two wheel/ground contact points as shown in figure 1.1.

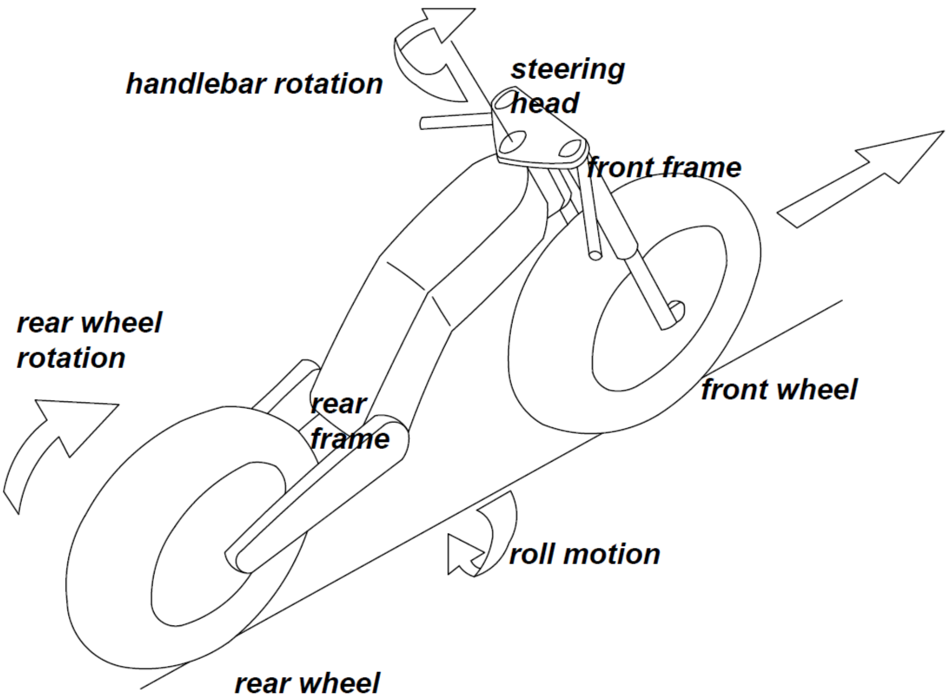


Figure 1.1: kinematic structure of a motorcycle

Each revolute joint inhibits five degrees of freedom in the spatial mechanism, while each wheel/ground contact point leaves three degrees of freedom free. Considering the pure rolling of tire on the road, it is easy to ascertain that each wheel, with respect to the fixed road can only rotate around

- The contact point on the wheel plane (forward motion)
- The intersection axis of the motorcycles and road planes (roll motion)
- The axis passing through the contact point and the center of the wheel (spin).

While driving the rider manages all three major movements according to his personal style and skill: the resulting movement of the motorcycle and its trajectory depend on the combination of the three motion related to the three degrees of freedom. All these considerations are under the hypothesis that the wheels move without any slippage, but in reality the tire movement is not just a rolling process.

The generation of longitudinal driving and braking forces requires some degree of slippage in both the longitudinal and lateral direction depending on the road condition. These considerations bring to the calculus of the number of degrees of freedom, which becomes seven:

- Forward motion of the motorcycle
- Rolling motion
- Handlebar rotation
- Longitudinal slippage of the front wheel (braking)
- Longitudinal slippage of the rear wheel (thrust or braking)
- Lateral slippage of the front wheel
- Lateral slippage of the rear wheel.

1.2 The geometry of motorcycles

This kinematic study refers to a rigid motorcycle, meaning that the vehicle is without suspensions with the wheels fitted to non-deformable tires and schematized as two toroidal solid bodies with circular sections as shown in figure 1.2

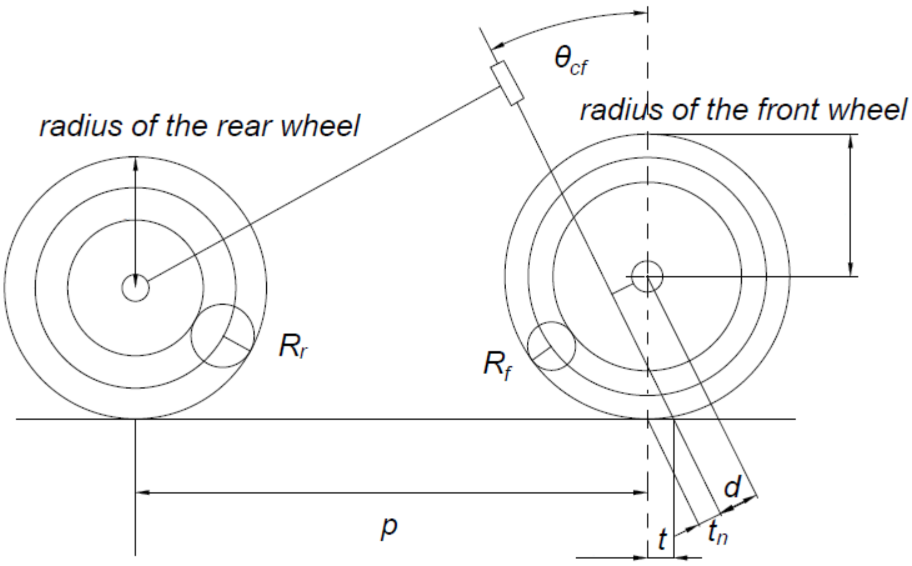


Figure 1.2: model of the geometry of a motorcycle

Motorcycles can be described using the following geometric parameters:

- ***p*** wheelbase
- ***d*** fork offset: perpendicular distance between the ais of the steering head and the center of the front wheel

- $\boldsymbol{\theta_{cf}}$ caster angle
- $\boldsymbol{R_f}$ The radius of the front wheel
- $\boldsymbol{R_r}$The radius of the rear wheel
- $\boldsymbol{r_f}$ radius of the front tire cross section
- $\boldsymbol{r_r}$ radius of the rear wheel cross section

The following equation describe how to find the correlation between the radius of the whole front wheel with the normal and the trail of the motorcycle:

$$t_n = R_f * \sin(\theta_{cf}) \tag{1}$$

$$t = R_f * tan(\theta_{cf}) \tag{2}$$

$$t_n = R_f * \sin\left(\theta_{cf}\right) - d \tag{3}$$

$$t = R_f * \tan\left(\theta_{cf}\right) - \frac{d}{cos(\theta_{cf})} \tag{4}$$

Equations (1) and (2) are valid in case of absence of front fork offset and equations (51) and (52) are valid in case of offset, with:

- $\boldsymbol{t_n}$ normal trail length;
- ***t*** mechanical trail length;
- ***d*** front fork offset, which corresponds to the perpendicular distance between the steering axis and the center of the wheel

The geometric usually used to describe the geometry of motorcycles are

- ***p*** wheelbase
- ***t*** the trail
- $\boldsymbol{\theta_{cf}}$ the caster angle

This parameters are measured with the motorcycle in a vertical position and the steering angle δ_f of the handlebars set to zero.

The wheelbase ***p*** is the distance between the contact points of the tires on the road, the caster angle $\boldsymbol{\theta_{cf}}$ is the angle between the vertical axis and the rotation axis of the front section, which is the axis of the steering head and ***t*** the trail is the distance between the contact point of the front wheel and the intersection point of the steering head axis with the road measured in the ground plane.

All these parameters together are important in defining the maneuverability of the motorcycle as it is perceived by the rider and it is not practical to examine the effects produced by only one of these geometric parameters because they are strongly related between them. The value of the wheelbase varies according to the type of vehicle considered and it ranges from $1200\ [mm]$ in case of small scooters to $1300\ [mm]$ for light motorcycle, to $1350\ [mm]$ for medium displacement motorcycles up to $1600\ [mm]$ for touring motorcycles with greater displacement of the engine.

In general an increase in the wheelbase without the variation of the other geometric parameters leads to:

- An unfavorable increase in the flexional and torsional deformability of the frame, parameters very important for the maneuverability of the vehicle (the more deformability the motorcycles gets, the less maneuverable it becomes)
- An unfavorable increase in the minimum curvature radius, since it makes it more difficult to turn on a path that has a small curvature radius
- In order to turn the motorcycle there must be an unfavorable increase in the torque to be applied to the handlebars
- A favorable decrease in the load transfer between the two wheels during the acceleration and braking phases, with a resulting decrease in the pitching motion; this makes forward and rearward flip-over more difficult
- A favorable reduction in the pitching movement generated by road unevenness
- A favorable decrease in the directional stability of the motorcycle

The trail and the caster angle are especially important because they define the geometric characteristics of the steering head and so the properties of maneuverability and directional stability of the motorcycles depend on them.

The caster angle varies according to the type of motorcycle: from $19°$ to $21° - 24°$ for competition or sport motorcycles, up to $27° - 34°$ for touring motorcycles. From a structural point of view a very small angle causes a lot of stress on the front fork during the braking phase, since the front fork is rather deformable, both flexionally and torsionally, small values of the angle will lead to greater stress and therefore greater deformations, which can cause dangerous vibration in the front assembly.

The value of $\boldsymbol{\theta_{cf}}$ caster angle is closely related to the value of trail, in general in order to have a good feeling for the motorcycle's maneuverability an increase in the caster angle must be linked to a corresponding increase in the trail.

The value of the trail depends on the type of motorcycle and its wheelbase, but usually it ranges from 75 $[mm]$ to 90 $[mm]$ for competition vehicles, to values of 90 $[mm]$ to 100 $[mm]$ in touring and sport motorcycles up to 120 $[mm]$ and beyond in purely touring motorcycles.

1.3 The importance of trail

One of the peculiarities of motorcycles is the steering system, whose function is essentially to produce a variation in the lateral force needed, for example to change the motorcycle's direction or to assure equilibrium.

According to this point of view the steering system could hypothetically be made up of two little rockets placed perpendicular to the front wheel, which could generate lateral thrusts (even if with high difficulties for the rider), meaning that it could perform the same function as the steering system.

From a geometrical point of view the classical steering mechanism is described by three parameters:

- $\boldsymbol{\theta_{cf}}$ the caster angle
- ***d*** front fork offset
- the radius of the front wheel

These parameters make it possible to calculate the value of the normal trail t_n, which is the perpendicular distance between the contact point and the axis of the motorcycle's steering head. This parameter is considered positive when the front wheel's contact point with the road plane is behind the point of the axis intersection of the steering head with the road itself, as it is possible to see in figure 1.2.

The trail measured on the road is related to the normal trail with the equation

$$t = t_n / \cos(\theta_{cf}) \tag{5}$$

The value of the trail is most important for the stability of the motorcycle, especially in rectilinear motion.

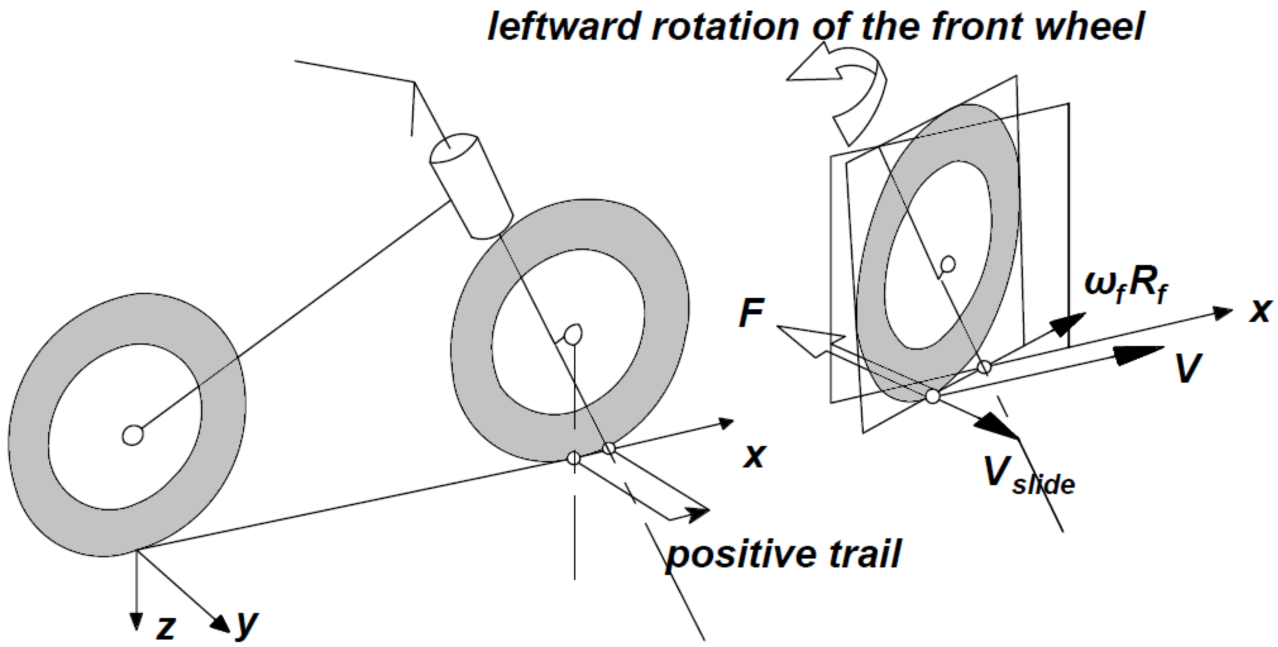


Figure 1.3: influence of positive trail

Considering a motorcycle driving a straight path at constant velocity V and an external disturbance causes a slight rotation of the front wheel to the left, without considering the fact that the vehicle would start to lean to the right, in this way the wheel contact point has velocity V in the same direction, which is divided in two components as shown in figure 1.3:

- the component $\omega_f R_f$, which represent the velocity due to rolling

- The component V_{slide} which represents the sliding velocity of the contact point

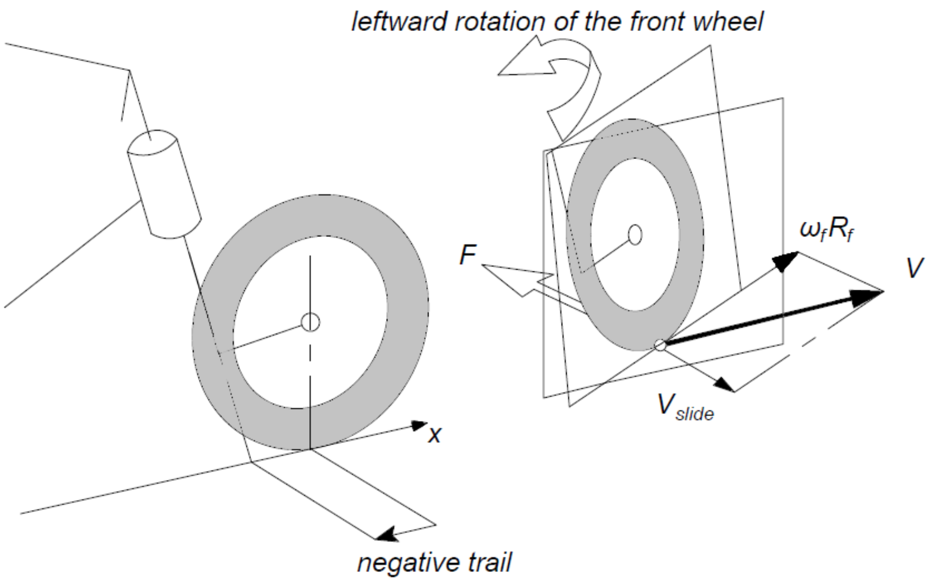


Figure 1.4: influence of negative trail

In this way a frictional force F acts on the tire parallel to V_{slide} in the opposite sense, meaning that the friction force F generates a moment that tends to align the front wheel because the trail is positive.

Considering a vehicle with a negative trail, meaning that the the contact point is in front of the intersection point of the steering axis with the road plane, the frictional force F is always in the opposite direction of the velocity V_{slide} which would lead to a moment around the steering head that would tend to increase the rotation to the left as it is possible to see in figure 1.4. The force F would amplify the disturbing effect, compromising the motorcycle's equilibrium.

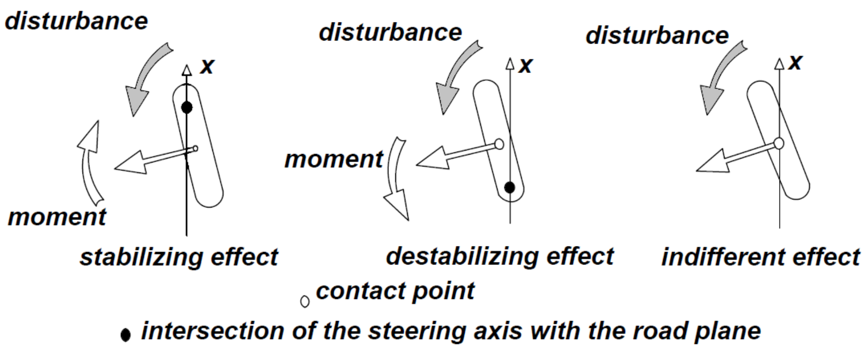


Figure 1.5: summary of the effects of trail

Small trail values generate small aligning moment of the lateral friction force, even if the driver has the impression that the steering movement is easy, the steering mechanism is very sensitive to the irregularities of the road and higher values of the trail (they can be reached with high values of the caster angle) increase the stability of the motorcycle's rectilinear motion, reducing maneuverability.

1.4 The center of gravity

The position of a motorcycle's center of gravity has a significant influence on the motorcycle's dynamic behavior, its position depends on the distribution and quantity of masses of the individual components of the motorcycle. The engine is the heaviest component and its location greatly influences the location of the motorcycle's center of gravity.

The longitudinal distance L_r between the contact point of the rear wheel and the center of gravity can be easily determined by measuring the total mass of the motorcycle and the loads on the wheels under static conditions (front load N_{sf} and rear load N_{sr}):

$$L_r = \frac{N_{sf}p}{mg} = p - \frac{N_{sr}p}{mg} \tag{6}$$

In general a motorcycle is characterized by the static loads that act on the wheels, expressed in a percentage formula

$$\frac{\%\,front\ load}{\%\,rear\ load} = \frac{N_{sf}/mg}{N_{sr}/mg} = \frac{L_r/p}{L_f/p} \tag{7}$$

The distribution of the load on the wheels under static conditions is generally greater on the fron wheel for racing motorcycles (50-57% front, 43-50% rear) and it is greater on the rear wheel in the case of touring motorcycles (43-50% front, 50-57% rear).

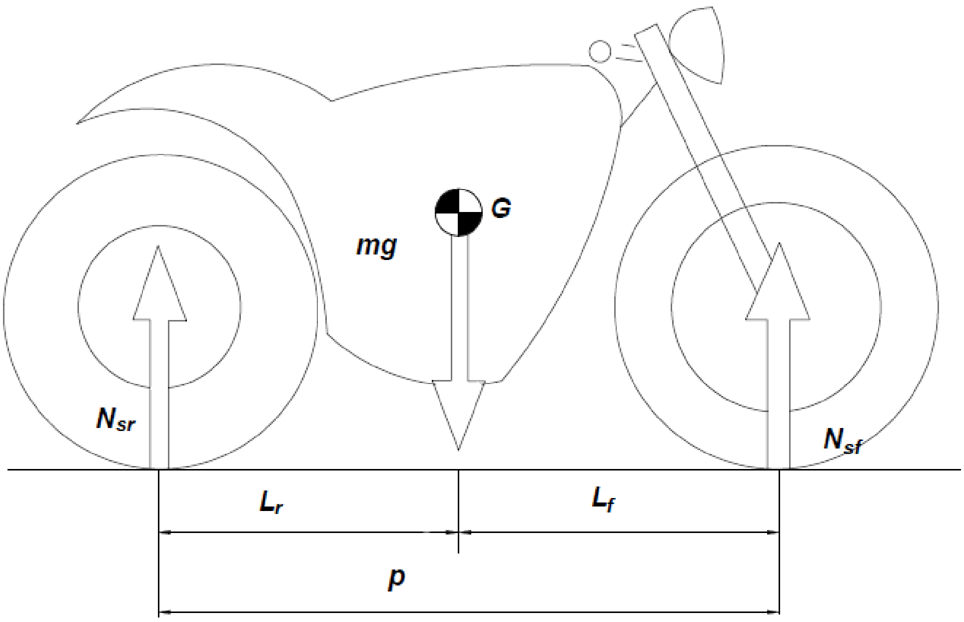


Figure 1.7: center of gravity longitudinal position

When the center of gravity is more forward, meaning that the front load is higher than 50%, wheeling the motorcycle becomes more difficult, or in other words there is an easier transfer of the power to the ground. This is one reason why usually racing motorcycles are more loaded on the front wheel, furthermore this more weight on the front partially compensates for the aerodynamic effects that unload the front wheel, especially at high velocities. When the position of the center of gravity is more towards the rear of the motorcycle, braking capacity is increased reducing the danger of the rear wheel wheeling or even forward flip over during a sudden stop with the front brake. Modern sport motorcycles tends to have a 50-50% distribution in order to perform well in both acceleration and braking phases. For a question of safety it is preferable to have longitudinal slip of the rear wheel during the acceleration phase rather than longitudinal slip of the front wheel during the braking phase. The ratio L_r/p without the rider varies from 0.35 to 0.51 with the smallest values for the small motorcycles while the highest for racing motorcycles. In general the position of the rider shifts the whole center of gravity towards the rear, as it is possible to see in figure 1.6, the presence of the rider increases the load on the rear wheel diminishing the percentage of load on the front wheel.

Once the longitudinal position of the center of gravity is found its height can be determined by measuring the load on one wheel, raising the front wheel of a known amount as shown in figure 1.8, brings to

$$h = \left(\frac{N_{sf}p}{mg} - L_r\right)\cot\left[\arcsin\left(\frac{H}{p}\right)\right] + \frac{R_f+R_r}{2} \tag{8}$$

The height of the center of gravity has a significant influence on the dynamic behavior of a motorcycle, especially in the acceleration and braking phases. A high center of gravity in acceleration leads to a larger load transfer from the front to the rear wheel, this change influences the quantity of driving force that can be applied to the rear wheel, but the fewer load on the front wheel makes the front wheel wheeling more probable.

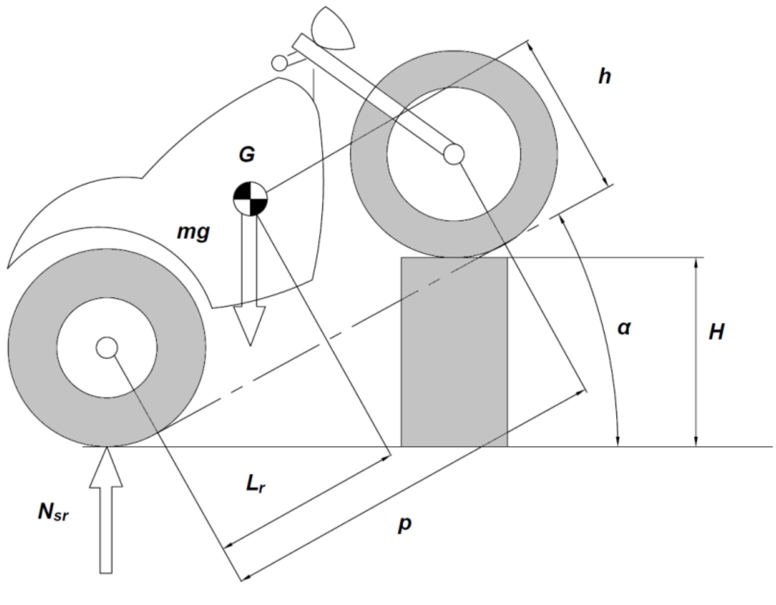


Figure 1.8: center of gravity height estimation

In braking an high center of gravity causes a greater load on the front wheel for the same principle quoted before, the load transfer occurs from the rear wheel to the front one. The greater load on the front wheel improves the braking characteristics of the motorcycle, but it also makes the forward flip-over more likely.

The optimal height of the center of gravity also depends on the driving/braking traction coefficient between the tires and the road plane. With low values of the driving/braking traction coefficient it is good to have an higher center of gravity in order to improve both the acceleration and braking capacities. With high values of the friction coefficient it is better to have a lower center of gravity in order to avoid the limit conditions of wheeling and forward flip-over.

The choice of the center of gravity position is therefore a compromise, it is necessary to take into account the purpose and the actual use that the motorcycle is going to have. All-terrain motorcycles are characterized by rather high centers of gravity, while sport and very powerful motorcycles are usually characterized by a lower center of gravity. The main effects of the position of the center of gravity are summarized in table 1.1

Forward center of gravity	The motorcycle tends to over-steer, the rear wheel tends to slip laterally when taking a turn
Rear center of gravity	The motorcycle tends to under-steer, the front wheel tends to slip laterally during a curve
High center of gravity	The front wheel tends to lift in acceleration and the rear wheel tends to lift in braking
Low center of gravity	The rear wheel tends to slip in acceleration and the front wheel tends to slip in braking

Table 1.1: effects of the position of the center of gravity

The height of the center of gravity of the motorcycle alone has values varying from 0.4 to 0.55 [m], but the presence of the rider raises the center of gravity to values ranging from 0.5 to 0.7 [m], the displacement of the center of gravity due to the presence of the rider depends on the relation between the mass of the rider and the mass of the motorcycle.

The ratio h/p without rider and with fully extended suspensions varies in the range $0.3 - 0.4$, the smallest values are valid for cruisers and scooter, while the highest for dual sport and endure type motorcycles.

1.5 Motorcycle equilibrium in steady state rectilinear motion

Considering a model of the motorcycle-rider system, as illustrated in figure 1.9 and assuming

- The rolling resistance force is zero
- The aerodynamic lift force F_l is also considered zero
- Since the road surface is flat, the force resisting the forward motion of the motorcycle is reduced to just the aerodynamic drag force F_d

The pressure center of the motorcycle in which is applied the drag force coincides with the system's center of gravity.

In addition to the drag force, the other forces acting on the motorcycles are

- The weight force mg that acts at the center of gravity
- The driving force S applied at the rear tire-ground contact point
- The vertical forces N_f and N_r exchanged between the tires and the road plane

The equations of equilibrium of a motorcycle make possible the computation of the vertical reaction forces N_f and N_r once the weight force, the driving force and the drag force are known.

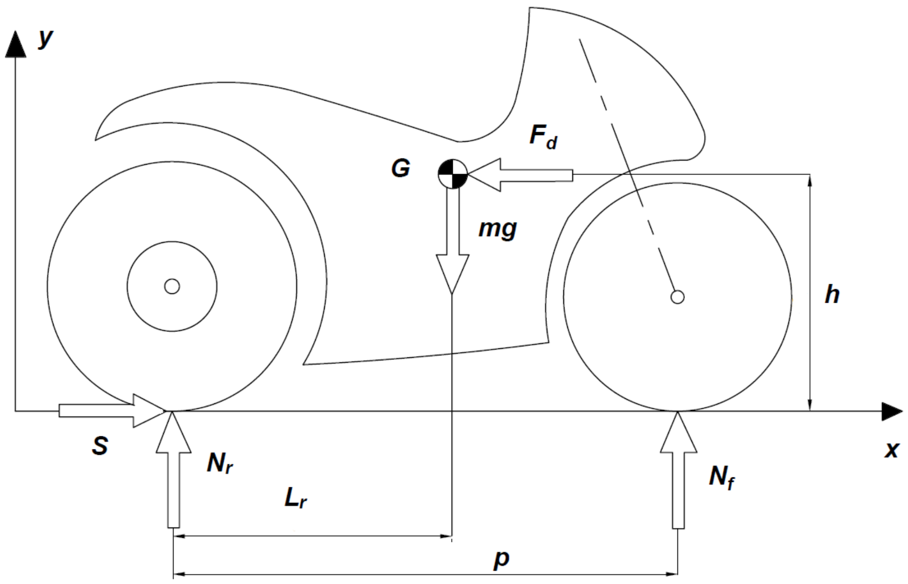


Figure 1.9: Forces acting on the motorcycle

The equilibrium of the horizontal forces is

$$S - F_d = 0 \tag{9}$$

Vertical forces

$$mg - N_f - N_r = 0 \tag{10}$$

Equilibrium of the moments around the center of gravity

$$Sh + N_f L_f - N_r L_r = 0 \tag{11}$$

The vertical forces exchanged between the tires and the front wheels are then found as

$$N_f = \frac{mgL_r}{p} - \frac{Sh}{p} \tag{12}$$

$$N_r = \frac{mgL_f}{p} + \frac{Sh}{p} \tag{13}$$

The vertical reaction forces of the tires are then composed by the static load, which depends on the distribution of the load between front and rear, while the second term is call load transfer, which is directly proportional to the drive force S and the height of the center of gravity and inversely proportional to the motorcycle wheelbase p. The load transfer refers to the fact that there is a decrease in the front wheel load and a corresponding increase in the load on the rear wheel, meaning that the load is "transferred" from the front to the rear wheel. The ratio h/p is higher in motorcycle with respect to that of other vehicles and it is usually in the interval $0.3 - 0.45$, obviously the higher ratio brings to an higher load transfer.

Figure 1.10 shows the load transfer angle, the weight force mg is equal to the sum of the static loads acting on the wheels N_{sf} and N_{sr}, the driving force S and the load transfer is applied to the rear wheel contact point with the ground, the load transfer is also applied at the front wheel in the opposite way with respect to the rear wheel. The direction of the resultant of the two driving force and of the load transfer at the rear wheel is inclined with respect to the road by the angle

$$\tau = \arctan h/p \tag{14}$$

In order for a motorcycle to maintain equilibrium this resultant force must be equal and opposite in sign to the resultant of the drag force and the load transfer of the front wheel.

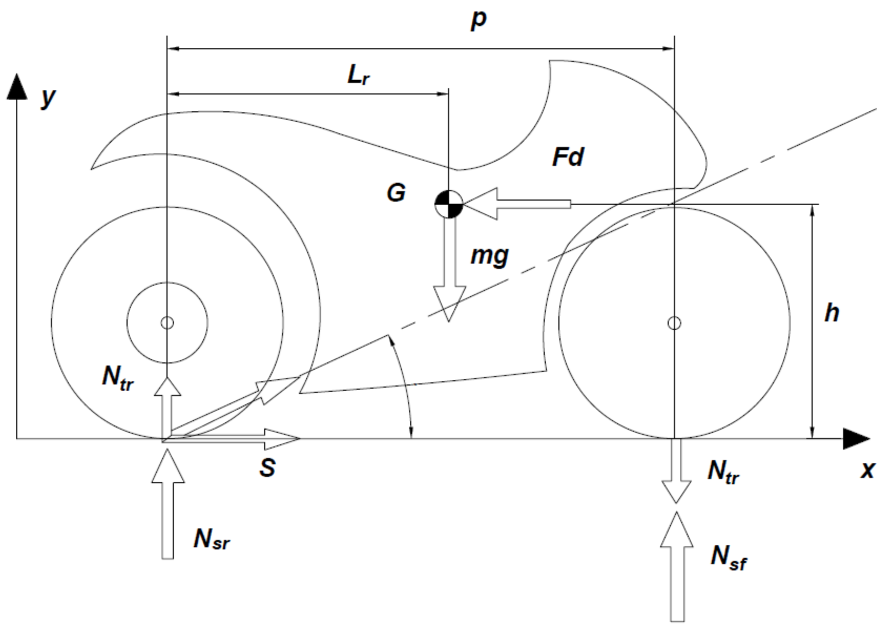


Figure 1.10: Load transfer angle

1.6 Steady turning

During steady turning motion the motorcycle can show can have neutral, under-steering or over-steering behavior, in order to maintain equilibrium the rider applies a torque at the handlebars that can be zero, positive if it is in the same direction of the handlebar rotation or negative if the torque is applied in the opposite direction to the rotation of the handlebar and these characteristics define the handling behavior of the motorcycle.

1.6.1 Ideal roll angle

The motorcycle in steady turning is subjected to both a restoring moment, generated by the centrifugal force that tends to return to a vertical position, and to a tilting moment generated by the weight force that tends to increase the motorcycle's inclination, which is called roll angle as illustrated in figure 1.11.

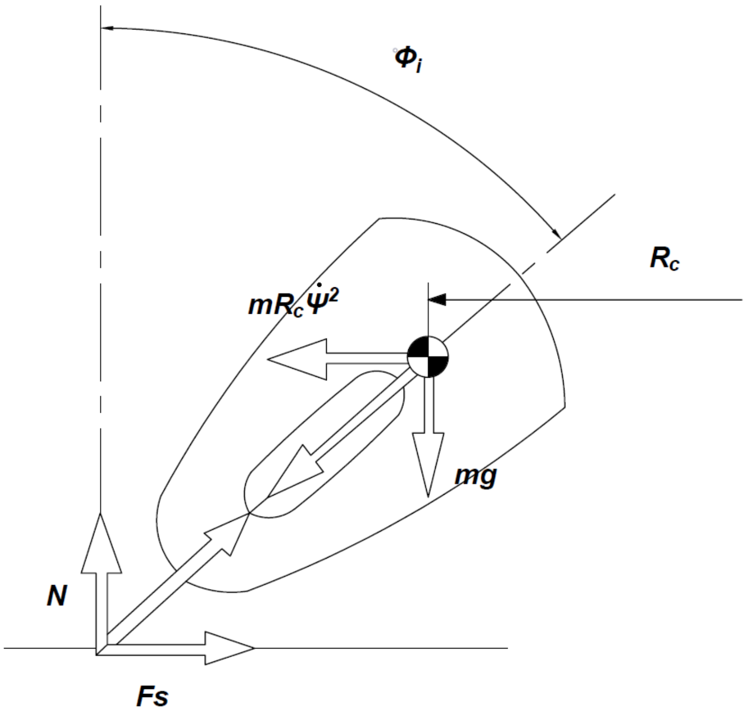


Figure 1.11: Ideal roll angle

The hypothesis in the following considerations are

- The motorcycle runs along a curve of constant radius at constant velocity (steady state conditions)
- The gyroscopic effects are considered negligible
- The cross section thickness of the tires is zero

By solving the equilibrium of the moments generated it is possible to derive the roll angle in terms of the turning radius R_c and of the forward velocity $V = \dot{\psi} R_c$, with $\dot{\psi}$ being the yaw rate of the motorcycle and the ideal roll angle is found as

$$\phi_i = \arctan \frac{R_c \dot{\psi}^2}{g} \tag{14}$$

In conditions of equilibrium the resultant of the centrifugal force and the weight force passes through the line joining the contact points of the tires and the road plane, but this happens only when the tires have zero thickness and the steering angle is very small, in reality if a non-zero steering angle is assigned the front contact point is displaced laterally with respect to the x-axis of the rear frame and the contact points of the tires is no more contained in the plane of the rear frame.

1.6.2 Effective roll angle

Considering a motorcycle with tires of thickness *2t* that runs the same turn radius R_c at the same velocity $V = \dot{\psi}R_c$, the roll angle which is necessary to reach the equilibrium of the moments exerted by the weight force and the centrifugal force is greater than the ideal one

$$\phi = \phi_i + \Delta\phi \tag{15}$$

The increase in the roll angle is found with

$$\frac{\sin\Delta\phi}{t} = \frac{\sin\phi_i}{h-t} \tag{16}$$

So equation 15 becomes

$$\phi = \phi_i + \Delta\phi = \arctan\frac{R_c\dot{\psi}^2}{g} + \arcsin\frac{t\sin\left(\arctan\frac{R_c\dot{\psi}^2}{g}\right)}{h-t} \tag{17}$$

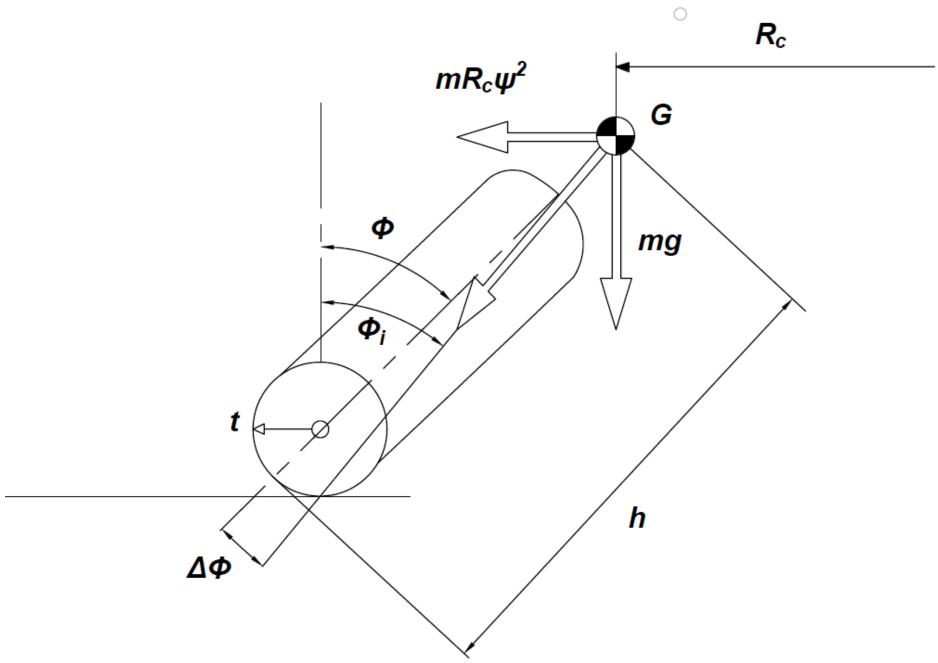


Figure 1.12: Effective roll angle

Equation 17 shows that $\Delta\phi$ increases both as the roll angle and the cross tire section radius increase and as the height of the center of gravity decreases, this means that the use of wide tires forces the rider to use greater roll angles with respect to the angle necessary with motorcycles using smaller tires. Another conclusion is that with the same cross section of the tires to describe the same turn with the same forward velocity a motorcycle with low center of gravity needs to be more inclined with respect to one with a higher center of gravity.The motorcycle roll angle on a turn is influenced by the rider's driving style, in fact by leaning with respect to the vehicle the rider changes its position of his center of gravity with respect to the motorcycle.

1.6.3 Wheel velocity in a curve

The velocity of the vehicle is represented by the forward velocity of the contact point of the rear wheel, so the yaw velocity is

$$\dot{\psi} = \frac{V}{R_c} \tag{18}$$

If the wheel slippage between the tires and the road surface in the forward direction of the wheels is not considered the spin velocity of the wheels in terms of the vehicle forward velocity, roll angle and steering angle are

$$\omega_r = \frac{V}{(R_r - r_r) + r_r \cos\phi} \tag{19}$$

$$\omega_f = \frac{V}{((R_f - r_f) + r_f \cos\beta)\cos \delta_f} \tag{20}$$

With the angle β being the camber angle of the motorcycle.

In reality during the thrust and braking phases there is always a longitudinal slippage between the rear wheel and the road plane, while in the front wheel there is slippage in the braking phase and under steady state condition the slippage is negligible because it is only due to rolling resistance.

It is important to note that with the same longitunal velocity the angular velocity of the wheels increases during turning with respect to the angular velocity of the wheels in straight running because the contact does not occur on the largest circumference of the wheels.

1.7 Torque applied to steering

The equilibrium of moments around the steering axis enables the evaluation of the torque M that the rider must apply to the handlebars to assure the motorcycle’s equilibrium in a curve, referring to steady turning. In transitory movement the torque the rider must apply is different from that calculated in steady state, especially if the variations in velocity and trajectory occur suddenly.

The torque applied by the rider is equal, but of opposite sign to the resultant of all the moments generated by the forces acting on the front section and the resultant torque is composed of six terms:

$$M = -M_{Pf} - M_{Cf} - M_{Nf} - M_{Ff} - M_{Wf} - M_M \tag{21}$$

In table 1.2 each component of the resultant torque is shown

M_{Pf}	Disaligning influence due to the weight force of the front section
M_{Cf}	Aligning influence due to the centrifugal force of the front section
M_{Nf}	Disaligning influence due to the normal load on the front wheel
M_{Ff}	Aligning influence due to the lateral force on the front wheel
M_{Wf}	Aligning influence due to the gyroscopic effect of the front wheel
M_M	Disaligning influence due to the twisting torque of the front tire

Table 1.2: components of the steering torque

Figure 1.13 describes the steering torque and all the moments which influence its characteristics, point A indicates the intersection of the steering axis with the normal line passing through the front tire contact point and the distance between point A and the contact point P_f represents the effective trail of the tire.

The torque exercised by the rider is by definition positive if it tends to increase the steering angle into the turn, this means that:

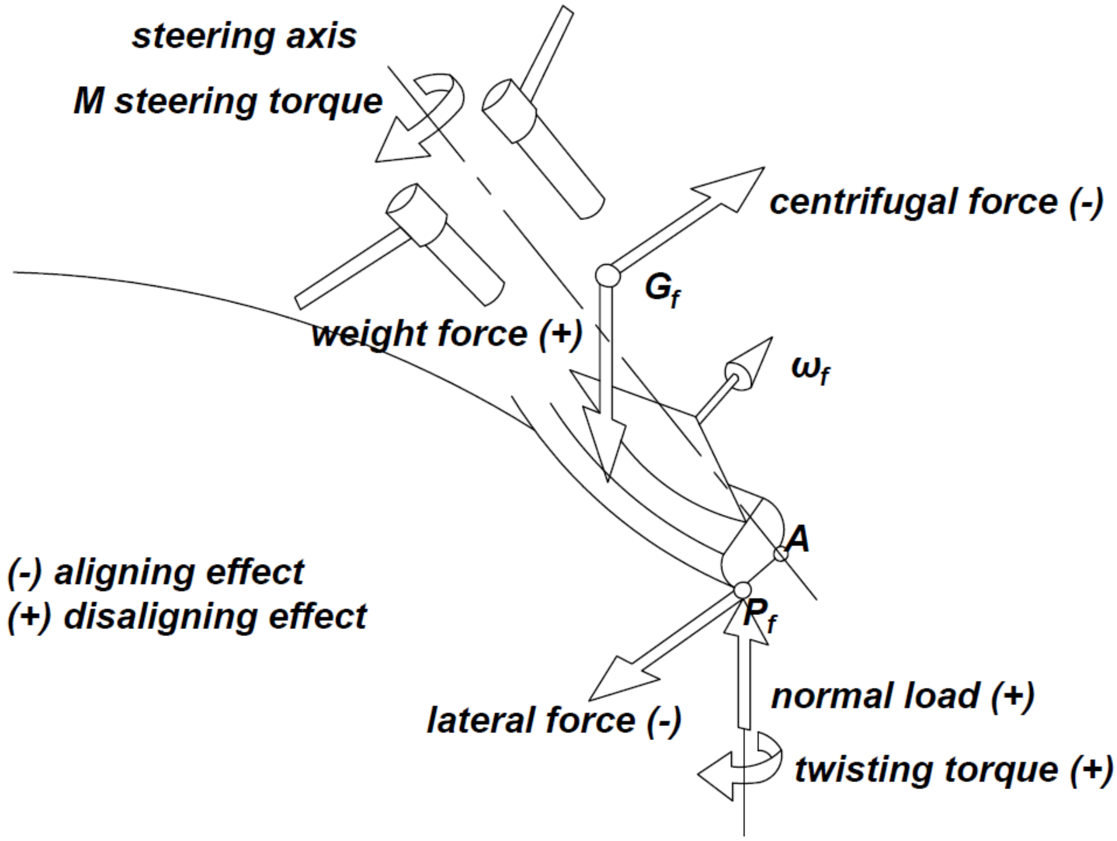


Figure 1.13: steering torque components

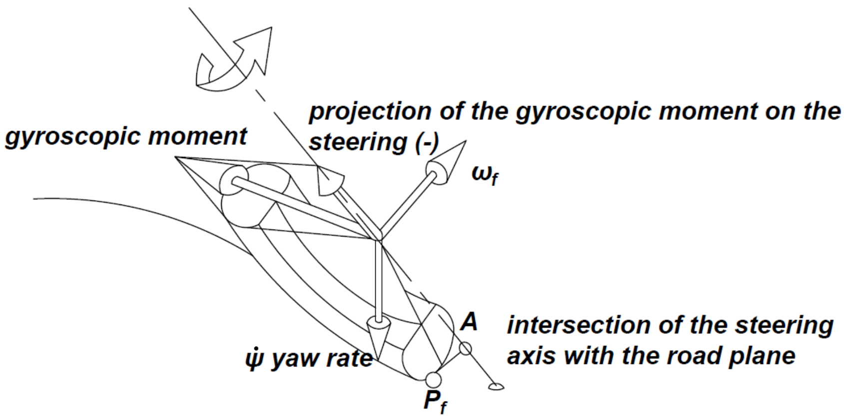


Figure 1.14: gyroscopic component of the steering torque

- At low velocities the steering torque is negative, therefore in steering the rider must block the handlebars, which otherwise tend to rotate further. When the values of the steering torque become strongly negative, the inclination and the entry into the turn becomes easier;

- With an increase in velocity the torque to be applied to the handlebars becomes positive, if the values remain high the torque generates in the rider the unpleasant sensation of driving a motorcycle that is hard to incline and to insert into tight turns.

The various contributions have the following effects:

- Vertical load: the vertical reactive force generates a positive moment of high value
- Lateral force: the lateral reactive force generates a high value negative moment of the same order of magnitude generated by the vertical load
- Front weight force: the moment is positive
- Centrifugal force: the moment is negative and of the same order of magnitude of the moment generated by the front weight force
- The gyroscopic effect generates a positive moment
- The twisting moment generates a disaligning effect that increases with the roll angle

For small roll angles the rider needs to apply a negative torque in order to obtain equilibrium, while for high roll angles the torque must be positive.

The maximum maneuverability is obtained when the torque necessary to assure equilibrium is almost zero, in this conditions in fact if the rider lets the handlebars free the motorcycle continues to round the set turn.

1.7.1 The influence of the motorcycle geometry on the steering torque

The steady turning behavior of a motorcycle is a function of the vehicle geometry, inertia and tire properties.

The first parameter considered is the normal trail, an increment in its value brings to lower steering torque needed to obtain equilibrium. This result can be explained considering the fact that when the trail increases the disaligning effect due to the front tire vertical load increases more than the aligning effect due to the lateral force, thus resulting in a more stable steering behavior.

In case of an increase in the caster angle $\boldsymbol{\theta_{cf}}$ the effect is that of having a more aligning effect, since the steering torque increases, this parameter is in fact relevant. Considering that the real steering head angle is influenced by the motorcycle’s attitude, depending on speed, mass distribution and suspension behavior and so particular attention should be paid when choosing the parameter during the design.

Increasing the front tire cross section radius $\boldsymbol{r_f}$ brings to a strong aligning effect, this is caused by the displacement of the front wheel contact point due to the roll angle.

The rider position is another parameter that influences the torque of the motorcycle, in fact a forward displacement of the rider’s center of mass has a slightly self-steering effect. If the rider moves remaining in the plane of symmetry of the motorcycle, the steering behavior does not change significantly. If the rider instead shows a lateral displacement towards the inside of the curve has a strongly aligning effect, in fact the steering characteristics of the motorcycle are strongly influenced by the driving style of the rider. The presence of a passenger alters the mass distribution of a motorcycle, the resulting effect is slightly aligning but the steady turning steering torque is not substantially changed.

The influence of the rider's lateral position of the steering behavior of the motorcycle of 0.05 [m] towards the center of the curve corresponds to a decrease in the roll angle of about $1°$.

Any modifications to the motorcycle in each of its parameter bring to a variation in the torque to be applied to the handlebars in order to obtain equilibrium, the influence of the main geometric and inertial parameters on the steering torque is evidenced in table 1.3. The parameters that most influence the torque value are the caster angle $\boldsymbol{\theta_{cf}}$, the front tire cross section radius $\boldsymbol{r_f}$, the height of the center of gravity and the normal trail $\boldsymbol{t_n}$.

	Rider displacement inside the curve of 0.05 $[m]$	*Strong aligning effect*
$\boldsymbol{\theta_{cf}}$	*Caster angle*	*Strong aligning effect*
$\boldsymbol{r_f}$	*Radius of the cross section of the front tire*	*Aligning effect*
$\boldsymbol{h}$	*Height of the motorcycle mass center*	*Aligning effect*
$\boldsymbol{t_n}$	*Front tire normal trail*	*Aligning effect*
$\boldsymbol{I_{wf}}$	*Front wheel spin inertia*	*Small aligning effect*
$\boldsymbol{L_r}$	*Distance motorcycle mass center-rear wheel*	*Small disaligning effect*
$\boldsymbol{t}$	*Mechanical trail*	*Disaligning effect*
$\boldsymbol{M_M}$	*Twisting torque of the front tire*	*Disaligning effect*

Table 1.3: effects of the increase of parameters on the steering behavior of a motorcycle

In case of aligning effect the steering angle tends to decrease, the rider must steer into the turn to counteract this effect. If the torque is negative in sign, its value must increase. When there is a disaligning effect the steering angle tends to increase, the rider must steer out of the turn to counteract this effect, and if the torque is negative its value must continue to decrease.

1.8 Modes and stability

The front and rear end of a motorcycle in motion can start to oscillate around the steering axis, even if the wheels are well balanced. Oscillations can be observed at certain speeds, especially if the front wheel is out of balance, they reach their maximum amplitude and then decrease as speed decreases until they disappear completely. At low speed it is possible to observe that the motorcycle tends to fall over sideways.

These observations of motorcycle dynamics show that there are three major modes:

- Capsize, a non-oscillating mode used and controlled by the rider;
- Weave, an oscillation of the entire motorcycle, but mainly the rear end;
- Wobble, an oscillation of the front end around the steering axis which does not involve the rear end in any significant way.

The rider's control task can be considered to involve either fixed control or free control, i.e. with or without their hands grasping the handlebars. With the steering rotation fixed the motorcycle-rider system is unstable at all speeds, like a capsizing ship. In the unconstrained condition the steering system is free to steer itself, potentially relieving the rider of the need to apply steering control action for stabilization. At very low speeds a motorcycle is unstable because of capsize, the weave mode is unstable up to $7 - 8$ $[m/s]$. Over this speed the vehicle usually enters into a stable zone, so that the rider could remove his hands from the handlebars without falling. As speed increases each mode may become unstable depending on the motorcycle's characteristics and the rider has to counteract these modes with a torque applied at the handlebars.

1.8.1 Capsize

This mode is deeply influenced by the rider action on the handlebars, so this mode easily shifts from the unstable to the stable zone.

Capsize is actually a mode used by the rider to roll the motorcycle, this action is achieved through the rider's effort to hold or move the steering head rotation to some non-equilibrium position. Since the motorcycle exists as an inverted pendulum, this mode is always unstable. The capsize mode consists mainly of a roll motion combined with a lateral displacement plus some less important steering and yaw moment and depends on a number of factors:

- Speed of the motorcycle
- Wheel inertia
- Position of the center of gravity
- Motorcycle mass
- Motorcycle roll inertia
- Caster angle
- Properties of the tires, primarily the cross sectional size of the tires, twisting torque and pneumatic trail of the front tire

To highlight the influence of some geometrical and inertial properties of the motorcycle on the capsize mode it is useful to analyze the fall motion of a motorcycle with the steering head locked, in this hypothetical case, within the limits of linear approximation, capsize can be expressed as an exponential law:

$$\phi = \phi_0 e^{t/\tau} \tag{22}$$

In this case τ is a positive time constant, therefore the capsize is always unstable. This time constant is a measure of how easily a motorcycle tends to lean over, racing motorcycles need a small time constant so that they can change their trajectory quickly, touring motorcycles need to roll more slowly, making them easier for the rider to control.

Capsize instability should not be viewed as a drawback since it enables the motorcycle to lean into the curves and then execute them correctly: the smaller the time constant the less lead time is needed to start leaning the motorcycle into the curve.

The simplified models, with the steering head locked and negligible gyroscopic effects, yield smaller time constant values than the ones which can be obtained by studying the complete model of a motorcycle. The simplified models clearly show how geometric and inertial properties affect the capsize time constant. Using a model with thin disk wheels as the one showed in figure 1.15 it is possible to understand the falling time of a motorcycle, since the mathematical modeling of motorcycle capsize is complicated by the presence of the steering head, gyroscopic effects and tire contact forces arising from the slip and camber angles.

The model follows the following assumptions:

- The motorcycle is moving in direction x at speed V

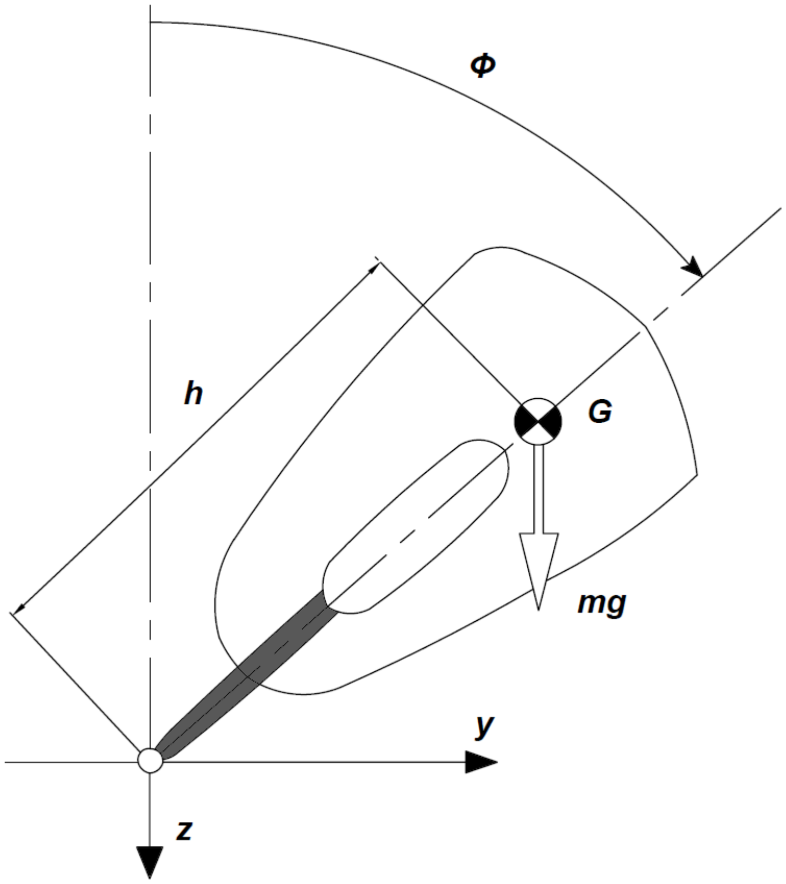


Figure 1.15: capsize model of the motorcycle with thin disk wheels

- The thickness of the cross section tires is null
- There is no slippage between the tires and the road
- The steering head is locked in place
- Gyroscopic effects are considered negligible

Based on these assumptions, capsize is a simple rotation of the motorcycle around the axis defined by the points in which the tires come into contact with the roadway and the equilibrium of moments with respect to the contact point gives the following equation

$$\left(I_{xg} + mh^2\right)\ddot{\phi} = mgh\sin\phi \tag{23}$$

Linearizing the equation around the vertical equilibrium position and introducing the solution $\phi = \phi_0 e^{st}$ it is possible to utilize the *Laplace transform* and obtain its solution, which is a real number and therefore corresponds to a non-oscillating motion:

$$s = \pm\sqrt{\frac{mgh}{I_{xg}+mh^2}} \tag{24}$$

The time constant τ can be found as the inverse of the positive real eigenvalue of equation 24

$$\tau = \sqrt{\frac{I_{xg}+mh^2}{mgh}} \tag{25}$$

The time constant τ is determined by the height of the center of gravity, mass of the motorcycle and the motorcycle's moment of inertia about the x-axis through the motorcycle's center of gravity

Using the radius of gyration ρ to express the motorcycle's moment of inertia ($I_{xg} = m\rho^2$) it is possible to modify equation 25 into

$$\tau = \sqrt{\frac{h}{g}}\sqrt{1+\frac{\rho^2}{h^2}} \tag{25}$$

Assuming that the mass and the center of gravity height are constant but it is possible to distribute differently the mass in order to modify the moment of inertia and so the radius of gyration ρ it is possible to see that the time constant increases as the radius of gyration increases. For a given radius of gyration instead it is possible to see that the time constant τ decreases as the height of the center of gravity increases until it starts to in crease. This means that once the radius of gyration is set the

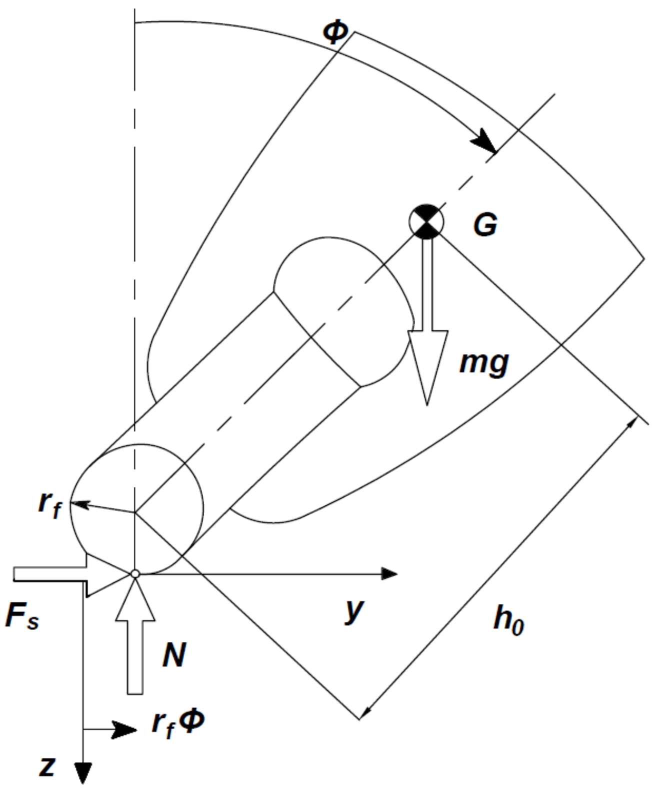


Figure 1.16: capsize model of the motorcycle with lateral rolling of the tire

time constant is at its lowest value when the height of the center of gravity is equal to the radius of gyration.

A second simplified model can be built from the first by removing the assumption that the wheels are thin disks, assuming a motorcycle with circular tire cross sections which do not slip laterally on the roadway during capsize. The equilibrium of the forces and moments of figure 1.16 are

$$\begin{cases} m\ddot{z}_g = -N + mg \\ m\ddot{y}_g = F_s \\ I_{xg}\ddot{\phi} = N(y_g - y) + F_s z_g \end{cases} \tag{26}$$

Under pure forward rolling conditions the system has just one degree of freedom, in this way y, y_g and z_g can be expressed in function of the roll angle

$$\begin{cases} y = \phi r_f \\ y_g = \phi r_f + h_0 \sin\phi \\ z_g = -r_f - h_0 \cos\phi \end{cases} \tag{27}$$

With mathematical substitutions it is possible to use the Laplace transform and obtain the following equation

$$[I_{xg} + m(h_0 + r_f)^2]s^2 - mgh_0 = 0 \tag{28}$$

The resulting time constant is

$$\tau = \sqrt{\frac{h_0}{g}}\sqrt{(1 + \frac{r_f}{h_0})^2 + \frac{\rho^2}{h^2}} \tag{29}$$

Equation 29 shows that the time constant increases with the radius of the tire cross section, so entering a curve with a motorcycle with large tires takes longer to lean with respect to one with small tires.

1.8.2 Wobble

Wobble is an oscillation of the front assembly around the steering axis that that can become unstable at fairly low to middle speeds. These oscillations resemble the ones obtained when an airplane lands, their typical frequency value range from $4\ [Hz]$ for heavy motorcycles up to $10\ [Hz]$ for lightweight motorcycles. Wobble frequency increases as the trail increases and the front frame inertia decreases, and it is determined by the stiffness and damping of the front tire. In the forward speed range from $10 - 20\ [m/s]$ wobble is slightly damped and can therefore become unstable, adding a steering damper increases the damping effect and so the stability of the system.

This mode can be thought of in complete isolation from the rear assembly motion and roll, the front end can be considered as a rigid body rotating around the steering axis while the rear frame is fixed.

1.8.3 Weave

Weave is an oscillation of the entire motorcycle, but mainly the rear end. The natural frequency of this side to side motion is zero when the forward speed is also zero and ranges from $0 - 4\ [Hz]$ at high speed. The factors that determine weave are:

- Position of the center of gravity of the rear assembly and that of the front assembly
- Wheel inertia
- Caster angle
- Trail

- Cornering stiffness of the rear tire

Weave is usually unstable at low speed up to $7-8\ [m/s]$, usually stable in the middle range and can become uncontrollable at high speed since its damping might decrease substantially and its natural frequency can become too high for the rider to control. The weave is generated by the coexistence of two unstable non-oscillating modes:

- Body capsize
- Steering capsize

Body capsize indicates the capsize of the entire motorcycle, the time constant with the steering free of motion decreases slightly, supposing that the vehicle is falling to the rider's right the steering geometry causes the vehicle to steer to the left, moving the contact point towards the rider's right, increasing in this way the gravitational torque with the whole vehicle capsizing less quickly.

Steering capsize is a capsize of the steering head due to the disaligning effect of both the front tire normal load and front frame weight force. The time constant of the steering capsize has values in the range of $0.1-0.2\ [s]$ for speed lower than $1\ [m/s]$.

1.9 Gyroscopic Moments

A gyroscopic effect is generated by a rigid body rotating around an axis which in turn is rotating around a second axis not parallel to the first one, the gyroscopic effect takes the form of a moment which is equal to the vector product of the angular momentum of the body around the first axis and the speed of rotation around the second axis.

Motorcycle dynamics incorporate a variety of gyroscopic effects, they can be classified as:

- Yaw gyroscopic effects
- Roll gyroscopic effects
- Steering gyroscopic effects

In the following section these effects are examined.

1.9.1 Gyroscopic effects generated by yaw motion

The first case considered is the one in which the gyroscopic effect is generated by the wheels during cornering, the front wheel is rotating at an angular speed ω_f and the motorcycle is travelling through a curve of radius R_c at a constant yaw rate $\dot{\psi}$.

The angular motion of the wheel generates a gyroscopic moment around the horizontal axis which has the effect of straightening the wheel

$$M_g = I_{wf}\omega_f\dot{\psi}\cos(\phi) \tag{30}$$

This equation can be considered valid if the $\dot{\psi}$ can be considered small with respect to the angular velocity ω_f of the front wheel and this assumption is verified in practice because the turning radius

is much greater than the wheel radius. The reference frame taken in consideration for the wheel is linked to the front fork. Motorcycle equilibrium is reached when the resultant of the weight force and the centrifugal force intersects the line joining the contact points of the two wheels, which both contribute to the total gyroscopic effect which will be

$$M_g = I_w \omega \dot{\psi} \cos(\phi) \tag{31}$$

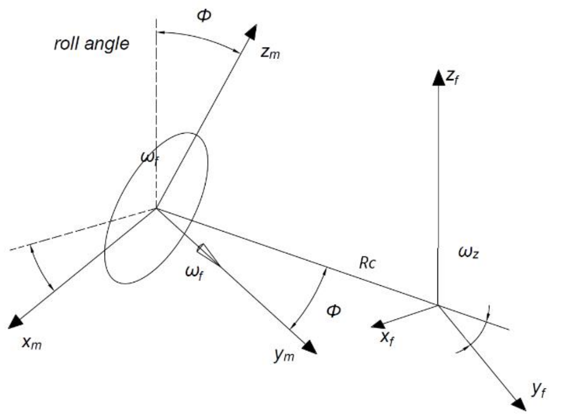


Figure 1.17: gyroscopic moment generated by the wheel rotational speed ω_f and the yaw motion ψ

Without considering the gyroscopic effect and assuming the wheels without any thickness the ideal roll angle for a motorcycle in steady state cornering is given by

$$\phi = \arctan(\frac{R_c \dot{\psi}^2}{g}) \tag{32}$$

The gyroscopic effect of the wheels during cornering is manifested by a righting moment and in order to counteract this effect and maintain equilibrium during the corner the rider can lean into the turn in a way that the resultant of the weight force and the centrifugal force generates a moment

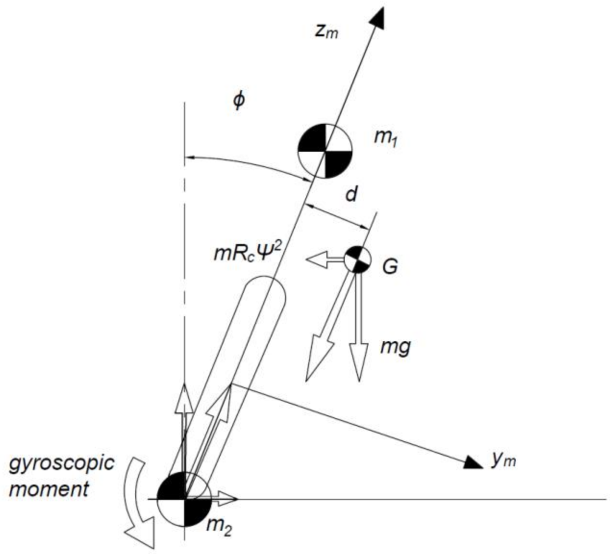


Figure 1.18: gyroscopic moment generated by the wheel rotational speed ω_f and the yaw motion ψ

equal and opposite to the gyroscopic moment of the two wheels:

$$M = -d\sqrt{(mg)^2 + \left(mR_c\dot{\psi}^2\right)^2} = -M_g \tag{33}$$

In this way the final lean angle will be greater than the ideal roll angle calculated on the assumption that the gyroscopic moment is zero.

In this case the righting moment generated by both the gyroscopic moment and the centrifugal force are offset by the overturning moment of the weight force, this means that the gyroscopic moment makes the actual roll angle greater than the ideal roll angle that would be achieved without this effect. The increase of roll angle $\Delta\phi$ needed to counterbalance the gyroscopic effect is given by

$$\Delta\phi = \arcsin\frac{d}{h} = \arcsin\frac{I_w\omega\dot{\psi}\cos(\phi+\Delta\phi)}{h\sqrt{(mg)^2+(mR_c\dot{\psi}^2)^2}} \tag{34}$$

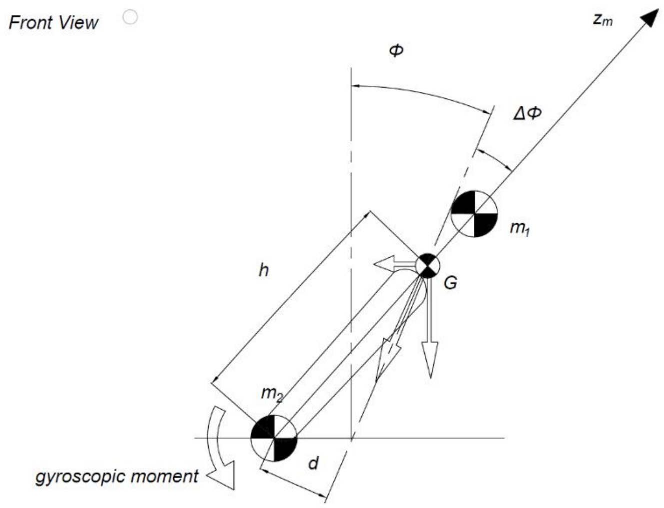


Figure 1.19: Increase in the roll angle ϕ caused by yaw gyroscopic effect

Since $\Delta\phi$ is smaller than ϕ it can be neglected in the computation and the equation (57) becomes

$$\Delta\phi = \arcsin\frac{d}{h} = \arcsin\frac{I_w\omega\dot{\psi}\cos(\phi)}{h\sqrt{(mg)^2+(mR_c\dot{\psi}^2)^2}} \tag{35}$$

The increase $\Delta\phi$ makes the motorcycle less maneuverable, since the motorcycle takes more time to reach the incrementally larger equilibrium roll angle.

1.9.2 Gyroscopic effects generated by transversally mounted engine

The gyroscopic effect generated by the engine is determined by the engine's rotational speed ω_m, which depends on what gear the motorcycle is in. Assuming a motorcycle in steady state cornering and neglecting the inertia of the front and rear wheels, the gyroscopic effect is generated by the rotation of the engine, its main shaft rotates in the same direction as the wheels.

The gyroscopic effect generated by the engine brings the driver to lean the motorcycle of an angle greater than the ideal roll angle that would be necessary if this effect were absent, the increase in roll angle is given by

$$\Delta\phi = \arcsin\frac{I^*{}_{wm}\omega_m\dot{\psi}\cos(\phi)}{h\sqrt{(mg)^2+(mR_c\dot{\psi}^2)^2}} \tag{36}$$

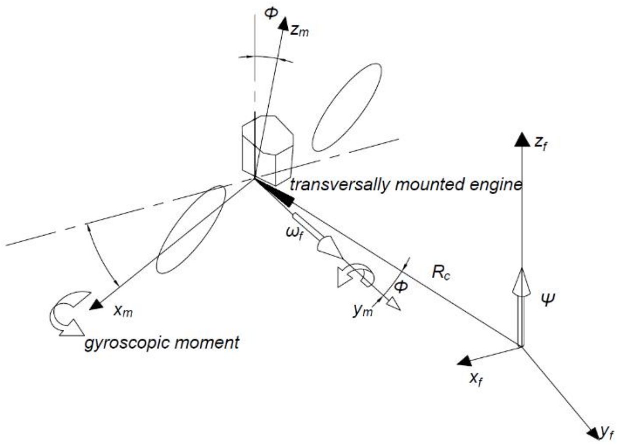


Figure 1.20: gyroscopic moment generated by the transversal engine rotational speed ω_m and the yaw motion ψ

The sign of the angle is positive if the engine is rotating in the same direction as the wheels and negative for a counter-rotating engine.

The term $I^*{}_{wm}\omega_m$ expresses the engine total angular momentum, incorporating the angular momentum of the drive shaft, transmission shafts and any other rotating shaft parallel to the rear wheel axis.

$$I^*{}_{wm}\omega_m = \Sigma\, I_{wf}\omega_f \tag{37}$$

The engine contribution should be added or subtracted (depending on the sign of the gyroscopic moment) to the contribution given by the wheels.

1.9.3 Gyroscopic effects generated by longitudinally mounted engine

If the engine is mounted longitudinally with the drive shaft rotating toward the outside of the curve and the motorcycle is taking a turn to the left with respect to the forward direction of motion, the gyroscopic moment will be around the y_m axis and it is equal to

$$M_g = -I^*{}_{wm}\omega_m\dot{\psi} \tag{38}$$

The gyroscopic moment has the effect of extending the front suspension and compressing the rear suspension to a greater degree, this brings the motorcycle to pitch backwards, when the motorcycle leans to the right the gyroscopic moment has the opposite effect, compressing the front suspension and extending the rear suspension.

If the drive shaft is rotating toward the inside of the curve the gyroscopic moment changes sign and becomes

$$M_g = I^*{}_{wm}\omega_m\dot{\psi} \tag{39}$$

The effect generated by the gyroscopic moment is increasing the load on the front suspension and decreasing the load on the rear suspension, but the moment generated by the suspension forces balances out the gyroscopic moment with the end result of the motorcycle slightly pitching forward.

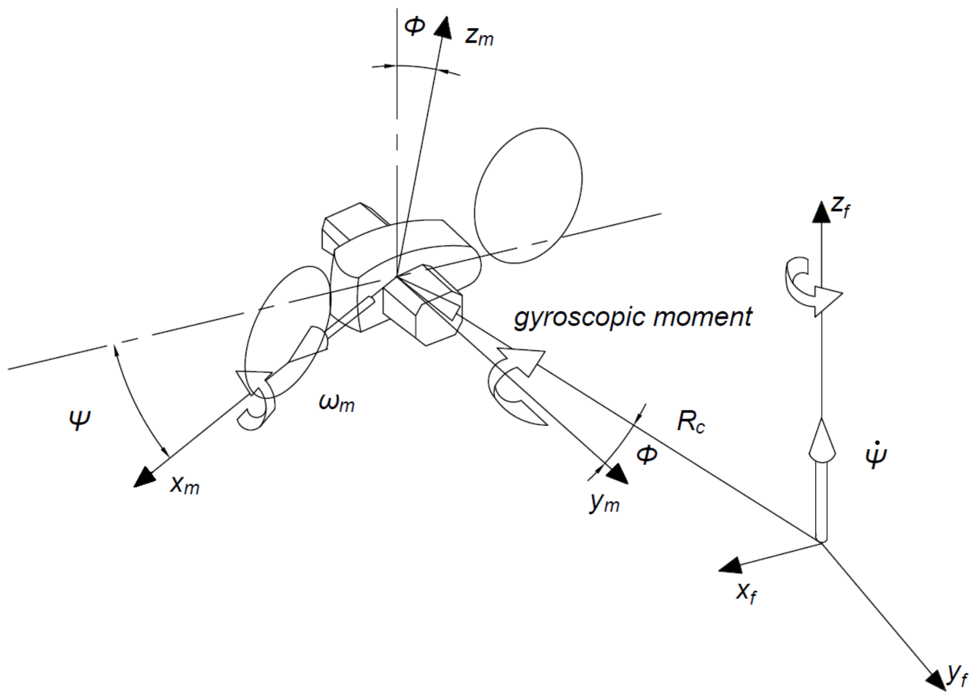


Figure 1.21: gyroscopic moment generated by the longitudinal engine rotational speed ω_m and the yaw

1.9.4 Gyroscopic effects generated by roll motion and front wheel

It is possible to look at the front wheel while the motorcycle is rolling to the right. The front wheel is revolving at an angular velocity ω_f and its motion coupled to the rolling rate $\dot{\phi}$ generates a gyroscopic moment M_g that acts on the front frame around an axis lying in the plane of the motorcycle and perpendicular to the longitudinal roll axis as shown in the figure. The equation of the gyroscopic moment generated is:

$$M_g = -I_{wf}\omega_f\dot{\phi} \tag{40}$$

The projection along the steering axis provides the beneficial moment around the steering axis

$$M_{gu} = -I_{wf}\omega_f\dot{\phi}\cos\vartheta_{cf} \tag{41}$$

This means that the gyroscopic moment has the effect of turning the steering head to the right, thereby helping the motorcycle enter the turn because increasing the steering angle δ_f reduces the

turning radius R_c. Furthermore when the roll rate $\dot{\phi}$ changes sign as the motorcycle returns to the vertical position the gyroscopic moment changes sign and it has the effect of reducing the steering angle δ_f helping the motorcycle exit the turn and return to rectilinear motion.

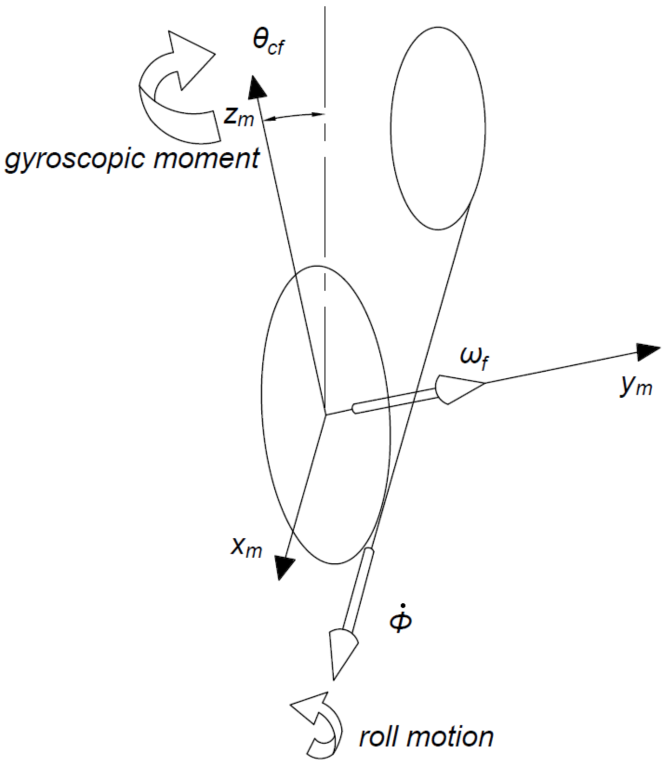


Figure 1.22: gyroscopic moment generated by the front wheel rotational speed ω_f and the roll rate $\dot{\phi}$

1.9.5 Gyroscopic effects generated by roll motion and both wheels

If the motorcycle is assumed to be a rigid body with the steering head locked in place, the gyroscopic moment of the wheel revolving at ω_f and ω_r during roll motion can be shown as a generation of a yawing moment as shown in the picture.

Considering a motorcycle rolling from left to right, the gyroscopic moment acting on the motorcycle is equal to

$$M_g = -(I_{wf}\omega_f + I_{wr}\omega_r)\dot{\phi} \tag{42}$$

This moment tends to make the motorcycle yaw to the right and is balanced by the lateral resistance exerted on the wheels by the ground. This means that the front lateral force slightly increases of ΔF, while the rear lateral force decreases by the same amount

$$M_g = -(I_{wf}\omega_f + I_{wr}\omega_r)\dot{\phi} = \Delta F\ p\cos\phi \tag{43}$$

When exiting the turn the motorcycle rolls from right to left and the gyroscopic moment reverses sign and in this way also the variation in the tire lateral forces ΔF changes sign.

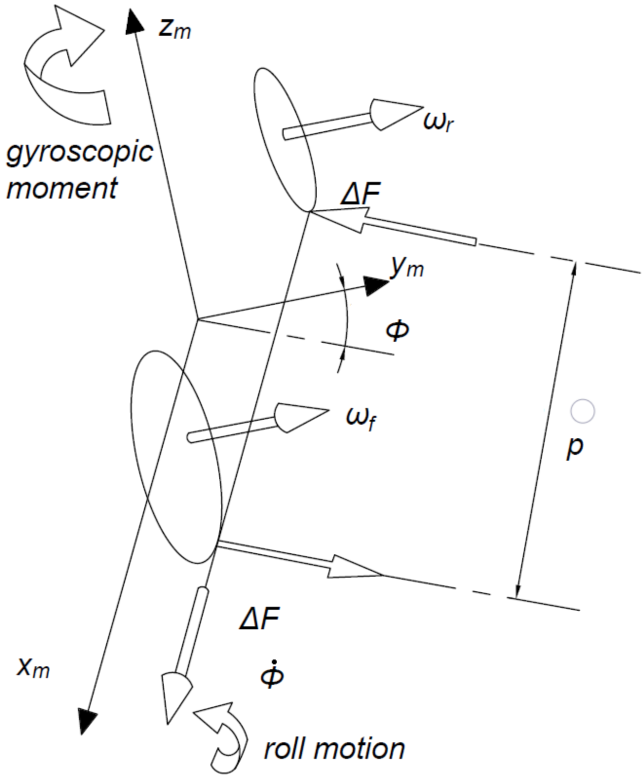


Figure 1.23: gyroscopic moment generated by the both wheels rotational speed ω_f and ω_f and the roll rate $\dot{\phi}$ with the steering head locked

1.9.6 Gyroscopic effects generated by steering

Since the wheel's direction of angular velocity ω_f is perpendicular to the steering head axis, turning the handlebars from right to left generates a gyroscopic moment around an axis perpendicular to both the steering axis and the axis of the front wheel, the equation expressing the moment is:

$$M_g = I_{wf}\omega_f\dot{\delta} \tag{44}$$

This has the effect of leaning the motorcycle over towards the right, the projection of the gyroscopic moment on the roll axis is expressed as

$$M_g = I_{wf}\omega_f\dot{\delta}\cos\vartheta_{cf} \tag{45}$$

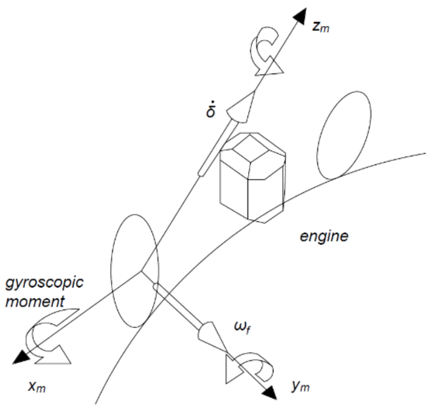


Figure 1.24: gyroscopic moment generated by the front wheel rotational speed ω_f and the steering rate $\dot{\delta}$

2 Automatic control system

Automatic control is essential in any field of engineering and science, in fact it is an important and integral part of space-vehicle systems, robotic systems, modern manufacturing systems and many industrial operations involving control of temperature, pressure, humidity, flow etc.

Some basic terminology is here explained:

- **Controlled variable and control signal or manipulated variable.** The *controlled* variable is the quantity or condition that is measured and controlled, the *control signal* is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable and it is usually the output of the system. Control means measuring the value of the controlled variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from a desired value.
- **Plants.** A plant can be a piece of equipment with the purpose of performing a particular operation
- **Processes.** Any operation to be controlled is considered a *process*
- **Systems.** A system is a combination of components that act together and perform a certain objective and this concept can be applied to abstract, dynamic phenomena such as those encountered in economics.
- **Disturbances.** A disturbance is a signal that tends to adversely affect the value of the output of a system, if a disturbance is generated within the system it is called *internal*, while an *external disturbance* is generated outside the system and is an output.
- **Feedback control.** It refers to an operation that in the presence of disturbances tends to reduce the difference between the output of a system and some reference input and dos so on the basis of this difference.

2.1 Closed-loop control versus open-loop control

A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control system.

- **Closed-loop control systems**. Feedback control systems are often referred to as closed-loop control systems. In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal is fed to the controller so as to reduce the error and bring the output of the system to a desired value. The term closed-loop control always implies the use of feedback control action in order to reduce system error.
- **Open-loop control systems.** Those systems in which the output has no effect on the control action are called open-loop control systems, in these systems the output is not measured or fed back for comparison with the input. To each reference input there corresponds a fixed operating condition and as a result the accuracy of the system depends on the calibration. In the presence of disturbances an open-loop control system can not perform the desired task, so these systems can be used only if the relationship between the input and output is known and if there are neither internal or external disturbances, so they are not feedback control systems.
- **Closed-loop versus open-loop control systems.** An advantage of the closed-loop control system is the fact that the use of feedback makes the system response relatively insensitive to external disturbances and internal variations in systems parameters. It is therefore

possible to use relatively inaccurate and inexpensive components to obtain the accurate control of a given plant and this is impossible in case of an open-loop system. From the point of view of stability the open-loop control system is easier to build because system stability is not a major problem, while in closed-loop systems it is a major problem.

For systems in which the inputs are known ahead of time and in which there are no disturbances it is advisable to use open-loop controls, closed-loop control systems have advantages only when unpredictable disturbances and/or unpredictable variations in system components are present.

The major advantages of open-loop control systems are the simplicity of construction and ease of maintenance, no stability problem is present, they are convenient when the output is hard to measure. Their disadvantages are the fact that when disturbances occur errors are present, so the output may become different from the desired one.

2.2 Mathematical modeling of control systems

A mathematical model of a dynamic system is defined as a set of equations that represents the dynamics of the system accurately, a system may be represented in many different ways and may have many mathematical models and the dynamics of many systems can be described in terms of differential equations.

Mathematical models may assume many different forms depending on the particular system and the particular circumstances, for example in optimal control problems it is advantageous to use state-space representations, while for the transient-response or frequency-response analysis of single-input single-output linear time invariant systems the transfer-function representation may be more convenient than any other.

A system is called linear if the principle of superposition applies, this principle states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. For linear system the response to several inputs can be calculated by treating one input at a time and adding the results.

A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamics system that are composed of linear time-invariant parameter can be described by linear time-invariant differential equations, these systems are called linear time-invariant systems or LTI systems. Systems which are represented by differential equations whose coefficients are functions of time are called linear time-varying systems.

In control theory the transfer functions are commonly used to characterize the input-output relationships of components or systems that can be described by linear, time-invariant, differential equations. The transfer function of an LTI system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input under the assumption that all the initial conditions are set to zero. Using the concept of transfer function it is possible to represent system dynamics by algebraic equations in the variable s.

- The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.
- The transfer function is a property of a system itself, independent of the magnitude and nature of the input function

- The transfer function includes the units necessary to relate the input to the output, however it does not provide any information concerning the physical structure of the system.
- If the transfer function of a system is known the output response can be studied for various forms of inputs with a view toward understanding the nature of the system.
- If the transfer function of a system is unknown it may be established experimentally by introducing known inputs and studying the output of the system, once established it gives a full description of the dynamic characteristics of the system.

In order to describe a control system it is useful to use a diagram called block diagram, it is a pictorial representation of the functions performed by each component and of the flow of the signals, differing from a purely abstract mathematical representation a block diagram has the advantage of indicating more realistically the signal flows of the actual system. In a block diagram all system variables are linked to each other through functional blocks. The functional block is a symbol for mathematical operations on the input signal to the block that produces the output. Figure 2.1 shows an element of the block diagram, the arrowhead pointing toward the block indicates the input and the arrowhead leading away from the block represents the output, each arrow is referred to as a signal.

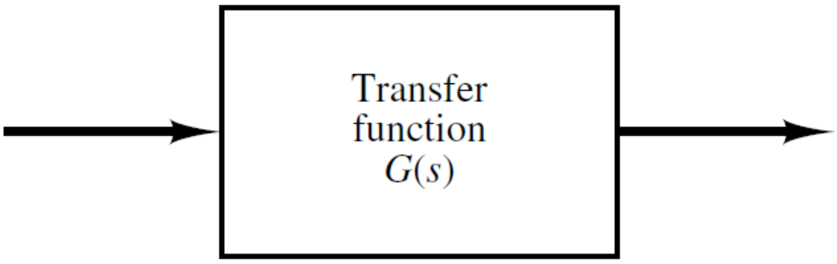


Figure 2.1: element of a block diagram

Figure 2.2 shows an example of a block diagram of a closed-loop system. The output $C(s)$ is fed back to the summing point, where it is compared with the reference input $R(s)$. The output of the block $C(s)$ is obtained by multiplying the transfer function $G(s)$ by the input to the block, $E(s)$. Any linear control system may be represented by a block diagram consisting of blocks, summing points and branch points.

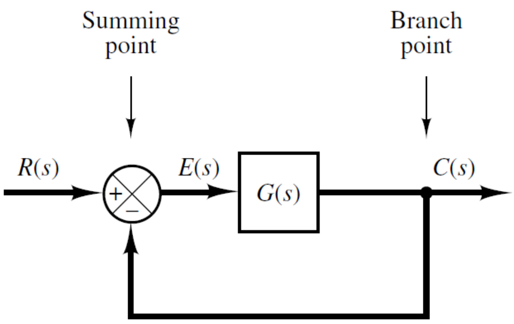


Figure 2.2: block diagram of a closed-loop system

When the output is fed back to the summing point for comparison with the input it is necessary to convert the form of the output signal to that of the input signal. This conversion is accomplished by the feedback element whose transfer function is $H(s)$ as shown in figure 2.3, the role of the feedback element is to modify the output before it is compared to the input.

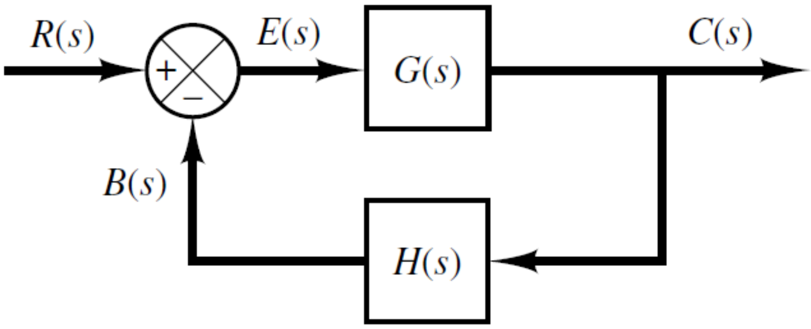


Figure 2.3: block diagram of a closed-loop system

2.3 Modeling in State Space

The modern trend in engineering system is toward greater complexity, due mainly to the requirements of complex tasks and good accuracy. Complex systems may have multiple inputs and multiple outputs and may be time varying, this new approach is based on the concept of state.

The state of a dynamic system is the smallest set of variables (called state variables) such that the knowledge of these variable at $t = t_0$ together with the knowledge of the input for $t \geq t_0$ completely determines the behavior of the system for any time $t \geq t_0$. The state variables of a dynamic system are the variable making up the smallest set of variables that determine the state of the dynamic system. If at least n variables $x_1, x_2, \dots, x_n$ are needed to completely describe the behavior of a dynamic system, this set of variables are a set of state variables. It is convenient to choose easily measurable quantities for the state variables, if this is possible, because optimal control laws will require the feedback of all state variables with suitable weighting. If n state variables are needed to completely describe the behavior of a given system, then these can be considered the n components of a vector x, which is called a state vector.

The n-dimensional space whose coordinate axes consist of the $x_1$axis, x_2 axis, …, $x_n$axis, where $x_1, x_2, \dots, x_n$ are state variables is called a state space.

The dynamic system must involve elements that memorize the values of the input for $t \geq t_0$. Since integrators in a continuous-time control system serve as memory devices, the outputs of such integrators can be considered as the variables that define the internal state of the dynamic system.

The system can be described by two equations that are function of the variables x, u, t

$$\dot{x}(t) = f(x, u, t) \tag{46}$$

$$y(t) = f(x, u, t) \tag{47}$$

Equation 46 is the state equation of the system while equation 47 is the output equation, if those equations involve time t explicitly, the system is called a time-varying system. Those two equations can be linearized about the operating state and they become

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{48}$$

$$y(t) = C(t)x(t) + D(t)u(t) \tag{49}$$

Where $A(t)$ is called the state matrix, $B(t)$ the input matrix, $C(t)$ the output matrix and $D(t)$ the direct transmission matrix.

If time is not involved in the computations, the system is called a time-invariant system and equations 48 and 49 become

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{50}$$

$$y(t) = Cx(t) + Du(t) \tag{51}$$

Equation 50 is the state equation of the linear time-invariant system and equation 51 is the output equation for the same system.

2.4 Control systems analysis in State Space

In order to analyze complex systems that can have many inputs and many outputs, it is essential to reduce the complexity of the mathematical expressions, as well as to resort to computers for most of the difficult computations necessary for the analysis and the state-space approach to system analysis is best suited from this point of view. Conventional control theory is based on the input-output relationship or transfer function, modern control theory is based on the description of system equations in terms of n first-differential equations, which may be combined into a first order vector-matrix differential equation. The use of vector-matrix notation greatly simplifies the mathematical representation of systems of equations.

The increase in the number of state variables, the number of inputs or the number of outputs does not increase the complexity of the equations. In fact the analysis of complicated multiple-input, multiple-output systems can be carried out by procedures that are only slightly more complicated than those required for the analysis of first-order scalar differential equations.

Considering a system defined by n derivates of the y with n parameters a_i and n derivatives of the input signals u it is possible to state

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \tag{52}$$

It is possible to represent the state space of the system in controllable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \vdots\,' \\ -a_n & & \cdots & & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \tag{53}$$

2.5 Controllability and Observability

A system is said to be controllable at time t_0 if it is possible by means of an unconstrained control vector to transfer the system from any initial state $x(t_0)$ to any other state in a finite interval of time. A system is said to be observable at time t_0 if, with the system in state $x(t_0)$, it is possible to determine this state from the observation of the output over a finite time interval. These two concepts play an important role in the design of control system in state-space, in fact the conditions of controllability and observability may govern the existence of a complete solution to the control system design problem. The solution to this problem may not exist if the system considered is not controllable. Although most physical systems are controllable and observable, corresponding mathematical models may not posses the property of controllability and observability, so it is necessary to know the conditions under which a system is controllable and observable.

Considering a continuous-time system

$$\dot{x}(t) = Ax + Bu \tag{54}$$

- x is a state vector
- u is a control signal
- A is an $n\ x\ n$ matrix
- B is an $n\ x\ 1$ matrix

This system is considered to be state controllable at $t = t_0$ if it is possible to construct an unconstrained control signal that will transfer an initial state to any final state in a finite time interval $t_0 \leq t \leq t_1$. If every state is controllable the system is said to be completely state controllable.

The condition for complete state controllability are defined assuming that the final state is the origin of the state space and that the initial time is $t_0 = 0$. The solution of equation 54 is

$$x(t) = e^{At} + \int_0^t e^{A(t-\tau)} Bu(\tau)d\tau \tag{55}$$

If the system is completely state controllable it is possible to find the controllability matrix given any initial state $x(0)$, it is necessary that the rank of the $n\ x\ n$ controllability matrix is of the rank n.

An alternative form of the condition for complete state controllability is considering the system defined in equation 54 with the same conditions for the previous case but the B matrix is in the form $n\ x\ r$, if the eigenvectors of A are distinct then it is possible to find a transformation matrix P such that

$$P^{-1}AP = D = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix} \tag{56}$$

If the eigenvalues of A are distinct, then the eigenvectors of A are distinct. Furthermore each column of the P matrix is an eigenvector of A associated with λ_i, with $i = 1,2,\ldots,n$. Defining $x = Pz$ it is possible to modify equation 54 and obtain

$$\dot{z}(t) = P^{-1}APz + P^{-1}Bu \tag{57}$$

The condition of complete state controllability is that the eigenvectors of A are distinct, then if and only if no row of $P^{-1}B$ has all zero elements the system is completely controllable. In order to have this condition the matrix $P^{-1}AP$ must be in diagonal form. If the A matrix doess not possess distinct eigenvectors the diagonalization is impossible, in this case it is possible to transform A into a Jordan canonical form.

Complete state controllability of the state is neither necessary nor sufficient condition for controlling the output of the system, so it is possible to define separately complete output controllability. Considering the system

$$\dot{x}(t) = Ax + Bu \tag{58}$$

$$y(t) = Cx + Du \tag{59}$$

- x is a state vector (n-vector)
- u is a control vector (n-vector)
- y is the output vector (m-vector)
- A is an $n\ x\ n$ matrix
- B is an $n\ x\ r$ matrix
- C is an $m\ x\ n$ matrix
- D is an $m\ x\ r$ matrix

The system is completely output controllable if and only if it is possible to construct an unconstrained control vector $u(t)$ that will transfer any given initial output $y(t_0)$ to any final output $y(t_1)$ in a finite time interval $t_0 \leq t \leq t_1$. The condition for complete output controllability is that the rank of the output controllability matrix is m.

A system is uncontrollable when it has a subsystem that is physically disconnected from the input.

A partially controllable system is said to be stabilizable if the uncontrollable modes are stable and the unstable modes are controllable.

Considering an unforced system

$$\dot{x}(t) = Ax \tag{60}$$

$$y(t) = Cx \tag{61}$$

With

- x is a state vector (n-vector)
- y is the output vector (m-vector)
- A is an $n\ x\ n$ matrix
- C is an $m\ x\ n$ matrix

The system is said to be completely observable if every state $x(t_0)$ can be determined by the observation of $y(t)$ over a finite time interval $t_0 \leq t \leq t_1$. The system is therefore completely observable if every transition of the state eventually affects every element of the output vector, the concept of observability is useful in solving the problem of reconstructing unmeasurable state variables from measurable variables in the minimum possible length of time.

The concept of observability is very important because the difficulty encountered with state feedback control is that some of the state variables are not accessible for direct measurement, with the result that it becomes necessary to estimate the unmeasurable state variables in order to construct the control signals. For the observability conditions the unforced system is considered because the matrices $A, B, C\ and\ D$ are known and $u(t)$ is also known, therefore they can be subtracted from the observed value of $y(t)$. In this way the output vector $y(t)$ is

$$y(t) = Cxe^{At}x(0) \tag{62}$$

If the system is completely observable the observability matrix $n\ x\ nm$ must be of rank n, or if this matrix has n linearly independent column vectors.

2.6 Control systems design in state space

The methods for the design of state-space system are based on the pole-placement method, observers, quadratic optimal regulator systems and robust control systems.

2.6.1 Pole placement

Pole placement is a design method to design control systems in state space. Assuming that all the variables of a system are measurable and available for feedback, this design technique begins with a determination of the desired closed-loop poles based on the transient-response and/or frequency response requirements. By choosing an appropriate gain matrix for state feedback it is possible to force the system to have closed-loop poles at the desired locations, provided that the original system is completely state controllable.

In the conventional approach to the design of a single-input single-output control system a controller is designed such that the dominant closed-loop poles have a desired damping ratio ζ and a desired undamped natural frequency ω_n. The present pole-placement approach specifies all closed-loop poles. The requirement is that the system is completely state controllable.

Considering a control system

$$\dot{x}(t) = Ax + Bu \tag{63}$$

$$y(t) = Cx + Du \tag{64}$$

Where

- x is a state vector (n-vector)
- u is a control signal (scalar)
- y is the output signal (scalar)
- A is an $n \ x \ n$ matrix
- B is an $n \ x \ 1$ matrix
- C is an $1 \ x \ n$ matrix
- D is a constant

The control signal is chosen to be

$$u = -Kx \tag{65}$$

The control signal u is determined by an instantaneous state, this scheme is called state feedback. The $1 \ x \ n$ matrix K is called the state feedback gain matrix, all the state variables are assumed to be available for feedback. This closed-loop system has no input, its objective is to maintain the zero output, since there may be some disturbances present the output will deviate from zero. The nonzero output is returned to the zero reference input because of the state feedback scheme of the system as shown in figure 2.4. It is possible to substitute equation 65 in equation 63, obtaining

$$\dot{x}(t) = (A - BK)x(t) \tag{66}$$

The solution of this equation is given by

$$x(t) = e^{(A-BK)t}x(0) \tag{67}$$

With $x(0)$ being the initial state caused by external disturbances.

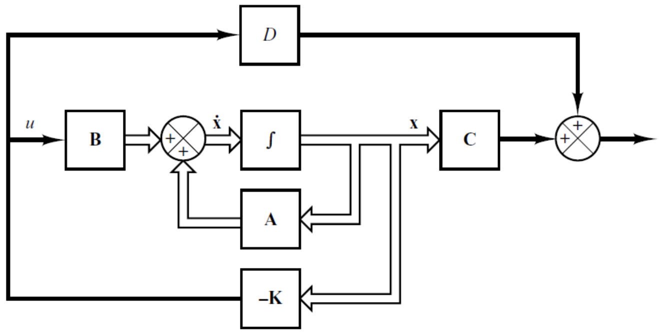


Figure 2.4: closed-loop control system with $u = -Kx$

The stability and transient-response characteristics are determined by the eigenvalues of the matrix $A - BK$. If matrix K is chosen properly the matrix $A - BK$ can be made an asymptotically stable matrix, and for all $x(0) \neq 0$ it is possible to make $x(t)$ approach 0 as t approaches infinity. The eigenvalues of the matrix $A - BK$ are called the regulator poles.

The necessary and sufficient condition for arbitrary pole placement is that the system should be completely state controllable.

When a system defined by equations 63 and 65, the feedback gain matrix that forces the eigenvalues of $A - BK$ to be the desired values can be determined following this procedure:

1. Check the controllability condition for the system
2. From the characteristic polynomial for matrix A $|sI - A| = s^n + a_1 s^{n-1} + \cdots + a_n$ determine the a_i values
3. Determine the transformation matrix T that transforms the system state equation into the controllable canonical form
4. Using the desired eigenvalues (desired closed-loop poles μ_i) write the desired characteristic polynomial $(s - \mu_1) \ldots (s - \mu_n) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n$ and determine the values of $\alpha_1, \ldots, \alpha_n$
5. The required state feedback gain matrix K can be determined as
   $K = [\alpha_n - a_n \;\vdots \cdots \vdots\; \alpha_1 - a_1]T^{-1}$

2.6.2 Quadratic Optimal regulator systems

An advantage of the quadratic optimal control method over the pole-placement method is that the former provides a systematic way of computing the state feedback control gain matrix, considering a system

$$\dot{x}(t) = Ax + Bu \tag{68}$$

It is possible to determine the K matrix of the optimal control vector

$$u(t) = -Kx(\mathrm{t}) \tag{69}$$

So as to minimize the performance index

$$J = \int_0^\infty (x'Qx + u'Ru)\,dt \tag{70}$$

Where Q is a positive definite Hermitian or real symmetric matrix and R is a positive-definite Hermitian or real symmetric matrix. Those two matrices determine the relative importance of the error and the expenditure of the energy, since the second term of equation 70 accounts for the expenditure of the energy of the control signals. The linear control law given by equation 69 is the optimal control law, therefore if the unknown elements of the matrix K are determined so as to minimize the performance index, then $u(t) = -Kx(\mathrm{t})$ is optimal for any initial state $x(0)$. Substituting equation 69 in equation 68 it is possible to obtain the matrix $A - BK$, which is assumed to be stable and its eigenvalues have negative real parts.

$$J = \int_0^\infty x'(Qx + K'RK)x\,dt \tag{71}$$

It is possible to set $x'(Qx + K'RK)x = -\frac{d}{dt}(x'Px)$, with P being a positive definite Hermitian or real symmetric matrix, in this way it is possible to state

$$(A - BK)'P + P(A - BK) = -(Q + K'RK) \tag{72}$$

If $A - BK$ is a stable matrix, a positive-definite matrix P exists in order to satisfy equation 72.

The performance index J can be evaluated as

$$J = \int_0^\infty x'(Qx + K'RK)x\,dt = -x'(\infty)Px(\infty) + x'(0)Px(0) \tag{73}$$

Since the eigenvalues of $A - BK$ are assumed to have negative real parts, $x(\infty) = 0$, equation 73 becomes

$$J = x'(0)Px(0) \tag{74}$$

To obtain the solution of the quadratic optimal control problem, the matrix R is written as $R = T'T$, with T as a nonsingular matrix. Equation 72 becomes

$$(A' - K'B')P + P(A - BK) + Q + K'T'TK = 0 \tag{75}$$

It is possible to write

$$A'P + PA - PBR'B'P + Q = 0 \tag{76}$$

Equation 76 is the Riccati equation, solving this equation it is possible to find the P matrix, if it exists the system is stable and the matrix $A - BK$ is stable. Substituting the P matrix into the equation $K = R^{-1}B'P$ it is possible to find the optimal matrix K.

Given any initial state $x(t_0)$ the optimal regulator problem is to find an allowable control vector $u(t)$ that transfers the state to the desired region of the state space and for which the performance index is minimized. For the existence of an optimal control vector $u(t)$ the system must be completely state controllable.

The system that minimizes the selected performance index is by definition optimal. The characteristic of an optimal control law based on a quadratic performance index is that it is a linear function of the state variables, which implies the need to feedback all state variables. If not all variables are available for feedback a state observer must be used in order to estimate unmeasurable state variables and use the estimated values to generate optimal control signals. When the optimal control system is designed in the time domain it is desirable to investigate the frequency response characteristics to compensate for noise effects. If the upper limit of integration in the performance index J is finite, the optimal control vector is still a linear function of the state variables with time varying coefficients.

3 Roll stability at low velocity

The problem of roll stability in motorcycles at low speed or in stationary situations is evaluated in this report because in those conditions there are no sufficient restoring forces to maintain the vehicle stationary. Following the article "Study of Riding Assist Control Enabling Self-standing in Stationary State" a model of the motorcycle having roll stability when the vehicle is stationary or at low speed with a steering control for self-standing assist was taken into account in order to represent the dynamics of the roll motion composed by a fixed point mass above the center of gravity and a movable point mass below the center of gravity on the ground.

3.1 Dynamic model

According to the model, the steering action allows the roll moment direction generated by the movable point mass to become the same as the direction generated by the ground contact point shift of the front tire, in this way the total roll moment is enough to restore the vehicle inclination only by steering control and because of this action it's possible to establish a self standing control for the stationary state.

The dynamic model used to study the conditions to enable roll stabilization by steering control is introduced following the assumptions that:

1. the whole vehicle is a rigid point mass with mass m at the center of gravity and moment of inertia I about the roll axis;
2. only the lateral movement of the vehicle's point mass and the moment of the grounding point of the cross-sectional profile of the front and rear tires are considered as the dominant elements that generate roll moment.

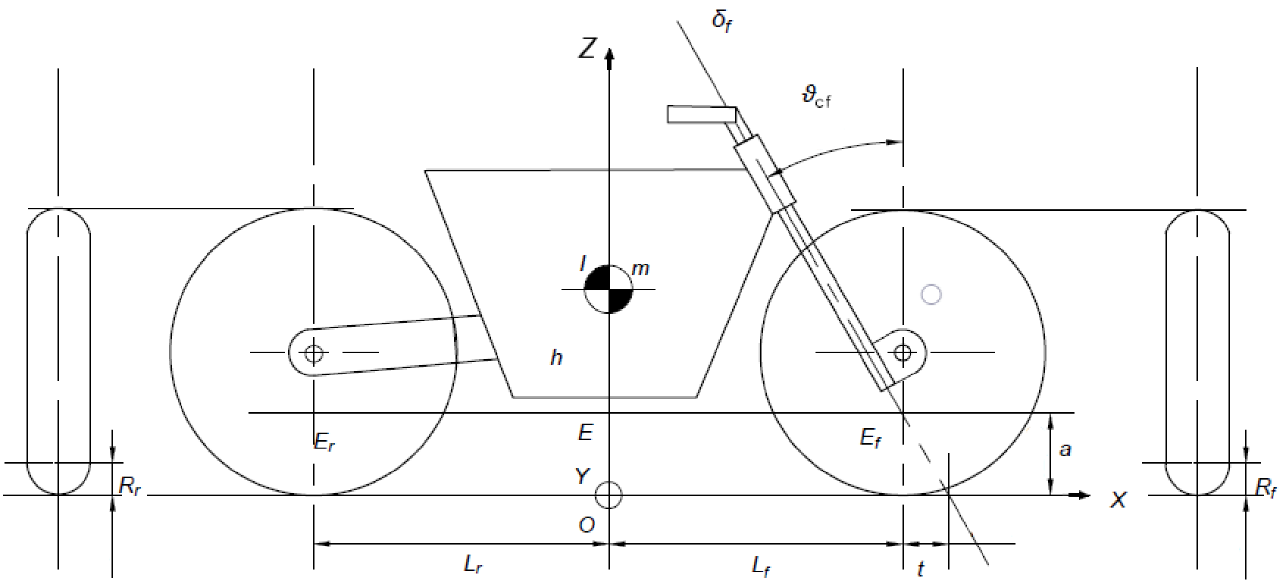


Figure 3.1: Dynamics model of a motorcycle

Figure 3.1 shows the dynamic model, the coordinates are set in this way:

- Both the front wheel steering angle and the roll angle are set to zero in the reference attitude.
- The origin of the coordinate system is placed in the projected point on the ground of the center of gravity of the vehicle in the reference attitude, from that point the Cartesian coordinate system has X, Y and Z axes in a right handed system.

- In the reference attitude, E_f is the point of intersection between the line that connects the center of the front wheel and the line of the steering axis.
- With a line parallel to the ground passing through E_f the point E is found with the intersection of the Z axis, while E_r is the point on the line directly below the center of the rear wheel.

After the definition of the relevant parameters it is possible to find the relationship between the height a and the trail length: $t = a \tan \theta_{cf}$.
Following these assumption the system can be converted to the equivalent system consisting of two point mass, the first is at a height above the center of gravity h with a mass m_1 and a height h', the second point mass is on the ground with mass m_2 and a height 0 as shown in figure 3.2.

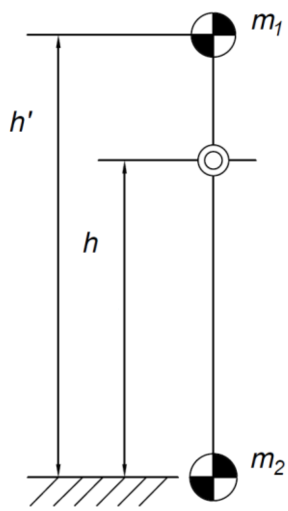


Figure 3.2: Approximate dynamics model of an equivalent two point mass system

With simple equilibrium equations it is possible to find the values of m_1, m_2 and h':

$$m_1 + m_2 = m \tag{77}$$

$$m_1(h' - h) = m_2 \tag{78}$$

$$m_1(h' - h)^2 + m_2 h^2 = I \tag{79}$$

The results from these three equations bring to

$$h' = h + I/(mh) \tag{80}$$

$$m_1 = \left(\frac{h}{h'}\right) m \tag{81}$$

$$m_2 = \left(1 - \frac{h}{h'}\right) m \tag{82}$$

From here it is possible to show the roll motion of a motorcycle in the approximate model of the two point mass system consisting of the two point masses in order to approximately represent a near-reference attitude. The initial angular momentum is considered to be zero and the change in angular momentum due to the gravity term is now ignored and the point masses are assumed to be movable only horizontally.

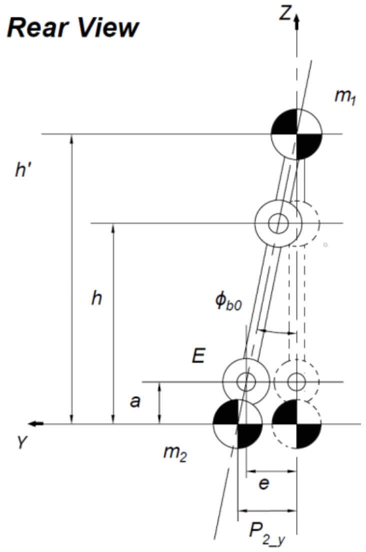


Figure 3.3: steering action and the resulting movement of the second point mass and associated roll angle change in the approximate dynamics model of an equivalent two point mass system

Under these assumption the position of the second point mass is solely determined by the steering angle and since any movement of the second point mass will not generate an angular momentum the first point mass remains stationary: if gravity terms are ignored the first point mass can be regarded as a fixed point for steering, resulting in a rotation about the first point mass in the roll direction. It is now possible to express the displacement of the second point mass along the Y-axis as P_{2_y}in function of the tilt angle in the roll direction as ϕ_{b0}, which is sufficiently small so that $\sin\phi_{b0} = \phi_{b0}$:

$$P_{2\_y} = h'\phi_{b0} \tag{83}$$

Also if the value of the steering angle is sufficiently small the roll angles of front and rear wheel can be approximated as follows:

$$\phi_f = -(\sin\theta_{cf})\delta_f + \phi_{b0} \tag{84}$$

$$\phi_r = \phi_{b0} \tag{85}$$

Then also the displacement of E_f, E_r and E in the Y-axis are expressed as follows:

$$e_f = -a\phi_f \tag{90}$$

$$e_r = -a\phi_r \tag{91}$$

$$e = \frac{L_r}{L_f+L_r}e_f + \frac{L_f}{L_f+L_r}e_r \tag{92}$$

The angle of inclination of the line that connects the point E and the second point mass is equal to the roll angle ϕ_{b0} of the vehicle, this means that P_{2_y} can be also given by

$$P_{2\_y} = e + a\phi_{b0} \tag{93}$$

This means that both P_{2_y} and ϕ_{b0} can be represented as function of δ_f

$$P_{2\_y} = \frac{L_r}{L_f+L_r} a\,(\sin\theta_{cf})\delta_f \tag{94}$$

$$\phi_{b0} = \frac{L_r}{L_f+L_r}\frac{a}{h'}(\sin\theta_{cf})\delta_f \tag{95}$$

From the above discussion the roll moment M_2 about the origin generated by the movement of the second point mass in the Y-axis direction can be represented as a function of δ_f because

$$M_2 = -gm_2P_{2\_y} \tag{96}$$

$$M_2 = -(1-\frac{h}{h'})mg\frac{L_r}{L_f+L_r} a\,(\sin\theta_{cf})\delta_f \tag{97}$$

The roll moment generated by the movement of the grounding points of front and rear wheels needs to be evaluated, first of all the displacement of the front and rear wheels' grounding point are found as:

$$P_{rpt\_fy} = -R_f\phi_f \tag{98}$$

$$P_{rpt\_ry} = -R_r\phi_r \tag{99}$$

And the vertical road surface reaction forces applied to the front and rear wheel grounding points can be represented as:

$$F_{rpt\_fz} = \frac{L_r}{L_f+L_r}mg \tag{100}$$

$$F_{rpt\_rz} = \frac{L_f}{L_f+L_r}mg \tag{101}$$

It is now possible to evaluate the roll moment M_{rpt} about the origin generated by the movement of the grounding points in the Y-axis direction, and it is represented as follows:

$$M_{rpt} = F_{rpt\_fy}P_{rpt\_fy} + F_{rpt\_rz}P_{rpt\_ry} \tag{102}$$

Also M_{rpt} can be represented as a function of δ_f

$$M_{rpt} = mg\frac{L_r}{L_f+L_r}\delta_f(R_f - \frac{a}{h'}R_g)\,(\sin\theta_{cf})\delta_f \tag{103}$$

$$R_g = \frac{L_r}{L_f+L_r}R_f + \frac{L_f}{L_f+L_r}R_r \tag{104}$$

It is now possible to find the total moment of inertia about the origin, which is represented as a function of δ_f

$$M_{sum} = M_2 + M_{rpt} = \frac{L_r}{L_f+L_r}\left(R_f - \frac{R_g+h'-h}{h\prime} a\right) mg(\sin\theta_{cf})\delta_f \tag{105}$$

Then M_{sum} can be represented as

$$M_{sum} = mgk_{sum}(a_{sum} - a)\ \delta_f \tag{106}$$

$$k_{sum} = \frac{L_r}{L_f+L_r}\frac{R_g+h'-h}{h\prime}\sin\theta_{cf} \tag{107}$$

$$a_{sum} = \frac{h\prime}{R_g+h'-h}R_f \tag{108}$$

When a motorcycle is rolled in a negative roll angle during a medium to high speed run it normally steers to the positive steering angle through the self steering effect which results in a positive moment that tends to pull up the vehicle body. This fact indicates that for a motorcycle in a stationary state to show the same roll and steering behavior as demonstrated at medium to high speed the coefficient $(a_{sum} - a)$ in the equation of M_{sum} must be a positive value, which means that a must be smaller than a_{sum} and this indicates that the smaller this coefficient becomes the larger roll moment will be generated. In order to obtain those results this means that the trail length t must be smaller than $a_{sum}\tan\theta_{cf}$, the trail length should be made as small as possible, even allowing negative values, to be advantageous in generating a sufficient roll moment for a motorcycle in stationary state to achieve a self-standing condition.

3.1.1 Control Law

It is important to establish a control law to ensure roll stability, so an equation of motion applicable to the two point masses system must be found. Here the gravitational forces are considered and so the angular momentum of the equivalent system will change according to them acting on the first point and second point masses and the road surface reaction forces on the grounding points. The position of the second point mass is basically determined by the steering angle and no angular momentum is generated by the second point mass, the angular momentum of the first point mass becomes the angular momentum of the equivalent system.

The roll angle ϕ' of the first point mass is defined as:

$$\phi' = \phi_{b\_act} - \phi_{b0} \tag{109}$$

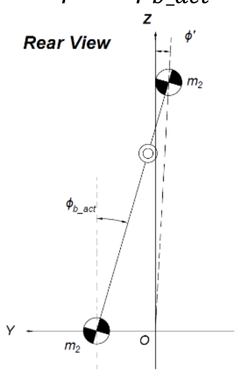


Figure 3.4: Roll angle of the first point mass in the approximate dynamics model of the equivalent two point masses system

When the vehicle is running in addition to the roll moments M_2, M_{rpt} another roll moment M_i composed by the Y-axis acceleration component and the centrifugal force component is generated and expressed as:

$$M_i = m_1 h' \dot{V_{oy}} + m_1 h' V_{ox} \omega_z \tag{110}$$

In this expression V_{ox} is the running velocity in the X-axis direction, V_{oy} is the traveling rate in the Y-axis direction and ω_z is the yaw rate about the Z-axis. V_{oy} and ω_z can be expressed in function of the actual steering angle δ'_f:

$$V_{oy} = \frac{L_r}{L_f + L_r} V_{ox} \tan \delta'_f \tag{111}$$

$$\omega_z = \frac{1}{L_f + L_r} V_{ox} \tan \delta'_f \tag{112}$$

A further approximation is applicable:

$$\tan \delta'_f \cong \delta'_f \cong (\cos \theta_{cf}) \delta_f \tag{113}$$

So it is possible to express both V_{oy} and ω_z in function of δ_f

$$V_{oy} = \frac{L_r}{L_f + L_r} V_{ox} (\cos \theta_{cf}) \delta_f \tag{114}$$

$$w_z = \frac{1}{L_f + L_r} V_{ox} (\cos \theta_{cf}) \delta_f \tag{115}$$

The equation of motion about the origin around the X-axis is given by:

$$m_1 h' \ddot{\phi}' = m_1 h' g \phi' + M_2 + M_{rpt} + M_i \tag{116}$$

Under the assumption that ϕ' is sufficiently small the displacement of the first point mass can be expressed as:

$$P_{1\_y} = -h' \phi' \tag{117}$$

The previous equation (116) can be now represented in this way:

$$\ddot{P_{1\_y}} = \frac{g}{h'} P_{1\_y} - \frac{g}{h} k_{sum} (a_{sum} - a) \delta_f - \frac{1}{L_f + L_r} V^2{}_{ox} (\cos \theta_{cf}) \delta_f - \dot{V_{oy}} \tag{118}$$

A new parameter $V_{by} = V_{oy} + \dot{P}_{1\_y}$ is introduced and it is possible to deform this equation into:

$$\dot{P}_{1\_y} = V_{by} - \frac{L_r}{L_f + L_r} V_{ox} (\cos \theta_{cf}) \delta_f \tag{119}$$

$$\dot{V}_{by} = \frac{g}{h'} P_{1\_y} - \frac{g}{h} k_{sum} (a_{sum} - a) \delta_f - \frac{1}{L_f + L_r} V^2{}_{ox} (\cos \theta_{cf}) \delta_f \tag{120}$$

From these equations it is possible to represent a state-space model, which is represented as follows:

$$\dot{x} = Ax + Bu \tag{121}$$

$$y = Cx \tag{122}$$

The states of the system are:

$$x = \left[P_{1\_y}\ V_{by}\delta_f\ \dot{\delta}_f\ \right] \tag{123}$$

The matrices are represented here below:

$$A = \begin{bmatrix} 0 & 1 & A_{13} & 0 \\ \frac{g}{h} & 0 & A_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{124}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{125}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{126}$$

The input is $u = \ddot{\delta}_f$.
The parameters inside the A matrix of the system A_{13} and A_{23} are:

$$A_{13} = -\frac{L_r}{L_f+L_r} V_{ox}\left(\cos\theta_{cf}\right) \tag{127}$$

$$A_{23} = -\frac{g}{h} k_{sum}(a_{sum} - a) - \frac{1}{L_f+L_r} V^2{}_{ox}\left(\cos\theta_{cf}\right) \tag{128}$$

From here the control law is defined as $u = Kx$.

3.2 First simulation

After setting all the parameters in a Matlab script the first step of the work was focused on the study of the state-space system, the eigenvalues of the A matrix and the poles of the system give the same results, in each case a positive value was found and that suggests that the system is obviously unstable and it needs to be controlled. Then a Simulink model was performed in order to represent the system, it brings to the expected results of instability of the system. Furthermore it was important to check if the system was controllable and observable, so the controllability and observability matrix were found in Matlab and their rank was inspected, with the result of finding out that the system is both controllable and observable.

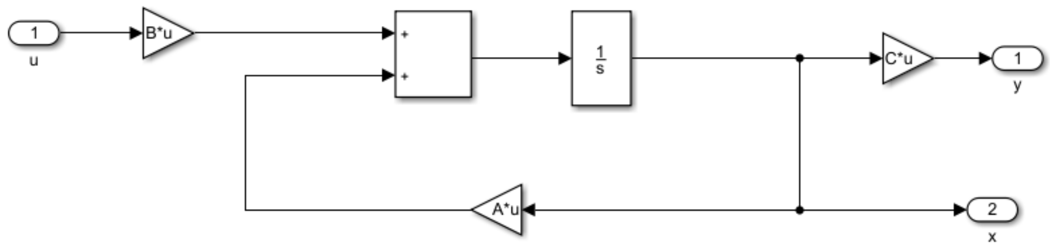


Figure 3.5: Simulink model representing the State-space system

Since the problem is the roll stability in motorcycles at low speed an initial speed considered is $V_{ox} = 0\ [\frac{km}{h}]$ and the results show the instability of the system, in figure 3.6 it is possible to observe the curves of the states of the system:

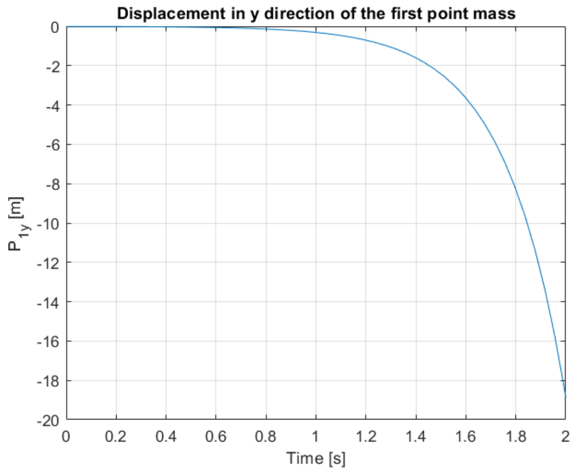


Figure 3.6(a): displacement of the first point mass in y direction, result of the simulation performed without a controller

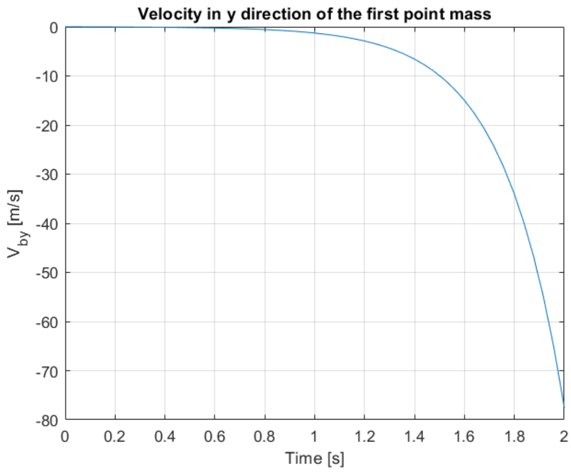


Figure 3.6(b): velocity of the first point mass in y direction, result of the simulation performed without a controller

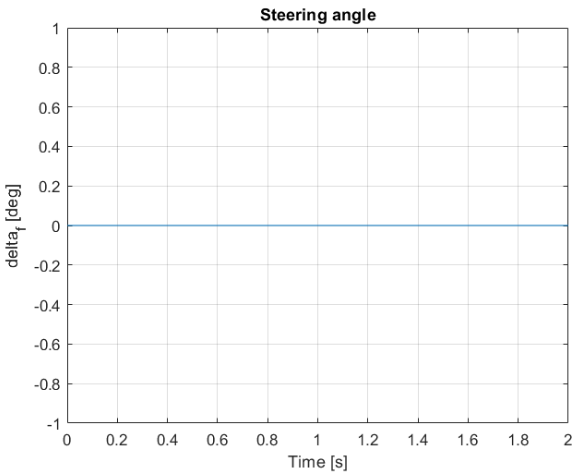


Figure 3.6(c): steering angle of the motorcycle δ_f, result of the simulation performed without a controller

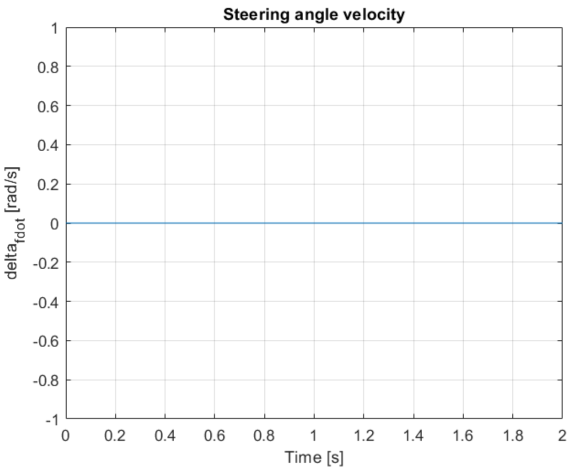


Figure 3.6(d): steering angle velocity $\dot{\delta_f}$, result of the simulation performed without a controller

The results obtained in the simulation show the instability of the system, in fact both the curve in figure 3.6(a) of the displacement of the first point mass and the curve in figure 3.6(b) of the velocity of the first point mass tend to $-\infty$, highlighting the fact that the motorcycle without an action on the steering system or without a driver putting their feet on the ground tends to fall at low velocities. Following these results a controller was designed in order to stabilize the system.

3.3 Design of the controller

After checking the instability of the system the poles were investigated, their results were $poles(sys) = [\,4.1115, -4.1115, 0, 0]$, the controller could be designed both by

- using the method of the second order dominant poles so that the desired eigenvalues are chosen in such a way that the controlled system becomes similar to a second-order system with desired damping and natural frequency
- using the method of the optimal control (Linear Quadratic Regulator, LQR)

The main advantage of pole placement technique is that the poles are placed at the desired location using state feedback gain matrix, the poles can be shifted and so it is possible to shape the closed loop characteristics of the system in order to meet the design requirement. Pole placement method can give the desired performance characteristics but it does not guarantee a robust system, while LQR gives the optimal solution considering the control signal with the advantage of having the system always stable and robust, but it only allows pole placement in a specific region and the pole that gives the desired performance may or may not be in the desired region.
The poles with the first method were placed in $[-8, -8, -8, -8]$ in Matlab via the command *acker* and the system is now represented in Simulink as shown in Figure 3.7.

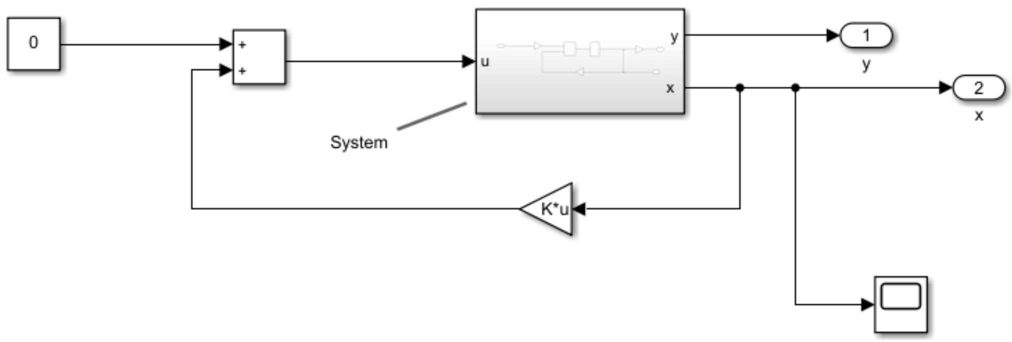


Figure 3.7: Simulink model of the system with a controller

The parameters of the motorcycle taken in consideration are listed on the table below:

L_f	Distance from the origin to the front wheel contact point with the ground	L_f=0.865 [m]
L_r	Distance from the origin to the rear wheel contact point with the ground	L_r=0.789 [m]
p	Wheelbase	p=1.654 [m]
R_f	Radius of the front tire cross sectional profile	R_f=0.109 [m]
R_r	Radius of the rear tire cross sectional profile	R_r=0.108 [m]

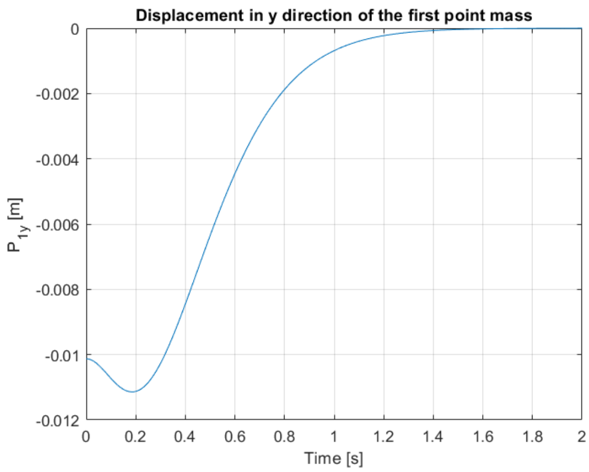
d	Front fork offset	d=0.0463 [m]
t	Front trail length	t=0.110 [m]
a	Height of the points E_f, E_r,E	a=0.2131 [m]
$\boldsymbol{\theta_{cf}}$	Front wheel caster angle	$\theta_{cf} = 27.3°$
m	Total mass including a driver of 75 Kg	m=270 [Kg]
I	Total roll inertia about the center of gravity	I=13 [Kg*m^2]
h	Height of the center of gravity	h=0.480 [m]
$\boldsymbol{\delta_f}$	Front wheel steering angle	$\delta_f = [-]$

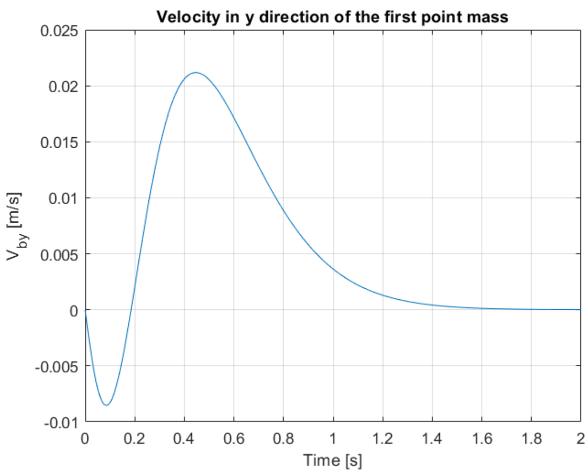
Table 3.1: List of parameters and their values

From this list it is possible to notice that some of the parameters taken in consideration in the simulation are not the ones usually used in traditional motorcycles, mostly for the radius of the front tire cross sectional profile which is large with respect to traditional motorcycles, expecially for a front tire. However the front tire was considered as a 218/45-17 and the value of the front fork offset $d = 0.0463\ [m]$ was chosen accordingly to the parameters of traditional motorcycles, which also correspond to having a trail of length $t = 0.110\ [m]$, then the simulation of the system was performed on Simulink.
In the first case no external forces were taken into account and the main assumptions for the computation were:

- $\phi_{b\_act} = 1°$;
- The initial condition of the state are $x0 = [P_{1\_y}\ V_{by}\delta_f\ \dot{\delta_f}\ ]$ with P_{1_y} and V_{by} set as the result of the computation of the previously discussed equations, δ_f and $\dot{\delta_f}$ set as 0.
- The initial longitudinal velocity of the vehicle was considered $V_{ox} = 0\ [\frac{km}{h}]$

In the case of the vehicle with a trail of $t = 0.110\ [m]$ it is possible to see that the results have very high values for what concerns the δ_f and $\dot{\delta_f}$ states, that is mainly due to the fact that with the trail set to $t = 0.110[m]$ the result of the equation (26) brings to a negative value. From the previous discussion and because of the correlation between t and a from the equation $= a \tan \theta_{cf}$, the a parameter should be smaller than a_{sum} but with those set of assumption the results are: $a_{sum} = 0.1758$, while $a = 0.2131$. In figure 3.8 it is possible to see the results of the simulation.





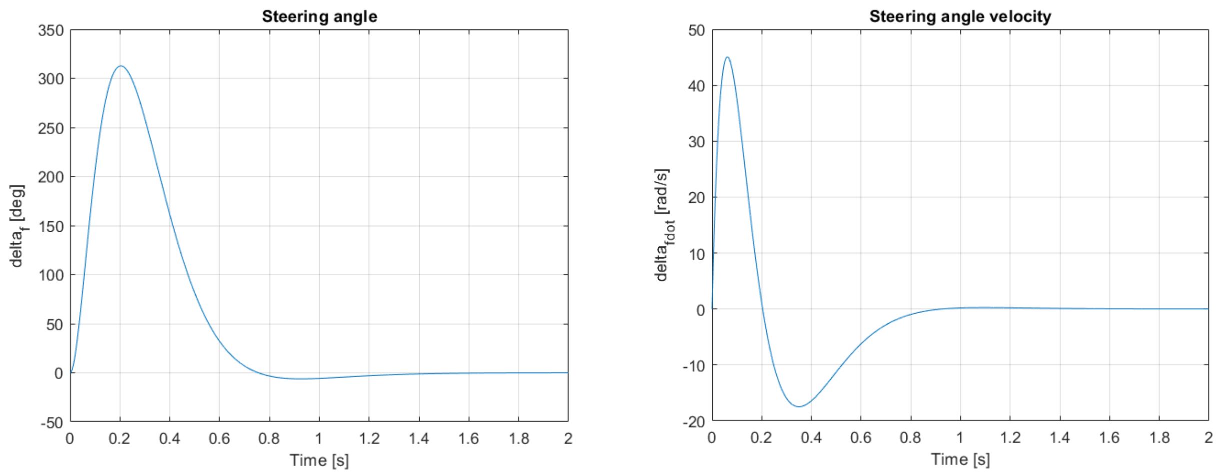


Figure 3.8: Results of the simulation with $t = 0.110\ [m]$ and $d = 0.0463\ [m]$

However the results show that the system can be controlled, by changing the values of the poles the control can act faster but with higher overshoot or slower but with a lower overshoot, in this case the compromise between performance and handling of the control are considered valid.
The following step was to investigate what happens if the value of the trail length was changed, the process was studied by changing the value of the front fork offset.

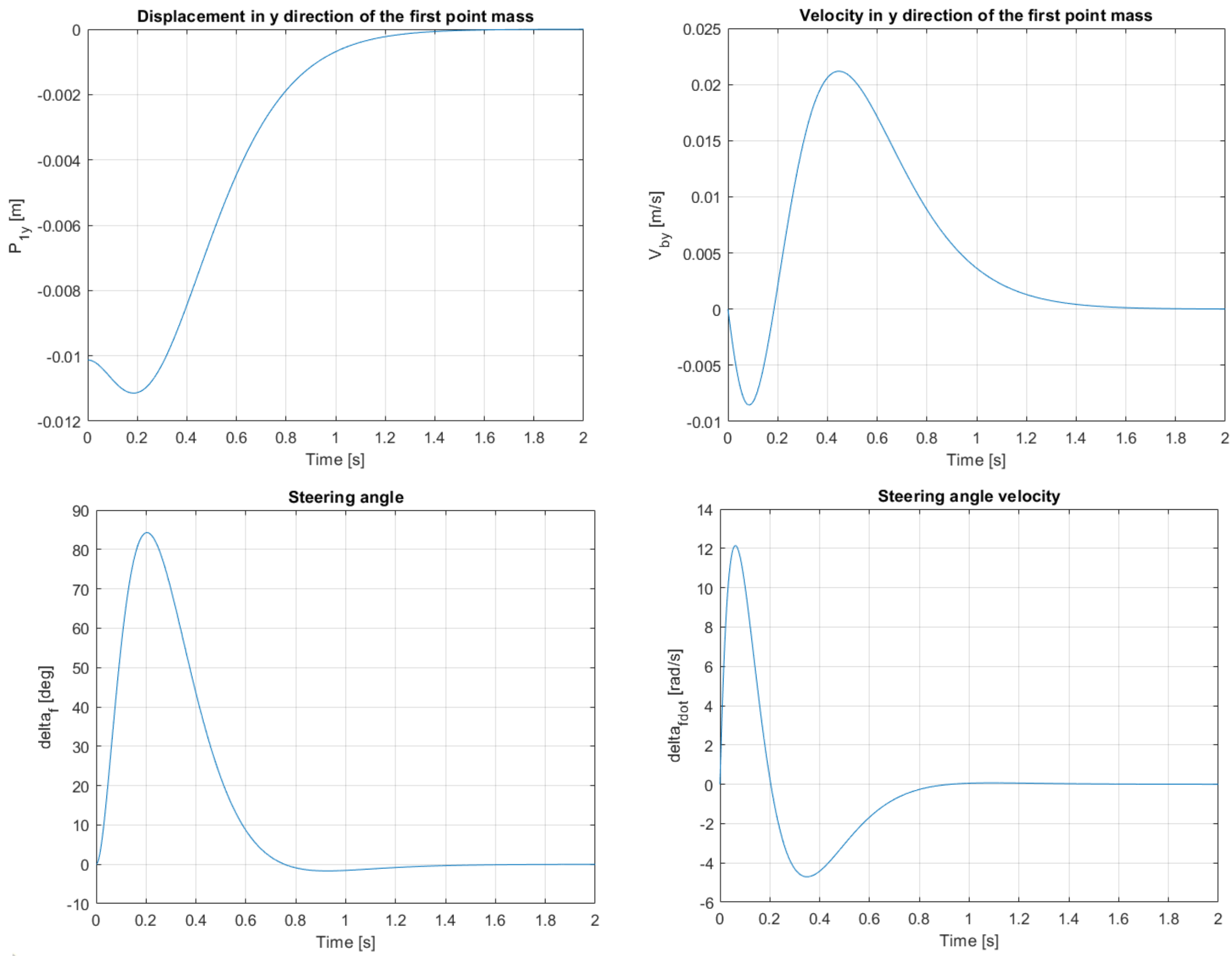


Figure 3.9: Results of the simulation with offset $d = 0[m]$ and $t = 0.1621\ [m]$

Since the results obtained in case of having a motorcycle with a mechanical trail $t = 0.110\ [m]$ did not bring to satisfactory results a simulation was performed considering the same parameters of the motorcycle but the front fork offset was set to zero. This annulment of the offset brought to a mechanical trail length of $t = 0.1621\ [m]$ and the results obtained are shown in figure (3.9).

The results obtained with these set of parameters show an improvement in the behavior of the δ_f and $\dot{\delta}_f$ curves even if the peak is still an high value, in fact in motorcycles the δ_f is usually in the range $\delta_f = \pm 35°$, the peak is in found at 0.2 seconds and its value is $\delta_f = 82°$, the problem with a vehicle without offset would become even greater when travelling from medium to high speed for what regards its stability. An interesting consideration is the fact that with this set of values $a = 0.3140\ [m]$ and it is bigger than $a_{sum} = 0.178\ [m]$, this brings to having the steering angle positive as in the case with mechanical trail $t = 0{,}110\ [m]$.

The following step was to set the front fork offset to a value of $d = 0.0774\ [m]$, with such a value the mechanical trail becomes $t = 0.075\ [m]$. This value of mechanical trail guarantees an important improvement in the results of equation (26) because it brings the difference $a_{sum} - a$ to a positive value in fact $a = 0.145\ [m]$. In figure 3.10 are shown the results of the simulation.

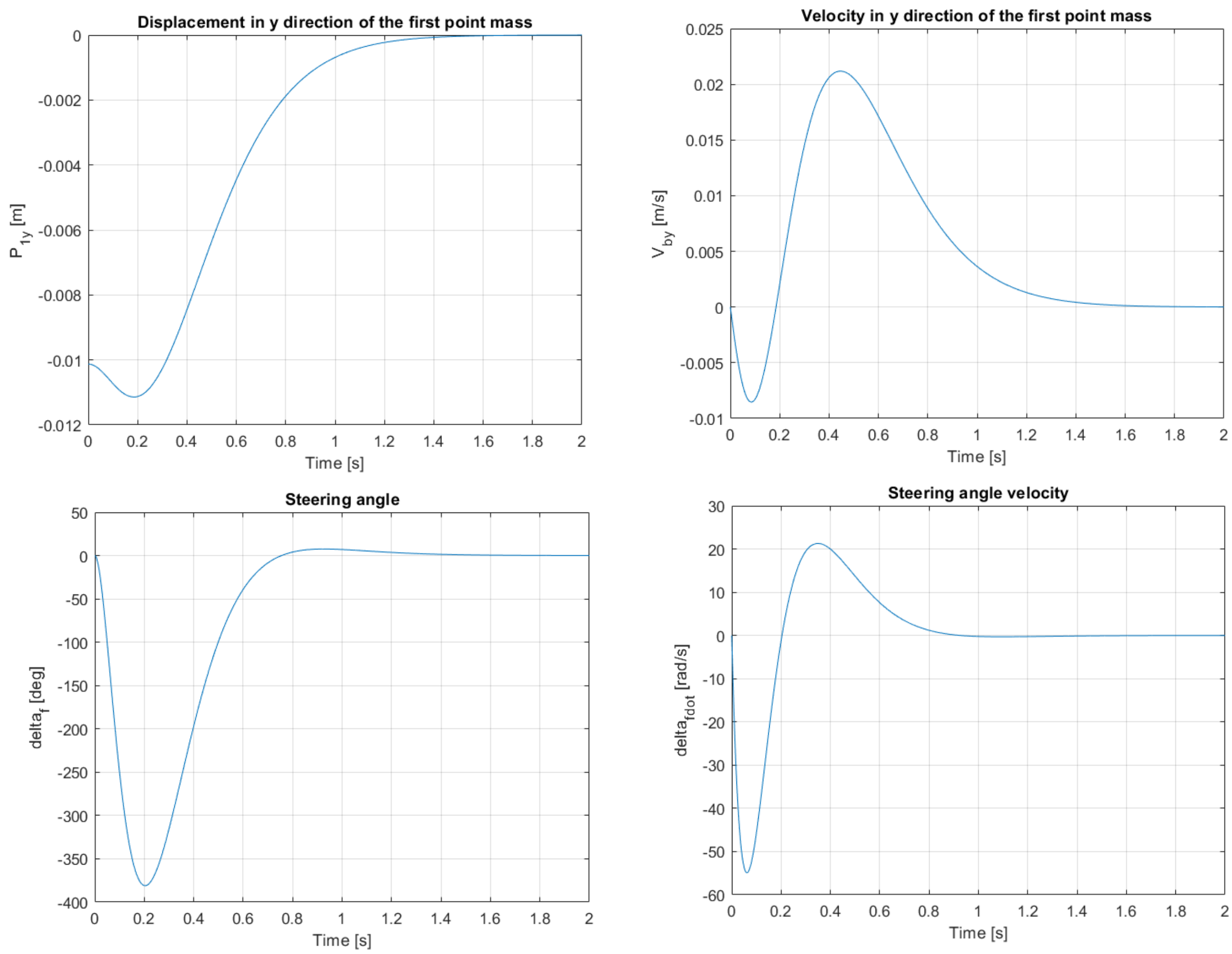


Figure 3.10: Results of the simulation with offset $d = 0.0774\ [m]$ and $t = 0.075\ [m]$.

The results obtained with those set of parameters show the negative value for the δ_f curve, the peak results to be $\delta_f = -380°$ which is of course an impossible value to reach for the steering of a motorcycle. For what concerns the curves of the position and velocity of the first point mass their behavior is the same as when they are set in the other configuration, with their peak values changing of some very small quantities.

After this configuration the front fork offset was set with a negative value in order to see if the peak of the δ_f curve would increase or decrease, it was set $d = -0.083\ [m]$ without varying the θ_{cf}. This is a theoretic consideration, in figure 3.12 this configuration is shown while in figure 3.11 it is possible to see the results obtained with the simulation.

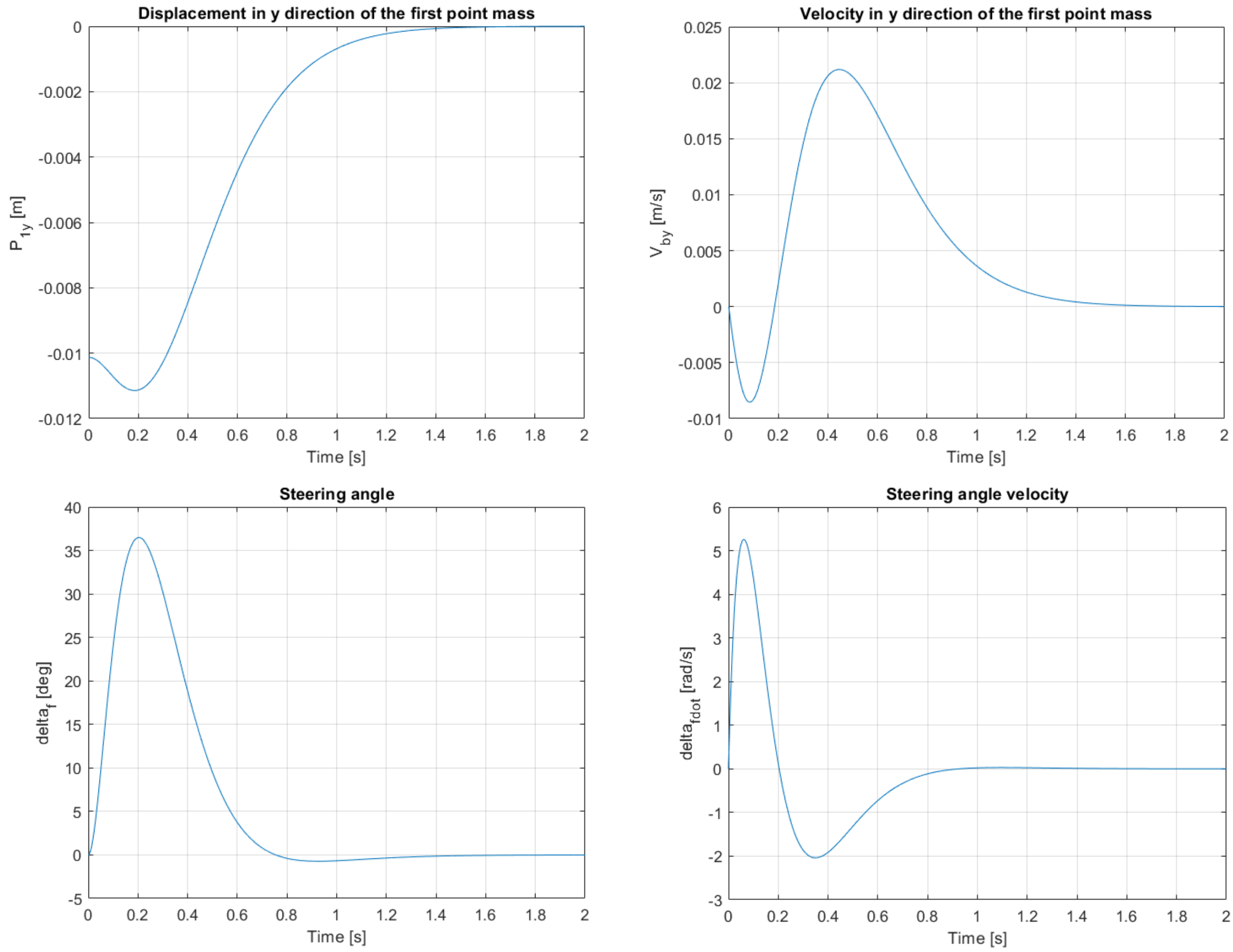


Figure 3.11: Results of the simulation with offset $d = -0.083\ [m]$ and $t = 0.255\ [m]$.

The peak of the δ_f curve in this case is $\delta_f = 36.5°$, this value is pretty close to the actual range of the steering of a traditional motorcycle. In order to implement this solution it would be necessary to mount the triple clamp of the front fork in the opposite way with respect to a traditional motorcycle and with a big value of offset, this could also bring to structural problems for what concerns the whole front fork assembly.

The following step was to implement the solution proposed by the SAE article, this means that the value of the mechanical trail is to be set in a negative value in order to control the system properly.

The value of the front fork offset needed in this configuration in order to have $t = -0.060\ [m]$ is $d = 0.1973\ [m]$ and the results obtained with the simulation using these parameters are shown in figure 3.13.

The state δ_f in this case show a negative behavior and its peak value is $\delta_f = -40°$, which is still pretty close to the steering range of a traditional vehicle. For what concerns the other states the

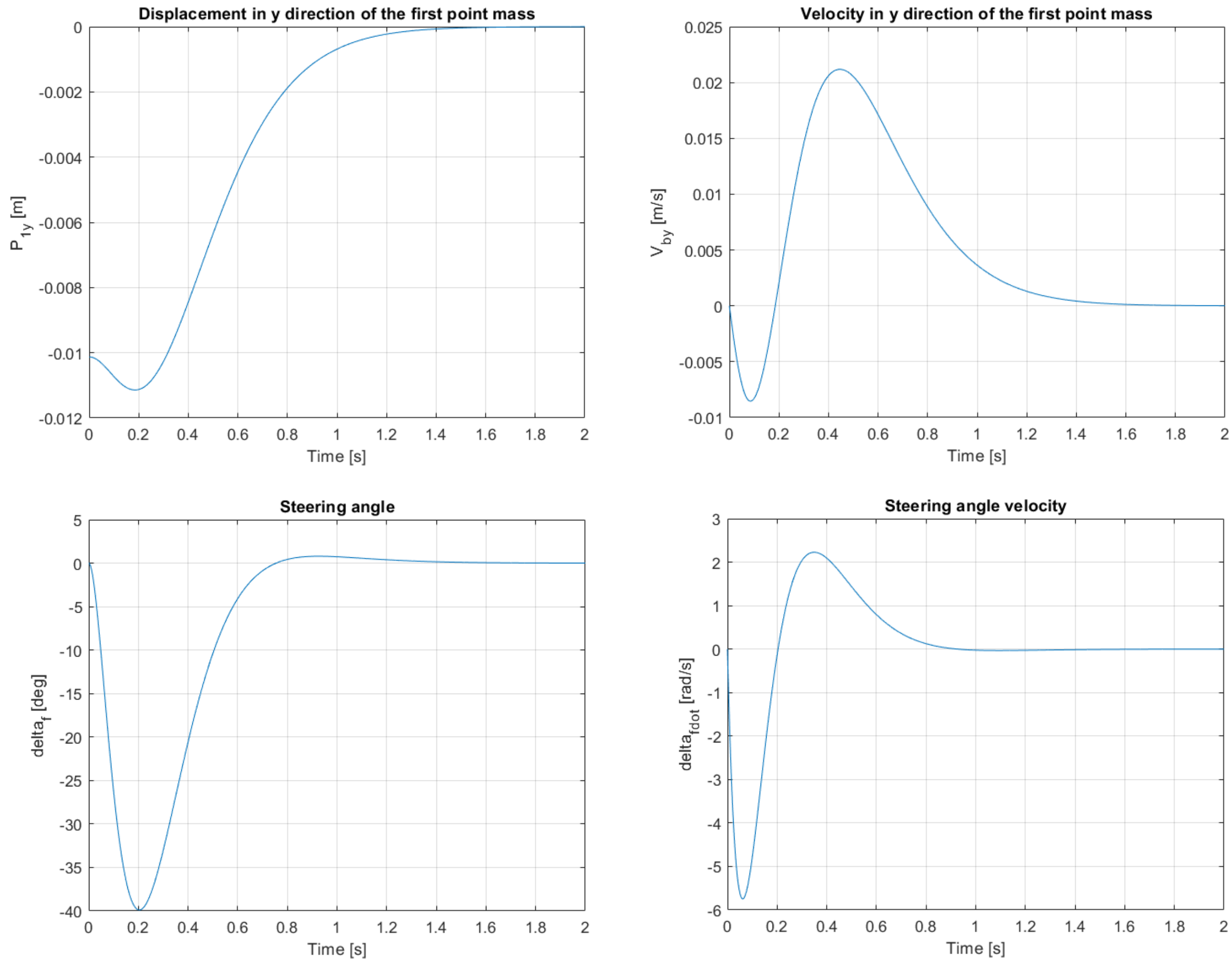


Figure 3.13: Results of the simulation with offset $d = 0.1973\ [m]$ and $t = -0.060\ [m]$.

same considerations for the other configurations are valid.

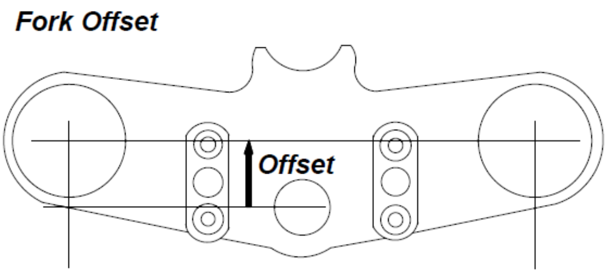


Figure 3.12: Typical triple clamp of a front fork with positive offset

The objective has become the improvement of the behavior of the system via the controller, so the desired poles were set to $[-6, -6, -6, -6]$ and the results with the same parameters as the ones set for the previous simulation are shown in figure 3.14.

The results show a slight decrease in the absolute value of the peak of the δ_f curve, in fact its value is $\delta_f = -39°$, but the peak value is reached in a longer time, in fact it is found at $Time = 0.3\ [s]$ while in the other simulations with the poles set at $[-8, -8, -8, -8]$ it was reached at $Time = 0.2\ [s]$. This means that the control acts in a slower way with the new set of desired poles but the overshoot of the curve is also lower.

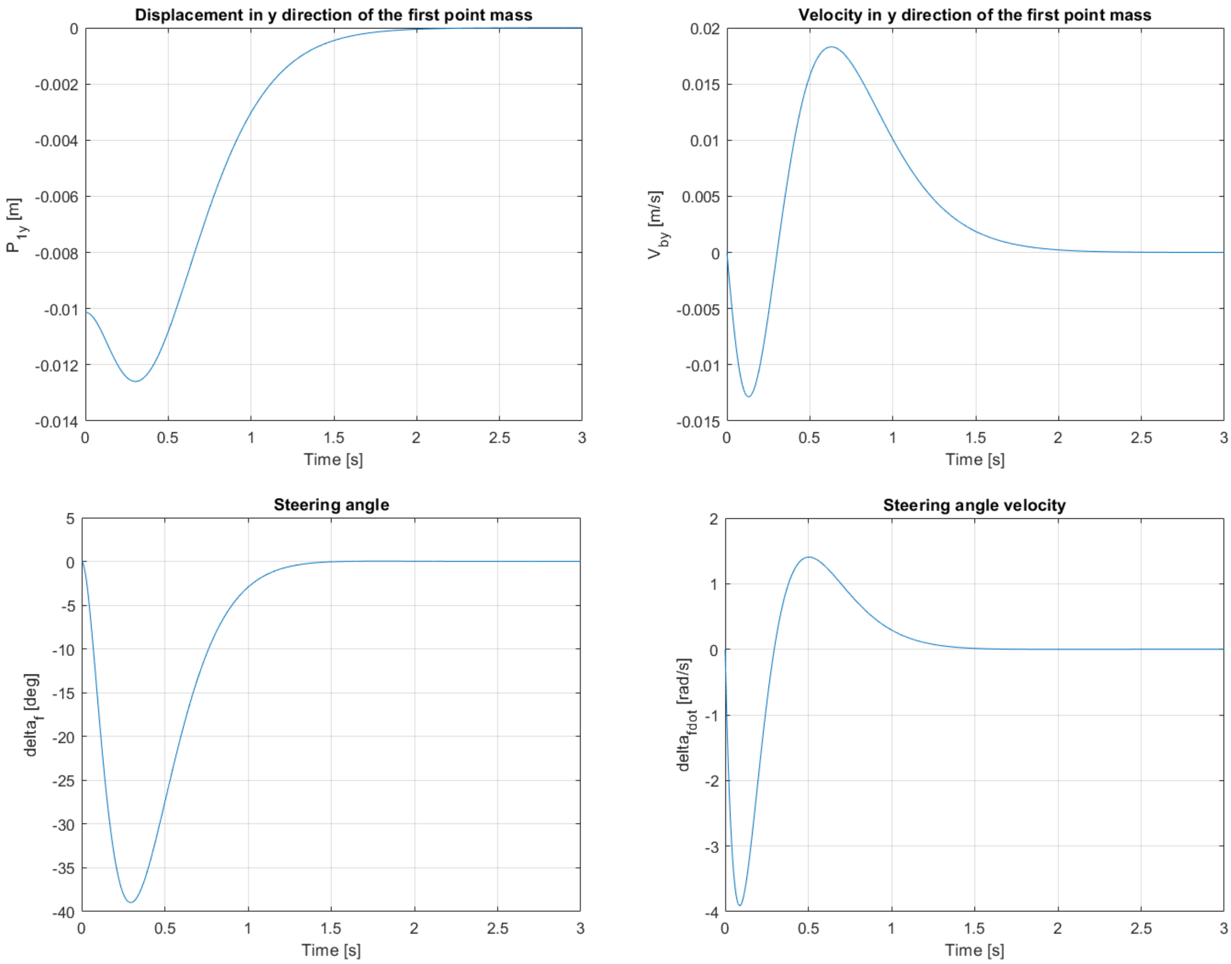


Figure 3.14: Results of the simulation with offset $d = 0.1973\ [m]$ and $t = -0.060\ [m]$ and the poles set at $[-6, -6, -6, -6]$

After these experiments it was decided to change the length of the wheelbase of the motorcycle, the value of L_f was set to $L_f = 0.665\ [m]$, this means having a total wheelbase of $p = 1.454\ [m]$ but the difference in the set L_f means that the position of the center of gravity is now shifted towards the front assembly of the motorcycle in the longitudinal position, this means that as stated in table 1.1 having the center of gravity positioned more forward will make the motorcycle tend to over-steer and the rear wheel tends to slip laterally when taking a turn.

The poles of the controller were set back to $[-8, -8, -8, -8]$ and the results obtained are shown in figure 3. 15.

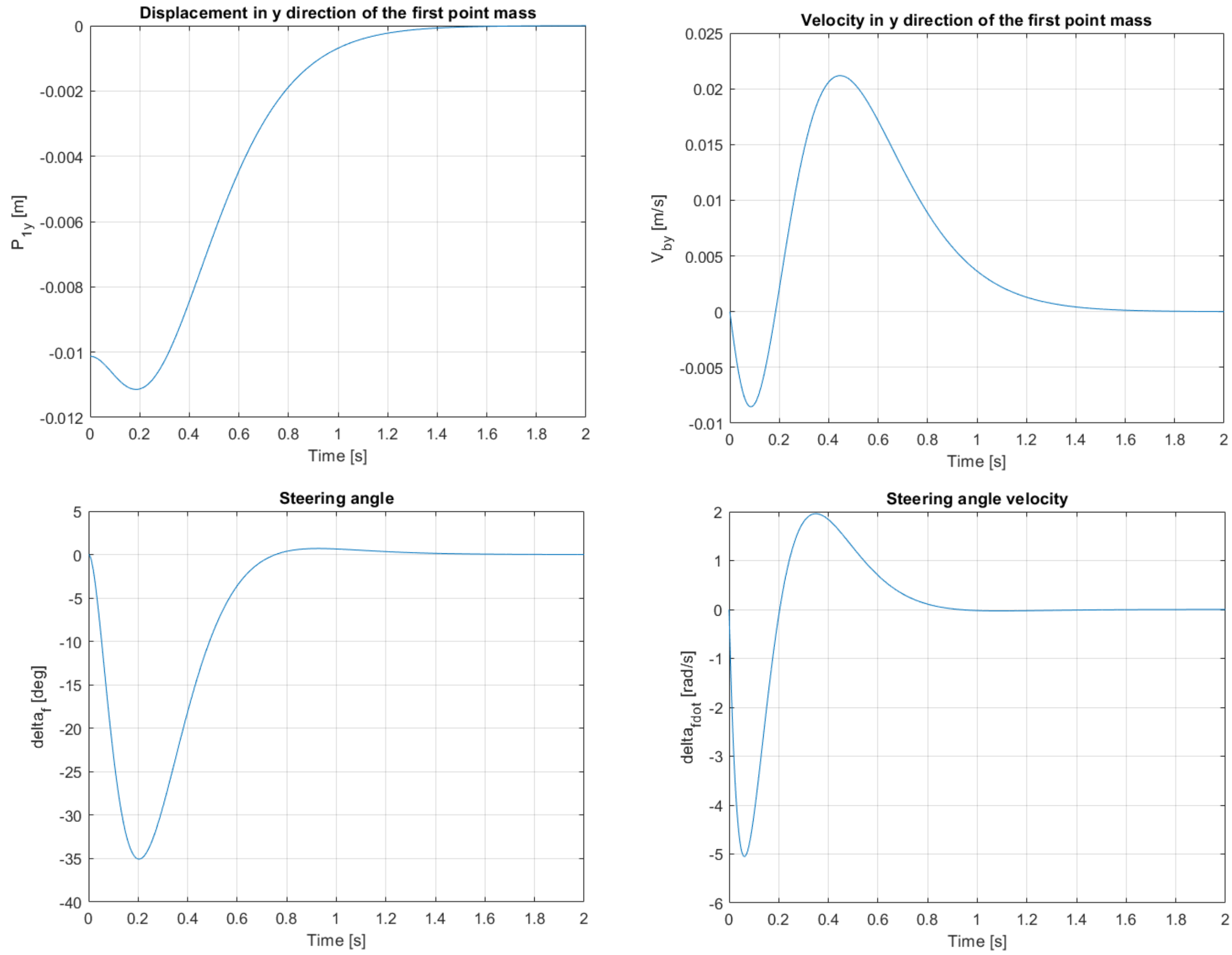


Figure 3.15: Results of the simulation with offset $d = 0.1973\ [m]$ and $t = -0.060\ [m]$ and the modified wheelbase with $L_f = 0.665\ [m]$

With this new configuration of the vehicle the curve of δ_f show the same behavior as the one obtained with the previous wheelbase, but its peak value is now $\delta_f = 35°$, now the value is in the range of the steering of a motorcycle and so the result is quite good.

3.4 LQR Control

In the previous discussion the difference between LQR and the pole placement technique was discussed, from now the simulations are computed with a controller using the LQR method. Using the same parameters for the motorcycle as the ones used before, the results obtained with the LQR are now analyzed. The first simulation was performed with the motorcycle set as the traditional motorcycle suggested by the SAE article, $t = 0.110\ [m]$ and $d = 0.0463\ [m]$ and the results are shown in figure 3.16.

The results of this simulations are not good, the control acts way too slow and the values obtained for the δ_f and $\dot{\delta}_f$ curves are really high, furthermore the peak value of the steering state is $\delta_f = 283°$. Furthermore there is an increase in the state of the position of the first point mass P_{1_y}, its peak value is now $P_{1\_y} = -0.0159\ [m]$ and it is found at $Time = 0.7\ [s]$ and in order to obtain the first point mass to get to the position $P_{1\_y} = 0\ [m]$ ten seconds of simulation are not enough.

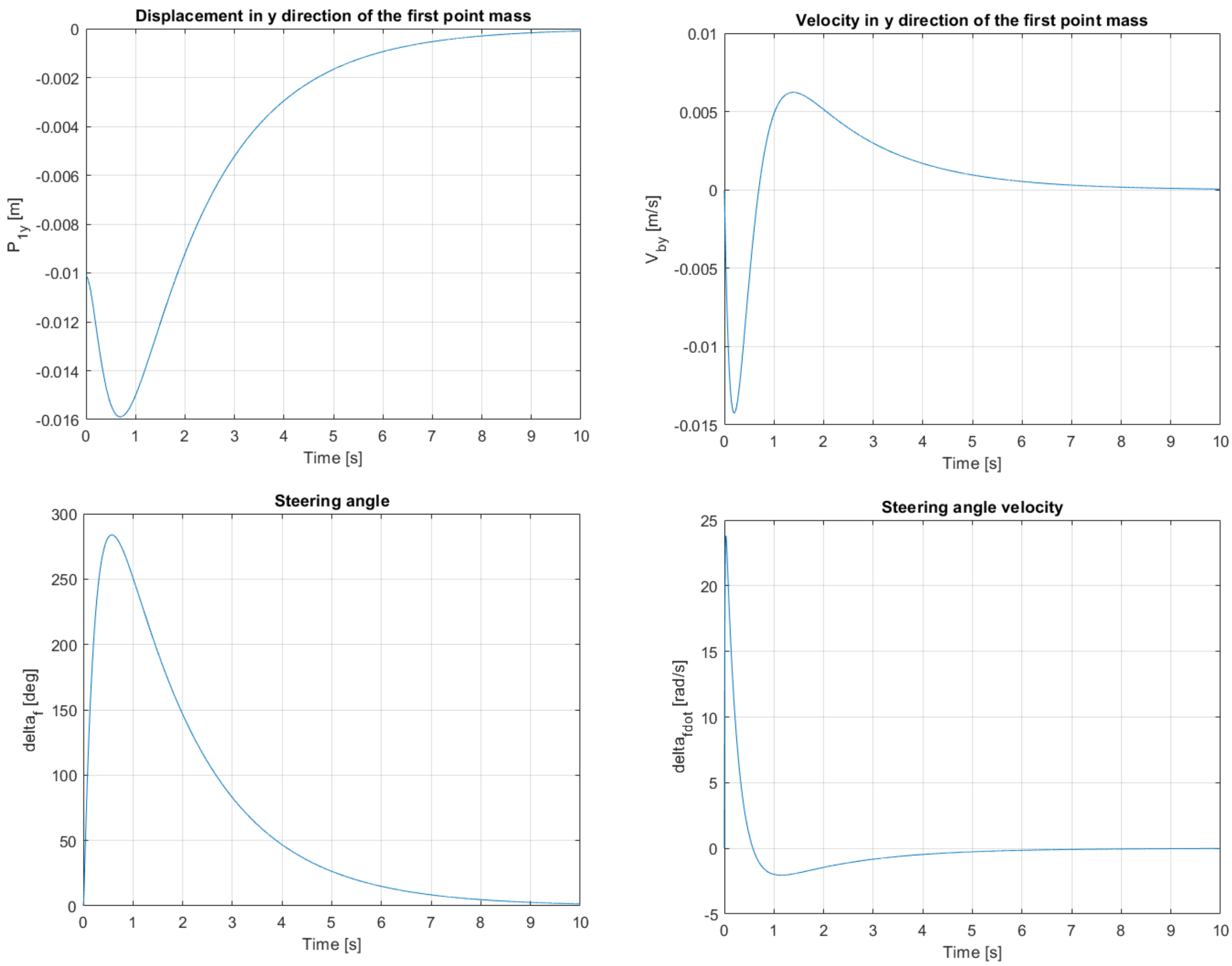


Figure 3.16: Results of the simulation with $d = 0.0463\ [m]$ and $t = 0.110\ [m]$ with the LQR control

However the simulation was then performed with the same parameters for the LQR control but changing the value of offset and of the mechanical trail of the motorcycle and the results obtained are now analyzed. The motorcycle with zero front fork offset is the first taken in consideration and in figure 3.17 it is possible to see the outcome of the simulation.

The curves obtained show a system controlled in a faster way with respect to the previous simulation, in fact the position $P_{1\_y} = 0\ [m]$ is reached right before the end of the simulation, which is set at $Time = 3\ [s]$. Furthermore the peak of the curve of δ_f is found at $Time = 0.3\ [s]$, with a value of $\delta_f = 70°$, which is not suitable for the steering of a classical motorcycle, but it is possible to compare it with the curve of δ_f in figure 10 in which the controller was set with the pole placement method and it is possible to see that the peak value is $\delta_f = 82°$. This means that the control even if acting a little slower with respect to the one used with the pole placement method has lower values for the curve. Another interesting result is concerned with the position of the first point mass, the curve has the same behavior as the ones obtained with the pole placement method and even the values obtained are similar as it is possible to see confronting figure 3.17 with figure 3.9.

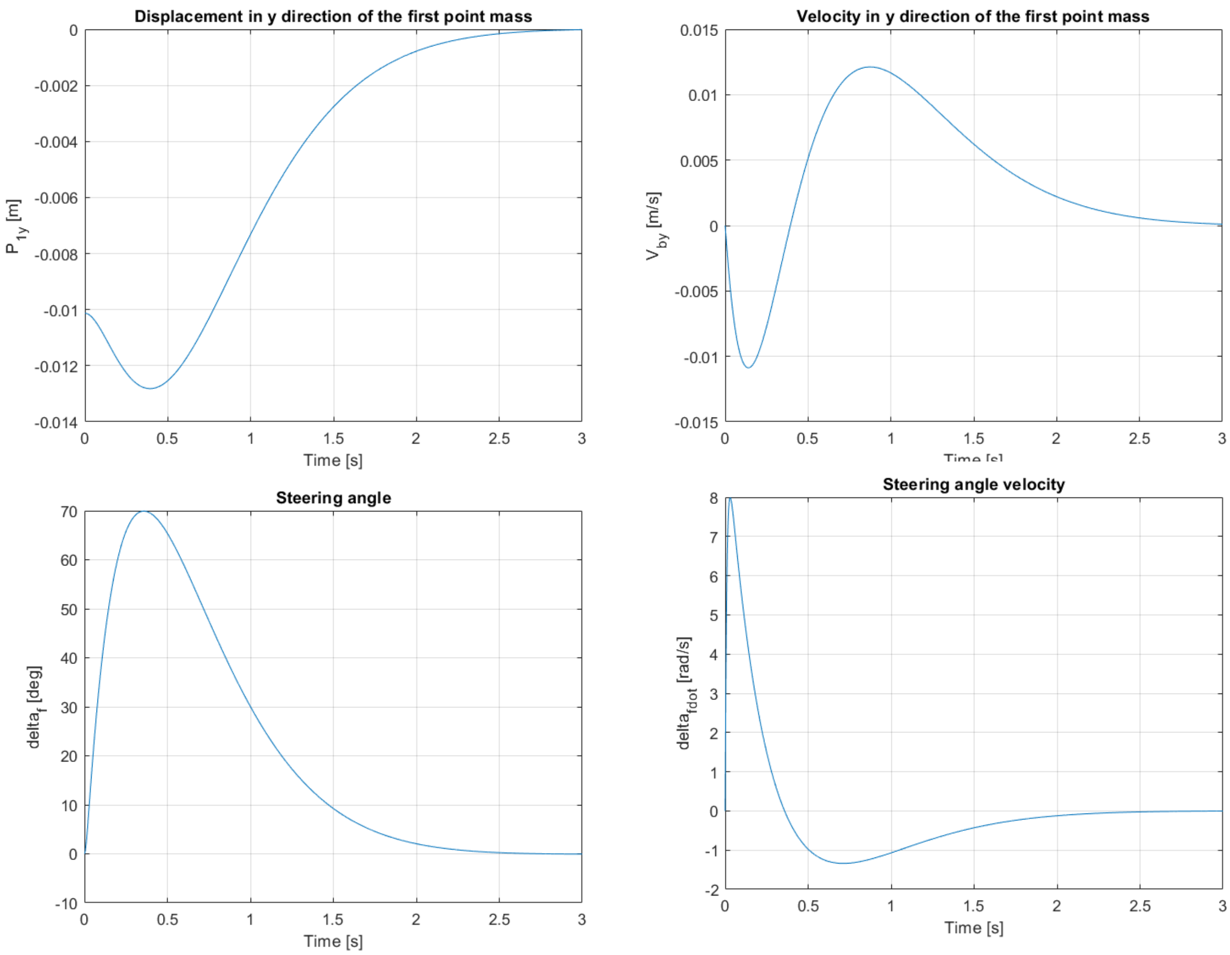
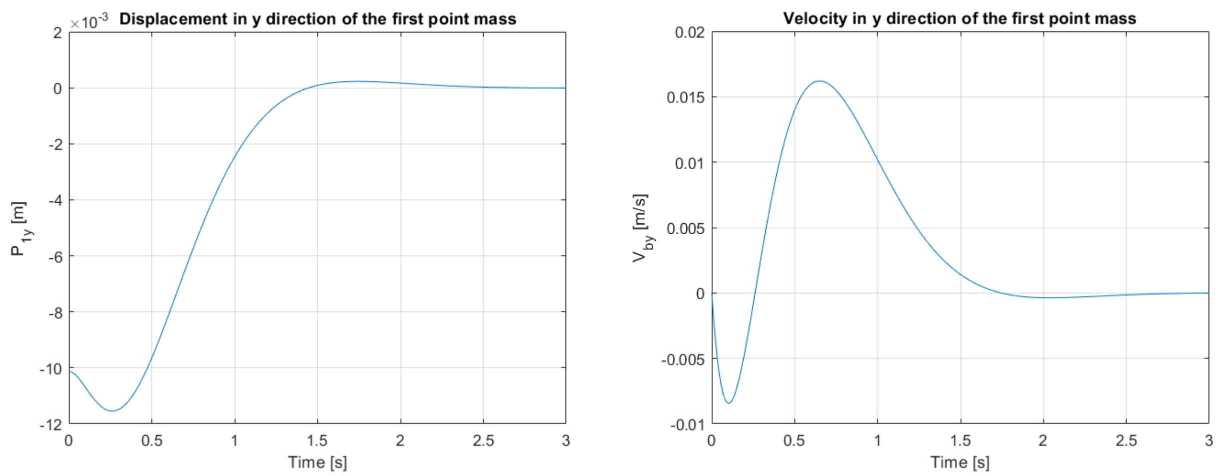


Figure 3.17: Results of the simulation with $d = 0\ [m]$ and $t = 0.1621\ [m]$ with the LQR control

The next simulation was performed considering the solution proposed by the SAE article, the mechanical trail was set with the negative value of $t = -0.060\ [m]$ and the results are shown in figure 3.18.The δ_f state in this configuration has the same behavior as the one obtained in figure 3.13, in which the peak of the curve was reached at $Time = 0.2\ [s]$ with a value of $\delta_f = -40°$ while with the LQR control the peak value is at $Time = 0.2\ [s]$ with $\delta_f = -33°$, this means that the steering of the motorcycle is able to perform with this type of controller (because $\delta_f \cong \pm 35°$) and the motorcycle can be balanced with just an action on the steering pad.



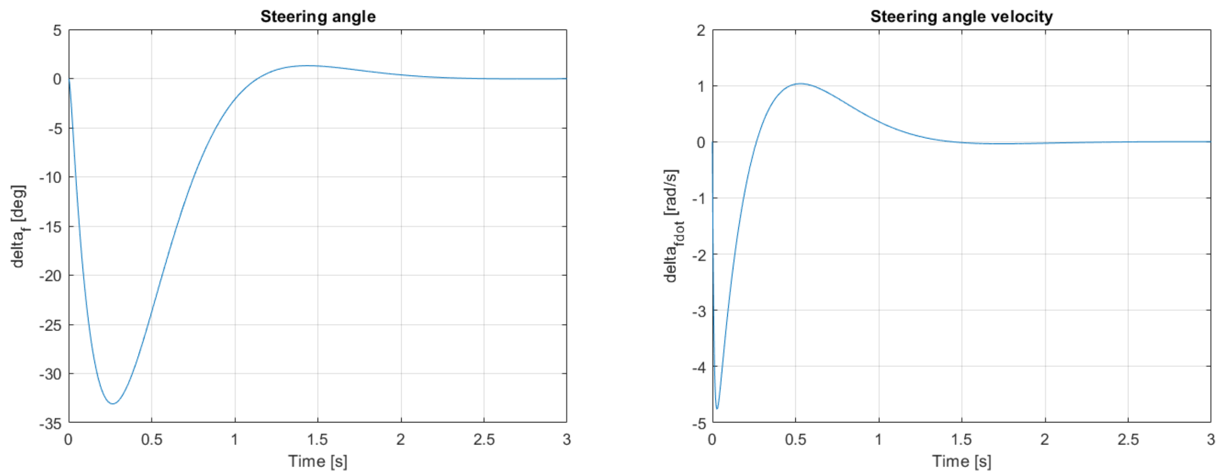
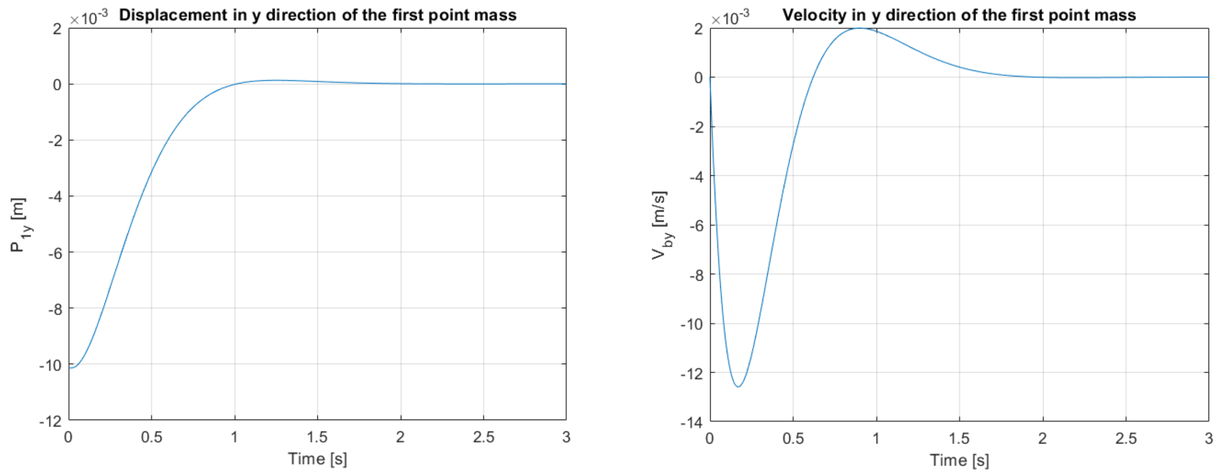


Figure 3.18: Results of the simulation with $d = 0.1973\ [m]$ and $t = -0.060\ [m]$ with the LQR control

3.5 Influence of velocity

All the previous simulations were performed assuming a longitudinal velocity of the motorcycle V_{ox} of $0\ [\frac{Km}{h}]$, the problem with roll stability at low velocities is studied in this work and it is interesting to see the influence of the increment of velocity in the longitudinal direction. When considering the state space model it is possible to see that equation (127) and (128) are influenced by V_{ox} and these two parameters influence the A matrix of the system $\dot{x} = Ax + Bu$. The parameters of the motorcycle considered in this computation are the ones that gives the solution shown in Figure 3.18, meaning that the LQR control is still the one in use, when increasing the longitudinal velocity to $V_{ox} = 1\ [\frac{Km}{h}]$ the results are shown in Figure 3.19. The results show a similar behavior for what concerns the curves of the velocity V_{by}, the steering angle δ_f and the steering angle velocity $\dot{\delta}_f$ as the one with $V_{ox} = 0\ [\frac{Km}{h}]$, what it is interesting to see is the difference in the curve of the position of the first point mass P_{1_y} that is not decreasing after the



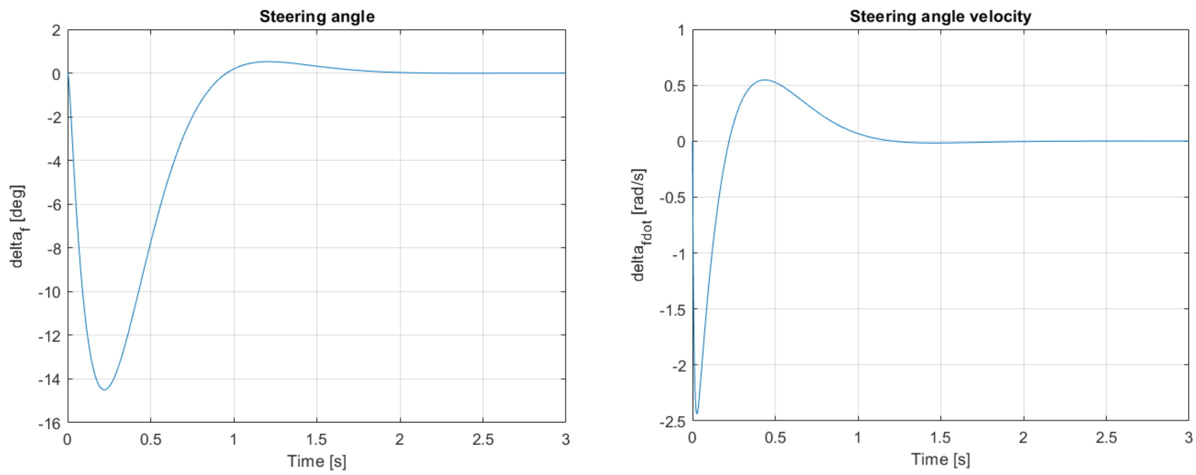


Figure 3.19: Results of the simulation with $d = 0.1973\ [m]$ and $t = -0.060\ [m]$ and $V_{ox} = 1\ [\frac{Km}{h}]$

In the following figures 3.20 and 3.21 the results obtained with $V_{ox} = 2\ [\frac{Km}{h}]$ and $V_{ox} = 4\ [\frac{Km}{h}]$ are reported and it is possible to see that they show the same behavior as the one with $V_{ox} = 1\ [\frac{Km}{h}]$.

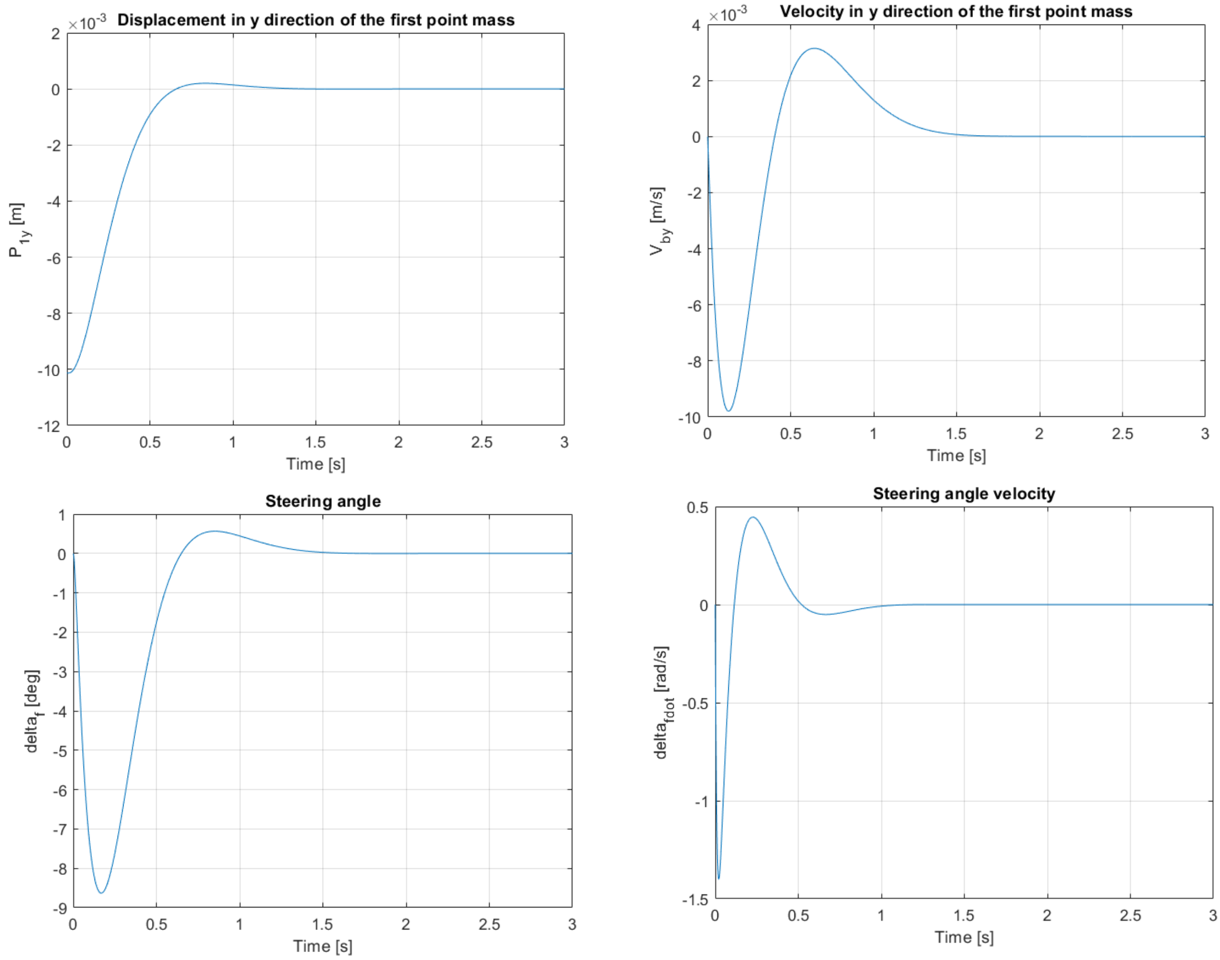


Figure 3.20: Results of the simulation with $d = 0.1973\ [m]$ and $t = -0.060\ [m]$ and $V_{ox} = 2\ [\frac{Km}{h}]$

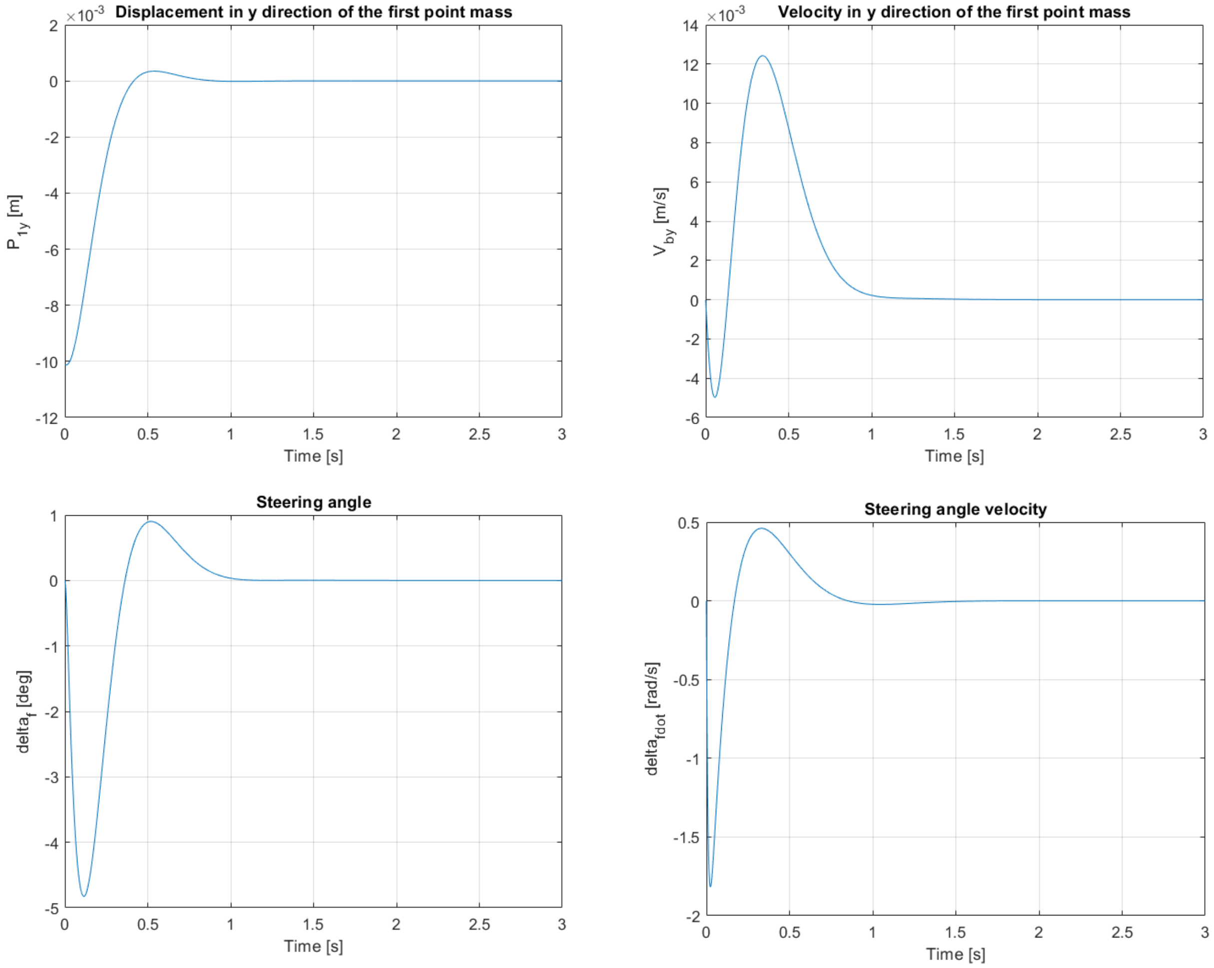


Figure 3.21: Results of the simulation with $d = 0.1973\ [m]$ and $t = -0.060\ [m]$ and $V_{ox} = 4\ [\frac{Km}{h}]$

The peak value for the steering angle $\delta_f = -8.67°$ with $V_{ox} = 2\ [\frac{Km}{h}]$, the position of the first point mass has basically the same behavior of the one with $V_{ox} = 1\ [\frac{Km}{h}]$ while its velocity reaches lower values for both the velocity of the first point mass and the steering velocity $\dot{\delta}_f$.

The same results are found when increasing the velocity up to $V_{ox} = 4\ [\frac{Km}{h}]$, now the peak value of the steering angle further decreases down to $\delta_f = -4.83°$ but a greater overshoot in the second part of the graph is found, the same behavior of a bigger overshoot is found in the position of the first point mass P_{1_y}. The curve of the velocity of the first point mass behaves differently from the one found with a $V_{ox} = 2\ [\frac{Km}{h}]$, in fact it shows a positive peak higher with respect to the negative part of the curve.

The results obtained with the increase of the velocity are those expected when considering the roll stability of a motorcycle, a motorcycle is a vehicle normally unstable when is travelling at low velocity especially when the velocity is null, bringing to the conclusion that the intervention of the driver is fundamental to obtain the roll stability, but with this type of control it is possible to stabilize the vehicle. Increasing the velocity brings to a good improvement in both the value of the steering angle δ_f to supply to the vehicle, furthermore even the position of the first point mass and its velocity have narrower curves with respect to those obtained at zero velocity.

After these computations it is possible to modify the equations found until now with the study of the influence of gyroscopic effects on the system, since the control should be able to act even when the vehicle is moving and in a motorcycle the presence of the wheels rotating generate some moments that needs to be taken in consideration.

3.6 Influence of gyroscopic effects on the simulation

3.6.1 Yaw motion

The gyroscopic effects are now considered in the following computations for the system, of course they can be considered only when the wheels are actually spinning with an effective rotational velocity ω_f and ω_r, so at $V_{ox} = 0\ [\frac{Km}{h}]$ these effects are actually not part of the problem.

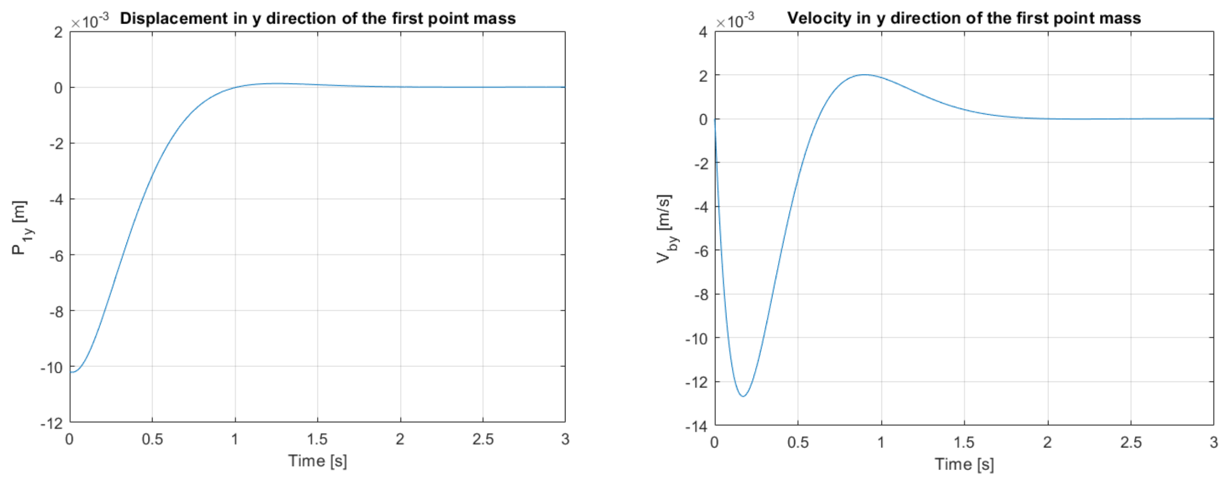
The first effect considered is the gyroscopic moment generated by the yaw motion of the vehicle, the motorcycle is imagined to be taking a curve with a small radius of curvature $R_c = 2\ [m]$ since the velocities taken in consideration are really small. Furthermore the effect of the gyroscopic moments given by the engine inertia are not considered because of the small velocities and the small contribution given by these effects, since the gyroscopic moments created by the engine are about $5\% - 15\%$ of the gyroscopic moments generated by the wheels. As shown in figure 1.15 the gyroscopic moment generated by the yaw motion increases the ideal roll angle of the vehicle so it is possible to consider this effect in the simulation by modifying equation (29) into

$$\phi' = \phi_{b\_act} - \phi_{b0} + \Delta\phi = \phi_{b_{act}} - \phi_{b0} + \frac{(I_{wf}\omega_f + I_{wf}\omega_f)\dot{\psi}\cos(\phi_{b_{act}})}{\sqrt{(mg)^2 + (mR_c\dot{\psi}^2)}^2} \tag{129}$$

For what concerns the value of $\dot{\psi}$, equation (70) comes in help

$$\dot{\psi} = \frac{V_{ox}}{R_c} \tag{130}$$

This brings to an increment in the roll angle of the vehicle, bringing to a slight difference in the results given by the simulation performed with this new effect taken into account. In the following figures the results at different velocities are shown.



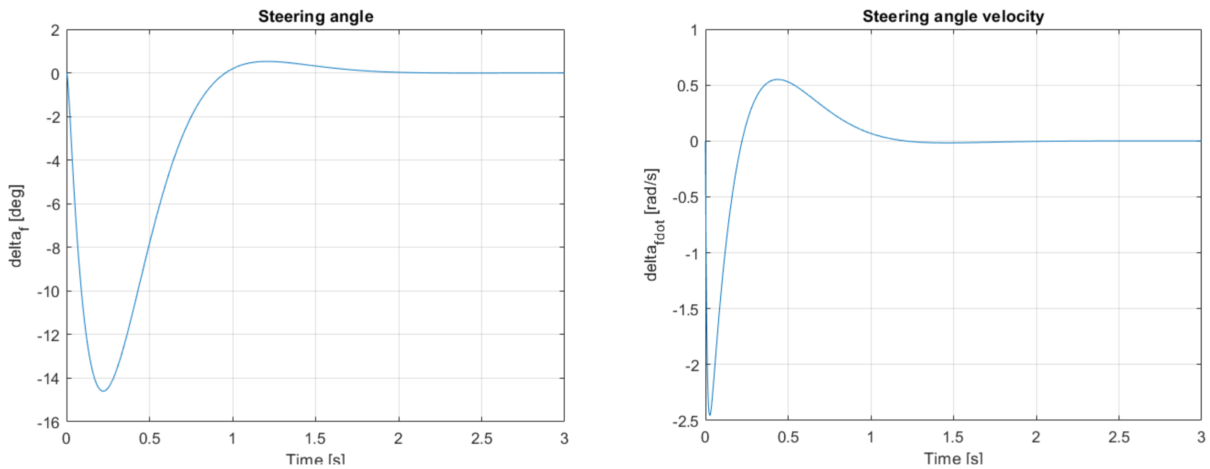


Figure 3.22: Results of the simulation with the yaw gyroscopic effects generated at $V_{ox} = 1\ [\frac{Km}{h}]$

At $V_{ox} = 1\ [\frac{Km}{h}]$ the effects generated are really small, in fact the graphs show the same behavior as the one shown by the same vehicle in figure 3.23 but there is a slight increment in the values obtained: the steering angle has its peak value found at $\delta_f = -14.60°$, the same slight increment is found in the peak value for the position of the first point mass, which decreases down to $P_{1\_y} = -0.0102\ [m]$.

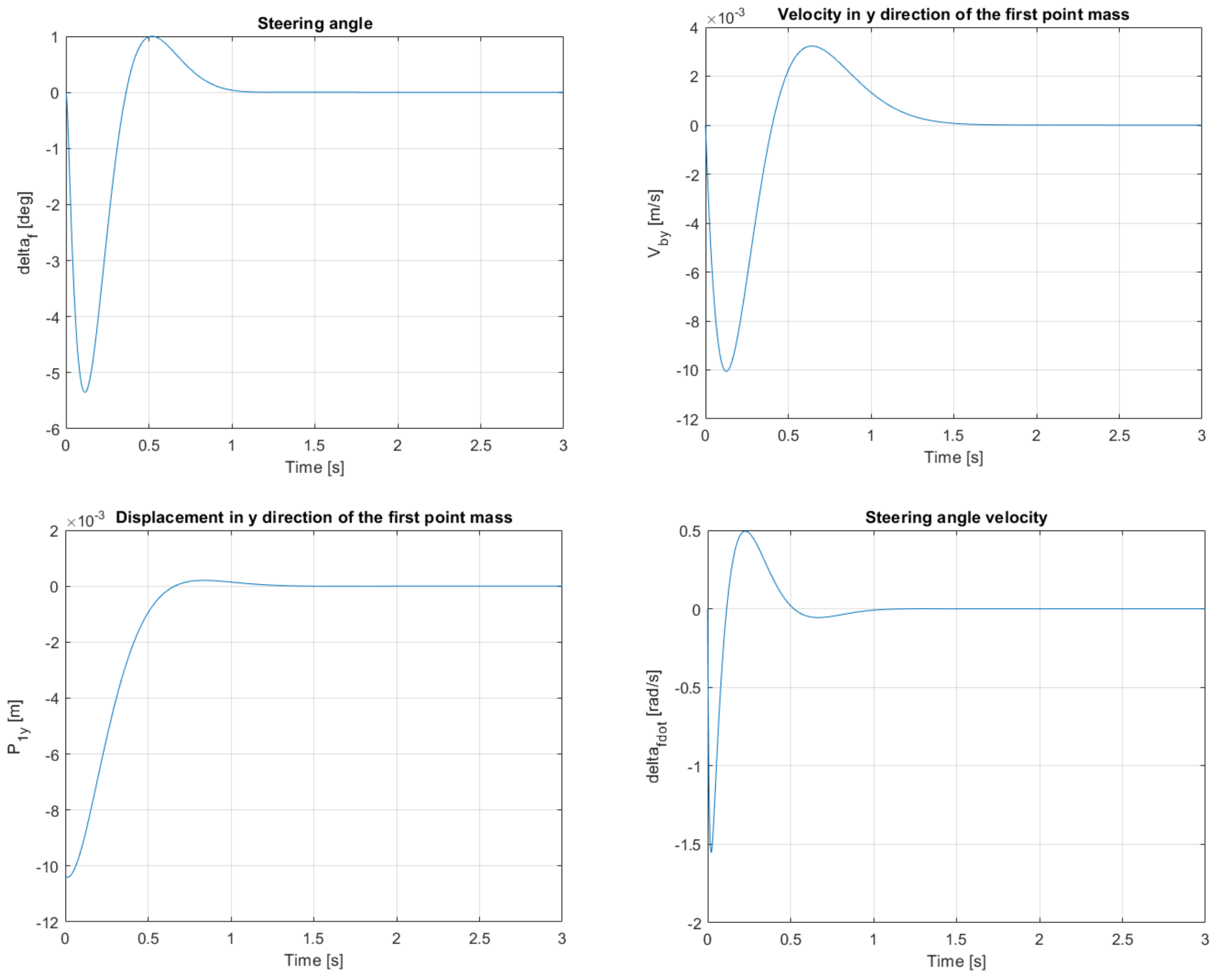


Figure 3.23: Results of the simulation with the yaw gyroscopic effects generated at $V_{ox} = 2\ [\frac{Km}{h}]$

Increasing the velocity up to $V_{ox} = 2\ [\frac{Km}{h}]$ the gyroscopic moment increases its effect on the roll angle obtained, it is possible to notice that the curves always maintain the same behavior as the one shown in figure 22 but the values are more divergent because of the increase in the yaw rate $\dot{\psi}$ (the same R_c is still considered). So while taking a curve with a $R_c = 2\ [m]$ at $V_{ox} = 2\ [\frac{Km}{h}]$ the peak of the curve to stabilize the vehicle increases up to $\delta_f = -8.88°$ with respect to the one obtained without considering the gyroscopic effect $\delta_f = -8.67°$. The same thing applies for both the position of the first point mass $P_{1\_y} = -0.0104\ [m]$ and its velocity V_{by} as it is possible to see in figure 3.24.

The same reasoning also applies for when the velocity is increased up to $V_{ox} = 4\ [\frac{Km}{h}]$ and in figure 3.25 it is possible to see the curves obtained with this simulation.

In this case the increase in the peak for the steering angle reaches up to $\delta_f = -5.35°$, meaning that when taking a turn at $V_{ox} = 4\ [\frac{Km}{h}]$ with a radius of curvature $R_c = 2\ [m]$ brings to an increase of the steering angle with respect to traveling with a straight path of almost $0.5°$ in order to maintain the stability in the same time necessary with the control system used. The increase in the displacement of the first point mass P_{1_y} is instead bigger than with the other velocities and it reaches a value of $P_{1\_y} = -0.0112\ [m]$.

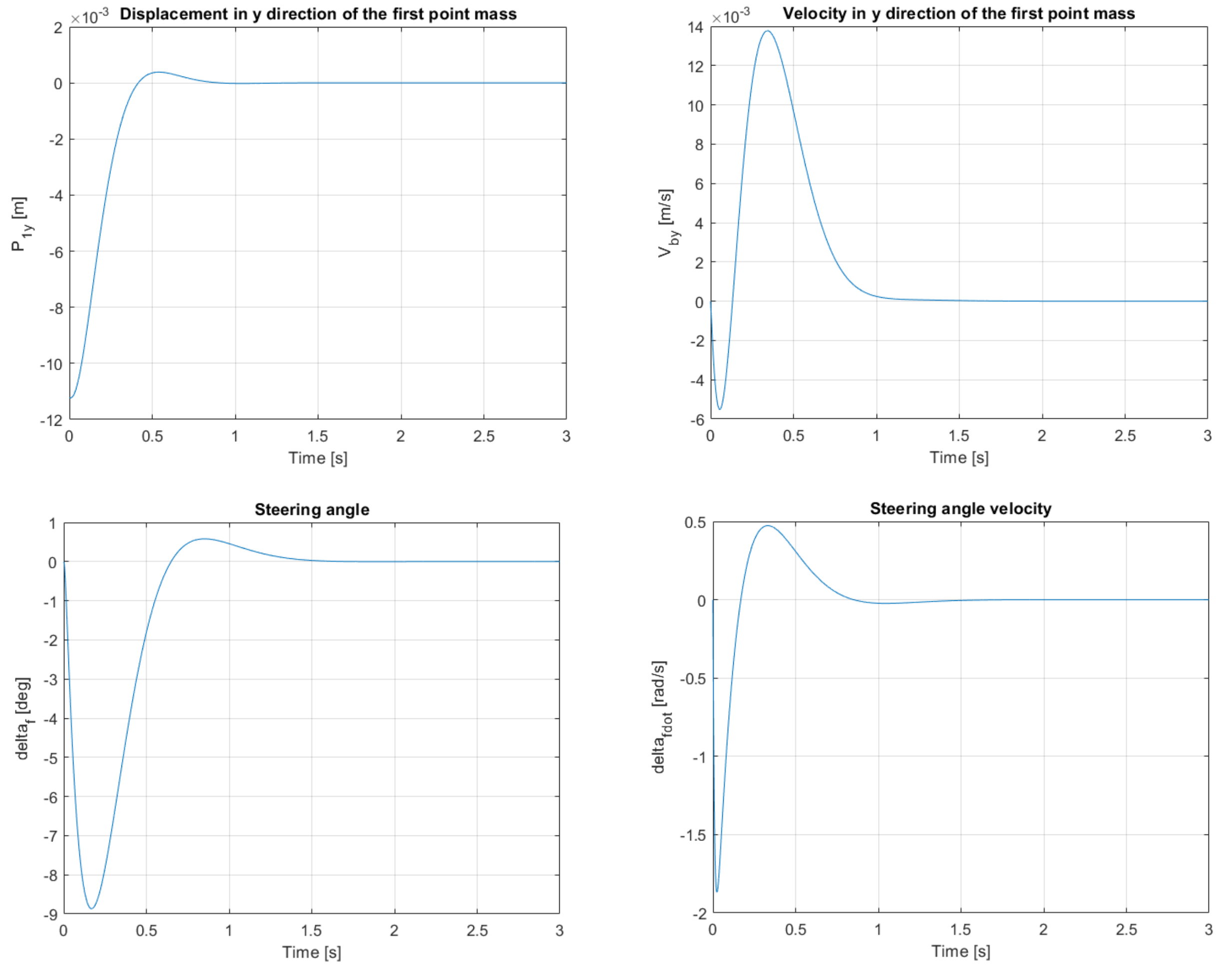


Figure 3.24: Results of the simulation with the yaw gyroscopic effects generated at $V_{ox} = 4\ [\frac{Km}{h}]$

3.6.2 Roll motion

As it is possible to see in figure 1.21 the roll motion $\dot{\phi}$ combined with the front wheel rotational speed ω_f generate a gyroscopic moment around the steering axis which contribute to the movement of the handlebar when the vehicle is running. In first approximation the effect is considered when the vehicle is running a straight trajectory and the inclination of the vehicle is reached with a constant roll rate $\dot{\phi} = 0.05\ [\frac{rad}{s}]$ which directly correlate to the velocity in the lateral direction of the first point mass, remembering that $V_{by} = V_{oy} + \dot{P}_{1\_y}$ and deriving equation (37) it is obtained

$$\dot{P}_{1\_y} = -h'\dot{\phi}' \tag{131}$$

$$M_{gu} = I_f \ddot{\delta}_f \tag{132}$$

In this way it is possible to express the angular acceleration obtained by the handlebar, considering that I_f is the moment of inertia of the steering handlebar

$$\ddot{\delta}_f = \frac{-I_{wf}\omega_f \dot{\phi} \cos\vartheta_{cf}}{I_f} \tag{133}$$

After these considerations it is possible to perform the simulations in order to see the effect of the gyroscopic moments due to the roll motion. The first velocity considered was $V_{ox} = 1\ [\frac{Km}{h}]$, figure 3.25 shows the results obtained

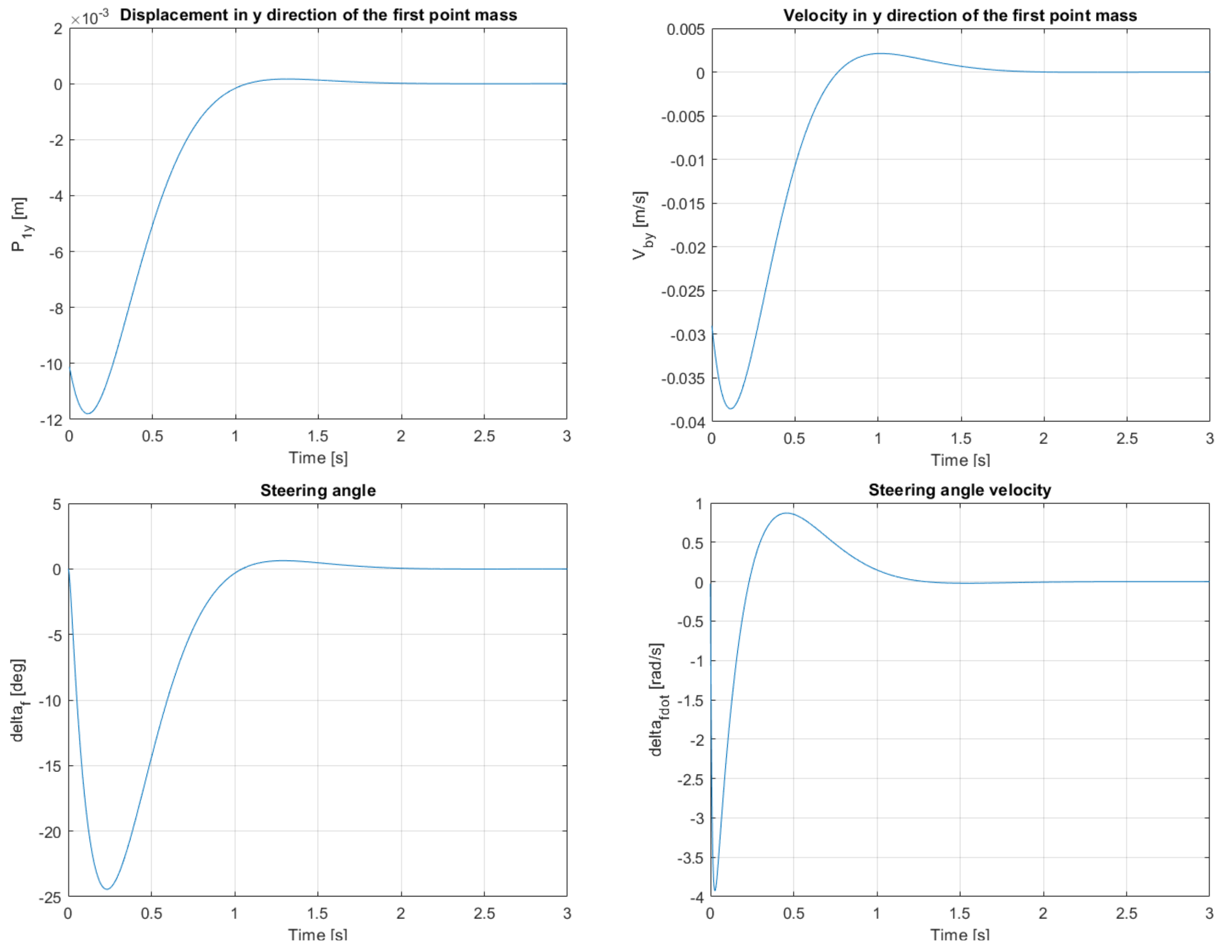


Figure 3.25: Results of the simulation with the roll gyroscopic effects generated at $V_{ox} = 1\ [\frac{Km}{h}]$

As before the simulations show a similar behavior to the results obtained without considering the gyroscopic effects, of course the major factor influencing these results is the constant roll rate which influences V_{by} that usually started at $V_{by} = 0\ [\frac{m}{s}]$, while now starts at $V_{by} = -0.029\ [\frac{m}{s}]$ but still the peak values are actually $\delta_f = -24.43°$, $P_{1\_y} = -0.0118\ [m]$. The difference in the steering angle is actually quite high with respect to the ones found considering the yaw motion or without considering any gyroscopic effects, in fact it is of about 10°.

Figure 3.26 shows the results obtained when increasing the velocity up to $V_{ox} = 2\ [\frac{Km}{h}]$, as before each state has basically the same behavior as the one obtained without considering the gyroscopic effects, the peak values obtained are now $\delta_f = -14.18°$, which is bigger than the one obtained before but the difference is now of about 6°, while the position of the first point mass is now found at $P_{1\_y} = -0.0112\ [m]$.

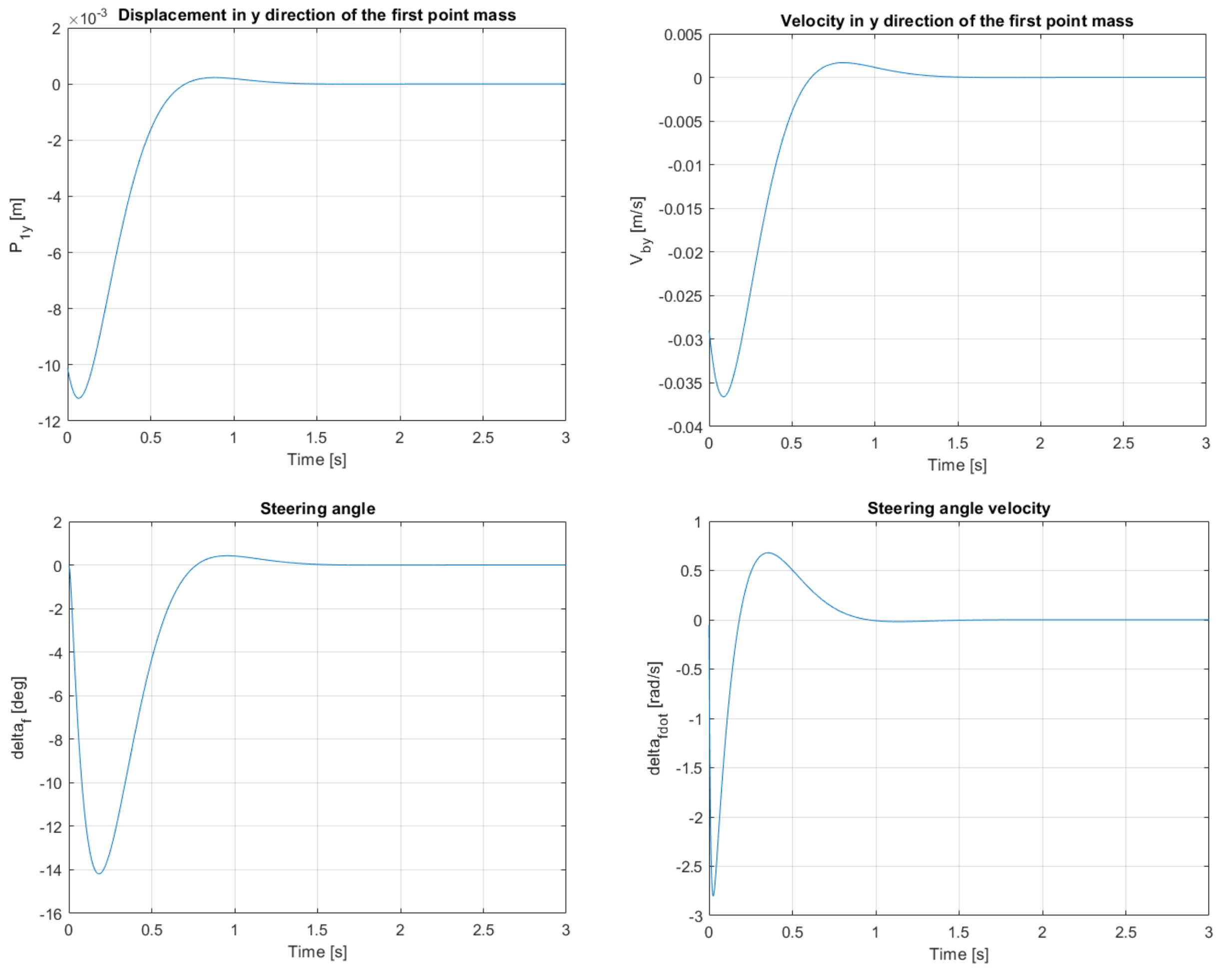


Figure 3.26: Results of the simulation with the roll gyroscopic effects generated at $V_{ox} = 2\ [\frac{Km}{h}]$

In figure 3.28 it is possible to find the results for when the velocity V_{ox} increases up to $4\ [\frac{km}{h}]$, this change in the velocity brings to a better stabilization of the vehicle, in fact even if the velocity of the first point mass starts at $V_{by} = -0.029\ [\frac{m}{s}]$, which is the same as the other found with the other velocities, the peak value for the steering angle state is found at $\delta_f = -7.46°$ with a difference of about 3° with respect to the one without gyroscopic effects. For what concerns the position of the first point mass its peak value is found at $P_{1\_y} = -0.0108\ [m]$.

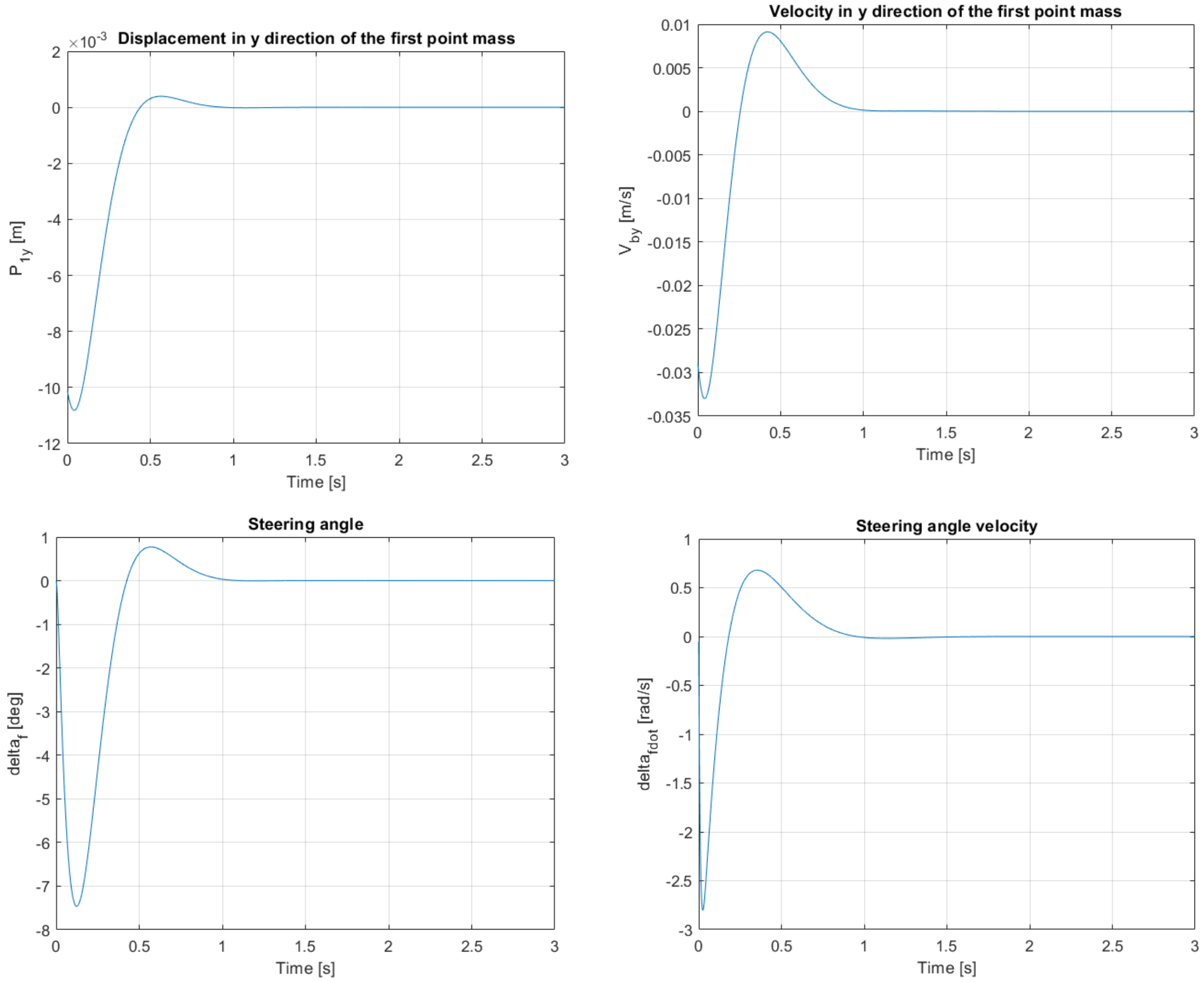


Figure 3.27: Results of the simulation with the roll gyroscopic effects generated at $V_{ox} = 4\ [\frac{Km}{h}]$

3.7 Steering velocity

As explained in the previous section the movement of the handlebars is another cause of gyroscopic effect, since the moment generated is in the same direction as the gyroscopic moment generated by the yaw motion it is possible to include its effect in equation (109), which now becomes

$$\phi' = \phi_{b\_act} - \phi_{b0} + \Delta\phi = \phi_{b_{act}} - \phi_{b0} + \frac{(I_{wf}\omega_f + I_{wf}\omega_f)\dot{\psi}\cos(\phi_{b_{act}})}{\sqrt{(mg)^2 + (mR_c\dot{\psi}^2)^2}} + \frac{I_{wf}\omega_f\dot{\delta}_f\cos(\vartheta_{cf})}{\sqrt{(mg)^2 + (mR_c\dot{\psi}^2)^2}} \quad (134)$$

If the vehicle is traveling in a straight path the influence of the yaw motion is not included in the computation and it is possible to see the effect of the steering velocity $\dot{\delta}_f$ on the vehicle. It is supposed that the motorcycle is travelling straight when a steering velocity $\dot{\delta}_f = 0.5\ [\frac{rad}{s}]$ is applied at a velocity $V_{ox} = 1\ [\frac{Km}{h}]$, in figure 36 the results of the simulation are shown, taking in consideration that the $\Delta\phi$ generated by the steering motion is quite small, given the fact that the velocities considered are small. The results show a behavior even in this case similar to the one obtained without considering the gyroscopic effects, the peak values for the curve $\delta_f = -14.74°$, that means that just a small increase in the steering angle is needed to stabilize the motorcycle in the same time.

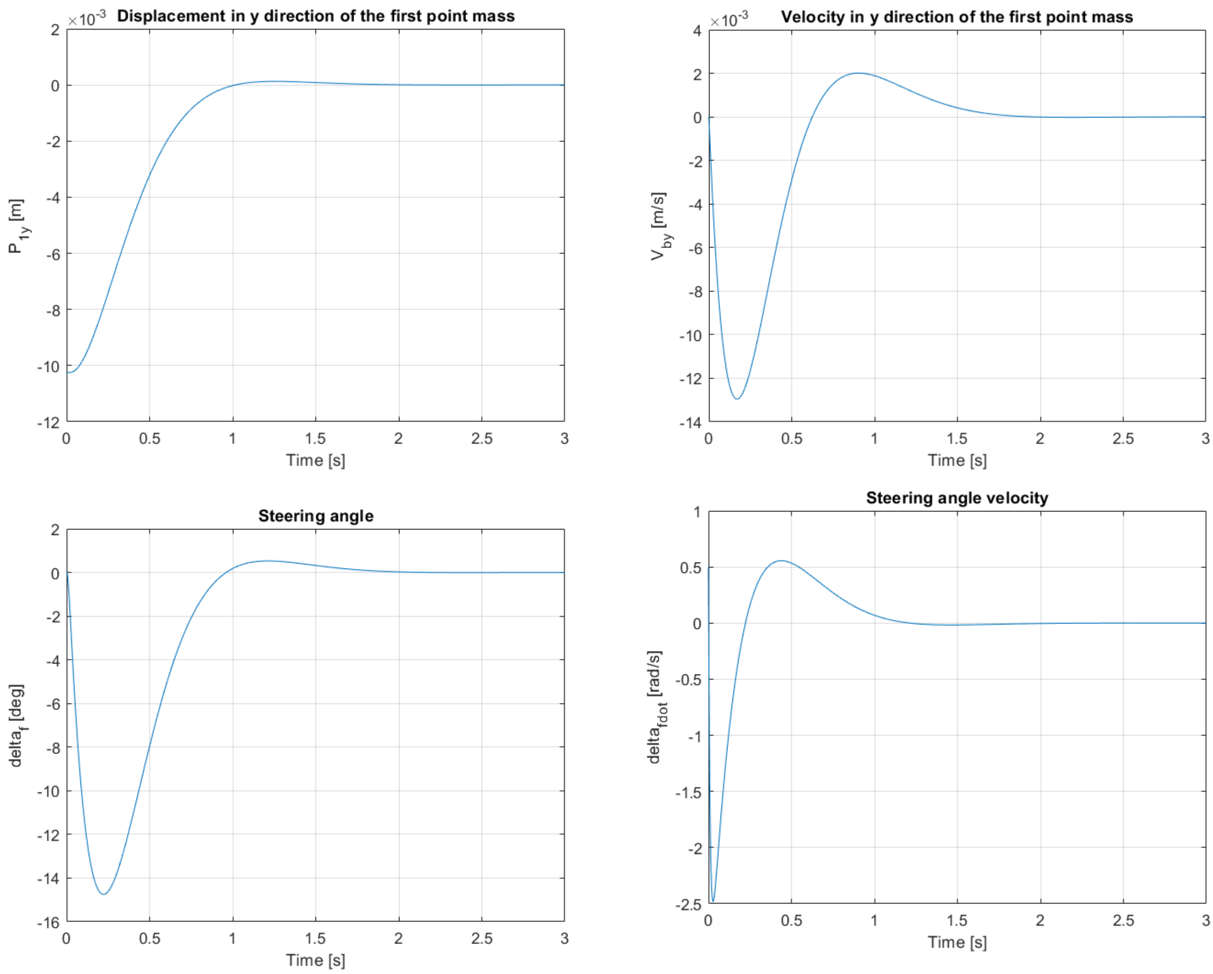


Figure 3.28: Results of the simulation with the steering gyroscopic effects generated at $V_{ox} = 1\ [\frac{Km}{h}]$

4 Conclusions and future works

The study of the stability of a motorcycle stability at standstill and low velocities was studied in this project, the results obtained show that it is possible to control the stability of motorcycles with an action taken on the handlebars of the steering head causing a restoring roll moment able to set the motorcycle in an upright position. The particular fact is that it was possible to obtain these results by modifying mostly the mechanical trail and setting it to a negative value, this type of action is possible only at low velocities since the mechanical trail is a parameter that influences the dynamic behavior of a motorcycle at all speed ranges, in fact its value is designed in order to have the best performances depending on the target of the motorcycle and usually it is set to be a positive value. This means that it is necessary to design a tool that is capable of modifying the inclination of the front fork of a motorcycle without changing the inclination of the steering axis, acting only at low velocity in order to have this type of configuration of motorcycle for low velocities, while returning to the designed inclination of the front fork at medium to high speed ranges in order to have a classical type of behavior for a motorcycle. This operation can be possible using a trail length changing mechanism, the trail length is changed by pivoting the front fork back and forth about the upper triple clamp with the steering axle fixed on the vehicle frame.

The influence of gyroscopic effects has been also studied, with the result of slightly influencing the results firstly obtained without the consideration of them.

Future studies may concern the implementation of the trail length mechanism in the performance of the vehicle in order to find out if the result of changing the trail from a negative to a positive value, reducing the amount of steering control as the speed is increased, can achieve a smooth transition to normal control characteristics of a motorcycle.

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Figure 3.27: Results of the simulation with the roll gyroscopic effects generated at $V_{ox} = 4\ [\frac{Km}{h}]$

Figure 3.28: Results of the simulation with the steering gyroscopic effects generated at $V_{ox} = 1\ [\frac{Km}{h}]$

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