MIRIAM WIECZOREK LOCIKS DE ARAUJO

OPTIMIZATION OF THE BRAZILIAN ENERGY SUPPLY STRUCTURE USING THE MODERN PORTFOLIO THEORY

Thesis presented to the Politecnico di Torino as a prerequisite to receive the degree of Master of Science in Engineering and Management.

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Dep. of Management and Production Engineering

Advisors: Anna Filomena Carbone
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Abstract

The Modern Portfolio Theory (MPT) was first developed to solve the problem of investment allocation in the financial sector. In short, it says that diversifying the types of investment decreases the overall risk of the portfolio, advising investors to do not put “all eggs in the same basket”.

In this thesis, the idea is to apply the same concept in the energy sector, treating each source of energy as sort of investment. The hypothesis is that one could create an optimization model to guide the decision of how much energy should be produced by each source in order to decrease the expected cost of production and associated risks.

In this sense, the purpose of the present thesis is to explore the composition of energy portfolios in Brazil, which tells about how much electricity is produced (or can be produced) by each source in the country. In order to do this, the evolution of the Brazilian energy supply structure and its projections for 2027, 2030 and 2050 are analyzed in terms of diversity and compared with efficient portfolios under the perspective of the MPT.

To achieve this goal, an optimization model is proposed, and its results (efficient portfolios) are used to evaluate projections for Brazilian energy supply structure in terms of expected cost of producing energy and risk (variance of cost). The optimization model was programed using MATLAB (R2018b) following MPT, using a computer with microprocessor AMD E1-1500 APU with Radeon™ HD, 1.48GHz and 4 GB RAM. The analysis of evolution of diversity in the Brazilian energy supply structure and its projections is made by calculating the entropy indices (Shannon-Weiner Index, Shannon’s Equitability and Herfindahl-Hirschman Index) for each portfolio throughout the years. The evolution of these indices indicates how is the development of the diversity of the Brazilian energy matrix in terms of diversity of energy source.

Keywords: Modern Portfolio Theory (MPT), entropy, energy supply structure, Brazil.

Resumo

A Teoria Moderna do Portfólio (MPT) foi desenvolvida pela primeira vez para resolver o problema de alocação de investimentos no setor financeiro. Em resumo, a teoria diz que a diversificação diminui o risco total da carteira, aconselhando os investidores a não colocar “todos os ovos na mesma cesta”.

Neste Trabalho de Conclusão de Curso, a ideia é aplicar o mesmo conceito no setor de energia, tratando cada fonte de energia como um investimento. A hipótese é que se pode criar um modelo de otimização para orientar a decisão de quanta energia deve ser produzida por cada fonte energética, a fim de diminuir o custo esperado de produção e os riscos associados.

Nesse sentido, o objetivo do presente trabalho é explorar a composição de portfólios de energia no Brasil, isto é: como a eletricidade está distribuída entre fontes no país, tanto em termos de geração em si como em termos de capacidade instalada. Para isso, a evolução histórica da estrutura de suprimento de energia brasileira e suas projeções para 2027, 2030 e 2050 são analisados em termos de diversidade com portfólios eficientes na perspectiva do MPT.

Para atingir esse objetivo, é proposto um modelo de otimização e seus resultados (portfólios eficientes) são utilizados para avaliar as projeções da matriz energética brasileira em termos de custo esperado da produção de energia e risco associado (variação de custo). O modelo de otimização foi programado com MATLAB (R2018b) seguindo MPT, usufruindo de um computador com microprocessador AMD E1-1500 APU com Radeon™ HD, 1.48GHz e 4 GB RAM.

A análise da diversidade da matriz e suas projeções é feita calculando índices de entropia (*Shannon-Weiner Index, Shannon’s Equitability* e *Herfindahl-Hirschman Index*) de cada portfólio. A evolução desses índices indica como está o desenvolvimento da diversidade na matriz energética brasileira em termos de diversidade de fonte energética.

Palavras-chave: Teoria Moderna de Portfólio (MPT), entropia, matriz energética, Brasil.

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Summary

1. Introduction

Under the threat of global climate change international arrangements such as the Paris Climate Agreement have taken place highlighting the energy planning relevance. In a broader context, stakeholders of different levels have been triggered to review the world's energy system. On one hand, there are the energy producers, who have to choose the amount of investment allocated to different projects [1]. On the other hand, there are the governments which can expedite the marketplace response to the threats of climate destabilization through regulation, limitation over greenhouse gas emissions, subsidies for alternative energy sources or related research and development activities. These actors are planning and heavily investing under excruciating uncertainty which makes essential the development of effective frameworks and tools to analyze the underlying risks, namely: financial, social, technological and environmental risks besides the uncertainty associated to production costs and security of supply.

The aim of the present thesis is to explore the composition of energy portfolios, specifically Brazilian electricity generation portfolios and installed capacity through an optimization model. It was used both the well established MPT and newly proposed entropy indexes. These tools are applied to analyze the historical and projected Brazilian energy supply structure in terms of cost, risk and diversity. The contribution of this work is to better understanding about the evolution of the Brazilian supply structure over the past decades and its possibilities for the future.

[1] This group includes some of the most important oil and gas producers which are surprisingly heavily investing in Renewable Energy Sources (RES). Royal Dutch Shell, Equinor and Total, are the companies which most distinguishes themselves by investing billions of dollars per year in RES in the last decade. Equinor have even changed its name as a commitment towards a future not based on oil exploitation: before it was called Statoil.

2. State of the Art

2.1. Modern Portfolio Theory (MPT)

The MPT developed by Markowitz (1952), is a well know methodology which aids the selection and prioritization of investments focusing on the balance between overall risk and return.

Markowitz (1952) recommends the use of the mean-variance rule both as a hypothesis to explain well-established investment behavior as a maxim to guide one's choice of portfolio. The mean-variance rule suggests that instead of analyzing only the expected return and ending up allocating all budget in a single investment which has provided high historical returns, it would be better to make balance between overall portfolio's return and its associated risk. Thus, the model's objective can be set in two different ways:

1. To minimize the overall risk given a minimum acceptable level of return or
2. To maximize expected return given a maximum acceptable level of risk.

In this work, only the first approach will be shown, since the second can be easily derived from the first one. The model (M1) related to the first approach is characterized be a quadratic function and is presented below.

Objective function:

$$\min_{x} V_P = \sum_{i \in J} \sum_{j \in J} x_i . x_j . \sigma_{ij} \qquad \text{Function 1}$$

Subject to:

$$\sum_{j \in I} x_j . \mu_j \geq R_{min} \qquad \text{Constraint 1}$$

$$\sum_{j \in I} x_j = 1 \qquad \text{Constraint 2}$$

$$0 \leq x_j \leq u_j \qquad \forall\, j \in I \qquad \text{Constraint 3}$$

Where:

μ_j is the expected return of asset j

x_j is the fraction of the budget allocated to asset j;

σ_{ij} is the covariance between returns of assets i and j;

I is the set of possible investments;

R_{min} is the minimum acceptable return;

u_j is the maximum acceptable investment on asset j.

Constraint 1 guarantees that the expected return is greater than or equal to a defined minimum limit (R_{min}); Constraint 2 defines that the sum of investments' fractions is equal to 1 – that is, it guarantees that all available capital will be invested; while Constraint 3 prevents short-selling of investments and limits each one of them to a certain ceiling (u_j).

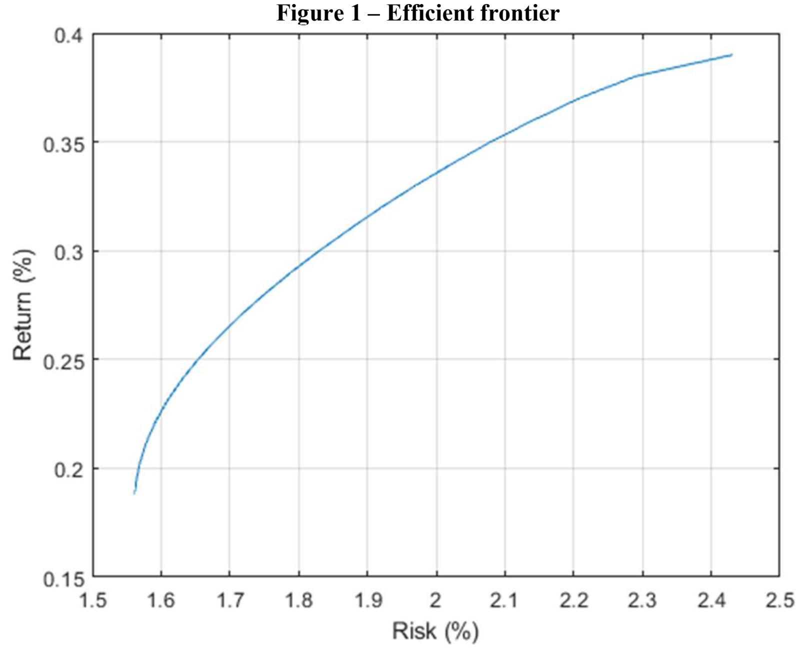
For the sake of completeness, follows an exercise adapted from the one proposed by Fávero & Belfiore (2013), as an example of the model's application on finance. This example is helpful to present relevant elements of the MPT. In this exercise, ten different stocks were analyzed; their name and respective Bovespa's code follows on Table 1. Their expected return and covariances were estimated based on time series concerning their daily return from 01/14/2009 to 01/13/2010. These estimates can be found on Table A1 and Table A2 in the Annex. Investments were limited to a maximum of $u_j = 30\%$ and the expected daily return was limited to be not smaller than $R_{min} = 0.25\%$. The resulting allocation of investment can also be found on Table 1. The standard deviation of daily returns achieved was 1.660%.

Table 1 – Stocks' names, Bovespa's codes and investment allocation.

Stock	Code	Allocation (x_j)
Banco Brasil ON	BBAS3	0.2295
Bradesco PN	BBDC4	0.0163
Eletrobrás PNB	ELET6	0.2339
Gerdau PN	GGBR4	0.0033
Itausa PN	ITSA4	0.0042
Petrobras PN	PETR4	0.1206
Sid Nacional ON	CSNA3	0.0049
Telemar PN	TNLP4	0.2194
Usiminas PNA	USIM5	0.1576
Vale PNA	VALE5	0.0102

The efficient frontier can be seen in Figure 1. The efficient frontier is defined as the set of portfolios that present the best possible return for each level of risk. In order to draw such graph, one could vary the level of expected return on the model and save the value of each correspondent risk. The minimum risk portfolio is characterized by a risk of 1.560% and return of 0.1878% (it can be found by the described model without the constraint on expected return).

Figure 1 – Efficient frontier



(Source: author's own calculation)

It is important to highlight the fact that this model already implies diversification in most of the cases. For example, one could consider a portfolio where only two alternatives of assets are available. Then the portfolio's variance would be given by Equation (1).

$$V_p = x_1^2.\sigma_1^2 + x_2^2.\sigma_2^2 + 2.x_1.x_2.\sigma_1.\sigma_2.\rho_{12} \tag{1}$$

Where:

V_p is the portfolio variance;

x_i is the weight associated to investment i, with $i = 1$ or 2;

σ_i is the standard deviation associated to investment i, with $i = 1$ or 2;

ρ_{12} is the correlation coefficient between assets 1 and 2.

The correlation coefficient is calculated by Equation 2.

$$\rho_{1,2} = \frac{cov(\mu_1 * \mu_2)}{\sigma_1 * \sigma_2} \tag{2}$$

Where:

ρ_{12} is the correlation coefficient between assets 1 and 2;

μ_i is the return associated to investment i, with $i = 1$ or 2;

$cov(\mu_1 * \mu_2)$ is the covariance between μ_1 and μ_2;

σ_i is the standard deviation associated to investment i, with $i = 1$ or 2.

.

By analyzing Equation 1, one can realize that the variance of a portfolio with two different assets is not higher than the one of a portfolio consisting of only one investment, since in the first case $\rho_{12} < 1$, while in the second $\rho_{12} = 1$, by definition (covariance between two identical series is equal to their variance, which is exactly the denominator value, in this case). Thus, in general, the greater the number of investments, the lower the variance of the portfolio's return.

Moreover, Markowitz (1952) considers that, since the model takes into account the covariance between assets, the MPT implies the “right kind” of diversification: not only the number of assets suggested is larger, but the assets to be chosen has provided significantly disconnected returns overtime. In this work (MARKOWITZ, 1952) he says that:

> […] in trying to make variance small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariances among themselves. We should diversify across industries because firms in different industries, especially industries with different economic characteristics, have lower covariances than firms within an industry.

Though generally MTP suggests a fairly diversified portfolio, one can conceive some exceptions. For example, a situation in which one asset has significant larger returns and lower variance in comparison with others. In this case, MTP would recommend that all budget should be allocated in this distinctive asset, which consists in an extreme case of undiversified portfolio.

Besides, the choice of variance as a risk measure can be considered questionable by some: it does not meet most of the requirements of coherent risk measures[2] and it is symmetric, penalizing equally returns above and below the expected value, although the first one is desirable while the second is not.

Another possible problem is that the historical variances, covariances and returns might not well represent their future value. Markowitz (1952) considers that the estimates based on historical data should be adjusted according to opinion of specialists, which is a rather subjective mean, considering that covariance is not easily (numerically) forecasted by experience.

[2] It does not meet with the requirements of monotonicity, translation invariance, positive homogeneity and subadditivity. These are not going to be explored here since it falls far beyond the scope of this work. More information about variance, its drawbacks and other risk measures can be easily found in the literature.

2.2. Application of MPT in Energy Planning

The MPT was primary developed for the financial sector, nevertheless it is possible to amplify the domain of its application by adapting the methodology to cope with different sectors' specificities.

As a matter of fact, the MPT has already been applied specifically to the energy planning problem, being widely studied besides being strongly criticized as it can be seen in the exhaustive review of the state of the art developed by deLlano-Paz *et al.* (2017).

The application of MPT in energy planning presumes that one could estimate the costs and risks associated to each technology (e.g. solar, wind, hydro) and use this information as an input of an optimization model to find the best energy matrix in terms of capacity and electricity generation by source and by territory. In this specific case, the optimization model is the one proposed in MPT, with some required adaptations.

In this section, the most significant adaptations required to apply MPT in energy planning are explained: mainly, how the estimates of risk and cost are calculated. Additionally, an example of a complete model is presented by the end of this section, model which will serve as a reference in the methodology of this thesis.

2.2.1. Portfolio Cost

To apply the MPT in the Energy Planning problem, the concept of expected return is replaced by a measure of expected cost of producing energy. The expected cost of the portfolio is given by the sum of the expected costs of each type of technology contained in the portfolio (see Equation 3).

$$E[C_p] = \sum_{t \in T} E[C_t] = E[C_{solar}] + E[C_{hydro}] + \cdots + E[C_{gas}] \tag{3}$$

Where:

$E[\cdot]$ refers to the calculation of the expected value of a variable;

C_p is the cost of the portfolio;

C_t is the cost embedded in the technology t;

T is the set of relevant technologies being considered.

The types of technology usually considered are nuclear, coal, natural gas, oil, wind, hydro, biomass and solar. It is possible to create subdivisions in these groups, differentiating small power plants from large ones, old from new or onshore from offshore. These subdivisions are justified by the fact that new/large plants are more efficient than old/small plants, for example. Besides, the type of cost which should be considered [3] and the risks associated to each subdivision might be different between subdivisions.

The expected cost of a technology is given by the sum of the expected cost of specific aspects for the given technology (see Equation 4). That is, the total cost associated to a source is assumed to be composed by supposedly known minor costs. The expected value of these minor costs can be estimated by analyzing related data-series, through simulation or estimated by experts' experience.

$$E[C_t] = \sum_{c \in C} E[C_{t,c}] = E[C_{t,Fuel}] + E[C_{t,O\&M}] + \cdots + E[C_{t,CO_2}] \qquad (\forall\, t \in T) \tag{4}$$

[3] For example, sunk costs should not be taken into account when evaluating old plants since they are already sunk. However, when evaluating new plants, sunk costs should indeed be considered, since investors must verify if the investment can be recovered with a reasonable margin.

Where:

$E[C_t]$ is the expected cost of technology t;

$C_{t,c}$ is the cost of the specific aspect c related to the technology t;

C is the set of relevant types of cost being considered (fuel, O&M, CO_2 or other costs).

The types of minor costs being considered vary from author to author. deLlano-Paz *et al.* (2014), for example, considers costs related to both production costs and externality costs. Where the former includes Fuel Price, Operation and Maintenance (O&M), Investment and Complementary costs; the latter includes indirect costs incurred by the society of environment (CO_2 emission, radioactivity, land use and others).

2.2.2. Portfolio Risk

Originally, the portfolio risk is assessed through the weighted sum of variance and covariance values of assets' returns, where the weights are the fractions of capital allocation into each asset. Conversely, when dealing with energy planning, the portfolio risk is given as the weighted sum of variances and covariance values of technologies' cost, where the weights represent the fraction of electricity generation (or capacity) of each technology in the portfolio (see Equation 5).

That is, instead of analyzing the covariance values of assets returns one should analyze the covariance of assets cost. Besides, the concept of capital allocation is replaced by a sort of production allocation (or capacity allocation): the fraction of each technology (x_i) could be measured either in terms of capacity or in terms of electricity generated.

$$\sigma_p^2 = \sum_{i \in T} \sum_{j \in T} x_i . x_j . \sigma_{ij} \qquad (5)$$

Where:

σ_p^2 is the portfolio risk;

x_i is the fraction of the technology i in relation to the overall portfolio;

σ_{ij} is the covariance between costs of technology i and j.

The variances and covariance values for each technology (σ_t^2 and σ_{ij} respectively) are given by the simple sum of the variances and covariance values of specific aspects, as follows in Equation 6 and 7 respectively. A sound explanation of these formulas can be found in the study developed by Costa *et al.* (2017).

$$\sigma_t^2 = \sum_{c \in C} \sigma_{t,c}^2 \qquad (\forall\ t \in T) \qquad (6)$$

$$\sigma_{ij} = \sum_{c \in C} \sigma_{ij}^c \qquad (\forall\ i, j \in T) \qquad (7)$$

Where:

$\sigma_{t,c}^2$ is the variance of costs of type c embedded in technology t and

σ_{ij}^c is the covariance of cost of type c between technologies i and j.

2.2.3. Example of MPT's Application

By aggregating all these adaptations, one way of writing the MPT to solve an Energy Planning problem is the given optimization model (M2):

Objective function:

$$\min_{x} \sigma_p^2 = \sum_{i\in\mathcal{T}} \sum_{j\in\mathcal{T}} x_i . x_j . \sigma_{ij} \;=\; \sum_{i\in\mathcal{T}} \sum_{j\in\mathcal{T}} x_i . x_j . \left(\sum_{c\in\mathcal{C}} \sigma_{ij}^{c}\right) \qquad \text{Function 2}$$

Subject to:

$$E[C_p] = \sum_{t\in\mathcal{T}} x_t . E[C_t] = \sum_{t\in\mathcal{T}} x_t . \left(\sum_{c\in\mathcal{C}} E[C_{t,c}]\right) = C_{target} \qquad \text{Constraint 4}$$

$$\sum_{t\in\mathcal{T}} \quad x_t = 1 \qquad \text{Constraint 5}$$

$$x_t \geq 0 \qquad (\forall\, t \in T) \qquad \text{Constraint 6}$$

Where:

σ_p^2 is the portfolio variance;

x_t is the fraction of the energy produced by technology t;[4]

σ_{ij} is the covariance between costs of technologies i and j;

$\mathcal{T}$ is the set of possible energy sources;

$\mathcal{C}$ is the set of types of costs;

σ_{ij}^{c} is the covariance between specific costs of technologies i and j;

$E[C_p]$ is the expected portfolio cost;

$E[C_t]$ is the expected cost of technology t;

$E[C_{t,c}]$ is the expected value of a specific aspect c in technology t;

C_{target} is a defined target for expected cost.

Constraint 4 guarantees that the portfolio expected cost is equal to a defined target (C_{target}); Constraint 5 defines that the sum of energy production fractions is equal to 1 while Constraint 6 prevents a sort of short-selling of energy production.

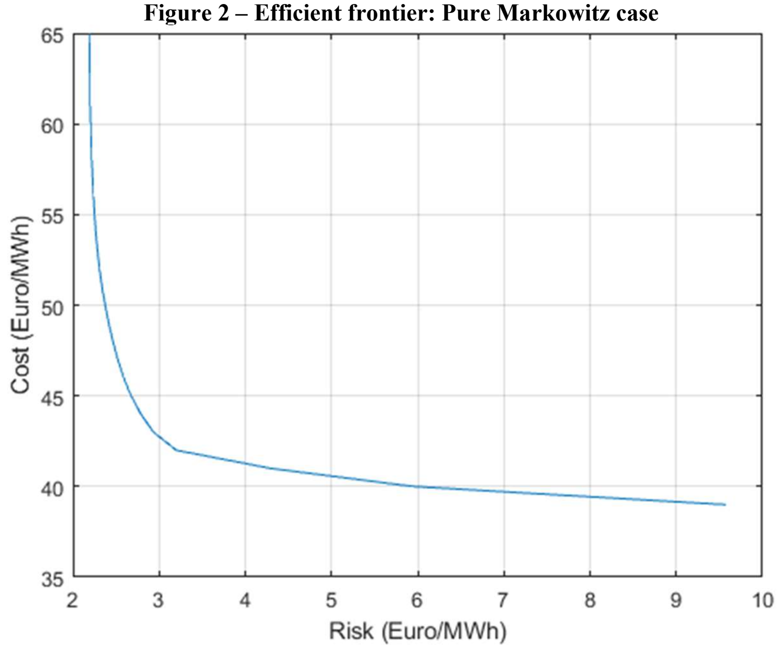
[4] x_t, x_i and x_j mean the same thing.

This model (M2) was used by deLlano-Paz *et al.* (2014). The "Pure Markowitz" scenario is replicated in this thesis, using the same model and data of the mentioned authors, in order to check the formulation, the meaning of each variable and the procedure of data collection.

In Annex (see from Table A3 to Table A6) it can be found a table presenting the set of possible energy sources, the set of type of costs and the related values of specific cost per type of technology. The annex also contains the cost correlation matrixes and cost standard deviation matrix.

The following graph (Figure 2) presents the efficient frontier for the concerning model (M2) and data (Table A3 to Table A6). One can notice that, when dealing with Energy Planning, the efficient frontier is given by the lower part of the attainable set (instead of the upper part, as usually occurs). This happens because the model deals with cost instead of returns.

Figure 2 – Efficient frontier: Pure Markowitz case



(Source: Author's own calculations)

2.3. MPT’s Advantages and Drawbacks

According to deLlano-Paz *et al.* (2017), MPT stands out in comparison to other methods due to both its easiness to be put on practice and the fact that it deals explicitly with the trade-off risk versus return. The study also concludes that some relevant critiques rely on limitations derived from the different nature of the assets being analyzed (financial versus real assets), though these can be surpassed with simulation techniques, demand-side models and inclusion of different parameters in the optimization model.

However, one of the main problems in the application of MPT relies on its tricky dependence on probabilistic estimates. Stirling (1994) for example, highlights that the probabilistic estimates embedded in this methodology tends to be opaque to critical examination, vulnerable to error, to unconscious bias or even manipulation.

2.4. Diversity Indexes (DI)

A diversity index is a mathematical measure of elements diversity in a given system. Such definition is broad since diversity is a property if any system whose elements may be apportioned into categories (STIRLING, 2009).As such, the application of these indexes is well known in fields like taxonomy, paleontology, archeology and conservation biology. In the latter field, for example, these indexes asses the species diversity in a given community.

In this work, the attention will be given to Shannon-Wiener Index, Shannon's Equitability and Herfindahl-Hirschman Index (HHI). More information about diversity indexes can be found in the work of Chuang & Ma (2013) or Stirling (2009).

The Shannon-Wiener Index accounts for both abundance and evenness of the element present in the concerning system. The *abundance*, also said as *variety*, is the number of different categories into which a system may be apportioned; the *evenness*, also said as *balance*, tells about the distribution of elements among categories. All else being equal, the grater the system's variety or balance, the greater the system's diversity.

The Shannon-Wiener Index (H) is calculated as follows in (8):

$$H = -\sum_{i \in I} p_i . \ln p_i \qquad (8)$$

Where p_i is the proportion of the category i in the system (*"how many elements in relation to the total elements are allocated in this category?"*) and I is the set of all categories available in the system.

A deriving index, Shannon's Equitability (E_H), express the evenness of a system. It can be calculated by dividing the current value of H by the maximum value that H would assume if the system where completely balanced (H_{max}). Thus, considering N as the total number of categories within the system, the index would be (see Equation 9):

$$E_H = \frac{H}{H_{max}} = \frac{\sum_{i \in I} p_i . \ln p_i}{\ln N} \qquad (0 \leq E_H \leq 1) \qquad (9)$$

Just like the Shannon-Wiener Index, the HHI measures abundance of categories and evenness in the distribution of elements among categories. However, the HHI decreases as diversity goes up – which can be counter intuitive. In order to avoid confusion, often transformations of HHI that increase with increasing diversity have been used instead. For example, the inverse Simpson index (1/λ) and the Gini–Simpson index (1 – λ). In order to compare HHI with the Shannon-Weiner Index, one can also use the inverse of H, maintaining the usual behavior of HHI.

Equation 10 shows how HHI (λ) is calculated. It can be interpreted that the index equals the probability that two elements randomly taken from the system of interest represent the same category.

$$\lambda = \sum_{i \in I} p_i^2 \qquad (0 \leq \lambda \leq 10{,}000) \tag{10}$$

In this thesis, one example from Beals, Gross & Harrell (2000) is adapted to a more generic case in order to illustrate the effects in the Shannon-Weiner Index and Shannon's Equitability when variety or balance changes in a given system.

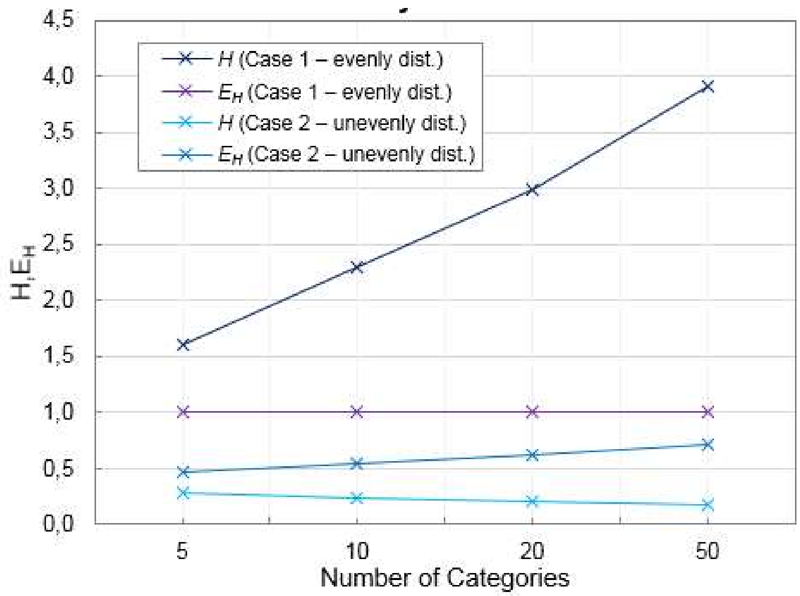
Considering four hypothetical systems with 100 elements. The systems are composed of 5, 10, 20 and 50 categories, respectively. For each system H and E_H have been calculated for two cases: one in which elements are distributed evenly among the different categories (maximum balance), and another in which one category has 90% of the elements, and the remaining elements are distributed evenly. Table 2 explains the systems' composition in each case, while results (values for H and E_H) are shown in Figure 3.

Table 2 – Hypothetical systems and its elements' distribution for cases 1 and 2

		Case 1	Case 2	
Total elements	**Nº of categories**	**Nº of elements in each categories**	**Nº of elements in the large category**	**Nº of elements in other categories**
100	5	20	90	2.5
//	10	10	90	~1.11
//	20	5	90	~0.52
//	50	2	90	~0.20

(Source: Author's own development)

Figure 3 – Diversity Indexes for hypothetical systems in cases 1 and 2



(Sources: Author's own development)

For the case in which elements are equally distributed among categories: E_H is constant and equal to one, as expected; H increases dramatically solely because of the increase in variety. Here one can notice that E_H is insensible variety, since it is not affected by changes in the number of categories in a system.

For the case in which one category makes up 90% of the system: clearly, for all systems E_H is lower if compared to the first case because of the decrease in balance. The index becomes even smaller as the number of existing categories goes up, as only the remaining 10% of the elements are distributed among the remaining categories – the fraction of elements allocated to the other categories becomes smaller and the overall inequality increases. H is also lower if compared to the first case. It does increase with higher numbers of existing categories, reflecting the growth of variety; however, it does it with a significant lower rate than in the first case, compensating the reduction on balance.

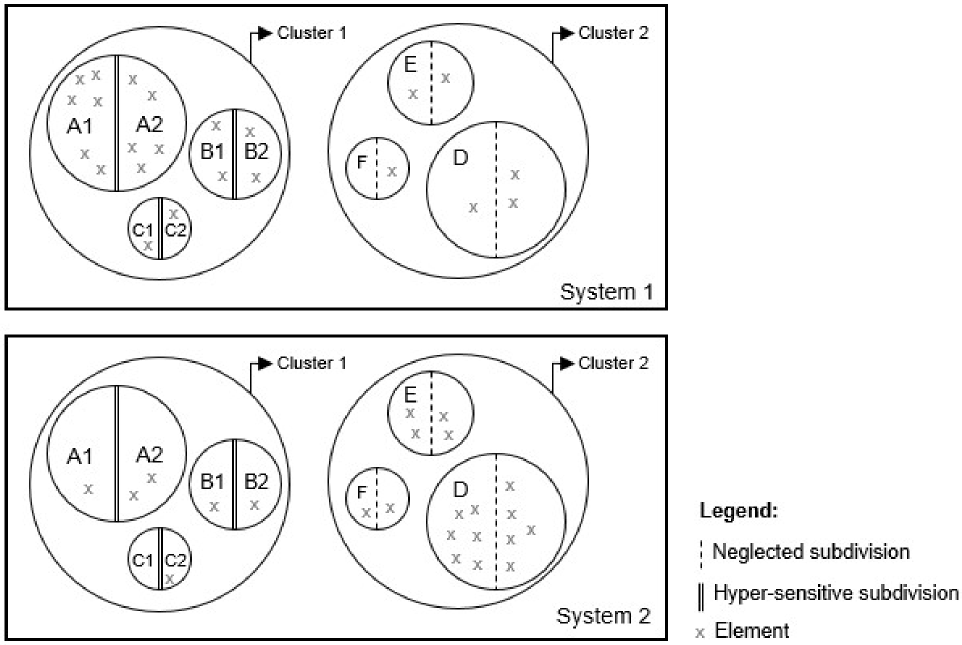
2.5. Disparity

Until now only two aspects of diversity have been discussed: variety and balance. However, according to Stirling (2009), there is a third aspect, which is the most important, yet most frequently neglected: *disparity* between categories. Disparity refers to the manner and degree in which categories may be distinguished. That is, it is the bases for establish the criteria to aggregate a set of elements into a category.

The Shannon-Wiener Index does not explicitly consider the disparity within a system. However, the first step before calculating the index is to define the existing categories: one must cluster the elements which are similar in given aspects.

This partition, though, can be quite subjective: one category can be partitioned in several subcategories increasing the diversity index without having any justifiable change in the system. In order to illustrate how this subjectivity could be problematic Figure 4 was elaborated. It illustrates a hypothetical example where the level of precision and sensitivity to differences are high when partitioning specific cluster leading to its subdivision into new categories, while comparable differences within another cluster are neglected remaining as a single category. Such a biased analysis would probably conclude that a system with more elements in the former cluster (more subdivided) is more diversified than a system with more elements in the latter cluster (less subdivided).

Figure 4 – Systems with comparable diversity and biased partitioning choice



(Source: author's own development)

In this example, although both systems are comparable in terms of diversity, the biased method of partitioning the elements into categories would suggest that System 1 has a greater diversity in comparison with System 2.

The disparity can be assessed with less sensitivity to arbitrary conventions by using covariances, when suited data are available. An alternative approach is based on a scalar distance measure between categories and other measures of diversity which explicitly accounts for disparity can be found in the work developed by Stirling (2009).

2.6. Diversity Indexes and Energy Planning

Stirling (2009) recognizes that energy planning depends on a set of unknowable information and proposes diversification as a robust response to these forms of uncertainty, ambiguity and ignorance. Such diversification has been quantified by measures of entropy. Doherty (2005), for example, uses Shannon-Wiener index and concludes that it enriches the robustness of the model by decreasing the dependence over the probability estimates. Some authors also propose the Shannon-Wiener index to evaluate the energy security[5].

Shannon-Wiener index is not the only one being used: HHI and other dozens of alternatives are studied. In fact, Chuang & Ma (2013), for example, review the concept of diversification and propose different indexes to give more weight to certain aspects such as the dependence on external energy supply (non-indigenous).

In Energy Planning, one deals with the energetic supply system, where the categories are the different type of sources (e.g. type of technology, origin) and the elements are the unit of energy produced (e.g. MWh). Therefore, the proportion p_i found in Shannon-Wiener Index formula (Equation 8) refers to the proportion of energy produced by the type of source i in the energy system (*"how much energy in relation to the total produced comes from this source?"*) and I refers to the set of all types of source available in the system.

Usually the application of the diversity index is to assess past energy portfolios or compare alternative future ones. Chuang & Ma (2013), for example, analyze the evolution of the Taiwanese energy supply structure, measuring its diversity through different indexes over time – including three new indexes created by themselves.

The evaluations with Shannon-Wiener Index and HHI developed by Chuang & Ma (2013) were replicated in this work, in order to create a better understanding with respect to these Indexes and their application in Energy Planning. The data used and calculation can be found in

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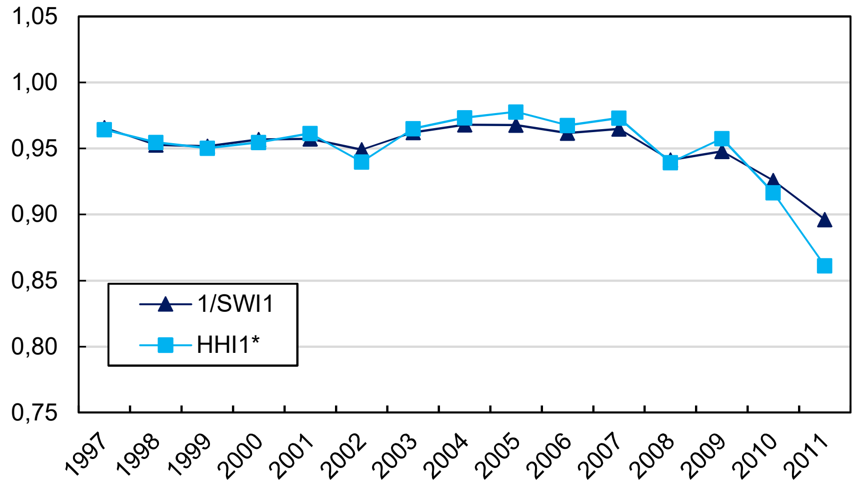
[5] Energy security: an uninterruptible supply of energy, in terms of quantities required to meet demand at affordable prices. Europe's Vulnerability to Energy Crises, World Energy Council 2008

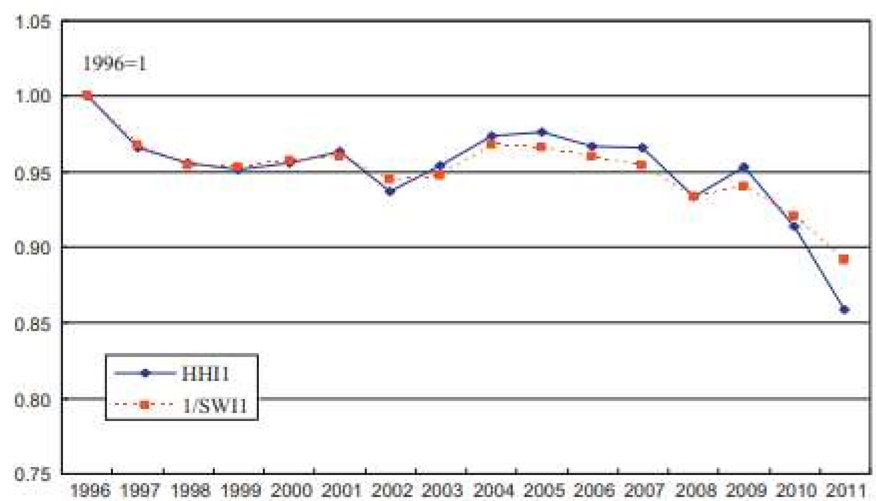
Table A7 in Annex 1. The graphical representation of the result found by the present author are shown in Figure 5 (compare with the one found by Chuang & Ma, in Figure 6).

Figure 5 – Exercise: HHI and Shannon-Weiner Index for the Taiwanese energy supply structure



(Source: Author's own calculation)

Figure 6 – Comparison of HHI and Shannon-Weiner Index for the Taiwanese energy supply structure

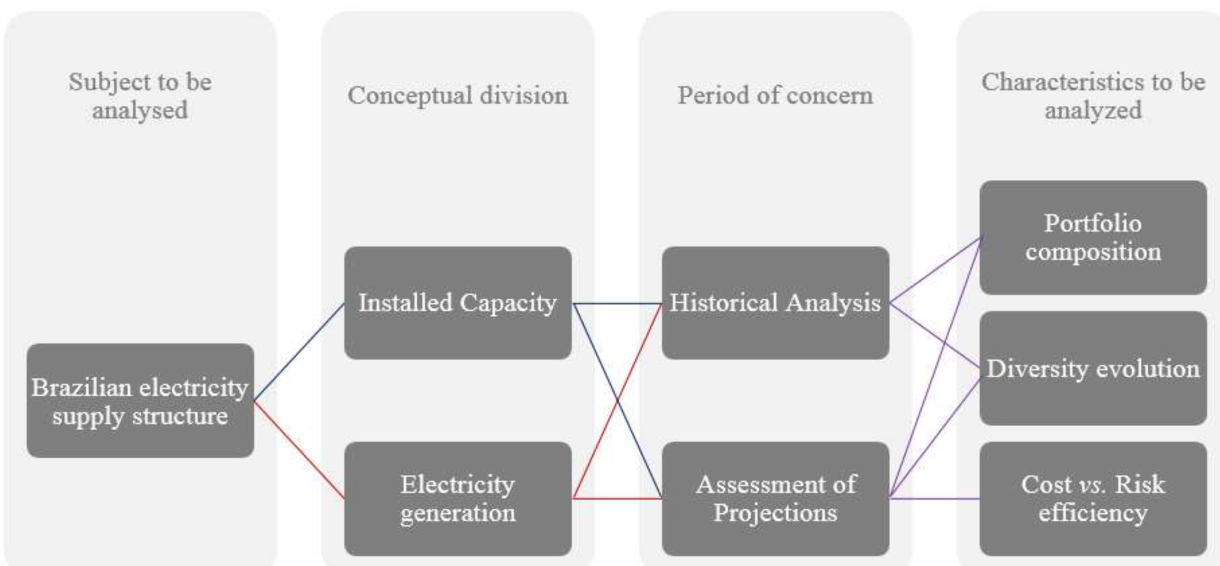


(CHUANG & MA, 2013)

3. Methodology

In order to provide an adequate diagnosis of the Brazilian energy supply structure, the methodology starts with its historical analysis in terms of technologic composition and diversity, offering a review on the system's evolution until recent years. Then, an evaluation of the projections for the matrix in terms of composition and diversity is performed, elucidating how the system might change from now on. Projections of electricity generation and capacity were also evaluated in terms of cost and risk, allowing the evaluation of the system's efficiency and possibly indicating an estimate for governmental risk aversion level and budget level acceptance. Finally, these same projections are compared with efficient portfolios, following the MPT. The methodology used to develop this thesis can be seen in Figure 7.

Figure 7 – Methodology



(Source: Author's own development)

3.1. Historical Analysis of Brazilian Electricity Matrix

For the historical analysis, it is considered the evolution between 2009 and 2017 of the installed capacity and electricity generation by source, all collected from official annual reports[6]. The data is presented in Table A8 and Table A9 in the Annex of this work.

Electricity generation tells about how much energy is actually produced by a certain source; it is typically measured in gigawatt hours (GWh). Installed capacity, on the other hand, tells about how much energy could be produced by a certain type of technology ("by source"); it is typically measured in gigawatts (GW).

All these series were analyzed in order to access the evolution of the Brazilian energy matrix's diversity. Firstly, the percentages of each source in the distribution of electricity generation and installed capacity were computed. Then, these percentages were used to calculate the Shannon-Weiner Index (*H*), Shannon's Equitability ($\boldsymbol{E_H}$) and HHI ($\boldsymbol{\lambda}$) for each time period and for each set of data. The output of this procedure consists in a collection of series representing the evolution of diversity in terms of energy sources for energy generation and installed capacity. Table 3 presents a rapid description of the methodology.

Table 3 – Series to be computed during the Historical Analysis of Brazilian Energy Matrix

		Output: time-series of territorial and technological diversity		
Data from annual reports	**Percentages (p_t)**	**Shannon Index (H_t)**	**S. Equitability (E_t)**	λ_t
Evolution of Electricity Generation by Source	p_t^{EG}	H_t^{EG}	E_t^{EG}	λ_t^{EG}
Evolution of Installed Capacity by Source	p_t^{IC}	H_t^{IC}	E_t^{IC}	λ_t^{IC}

(Source: Author's own development)

[6] 2014 and 2018 Statistical Yearbook of electricity, developed by the Energy Research Company (*Empresa de Pesquisa Energética* – EPE).

3.2. Assessment of Projections for the Brazilian Electricity Matrix

In order to examine the projections of the Brazilian Energy Matrix, the forecasted electricity capacity and generation by source for the years 2020, 2030 and 2040 provided by WEO 2016[7] were evaluated in terms of composition, diversity, risk and cost. Such calculation of risk and cost is in accordance to the model proposed by deLlano-Paz (2014), based on MPT. The evaluation in terms of technology diversity uses the Shannon-Weiner Index ($\boldsymbol{H}$), Shannon's Equitability ($\boldsymbol{E_H}$) and HHI ($\boldsymbol{\lambda}$). Table 4 presents the methodology.

Table 4 – Methodology of Projections Assessment

Data		**Risk-Cost evaluation**			**Technology diversity evaluation**		
Source: WEO 2016		**Method proposed by de Llano Paz (2014)**			H	E_H	λ
Projection of electricity generation by source	2020	Calculate:	$E[C_{EG\_20}]$	$\sigma^2_{EG\_20}$	H_{EG_20}	E_{EG_20}	λ_{EG_20}
	2030	//	$E[C_{EG\_30}]$	$\sigma^2_{EG\_30}$	H_{EG_30}	E_{EG_30}	λ_{EG_30}
	2040	//	$E[C_{EG\_40}]$	$\sigma^2_{EG\_40}$	H_{EG_40}	E_{EG_40}	λ_{EG_40}
Projection of installed capacity by source	2020	//	$E[C_{IC\_20}]$	$\sigma^2_{IC\_20}$	H_{IC_20}	E_{IC_20}	λ_{IC_20}
	2030	//	$E[C_{IC\_30}]$	$\sigma^2_{IC\_30}$	H_{IC_30}	E_{IC_30}	λ_{IC_30}
	2040	//	$E[C_{IC\_40}]$	$\sigma^2_{IC\_40}$	H_{IC_40}	E_{IC_40}	λ_{IC_40}

(Source: Author's own development)

[7] WEO 2016 can be freely assessed through internet. Its data does not truly reflect Brazilian current system, since it refers to the period when Dilma Rousseff was the president of Brazil. Although there is a new version of WEO's annual report (WEO 2018), it is not freely available. Besides, WEO 2018 also does not offer projections based on the current government: it refers to the transitional period that has followed ex-president Dilma's impeachment. Therefore, in this thesis, WEO 2016 is used as a source, offering an analysis of the projections under Dilma's government. The same methodology could be applied to projections based on more recent data, when these projections are made available.

The projections for electricity generation and installed capacity by source are presented in Table 5.

Table 5 – Projected Electricity Generation and Installed Capacity

	Generation [TWh]				Capacity [GW]	
	2020	**2020**	**2030**	**2040**	**2030**	**2040**
Total	**636,00**	**170,00**	**218,00**	**273,00**	**835,00**	**1069,00**
Coal	24,00	5,00	5,00	4,00	22,00	22,00
Oil	14,00	8,00	7,00	7,00	12,00	12,00
Gas	55,00	17,00	23,00	33,00	64,00	126,00
Nuclear	26,00	3,00	4,00	5,00	31,00	39,00
Hydro	415,00	106,00	128,00	156,00	540,00	648,00
Bioenergy	48,00	14,00	17,00	19,00	59,00	70,00
Wind	49,00	14,00	24,00	31,00	88,00	119,00
Geothermal	0,00	0,00	0,00	0,00	0,00	0,00
Solar PV	5,00	3,00	10,00	17,00	18,00	30,00
CSP	0,00	0,00	0,00	1,00	1,00	3,00
Marine	0,00	0,00	0,00	0,00	0,00	0,00

(Source: WEO 2016)

The first step is to calculate ratio between the amount of energy generated by each source and the total energy generated in that period (see Equation 11). Then – considering that one has estimates for correlation matrixes, mean and standard deviations of associated costs – it is possible to calculate estimates for risk and total cost of each projected portfolio total cost in Equation 12 and variance in Equation 13.

$$x_{t,p} = \; G_{t,p} \,/\, \textstyle\sum_{t\in T} G_{t,p} \tag{11}$$

$$E[C_p] = \textstyle\sum_{t\in T} x_{t,p}.E[C_t] = \textstyle\sum_{t\in T} x_{t,p}.\left(\textstyle\sum_{c\in C} E[C_{t,c}]\right) \tag{12}$$

$$\sigma_p^2 = \textstyle\sum_{i\in T}\textstyle\sum_{j\in T} x_{i,p}.x_{j,p}.\sigma_{ij} \; = \textstyle\sum_{i\in T}\textstyle\sum_{j\in T} x_{i,p}.x_{j,p}.\textstyle\sum_{c\in C}\sigma_{ij}^c \tag{13}$$

Where:

$x_{t,p}$ is the fraction of the electricity generated by technology t in the projection p;

$G_{t,p}$ is the electricity generated by technology t in the projection p;

$E[C_p]$ is the expected portfolio cost for generating electricity;

$E[C_t]$ is the expected cost of electricity production by technology t;

$E[C_{t,c}]$ is the expected cost of a specific aspect c of technology t;

T is the set of technologies being considered (solar, wind etc.);

C is the set of specific aspects in cost being considered (CO_2, O&M etc.);

σ_p^2 is the portfolio variance (variance in cost for generating electricity);

σ_{ij} is the covariance between costs of technologies i and j;

σ_{ij}^c is the covariance between specific costs c of technologies i and j;

The same the estimates for correlation matrixes, mean and standard deviations of associated costs ($E[C_{t,c}]$ and σ_{ij}^c) applied in the work of deLlano-Paz (2014) are used in this thesis (see Annex 1 from Table A3 to Table A6).

An analogous procedure is performed with the data concerning electricity capacity (instead of electricity generation). The same estimates for the electricity generation problem ($E[C_{t,c}]$ and σ_{ij}^c) are used to solve the corresponding model with electricity capacity data.

To perform this procedure – estimation of risk and cost of projected electricity capacity by source – the fractions ($x_{t,p}$) must be measured again. However, now the fractions are measured as the ratio between the electricity capacity provided by a technology ($K_{t,p}$) and the total capacity provided by the system (see Equation 14).

$$x_{t,p} = \; K_{t,p} \,/\, \textstyle\sum_{t \in T} K_{t,p} \qquad (14)$$

Then, by using the same estimates for correlation matrixes, mean and standard deviations of associated costs applied previously, it is possible to calculate estimates for risk and total cost of each projected portfolio. The equations for cost and risk are the same as in the previous case: estimation total cost can variance can be found in Equation 12 and Equation 13, respectively.

3.3. Efficient Portfolios

The WEO (2016) provides six different projections (see Table 6).

Table 6 - Projections borrowed from WEO 2016

Projection of Electricity Generation by source			Projection of Installed Capacity by source		
2020	2030	2040	2020	2030	2040

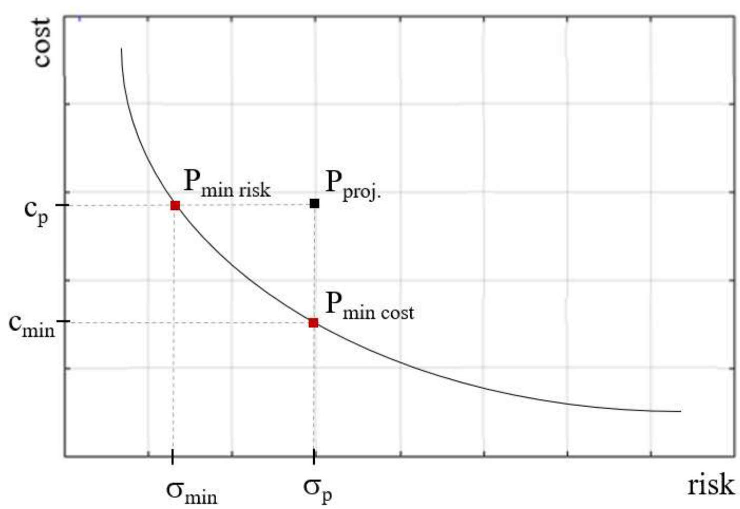
(Source: Author's own development)

For each one of these projections two efficient portfolios were calculated:

- **Portfolio I:** same cost but lower level of risk
- **Portfolio II:** same level of risk but at a lower cost.

Thus, in the end there are twelve efficient portfolios: two for each projection from WEO (2016). In this way, one can compare each projection with its optimized versions and see what would have to change in the energy production to reduce either the cost or the risk (see Figure 8).

Figure 8 – Optimized portfolios of a projection



(Source: Author's own development)

To find such portfolios, the optimization models were written on MATLAB (R2018b) following the MPT, using a computer with microprocessor AMD E1-1500 APU with Radeon^{TM} HD, 1.48GHz and 4 GB RAM. Like in the historical evaluation, the same correlation matrixes, estimates for mean and standard deviations of total cost from deLlano-Paz (2014) are used here (see from Table A3 to Table A6 in the Annex).

In order to find the first type of portfolio – same cost but lower level of risk – it was applied the same optimization model presented in Section 2.2 (model M2). In the present section the model is presented again, modifying only the definition of x_t, that now depends on whether the portfolio is based on electricity capacity or electricity generation data (model M3).

The objective function is given by Function 3: it is the minimization of portfolio variance. The decision variable is x_t, the fraction of the energy produced (or capacity provided) by technology t.

Objective function:

$$\min_x \sum_{i\in T}\sum_{j\in T} x_i . x_j . \sum_{c\in C} \sigma_{ij}^c \qquad \text{Function 3}$$

Subject to:

$$E[C_p] = \sum_{t\in T} x_t . E[C_t] = \sum_{t\in T} x_t . \left(\sum_{c\in C} E[C_{t,c}]\right) = C_{target} \qquad \text{Constraint 7}$$

$$\sum_{t\in T} x_t = 1 \qquad \text{Constraint 8}$$

$$x_t \geq 0 \qquad (\forall\, t \in T) \qquad \text{Constraint 9}$$

Where Constraint 7 restraints the budget; Constraint 8 defines that the sum of fractions related to electricity production (or capacity provided) by source is equal to 1 and Constraint 9 prevents a sort of short-selling of energy production (or capacity provided).

Usually, Constraint 7 is relaxed: an inequality is used limiting the expected costs to be not greater than a target. However, since the problem is to find a portfolio with lower risk but *at the same cost*, an *equality* is used, forcing the expected cost of the portfolio to be equal to a target (C_{target}).

The target (C_{target}) changes according to the projection on focus. That is, when looking for an optimized version of a projection one must set the target cost as equal to the expected cost of this same projection (see Equation 15 and Table 7).

$$C_{target} = E[C_{proj.}] \qquad (15)$$

Table 7 – Target cost used: expected cost of projected portfolio

	Projection of Electricity Generation by source			Projection of Installed Capacity by source		
	2020	2020	2030	2040	2030	2040
$C_{target} =$	$E[C^{EG}_{2020}]$	$E[C^{IC}_{2020}]$	$E[C^{IC}_{2030}]$	$E[C^{IC}_{2040}]$	$E[C^{EG}_{2030}]$	$E[C^{EG}_{2040}]$

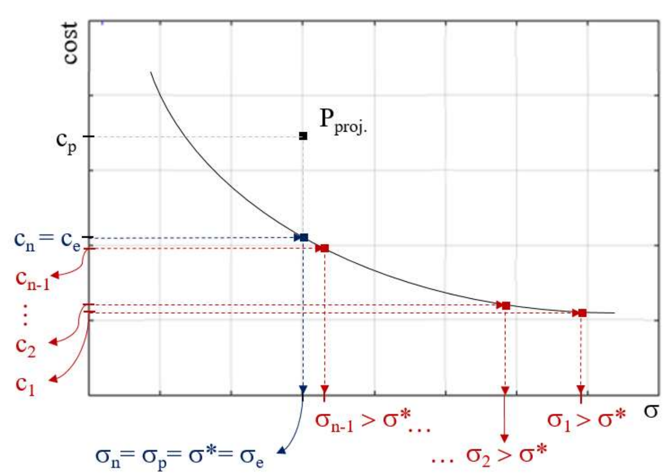
(Source: Author's own development)

To find the second type of portfolio – same level of risk but lower cost – one could try to adapt the previous model by simply switching the Constraint 7 and Function 3, transforming the objective function into a minimization of portfolio's expected cost and the constraint into a limitation of risk. However, this adaptation would lead to a non-linear constraint since the measure of risk used is variance, which is quadratic. In this thesis, such situation is avoided since the optimization function used (*fmincon*) requires the use of linear constraints only.

In order to overcome this limitation of *fmimcon*, another approach of modeling this problem was used. Therefore, the optimization function *fmincon* is maintained and the problem is modelled in such a way that all the constraints are linear – the only non-linearity is contained in the objective function.

The approach consists in performing several minimizations of risk varying the limitation on budget looking for a portfolio whose minimized risk matches the restriction on risk (with some acceptable approximation). This portfolio is close to optimal one. Follows Figure 9 with a representation of the mechanism.

Figure 9 – Optimization procedure to find minimum cost portfolio



(Source: Author's own development)

4. Results

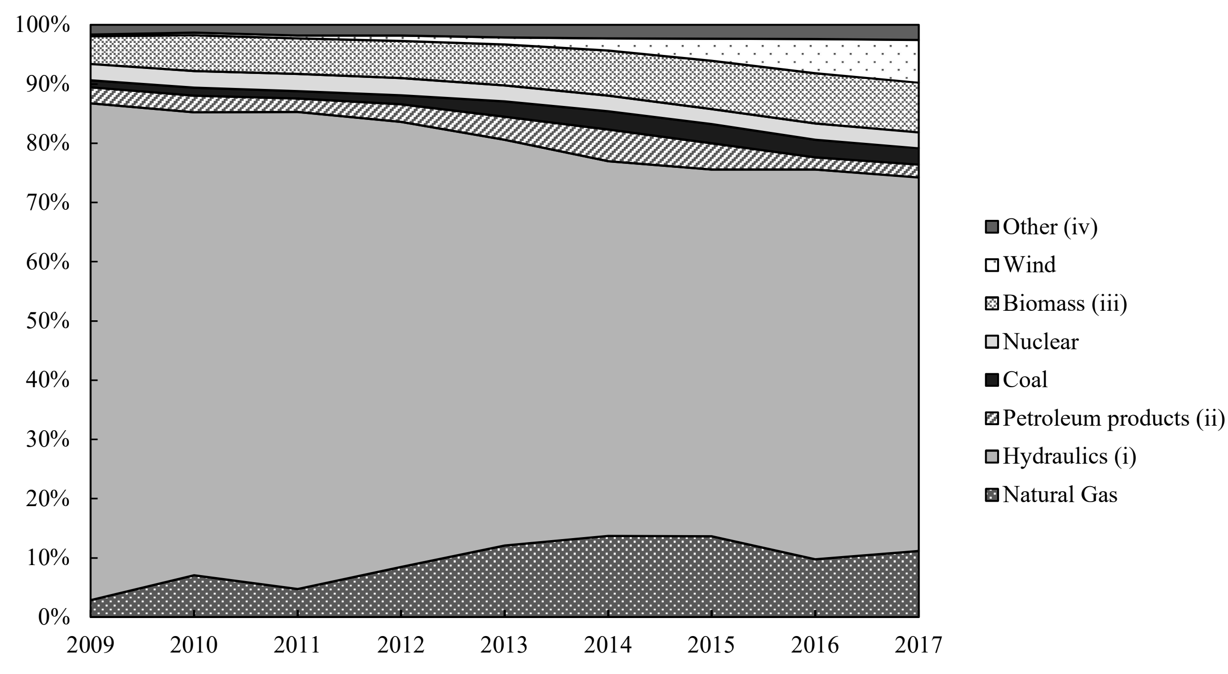
4.1. Historical Analysis of Brazilian Electricity Matrix

4.1.1. Historical Analysis of Brazilian Electricity Generation by Source

Figure 10 presents how much electricity each source produced in relation to the total electricity generated in that year. Conformingly to the procedure described in the methodology, the data used to build this figure was taken from Statistical Yearbook of Electricity, from EPE (2014, 2018). In this figure, one can easily notice the importance of hydraulic power plants in Brazil, which produces more than 70% electricity. The fluctuations in hydraulic energy generation is mainly compensated by natural gas: when hydraulic energy production decreases, the consumption of natural gas increases. It is also possible to notice that there is a progressive increase in wind energy generation.

It is important to keep in mind that this figure examines the relative electricity production by source, not the absolute: biomass, nuclear, coal and natural gas energy presented a significant increase on their production although their relative production may seem unchanged.

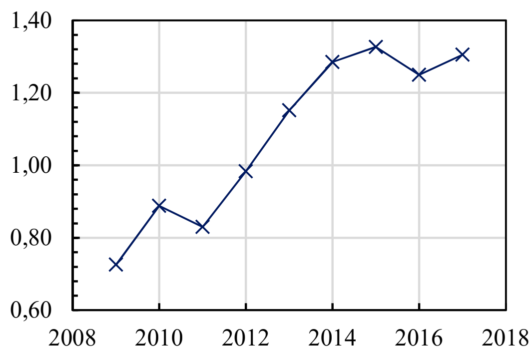
Figure 10 – Electricity Generation by Source (2009-2017)



(Source: Author's own calculation)

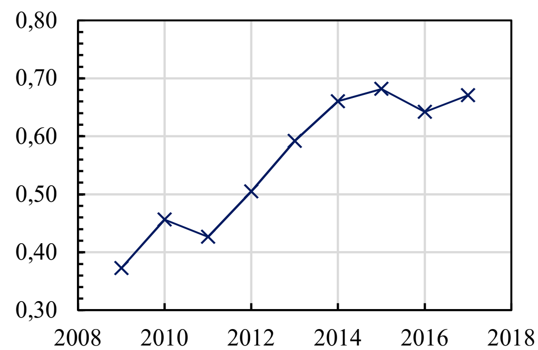
The following figures (from Figure 11 to Figure 13) show the diversity of electricity generation by source from 2009 until 2017. The evolution in terms of diversity is measured again through the Shannon-Weiner Index, HHI and Shannon's Equitability.

Figure 11 – Evolution of electricity generation: Shannon-Weiner Index



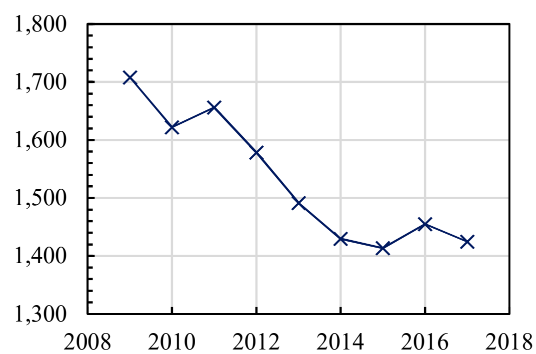
(Source: Author's own calculation)

Figure 12 – Evolution of electricity generation: Shannon's Equitability



(Source: Author's own calculation)

Figure 13 – Evolution of electricity generation by source: HHI



(Source: Author's own calculation)

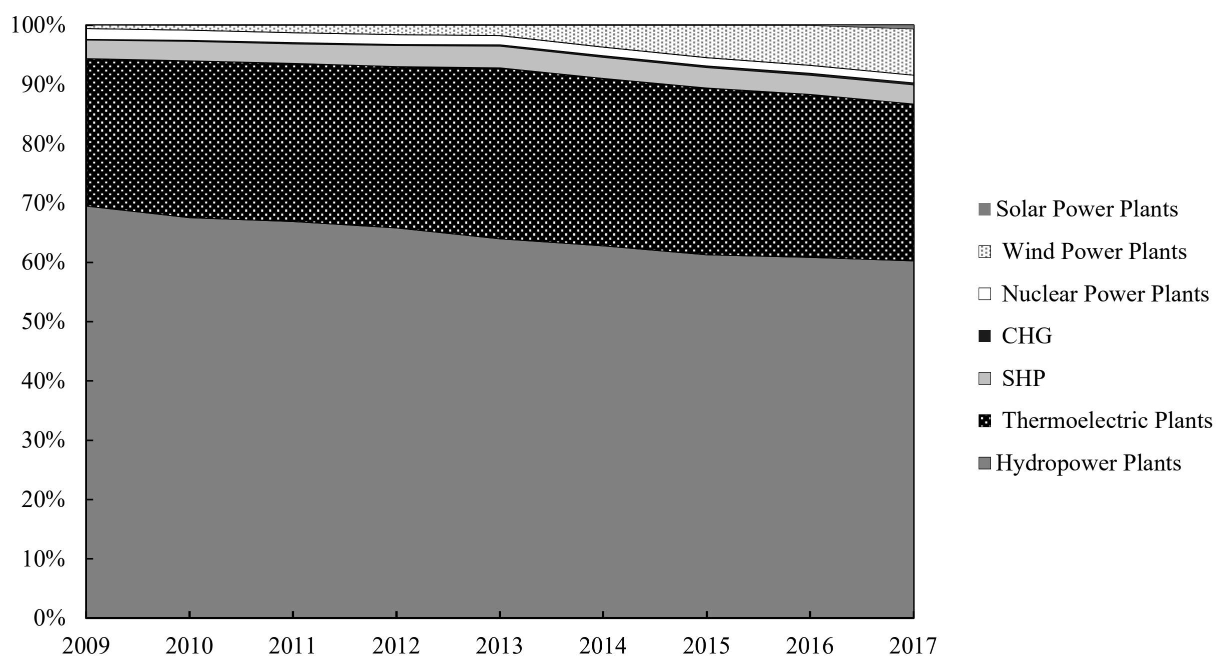
One can notice that the Shannon-Weiner Index and the Shannon's Equitability vary in the same manner in this case. This happens because of the partitioning method used: one cannot see exactly how many sources are used to generate energy (see Table A9 in the Annex) since it is not possible to assess which sources within the category "Others" are really activated during each year. This inadequacy in the nomenclature limits the interpretation of Shannon's Equitability: the equitability counts only the more prominent categories.

The Shannon-Weiner Index and HHI provide comparable results: diversity in term of energy generation by source steadily improves, with two exceptions: the years 2010-2011 and 2015-2016. Looking at Figure 10, one realizes that during these two periods the relative electricity production by Hydraulic Power Plants increased (major source of energy), compensated by a decrease in natural gas electricity generation. The general improvement in the diversity is due to the growth of electricity production by wind, biomass, natural gas and coal energy.

4.1.2. Historical Analysis of Brazilian Installed Capacity by Source

Figure 14 presents how much capacity each source has in relation to the total capacity available during each year. Again, the data used to build this figure was taken from 2014 and 2018 Statistical Yearbook of Electricity. It is possible to note that hydropower plants provide the larger share of the installed capacity throughout the years, followed by thermoelectric plants. The slight decrease in the relative importance of hydropower plants is mainly due to an increase in the installed capacity of wind power plants. Nuclear and solar power plants play a marginal role, offering less than 2% of the total installed capacity (each).

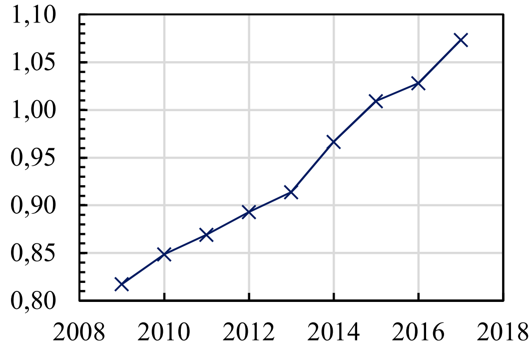
Figure 14 – Installed Capacity by Source (2009-2017)



(Source: Author's own calculation)

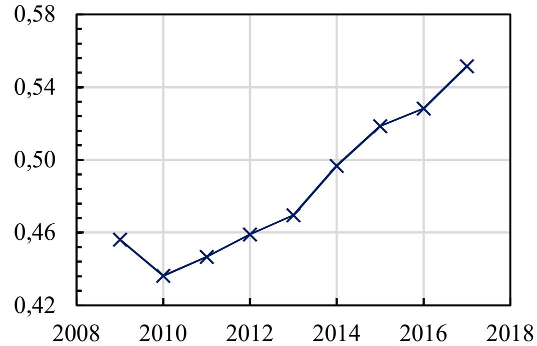
The following figures (from Figure 15 to Figure 17) show the evolution of installed capacity by source from 2009 until 2017. The evolution is in terms of diversity, measured by the Shannon-Weiner Index, HHI and Shannon's Equitability.

Figure 15 – Evolution of installed capacity: Shannon-Weiner Index



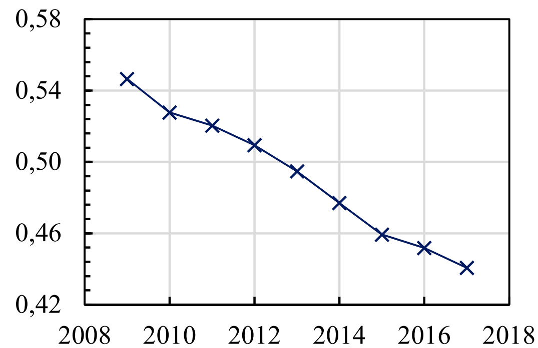
(Source: Author's own calculation)

Figure 16 – Evolution of installed capacity: Shannon's Equitability



(Source: Author's own calculation)

Figure 17 – Evolution of installed capacity: HHI



(Source: Author's own calculation)

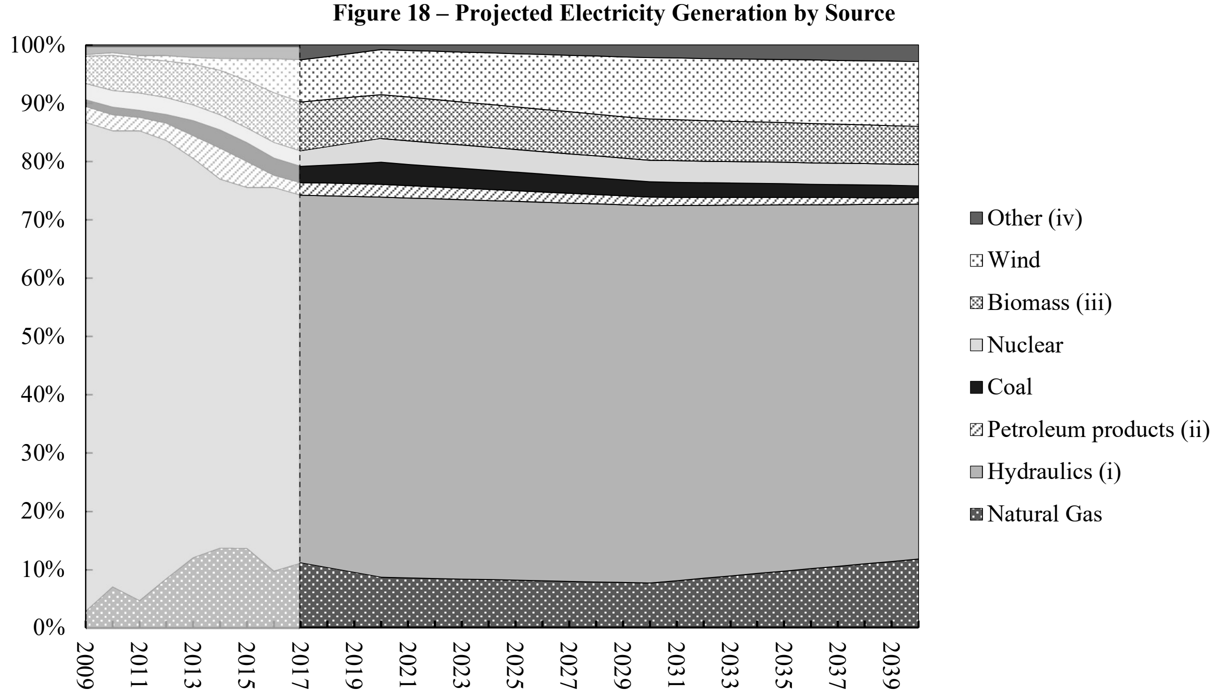
HHI and Shannon-Weiner Index consistently improve with small changes in the rate at which diversity increases. Conversely, Shannon's Equitability shows a significant decrease between 2009 and 2010. This decline happens because in 2010 it was recorded for the first time a non-zero capacity for solar energy. As the number of sources augmented, the index responded with a decline reflecting the higher inequality in capacity distribution.

4.2. Assessment of Projections for the Brazilian Electricity Matrix

4.2.1. Projected Electricity Generation by Source

Figure 18 presents how much electricity each source produced or is expected to produce in relation to the total electricity generated in that year. The figure includes the historical analysis already presented in Figure 10 (concerning the period 2009-2017) complemented by the projections for 2020, 2030 and 2040 (taken from WEO 2106). The percentages of each source can be found in Table A10 in the Annex. In order to complement the graph, years between projections and years between 2017 and 2020 received interpolated values – simply a linear interpolation between the values from the last year with data and the next projection.

Figure 18 – Projected Electricity Generation by Source



(Source: Author's own calculation)

The most notable changes that the projections show in relation to the past are:

I. Reduction of relative importance of natural gas electricity production, followed by its increase – trespassing in 2040 by 6% the 2017's level;

II. Increase followed by decrease of relative importance of electricity generation by hydropower plants (difference of -3% between 2017 and 2040);

III. Reduction of relative importance of petroleum derivatives, coal and biomass for generating electricity (-48%, -25% and -22% from 2017 to 2040);

IV. Increase of relative importance of nuclear, wind and other alternative sources of energy (36%, 54% and 10% from 2017 to 2040).

Although the dimension of growth and reduction shown in point III and IV seem to be expressive, in Figure 18 they are almost null, given the huge relative importance of hydroelectric energy in the Brazilian energy matrix.

Another interesting point to be noticed is that there is a jump between 2017 and 2020, with drastic changes in the relative production of nuclear, coal and natural gas energy. This is because the 2020 projection was calculated in 2016, and as time goes by the difference between projection and reality materializes so that the projection seems to be unreachable. This is not to say that the projection is bad, it is just inaccurate as most of projections. In reality, situations where projections really predict what will happen in the future are rather rare.

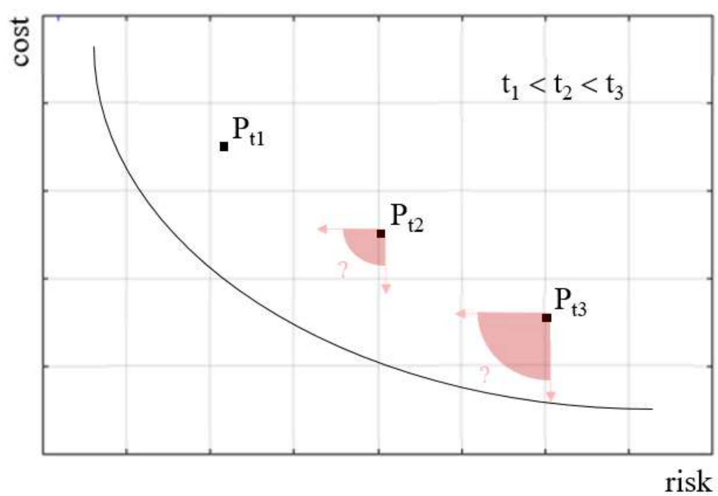
In addition, it can be noted that the 2020 projection differs slightly from the rest of the projections (mainly for coal, nuclear and “others”). That is, aside from the fact that this projection is a bit old, it seems to be inconsistent with the other projections. It seems that to get to the 2040 scenario, one would not have to go through the 2020 scenario. In fact, if 2020 projection data were not included in the Figure 18, the transition from 2017 to 2030 or 2040 would be much smoother. And this would happen not only because of the temporal distance between 2017 and 2030 (or 2040) but also because of the similarity between the percentages of electricity generation of each source, which are similar in those years.

This dissonance also does not necessarily indicate a projection error. In fact, the possible effects of an uneven transition can explain it: the development of various sources could be planned for the long term but nuclear sources and coal plants could be prioritized by 2020. This would increase the relative importance of these sources in the short term, which in the future could diminish and even lower than 2017, with the development of other energy sources.

After the examination of the portfolios composition, the projections were evaluates in terms of efficiency, to estimate how much cost efficient and risky each projected portfolio is[8].

It is important to highlight the fact that, in order to simplify the problem the formulae used in this work to calculate cost and risk contain a reduced number of variables. Part of this simplification is based on the assumption that estimates do not need vary over time. That is, the same estimates are used to analyze all the projections over time – independently if a projection is for 2020, 2030 or 2040 – instead of using estimates specifically drafted to each period. It would be sensible to use specific estimates to each projection since the power plants' efficiency is expected to increase over time because of either learning economies (in case of old plants) or technological improvements (in case of new plants). By improving efficiency, it is expected a reduction in both risk and cost. Consequently, the results found in this thesis increasingly overestimate the cost and risk portfolios as they become more distant in the time horizon (see Figure 19).

Figure 19 – Growing overestimation of cost and risk for portfolios



(Source: author's own development)

Besides, the methodology used in this work does not consider any type of distinction within technologies. This choice probably underestimates the differences in risk and cost associated to individual plants within the same category. For example, it is assumed that all solar power plants have the same cost and risk when it is known that they might differ quite a lot. Of

[8] As described in the methodology, the result found in Table 8 and Figure 20 are the result of calculations based on data provided by WEO 2016 (the percentage of each source) and data provided by deLlano-Paz (2014) (expected cost, variances and covariance matrixes).

course, it is expected that some underestimated values compensate some overestimated ones. However, it would be better to model the problem considering the data of each individual plant.

Evidently, all these simplifications were applied since the estimates needed ($E[C_{t,c}]$ and σ_{ij}^{c}) cannot be easily found: the problem can only be solved within a reasonable time and budget if some information is marginalized; the price for this choice is some imprecision and limited interpretation.

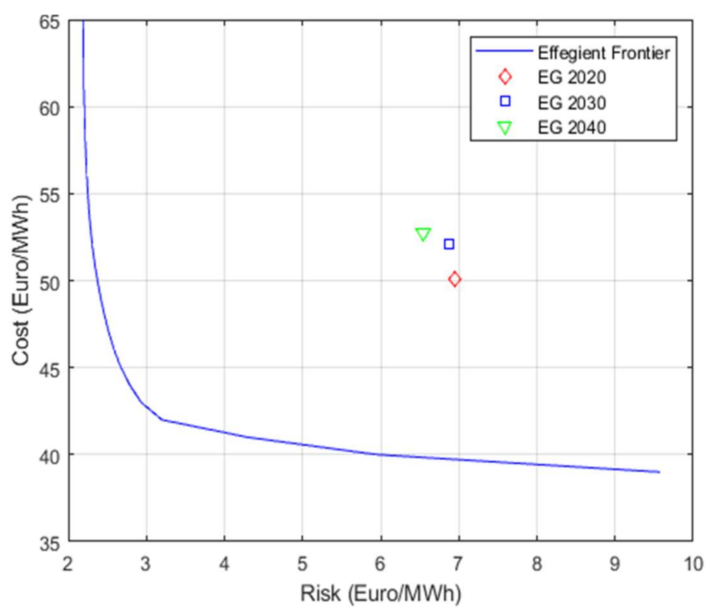
Table 8 and Figure 20 show the result of this evaluation, presenting how projected electricity generation behave with respect to their risk and cost.

Table 8 – Risk and cost of projected portfolios: electricity generation

	2020	2030	2040
Cost [Euro/MWh]	50.0708	52.0630	52.7836
Risk [Euro/MWh]	6.9530	6.8833	6.5446

(Source: Author’s own development)

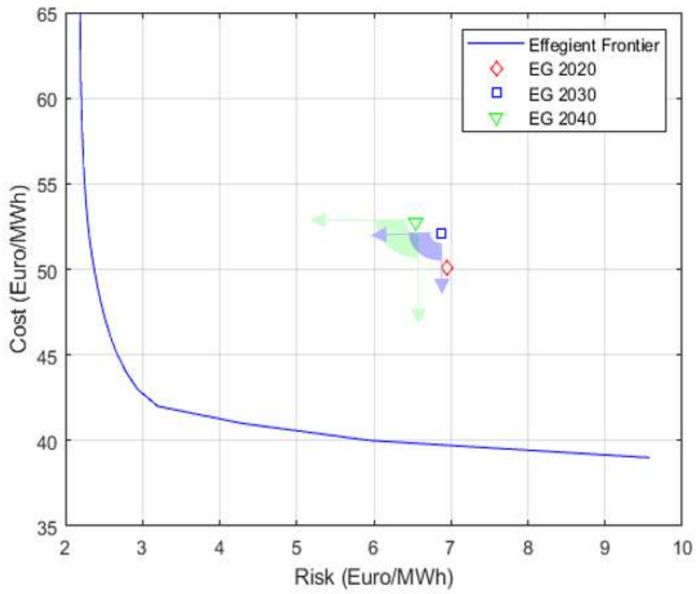
Figure 20 – Risk vs. cost of projected portfolios: electricity generation



(Source: Author's own calculation)

The first thing to note is that projections are not efficient portfolios: no projection is an efficient border point, and, seemingly, it is not close to the border. The apparent trend observed is a slight decrease in portfolio risk accompanied by a minor increase in portfolio cost. This trend, however, might be only an effect of the overestimation of future costs and risks, as previously explained (see Figure 21). That is, since the projections are quite close to each other in the risk *versus* cost frame, if the effects of economies of learning and use of new technologies were considered, the observed trend possibly would indicate an approximation to the efficient frontier.

Figure 21 – Effect of overestimation of cost and risk in future portfolios

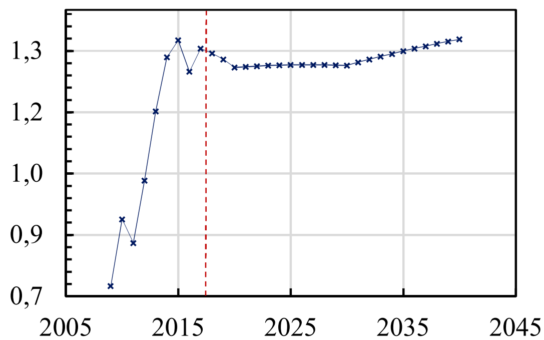


(Source: Author's own development)

The acceptable budget is approximately 50 euros/MWh and the risk acceptance level is equivalent to a standard deviation of approximately 7 euros/MWh.

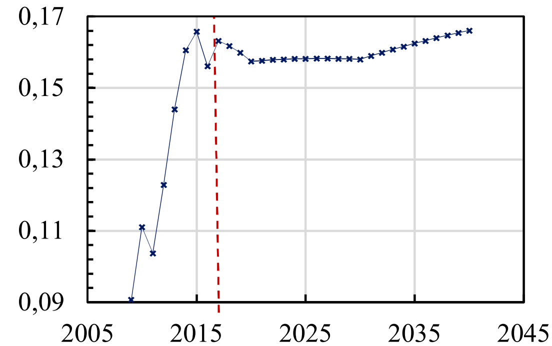
Figure 22 to Figure 24 show the diversity in the projected portfolios for 2020, 2030 and 2040. The portfolios project electricity generated by each source. The evolution in terms of diversity is measured again through the Shannon-Weiner Index, HHI and Shannon's Equitability. These figures contain also an interpolation for those years without projection – years between 2017-2020, 2020-2030 and 2030-2040.

Figure 22 – Projected electricity generation: Shannon-Weiner Index



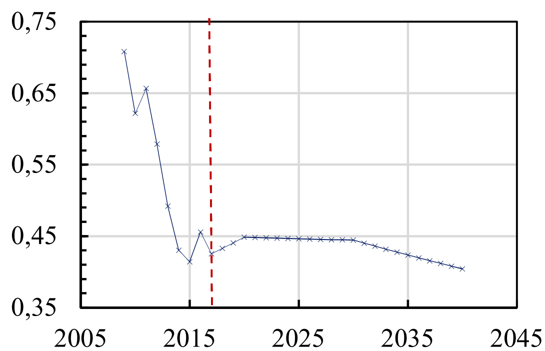
(Source: Author's own calculation)

Figure 23 – Projected electricity generation: Shannon's Equitability



(Source: Author's own calculation)

Figure 24 – Projected of electricity generation: HHI



(Source: Author's own calculation)

Projections indicate a stabilization of matrix diversity, in contrast to the significant growth in diversity observed since 2009. Between 2017 and 2020, even a slight decrease in diversity is projected - explained by the rise in relative importance of hydropower in the period (from 63.1 to 65.25% of the electricity generated). From 2020 to 2040, diversity increases again, slightly surpassing the level of diversity observed in 2017.

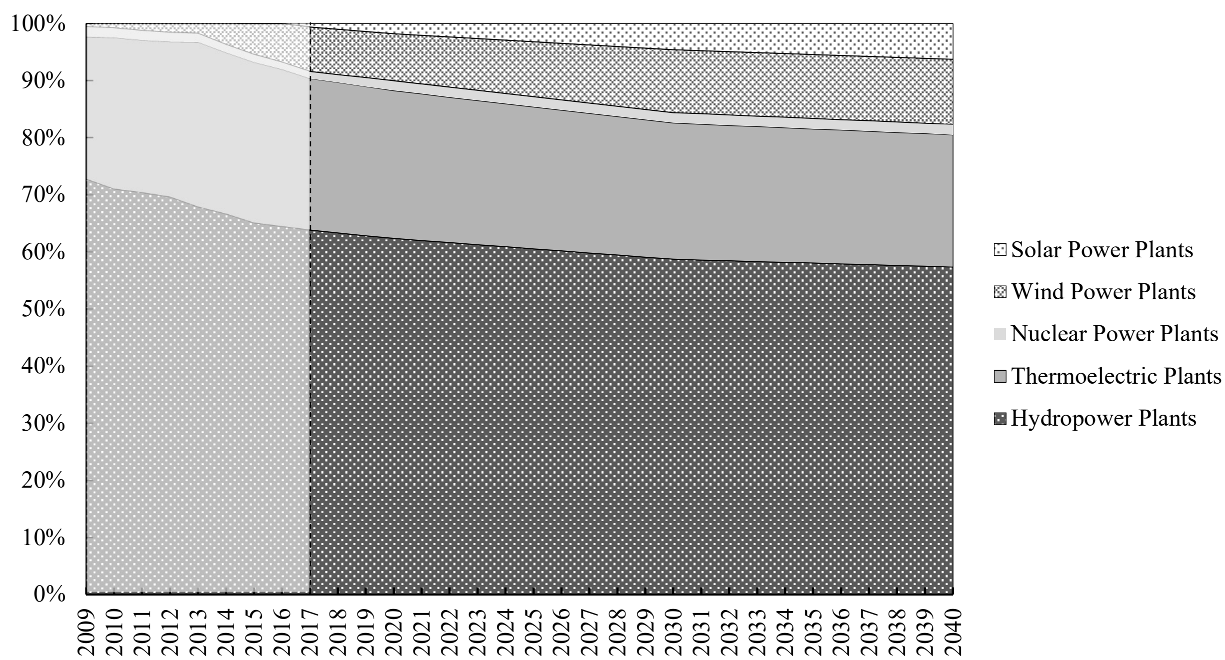
The Shannon's Equitability has the same shape as the Shannon-Weiner Index. This is because all categories are "activated" from the beginning. One should notice that one of the categories is "Other" which includes more than one energy source and acts as a sort of "black box": we do not know what happens within that category. That is, the electricity generated by this category may come from one or several energy sources. This difference, hidden within this category, would impact Shannon-Weiner Index, Shannon's Equitability and HHI if it was explicit.

It is important to note that this does not invalidate the indices that have been calculated in this work, rather it limits its interpretation. That is, one must remember when analyzing this data that only the most expressive categories were explicitly considered – conversely categories representing less than 0.3% of the electricity generated were included in the category named "Other".

4.2.2. Projected Installed Capacity by Source

Figure 25 presents how much capacity was installed or was expected to be installed for each source in relation to the total available capacity in that year. The figure includes the historical data already presented in Figure 14 (contemplating the years 2009-2017) complemented by the projections for 2020, 2030 and 2040 (taken from WEO 2016). The percentages of each source can be found in Table A11 in the Annex. In order to complement the graph, years between projections and the years between 2017 and 2020 received interpolated values.

Figure 25 – Projected Installed Capacity by Source



(Author's own calculation)

The most notable points are:

1) The installed capacity graph has smoother transitions between projections compared to the electricity generation graph (Figure 18).
2) There is a small decrease in the importance of hydropower and thermoelectric power (decreased by 10% and 13% respectively, compared to 2017 percentages);
3) The relative importance of nuclear, wind and solar energy is projected to increase until 2040 (by 45, 45 and 946% if compared to 2017 percentages).

Here, once again, it can be seen that, although the growth mentioned in item 3 is significant, when it is calculated based on the initial values of 2017, in the graph these changes are almost insignificant, given the weight of hydropower in matrix.

Projections for installed capacity go in the same direction as projections for electricity generation. Both of them point out to a slight decrease in the importance of hydropower and thermoelectric energy (derived from petroleum, coal, biomass and natural gas) and indicate an increase in the importance of wind, solar and nuclear energy.

After the examination of the portfolios composition, the projected installed capacity was evaluated in terms of efficiency. It is important to highlight the fact that, in order to perform this evaluation, a number of simplifications were used, limiting the results' interpretation/precision. For example, the same values of $E\left[C_{t,c}\right]$ and σ_{ij}^{c} are used for both electricity generation and installed capacity[9], which is an enormous simplification: it is not known how much costs and risks differ when comparing capacity and generation, therefore, by assuming that they simply are the same, there is the possibility that quite significant divergences are being neglected.

Consequently, the procedure provides only a rough estimation of how the risk and cost may evolve between projections, purely giving an idea on whether the risks or cost associated may increase or decrease over time. Clearly, the results found by analyzing the electricity capacity are far less precise than the ones found by analyzing the electricity generation. However, it may be useful to compare the behavior of risk and cost of both analyses to see if the results corroborate or, maybe, strongly contradict each other.

[9] One possible interpretation of the outcomes originated by this analysis is the cost and risk associated to the matrix if all installed capacity were being used to generate electricity.

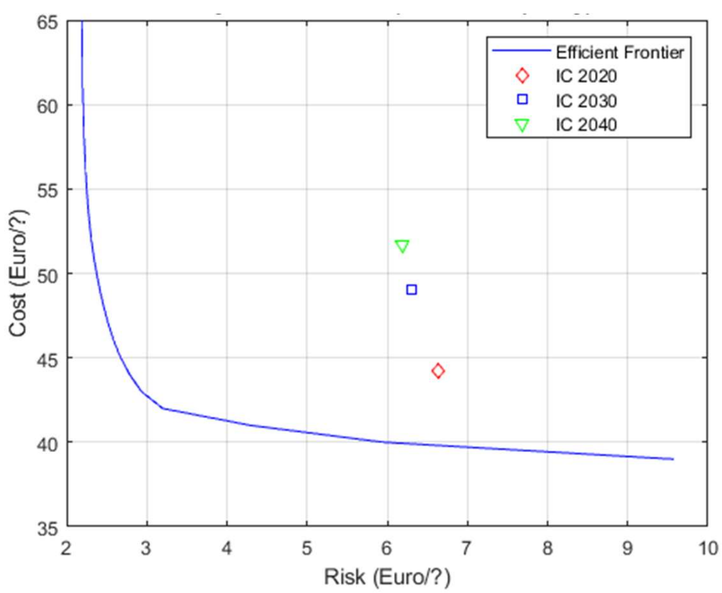
Table 9 and Figure 26 show the outcomes of this analysis: how projected installed capacity behave in relation to portfolio risk and cost under the models' perspective.

Table 9 – Risk vs. Cost of projected portfolios: installed capacity

	2020	2030	2040
Cost [Euro/?]	44.2414	49.0320	51.6833
Risk [Euro/?]	6.6351	6.2985	6.1879

(Source: Author's own development)

Figure 26 – Risk vs. cost of projected portfolios: installed capacity



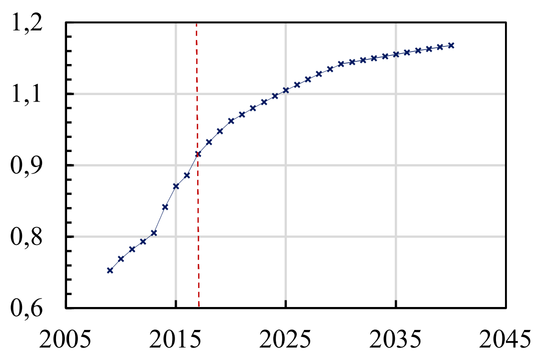
(Source: Author's own calculation)

Again, none of the projections is an efficient portfolio. In fact, the projection for 2020 is closer to the efficient frontier and the apparent trend is to move away from the frontier over the years (2020-2040). This trend translates into a small drop in portfolio risk accompanied by a significant escalation in portfolio cost. Again, one must remember that risk and cost might be more overestimated for portfolios that are more distant in the time horizon. In this sense, this apparent trend might be only an effect of the learning effect's and technology improvements' disregard. In this case, however, the portfolios are more distant to each other; therefore, it would be unlikely to observe an opposite trend (approximation of the efficient frontier) purely due to learning effects/technology improvements.

The apparent acceptable budget is up to 52 euros/MWh by 2040 and the risk acceptance level is equivalent to a standard deviation of 6 to 7 euros/MWh.

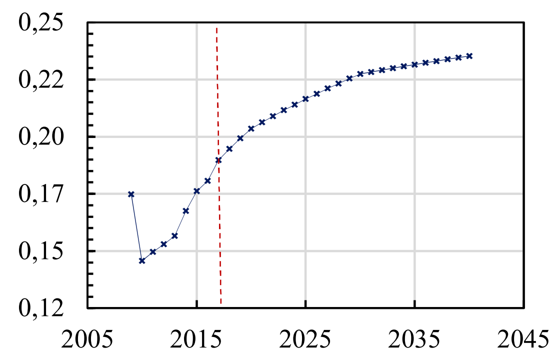
The following figure (from Figure 27 to Figure 29) show the diversity in the projected portfolios for 2020, 2030 and 2040, where the portfolios project installed capacity for each source. Note that the limitation in interpreting the results seen in the previous analysis **<u>is not</u>** applicable in this evaluation.

Figure 27 – Projection of installed capacity: Shannon-Weiner Index



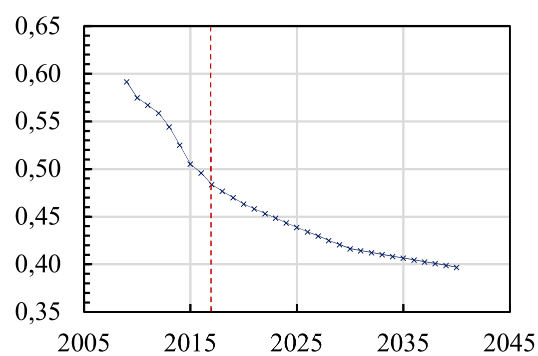
(Source: Author's own calculation)

Figure 28 – Projection of installed capacity: Shannon's Equitability



(Source: Author's own calculation)

Figure 29 – Projection of installed capacity: HHI



(Source: Author's own calculation)

One can see that the increase in diversity was more rapid between 2009 and 2017. That is, projections point to a slower increase in the level of diversity by 2040.

Another interesting point to be analyzed is the first value of the Shannon's equitability. This index begins the observation period at a medium-low level and drops sharply the following year. This probably happened because in the year of the fall (2010) there was the first non-zero value of installed capacity for solar energy.

4.3. Efficient Portfolios

4.3.1. Efficient Portfolios for Electricity Generation

Table 10 presents the efficient portfolios that are equivalent to the projected portfolios of electricity generation by source. The table also contains information about the overall cost and return of the portfolio.

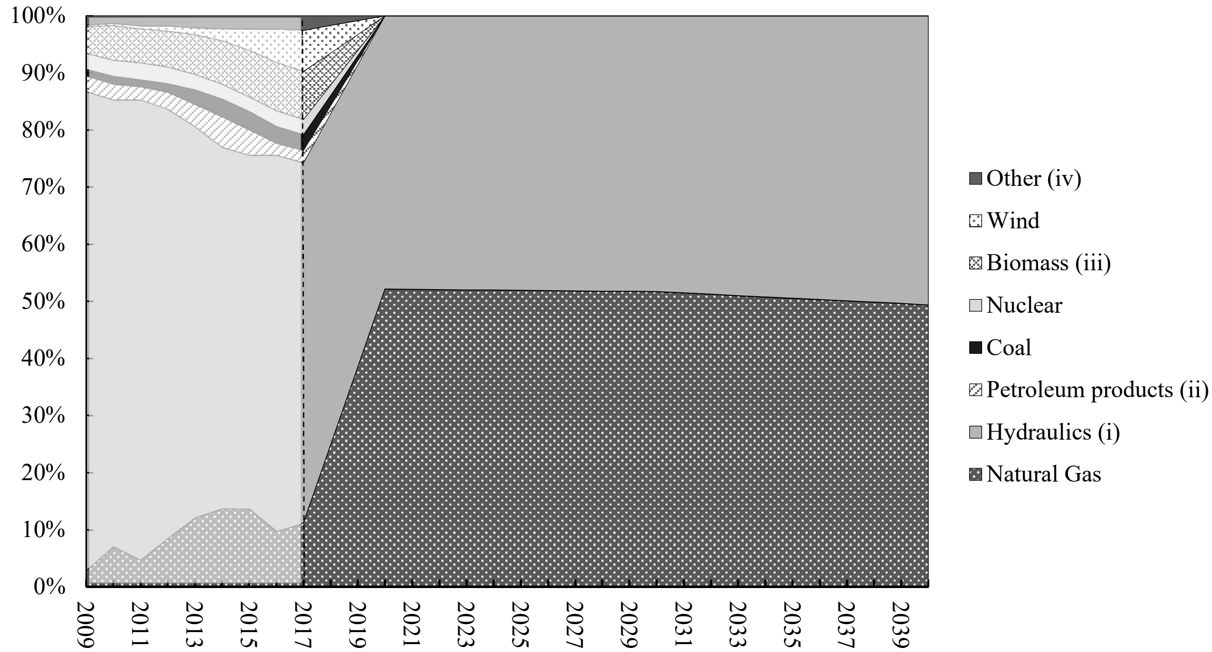
Table 10 – Efficient portfolios for electricity generation

	Cost Minimization			Risk Minimization		
	2020	**2030**	**2040**	**2020**	**2030**	**2040**
Cost	40.51	40.54	40.70	50.07	52.06	52.78
Risk	6.95	6.88	6.54	2.78	2.66	2.62
Nuclear	0,00%	0,00%	0,00%	7,39%	7,44%	7,38%
Coal	0,00%	0,00%	0,00%	7,25%	7,27%	7,20%
Coal (CCS)	0,00%	0,00%	0,00%	1,71%	3,01%	3,28%
Natural gas.	52,13%	51,70%	49,37%	14,19%	12,75%	12,26%
Nat. gas (CCS)	0,00%	0,00%	0,00%	3,59%	4,02%	4,07%
Oil	0,00%	0,00%	0,00%	0,00%	0,27%	0,65%
On-shore wind	0,00%	0,00%	0,00%	9,30%	9,73%	9,85%
Large hydro	36,67%	36,21%	33,76%	4,60%	4,19%	4,07%
Small hydro	11,19%	12,09%	16,87%	46,33%	43,29%	42,42%
Off-shore wind	0,00%	0,00%	0,00%	4,21%	5,48%	5,84%
Biomass	0,00%	0,00%	0,00%	1,45%	2,56%	2,97%
Solar PV	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

(Source: Author's own calculation)

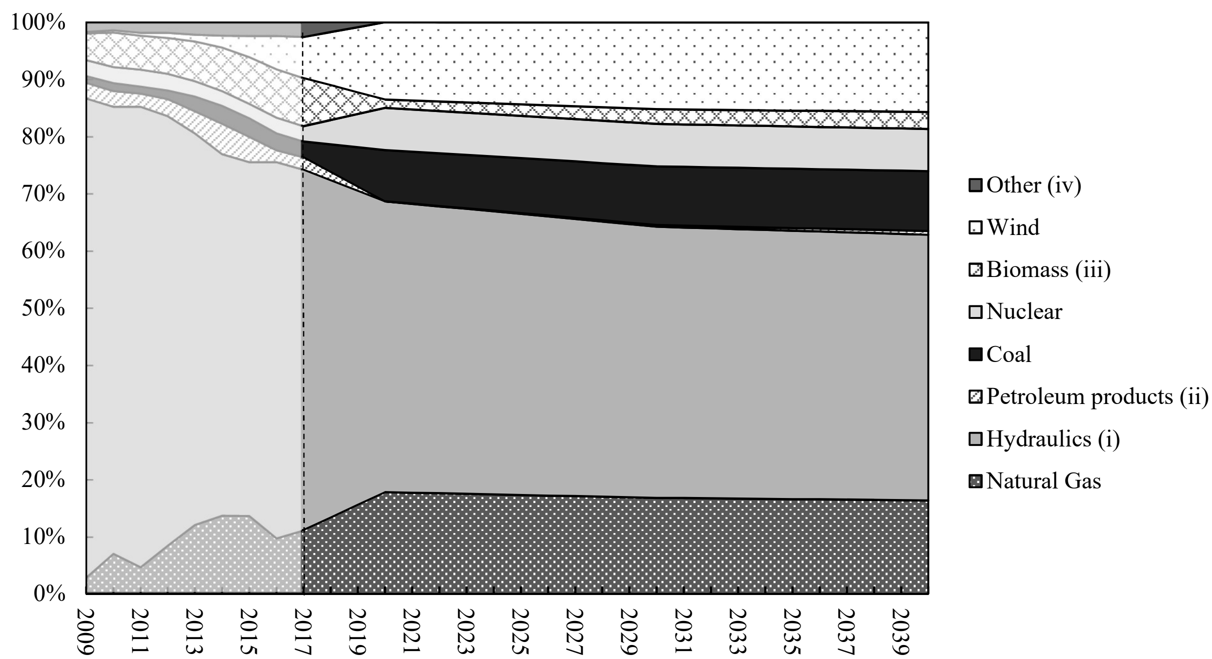
The following figures (Figure 30 and Figure 31) illustrate the distribution of energy generation between sources contained in Table 10, in addition to the historical data already presented in Figure 10 (from 2009 to 2017). These two figures should be compared to Figure 18 in order to understand how different the efficient portfolios are in relation to the projected portfolios.

Figure 30 – Electricity Generation by Source: Cost Minimization



(Source: Author's own calculation)

Figure 31 – Electricity Generation by Source: Risk Minimization



(Source: Author's own calculation)

The first thing one can notice in this result is that minimum-cost portfolios have an extremely reduced number of sources generating electricity. This indicates that, although diversification might be important to achieve efficient portfolios, in theory, there is the possibility to have an efficient portfolio extremely undiversified if the focus is on cost minimization rather than risk minimization. However, the "efficiency" of this portfolio might not be applicable in the real world: it is quite unimaginable to consider a country as big as Brazil depending on only two different sources of energy to generate electricity. Probably the energy security would be hugely diminished: any problem with the level of water in the reservoirs would considerably escalate costs of electricity production. Besides, probably the amount of natural gas required would not be completely internally acquired, generating a dependence on exportation and vulnerability to its costs fluctuations.

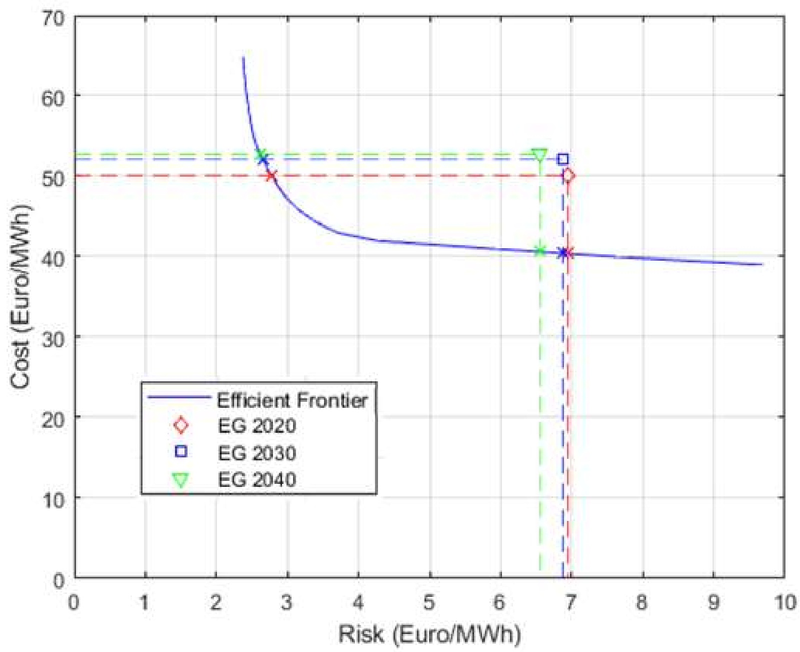
On the other hand, minimum-risk portfolios suggest the generation of electricity by a larger number of different sources. Further, they suggest a more significant reduction of the hydraulic generation's importance over the next decades, which also improves the system diversity. This result is quite intuitive, considering the expected influence of diversity on portfolio risk.

One interesting similarity observed is that both models (minimization of risk or cost) still suggests hydro energy as one of the main source for electricity generation. It is true that all portfolios consistently suggest a decrease of this importance, however none of them suggest a level lower than 46%. Another point in common between the optimal portfolios is the importance given to natural gas: both models indicate that this source of energy should prioritized, together with hydro energy. This similarity might indicate that these sources have a great balance of cost efficiency and risk in comparison to the other sources.

Differently from the minimum-cost portfolios, sources based on wind, nuclear and coal energy also have significant importance on minimum-risk portfolios. This result suggests that these sources might be relatively more expensive than the others, but including them on the portfolio might reduce the overall risk in producing electricity.

Figure 32 shows how these portfolios are arranged along the efficient frontier. Note that in order to minimize the portfolios' risk it is necessary to change the portfolios' composition – in a relatively small variation, if we consider the historical changes that had already taken place in the matrix – so that the estimated production risk falls around 4 euros/MWh (a reduction of approximately 57%). On the other hand, to minimize the portfolios' cost, it is necessary a quite drastic change in the matrix (possibly an unfeasible one) so that the estimated production costs falls around 10 euros/MWh (a reduction of approximately 20%).

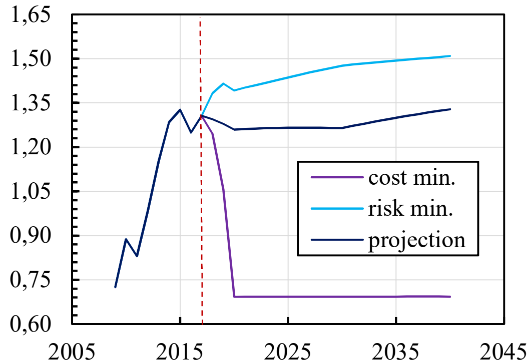
Figure 32 – Efficient Portfolios for Electricity Generation



(Source: Author's own calculation)

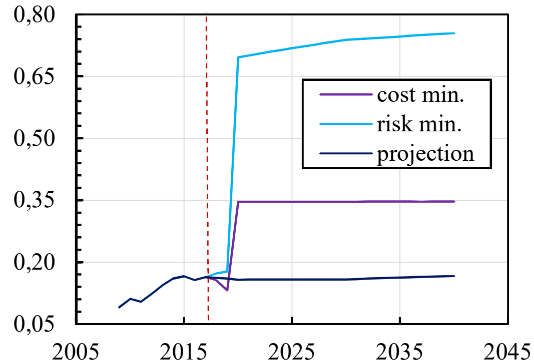
After assessing the efficient portfolios' composition, cost and risk, the efficient portfolios' diversity was evaluates. The following figures (from Figure 33 to Figure 35) show how diversity would vary in a scenario where efficient portfolios materialized rather than projected portfolios. It is important to note that the calculated efficient portfolios refer to the years 2020, 2030 and 2040 only: from 2009 to 2017, the historical series is repeated in the charts for comparison purposes. The darker line in the figures shows the diversity in projected portfolios (inefficient ones) to understand whether efficient portfolios show greater or less diversity than the projected ones (and how large is this difference).

Figure 33 – Comparison of efficient portfolios and projections for electricity generation: Shannon Index



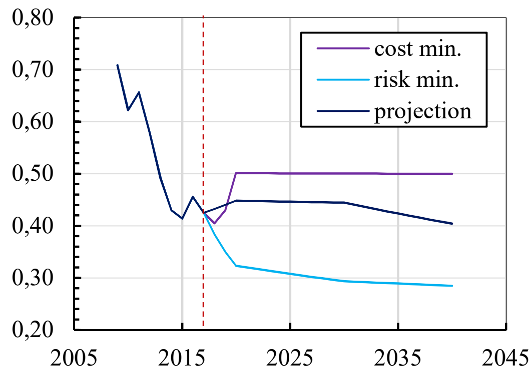
(Source: Author's own calculation)

Figure 34 – Comparison of efficient portfolios and projections for electricity generation: S. Equitability



(Source: Author's own calculation)

Figure 35 – Comparison of efficient portfolios and projections for electricity generation: HHI



(Source: Author's own calculation)

One can observe that increasing portfolio efficiency by MPT is not always accompanied by an improvement in portfolio diversity. In fact, the diversity indexes corroborates with what was previously discussed: in this model risk minimization leads to diversity improvement, while cost minimization leads to diversity detractions. The only exception observed is on Shannon's Equitability: since the minimum-cost portfolios divide almost equally the electricity generated between sources, their Shannon's Equitability is larger than the projected portfolios' ones.

Another relevant point to be observed is that, although the diversity index has been improved in the minimum-risk portfolios, it clearly is not the optimal diversity condition – characterized by equal fractions for each energy source – instead only a few energy sources dominates the portfolio composition. Besides, there is a reduction in the number of sources generating electricity – which also goes against the idea of maximizing diversity index. Therefore, minimization of risk using MPT does not implicates maximization of diversity.

4.3.2. Efficient Portfolios for Installed Capacity

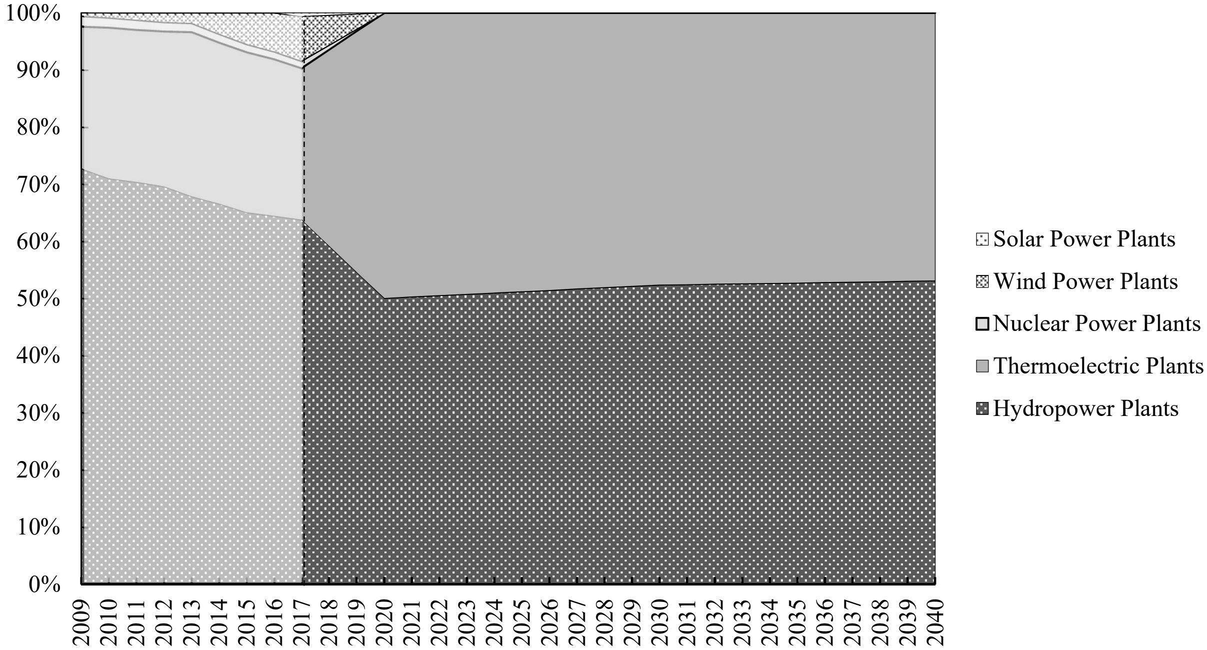
Table 11 presents the efficient portfolios which are equivalent to the projected portfolios of installed capacity by source. The table also contains information about the overall cost and return of the efficient portfolios. Figure 37 and Figure 36 illustrate the distribution of installed capacity between sources contained in Table 11, in addition to the historical data already presented in Figure 14 (from 2009 to 2017). These two figures should be compared to Figure 25 in order to understand how different the efficient portfolios are in relation to the projected portfolios.

Table 11 – Efficient portfolios for projections concerning installed capacity

	Cost Minimization			Risk Minimization		
	2020	**2030**	**2040**	**2020**	**2030**	**2040**
Cost	40.51	40.54	40.70	50.07	52.06	52.78
Risk	6.95	6.88	6.54	2.78	2.66	2.62
Nuclear	0,00%	0,00%	0,00%	4,55%	7,33%	7,46%
Coal	0,00%	0,00%	0,00%	5,25%	7,20%	7,30%
Coal (CCS)	0,00%	0,00%	0,00%	0,00%	0,93%	2,86%
Natural gas.	49,95%	47,63%	46,90%	20,97%	14,95%	13,01%
Nat. gas (CCS)	0,00%	0,00%	0,00%	0,00%	3,31%	3,99%
Oil	0,00%	0,00%	0,00%	0,00%	0,00%	0,07%
On-shore wind	0,00%	0,00%	0,00%	2,69%	9,05%	9,67%
Large hydro	34,37%	31,91%	31,15%	6,85%	4,83%	4,25%
Small hydro	15,68%	20,46%	21,95%	59,70%	48,03%	43,75%
Off-shore wind	0,00%	0,00%	0,00%	0,00%	3,50%	5,29%
Biomass	0,00%	0,00%	0,00%	0,00%	0,88%	2,34%
Solar PV	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

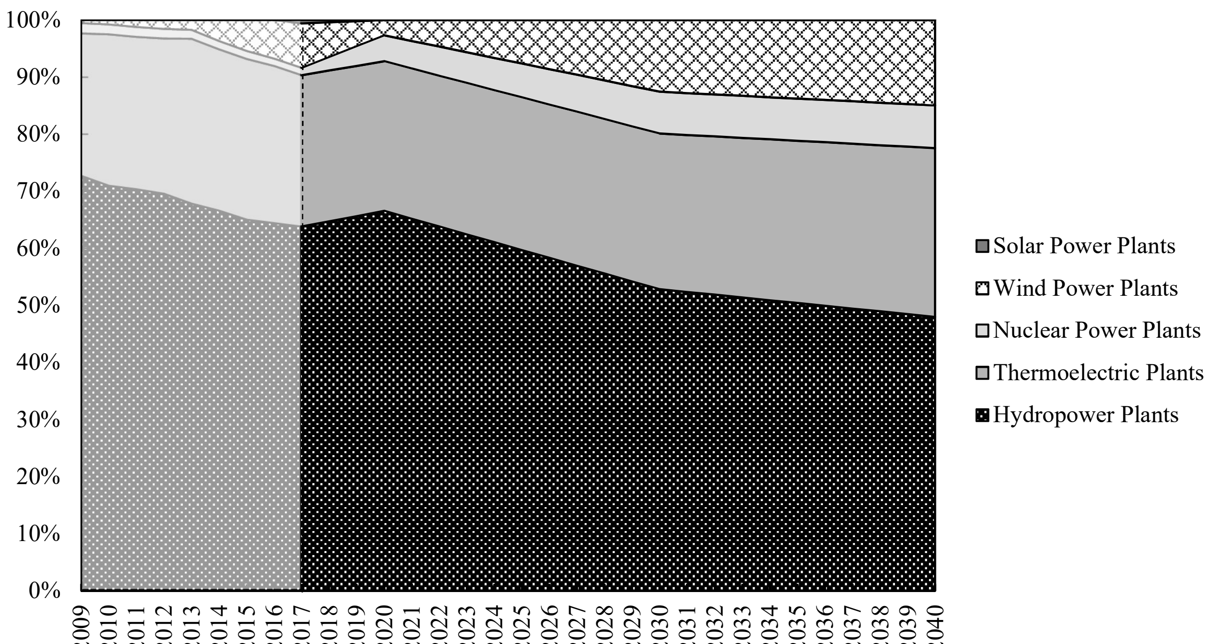
(Source: Author's own calculation).

Figure 36 – Installed Capacity by Source: Cost Minimization



(Source: Author's own development)

Figure 37 – Installed Capacity by Source: Risk Minimization



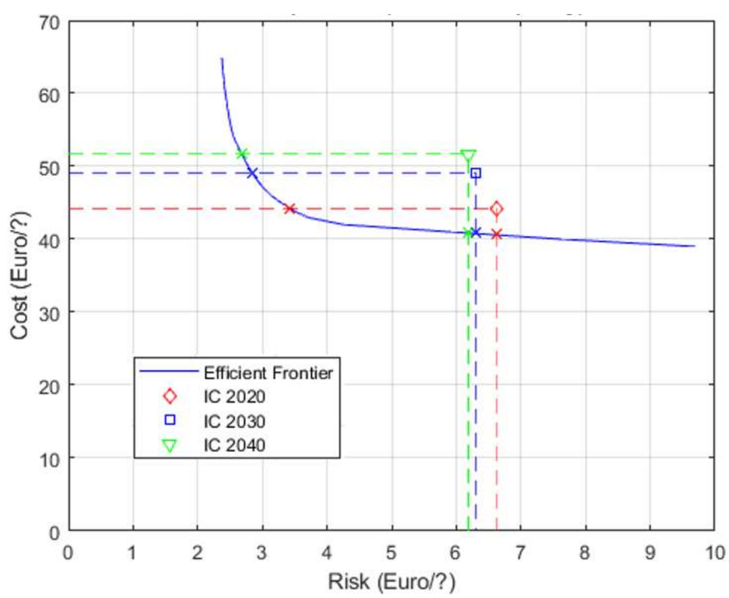
(Source: Author's own development)

As previously discussed, the discussion of the results found in this section ("4.3.2 Efficient Portfolios for Installed Capacity") is highly limited, since the severity of the simplification used. A possible interpretation is that the optimal portfolios found here indicate the optimal installed capacity distribution in a scenario where the maximum capacity is used.

Clearly, the results for installed capacity go in the same direction of the results for electricity generation. This is not a surprise since both problems (electricity generation and installed capacity) used the same covariance matrixes and the same values of expected costs. The only input that changed from one model to the other was the allocation of projected portfolios ($x_{t,p}$), however even this input does not vary much numerically.

The following figure (Figure 36) shows where these optimal portfolios are positioned in the efficient frontier.

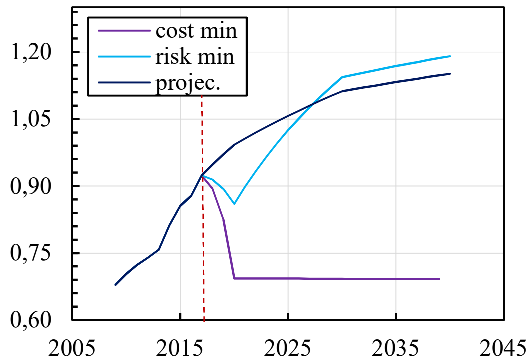
Figure 36 – Efficient Portfolios for Installed Capacity



(Source Authors own calculation)

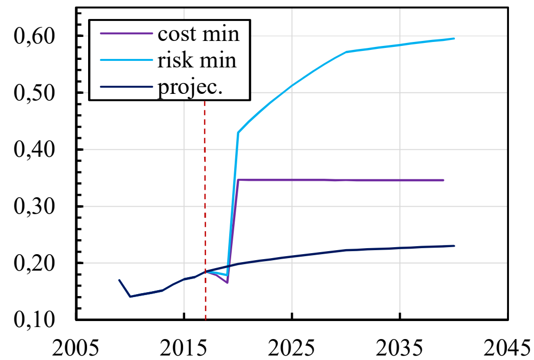
The following figures (from Figure 37 to Figure 39) show how diversity would vary in a scenario where efficient portfolios materialized rather than projections. Again, the calculated efficient portfolios refer to the years 2020, 2030 and 2040 only: from 2009 to 2017, the historical series is repeated in the charts for comparison purposes.

Figure 37 – Comparison of efficient portfolios and projections for electricity generation: Shannon Index



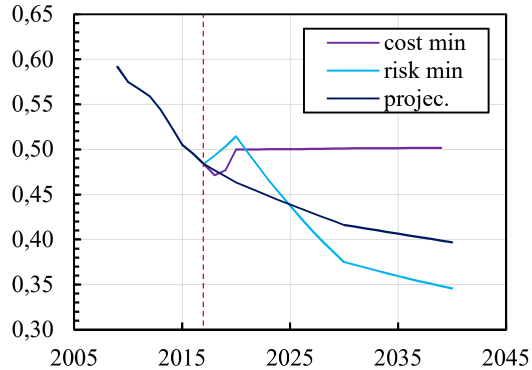
(Source: Author's own calculation)

Figure 38 – Comparison of efficient portfolios and projections for electricity generation: S. Equitability



(Source: Author's own calculation)

Figure 39 – Comparison of efficient portfolios and projections for electricity generation: HHI



(Source: Author's own calculation)

When analyzing the diversity of efficient portfolios for installed capacity, one can notice that the results also are similar to the ones observed when analyzing the diversity of efficient portfolios for electricity generation. That is, the minimum-cost portfolios generally is accompanied by worsened diversity indexes, while minimum-risk portfolio is accompanied by ameliorated diversity indexes. However, for installed capacity there are two more exceptions (besides the Shannon’s equitability of minimum-cost portfolios, which was already discussed): the Shannon-Weiner index and HHI for the 2020 efficient portfolio. This efficient portfolio, specifically, suggests and increase of installed capacity of hydropower plants, which is already dominant in the matrix, diminishing diversity in the system.

5. Conclusion

Clearly, the results reaffirmed the historical importance of hydropower plants and thermoelectric plants for the matrix. Further, it concluded that the slight decrease in the relative importance of hydropower plants throughout the recent years is mainly due to an increase in the installed capacity and electricity generation by wind power plants. Nuclear and solar power plants still play a marginal role, offering less than 2% of the total installed capacity (each).

Historically, diversity in installed capacity has consistently improved with small changes in the rate at which indexes changed. Energy generation's diversity has also presented a quite consistent improvement, with only two exceptions (2010-2011 and 2015-2016). The general improvement in the diversity is mainly due to the growth of electricity production by wind, biomass, natural gas and coal energy.

Projections for electricity generation and for installed capacity go in the same behavior: both point out to a slight decrease in the importance of hydropower and thermoelectric energy and indicate an increase in the importance of wind, solar and nuclear energy. The results indicate none of the projections is an efficient portfolio. Besides, by analyzing diversity index, projections indicate either a stabilization of matrix diversity (electricity generation) or a slower increase in the level of diversity (installed capacity), in contrast to the significant growth in diversity observed from 2009 to 2017.

When analyzing the efficient portfolios, it was concluded that minimum-risk portfolios suggest a more feasible scenario than minimum-cost portfolios, since the diversity offered by the latter is extremely low. Minimum-risk portfolios suggest a high importance to hydraulic, natural gas energy, wind, nuclear and coal electricity generation; in contrast, biomass, petroleum products and other sources of energy have low or zero importance.

Lastly, it was concluded that increasing portfolio efficiency by MPT is not always accompanied by an improvement in portfolio diversity: in this model, risk minimization might lead to diversity improvement, while cost minimization leads to quite drastic diversity detractions. Besides, although the diversity index has been enhanced in the minimum-risk portfolios, it clearly is not the optimal diversity condition. Meaning that, in this model, minimization of risk does not necessarily implicates in maximization of diversity.

6. Suggestions for Future Works

As a proposal for future works, it is suggested:

1) The construction of a model following MPT with data specifically tailored to Brazil (expected cost data and covariance matrix), building an efficient frontier which fits better to the country's possibilities;

2) Use specific estimates for the period of each portfolio, considering the effects of learning economies and technology evolution, making the projections for 2030 and 2040 more accurate;

3) Use specific estimates to calculate installed capacity optimization, instead of applying the same estimates used for electricity generation;

4) Use more recent projections, calculated more recently;

5) Consider the real possibilities of expansion of each source, in order to avoid that the model suggestion exceeds a limit imposed by the local geography;

6) Lastly, verify how the electricity distribution networks, the demand side and the extent of power of each stakeholder fit into this effort to optimize the choice of matrix.

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Annex

Table A1 – Estimates of expected daily return and risk

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
μ_j	3,94262E-03	2,59873E-03	1,16624E-03	3,57215E-03	2,55063E-03	2,28059E-03	3,25907E-03	1,91055E-03	3,31055E-03	2,89873E-03
σ_j	2,50601E-02	2,19063E-02	1,94651E-02	2,95035E-02	2,43129E-02	2,00720E-02	2,63274E-02	2,15657E-02	2,67730E-02	2,48598E-02

(Source: Authors' own calculation)

Table A2 – Estimate of covariance matrix

	X_1	X_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	6,28007E-04	3,88092E-04	1,70805E-04	4,45088E-04	4,14598E-04	3,16584E-04	4,09129E-04	2,34718E-04	3,11113E-04	3,64700E-04
x_2	3,88092E-04	4,79885E-04	1,92923E-04	4,41810E-04	4,64833E-04	3,24722E-04	3,87803E-04	2,22322E-04	3,18211E-04	3,72084E-04
x_3	1,70805E-04	1,92923E-04	3,78892E-04	2,70694E-04	2,26893E-04	1,46174E-04	2,24935E-04	1,47576E-04	1,76554E-04	1,86610E-04
x_4	4,45088E-04	4,41810E-04	2,70694E-04	8,70457E-04	4,60847E-04	4,31390E-04	6,27634E-04	2,75395E-04	5,21204E-04	5,66950E-04
x_5	4,14598E-04	4,64833E-04	2,26893E-04	4,60847E-04	5,91115E-04	3,52595E-04	4,24195E-04	2,64804E-04	3,50495E-04	4,22760E-04
x_6	3,16584E-04	3,24722E-04	1,46174E-04	4,31390E-04	3,52595E-04	4,02885E-04	3,95311E-04	2,03793E-04	3,16620E-04	3,89222E-04
x_7	4,09129E-04	3,87803E-04	2,24935E-04	6,27634E-04	4,24195E-04	3,95311E-04	6,93130E-04	2,43416E-04	5,13592E-04	5,05323E-04
x_8	2,34718E-04	2,22322E-04	1,47576E-04	2,75395E-04	2,64804E-04	2,03793E-04	2,43416E-04	4,65078E-04	1,40432E-04	2,39989E-04
x_9	3,11113E-04	3,18211E-04	1,76554E-04	5,21204E-04	3,50495E-04	3,16620E-04	5,13592E-04	1,40432E-04	7,16795E-04	4,09241E-04
x_{10}	3,64700E-04	3,72084E-04	1,86610E-04	5,66950E-04	4,22760E-04	3,89222E-04	5,05323E-04	2,39989E-04	4,09241E-04	6,18008E-04

(Source: Authors’ own calculation)

Table A3 Costs by technology [€/MWh]

		Possible energy sources (T)											
		Nuclear	Coal	Coal (CCS)	Natural gas	Natural gas (CCS)	Oil	On-shore wind	Large Hydro	Small Hydro	Off-shore wind	Biomass	Solar PV
Type of costs (C)	Investment	9.17	8.24	14.42	9.89	20.67	23.58	26.67	26.63	29.96	28.57	20.44	170.21
	O&M	10.24	9.89	21.63	9.89	20.67	16.27	22.00	11.98	12.98	33.21	9.20	29.79
	Fuel	7.48	15.75	20.79	10.11	11.78	39.66	0.00	0.00	0.00	0.00	66.93	0.00
	Complement.	3.15	N/A	19.07	N/A	9.25	N/A	12.03	N/A	N/A	12.03	N/A	12.03
	CO_2	N/A	18.35	2.52	8.90	1.22	13.66	N/A	N/A	N/A	N/A	0.05	N/A
	SO_2	N/A	0.58	0.17	0.07	0.08	0.44	N/A	N/A	N/A	N/A	1.20	N/A
	NO_x	N/A	1.51	1.44	2.11	2.35	1.13	N/A	N/A	N/A	N/A	3.30	N/A
	PM	N/A	0.27	0.22	0.03	0.03	0.20	N/A	N/A	N/A	N/A	5.46	N/A
	Radioactivity	4.16	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	Land use	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	3.43	N/A
	Accident plant	23.00	0.06	0.06	0.09	0.09	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Total cost by tech.		**57,2**	**54,7**	**80,3**	**41,1**	**66,2**	**94,9**	**61,7**	**38,8**	**43,3**	**74,9**	**109,5**	**212,6**

(DELLANO-PAZ et al, 2014)

Table A4 Standard deviation by technology [€/MWh]

		Possible energy sources (T)											
		Nuclear	Coal	Coal (CCS)	Natural gas	Natural gas (CCS)	Oil	On-shore wind	Large Hydro	Small Hydro	Off-shore wind	Biomass	Solar PV
Type of costs (C)	Investment	2.11	1.90	3.32	1.48	3.10	5.42	1.33	10.12	3.00	2.86	4.09	8.51
	O&M	0.56	0.53	1.17	1.04	2.17	3.94	1.76	1.83	1.99	2.66	0.99	1.01
	Fuel	1.80	2.20	2.91	1.92	2.24	9.92	N/A	N/A	N/A	N/A	12.05	0.00
	Complement.	0.29	N/A	5.00	N/A	5.00	N/A	6.07	N/A	N/A	6.07	N/A	6.07
	CO_2	N/A	4.77	0.66	2.31	0.32	3.55	N/A	N/A	N/A	N/A	0.01	N/A
	SO_2	N/A	3.13	3.13	3.13	3.13	3.13	N/A	N/A	N/A	N/A	3.13	N/A
	NO_x	N/A	3.26	3.26	3.26	3.26	3.26	N/A	N/A	N/A	N/A	3.26	N/A
	PM	N/A	2.65	2.65	2.65	2.65	2.65	N/A	N/A	N/A	N/A	2.65	N/A
	Radioactivity	2.39	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0.00	N/A
	Land use	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	1.07	N/A
	Accident plant	6.64	0.14	0.14	0.04	0.04	0.14	N/A	N/A	N/A	N/A	N/A	N/A
Standard Dev. by tech.		**7.61**	**7.68**	**8.59**	**6.31**	**8.48**	**13.54**	**6.46**	**10.29**	**3.59**	**7.21**	**13.84**	**10.50**

(DELLANO-PAZ et al, 2014)

Table A5 – O&M correlation coefficients

	Possible energy sources (T)											
	Nuclear	Coal	Coal (CCS)	Natural gas	Natural gas (CCS)	Oil	On-shore wind	Large Hydro	Small Hydro	Off-shore wind	Biomass	Solar PV
Possible energy sources (T)												
Nuclear	1.00	0.00	0.00	0,24	0,24	-0,17	-0,07	-0,41	-0,41	-0,07	0,65	0,35
Coal	0.00	1.00	1.00	0,25	0,25	-0,18	-0,22	0,03	0,03	-0,22	0,18	-0,39
Coal (CCS)	0.00	1.00	1.00	0,25	0,25	-0,18	-0,22	0,03	0,03	-0,22	0,18	-0,39
Natural gas.	0,24	0,25	0,25	1.00	1.00	0,09	0.00	-0,04	-0,04	0.00	0,32	0,05
Nat. gas (CCS)	0,24	0,25	0,25	1.00	1.00	0,09	0.00	-0,04	-0,04	0.00	0,32	0,05
Oil	-0,17	-0,18	-0,18	0,09	0,09	1.00	-0,58	-0,27	-0,27	-0,58	0,01	-0,04
On-shore wind	-0,07	-0,22	-0,22	0.00	0.00	-0,58	1.00	0,29	0,29	1.00	-0,18	0,05
Large hydro	-0,41	0,03	0,03	-0,04	-0,04	-0,27	0,29	1.00	1.00	0,29	-0,18	0,30
Small hydro	-0,41	0,03	0,03	-0,04	-0,04	-0,27	0,29	1.00	1.00	0,29	-0,18	0,30
Off-shore wind	-0,07	-0,22	-0,22	0.00	0.00	-0,58	1.00	0,29	0,29	1.00	-0,18	0,05
Biomass	0,65	0,18	0,18	0,32	0,32	0,01	-0,18	-0,18	-0,18	-0,18	1.00	0,25
Solar PV	0,35	-0,39	-0,39	0,05	0,05	-0,04	0,05	0,30	0,30	0,05	0,25	1.00

(DELLANO-PAZ et al, 2014)

Table A6 – Fuel and CO_2 correlation coefficients

	Nuclear	Coal	Natural gas	Oil	Biomass	CO_2
Nuclear	1.00	0.97	0.99	0.88	-0.31	0.89
Coal	0.97	1.00	0.92	0.97	-0.53	0.99
Natural gas.	0.99	0.92	1.00	0.79	-0.15	0.97
Oil	0.88	0.97	0.79	1.00	-0.72	0.92
Biomass	-0.31	-0.53	-0.15	-0.72	1.00	-0.40
CO_2	0.89	0.99	0.97	0.92	-0.40	1.00

(DELLANO-PAZ et al, 2014)

Table A7 – Taiwanese energy supply structure – case replication (Source: Author's own calculation).

	p i * 100 = Energy Supply %											
	Coal & deriv.	Crude Oil & deriv.	Natural Gas	Waste and Biofuel	Hydro.	Nuclear	Wind	Solar	HHI1	SWI1	1/SWI1	HHI1*
1997	28,40	52,47	5,76	0,54	0,58	12,17	0,00	0,08	0,374	1,180	0,966	0,964325
1998	27,81	52,41	6,55	0,63	0,66	11,86	0,00	0,08	0,370	1,197	0,953	0,954795
1999	28,70	51,81	6,33	0,83	0,51	11,75	0,00	0,08	0,369	1,198	0,952	0,950271
2000	29,69	51,59	6,35	0,92	0,43	10,94	0,00	0,08	0,370	1,191	0,957	0,954657
2001	30,50	51,60	6,54	1,19	0,46	9,65	0,00	0,08	0,373	1,191	0,957	0,961427
2002	30,85	50,39	6,98	1,21	0,24	10,26	0,00	0,08	0,365	1,201	0,949	0,939791
2003	30,34	51,85	6,71	1,38	0,24	9,41	0,00	0,07	0,374	1,185	0,962	0,965071
2004	30,12	52,34	7,36	1,26	0,23	8,61	0,00	0,07	0,378	1,178	0,968	0,97336
2005	29,63	52,78	7,35	1,24	0,28	8,63	0,01	0,07	0,379	1,178	0,968	0,97778
2006	30,12	52,10	7,73	1,23	0,29	8,44	0,02	0,07	0,375	1,186	0,962	0,967581
2007	30,06	52,38	7,79	1,22	0,29	8,15	0,03	0,07	0,378	1,181	0,965	0,973184
2008	30,29	50,76	8,76	1,29	0,30	8,49	0,04	0,08	0,364	1,211	0,941	0,939341
2009	28,24	52,56	8,73	1,25	0,26	8,82	0,06	0,08	0,372	1,203	0,948	0,957654
2010	29,52	50,05	10,34	1,24	0,28	8,43	0,07	0,08	0,356	1,231	0,926	0,916505
2011	31,66	46,08	11,72	1,30	0,28	8,78	0,11	0,08	0,334	1,272	0,896	0,861329
2012	30,02	47,79	12,06	1,30	0,38	8,26	0,11	0,08	0,340	1,266	0,900	0,876446
2013	30,54	47,38	11,88	1,27	0,36	8,36	0,13	0,08	0,339	1,267	0,900	0,8738
2014	29,57	48,35	12,13	1,20	0,28	8,26	0,13	0,08	0,343	1,257	0,907	0,883766
2015	29,67	48,16	13,19	1,23	0,29	7,22	0,16	0,08	0,343	1,256	0,907	0,883358
2016	29,34	48,90	13,66	1,19	0,43	6,25	0,17	0,08	0,348	1,248	0,914	0,896737
2017	30,17	48,45	15,15	1,15	0,36	4,43	0,22	0,08	0,351	1,227	0,929	0,904197

(Source: Author's own calculation)

Table A8 – Brazil installed capacity by source [GW]

	2009	2010	2011	2012	2013	2014	2015	2016	2017
Total	**110,44**	**116,38**	**117,14**	**121,10**	**126,74**	**133,91**	**140,86**	**150,34**	**157,11**
Hydropower Plants	76,78	78,61	78,37	79,75	81,13	84,09	86,37	91,50	94,66
Thermoelectric Plants	27,48	30,78	31,24	32,91	36,53	37,83	39,56	41,27	41,63
SHP	3,40	3,87	3,87	4,30	4,62	4,79	4,89	4,94	5,02
CHG	0,17	0,19	0,22	0,24	0,27	0,31	0,40	0,48	0,59
Nuclear Power Plants	2,01	2,01	2,01	2,01	1,99	1,99	1,99	1,99	1,99
Wind Power Plants	0,60	0,93	1,43	1,89	2,20	4,89	7,63	10,12	12,28
Solar Power Plants	0,00	0,00	0,00	0,00	0,01	0,02	0,02	0,02	0,94

(EMPRESA DE PESQUISA ENERGÉTICA, 2014), (EMPRESA DE PESQUISA ENERGÉTICA, 2018)

Table A9 – Brazil electricity generation by source [TWh]

	2009	2010	2011	2012	2013	2014	2015	2016	2017
Total	**466,16**	**515,80**	**531,76**	**552,50**	**570,83**	**590,54**	**581,23**	**578,90**	**587,96**
Natural Gas	13,33	36,48	25,10	46,76	69,00	81,07	79,49	56,48	65,59
Hydraulics	390,99	403,29	428,33	415,34	390,99	373,44	359,74	380,91	370,91
Petroleum products	12,72	14,22	12,24	16,21	22,09	31,53	25,66	12,10	12,73
Coal	5,43	6,99	6,49	8,42	14,80	18,39	18,86	17,00	16,26
Nuclear	12,96	14,52	15,66	16,04	15,45	15,38	14,73	15,86	15,74
Biomass	21,85	31,21	31,63	34,66	39,68	44,99	47,39	49,24	49,39
Wind	1,24	2,18	2,70	5,05	6,58	12,21	21,63	33,49	42,37
Other	7,64	6,92	9,61	10,01	12,24	13,54	13,73	13,81	14,98

(EMPRESA DE PESQUISA ENERGÉTICA, 2014) and (EMPRESA DE PESQUISA ENERGÉTICA, 2018)

Table A10 – Percentage of projected electricity generation by source

	2020	2030	2040
Coal	3.77%	2.63%	2.06%
Oil	2.20%	1.44%	1.12%
Gas	8.65%	7.66%	11.79%
Nuclear	4.09%	3.71%	3.65%
Hydro	65.25%	64.67%	60.62%
Bioenergy	7.55%	7.07%	6.55%
Wind	7.70%	10.54%	11.13%
Geothermal	0.00%	0.00%	0.00%
Solar	0.79%	2.16%	2.81%
CSP	0.00%	0.12%	0.28%
Marine	0.00%	0.00%	0.00%

(Source: Author's own calculation)

Table A11 – Percentage of projected Installed Capacity by source

	2020	2030	2040
Coal	2,94%	2,29%	1,47%
Oil	4,71%	3,21%	2,56%
Gas	10,00%	10,55%	12,09%
Nuclear	1,76%	1,83%	1,83%
Hydro	62,35%	58,72%	57,14%
Bioenergy	8,24%	7,80%	6,96%
Wind	8,24%	11,01%	11,36%
Geothermal	0,00%	0,00%	0,00%
Solar	1,76%	4,59%	6,23%
CSP	0,00%	0,00%	0,37%
Marine	0,00%	0,00%	0,00%

(Source: Author's own calculation)