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Master of science in Mechatronic engineering

Master Degree Thesis

Learning from humans to improve Socially-Aware motion planning

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Abstract

In the near future, one of the most likely scenarios is the daily coexistence of humans and robots. This scenario will be witnessed, for instance, in assistive robots for seniors, cleaning robots, reception robots in public or private shops.
Although there already exist technological tools to perform a secure motion in a crowded environment, not too many authors have studied if a robot trajectory would be pleasant and accepted by humans. The thesis fits in this scenario and is focused on the trajectory generation for a mobile robot in order to be acceptable and not annoying humans.
How can we reach this ambitious goal?
Humans are more successful in planning a collision-free trajectory with mutual avoidance manoeuvres in a populated environment than any motion planning algorithm developed so far.
While humans can easily deal with predicting the motion of surrounding people, robotic systems are still facing problems.
Moreover, humans tend to attribute intentions and consciousness to non-human entity. In literature, this behaviour is named *anthropomorphizing*. Here, we leverage anthropomorphizing to improve human-robot interaction and, thus, raise the acceptance for humans. This is achieved by attempting to design the robot motion in a *human-like* manner.
For these reasons, we want to improve the robot navigation starting from the study of humans' decisions.
The majority of the approaches concentrate their attention to predict human motion individually without considering the interaction between humans.
Here, we model the motion of humans considering the interaction between and with pedestrians using a game-theoretic approach.
Generally, many motion planning approaches at the state of the art have a *reactive* behaviour, i.e. they avoid obstacles without considering the prediction of human motion and the interaction with them.
Game theory has a lot of advantages over reactive methods, in fact, it can model the mutual anticipation of the influence of other agents and adapt their own decisions based on the possible actions of others.
In this thesis, non-cooperative game theory is applied to predict the decision of multiple humans that interact with each other during navigation.
Hence, the concept of Nash equilibrium in dynamic games is applied to solve our model.
In the last decades, some scientists studied human motion from a game-theoretic point of view, but their models comprised two people. Moreover, static obstacles

and groups of people were not accounted for. Here, we have reformulated the problem considering a different cost function, extending the model considering multiple people and detecting groups of them, as well as evaluating patterns of *natural* interaction between humans and static objects.
The model has been validated with real-world surveillance videos, *qualitatively* comparing real trajectories and the output of our model.
In the second part of the dissertation, we used the model previously developed, to create a human-like trajectory for an autonomous robot that navigates among multiple humans, considering the robot as a player of the game.
In the end, with a variation of the Turing test, we tried to evaluate *quantitatively* whether the final motion robot planning is socially acceptable for humans. This quality has been measured considering the *human likeness* of the trajectory generated by our motion planner.
50 volunteers participated in our test and the collected data validates the proposed approach.

Chapter 1

Introduction and motivation

The adoption of robots in our daily life causes an important issue about the necessity of humans to interact with them. Generally, in industrial applications, robots are completely separated from the humans' workspace, but this is not the case in the near future applications, where robots will share the same environment with humans. In order to accomplish a diverse set of objectives, navigation is an essential task for autonomous mobile robots. As a matter of fact, to improve the collaboration with humans, the robot should ensure:

- *human-likeness motion*: i.e., the robot's trajectory should follow a smooth behaviour similar to human motion;
- *safe motion*: i.e., the robot does not harm the human in a physical or psychological way;
- *reliable and effective motion*: i.e., the robot executes the task adequately and reaches the goal considering its motion limit (i.e. minimum turning radius of the robot, maximum linear and angular velocities, etc.);
- *interaction awareness*: i.e., the robot anticipates the human mutual collision avoidance.

If a robot satisfies the above conditions, its interaction with humans becomes easier and more intuitive for humans.
In particular, our object is to design a robot's motion that is acceptable for humans. How to measure the social acceptance of a robot?
Humans attribute sometimes, intentions and consciousness to non-human agent [55]. The term to explain this behaviour is *anthropomorphizing*. In this thesis, we use anthropomorphizing to improve human-robot interaction [50]. This is achieved by attempting to design the robot motion in a *human-like* manner.
This perspective opens up important research opportunities.
Since the robot should move like a human and should "know" the intentions of

humans, a model of human motion behaviour is necessary to compute a robust socially-aware motion planning.

Definition 1 *Socially-aware navigation is the strategy exhibited by a social robot which identifies and follows social conventions in order to preserve a comfortable interaction with humans. The resulting behaviour is predictable, adaptable and easily understood by humans [41].*

What are the advantages of modelling human's intentions from the robotic point of view?
A first benefit in predicting the human motion is that humans are no longer recognised as dynamical obstacles but as social entities that interact with other pedestrians.
In addition, the output of a human motion model can be used as a source of information that allows robot to adapt its motion according to the predicted human motion. Robot can promptly respond in a safe manner (Figure 1.1). Moreover, robot can use their predictions of human motion to generate its own motion planning, in order to achieve a given task. In this way, the "intention" of the robot becomes more intelligible and natural to be predicted by humans [5]. Then, this, increases the social acceptance of robots in daily life.
Last but not the least, it may happen that the robot locks in a place (or executes unnecessary manoeuvres) while trying to avoid collisions with humans, since all available trajectories are unsafe, it remains stuck in a deadlock. The name of this situation is "freezing robot problem" (FRP), which can be avoided, for example, by implementing an socially-aware motion navigation [48].

Figure 1.1: Socially-aware navigation in a crowd of humans [49].

From the literature, it is possible to summarise the requirements that socially-aware navigation should have [23]:

- *Respect personal space.* When humans interact with others, they feel annoyed if others are too close or too far away from their own personal space (the proxemic interpersonal distance is shown in Table 4.1 and the personal space in Figure 1.2**a**).
  In particular, if the distance to someone is excessive, it indicates a dislike. In this regard, also the robot must keep an appropriate social distance with humans to avoid fear and discomfort.
- *Respect activity spaces.* Robot should avoid the space where humans can perform actions. In the related work [29], there is not a precise definition of the shape of the *activity space* because it depends on the type of humans activities.
- *Respect group of agents zone.* In the literature, the area shared among interacting people is called O-space [20], as shown in Figure 1.2**b**. In general, the geometrical shape of the O-space depends on the posture and orientation of humans. In [20] the authors show that, generally, humans are placed in a circle.
- Avoid *weird motion* or *noises* that could cause a distraction for humans.
- Modulate the *velocity* based on the distance from humans or maintain the same speed similar to human walking speeds. We used the latter situation to increase the possibility of anthropomorphizing the robot as in [34].

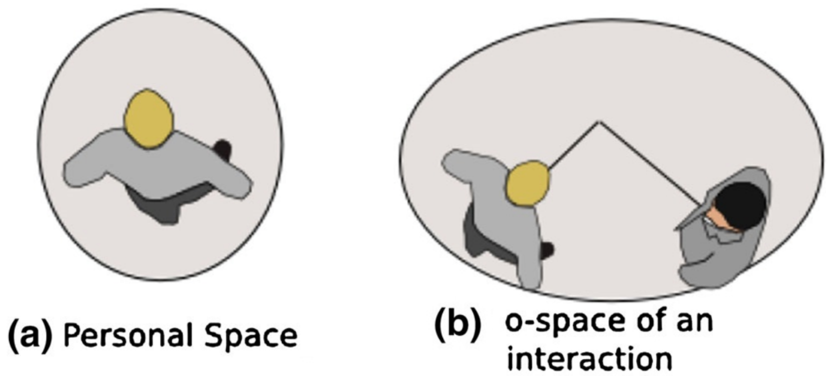


Figure 1.2: During navigation, the robot should take into account: **a**) Personal space; **b**) O-space of an interaction [41].

The following section starts with the presentation of our human motion model and proceeds with the description of the requisites that have been implemented among of those presented above, to achieve the goal of the socially-aware navigation.

1.1 Our approach

Game theory is a branch of mathematics that is useful to model situations in which players make decisions that are interdependent with other participants. Indeed, game theory is a powerful tool to investigate optimal decision-making by rational players considering the other player's possible decisions.
Before proceeding with the dissertation, for more clearness, it is necessary to underline that from now the terms *player, pedestrian, person* and *human* are used as synonyms.
Moreover, with the term *agent*, we denote any mobile entity, either human or robot.
This study uses a game-theoretic approach to model the decision process of pedestrians in a dynamic environment. Players are considered as *rational agents*, i.e. they try to maximise their profits.
In general, we adopt a *microscopic* approach, i.e. we focus on predicting both the interaction and the decision processes between humans in a crowd without considering the overall movement of the crowd (macroscopic analysis).
In this dissertation the interaction-aware decision making is modelled as a non-cooperative, dynamic and non-zero-sum game (this nomenclature will be explained in more detail in Chapter 3).
The pedestrians respect the other personal zones by maintaining a certain distance based on the relationship between players.
In our approach, we consider the achievement of the Nash equilibrium [35] as the solution of the game. The Nash equilibrium is the best response for all players, though we give a more formal definition of Nash equilibrium in Chapter 3.
Generally, many human motion approaches at the state of the art have a reactive behaviour, i.e. they avoid obstacles without considering the human motion and the interaction with them [16, 17].
Our approach overcomes the reactive methods (see Section 2.1.1), since with game theory we can predict the motion of other pedestrians. Thus, each player can adapt its own decisions accordingly to the others strategies. In this way, our model is considered *predictive*.
Recently (see Section 2.1.4), some authors studied human motion from a game-theoretic point of view considering only two players, without focusing on the recognition of groups, and avoiding a static obstacle in a human-like way [49].
Here, we have reformulated the problem considering a different cost function, extending the model considering multiple people and detecting groups of them, as well as evaluating patterns of *natural* interaction between humans and static objects.
The group recognition is important because if a robot identifies a group, it avoids to move through or too close to it, thus, increasing the quality of the interaction with humans.

Based on the prediction from our model, we created a socially-aware motion planning for an autonomous robot to navigate in a crowd. Thus, since our trajectory is inspired by our human motion model, during navigation the robot satisfies some of the requirements that we described in the previous section. In particular, the robot does not invade the human personal zone and conserves the same speed as a human. Moreover, the robot avoids weird motion because the cost function has a term connected to the smoothness of the final trajectory.
The respect of the activity space is out of the scope of this dissertation, since we are studying human motion in urban space, without involving specific activities.
About the recognition of the O-space shared by a group of people that are stuck in a place, the robot identifies that group as a static obstacle, thus, it will maintain a certain safe distance and it will avoid to pass through it.
In a nutshell, the final objects of this thesis are:

1. Modelling the intention of humans in populated environment using game theory.

2. Improving existing motion planning for mobile robots, based on the model of human behaviour described in the previous point.

1.2 Thesis organisation

The rest of the thesis is organised as follows.
Chapter 2 reviews models of pedestrian motion and the state of the art of reactive robot navigation. The latter, will be used to implement the non-player robot for the videos that we designed for the final experiment.
Chapter 3 illustrates the preliminaries on game theory. The detailed description of our model is shown in Chapter 4. In Section 5, we discuss and validate the game theoretical model through real-world surveillance videos. Besides, we describe the setup and details of our variation of Turing experiment used to validate the human motion model and the human likeness of our socially-aware motion planner.
Chapter 6 reports results coming from the collected data analysis of our experiment.
Finally, Chapter 7 draws the conclusions of this dissertation and devises future avenues of research.

Chapter 2

Related work

When a mobile robot moves in a dynamic scenario, where humans are goal-directed, the sensors perceiving the environment and detect the human positions.
If the robot estimates the human motion with a realistic model, the robot can navigate safely and comfortably with humans.
Thereby, a performing socially-aware navigation is strictly connects to a deep research in human motion models.

2.1 State of art of human motion prediction

Lots of modelling and simulating pedestrian motion have been developed in the last years in different areas to reach different objects such as: simulating evacuation movements to design a safely public or private building, human motion analysis for socially-aware robot navigation or gaming computer animation.
In general, the research studies about predict human decision in a crowd can be divided into two big approaches: macroscopic and microscopic.
The *macroscopic* approach is normally used in the case of large crowd considering the group of people as a whole. The *macroscopic* model is less computationally intense than the *microscopic* model, because it considers fewer details among individuals interactions and between individuals and the environment.
On the other hand, the *microscopic* description focuses on predicting the interaction and the decision processes between humans in a crowd without considering the overall movement of the crowd.
In this dissertation, since one of the goals is find a good model that is able to predict the human motion in a crowd, we will take into account only the *microscopic* approach.
We present in the following section, the state of the art of human motion prediction considering an overview of the reactive models, predictive planners, learning-based strategies and game theory models.

In the scientific literature, there is not a stringent separation between the geometric reasoning approach and the learning technique, because both models have things in common and can be combined to obtain a hybrid approach.
Nevertheless, in this thesis, for simplicity, we try to divide the publications considering the approach they focus on.
For each class, we consider the advantages and disadvantages and we motivate the choice of game theory.
In the end, we present an overview of robot navigation because it will be useful for the test explained in Chapter 5.

2.1.1 Reactive model

Pioneering work of modelling humans as reactive particles is the social force method [16]. In this model, the agents navigate in the environment considering attractive forces that guide them towards the destination and repulsive forces that ensure collision-avoidance depending on their relative distances (Figure 2.2a). It generates plausible patterns about global motion. On the other hand, in local level, individual trajectory is not human-like.
In most of the available research [2], the scientists assume a static world, where the human prediction is simplified with the assumptions of proceeding to walk straight with current direction and speed (Figure 2.2b).
One of the well-known models based on grid motion decisions is the Cellular Automaton (CA). This technique uses a discrete representation for the environment and the human decision motion. The first research about CA is conducted by Tadokoro et al. [46]. Human motion is predicted using a probability distribution maps of movement in the near future, considering the status of neighbouring cells. The main idea is visualised in Figure 2.2c, where the darkness of grid maps represents the likelihood of the transition.
Schadschneider [44] proposes the concept of floor field to improve the CA technique. Starting from the CA grid map, Schadschneider adds a second grid of cells that can be static or dynamic. The first one does not depend on time and it is used to model the most attractive region in the map (for instance an emergency exit). On the other hand, the dynamic floor is adopted to take into account the interaction between agents.
In contrast to this discrete approach, there are several instances of continuum models. For example, Hoeller et al. [17] define the human motion as the combination of attractive and repulsive potential fields. The first one guides the pedestrian to the possible destination and the other one is useful to avoid collision with obstacles. Since the true destination of a person is unknown, Hoeller defines an attractor that guess the possible destinations (Figure 2.2d).

The models examined so far consider the pedestrians as *passive* subjects that react only to external forces (Figure 2.1a). In this way, they ignore the *interactive* nature of humans. Indeed, pedestrians are *active* agents that, during navigation, repeat decision-making progress based on the surrounding humans prediction. This aspect is taken into account in the predictive planner (as shown in Figure 2.1b) and in the game theory models.

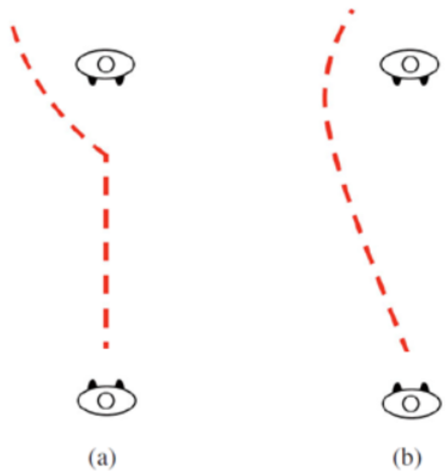


Figure 2.1: **a**) Reactive based planning: the pedestrian changes its direction when another human appears in his way; **b**) Predictive planning: the human first predicts the motion of the other agent and then, it calculates its path considering the mutual avoidance manoeuvres in advance.

2.1.2 Predictive planners

Paris et al. [36], based on Fiorini and Shiller [8] approach, create a velocity-based model that improves the Fiorini's work. The main advantage of Paris' technique is the resolution of the oscillation between velocities due to the lack of the anticipation of the surrounding agents. With this method, the reference entity is not repelled by neighbouring pedestrians and static object, but humans actively find a free path through the crowd. The considered agent computes the path planning evaluating: the reachable space regarding all directions and a limited set of velocities, simultaneously, it researches for the possible collision with the surrounding pedestrians, thus, it finds the optimal path for the near future.
Then, Karamouzas et al. [19], based on the Paris' approach, tried to reduce the computational effort focusing more on upcoming collisions.
In the last year, Warren [54] builds a model where the pedestrian motion is computed considering the superposition of 3 frameworks: the closest, the furthest and

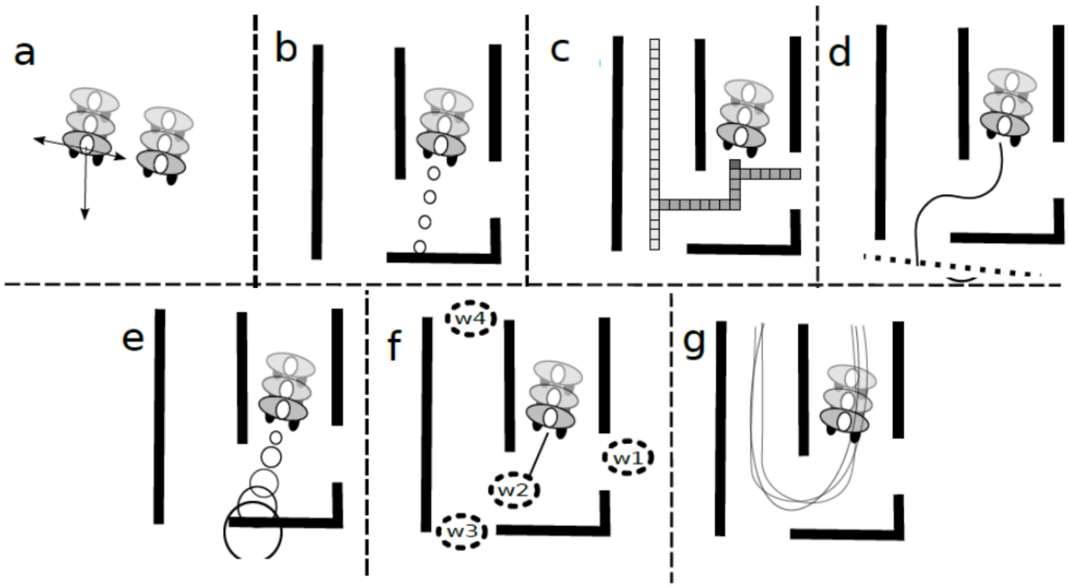


Figure 2.2: Example of human motion prediction in literature: **a**) Social Force model; **b**) Linear; **c**) Probable state transition in a grid map; **d**) Using potential field; **e**) Growing uncertainty; **f**), **g**) Machine Learning technique [23].

the intermediate zone.
The closest pedestrians area is the repulsion zone that corresponds to the personal Hall space [13], the furthest area is the zone where pedestrians move toward neighbours, instead, the intermediate zone is the alignment area where the agent matches the speed and the heading directions of neighbours.
In continuous models, the increasing prediction uncertainty is commonly represented as in Figure 2.2e. In this regard, Trautman et al. [48] study dense human crowds and evaluate the pedestrian motion considering the Gaussian process. The authors start from the "freezing robot problem", where the motion planner cannot compute safe decision because the environment is too crowded, thus, the robot freezes in a place or performs unnecessary motions. In order to solve this problem, the robot predicts human cooperation with interacting Gaussian processes.
The predictive methods adhere to the rules of human motion navigation. For this reason, predictive humans planners are more feasible and incorporate much social information than the reactive one.
On the other hand, the pedestrian prediction requires much computational effort, especially when the crowd density becomes significant.
Furthermore, since humans take stochastic decisions, the long-time prevision is significantly uncertain.

2.1.3 Learning approach

In the previous approaches, predictions are based on how pedestrians behave in general. In learning strategy, previsions are estimated considering the observations of humans *individually* in particular situations and environments.
Recently, the *machine learning technique* is becoming significantly popular due to the advantage of improving over time and adapting to special circumstances.
The greatest computational effort occurs offline with the training process that needs a large amount of data, especially when the number of pedestrians increases.
Bennewitz et al. [3] create an algorithm that has a set of trajectories as input. As start and target points, they identify the so called "resting place", where the pedestrians normally stop and stay for a certain amount of time.
The model learns observing human motion and clustering the trajectories into a set of motion patterns through the use of Expectation-Maximisation algorithm (Figure 2.2f).
Foka et al. in [9] forecast trajectories and velocity of humans using neural network approach. This technique is useful to predict non-linear behaviours for one step ahead prediction (Figure 2.2g).
More recent works have developed models with a recurrent neural network (RNN) to predict future human action as in [45]. However, this approach does not consider the behaviour of other people in proximity, for this reason, Alahi et al. [1] propose a social LSTM (Long short-term memory) that combine the forecasting of all agents inside a crowd and the common sense rule in a shared environment with a "Social pooling of hidden states". Nevertheless, this method evaluates pedestrian prediction near the considered human without analysing interactions between all agents. Furthermore, Alahi et al. have as output a single trajectory that is the "average behaviour".
In this regards, Gupta et al. [12] try to overcome these restrictions and build a model with Generative Adversarial Networks (GANs), that is also more competitive about computational complexity.
Liang et al. [27] outperform the social LSTM model and the social GAN, forecasting the future path and the possible activity in videos.
The main advantages of the machine learning technique are the natural behaviour trajectory because the model is trained using real trajectory data, and the capability to be accurate also in a complex crowded settings.
Learning structure incorporates the influence of other pedestrians considering the forecasting state of other agents (*interaction-awareness*).
Nevertheless, the main problem is the scenario generalisation, indeed if the environment changes, the model should be trained again. This approach should deal with the generalisation problem efficiency.

2.1.4 Game theory modelling

The game theory has been tested in different scenarios to demonstrate the capability of modelling cooperative behaviour of rational decision players. Some applications of game theory are: biology, computer science, psychology, economics, political science, evacuation process, electric power market, etc.
Despite the corroborated ability of the game theory to model different types of situations, in field of human motion prediction the literature is limited.
The pioneer of human motion prediction with the game theory is Hoogendoorn et al. [18]. The study analyses the pedestrian navigation in a simulated environment. In particular, the authors adopt *differential game* to model the human motion and describe pedestrians as optimal feedback oriented controller that tries to reach their goals minimising the cost of navigation, supposing the motions of the other agents. However, the game solution is not an equilibrium but is computed as an optimal control problem based on a pedestrian cost function that takes into account also the running cost of the other agents.
In another study, game theory is joined with Cellular Automata [47] and after some years, Mesmer and Bloebaum [32] present a model where human decision navigation is modelled considering game theory and velocity obstacle. Though, both mix methods are modelled for an emergency evacuation situation.
Turnwald et al. [49] analyse the interaction during human navigation in a microscopic way. They study five different cost functions for a non-cooperative game and evaluate the best result with a real experiment (the experimental players are only two). The most suitable cost function is related to the length of the trajectory. However, the main framework of this work is the examination and validation of the Nash equilibrium for human motion. In other words, they demonstrate that all players choose the trajectory that generate a Nash equilibrium.
More recently, Ma et al. [31] combine the game theory and the deep learning approach to forecast future trajectories of multi-agents. The authors used the Brown's fictitious game to predict the long-term navigation considering the interaction with the other players, instead, to customize the pattern for each pedestrian they used the learning approach from a *single* image.
A similar procedure is adopted in [40], where pedestrian motion is modelled as two games: the first is played with the closest agent in the visibility zone and the other one with all surrounding humans modelled as the learning-based game. The game with the nearest person is modelled as a static, non-cooperative and non-zero-sum game. The method is validated considering a microscopic and macroscopic approach.
In another work, Roy et al. [43] investigate the avoidance technique of two interacting pedestrians with the Fokker-Plank Nash game. The differential game is solved considering the scenario as an optimal control problem, namely, each player tries to minimise the collision cost function.
So far, we have shown an overview of methods considering the interaction with a

limited number of players. If the amount of pedestrians increases, the computational cost will become extremely hard to manage. To surmount this problem, some works adopt the mean field game to handle a big crowd as in [6] and [37].

2.1.5 Proposed approach

Our goal is to study and model the human motion based only on game theory.
Game theory has a lot of benefits over the methods presented above. Specifically, game theory overcomes the reactive method considering the possible pedestrian motion in advance. The learning approach is notably promising because can customise the trajectory of each pedestrian collecting human motion data from the sensor. This is at the same time a downside, because the model is not versatile for all scenarios but it depends on trial data. With game theory, human motion is not customised for each player, but the method is adaptable and can be used in a new environment without passing through the training data.
The exclusive work that could be compared with our scenario is the Turnald et al [49] method. Nonetheless, we overcome that model because we increase the number of players, that in [49] were two, and we build a different cost function. In addition, we also modelled the interaction between humans and static object considering the avoidance in a *natural* (i.e. in a human-like) way.
In our model, we consider also the group recognition that improves the computational performance of the game theory algorithm. The last two features are the main differences between the Turnald's work [49].

2.2 Robot navigation

Typically, the robot navigation can be summarised as follows:

- *Localisation*: is the method of figuring out where the robot is on the map;
- *Mapping*: is the process where the robot creates the map of the environment, starting from the knowledge of its pose (localisation);
- *Planning*: the process where the robot finds a sequence of valid configurations in order to reach its final goal;
- *Motion control*: moves the robot considering the path planning as a reference, to reach the given goal;

Robots use *active* localisation that combines the measurements from odometry (proprioceptive sensor) and from exteroceptive sensors through probabilistic filters. The most famous approaches are: extended Kalman filter and Particle filters (Monte Carlo Localisation methods). About mapping is possible to use two main methods: landmarks and occupation grids. The first one builds stochastic maps with a probabilistic description of static obstacles. Indeed, occupation grids create a map where each cell is associated with the occupation probability.
In general, when a robot is moving in an unknown environment prefers to not solve the two problems *separately* but *concurrently*.
For this reason, in most cases, robots use SLAM (Simultaneous Localisation And Mapping).
The analysis of localisation and mapping is out of this work.
We did the hypothesis that the robot has the MAP with *static obstacle* before it starts the navigation. Then, the robot uses the SLAM technique to localise itself and map the positions of *dynamic obstacles* (Figure 2.3a) and finally, the robot utilises our socially-aware motion planning.
On the other hand, we study also the non-player robot navigation (i.e. the reactive motion) to compare the two scenarios with a variation of the Turing test (Figure 2.3b).
The next section gives an outline of the state of the art motion controller for robotics, that we will use for the implementation of the non-player robot.

2.2.1 Motion controller review

The complementary framework of path planning with the motion controller (Figure 2.3**a**) is only the obstacle avoidance problem (Figure 2.3**b**).
The final goal is to reach the robot target considering the obstacles detection with sensors. In this way, the robot adapts its motion in a *local* manner.

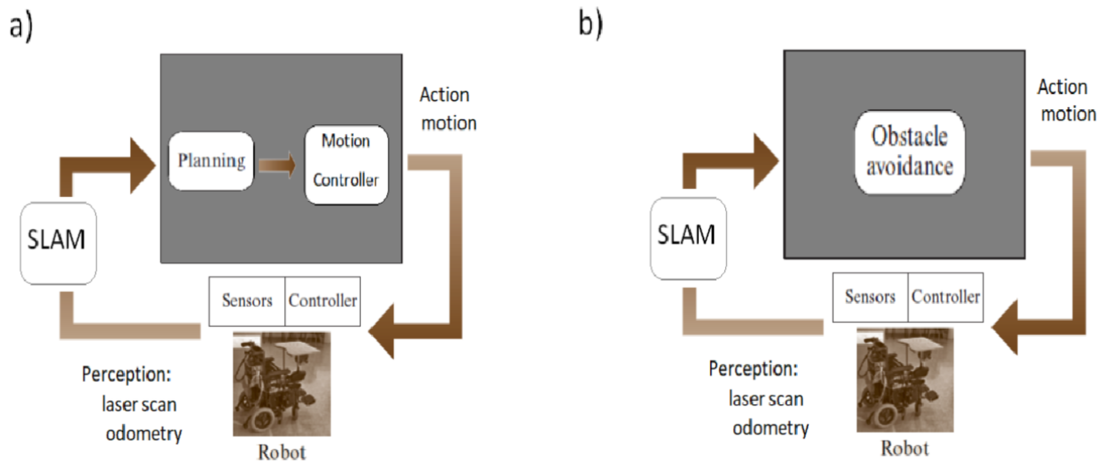


Figure 2.3: Overview of robot navigation: **a**)Player robot technique; **b**)Non-player robot approach.

One of the first related work for motion controller problem is developed by Khatib [21] with time-varying artificial potential field to avoid obstacles in real-time.
The robot navigates toward the target considering the superimposition of two types of potential field: repulsive from the obstacle and attractive from the goal pose.
In [30], Lumelsky et al. create a Bug algorithm where the robot follows the perimeters of the obstacles if the latter are in the way toward the final target.
Boreinstein et al. [4] use the Vector Field Histogram algorithm (VFH), to compute a polar obstacle density histogram over the sensor angular sector. To select the final output direction, the algorithm studies each histogram and compares them with a given threshold. The set of available directions (*candidate valley*) are the histograms below the threshold and the final strategy is chosen considering three heuristics that depend on the target position.
The main problems of this technique are: the oscillations between positions in the case of environments with a lot of narrows, the absence of smooth trajectory and the neglect of the robot dynamics and kinematics.
To solve these problems, after some years, the same authors develop VFH+ [51] and VFH* [52] that are enhanced versions of VFH.
In VFH+ algorithm there are two phases: first, it is necessary to read from the sensor the angles and the distance between the obstacle and the actual position of the robot to build a polar histogram for obstacle locations. In the second phase, the algorithm generates a masked histogram based on two hysteresis thresholds that solve the indecisive robot behaviour of the basic VFH.
The VFH+ algorithm considers a set of steering directions based on a cost function

(based on the smoothness of the navigation and the goal-distance) and chooses the free direction with minimal cost. It is possible to set also the *Robot radius* that takes into account the robot width.
Further work, related to obstacle avoidance, is proposed in [10] with the Dynamic-Window Approach (DWA), where the motion commands are selected from the space of velocities.
In [33], Minguez et al. develop a motion controller able to have a good performance in an unknown and cluttered environment that is improved in [7].

So far we have considered only motion controllers that have *local* approach to the environment. This is a limitation because an optimal local solution does not guarantee the best solution for the final general path.
For this reason, some authors try to close the gap between global path planning and local motion control. For instance, Quinlan and Khatib [38] present the Elastic Band approach, where the algorithm deforms in real-time the general path planning subject to internal and external force to obtain a smooth path and an appropriate distance from obstacles. Rosmann et al. [42] expand that method with the Timed-Elastic-Band.
However, the reason behind the research about motion robot control is to find a reasonable state of the art obstacle avoidance to do the comparison with our socially-aware trajectory planning.
We chose the VFH+ algorithm to implement the non-player robot because there is a good trade-off between performance and ease of implementation.
As a matter of fact, in two parts of our experiment, we showed to volunteers some videos with our player-robot and non-player robot to understand if they recognise the difference or not.

Chapter 3

Background on Game Theory

The purpose of this thesis section is to give to the reader the main concepts about game theory, in order to work out the human motion model developed in Chapter 4. For completeness and clarity, at the end of this chapter is given also an example of a simple navigation model with game theory.

3.1 Overview

Game theory is a vast discipline that has been studied for decades and has been used in different fields as: economics, politics, biology, evacuation process, military strategy, human motion forecasting, etc.
Game theory creates a mathematical model to study the strategy of rational decision-maker. The pioneer on this field was Von Neumann that published in 1928 the general theory for solving the zero-sum cooperative game. This work has been improved in 1944 with [53]. Nevertheless, the most famous concept about game theory was developed in 1950 by John Nash [35] with the Nash equilibrium theory in non-cooperative game.
In the following section there is the most widely used terminology in game theory useful to understand the following chapters.

3.1.1 Terminology

In a game, the participants are the *players*. Each player has an *action set* that is used to make the decision during the game. The number of times in which the player are called to make decision are the *stages*. The information about all players in that particular stage is the *state* of the game.
Furthermore, a *strategy* in game theory, means a procedure in which the player decides what to do for all situations throughout the game. In other words, when the game starts, the player specifies the *action* that will take for each possible situation

of the game. In particular, the specification can be *deterministic* (*pure strategy*), or the specification can be gives as a probability distribution for every *action* of the action set (*mixed strategy*).
For each player is defined a *cost function* that guides the choice of a certain action during the game. The players have a *rational* behaviour, it means that they try to maximise their profit (i.e. minimise the expected cost).
To study the interaction among players there are a different class of games that are presented in more details in the following section.

3.1.2 Game types

Games can be classified according to certain attributes, but in the following, we present only a summary of the common types of games [26] that we will use to model the interaction and the behaviour of pedestrians in a crowded environment.

Non-cooperative or cooperative:
In *non-cooperative* games the focus is set on the individual player that tries to maximise the own profit. It does not mean that the players do not cooperate, but they collaborate if the coalition can help them to reach their individual interest [26]. If the game leads to a situation where all players maximise their profits, the game reaches the *equilibrium* point, that in *non cooperative game* is the *Nash equilibrium.* On the other hand, in *cooperative games* the unit is the group and the players put the interest of the coalition before their own.

Zero-sum or Non-zero-sum
In *zero-sum* game the gain of one player corresponds to the same amount of loss of the other player. Thus, the sum of the payoffs of all players is zero. This situation is the most extreme circumstance of conflicting interest.
Meanwhile, in *non-zero-sum* game the player not wins the same amount that the other player loses.

Static and dynamic game
In the *static* case the decisions of all players are taken *simultaneously.* Considering the pedestrians scenario, if the condition is modelled as *static* game, the agents observe the situation and react *instinctively.* Instead, in *dynamic game*, the decisions are taken *sequentially* [49]. In this last condition, it is necessary to describe the amount of information that each player has about the current and previous state of the other players.

Perfect information game:
If each player, when makes a decision, knows the previous *actions* of all players, the

game is called *perfect information* game.

Finite game:
If the number of players and the corresponding actions are limited the game is called *finite*.

3.1.3 Nash equilibrium of Non-cooperative game

The most famous solution for non-cooperative games is the *Nash equilibrium* concept [35]. The equilibrium represents a game strategy where all players find a balance between the self interests of all the agents. In other words:

"A Nash equilibrium is a combination of strategies where no agent can reduce its own cost by changing its action if the other agents stick to their actions. A Nash equilibrium is the best response for everyone [50]."

From now on we indicate the Nash equilibrium with an asterisk

$$s_i^{j*} = (s_1^{j*}, ..., s_N^{j*})$$

this is the combination of the optimal strategies of all payers (*subscript* i). The *superscript* j refers to the *stage* of the game.
In mathematical terms it is possible to define the Nash equilibrium as follows:
the N-tuple (a set on N items, where each item is correlated to a different player [35]) of strategies s_i^{j*} is a Nash equilibrium if the following N inequalities are satisfied [49]:

$$\begin{gathered}
J_1(s_1^{j^*}, s_2^{j^*}, ..., s_N^{j^*}) \le J_1(s_1^{j}, s_2^{j^*}, ..., s_N^{j^*}) \\
J_2(s_1^{j^*}, s_2^{j^*}, ..., s_N^{j^*}) \le J_2(s_1^{j^*}, s_2^{j}, ..., s_N^{j^*}) \\
\vdots \\
J_N(s_1^{j^*}, s_2^{j^*}, ..., s_N^{j^*}) \le J_N(s_1^{j^*}, s_2^{j^*}, ..., s_N^{j})
\end{gathered}$$

where:
J_i is the cost function for the i-th player.

3.2 Navigation example with game theory

One of the simplest navigation decision problem scenario of two players is shown in Figure 3.1. Two pedestrians, P_1 and P_2, walk toward each other. In this example, for simplicity, the game is modelled as static, non-cooperative and non-zero sum.

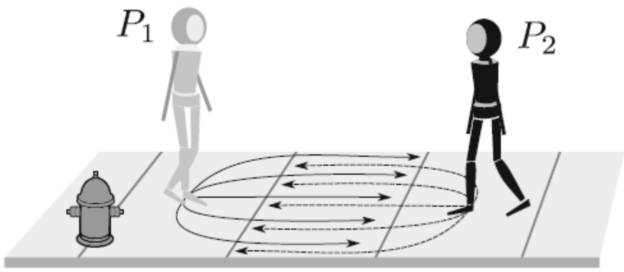


Figure 3.1: Example of human motion and interaction-aware navigation [49].

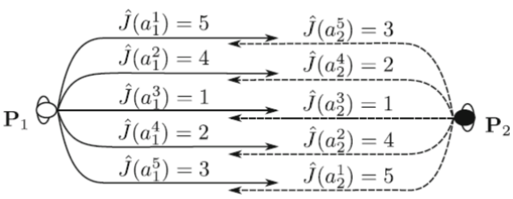
For each player (P_i) in Figure 3.2a is represented the *action* set $(a_i^1, a_i^2..., a_i^5)$ for the *stage j* and the relative cost for each trajectory. The cost for each action (in this case coincide with a trajectory), is computed considering the length of the path and crossing right is more convenient than passing left. If the two trajectories collide the cost is infinite (Figure 3.2b). Each cell of the table in Figure 3.2b contains the cost pairs $J_1(a_1^k) \mid J_2(a_2^k)$. According to the definition in Section 3.1.3, it is possible to compute the Nash equilibrium at the j-th *stage* of the game as:

$$J_1(s_1^{j^*}, s_2^{j^*}) = min(J_1(a_1^k, a_2^{k^*}))$$
$$J_2(s_1^{j^*}, s_2^{j^*}) = min(J_2(a_1^{k^*}, a_2^k))$$

In this example, we find four Nash equilibrium that is highlighted with a circle in Figure 3.2b.
How to calculate the Nash equilibrium by looking only at the table?
At the same time 2 conditions should be observed:

- the cost column J_1 is less or equal than all the other same column cells.
- the cost row J_2 is less or equal than all the other same row cells.

This scenario was tested in [49] with a real experiment and the result was that the two players during navigation adopted one of the Nash equilibrium solutions. Nevertheless, this example originates the question about which equilibrium a player should choose. It is possible to select the equilibrium trajectories reasonably. For instance, for each player is computed the Nash equilibrium at each time step. In order to select in the current time (t) the right equilibrium, the player should consider the set of Nash equilibrium in the previous time step $(t-1)$ and in the current time (t) but also the set of *observed* trajectory in the previous time step $(t-1)$.
First of all, the player considers which observed trajectory is similar to the Nash



a)

$P_1 \backslash P_2$	a_2^1	a_2^2	a_2^3	a_2^4	a_2^5
a_1^1	5|5	5|4	5|1	∞	∞
a_1^2	4|5	4|4	∞	∞	∞
a_1^3	1|5	∞	∞	∞	1|3
a_1^4	∞	∞	∞	2|2	2|3
a_1^5	∞	∞	3|1	3|2	3|3

b)

Figure 3.2: **a**)Situation in Figure 3.1 modelled as *static* game; **b**)The action of each player and the respective cost function is shown. If the two players collide the cost is infinite. The Nash equilibrium is circled [49].

equilibrium in the previous time step, then, compares the chosen equilibrium with the equilibrium set in the current time.
The main problem of this approach is the computation and evaluation of all equilibrium each time for all players. For this reason, we modelled the human motion behaviour considering a *sequential best response*, it means that the game is dynamic and the solution is calculated considering the current observation of all agents and, thus, the Nash equilibrium concept (this concept will be explained in the Chapter 4).
The best response strategy in our model is *unique* and overcomes the *static* approach.

Chapter 4

Model of human motion prediction

4.1 Social behaviour

Socially-aware planning combines the typical information used to perform the motion planning (composed by the abstraction of the environment coming from the proprioceptive and/or exteroceptive robot's sensor) and the social conventions connected to the society in which the robot will move.

Definition 2 *The* social conventions *are behaviours created and accepted by the society that help humans to understand intentions of others and facilitate the communication [41].*

In order to generate a safe and comfortable strategy for robot navigation in a populated humans environment, it is essential to consider the main aspect of the social conventions (Definition 2) during human motion.
In this regard, it is interesting to explore, during navigation, how humans manage their surrounding area and, thus, the distance that human mutually respects to prevent emotional discomfort.

4.2 Proxemic

The concept of "Proxemic" was proposed by Edward T.Hall in [13] for human-human interaction scenario.

Definition 3 Proxemics *is the study of spatial distances individuals maintain in various social and interpersonal situations. These distances vary depending on environment or cultural factors [41].*

Hall [13] studied the existence of particular unwritten rules that humans adhere during the interaction depending on their relationship. He observed different social distances that individuals maintain from others, as shown in Table 4.1.

Zone	Distance	Intention or Relationship
Intimate	$\leq$ 0.45 m	Embracing, touching, whispering
Personal	0.45 - 1.2 m	Friends
Social	1.2 - 3.6 m	Acquaintances and strangers
Public	>3.6m	Public speaking

Table 4.1: Space around a person considering social interaction according to the Hall's study [13].

Based on the Theory of Mind [22], during navigation humans maintain a certain space for themselves as those they imagine others would prefer.
It should be noted that our human motion model takes into account the proxemic for all players, groups included.

4.3 Personal space

Definition 4 *A* personal space *is the region around humans that they actively maintain into which others cannot intrude without causing discomfort [14].*

An example of people that respect the individual personal space is shown in Figure 4.1, in which the personal space is illustrated using a blue circle and considering the Hall's theory.

Figure 4.1: Situation in which humans respect personal space (blue circle)[41].

In the scientific literature different shapes of personal space (Figure 4.2) have been proposed.
Concentric Circle (Figure 4.2**a**). As shown in Table 4.1, it is possible to classify the space around a human in four specific zones. It is necessary to highlight that

the distances between people are not rigid and vary with the culture, age, type of relationship and context. Indeed, in some cultures the physical contact is avoided, instead of others that are more tolerant. For this reason, it is very important to underline that the zones proposed by Hall are refereed to US citizens.

Egg shape (Figure 4.2**b**). Humans are more exigent regarding the respect of the frontal area. Thus, the invasion of the frontal zone is considered as uncomfortable [15].

Ellipse shape (Figure 4.2**c**). One of the most famous approach to represent human motion behaviour is the Social Force Model [16]. In this model, there are two types of forces: attractive and repulsive. The first one guides humans to the destination. On the other hand, the repulsive force is used to avoid collisions between pedestrians. The potential repulsive force is modelled as a monotonic decreasing function with equipotential lines having the form of an ellipse directed to the direction of motion.

Asymmetric shape (Figure 4.2**d**). After physical experiments and virtual simulations, the researchers in [11] concluded that the size of the personal space does not change with the walking speed during the circumvention of a static obstacle. In particular, the personal zone is smaller in the pedestrian's dominant side.

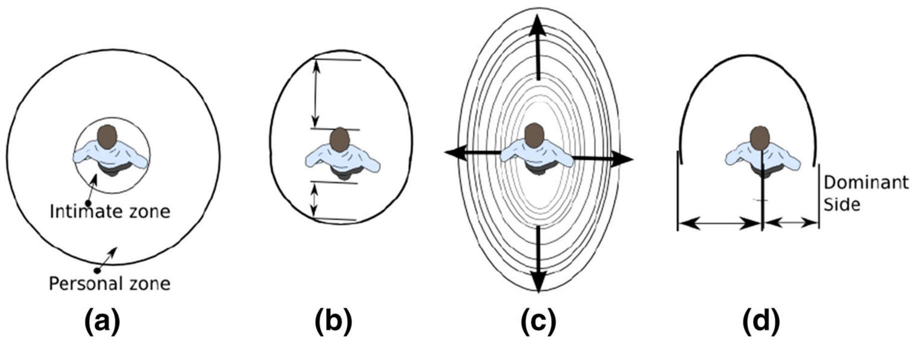


Figure 4.2: Different forms of surrounding personal area: **a**) Concentric circles [13]; **b**) Egg shape [15]; **c**)Ellipse shape [16]; **d**)Asymmetric shape [11].

In our human motion model we designed the personal space as a *concentric circle* (Figure 4.2**a**) considering a *social* zone greater than 1.2m diameter, as shown in Table 4.1.

4.4 Modelling navigation as a game

During navigation, human observes the environment and reacts to people and generic obstacles, taking *sequential* decisions. To model this condition, we use a *dynamic* game. In fact, in this scenario the player chooses the strategy after observing the actions of other players.
It is necessary to highlight that each player has *perfect* information, i.e. knows the current and previous actions of all players.
Each pedestrian wants to reach its own goal, thus, the game is *non-cooperative*. Further, if a pedestrian "wins", it is not necessary that another player loses the same amount, hence, the game is *non-zero-sum*.
Each player has a finite number of actions included in the *action set*. The proposed approach uses an action set with 7 actions.
The solution used to solve this game is based on the Nash equilibrium, that is explained in Section 3.1.3.
The solution of the *dynamic* game, with only two players, is presented in Figure 4.3. The player1 observes the player2 and notes that the player2 is moving in a particular direction. Then, the player1 hypothesis that the player2 will move in the same direction for the following time steps (T). This hypothesis is sketched with "Initial Strategy2" in Figure 4.3. Thus, the player1 solves its optimisation problem and the output is the "Strategy1". Subsequently, a control action is necessary to verify if the actual *total* strategy is the same computed in the previous iteration. During the first iteration the total strategy is completely composed by the actual directions of the players.
If the Nash equilibrium is not reached, the player2 solves its optimisation problem considering the *Strategy1*. The output of the optimisation problem is the Strategy2 and, then, the same control strategy previously presented is applied.
The solution is achieved if the process reaches the Nash equilibrium.
Resuming, the game is *dynamic, finite, perfect information, non-cooperative and non-zero sum* and is solved with a *Nash equilibrium*.

In the following, we present in details the optimisation problem solved for each pedestrian.
Furthermore, in general, the model considers three situations:

- single pedestrian that interacts with all people
- recognition and resolution of the game considering a group of people
- interaction between human and a static object.

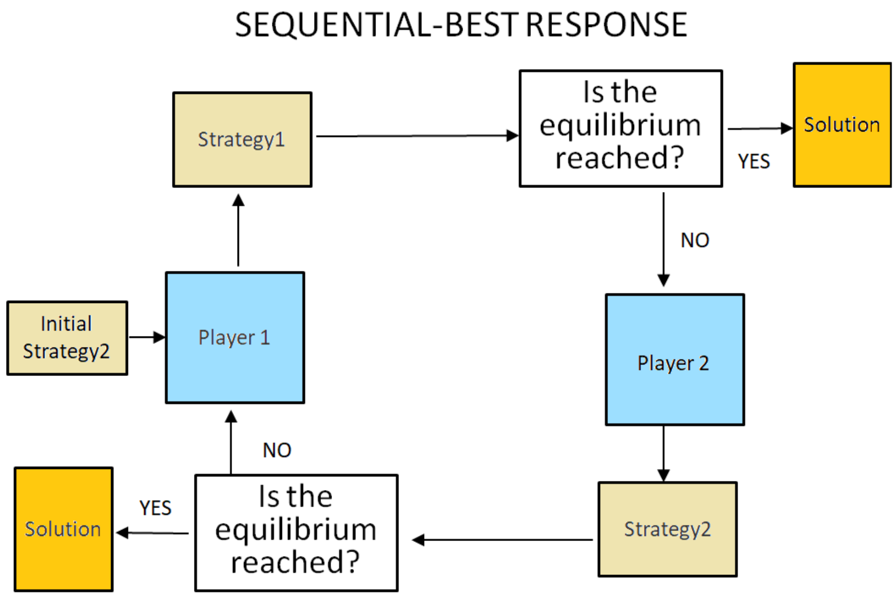


Figure 4.3: Resume of a *dynamic* game solution with two pedestrians.

4.5 Optimisation problem

To produce a human-like prediction we use a constrained optimisation problem composed by a cost function subject to a set of constraints.
A *cost* or *objective function* is a mathematical function that must be maximised or minimised with respect to some variables, in order to search for the optimal [1] solution of the problem.
Constraints are mathematical concepts used to limit the variables evaluated by the optimisation. Constraints are *hard* or *soft*: *hard constraint* is an absolute limit that cannot be overcame; while *soft* constraint allows to relax the inflexible constraints acting a penalty to the objective function.
The standard form of an optimisation problem is:

$$\begin{aligned} \underset{x}{\text{minimise}} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0, \; i = 1, \dots, m. \\ & h_j(x) = 0, \; j = 1, \dots, p. \end{aligned}$$

where:

- $f(x)$ is the objective function to be minimised over the vector x with n-variable
- $g_i(x) \leq 0$ are m **inequality** constraints
- $h_j(x) = 0$ are p **equality** constraints
- $m \geq 0$ and $p \geq 0$. If m and p are equal to zero, the optimisation problem is unconstrained.

4.6 Constraints and objective function

We started assuming that pedestrians are always *goal-directed*, i.e. their motion is always directed toward the goal. In this regard, an element of the objective function (see Equation 4.1) is modelled minimising the overall path length; in this way, the resulting path planned by people is the shortest one.

$$\underset{\varepsilon_i}{\text{minimise}} \quad \gamma_1 || p_i(t) - p_i^* || \tag{4.1}$$

where:

[1] *Optimal solution* is the best feasible result for a given problem

- γ_1 is a vector containing weighting factors.
  In the simulation, we chose $\gamma_1 = [0.6; 0.7; 0.8; 1]$, considering the total prevision time equal to 4 steps.
  We chose those weighting factors because more the player is closer to the goal more this term acquires importance.

- ε_i is a vector that contains the future directions (θ_i) of the *i-th* agent from 1 to T_{Prev} (total prevision time).

$$\varepsilon_i = \begin{bmatrix} \theta_i(1) \\ \theta_i(2) \\ ... \\ ... \\ \theta_i(T_{Prev}) \end{bmatrix}$$

- $p_i(t)$ is the predicted position of human computed as follows:

$$p_i(t) = p_i(t-1) + \theta_i(t-1)v\Delta t \tag{4.2}$$

  where:
  Δt is the time between two time instants.

- p_i^* is the target position computed as:

$$p_{Est}^*(t) = p_{Start} + \theta_i(0)T_{Prev}v \tag{4.3}$$

$$p_i^*(t) = p_i^*(t-1)(1-\alpha) + p_{Est}^*(t)\alpha \tag{4.4}$$

  where:

- $\theta_i(0)$ is the observed direction of the player i.

- v is the velocity of the player.

- p_{Start} is the initial player position at time instant zero.
  In order to compute $p_i^*(1)$, α is set equal to 1.

In our simulation, we set α equal to 0.7 when the time is different to 1. In this way, the target computed in the previous time step counts only 30% of the final target.
An example is shown in Figure 4.4.

Moreover, we considered that people prefer to move in a straight line and would avoid to change their direction every time. For this reason, we added in the objective function a term that takes into account the *smoothness* of the future trajectory (Equation 4.5).

$$\underset{\varepsilon_i}{\text{minimise}} \quad \gamma_2 \sum_{t=1}^{t=T} |\theta_i(t) - \theta_i(t-1)| \tag{4.5}$$

where:

- γ_2 is a vector containing the weighting factors associated to this part of the objective function. In our simulation, γ_2 assumes the following value:

$$\gamma_2 = 1 - \gamma_1$$

- $\theta_i(t)$ and $\theta_i(t-1)$ are the direction at time t and $(t-1)$, respectively.

In summary, this term penalises changes in motion direction.
Moreover, since people avoid also obstacles with a *smooth* motion, we assign a cost function that penalises small distance between pedestrian and obstacle (Equation 4.6).

$$\underset{\varepsilon_i}{\text{minimise}} \quad \frac{\rho}{|p_i(t) - y|} \tag{4.6}$$

where:

- ρ is a weighting factor. In our simulation we chose ρ equal to 7500.
- $p_i(t)$ is the human position at time t.
- y is the closest obstacle point to the $p_i(t)$ player position.

The overall objective function is:

$$\underset{\varepsilon_i}{\text{minimise}} \quad \gamma_1 ||p_i(\varepsilon_1) - p_i^*|| + \gamma_2 \sum_{t=1}^{t=T} |\theta_i(t) - \theta_i(t-1)| + \frac{\rho}{|p_i(t) - y|} \tag{4.7}$$

The weighting factors imply the contribution of each term of the objective function adjusting the output of the optimisation problem.
In Figure 4.5a is shown a human cost map in a static environment considering only the first and the last term of the cost function.

In order to model the region around humans as personal spaces, that should not be violated, we added an hard constraint with the following structure:

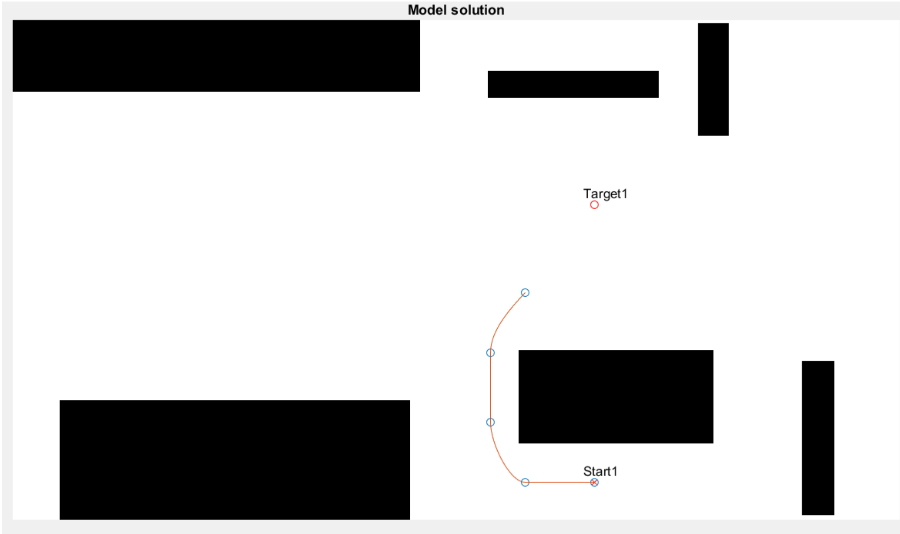
$$\begin{aligned} |p_i(t) - p_j(t)| \geq \beta \;\; \forall t \\ i, j \in 1...N, i \neq j \end{aligned} \tag{4.8}$$

N is the number of players and β is the minimum distance allowed between two pedestrians.
Equation 4.8 is a *collision avoidance constraint*. In this way, the distance between pedestrians i and j must be greater or equal to a certain value β. The latter is chosen according to the Hall observation [13]. As a result, the personal space is a circle around human as in Figure 4.2**a**.

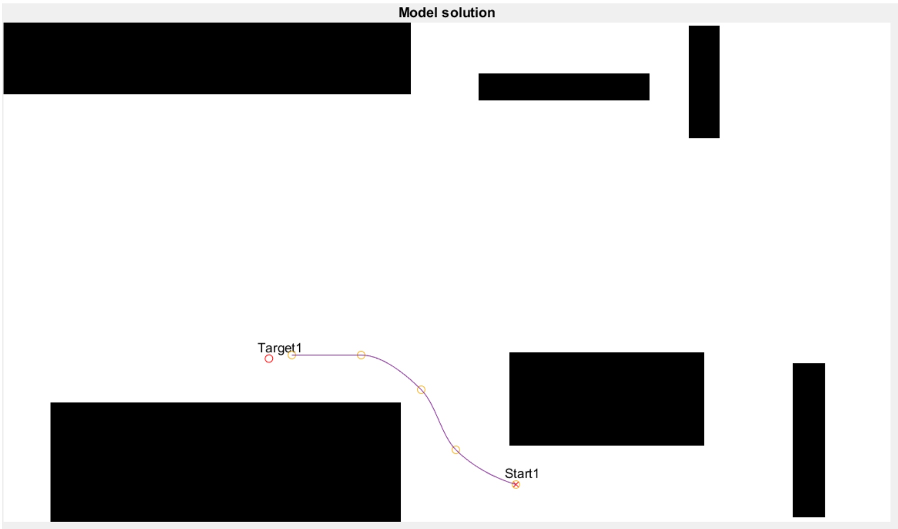
Each pedestrian can choose the future direction according to a finite set of options. Hence:

$$\theta_i \in \Theta_i$$

where Θ_i is a set of possible angles identified inside the human's visibility zone.
In the simulation, we selected 7 options. In particular, the player can choose between the actual position plus 0, -30, -60, -90, 30, 60 or 90 degrees. We selected these *relative* angles because is a good trade-off between good performance and reasonable computational complexity of the algorithm.



a)



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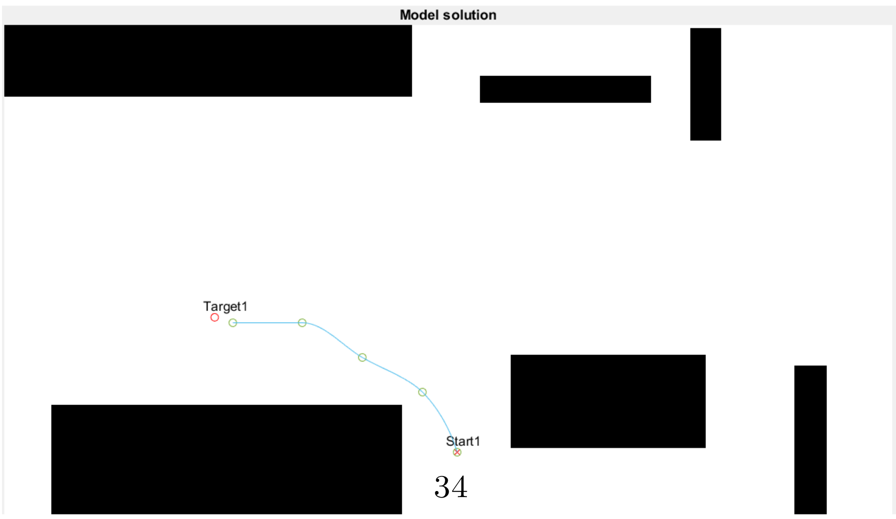
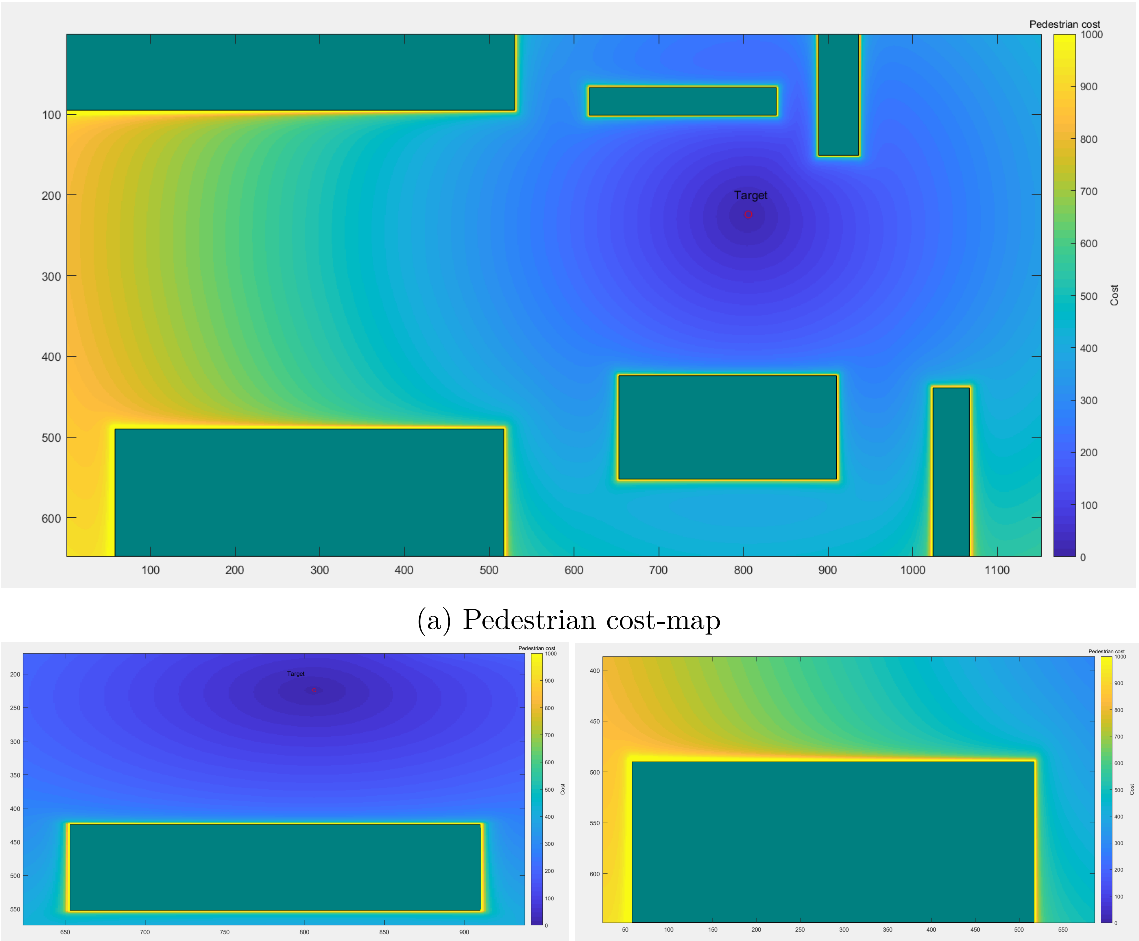


c)

Figure 4.4: Weight target at different time instant.



(a) Pedestrian cost-map

(b) Zoom of the target position

(c) Zoom of the obstacle

Figure 4.5: *Cost map of human in a static environment.* Blue areas are with the smallest cost, while yellow areas are with the highest cost, i.e. with a cost equal or greater than 1000.

4.7 Motivation for the choice of a discrete approach

Before assuming continuous solutions for the optimisation problem and, thus, for the game, it is necessary to classify the problem.
The outputs of the model, in this case, are the *directions* and the *velocities*, for each pedestrian for the total prevision time.
Considering these quantities unknown and using the model shown in Section 4.6, the optimisation problem is *non-linear* and *non-convex*.
Unfortunately, these types of problems are particularly difficult to solve. Non-convex optimisation problems have multiple local optimal points and, therefore, the solution depends mainly on the starting point given as input. Further, is not always guaranteed by the optimisation a *feasible* solution, due to the initial condition or the unknown variables constraints.
We have analysed many Matlab mathematical programming solver at the state of the art to solve this type of problem.
In [39], an interesting comparison (Figure 4.6) between *fmincon* solver, already included in Matlab, and *Knitro*, a commercial non-linear optimisation solver, is presented.
In Figure 4.6**b**, Knitro outperforms fmincon, but it does not ensured a global optimum solution. To improve the performance, Knitro offers a multi-start (MS) feature where it is possible to set n_{MS} different initial conditions. In this way, the solver finds the best solution from the n_{MS} starting points.
On the other hand, Knitro with MS increases enormously the execution time compared to the fmincon and Knitro without MS (Figure 4.6**a**).
Thus, we solved our problem with both of these solvers to reach and find a reliable solution. In Figure 4.7 the result considering the same starting point for both solvers is summarised.
Curiously, with Knitro without MS, we found an *unfeasible* solution (Figure 4.7**a**), while, in contrast with *fmincon* the solution is feasible. We tried with different initial pedestrian position and different velocities constraints but with Knitro, the solution is always *unfeasible*.
To overcome this problem, we tried to use the *multi-start* function, but with the student trial version of Knitro we cannot exploit this feature.
Then, we focused on the *fmincon* solver, even if we discovered two main problems. First of all, with the same optimisation problem shown in Section 4.6, and considering, in addition, velocity constraints, we found a maximum deceleration if the pedestrian wants to avoid the other players, and, on the opposite, a maximum acceleration in the remaining cases, due to the cost function that searches for the shortest and fast path.
Thus, the final trajectory is not human-like. For these reasons, we decided to model

the human motion with a *discrete* approach.

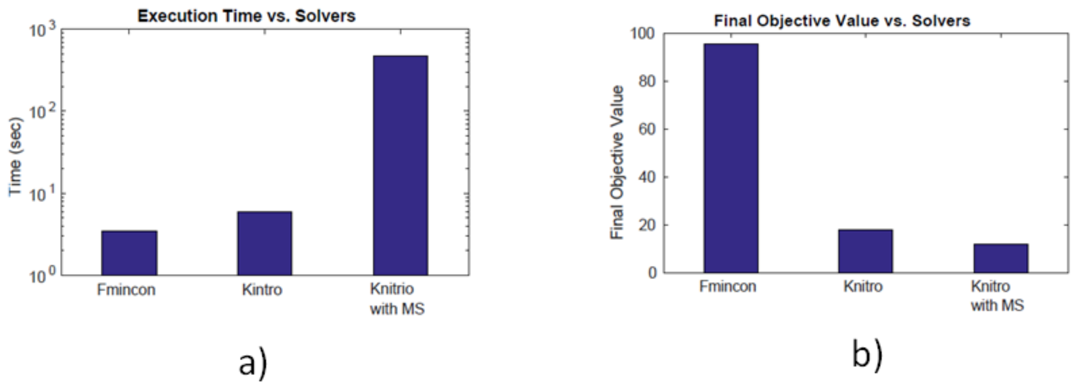


Figure 4.6: Comparison between the performance of *Knitro* and *fmincon* [39].

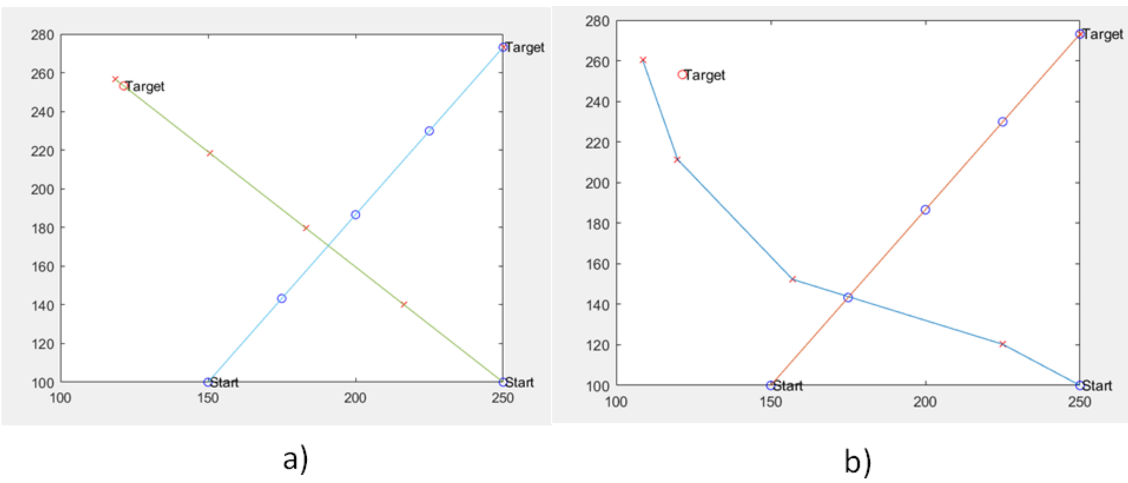


Figure 4.7: Simulation results: **a**) Unfeasible solution with *Knitro*; **b**) Feasible solution with *fmincon*. In both graphs the axis identify the two-dimensional Cartesian space.

4.8 Algorithm

The algorithm, to solve the problem presented in Section 4.6, was implemented and simulated in Matlab.
The pseudo-code is summarised in Algorithm 1.
First, some initialisation procedures are performed before computing the game solution. The environment (***AcquisitionImage***) and the initial pedestrians data (***PedestrianSelection***, ***InitialData***) are acquired. Then, the algorithm tries to recognise the group of people and save them in the *group* vector (***GroupControl***). This process is useful to improve the computational complexity of the algorithm.
However, the main part of the code is the following.
The algorithm considers each pedestrian n (Line 8) and saves the strategy inside the variable $\varepsilon_t(n)$. If the Nash equilibrium (Line 35) is *not* reached, the pedestrian strategy is computed again (Line 7), otherwise the final game solution is achieved and saved. This process is repeated for T time steps.
In more details, for each player n, we evaluate if it belongs to one group. If n is included in *group* and is not the first group-player, the algorithm extends the strategy of the first group component to the n-player. Otherwise, the code estimates the final target of the n-player (***ComputationalTarget***) and, thus, computes $p_{FirstEst}$, that is the first estimated trajectory (an example is shown in Figure 4.8) for all players (***FirstEstimation***).
Afterwards, based on $p_{FirstEst}$, the trajectory intersection between the n player and the other agents, $player_{Inter}$, is computed (***Intersection***).
Moreover, also the obstacle collision is evaluated, considering the first estimated trajectory (examples in Figures 4.13**a** and 4.11**a**). Thereby, based on these information, it is possible to divide the solutions into three main cases:
(i) If the estimated n-player trajectory does not collide with any obstacle and any other player, the solution for the n-player is with: constant direction and constant velocity (Line 14);
(ii) If the n-player trajectory collides with the trajectory of other players (line 18), the algorithm solves the optimisation problem (***OptGame***) and computes a temporary strategy ε_{temp1}. Furthermore, another strategy (ε_{temp2}) is computed, considering a constant direction, but with the possibility to decelerate (***DecelTree***).
Then, comparing the cost connected to the optimisation problem and the one coming from the ***DecelTree*** function, the code computes the strategy with the minimum cost. Examples of this scenario are presented in Figures 4.11 and 4.12;
(iii) In the remaining case, i.e. when the pedestrian collides *only* with an obstacle (example Figure 4.13**a**), the algorithm solves the optimisation problem only once (when f is equal to 1) in order to improve the performance of our algorithm.

Algorithm 1 Main

$MAP \leftarrow$ **AcquisitionImage**$(MapImage)$
$p_{Start}, angles, players \leftarrow$ **PedestrianSelection**(MAP)
$n_{MAX}, T, v, \Delta t, Choice, \beta, T_{prev} \leftarrow$ **InitialData**$()$
$group \leftarrow$ **GroupControl**$(p_{Start}, angles, players)$
for t = 1 : T **do**
    exit=2;
    for f = 1 : n_{MAX} **do**
        for n = 1 : players **do**
            if $n \neq group$ **then**
                $p^* \leftarrow$ **ComputationalTarget**$(v, T_{Prev}, p_{Start}, angles, players)$
                $p_{FirstEst} \leftarrow$ **FirstEstimation**$(players, p_{Start}, T_{Prev}, v, angles, \Delta t)$
                $player_{Inter} \leftarrow$ **Intersection**$(p_{FirstEst}, \beta, players, n, T_{Prev})$
                $obst \leftarrow$ **Obstacle**$(p_{FirstEst}, MAP)$
                if obst==0 $\wedge$ isempty$(player_{Inter})$==1 **then**
                    $p_{Prev}(n) = p_{FirstEst}(n)$
                    $\varepsilon_t(n)$ =angles(n)
                end if
                if isempty$(player_{Inter})$ == 0 **then**
                    $Cost_1, \varepsilon_{temp1}, p_{Temp} \leftarrow$ **OptGame**$(p_{Start}, \varepsilon_t, p_{Prev}, \Delta t, Beta, p^*, MAP, angles)$
                    $Cost_2, \varepsilon_{temp2}, p_{TempDecel} \leftarrow$ **DecelTree**$(p_{Start}, \varepsilon_t, p_{Prev}, \Delta t, p^*, MAP, angles)$
                    if $Cost_1 > Cost_2$ **then**
                        $p_{Prev} = p_{TempDecel}$
                        $\varepsilon_t(n) = \varepsilon_{temp2}$
                    else
                        $p_{Prev}(n) = p_{Temp}$
                        $\varepsilon_t(n) = \varepsilon_{temp1}$
                    end if
                end if
                if $obst == 1 \wedge isempty(player_{Inter}) == 1) \wedge f == 1$ **then**
                    $\varepsilon_t, p_{Prev} \leftarrow$ **OptGame**$(p_{Start}, \varepsilon_t, p_{Prev}, \Delta t, Beta, p^*, MAP, angles)$
                end if
            else
                $p_{Prev}, \varepsilon_t \leftarrow$ **UpdateGroupStrategy**$(\varepsilon_t, v, T_{prev}, p_{Start})$
            end if
            if $\varepsilon_t == \varepsilon_{t-1}$ **then**
                $exit = 1;$
                break
            else
                $\varepsilon_{t-1} = \varepsilon_t$
            end if
        end for
        if exit==1 **then**
            break
        end if
    end for
    $angles, p_{Start} \leftarrow$ **UpdateInitialData**$(\varepsilon_t, p_{Prev})$
end for

4.8.1 Function explanation

AcquisitionImage. This function takes as input the *MapImage*, that is the full-colour image of the environment, and returns the occupancy grid denoted as MAP defined as a matrix, in which 0 denotes occupied cell and 1 a free cell.
PedestrianSelection. This function selects pedestrians in the environment (p_{Start}) and choose the initial direction (*angles*) of each agent. The function also counts the number of pedestrians that is given as *players* variable.
InitialData. In order to use the starting data in a practical way, this function gives: n_{MAX} the maximum iteration for the following *for loop*, T total number of time steps, Δt the time between two time instants, v mean velocity for each pedestrian, *Choice* is the vector connected to the player action set, β is the minimum distance between players and T_{prev} is the prevision time steps of pedestrians.
GroupControl. If some pedestrians are close to each other and with similar initial directions, the function classifies them as *group*.
ComputationalTarget. This function computes the weight target (as shown in Section 4.6) for all pedestrians considering the initial data.
In our simulation, we set T_{Prev} equal to 4 time steps and the targets were computed considering 70% of the estimated target at time instant t and the remaining 30% of the goal computed at the instant $(t-1)$.
FirstEstimation. Based on the initial data, this function computes a first pedestrians' estimation ($p_{FirstEst}$) assuming a constant direction and a constant velocity for all players, as shown in Figure 4.8.
Intersection. Considering $p_{FirstEst}$ previously computed, the function evaluates if the first estimated trajectory of the player n intersects or is too close to other trajectories and save the results in $player_{Inter}$.
Obstacle. Given the map of the environment (MAP) and $p_{FirstEst}$, this function analyses if the first estimated trajectory collides with an obstacle or not, and saves the result in *obst*. The variable *obst* is a binary variable: 1 represents the presence of an obstacle; 0 otherwise.
OptGame. This function solves the optimisation problem evaluating all possible strategy combination and considering the constraints. Indeed, starting from the initial *angles*, the function builds a choice tree and extends only the branches that satisfy the personal space constraint. The best strategy (ε_{temp1}) has a constant velocity, minimum cost ($Cost_1$) and satisfy all constraints.
DecelTree. In parallel with the resolution of the optimisation problem, *DecelTree* builds another choice tree assuming a constant direction, but evaluating an incremental deceleration (40%, 50%, 60%, 70%) considering the current velocity. The strategy solution, the corresponding position and the cost are saved in ε_{temp2}, $p_{TempDecel}$ and $Cost_2$, respectively.
UpdateGroupStrategy. Given the matrix with all pedestrians' strategies ε_t at time t and the other initial data (v, T_{prev}, p_{Start}), this function extends the strategy

of the fist group player to the other members of the group.
UpdateInitialData. Considering the pedestrian strategy matrix ε_t and the position prevision p_{Prev}, *UpdateInitialData* upgrades the variables *angles* and p_{Start} for all players.
In the following subsections are shown and analysed some simulation results coming from the Matlab code.

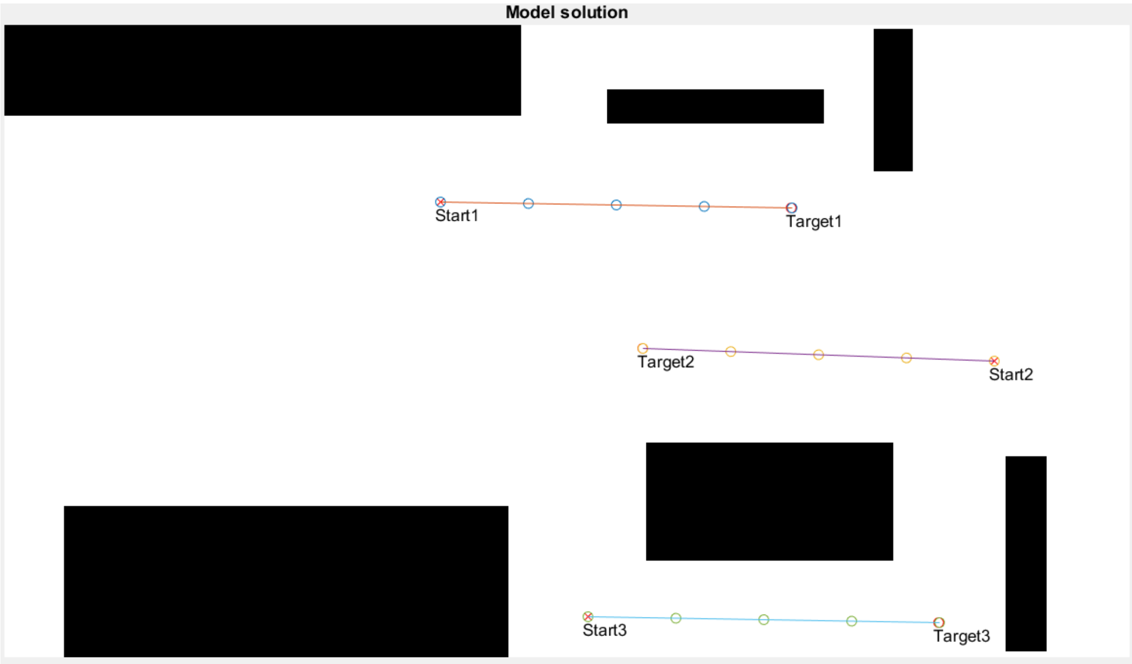
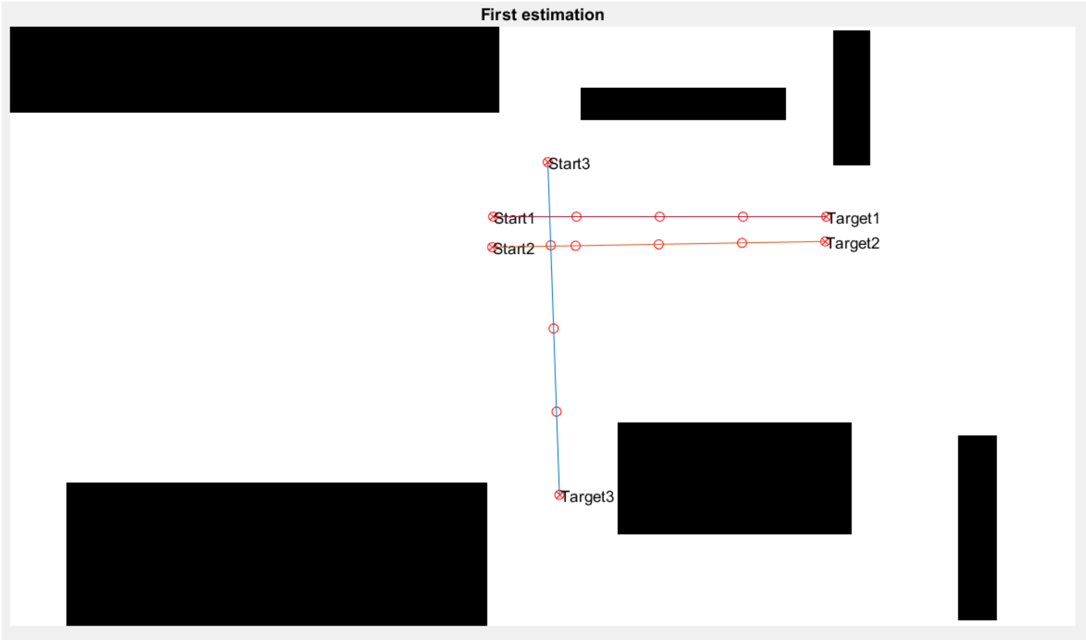


Figure 4.8: Simulation results in which agents do not intersect obstacles and other pedestrians trajectories.

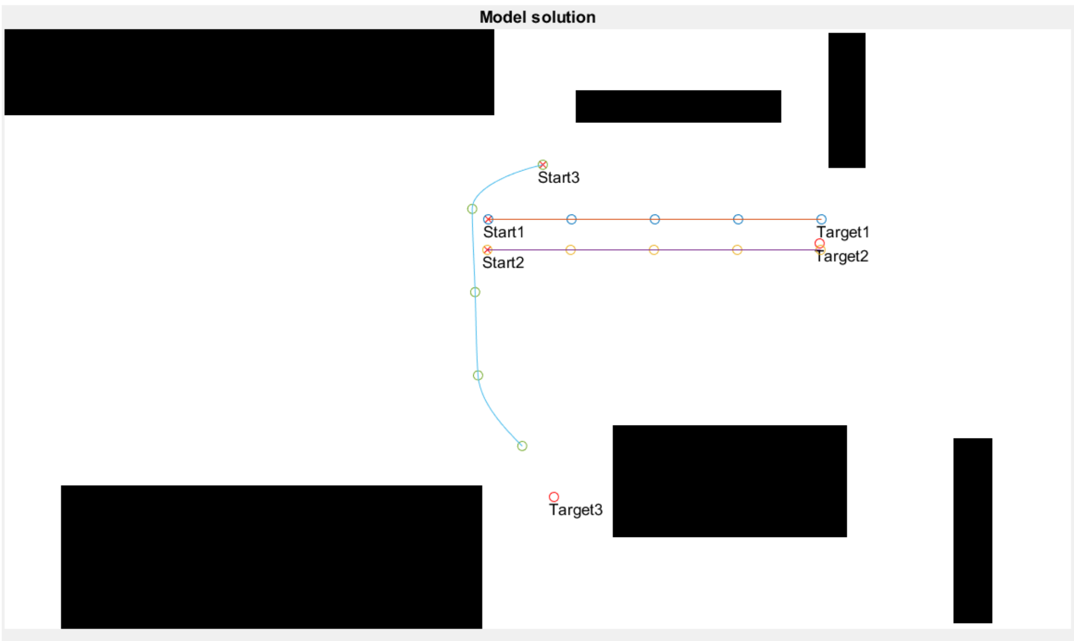
4.8.2 Group of people

The code is able to recognise the groups of people that are moving in the environment (MAP) with the function ***GroupControl***.
Some examples are shown in Figures 4.9 and 4.10.
In particular, in Figure 4.9, the algorithm classifies the players one and two as belonging to one group, and the third one avoids the group.
In Figure 4.10, the situation is similar to Figure 4.9, but the group consists in 3 people. Similarly to the previous scenario, the fourth pedestrian avoids the group.
In both figures, the circles over the trajectories are the estimated positions that are the outputs of the simulation.
Why is it so important to recognise groups?
For example, if in Figure 4.9 the pedestrian 1 and pedestrian 2 were classified as

two different players, the algorithm would applied the personal space constraint for each of them and, as a consequence, the final trajectory would be calculated to avoid each other. This happened because normally, when pedestrians navigate in a group, the distance between components inside the group is lower than the β parameter.
Further, the group recognition improves the computational complexity of the algorithm. Indeed, if n is included in group and is not the first group-player, the algorithm extends the strategy of the first group component to the n-player without solving the game.

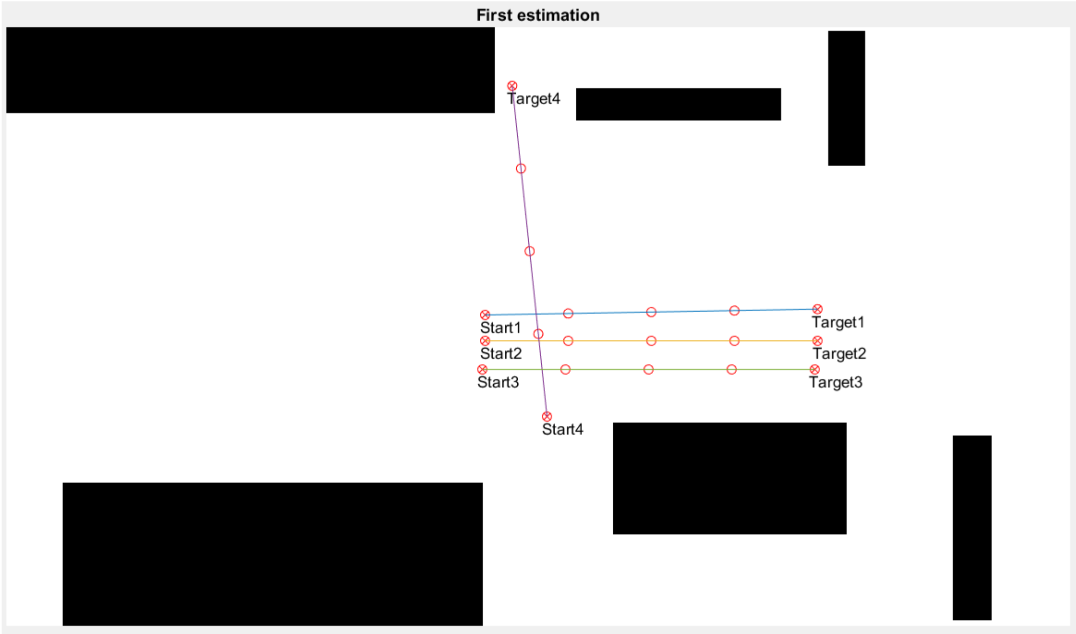


a)

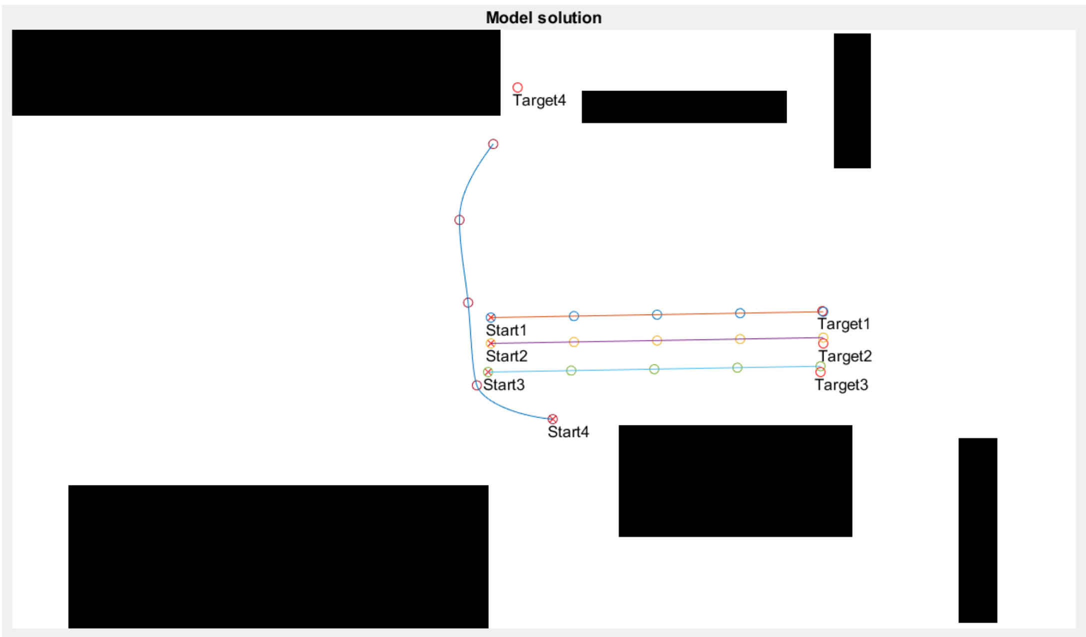


b)

Figure 4.9: Simulation results: **a**) First estimation and group recognition; **b**) Model solution.



a)



b)

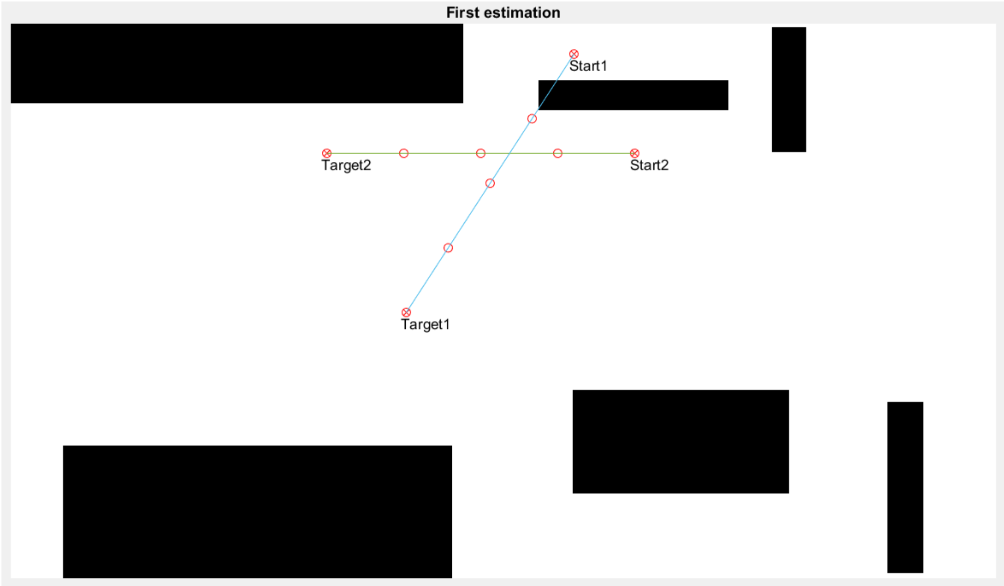
Figure 4.10: Simulation results: **a**) First estimation and group recognition; **b**) Model solution.

4.8.3 Game solution

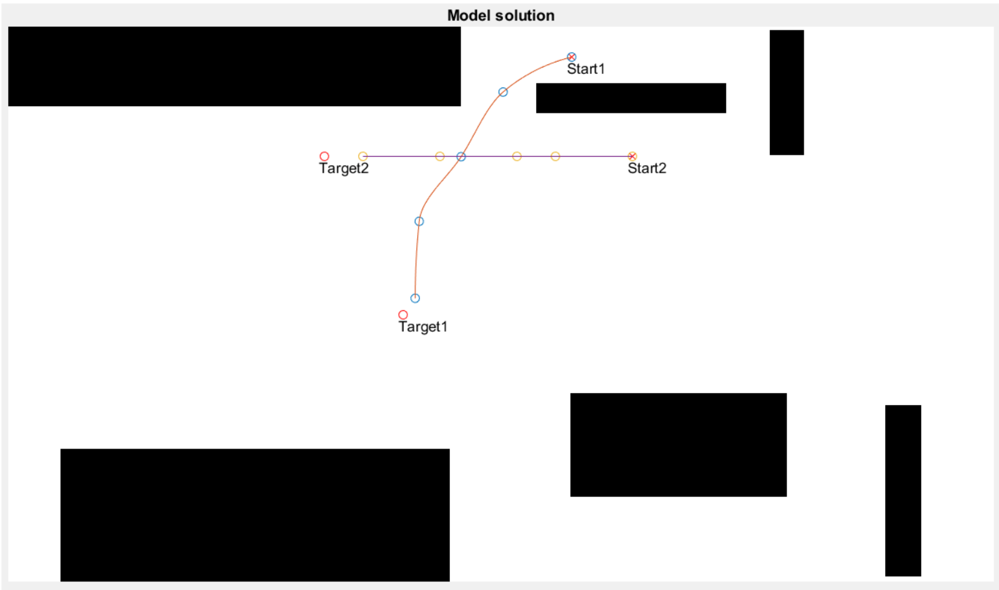
Figures 4.11 and 4.12 show the solution of the game considering the intersection with other pedestrians.
In Figure 4.11**a**, the first pedestrian intersects the obstacle and the trajectory of the second player. The solution of this scenario is shown in Figure 4.11**b**, where the first pedestrian avoids the obstacle but is still goal-oriented, and the second player decelerates to pass the first agent and, then, continues with his previous speed.
On the other hand, in Figure 4.12**a**, two couples of trajectories intersect. The solution is presented in Figure 4.12**b**, where the first pedestrian decelerates to pass the second one and the third player avoids the fourth agent.

4.8.4 Human-static object interaction

The last analysis is the intersection of the first estimated trajectory with obstacle (Figure 4.13**a**).
The output trajectory is smooth and is not excessively close to the obstacle (Figure 4.13**b**), because of the cost function term (explained in Section 4.6).

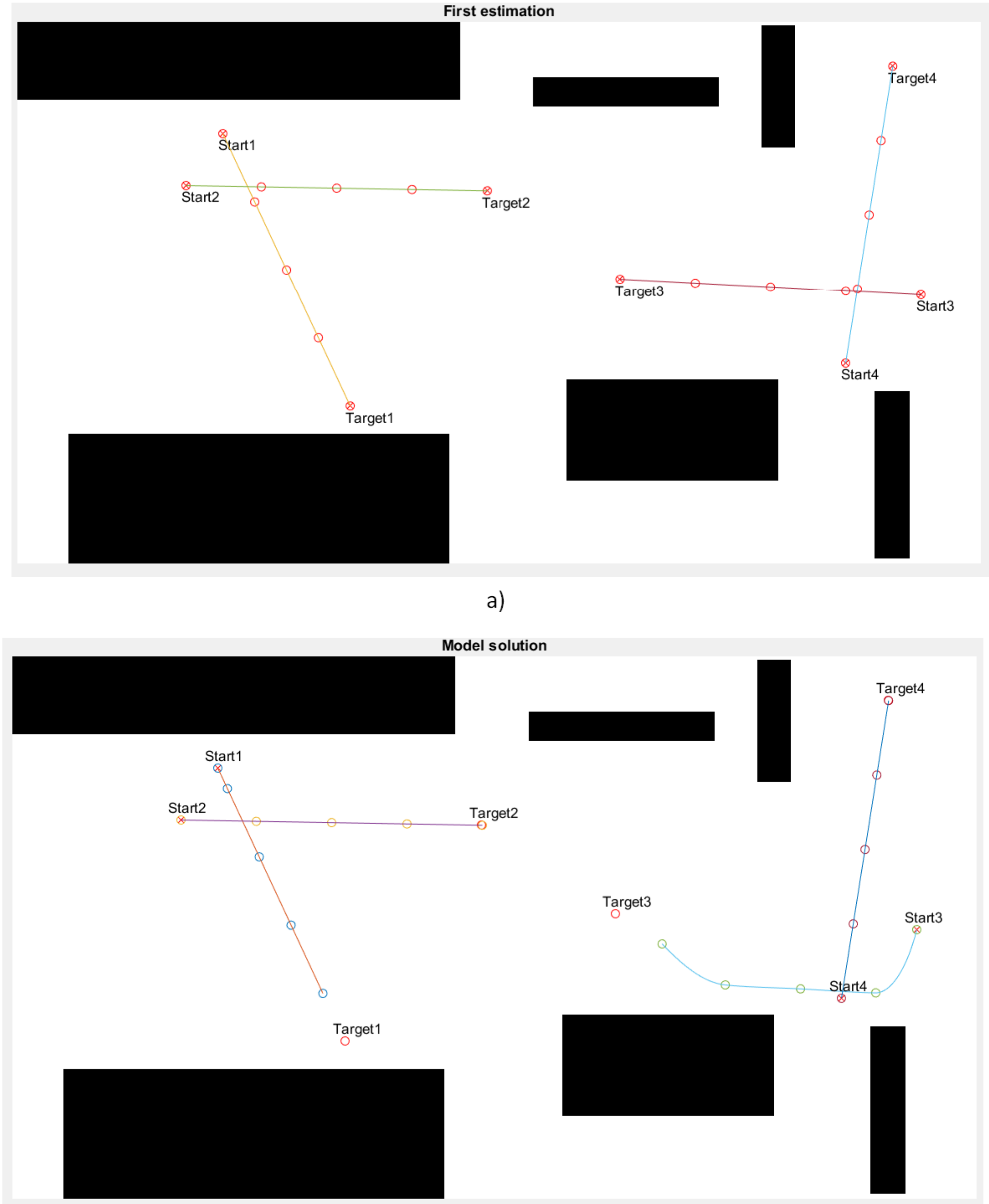


a)



b)

Figure 4.11: In **a**) the *first estimation* with the obstacle and intersection recognition; In **b**) the *model solution* with game theory.



b)

Figure 4.12: In **a**) the *first estimation* with the intersection recognition. In **b**) the *model solution* with game theory.

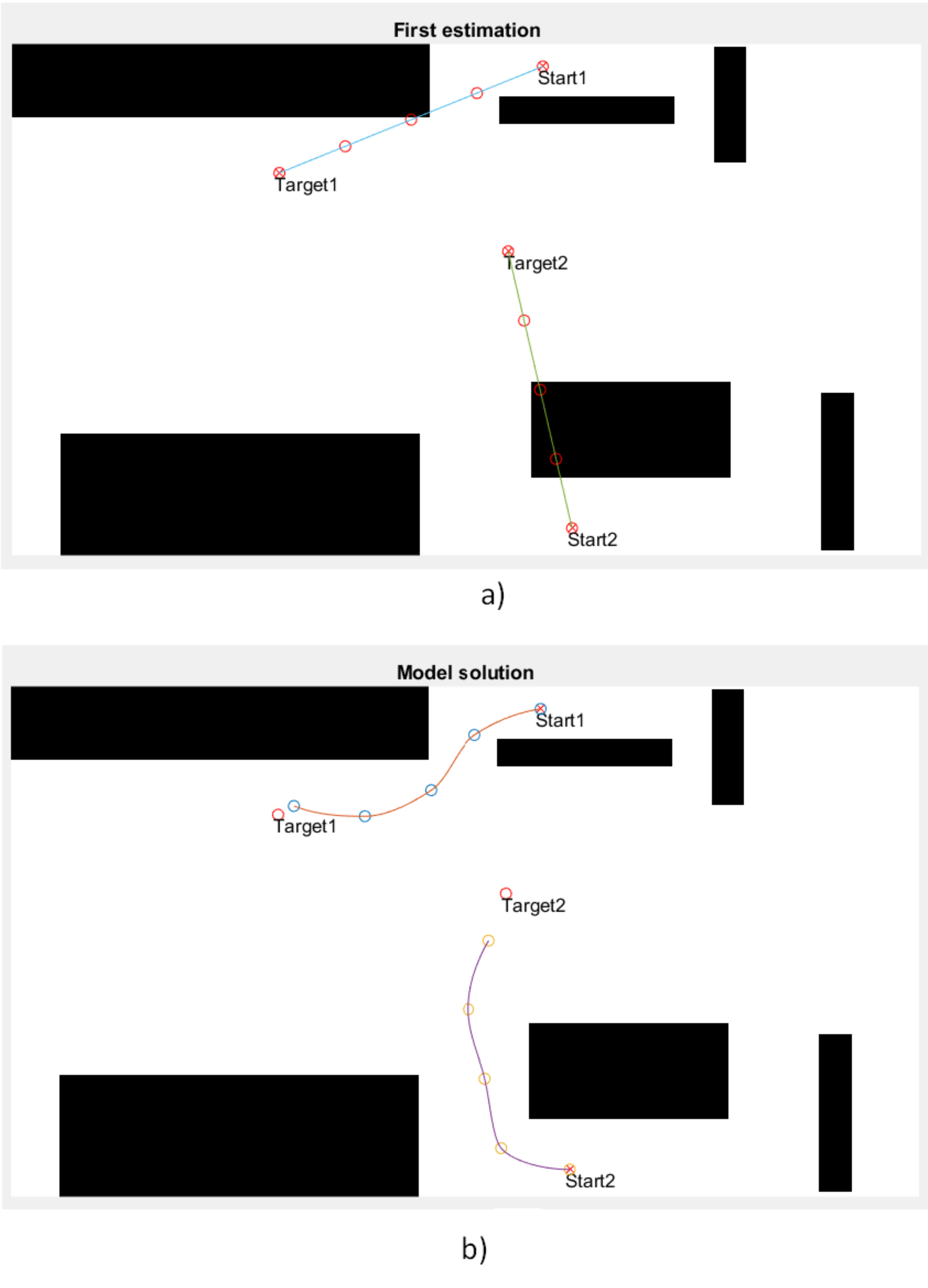


Figure 4.13: Simulation results: **a**) First estimation and obstacles recognition; **b**) Model solution.

Chapter 5

Model validation and experiment

In this chapter, we discuss what kind of methods we adopted to validate our work. In the first paragraph, we explain how we validated the human motion model based on game theory with real-world surveillance videos. Afterwards, we present the setup of the variation of the Turing test that we have done to validate our human motion model and to understand the social acceptance of the player robot.

5.1 Human motion model validation with videos

The surveillance videos that we used to validate the model are completely open-source [25]. In those videos, multiple pedestrians walk in an urban environment and actively avoid each other during the navigation.
The *qualitative* comparison between real-trajectories and the output of the game theory model are summarised in Figures from 5.1 to 5.4.

It is evident, in all cases (Figures 5.1-5.4), there are some differences but in the following, we tried to explain the reasonable motivations and the possible future improvement.
In general, the collision-free trajectories computed by our model are similar to the real one but with an excessive reaction to the dynamic obstacles. For instance, in Figure 5.2, the pedestrian with the green trajectory would avoid the blue one. The output of our model gives a green collision-free trajectory but with a high distance from the surrounding pedestrians during the last instances of the forecasting. The same situation is shown in Figure 5.3 (red trajectory) and in Figure 5.4 (blue trajectory). The main reason behind this phenomenon is that the model that we created is *discrete*.
Precisely each pedestrian has an action set composed of 7 possible directions in his *visibility* zone. The player can choose one of those actions to minimise the cost function, considering the constraints and, thus, the all strategies of other players.
The first possible action, in order to avoid the obstacle, is the actual direction plus

Figure 5.1: Validation with surveillance videos. *On the left*: the real trajectories are shown; *On the right*: Trajectories output of the game theory model.

Figure 5.2: Validation with surveillance videos. The scenario is the same described in Figure 5.1.

or minus 30 degrees. Clearly, this choice cannot give us a precise result, but we selected 7 directions because they are the best trade-off between acceptable trajectory and computational complexity of the algorithm.
In future, if we develop a more performing algorithm, surely, we will introduce for each pedestrian as many directions as possible and, thus, will have more accurate results.
However, in Figure 5.1, the yellow predicted trajectory is completely different from the real one. This specific pedestrian has a special motion, indeed, watching the video it is easy to deduce that the pedestrian is waiting for someone/something, but obviously, the proposed model can not manage this situation.

Figure 5.3: Validation with surveillance videos. The scenario is the same described in Figure 5.1.

Figure 5.4: Validation with surveillance videos. The scenario is the same described in Figure 5.1.

The human does not have a specific target to reach and then the goal that we estimated is useless and therefore also the trajectory.
Furthermore in general, we observed that the long-term prediction is less reliable than the short-term one, most likely because we do not know the real human target but we estimated the goal as we explained in Chapter 4. This is not a real issue for our scenario, because we have assumed that every 1 second we compute again the trajectories for all pedestrians, thus, the most important prediction for our purpose is the short-term one.

5.2 Test

5.2.1 Evaluation method

With this experiment, we want to validate our human motion model based on game theory and understand the social acceptance of the player robot.
How to measure the social acceptance of the robot?
Humans, attribute sometimes intentions and consciousness to non-human agent [55]. The term to explain this behaviour is *anthropomorphizing*. We can use anthropomorphizing to improve human-robot interaction during the robot motion [50].
This is reached by attempting to design the motion of the robot in a *human-like* manner [34].
Thus, to measure the acceptance, during the experiment, we evaluate the human-likeness of the player robot motion.

Definition 5 *"**Human-like motion planning** consists of planning collision-free motions for one or more agents such that they behave equivalent to, or indistinguishable from, a human [50]".*

To reach the goals of the experiment, we formulate an experiment composed of four sections (Figure 5.6).
We started with a training phase, in order to give to the participant a general idea of the test scenario (Figure 5.5).

Figure 5.5: Training **a**) First section training; **b**) Training with arrows.

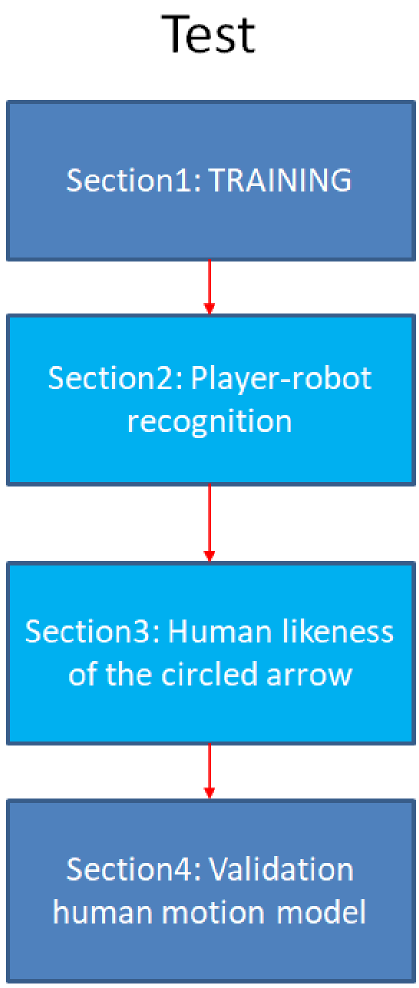


Figure 5.6: Overview of the experiment.

From the first step onwards each participant watches only arrows that move as pedestrians in the same environment of the training phase but without the visualisation of the urban space (Figure 5.7).
We decided to not insert the map, in the majority of the videos, to prevent participants from being bias by the environment. Instead, we want that the attention of the test participants is focused on the agent motion, essential for our purposes.
The four phases have been designed respecting the rule of the "funnel", that is, starting from the general and arriving at the particular.
As a matter of fact, the second and the third phase of the experiment are used to validate our socially-aware motion planning creating videos with real humans and a robot (that is not distinguishable from the others arrows) that moves considering our artificial walking motion or a reactive motion planner (non-player robot) based

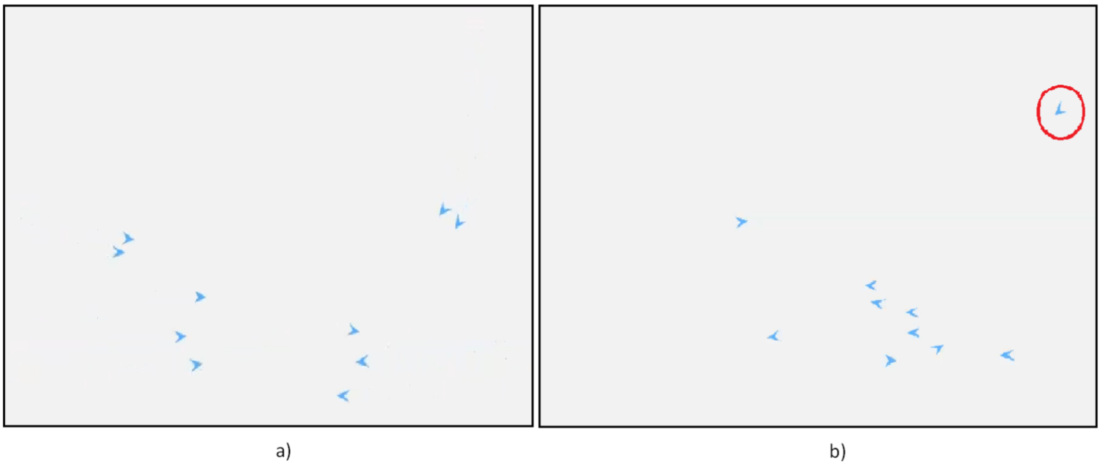


Figure 5.7: Arrows that are moving as agents in the same training urban environment. **a**) Second and fourth section set-up; **b**) Third phase set-up.

on the Enhanced Vector Field Histogram (VFH+) algorithm.
For more clarity and completeness, in Figure 5.8 is shown the main difference between the two algorithms.

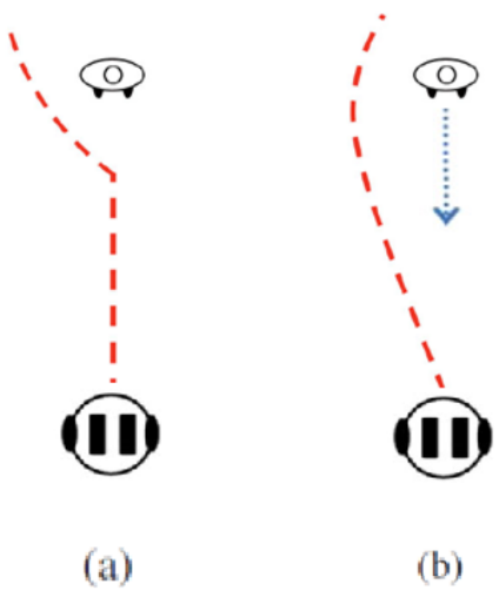
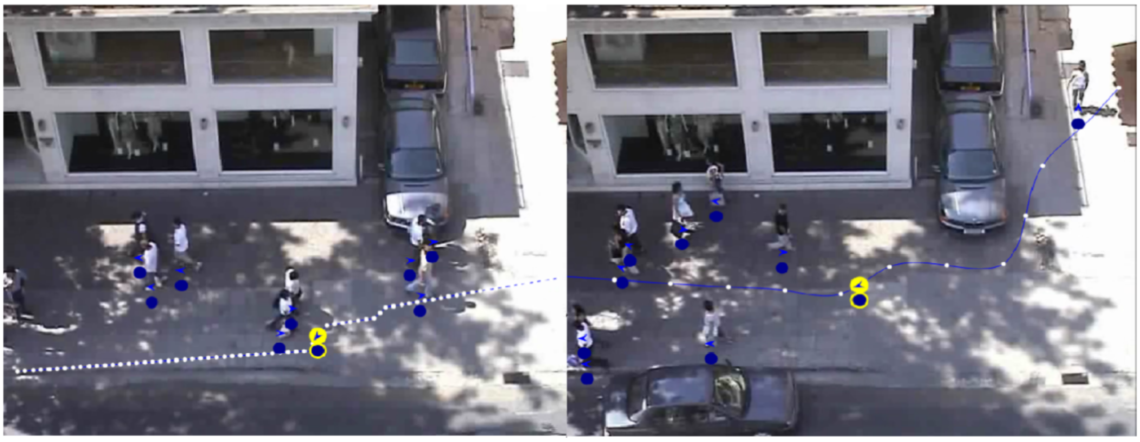


Figure 5.8: **a**) Non-player robot behaviour (VFH+); **b**) Player robot behaviour.

In Figure 5.8**a**, the navigation of the robot is with a reactive planner (VFH+) and it does not forecast the trajectory of pedestrian. On the other hand, in Figure 5.8**b**, the robot predicts the pedestrian trajectory and in advance avoids the agent. The implementation result of the two algorithms in an urban environment is shown in Figure 5.9, where the robot trajectory is highlighted.
It is evident that in the VFH+ case (Figure 5.9**a**) the yellow arrow (non-player robot) avoids pedestrian only if it is near to the human, on the contrary, in player robot scenario (Figure 5.9**b**) the yellow arrow predicts human motion and avoids the pedestrian in advance.
Another difference between the two algorithms is that in the VFH+ simulation, the robot can move with a constant velocity that has been set as a human velocity, conversely, the player robot starts with a constant human velocity (in order to be more likely anthropomorphized) but can decrease its speed if necessary.
In particular, in the third section of the test, we want to concentrate the attention of participants to one of the arrows in the video that can move as real humans, non-player robot or player robot (as shown in Figure 5.7**b**).
In the last section of the experiment, we want to validate the human motion model doing the comparison between videos. The motions in the videos are generated based only on real humans or considering only agents controlled by our motion planner.



a) b)

Figure 5.9: Robot in urban space: the yellow arrow, in both images, represents the robot position at the same instant of pedestrians (blue points). **a**) Non-player robot (VFH+); **b**) Player robot.

During the experiments, participants watch 25 videos (20 seconds each) and answer some questions. In general, the answer should be binary, it means or yes or not, but in some particular case, we ask to express the opinion on a Likert scale [28]

from 1 to 5.
The study takes at least 8 minutes to complete successfully. The unique two rules of the experiment are: (i) the participant can pause the video if watch something abnormal (ii) cannot ask to watch again.
The following section presents in more details the questions in all steps with the experimental video setup designed as a variation of the Turing test.

5.2.2 Experimental setup

General questions
For each person we asked: gender, age and their level of professional experience with the robotics field on a Likert scale from 1 (no experience) to 5 (expert).

First section: Training
The videos showed pedestrians walking in a metropolitan space only in this section (Figure 5.5a). In order to pass gradually to the format shown in Figure 5.7, we designed a situation like in Figure 5.5b.

Second section: Validation and recognition of human likeness
In this case, we exhibited 9 videos in random way. Three of those had only real humans, other three had real humans with our player robot and the remaining had real humans and the non-robot player that was moving with a reactive planner (VFH+).
We asked the participant to press pause and point out the arrow that was moving abnormally. Then, we asked the following question:

1. Please, rate the degree of the naturalness motion of the arrow on a scale from 1 (completely unnatural) to 5 (completely natural).

With *naturalness*, we denoted the *human-likeness* of the arrow motion.

Third section: Recognition of human likeness of a circled arrow
In this phase we showed the same 9 videos of the previous section but in different order, with the addition of a circled arrow that the participant should follow in the video. The questions we asked the participants were:

1. Is the moving arrow a human?

2. Please, rate the degree of the naturalness motion of the arrow on a scale from 1 (completely unnatural) to 5 (completely natural).

Fourth section: Validation of the human motion model
In this phase we showed 4 videos in a random way. Two were real pedestrians and

the remaining were the output of our human motion model. After the videos we asked:

1. Are the moving arrows humans?

2. Please, rate the degree of the naturalness motion of *all* arrows on a scale from 1 (completely unnatural) to 5 (completely natural).

Chapter 6

Test results

In this chapter, we summarise the main results of our study, in particular, the principal outcomes of the variation of the Turing test.
50 volunteers participated in our test. At the beginning of the experiment, we asked participants the general questions about gender, age and the level of professional experience in the robotic field on a scale from 1 (no experience) to 5 (expert). These information are summarised in Table 6.1 and in Figures 6.1-6.2.
Most of the participants were men aged 19-28 with no experience in the robotics field. For each human, we showed 25 videos and asked the questions described in detail in Chapter 5.
In the following sections, we describe and analyse the results for each phase of the experiment and, at the end, we discuss and compare the test results of different sections.

General information	Collected data
Number of participants	50
Gender	30% Female and 70% Male
Age	from 19 to 58

Table 6.1: General information collected from the participants.

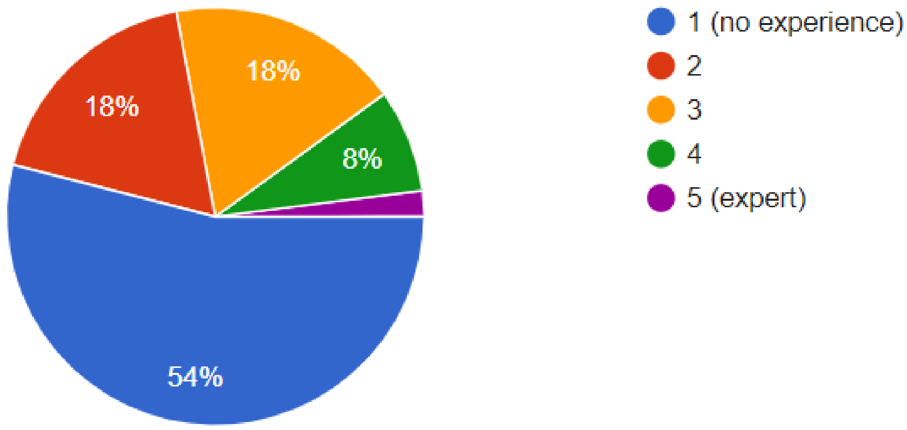


Figure 6.1: Level of experience of the participants in robotics.

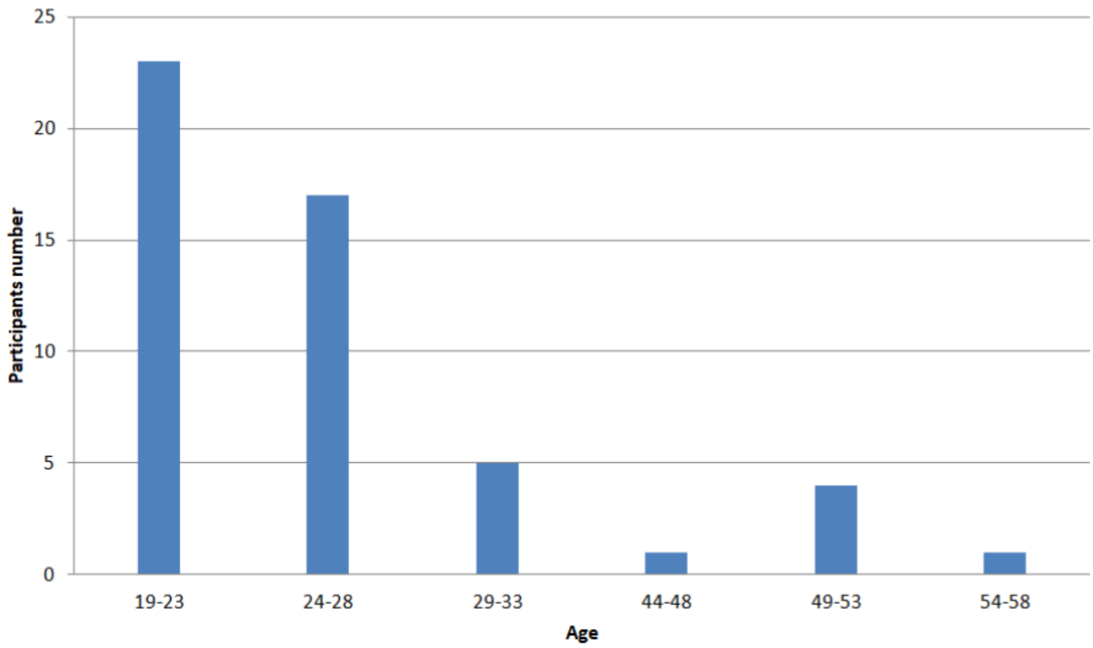


Figure 6.2: Distribution of participants' age.

6.1 *Second section test* result

In this phase, the participant should indicate the arrow that in his opinion moves in a strange way. If he/she has found the right arrow, we asked the degree of naturalness of the arrow's movement on a scale from one (completely unnatural) to five (completely natural).
We showed 9 videos and, in the following, we present the result for each video, grouping them into 3 categories: only humans, humans with non-player robot, humans with player robot.

6.1.1 Surveillance videos

In this section, we have summarised the recognition of at least one agent with a weird behaviour in videos with only real humans (Figure 6.3).

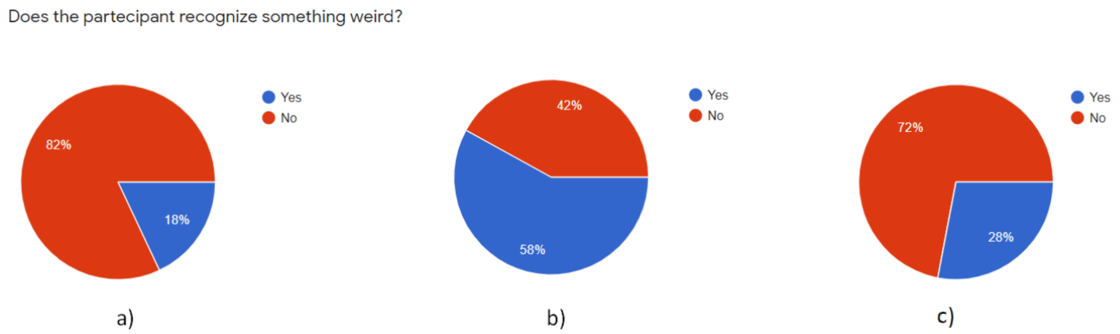


Figure 6.3: Surveillance videos recognition.

In Figure 6.3**a** and 6.3**c**, we collected the answer we expected, indeed, the majority of the participants (82% in Figure 6.3**a** and 72% in Figure 6.3**c**) did not recognise strange arrow movement.
Instead, in the Figure 6.3**b** more than half (58%) participants identified something weird, also in this case this is a trend that we supposed to achieve. In particular, the second videos surveillance has been selected to test the focus of our participants, in fact in that videos, there is an arrow that suddenly changes the direction of travel, perhaps because it has changed the final goal during the navigation.
Despite the strange behaviour, in the 6.2.1, more than half of the participants classified the same arrow as human because of its natural and smooth motion.

6.1.2 Humans with non-player robot

In this part, there is an overview of the recognition of a non-player robot (controlled with the VFH+ algorithm) in a crowd of humans (left panels of Figure 6.4).
In addition, in Figure 6.4 on the right, is represented the degree of naturalness of the recognised non-player robot arrow on a Likert scale from 1 (completely unnatural) to 5 (completely natural).

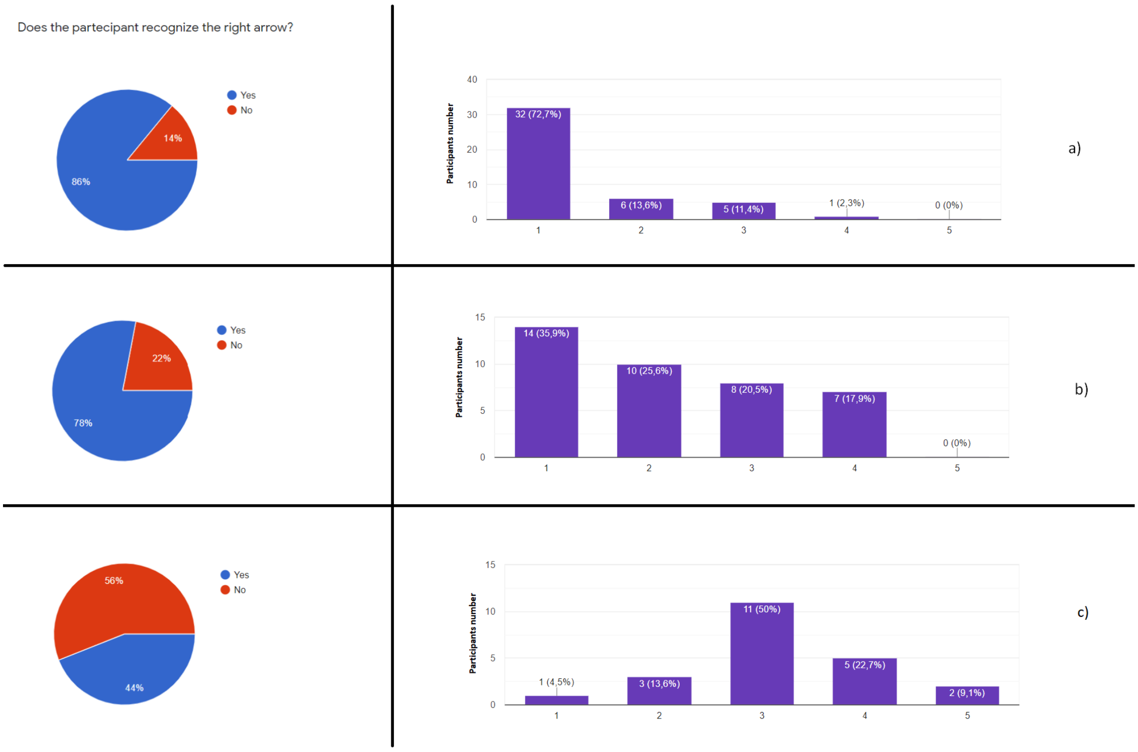


Figure 6.4: *On the left*: recognition of the *non-player* robot; *On the right*: naturalness of the recognised non-player robot on a Likert scale from 1 (completely unnatural) to 5 (completely natural). Labels (a), (b) and (c) are used to identify the specific video.

In the first video, Figure 6.4**a**, we placed a *reactive* robot that is easy to recognise because it has a non-smooth trajectory and it changes directions several times during navigation. Thus, as we expected, 86% of the participants identify the correct arrow and assign the minimum of naturalness.
In the second video, Figure 6.4**b**, the *non-player robot* changes its movement less time than the previous case and the final trajectory is not completely smooth.
As we supposed, the second video is less discernible than the first case (the second case is noted by 78%, in contrast, the first video by 86%) with a greater distribution of naturalness than the preceding instance.

In the last situation, Figure 6.4**c**, only 44% of participants distinguish the *non-player* robot from the real humans. This is a particular tricky scenario, indeed, the *reactive* arrow does not change its directions quickly, but avoids the other arrows only when it arrives in front of them. The reason why it was not recognised by the remaining 56% of participants will become clear by looking at the results of the section 6.2.2.
In general, the non-player robot is most of the time recognised for its strange movement, except for the third video which is a special scenario.
The remaining 14% for the first case and 22% for the second, they did not identify something weird. A possible reason for this result is: they were distracted by the other arrows' movement.

6.1.3 Humans with player robot

This result section is one of the most important because we show the outcomes of videos with real humans and our player-robot.
As in the previous sections, in Figure 6.5 there is a summary of the recognition result of the robot on the left, and an overview of the naturalness for the participants that identified the correct arrow on the right.
The degree of *naturalness* is based on a Likert scale from 1 (completely unnatural) to 5 (completely natural). It is evident in orange, in Figure 6.5, that the majority of participants did not identify the arrow that was moving with our socially-aware motion planning. Thus, the player robot easily blends into the real human crowd.
Moreover, if the player robot is identified (12%, 6%, 28% respectively), the degree of naturalness is still medium-high.
We would also highlight that the player robot in the first two videos (Figure 6.5**a**, 6.5**b**) was created with the same simulation parameter (same time delta equal to 1.2 second), in fact we collected similar results.
In contrast, in the third case, (Figure 6.5**c**) the player robot computes its trajectory considering a smaller time delta (0.7 second). This last project choice influences the final trajectory of the robot and, thereby, also the recognition during the experiment. In particular, the smaller is the forecast time, the lower is the readiness to avoid the obstacle during navigation. Probably, for this reason in the third case (Figure 6.5**c**) 28% of people identified our player robot.

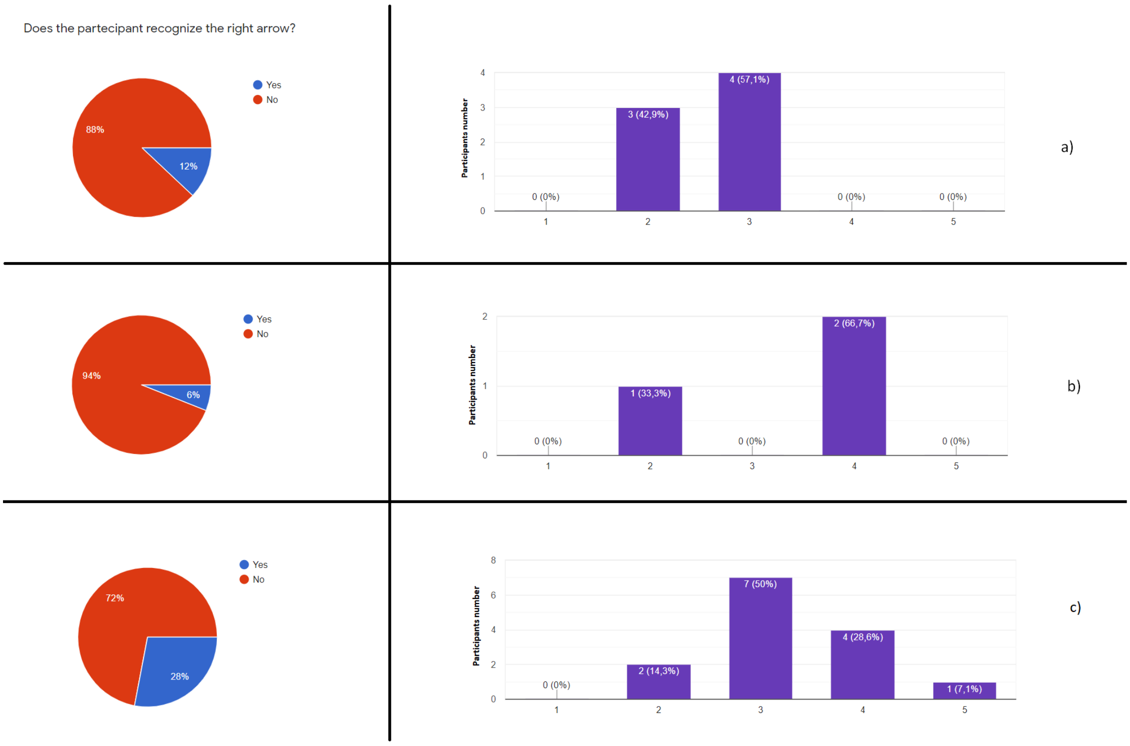


Figure 6.5: *On the left*: recognition *player* robot; *On the right*: naturalness of the recognised player robot. Labels (a), (b) and (c) are used to identify the specific video.

6.2 *Third section test* result

The videos, showed in this part, were the same of the *second* section but we integrated a circled arrow to focus the human attention on a specific agent/arrow.
In the following, we present the main result grouping the outcome, again, as: only humans, humans and *non-player* robot, humans and *player* robot.
The sequence in which the results are presented is the same as the previous section, in order to compare the outcomes easily. Clearly, the order of analysis of the results does not reflect the procedure in which we showed the videos to the participants.

6.2.1 Surveillance videos

In Figure 6.6 is shown the main result of the circled arrow that was moving has a real pedestrian.

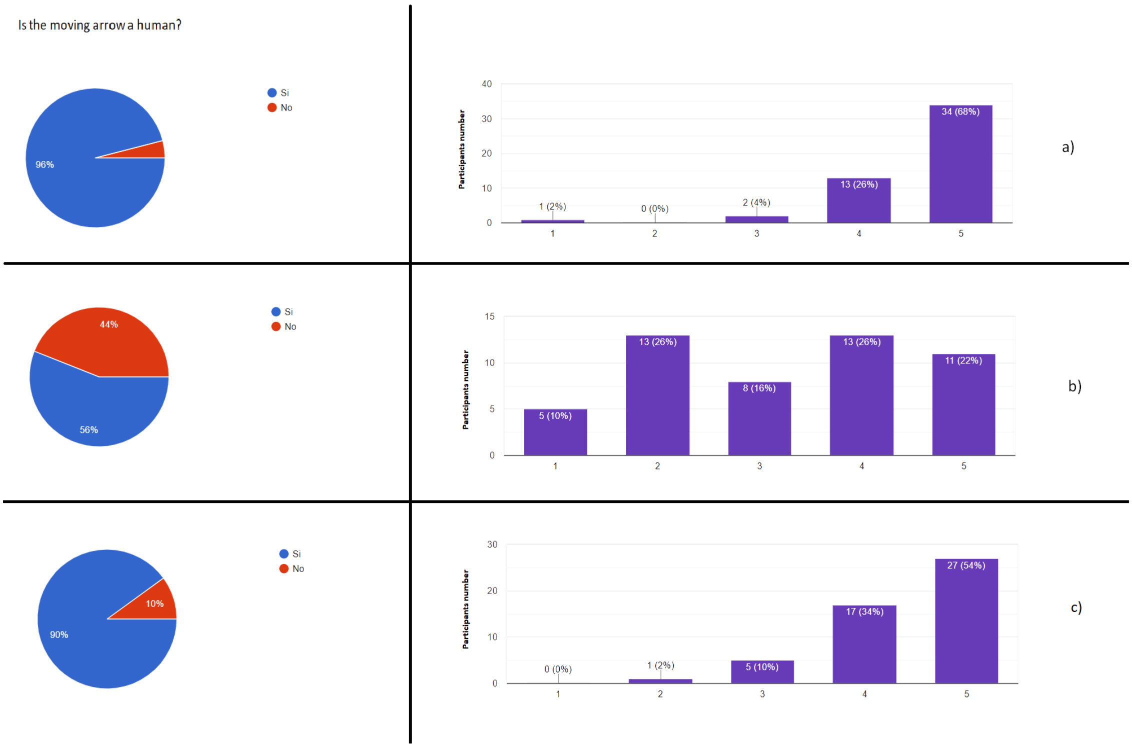


Figure 6.6: *On the left*: recognition of the circled arrow as human; *On the right*: naturalness of the circled arrow on a Likert scale from 1 (completely unnatural) to 5 (completely natural). Labels (a), (b) and (c) are used to identify the specific video.

As we supposed, the real human is perceived mostly as a pedestrian with 96%, 56% and 90%. However, also the naturalness is significantly high for the first and third case (Figure 6.6**a** and 6.6**c**), with more than 50% with the highest value of the Likert scale. Further discussion is necessary for the second video (Figure 6.6**b**).
As we have explained in Subsection 6.1.1, the second video is different from the other, due to the weird behaviour of the pedestrian. Nevertheless, more than half (56%) of humans classified that strange trajectory as likely for people.
The most frequent comment to justify that answer was: "it could be a human because it has too unpredictable behaviour".
The naturalness is distributed by all the Likert's scale (Figure 6.6**b**), probably because of the unconventional nature of the arrow chosen.

6.2.2 Humans with circled *non-player robot*

The overview of this section results is shown in Figure 6.7.

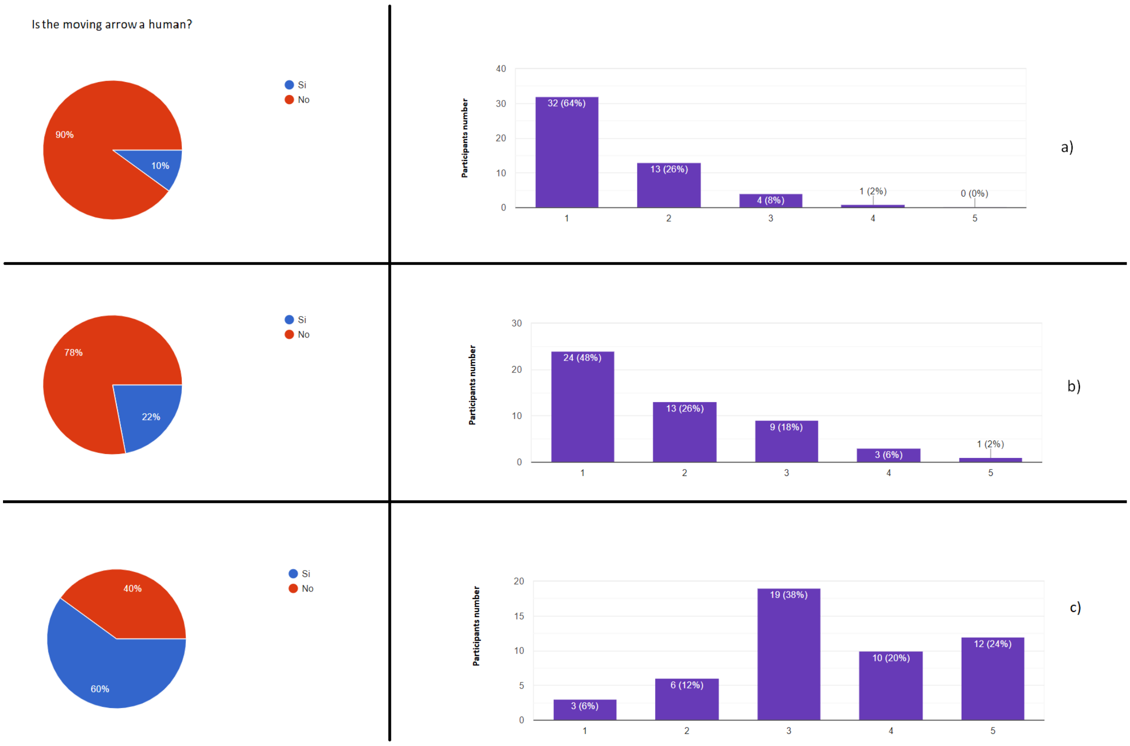


Figure 6.7: Each image row is labelled by a letter (it identifies the results of the same videos). *On the left*: the recognition of the circled non-player robot as human; *On the right*: the degree of naturalness of the circled arrow.

As we assumed, in the first two cases (Figure 6.7**a**, 6.7**b**) the non-robot player is perceived as an *artificial* agent (90% and 78%) with a motion behaviour very far from being natural. These outcomes are consistent with the results in Subsection 6.1.2, in fact the *qualitative* distribution of the naturalness shown in Figure 6.4**a** and 6.4**b** is very similar with the Figure 6.7**a** and 6.7**b**.
However, the third case, has achieved results in contrast to what we expected (Figure 6.7**c**). 60% of participants consider as human the non-player robot arrow with a medium-high naturalness level. This result is very interesting and strange but can explain why in the previous section (6.1.2) 56% of the participants do not recognise the exact arrow.
As we said above, in this third video the robot avoids the pedestrians when it arrives in front of them, but in general, it does not have sudden changes of direction.
For this reason, the main observation that the participants said to justify the human choice was: "Probably the pedestrian is looking at the smartphone and does not

notice the other pedestrian before arrives in front of them".

6.2.3 Humans with circled *player robot*

In these videos, the humans follow an arrow that is moving considering our algorithm. In Figure 6.8 is shown a result summary of this part.

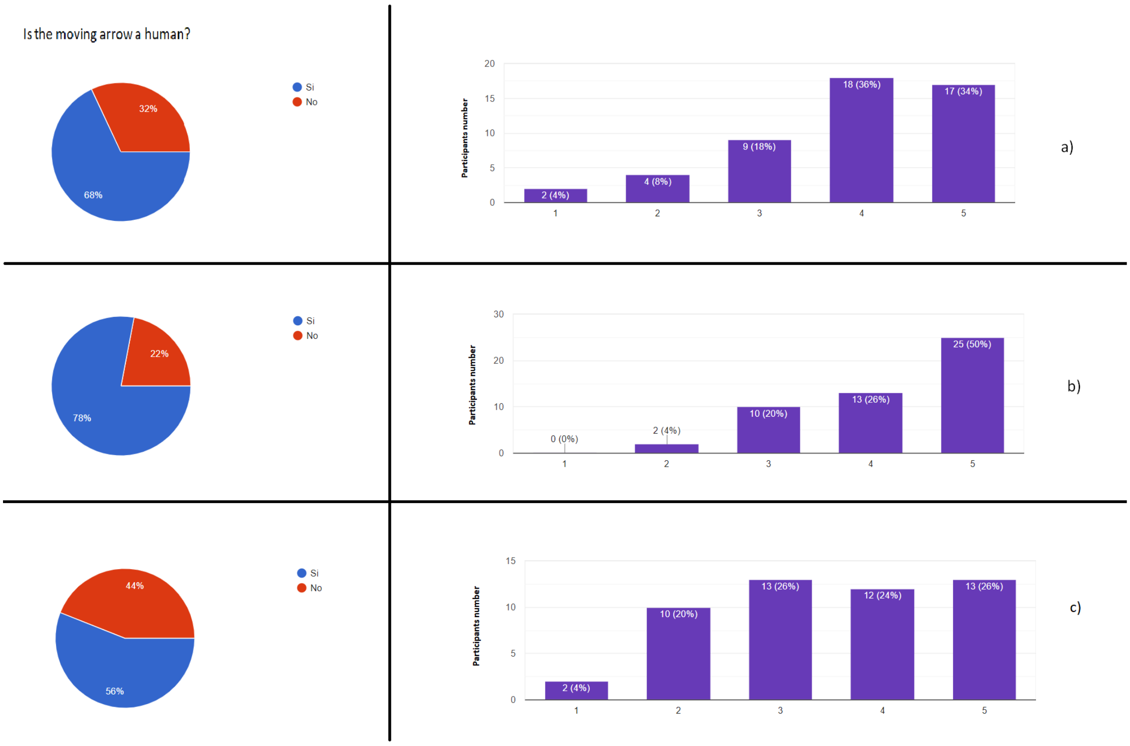


Figure 6.8: *On the left*: the recognition of the circled player robot as human; *On the right*: the degree of naturalness of the circled arrow. The image row is labelled by a letter in order to identify easily the final results of each video.

In all videos the majority of the participants classified out player robot as a human with 68%, 78% and 56% respectively.
As we highlighted in the Subsection 6.1.3, in the first two cases (Figure 6.8**a** and 6.8**b**) the robot has equal parameters. This similar design choice influences also comparable results.
On the other hand, the results of the third video (Figure 6.8**c**) deviate somewhat from the first two but the majority of people granted a medium-high naturalness. This fundamental consideration can be generalised to all videos with circled robot player. Indeed, comprehensively, the naturalness assigned to the socially-aware robot is medium-high, even if the robot is identified as a non-human entity (Figure 6.9).

Why do humans recognise the robot as something artificial but consider its navigation as natural? One of the possible explanation for this phenomena is that the arrow movements was too perfect to belong to a human being because it does not have the uncertainty and the unpredictability like a real pedestrian, but despite everything, the movement is perceived as natural because it is quite credible.
This hypothesis could be confirmed by looking at the graphs in Figure 6.9.
In fact, the first two videos have more performing robot than the third one and this is visible also in the results in Figure 6.9**a** and 6.9**b** with 71% and 83% of medium-high naturalness.

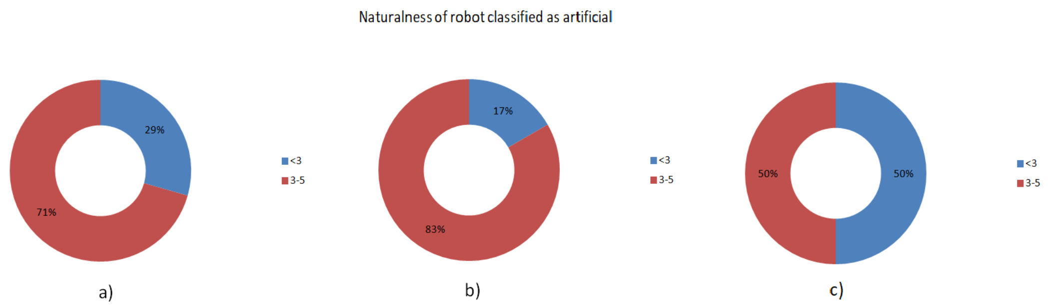


Figure 6.9: Graphs show the percentage of naturalness for the player robot that was classified as an *artificial* arrow for the three videos.

6.3 *Fourth section test* result

This last section aims to validate the human likeness of our human motion model. The arrows in the videos are or *all* real humans or *totally* artificial agents.
The results are summarised in the following.

6.3.1 Surveillance videos

Figure 6.10 shows that humans recognise successfully the movement of real pedestrians with 78% and 86%. Moreover, also the naturalness is consistent with the previous result having a high degree of spontaneity.

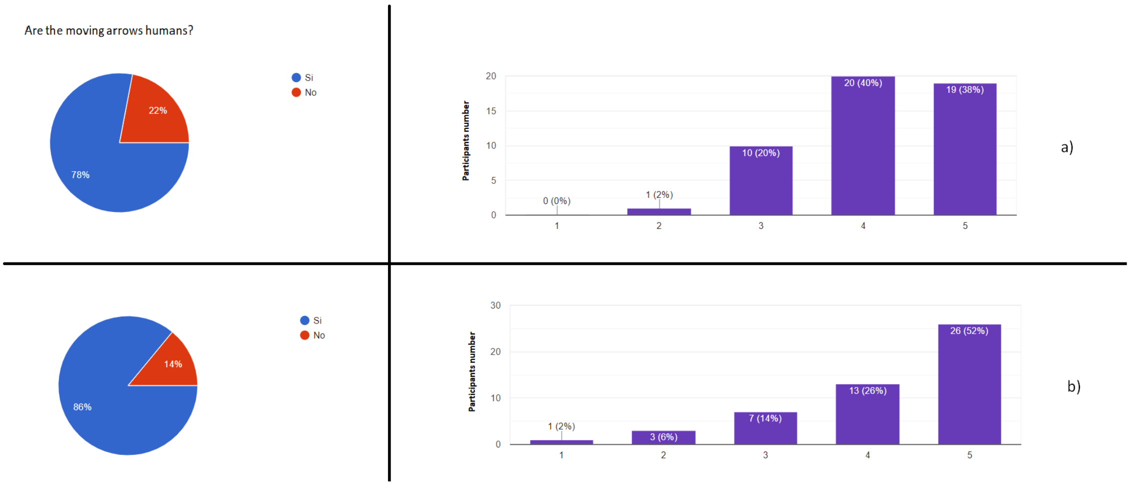


Figure 6.10: *On the left*: arrows categorized as human; *On the right*: naturalness of arrows on a Likert scale from 1 (completely unnatural) to 5 (completely natural).

6.3.2 Human motion model

In this section, we tried to validate the human likeness of our model.
The results are presented in the Figure 6.11. In this case, an high percentage of humans identify the arrows as *artificial* (50% and 42%), though the naturalness is weirdly medium-high.

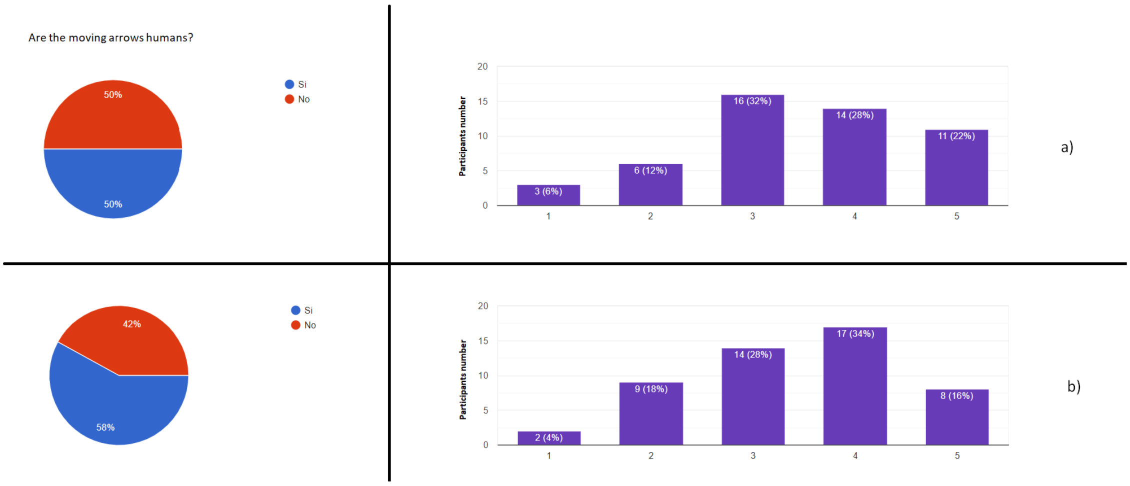


Figure 6.11: *On the left*: in orange is shown how many participants identified the arrows' movement as human motion; *On the right*: the naturalness of *all* arrows on a Likert scale from 1 (completely unnatural) to 5 (completely natural).

This result could be prove considering that the movement of all arrows is too

tidy and coordinated to be real (especially the grouped arrows).
However, in general, the trajectories are considered *natural* because they are quite credible. Considering that our model is discrete, we expected that the participants largely recognised the scenario as not-human. But it was less evident that the same trajectories are catalogued as human-like.
Overall, we collected interesting outcomes, but we would highlight that the final goal of this section was to validate the model and, then, use it for the robot trajectory.

6.4 Discussion

Although most of the participants have no experience in the robotic field (54%), surely everyone found themselves navigating through a crowd daily. Thus, the working environment is very well known to all human beings, in fact, every day humans predict in a completely natural way the behaviour of the other pedestrians even just looking a few moves.
Comprehensively, the results are consistent with what we expected. In particular, the participants did not distinguish between our planner and real human but detected the difference with the non-player robot (controlled with the VFH+ algorithm).
In the third section, considering the videos with the circled player robot, we collected the main result of the thesis. The player robot is classified, globally, as a human with a high naturalness.
However, it is important to highlight an interesting and unexpected result. The minority of participants that identified the player robot as an artificial pedestrian still assign a medium-high degree of human-likeness.
Although the environment is aseptic and participants do not have great competence on the robotic subject, it is incredible how the arrow is widely classified as not human but with a maximum naturalness.
We tried to explain this unexpected result comparing the outcomes of different sections. The player-robot arrow sometimes is classified as artificial because, clearly, the movement is the output of a discrete model and the final trajectory is excessively smooth and predictable. In contrast, unpredictable and imperfect trajectories are perceived as human beings (like in Figure 6.7**c** and Figure 6.6**b**).
The other goal of this thesis is reached in the fourth section of the experiment, where we tried to validate the human motion model based on game theory.
The players' arrows were identified as *artificial* only by 50% and 42% with a medium-high naturalness of motion (Figure 6.11). In this case, the result can be explained by considering that the arrows' movement is overly ordered compared to the real case but with natural movement anyway.

Chapter 7

Conclusions and future works

In this thesis, we studied the human-robot interaction focusing on the human behaviour. The main goal was to plan a trajectory for an autonomous robot that is pleasing and acceptable for humans. How to reach this ambitious goal?
We first created a model that was useful to predict the human decision process during motion in a crowded place. The model is based on a *dynamics, non-cooperative and non-zero sum* game and it is solved with the Nash equilibrium.
The *qualitative* validation, with open source surveillance videos, gave us promising results.
Subsequently, we created the motion planner for an autonomous robot based on our model, considering the robot as a player.
With the experiment, we tried to *quantitative* validate the effectiveness of the methodologies designed for the *player* robot. The participants of our test did not distinguish the human motion and the output trajectory of our model.
In contrast, they recognised as an artificial entity the non-player robot that was moved using a *reactive* method (VFH+).
In the third section of the test, the majority of the participants classified the *player* robot as a human with a medium-high human-likeness. Nevertheless, from the literature [34], we found that if a robot moves like a human, is more likely to be anthropomorphized, and this situation increases the acceptance between humans and robots. Thus, with our experiment, we reached the *ambitious goal* that we presented above.
However, it is important to highlight an interesting and unexpected result. The minority of participants that identified the *player* robot as an artificial pedestrian still assign a medium-high degree of human-likeness. It is incredible how, although the environment is aseptic and participants do not have great competence on the robotic subject (54%), the arrow is classified as not human, but with a maximum naturalness. We tried to explain this unexpected result considering that humans daily navigate in a crowd and have developed predicting skills about surrounding people, only looking a few moves. These abilities also come out in the analysis of

the arrows' movement and most likely they have figured out that the final *player* robot trajectory was excessively smooth and predictable.
Our human motion model based on game theory could have promising applications, not only in the autonomous robotic field, but also in computer games or virtual reality applications to simulate humans realistic motions.
Soon, this model could be useful also for autonomous cars to increase humans safety. Clearly, our model could be improved, for example increasing the number of actions for each player and developing another performing algorithm to solve the game in real-time. Other types of solution for the game could be investigated and compared with our approach.
Another possible improvement of the proposed work could be the combination of the learning approach with the game theory. Indeed, learning algorithms have a great performance, in particular, they have good customisation for every pedestrian based on the data collected during the training phase.
In general, our motion planning method shows very promising results for robots that navigate in a shared environment with humans. Some examples are: assistive robots for seniors, cleaning robots, receptionist robots in public or private shop, delivery robots, to name a few.
Based on our test results, we can affirm that in these scenarios our *player* robot increases the acceptance and the cooperation with humans.
Moreover, one of the boundaries of our experiment, is the limited number of participants (only about 50) and also the restricted number of the videos watched. Could be interesting to repeat the same test with a bigger number of participants (at least 150), and at least 10 videos per types for each section to additionally corroborate this thesis.
Further application is in a scenario where multiple robots are coordinated and planned with our game-theoretic trajectory planning as in LaValle [24] works, or, most recently, with Zhu et al. project [56]. Additionally, in that condition, might be interesting to consider and study the uncertainties in the information of other players, like in a Bayesian game.
Moreover, future works will principally focus on the implementation of our socially-aware trajectory planner in a real robotic platform, for example, in an urban scenario like in Figure 5.5a. Therefore, ask humans to judge the rank of safety and naturalness of robot trajectory.

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