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PRICING ALGORITHMS: IMPLICATIONS FOR COMPETITION

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To my family

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INTRODUCTION

The role of algorithms in modern society is increasingly disruptive. We are aware of the fact that algorithms are part of our daily lives and are able to influence our activities, including our purchases, our preferences, and our choices, even predicting what we would be willing to do in the future and obviously by tracing our actions in order to have an increasingly realistic description of our personality. On the other hand, it is undeniable that the use of algorithms has led to an evolution of many of the processes that take place in the modern economy both from the business point of view, therefore efficiency, higher quality, automation and shorter development times, but also for end consumers, helping them in their choice by expanding the range of products offered, and in other phases of the user experience such as purchasing, increasing ease of use itself. However, over the years, a common thought has emerged from the rapid evolution of new technologies, that is, how human decision-making process will be influenced in the near future by this phenomenon and the implications that this entails from the point of view of competition and consumer.

The development of such relevant algorithms derives from the evolution of the technology on which these are implemented that is artificial intelligence. There is no doubt that the latter has revolutionized economic dynamics and will bring further changes and improvements in many industries and areas of daily life. There are numerous articles that report on the use and impact of this technology. According to the World Economic Forum (2018), thanks to the use of AI there will be 58 million new jobs by 2022, according to Netflix the use of machine learning will allow the platform to save 1000 million dollars a year thanks to the effectiveness and efficiency of its algorithms in terms of suggestions, research, and services on the platform. Not only the streaming giant will benefit from AI, but also Amazon will be able to reduce the budgetary weight of operating costs by 20% (McKinsey, 2017) and about 67% of the world's major companies will implement AI by 2021 (MMC, 2019). Artificial intelligence is not only synonymous with cost savings or increased profits but can be used to eradicate diseases and poverty. A noteworthy example is the recent use of AI for the

synthesis of a drug against fibrosis. The algorithm took 46 days, compared to 8 years (on average) compared to human researchers and investment of 150,000 dollars against millions of dollars that are normally invested for such research (MIT, 2019).

However, all these benefits have alarmed scholars, politicians, and economists by realizing the power of this tool which can also prove to be uncontrollable and difficult to monitor. Therefore, the first conferences and the first studies on the different consequences that may occur were held following the use of these technologies. The factors most often taken into consideration are those that undermine the dynamics of the markets, therefore the maintenance of competition and the final consumer, social welfare and data protection. In recent years the competent authorities have been protagonists in questioning the current regulations and laws in force, so as to open a comparison, as broad as possible, to provide the imminent arrival of these technologies that have shown to be in able to modify economic dynamics in a global manner with disarming ease.

Some of the problems that could emerge from the use of algorithms accompanied by artificial intelligence are new forms of collusion, the personalization of prices and predatory prices.

The modern economic scenario indicates that algorithms are changing the competitive landscape by facilitating anticompetitive practices in ways that do not necessarily require the achievement of an explicit agreement or that may not require any human interaction. Economic literature states that algorithms can increase market transparency, increase the frequency of interaction and therefore increase the likelihood of collusion. Furthermore, with the collection and analysis of consumer data, algorithms can apply discriminatory pricing strategies that are difficult to detect and often based on factors that segment the population in an illegal and anti-ethical way. Finally, the algorithms can be trained to monitor the external environment and begin price wars with competitors if the predatory strategy is effective and with a good chance of success.

The paper aims to investigate the aforementioned consequences, analyzing the available literature on models and analyzes carried out by scholars, as well as supporting the considerations with empirical examples benefiting in reality and obviously detected by the competent authorities. Therefore, the first chapter intends to introduce the basic concepts of artificial intelligence and algorithm, with particular attention to price algorithms, which are analyzed by category based on their mode of operation. The core of the paper is the second chapter which analyzes the three phenomena that the use of price algorithms can develop. The concepts of collusion, price customization, and predatory pricing are defined from a theoretical point of view, to then analyze what are the factors that normally favor these legally prosecutable practices and how the use of algorithms facilitates the application of these practices by making the market more favorable and the companies more inclined to apply them. The analysis highlights the reasons that lead companies to apply these practices, to refine the effectiveness of the algorithms and what are the problems with which the authorities must confront. Finally, some space is given to a summary analysis of the legislation, of the laws in force, highlighting the critical points and which are the roads that must be followed to face future situations so as not to be caught unprepared in the event of market failure, and exploitation of data to the detriment of consumers.

CHAPTER I

Artificial Intelligence, Machine Learning, and Deep Learning

Artificial Intelligence is just one of the many frontiers of knowledge that humanity is trying to study and handle following the countless discoveries that have been the protagonists of technological changes so disruptive as to create real historical revolutionary periods. This phenomenon is rapidly expanding and the researchers, the universities, as well as the companies are competing to identify new algorithms and new solutions optimized to respond to certain real problems, in order to carry out the activities without the help of men increasing the efficiency of machines and cutting costs.

The term Artificial Intelligence (hereinafter "AI") is coined by John McCarthy, an American computer scientist, during the Dartmouth Conference in 1956. According to his definition, AI is "The science and engineering of making intelligent machines, especially intelligent computer programs". From this moment the AI obtained its current name, its mission, its creators and precursors, giving, in fact, the birth of this academic discipline. From its birth, this phenomenon saw several waves of optimism and less profitable moments, characterized by different methods, approaches, and successes that have led this technological frontier to the present day. In the following two decades, researchers and academics were astonished by the first results obtained in this area: computers that solved algebra problems, demonstrations of geometry theorems and learning to speak English, so much so that the first investments from both the public and private sectors grew out of all proportion. After a negative parenthesis with little relevant studies and strong skepticism about the actual potential of this technology, AI had a further boom in the 1980s with the advent of expert systems¹. Despite this, until 1993 there were no further improvements or applications and only in the

¹An expert system is a program that answers questions or solves problems about a specific domain of knowledge, using logical rules that are derived from the knowledge of experts (Britannica, 2016).

90s, AI took off in a disruptive way in the technology industry. Some of the success was due to increasing computer power and some was achieved by focusing on specific isolated problems and pursuing them with the highest standards of scientific accountability. In the first decades of the 21st century, access to large amounts of data (known as "big data"), faster computers and advanced machine learning techniques were successfully applied to many problems throughout the economy. In fact, McKinsey Global Institute estimated in their famous paper "Big data: The next frontier for innovation, competition, and productivity" that "by 2009, nearly all sectors in the US economy had at least an average of 200 terabytes of stored data". By 2016, the market for AI-related products, hardware, and software reached more than 8 billion dollars, and the New York Times reported that interest in AI had reached a "frenzy". The applications of big data began to reach into other fields as well, such as training models in ecology and for various applications in economics. Advances in deep learning (particularly deep convolutional neural networks and recurrent neural networks) drove progress and research in image and video processing, text analysis, and even speech recognition (Nature, 2015).

Artificial Intelligence is a branch of computer science and the main problems it aims to deal with are related to the representation of knowledge, reasoning and problem solving, planning, learning, natural language processing, perception, motion and manipulation, social and general intelligence. Knowledge engineering is a core part of AI research. Machines can often act and react like humans only if they have abundant information relating to the world. Artificial intelligence must have access to objects, categories, properties, and relations between all of them to implement knowledge engineering. Initiating common sense, reasoning and problem-solving power in machines is a difficult and tedious task. Machine perception deals with the capability to use sensory inputs to deduce the different aspects of the world, while computer vision is the power to analyze visual inputs with a few sub-problems such as facial, object and gesture recognition. Robotics is also a major field related to AI. Robots require intelligence to handle tasks such as object manipulation and navigation, along with subproblems of localization, motion planning, and mapping. Finally, Machine Learning consists of the more advanced techniques and models that enable computers to figure things out from the data and deliver AI applications. ML is the science of getting computers to act without being explicitly programmed (Standford University). Learning without any kind of supervision requires an ability to identify patterns in streams of inputs, whereas learning with adequate supervision involves classification and numerical regressions. Classification determines the category an object belongs to and regression deals with obtaining a set of numerical input or output examples, thereby discovering functions enabling the generation of suitable outputs from respective inputs. Mathematical analysis of machine learning algorithms and their performance is a well-defined branch of theoretical computer science often referred to as computational learning theory.

Machine Learning

Following this brief and general presentation on this huge subset of newly developed computer science, it is possible to identify two further branches of artificial intelligence. They are Machine Learning and Deep Learning. Also, in this case, these technologies will be treated in a relatively in-depth way in order to concentrate the analysis on the main implications of their use and less on their technical aspects.

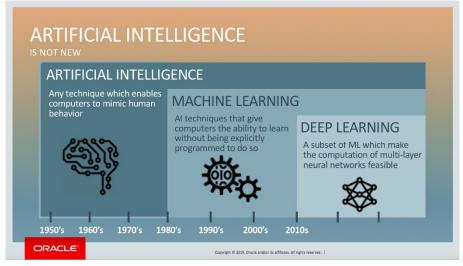


Figure 1 - Relationship between these areas (source: https://blogs.oracle.com/bigdata/difference-ai-machine-learning-deep-learning)

The birth of Machine Learning was a consequence of the need to deal with increasingly complex problems that were impossible to solve by writing fixed codes and static algorithms that do not fit, for examples, with the recognition of images or extraction of meaning from a text. The solution to these new problems stemmed from the emulation of the cognitive process of human beings, rather than just their behavior as in fact the AI sets itself as its main objective. The principle on which Machine Learning is based is essentially to feed the algorithms with huge amounts of data to allow them to train themselves to recognize and understand what kind of problem they are facing and propose an effective solution.

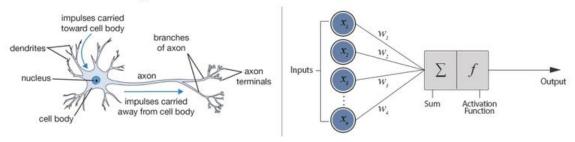
In Machine Learning there are different algorithms that generally fall into three different categories: Supervised Learning, Unsupervised Learning and Reinforcement Learning.

- Supervised learning: Involves an output label associated with each instance in the dataset. This output can be discrete/categorical or real-valued. This model assumes the use of data already tagged in order to train the algorithm teaching it how to discriminate between the entities that make up the sample set. A practical example may be the training of an algorithm that filters e-mails to identify spam. Techniques such as linear or logistic regressions and decision tree classification fall under this category of learning. The regression is used when it is necessary to predict the evolution of future trends or of certain characteristics not currently present on the market such as the cost of a house that has X bathrooms and an extension of at least Y square meters. Instead, the classification is used when it is necessary to categorize a certain observation in a given group.
- Unsupervised learning: This type of algorithm is used when no targeted data is available, and the result of the analysis is not known in advance. They are algorithms that try to learn independently, mostly by clustering and dividing data into different sets. The main family types are clustering and association, the latter is used to carry out market research on consumer trends, or for recommendations that the famous e-commerce sites highlight after a purchase.
- Reinforcement learning: In this case systems are trained by receiving virtual "rewards" or "punishments", essentially they learn by trial

and error. This strategy is built on observation and trial & error to achieve goals or maximize reward. The agent makes a decision by observing its environment. If the observation is negative, the algorithm adjusts its weights to be able to make a different required decision the next time. Reinforcement learning algorithms try to find the best ways to earn the greatest reward. Rewards can be winning a game, earning more money or beating other opponents. Due to its intrinsic structure, this type of algorithm is used more in an economic environment because it aims to maximize the rewards that for a company means to maximize profits. This is, they adapt their behavior based on past actions, more often adopting actions that led to a reward and less often to those that led to a failure. In this way, the algorithms can learn a policy and a sequence of actions that lead them to an optimal solution or that approximates the optimal one. There are many types of reinforcement algorithms, but since the area of interest of this paper is the economic one, it is important to consider those that keep the memory of their actions and those of their opponents, given that memoryless algorithms are not able to punish rivals for past desertions. The family of algorithms most used is precisely that of Q-learning. This choice is since this type of algorithm is constructed to maximize the present value of flows deriving from future rewards in a repeated choice problem. Moreover, they are very popular among computer scientists and simple to set up since they depend on a few variables that can also have an economic meaning (Calvano et al., 2019).

Deep Learning

Another field of Machine Learning is Deep Learning. It is the third tier of the two, AI and Machine Learning, and uses multi-level techniques and methodologies to build different solutions. Deep Learning is also used in an industrial environment in competitive contexts because it has great flexibility in its use and very large potentials that can be completely customized for each company. Deep Learning uses multi-layered artificial neural networks to deliver high accuracy in tasks such as object detection, speech recognition, and language translation. The peculiarity of this area of Machine Learning is that it can automatically learn, extract, translate features from data sets such as images, video or text, without introducing traditional hand-coded code or rules. Due to its intrinsic functioning, neural networks require much more power at the hardware level, in particular, GPUs and CPUs with higher performances than other systems that use algorithms based on other technologies. The main feature of this typology of algorithms is that they replicate the functioning of the human brain, in particular of the biological neural network composed of billions of neurons, each of which is connected with thousands more.



Biological Neuron versus Artificial Neural Network

Figure 2 - Visual comparison between biological neuron and artificial neural network (source: https://www.datacamp.com/community/tutorials/deep-learning-python)

Biologically speaking, each of these neurons receives electrochemical signals and passes these messages to other neurons. Deep Learning is inspired by the functionality of our brain cells called neurons which lead to the concept of artificial neural networks (ANN). ANN is modeled using layers of artificial neurons to receive input and apply an activation function along with a human set threshold. As in other areas, scientists have tried to replicate the functioning of nature also in this case, despite not having a deep and detailed knowledge of the functioning of our neurons.

In the most basic feedforward neural network, there are five main components to artificial neurons. Input nodes are associated with a numerical value, which can be any real number. An example could be a one-pixel value of an image; connections that each of them departs from the input node and has a weight (w) associated with it and this can be any real number. The ANN runs and propagates millions of times to optimize these "w" values. Next, all the values of the input nodes and weights of the connections are brought together. They are used as inputs for a weighted sum. This result will be the input for a transfer or activation function. Just like a biological neuron only fires when a certain threshold is exceeded, the artificial neuron will also only fire when the sum of the inputs exceeds a threshold. These are parameters set by humans. As a result, the output node, which is associated with the function of the weighted sum of the input nodes.

Since this is an area of Machine Learning, the algorithms based on Deep Learning are also trained using the techniques of Supervised, Unsupervised and Reinforcement Learning. Furthermore, there is another point of diversity with the other types of algorithms, namely the presence of the concept of depth. Depth is the number of node layers where there is more than one hidden layer thus need for more computation power for forward/backward optimization while training, testing and eventually running these ANNs. Among the layers, it is possible to distinguish an input layer, hidden layers, and an output layer. The layers act as biological neurons. The outputs of one layer serve as the inputs for the next layer. The computational complexity of an algorithm based on Deep Learning derives precisely from the number of levels it has, in other words, the more these levels are, the more the algorithm is intelligent. These types of algorithms are used for image recognition, including human faces, individuals and tumors. Furthermore, there are variants "with memory" that allow the algorithm to extract the meaning from a text and variants composed of two types of neural networks: a generating network and a discriminating network. The discriminator is trained to recognize certain types of images, while the generator must learn following the behavior of the discriminator based on its output. Once a certain level of training is reached, the discriminator will no longer be able to discriminate between a real image and one created by the generator. This type of algorithm is used for increasing the resolution of an image, recreating popular images or paintings or generating an image from text, producing photo-realistic depictions of product prototypes, generate realistic speech audio of real people as well as producing fashion/merchandise shots.

Algorithms: concepts and definitions

Having introduced the concepts of Artificial Intelligence and the related areas of interest for this paper, it is necessary to introduce the concept of Algorithm and its uses in today's market, as well as to present the subfamily concerning the Price Algorithms combined with the use of the technologies presented above.

Algorithms are used for calculation, data processing, and automated reasoning. There is no one precise definition of an algorithm that has been universally adopted. Instead, there are numerous formal and informal definitions that have been included within the literature. For the purposes of this analysis, it is possible to describe this concept as any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values as output (Cormen et al., 2001). Algorithms have been developed to solve a wide range of practical applications. This includes algorithms that complete simple tasks such as ordering a series of unordered numbers, to complex algorithms that enable digital encryption, internet communication, and the management of scarce resources. Many of these activities have replaced man in performing these processes in order to avoid wasteful situations from the health point of view, but with the advent of Artificial Intelligence and Machine Learning the algorithms have reached a higher level of complexity accompanied obviously from an ability to solve much more complex problems, make forecasts and make decisions efficiently.

Given their importance for modern society, the use of algorithms is becoming increasingly massive, so much so that Stucke and Ezarchi (2016) introduced for the first time the concept of "algorithmic business" referring to the use of the latter for decisions not necessarily merely trivial. Among these, it is possible to find the predictive analysis that involves algorithms in order to elaborate results that can serve as forecasts for the future, based on historical series and data collected in the past. This type of model is widely used to estimate demand, evaluate price changes, predict variations in exchange rates, analyze stock trends and countless other factors that can influence a particular business. In this way, companies have the ability to act more effectively according to corporate strategies and develop innovative and customized services that otherwise would not have existed. Furthermore, the algorithms can be safely used to optimize a company's internal business processes by reducing production and transaction costs, but also by segmenting consumers based on their characteristics gaining an economic advantage. This ability arises from the ability to quickly analyze huge amounts of data at a lower cost in a timely manner compared to how a human could do by achieving the same result.

The use of algorithms is not limited to a specific industry, but rather is affecting all sectors, bringing numerous benefits to companies that use them. Obviously, there are specific sectors where this type of innovation, together with artificial intelligence and machine learning are revolutionizing the market itself. Some areas where the introduction of these new algorithms affect in a disruptive way compared to the techniques of the recent past are the health market thanks to the increasingly sophisticated images recognition software, useful in tracking cancers and assist during delicate surgical procedures (Hemsoth, 2016), civil and mechanical engineering sector where deep learning can predict the response of buildings under certain conditions or earthquakes (Suryanita et al., 2016) and forecast the traffic conditions on a street (Lee and Teknomo, 2016). Moreover, in financial markets where algorithms are been designed to understand the exact time when buy or sell a stock (Er and Hushmat, 2016) and predict corporate bankruptcy (Jones et al., 2016), acting more efficiently than traditional methods. This innovation process, which is affecting almost all industrial sectors, is opening a vicious circle about the development of increasingly functional and efficient algorithms and companies have greater incentives to explore these technologies trying to reap more advantages than competitors.

Pricing algorithms

The concept of algorithm encompasses innumerable applications and uses that companies can use to increase the performance of their business, but the focus of this paper lies on the family of price algorithms, which use prices as input to return prices as output using different processing procedures that depend on the type of algorithm itself. They are able to change the competitive landscape in which many companies operate and the ways in which they make commercial and strategic decisions. Nowadays, a growing number of companies are using algorithms to improve their pricing models and predict market developments. Algorithms can also have a positive impact on consumers and on general social well-being. In fact, these allow access to multiple information more quickly and efficiently, considering consumer preferences. However, when the new technological tools radically revolutionize the way of acting and the interaction of the operators, there is the danger that some market players use their greater power in the service of private interests, which do not correspond to the common objectives of the society.

The ways in which these types of algorithms can be applied in the market are manifold and are the study of this elaborate. It is possible to find algorithms that facilitate anticompetitive agreements, algorithms that use huge amounts of data to generate personalized prices for each consumer or group of consumers, determining, in all cases, repercussions from the point of view of welfare, policy, and regulation.

In the first case, in situations where explicit communication would be necessary to reach an agreement, algorithms can create automatic mechanisms. This favors the implementation of a common policy and the monitoring of the behavior of the various companies, without the need to go through human interaction. In other words, algorithms allow companies to replace tacit coordination with an explicit cartel. While in the second case, the algorithmic approach helps to segment or personalize certain elements of the price waterfall², such as discounts and special offers. It also accommodates differences in consumer tastes, price sensitivities, and other statistical information on potential buyers. Moreover, since these algorithms are driven by millions of data found on the market, they enable to optimize the price strategies to be implemented, customizing them for the different product categories. The data-driven approach helps to determine near-optimal pricing parameters and discover missed opportunities. For example, an algorithmic promotion management system can

² Price Waterfall is the cascade that leads from the gross list price to the net list price, and provides a measurement of the transaction price achieved, going beyond the simple price published on the price list.

suggest new promotions that can improve the performance of the baseline promotion calendar.

The analysis will focus on how the use of algorithms increases the risk of an anticompetitive agreement, presenting a list of the roles that these can play in maintaining a collusion result. The types of algorithms analyzed here are monitoring algorithms, parallel algorithms, signalling algorithm, self-learning algorithm.

Monitoring algorithms

The most obvious way in which algorithms facilitate anticompetitive agreements is by monitoring the behavior of competitors. This may include gathering information and filtering data to detect deviations from the agreed policy and, if necessary, planning for immediate retaliation. The collected data can be analyzed and associated with a price calculation algorithm, which activates an automatic response in the event of deviations from the agreed price. The triggering of a possible price war is therefore unlikely (see Figure 3). Given the speed with which the algorithms are able to detect and sanction any discrepancies, companies naturally have no interest in deviating from the agreement. Thus, unlike traditional cartels, it is rare to see price wars between algorithms, except if done intentionally or by an error triggered by the algorithm itself.

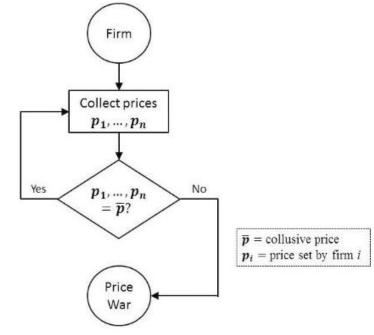


Figure 3 - Monitoring algorithm's illustration

An example of a real application of this type of algorithm can be found in the service offered by Amazon "Match Low Price" for third-party vendors using the platform. This service allows the seller to monitor the actions that the competitors perform for their products, effectively canceling the advantage of a slight deviation to increase sales volumes given the elevated reactivity of the algorithms, without which the advantage would have been realized. A case is what happened to the price of the book "The Making of a Fly" on Amazon in 2011. This textbook on developmental biology reached a peak price of \$23 million. This price was the result of two sellers pricing algorithms. The first algorithm automatically set the price of the first seller for 1.27059 times the price of the second seller. The second algorithm automatically set the price of the second seller at 0.9983 times the price of the first seller. This resulted in the price spiraling upwards until one of the sellers spotted the mistake and repriced their offer to \$106.23 (Eisen, 2011). This example appears to have been the result of a lack of "sanity checks" within the algorithms, rather than any anti-competitive intent. However, it demonstrates how the lack of human intervention in algorithmic pricing may lead to unintended results.

In conclusion, monitoring algorithms can facilitate illegal agreements and make collusion more efficient, avoiding completely unnecessary price wars. However, explicit communication remains necessary during the implementation of the cartel, in this way the authorities that supervise the correct execution of the agreements between the operators can continue to use the normal techniques of detection.

Parallel algorithms

One of the difficulties in maintaining a cartel in a highly dynamic market derives from the fact that continuous changes in supply and demand require frequent adjustments to prices, production, and other commercial factors. The algorithm makes it possible to automate the decision-making processes of companies, overcoming the classic methods of negotiation (meetings, phone calls, e-mails), so that prices react simultaneously to any change in the market, thus applying a form of conscious parallelism. This reduces the risks associated with identifying the "cartel" parts. Today these processes are used, in particular by airlines, hotel reservation services, and transmission system operators, to efficiently adapt the offer to fluctuations in demand. However, a problem could arise if several companies started using the same dynamic pricing algorithm, programming it to avoid competition and set prices at an anti-competitive level. Such an algorithm would allow interested companies not only to act in collusion but also to ensure that their prices react automatically to market developments, without the need for further communications.

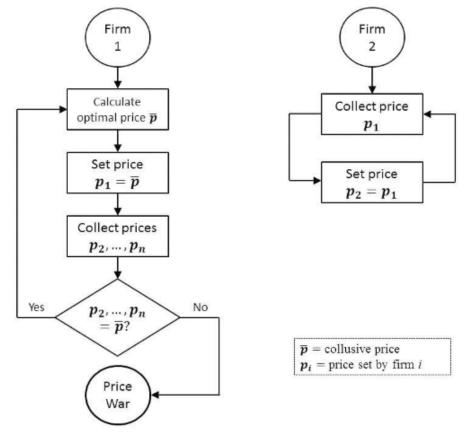


Figure 4 - Parallelism algorithm's illustration

In 2015, the US Department of Justice blamed David Topkins, an Amazon market seller, for coordinating the price of posters sold online with other sellers, between September 2013 and January 2014. According to information published in the inquiry by the Ministry of Justice, David Topkins and his partner had designed and exchanged dynamic pricing algorithms programmed to act in accordance with a cartel agreement. On this occasion, the deputy minister of justice said: "*We will not tolerate anticompetitive conduct, whether it is a complex pricing*

algorithm. American consumers have the right to a free and fair marketplace online, as well as in brick and mortar businesses".

The Topkins' case remains, to our knowledge, the only case of algorithmic agreement detected by a competition authority and which gave rise to criminal proceedings. Therefore, it represents a symbolic case among academics and professionals on the risks related to the use of algorithms.

While sharing pricing algorithms with rivals is a clear violation of competition rules, there may be different ways of coordinating behavior without involving any kind of explicit communication. For example, coordination situations can arise if companies outsource the creation of the algorithms, they would like to use at the same supplying company. This can create a sort of "hub and spoke" scenario where the co-ordination is actually caused by the use of the same "hub" for the development of the same algorithms or similar versions, used to determine the pricing strategies of the companies themselves (Ezrachi and Stucke, 2015). Similarly, it is possible to obtain a collusive result in the scenario that different companies use price algorithms to follow the market leader (tit-fortat strategy), which in turn could turn out to be responsible for fixing prices above the competitive level (see Figure 4, where firm 1 is the leader and firm 2 is the follower).

Signalling algorithms

In highly changeable and heterogeneous markets where the companies that are part of them differ from each other in terms of size, business strategies, products offered, tacit collusion can be quite complicated to apply among operators. In order to avoid explicit communication, companies may attempt to reveal an intention to collude and coordinate more complex cooperative strategies through signalling and unilateral price announcements. As recognized by Judge Posner during the course of an antitrust litigation: "*If a firm raises the price in the expectation that its competitors will do likewise, and they do, the firm's behaviour can be conceptualized as the offer of a unilateral contract that the offerees accept by raising their prices*".

The use of this type of mechanism forces the authorities to constantly evaluate the positive and negative effects on market competitiveness and there is no clear guideline that institutions can strictly follow. Generally, the greater transparency in the market is encouraged by the authorities, but this can lead to undesirable effects, such as the alignment of behavior by other companies in order to take advantage of it in a collusive manner. However, signalling often presents itself as a cost to the company that applies it. Whenever a firm increases the price to indicate an intention to collude, if most competitors do not receive the signal or intentionally decide not to react, the signalling firm loses sales and profits. This risk could lead to waiting for competitors to signal, eventually leading to delay or even failure to coordinate (Harrington and Zhao, 2012). Algorithms can reduce or even completely eliminate the cost of signalling, allowing companies to automatically and quickly activate certain measures. For example, a company can plan overnight price changes which, although they have no particular impact on sales, can still send a signal to the competition. Alternatively, a company can use algorithms to disclose a large amount of data. This procedure represents a codified way of proposing and negotiating price increases, as happened in the case of US airlines (Borenstein, 1994). Indeed, in the 1990s, the United States Department of Justice investigated the application of tariffs in the airline industry which followed a collusive mechanism resulting from the exchange of information between the parties and the use of signalling systems. The flight companies used to send information about the routes they offered including price, airports of origin and destination of a flight, with the Airline Tariff Publishing Company (ATPCO). This information was public and updated in real-time so that the companies announced the rates they would set in the near future and if these signals were accepted by rivals, the rates were set but at a price obviously higher than a competitive context. According to the DOJ's case, it was the existence of a fast data exchange mechanism to monitor tariffs and react rapidly to price changes that enabled companies to collude without explicitly communicating.

Another example that occurred in the 2000s in Italy concerns the illicit agreements between insurance companies. The information exchange was carried out by more than 40 companies in the car insurance sector and gave rise to a

complex and articulated concerted practice between competing companies through the exchange of sensitive information on the prices of RCA policies. This was a serious violation of competition law, given the relevance, detail and frequency of the information exchanged - concerning, among other things, the commercial and contractual conditions actually applied by each company to its customers - as well as the importance and the number of companies involved, which represent around 80% of the car insurance market. This information circuit, implemented in an institutionalized form by the companies through a third company, in particular by adhering to specific ad hoc observatories, was able to determine higher commercial premiums than those that would be registered in a competitive market (AGCM, 2000).

These two examples show that signalling can be very effective not only in establishing a cartel but also in supporting negotiation between companies whose interests are not necessarily aligned. When empowered with technologically advanced algorithms, this informal negotiation process may become even faster and more efficient.

Figure 5 shows the process of a signalling algorithm that highlights how the terms of the collusion are negotiated and established before setting the price

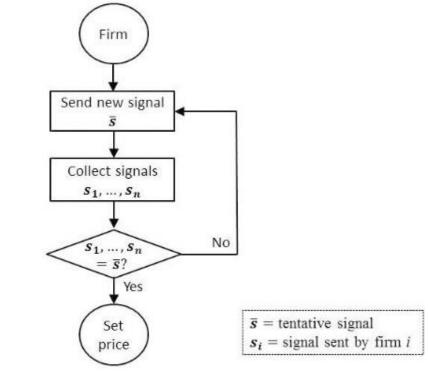


Figure 5 - Signalling algorithm's illustration

uniformly. As portrayed in the flowchart, each firm continuously sends new signals (for instance, offers to raise prices) and monitors the signals sent by the other competitors. When all players finally reach an agreement by sending the same signal, they fix the agreed price until a new successful negotiation takes place.

Self-learning algorithms

Finally, the most complex and sophisticated way to obtain collusive results consists in the use of the self-learning algorithm, which allows reaching a monopoly result, without even the competitors having to explicitly program their own algorithms to achieve this goal. In other words, some algorithms with powerful predictive capabilities, constantly adapting to decisions made by other market operators, are able to establish agreements without any human intervention being necessary.

The way this type of algorithm will actually achieve a collusive result is still not entirely clear. However, when market conditions are favorable to the trade agreement, it is very likely that the algorithms, which learn faster than humans, through repeated trial and error, reach a cooperative equilibrium. Game theory was also used to analyze the aptitude of machine learning to obtain cooperative results. Hingston et Kendall (2004), in particular, conceived an evolutionary game scenario, in which a group of adaptive and non-adaptive agents plays the Iterated Prisoner's Dilemma. In this simulation the former gets a better result than the latter, determining an interpretation of the positive role played by the self-adapting modalities, compared to the non-adaptive ones. More recently, Agrawal and Jaiswal (2012) proposed a machine learning algorithm for the Iterated Prisoner's Dilemma that is shown to perform better than the tit-for-tat strategy.

At present, it is difficult to see if the self-learning algorithms are already leading to situations of collusion in the markets and even less to detect certain situations punishable in the form and in the result given that the establishment process is completely hidden. In fact, by entrusting to the machine learning business decisions, managers not only avoid any kind of explicit communication, but also the creation of any means or structure, such as signalling mechanisms, which could be identified by the authorities as facilitating practices.

In the case that companies begin to use deep learning algorithms to set prices or other variables automatically, collusion would become even more optic to be identified with current standards and identification tools. The operating mode of this type of algorithm can be described using the concept of "black box" (see Figure 6), since it processes the raw data in a complex, accurate and fast way (not unlike the human brain), achieving optimal results without revealing the criteria that underlie the decision-making process. Therefore, companies will actually be able to achieve collusive results without being aware of them. But this does not exclude possible responsibility imputations in case of violations resulting from the functioning of a self-learning algorithm.

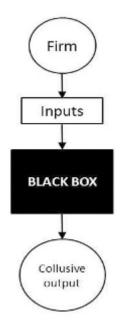


Figure 6 - Self-learning algorithm's illustration

In order to clarify the differences and characteristics of the types of algorithms just analyzed, Table 1 highlights the role that these algorithms can play in implementing a collusive situation.

Table 1 - Key points for each type of algorithm in implementing collusion

Role in implementing collusion	
Monitor the behavior of competitors by collecting data and	
eventually punish their actions.	
Alignment of behaviors between the parties by sharing and	
monitoring decision variables.	
Make sensitive information public to find a shared strategy	
with competitors.	
Adaptation of actions based on the behavior of competitors	
in order to maximize profits.	

Role in implementing collusion

The use of pricing algorithms in real cases

In addition to the examples presented above, there are several cases of use of this family of algorithms by well-known companies, as well as leaders in the market in which they do business. Amazon is once again the marketplace where innovations are implemented promptly, so much so that according to a study carried out by Chen et al. (2016), sellers who use price algorithms get better sales performance than those who do not use them. Not only they are able to be more competitive on the price, but they receive more feedback, they complete greater sales volumes, obtaining a higher ranking in the list among the top sellers. In this way, a seller gains access to the "Buy Box" for a particular product (Figure 7). This amazon feature is very important for the sellers and once the position in the box is reached, the number of purchases for that seller grows even more, increasing the revenue due to the position is not conquered by the low price fixed by the seller, but it comes from the overall rank and so the price could be higher than competitors one. Moreover, to encourage switching from customers, sellers may use algorithms in order to compete. Users of algorithms represent around 2-3% of total sellers on the marketplace and around 40% of those sellers who change their product at least 20 times over its lifespan (Chen et al. 2016).



Figure 7 - Amazon Buy Box example

Another example of an important use of price algorithms can be found in the sharing economy giant focused on public transport: Uber. The peculiarity of this service lies in the complexity of the pricing. Price for a ride is set by Uber's algorithm and cannot be influenced by the driver who gets the ride. The variables taken into consideration are obviously the distance and the time used, but at times when there is scarcity of supply (few uber drivers) or there is a peak in demand, the tariff is multiplied by a corrective factor (Figure 7) that takes into account this disparity between supply and demand (Uber Assistance, n/a). As such, Uber's



pricing algorithm aims to continually balance supply and demand in the short run. As the price rises, more demand-sensitive customers will be able to reduce demand and the higher prices attract drivers to areas where the surge is active. The base rate acts as a price floor: prices do not fall further even if demand is extremely low - the surge multiplier can never fall below 1.

Staying in the sharing economy market, another operator for short-term housing rentals is the worldwide platform Airbnb. The business model of this platform is particularly complex, starting from the pricing. Hosts can set prices freely, but Airbnb recommends prices according to an algorithm that incorporates machine learning. The price recommendations are based on such as location, the property's occupancy rate, the booking duration, the size of the accommodation, the time of year, and competitors' prices and availability. Recommendations vary over time - for example, to take account of local events - and are regularly updated. Airbnb's pricing strategy aims to maximize the value of bookings by ensuring that prices are optimal to both parties, and by providing enough incentives for hosts to list their available space on the platform. In fact, the platform's strategy is therefore threefold: to maximize the number of transactions; to ensure that listings are optimally priced; to ensure participation on the platform by both hosts and guests. To maintain this strategy as the core of its business, Airbnb needs to constantly seek the right balance between charges, prices, and percentages to be applied to both hosts and guests in different weights and ways, using very sophisticated price (and not only) algorithms.

Pro-competitive effects of pricing algorithms on the market

The use of price algorithms has many implications in the market, many of which are also negative both from the point of view of consumers and companies, affecting competition in the market. The discussion of the negative effects, the possible repercussions and any measures that will have to be applied will be the objective of the next chapter, while the main benefits that price algorithms and their use can bring with them will be analyzed below.

Timely price change

Algorithms can be faster and better at correctly identifying changing market conditions such as demand shocks and cost changes. This enables companies to adjust prices more quickly to the efficient price level. This makes possible to respect the capacity constraints, where present, and reduce excess demand or excess supply. Furthermore, agility in changing prices can help whenever a company sees a reduction in production costs and can afford to possibly lower the selling price becoming more competitive. In the case of platforms, the reactivity in updating prices allows them to extract more value from the users of their services, also because they normally have enormous amounts of data on users that they can easily use to maximize the profitability of the platform. In this sense, a platform can use an algorithm to set prices that are much closer to competitive prices than they would be in a scenario where users set the prices.

Cost reduction

Algorithms can monitor the market and adjust prices at a very low marginal cost. Limited human involvement reduces staff costs and can reduce the possibility of behavioral distortions (such as the tendency of people to prefer to avoid a loss compared to the acquisition of a gain of the same entity, defined as "loss aversion"). However, initially setting up the algorithm and verifying that it is behaving "well" can be expensive. Smaller suppliers can reduce this initial cost by purchasing a software subscription, such as Feedvisor³, PricingPro⁴, Intelligence Node⁵. These long-term cost reductions can eventually be passed on to consumers in the form of lower prices.

Lower barriers to entry

The high reliability of the pricing algorithms allows reducing the amount of knowledge of a specific market required to be competitive within it or to get in as a new operator. Furthermore, this allows operators to expand their product portfolio despite not having adequate knowledge of them. In this way, the increase

³ https://feedvisor.com/

⁴ https://mypricingpro.com/catalog/

⁵ https://www.intelligencenode.com/

of companies entering a sector, rather than the expansion and diversification of the product portfolio of a specific company, is encouraged using these algorithms that facilitate the operations and business strategies of the agents.

The hidden shadow of this technology

The positive effects analyzed above are of considerable importance and pricing algorithms allow to exploit the information contained in the data, bringing innovation and making the market even more efficient.

However, a doubt arises about the effectiveness of these tools for market operators. In fact, these tools are able to profoundly modify the market and the strategies that are part of it, granting those who possess these instruments a power that is difficult to match if used to take advantage and to increase completely private purposes. In the following paragraphs, we will deal with what may be the main problems and inefficiencies that can manifest themselves in a competitive situation with the use of algorithms and self-learning technologies.

CHAPTER II

Algorithms and collusion

Given the great potential that automated algorithms can bring to the digital economy and in the ever-changing markets, thanks to their capacity to process the data collected, innovation, efficiency, and lowering of costs are a direct consequence of their application. However, although these new technological tools promote competition and revolutionize the way of doing business and how companies interact with each other, there is the risk that some market players exploit their increased power to reach goals and interests that are not aligned with those of the society. Therefore, it is of primary importance the need to understand the risks that the massive use of this family of algorithms can cause and identify solutions that are compatible with the incentives to constantly innovate, also from the technological point of view.

In this section the concept of collusion and its definitions are presented from a theoretical point of view and, in addition, a survey of the literature is provided on the impact of price algorithms on the modern economy, as well as the ways these influence it.

The concept of collusion

In the literature, the term "collusion" commonly refers to a form of coordination among competing firms with the objective of raising profits to a higher level than the non-cooperative equilibrium, resulting in a deadweight loss (Green et al., 2013). In other words, collusion is a strategy to maximize profits shared among competitors in the same sector. The parties involved to achieve and maintain the collusive equilibrium overtime must put into practice a structure that governs their interactions, in order to agree on a common policy, monitor the behavior of other players, and possibly implement actions that go to punish deviant behavior. There are two forms of collusion in the literature, explicit and tacit (Green et al., 2013).

- Explicit collusion involves direct interactions between the parties in order to agree on the optimal price level or any other variable to be maintained over time. In other words, this type of collusion involves explicit agreements resulting from oral or written forms of interactions.
- Tacit collusion, on the other hand, is achieved without the aid of explicit agreements between the parties, but so that each player decides his own price strategy that maximizes profits regardless of that of his opponent. This practice is more complex to implement, but the transparency of the market and the low number of competitors makes this form of collusion applicable without companies coming into contact explicitly.

One of the problems of detecting collusion is that, under certain market conditions (i.e. transparent markets with few sellers and homogeneous products), supra-competitive pricing strategies may be the normal result of the rational economic behavior of each company on the market. It is for this reason that tacit collusion is not part of the behavior punishable according to competition laws, but, from a policy point of view, this form can have consequences that are not entirely desirable given that companies may benefit to the detriment of consumers who they would be damaged as if it were an explicit agreement. However, between explicit collusion (which should always be considered illegal under the competition rules) and mere conscious parallelism (which should fall outside the scope of competition law as it does not involve any form of coordination between competitors), there is a gray area of corporate behavior that goes beyond conscious parallelism but at the same time does not involve an explicit agreement between competitors. This is a situation that can emerge in an oligopolistic market where competitors are able to coordinate prices obtaining a result similar to tacit collusion involving facilitating practices that allow more effective and simple coordination.

Relevant factors for collusion and the impact of algorithms on them

Several studies have identified the most relevant factors that facilitate and increase collusive behavior in a given sector (Ivaldi et al., 2003). These factors can be divided into structural characteristics, demand-side characteristics and supply-side characteristics. Below is discussed how these factors are influenced by the use of algorithms from a structural point of view and from that supply and demand side by modifying the sectors making it more favorable to collusion.

Structural characteristics

The number of firms and entry barriers are two factors that greatly influence competition within a sector. A large number of companies operating in one sector makes it difficult to identify a common strategy among all of them and, moreover, reduces the incentive to collude as the players would receive only a small part of the gains obtained colluding explicitly or tacitly. Similarly, the absence of entry barriers causes an increase in incentives to deviate and attracts new operators in the event of an increase in profits, in this sense the collusion would become difficult to apply.

In general, it is not very clear how the families of algorithms analyzed influence these structural factors. Some of the most typical sectors, where algorithms are used to set dynamic prices, segment consumers or improve product quality, have a limited number of major players, such as search engines, online markets, discount stores, booking agencies, airline companies, and social networks. However, many of these industries are also characterized by natural barriers to entry, such as economies of scale, economies of scope and network effects, which allow companies to grow, collect large amounts of data and develop more accurate algorithms. Therefore, it is difficult to say whether the algorithms are the cause or the effect of barriers to entry. Concerning the number of operators in a sector, it is possible to state that the use of algorithms makes this factor less relevant for collusion. In a traditional situation, collusion is more easily achieved if there are few players in the sector since it is easier to find the terms for coordination, monitor the actions of competitors and implement punitive mechanisms. However, the algorithms allow to coordinate, monitor and punish competitors even though the sector is less concentrated thanks to their processing capacity and speed in collecting and analyzing data. In other words, a small number of companies is an important but not necessary condition for algorithmic collusion (Ivaldi et al., 2003).

Other two important structural features are market transparency and frequency of interaction, which make both industries more prone to collusion. While transparent markets allow companies to monitor each other's actions and detect deviations from an agreement, frequent interactions allow them to quickly retaliate and aggressively punish any deviators. Unlike the number of companies and entry barriers, it is very likely that algorithms enhance these two factors for collusion, thus constituting a threat to competition (Ivaldi et al., 2003).

Regarding market transparency, the use of price algorithms as a significant tool in the context of corporate strategy is effective if accompanied by a precise and real-time analysis of the data that characterize the market. Therefore, besides the importance of the availability of the information available on the market, the application of algorithms capable of automatically making decisions starting from the collection and analysis of data, without the need for human intervention, is fundamental. Furthermore, the use of these algorithms by some companies that seek to obtain an "algorithmic competitive advantage", pushes the remaining operators to adopt the same techniques to reduce the technological gap. In this way, the result is an industry where all operators monitor, collect and analyze data, making the market more transparent facilitating collusion.

Concerning the frequency of interaction, the digitization of businesses and the massive use of technologies has revolutionized the speed of making business decisions. Unlike brick and mortar businesses where price adjustments can prove costly and time-consuming to apply, in digital business prices can be changed in real-time without constraints on the number of updates. This allows companies to react immediately to the actions that competitors perform, in particular, to retaliate in the event of deviations from collusion. A real example, verified in 2013, once again concerns the e-commerce giant Amazon. In fact, it was found that the ecommerce site implemented more than two million price changes per day when in the same period of time Best Buy and Walmart were able to change the price approximately 50,000 times in a month (Profitero, 2013). Furthermore, with the combined use of machine learning, algorithms can accurately predict rivals' actions by anticipating any form of deviation before they are actually implemented.

Although market transparency and frequency of interactions have already been studied previously and identified as factors facilitating collusion, with technological improvements they are becoming again relevant from a policy point of view. In fact, thanks to the ability to process all data available on the market accompanied by modern algorithms, companies can monitor, analyze and predict the actions of competitors, facilitating collusive practices and the establishment of sustainable supra-competitive price equilibrium (Autorité de la Concurrence and Bundeskartellamt, 2016).

Demand and supply characteristics

Collusion in an industry can also be influenced by demand factors. In particular, the stagnation of demand due to the presence of business cycles may hinder collusion. Firms are inclined to deviate to increase profits in periods when demand is high and to reduce costs deriving from retaliation in periods when demand is low. It appears that the use of price algorithms by firms does not have significant effects on the collusion deriving from the demand-side factors (Ivaldi et al., 2003).

Regarding supply factors, these can play a greater role in the sustainability of collusive agreements. One of the most important features in supply-side factors is innovation. This effectively reduces the value that collusion can bring, given that price algorithms make it possible to be even more effective and precise in facing competition in an industry, becoming a source of competitive advantage. Furthermore, the efficiency of these algorithms can lead to considerable cost savings compared to companies that do not use them, causing cost asymmetry within the same sector. With this, it is possible to conclude that some supply characteristics counterbalance the effects deriving from the structural factors that favor collusion.

Algorithmic collusion in the literature

Many of the ideas discussed so far come from studies and researches concerning the concept of collusion deriving from the application of artificial intelligence and algorithms even more at the center of studies and academic conferences. In addition to scholars, this type of topic has been addressed by major newspapers such as the Harvard Business Review (Stucke et Ezrachi, 2016), the Economist (n/a, 2017), the American Bar Association Journal (Deng, 2018), and the Financial Times (Lynch, 2017) giving visibility to this topic.

In the following paragraphs, several studies are presented which analyze the behavior of algorithms in certain situations. It is important to remark that the literature is quite vast and characterized by numerous contributions that vary according to the hypotheses applied, the method used, and the technologies implemented. The choice of factors has a considerable impact on the results obtained by a study, therefore it is not possible to determine with extreme certainty that the use of algorithms, in particular, those of price, can lead to situations of collusion and, moreover, a common factor emerges from the studies which corresponds to the need for communication to facilitate collusion between the parties. In order to be a little more precise, the economic literature has investigated in different ways the problem of coordination in oligopolistic markets or in similar situations, such as the prisoner's dilemma. The most common approach used to analyze collusion is that of a framework of repeated games since the interaction between players lasts over time, and obviously, the degree of complexity of the problem, the type of problem implemented are factors that change from one study to the other and constitute the literature in this complex topic.

Initial stages of algorithms and repeated games

The first models of algorithms programmed to play the repeated prisoner's dilemma are dated in the 1980s. These models have been developed to analyze players' strategies using simple algorithms in a context of limited rational behavior (Marks, 1992). These simple algorithms consist of a set of states, an input and an output function and a transition function. The automaton receives an input, eventually changes its state according to the transition function and gives an

output. It has been shown that in this type of framework a collusive situation does not correspond to an equilibrium (Marks, 1992). This is mainly due to the introduction of a factor that increases the complexity of the models. In particular, Rubinstein (1986) imposed a cost each new state and the player had to pay a sum composed of the amount of costs for each state that the algorithm changed in the repeated game. In other words, the algorithm has the objective of maximizing profits and minimizing the number of states, therefore it has emerged that only the cases in which the algorithm is able to avoid redundant state changes are able to form coordination between the parties (Rubinstein, 1986). Moreover, Tit-for-Tat and Grim strategies do not reach equilibrium since the punitive states are not used in equilibrium. A different model has been analyzed by Cho (1994) using a neural network of modest dimensions, applying the perfect folk theorem, showed that all possible outcomes can constitute an equilibrium in a repeated game.

The research on bounded rationality in economics has shown that the algorithms were not able to learn from past experiences (Marks, 1992). In this sense, this factor has been implemented and considered in different ways in the games studied by contributions in the literature. Even in finite repeating games, the concept of learning has been implemented. See, for example, Miller's contribution (1996) where he uses a genetic algorithm to study whether a coordinating situation can emerge in a repeated prisoner's dilemma. In his model, the genetic algorithm randomly recombines strategies and mutations in the succession of repetitions. Success strategies, i.e. those that have a high payoff, are more likely to be performed in the next repetition. In fact, it is a mechanism of recombination, modification, and selection. This study showed that collusion is a result of equilibrium in a finite repetition game. However, it emerged that the degree of cooperation is influenced by the degree of transparency of the system, in particular, if the opponent's action has not been perfectly observed, the probability of cooperation is considerably reduced.

With the contribution of Stimpson et al. (2001), it is possible to identify another way to obtain coordination in a repeated prisoner's dilemma with the use of satisficing learning. This type of learning assumes that in each period a player compares the current payoff with a previously calculated level. If the payoff obtained is greater or equal to the benchmark, the action performed will be repeated in the following period, otherwise, the benchmark is calculated taking into account the payoff received and the previous benchmark. Simulations have shown that in many cases agents hardly collude and this type of learning converges to a one-shot Nash equilibrium.

Reinforcement learning in games

A family of algorithms, previously presented, and increasingly used to study the collusion between automata in the game theory, is reinforcement learning. Some contributions using this technology state that it is not particularly helpful in facilitating collusion. Examples are the studies addressed by Erev et Roth (1999, 2001) where this type of learning is applied to the repeated prisoner's dilemma and they show that reinforcement learning, in general, does not lead to a collusive result. Another model that uses reinforcement learning with a finite-stage algorithm is in Hanaki et al.'s contribution (2005). The model is based on a tworepetition game and therefore 26 game strategies are available. In the first period, the algorithm explores all the strategies, while in the second period it limits its attention to those that were successful in the first. The simulation results show that cooperation is the outcome in the case of limited complexity.

An evolution of the reinforcement learning used in recent years by numerous scholars is the Q-learning applied to the prisoner's dilemma. In general, from these studies, it emerges that a collusive behavior is a possible result of these models, under certain conditions and assumptions about the player's memory, the learning rate and the transparency of the environment which also includes the ease of acquiring information (Banerjee et Sen, 2007).

Taking into consideration more recent contributions that analyze the same topics by applying models with different factors and characteristics, it is possible to mention the studies by Leibo et al. (2017) and Crandall et al. (2017).

In Leibo et al. (2017) a deep neural network is implemented and the agents learn to deal with two types of social situations characterized by payoffs very similar to those present in the prisoner's dilemma. In the first case, known as a gathering game, players must collect resources and have the possibility to exclude other players from the game. The algorithm provides a reward of 1 when a player collects the resource, defined as an apple. This is removed from the map and then repositioned after N steps. Players can eliminate their opponent for a certain period of time, but there is no reward for this type of action. The only motivation that drives a player to eliminate the opponent is the competition that subsists between them. Furthermore, in order to understand the competitive environment of this model, some variables such as the rate at which apples are regenerated and a player's exclusion period after being eliminated have been modified during a test that lasted forty million steps. In this case, it has emerged that players learn to cooperate if the resources are satisfactory, while if the resources are scarce the players act selfishly and try to remove other players. In the second scenario, known as hunting game, two players, the wolves, must hunt a third player, the prey. If one of the wolves "hunts" the prey, the other wolves receive a reward that depends on the number of wolves that make up the game. However, if two wolves capture the prey together, they both receive a higher reward than the previous one. Therefore, it appears that cooperation is more difficult to achieve in the hunting game than in the gathering game because in the latter the players are not influenced by the actions of others. In the hunting game, collusion requires cooperation between the parties since the result depends also on the behavior of a partner.

In the contribution of Crandall et al. (2017), there is an analysis related to the implementation of algorithms able to form situations of cooperation with other algorithms but also with humans. These algorithms should show a greater capacity for processing in different contexts, should be able to establish cooperative relations with other algorithms or humans without having prior knowledge of them, should be able to prevent exploitative behavior and determine the possibility of cooperating with another actor who may not be inclined to this type of agreement. Scholars have considered twenty-five different algorithms in terms of complexity, the adopted strategy, and the implemented technology. The simulations showed that even rather complex algorithms based on deep neural networks implementing reinforcement learning do not cooperate in a repeated prisoner's dilemma. The most successful algorithm was the expert algorithm⁶, which showed greater effectiveness if it was able to communicate with the counterpart by sending signals.

In addition to the application of reinforcement learning to the prisoner's dilemma and, in general, to simple situations two players-two strategies, there are many contributions that apply this method to oligopoly models. Some of the contributions that need to be mentioned are Izquierdo et Izquierdo (2015), Tesauro et Kephart (2002), Kaymak et Waltman (2008), Kimbrough et Lu (2003).

In these models, it has been shown that agents may arrive at a collusion situation as a result of the game. However, the result always depends on the factors and details that are used in the learning process. Izquierdo et Izquierdo (2015)'s contribution shows that a simple reinforcement learning model, such as "Win-Continue, Lose-Reverse" leads to a cooperative outcome. Despite this, the result does not show high robustness as the system variables vary such as the company's costs or a change in the profit function so that these changes lead to the non-coordinated Nash-Cournot equilibrium. This situation occurs because these perturbations make the coordination between the parties more difficult and in fact prevent a collusion situation within the game.

Kaymak et Waltman (2008) have shown how the simulations performed by a computer of a Q-learning model applied to a Cournot oligopoly lead to a result of coordination between the parties, even in the case with more than two players and these have no memory. However, the degree of coordination is a decreasing function with the number of players in the simulation. This model is characterized by the possibility of exploring the environment and cooperation is influenced by the value that this factor has due to it represents the degree of learning. In general, complete coordination as equilibrium is not achieved. Another contribution,

⁶ An expert algorithm can be described as follows: before the beginning of the game, the algorithm uses the description of the game to create a set of experts which are described as a strategy or a learning algorithm that defines their behavior in the game. The algorithms determine the highest payoff that each expert should reach, and these payoffs are then compared to the level set by the mother algorithm. The latter identifies and chooses the experts who can ensure a payoff in line with that set by it. Subsequently, one of these experts is chosen to play several rounds of the game. From the payoffs obtained in these rounds, the benchmark payoff is recalculated and the identification and selection processes are again implemented.

deriving from Tesauro et Kephart (2002), about a price-setting duopoly with fixed production, where companies follow a Q-learning algorithm, shows a possible convergence at prices that are higher than the level of those in competition. Remaining in a situation of Bertrand oligopoly, Calvano et al. (2019) show that the algorithms, learn to collude by setting supra-competitive prices without communicating with each other. This situation of coordination is strongly reinforced by a classic collusive strategy that punishes every deviation by an actor, in a finite time, returning to cooperation gradually. Furthermore, the researchers assert that this model is robust to cases of asymmetry of costs among actors, demand asymmetry and changes in the number of players. Therefore, being one of the most recent contributions, it could be an alarm bell as computational power and complexity management are becoming more and more easily managed by today's computers, without having to face huge investments as this case study represents. Obviously, even in this case, despite the proven robustness carried out by modifying some factors that influence collusion, it is necessary to go further into the results in a more realistic context, for example, modifying them together and increasing the complexity of the environment. Moreover, the speed of learning and the use of different algorithms among the actors are completely consistent and realistic factors that increase the complexity of the problem even more and could drastically change the result.

Another paper that has made a significant contribution to literature is Salcedo (2015). Also this case is a duopoly model with price competition and a homogeneous product described by a finite repetition algorithm without the use of reinforcement learning. The model is characterized by the possibility of observing the prices set by the opponent and his behavior at every step. In this way, each algorithm is able to reconstruct the algorithm used by the opponent and modify its structure, adapting itself. This type of decoding can be seen as a kind of communication between the parties. The pricing behavior cannot be changed between one revision period and another. If the two algorithms start by fixing competitive prices, at the first revision, an algorithm can modify its behavior so that it contains the possibility of matching a higher price if the opponent plays a higher price than the competitive one. In this way, the information is decoded by the other algorithm which in turn adjusts its behavior. This change is also perceived by the first algorithm and the two set higher prices than the initial rounds. In fact, the firms increase their prices so as to maximize joint profits. It is easy to see how in this model the only possible result is collusion. However, it is not true tacit collusion but contains elements, such as decoding, which actually allow explicit communication between the operators involved. If this type of information exchange did not exist, the collusive result would not manifest itself.

From all the contributions just analyzed, it appears that a possible result of these models is a collusive form between the parties involved. However, it is not always simple and immediate to arrive at this type of outcome and often the result depends on countless factors. First of all, whether cooperation is achieved depends on the hypotheses considered by the model and the type of algorithm used. It often turns out that cooperation is achieved by simple algorithms, rather than algorithms based on machine learning or deep learning. Moreover, as already mentioned, these models are implemented in almost static environments, in which only some variables can be modified according to a certain logic, while in reality, the system would become much more complex, with non-trivial dynamics. For example, in the previous models, the cooperation is achieved with the use of two main actions, such as cooperate and deviate, while in a real context actions can involve prices, quantities, qualities, etc. Another important factor is that in these models the actors exploit the same algorithm and the same logic, but it would be interesting to study what would happen if the actors used different algorithms and how easily they could achieve a collusive result.

Empirical evidence

The results obtained from the contributions analyzed above show that the tacit collusion between the algorithms, in particular the pricing ones, is a verifiable outcome without too much difficulty. In this sense, however, there are further papers that test the algorithms forced to work in more complex environments characterized, for example, by a number of players that increases in different simulations (Huck et al., 2001) and the possibility of communicating with the counterparts to establish some collusive agreement (Engel, 2015).

One of the fundamental aspects that influence the collusion is the number of operators in the market, not for nothing, the antitrust authorities monitor very carefully the operations of merger and acquisition between the companies. Huck et al. (2001) demonstrated this theory in their contribution that replicates an oligopoly model with a growing number of operators. The result is that with two firms it is possible that some form of tacit collusion occurs, but with a greater number of operators, the outcome is a competitive output. A more recent paper by Horstmann et al. (2016) confirmed that in an oligopolistic market tacit collusion is more common with two firms, while as the number of companies increases the likelihood of collusion occurs drastically, everything else being equal. These results are valid for both Bertrand and Cournot oligopoly models.

Not only the number of actors is relevant in order to create collusive behavior. In fact, many studies have shown that communication between agents is an important factor for a collusive outcome to occur, even in the presence of more than two firms in the simulation. This is because the communication between the parties, although not binding, makes the coordination between the parties much easier and therefore also a collusive result is more likely. One of the first contributions that analyzed the influence of communication in a market is Friedman (1967). The author has highlighted how the prices are higher if the players can communicate by sending non-binding messages or signals than situations in which communication is not permitted. Moving to a more recent contribution, Engel (2015) also confirms the significant influence that communication has on facilitating coordination and simplifying collusion situations. Furthermore, the importance of communication for a collusive outcome has not only been demonstrated with simulations of oligopolistic models but also by the Airline Tariff Publishing Case (Borenstein, 1999), analyzed above, where the exchange of information regarding routes' fares constituted the vehicle for establishing a collusive form between the airline companies.

Fonseca and Normann (2012) analyzed the difference in the impact of communication as the number of companies in the market varied. Their contribution has shown that the influence of communication as a factor facilitating collusion decreases as the number of companies in the market increases. While in

a duopoly communication is generally not necessary to obtain a collusive result, in large markets the possibility of communicating, in fact, does not favor collusion, given the complexity of the entire system. However, in medium-sized situations communication can drastically change the result. Precisely in this situation, without communication the result is competitive, while with communication firms are able to maintain a certain degree of coordination.

Therefore, the possibility of initiating communication between the parties appears to be a very important factor in order to establish a collusive situation in oligopoly. However, it is possible to state that this type of result may also depend on the ability of the agents or algorithms to communicate and on the number of operators present in the market. Therefore, it is necessary to understand if the algorithms are able to communicate with each other or if they are able to learn to communicate despite the differences that characterize them and the absence of an explicit pre-configured communication protocol.

Algorithms and personalized pricing

The use of pricing algorithms has not only implications about collusion and its forms, but with the increase in the amount of data available on the market, the so-called Big Data, and the ease of getting them into possession causes another very important phenomenon, that is the pricing personalization.

The term personalized pricing does not have a clear and shared definition in the economic literature and therefore it is not uncommon for it to be used improperly as the concept of price discrimination or even as the term dynamic pricing. One of the definitions that emerges from the contributions of the literature defines the practice of personalized pricing as the phenomenon in which firms use information observed, voluntarily or deduced, or gathered about the consumers' behavior, characteristics, and conditions, to set prices in a precise and targeted way for each individual or group of individuals, based on their willingness to pay for a particular good. From this definition, two important characteristics come up. The first is linked to which subjects are affected by this practice, i.e. the consumers. Therefore, the focus shifts to companies that have a business-to-consumer relationship. The second highlights that this practice is based exclusively on the habits and actions that consumers show during the purchase phase. In this way, it is possible to discriminate this phenomenon from those, for example, of dynamic pricing and discrimination pricing. In fact, for completeness, dynamic prices are found when prices vary based on variables that are not related to consumers and their characteristics. Variables such as the available offer, competitors' prices, the seasonality of the product. In this scenario, all consumers are subjected to the same price, without any discrimination between them. On the other hand, personalized prices, at the limit, are tailored to the individual consumer based on their characteristics and preferences (Tringale, 2018). Furthermore, considering this definition it is possible that the phenomenon of personalized pricing may be associated with a different form from those that characterize the price discrimination theory. In fact, often price customization is associated with firstdegree price discrimination, but it is possible to theorize a scheme in which consumers are not charged their entire willingness to pay, but only a percentage of it. Moreover, when information does not allow discrimination on an individual

basis, price personalization may discriminate groups of people, such as thirddegree price discrimination. It is also conceivable that companies may tailor both prices and products for each consumer, resulting in second-degree discrimination.

For completeness, the definitions of the three degrees of price discrimination identified by Pigou (1920) are given below.

- First-degree price discrimination: called also perfect price discrimination, it is the form of discrimination where every consumer is charged with his or her entire willingness to pay. In this scenario, companies must be able to perfectly observe all the relevant features for each consumer and set prices appropriately.
- Second-degree price discrimination: it is a discrimination that requires a company to offer different versions of a product (also in term of quantity and quality) at different prices, leaving the consumer the choice that is closest to his tastes and preferences. In this case, the importance of collecting information concerning the consumer is less visible.
- Third-degree price discrimination: it occurs when sellers fail to perfectly observe the characteristics of consumers and therefore discriminate based on attributes that characterize consumer groups.

Therefore, in light of these considerations, it is possible to define the concept of personalized pricing as any practice of price discrimination of the final consumer based on the characteristics owned and on the actions carried out, resulting in a price as an increasing function with the consumers' willingness to pay. As can be seen, these practices are much more common in digital markets than in brick-and-mortar scenarios because there is the possibility of collecting a higher amount of data with a better representation of the consumer.

Personalized pricing's implementation

Having discussed the definition of personalized price and the differences with other economic concepts, it is good to analyze how this practice is implemented within the company strategies. In general, there is no shared method among companies because there are substantial differences in terms of process and product and, moreover, the same market does not seem to be so clear and transparent, making the practices opaque to the eyes of external agents. Nevertheless, it is possible to identify common principles established by companies to personalize prices.

There are at least three steps that must be followed to implement personalized pricing. First, the company must collect data concerning consumers' characteristics. Second, the company must estimate the willingness to pay of each consumer from the data collected. Third, based on this estimate, the company must set an optimal price for each consumer.

In particular, data collection is a fundamental step for the successful implementation of personalized pricing. In this step, the company must identify all the variables that influence consumers' purchasing decisions, which can be classified into three macro-categories: information voluntarily given by consumers, data directly observed by the company, data deduced from consumer behavior. About the information that the consumer voluntarily provides, this can be found by a company thanks to the opening of an account where personal information is saved, such as name, address, date of birth, credit card number, history of past purchases, gender, etc. Concerning the data provided unintentionally, companies can recognize a consumer based on their IP address or using cookies with a unique identifier. In this case may be collected data about the place from which the consumer is browsing, the type of connection (fixed, mobile), the internet provider and the speed of navigation. About the third category of information, this can be found through affiliations with advertisements or other sites that the consumer visits. The use of cookies and other technologies may identify the consumer and trace his or her actions while browsing having an increasingly complete profile related to his or her preferences and personal characteristics.

Once personal data has been collected, it is necessary to estimate the willingness to pay for each consumer based on the personal factors that influence this characteristic, such as type of employment and past purchases. In this step, it is difficult to estimate a quantity that cannot be directly observed but must be

constructed by observing the behavior of the consumer when he or she visits the company website, how he or she behaves in front of the prices that it has set for the products. In this way, the willingness to pay for each consumer can be estimated directly from the registered behavior.

Finally, the last step concerns setting a price that maximizes profits. Contrary to the theory that plans to allocate to consumers all their willingness to pay, in reality, this practice is not always convenient and possible. This is because the estimate made in the second step can be erroneous, overestimating what is the real willingness to pay by the consumer, risking the loss of a possible buyer, moreover, the competition does not allow companies to set prices that may result to be greater than those that others operators fix in the market.

Favorable conditions in the market for personalized pricing

Another important issue that should be addressed concerns the circumstances in which price personalization is favored in the market. According to Varian (1989), there are three necessary conditions for which price discrimination can occur.

- Clear identification of consumer's value: The fundamental condition is that companies are able to apply all the methods necessary to obtain an estimate of the willingness to pay as much as possible truthfully in order to set an optimal price both for consumers and for them. The advent of Big Data and greater computational power has certainly increased the ease of obtaining these estimates, but, in general, the personalized pricing is more easily observable in digital markets where data is highly concentrated thanks to network effects, economies of scale and scope.
- Absence of arbitrage: Price customization can only be effective if arbitrage is not possible, i.e. a secondary market of goods is not intrinsically possible where, for example, consumers with low willingness to pay can resell goods at a higher price to consumers with high willingness to pay. The sectors in which this is naturally guaranteed are those that sell online services for offline use, such as

flight and hotels bookings, given that the tickets are usually not transferable, or even the sectors of digital products such as movies, e-books because they are protected by technologies that guarantee unique access. However, it is more difficult to avoid arbitrage in the case of sales of tangible and durable goods.

Market power: The presence of personalized pricing requires a minimum level of market power, given that in case of perfect competition, prices would equal the marginal costs for all consumers. Therefore, the practice of personalized pricing is favored in markets with a certain degree of economies of scale, economies of scope, network effects, entry costs and switching costs, granting companies market power by being able to set prices above of the marginal cost curve.

The effect of pricing algorithms on personalized prices

As emerged from the discussion carried out so far in this topic, the customization of prices has existed in many sectors even without the use of price algorithms. In fact, the examples that can be mentioned are personalized discounts as indirect differentiation or price differentiation in the insurance and credit sectors. However, with the continuous expansion of price algorithms and the growing availability of consumers' data, companies are increasingly inclined to offer personalized prices to each consumer, with the limitation of being able to reconstruct and identify each consumer with their own characteristics and fix a unique optimal price. All the information collected by the companies is used to model and predict the willingness to pay of each consumer, in order to approximate the first-degree price discrimination as much as possible.

As can be supposed, this practice has two conflicting consequences from an economic-social point of view. The positive aspect is that with the increase in the availability of information combined with the computational capacity of algorithms, it is possible to obtain a better understanding of consumers' demand and preferences which implies the possibility, for companies, to offer a wider offer of products based on the price-quality trade-offs of individual consumers.

Furthermore, companies can recognize the cost of the service for each consumer and consequently set a price suitable for buyers with a low cost of the service, which otherwise would not be served by the market. However, there are some areas where cross-subsidization is inevitable and beneficial for some consumer groups. In these cases, to protect certain categories of consumers, cross-subsidization is completely legal and regulated by policymakers and it would be put at risk with the ability to extrapolate (almost) all the willingness to pay of consumers. Examples are the sectors of electricity, internet connection, and water supply.

From the contribution of Acquisti et Varian (2005), it is possible to study how a price algorithm can set an optimal price in certain circumstances. In particular, the authors evaluate the outcome of algorithms that use consumers' past purchases in a model where consumers can take countermeasures and protect their privacy. From this contribution, it emerges that a monopolist, by fixing the prices of products using the history of purchases, is not optimal to totally discriminate consumers, for example by attributing the entire willingness to pay that characterizes them. In fact, it is more effective to give some benefits to consumers to extrapolate further information by offsetting the fact that these data are exploited by the company. However, the result shows that there are conflicting results among consumers who use countermeasures to hide their data and those who do not protect themselves.

The results do not assess whether this practice is efficient even in oligopoly situations with a few companies, in which the increase in individual profits may not correspond to an increase in welfare for companies together. In fact, in the case of monopolies, price discrimination transfers part of the welfare of consumers to the welfare of companies, while in the case of oligopolies price discrimination can favor consumers by increasing competition (OECD, 2016).

It is possible to find business strategies that focus less and less on maximizing profits statically, but which involve price strategies to increase market power and the consumer base. This type of strategy involves the screening of consumers discriminating between new ones, those with a high willingness to pay and other personal characteristics. Digital markets favor this process as it is normally necessary to create an account to use a site and product prices are available to consumers who indirectly give their navigation information to companies (OECD, 2016).

Another important factor is the partitioning. Companies, with the help of pricing algorithms, segment their consumers according to their characteristics and their behavior in order to identify and capture the most profitable segments, leaving the subjects with the lower willingness to pay to the competition. This practice makes it possible to obtain greater market power by exploiting the most profitable segments, but, at the same time, abusing consumers who are unaware of their disadvantageous position compared to sellers. Moreover, always considering the phenomenon of partitioning, pricing algorithms can exclude rivals from the market knowing their characteristics and lowering, to the nullification point, the switching costs.

Chen and Zhang (2009) point out that in the dynamics of the markets, price competition as a way to reach the greatest number of customers is compensated by the allocation of higher prices to benefit from customer loyalty excluding consumers with a lower willingness to pay. Therefore, dynamic targeted pricing can expand the market and improve social welfare, but the consequences that can be obtained in the event that companies are able to collect more and more data and develop increasingly sophisticated tools to discriminate consumers precisely remain opaque.

Implications on the distribution of consumer surplus by price algorithms

The issue of surplus distribution remains one of the most monitored topics by policymakers. For example, actions could occur that undermine the equal opportunities of consumers who are discriminated on the basis of ethnic factors by increasingly powerful and effective algorithms. Therefore, it is important to analyze how the pricing algorithms interact with the already existing discrimination processes, given that they do nothing but increase this problem exponentially. Consumers themselves may have concerns about the use of algorithms and the phenomenon of personalized pricing. Even those who benefit from this practice by paying less for a product may prove to be uncomfortable

about the inequality of treatment between the different consumers and the possibility that companies may change prices based on their willingness to pay. Therefore, there can be great discomfort with price differentiation when this practice is not clearly linked to the different cost of service. The consumers' attitude to detect and not appreciate this behavior, has slowed down the spread of this practice in the market. For example, in 2000 it was reported by various newspapers (BBC News, 2000) that Amazon offered products at different prices for different users. Users have noticed how the price of a particular product went down if the cookies from their own computer were eliminated and, again, how the products presented lower prices to new users than those who already had an account (Salkowski, 2000). Amazon immediately abolished these practices, justifying that they were normal random discounts and granted refunds to users who paid above average. Furthermore, the CEO, Jeff Bezos, said in a conference that they had never price discriminated based on demographic factors (Amazon, 2000). Another case that emerged in 2012 concerned the travel rate aggregator website and hotel room search engine of Orbitz.com. According to the Wall Street Journal (2012), Orbitz found that Mac owners are willing to spend up to 30% on hotel rooms compared to Windows users. To take advantage of it, the company launched that defines a "predictive analysis" initiative in an attempt to increase profits. Furthermore, according to their analysis, Mac users are more likely to book a room in a more luxurious hotel than other users and, for the same hotel, Mac users prefer more expensive rooms than others. This example highlights how a company that collects user data is able to find factors that allow users to be discriminated against to its advantage. However, the company had justified itself by claiming that it was not showing the same room to different users at different prices, but more luxurious rooms at higher prices. Despite this, Orbitz is choosing to show more expensive rooms to a particular group of users in order to increase profits and, in fact, it is a discriminatory form to the detriment of users. A further survey carried out in 2012 by the same New York newspaper found that Staples.com website shows different prices to consumers based on their location. In particular, the site takes into account the competitors' locations and if they are within 20 miles from the consumer, the latter sees a discounted price compared to other consumers. However, a side effect has emerged from the use of these pricing

methods. In fact, the analyzes and tests carried out showed that the areas with the greatest discounts were those with the highest income, while the areas with lower incomes tended to see higher prices. Also in this case, it is possible to identify this phenomenon as discriminatory prices on a consumer characteristic in order to increase company profits.

As already mentioned, there are markets, such as insurance and credit, where prices are set by algorithms that take into account the personal characteristics of the user. In situations where prices reflect costs then the practice of price customization is efficient. However, questions of transparency and validity have emerged in situations where the factors that predict the cost for each consumer are also influenced by characteristics such as ethnicity, age, gender, given that this type of procedure is completely illegal and discriminatory. An example brought to light by Edelmann et Luca (2014) is the phenomenon recorded on the Airbnb's platform. In fact, the two scholars have analyzed the owners of the announcements in New York City and the relative prices coming to discover that on average the prices of the black owners are around 12% lower than those that fix the non-blacks for an equivalent rent. Given that Airbnb suggests rates to owners, it is possible to assume that the algorithms of the platform itself consider characteristics that go against morality and fuel racial discrimination.

Price algorithms can be modeled on the price decisions history. Therefore, discrimination on how decisions were made in the past can be repeated in the present and in the future by the algorithms themselves. Moreover, a further problem arises in the difficulty of accusing the use of illegal practices. In fact, the discriminations carried out by algorithms are more difficult to prove since the decision mechanisms are less transparent and should be free of preconceptions and prejudices as opposed to humans. This is not necessarily a criticism of the use of algorithms as they may not be worse than the humans they replaced, but there may be scope for them to tackle this type of problem rather than inherit it.

Self-reinforcement example

One last observation on the use of price algorithms concerns how it is possible to modify the intrinsic structure of the algorithms with a normal reinforcement learning model.

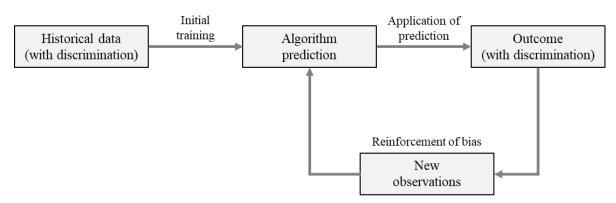


Figure 9 - Scheme of learning by reinforcement with presence of discrimination

As it can be seen from Figure 9, if a business process, such as setting prices, was done in a discriminatory way with human intervention, these decisions can be part of the training bases on which the algorithms that exploit machine learning learn and become operational. The problem also propagates in future decisions, if this algorithm takes the place of humans in setting these prices. In this way, consumers continue to observe personalized prices on a basis that in some cases cannot be accepted ethically. Moreover, due to their computational capacity, algorithms can succeed in decoding the training base of the other algorithms becoming increasingly precise and efficient but at the same time also extremely discriminatory.

This observation is important because the algorithms are replacing in many tasks humans in the business activity. In markets where the human being still dominates, the figure of the policymaker manages to protect vulnerable consumer categories. With the advancement of algorithms, on the other hand, algorithms must somehow be understood to be able to intervene in an accurate manner, trying to reduce their impact on social well-being. However, as already mentioned, the algorithms completely eliminate the preconceptions that are characteristic of the human mind and, therefore, it is difficult to intervene on this point in particular.

Algorithms and predatory pricing

Another phenomenon of particular importance linked to the use of price algorithms concerns the predatory prices which is part of the family of exclusionary conducts that a market leader can implement to maintain its primacy.

In general, the antitrust authorities monitor and sanction collusive and exclusionary practices. The latter can be implemented by a company (or group of companies) by considerably raising costs or drastically lowering product prices, even below marginal costs, effectively reducing profits or even bringing to losses so that new operators in the market go out, definitively abandoning the business.

A company can be accused of predatory pricing when its price level is inexplicably low, resulting in lower than some cost measures or because it generates low profits. At first glance, it seems that the consumer benefits from this practice, given that it offers the same products as the competition at a bargain price and can be considered as an index of good application of the concept of competition, reaching almost perfect competition levels. However, this practice is an abuse of a dominant position. In fact, the predator offers its goods and services at an extremely low price in order to reach a long-term goal. With this behavior, the predator company tries to discourage the entry of a new rival into the market or to lead it out to obtain a dominant position and in the long run recover the lost profits during the price war. Here comes the problem that brings with it this practice from the point of view of fair competition and the consumer.

Market characteristic of predatory pricing

In order to detect predatory pricing, it is necessary to consider numerous economic and non-economic factors that characterize the market to avoid false negatives and false positives given that the resulting accusation would be very heavy for the defendant company.

Types of costs involved in predatory pricing

An important economic factor for assessing predatory prices is the concept of cost and the various components that are directly involved in the economic analysis in the practice of predatory pricing. In general, fixed costs are those independent of production, that is, they do not vary with the output produced. Some examples are interest on debt, depreciation, etc. The variable costs, instead, are those that vary with the output produced, for example, raw materials, energy, and the workforce. The sum of these two types of costs gives the total costs. The average variable costs are the variable costs to produce a unit of product, i.e. the total variable costs divided by the output units. The average total costs are the total costs involved in the production of a unit calculated by dividing the total and variable costs with the number of units produced. Marginal costs are the change in total costs due to the increase or decrease in the output of a unit. Fixed costs do not take part in the computation of marginal costs since they do not vary with production changes.

Market conditions

Predatory pricing can be an effective and successful strategy only if market conditions allow it. In particular, the optimal market structure corresponds to the presence of a dominant company or a group of companies that act jointly and have a relevant market share and, in addition, barriers to entry (Bolton et al., 1999).

About dominance, theoretically, any company could start a practice of predatory pricing. However, in reality, only a dominant company has the capacity and the ability to proceed with this strategy. First, a large amount of money is needed to cover sales at a price lower than costs, which is easier to do in a large company. Second, it would make little sense for a company to sustain these losses and invest its capital to support this strategy when the market remains relatively competitive and recovery expectations are minimal. On the contrary, a company with a dominant position in the market would aim to obtain a monopoly position and succeed in establishing prices for the whole market. Moreover, when analyzing the level of dominance of a firm, the market must be scrutinized taking into account the relevant product and the geographic market by examining the potential demand and the degree of substitution of products and services. Another important issue is that the predatory pricing strategy can not only take place in markets where the predator occupies a dominant position but also in adjacent markets for the purpose of deterring the entry or expansion of rivals into other markets where the predator operates. Even if the prey has a significant market power in which the practice of predatory pricing is implemented, the economic strength of the predator may derive from the position it holds in other markets.

Concerning entry barriers, these are critical to the success of the strategy as a victim of this practice or other potential market entrants would immediately enter the market as soon as the predator raises prices. Consequently, the absence of this type of barrier would reduce the power of the predator and even more the effectiveness of the predatory pricing strategy. In fact, the predator would not be able to raise prices at the monopoly level since it would attract new competitors to the market and, therefore, it would not be able to recover the accumulated losses due to the initial price war. Entry barriers exist when a new entrant in the market has to face costs that the incumbent has already incurred or does not have to face. The most frequent are the sunk costs, that are fixed costs for an investment like the construction of infrastructures. In this way, the entrant has to face these costs and, moreover, the risk of price undercutting carried out by the predator. This allows the incumbent to raise prices above the competitive level. It is possible to identify a sub-category of entry barriers in the re-entry barriers. These types of barriers exist when a firm has left a market and in trying to enter it again it must face costs to try to begin the business again. Some examples of this type are marketing campaigns to recover the image damage due to the exit from the market. In the absence of these barriers, operators would be able to cancel the benefits of the predator's strategy since it could not raise prices at the monopoly level having to compete again with another operator.

A further determining factor for the success of the strategy in analysis concerns the financial resources of firms. This is because in the first phase of the practice of predatory pricing, competitors suffer heavy losses and only those with financial resources capable of supporting this effort can survive. In fact, in general, predators have this kind of economic availability in such a way as to drive rivals out of the market. However, this price war can prove to be rather arduous to the capital market, in fact, rivals can sustain this kind of situation for longer than foreseen by the predator (Milgrom et al., 1990).

The logic of predatory pricing

Not all scholars are convinced of the effectiveness of the practice of predatory pricing by arguing that this is an irrational and not very pragmatic strategy. However, in recent years, with the study of cases and the modern economy, some factors have been identified that influence the application of this strategy and make it a valid alternative to undermine competition.

Recovery of losses

The predator, before applying this practice, must have good expectations of recovering the initial losses that the price war with his rivals brings with it. Without this expectation, the application of predatory pricing is not a reasonable choice (Elzinga et al., 2001). Therefore, the recovery of losses is also the predator's long-term goal which obviously aspires to increase profits compared to when rivals were present in the market. From the consumer's point of view, they are affected by this war in a positive way as they see the prices of products fall. Furthermore, it is possible that the predator does not have enough market power and is unable to raise the price on a monopoly level, benefiting consumers in this case as well (Korah, 2000). This factor does not only concern the recovery of losses but can be seen as a reputational benefit that occurs not only in the market where predatory prices have taken place but also in other markets where the predator is active (Hovenkamp, 2001).

Mergers

Some scholars argue that an acquisition or merger with the rival in the market would be more effective in obtaining the monopoly position (McGee, 1980). This would avoid harmful losses to the company that would like to apply predatory prices. However, obstacles to this alternative emerge. First, the rival could oppose the merger to maintain its independence, second, this type of operation must be approved by the authorities that monitor competition, while the predatory pricing strategy is, although illegal, more difficult to detect and to prove.

Application of predatory prices in different industries

As demonstrated above, predatory pricing will occur when market conditions are favorable, i.e. when the predator holds a dominant position and high entry barriers that make it possible to recover the losses incurred. This is particularly true when the regulatory framework and the structure itself of the sector favor the occurrence of the practice. These conditions are still present in unregulated sectors where the last monopolists hold a dominant position so as to discourage entry into that sector by other rivals. In addition to these cases, there are sectors where regulation has taken place by separating legal privileges from state-owned companies, but the evolution of the structure of the market itself is slow to occur due to the still present power and the dominant position occupied by monopolists. In general, network industries provide an environment conducive to the practice of predatory pricing. An example is the telecommunications industry where infrastructure investments are enormous and can be considered as sunk costs. Another example is the software industry where companies try to impose their standard by distributing their software mostly for free trying to exploit network economies and forcing rivals to exit the market because they do not have the standard that emerged spontaneously from the market. In this case, the losses are recovered as soon as the standard is established (Niels et al, 2000).

Predatory pricing evaluations

The argument about predatory pricing is relatively recent as the first analyzes and methods to recognize this practice only in the 1980s appear in the literature. Over time, different methods and tools have been presented to conduct the analyses necessary to detect this type of conduct. The objective of these techniques is to offer the best method to achieve a balance between predatory prices and the protection of competition in an increasingly complex and opaque environment. These approaches face two major problems. First, the instrument must be able to identify a situation in which rivals are expelled from the market due to the efficiency of an operator, thus rewarding meritocracy. Second, the tool, at the same time, must grasp the behaviors that exploit the dominant position taken by an operator to exclude rivals from the market with illegal conduct. The balance between these two extremes is difficult to achieve, so much so that over the years many techniques have been proposed, each of which has pros and cons.

No regulation approach

The scholars of the Chicago school of thought are those who say that this practice is so rare that no regulation of any kind is considered necessary. Indeed, Bork (1979) states that it is a self-dissuasive practice for those who would like to apply it. Not only that, Easterbrook (1981) adds that any rule would damage the market much more precisely because of the risk of incurring false positives and the difficulty that vigilant institutions would encounter in distinguishing various situations. However, given the great literature that justifies this type of practice, it is difficult to deny its existence in modern markets, even if the possibility of incurring false positives and false negatives must be absolutely recognized.

Price-cost tests

Many contributions in the literature of predatory prices have suggested an approach that detects the use of this practice when the prices charged for a product or service do not cover some measures of its cost, using the relationships between the dominant company's prices to its costs as a first analysis tool. The problem arises among scholars about which type of cost must be considered in order to effectively identify this practice, possibly eliminating the probability of error.

Areeda-Turner's test

The most widely used approach for detecting the use of predatory pricing is the test devised by Areeda and Turner (1975). The tool focuses on short-term costs and assumes that prices are predators if they are below the short-term marginal cost of supplying the product or service unless it is greater than average total costs. Given that marginal costs are rather complicated to identify, the tool may use the average variable costs as a practical approximation. The advantage of this tool, focusing only on the price-cost relationship, is its simplicity given that it avoids complicated analyzes of the predator's actions. On the other hand, it has been criticized for this, avoiding to consider in a broader way other economic factors and strategic aspects of which the use of this practice is characterized.

Long-term cost-based rules

Another completely different vision from the previous one was introduced by Posner (1976) who considers long-term costs as the correct factor for comparison with prices, given that the predator pricing at marginal costs can easily eliminate even more efficient rivals, who have no possibility of sustaining losses in the short term. Furthermore, it has been criticized that the test with short-term costs does not consider the true objective of the companies, i.e. the maximization of long-term profits. Finally, it can be discussed that short-term marginal costs are not a well-founded parameter that represents an optimal allocation of resources given that, considering market imperfections, the difference between this type of cost and the price does not necessarily reflect the opportunity cost of the sacrificed resources and that the marginal cost of the company is determined by a past investment decision (Koller, 1979).

Performance tests

The following tests continue to focus on the long-term period but analyze the predator's performance following the exit of the rival from the relevant market. Williamson's (1977) contribution focuses on the output of the alleged predator. In general, a company confronting an incoming potential could increase the output produced without lowering the price below marginal costs, or, alternatively, it could decrease the output by raising prices until the entry of the new operator occurs, maximizing profits at that capacity level. If the output of a company remained constant or lower in facing a situation of probable entry of a new operator, it would indicate an absence of predatory pricing strategies. The probable negative effects could be avoided by limiting the production of the incumbent for a determined period, in order to avoid a dizzying lowering of prices. However, this detection tool involves the analysis of complex rules and therefore does not simplify the tools analyzed above. Furthermore, the possible introduction of a constraint on production could damage competition given the characteristic of the market to be dynamic (Baumol, 1979). Another approach linked to performance is the one proposed by Baumol (1979), in which any type of price cut is required in response to the entry of a new operator in the market for a period of five years that heavily influences the economic performance of agents in the market. However, even in this case, it is difficult to monitor the variables in question for the authorities, given that the predator can change price based on variations in demand and business costs. Furthermore, the rule could only be effective after the exit of the rival which, therefore, means that the predator has already undermined competition in the market.

Rule of reason tests

The reason tests rule abandons the tools based exclusively on the analysis of costs and their relationship with the price, to the aim of detecting predatory prices with all possible evidence in hand. Scherer (1976), with his contribution, proposed the use of a large number of variables that are influenced by the predator's conduct from both an internal and external economic point of view. He asserts that tests based on short-term costs do not have the ability to consider effects in the long run and in general, tests based on cost analysis generate passive behavior by the dominant company and chronic excess capacity.

However, even if this type of approach is completer and more exhaustive and reduces the possibility of error in cases of a false positive or a false negative, in practice it is almost unusable due to the enormous amount of information and data that the authorities should keep in consideration. Moreover, with the absence of clear economic parameters companies have no reference for the lawfulness of their pricing strategies if they engaged in price competition. In other words, it is possible to state that Areeda and Turner's test has weak economic bases, but it is used from a legal point of view, while Scherer's test has good economic bases but little juridically exploited.

Structural tests

The structural tests aim to reduce costs, efforts, and errors by using the aspects of the rules described above but analyzing, firstly, the relevant market and then limiting the investigations to markets that present favorable conditions for successful development of a strategy of predatory prices. In fact, this test is characterized by two levels of analysis that occur sequentially. This approach was devised by Joskow and Klevorick (1979) and predicts that where shares are assimilated at predatory prices, the structure of the market is analyzed with a price-

cost analysis of the operators and their strategies. In the first analysis, the market power and the market share of the alleged predator are studied, moreover also the presence of barriers to entry is part of the initial study, as well as the dynamics of competitors and probable entrants. Only if the instances of predatory pricing were verified, the test proceeds with the second-level analysis incorporating price-cost tests very similar to those analyzed in the previous paragraphs, such as Areeda-Turner tests and the reason tests rule. In this way, different scenarios can be distinguished. Prices below average variable costs are considered predatory unless the alleged predator demonstrates that its pricing strategy is justified by overcapacity. Prices positioned between average total costs and average variable costs are considered predatory unless the company proves that the sector is in decline or that the entry of a new operator has reduced prices. Prices above average total costs are deemed legal unless a price war in response to entry has ceased within two years without cost or demand reasons (Joskow et al., 1979). This process, which also focuses on market conditions, allows the authorities to save complex and costly analyzes that should be conducted in any case taken into consideration.

Predatory pricing application

In literature, scholars have identified numerous strategies that make this practice rational and usable in the market by firms. Predatory or exclusionary strategies can be of different types, for example, predatory investments, extreme product differentiation, predatory advertising, predatory product innovation. The analysis presented below mainly concerns predatory practices that involve the price of products or services.

The contributions that analyze this practice try to replicate the characteristics of the external world in the models, considering the factors of imperfection, asymmetric information, and dynamism of the economic context. In fact, in general, the incumbent has a position of advantage over potential entrants regarding the knowledge of the costs and other relevant economic factors due to its experience in the market (Bolton et al., 1999). This information asymmetry

allows the incumbents to lower prices or increase output by trying to influence the behavior of rivals and undermining their desire to enter the market.

In this way, the incumbents who want to protect their market share can take advantage of this information asymmetry. In fact, the predator can build a reputation for aggressive behavior by applying predatory prices, or it can transmit and disclose untruthful signals about the characteristics of the market, such as business costs and demand, describing the market as an unfavorable context for a business opportunity. Despite the meager literature related to this practice and the pricing algorithms, it is not possible to exclude that such implementation can actually take place. In fact, it is a matter of programming the algorithms differently, keeping their abilities and functions unchanged as seen in the initial paragraphs. In a context increasingly characterized by artificial intelligence, discriminating information that is collected in the market is quite difficult and algorithms, instead of being considered a "black box", can be programmed to modify the outputs that the opponents can intercept, providing them data that could be decisive for the implementation of business strategies favoring the predator and its market share.

Some strategies that can be implemented are the reputation effect, cost signaling, test market, and signal jamming. The cost and demand signaling are tailored to induce the prey to believe the demand in the market is low. In the case of the cost signalling, the predator drastically lowers the prices, making the prey think that it is much more efficient and has lower costs than them. In the case of signal jamming, the predator reduces prices to distort the market test by influencing the strategic choices of the preys to its advantage. Therefore, with the use of algorithms the application of this practice is even more simple and effective, and, of course, less transparent. On the other hand, the detection of this practice by the authorities does not require further action or modification, but the fact remains that the incumbents have an advantage in using the algorithms to set prices and manipulate the data that would affect the choices of other operators.

CHAPTER III

Competition law enforcement, policy implication, consumer protection

The current expansion of big data, machine learning, artificial intelligence, and algorithms is introducing countless changes in the markets, especially in digital ones, with many implications for the authorities that monitor the market and for the regulations in force in the various economic areas. Obviously, technological evolution certainly has positive consequences such as cost reduction, increased quality offered, better allocation of resources; however, such as emerged from the analysis of the previous chapter, there are some incorrect practices that can be favored by the use of this type of technologies as collusion, price discrimination and exclusionary practices, including predatory pricing. Despite the use of algorithms and the practice of automating many of the business processes by collecting and analyzing a great amount of data are increasingly widespread, there is no great empirical evidence of the direct consequences that the algorithms have on the current price level and competitive practices within the various industries. However, not in all sectors there is the same degree of use of these technologies and the economic dynamics themselves can be completely different, also influencing the effectiveness of the use of the algorithms and their diffusion, but, however, remains of fundamental importance a continuous monitoring of the intrinsic dynamics in the markets since it is possible to assume that the use of these technologies will be more common and their computational power will be higher and with it also the risk of seeing these practices intensify in economic spaces.

Analyzing these phenomena from the regulatory point of view, it is possible to distinguish scenarios in which the algorithms amplify and facilitate the behaviors that are already regulated by the current legislation and those in which the algorithms can create risky situations which are not at all considered by the current regulations. About the first scenario, the algorithms do nothing but put into practice the actions that would otherwise be performed by humans. In this case, it is possible that the detection of these behaviors is more difficult, even in probative terms, but the authorities can continue to use the already existing rules on anticompetitive agreements, concerted practices, exclusionary practices, and price discrimination. However, they are left with the burden of understanding how these algorithms work and how they facilitate these illegal practices. On the other hand, the scenarios in which the algorithms operate in a totally unregulated way are more dangerous. In these cases, the algorithms may establish a form of tacit collusion (not punished directly by the law) or without using any means as a facilitating practice, forcing the authorities to find both ways to detect these practices and to regulate these behaviors by punishing them.

Collusion's implications

Focusing on the analysis on the phenomenon of collusion, the scenario that is more complex is certainly the case in which the algorithms carry out actions that go beyond the current regulations, creating a risk for the community that is not indifferent. In these circumstances, these algorithms can implement interdependent actions without the need for explicit communication, increasing the risk of tacit collusion with direct consequences on fixed prices. It is not a completely new situation. In fact, it is known that in some markets characterized by high concentration and good transparency, the actions of each firm have a significant impact on the choices of the other operators. Therefore, after some periods of repeated iterations, companies understand that combining their actions can achieve better results and performance. In other words, the structure of some markets is such that, following some periods of iterations, the best strategy is to act in a coordinated way, arriving at fixing monopoly prices.

The current regulatory body does not give much weight to the case of tacit collusion considering it difficult to apply given the specific conditions that favor its implementation, such as very high concentration (often a duopoly is necessary for real contexts), a high degree of transparency and presence of entry barriers. However, the algorithms could be used as catalysts for the implementation of this practice by providing new methods not foreseen by the current standards. In fact, they could directly facilitate a non-competitive balance by working as tools that eliminate the need for explicit communications or direct interactions between competitors. In fact, algorithms can be considered as intermediaries between companies, collecting and processing data and responding quickly to the actions of competitors and, given their accuracy, speed and precision, they may achieve to manage collusive situations better than humans (Mehra, 2015).

The considerations just made lead to reflect on whether the tacit collusion could become a more common phenomenon with the continuous evolution of the algorithms. In this case, the consumer would be the subject concerned with very negative consequences and, therefore, the authorities should ask themselves if a review of the rules is necessary in order to adapt them to the modern and digital context.

Possible approaches to algorithmic collusion

In order to prevent and detect illegal conduct that aims to coordinate actions between operators, in addition to significant legislative interventions, the authorities can use traditional measures to alleviate and, at the most, solve the problems inherent in this phenomenon.

In particular, a first tool that is used by antitrust agencies is market analysis. In fact, when there are signs that the market is not working well but there are no indications that lead to the presence of coordination practices among operators, the authorities can concentrate on the dynamics involved in the market and understand why the market is failing. With regard to algorithmic collusion, the supervisory authorities could verify whether the algorithms implement coordination actions and, in the case, identify the circumstances and the sectors in which the conditions favor this practice. In this way, a market study can lead to results that help to identify the characteristics that directly influence collusion and improve the tools for detection. Furthermore, the market analysis can lead to suggestions and improvements applicable to the legislation in terms of legal and structural restrictions, as well as the possibility to start an investigation procedure, but also to recommendations to companies by clarifying the limits of the practices and standardizing the principles of competition.

Another approach that antitrust authorities can use to establish a tool to act against collusive practices is through more stringent control over mergers in the market. In fact, in addition to accurately investigating the case in which the market turns into a duopoly, the institutions should pay the same attention to the other cases in which the number of companies in the market decreases. In this way, the authorities can study the risk of future coordination in more complex situations compared to the duopoly in which the use of algorithms facilitates the initiation of illegal practices.

Finally, the authorities responsible for controlling competition could make tacit collusion difficult to enforce by placing restrictions on the behavior of the oligopolists that would undermine the normal dynamics of the market. Indeed, it could be argued that relying on certain algorithms would directly imply the alleged use of practices facilitating collusion and therefore the application of the competition law and the opening of an investigation procedure. Another tool could be the use of audit mechanisms to monitor algorithms in order to ensure that these are not programmed to start collusive or coordination practices. However, according to Ezrachi and Stucke (2016), such an instrument would fail to give reliable results since the algorithms do not have lines of code that impose or facilitate collusion, but rather aim at maximizing profit and the improvement of this tool would hardly be able to keep up with that of the algorithms themselves thanks to their speed of adaptation to the external context, finally it could be difficult to prevent the algorithms from accessing information in the public domain by limiting their computational power.

In addition to the exploitation of traditional tools trying to increase their power of detection, with the continuous diffusion of algorithms in various sectors, scholars and politicians share the idea that political, regulatory and institutional intervention is also necessary to face this phenomenon in continuous expansion.

In the government sphere, some scholars have proposed the introduction of new institutions to control and regulate the modern digital economy. One of these is Gawer (2016), which suggests establishing a global digital regulator in charge of Internet and data supervision. Scherer (2016) has instead proposed an institution for the control of artificial intelligence and its market, releasing certificates of suitability that ensure the correctness of these machines so that they do not jeopardize competition in the market. However, the introduction of these types of institutions is still an open question due to the intrinsic complexity of the subject. In general, in fact, governments have had a market-oriented approach, trying not to limit or damage the natural evolution of these technologies that has certainly brought benefits to the entire economy but also to the consumers themselves who have seen the number of services available increase. Therefore, the OECD (2009) states that the policy, before making any decision about the introduction of regulatory bodies and unnecessary regulations that would slow down the innovative process of companies looking for increasingly efficient tools, should always evaluate the competitive impact of these market rules in order to evaluate benefits and problems that an implementation could involve.

Further regulatory actions recently discussed focus on the issue of transparency of algorithms that are implemented by companies. In this sense, Chancellor Angela Merkel also expressed her doubts and concerns about this issue by referring to the two giants Facebook and Google during the Munich conference (2016). The Chancellor, in fact, expresses frankly to them to reveal their algorithms, not only referring to those of price, to guarantee transparency and provide awareness to users who use their services. She also adds: "These algorithms, when they are not transparent, can lead to distortion of our perception, they are narrow our breadth of information" (BBC, 2016). Along these lines also the European Commissioner Vestager (2017) is present. He delves into the issue by stating that companies must program their algorithms in accordance with data protection and competition directives. However, it is clear that the request to publish the source code of these algorithms might not be a measure of transparency since the machines under consideration are capable of learning with experience and, in fact, the code would turn out to be a black box with the eventual implicit prejudices difficult to detect. Complete transparency could be obtained through an entity that knows how to explain the reason for a specific action on the part of the algorithm, but, once again, an explanation would be difficult to obtain given that most of the algorithms make decisions autonomously without the need for specific programming. Furthermore, it is difficult to identify which authority is best placed to monitor these types of algorithms since companies operating in digital markets embrace various regulations, such as privacy, transparency, data protection,

intellectual property protection, consumer protection, and competition. Even in the case of authorities formed by a set of institutions, it would be difficult to coordinate activities and manage them effectively. Finally, the territorial factor is also important given that almost all digital companies operate in different parts of the world, subjecting themselves to different regulatory regimes.

To date, it is still unclear whether the creation of regulations to prevent and regulate collusive practices between algorithms has direct and negative consequences for market dynamics. However, having ascertained that the use of algorithms has become a technological standard and that it has a not inconsiderable speed in the evolution becoming more and more effective, it is necessary to reflect on which typology of norms it is good to concentrate for the future in case there were situations of tacit collusion or facilitating practices.

A first factor that could be considered to regulate is prices. In fact, precisely because the algorithms can set prices well above the competitive level more easily than humans, we could think of setting maximum achievable price levels. However, the limitation of prices is an obstacle to competition, in fact, not only reduces the incentives to innovation or to supply products with higher quality, but could result in a focal point for the collusion of the various firms that otherwise would have fought to stay in the market. Secondly, one could think of introducing policies with the aim of modifying the structure of digital markets by regulating certain factors that favor collusion such as transparency and frequency of interaction. In this sense, we could think about introducing secret discount systems or imposing restrictions on information that can be published online; while, with regard to the frequency of interaction, constraints of delays in the adjustment of prices could be introduced with respect to competitors. Even in these cases, there would be a negative impact on competition, reducing the information available also for consumers and introducing voluntary asymmetries about prices and market demand-supply (Ezrachi et al., 2017). A third possibility is covered by the introduction of regulation for the algorithms' design. In this sense, the authorities could envisage regulations that constrain and limit the ways in which algorithms can be designed. Therefore, algorithms could be designed to not react to the most recent price changes or to changes introduced by specific companies, while taking

into consideration the average market prices. This solution is less invasive than the previous ones but still remains influential from the point of view of competition and incentives for innovation.

Finally, it remains clear that the regulatory issue about tacit collusion and non-traditional practices that algorithms can use must be addressed by all the authorities involved, including jointly through international agencies and collaborations between different countries to have the broadest scope possible.

Price discrimination's implications

By continuing to analyze the phenomena that the use of pricing algorithms can generate in the market, there is the practice of pricing personalization. This practice can be detected and pursued by different instruments and regulations that the authorities have for monitoring competition within the markets, in fact it can be analyzed from different points of view such as the abuse of a dominant position by a company, as an unfair practice and ethical issues such as respect for privacy and the absence of discrimination based on factors such as origin, religion, language, etc.

In order to consider personalized prices as a practice of abuse of a dominant position, it is necessary to check whether this category is considered a practice of abuse and that the competition authority must therefore intervene. This concern raises a big question because the most widespread legislation among the various economic spaces considers a practice that undermines the competitors and not final consumers directly as an abuse of dominant position. However, it is possible to hypothesize this practice as an abuse of exploitation by demonstrating that the prices that some consumers pay are excessively high for reasons unrelated to costs; or as an abuse of exclusion in cases where some companies use their pricing strategies to target all consumers with a higher willingness to pay, leaving rivals with less willingness to pay, which is called selective pricing (Reed, 2014). In the case of a scenario in which the customization of prices falls into a category of abuse, it is important to identify the circumstances in which a particular situation of this kind is found to be anti-competitive. In fact, personalized prices should not be considered *a priori* harmful and their effects should be studied on a case-by-

case basis, given that the economic literature emphasizes that this practice is not necessarily harmful to the surplus of consumers, but, on the contrary, can increase the welfare of them compared to uniform prices (Dethmers et al., 2017).

Concerning the affinity between unfair practices and the customization of prices through consumer protection regulations, a scenario with many questions opens up again, which certainly needs further investigation to deal with future situations.

Also in this case, it is difficult to establish whether the personalized prices can be defined as unfair given that the criteria and regulations vary according to the jurisdiction under which a phenomenon is studied. Considering this aspect, it is possible to analyze the normative body of various countries in order to understand whether with the current legislation there is the possibility of framing the personalization of prices as an unfair practice.

In the United States, for example, in order for personalized prices to be considered unfair practices the conduct must create substantial damage, be difficult to avoid, and not offset by the effects on consumer surplus. These conditions are hardly verified because normally the customization of the prices brings benefits for the consumers, increasing the surplus. However, situations may arise in which this practice is systematically harmful and therefore qualified as an unfair practice (OECD, 2016). In Ontario, a province of Canada, the laws regarding unfair practices consider personalized prices as if "the price significantly exceeds the price at which similar goods or services are available to similar consumers". Therefore, personalization of price can be considered unfair practice when a personalized price involves a payment substantially higher than for other consumers (OECD, 2016).

Contrary to seeking definitions and rules governing personalized prices in order to identify them as unfair practices, it may be more effective to define certain circumstances in which personalized prices would automatically be considered an unfair practice. One possibility would be to consider any price customization scheme that charges specific categories of vulnerable consumers according to certain factors such as age, educational level, educational level, etc. to be illegal. An additional approach that could be useful to consider is to qualify any nontransparent custom price as allowing a consumer the option of giving up. Transparency is fundamental because consumers use it to limit the knowledge of companies about themselves, protecting their data tracing the purpose of pricesetting algorithms. Following these considerations, the regulations on unfair practices are a significant tool to contrast non-transparent forms of personalized prices or those that consumers cannot avoid.

The last two tools that the authorities can use to limit and monitor the use of personalized prices are privacy and data protection laws, and anti-discrimination laws.

In an increasingly strong digital economy, privacy and data protection are of great importance given that companies collect data in order to offer consumers products and services as close as possible to their characteristics and preferences. Although these regulations do not have the specific tools to directly regulate the practice of personalized prices, they are equally useful given that they regulate certain phases of the personalization process such as collection, processing, and storage of data. In OECD countries, if a company wants to collect consumer data, it must disclose the reasons it aims to achieve with them, and of course, this definition also includes data relating to price customization. An example of legislation that can be used in the analysis of personalized prices is that of profiling which turns out to be "any form of automated processing of personal data consisting of the use of personal data to evaluate certain personal aspects relating to a natural person, in particular to analyze or provide aspects concerning the performance of such a natural person at work, economic situation, health, personal preferences, interests, reliability, behavior, position or movements" (GDPR). Under the new GDPR regulation, companies are authorized to profiling users only if it is necessary for the business activity, is authorized by the authorities, or users consent it, but there are also other requirements provided by the law in order to guarantee the most transparent scenario possible.

Finally, it is important to consider the anti-discrimination laws that safeguard the principle of equality which, while not prohibiting personalized prices, limit companies to discriminate against consumers according to principles that are contrary to human rights. In fact, many of the regulatory bodies prohibit discrimination of consumers according to factors such as gender, age, religion, race, political and sexual orientation, nationality, disability, regardless of whether these factors are relevant to analyze the willingness to pay of a group of consumers as it would risk fragmenting society and bringing negative effects.

Furthermore, the jurisprudence states that the price algorithms that use the company must be designed in such a way that they average the same price to all consumer groups, effectively eliminating any form of discrimination (OECD, 2016). This imposition could be attractive at first sight but would allow or even oblige companies to collect sensitive data on consumers in order to be able to recognize the various groups that make up the market and therefore to strengthen discrimination indirectly. Therefore, given the above considerations, a more drastic approach could be envisaged which prohibits the collection of sensitive information by companies in order to avoid price customization on these factors avoiding discriminatory and anti-ethical behavior. These options could help to guarantee the absence of price customizations that involve discriminatory factors, resulting in illegal and obviously unfavorable practices for the society.

Predatory pricing's implications

Considering the various methods of analysis present in the literature of the predatory prices discussed above, this phenomenon is still extremely delicate, devoid of a common regulation between the various economic spaces which presents significant differences between them and related divergences in phases of detection and accusation. Furthermore, the authorities often base their judgment on the jurisprudence created in their own economic space, which further reduces the homogeneity of sentences, as well as penalties. Therefore, the question emerges about the evolution of the discipline concerning predatory pricing that pushes for a regulation shared between the various countries to face also this problem that can emerge from the use of algorithms.

Reporting the characteristics that emerged over the years following numerous judgments and analyzes for both parties, it is possible to define the US approach more prone to market protection than to the protection of competitors, considering the very low prices of the first period a favorable factor for the final consumers. The different constitution of the European Union, on the other hand, requires greater attention to this phenomenon, in fact, the European authorities are inclined to protect competition and the profitability of small companies, as well as the consumers themselves. This reflects the interest at EU level to address Europe's economic fragmentation and thus foster greater integration between its component countries.

It is really complicated to develop an approach to identify predatory prices that takes into account a part of legal certainty and on the other hand leaves a margin for case-by-case assessment given the numerous factors that are involved in the implementation and analysis of this practice. The objective of defining a regulatory system that will allow it will be a task of the authorities in the near future, possibly in a coordinated way. Here it is possible, however, to analyze some of the factors that most influence this phenomenon on which it is reasonable to lay the foundations for future legislation. It is shared among scholars and institutions that both an analysis based on cost-price and a structural one are necessary (OECD, 1989), given that a completely unbalanced analysis on costs would be too limiting and would leave out significant results for the purpose of obtaining the result obtainable only alongside the structural analysis of the market and the internal context of companies (OECD, 1989). However, a cost analysis is very important to avoid false positives and false negatives. In fact, considering prices below average variable costs as predators would be a constraint for some sectors characterized by excess capacity as in the case in which the risk subsists that a product becomes obsolete or perishes. In these cases, these prices do not necessarily imply that the company adopts behaviors to fix predatory prices. Even in the software industry, there may be a similar scenario in which variable costs tend to zero thanks to the virtually zero costs of replicating the software. Therefore, tools have been devised that consider incremental costs instead of variable average costs (Bolton et al, 1999).

However, the cost analysis presents two main obstacles. First, it is necessary to identify the appropriate cost measure for the analysis; second, it is crucial to calculate the costs identified and it could be a non-trivial activity considering the complexity of real businesses. It is more convenient to switch to other detection methods before performing the cost-price analysis to detect the presence of predatory prices, also because a thorough and detailed analysis on these factors would require a really long time, which would be ineffective in acting promptly against possible abuses in the various industries. Therefore, as the first step of the analysis, we could consider analyzing the structure of the market, the prospects for recovery of lost profits and the intention of the alleged predator in setting this price level. In this way, it would be possible to discriminate and focus on situations in which economic conditions favor predatory practices and where market dynamics make anticompetitive conditions more probable. As already mentioned, recoupment is possible where there are entry barriers to the market. If the probability of recovery were not high, a company would not concentrate in applying a predatory strategy as it would not benefit from it, risking not to recover the initial investment, let alone to obtain supra-competitive profits. In considering the intentions of the alleged predator there is the disadvantage that the elimination of rivals is inherent in competition. This condition is defined by economists as "intention to exclude" to refer to behaviors that would not be sensitive if exclusion were not manifested. Therefore, a correct strategy to analyze whether prices should be considered predators could be the combination of a test that analyzes the conditions of the market, the structure and the intention of the alleged predator to verify if the requests that favor this practice are present and, only later, it is possible to proceed with an analysis of the cost-price relationship that is often difficult to analyze.

Finally, it is possible to make a final consideration on predatory prices given the complexity and uncertainty present to evaluate them. Since a predatory price of companies often involves a selective and therefore discriminatory price, the jurisprudence has been able to avoid the formalization of new rules and instruments which, as just seen, are not trivial and would require a considerable commitment by the authorities, but , instead, consider predatory prices as discriminatory and therefore detectable analysable with the laws on the abuse of dominant power (Article 82 of the EC Treaty, ex Article 86) and all the other regulations seen in the previous paragraph with the due implications and consequences.

CONCLUSION

The evolution of the digital economy and the technological tools it uses are rapidly and radically changing the economic dynamics, as well as the tools that the authorities use to monitor and analyze the behaviors and strategies implemented by operators in the market to guarantee social welfare and promote competition. The focus of the discussions and current perplexities of the supervisory authorities is the attention on the consequence that the integration of the algorithms in the business choices can create risks for the competition that cannot be undervalued. Without neglecting the undeniable positive effects that the algorithms bring with their use, this paper focused the attention on the impact of the algorithms on collusion, on personalized prices and on predatory prices, identifying some preliminary answers for the treatment of these phenomena in the short-term period.

From the previous analysis, it emerged that the algorithms are able to modify the dynamics of the market, increasing the transparency and the frequency of interaction that allows companies to react quickly and precisely. These factors create an environment conducive to the establishment of collusion practices, making them stable. Furthermore, with the characteristics of algorithms, such as monitoring, sending signals to competitors, or maximizing joint profits through deep learning, companies could achieve satisfactory results with the sole use of tacit collusion. To remain in the theme of collusion, from the point of view of the application of the actions that lead to this practice, it is necessary to distinguish between the cases in which the algorithms are used by the operators as an auxiliary tool to a collusive agreement that falls within the traditional area of application of the rules on anticompetitive conduct, and the cases in which the algorithms allow companies to align in what looks like parallelism, a conduct that is not illegal per se according to the regulations in force. As can be seen, in the first case the agencies must identify the possible anti-competitive cases and understand the technology in order to gather sufficient evidence for the accusatory system, while in the second scenario the traditional rules on anti-competitive agreements cannot be applied.

Therefore, to deal with these difficulties, authorities can use traditional tools and, first of all, conduct a market analysis in order to assess whether algorithmic collusion is a commonly observed phenomenon and under what circumstances this occurs. So, if there is a result that demonstrates the presence of a problem in the competition, the authorities can look for solutions on monitoring market concentration or limit the use of algorithms that facilitate collusion. However, with the increasing number of cases or evidence demonstrating the expansion of this phenomenon due to algorithms, the authorities can envisage a review of the rules, redefining the fundamental concepts such as tacit collusion and agreement to be effective in the case of necessity.

Regarding personalized prices, this practice has always been present in the market, but the advent of algorithms has drastically improved the precision with which companies can reconstruct consumers' preferences and characteristics. Precisely for this reason, in most cases, consumers are opposed to this practice, even if from an economic point of view it can increase collective welfare. However, the most considerable concern stems from the effects this practice has on competition but above all on the protection of consumer data. In fact, a personalization of prices requires numerous data that the consumer grants to companies, even of a strictly personal nature such as gender, religion, origin and this can have completely illegal discriminatory consequences. Therefore, close coordination between the authorities is necessary to detect these practices and regulate them so that they can bring benefits to the community, not only from a theoretical point of view.

A more unstable scenario appears in the analysis of predatory prices as scholars and politicians continue to look for the best way to monitor and regulate this type of practice as homogeneously as possible. Despite this, to date, no agreement has been found on how to tackle the phenomenon by facing it differently depending on the economic space in which it is located. On the other hand, there is a general agreement between scholars and the jurisdiction that requires an analysis of the company's costs in order to relate them to the prices it sets. However, this test is not sufficient to avoid false negative and false positive errors and a two-level test is therefore considered necessary. The latter includes a market screening, and if the predatory practice seems to be highly probable, it is possible to proceed with the second analysis which involves a cost-based test and an investigation into compliance with competition laws. However, an innovative push is needed that allows the coordination between authorities of the various economic spaces because at this moment there is still a marked discrepancy between the various regulations in terms of application, detection, and sanctions.

Finally, this paper deals with some regulatory approaches that could be considered in the future to address the phenomena just described, such as price regulation, policies to hinder tacit collusion, limits to the collection of consumer data, an introduction of a standard for the design of algorithms. However, at this stage, there are still concerns about the introduction of regulatory changes as they could have serious adverse effects on competition by overcoming the expected benefits. Moreover, given the multiplicity of factors involved and the expected consequences, the study and renewal of the regulatory body should be implemented by considering various authorities that guarantee data protection, competition, IT experts, especially on artificial intelligence. In conclusion, although the risks related to the implementation of the algorithms are obvious, the situation is still blurred by the uncertainty and complexity of the phenomenon. A lack of intervention or too invasive regulation would lead to considerable costs both for the community and for companies. Beyond the actions that will be taken in the future, these should always be anticipated by an in-depth analysis of the context and characterized by a cautious approach.

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