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MASTER'S DEGREE THESIS

Master of Science in Energy and Nuclear Engineering

***Process analysis and demand forecasting in
the automotive sector***

The Eaton Italy Case

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Special Thanks

Here we are. I am finally at the end of my thesis and of these beautiful years of University in which I believe I grow up professionally, but also as a person. I want to spend one page of my thesis to talk about all the actors that supported me during my 6 years-long travel. It has really been a stimulant but tough voyage during which I have not only had the possibility of moving from the adolescence to the adulthood, but also to physically visit a lot of places.

If I had to thank one by one all the people that somehow contributed to my success, I would need just one thesis on that. I'll try to group some of them together hoping not to forget anybody.

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Now it is time to talk about my **parents**. I don't even have the words to summarize all they did for me in these 6 years. They have always been a reference point for all. Each time I was facing some difficulties or uncertainties, they were there to support me; especially during my last year abroad. When I was down, they were there to pull me up, when I was starting to fly around they were there to keep me with my feet on the ground. They are the perfect masters of life and they are the perfect demonstration that nothing is impossible if you really are willing to put all yourself in it. I hope one day to be able to do for you what you did for me. THANK YOU.

A special mention must be done for all my **friends**. Everybody in his way helped me to become the person that I am today, even those I met once in my life.

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sure this will take time and a lot of effort therefore I need your trust. After 6 years, she still believes in me and everyday does her best to sustain me.

I have already thanked my parents some paragraphs before, but a special mention has to be made for my **father**: without him, all this would not have been possible. His patience and his kindness in helping me when I was completely lost in some moments of my life was absolutely essential to drive me where I am now. He always taught me how to keep calm even when the situation was very complicated and, more than all, how to always see the glass always half full and not half void. THANK YOU.

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Introduction

The following thesis comes from a five months internship experience in the Sales Department at the EATON EMEA headquarter located in Turin, from mid April until the end of August 2018.

High quality, competitive production and Best in Class products have always been the main objectives of the EATON brand. The demand forecasting gives a fundamental contribution to the sustainable and profitable growth of the company.

The necessity of this work is born from the observation of a strong demand variability, especially in a transition period like the one in which we are. Moreover, the circulation restrictions imposed to the Diesel engines inside the cities and the new CO2 emissions standards (that all the autovehicles must respect starting from 2020) are heavily impacting the productive choices of the Car Manufacturers and the decision of the final consumer.

In addition, the hybrid motorization trend has to be taken into account. There are at least five or six possible different configurations that, based on the different electrification grade of the motopropulsion, can drive the final consumer's choices.

For these reasons, the demand forecasting optimization should allow the company to have a good indication of the products (and the relative quantity) that are going to be required by the market in the near future. In this way, it will be possible to plan the production and the delivery of the products well in advance.

The general objective of this thesis is to evaluate the company previsional process in order to detect some possible issues and to propose some corrective actions and useful information to improve the process itself. The chapters' order wants to reflect the methodology used in the analysis and the steps followed to get the final solution proposed in accordance with the company and the objective declared before.

The thesis can be segmented in three different parts.

The first one is based on the literature review of the Supply Chain aspects, the demand forecasting process and the related techniques.

In the second part, the current situation of the demand forecasting in the company has been studied. Subsequently, it has been evaluated the output of the analysis made by the actual model and compared with the one obtained by the other forecasting techniques; such as the moving average, the weighted moving average, the exponential smoothing, the exponential smoothing with trend and the ensemble method.

To that end, a mix of engines typologies and of significative markets has been identified. For each of them it has been determined the best forecasting technique based on the characteristics of the relative demand.

The third part of the thesis is dedicated to the evaluation of the results obtained with these forecasting methods. Comparing them with the values obtained by the company with the current methods some useful hints to improve the process in use have been gathered.

CHAPTER 1 – The Forecast

In this chapter, it will be explained the importance of using market forecasts in companies: it is at the base of all the strategic decisions and planning for the supply chain team, for the sales team, for the business development team and for the financial team.

1.1 Why is it important to forecast the future demand?

Every industrial company that operates in the field of supplying raw materials, the distribution of tangible goods or services in the markets is daily required to make estimates regarding the extent of future commercial demand that will presumably be expressed by the users of these goods or services. The business process of defining the time schedule of demand, relating to all the products in the range marketed by the company and for all customers and the types of distribution channels, plays a key role in supporting and nurturing other business processes of planning of use of the resources available for the physical realization and distribution of the products on the sales markets (Milanato, 2008).

Demand forecasting moves the operational, tactical and strategic planning that guides the entire company, from the formulation of the strategy, to the financial forecasts, to the quantities required for distribution.

The role and contribution of forecasts towards short-term planning at the operational level is quite intuitive. Forecasts are "the most obvious value" - the best possible estimate of the future. They are calculated by identifying and extrapolating established models or existing relationships. Their accuracy depends on the extent to which the future turns out to be a continuation of the past. Therefore, if it is believed that this will not happen, changes must be made based on the singular or collective judgment to correct the extrapolated forecasts.

At the same time, uncertainty about the future is accepted and measured. Moreover, this uncertainty is incorporated into the planning through the establishment of security stocks or other buffering mechanisms. Higher flexibility, smaller production lots or just-in-time productions could be adopted to minimize the negative effects of forecasting errors and respond to the future as efficiently as possible.

Recently, the results of a research conducted by a leading US consulting firm on a large sample of companies have shown how, for 85% of the "best-in-class" companies

in terms of logistics performance (ie, for the more shrewd and better organized companies, even if this is also valid for 68% of the remaining companies), the most important strategic action to reduce stocks and at the same time improve the level of service is the "analysis and understanding of the demand phenomenon ". capacity that must naturally offer the possibility of achieving a "reliable forecast" (LogisticaEfficiente 2014).

In the Figure 1.1 it is shown that, only in case of products tailored on a specific demand, it is possible not to have a precise forecast. Instead, for the vast majority of companies, it is essential to have a proper schedule of the future actions to take.

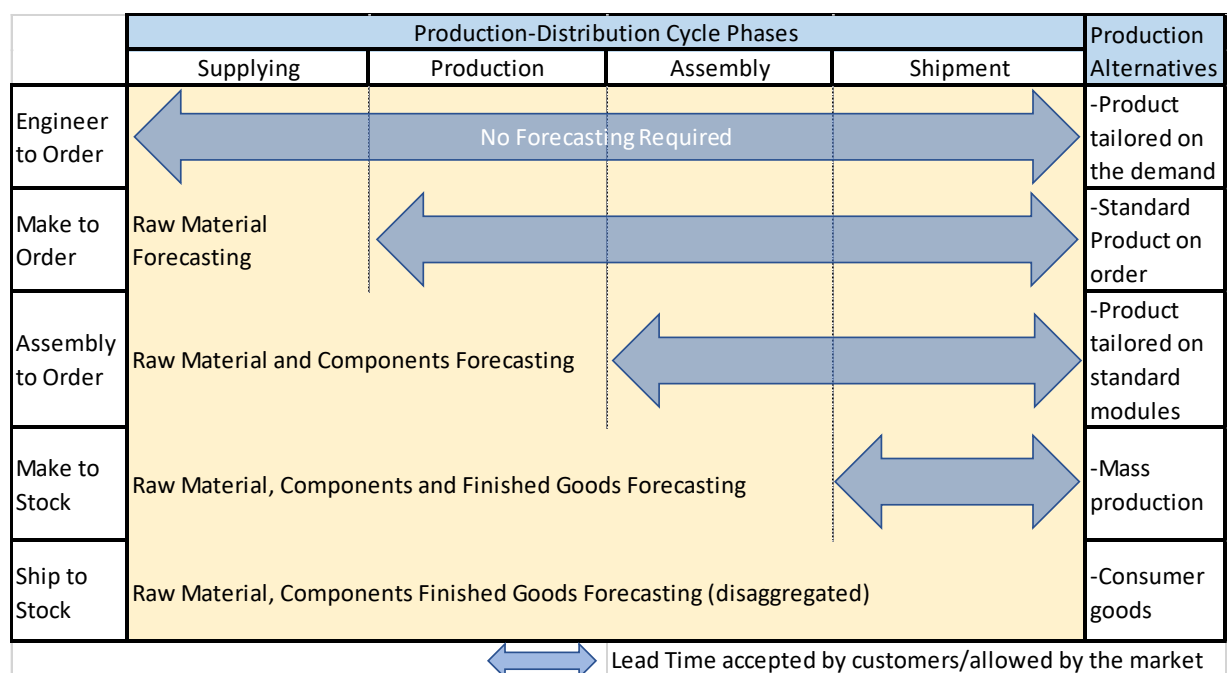


Figure 1.1 - Various types of production and distribution processes (Dallari 2009)

1.2 Which are the benefits of the demand forecasting

The information, in the form of sales forecasts and sales orders, comes from the end customers and is transformed into operational production and / or procurement plans.

The activities involved, carried out within each Supply Chain company, can be brought back to two main flows:

- **The Physical Flow of Goods;**
- **The Information Flow on Needs.**

The physical flow of the *added value* represents the operational aspect of the logistic activity: handling and storage of raw materials and finished products generate value as they make the stocks available in the times, places and quantities required.

This allows the connection of the company with suppliers and customers. Both flows (the physical flow of value-added stocks and that of information on needs) must exist simultaneously. In particular, the logistics system is made up of five integrated and coordinated areas (Luceri, 1996)

1. Plant Structure;
2. Demand Forecasting and Orders Management;
3. Transports;
4. Stocks Management;
5. Physical Storage of Goods.

1.2.1 Supply management

The forecast of customer orders is the most important information in terms of logistics planning. Knowing how to predict the final demand is indeed fundamental for managing the company's production and procurement activity. In this area, the factors indicative of a good effectiveness and efficiency of the logistics system are, in addition to the forecasting capacity, the speed of dissemination of information between the various business functions and their reliability.

At the operational level, the effect of precise forecasts can be fundamental. By accurately forecasting production needs, raw materials, semi-finished products and finished sub-components can be purchased at very advantageous conditions compared to last-minute purchases.

Similarly, from a logistical point of view, services can be obtained at lower costs through long-term contracts than through short-term agreements.

However, these contracts can only work when the demand can be accurately predicted.

Perhaps the most important impact of forecasting accuracy is on the level of stocks present (Luceri, 1996).

1.2.2 Stock and production management

The function of the stocks is to make the various operations that make-up the logistics cycle independent, guaranteeing the availability of the products in the places, times and quantities desired by the various levels of the Supply Chain.

The absence or incorrect sizing of these stocks would in fact require a prompt response upstream, in the production chain, following any change in demand, generating production inefficiencies (line stops and sudden and more frequent lot changes). Furthermore, this level of reactivity is not always operationally manageable and / or economically convenient.

The reference basis for the planning of supplies and stocks is the forecast of future demand. A correct forecast will allow to minimize the costs related to the management of the stocks, which involves the determination of the frequencies and the quantities of reorganization that reduce to the minimum the total cost of the stocks in the respect of the restrictions set by the company in terms of service level.

Demand forecasting moves the operational, tactical and strategic planning that guides the entire company, from the formulation of the strategy, to the financial forecasts, to the quantities required for distribution.

It is possible to classify the different types of forecast based on the time horizon covered:

1. **Long Term**, over 24 months, where forecasts are formulated that act as a support to managerial decisions regarding the business development plans: company purchases, construction of new factories, increase in production capacity, etc. These are **strategic** decisions.
2. **Medium Term**, between 12 and 24 months, where forecasts are built to support decisions relating to aggregate production plans: definition of production volumes by product family, definition of daily work shifts, use of layoff funds, etc. These are **tactical** decisions.
3. **Short Term**, up to 12 months, where forecasts represent support for **operational** decisions such as the use of new suppliers and / or third parties and overtime.

The role and contribution of forecasts towards short-term planning at the operational

level is quite intuitive. Forecasts are "the most obvious value" - the best possible estimation of the future.

Their accuracy depends on the extent to which the future turns out to be a continuation of the past.

At the same time, uncertainty about the future is accepted and measured. Moreover, this uncertainty is incorporated into the planning through the establishment of security stocks or other buffering mechanisms in the production line (Wacker et al, 2002).

1.2.3 Investments and resources management

Always considering the short-term, the demand forecasting can be used also to analyse the financial situation during the year based on aggregate projections.

The prevision includes all the relevant flows in the balance sheet, income statement and cash flow statement. Moreover, it has to include total revenues: the total amount of sales, from quantitative and monetary point of view, and the totality of costs. As the forecast objective is to contribute at the definition of the global results projection, the level of detail is not excessive.

For example, at this stage it is not so important to know in which city the product has been sold as well as it is not too relevant to the color and the analytical characteristics of the product itself. Instead, on the other hand, in the preparation for the logistics plans, the specific product features, the specific product model, the customer site and the shipment date are very important for the planning of the service. Who oversees the forecasting has to prepare a projection for each warehouse as, even though the prevision results to be completely right, if the company is not able to satisfy the request of a specific customer in terms of products, site and time, the entire logistics system results inefficient. This could lead the company to the risk losing orders or to incur in additional costs.

Even in the medium term, the forecasts role in the financial planning is well defined, despite the uncertainty is higher due to economic cycles or unexpected events. Generally speaking, the financial estimations in the medium-term are normally based on the average of the past recessions and improved by interpreting the particular circumstances of each economic cycle.

The sales forecasts are based on the future projection of customer needs. Moreover, these are directly incorporated in the annual sales and production plans after having been sharpened by the sales offices at the different regional levels. Finally, these predictions are used to define the operational activities, among which there are the distribution needs, the production program (MPS) and the materials need (MRP) and the need/training need of labour force.

The demand forecast in the medium-long term is also the essential basis for deciding which investments to make to upgrade or renovate existing production plants and / or in which "Low Cost" countries to evaluate any increase in supplies or production.

The function of long-term forecasts is less obvious, despite their role in planning and defining the long-term strategy is as critical as that of forecasts at the operational and financial level. Long-term forecasts are needed to understand what will happen and to assess the size and directions of changes and their impact. In addition, they are essential to identify potential opportunities and dangers of the competitive and economic environment. This is the aspect where forecasts can provide real strategic advantages and where much can be done to improve their importance at the executive level.

Inaccurate forecasts cause inefficiencies in production planning since the chain generates difficulties in purchasing, financing and planning. Managers make decisions based on forecasting, but in an increasingly competitive world, product configurations change rapidly.

These changes are reflected in the specific forecast of the product to be inaccurate and require the modification of the allocation of resources between the products. At a strategic level, there are specific time horizons to decide on resource commitments and when changes are made within these horizons, a bad allocation of resources can generate huge economic and financial inefficiencies. Forecasting future demand is increasingly becoming a strategic boundary for companies that interact with the market (Sanders et al, 1989).

1.3 What is it necessary to do

The correct prediction of major future changes is an area of common interest between those who make the forecasts and those who develop strategic plans and for this requires a high collaboration between the two parties. The key point is to define how correct long-term forecasts can be formulated and how they can actually be incorporated into the corporate strategy (Makridakis, 1996).

It is also frequent to find situations in which companies are working in "watertight compartments". In such a case, a business function (for example logistics), could perhaps produce an interesting statistical forecast, that is not then used as an element of comparison and validation with all the others business functions, but which instead is only used "locally" by the function that produced it. To properly manage a business, it is always better to link all the information gathered from the different functions in order to provide a valid picture of the situation from different points of view.

In conclusion, to try to optimize the logistic-productive performances it is necessary to apply a business process that implements a coherent and coordinated action of all the various business functions aimed at forecasting the future customer demand on which to base all the decisions, called **Demand Planning**.

In the Figure 1.2 it is represented how, through the demand forecast, it is possible to generate the Demand Plan and how this is strictly related with the Business Plan, the Business Objectives, the Production Plan and the Production Constraints.

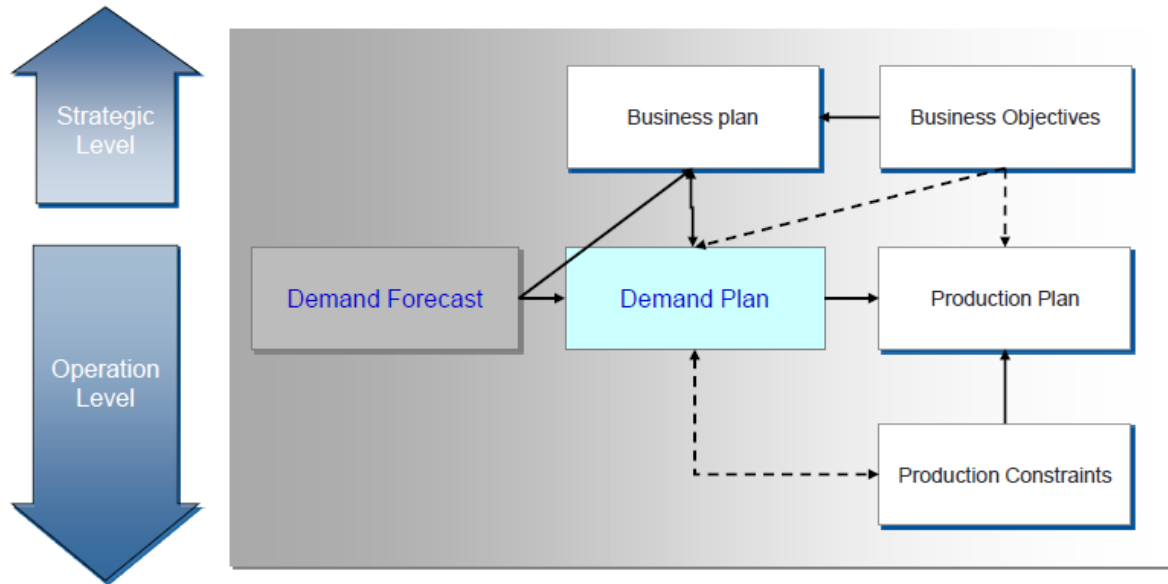


Figure 1.2 - Demand Planning Scheme (Logistica Efficiente 2014)

The term **Demand Planning** is intended to define the set of business processes, management methods and quantitative techniques designed to support the definition of company demand forecasting, appropriately placed in the Supply Chain (Milanato, 2008).

The management reasons that justify the adoption of an integrated process of forecasting commercial demand can be divided into external and internal.

The first are aimed at the level of customer satisfaction, the second concerns the efficient use of resources throughout the logistics process.

Summarizing, the various areas that benefit from a correct forecast of the demand are:

- *customer management*: customer service, maximized by guaranteeing product availability, when requested and in the quantities requested;

- *supplier management*: possibility of negotiating at favorable prices and the availability of raw materials over time, as well as the methods and timing of supply;
- *collaborative management*: efficient planning with supply chain partners by sharing accurate and truthful supply and demand plans;
- *operations management*: efficient preparation of employment plans for internal resources (production, storage, transport) by choosing the best production and logistics alternatives;
- *inventory management*: accurate monitoring of stocks throughout all levels of the process, in order not to dispose of excessive quantities of unsold and expensive finished products in terms of stock keeping in warehouses;
- *resource management*: efficient planning and time development of investments in product, process, storage, transportation, human capital and know-how technologies;
- *finance management*: efficient use of cash liquidity over time, efficient planning of financial resources to be found and provided to investment projects for the enhancement of physical activities and supply chain management.

In other words, the generation of demand forecasts plays a vital role in the company's logistic-production process as it represents the main input to implement future business strategies. These can be operational (less than twelve months), tactical (with an outlook between the year and the three) and finally long-term strategical (with a vision that exceeds three years).

The Figure 1.3 shows who is entitled to forecast what.

The main company functions carry out forecasts with different objectives, aggregation levels, units of measurement, reference periods and forecast horizons

	Marketing/ Sales	Production/ Purchase	Accounting/ Finance	Logistics
Previsional Needs	New products analysis Consumption trends New commercial channels Pricing policies Promotion effect Sales targets	Production plan Production capacity Investments Labor Cost of raw materials Supply	Cost of money Cash availability Exchange rates Profit and Losses CAPEX	Stock handling Delivery plan People in the warehouses Warehouse dimension
Aggregation Level	Article, family	SKU, article	Entire company, division, family	SKU
Period and Horizon of Reference	Annual, with monthly and trimestral update	1-6 months / 1-5 years with weekly/monthly update	Annual, with monthly/trimestral update	1-4 weeks/1-2 years with weekly/daily updates

Figure 1.3 - Who should predict and what in the company (Dallari 2009)

1.4 Results of Poor Forecasting

The definition of a forecast of an inaccurate demand:

- Will lead to an increase in the likelihood of incurring stockout periods, where real demand is higher than forecasts made by the company;
- Induces to increase the level of safety stock of finished products, to be kept at the warehouses of the production plants and the deposits of the logistic network in order to protect itself against the underestimated demand;
- Negatively affects the performance of the service level as the company will be forced to pay late delivery penalties to customers, defined backlog or deferred delivery costs;
- Involves frequent revisions to the short-term operational plans, in order to compensate for the lack of materials at the plants and warehouses of the supply chain, modifying the production plans (going to redefine the quantities and production methods already programmed, re-equipping the plants productive and splitting of the lots) and of distribution (modifying the composition of the loading units, setting up transport loads in a non-efficient way from the point of view of volumetric and spatial occupation).

In Figure 1.4 it is possible to see which the effects of over-forecasting or under-forecasting are for a company.

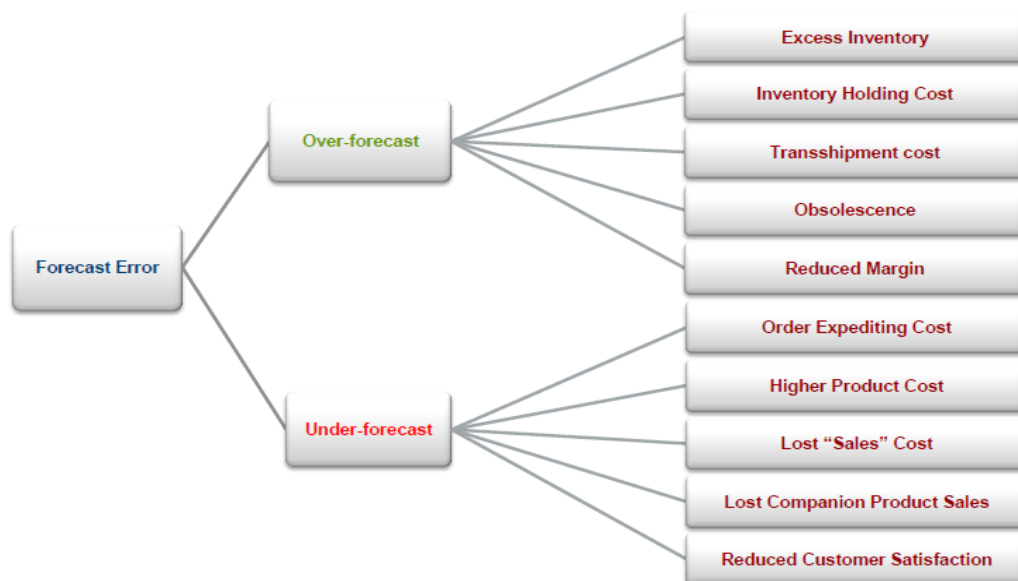


Figure 1.4 – Effects of Over-forecasting and Under-Forecasting (Kenneth 2003)

Mistakes that are made during sales forecasting can generate two types of inefficiencies such as underforecasting and overforecasting (Wacker, 2002).

The first error consists in underestimating the extent of the demand, leading to the loss of sales opportunities on the market, not providing adequate support to the purchase will expressed by the customer generating a stockout cost. The lack of profit margin is accompanied by the reduced level of service offered. This leads to a reduced appeal to the market. The underestimation of demand also generates the under-use of production plants, storage and distribution, deteriorating the profitability of the investments made in the logistics-production process.

The second error consists in overestimating the extent of the demand, leading to the placing on the market of an excess quantity of products, difficult to place over time. Unsold products must also be associated with the costs associated with their physical perishability and the consequent disposal. Moreover, also technological obsolescence costs have to be taken into account. These are present in the companies that operate in the automotive market due to the impossibility of selling the products within a certain time limit since they would be inadequate for the customer (such as the electronic control unit software).

The forecast overestimation of the demand induces the logistic-distributive system to increase the ordered volumes and to keep huge quantities of product in stock at the warehouses for long periods determining the onset of maintenance costs on stock of "immobilized" working capital. It is also necessary to consider the inefficiency generated in the production plants as the operating machines could be overused due to the excess of production at the expense of expected products with a higher level of accuracy.

In Figure 1.5 it is possible to find a summary of the error propagation laws.



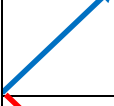

Forecast Accuracy:	
	INCREASING THE PRODUCT AGGREGATION LEVEL The forecast done by family products is more accurate than the one done by singular products
	INCREASING THE TIME AGGREGATION LEVEL The monthly based forecast is more accurate than the weekly based one
	INCREASING THE SPATIAL AGGREGATION LEVEL For example, the forecast made considering the entire Italian area is more accurate than the one made for each individual region
	INCREASING THE PREVISIONAL HORIZON The more I set my horizon far from the actual year, the higher will be the inaccuracy

Figure 1.5 – How the accuracy of the forecasts in terms of aggregation and time varies (Dallari 2009)

CHAPTER 2 – Demand Forecasting Techniques

In this chapter it is reported the description of the main market forecasting methods together with the analysis of the historical data series and the errors that can be committed by using these methodologies.

2.1. Preamble

The growth in demand for forecasting economic variables by market operators was dictated by the increase in the complexity of market phenomena, by the suddenness of changes in scenarios, and by the increase in international competition. In the company operating reality, forecasts on the future trend of sales and global demand play a fundamental role. They provide a fundamental help both in the short term to organize resources and business functions, and in the long term to decide investment plans (Chase et al, 2012).

In general, in all cases in which the delivery time requested by the customer (also known as Delivery Lead Time) is less than the production and distribution time (Total Lead Time), it is necessary to make a forecast to anticipate some of the necessary activities to serve the customer. The forecast must cover the Supply Chain activities that cannot be carried out on order, and therefore those for which the sum of the lead times is greater than the delivery lead time that the customer requires (Brandimarte et al, 2004).

There are many opinions regarding the usefulness and validity of forecasting. A discipline that aims to predict an uncertain future can easily be discredited and is therefore defined as unrealistic and implementable in organized and complex corporate structures. In recent years technological and scientific advances in forecasting have been made; nowadays it is possible to foresee multiple phenomena, such as the fall of a beam, the trajectory of a satellite, weather forecasts, etc. The need to make predictions goes hand in hand with the attempt by many managers to reduce the company's dependence on the random data and to link corporate choices to scientific data.

Currently the business functions that benefit from forecasting include **scheduling, acquisition and determination of resource requests**.

The *scheduling* requires an efficient use of company resources; in this sense, identifying the future trend in product demand to define and allocate the appropriate

resources is extremely helpful. Compared to the acquisition of resources, the forecast of future demand can bring benefits since the lead-time for finding resources on the market can vary from a few days to some years.

Therefore, we use forecasting techniques to better identify future requests for resources such as the acquisition of raw materials, the hiring of personnel or the purchase of new production lines.

Organizations must determine what resources are needed to implement the strategic plan outlined by the forecasting process; these decisions depend on market opportunities, environmental factors and the internal development of financial, productive and technological resources. In general, all strategic decisions require good forecasts and a management that knows how to interpret the data and define appropriate tactical-strategic plans.

The categories dealt with are typical of the short, medium and long term. This range of different terms requires the company to implement a plan of diversified approaches to predict uncertain data. In this perspective, one of the necessary requirements of the company is to possess the knowledge of the macro-environment information and to have the ability to analyze some specific areas. These include the identification and definition of forecasting problems, the application of a series of forecasting methods, the procedures to select appropriate methods for each specific situation, as well as the organizational support for the application and use of formalized methods of future demand forecasting.

Before starting to deal with the different forecasting methods it is necessary to introduce some parameters that allow to fully define the object of forecasting. The object of forecasting techniques is often represented by demand, which needs to be qualified in a more precise manner. We must essentially define:

- The **time bucket**, that is the unit of time, represented by an indivisible amount of time, for which one wishes to foresee. This unit can be day, week, month or even year.
- The **forecast horizon** consists of the advance with which we want to make demand estimates. In other words, given a one-month time bucket, it must be understood whether it is necessary to forecast the demand for the following month rather than for the eleventh month. In reality, it is often essential to forecast demand in a range of forecasting horizons.
- The **revision frequency** is the parameter that identifies the frequency with which the forecasts are updated. The forecasting methods can be defined as “rolling”, if the updates are made at the end of each time bucket and “non-

rolling”, if adjustments are made at the end of the forecast horizon considered. In the first case, the company will always have a 12-month horizon available, in the second case, the company would have a range of time ranging from one to twelve months. The costs associated with updating the forecast rolling process will be higher, as more forecasts will be made per year (Brandimarte et al, 2004).

Furthermore, it must be emphasized that the forecasts should be updated if there are improvements to the information available. In summary, for the forecast frequency there is a trade-off between the cost of this activity and the actual availability of additional information with which to update the latest estimations made.

- The **product** turns out to be the definition of the product or set of products to which the organization refers. Indeed, to foresee the demand for a specific product is more difficult than to foresee the aggregate demand for similar products.
- The **market** is the parameter that defines the market or the geographical area to which the organization refers. In the global market, the forecast of the aggregate demand of a product is very different from the single point of sale. There the volumes are affected by a greater variability given by uncontrollable factors that could radically change the sales trend (Brandimarte et al, 2004).

Management should pay close attention to the most appropriate definition of the parameters just defined. The search for reliable forecasts could lead to a simplistic view of the problem; in other words, organizations could be enticed into defining very large time buckets, short horizons and large product aggregations for large markets. The configuration of the forecasting process described above presents a better performance since it minimizes possible errors.

In the presented way doesn't give importance to the relationship between forecasting processes and decision processes. Forecasting makes sense only if it is included within a corporate decision-making process that identifies its context, parameters and objectives. To correctly define a forecasting problem, it is necessary to understand the decision-making process that the forecast offers to support.

The forecast analysis of the demand can be conducted in a qualitative way, which aims to describe and interpret the behavior of the client, or quantitative, which measures the degree of diffusion of the product in relation to the overall potential demand. In particular, the latter allows to calculate:

1. Current and Potential Demand
2. Market Share
3. Demand Elasticity

4. Sales Forecasts

The first three points are included in the "marketing plan", the last dimension includes the crucial activities outlined by the company management through forecasting methods that can be divided into two large families (figure 2.1):

- **Subjective Qualitative Methods**
- **Objective Qualitative Methods (or Quantitative)**

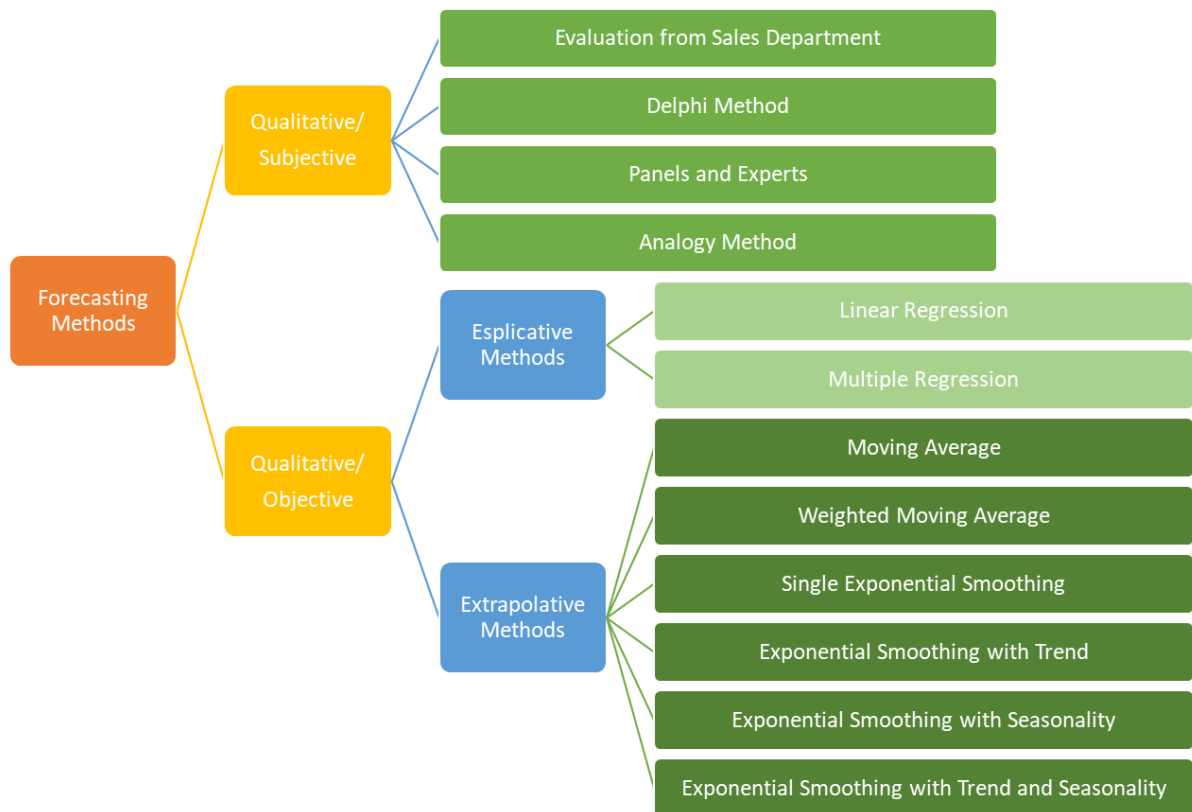


Figure 2.1 - Overview of demand forecasting systems (Basile 2005)

2.2. Qualitative methods

The qualitative forecasting methods can be well adapted to very unstable contexts, such as the launch of new products or long-term forecasting. These methods are very flexible as they do not require formalizing any relationship between the information deemed relevant and the forecast.

Below we will examine the main subjective methods useful for estimating future demand: the evaluation of the sales department, the Delphi method, the panel of experts and the analogy method (Milanato, 2008).

The **evaluation of the sales department** guarantees plausible information to outline future sales scenarios as it refers to the data collected by the sales force. This is a simple method that uses networking and dialogue with customer representatives.

The advantages it guarantees are manifold because the forecasts are sufficiently reliable and inexpensive and the sales force staff is integrated into the forecasting process. The implementation of the forecasting process integrated with the personnel brings various disadvantages including the dispersion of resources, since the time devoted to forecasts by the sales force is time taken away from the actual marketing activity. By interpreting the forecasts as objectives, the sellers prudently expect less sales than those they believe they can make.

Finally, sales force forecasts are often mere projections, the result of pure statistics and not projected into a future dynamic, with the risk of being translated more into reports than into forecasts.

1. The **Delphi method** develops along a systematic and organized process of questions and aggregation of the answers that are configured as expert judgments (Riggs, 1983).

This method was created to fill a gap in forecasting techniques and to experiment with a retro-deductive approach (first the statement, then the data and the arguments to support it).

In this way it is possible to give an answer to those problems that cannot be faced either through the deductive approach of the predictions, or through the inductive one of the projections, trying to subject the subjective judgment and the intuition of the experts to a more rigorous treatment. The term informed judgment was the expression used to indicate something that was halfway between scientific knowledge and speculation. It is a system that involves the participation of selected technical experts of the sector, to whom a questionnaire is submitted to be filled in anonymously, concerning the specific area of the market to be investigated; the evaluations collected are

processed and communicated to the respondents who produce new judgments based on those previously formulated, in order to obtain an integrated analysis. Rounds generally range from a minimum of three to a maximum of six.

This method is effective mainly for long-term forecasts, for which it is difficult to use mathematical-statistical techniques; moreover, the validity of the judgment is dictated by the opinion of the experts. However, the methodology is very expensive and ineffective for an operational forecasting horizon.

2. The **panel of experts** presents, similar to the Delphi method, a set of individuals external to and internal to the organization. While the method described above excludes the possibility of conditioning among the participating subjects, the panel of experts is based on a closer interaction between the subjects in order to achieve valid forecasts through a progressive consensus.
3. The **method of analogy** is a method used to estimate the likely evolution of a product or market for which there is not enough information available or it is not appropriate to carry out specific research "in the field", for reasons of competitive prudence. The technique consists in the selection of one or more products similar to the one under examination and to seek, through the analysis of historical series, the development perspectives, using appropriate corrections for the projections on the examined product. The effectiveness of the model is linked to the degree of similarity of the product tested to the one examined.

2.3. Quantitative methods

These methods require the use of mathematical models applied to time series, formalizing explicit hypotheses about the behavior of the demand. Therefore, they are less flexible, but being able to manage many products or many markets with limited use of resources, are more efficient and often have a good consistency with respect to qualitative methods.

In reality, in most cases, time series refer to sales data and not to demand data. Sales and demand coincide in the event that the demand is less than or equal to the production and distribution capacity, while they may differ considerably if there is an unanswered demand due to a logistic-productive system not properly sized. In this case, it will be necessary to correct the data obtained with qualitative procedures. The integrated use of quantitative and qualitative forecasting methods can lead to superior results compared to the exclusive use of a forecasting methodology. The choice of the most suitable quantitative techniques to predict the future trend of demand depends on both the nature of the product or service, and the quantity and quality of the information available (Chase et al, 2012).

Within the family of quantitative methods, it is possible to identify two subgroups classified in:

- **Explanatory Methods**
- **Methods based on historical series**

2.3.1. Explanatory methods

The explanatory (or causal) methods mathematically express the cause-effect relations between the forecasted variables and their explanatory variables (Hanke and Reitsch, 1995).

The attempt of the causal model is to define an activity through a system of mathematical equations that identify the relationships between the dependent variable (for example sales) and the variables that could influence it. Mathematically the model can be expressed in the following form:

$$y = f(x_1, x_2, \dots, x_n) \quad (2.1)$$

This relationship explains how the values of the variable y are directly related to those of the independent variable x .

The definition of the relations and consequently of the variables are based on experience, judgment or theory. The causal model takes into account every element

of the system dynamics and being a statistical association, it can indicate the existence of a causal correlation between the variable under study and the observed phenomenon. However, the mere presence of an association does not necessarily prove the existence of a cause-effect relationship between the two variables found to be associated. Initially, it may therefore be necessary to formulate hypotheses on some of the relationships and then test them on the basis of the actual trend observed.

Usually, as the available knowledge grows, the model is continually updated and revised, so as to be able to seize turning points and to prepare long-term forecasts.

The amount of equations and independent variables used in the model can vary considerably as this number is closely linked to the time range of the volume of available historical data.

The variables that can be used can be classified as endogenous and exogenous: the former are variables whose value is explained by the model, ie they are under control of the person responsible for implementing the demand forecast or defined reviewer, the latter instead, assume a value independent of the balance represented in the model itself and can neither be modified nor controlled by the organization as they depend on the influence of the external environment.

The steps needed to build a model are:

1. Identify all the factors that could be classified as variables;
2. Highlight all the existent relations among variables;
3. Speculate the functional form;
4. Estimation of the parameters to be used in the equation;
5. Test the model on past data that are already known (method validation);
6. Prediction of future values of exogenous variables and their substitutions in the model;
7. Equation resolution to determine sales forecast values.

An example of a causal method is represented by the linear regression, a statistical method that allows us to estimate, using empirical data, the linear relationship between an assumed dependent variable y and an assumed independent variable x .

Simple linear regression model

Extrapolative models are often used by many companies, but suffer from an intrinsic weakness, represented by the fact that demand depends only on time. On the contrary, in many contexts, the demand could be a function of other variables, such as for example advertising expenditure, weather, the number of sales points that are currently distributing the product in question or the economic trends (Brandimarte et al, 2004).

In the first place, the two series of data must be reported in a Cartesian plan, representing the dispersion diagram. The second step is aimed at determining the equation of a line (regression line) or of a curve so as to know the link between the two variables considered. The expression generated by the linear causal model is composed as follows:

$$Y_i = \alpha + \beta x_i + \varepsilon_i \quad (2.2)$$

The least squares method is used to determine the values of the coefficients α and β of the regression line. This method proves to be useful as it measures the accuracy with which the model represents empirical data and is easily manageable with respect to absolute value measurements (Vicario et al, 2008).

As a first step we need to find the equation of a curve in which the e_i gaps between the estimated \hat{Y}_i points (in the regression line) and the real Y_i points (in the scatter diagram) are as close as possible. For the development of calculations, it is more useful to consider the squares of errors, e_i^2 , rather than simple errors.

The total error will be given by the following formula:

$$e = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (2.3)$$

It is necessary to define previously the most suitable curve (straight line, parabola, exponential, logarithmic). The choice is made through a preliminary analysis of the trend of the data dispersion cloud so as to identify the curve that most closely approximates the trend of the same.

The simple linear regression model assumes that the link between the independent variable and the dependent variable is:

$$Y = a + bX \quad (2.4)$$

To calculate the coefficient b , you can apply the following formula:

$$b = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \quad (2.5)$$

The least squares method shows that the regression line of a diagram of dispersion passes through the midpoint (\bar{x}, \bar{y}) , this observation is useful for estimating the coefficient a , in fact, setting \bar{x} and \bar{y} in the equation instead of X and Y will result in

$$\bar{Y} = a + b\bar{X} \quad \text{from which} \quad a = \bar{Y} - b\bar{X}$$

By replacing the values a and b found, it is possible to determine the regression line. The question that arises concerns the degree of correlation that exists between the variables of the model.

In the demand forecasting phase, the detection of the degree of correlation is extremely important, as it provides a discretionary assessment of the reliability of the two variables and their correlation. It is necessary to estimate and evaluate whether the regression has a good explanatory capacity, i.e. if the variable X succeeds in effectively explaining the Y .

To rationalize the concept of correlation intensity the following formula is used:

$$r = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}} \quad (2.6)$$

The sample correlation coefficient is a dimensionless measure and is included between the values -1 and 1. Measure if a positive deviation of the observation of Y from its sample mean \bar{Y} corresponds to a positive (or negative) shift of X and its sample mean \bar{X} .

As for the forecast, a correlation coefficient close to the zero value means that the

considered independent variable does not help in identifying the future trend of the demand. Indeed, there does not seem to be any particular relationship between X and Y.

On the contrary, the presence of a strong correlation indicates that the variables considered have intimately related oscillations and, therefore, the independent variable could be very useful in identifying the trend of that employee.

In figures 2.2, 2.3 and 2.4 the extreme cases are represented in which there is perfect positive correlation (the correlation coefficient assumes values equal to 1), perfect negative correlation (the correlation coefficient takes values equal to -1) and null correlation with values of the correlation coefficient that are around 0.

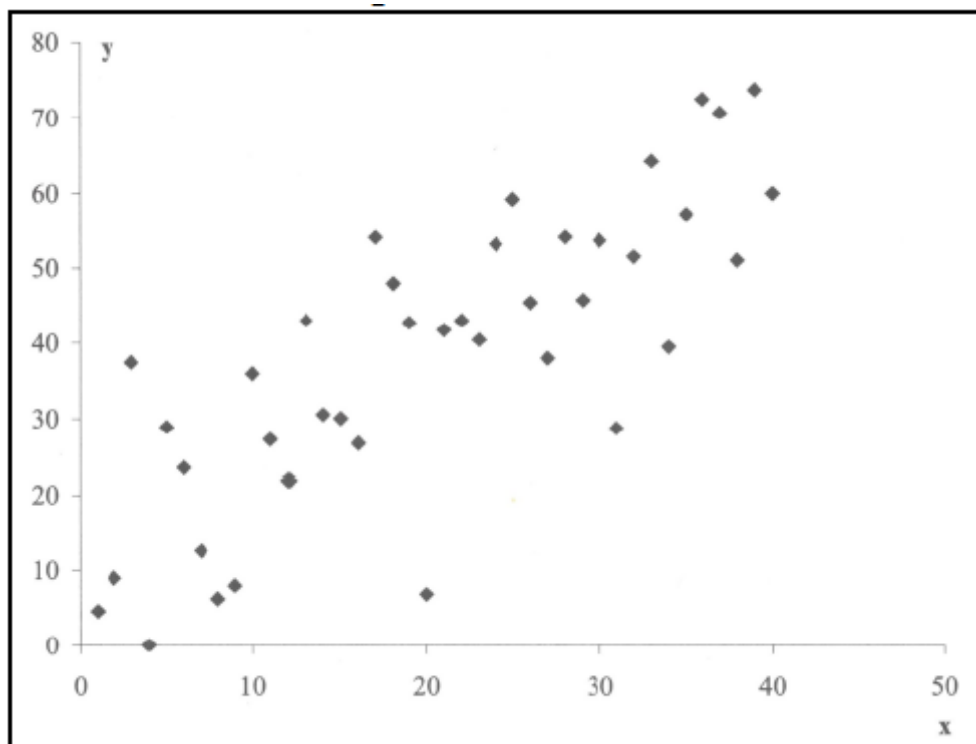


Figure 2.2: Positive Correlation (Brandimarte et al 2004)

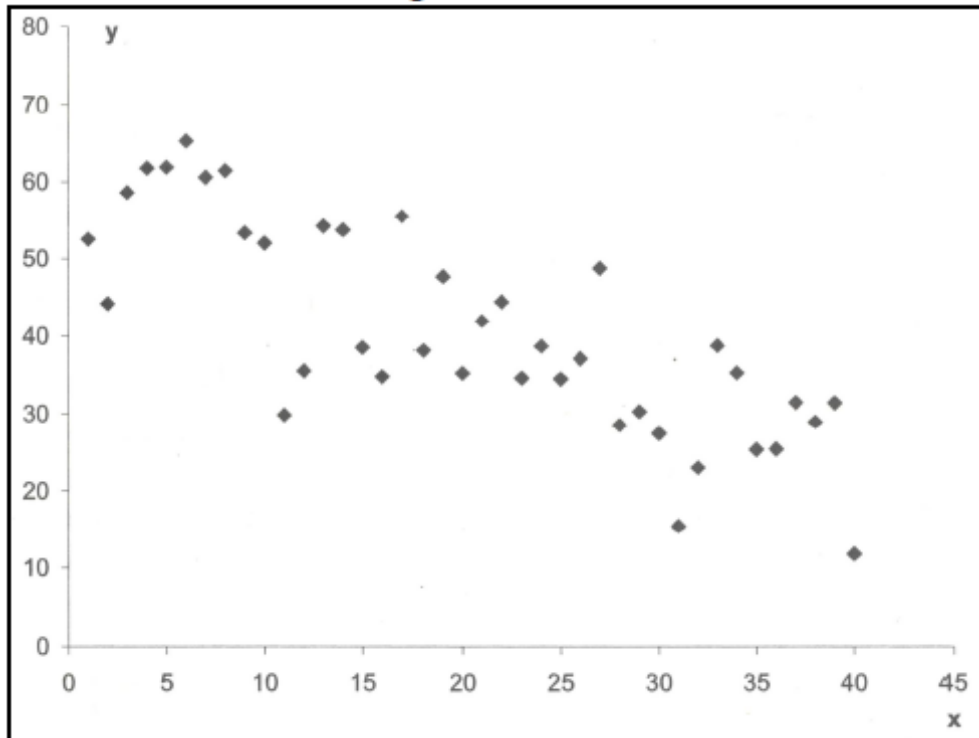


Figure 2.3: Negative Ccorrelation (Brandimarte et al 2004).

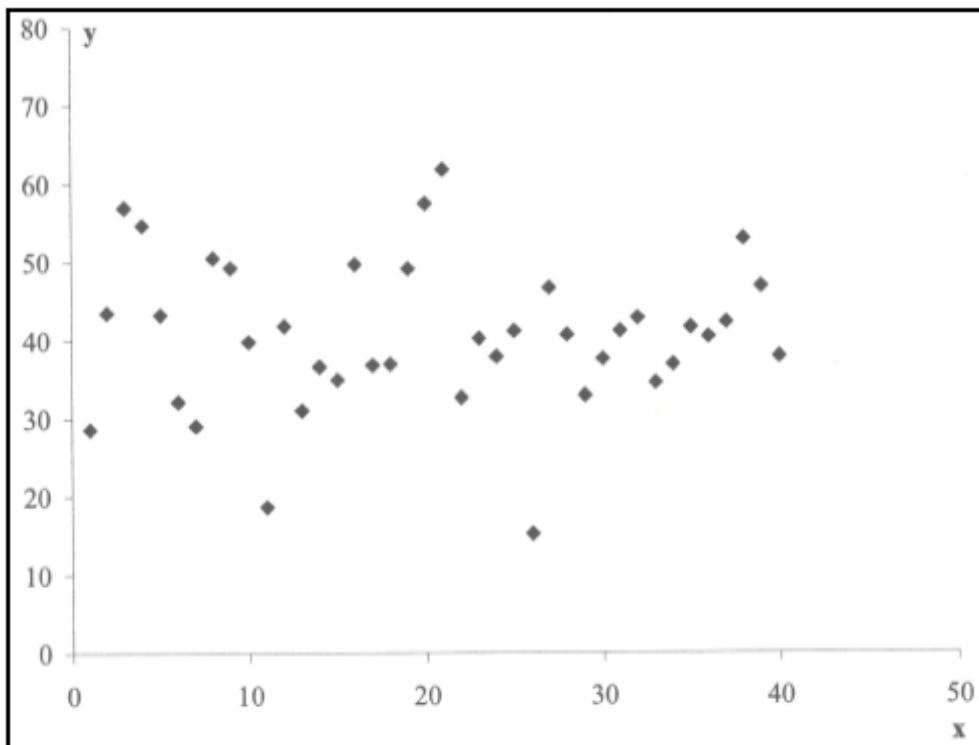


Figure 2.4: Null Correlation (Brandimarte et al 2004)

Multiple linear regression model

Often, for the explanation of the sales trend of a product or a family of products, we tend to measure the correlation, rather than between two variables, between several quantities using a multiple regression model (Cooley et al, 1971).

The multiple regression model is used to explain the relationship between a Y variable, called an endogenous variable, and one or more independent explanatory variables, which in terms of function will have:

$$y = f(x_1, x_2, x_3, \dots, x_n) + \varepsilon \quad (2.7)$$

which indicates the existence of a functional link between the dependent variable and the regressors, represented by the component $f(x_1, x_2, x_3, \dots, x_n)$ called systematic. In addition to the systematic component, another denomination is added.

In the multiple regression model, it is assumed that each observed value of the dependent variable can be expressed as a linear function of the corresponding values of the explanatory variables, plus a residual term that translates the model's inability to accurately reproduce the observed reality.

The functional link in theory can be of any type; however, in practice we prefer to use the model in its simplest form, i.e. the linear one that follows this formula:

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots + \beta_n x_n \quad (2.8)$$

where $\alpha, \beta_1, \beta_2, \dots, \beta_n$ are coefficients to be estimated that measure the influence of the respective independent variables on the dependent variable.

The least squares method offers, also in this case, an accuracy performance between the estimated and the actual demand.

If there are more variables to be related, it is necessary to use electronic data processing, using a program (Statistical Package for Social Science and Excel) able to estimate the aforementioned parameters. The calculation is implemented by entering the values of the historical series of the various variables, determining the individual coefficients and the set of statistical tests that allow to evaluate the confidence level and the reliability of the results obtained.

For the parameter estimation the only necessary assumption concerns the linear independence between the n explanatory variables, since if there was a perfect linearity between two variables it would mean that the information contained in it are already present in the data set through the other variables. Therefore, the elimination of the related variable would not lead to a substantial loss of information.

Once the model has been estimated, it is possible to calculate the goodness of adaptation through the multiple correlation index, with values that vary between 0 and 1. As already observed for the simple linear regression model, also in this case the property of the decomposition of the total variance, where the total sum of the squares (SQT) can be broken down into the sum of the squares of the regression (SQR) and in the sum of the squares of the errors (SQE), holds:

$$\sum_{i=1}^n (Y_i - \bar{Y})^2 = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2 + \sum_{i=1}^n \widehat{e}_i^2 \quad (2.9)$$

The multiple correlation coefficient is given by:

$$R^2 = \frac{SQR}{SQT} = 1 - \frac{SQE}{SQT} \quad (2.10)$$

This index is an extension of the coefficient of determination to the case of two or more explanatory variables, indicating the proportion of variability of Y explained by the explanatory variables through the regression model.

When a new variable is added to the explanatory variables of the regression model, the sum of the squares of the errors (SQE) does not increase and normally the estimated values \hat{y}_i are closer to the observed values y_i .

This index measures the degree to which the interpolating function found represents the observed phenomenon (Borra et al, 2004).

Multiple regression could also have a non-linear relationship so that a parabolic surface is identified which interprets the variable sought.

To conclude, the hypotheses underlying the various regression models are:

- presence of an appropriate model;
- interdependence between error terms (ε_i);
- distribution with a Normal (approximately) of error terms;
- same variance for error terms.

If the model is not correctly specified, you can run into the specification error. This error causes distortion in the coefficient estimates and, consequently, affects all statistical inference procedures.

The assumption of homoschedaticity, in which it establishes that the error terms all have the same variance, is the hypothesis that is most violated. The coefficient estimates, without the assumption of homoschedaticity, continue to be undistorted; however, the variance of the estimates will be underestimated leading to the invalidity of the hypothesis tests.

These models, even with the aforementioned assumptions, are frequently used, with satisfactory results for forecasting the future demand required by the market. To create and update the model, an analyst is required who has not only knowledge of the methodological, statistical and mathematical procedure, but also a repeated experience in the field. The analyst's experience can provide the culture and sensitivity necessary to be able to represent and analyze the variables of the model (Vicario et al, 2008).

2.3.2 Methods based on historical series

The extrapolative methods are based on the analysis of historical series, sequences of values assumed by a measurable quantity at certain instants of time t , generally uniformly distributed. These forecasting methods try to extrapolate the future trend of the demand starting from the study of its historical trend.

«*The past contains the present; the present contains the future*». This sentence is the one that best summarizes the basic hypothesis of the method among all the attempts at definitionextrapolation of the trend by analyzing a series of sales results. (Montgomery et al, 1976).

Extrapolative models are named after the fact that the time series are analyzed over a time horizon, describing sales curves, year by year, month by month, in order to intuit the repetitiveness of trends. In this way, they can contribute to building a coherent forecasting model.

Often, in the analysis of the historical series of sales, a superficial approach of data that seems to articulate over time with a nervous trend, can create difficulties in

modeling a coherent forecast. Statistical analysis, on the other hand, provides a methodology that shows a regularity in the distribution of sales data.

Thanks to the statistical analysis we realize that the fluctuations of a historical series follow a certain behavior that develops periodically with regularity.

These methods have a widespread application in the industrial field due to their ease of use but, the validity of the results turns out to be such only to the extent that their simple assumptions prove realistic. In the process of analyzing and diagnosing a market, the statistical examination of the historical series thus becomes of fundamental importance for the search for the components of variation (Montgomery et al, 1976).

The temporal successions are subject to forces of different nature, such as random and non-random factors, which often influence their performance. Indeed, the analysis of the historical trend of sales is studied through the time variable but, as can be seen, the irregularities of the trend depend on several variables that cause direct distortions with the considered quantity. The variables that mostly influence the historical trend are the economic factors, which act in the long term, the seasonal factors, in the medium-short term on an ongoing basis and finally, the unforeseeable accidental factors, which are not repeated continuously.

From the statistical analysis of a large number of economic cycles it was found that a historical series is influenced by some fundamental components such as (Siciliano, 2010):

- general trend;
- seasonal fluctuation;
- cyclical economic movement;
- occasional fluctuation;

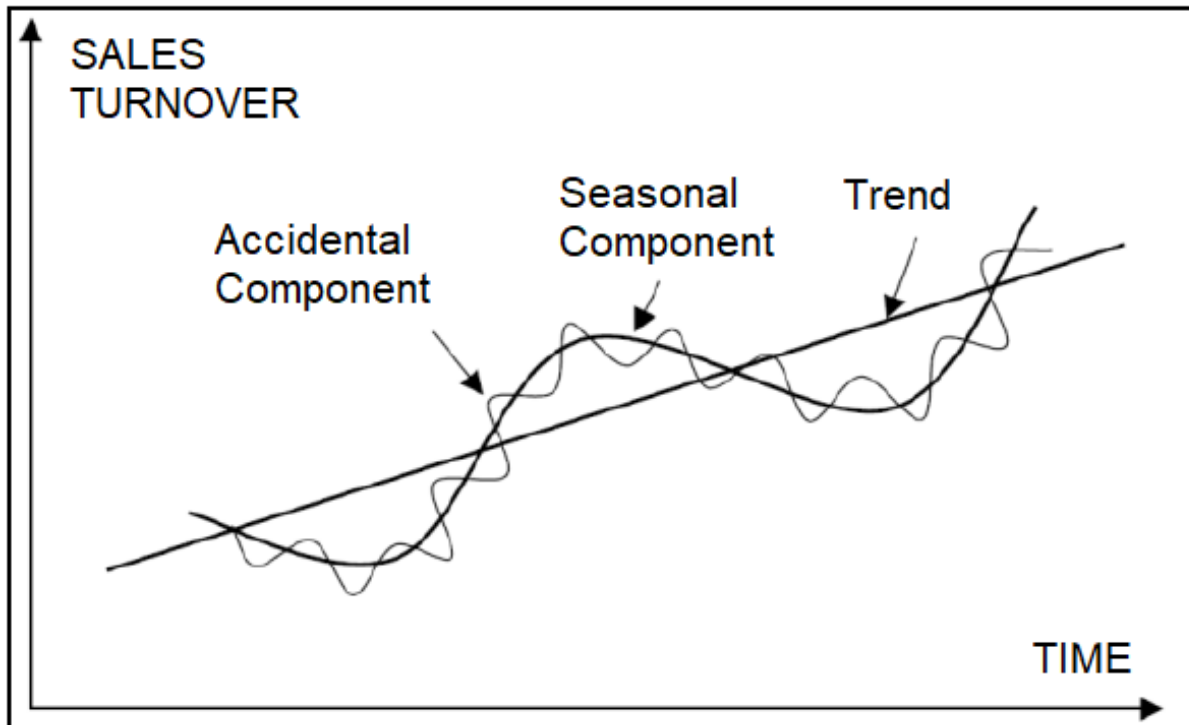


Figure 2.5 - Fundamental components that influence a historical series (Lewandosky, 1993)

The first cause of non-stationarity of the demand is represented by the presence of the trend, identified by a growing line, which could also be decreasing, in a constant manner. Growth can be both linear (constant in absolute terms) and exponential (constant in percentage terms).

A second cause of non-stationarity is the so-called seasonal fluctuation, which presents a cyclical trend around the trend line, just as the accidental nature fluctuates around the seasonal variation (figure 2.5).

The **general trend** is evident when data are available on a sufficiently wide horizon, since it is possible to note the long-term trend that can be constant (in this case we speak of stationarity), increasing or decreasing (figure 2.5).

Knowledge of the trend behavior is essential to proceed with the detection and forecasts of market development. Moreover, in the internal company analysis phase, it is necessary to structure the corporate strategic plan.

The regularity of many historical series is caused by major **seasonal phenomena**. The "natural" factors are identified, if determined by the influence of the seasons and "artificial or conventional", when the demand for a product depends on the influence of the institutions, habits, regulations or laws and on customs.

Seasonal fluctuations therefore occur periodically, every year, following a constant time pattern. The presence of these seasonal fluctuations often causes the company problems of a productive and financial nature. It is therefore necessary to carefully detect every significant movement (figure 2.5) (Mills, 1995).

In addition to the components of trends and seasonality, we must also consider the **economic performance** of each nation. These are cyclical movements that alternate periods of great productivity with periods of production decline, periods of contraction and economic development. The trend described is highlighted by a sinusoidal curve around a trend. The economic cycle is normally characterized by a phase of maximum expansion and one of recession in which the macroeconomic system regresses, penalizing the entire productive sector of the country. Analyzing the economic cycle and identifying the phase, in which the economy presents itself, becomes relevant in the process of forecasting market trends to understand the shorter or more distant variations of trend reversal, ie the upper and lower turning points (Ravazzi, 2012).

Finally, there are all those factors which, although they are particularly difficult to identify and predict with forecasting techniques, can have a decisive impact on market demand trends. These components are generally referred to as **occasional factors**. Some occasional fluctuations have a positive influence on the development of the turnover or on the market in general, others, instead, can have an opposite and compressing effect. When the occasional component is preponderant, although it rarely occurs, the trend in demand is influenced. These influences are usually temporary and can be caused either by catastrophic natural events, or by prolonged strikes, by promotional actions of competition, by a particular corrective measure adopted by the central authorities, etc.

Therefore, it is necessary to identify and isolate the nature of the occasional component, since it could compromise the forecasting model of the time series. (Ravazzi, 2012).

In the historical series, however, the factors of trend and seasonality prevail; the perturbation due to the occasional component is minimal and being a random component, it is difficult to identify. The cyclical component, on the other hand, is carefully analyzed in the long-term forecast plans as it is highlighted in a longer time horizon.

Analyzing a historical series means recognizing and identifying the components within the available data sequence. The elementary components, generally found in the historical series, are identified in (Siciliano, 2010):

- **Trend, $T(t)$** : represents the long-term trend of the series. It can be represented by a simple mathematical function, which in the case of the line is identifiable in

the linear regression of z with respect to t .

- **Cyclical component, $C(t)$:** represents the alternation of phases between two periods, therefore easily confused with the trend. For this reason, this component is usually associated with the trend.
- **Seasonal component, $S(t)$:** represents the regular repetition of effects, by series with surveys lower than a year (month, quarters, ...).
- **Random or erratic component, $R(t)$:** represents the randomness of the behavior of the series without the other components. For this purpose, two models are used:
 - **Additive model:** it is assumed that the Y series can be expressed as the sum of the components $Y=C+T+S+R$;
 - **Multiplicative model:** it is assumed that the Y series can be expressed as a product of the components $Y=C \cdot T \cdot S \cdot R$.

The fundamental difference between the two models consists in the units of measurement to be attributed to the various components. In the additive model, each component must be expressed in the unit of measurement of the time series, while in the multiplicative model, having established a component in absolute value, the other components must be expressed through "seasonal indices", dimensionless numbers that represent multiplicative factors with respect to the value middle of the historical series (Siciliano, 2010).

Simple moving average

The calculation of moving averages is a first forecasting method. The fundamental assumption on which this approach is based is that of having a stationary trend demand. In fact, if the phenomenon considered is animated by a tendential movement, seasonal fluctuations and accidental variations, the moving averages are not a good approximation model.

More formally, it is assumed that the demand is generated by a process of the type:

$$Y_t = \bar{d} + \epsilon_t \quad (2.11)$$

where \bar{d} is the expected demand, unknown parameter and object of estimation, and ϵ_t defined as noise term such that $E[\epsilon_t]$ must be null. In essence, we do not expect the expected demand to remain truly constant, but that the variations are slow and not sudden over time.

With the moving average method, the expected sales value at time $t + 1$ is equal to the average value of sales calculated at the end of period t , that is:

$$F_{t,h} = \sum_{i=t-k+1}^t \frac{Y_i}{k} \quad \forall h \quad (2.12)$$

where: Y_i is the observation in the period t of the variable to be forecast, k the time interval considered chosen so as to minimize the errors according to the needs of the operator $F_{t,h}$ the forecast made in the period t with forecast horizon h , therefore, for the period $t + h$, with $h = 1, 2, 3, \dots$;

In using this forecasting method, it is necessary to choose the k parameter, that is the number of observations that are used to generate the forecast. In choosing this parameter it is necessary to analyze the trade-off between:

- The ability of the method to filter the noise, ie not to be conditioned by

- observations of the demand much higher or lower than the average;
- The ability of the method to react immediately to any increases or decreases in average demand.

In the case where a high k value is chosen, the method will have a high inertia and therefore, it will be little influenced by an observation of demand not in line with the current average. However, it will also be very slow to sense any increases or decreases in the average level of demand. An opposite result will occur in the case of a very low k value, with a much more reactive forecast as it is influenced only by the last observation.

The amplitude of the moving average range varies in the number of its terms depending on the purposes to be achieved.

The method is so called because it consists in the construction of a new series, obtained by replacing the original data of the historical series with a series of averages of the original data. Using the method of moving averages, as k increases, it is possible to eliminate passing fluctuations and to be able to better distinguish the average trend of demand.

This model of forecasting, while on the one hand satisfies the criterion of simplicity and easy intuition, however, turns out to be very simplistic and with some limits. In fact, in this procedure, the weight given to the last k observations is always $1 / k$, while the previous observations are not considered in any way (Brandimarte et al, 2004).

Weighted moving average

To overcome the limits described above, the weighted moving average method is used, in which the demand forecast is obtained by attributing a higher weight to the more recent observations than to the more remote ones.

In this sense, the attribution of the weights is carried out in order to allow the demand, in relation to more recent periods, to influence the forecast in a higher way than the data referring to more distant periods in time. This allows you to react promptly to changes in the market. In analytical form the weighted moving average is expressed as follows:

$$F_{t,h} = \frac{\sum_{i=1}^t (W_{t+1-i} \cdot Y_{t+1-i})}{\sum_{i=1}^t W_{t+1-i}} \quad (2.13)$$

where: W_t = relative weight of the period t .

Therefore, the advantage of mobile weighing averages consists in the possibility of representing an importance value on the periods considered in the average (Mills, 1995).

Exponential smoothing

The exponential smoothing is a scientific method of prediction based on data derived from previous experiences. It is similar to the weighted moving average in which the demand values are multiplied by a weight that decreases exponentially, moving towards the less recent values of the demand.

Exponential smoothing tends to increase the forecast when high demand levels are observed, to reduce it when low demand levels are observed. Therefore, this forecasting method updates the previous forecasts based on the last observations of the application. The analytical relationship proposed by the basic model, the simple exponential smoothing (Brown, 1962) is the following:

$$F_{t,h} = \alpha Y_t + (1 - \alpha)F_{t-1,h} \quad 0 \leq \alpha \leq 1 \quad \forall h \quad (2.14)$$

where: $F_{t,h}$ is the forecast made in the period t with forecast horizon h , therefore, for the period $t + h$, with $h = 1, 2, 3, \dots$; Y_t is the observation in the period t of the variable to be forecast.

In this forecasting method, α , called dispersion coefficient, is a parameter ($0 \leq \alpha \leq 1$) that defines the reactivity of the forecasting model. The weighting coefficient plays a role very similar to what the parameter k defines in the case of the moving average. In fact, depending on α we change the weight attributed to the last observation of the application and to the last forecast produced. High values of α allow, in the presence of a series characterized by a limited variability, to quickly correct any errors made in the forecast.

The choice of the coefficient α , as is more generally the smoothing parameters, cannot be performed once in a while, but must be updated as the observed demand changes. In this perspective the value of the coefficient during a change in demand will be increased, and the value will be reduced when the average demand is stationary and with some oscillation around its average level.

The **tracking signal (TS_t)** defined as follows is used to support the optimal choice of the α coefficient level:

$$TS_t = \alpha' \frac{e_t}{Y_t} + (1 - \alpha')TS_{t-1} \quad (2.15)$$

$$0 \leq \alpha' \leq 1 \quad -1 \leq TS_t \leq 1$$

where: α' = dispersion coefficient associated with TS_t and e_t = forecast error for the period t .

The tracking signal is practically a smoothed average of the mistakes made in the previous periods. The assumption underlying this instrument is that, if demand is relatively stable, the forecast will not be diverted, but, in the worst case, inaccurate because one is not able to grasp the fluctuations in demand around its average. For this reason, errors (smoothed) will tend to disappear and TS_t will tend to be null.

On the contrary, if the demand starts to increase (or decrease) the exponential smoothing provides conservative (optimistic) estimates of the demand. In this case the errors tend to have all the same sign and therefore, to add up rather than to cancel. Thus, the TS_t signals an increase (decrease) in the demand deviating significantly from zero. For this reason, TS_t can be used to decide when to increase α (in the case of TS_t significantly different from zero) or to reduce it (in the case where TS_t settles again around zero) (Brandimarte et al, 2004).

Exponential smoothing, like the moving average model, is designed to operate in fairly simple conditions in which demand is statistically stationary. When the change is persistent, that is when there is a continuous growth of the trend, the exponential smoothing will resist even this change and the predictions obtained with this method will always be behind the reality since the method reacts slowly to the changes.

Exponential smoothing with trend

To overcome the limitations of the model, we need to use the exponential smoothing with trend model.

However, in order to work, this method needs an initial forecast starting from which to derive the following ones, the procedure described is commonly called "initialization of the forecast". To initialize the model there are different approaches to choose $F_{t-l,h}$

1. First, you can choose to start with a forecast of zero. In this case, however, the initial level of the forecast will certainly be diverted.
2. Secondly, it is possible to assume $F_{t-l,h} = Y_{t-l+1}$, i.e. to set the first forecast equal to the first observation of the question. This approach allows to obtain an initial value of the non-deviated forecast, as described in the previous case, but which could be significantly different from the average demand since it is based on a single observation.
3. To mitigate one of the problems of the previous approach it is possible to use the average of the first l periods to initialize the forecast:

$$F_{t-l,h} = \sum_{i=t-l+1}^{t-l+l} \frac{Y_i}{l} \quad (2.16)$$

In this case the initialization is based on l periods rather than on one and, therefore, better represents the average of the demand.

In the case of a continuous and persistent variation it is necessary to correct the method to mitigate the phase shift between reality and forecast. Simple exponential smoothing, in fact, is not able to capture the trend in a historical series and for this reason it is extended to incorporate the trend component T_t .

In this model it is necessary to estimate two parameters and not, as in the previous case, one because the underlying hypothesis about the behavior of the demand is more complex. The parameters that need estimation are respectively:

- B_t , the basic level reached by the demand at period t (smooth average of the demand).
- T_t , the level of the growth or decrease trend that demand has reached in the t period.

This new model, smoothing with linear tendency, (Brown, 1962) is defined by the following relation:

$$F_{t,h} = B_t + hT_t \quad (2.17)$$

Essentially, the expected demand for the $t + h$ period is equal to the base reached in the period t (B_t) plus h times the growth in demand expected in a single period (T_t).

The logic that uses this model to update the two parameters that characterize it is defined by smoothing. For the coefficient B_t , as in the previous case, to update the previous estimates we use the last demand observation. Formally, the following formula is used to calculate the coefficient B_t :

$$B_t = \alpha Y_t + (1 - \alpha)(B_{t-1} + T_{t-1}) \quad 0 \leq \alpha \leq 1 \quad (2.18)$$

where, α is the dispersion coefficient, which determines the reactivity of the model.

Instead, to calculate the trend coefficient the last value of the observed trend T_{t-1} is updated with the last observation of the growth level (or decrease) of the variable to be predicted, ie $B_t - B_{t-1}$:

$$T_t = \beta(B_t - B_{t-1}) + (1 - \beta)(T_{t-1}) \quad 0 \leq \beta \leq 1 \quad (2.19)$$

where, β is a second smoothing coefficient associated with the trend.

The smoothing coefficient of the trend makes it possible to decouple the speed with which B_t and T_t are updated. The consequences of choosing a high rather than a low coefficient value are almost similar to those previously discussed for the coefficient α . The value of the trend factor will depend on both coefficients described.

Indeed, with a low α value, the difference between $B_t - B_{t-1}$ tend to be equal to T_{t-1} and the trend factor tend to evolve slowly. This model, as already mentioned above, can manage stationary demand conditions.

However, in order to work, this method needs an initial forecast starting from which to derive the following ones, the procedure described is commonly called "initialization of the forecast". The following approaches exist to initialize the model (Brandimarte et al, 2004):

1. First, it is possible to exploit the linear regression (described in paragraph 2.3.1) which, through interpolation, is able to highlight the parameters a and b of a line $y = a + bt$. The two estimated parameters can be used to initialize the base and the trend factor at instant zero, $B_0 = a$ and $T_0 = b$
2. Secondly, it is possible to use methods that only analyze the average level of demand and the average of the trends observed during the first l periods. It is possible to initialize the level of the trend by putting it equal to the average of the increments observed:

$$T_0 = \frac{Y_l - Y_1}{l - 1} \quad (2.20)$$

This method does not exploit all the information contained in the l observations of the application and limits itself to using the first and the last of these.

After initializing T_0 it is possible to initialize the base as follows:

$$B_0 = \frac{\sum_{i=1}^l (Y_i - i \cdot T_0)}{l} \quad (2.21)$$

However, despite the capacity described, the method has some limitations. With the increase in the forecast horizon, the model increases its sensitivity with respect to a possible error in the estimation of the trend factor. The assumption underlying the model, for which the trend observed in the past will continue also into the future, could be put into crisis when the market should present trend reversals.

Furthermore, using the above model can sometimes lead to incorrect forecasts using a fully linear demand forecast. To overcome this problem, a model is used in which the trend is modeled as a multiplicative factor. For completeness, the multiplicative exponential smoothing model is formally illustrated (Brandimarte et al, 2004):

$$F_{t,h} = B_t(T_t)^h \quad (2.22)$$

$$B_t = \alpha(Y_t) + (1 - \alpha)B_{t-1} \cdot T_{t-1} \quad (2.23)$$

$$T_t = \beta \frac{B_t}{B_{t-1}} + (1 - \beta)T_{t-1} \quad (2.24)$$

Exponential smoothing with seasonality

As highlighted above, a second cause of non-stationarity in the demand is seasonal fluctuation. In many sectors the trend is stable, but consumption changes from period to period due to seasonal variations. The effect of seasonal fluctuation can be modeled using a coefficient that attributes a multiplicative factor.

In adapting the forecasting model, it is necessary, at first, to identify the periodicity to be analyzed, ie the extension of the most important "season" for the purpose being pursued. It should be emphasized that the choice of the correct duration of the season can be reinforced by the preliminary analysis of the historical series data.

The parameters that qualify the model with seasonality are the average level of demand B_t and the factor that identifies the season S_t . The average level of demand will be the only parameter that will change over time based on the progress of the series. For the seasonality factor, there will be as many factors as the number of periods making up a season. The seasonality factor could take on also a monthly value, in order to capture the fluctuations that occur in a given period of the year. The model of demand behavior identified by this forecasting method is the following:

$$F_{t,h} = B_t \cdot S_{t+h-s} \quad \text{for} \quad h \leq s \quad (2.25)$$

or more generally, to include the possibility of foreseeing a horizon that exceeds a single "season":

$$F_{t,h} = B_t \cdot S_{t+h-\lceil \frac{h-1}{s} + 1 \rceil \cdot s} \quad (2.26)$$

In practice, the forecast made at period t for the period $t + h$, on the one hand takes into account the most recent estimate of the average level of demand B_t and, on the other, applies a suitable seasonal factor for this period $t + h$.

To better understand the average demand trend, it is mandatory to update the estimate made for the demand level B_{t-1} , with more recent observation of the variable to be forecast, adjusted for the "non-stationary causes". Formally it can be expressed as follows:

$$B_t = \alpha \frac{Y_t}{S_{t-s}} + (1 - \alpha)B_{t-1} \quad 0 \leq \alpha \leq 1 \quad (2.27)$$

The seasonality factor tries to guess how much the expected demand of a specific month is lower or higher than the average variable. In order to update the estimate of the seasonal factor it is necessary to compare the last observation of demand Y_t with the average trend of the same B_t . This factor can be calculated as follows:

$$S_t = \gamma \frac{Y_t}{B_t} + (1 - \gamma)S_{t-s} \quad 0 \leq \gamma \leq 1 \quad (2.28)$$

where γ is a smoothing coefficient.

As previously described, the coefficients α and γ define the speed with which the factors B_t and S_t are updated conditioning, on the one hand, the ability to react to changes and, on the other, the ability to filter out the noises encountered.

Also this forecasting method is recursive and, therefore, must be initialized to be used. In this case, initialization requires the estimation of s factors seasonality and a basic level. Thus, this forecasting method can never be used if no information is available on at least one entire demand season.

To better apply the method, it is advisable to use a number of periods l which we assume to be a multiple of s so that we can use "whole seasons". To initialize B_0 it is possible to use the simple average of the question observations because the effects of seasonality will not be perceived in any way by adding l/s seasons.

It will therefore be possible to initialize the seasonality of a given month considering the ratio between the average demand of all the months of the same period determined included in the sample l used for initialization and the average level of demand B_0 :

$$B_0 = \frac{\sum_{i=1}^l Y_i}{l} \quad (2.29)$$

$$S_{j-s} = \frac{\sum_{k=0}^{l/s-1} Y_{j+ks}}{B_0 \cdot l/s} \quad (2.30)$$

The forecasting method described uses a large set of parameters to estimate and requires an equally large set of data, which could lead to distortions since to analyze the time series we are forced to consider moments of time that could be remote and, therefore, of little significance to predict the current seasonality (Brandimarte et al, 2004).

Exponential smoothing with trend and seasonality

In addition to considering the factors of trend and seasonality as exclusive elements, it is possible to develop a model defined as "smoothing with trend and seasonality" that themjointly considers (Winters, 1960).

Formally modeled in the following mode:

$$F_{t,h} = (B_t + hT_t)S_{t+h-s \cdot [\frac{h-1}{s+1}]} \quad (2.31)$$

In other words, the growth (or decrease) that is expected during the h periods following the level reached by the B_t base is added, achieving the value that would be convenient to expect at the $t + h$ period in the absence of seasonality. To reach a punctual forecast of demand in the $t + h$ period, seasonality is considered as a multiplicative factor that makes it possible to estimate whether, in the period considered, it would be convenient to expect a demand lower or higher than the general trend.

In this model it is necessary to estimate as many as three parameters B_t , T_t and S_t . To identify the average trend of the demand we must combine the considerations expressed for the two methods previously illustrated, since we must both seasonally adjust the question, and add the trend factor T_{t-1} :

$$B_t = \alpha \frac{Y_t}{S_{t-s}} + (1 - \alpha)(B_{t-1} + T_{t-1}) \quad 0 \leq \alpha \leq 1 \quad (2.32)$$

Instead, as regards the factors of trend and seasonality, it is possible to take advantage of exactly the same equations described in the previous models.

This forecasting method is also recursive and must be initialized. It takes $s + 1$ periods to initialize it because to be able to estimate the trend it is necessary to compare periods characterized by the same seasonality. If only this minimum set of information is available, the smoothing can be initialized as follows:

$$T_0 = \frac{Y_{s+1} - Y_1}{s} \quad (2.33)$$

Analyzing the difference between the only two comparable observations regarding their seasonality.

This methodology poses a significant problem: T_0 is affected by seasonality, since both observation Y_1 and observation Y_{s+1} are affected by seasonality, consequently their difference is also affected by it. If, instead, you have $2s$ periods available to calculate the initial trend the following formulation is used:

$$T_0 = \frac{1}{s^2} \sum_{i=1}^s (Y_{s+i} - Y_i) \quad (2.34)$$

Using this estimate of the initial trend, the effect of the trend is separated from the observations of the demand, deriving both the basis and the seasonal factors. If you have used $s + 1$ demand observations to initialize the parameters, you can estimate the base as follows:

$$B_0 = \frac{\frac{[(Y_{s+1} - (s + 1)T_0) + (Y_1 - T_0)]}{2} + \sum_{i=2}^s (Y_i - iT_0)}{s} \quad (2.35)$$

In this case it is first necessary to remove the trend factor from all the $s + 1$ question observations. Subsequently, it is necessary to calculate the average demand in each period of the season and finally it is possible to calculate the average demand which allows to obtain a base value adjusted for both trends and seasonality.

In the case in which instead $l = k \cdot s$ observations are used (with k integer positive) for the initialization, this problem is avoided because, considering entire seasons, the effect of seasonality is mediated and, therefore, it is possible to use the following formulation:

$$B_0 = \frac{\sum_{i=1}^l (Y_i - iT_0)}{l} \quad (2.36)$$

The seasonality factors will instead be calculated as the average of the demand observations, without the trend influence, which share the same seasonality, divided by the base B_0 . Thus, distinguishing the cases of $l = s + 1$ and the multiple of s :

$$S_{j-s} = \frac{\sum_{k=0}^{l/s-1} (Y_{j+ks} - (j + ks)T_0)}{2B_0} \quad \text{for } j = 1 \text{ e } l = s + 1 \quad (2.37)$$

$$S_{j-s} = \frac{(Y_j - jT_0)}{B_0} \quad \text{for } j \neq 1 \text{ e } l = s + 1 \quad (2.38)$$

$$S_{j-s} = \frac{\sum_{k=0}^{l/s-1} (Y_{j+ks} - (j + ks)T_0)}{l/s \cdot B_0} \quad \text{for } j = 1, \dots, s \text{ and } l \text{ multiple of } s \quad (2.39)$$

Finally, as regards both the choice of parameters and the limits of this model, obviously, the considerations made for the trend damping model and the seasonally based one are valid.

2.4 Analysis of the forecasting error

To measure the performance of the forecasting process it is necessary to understand which is the nature of the forecast made. The forecasts are modeled on the basis of known information, relating to the actual conditions of the general state of the company, of the sector in which it operates. It is necessary to verify the accuracy of the estimate based on the comparison of the results had with the real ones, going to analyze the obtained errors (Brandimarte et al, 2004).

These differences, which represent the variance or the margin of error between actual and expected, can derive from an imperfect forecasting procedure, from unpredictable disturbing events, from factors internal or external to the company that have influenced sales and developments. Activating a periodic procedure that checks and rectifies the forecasting process is of fundamental importance.

The procedure that can be activated can be divided into a series of steps which can be summarized as follows:

1. Review and periodic check of the actual demand trend, for example weekly, monthly, quarterly, etc. in relation to the demand considered.
2. Annual control of forecasts, implementing a revision process of forecasts of a longer time horizon;

3. Review of the model adopted by the organization through a systematic control of its goodness to represent the future demand of the sector. The purpose of the revision of the forecast models, in particular the short-term ones, is to understand whether, with current information, the model adopted still manages to have excellent performance.

It could indeed happen that, due to disturbing factors, the trend shown suddenly changes, generating significant deviations between expectations and real developments in demand. For this reason, it is necessary to activate a systematic control process, rectifying the model in use.

In organizations, the forecasting process and the planning and control process should be a joint and continuous procedure. It is therefore necessary to analyze the deviations that have occurred to determine whether these differences fall within the tolerance margins deemed acceptable by management and of natural occurrence or unusual.

Uncertainty in the future makes it more important than ever for the company to carry out an effective and periodic control of sales and their relationships with environmental events that are more capable of generating a disturbance.

It is therefore entirely physiological to observe differences between real and expected demand, but it is advisable to assess whether these differences are those expected in the implantation phase. The analyst's task is to control that the data remains within a control interval that represents the possible error gap.

The identification of the deviations is carried out by graphically projecting the dispersion diagrams of the historical series of sales and expected data. This projection facilitates the analysis and the arithmetic measurement of the variance, ie of the margins of error between the real results and those expected, and the temporal phases in which the deviation between the data has occurred or increased. The detection of the margin of error is carried out both in absolute and relative terms, in order to more accurately control the weight and the sense of the variation (Brandimarte et al, 2004).

2.4.1 Performance and accuracy indicators

The various forecast models are able to represent the expected sales trend with different degrees of accuracy and reliability. Therefore, performance indicators are introduced to assess the soundness of the forecast (Chase et al, 2012).

A first indicator of performance is simply the average of forecast errors, is the **Mean Error (ME)**

$$ME = \frac{1}{n} \sum_{t=1}^n e_t \quad (2.40)$$

where: $e_t = Y_t - F_t$ and n = periods of the sample test

This is an indicator of deviance of the forecast that limits itself to measuring if, on average, the forecast underestimates or overestimates the real demand. Furthermore, this indicator tends to eliminate positive and negative errors. It is therefore necessary to add accuracy indicators, which differ from the ME since positive and negative errors are added rather than offset. A first indicator of accuracy is the **MAD (Mean Absolute Deviation)** which uses the absolute value to add positive and negative values:

$$MAD = \frac{1}{n} \sum_{t=1}^n |e_t| \quad (2.41)$$

A second indicator of accuracy is the **RMSE (Root Mean Square Error)**, that is the root of the mean square error) which, unlike the MAD, raises the error squared to add errors:

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n e_t^2} \quad (2.42)$$

This indicator, using quadratic errors, tends to provide indications from which it is possible to derive directly an estimate of the variance of the demand.

Furthermore, while the MAD assigns a uniform weight to all errors, the RMSE tends to weigh more heavily the preponderant errors. In other words, this indicator will reward a forecasting method that leads to uniform errors over time rather than one that, although accurate in many periods, can sometimes cause significant errors.

The indicators discussed so far measure the forecast error with the same scale with which the demand is calculated, thus generating a possible weakness. The use of the same scale it influences the value of the indicators as it is complex to analyze the performance results obtained. In other words, with these indicators it is difficult to compare the accuracy of the forecast of products (or markets) that have substantially different levels.

To address the gaps in the indicators described, type indicators are often used relative measure of the forecast error with respect to the demand.

The indicators used are respectively the **MPE (Mean Percentage Error)** and the **MAPE (Mean Absolute Percentage Error)**, which are respectively indicators of deviation and accuracy. Formally the indicators can be expressed as follows:

$$MPE = \frac{1}{n} \sum_{t=1}^n \frac{e_t}{Y_t} \quad (2.42)$$

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|e_t|}{Y_t} \quad (2.43)$$

These indicators, compared to those previously exposed, are not influenced by the scale with which the demand is measured. It is easier to compare the soundness of the forecast of products (or markets) qualified by a different level of demand.

A weak point that limits its use consists in the non-use if the application turns out to be null; in these cases, it is impossible to calculate the percentage error committed. In addition to being unable to be used with a zero demand, even in cases where the demand is never null, there could be significant distortions in the event that the demand presents high fluctuations. It is necessary to develop indicators that consider errors made in periods of high or low demand to be the same. These, practically, go to relate the ME, the MAD and the RMSE to the average demand of the product / market:

$$ME\% = \frac{ME}{\bar{Y}} \quad (2.44)$$

$$MAD\% = \frac{MAD}{\bar{Y}} \quad (2.45)$$

$$RMSE\% = \frac{RMSE}{\bar{Y}} \quad (2.46) \quad \text{Where: } \bar{Y} = \frac{1}{n} \sum_{t=1}^n Y_t \quad (2.47)$$

These indicators are able to assess the effectiveness of forecasts in relation to average demand. However, it must be emphasized that a very variable and a little variable demand does not mean that the quality of the process is identical in both cases. In other words, it is profitable to consider not only the soundness of the forecast, but also to relate it to the intrinsic difficulty of the forecasting process. To connect the two components, we use a last indicator called the **U statistic of Theil** (or **Theil's U-Statistic**).

This indicator can be calculated using the following formula:

$$U = \sqrt{\frac{\sum_{t=1}^{n-1} \left(\frac{F_{t+1} - Y_{t+1}}{Y_t} \right)^2}{\sum_{t=1}^{n-1} \left(\frac{Y_t - Y_{t+1}}{Y_t} \right)^2}} \quad (2.48)$$

where: Y_t is the demand observation in the period t and F_t is the demand forecast

made in the period t with a forecast horizon h .

By looking at the two terms at denominator and at numerator, we can provide an interpretation of the U statistic. The term $F_{t+1} - Y_{t+1}$ represents the error made at time $t + 1$, while the term $Y_t - Y_{t+1}$ represents the error made at time $t + 1$ if a trivial forecasting method is used for which the forecast at time $t + 1$ is equal to the demand at time t (also called naive forecast). Thus, the U statistic compares the prediction error of the method in analysis with the one that a simplistic method involves.

In the case in which the prediction method adopted generates errors greater than those of the naive method, the U statistic will be greater than one. If, on the other hand, the analyzed forecasting method provides performances equal to those of the reference naive method, the U statistic will be equal to one.

Finally, if the analyzed forecasting method provides much more precise indications than those of a naive method, there will be a U statistic close to zero. Therefore, the U statistic does not evaluate the forecast error, but rather the level of the error with respect to the intrinsic "difficulties" of the demand (Brandimarte et al, 2004).

CHAPTER 3 – The Context

The models previously explained in the chapter 1 will be applied to the Eaton data, and so in this chapter a brief introduction into the Eaton world to better settle the context is made. Below it is possible to find which products do they manufacture, in which market do they operate and how do they currently make the forecasts.

3.1 Who is EATON and what it does

Eaton produces a broad range of products and services, from fuel-efficient systems, to Power Chain management tools and components that guide commercial aircraft. Eaton is always looking for new ways to increase the value of the products and services it offers the customers.

More precisely, Eaton operates in these sectors:

- **Aerospace:** Eaton is a leading producer of aerospace systems and components
- **Electrical:** Eaton is a leading producer of the world's electrical equipment and systems
- **Filtration:** Eaton combines worldwide engineering, manufacturing, technical sales support, and customer service with the optimum manufacturing and industrial filtration solutions.
- **Hydraulics:** Eaton is a leading producer of Hydraulics components and systems
- **Industrial Clutches and Brakes:** Eaton is a world's leading producer of Industrial Clutches and Brakes
- **Plastic Extrusion:** Eaton has more than 40 years of experience and expertise in plastic profile extrusion
- **Named Products:** Eaton product names such as Cutler-Hammer and Vickers are known around the world

- **Vehicle:** Products and systems that are designed to improve a vehicle's overall efficiency, performance and power

The internship experience, on which the thesis is based, has been done in Turin, the EMEA Headquarter for the Vehicle division. In this sector, the company provides automotive and commercial vehicle manufacturers worldwide with products and systems that are designed to improve a vehicle's overall efficiency, performance and power (such as emission control components, engine valves, valvetrain systems and superchargers) as well as offering driveline expertise through its transmission, clutch and torque management products and systems.

The Vehicle Group has 49 facilities located on six continents, it operates four regional technical centers in the United States, Czech Republic, China and India, and employs approximately 15,000 employees.

A quick overview of the company's product portfolio follows.

- **Transmissions:** Manual and automatic transmissions for light-, medium-, and heavy-duty vehicles
- **Supercharger Products:** These products can power passenger cars to commercial vehicles from 1-liter to 6-liter engines (Figure 3.1)



Figure 3.1 – Supercharger

- **Advanced Machining:** Eaton offers complex machining and assembly processing
- **Engine Valves, lifters and Valve Actuation:** Eaton has been one of the largest producers of both valve and valve actuation products for over 75 years (Figure 3.2)



Figure 3.2 – Valves and Valves Actuators

- **Fuel Emission Control:** As a leader in fuel emissions management, Eaton continues to develop value driven fuel venting solutions to exceed the world's most demanding emissions regulations (Figure 3.3)



Figure 3.3 – Fuel Valves

- **Integrated Powertrains:** Eaton collaborates with leading truck and engine manufacturers to develop powertrains that meet today's demanding standards (Figure 3.4)



Figure 3.4 – Integrated Powertrain

- **Plastics:** Eaton specializes in the design and manufacture of high quality, functional plastic moulded components and assemblies for use in stringent automotive engineering applications
- **Market Applications:** On-road and vocational vehicle products and solutions for OEMs and aftermarket applications
- **Resources:** Literature, tools, calculators, news and videos that support vehicle products
- **Clutches:** Eaton offers a vast range of clutches for all needs (Figure 3.5)



Figure 3.5 – Clutches

- **Differentials and Tracton Control:** Eaton performance differentials are made to maximize the wheel traction
- **Aftermarket:** Products and solutions that increase performance and extend the life of your vehicle
- **Fluid Conveyance:** Eaton is a global supplier of high-quality air conditioning, active suspension, power steering and a wide variety of other fluid connection products (Figure 3.6)



Figure 3.6 – Hoses to convey fluids

- **Hybrid Power Systems:** Eaton's hybrid power system can provide significant fuel savings and can reduce vehicle fuel emissions
- **Lubricants:** Roadranger Synthetic Products have a unique formula containing high quality raw materials and additives to offer the best heavy-duty truck drive system performance and economy
- **Service Tools:** Eaton offers a full range of diagnostic and special service tools
- **Expertise:** Eaton expertise provides customers and industry professionals with innovative products and system knowledge
- **Purchase and Support:** This support can help customer to discover where to buy products and services, warranty information, and support resources

3.2 How is it done today the demand forecasting in EATON

During this experience, the author worked close to the Italian Salas manager. The core business was valves and valve trains for all the Italian customers.

Currently, within the Sales department, the evaluation of the future customer needs is done with the Subjective Qualitative method.

In fact, Each Sales Manager, responsible for the sales of a certain Segment, (Automotive Gasoline & Diesel, Truck etc.) in a certain market (Emea, Apac, Nafta etc.) estimates future demand for its products for the next period independently. The hypothesis underlying this method, although not always true, is that the people closest to the customer know his future needs better than anyone else. This information is subsequently aggregated to obtain a global forecast for each geographical area or product family.

The main source of the data is represented by the medium / long-term planning coming from the Marketing Department. In fact, this function must be able to influence or modify the proposed projections, based on the knowledge that it has of the future trend of:

- Customer initiatives
- Scheduled promotions
- Forecast of acquisition of a large customer order
- Deadlines related to customer initiatives

- Modification of budget-related deadlines
- Changes in the macroeconomic trend

More generally, of all the information that can influence volumes and sales mix in the medium term.

This forecasting method allows marketers to focus their attention on improving the quality of forecasts by adding the value deriving from their specific knowledge on future sales.

Two other important departments for the demand forecasting in the company are: logistics and direct purchases. Thanks to their collaboration, it is possible to have visibility on the 'Long Time Business Plan' reports that contain the forecasts for the next 4 or 5 years (even though they are increasingly random with the period going away).

To all this is added the experience of the Sales Managers. In fact, he reviews the numbers and extrapolates what will be the Budget on which he commits the Company's productivity and investments and on which his results will be evaluated according to the market rumors, the technological evolution in progress, the previous sales trend, the reliability of the data obtained from the customers in the past years and mediating the estimates among the different Competitor Customers.

Finally, each Sales Manager, thanks to the aggregate picture provided by the mixture of all the projections coming from the different internal and external sources, can take the best decision on the quantities to forecast for the future years.

It is necessary to put together all the information as a lot of factors, that are not always totally visible by the Sales department, can influence the demand. Among these there are:

- **Company Factors:** such as sales data, price and promotion, service level, quality and budget (almost totally visible by the sales manager)
- **Marketplace Factors:** such as consumer perception, demographics, competition, innovation and random factors (partially visible by the sales manager)
- **Environment Factors:** such as regulation, economy, business cycle, wealth conditions (partially visible by the sales manager)

3.3 Issues of the current methodology

Associated to this way of doing forecasting there are some possible pitfalls:

1. Unexpected Occurrences

All qualitative forecasts assume that certain market characteristics that existed in the past will exist in the future. Unfortunately, during each operating period, the market can be affected positively or negatively by unanticipated occurrences. For example, the unexpected death of Sergio Marchionne happened in July 2018 completely changed the cards on the table for the company. In fact, his policy has always been oriented versus high efficient Diesel engines and all the strategic decision taken inside FCA were in line with that vision. At that time, there was no hybrid or electrical solution development; there were just some ideas for the future on that kind of vehicles. Then, all in a sudden, he had to be replaced by a new CEO that decided to maintain some of the Marchionne's strategies, but also to start exploring the electric and hybride world. For a company like EATON, as they need to be in close contact with the OEMs, it was not so easy to get the information about the future production plans in such a chaotic situation.

2. Invalid Expert Opinions

Unfortunately, if the opinion of one person, whose view prevails, is incorrect, the forecast is incorrect. In addition, the most recent operational results can overly influence individuals, who then create overly pessimistic or optimistic forecasts.

3. Forecaste Bias

A company uses qualitative forecasting techniques to attempt to approximate customer demand using "soft information," such as personal opinions. In doing so, the company analyzes previous demand patterns while making allowances for current market conditions. Unfortunately, it's difficult to eliminate the forecaster's personal bias from the data that underlies the forecast.

CHAPTER 4 – Application of Forecasting Models

In this chapter, the most common quantitative models, described in the Chapter 1, will be applied to the Eaton's available historical data. The forecasts obtained through these methodologies are then compared with the qualitative data obtained by the company. The aim is to understand if it is possible to help the Sales Managers in coping with their job thanks to the help of some statistical tools.

4.1 How is it possible to identify the best methodology

The Figure 4.1 represents how does it work the forecasting technique. Infact, there is a first period with the Effective Demand that is used to settle the model (D_t). The second component is the forecast (the future values sought, P_{t+m}) done over a forecasting horizon (m).

Symbology Adopted:

- EFFECTIVE DEMAND for the period t : D_t
- FORECAST made at the end of the period t for the period $t+m$: P_{t+m}
- FORECAST HORIZON: m

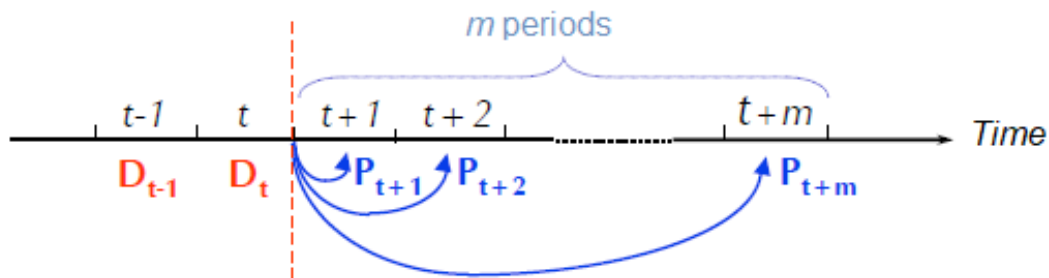


Figure 4.1 – Effective Demande & Forecast over time - symbology (Dallari 2009)

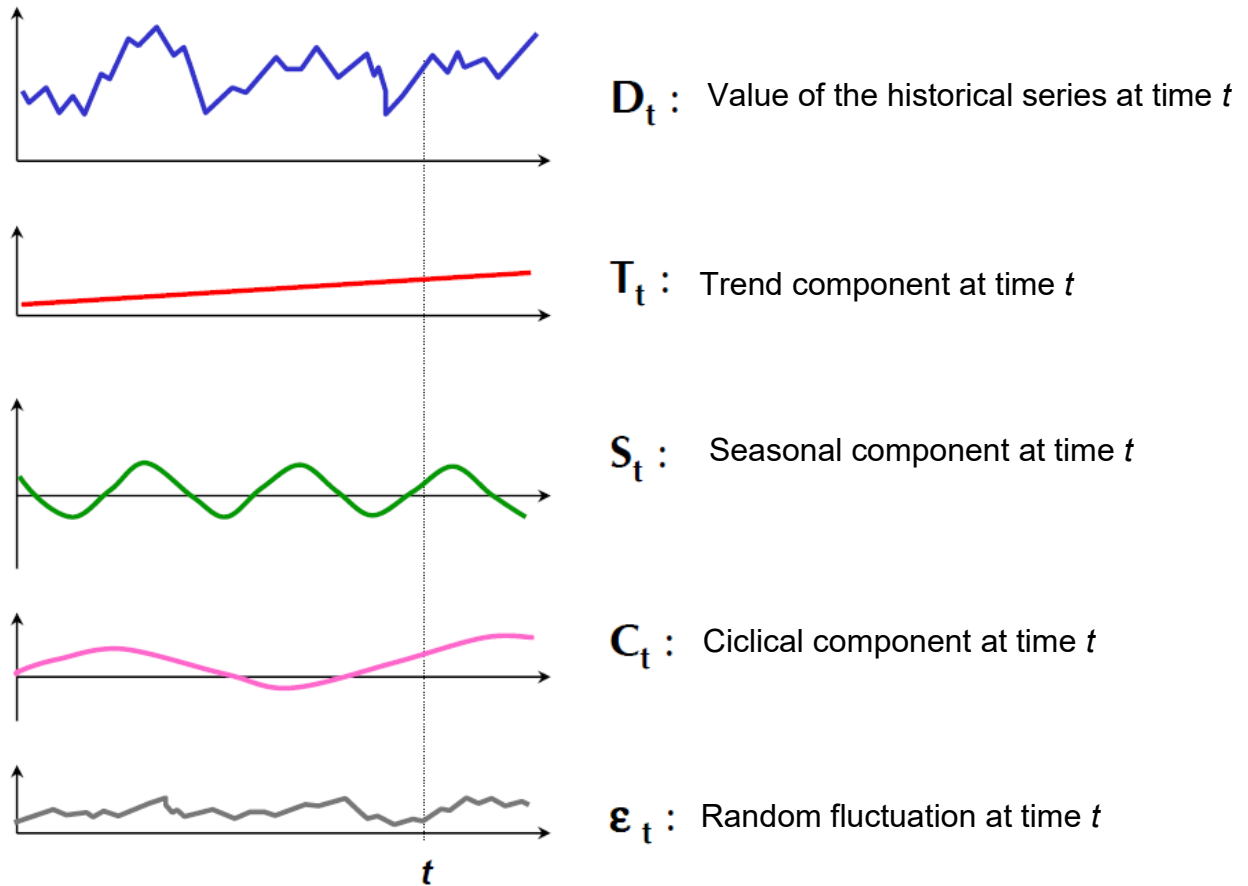


Figure 4.2 - Components of a Historical Series at time “ t ” (Dallari 2009)

In Figure 4.2 it is possible to find a graphical representation of the different components that are possible to find in analysing historical data.

The equation 4.1 shows how the different components of the historical analysis are linked together. Infact, the value of the historical time series in the instant t is a function of the Trend at the time t , of the Season at the time t and of the Cyclicity always at the instant t with the addition of the Random component.

$$D_t = f(T_t, S_t, C_t) + \epsilon_t \quad (4.1)$$

Systematic Components
Random Component

Before formulating the sales forecasts, it is necessary to analyse the past trend of the historical series in order to identify the existence of any Trend or Seasonality components.

Looking at the R^2 value derived from the Trend analysis, it is possible to make the first cut of the methodologies to apply. In fact, in case R^2 is higher than 0,5 only a couple of methods can be applied: the Double Exponential Smoothing (called in Chapter 2 Exponential Smoothing with Trend) and the Trend Forecast (meaning to forecast future data based on historical values through the trend line). It could be interesting to check, in case the R^2 value is close to 0,5 (for example 0,6 or 0,7), also the Single Exponential Smoothing methodology (called in Chapter 2 Exponential Smoothing). It could be the case that, due to the fact that the trend component is not so highlighted, this last technique is the best to approximate the future trends.

In case the R^2 is between 0 and 0,5, the best methodology has to be selected among the Moving Average, the Weighted Moving average and the Single Exponential Smoothing.

Once it has been identified the right methodology to apply, to select which will be the best one it is important to look at the **Standard Deviation (σ)** value obtained with the following formula:

$$\sigma_k = \sqrt{\frac{\sum_{j=k+1}^t (P_j - X_j)^2}{t - (k + 1)}} \quad (4.2)$$

Where: P_j is the forecasted demand for the period j , X_j is the effective demand in the period j , t is the number of real data available and k is the number of real data available not considered for creating the model.

The method that can follow at best the trend of the past will have the lowest standard deviation values and so, it will be the best to represent the series also in the future.

Finally, there is one more methodology that can be used to do forecasting. This will be illustrated here below, but it will not be applied: the **DECOMPOSITION METHOD**. This is used to identify the main components in which a time series can be divided. First of all, it requires to identify the model of representation of the historical series:

1. Additive: **$D_t = T_p + S_t + C_t + \varepsilon_t$**

2. Multiplicative: $D_t = T_p \cdot S_t \cdot C_t \cdot \varepsilon_t$

More precisely, this second technique requires unbundling one at a time the main components of the historical series following this procedure:

- Determination of the joint component of trends and cyclicality by calculating the moving average MM_t :

$$T_t \cdot C_t \approx MM_t$$

- Determination of the seasonal component through the calculation of seasonality coefficients:

$$S_t \cdot \varepsilon_t = \frac{D_t}{T_t C_t} \approx \frac{D_t}{MM_t}$$

- Purification of the seasonality component of the effect of random ε_t fluctuations as an average of the $S_t \cdot \varepsilon_t$ values over the different seasons
- Seasonal adjustment of the time series obtained by dividing each value of the series by the corresponding seasonal coefficient:

$$\frac{D_t}{S_t} = T_t \cdot C_t \cdot \varepsilon_t$$

- Determination of the trend component through the identification of a regression curve (for example linear) of the seasonally adjusted values of the series as a function of time:

$$T_t = a + b + t$$

- determination of cyclical factors through the removal of the historical series of the components of seasonality and causality (through the moving average) and of the trend component (through the regression):

$$C_t = \frac{MM_t}{T_t}$$

Moreover, the indicator used to represent the forecast accuracy is the **RMSE** :

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n e_t^2}$$

The Root Mean Square Error (RMSE) is the standard deviation of the residuals (prediction errors). Residuals are a measure of how far from the regression line data points are and so RMSE is an indicator of how spread out these residuals are. In other words, it shows how concentrated the data are around the line of best fit.

Furthermore, the RMSE tends to weigh more heavily the preponderant errors. In other words, this indicator will reward a forecasting method that leads to uniform errors over time rather than one that, although accurate in many periods, can sometimes cause significant errors.

4.2 World – All car manufacturers

4.2.1 On which data the historical series is based

First of all, it is important to stress the fact that, since Eaton is a company that produces components mainly related with the engines, the main metric used for all the analysis reported below is the number of engines, independently from the car model on which these engines are or will be installed. Instead, a strong distinction is related to the type of fuel used. Infact, changing the propellant has a lot of implications in terms of raw materials used and of physical conformation of the components.

To perform the study, two different macro areas have been considered: the entire world and EMEA. This has been done mainly to understand if it is possible to apply the same forecasting methodologies independently on the area considered or if there are some specific characteristics that impose to adopt a different technique according to each macro area. Finally, talking about car manufacturers, two analysis

have been performed: one including all the brands and another including only FCA data.

A first big distinction has been made for all the areas analysed. In fact, only the trends for gasoline and diesel were considered. This is mainly due to the fact that there were more data to support the analysis and because these two engine typologies still represent the most relevant source of revenues for the largest part of the car manufacturers.

The first historical series considered is the most comprehensive one: all the car manufacturers in all the world. This first analysis can be useful to understand in which direction the entire world is moving. It is possible to find a summary table of the data considered for the hystorical analysis in Table 4.1.

The historical data used for the analysis start in 2010 and have been updated annually. The expected forecast horizon goes from 2019 to 2022.

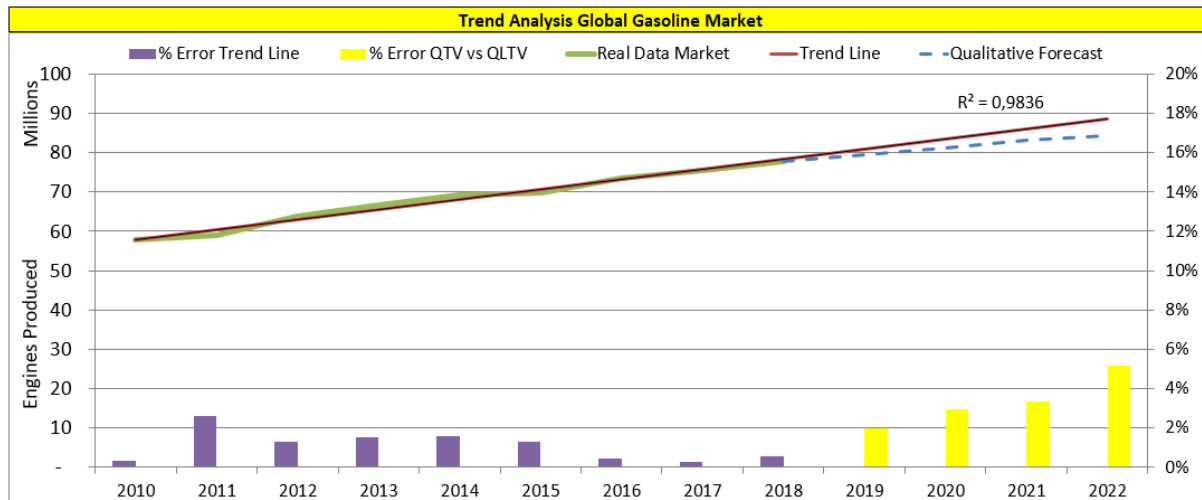
		Real Data (Unit of Global WW All Brands Engines)								
Market	Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Global Gasoline Market		57.715.766	58.895.403	63.841.987	66.568.090	69.198.237	69.761.924	73.552.727	75.563.027	77.879.101
Global Diesel Market		16.186.993	17.494.113	17.223.360	17.637.821	17.709.221	18.434.586	18.797.056	18.463.108	18.090.698

Table 4.1 - Global Engines Produced by All Brands

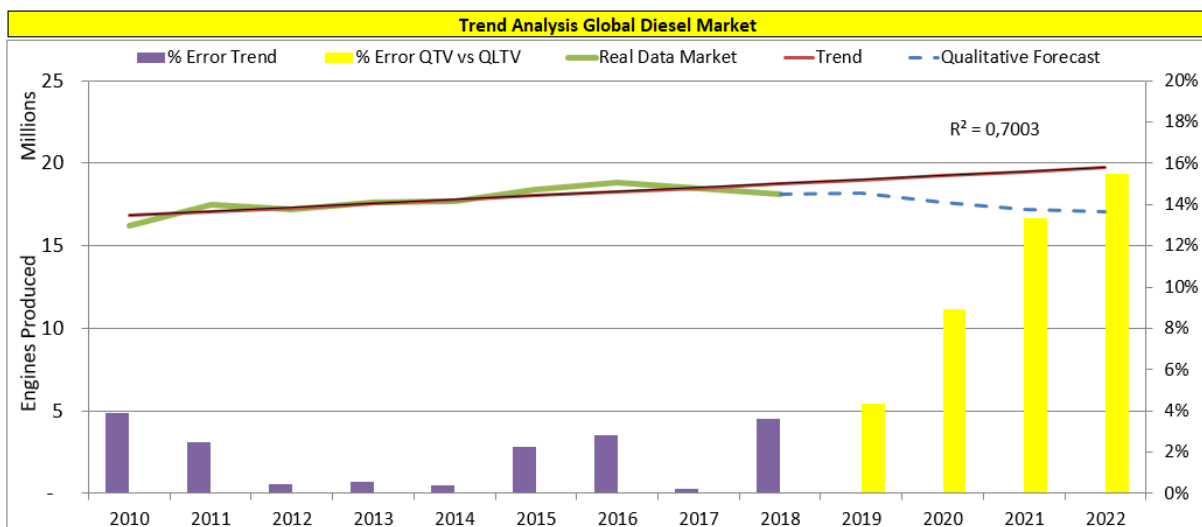
As it is possible to notice, there have always been sold more Gasoline than diesel cars all around the world.

4.2.2 How the model can be optimized

First of all, to understand which the best model to use for the forecasting was, a trend analysis has been performed. This is useful as the Holt method can only be applied if a clear trend is visible ($R^2 \geq 0,5$). Below the graphs of the study done (Appendix 4.1 for the data used to make it).



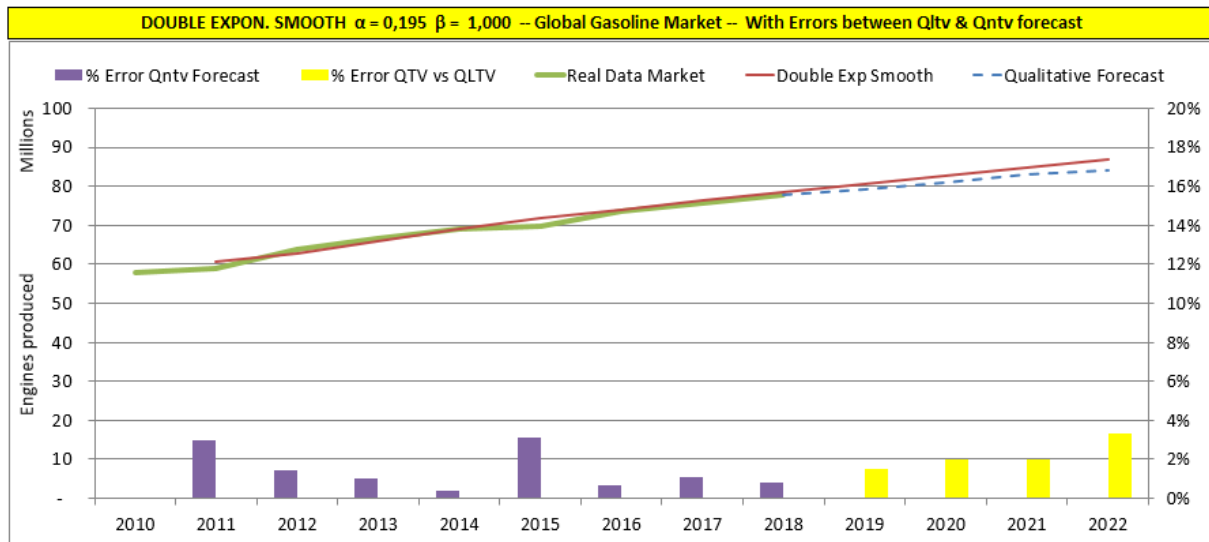
Graph 4.1 - Trend Analysis for Global Gasoline Market



Graph 4.2 – Trend Analysis for Global Diesel Market

It is possible to see that in both cases (Graph 4.1 and Graph 4.2) the R^2 is higher than 0,5 and so the choice of the best method has to be made between the Holt one and the Trend one. For the Diesel case, also the Brown method will be taken into consideration as the trend is not as clear as in the Gasoline case; we expect in fact that also this third method could lead to interesting results.

It is possible to find the summary graphs of the Gasoline cases in the Graph 4.3 and Graph 4.4 and in the Appendix 4.2 there is the table with the data used to create them.



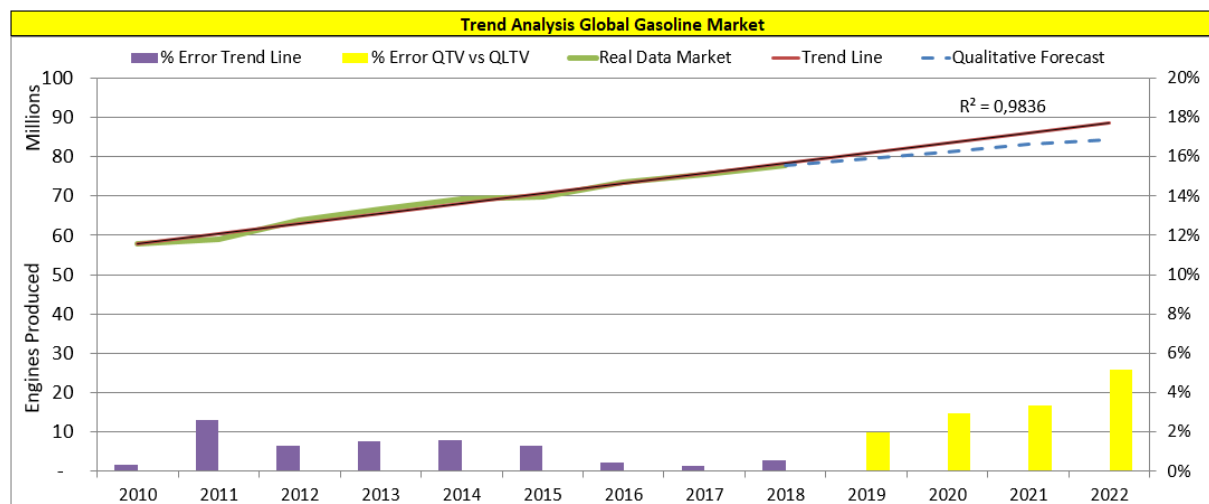
Graph 4.3 - Analysis of Global Gasoline Market with Holt Method

Summary Table - Double Exp Smoth.			
	alpha min	beta min	Sigma (Unit Engines)
Gasoline	0,19	1,00	1.236.215

Table 4.2 – Best Alpha and Beta values to minimize the Sigma

In Table 4.2 it is possible to find the coefficients of the Double Exponential Smoothing and the related Sigma.

Alpha and Beta values have been obtained thanks to the Excel solver in order to minimize the standard deviation. It is possible to see in Graph 4.3 that, as expected, this methodology can approximate pretty well the past and it is quite close to the future prediction obtained by the company with a qualitative approach.



Graph 4.4 - Analysis of Global Gasoline Market with Trend Method

In Graph 4.4, it is possible to find the forecast done with the Trend methodology. Even this second technique seems to approximate well the real demand.

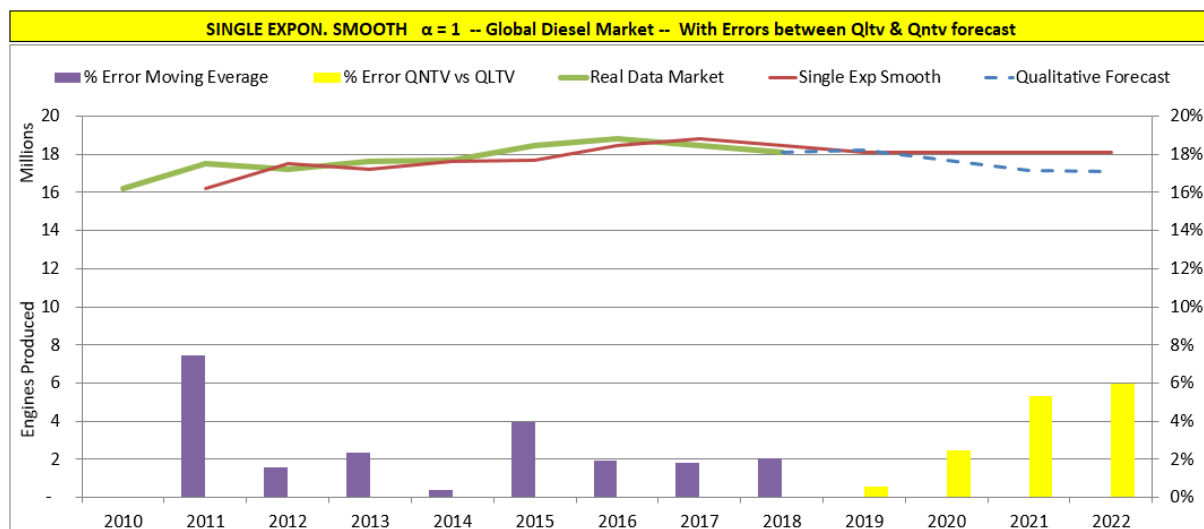
Thanks to the standard deviation comparison, it has been possible to identify the best methodology to use in Table 4.3:

Gasoline Market	Double Exp Smooth	Trend
Sigma (Unit of Engines)	1.236.215	903.163

Table 4.3 – Best method with lower Sigma for Gasoline Market

Moving now to the Diesel case, below there are reported the two graphs obtained with the Brown, Graph 4.5, and Holt methods, Graph 4.6 (Appendix 4.3 for the data used).

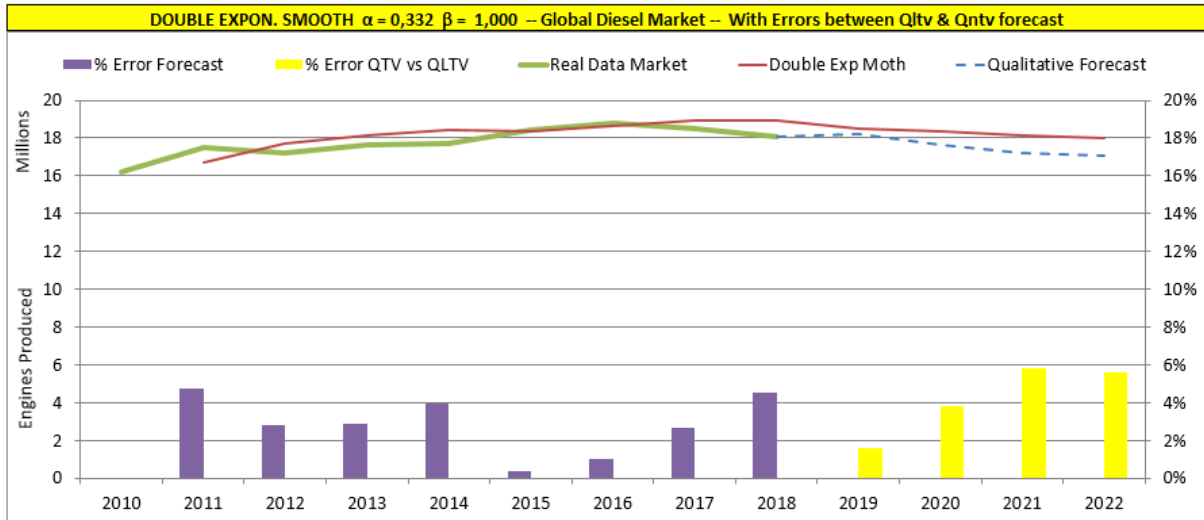
Both graphs represent the situation with the best Alpha and Beta to have the lowest Sigma possible as shown in Table 4.4 and Table 4.5.



Graph 4.5 - Analysis of Global Diesel Market with Single Exponential Smoothing Method

Summary Table - Simple Exp Smooth					
	Alpha = 0,4	Alpha = 0,45	Alpha = 0,5	Alpha=0,94	Alpha = 1
Sigma Diesel (Unit of Engines)	5.277.278	4.673.336	4.142.139	957.488	639.921

Table 4.4 – Alpha values and the related Sigma for Single Exponential Smoothing on Diesel Market

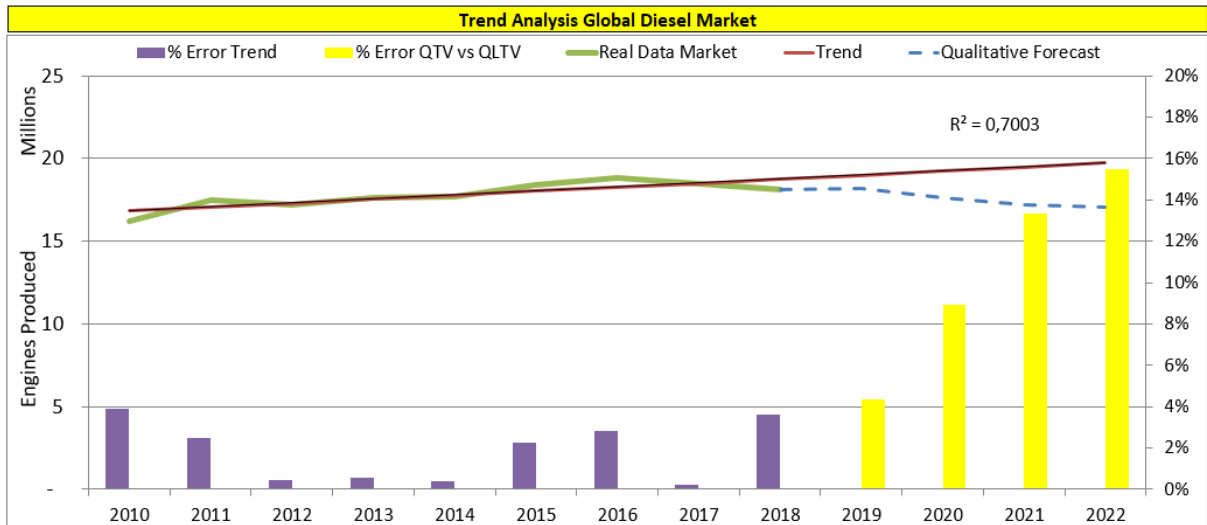


Graph 4.6 - Analysis of Global Diesel Market with Double Exponential Smoothing Method

Summary Table - Double Exp Smoth.			
	alpha min	beta min	Sigma (Unit Engines)
Diesel	0,33	1,00	608.776

Table 4.5 – Best Alpha and Beta values to minimize the Sigma

Also in this case, Alpha and Beta values of Tab 4.5 and Alpha of Tab 4.4 have been obtained thanks to the Excel solver in order to minimize the standard deviation.



Graph 4.7 - Analysis of Global Diesel Market with Trend Method

Comparing these three different methodologies, the Trend has revealed to be the most suitable to minimize the standard error (see Table 4.6).

Diesel Market	Simple Exp Smooth	Double Exp Smooth	TREND
Sigma (Unit of Engines)	639.921	608.776	431.967

Table 4.6 – Best method with lower Sigma for Diesel Market

To summarize, considering the entire world with all the car manufacturers, it is possible to see that the Gasoline market and the Diesel market are better represented by the Trend method.

4.2.3 Which information are provided by the optimized model

Looking at the results it is possible to see that the relative errors between the qualitative forecast and the values obtained by the model for the Gasoline market are lower than those for the Diesel market. This can be explicable as the two demands are quite different from each other. The Gasoline market is more stable and, in the previous years, has been continuously growing while the Diesel one, especially from 2018, has entered into a period of crisis and uncertainty. This means that it is harder for the model to react at this unexpected decrease, leading to a worse quality of the market forecast. In both cases, the relative errors are quite low (at maximum around 6%) and so these methods can be used to have a first broad picture of how the automotive world is going to evolve in the next years. This last aspect will be treated better in the next section.

4.2.4 Comparison with the data extracted with qualitative forecasting

In Table 4.7 there are reported the data for the next four years obtained by the company using the qualitative methodology for the same market examined in the paragraph 4.2.1.

		Qualitative Forecast (Unit of Global WW All Brands Engines)			
Market \ Year		2019	2020	2021	2022
Global Gasoline Market		79.333.268	81.052.648	83.188.263	84.199.814
Global Diesel Market		18.197.161	17.653.014	17.175.487	17.069.507

Table 4.7 – Qualitative data for All Brands Global Market

In this section, a deeper analysis on the differences between qualitative and quantitative forecasts is performed. Here it is possible to find the data from which the graphs related to the Gasoline market have been done.

In Table 4.8 and Table 4.9 it is possible to find the deviation from the consolidated values and from the future forecast for the Gasoline market adopting the Double Exponential Smoothing Methodology and Trend Methodology.

	Year	Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	-1.771.138	3.136.928.634.285	3,01%		
	2012	914.217	835.792.474.296	1,43%		
	2013	678.392	460.215.503.326	1,02%		
	2014	260.460	67.839.433.693	0,38%		
	2015	-2.193.278	4.810.468.607.391	3,14%		
	2016	-515.304	265.538.486.748	0,70%		
	2017	-844.161	712.607.220.603	1,12%		
	2018	-638.902	408.195.263.886	0,82%		
Forecast	2019				1.211.136	1,53%
	2020				1.642.527	2,03%
	2021				1.657.683	1,99%
	2022				2.796.903	3,32%

Table 4.8 - Gasoline Market – Double Exponential Smoothing Data

	Year	Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	-174.606	30.487.193.154	0,30%		
	2011	-1.549.494	2.400.932.448.000	2,63%		
	2012	842.564	709.914.618.358	1,32%		
	2013	1.014.142	1.028.483.748.263	1,52%		
	2014	1.089.763	1.187.584.364.847	1,57%		
	2015	-901.075	811.936.135.601	1,29%		
	2016	335.203	112.360.768.149	0,46%		
	2017	-209.023	43.690.554.145	0,28%		
	2018	-447.474	200.233.239.217	0,57%		
Forecast	2019				1.547.833	1,95%
	2020				2.382.978	2,94%
	2021				2.801.889	3,37%
	2022				4.344.863	5,16%

Table 4.9 - Gasoline Market – Trend

The column % Error represents the distance in % of the real data from the forecasted ones. Instead, the column % Error Qtv vs Qltv represents how close is the prevision performed by the model adopted to the qualitative forecast made by the company. As already said, the errors are very low in these cases. The Trend model wins against the others, as, on average, it is able to better approximate the real data from 2010 to 2018.

Moving to the Diesel market, the tables reported below refer to the Brown (Table 4.10), Holt (Table 4.11) and Trend (Table 4.12) methods respectively

	Year	Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	1.307.120	1.708.562.694.400	7,47%		
	2012	-270.753	73.307.187.009	1,57%		
	2013	414.461	171.777.920.521	2,35%		
	2014	71.400	5.097.960.000	0,40%		
	2015	725.365	526.154.383.225	3,93%		
	2016	362.470	131.384.500.900	1,93%		
	2017	-333.948	111.521.266.704	1,81%		
	2018	-372.410	138.689.208.100	2,06%		
Forecast	2019				106.463	0,59%
	2020				437.684	2,48%
	2021				915.211	5,33%
	2022				1.021.191	5,98%

Table 4.10 - Diesel Market – Single Exponential Smoothing Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	823.511	678.169.818.114	4,71%		
	2012	-477.988	228.472.172.789	2,78%		
	2013	-503.132	253.141.395.656	2,85%		
	2014	-695.845	484.200.073.815	3,93%		
	2015	60.583	3.670.352.495	0,33%		
	2016	182.708	33.382.096.040	0,97%		
	2017	-492.850	242.901.520.172	2,67%		
	2018	-818.728	670.316.188.550	4,53%		
Forecast	2019				285.498	1,57%
	2020				674.858	3,82%
	2021				997.597	5,81%
	2022				948.789	5,56%

Table 4.11 - Diesel Market – Double Exponential Smoothing Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	-630.494	397.522.095.575	3,90%		
	2011	435.527	189.683.854.834	2,49%		
	2012	-76.325	5.825.546.332	0,44%		
	2013	97.036	9.416.056.456	0,55%		
	2014	-72.663	5.279.911.569	0,41%		
	2015	411.603	169.416.727.767	2,23%		
	2016	532.973	284.060.502.981	2,84%		
	2017	-42.074	1.770.229.891	0,23%		
	2018	-655.583	429.789.681.767	3,62%		
Forecast	2019				790.220	4,34%
	2020				1.575.466	8,92%
	2021				2.294.093	13,36%
	2022				2.641.172	15,47%

Table 4.12 - Diesel Market – Trend Data

As it is possible to see, the relative errors are higher but still acceptable. As already mentioned before, also in this case the Trend method wins thanks to a lower Sigma.

4.2.5 Is it the model able to correctly interpret the past and so can be used for the forecasting activity?

For the Gasoline and Diesel markets it has been possible to find a good method able to follow the past and to estimate the future precisely. Looking at the company perspective, this typology of analysis is useful to understand if it is the case to change something in the product portfolio in order to follow the future trends or if it is better to keep on investing in the actual products as it is likely to see, also in the future, a demand growth.

4.2.6 Considerations

In the specific case, for the Gasoline market there is no doubt that it will grow and so the actual offer has to be kept as it is with some adjustments based on the new engine development. The first alarm bell should ring looking at the Diesel trend. It seems that something is going to change in the future for this market all around the globe. This is the reason why a deeper analysis on the European market has been done. In the last years, EMEA has been the region most affected by the strict Diesel regulations and so some very interesting changes in that area should be expected.

4.2.7 Comparative tables

Below it is possible to find some synthetic tables to compare all the methods used for the analysis. In Table 4.13, it is possible to see the Best Sigma for all the methodologies adopted. Instead, in Table 4.14 the Gasoline forecasts with the Double Exponential Smoothing and the Trend methodologies are reported and compared with the qualitative numbers. Finally, in Table 4.15 the forecasts for the Diesel market with Single Exponential Smoothing, Double Exponential Smoothing and Trend methodologies are shown and again compared with the qualitative numbers.

Comparative Methods Table - World All Brands			
	Simple Exp Smooth	Double Exp Smooth	Trend
Sigma Gasoline (Unit Engines)		1.236.215	903.163
Sigma Diesel (Unit Engines)	639.921	608.776	431.967

Table 4.13 – Best Sigma for Gasoline and Diesel Global Market

	Year	Global Gasoline Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Double Exponential Smoth (Unit of Engines)	Trend (Unit of Engines)
Real Data Market	2010	57.715.766	77.879.101		57.890.372
	2011	58.895.403		60.666.541	60.444.897
	2012	63.841.987		62.927.770	62.999.423
	2013	66.568.090		65.889.698	65.553.948
	2014	69.198.237		68.937.777	68.108.474
	2015	69.761.924		71.955.202	70.662.999
	2016	73.552.727		74.068.031	73.217.524
	2017	75.563.027		76.407.188	75.772.050
	2018	77.879.101		78.518.003	78.326.575
Forecast	2019		79.333.268	80.544.404	80.881.101
	2020		81.052.648	82.695.175	83.435.626
	2021		83.188.263	84.845.946	85.990.152
	2022		84.199.814	86.996.717	88.544.677

Table 4.14 – All Methods used for All Brands Global Gasoline Market vs Qualitative Forecast

	Year	Global Diesel Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Single Exponential Smoth (Unit of Engines)	Double Exponential Smoth (Unit of Engines)	Trend (Unit of Engines)
Real Data Market	2010	16.186.993	18.090.698			16.817.487
	2011	17.494.113		15.196.976	16.670.602	17.058.586
	2012	17.223.360		17.353.617	17.701.348	17.299.685
	2013	17.637.821		17.231.327	18.140.953	17.540.785
	2014	17.709.221		17.612.959	18.405.066	17.781.884
	2015	18.434.586		17.703.334	18.374.003	18.022.983
	2016	18.797.056		18.389.862	18.614.348	18.264.083
	2017	18.463.108		18.772.151	18.955.958	18.505.182
	2018	18.090.698		18.482.009	18.909.426	18.746.281
Forecast	2019		18.197.161	18.114.631	18.482.659	18.987.381
	2020		17.653.014	18.114.631	18.327.872	19.228.480
	2021		17.175.487	18.114.631	18.173.084	19.469.580
	2022		17.069.507	18.114.631	18.018.296	19.710.679

Table 4.15 – All Methods used for All Brands Global Diesel Market vs Qualitative Forecast

4.3 EMEA – All car manufacturers

4.3.1 On which data the historical series is based

As anticipated before, in this second part of the analysis all car manufacturers have been included, but the focus is shifted on the EMEA market.

Also in this case, the big distinction between Gasoline and Diesel engines has been maintained for the reasons mentioned before.

This second analysis can be useful to understand in which direction the EMEA market is moving. Before to start to dig into the data, it is already possible to expect a decrease in the demand of Diesel engines due to the strict regulation on CO₂ emissions imposed by the European Union. In Table 4.16 it is possible to find a summary of the data considered for the historical analysis.

The historical data used for the analysis start in 2010 and have been updated annually. The expected forecast horizon goes from 2019 to 2022.

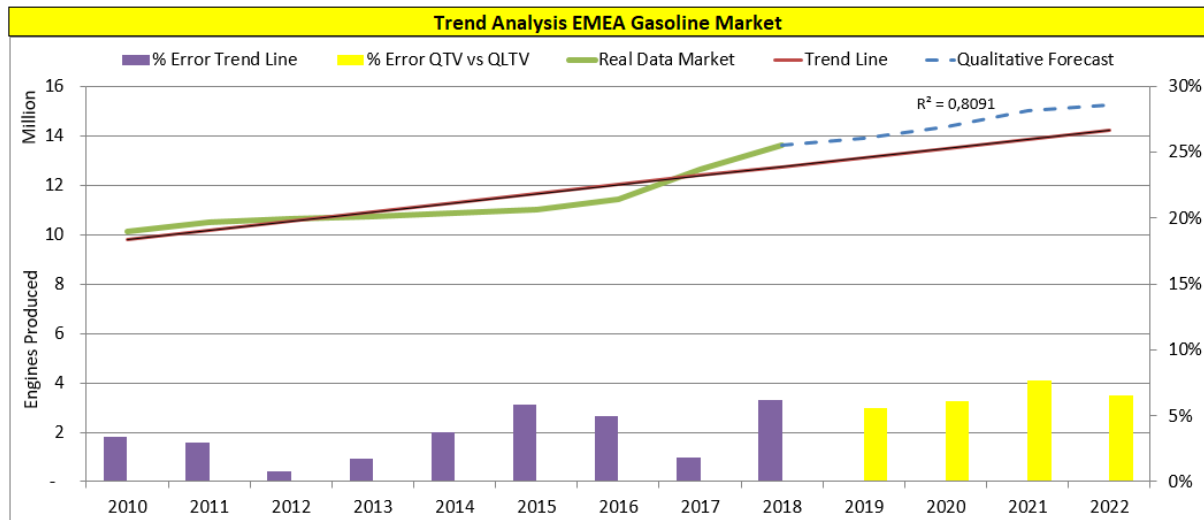
		Real Data (Unit of EMEA All Brands Engines)								
Market	Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
EMEA Gasoline Market		10.138.834	10.474.762	10.614.814	10.717.087	10.868.503	10.997.017	11.439.617	12.616.441	13.598.381
EMEA Diesel Market		8.885.008	9.694.389	8.658.131	8.743.088	9.212.051	9.880.366	9.991.764	9.477.686	8.958.582

Table 4.16 - EMEA Engines Produced by All Brands

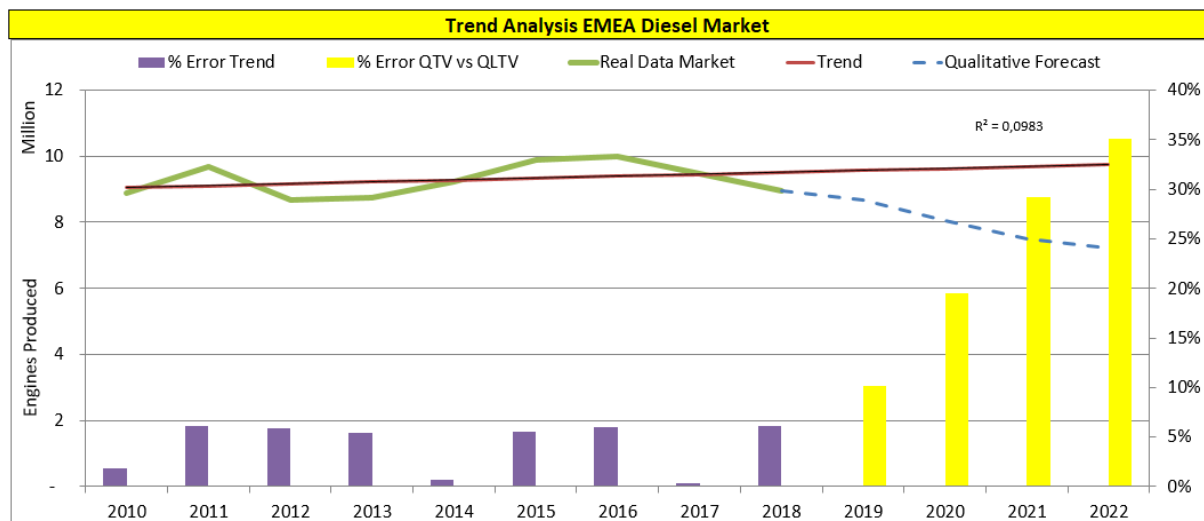
As it is possible to notice, also in EMEA there have always been sold more Gasoline cars.

4.3.2 How the model can be optimized

First of all, to understand which the best model to use for the forecasting was, also in this case, a trend analysis has been performed. This is useful as the Holt method can only be applied if a clear trend is visible ($R^2 \geq 0,5$). Below the graphs of the study done.



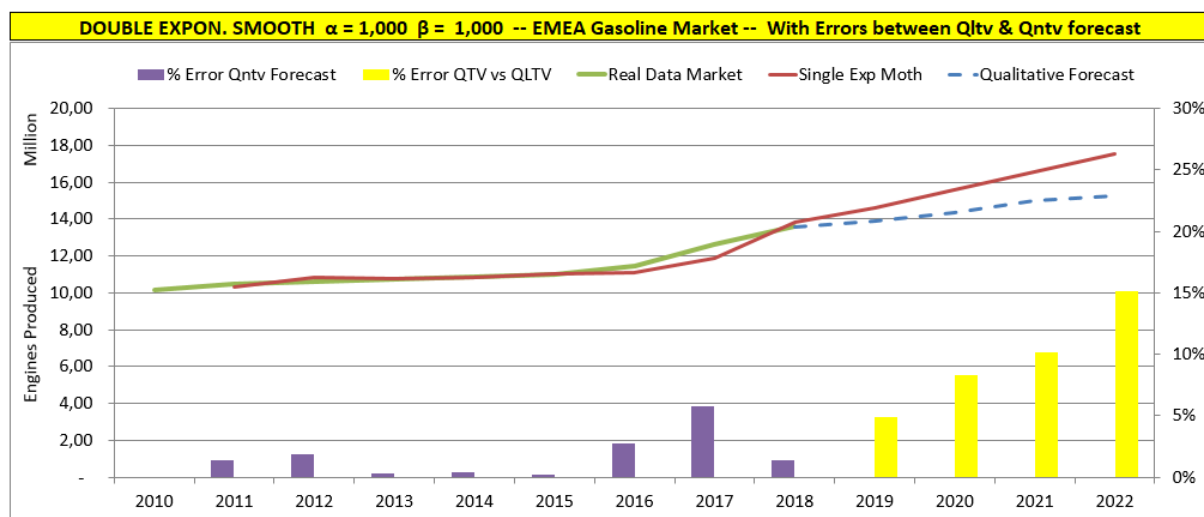
Graph 4.8 – Trend Analysis for EMEA Gasoline Market



Graph 4.9 – Trend Analysis for EMEA Diesel Market

It is possible to see that, in the first graph (Graph 4.8), the R^2 is higher than 0,5 and so the choice of the best method to use is between the Holt one and the Trend one, as in the first case. Instead, for the Diesel case (Graph 4.9), the R^2 is close to zero. This means that the data analysed have not a trend and so the Holt method is not the right one to use. In this case, to find the best forecasting technique, the Simple Exponential Smoothing, the Moving Average and the Weighted Moving Average have been applied. The data from which these graphs have been obtained are under the Appendix 4.4.

In Graph 4.10 and Graph 4.11 it is possible to find the summary graphs for the Gasoline cases and in the Appendix 4.5 the table with the data used to make them.

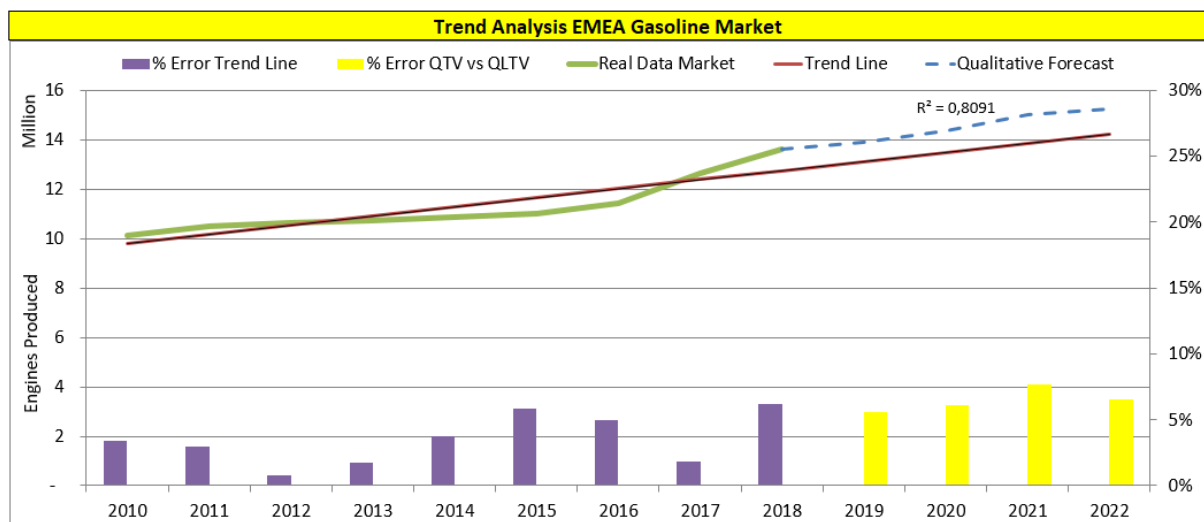


Graph 4.10 - Analysis of EMEA Gasoline Market with Holt Method

Summary Table - Double Exp. Smoth.			
	alpha min	beta min	Sigma (Unit Engines)
Gasoline Emea	1,00	1,00	324.906

Table 4.17 – Best Alpha and Beta values to minimize the Sigma

The Alpha and Beta values of Table 4.17 have been obtained thanks to the Excel solver in order to minimize the standard deviation. It is possible to see that, as expected, this methodology can approximate pretty well the past and it is quite close to the future prediction obtained by the company through a qualitative approach.



Graph 4.11 - Analysis of EMEA Gasoline Market with Trend Method

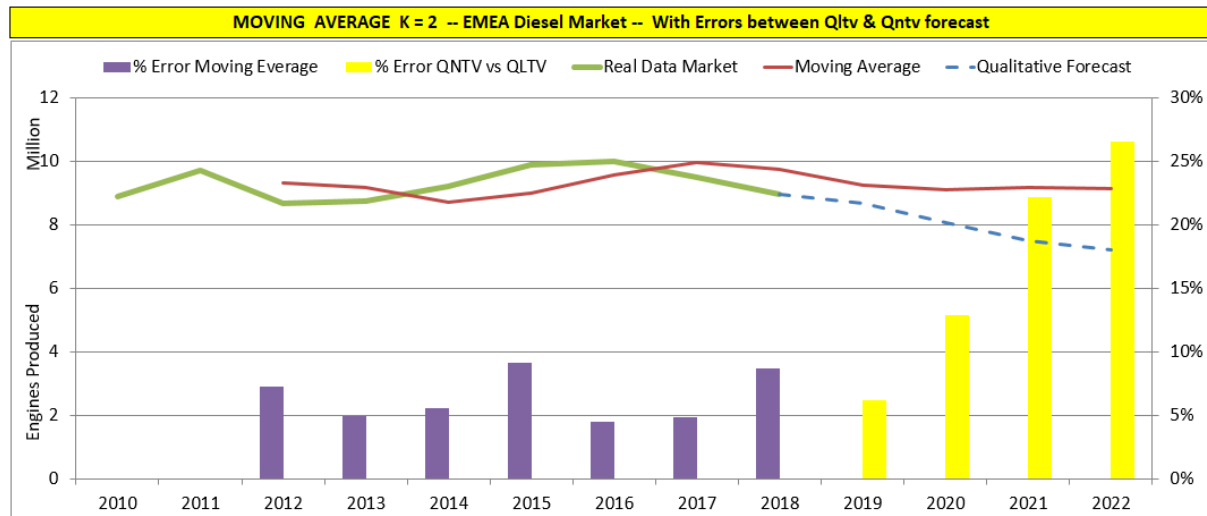
The chart above (Graph 4.11) represents the forecast done with the Trend methodology. Even this second technique seems to approximate well the real demand.

Thanks to the comparison of the σ_k values, it has been possible to identify the best methodology to use (Table 4.18):

Gasoline Market	Double Exp Smoth	Trend
Sigma (Unit Engines)	324.906	492.048

Table 4.18 – Best method with lower Sigma for Gasoline Market

Talking now about to the Diesel case, below are reported the three graphs obtained with the Moving Average (Graph 4.12), the Weighted Moving Average (Graph 4.13), and the Brown methods (Graph 4.14):

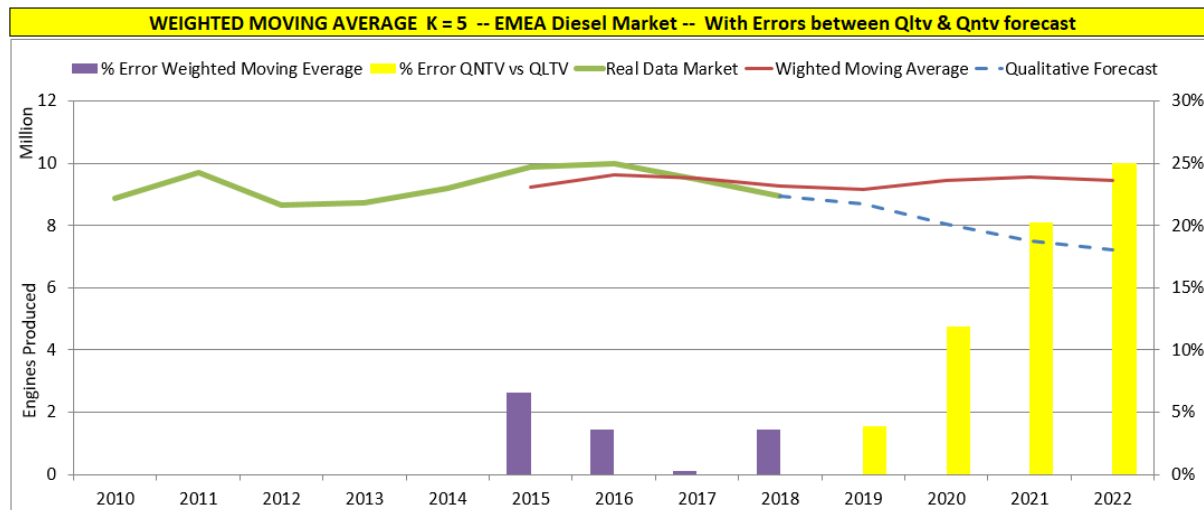


Graph 4.12 - Analysis EMEA Diesel Market with Moving Average Method

Summary Table - Moving Average				
Diesel Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	667.602	692.824	691.379	721.681

Table 4.19 – “K” values and related Sigma for Moving Average on Diesel Market

The Moving Average analysis has been performed with K=2, K=3, K=4 and K=5 (Table 4.19). The Graph 4.12 represents the solution that minimizes the standard deviation. Under the Appendix 4.6 it is possible to find the data from which this graph has been obtained as well as the results with the other Ks.



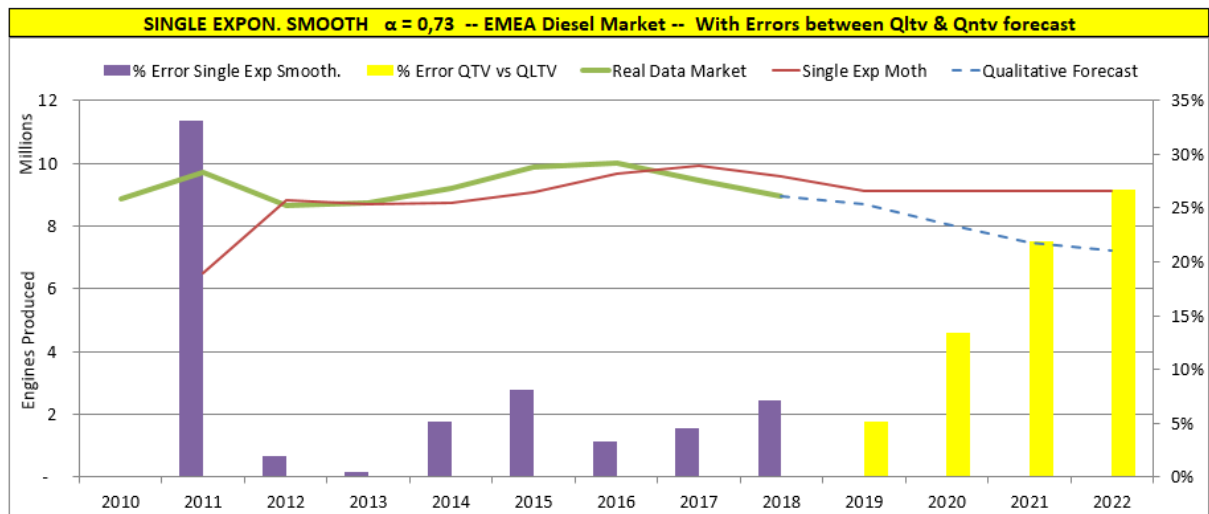
Graph 4.13 - Analysis EMEA Diesel Market with Weighted Moving Average Method

Summary Table - Weighted Moving Average				
Diesel Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	614.071	492.386	473.537	469.004

Table 4.20 – “K” values and related Sigma for Weighted Moving Average on Diesel Market

Again in this case, the study has been performed with K=2, K=3, K=4 and K=5 (see Table 4.20). In Graph 4.13, it is reported only the result able to minimize the standard deviation. Moreover, to distribute properly the weights among the K elements considered, the Excel solver has been used.

In the Appedix 4.7 it is possible to find the data as well as the other graphs with different Ks.



Graph 4.14 - Analysis EMEA Diesel Market with Single Exponential Smoothing Method

Summary Table - Simple Exp. Smoot.	
Diesel Market	Alpha = 0,73
Sigma (Unit Engines)	476.147

Table 4.21 – Best Alpha to minimize the Sigma for Simple Exponential Smoothing on Diesel Market

The Alpha value reported in the Graph 4.14 has been obtained thanks to the Excel solver in order to minimize the standard deviation (see Table 4.21). The Appendix 4.8 shows the data used for the analysis.

By comparing these different methodologies, it has been possible to see which one was best representing the expected trend. It is possible to notice that, even though not by far, the Weighted Moving Average wins over the others as shown in Table 4.22.

Diesel Market	Moving Average	Weighted Mov. Avg.	Simple Exp Smoth
Sigma (Unit Engines)	667.602	469.004	476.147

Table 4.22 – Best method with lower Sigma for Diesel Market

To summarize, considering the EMEA region with all the car manufacturers, it is possible to see that the Gasoline market is better represented by the Holt method while the Diesel market trend is better approximated by the Weighted Moving Average method with K=5.

4.3.3 Which information are provided by the optimized model

Looking at the results, it is possible to see that the relative errors between the qualitative forecast and the values obtained by the model for the Gasoline market are lower than those for the Diesel market. This can be explicable as the two markets are quite different from each other. The Gasoline market is more stable and in the previous years has been continuously growing while the Diesel one, especially from 2018, has entered a crisis and a period of uncertainty mainly due to the demonization of this typology of combustion engine. This means that it is harder for the model to react at this unexpected decrease leading to a worse quality of the market forecast. In the first case, the relative error for the first year is quite low. In the second case, the misalignment with the hypotetic demand is higher than the Gasoline scenario.

4.3.4 Comparison with the data extracted with qualitative forecasting

In Table 4.23 there are reported the data for the next four years obtained by the company using the qualitative methodology for the same market examined in the paragraph 4.3.1.

		Qualitative Forecast (Unit of EMEA All Brands Engines)			
Market	Year	2019	2020	2021	2022
EMEA Gasoline Market		13.898.816	14.367.831	15.018.154	15.228.306
EMEA Diesel Market		8.680.575	8.051.624	7.491.548	7.207.753

Table 4.23 – Qualitative data for EMEA All Brands market

In this section, a deeper analysis on the differences between qualitative and quantitative forecasts is performed. Here, it is possible to find the data from which the graphs related to the Gasoline market have been created.

In the Table 4.24 and Table 4.25 is possible to find the deviation from the consolidated values and from the future forecast for the Gasoline market adopting the Double Exponential Smoothing Methodology and Trend Methodology.

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	143.177	20.499.653.329	1,37%		
	2012	-195.876	38.367.407.376	1,85%		
	2013	-37.779	1.427.252.841	0,35%		
	2014	49.143	2.415.034.449	0,45%		
	2015	-22.902	524.501.604	0,21%		
	2016	314.086	98.650.015.396	2,75%		
	2017	734.224	539.084.882.176	5,82%		
	2018	-194.884	37.979.773.456	1,43%		
Forecast	2019				681.505	4,90%
	2020				1.194.430	8,31%
	2021				1.526.047	10,16%
	2022				2.297.835	15,09%

Table 4.24 – Gasoline Market – Double Exponential Smoothing Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	344.412	118.619.518.594	3,40%		
	2011	310.460	96.385.718.611	2,96%		
	2012	80.633	6.501.703.983	0,76%		
	2013	-186.973	34.958.979.596	1,74%		
	2014	-405.437	164.378.800.581	3,73%		
	2015	-646.802	418.352.705.030	5,88%		
	2016	-574.081	329.569.287.980	5,02%		
	2017	232.863	54.225.360.472	1,85%		
	2018	844.924	713.896.640.880	6,21%		
Forecast	2019				775.480	5,58%
	2020				874.615	6,09%
	2021				1.155.059	7,69%
	2022				995.332	6,54%

Table 4.25 – Gasoline Market – Trend Data

The column % Error represents the distance in % of the real data from the forecasted ones. Instead, the column % Error Qtv vs Qltv represents how close is the prevision performed by the model adopted to the qualitative forecast made by the company. Looking a bit more in details to the numbers, it is easy to notice that, for the first year, the errors are quite low in both models, while from 2020 they tend to grow.

Moving to the Diesel market, the tables reported below refer to the Moving Average (Table 4.26), Weighted Moving Average (Table 4.27) and Single Exponential Smoothing (Table 4.28) respectively.

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012	-631.568	398.877.507.056	7,29%		
	2013	-433.172	187.637.981.584	4,95%		
	2014	511.442	261.572.407.922	5,55%		
	2015	902.797	815.041.520.412	9,14%		
	2016	445.556	198.519.703.580	4,46%		
	2017	-458.379	210.111.307.641	4,84%		
	2018	-776.143	602.397.956.449	8,66%		
Forecast	2019				537.559	6,19%
	2020				1.036.734	12,88%
	2021				1.661.698	22,18%
	2022				1.913.049	26,54%

Table 4.26 – Diesel Market – Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012					
	2013					
	2014					
	2015	651.356	424.264.902.015	6,59%		
	2016	358.996	128.878.478.244	3,59%		
	2017	-30.497	930.066.488	0,32%		
	2018	-325.303	105.822.196.375	3,63%		
Forecast	2019				487.570	5,62%
	2020				1.401.723	17,41%
	2021				2.074.560	27,69%
	2022				2.236.094	31,02%

Table 4.27 – Diesel Market – Weighted Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	3.209.701	10.302.179.572.319	33,11%		
	2012	-169.145	28.609.926.569	1,95%		
	2013	39.262	1.541.496.513	0,45%		
	2014	479.570	229.987.150.617	5,21%		
	2015	797.873	636.600.774.485	8,08%		
	2016	326.946	106.893.971.856	3,27%		
	2017	-425.752	181.264.880.090	4,49%		
	2018	-634.123	402.111.489.010	7,08%		
Forecast	2019				449.318	5,18%
	2020				1.078.269	13,39%
	2021				1.638.345	21,87%
	2022				1.922.140	26,67%

Table 4.28 – Diesel Market – Single Exponential Smoothing Data

As it is possible to see, the three methodologies are quite similar for the first year of forecasting, while none of them is so good from 2020 on. This can be remarked as a weak point of these models. In case of unpredicted behavior, they are not able to provide significant results apart from the first year.

4.3.5 Is it the model able to correctly interpret the past and so can be used for the forecasting activity?

As just discussed, these methodologies, especially for the Diesel case, can be applied only in the short term. In any case, they could be useful in order to understand at least the direction of the specific market. EMEA has been for decades the leader market for the automotive sector, but now, mainly due to the every day stricter regulations in terms of emissions, it seems to slow down a bit.

4.3.6 Considerations

In the specific case, for the Gasoline market there are no doubts that it will grow and so the actual offer has to be kept at least as it is with some adjustments based on the new engine developed. A second alarm bell should ring looking again at the Diesel trend. It clearly seems that something is going to change in the future for this market, especially in EMEA. After this second analysis, the first doubt regarding a possible crisis for diesel engine becomes a certainty. Even FCA, the biggest supporter of high efficiency diesel engines, started to promote new hybrid solutions and it is planning to launch its first electrical car in the next years. This is the reason why a deeper analysis on FCA portfolio has been performed in chapters 4.4 and 4.5.

4.3.7 Comparative tables

Below is possible to find some synthetic tables to compare all the methods used for the analysis. In Table 4.29 it is possible to see the Best Sigma for all the methodologies adopted. Instead, in Table 4.30 the Gasoline forecasts with the Double Exponential Smoothing and the Trend methodologies are reported and compared with the qualitative numbers. Finally, in Table 4.31 the forecasts for the Diesel market with Moving Average, Weighted Moving Average and Single Exponential Smoothing methodologies are shown and again compared with the qualitative numbers.

Comparative Methods Table - EMEA All Brands					
	Moving Average	Weighted Mov. Avg	Simple Exp Smoth	Double Exp Smoth	Trend
Sigma Gasoline (Unit Engines)				324.906	492.048
Sigma Diesel (Unit Engines)	667.602	469.004	476.147		

Table 4.29 – Best Sigma for Gasoline and Diesel EMEA Market

	Year	Global Gasoline Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Double Exponential Smoth (Unit of Engines)	Trend (Unit of Engines)
Real Data Market	2010	10.138.834	13.598.381		
	2011	10.474.762		10.331.585	9.794.422
	2012	10.614.814		10.810.690	10.164.302
	2013	10.717.087		10.754.866	10.534.181
	2014	10.868.503		10.819.360	10.904.060
	2015	10.997.017		11.019.919	11.273.940
	2016	11.439.617		11.125.531	11.643.819
	2017	12.616.441		11.882.217	12.013.698
	2018	13.598.381		13.793.265	12.383.578
Forecast	2019		13.898.816	14.580.321	12.753.457
	2020		14.367.831	15.562.261	13.123.336
	2021		15.018.154	16.544.201	13.493.216
	2022		15.228.306	17.526.141	13.863.095

Table 4.30 – All Methods used for EMEA Gasoline Market vs Qualitative Forecast

	Year	EMEA Diesel Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Moving Average K=2 (Unit of Engines)	Weighted Moving Average K = 5 (Unit of Engines)	Single Exponential Smoothing (Unit of Engines)
Real Data Market	2010	8.885.008	8.958.582			
	2011	9.694.389				6.484.688
	2012	8.658.131		9.289.699		8.827.276
	2013	8.743.088		9.176.260		8.703.826
	2014	9.212.051		8.700.610		8.732.481
	2015	9.880.366		8.977.570	9.229.010	9.082.493
	2016	9.991.764		9.546.209	9.632.768	9.664.818
	2017	9.477.686		9.936.065	9.508.183	9.903.438
	2018	8.958.582		9.734.725	9.283.885	9.592.705
Forecast	2019		8.680.575	9.218.134	9.168.145	9.129.893
	2020		8.051.624	9.088.358	9.453.347	9.129.893
	2021		7.491.548	9.153.246	9.566.108	9.129.893
	2022		7.207.753	9.120.802	9.443.847	9.129.893

Table 4.31 – All Methods used for EMEA Diesel Market vs Qualitative Forecast

4.4 World - FCA

4.4.1 On which data the historical series is based

As just said, in this third part of the analysis only global FCA data have been included.

Also in this case, the big distinction between Gasoline and Diesel engines has been maintained for the reasons mentioned at the beginning of this chapter.

This third scenario can be useful to understand in which direction the FCA market is moving all around the world. In Table 4.32 is possible to find a summary of the data considered for the hystorical analysis.

The historical data used for the analysis start in 2010 and have been updated annually. The expected forecast horizon goes from 2019 to 2022.

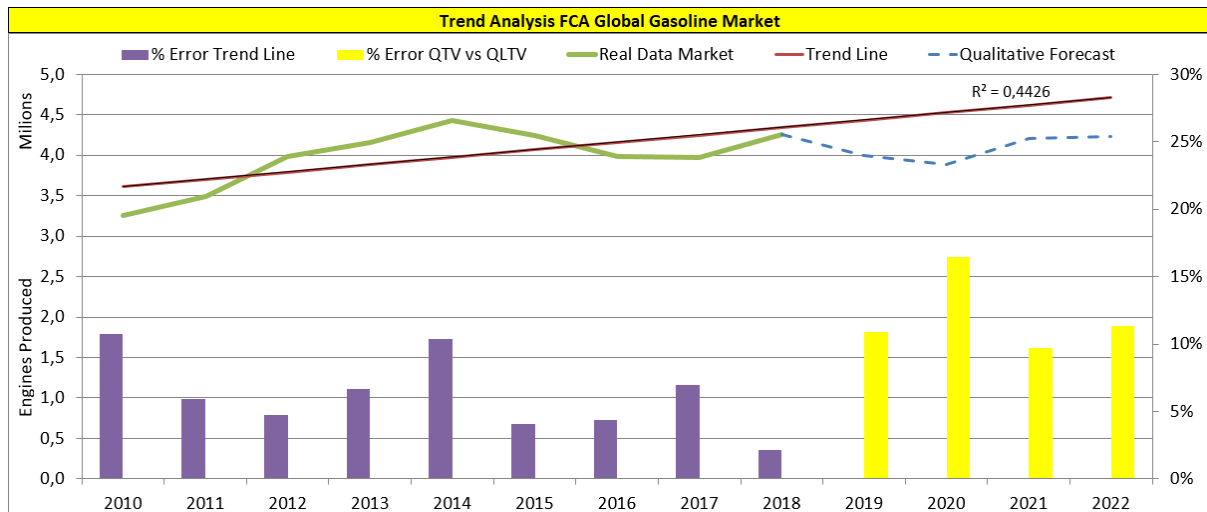
		Real Data (Unit of Global WW FCA Brand Engines)								
		2010	2011	2012	2013	2014	2015	2016	2017	2018
Market	Year									
FCA Global Gasoline Market		3.259.749	3.494.046	3.979.325	4.160.783	4.433.885	4.240.389	3.984.784	3.971.695	4.251.267
FCA Global Diesel Market		1.235.258	1.312.579	879.706	867.916	891.915	1.061.035	1.202.179	1.193.988	1.142.882

Table 4.32 – Global WW Engines Produced by FCA Brand

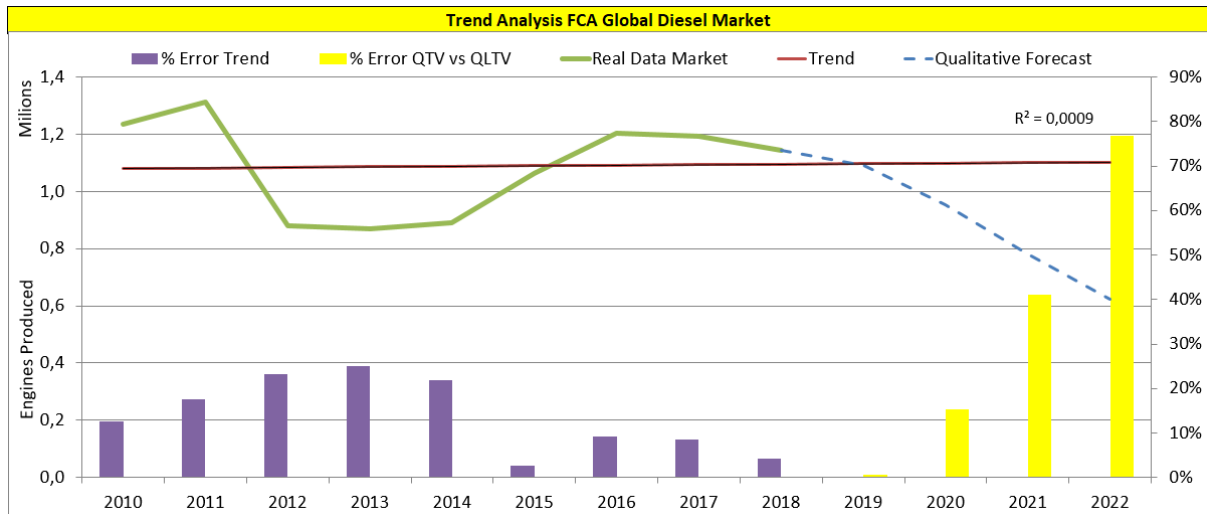
As it is possible to notice, also FCA is used to sell more Gasoline rather than Diesel vehicles.

4.4.2 How the model can be optimized

Again in this case, to understand which the best model to use for the forecasting was, a trend analysis has been performed. This is useful to understand if the Holt method can be applied or not. Below it is possible to find the graphs of the study performed and at the Appendix 4.9 the data used to make the graphs.



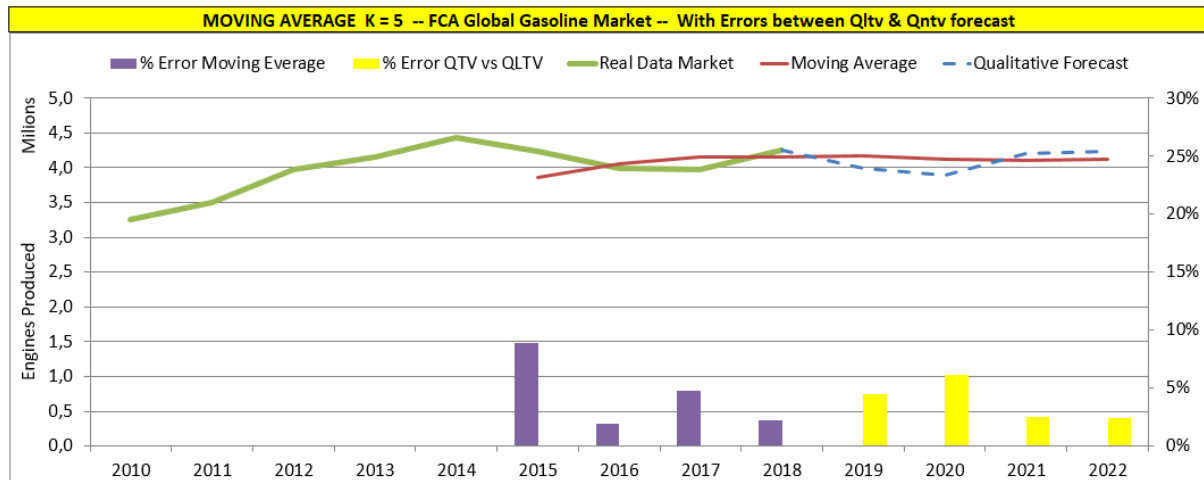
Graph 4.15 - Trend Analysis for Global WW FCA Gasoline Market



Graph 4.16 – Trend Analysis for Global WW FCA Diesel Market

It is possible to notice that in both cases the R^2 is lower than 0,5 (see Graph 4.15 and Graph 4.16) and so the Holt method cannot be applied for doing the forecasting. In particular, for the Diesel case, the R^2 is close to zero. In these cases, to find the best forecasting technique, the Simple Exponential Smoothing, the Moving Average and the Weighted Moving Average have been applied to both markets.

Focusing now on the Gasoline case, in Graph 4.17, Graph 4.18 and Graph 4.19 it is possible to find the summary of the best solutions for each method considered.

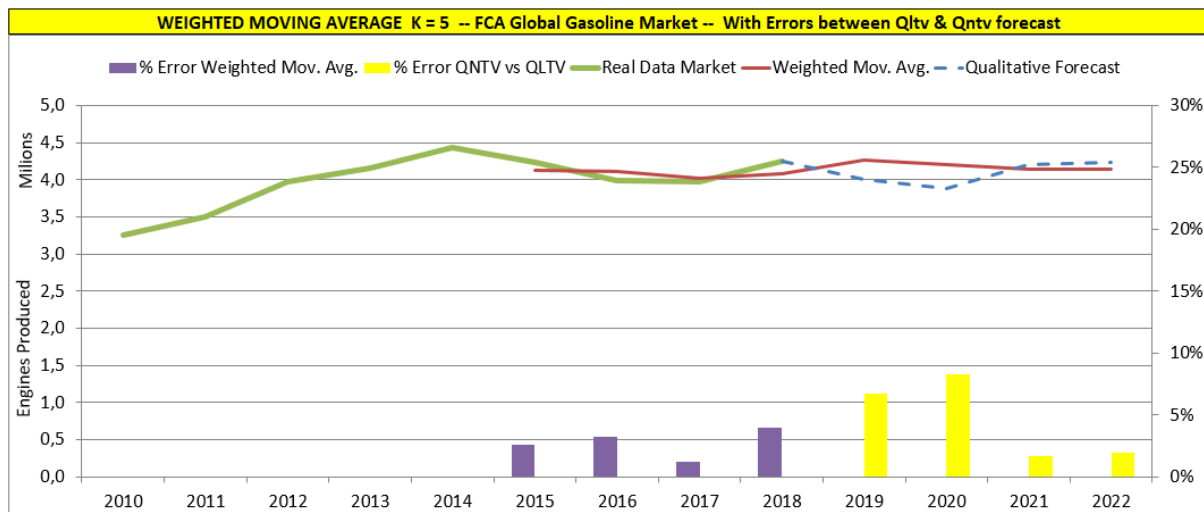


Graph 4.17 - Analysis of FCA Global Gasoline Market with Moving Average Method

Summary Table - Moving Average				
Gasoline Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	386.639	408.285	407.930	251.959

Table 4.33 – “K” values and the related Sigma

The Moving Average analysis has been performed with K=2, K=3, K=4 and K=5. The Table 4.33 reported above is the solution that minimizes the standard deviation. Under the Appendix 4.10 it is possible to find the data from which this graph has been obtained as well as the results with the other Ks.

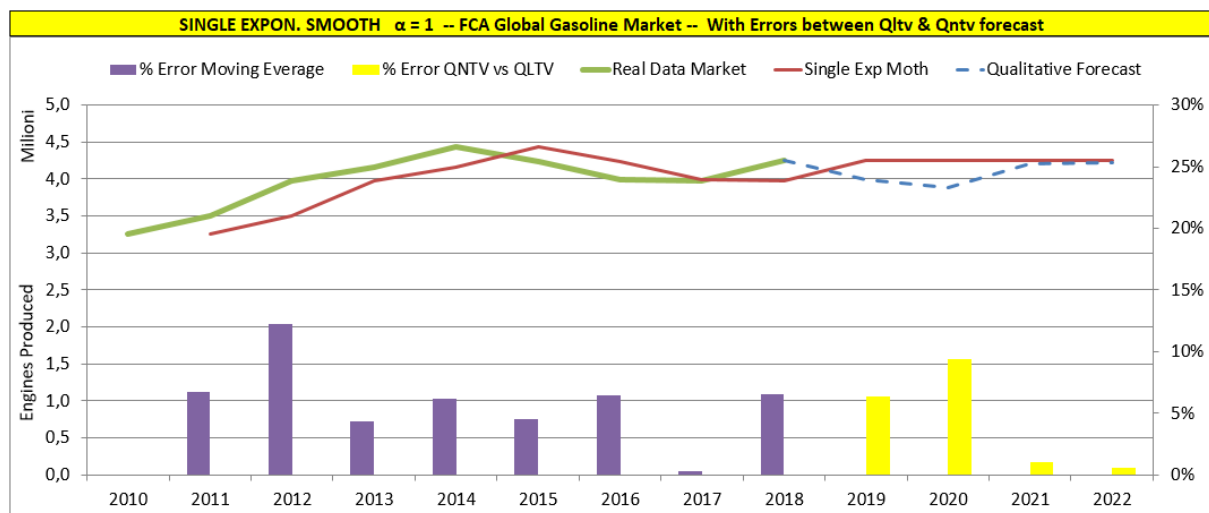


Graph 4.18 - Analysis of FCA Global Gasoline Market with Weighted Moving Average

Summary Table - Weighted Moving Avg				
Gasoline Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	295.554	240.257	248.671	141.033

Table 4.34 – “K” values and the related Sigma

Also the Weighted Moving Average analysis has been performed with K=2, K=3, K=4 and K=5. Looking at the Table 4.34 reported above, it is possible to find solution that minimizes the standard deviation. Under the Appendix 4.11 it is possible to find the data from which this graph has been obtained as well as the results with the other Ks.



Graph 4.19 - Analysis of FCA Global Gasoline Market with Single Exponential Smoothing

Summary Table - Single Exp Smooth				
Gasoline Market	Alpha = 0,4	Apha=0,6	Alpha=0,8	Alpha = 1
Sigma (Unit Engines)	936.736	534.542	342.384	273.630

Table 4.35 – Alpha values and the related Sigma for Single Exponential Smoothing on Gasoline Market

For the Single Exponential Smoothing, the best Alpha value has been obtained with the Excel solver. In the Table 4.35, it is possible to find the Sigma obtained with some Alpha values, but Alpha=1 is the solution that minimizes the standard deviation. Under the Appendix 4.12 it is possible to find the data from which this graph has been obtained.

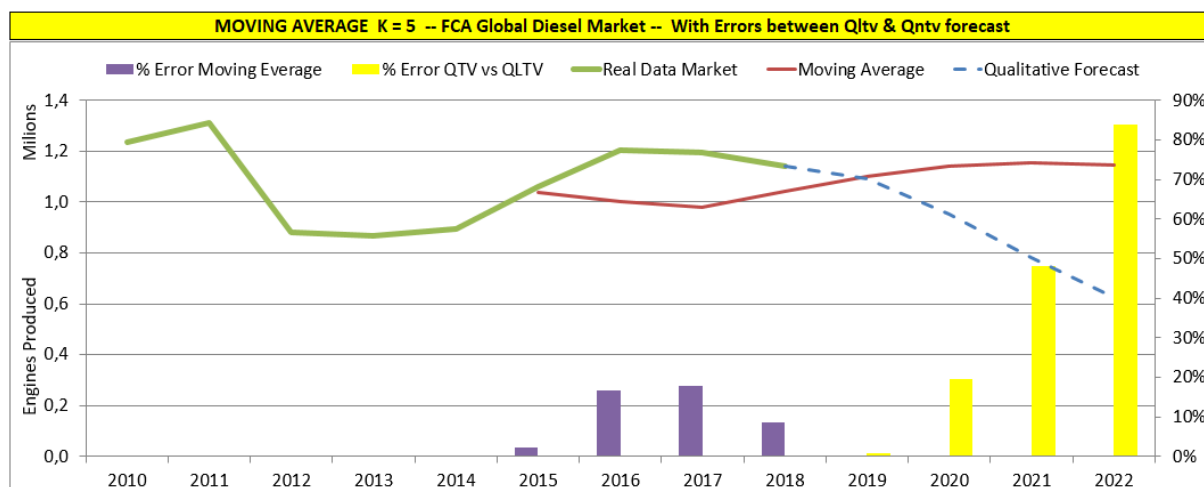
It is possible to see that, as expected, these methodologies can approximate quite well the past and they are very close to the future prediction obtained by the company through a qualitative approach. The σ_k values obtained are reported in Table 4.36:

Gasoline Market	Moving Average	Weighted Mov. Avg	Simple Exp Smoth
Sigma (Unit Engines)	251.959	141.033	273.630

Table 4.36 – Best method with lower Sigma for Gasoline Market

The Weighted Moving Average methodology with a K=5 is the best one to predict the future demand as it gives the lowest standard deviation value.

Talking now about the Diesel case, below there are reported the three graphs obtained with the Moving Average (Graph 4.20), the Weighted Moving Average (Graph 4.21) and the Brown methods (Graph 4.22).

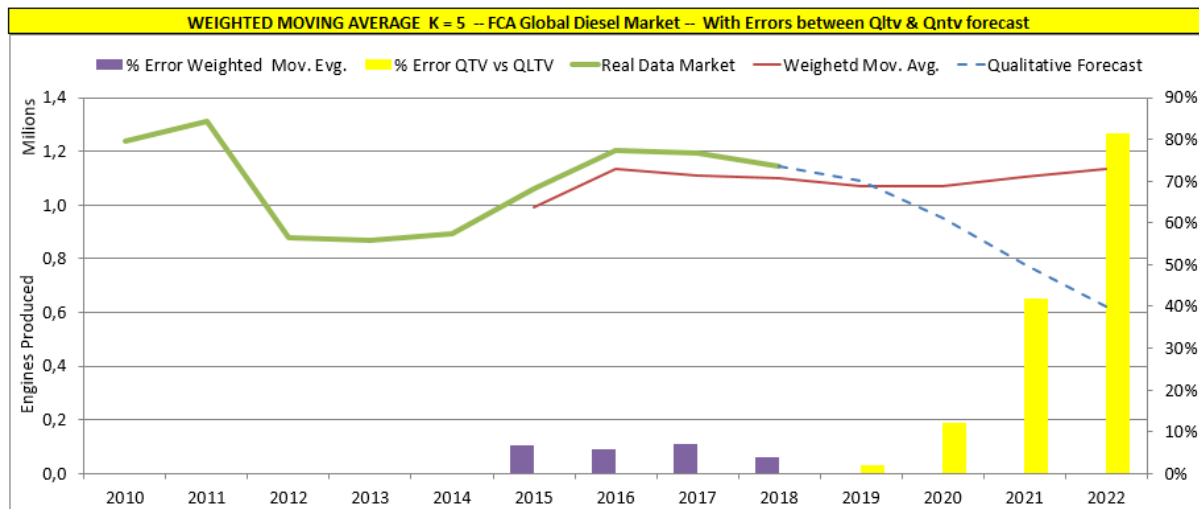


Graph 4.20 - Analysis Global WW FCA Diesel Market with Moving Average Method

Summary Table - Moving Average				
Diesel Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	223.048	206.677	196.026	178.723

Table 4.37 – “K” values and the related Sigma for Moving Average on Diesel Market

The Moving Average analysis has been performed with K=2, K=3, K=4 and K=5 (Table 4.37). In the Graph 4.20, it can be found the solution that minimize the standard deviation. Under the Appendix 4.13 it is possible to find the data from which this graph has been obtained as well as the results with the other Ks.



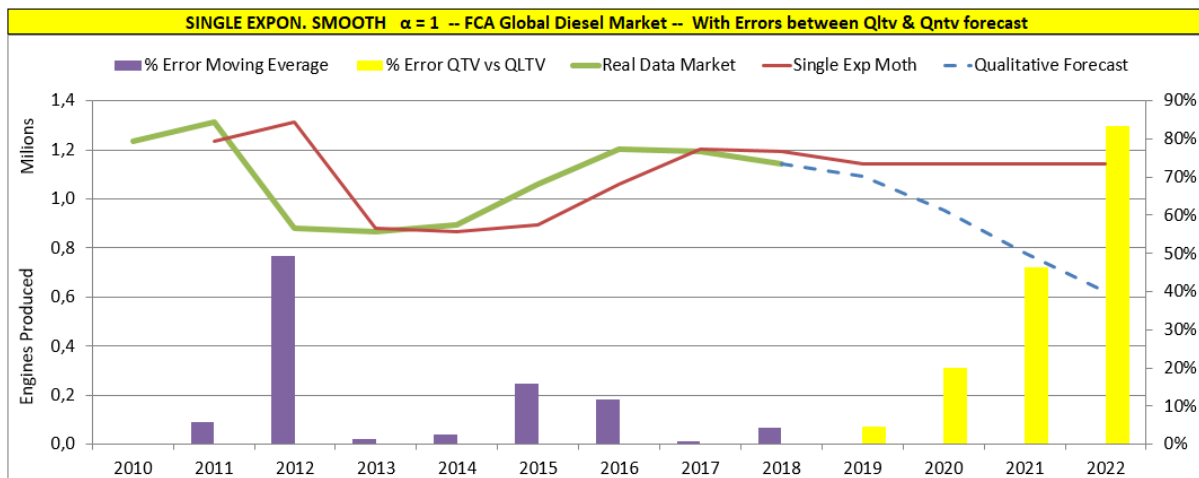
Graph 4.21 - Analysis Global FCA Diesel Market with Weighted Mov Avg Method

Summary Table - Weighted Moving Avg				
Gasoline Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	199.703	101.899	102.678	78.858

Table 4.38 -- "K" values and the related Sigma for Weighted Moving Average on Diesel Market

Again in this case, the study has been performed with K=2, K=3, K=4 and K=5 (see Table 4.38). In the Graph 4.21, it is reported only the result able to minimize the standard deviation. Moreover, to properly distribute the weights among the K elements considered, the Excel solver has been used.

In the Appedix 4.14 it is possible to find the data as well as the other graphs with different Ks.



Graph 4.22 - Analysis Global WW FCA Diesel Market with Single Exponential Smoothing Method

Summary Table - Single Exp Smooth				
Diesel Market	Alpha = 0,4	Apha=0,6	Alpha=0,8	Alpha = 1
Sigma (Unit Engines)	338.111	254.308	210.637	187.188

Table 4.39 – Alpha values and the related Sigma for Single Exponential Smoothing on Diesel Market

The Alpha value of the Graph 4.22 has been obtained thanks to the Excel solver in order to minimize the standard deviation (see Table 4.39). The Appendix 4.15 shows the data used for the analysis.

Comparing these different methodologies, it has been possible to see which one was the most suitable to represent the expected trend. It is clearly noticeable that the Wighted Moving Average with K=5 wins over the others as shown in Table 4.40.

Diesel Market	Moving Average	Weighted Mov. Avg	Simple Exp Smoth
Sigma (Unit Engines)	178.723	78.858	187.188

Table 4.40 – Best method with lower Sigma for Diesel Market

To summarize, considering the entire world with only FCA as car manufacturer, it is possible to see that the Gasoline market as well as the Diesel market are better represented by the Weighted Moving Average method with the same K value (i.e. 5).

4.4.3 Which information are provided by the optimized model

Looking at the results, it is possible to see that the relative errors between the qualitative forecast and the values obtained by the model for the Gasoline market are lower than those for the Diesel market. As already said, this can be explicable as the two markets are quite different from each other. The Gasoline market is more stable and in the previous years has been continuously growing while the diesel market, especially from 2018, has entered a crisis and a period of uncertainty mainly due to the strict regulation on CO₂ emissions. This is even more true looking at FCA data. They based the largest part of their sales on high efficiency diesel engines and now they are suffering more than the average to keep up with the new Electric Vehicle-trends. Moreover, as already discussed for the general trends, it is harder for the model to react at this unexpected Diesel decrease leading to a worse quality of the market forecast. In the first case, the relative errors are quite low for all the

years forecasted. In the second case, the misalignment with the hypotetic demand is much higher than the Gasoline scenario.

4.4.4 Comparison with the data extracted with qualitative forecasting

In Table 4.41 there are reported the data for the next four years obtained by the company using the qualitative methodology for the same market examined in the paragraph 4.4.1.

		Qualitative Forecast (Unit of Global WW FCA Engines)			
Market	Year	2019	2020	2021	2022
FCA Global Gasoline Market		3.998.178	3.885.718	4.207.740	4.227.782
FCA Global Diesel Market		1.091.045	952.729	780.303	623.376

Table 4.41 – Qualitative data for EMEA All Brands market

In this section, a deeper analysis on the differences between qualitative and quantitative forecasts is performed. Here, it is possible to find the data from which the graphs related to the Gasoline market have been created.

In the Table 4.42, Table 4.43 and Table 4.44 it is possible to find the deviation from the consolidated values and from the future forecast for the Gasoline market adopting the Moving Average Methodology, the Weighted Moving Average Methodology and the Simple Exponential Moving Methodology.

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012					
	2013					
	2014					
	2015	374.831	140.498.578.426	8,84%		
	2016	-76.902	5.913.856.083	1,93%		
	2017	-188.138	35.395.982.299	4,74%		
	2018	92.960	8.641.524.416	2,19%		
Forecast	2019				178.226	4,46%
	2020				239.190	6,16%
	2021				105.928	2,52%
	2022				102.565	2,43%

Table 4.42 – Gasoline Market – Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012					
	2013					
	2014					
	2015	110.016	12.103.463.472	2,59%		
	2016	-130.026	16.906.856.215	3,26%		
	2017	-47.794	2.284.313.079	1,20%		
	2018	168.452	28.376.199.919	3,96%		
Forecast	2019				268.847	6,72%
	2020				322.176	8,29%
	2021				69.056	1,64%
	2022				82.956	1,96%

Table 4.43 – Gasoline Market – Weighted Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	234.297	54.895.084.209	6,71%		
	2012	485.279	235.495.707.841	12,20%		
	2013	181.458	32.927.005.764	4,36%		
	2014	273.102	74.584.702.404	6,16%		
	2015	-193.496	37.440.702.016	4,56%		
	2016	-255.605	65.333.916.025	6,41%		
	2017	-13.089	171.321.921	0,33%		
	2018	279.572	78.160.503.184	6,58%		
Forecast	2019				253.089	6,33%
	2020				365.549	9,41%
	2021				43.527	1,03%
	2022				23.485	0,56%

Table 4.44 – Gasoline Market – Single Exponential Smoothing Data

The tables reported above refer to the Moving Average, Weighted Moving Average and Single Exponential Smoothing respectively. As it is possible to see, the three methodologies are quite good in general and can be used also for long term forecasting as the series are quite stable in time.

The column % Error represents the distance in % of the real data from the forecasted ones. Instead, the column % Error Qtv vs Qltv represents how close to the qualitative forecast made by the company is the prevision performed by the applied models.

Moving to the Diesel market, the tables reported below refer to the Moving Average (Table 4.45), Weighted Moving Average (Table 4.46) and Single Exponential Smoothing (Table 4.47) methodologies respectively.

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012					
	2013					
	2014					
	2015	23.560	555.083.024	2,22%		
	2016	199.549	39.819.723.581	16,60%		
	2017	213.438	45.555.694.469	17,88%		
	2018	99.475	9.895.355.205	8,70%		
Forecast	2019				7.355	0,67%
	2020				186.968	19,62%
	2021				375.126	48,07%
	2022				522.703	83,85%

Table 4.45 – Diesel Market – Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012					
	2013					
	2014					
	2015	70.121	4.916.995.432	6,61%		
	2016	68.614	4.707.935.990	5,71%		
	2017	84.790	7.189.364.218	7,10%		
	2018	42.913	1.841.512.334	3,75%		
Forecast	2019				20.526	1,88%
	2020				115.055	12,08%
	2021				326.232	41,81%
	2022				508.375	81,55%

Table 4.46 – Diesel Market – Weighted Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	77.321	5.978.537.041	5,89%		
	2012	-432.873	187.379.034.129	49,21%		
	2013	-11.790	139.004.100	1,36%		
	2014	23.999	575.952.001	2,69%		
	2015	169.120	28.601.574.400	15,94%		
	2016	141.144	19.921.628.736	11,74%		
	2017	-8.191	67.092.481	0,69%		
	2018	-51.106	2.611.823.236	4,47%		
Forecast	2019				51.837	4,75%
	2020				190.153	19,96%
	2021				362.579	46,47%
	2022				519.506	83,34%

Table 4.47 – Diesel Market – Single Exponential Smoothing Data

As it is possible to see, the first methodology is the best one for the short term, while for the years after 2019 it becomes the worst one. As already said in the EMEA analysis with all car manufacturers, this inability to follow an unpredicted path can be noticed as a weak point of these models.

4.4.5 Is it the model able to correctly interpret the past and so can be used for the forecasting activity?

To summarize, these methodologies can be applied for the long-term forecasting only for the Gasoline case as there are no strong variations of the series. For what concerns the Diesel case, it would be better to adopt these forecasts only for the short-term decisions. In any case, they could be useful in order to understand at least the direction in which the specific car manufacturer is going to drive the company.

4.4.6 Considerations

In the specific case, for the Gasoline market there are no doubts that it will grow and so the actual offer has to be kept as it is with some adjustments based on the new engine developed (48V Hybrid, Mid Hybrid, Full Hybrid or Plug in Hybrid). It clearly seems that something is going to change in the future for FCA. In fact, after this third analysis, it is possible to see how the biggest producer of Diesel engines is going to react at the new environmental issues (namely, the increase of CO₂ particles). The last analysis will move the focus on the EMEA market for FCA. This because the business related with FCA represents a big chunk of EATON revenues and, at this moment, EMEA represents the most evolving market. Understanding how this big company is going to behave in that market it is for sure the most difficult prevision to make. Therefore, it would be good to have a support from these forecasting methods in order to be able to react in time.

4.4.7 Comparative tables

Below it is possible to find a sintethic table to compare all the methods used for the analysis. In Table 4.48 it is possible to see the Best Sigma for all the methodologies adopted. Instead, in Table 4.49 the Gasoline forecasts with the Moving Average, Weighted Moving Average and Single Exponential Smoothing methodologies are reported and compared with the qualitative numbers. Finally, in Table 4.50 the forecasts for the Diesel market with Moving Average, Weighted Moving Average and Single Exponential Smoothing methodologies are shown and again compared with the qualitative numbers.

Comparative Methods Table - Global WW Fca Engines			
	Moving Average	Weighted Mov. Avg.	Simple Exp Smoth
Sigma Gasoline (Unit Engines)	251.959	141.033	273.630
Sigma Diesel (Unit Engines)	178.723	78.858	187.188

Table 4.48 – Best Sigma for Gasoline and Diesel FCA Global Market

	Year	FCA Global Gasoline Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Moving Average K=5 (Unit of Engines)	Weighted Moving Avg K=5 (Unit of Engines)	Single Exponential Smoth (Unit of Engines)
Real Data Market	2010	3.259.749	4.251.267			
	2011	3.494.046				3.259.749
	2012	3.979.325				3.494.046
	2013	4.160.783				3.979.325
	2014	4.433.885				4.160.783
	2015	4.240.389		3.865.558	4.130.373	4.433.885
	2016	3.984.784		4.061.686	4.114.810	4.240.389
	2017	3.971.695		4.159.833	4.019.489	3.984.784
	2018	4.251.267		4.158.307	4.082.815	3.971.695
Forecast	2019		3.998.178	4.176.404	4.267.025	4.251.267
	2020		3.885.718	4.124.908	4.207.894	4.251.267
	2021		4.207.740	4.101.812	4.138.684	4.251.267
	2022		4.227.782	4.125.217	4.144.826	4.251.267

Table 4.49 – Methods used for Global FCA Gasoline Market vs Qualitative Forecast

	Year	FCA Global Diesel Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Moving Average K=5 (Unit of Engines)	Weighted Moving Avg K=5 (Unit of Engines)	Single Exponential Smoth (Unit of Engines)
Real Data Market	2010	1.235.258	1.142.882			
	2011	1.312.579				1.235.258
	2012	879.706				1.312.579
	2013	867.916				879.706
	2014	891.915				867.916
	2015	1.061.035		1.037.475	990.914	891.915
	2016	1.202.179		1.002.630	1.133.565	1.061.035
	2017	1.193.988		980.550	1.109.198	1.202.179
	2018	1.142.882		1.043.407	1.099.969	1.193.988
Forecast	2019		1.091.045	1.098.400	1.070.519	1.142.882
	2020		952.729	1.139.697	1.067.784	1.142.882
	2021		780.303	1.155.429	1.106.535	1.142.882
	2022		623.376	1.146.079	1.131.751	1.142.882

Table 4.50 – Methods used for Global FCA Diesel Market vs Qualitative Forecast

4.5 EMEA - FCA

4.5.1 On which data the historical series is based

As just said, in this fourth part of the analysis the focus has been moved to the FCA performances in EMEA.

Also in this last case, the big distinction between Gasoline and Diesel engines has been maintained for the reasons mentioned at the beginning of this chapter.

This fourth analysis can be useful to better understand how FCA is reacting to all the changes going on in the European environment. It is possible to find a summary table of the data considered for the historical analysis in Table 4.51.

The historical data used for the analysis start in 2010 and have been updated annually. The expected forecast horizon goes from 2019 to 2022.

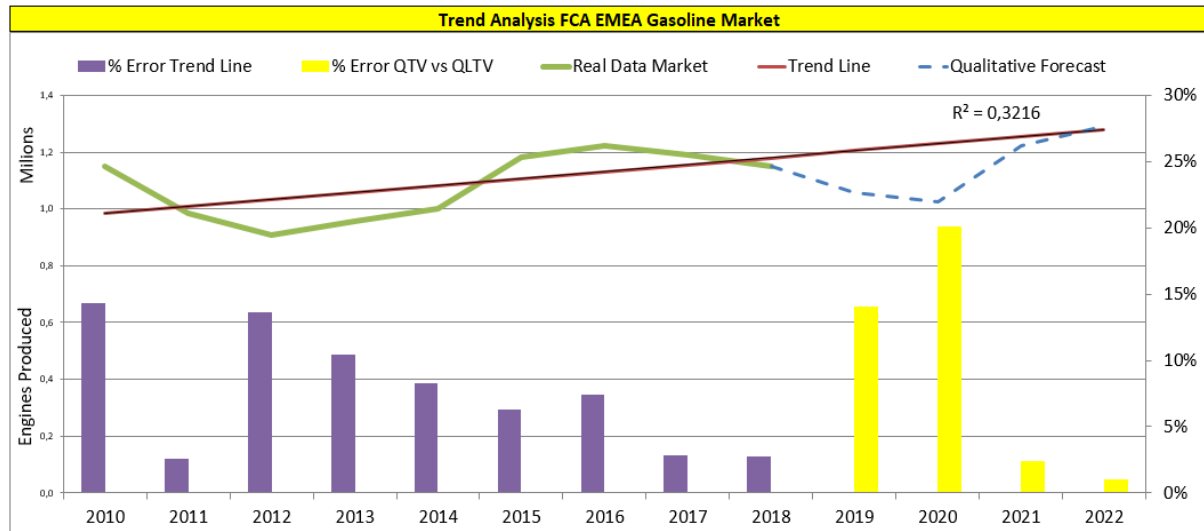
		Real Data (Unit of EMEA FCA Brand Engines)								
Market	Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
FCA EMEA Gasoline Market		1.148.873	983.405	909.116	958.100	999.534	1.180.454	1.221.956	1.189.797	1.148.994
FCA EMEA Diesel Market		851.787	884.084	623.047	624.170	648.792	752.248	863.308	885.524	803.042

Table 4.51 – EMEA FCA Engines Produced

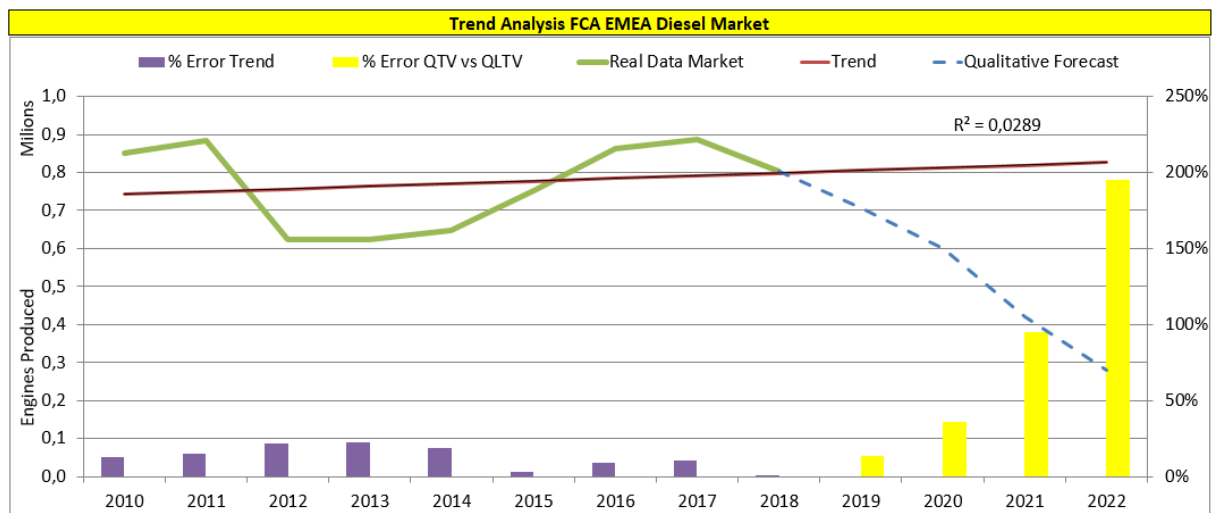
As it is possible to notice, also in this last case the Gasoline engines are higher in volumes compared to the Diesel ones.

4.5.2 How the model can be optimized

The trend analysis has been applied also in this last case to assess which was the best model to use for the forecasting. This is useful to understand if the Holt method can be applied or not. Below the graphs of the study performed and at the Appendix 4.16 the data from which they have been extrapolated.



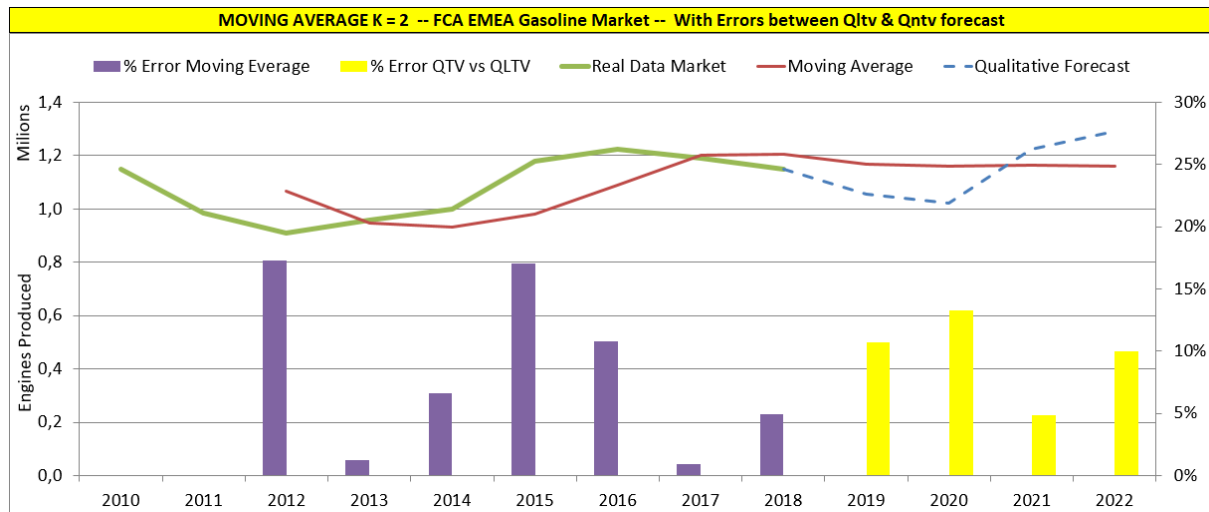
Graph 4.23 – Trend Analysis for EMEA FCA Gasoline Market



Graph 4.24 - Trend Analysis for EMEA FCA Diesel Market

It is possible to see that, in both cases (Graph 4.23 and Graph 4.24), the R^2 is lower than 0,5 and so the Holt method cannot be applied for doing the forecasting. In particular, for the Diesel case, the R^2 is again close to zero. In these cases, to find the best forecasting technique, the Simple Exponential Smoothing, the Moving Average and the Weighted Moving Average have been applied to both markets.

Focusing now on the Gasoline case, in the Graphs 4.25, 4.26 and 4.27 it is possible to find a summary graph of the best solution for each method considered.

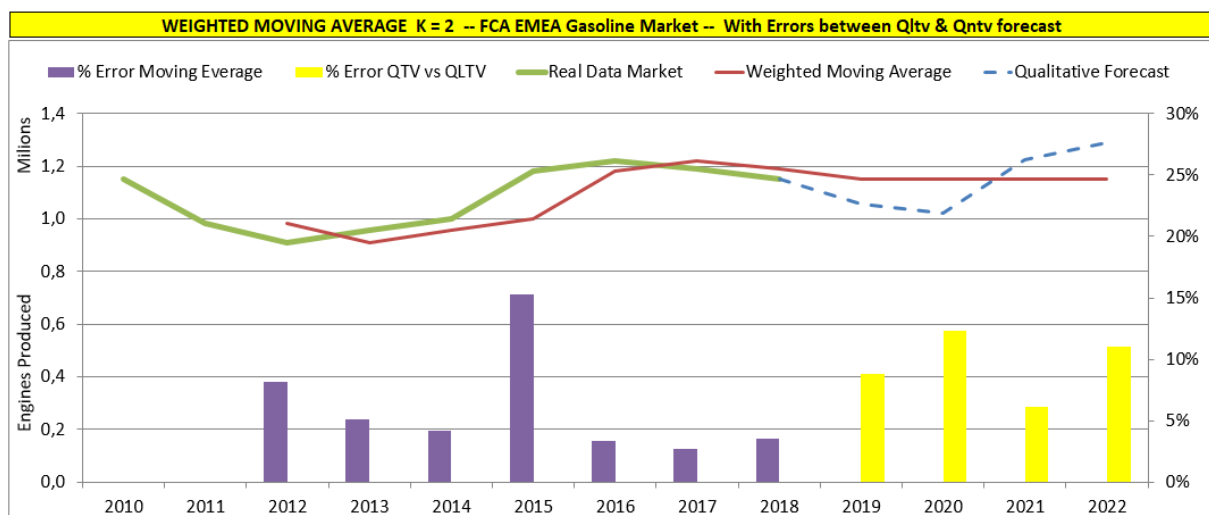


Graph 4.25 - Analysis of EMEA FCA Gasoline Market with Moving Average Method

Summary Table - Moving Average				
Gasoline Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	122.868	136.023	159.382	181.868

Table 4.52 – “K” values and the related Sigma

The Moving Average analysis has been performed with K=2, K=3, K=4 and K=5 (Table 4.52). The picture reported above is the solution that minimizes the standard deviation. Under the Appendix 4.17 it is possible to find the data from which this graph has been obtained as well as the results with the other Ks.

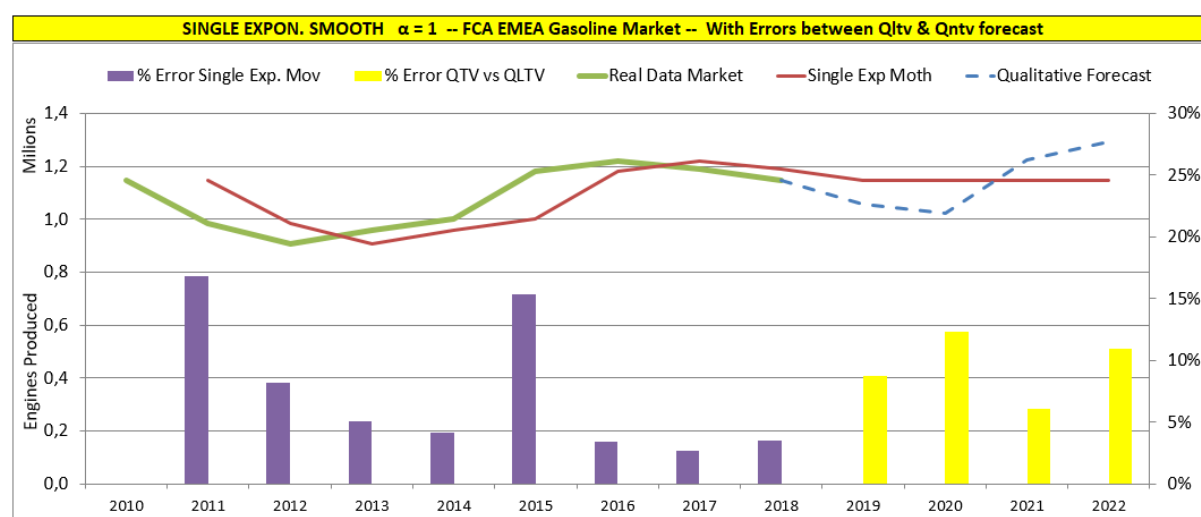


Graph 4.26 - Analysis of EMEA Gasoline Market with Weighted Moving Average Method

Summary Table - Weighted Moving Avg				
Gasoline Market	K=2	K=3	K=4	K=5
Sigma (Unit of Engines)	88.307	90.851	97.992	100.394

Table 4.53 – “K” values and the related Sigma

The same logic has been applied to the Weighted Moving Average. In fact, the analysis has been performed with K=2, K=3, K=4 and K=5 (Table 4.53). The picture reported above is the solution that minimizes the standard deviation. Under the Appendix 4.18 it is possible to find the data from which this graph has been obtained as well as the results with the other Ks.



Graph 4.27 - Analysis of EMEA FCA Gasoline Market with Single Exp. Method

Summary Table - Single Exp Smooth				
Gasoline Market	Alpha = 0,4	Apha=0,6	Alpha=0,8	Alpha = 1
Sigma (Unit of Engines)	187.729	100.865	86.521	81.756

Table 4.54 – Alpha values and the related Sigma for Single Exponential Smoothing on Gasoline Market

For the Single Exponential Smoothing, the best Alpha value has been obtained with the Excel solver (Table 4.54). The chart reported above (Graph 4.27) is the solution that minimizes the standard deviation. Under the Appendix 4.19 it is possible to find the data from which this graph has been obtained.

It is possible to see that, as expected, these methodologies can approximate quite well the past and they are able to make predictions very close to the ones obtained

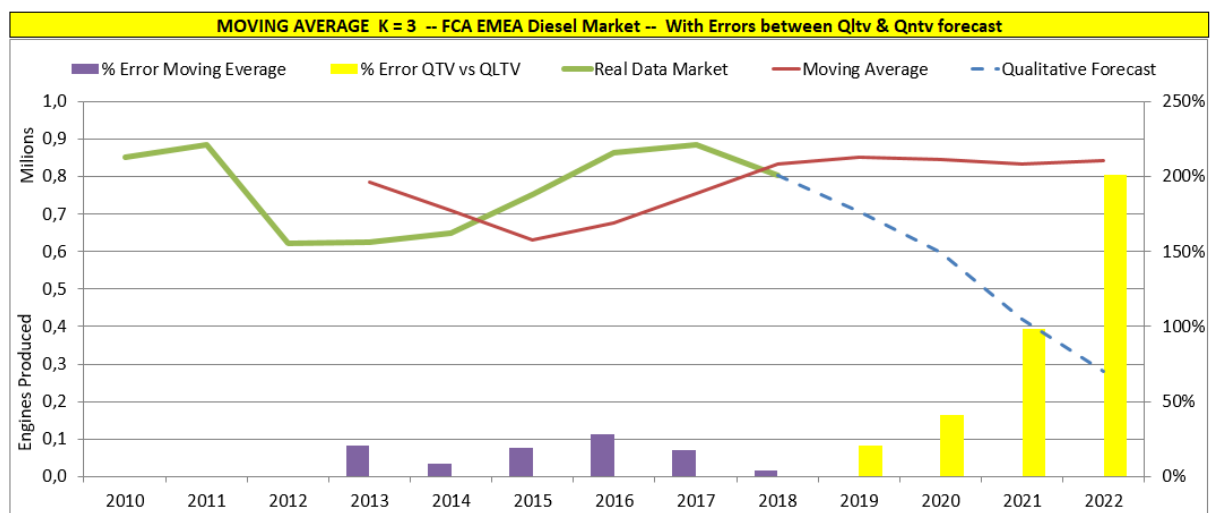
by the company through a qualitative approach. The σ_k values obtained are reported below:

Gasoline Market	Moving Average	Weighted Moving Avg	Simple Exp Smoth
Sigma (Unit of Engines)	122.868	88.307	81.756

Table 4.55 – Best method with lower Sigma for Gasoline Market

The Single Exponential Smoothing methodology is the best one to predict the future demand as it gives the lowest standard deviation value.

Moving now to the Diesel case, below it is possible to find the graphs obtained with the Moving Average (Graph 4.28), the Weighted Moving Average (Graph 4.29) and the Brown methods (Graphs 4.30).

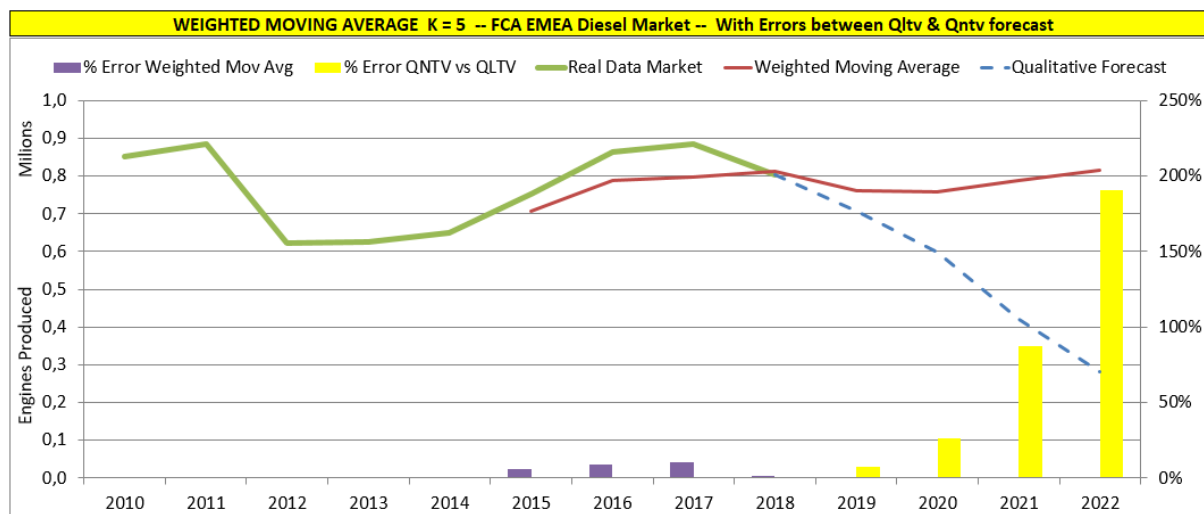


Graph 4.28 - Analysis of EMEA FCA Diesel Market with Moving Average Method

Summary Table - Moving Average				
Diesel Market	K=2	K=3	K=4	K=5
Sigma (Unit Engines)	146.281	140.009	141.527	142.783

Table 4.56 – “K” values and the related Sigma

The Moving Average analysis has been performed with K=2, K=3, K=4 and K=5. Above, among the different solutions found, there is also the one that can minimize the standard deviation (Table 4.56, K=3) that has also been graphed (Graph 4.28). Under the Appendix 4.20 it is possible to find the data from which this graph has been obtained as well as the results with the other Ks.



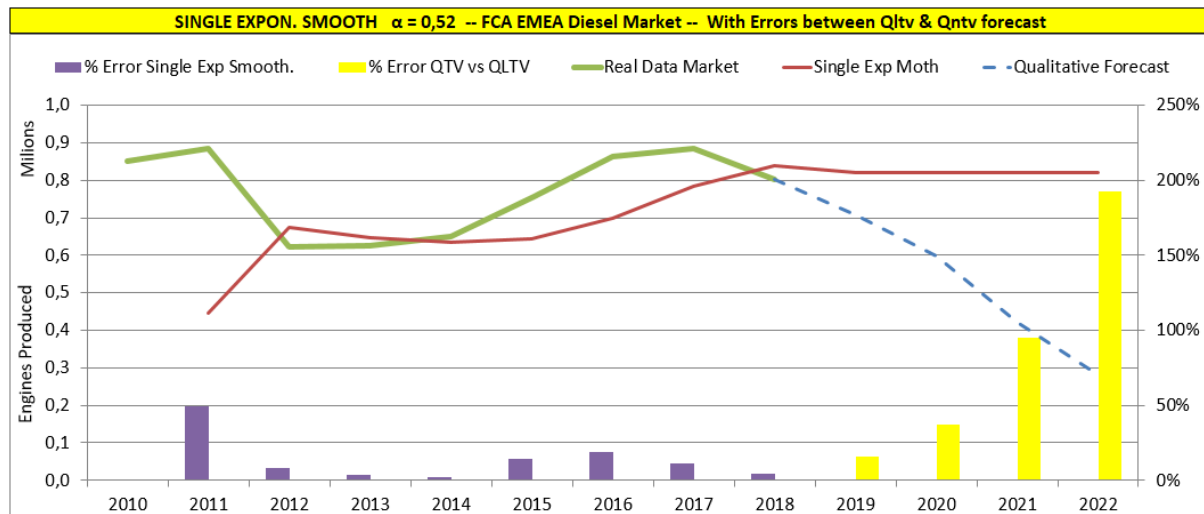
Graph 4.29 - Analysis of EMEA FCA Diesel Market with Weighted Mov. Avg. Method

Summary Table - Weighted Moving Avg				
Diesel Market	K=2	K=3	K=4	K=5
Sigma (Unit of Engines)	128.506	78.666	82.450	72.492

Table 4.57 – “K” values and the related Sigma

As for the other cases, also for the Weighted Moving Average the study has been performed with K=2, K=3, K=4 and K=5 (Table 4.57). In Graph 4.29, it is reported only the result able to minimize the standard deviation. Moreover, to properly distribute the weights among the K elements considered, the Excel solver has been used.

In the Appedix 4.21 it is possible to find the data as well as the other graphs with different Ks.



Graph 4.30 - Analysis of EMEA FCA Diesel Market with Single Exponential Smoothing Method

Summary Table - Single Exp Smooth					
Diesel Market	Alpha = 0,4	Alpha = 0,45	Alpha = 0,5	Alpha = 0,52	Alpha = 0,55
Sigma (Unit of Engines)	107.451	93.628	87.890	87.374	88.026

Table 4.58 – Alpha values and the related Sigma for Single Exponential Smoothing on Diesel Market

The Alpha value in Graph 4.30 has been obtained thanks to the Excel solver in order to minimize the standard deviation. In Table 4.58 there are some values of Sigma for different Alpha. The Appendix 4.22 shows the data used for the analysis.

Comparing these different methodologies, it has been possible to see which one was best to represent the expected trend. It is clearly noticeable that the Wighted Moving Average with K=5 wins over the others (see Table 4.59).

Diesel Market	Moving Average	Weighted Moving Avg	Simple Exp Smoth
Sigma (Unit of Engines)	140.009	72.492	87.374

Table 4.59 – Best method with lower Sigma for Diesel Market

To summarize, considering the EMEA world with only FCA as car manufacturer, it is possible to see that the Gasoline market is better represented by the Brown method while the Diesel scenario is better represented with the Weighted Moving Average technique.

4.5.3 Which information are provided by the optimized model

Looking at the final results, it is possible to understand that the relative errors between the qualitative forecasts made by the company and the values obtained by the model for the Gasoline case are much lower than those for the Diesel one. This can be explicable as the two markets are quite different from each other as already said. This is even more true looking at FCA data in EMEA. They based the largest part of their sales on Diesel engines and now they are suffering more than the average to keep up with the new EV-trends. Moreover, as already discussed for the general trends, it is harder for the model to react at this unexpected Diesel decrease leading to a worse quality of the market forecast. Especially for the Diesel data, it would be better to rely on them just for the first year of the forecast to have a first picture of the short-term volumes.

4.5.4 Comparison with the data extracted with qualitative forecasting

In Table 4.60 it is possible to find the data for the next years obtained by the company using the qualitative approach.

		Qualitative Forecast (Unit of EMEA FCA Brand Engines)			
Market	Year	2019	2020	2021	2022
FCA EMEA Gasoline Market		1.056.309	1.023.160	1.223.647	1.290.835
FCA EMEA Diesel Market		707.210	598.808	420.412	280.164

Table 4.60 – Qualitative data for EMEA FCA market

In this section, a deeper analysis on the differences between qualitative and quantitative forecasts is performed.

In the Table 4.61, Table 4.62 and Table 4.63 it is possible to find the deviation from the consolidated values and from the future forecast for the Gasoline market adopting the Moving Average Methodology, Weighted Moving Average Methodology and Single Exponential Smoothing Methodology respectively.

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012	-157.023	24.656.222.529	17,27%		
	2013	11.840	140.173.760	1,24%		
	2014	65.926	4.346.237.476	6,60%		
	2015	201.637	40.657.479.769	17,08%		
	2016	131.962	17.413.969.444	10,80%		
	2017	-11.408	130.142.464	0,96%		
	2018	-56.883	3.235.618.806	4,95%		
Forecast	2019				113.087	10,71%
	2020				136.035	13,30%
	2021				59.352	4,85%
	2022				129.090	10,00%

Table 4.60 – Gasoline Market – Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012	-74.289	5.518.854.070	8,17%		
	2013	48.984	2.399.433.141	5,11%		
	2014	41.434	1.716.777.145	4,15%		
	2015	180.920	32.732.049.993	15,33%		
	2016	41.502	1.722.416.977	3,40%		
	2017	-32.159	1.034.200.500	2,70%		
	2018	-40.803	1.664.883.844	3,55%		
Forecast	2019				92.685	8,77%
	2020				125.834	12,30%
	2021				74.653	6,10%
	2022				141.841	10,99%

Table 4.61 – Gasoline Market – Weighted Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	-165.468	27.379.659.024	16,83%		
	2012	-74.289	5.518.855.521	8,17%		
	2013	48.984	2.399.432.256	5,11%		
	2014	41.434	1.716.776.356	4,15%		
	2015	180.920	32.732.046.400	15,33%		
	2016	41.502	1.722.416.004	3,40%		
	2017	-32.159	1.034.201.281	2,70%		
	2018	-40.803	1.664.884.809	3,55%		
Forecast	2019				92.685	8,77%
	2020				125.834	12,30%
	2021				74.653	6,10%
	2022				141.841	10,99%

Table 4.62 – Gasoline Market – Single Exponential Smoothing Data

Starting from the Gasoline market, the tables reported above refer to the Moving Average (Table 4.60), Weighted Moving Average (Table 4.61) and Single Exponential Smoothing (Table 4.62) respectively. As it is possible to see, the three methodologies are quite good in general and can be used also for long term forecasting as the series are quite stable in time.

The column % Error represents the distance in % of the real data from the forecasted ones. Instead, the column % Error Qtv vs Qltv represents how close to the qualitative forecast made by the company is the prevision performed by the applied models.

Moving to the Diesel market, again the tables reported above refer to the Moving Average (Table 4.63), Weighted Moving Average (Table 4.64) and Single Exponential Smoothing (Table 4.65) respectively.

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012					
	2013	-162.136	26.288.082.496	20,62%		
	2014	-61.642	3.799.695.069	8,68%		
	2015	120.245	14.458.860.025	19,03%		
	2016	188.238	35.433.544.644	27,88%		
	2017	130.741	17.093.296.242	17,32%		
	2018	-30.651	939.504.235	3,68%		
Forecast	2019				143.415	20,28%
	2020				247.589	41,35%
	2021				412.943	98,22%
	2022				563.295	201,06%

Table 4.63 – Diesel Market – Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011					
	2012					
	2013					
	2014					
	2015	46.432	2.155.910.728	6,17%		
	2016	74.025	5.479.756.104	8,57%		
	2017	89.709	8.047.665.729	10,13%		
	2018	-9.064	82.154.091	1,13%		
Forecast	2019				52.501	7,42%
	2020				158.806	26,52%
	2021				366.893	87,27%
	2022				534.732	190,86%

Table 4.64 – Diesel Market – Weighted Moving Average Data

	Year	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010					
	2011	439.086	192.796.269.795	49,67%		
	2012	-51.342	2.636.043.962	8,24%		
	2013	-23.397	547.403.105	3,75%		
	2014	13.448	180.860.568	2,07%		
	2015	109.879	12.073.303.373	14,61%		
	2016	163.535	26.743.637.104	18,94%		
	2017	100.315	10.063.195.075	11,33%		
	2018	-34.574	1.195.378.262	4,31%		
Forecast	2019				112.344	15,89%
	2020				220.746	36,86%
	2021				399.142	94,94%
	2022				539.390	192,53%

Table 4.65 – Diesel Market – Single Exponential Smoothing Data

As it is possible to see, the second methodology is the best one for the short term. For the years after 2019 it remains the best one, but with not very reliable values. As already said in the EMEA analysis with all car manufacturers and in the previous one, this inability to follow an unpredicted path can be noticed as a weak point of these models.

4.5.5 Is it the model able to correctly interpret the past and so can be used for the forecasting activity?

To summarize, these methodologies can be applied for the long-term forecasting only for the Gasoline case as the series variation are smaller. Talking about the Diesel case, it would be better to adopt these forecasts only for the short-term decisions. In any case, they could be useful in order to understand at least the general plans of the specific car manufacturer.

4.5.6 Considerations

In this case, as already said before, for the Gasoline market there are no doubts that it will grow and so the actual offer has to be kept as it is with some adjustments based on the new engine developed. It clearly seems that FCA will be subjected to big changes in the next years. After this third analysis, it is possible to see how the biggest producer of Diesel engines is going to react at the new environmental issues in EMEA (the strictest market in terms of regulation). Having this analysis in mind, it is not difficult to say that it would be better for EATON to focus more on products to be used in Gasoline engines. Moreover, they need to be good in understanding this transitory period and, by cooperating with FCA, to propose some competitive products in line with the new EV trends. Finally, it could be a good idea not to abandon completely the Diesel engines as, if the hybridization on these power units will start to develop, there will be the possibility of seeing again a great growth in that segment.

4.5.7 Comparative tables

Below it is possible to find a synthetic table to compare all the methods used for the analysis. In Table 4.66, it is possible to find the Best Sigma for all the methodologies adopted. Instead, in Table 4.67 the Gasoline forecasts with the Moving Average, Weighted Moving Average and Single Exponential Smoothing methodologies are reported and compared with the qualitative numbers. Finally, in Table 4.68 the forecasts for the Diesel market with Moving Average, Weighted Moving Average and Single Exponential Smoothing methodologies are shown and again compared with the qualitative numbers.

Comparative Methods Table - EMEA FCA			
	Moving Average	Weighted Moving Avg	Simple Exp Smoth
Sigma Gasoline (Unit of Engines)	122.868	88.307	81.756
Sigma Diesel (Unit of Engines)	140.009	72.492	87.374

Table 4.66 – Best Sigma for Gasoline and Diesel FCA EMEA Market

	Year	FCA EMEA Gasoline Market (Unit Engines)	Qualitative Forecast (Unit Engines)	Moving Average K=2 (Unit Engines)	Weighted Moving Avg K=2 (Unit Engines)	Single Exponential Smoth (Unit Engines)
Real Data Market	2010	1.148.873	1.148.994			
	2011	983.405				1.148.873
	2012	909.116		1.066.139	983.405	983.405
	2013	958.100		946.261	909.116	909.116
	2014	999.534		933.608	958.100	958.100
	2015	1.180.454		978.817	999.534	999.534
	2016	1.221.956		1.089.994	1.180.454	1.180.454
	2017	1.189.797		1.201.205	1.221.956	1.221.956
	2018	1.148.994		1.205.877	1.189.797	1.189.797
Forecast	2019		1.056.309	1.169.396	1.148.994	1.148.994
	2020		1.023.160	1.159.195	1.148.994	1.148.994
	2021		1.223.647	1.164.295	1.148.994	1.148.994
	2022		1.290.835	1.161.745	1.148.994	1.148.994

Table 4.67 – All Methods used for Gasoline EMEA FCA Market vs Qualitative Forecast

	Year	FCA EMEA Diesel Market (Unit Engines)	Qualitative Forecast (Unit Engines)	Moving Average K=3 (Unit Engines)	Weighted Moving Avg K=5 (Unit Engines)	Single Exponential Smoth (Unit Engines)
Real Data Market	2010	851.787	803.042			
	2011	884.084				444.998
	2012	623.047				674.389
	2013	624.170		786.306		647.567
	2014	648.792		710.434		635.344
	2015	752.248		632.003	705.816	642.369
	2016	863.308		675.070	789.283	699.773
	2017	885.524		754.783	795.815	785.209
	2018	803.042		833.693	812.106	837.616
Forecast	2019		707.210	850.625	759.711	819.554
	2020		598.808	846.397	757.614	819.554
	2021		420.412	833.355	787.305	819.554
	2022		280.164	843.459	814.896	819.554

Table 4.68 – All Methods used for Diesel EMEA FCA Market vs Qualitative Forecast

To conclude this analysis, below (Table 4.69) it is possible to find, for each scenario analysed, which was the best methodology adopted to forecast the future sales and the suggestion for the company if it would be better to apply that model on the long-term or on the short-term.

Scenario	Fuel Type	Best Method	Applicability
All Car Manufacturers <i>All the World</i>	Gasoline	Trend	Long-term / Short-term
	Diesel	Trend	Short-term
All Car Manufacturers <i>EMEA</i>	Gasoline	Double Exp Smooth	Short-term
	Diesel	Weighted Moving Average	Short-term
FCA <i>All the World</i>	Gasoline	Weighted Moving Average	Long-term / Short-term
	Diesel	Weighted Moving Average	Short-term
FCA <i>EMEA</i>	Gasoline	Single Exp Smooth	Long-term / Short-term
	Diesel	Weighted Moving Average	Short-term

Table 4.69 – Applicability of Each Scenario Analysed

CHAPTER 5 - ARIMA

In this chapter a deeper analysis on the Arima (that belongs to the world of the Autoregressive models) methodology will be performed. Infact, it is possible to say that the more the model becomes complicated and structured, the more it will be able to better interpret the future trends. In this case, thanks to this technique, the autoregressive (AR) components as well as the moving average (MA) components are taken into account. More precisely, the 'AR' can be considered as the part able to model the "change since last time": The 'MA' part instead is able to capture smoothed trends in the data: Finally, the 'I' in ARIMA helps to make the data stationary

5.1 Autoregressive models: Introduction

The autoregressive models are a very useful tool to deal with the problem of forecasting in relation to an annual historical series. A strong correlation is often observed between consecutive values of a series; in this case we speak of autocorrelation, of the first order when we consider adjacent values, of the second order if we refer to the relationship between the values of the series at a distance of two periods and, in general, of the p -th order if the values considered "distant" from each other p periods. The autoregressive models allow precisely to exploit these dependency bonds to obtain useful predictions of the future behavior of the series (Levine, 2002).

In the following equations are reported three important autoregressive models:

First Order Autoregressive Model

$$Y_i = A_0 + A_1 Y_{i-1} + \delta_i \quad (5.1)$$

Second Order Autoregressive Model

$$Y_i = A_0 + A_1 Y_{i-1} + A_2 Y_{i-2} + \delta_i \quad (5.2)$$

P-th Order Autoregressive Model

$$Y_i = A_0 + A_1Y_{i-1} + A_2Y_{i-2} + \dots + A_pY_{i-p} + \delta_i \quad (5.3)$$

Where:

Y_t = observed value at the time i

Y_{t-1} = observed value at the time $i-1$

Y_{t-2} = observed value at the time $i-2$

Y_{t-p} = observed value at the time $i-p$

A_0 = constant to be estimated with the least squares method

A_1, A_2, \dots, A_p = autoregressive parameters to estimate with least squares method

δ_t = component of non-autocorrelated error, of zero mean and with constant variance

Choosing between different levels of autoregressive models means establishing the extent of the relationships between delayed observations with which one intends to work. The autoregressive model of the first order involves only the relations between consecutive variables of the historical series, in the autoregressive model of the second order, in addition to the relations between consecutive observations, is also taken into account the link between delayed observations of two periods, and so on. This logic can be applied up to the autoregressive model of the p -th order which involves all the relations between variables that are 1, 2, ..., p periods apart (Levine, 2002).

The choice is therefore not easy; there is also a trade-off between the simplicity of the lower order models and the possible greater explanatory capacity of those of a higher order. The length of the series (n) must also be taken into account with respect to which p , the order of the model, must not be excessively high.

Once the model has been chosen and the least squares method has been applied to estimate the parameters, it is necessary to define criteria that allow to assess the adaptability of the chosen model. One possibility is to estimate a model with a fairly high number of parameters, to then determine whether it is appropriate to eliminate some of them. In practice it is a matter of solving a problem of verification of hypotheses on the significance of the parameters that are gradually found at the last order of the model. In an autoregressive model of order p we will therefore make the following hypotheses on the parameter A_p (autoregressive parameter of maximum order).

H0: $A_p = 0$ (the maximum order parameter is equal to zero)

H1: $A_p \neq 0$ (the maximum order parameter is equal to zero)

The t test for the significance of the autoregressive parameter of maximum order is:

$$t = \frac{a_p - A_p}{S_{a_p}} \quad (5.4)$$

Where:

a_p = estimate of the autoregressive parameter of maximum order A_p

S_{a_p} = standard deviation of a_p

It can be shown that this test follows a t Student distribution with $n-2p-1$ degrees of freedom. Therefore, once the significance level α has been set, the null hypothesis must be rejected if the observed value of the statistic test is higher in module than the critical value t_{n-2p-1} of the corresponding t Student distribution. The following decision rule is then reached:

Refuse H_0 if $t > t_{n-2p-1}$ or $t < -t_{n-2p-1}$;

Otherwise accept H_0

If the observed value of the statistic test leads us to not reject the null hypothesis $A_p = 0$, we must conclude that the analyzed model contains an excessively high number of parameters. The autoregressive component of maximum order is therefore discarded and, once the new model has been determined, the procedure must be repeated on parameter A_{p-1} , which represents the new autoregressive parameter of maximum order.

The procedure continues until the null hypothesis is rejected. When this happens, the analyst can be sure of the significance of the last autoregressive parameter and can therefore use the selected model for forecasting purposes (Anderson et al, 2016).

Once the optimal number of autoregressive components has been identified with the method described above, it is possible to proceed with the estimation of the parameters.

The Estimated Autoregressive Model of the p -th Order

$$\hat{Y}_i = a_0 + a_1 Y_{i-1} + a_2 Y_{i-2} + \dots + a_p Y_{i-p} \quad (5.5)$$

Where:

- \hat{Y}_i = estimated value at the time i
- Y_{i-1} = observed value at the time $i-1$
- Y_{i-2} = observed value at the time $i-2$
- Y_{i-p} = observed value at the time $i-p$
- $a_0, a_1, a_2, \dots, a_p$ = estimated parameters

5.2 Use of the autoregressive model for forecasting purposes

As expressed by the equation 5.6, it is possible to observe that, when an autoregressive model of the highest order for forecasting purposes it is applied, the number of observations that come into play in the forecast is always equal to p , regardless of the distance j in the future of the value we want to predict. So, if $p = 3$, a forecast of j periods after the instant n will be based solely on the values observed in the years $n, n - 1, n - 2$.

$$\hat{Y}_{n+j} = a_0 + a_1 \hat{Y}_{n+j-1} + a_2 \hat{Y}_{n+j-2} + \dots + a_p \hat{Y}_{n+j-p} \quad (5.6)$$

Where:

- $a_0, a_1, a_2, \dots, a_p$ = estimated parameters
- j = number of future years
- \hat{Y}_{n+j-p} = forecast done at the instant n for the instant Y_{n+j-p} se $j - p > 0$
- \hat{Y}_{n+j-p} = observed value of Y_{n+j-p} if $j - p \leq 0$

Applying the previous equation, we get the following one-year forecast:

$$\hat{Y}_{n+1} = a_0 + a_1 Y_n + a_2 Y_{n-1} + a_3 Y_{n-2} \quad (5.7)$$

The one-year forecast comes into play in determining the two-year forecast:

$$\hat{Y}_{n+2} = a_0 + a_1 \hat{Y}_{n+1} + a_2 Y_n + a_3 Y_{n-1} \quad (5.8)$$

Proceeding iteratively, forecasts are obtained for subsequent years:

$$\hat{Y}_{n+3} = a_0 + a_1\hat{Y}_{n+2} + a_2\hat{Y}_{n+1} + a_3Y_n \quad (5.9)$$

$$\hat{Y}_{n+4} = a_0 + a_1\hat{Y}_{n+3} + a_2\hat{Y}_{n+2} + a_3\hat{Y}_{n+1} \quad (5.10)$$

And so on.

5.3 Analysis of an annual historical series through autoregressive models

In order to perform the analysis with the autoregressive models properly, the following list has been created:

1. Choose the p order of the initial model.
2. Estimate a multiple regression model with the predictors represented by the delayed Y variables.
3. Perform a test for the significance of the autoregressive parameter of maximum order A_p .
 - a. If the test leads to reject the null hypothesis, the model with p predictors must be chosen to represent the series and to make predictions
 - b. If the test leads to accept the null hypothesis, the last predictor must be discarded. Now consider the model with one less regressor. Check the significance of the autoregressive maximum order parameter of the new model. The procedure continues until a model is identified whose maximum-order autoregressive parameter is significant.
4. The model thus selected can be used to interpolate the observations and to predict future values of the series (Levine, 2002).

5.4 Forecasting Model Selection

Once different models have been applied, it is important to have some criteria to understand which is the most suitable for our case. The guidelines for this choice can be summarized as follows:

1. Residues Analysis.
2. Measurement of the magnitude of the residual error through the squared differences method.
3. Measurement of the magnitude of the residual error through the method of differences in absolute value.
4. Application of the parsimony principle.

Residues Analysis

The residues of the model are obtained as the difference between the observed values and the values obtained with the model itself ("interpolated"): $(Y_i - \hat{Y}_i) = e_i$. From the residue chart it is possible to evaluate the model's ability to capture the different components of the time series.

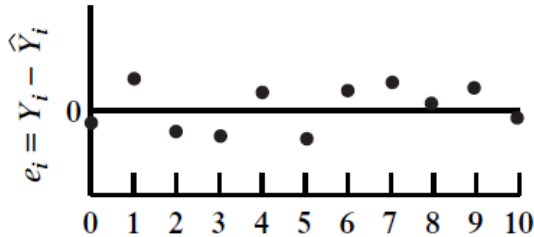


Figure 5.1 - Residues with random trend
(A) Adequate model (Levine 2002)

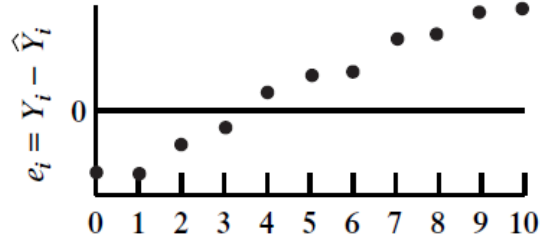


Figure 5.2 - Residues with systematic trend
(B) Model unable to capture the trend
(Levine 2002)

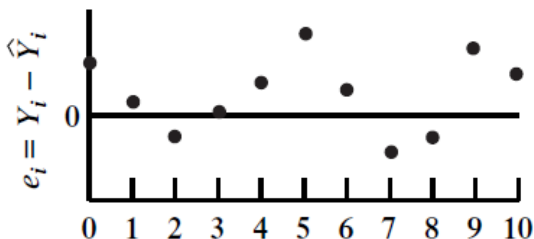


Figure 5.3 - Residues with systematic trend
(C) Model unable to capture the cyclical component (Levine 2002)

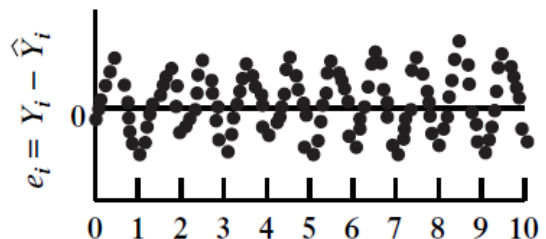


Figure 5.4 - Residues with systematic trend
(D) Model unable to capture the seasonal component (Levine 2002)

When the model appropriately interpolates the observations, the residuals take on the typical random trend exemplified in box A of the Figure 5.1. Instead, in the boxes from B to D there are reported residues following a systematic trend, signaling the inadequacy of the respective models to capture the trend components (Figure 5.2), cyclical (Figure 5.3) or seasonal (Figure 5.4) of the series (Levine, 2002).

Measurement of the Magnitude of the Residual Error Through the Method of Squared Differences and Differences in Absolute Value

Suppose the choice is between two models that, from the point of view of the residual trend, seem to explain our historical series just as well. In this case we need another method to help us in choosing the model. The reference is to the method, based on the principle of least squares, of the standard error of the S_{YX} estimate, which can be calculated as the sum of the squared differences between the observed values and those interpolated using the model. Obviously, if the model perfectly interpolates the observations, the value of this indicator will be zero; the value of the index tends to increase considering models less and less suitable for the representation of the series.

Some authors consider this index inadequate because, based on quadratic deviations, it leads to an excessive penalization of models in which there are single errors of prediction that are particularly high. It is therefore considered more reliable an index that involves the differences in absolute value between observed values and expected values, the *MAD* (absolute mean deviation).

The *MAD* therefore represents a measure of the quality of the model. Lower values of the index correspond to models that better interpolate the observations. This there is another criterion available that allows us to examine alternative models for the same historical series: we will choose the model with minimum *MAD*.

Parsimony Principle

When different models seem to be equivalent based on the criteria described above, in the choice it must be kept in mind one last intuitive consideration: for the same performance, the simplest model (principle of parsimony) must be preferred.

5.5 Introduction to ARIMA Models

ARIMA is the abbreviation for 'Auto Regressive Integrated Moving Average'. It is actually a class of models that 'explains' a given time series based on its own past values, that is, its own lags and the lagged forecast errors, so that equation can be used to forecast future values.

Any 'non-seasonal' time series that exhibits patterns and is not a random white noise can be modeled with ARIMA models.

An ARIMA model is characterized by 3 terms: p , d , q . More precisely:

p is the order of the Autoregressive term

q is the order of the Moving Average term

d is the number of differencing required to make the time series stationary

5.6 What does the p , d and q in ARIMA Model Mean

The first step to build an ARIMA model is to make the time series stationary. This because, term 'Auto Regressive' in ARIMA means it is a linear regression model that uses its own lags as predictors. Linear regression models work best when the predictors are not correlated and are independent of each other. The most common approach to make a series stationary is to difference it. That is, subtract the previous value from the current value.

Sometimes, depending on the complexity of the series, more than one differencing may be needed. The value of d , therefore, is the minimum number of differencing needed to make the series stationary. And if the time series is already stationary, then $d = 0$. Finally, the ' p ' is the order of the 'Auto Regressive' (AR) term. It refers to the number of lags of Y to be used as predictors. And ' q ' is the order of the 'Moving Average' (MA) term; it refers to the number of lagged forecast errors that should go into the ARIMA Model (Anderson et al, 2016).

5.7 What are AR and MA Models

The actual mathematical formula for AR and MA models can be described as follows.

A pure **Auto Regressive (AR only) model** is one where Y_t depends only on its own lags. That is, Y_t is a function of the 'lags of Y_t '.

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \epsilon_t \quad (5.11)$$

Where:

Y_{t-1} is the lag1 of the series

β_1 is the coefficient of lag1 that the model estimates

α is the intercept term, also estimated by the model

Likewise, a pure **Moving Average (MA only) model** is one where Y_t depends only on the lagged forecast errors (Faculty of Economic Informatics).

$$Y_t = \alpha + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \dots + \phi_q \epsilon_{t-q} \quad (5.12)$$

Where:

- The error terms are the errors of the autoregressive models of the respective lags
- The errors ϵ_t and $\epsilon_{(t-1)}$ are the errors from the following equations

$$Y_t = \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_0 Y_0 + \epsilon_t \quad (5.13)$$

$$Y_{t-1} = \beta_1 Y_{t-2} + \beta_2 Y_{t-3} + \dots + \beta_0 Y_0 + \epsilon_{t-1} \quad (5.14)$$

That was AR and MA models respectively.

An ARIMA model is one where the time series was differenced at least once to make it stationary and you combine the AR and the MA terms. The equation follows:

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \dots + \phi_q \epsilon_{t-q} \quad (5.15)$$

Trying to summarize the ARIMA model in words, it would be possible to say:

Predicted Y_t = Constant + Linear combination Lags of Y (upto p lags) + Linear Combination of Lagged forecast errors (upto q lags)

5.8 ARIMA Processes

Here below a brief introduction into the ARIMA process. The first point is related to the Auto-Regressive component while the second point will explain better the Moving average part.

1. Auto-regressive Process: ARIMA (1,0,0):

$$Y_t = \phi Y_{t-1} + e_t \quad \text{or} \quad Y_t = \theta + \phi Y_{t-1} + e_t \quad (5.16)$$

(5.17)

Bound of Stationary: the absolute value of $\phi < 1$, ($-1 < \phi < 1$).

If $\phi = 1$, it becomes ARIMA (0,1,0) which is non-stationary. If $\phi > 1$, the past values of Y_{t-k} and e_{t-k} have greater and greater influence on Y_t , it implies the series is non-stationary with an ever-increasing mean. To sum up, If Bound of Stationary does not hold, the series is not autoregressive; it is either drifting or trending, and first-difference should be used to model the series with stationary.

Autoregressive Process: ARIMA (p,0,0)

$$Y_t = \theta + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t \quad (5.18)$$

Or

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t \quad (5.19)$$

2. Moving Average Process: ARIMA (0,0,1)

$$Y_t = \theta e_{t-1} + e_t \quad \text{or} \quad Y_t = \alpha + \theta e_{t-1} + e_t \quad (5.20) \quad (5.21)$$

Writing an ARIMA (0,0,1) process at two points in time,

$$Y_t = \theta e_{t-1} + e_t \quad (5.22)$$

$$\text{And} \quad Y_t = \theta e_{t-2} + e_{t-1} \quad (5.23)$$

When substitute the expression for e_{t-1} into the expression for Y_t ,

$$Y_t = e_t + \theta(Y_{t-1} + \theta e_{t-2}) = \theta Y_{t-1} + e_t + \theta^2 e_{t-2} \quad (5.24)$$

And continuing this substitution back into time,

$$Y_t = e_t + \sum_{i=1}^{\infty} \theta^i Y_{t-i} \quad (5.25)$$

Therefore, an ARIMA (0,0,1) can be expressed identically as the infinite sum of exponentially weighted past observation of the process. Extension to any ARIMA (0,0,q) can also be expressed as an infinite series of exponentially

weighted past observations. Given this relationship, the values of moving average parameters must be constrained between -1 and 1 ($-1 < \theta < 1$).

Bound of Invertibility: The absolute value of θ is less than 1 ($-1 < \theta < 1$). If not hold, the model is non-stationary.

Moving Average Process: ARIMA (0,0,q)

$$Y_t = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_q e_{t-q} \quad (5.26)$$

The important feature of such ARIMA (0,0,q) is that the variables of e_{t-1} to e_{t-q} are unobserved and have to be estimated using the available sample data. In practice, it is usual to keep q at a small value, and it is often set at 1 or 2 (Faculty of Economic Informatics).

3. Integrated Process: ARIMA (0,1,0)

- Random Walk Process: ARIMA (0,1,0):

$$Y_t = Y_{t-1} + e_t \rightarrow Y_t - Y_{t-1} = e_t \rightarrow \Delta Y_t = e_t \quad (5.27)$$

All future values are expected to equal the last known actual value

- Deterministic Trend Process: ARIMA (0,1,0):

$$Y_t = Y_{t-1} + T + e_t \rightarrow \Delta Y_t = T + e_t \quad (5.28)$$

Where:

T is the trend

$$Y_{t+m} = Y_{t+m-1} + mT + e_t \rightarrow \Delta Y_{t+m} = mT + e_t \quad (5.29)$$

Where:

m is the forecast horizon

4. ARIMA (p,0,q):

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q} \quad (5.30)$$

5. ARIMA (p,1,q):

$$\Delta Y_t = \phi_1 \Delta Y_{t-1} + \phi_2 \Delta Y_{t-2} + \dots + \phi_p \Delta Y_{t-p} + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q} \quad (5.31)$$

5.9 ARIMA Model Application

To understand if the application of these models can really be beneficial to the future forecasts, a practical example has been done. Thanks to an Excel library **xlstat**, it has been possible to perform an ARIMA analysis on the data gathered. In particular, it has been decided to analyse the EMEA market with all the car manufacturers. In Figure 5.5 it is possible to find a picture of the interface the excel tool has.

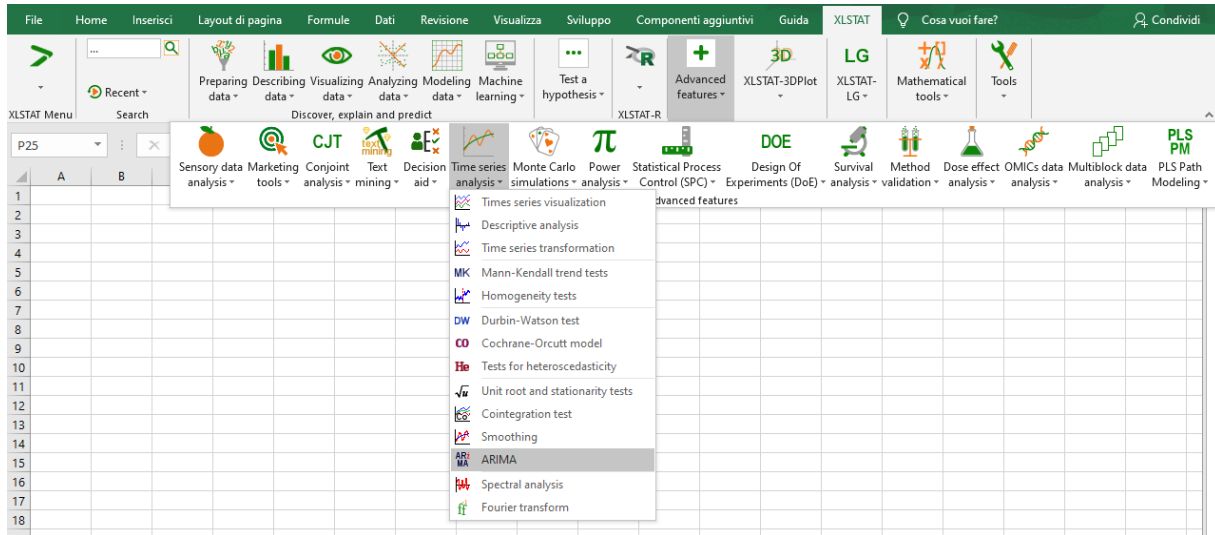


Figure 5.5 – xlstat excel interface

In the Graphs 5.1 and 5.3, it is possible to find the main results of the study. Thanks to the solver provided with the excel tool, it has been possible running a lot of simulations, but the results which will be shown later are just the ones able to provide

the best solution (lowest σ_k). During each trial, it was the internal solver that was optimizing the three variables (p , d , q), having as its purpose that of looking for the minimum σ_k (Figure 5.6).

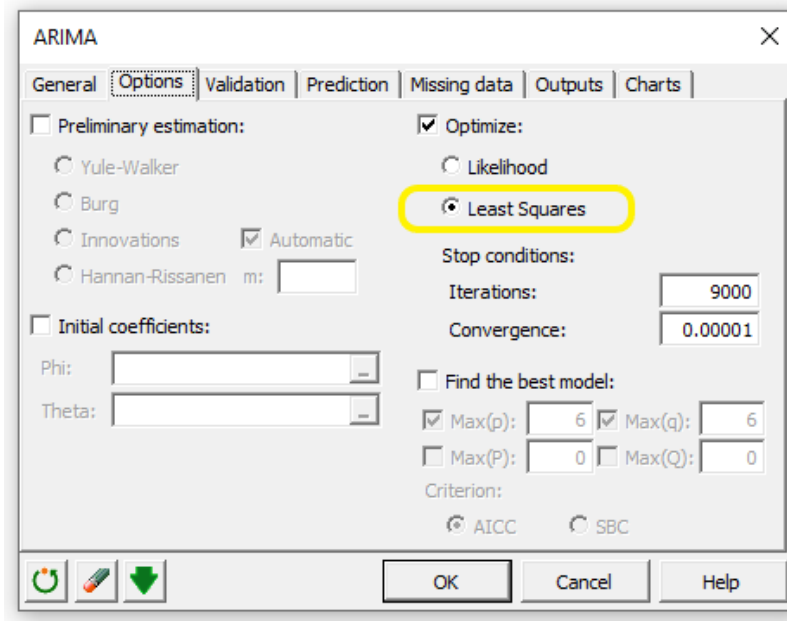
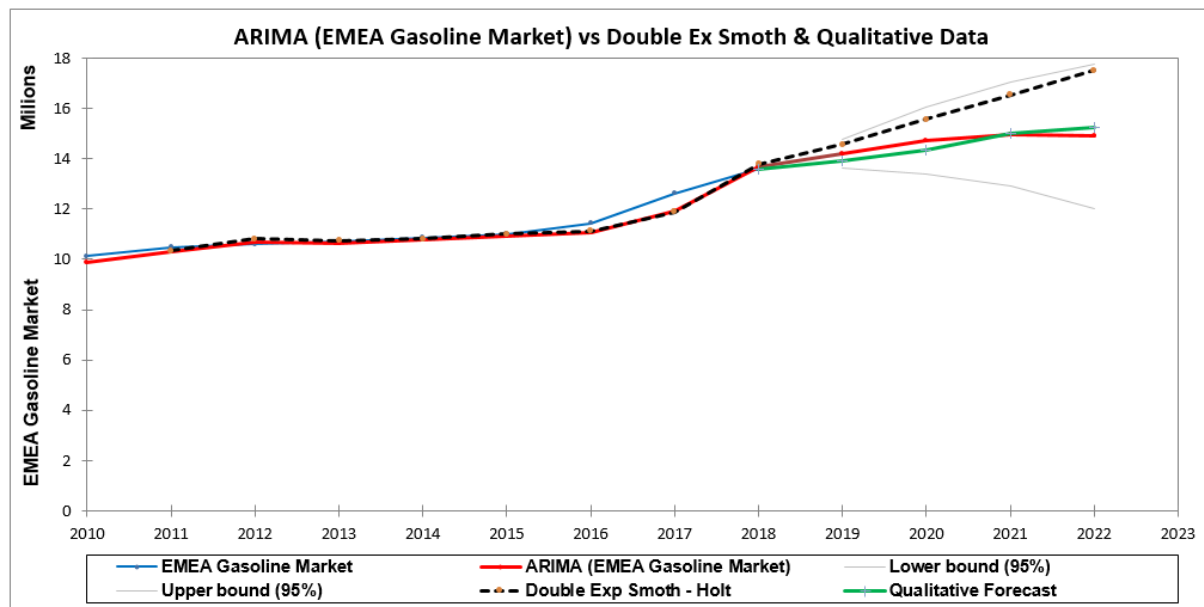
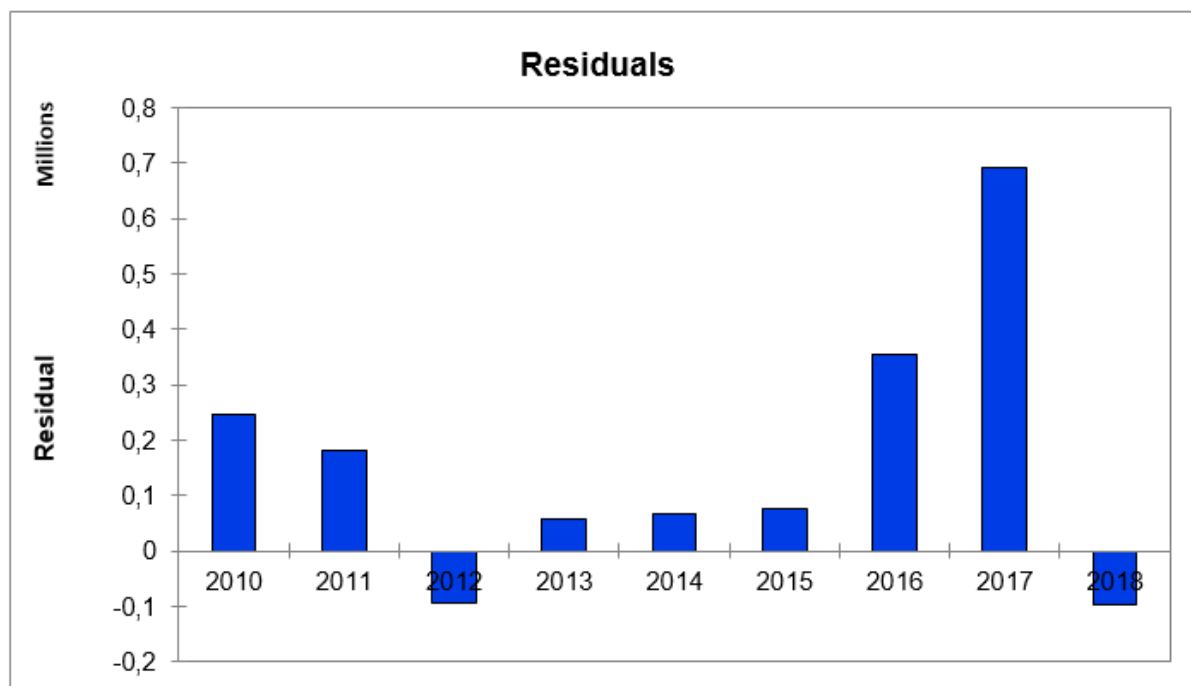


Figure 5.6 – xlstat solver

Starting from the Gasoline case, the best method found to optimize the model was the Double Exponential Smoothing. The results of this model have been compared with those coming from the ARIMA (1,0,5) one: where 1 represents the Autoregressive Part, 0 represents the stationarity of the model (and so the grade of the derivative) and finally the 5 represents the periods over which the Moving Average works.



Graph 5.1 - EMEA - All Car Manufacturers - Gasoline Case – Double Exponential Smoothing and ARIMA (1,0,5)



Graph 5.2 - EMEA - All Car Manufacturers - Gasoline Case – Residuals

In the Graph 5.1, it is represented the Double Exponential Smoothing behaviour (as it has already been discussed in the chapter 4) in comparison with the ARIMA results obtained. Instead, in the Graph 5.2, it is possible to find the residuals values obtained by applying the model. As discussed before, these seem to have a random trend

and so the ARIMA model used should be the right one to represent the future demand for this specific market.

The difference is clear even just by looking at the differences between the line of the Double Exponential (black dotted) and the line of the ARIMA forecast (the solid red one): the second computation is more accurate than the first one.

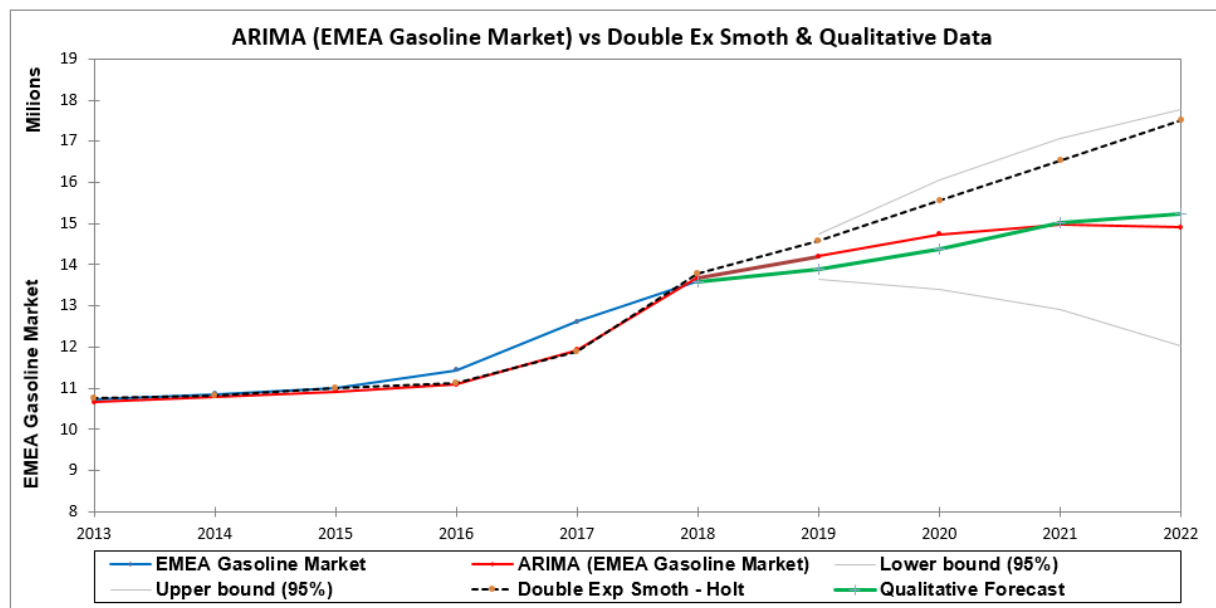
Looking now at the σ_k values (Table 5.1), it will be clearer the difference between the methodologies adopted in chapter 4 and the current one.

Comparative Methods Table Gasoline - EMEA All Brands			
	Double Exp Smoth	Trend	ARIMA
Sigma Gasoline (Unit Engines)	324.906	492.048	284.378

Table 5.1 – Comparative Table EMEA – All Car Manufacturers - Gasoline

As already anticipated just by looking at the graphic result, the ARIMA (1,0,5) model is better (meaning that it has a smaller σ_k) in forecasting the future than the Double Exponential Smoothing.

In the Graph 5.3 here below, there is a zoomed graph of the comparison between the two methodologies:



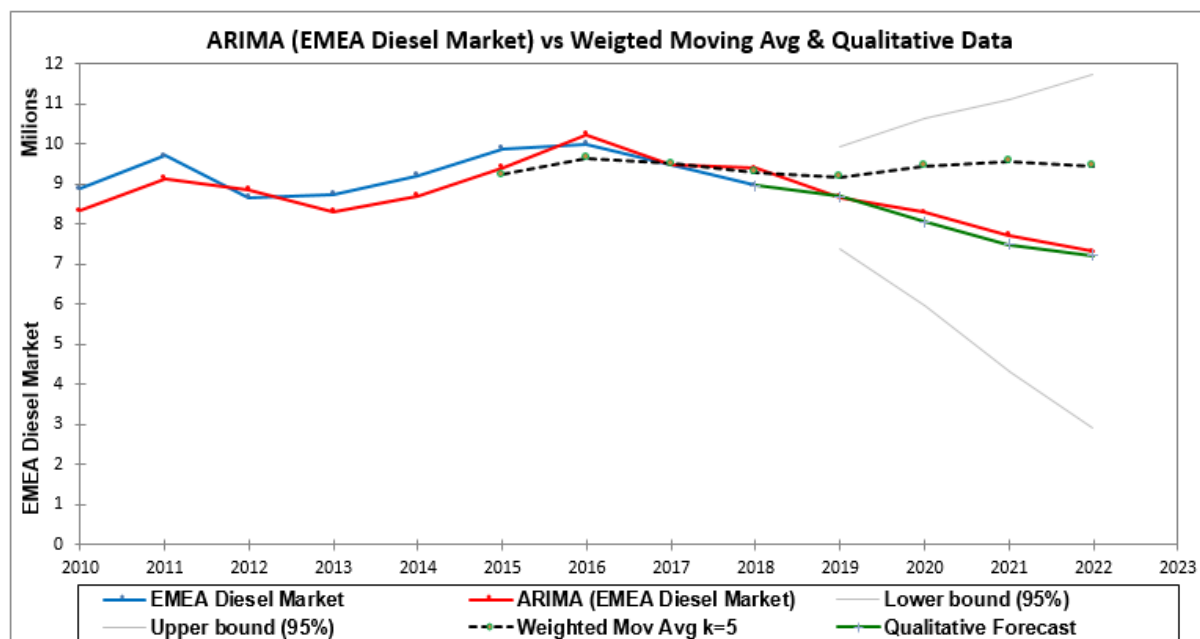
Graph 5.3 - EMEA - All Car Manufacturers - Gasoline Case – Double Exp. Smooth. and ARIMA (1,0,5) zoom

In the Table 5.2, it is possible to find the data used to create the Graph 5.3.

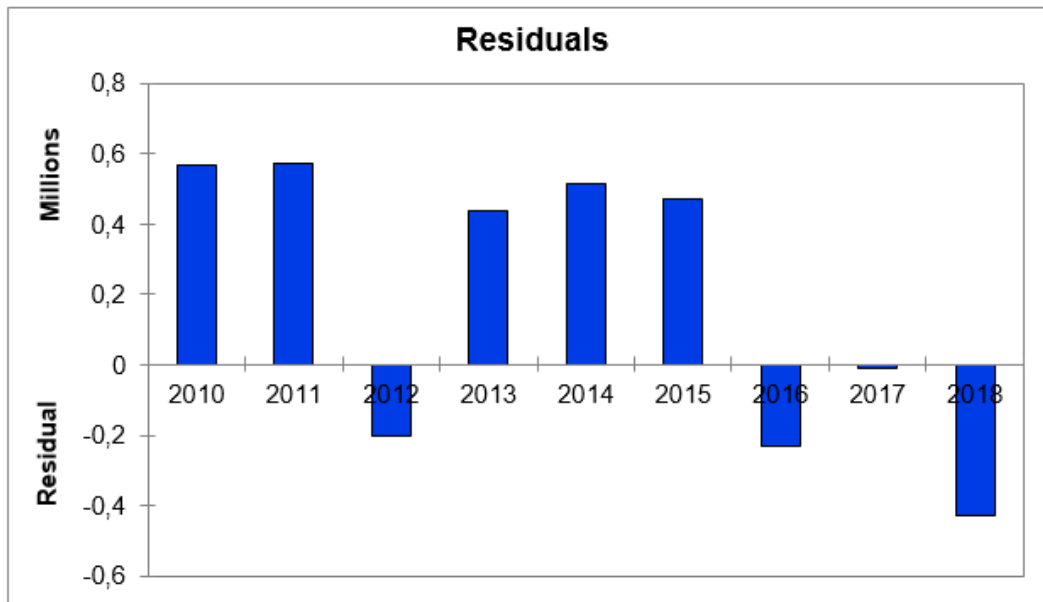
Observations	EMEA Gasoline Market	ARIMA(EMEA Gasoline Market)	Residuals	Standard error	Lower bound (95%)	Upper bound (95%)
2010	10.138.834	9.893.187	245.647			
2011	10.474.762	10.292.595	182.167			
2012	10.614.814	10.708.605	- 93.791			
2013	10.717.087	10.658.992	58.095			
2014	10.868.503	10.802.018	66.485			
2015	10.997.017	10.920.126	76.891			
2016	11.439.617	11.084.038	355.579			
2017	12.616.441	11.926.361	690.080			
2018	13.598.381	13.694.083	- 95.702			
2019		14.198.812		287.346	13.635.625	14.762.000
2020		14.738.898		676.845	13.412.305	16.065.491
2021		14.984.088		1.057.487	12.911.451	17.056.725
2022		14.905.305		1.464.046	12.035.828	17.774.783

Table 5.2 - EMEA - All Car Manufacturers – Gasoline - Arima (1,0,5)

Moving now to the Diesel case, the best method found to optimize the model was the Weighted Moving Average with K=5. The results of this model have been compared with those coming from the ARIMA (1,0,5) one. The graphic result is shown in Graph 5.4.



Graph 5.4 - EMEA - All Car Manufacturers - Diesel Case – Weighted Moving Average – K=5 and ARIMA (1,0,5)



Graph 5.5 - EMEA - All Car Manufacturers - Diesel Case – Residuals

In Graph 5.4, it is represented the Weighted Moving Average with K=5 behaviour (as it has already been discussed in the chapter 4) in comparison with the ARIMA results obtained. Instead, in Graph 5.5, it is possible to find the residual values obtained by applying the model. Also for this case, these seem to have a random trend and so the ARIMA model used should be the right one to represent the future demand for this specific market. It is also interesting to notice that, especially for the Diesel market that has been heavily affected by the EU regulation on CO₂ emissions, with this model the future forecast seems to be quite accurate if compared with what it has been extrapolated qualitatively.

The simpler model was able to provide a good forecast for the first year (2019), while it was almost unable to follow the forecasted trend for the next year (2020-2022). Using a more complex model has been beneficial for the accuracy in the forecast for the years farer from the current one.

Looking again at the graphs, the difference between the Wighted Moving Average with K=5 (black dotted) and the line of the ARIMA forecast (the solid red one) is clear: the second computation is more accurate than the first one.

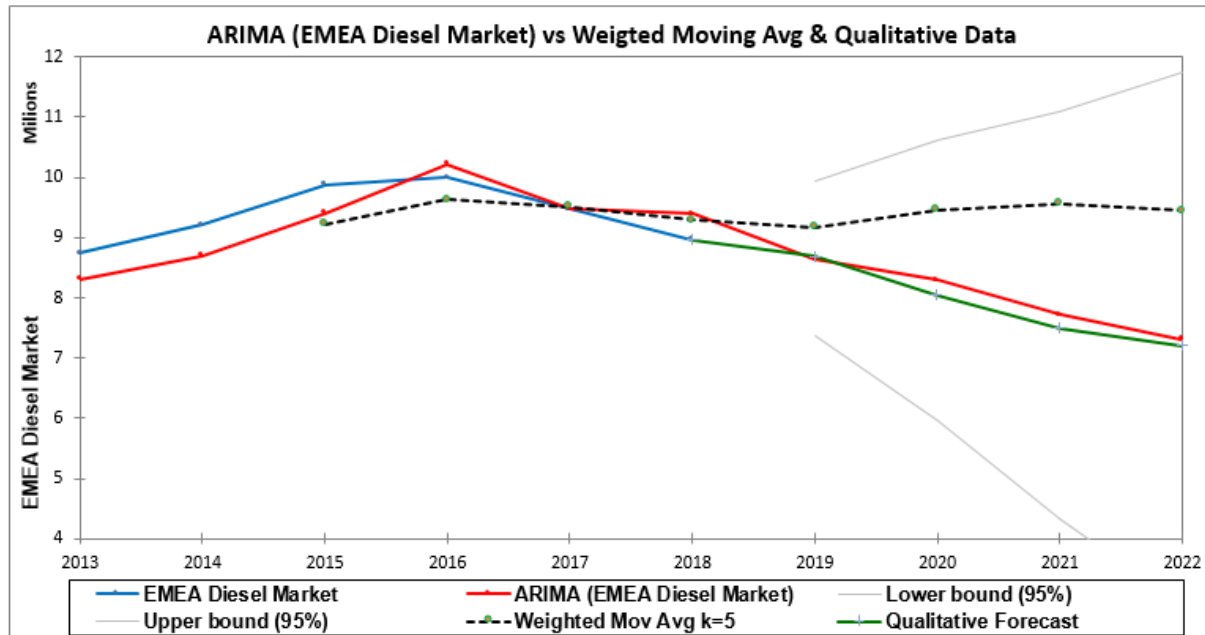
Looking now at the σ_k values (Table 5.3), it will be clearer the difference between the methodologies adopted in chapter 4 and the current one.

Comparative Methods Table Diesel - EMEA All Brands				
	Moving Average	Weighted Mov. Avg	Simple Exp Smoth	ARIMA
Sigma Diesel (Unit Engines)	667.602	469.004	476.147	422.397

Table 5.3 – Comparative Table EMEA – All Car Manufacturers - Diesel

As already anticipated just by looking at the graphical output, the ARIMA (1,0,5) model is better in forecasting the future than the Weighted Moving Average with K=5.

In the Graph 5.6 here below, there is a zoomed picture of the comparison between the two methodologies:



Graph 5.6 - EMEA - All Car Manufacturers - Diesel Case – Weighted Moving Average – K=5 and ARIMA (1,0,5) zoom

In the Table 5.4 it is possible to find the data used to create the graph in Graph 5.6.

Observations	EMEA Diesel Market	ARIMA (EMEA Diesel Market)	Residuals	Standard error	Lower bound (95%)	Upper bound (95%)
2010	8.885.008	8.318.455	566.553			
2011	9.694.389	9.123.091	571.298			
2012	8.658.131	8.860.115	- 201.984			
2013	8.743.088	8.306.623	436.465			
2014	9.212.051	8.696.570	515.481			
2015	9.880.366	9.407.359	473.007			
2016	9.991.764	10.223.289	- 231.525			
2017	9.477.686	9.485.414	- 7.728			
2018	8.958.582	9.387.527	- 428.945			
2019		8.652.756		652.718	7.373.452	9.932.060
2020		8.294.689		1.187.480	5.967.271	10.622.107
2021		7.715.043		1.727.210	4.329.774	11.100.311
2022		7.316.910		2.254.900	2.897.387	11.736.433

Table 5.4 - EMEA - All Car Manufacturers - Diesel – Arima (1,0,5)

At the end, the analytical comparison between the Qualitative data and the ARIMA forecasts is very impressive (Table 5.5 for the Gasoline market, Table 5.6 for the Diesel market):

Observations	EMEA Gasoline Market	ARIMA (EMEA Gasoline Market)	Qualitative Data Gasoline Market	Absolute Forecast Error Qntv vs Qltv	Error % Qntv vs Qltv
2010	10.138.834	9.893.187			
2011	10.474.762	10.292.595			
2012	10.614.814	10.708.605			
2013	10.717.087	10.658.992			
2014	10.868.503	10.802.018			
2015	10.997.017	10.920.126			
2016	11.439.617	11.084.038			
2017	12.616.441	11.926.361			
2018	13.598.381	13.694.083			
2019		14.198.812	13.898.816	299.996	2,1%
2020		14.738.898	14.367.831	371.067	2,5%
2021		14.984.088	15.018.154	34.066	0,2%
2022		14.905.305	15.228.306	323.001	2,2%

Table 5.5 - EMEA - All Car Manufacturers - Gasoline – Residuals Arima vs Qualitative Data

Observations	EMEA Diesel Market	ARIMA (EMEA Diesel Market)	Qualitative Data Diesel Market	Absolute Forecast Error Qntv vs Qltv	Error % Qntv vs Qltv
2010	8.885.008	8.318.455			
2011	9.694.389	9.123.091			
2012	8.658.131	8.860.115			
2013	8.743.088	8.306.623			
2014	9.212.051	8.696.570			
2015	9.880.366	9.407.359			
2016	9.991.764	10.223.289			
2017	9.477.686	9.485.414			
2018	8.958.582	9.387.527			
2019		8.652.756	8.680.575	27.819	0,3%
2020		8.294.689	8.051.624	243.065	2,9%
2021		7.715.043	7.491.548	223.495	2,9%
2022		7.316.910	7.207.753	109.157	1,5%

Table 5.6 - EMEA - All Car Manufacturers - Diesel – Residuals Arima vs Qualitative Data

Finally, in Table 5.7 there is the analytical comparison between Arima and the other methods for the EMEA Gasoline Market while in Table 5.8 it is possible to find the comparison on the EMEA Diesel market.

	Year	Global Gasoline Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Double Exponential Smoth (Unit of Engines)	Trend (Unit of Engines)	Arima (Unit of Engines)
Real Data Market	2010	10.138.834				9.893.187
	2011	10.474.762		10.331.585	9.794.422	10.292.595
	2012	10.614.814		10.810.690	10.164.302	10.708.605
	2013	10.717.087		10.754.866	10.534.181	10.658.992
	2014	10.868.503		10.819.360	10.904.060	10.802.018
	2015	10.997.017		11.019.919	11.273.940	10.920.126
	2016	11.439.617		11.125.531	11.643.819	11.084.038
	2017	12.616.441		11.882.217	12.013.698	11.926.361
	2018	13.598.381	13.598.381	13.793.265	12.383.578	13.694.083
Forecast	2019		13.898.816	14.580.321	12.753.457	14.198.812
	2020		14.367.831	15.562.261	13.123.336	14.738.898
	2021		15.018.154	16.544.201	13.493.216	14.984.088
	2022		15.228.306	17.526.141	13.863.095	14.905.305

Table 5.7 - EMEA - All Car Manufacturers - Gasoline – Arima vs Other Methods

	Year	EMEA Diesel Market (Unit of Engines)	Qualitative Forecast (Unit of Engines)	Moving Average K=2 (Unit of Engines)	Weighted Moving Average K = 5 (Unit of Engines)	Single Exponential Smoothing (Unit of Engines)	Arima (Unit of Engines)
Real Data Market	2010	8.885.008	8.958.582				8.318.455
	2011	9.694.389				6.484.688	9.123.091
	2012	8.658.131		9.289.699		8.827.276	8.860.115
	2013	8.743.088		9.176.260		8.703.826	8.306.623
	2014	9.212.051		8.700.610		8.732.481	8.696.570
	2015	9.880.366		8.977.570	9.229.010	9.082.493	9.407.359
	2016	9.991.764		9.546.209	9.632.768	9.664.818	10.223.289
	2017	9.477.686		9.936.065	9.508.183	9.903.438	9.485.414
	2018	8.958.582		9.734.725	9.283.885	9.592.705	9.387.527
Forecast	2019		8.680.575	9.218.134	9.168.145	9.129.893	8.652.756
	2020		8.051.624	9.088.358	9.453.347	9.129.893	8.294.689
	2021		7.491.548	9.153.246	9.566.108	9.129.893	7.715.043
	2022		7.207.753	9.120.802	9.443.847	9.129.893	7.316.910

Table 5.8 - EMEA - All Car Manufacturers - Diesel – Arima vs Other Methods

If the Error expressed in percentage is very low (ref. Table 5.7 and Table 5.8, last column on the right), it means that a very precise method, able to follow the Qualitative forecasts, has been identified. All the analysis is based on the assumption that the numbers provided by the Sales Managers are good because, if this was not the case, why to compare the results of a statistical model with some random numbers? To support this assumption, it is possible to say that in the last years the company has managed to grow and increase its turnover year over yer. As this is its standard methodology, it is possible to assume that there is something good in it. Moreover, the salary of each Sales Manager has a variable part related with the accuracy of their forecast. This is the right incentive for pushing them in gathering the best numbers for the future sales.

CHAPTER 6 - Conclusions

In this chapter, it will be discussed the utility, the limits, the applicability and the possible future steps of the methods seen and analysed before.

6.1 Which are the advantages of the quantitative forecasting compared with the current methodology?

It is true that no one can better know the business than the Sales Manager, but there is also the possibility this could lead to overly optimistic projection. For this reason, it would be good to add some quantitative methods for forecasting the future in order to have a more objective picture. Here below some good reasons why adopting these techniques can be helpful.

- Quantitative forecasting can help you in giving more importance to the more recent data. This could allow the company to detect trends that might provide better forecasts. While, on the contrary, the qualitative methods rely on the experience and feedback from external stakeholders (such as suppliers and customers).
- It could be the case that in the past a poor performance occurred. In that case it would be better to understand if it was an anomaly or if could be a recursive event. For the Sales department it could be quite difficult to correctly understand what happened in the past. Using objective, quantifiable historical data it is possible to create sales, revenue or expense projection based on the entire history.
- Adding a quantitative forecast could help to provide more coherent numbers to the senior management. Each forecast could be supported by a numerical methodology and not only led by what customers, suppliers and managers feelings suggest.
- The possibility to see all the data on a graph, could help managers to find patterns that help them to make more accurate projections. For example, in this way could be easy to identify if sales from your top product rose last year, but not as much as in previous years. This could

be a sign that the marketplace has been saturated and an increase in sales of that product for the next years shouldn't be expected.

- To propose action plans for the future, the possibility of having hard numbers in support can increase the likelihood to get what demanded (CAPEX increase for example) as senior management can feel more comfortable with data obtained by a mathematical model.

6.2 Limits in using the suggested models

As just said, the use of these methodologies can help the company in making better predictions about the future, but there are some limitations.

- They are not able to predict unexpected events. For example, talking about the diesel case, not even the best model could have been able to predict that dramatic drop in such a short time. Of course, that has been further enhanced, for example in the FCA case, by the death of Sergio Marchionne (former CEO of the company), but still it was happening. In these cases, only the combined action of quantitative and qualitative forecasts could have been able to smooth the big drop.
- The data available. Obviously, the higher the number of data available, the better it is for the forecast reliability. For sure, doing the same procedure in the next years with more data will provide a better result. Moreover, it would be good to understand if there is a stagionality component in each year in order to being able to forecast month by month the possible demand. For this reason, it would be better to have the breakdown of the annual data into, at least, quarterly if not monthly data.
- Each macro region has its own model. This means that there is not a standard approach to analyse the entire database, but, case by case, a different analysis has to be done. Moreover, this does not mean that once the best model for a specific geographical area is identified, then it will be the same forever. In fact, by adding more numbers it could be possible that the Trend changes and so the models previously identified as the best ones for that area are not valid anymore. It is really essential to continuously update the data and the model at the same time and this requires a lot of time and resources that maybe the company is not willing to invest.

- The models used are the most known in literature. Infact, the Trend Methodology (meaning the use of the trend line based on the consolidated data to forecast the future demand), the Moving Average, the Weighted Moving Average, the Single Exponential Smoothing, teh Double Exponential Smoothing and the Triple Exponential Smoothing have been used for doing all the analysis. There are more recent and complex mathematical functions that, maybe, can provde better results.

6.3 Possible evolutions

This thesis had the scope of identifying if a classical mathematical method could have been used to support the Sales Department. For this reason, specific macro areas have been selected. Established that, with some limitations, these models could provide a good help in the forecasting, it would be interesting to apply these reasonings to each Part Number. In fact, in that case it would be possible to limit the propagation of error and, theoretically, a good estimation of future volumes could be obtained.

In this way, using the future projections for all the Part Numbers of the Salese Manager portfolio, it should be easier to decide in which plant to invest and for which product.

6.4 Corporate applicability

All this said, how to suggest the company to use these results?

Comparing the data obtained by the model whith those forecasted in a qualitative way, it is possible to see that the difference is not so high. The suggestion for the company is to support the current qualitative approach with some more quantitative analysis. In this way, integrating the “hard numbers” inside the forecasting process, especially for the first drafts of the Profit Plan/ Strategic Plan, it could lead the Sales Managers to have an order of magnitude of the future demand quickly, without spending too much time in contacting the customers. In fact, the first two budgets drafts must be done in July and August respectively, periods in which usually the clients have not yet a clear idea of the future sales volumes. This allows the Sales Managers to save some time in these months and so to have more freedom for doing a deeper analysis with the customers about the foreseen future trends.

6.5 Future steps the company could make, taking inspiration from this thesis

This core part of this analysis has been conducted applying the most common forecasting techniques. If the company would like to have more reliable information on the future trends, it would be better to implement some more complex techniques. For example, it could be interesting to exploit the Autoregressive methodologies, in particular the ARIMA one (as already discussed in Chapter 5). Infact, these models are more flexible than other statistical techniques such as exponential smoothing or simple linear regression.

To conclude, the ARIMA models can be considered somehow in the middle between being simple enough to not overfit and being flexible enough to capture some of the types of relationships hidden in the data.

APPENDIX

	Year	Period	Global Gasoline Market	Qualitative Forecast	Quantitative Forecast		Error (Unit Engines) $Real Data_t - Forecast_t$	Abs. Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Abs. Forecat Error $Qntv vs Qltv$ (Unit Engines)	Error % $Qntv vs Qltv$
Real Data Market	2010	1	57 715 766	77 879 101		57 890 372	-174 606	30 487 193 154	0,30%		
	2011	2	58 895 403			60 444 897	-1 549 494	2 400 932 448 000	2,63%		
	2012	3	63 841 987			62 999 423	842 564	709 914 618 358	1,32%		
	2013	4	66 568 090			65 553 948	1 014 142	1 028 483 748 263	1,52%		
	2014	5	69 198 237			68 108 474	1 089 763	1 187 584 364 847	1,57%		
	2015	6	69 761 924			70 662 999	-901 075	811 936 135 601	1,29%		
	2016	7	73 552 727			73 217 524	335 203	112 360 768 149	0,46%		
	2017	8	75 563 027			75 772 050	-209 023	43 690 554 145	0,28%		
	2018	9	77 879 101			78 326 575	-447 474	200 233 239 217	0,57%		
Forecast	2019	10		79 333 268	80 881 101	80 881 101				1 547 833	1,95%
	2020	11		81 052 648	83 435 626	83 435 626				2 382 978	2,94%
	2021	12		83 188 263	85 990 152	85 990 152				2 801 889	3,37%
	2022	13		84 199 814	88 544 677	88 544 677				4 344 863	5,16%

4.1 World - All Car Manufacturers - Gasoline Case - Trend Data

	Year	Period	Global Diesel Market	Qualitative Forecast	Quantitative Forecast		Error (Unit Engines) $Real Data_t - Forecast_t$	Abs. Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Abs. Forecat Error $Qntv vs Qltv$ (Unit Engines)	Error % $Qntv vs Qltv$
Real Data Market	2010	1	16 186 993	18 090 698		16 817 487	-630 494	397 522 095 575	3,90%		
	2011	2	17 494 113			17 058 586	435 527	189 683 854 834	2,49%		
	2012	3	17 223 360			17 299 685	-76 325	5 825 546 332	0,44%		
	2013	4	17 637 821			17 540 785	97 036	9 416 056 456	0,55%		
	2014	5	17 709 221			17 781 884	-72 663	5 279 911 569	0,41%		
	2015	6	18 434 586			18 022 983	411 603	169 416 727 767	2,23%		
	2016	7	18 797 056			18 264 083	532 973	284 060 502 981	2,84%		
	2017	8	18 463 108			18 505 182	-42 074	1 770 229 891	0,23%		
	2018	9	18 090 698			18 746 281	-655 583	429 789 681 767	3,62%		
Forecast	2019	10		18 197 161	18 987 381	18 987 381				790 220	4,34%
	2020	11		17 653 014	19 228 480	19 228 480				1 575 466	8,92%
	2021	12		17 175 487	19 469 580	19 469 580				2 294 093	13,36%
	2022	13		17 069 507	19 710 679	19 710 679				2 641 172	15,47%

4.1 World - All Car Manufacturers - Diesel Case - Trend Data

	Year	Global Gasoline Market	Qualitative Forecast	Base Ei	Trend Ti	Quantitative Forecast (Unit Engines)	Error (Unit Engines) $Real Data_t - Forecast_t$	Abs. Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Abs. Forecat Error $Qntv vs Qltv$ (Unit Engines)	Error % $Qntv vs Qltv$
Real Data Market	2010	57 715 766	77 879 101	57 715 766	2 950 775						
	2011	58 895 403		60 321 768	2 606 002	60 666 541	-1 771 138	3 136 928 634 285	3,01%		
	2012	63 841 987		63 105 733	2 783 965	62 927 770	914 217	835 792 474 296	1,43%		
	2013	66 568 090		66 021 755	2 916 022	65 889 698	678 392	460 215 503 326	1,02%		
	2014	69 198 237		68 988 479	2 966 724	68 937 777	260 460	67 839 433 693	0,38%		
	2015	69 761 924		71 528 255	2 539 776	71 955 202	-2 193 278	4 810 468 607 391	3,14%		
	2016	73 552 727		73 967 721	2 439 466	74 068 031	-515 304	265 538 486 748	0,70%		
	2017	75 563 027		76 242 862	2 275 141	76 407 188	-844 161	712 607 220 603	1,12%		
	2018	77 879 101		78 393 633	2 150 771	78 518 003	-638 902	408 195 263 886	0,82%		
Forecast	2019		79 333 268			80 544 404	80 544 404			1 211 136	1,53%
	2020		81 052 648			82 695 175	82 695 175			1 642 527	2,03%
	2021		83 188 263			84 845 946	84 845 946			1 657 683	1,99%
	2022		84 199 814			86 996 717	86 996 717			2 796 903	3,32%

4.2 World - All Car Manufacturers - Gasoline Case - Double Exponential Smoothing Data

			Alpha		1,00					
	Year	Global Diesel Market	Qualitative Forecast	Quantitative Forecast		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	16 186 993	18 090 698							
	2011	17 494 113			16 186 993	1 307 120	1 708 562 694 400	7,47%		
	2012	17 223 360			17 494 113	-270 753	73 307 187 009	1,57%		
	2013	17 637 821			17 223 360	414 461	171 777 920 521	2,35%		
	2014	17 709 221			17 637 821	71 400	5 097 960 000	0,40%		
	2015	18 434 586			17 709 221	725 365	526 154 383 225	3,93%		
	2016	18 797 056			18 434 586	362 470	131 384 500 900	1,93%		
	2017	18 463 108			18 797 056	-333 948	111 521 266 704	1,81%		
	2018	18 090 698			18 463 108	-372 410	138 689 208 100	2,06%		
Forecast	2019		18 197 161	18 090 698	18 090 698				106 463	0,59%
	2020		17 653 014	18 090 698	18 090 698				437 684	2,48%
	2021		17 175 487	18 090 698	18 090 698				915 211	5,33%
	2022		17 069 507	18 090 698	18 090 698				1 021 191	5,98%

4.3 World - All Car Manufacturers - Diesel Case – Simple Exponential Smoothing Data

								Alpha =	0,33	Beta =	1,00	
	Year	Global Diesel Market	Qualitative Forecast	Base Ei	Trend Ti	Quantitative Forecast		Error (Unit Engines)	Abs. Squared Error	Error %	Abs. Forecat Error	Error %
								$Real\ Data_t - Forecast_t$	$ABS(Error_t)^2$	$\frac{Forecast}{vs\ Real\ Data}$	$Qntv\ vs\ Qltv$ (Unit Engines)	$Qntv\ vs\ Qltv$
Real Data Market	2010	16 186 993	18 090 698	16 186 993	483609							
	2011	17 494 113		16 944 170	757177		16 670 602	823 511	678 169 818 114	4,71%		
	2012	17 223 360		17 542 561	598391		17 701 348	-477 988	228 472 172 789	2,78%		
	2013	17 637 821		17 973 814	431252		18 140 953	-503 132	253 141 395 656	2,85%		
	2014	17 709 221		18 173 908	200094		18 405 066	-695 845	484 200 073 815	3,93%		
	2015	18 434 586		18 394 128	220220		18 374 003	60 583	3 670 352 495	0,33%		
	2016	18 797 056		18 675 043	280915		18 614 348	182 708	33 382 096 040	0,97%		
	2017	18 463 108		18 792 235	117192		18 955 958	-492 850	242 901 520 172	2,67%		
	2018	18 090 698	18 637 447	-154788		18 909 426	-818 728	670 316 188 550	4,53%			
Forecast	2019		18 197 161			18 482 659	18 482 659				285 498	1,57%
	2020		17 653 014			18 327 872	18 327 872				674 858	3,82%
	2021		17 175 487			18 173 084	18 173 084				997 597	5,81%
	2022		17 069 507			18 018 296	18 018 296				948 789	5,56%

4.3 World - All Car Manufacturers - Diesel Case - Double Exponential Smoothing Data

Year	Period	EMEA Gasoline Market (Unit Engines)	Qualitative Forecast (Unit Engines)	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1	10 138 834		9 794 422	344 412	118 619 518 594	3,40%	
	2011	2	10 474 762		10 164 302	310 460	96 385 718 611	2,96%	
	2012	3	10 614 814		10 534 181	80 633	6 501 703 983	0,76%	
	2013	4	10 717 087		10 904 060	-186 973	34 958 979 596	1,74%	
	2014	5	10 868 503		11 273 940	-405 437	164 378 800 581	3,73%	
	2015	6	10 997 017		11 643 819	-646 802	418 352 705 030	5,88%	
	2016	7	11 439 617		12 013 698	-574 081	329 569 287 980	5,02%	
	2017	8	12 616 441		12 383 578	232 863	54 225 360 472	1,85%	
	2018	9	13 598 381	13 598 381	12 753 457	844 924	713 896 640 880	6,21%	
Forecast	2019	10		13 898 816	13 123 336			775 480	5,58%
	2020	11		14 367 831	13 493 216			874 615	6,09%
	2021	12		15 018 154	13 863 095			1 155 059	7,69%
	2022	13		15 228 306	14 232 974			995 332	6,54%

4.4 EMEA - All Car Manufacturers - Gasoline Case - Trend Data

	Year	Period	EMEA Diesel Market (Unit Engines)	Qualitative Forecast (Unit Engines)	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error (Unit Engines) ¹	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1	8 885 008	8 958 582		9 047 981	-162 973	26 560 104 567	1,83%		
	2011	2	9 694 389			9 105 460	588 929	346 837 883 990	6,07%		
	2012	3	8 658 131			9 162 938	-504 807	254 830 522 313	5,83%		
	2013	4	8 743 088			9 220 417	-477 329	227 843 223 513	5,46%		
	2014	5	9 212 051			9 277 896	-65 845	4 335 578 657	0,71%		
	2015	6	9 880 366			9 335 375	544 991	297 015 232 469	5,52%		
	2016	7	9 991 764			9 392 854	598 910	358 693 414 355	5,99%		
	2017	8	9 477 686			9 450 333	27 353	748 205 148	0,29%		
	2018	9	8 958 582			9 507 812	-549 230	301 653 055 875	6,13%		
Forecast	2019	10		8 680 575	9 565 290	9 565 290				884 715	10,19%
	2020	11		8 051 624	9 622 769	9 622 769				1 571 145	19,51%
	2021	12		7 491 548	9 680 248	9 680 248				2 188 700	29,22%
	2022	13		7 207 753	9 737 727	9 737 727				2 529 974	35,10%

4.4 EMEA - All Car Manufacturers - Diesel Case - Trend Data

	Year	EMEA Gasoline Market	Qualitative Forecast	Base Ei	Trend Ti	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error (Unit Engines) ¹	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	10 138 834	13 598 381	10 138 834	192 751							
	2011	10 474 762		10 474 762	335 928		10 331 585	143 177	20 499 653 329	1,37%		
	2012	10 614 814		10 614 814	140 052		10 810 690	-195 876	38 367 407 376	1,85%		
	2013	10 717 087		10 717 087	102 273		10 754 866	-37 779	1 427 252 841	0,35%		
	2014	10 868 503		10 868 503	151 416		10 819 360	49 143	2 415 034 449	0,45%		
	2015	10 997 017		10 997 017	128 514		11 019 919	-22 902	524 501 604	0,21%		
	2016	11 439 617		11 439 617	442 600		11 125 531	314 086	98 650 015 396	2,75%		
	2017	12 616 441		12 616 441	1 176 824		11 882 217	734 224	539 084 882 176	5,82%		
	2018	13 598 381		13 598 381	981 940		13 793 265	-194 884	37 979 773 456	1,43%		
Forecast	2019		13 898 816			14 580 321	14 580 321				681 505	4,90%
	2020		14 367 831			15 562 261	15 562 261				1 194 430	8,31%
	2021		15 018 154			16 544 201	16 544 201				1 526 047	10,16%
	2022		15 228 306			17 526 141	17 526 141				2 297 835	15,09%

4.5 EMEA - All Car Manufacturers - Gasoline Case – Double Exponential Smoothing Data

		K =		2					
	Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	8 885 008	8 958 582						
	2011	9 694 389							
	2012	8 658 131		9 289 699	-631 568	398 877 507 056	7,29%		
	2013	8 743 088		9 176 260	-433 172	187 637 981 584	4,95%		
	2014	9 212 051		8 700 610	511 442	261 572 407 922	5,55%		
	2015	9 880 366		8 977 570	902 797	815 041 520 412	9,14%		
	2016	9 991 764		9 546 209	445 556	198 519 703 580	4,46%		
	2017	9 477 686		9 936 065	-458 379	210 111 307 641	4,84%		
	2018	8 958 582		9 734 725	-776 143	602 397 956 449	8,66%		
Forecast	2019		8 680 575	9 218 134	9 218 134			537 559	6,19%
	2020		8 051 624	9 088 358	9 088 358			1 036 734	12,88%
	2021		7 491 548	9 153 246	9 153 246			1 661 698	22,18%
	2022		7 207 753	9 120 802	9 120 802			1 913 049	26,54%

t = 9

$\Sigma(X_j - P_j)^2$	2 674 158 384 645
t-(k+1)	6
σ_k	667 602

4.6 EMEA - All Car Manufacturers - Diesel Case – Moving Average Data_K=2

		K =		3					
	Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	8 885 008	8 958 582						
	2011	9 694 389							
	2012	8 658 131							
	2013	8 743 088		9 079 176	-336 088	112 955 143 744	3,70%		
	2014	9 212 051		9 031 869	180 182	32 465 433 003	1,99%		
	2015	9 880 366		8 871 090	1 009 276	1 018 638 044 176	11,38%		
	2016	9 991 764		9 278 502	713 262	508 743 156 152	7,69%		
	2017	9 477 686		9 694 727	-217 041	47 106 795 681	2,24%		
	2018	8 958 582		9 783 272	-824 690	680 113 596 100	8,43%		
Forecast	2019		8 680 575	9 476 011	9 476 011			795 436	9,16%
	2020		8 051 624	9 304 093	9 304 093			1 252 469	15,56%
	2021		7 491 548	9 246 229	9 246 229			1 754 681	23,42%
	2022		7 207 753	9 342 111	9 342 111			2 134 358	29,61%

t = 9

$\Sigma(X_j - P_j)^2$	2 400 022 168 856
t-(k+1)	5
σ_k	692 824

4.6 EMEA - All Car Manufacturers - Diesel Case – Moving Average Data_K=3

			K =		4				
	Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	8 885 008	8 958 582						
	2011	9 694 389							
	2012	8 658 131							
	2013	8 743 088							
	2014	9 212 051			8 995 154	216 897	47 044 308 609	2,35%	
	2015	9 880 366			9 076 915	803 451	645 533 911 127	8,13%	
	2016	9 991 764			9 123 409	868 355	754 040 406 025	8,69%	
	2017	9 477 686			9 456 817	20 869	435 504 727	0,22%	
	2018	8 958 582			9 640 467	-681 885	464 966 812 283	7,61%	
Forecast	2019			8 680 575	9 577 100			896 525	10,33%
	2020			8 051 624	9 501 283			1 449 659	18,00%
	2021			7 491 548	9 378 663			1 887 115	25,19%
	2022			7 207 753	9 353 907			2 146 154	29,78%

t = 9

$\sum (X_j - P_j)^2$	1 912 020 942 770
t-(k+1)	4
σ_k	691 379

4.6 EMEA - All Car Manufacturers - Diesel Case – Moving Average Data_K=4

			K =		5				
	Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	8 885 008	8 958 582						
	2011	9 694 389							
	2012	8 658 131							
	2013	8 743 088							
	2014	9 212 051							
	2015	9 880 366			9 038 533	841 833	708 682 126 423	8,52%	
	2016	9 991 764			9 237 605	754 159	568 755 797 281	7,55%	
	2017	9 477 686			9 297 080	180 606	32 618 527 236	1,91%	
	2018	8 958 582			9 460 991	-502 409	252 414 803 281	5,61%	
Forecast	2019			8 680 575	9 504 090			823 515	9,49%
	2020			8 051 624	9 562 498			1 510 874	18,76%
	2021			7 491 548	9 498 924			2 007 376	26,80%
	2022			7 207 753	9 400 356			2 192 603	30,42%

t = 9

$\sum (X_j - P_j)^2$	1 562 471 254 221
t-(k+1)	3
σ_k	721 681

4.6 EMEA - All Car Manufacturers - Diesel Case – Moving Average Data_K=5

				K =	2				
	Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	8 885 008	8 958 582						
	2011	9 694 389							
	2012	8 658 131		9 606 948	-948 817	900 254 354 163	10,96%		
	2013	8 743 088		8 770 082	-26 994	728 680 435	0,31%		
	2014	9 212 051		8 733 910	478 141	228 619 048 529	5,19%		
	2015	9 880 366		9 161 387	718 979	516 930 718 779	7,28%		
	2016	9 991 764		9 808 165	183 599	33 708 494 702	1,84%		
	2017	9 477 686		9 979 729	-502 043	252 047 404 902	5,30%		
	2018	8 958 582		9 533 224	-574 642	330 213 307 561	6,41%		
Forecast	2019		8 680 575	9 014 663				334 088	4%
	2020		8 051 624	9 008 604				956 980	12%
	2021		7 491 548	9 009 259	9 009 259			1 517 711	20%
	2022		7 207 753	9 009 188	9 009 188			1 801 435	25%

t = 9

$\sum(X_j - P_j)^2$	2 262 502 009 070	Weight	
t-(k+1)	6	w(l-1)	0,89
σ_k	614 071	w(l-2)	0,11

4.7 EMEA - All Car Manufacturers - Diesel Case – Weighted Moving Average Data_K=2

			K =		3					
	Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	8 885 008								
	2011	9 694 389								
	2012	8 658 131								
	2013	8 743 088			8 669 065	74 023	5 479 426 985	0,85%		
	2014	9 212 051			8 788 934	423 117	179 028 078 126	4,81%		
	2015	9 880 366			9 185 356	695 010	483 038 874 038	7,57%		
	2016	9 991 764			9 825 557	166 207	27 624 654 993	1,69%		
	2017	9 477 686			9 954 187	-476 501	227 053 595 137	4,79%		
	2018	8 958 582			9 497 092	-538 510	289 993 338 062	5,67%		
Forecast	2019			8 680 575	9 008 374	9 008 374			327 799	3,78%
	2020			8 051 624	9 030 991	9 030 991			979 367	12,16%
	2021			7 491 548	9 027 502	9 027 502			1 535 954	20,50%
	2022			7 207 753	9 026 580	9 026 580			1 818 827	25,23%

t = 9

$\sum(X_j - P_j)^2$	1 212 217 967 341	Weight	
t-(k+1)	5	w(l-1)	0,95
σ_k	492 386	w(l-2)	0,00
		w(l-3)	0,05

4.7 EMEA - All Car Manufacturers - Diesel Case – Weighted Moving Average Data_K=3

			K =		4				
Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) $Real Data_t - Forecast_t$	Absolute Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	8 885 008							
	2011	9 694 389							
	2012	8 658 131							
	2013	8 743 088							
	2014	9 212 051		8 786 490	425 561	181 102 495 217	4,62%		
	2015	9 880 366		9 359 558	520 808	271 240 545 517	5,27%		
	2016	9 991 764		9 506 585	485 179	235 398 596 901	4,86%		
	2017	9 477 686		9 609 897	-132 211	17 479 733 990	1,39%		
Forecast	2018	8 958 582	8 958 582	9 396 450	-437 868	191 728 506 299	4,89%		
	2019	8 680 575	9 240 480	9 240 480				559 905	6,45%
	2020	8 051 624	9 470 236	9 470 236				1 418 612	17,62%
	2021	7 491 548	9 472 514	9 472 514				1 980 966	26,44%
	2022	7 207 753	9 315 345	9 315 345				2 107 592	29,24%

t = 9

$\sum(X_j - P_j)^2$	896 949 877 923	Weight	
t-(k+1)	4	w(l-1)	0,69
σ_k	473 537	w(l-2)	0,00
		w(l-3)	0,00
		w(l-4)	0,31

4.7 EMEA - All Car Manufacturers - Diesel Case – Weighted Moving Average Data_K=4

			K =		5				
Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) $Real Data_t - Forecast_t$	Abs. Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Abs. Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	8 885 008							
	2011	9 694 389							
	2012	8 658 131							
	2013	8 743 088							
	2014	9 212 051							
	2015	9 880 366		9 229 010	651 356	424 264 902 015	6,59%		
	2016	9 991 764		9 632 768	358 996	128 878 478 244	3,59%		
	2017	9 477 686		9 508 183	-30 497	930 066 488	0,32%		
Forecast	2018	8 958 582	8 958 582	9 283 885	-325 303	105 822 196 375	3,63%		
	2019	8 680 575	9 168 145	9 168 145				487 570	5,62%
	2020	8 051 624	9 453 347	9 453 347				1 401 723	17,41%
	2021	7 491 548	9 566 108	9 566 108				2 074 560	27,69%
	2022	7 207 753	9 443 847	9 443 847				2 236 094	31,02%

$\sum(X_j - P_j)^2$	659 895 643 122	Weight	
t-(k+1)	3	w(l-1)	0,63
σ_k	469 004	w(l-2)	0,00
		w(l-3)	0,00
		w(l-4)	0,17
		w(l-5)	0,20

4.7 EMEA - All Car Manufacturers - Diesel Case – Weighted Moving Average Data_K=5

			Alpha	0,73					
	Year	EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	8 885 008	8 958 582						
	2011	9 694 389		6 484 688	3 209 701	10 302 179 572 319	33,11%		
	2012	8 658 131		8 827 276	-169 145	28 609 926 569	1,95%		
	2013	8 743 088		8 703 826	39 262	1 541 496 513	0,45%		
	2014	9 212 051		8 732 481	479 570	229 987 150 617	5,21%		
	2015	9 880 366		9 082 493	797 873	636 600 774 485	8,08%		
	2016	9 991 764		9 664 818	326 946	106 893 971 856	3,27%		
	2017	9 477 686		9 903 438	-425 752	181 264 880 090	4,49%		
	2018	8 958 582		9 592 705	-634 123	402 111 489 010	7,08%		
Forecast	2019		8 680 575	9 129 893				449 318	5,18%
	2020		8 051 624	9 129 893				1 078 269	13,39%
	2021		7 491 548	9 129 893				1 638 345	21,87%
	2022		7 207 753	9 129 893				1 922 140	26,67%

t =	9
J =	2

$\sum (X_j - P_j)^2$	1 587 009 689 141
t-(k+1)	7
σ_k	476 147

4.8 EMEA - All Car Manufacturers - Diesel Case – Single Exponential Smoothing Data

	Year	Period	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1	3 259 749	4 251 267	3 609 133	-349 384	122 069 194 984	10,72%		
	2011	2	3 494 046		3 700 625	-206 579	42 675 050 800	5,91%		
	2012	3	3 979 325		3 792 118	187 207	35 046 539 892	4,70%		
	2013	4	4 160 783		3 883 610	277 173	76 824 776 458	6,66%		
	2014	5	4 433 885		3 975 103	458 782	210 481 331 330	10,35%		
	2015	6	4 240 389		4 066 595	173 794	30 204 375 677	4,10%		
	2016	7	3 984 784		4 158 087	-173 303	30 034 041 493	4,35%		
	2017	8	3 971 695		4 249 580	-277 885	77 219 909 582	7,00%		
	2018	9	4 251 267		4 341 072	-89 805	8 064 953 990	2,11%		
Forecast	2019	10		3 998 178	4 432 564				434 386	10,86%
	2020	11		3 885 718	4 524 057				638 339	16,43%
	2021	12		4 207 740	4 615 549				407 809	9,69%
	2022	13		4 227 782	4 707 042				479 260	11,34%

4.9 World - FCA - Gasoline Case – Trend Data

	Year	Period	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) $Real Data_t - Forecast_t$	Absolute Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1	1 235 258		1 079 976	155 282	24 112 458 115	12,57%		
	2011	2	1 312 579		1 081 856	230 723	53 233 133 492	17,58%		
	2012	3	879 706		1 083 736	-204 030	41 628 132 084	23,19%		
	2013	4	867 916		1 085 616	-217 700	47 393 086 814	25,08%		
	2014	5	891 915		1 087 495	-195 580	38 251 666 787	21,93%		
	2015	6	1 061 035		1 089 375	-28 340	803 163 157	2,67%		
	2016	7	1 202 179		1 091 255	110 924	12 304 148 566	9,23%		
	2017	8	1 193 988		1 093 135	100 853	10 171 381 397	8,45%		
	2018	9	1 142 882	1 142 882	1 095 015	47 867	2 291 294 365	4,19%		
Forecast	2019	10		1 091 045	1 096 894				5 849	0,54%
	2020	11		952 729	1 098 774	1 098 774			146 045	15,33%
	2021	12		780 303	1 100 654	1 100 654			320 351	41,05%
	2022	13		623 376	1 102 534	1 102 534			479 158	76,86%

4.9 World - FCA - Diesel Case – Trend Data

			K =		2					
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	3 259 749	4 251 267							
	2011	3 494 046								
	2012	3 979 325		3 376 898	602 428	362 918 892 756	15,14%			
	2013	4 160 783		3 736 686	424 098	179 858 689 506	10,19%			
	2014	4 433 885		4 070 054	363 831	132 372 996 561	8,21%			
	2015	4 240 389		4 297 334	-56 945	3 242 733 025	1,34%			
	2016	3 984 784		4 337 137	-352 353	124 152 636 609	8,84%			
	2017	3 971 695		4 112 587	-140 892	19 850 414 772	3,55%			
	2018	4 251 267		3 978 240	273 028	74 544 015 756	6,42%			
Forecast	2019		3 998 178	4 111 481				113 303	2,83%	
	2020		3 885 718	4 181 374	4 181 374			295 656	7,61%	
	2021		4 207 740	4 146 428	4 146 428			61 313	1,46%	
	2022		4 227 782	4 163 901	4 163 901			63 881	1,51%	

t = 9

$\sum(X_j - P_j)^2$	896 940 378 986
t-(k+1)	6
σ_k	386 639

4.10 World - FCA - Gasoline Case – Moving Average Data_K=2

			K =		3				
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) $Real Data_t - Forecast_t$	Absolute Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	3 259 749	4 251 267						
	2011	3 494 046							
	2012	3 979 325							
	2013	4 160 783							
	2014	4 433 885		3 577 707	583 076	339 978 010 493	16,30%		
	2015	4 240 389		3 878 051	555 834	308 951 065 000	14,33%		
	2016	3 984 784		4 191 331	49 058	2 406 687 364	1,17%		
	2017	3 971 695		4 278 352	-293 568	86 182 366 336	6,86%		
Forecast	2018	4 251 267		4 219 686	-247 991	61 499 536 081	5,88%		
	2019		3 998 178	4 065 623	185 644	34 463 818 499	4,57%		
	2020		4 069 249	4 069 249				71 071	1,78%
	2021		3 885 718	4 097 404	4 097 404			211 686	5,45%
	2022		4 207 740	4 139 306	4 139 306			68 434	1,63%
								125 796	2,98%

t = 9

$\sum(X_j - P_j)^2$	833 481 483 773
t-(k+1)	5
σ_k	408 285

4.10 World - FCA - Gasoline Case – Moving Average Data_K=3

			K =		4				
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) $Real Data_t - Forecast_t$	Absolute Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	3 259 749	4 251 267						
	2011	3 494 046							
	2012	3 979 325							
	2013	4 160 783							
	2014	4 433 885		3 723 476	710 409	504 681 302 486	16,02%		
	2015	4 240 389		4 017 010	223 379	49 898 289 331	5,27%		
	2016	3 984 784		4 203 596	-218 812	47 878 472 532	5,49%		
	2017	3 971 695		4 204 960	-233 265	54 412 676 858	5,87%		
Forecast	2018	4 251 267		4 157 688	93 579	8 756 982 452	2,20%		
	2019		3 998 178	4 112 034	4 112 034			113 856	2,85%
	2020		3 885 718	4 079 945	4 079 945			194 227	5,00%
	2021		4 207 740	4 103 735	4 103 735			104 005	2,47%
	2022		4 227 782	4 136 745	4 136 745			91 037	2,15%

t = 9

$\sum(X_j - P_j)^2$	665 627 723 658
t-(k+1)	4
σ_k	407 930

4.10 World - FCA - Gasoline Case – Moving Average Data_K=4

				K =		5				
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Abs. Squared Error <i>Abs(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	3 259 749	4 251 267							
	2011	3 494 046								
	2012	3 979 325								
	2013	4 160 783								
	2014	4 433 885								
	2015	4 240 389		3 865 558	374 831	140 498 578 426	8,84%			
	2016	3 984 784		4 061 686	-76 902	5 913 856 083	1,93%			
	2017	3 971 695		4 159 833	-188 138	35 395 982 299	4,74%			
	2018	4 251 267		4 158 307	92 960	8 641 524 416	2,19%			
Forecast	2019		3 998 178	4 176 404	4 176 404				178 226	4,46%
	2020		3 885 718	4 124 908	4 124 908				239 190	6,16%
	2021		4 207 740	4 101 812	4 101 812				105 928	2,52%
	2022		4 227 782	4 125 217	4 125 217				102 565	2,43%

t = 9

$\sum(X_j - P_j)^2$	190 449 941 224
t-(k+1)	3
σ_k	251 959

4.10 World - FCA - Gasoline Case – Moving Average Data_K=5

			K =		2					
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	3 259 749	4 251 267							
	2011	3 494 046								
	2012	3 979 325			3 494 046	485 279	235 495 741 529	12,20%		
	2013	4 160 783			3 979 325	181 458	32 927 020 110	4,36%		
	2014	4 433 885			4 160 783	273 102	74 584 724 981	6,16%		
	2015	4 240 389			4 433 885	-193 496	37 440 684 970	4,56%		
	2016	3 984 784			4 240 389	-255 605	65 333 894 491	6,41%		
	2017	3 971 695			3 984 784	-13 089	171 320 885	0,33%		
Forecast	2018	4 251 267		3 971 695	279 572	78 160 525 245	6,58%			
	2019		3 998 178	4 251 267	4 251 267				253 089	6,33%
	2020		3 885 718	4 251 267	4 251 267				365 549	9,41%
	2021		4 207 740	4 251 267	4 251 267				43 527	1,03%
	2022		4 227 782	4 251 267	4 251 267				23 485	0,56%

t = 9

$\sum(X_j - P_j)^2$	524 113 912 211	Weight	
t-(k+1)	6	w(l-1)	1,00
σ_k	295 554	w(l-2)	0,00

4.11 World - FCA - Gasoline Case – Weighted Moving Average Data_K=2

			K =		3					
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	3 259 749	4 251 267							
	2011	3 494 046								
	2012	3 979 325								
	2013	4 160 783			3 979 325	181 458	32 927 005 764	4,56%		
	2014	4 433 885			4 160 783	273 102	74 584 702 404	6,56%		
	2015	4 240 389			4 433 885	-193 496	37 440 702 016	4,36%		
	2016	3 984 784			4 240 389	-255 605	65 333 916 025	6,03%		
	2017	3 971 695			3 984 784	-13 089	171 321 921	0,33%		
		4 251 267		3 971 695	279 572	78 160 503 184	7,04%			
Forecast	2019		3 998 178	4 251 267	4 251 267				253 089	6,33%
	2020		3 885 718	4 251 267	4 251 267				365 549	9,41%
	2021		4 207 740	4 251 267	4 251 267				43 527	1,03%
	2022		4 227 782	4 251 267	4 251 267				23 485	0,56%

t = 9

$\sum(X_j - P_j)^2$	288 618 151 314	Weight	
t-(k+1)	5	w(l-1)	1,00
σ_k	240 257	w(l-2)	0,00
		w(l-3)	0,00

4.11 World - FCA - Gasoline Case – Weighted Moving Average Data_K=3

			K =		4					
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	3 259 749	4 251 267							
	2011	3 494 046								
	2012	3 979 325								
	2013	4 160 783								
	2014	4 433 885			4 102 706	331 179	109 679 588 707	7,47%		
	2015	4 240 389			4 373 307	-132 918	17 667 113 521	3,13%		
	2016	3 984 784			4 223 562	-238 778	57 014 859 483	5,99%		
	2017	3 971 695			3 996 128	-24 433	596 981 128	0,62%		
		4 251 267		4 001 486	249 781	62 390 579 547	5,88%			
Forecast	2019		3 998 178	4 250 566	4 250 566				252 388	6,31%
	2020		3 885 718	4 233 435	4 233 435				347 717	8,95%
	2021		4 207 740	4 216 564	4 216 564				8 824	0,21%
	2022		4 227 782	4 218 801	4 218 801				8 981	0,21%

t = 9

$\sum(X_j - P_j)^2$	247 349 122 385	Weight	
t-(k+1)	4	w(l-1)	0,94
σ_k	248 671	w(l-2)	0,00
		w(l-3)	0,00
		w(l-4)	0,06

4.11 World - FCA - Gasoline Case – Weighted Moving Average Data_K=4

			K =		5				
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	3 259 749							
	2011	3 494 046							
	2012	3 979 325							
	2013	4 160 783							
	2014	4 433 885							
	2015	4 240 389		4 130 373	110 016	12 103 463 472	2,59%		
	2016	3 984 784		4 114 810	-130 026	16 906 856 215	3,26%		
	2017	3 971 695		4 019 489	-47 794	2 284 313 079	1,20%		
	2018	4 251 267	4 251 267	4 082 815	168 452	28 376 199 919	3,96%		
Forecast	2019		3 998 178	4 267 025	4 267 025			268 847	6,72%
	2020		3 885 718	4 207 894	4 207 894			322 176	8,29%
	2021		4 207 740	4 138 684	4 138 684			69 056	1,64%
	2022		4 227 782	4 144 826	4 144 826			82 956	1,96%

t = 9

$\sum(X_j - P_j)^2$	59 670 832 685	Weight	
t-(k+1)	3	W(l-1)	0,70
σ_k	141 033	W(l-2)	0,00
		W(l-3)	0,00
		W(l-4)	0,20
		W(l-5)	0,10

4.11 World - FCA - Gasoline Case – Weighted Moving Average Data_K=5

			Alpha		1,00				
	Year	FCA Global Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	3 259 749							
	2011	3 494 046		3 259 749	234 297	54 895 084 209	6,71%		
	2012	3 979 325		3 494 046	485 279	235 495 707 841	12,20%		
	2013	4 160 783		3 979 325	181 458	32 927 005 764	4,36%		
	2014	4 433 885		4 160 783	273 102	74 584 702 404	6,16%		
	2015	4 240 389		4 433 885	-193 496	37 440 702 016	4,56%		
	2016	3 984 784		4 240 389	-255 605	65 333 916 025	6,41%		
	2017	3 971 695		3 984 784	-13 089	171 321 921	0,33%		
	2018	4 251 267	4 251 267	3 971 695	279 572	78 160 503 184	6,58%		
Forecast	2019		3 998 178	4 251 267	4 251 267			253 089	6,33%
	2020		3 885 718	4 251 267	4 251 267			365 549	9,41%
	2021		4 207 740	4 251 267	4 251 267			43 527	1,03%
	2022		4 227 782	4 251 267	4 251 267			23 485	0,56%

t = 9
J = 2

$\sum(X_j - P_j)^2$	524 113 859 155
t-(k+1)	7
σ_k	273 630

4.12 World - FCA - Gasoline Case – Single Exponential Smoothing Data

			K =		2					
	Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 235 258	1 142 882							
	2011	1 312 579								
	2012	879 706		1 273 919	-394 213	155 403 495 156	44,81%			
	2013	867 916		1 096 143	-228 227	52 087 335 302	26,30%			
	2014	891 915		873 811	18 104	327 754 816	2,03%			
	2015	1 061 035		879 916	181 120	32 804 273 280	17,07%			
	2016	1 202 179		976 475	225 704	50 942 295 616	18,77%			
	2017	1 193 988		1 131 607	62 381	3 891 389 161	5,22%			
	2018	1 142 882			1 198 084	-55 202	3 047 205 602	4,83%		
Forecast	2019		1 091 045	1 168 435	1 168 435			77 390	7%	
	2020		952 729	1 155 659	1 155 659			202 930	21%	
	2021		780 303	1 162 047	1 162 047			381 744	49%	
	2022		623 376	1 158 853	1 158 853			535 477	86%	

t = 9

$\Sigma(X_j - P_j)^2$	298 503 748 934
t-(k+1)	6
σ_k	223 048

4.13 World - FCA - Diesel Case – Moving Average Data_K=2

			K =		3					
	Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1 235 258	1 142 882							
	2011	1 312 579								
	2012	879 706								
	2013	867 916		1 142 514		-274 598	75 404 244 669	24,03%		
	2014	891 915		1 020 067		-128 152	16 422 935 104	12,56%		
	2015	1 061 035		879 846		181 189	32 829 574 514	20,59%		
	2016	1 202 179		940 289		261 890	68 586 546 693	27,85%		
	2017	1 193 988		1 051 710		142 278	20 243 124 136	13,53%		
	2018	1 142 882		1 152 401		-9 519	90 605 015	0,83%		
Forecast	2019		1 091 045	1 179 683	1 179 683				88 638	8,12%
	2020		952 729	1 172 184	1 172 184				219 455	23,03%
	2021		780 303	1 164 916	1 164 916				384 613	49,29%
	2022		623 376	1 172 261	1 172 261				548 885	88,05%

t = 9

$\Sigma(X_j - P_j)^2$	213 577 030 132
t-(k+1)	5
σ_k	206 677

4.13 World - FCA - Diesel Case – Moving Average Data_K=3

			K =		4					
	Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 235 258	1 142 882							
	2011	1 312 579								
	2012	879 706								
	2013	867 916								
	2014	891 915			1 073 865	-181 950	33 105 711 525	20,40%		
	2015	1 061 035			988 029	73 006	5 329 876 036	6,88%		
	2016	1 202 179			925 143	277 036	76 748 945 296	23,04%		
	2017	1 193 988			1 005 761	188 227	35 429 309 416	15,76%		
	2018	1 142 882			1 087 279	55 603	3 091 665 808	4,87%		
Forecast	2019		1 091 045	1 150 021	1 150 021				58 976	5,41%
	2020		952 729	1 172 268	1 172 268				219 539	23,04%
	2021		780 303	1 164 790	1 164 790				384 487	49,27%
	2022		623 376	1 157 490	1 157 490				534 114	85,68%

t = 9

$\sum (X_j - P_j)^2$	153 705 508 080
t-(k+1)	4
σ_k	196 026

4.13 World - FCA - Diesel Case – Moving Average Data_K=4

				K =		5				
	Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines)	Abs. Squared Error	Error %	Absolute Forecast Error	Error %
						$Real\ Data_t - Forecast_t$	$Abs(Error_t)^2$	$Forecast\ vs\ Real\ Data$	Qntv vs Qltv (Unit Engines)	Qntv vs Qltv
Real Data Market	2010	1 235 258	1 142 882							
	2011	1 312 579								
	2012	879 706								
	2013	867 916								
	2014	891 915								
	2015	1 061 035			1 037 475	23 560	555 083 024	2,22%		
	2016	1 202 179			1 002 630	199 549	39 819 723 581	16,60%		
	2017	1 193 988			980 550	213 438	45 555 694 469	17,88%		
	2018	1 142 882		1 043 407	99 475	9 895 355 205	8,70%			
Forecast	2019		1 091 045	1 098 400	1 098 400				7 355	0,67%
	2020		952 729	1 139 697	1 139 697				186 968	19,62%
	2021		780 303	1 155 429	1 155 429				375 126	48,07%
	2022		623 376	1 146 079	1 146 079				522 703	83,85%

t = 9

$\sum (X_j - P_j)^2$	95 825 856 279
t-(k+1)	3
σ_k	178 723

4.13 World - FCA - Diesel Case – Moving Average Data_K=5

			K = 2						
Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) $Real Data_t - Forecast_t$	Absolute Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 235 258							
	2011	1 312 579							
	2012	879 706		1 312 114	-432 408	186 977 019 443	49,15%		
	2013	867 916		882 307	-14 391	207 102 129	1,66%		
	2014	891 915		867 987	23 928	572 556 667	2,68%		
	2015	1 061 035		891 771	169 264	28 650 371 085	15,95%		
	2016	1 202 179		1 060 019	142 160	20 209 524 374	11,83%		
	2017	1 193 988		1 201 331	-7 343	53 918 110	0,61%		
Forecast	2018	1 142 882	1 142 882	1 194 037	-51 155	2 616 856 329	4,48%		
	2019		1 091 045	1 143 189				52 144	5%
	2020		952 729	1 143 187				190 458	20%
	2021		780 303	1 143 187				362 884	47%
	2022		623 376	1 143 187				519 811	83%

t = 9

$\sum(X_j - P_j)^2$	239 287 348 136	Weight	
t-(k+1)	6	w(l-1)	0,99
σ_k	199 703	w(l-2)	0,01

4.14 World - FCA - Diesel Case – Weighted Moving Average Data_K=2

			K = 3						
Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) $Real Data_t - Forecast_t$	Absolute Squared Error $ABS(Error_t)^2$	Error % $Forecast vs Real Data$	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 235 258							
	2011	1 312 579							
	2012	879 706							
	2013	867 916		879 706	-11 790	139 004 100	1,34%		
	2014	891 915		867 916	23 999	575 952 001	2,77%		
	2015	1 061 035		891 915	169 120	28 601 574 400	18,96%		
	2016	1 202 179		1 061 035	141 144	19 921 628 736	13,30%		
	2017	1 193 988		1 202 179	-8 191	67 092 481	0,68%		
Forecast	2018	1 142 882	1 142 882	1 193 988	-51 106	2 611 823 236	4,28%		
	2019		1 091 045	1 142 882				51 837	4,75%
	2020		952 729	1 142 882				190 153	19,96%
	2021		780 303	1 142 882				362 579	46,47%
	2022		623 376	1 142 882				519 506	83,34%

t = 9

$\sum(X_j - P_j)^2$	51 917 074 954	Weight	
t-(k+1)	5	w(l-1)	1,00
σ_k	101 899	w(l-2)	0,00
		w(l-3)	0,00

4.14 World - FCA - Diesel Case – Weighted Moving Average Data_K=3

			K =		4					
	Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 235 258								
	2011	1 312 579								
	2012	879 706								
	2013	867 916								
	2014	891 915			916 564	-24 649	607 564 525	2,76%		
	2015	1 061 035			947 624	113 411	12 861 972 923	10,69%		
	2016	1 202 179			1 037 021	165 158	27 277 084 963	13,74%		
	2017	1 193 988			1 157 912	36 076	1 301 485 634	3,02%		
2018	1 142 882	1 142 882		1 153 984	-11 102	123 251 656	0,97%			
Forecast	2019			1 091 045	1 132 043	1 132 043			40 998	3,76%
	2020			952 729	1 141 331	1 141 331			188 602	19,80%
	2021			780 303	1 148 305	1 148 305			368 002	47,16%
	2022			623 376	1 147 586	1 147 586			524 210	84,09%

t = 9

$\sum(X_j - P_j)^2$	42 171 359 700	Weight	
t-(k+1)	4	w(l-1)	0,87
σ_k	102 678	w(l-2)	0,00
		w(l-3)	0,00
		w(l-4)	0,13

4.14 World - FCA - Diesel Case – Weighted Moving Average Data_K=4

			K =		5					
	Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 235 258	1 142 882							
	2011	1 312 579								
	2012	879 706								
	2013	867 916								
	2014	891 915								
	2015	1 061 035			990 914	70 121	4 916 995 432	6,61%		
	2016	1 202 179			1 133 565	68 614	4 707 935 990	5,71%		
	2017	1 193 988			1 109 198	84 790	7 189 364 218	7,10%		
Forecast	2018	1 142 882			1 099 969	42 913	1 841 512 334	3,75%		
	2019		1 091 045	1 070 519	1 070 519				20 526	1,88%
	2020		952 729	1 067 784	1 067 784				115 055	12,08%
	2021		780 303	1 106 535	1 106 535				326 232	41,81%
	2022		623 376	1 131 751	1 131 751				508 375	81,55%

t = 9

$\sum(X_j - P_j)^2$	18 655 807 974	Weight	
t-(k+1)	3	w(l-1)	0,71
σ_k	78 858	w(l-2)	0,00
		w(l-3)	0,00
		w(l-4)	0,00
		w(l-5)	0,29

4.14 World - FCA - Diesel Case – Weighted Moving Average Data_K=5

			Alpha	1,00					
	Year	FCA Global Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) $Real\ Data_t - Forecast_t$	Abs. Squared Error $ABS(Error_t)^2$	Error % $Forecast\ vs\ Real\ Data$	Abs. Forecast Error $Qntv\ vs\ Qltv$ (Unit Engines)	Error % $Qntv\ vs\ Qltv$
Real Data Market	2010	1 235 258	1 142 882						
	2011	1 312 579		1 235 258	77 321	5 978 537 041	5,89%		
	2012	879 706		1 312 579	-432 873	187 379 034 129	49,21%		
	2013	867 916		879 706	-11 790	139 004 100	1,36%		
	2014	891 915		867 916	23 999	575 952 001	2,69%		
	2015	1 061 035		891 915	169 120	28 601 574 400	15,94%		
	2016	1 202 179		1 061 035	141 144	19 921 628 736	11,74%		
	2017	1 193 988		1 202 179	-8 191	67 092 481	0,69%		
Forecast	2018	1 142 882	1 142 882	1 193 988	-51 106	2 611 823 236	4,47%		
	2019		1 091 045	1 142 882				51 837	4,75%
	2020		952 729	1 142 882				190 153	19,96%
	2021		780 303	1 142 882				362 579	46,47%
	2022		623 376	1 142 882				519 506	83,34%

t =	9
J =	2

$\sum(X_j - P_j)^2$	245 274 646 124
t-(k+1)	7
σ_k	187 188

4.15 World - FCA - Diesel Case – Single Exponential Smoothing Data

	Year	Period	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines)	Absolute Squared Error	Error %	Absolute Forecast Error $Qntv\ vs\ Qltv$ (Unit Engines)	Error % $Qntv\ vs\ Qltv$
Real Data Market	2010	1	1 148 873	1 148 994	984 401	164 472	27 050 907 206,56	14,32%		
	2011	2	983 405		1 008 863	-25 458	648 108 066,80	2,59%		
	2012	3	909 116		1 033 325	-124 209	15 427 759 752,82	13,66%		
	2013	4	958 100		1 057 786	-99 686	9 937 318 533,21	10,40%		
	2014	5	999 534		1 082 248	-82 714	6 841 550 653,44	8,28%		
	2015	6	1 180 454		1 106 709	73 745	5 438 290 610,72	6,25%		
	2016	7	1 221 956		1 131 171	90 785	8 241 952 539,04	7,43%		
	2017	8	1 189 797		1 155 632	34 165	1 167 222 170,80	2,87%		
	2018	9	1 148 994		1 180 094	-31 100	967 205 853,34	2,71%		
Forecast	2019	10		1 056 309	1 204 556				148 246,50	14,03%
	2020	11		1 023 160	1 229 017				205 857,07	20,12%
	2021	12		1 223 647	1 253 479				29 831,63	2,44%
	2022	13		1 290 835	1 277 940				12 894,80	1,00%

4.16 EMEA - FCA - Gasoline Case – Trend Data

	Year	Period	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1	851 787	803 042		742 804	108 983	108 983	12,79%		
	2011	2	884 084			749 770	134 314	134 314	15,19%		
	2012	3	623 047			756 736	-133 689	133 689	21,46%		
	2013	4	624 170			763 701	-139 531	139 531	22,35%		
	2014	5	648 792			770 667	-121 875	121 875	18,78%		
	2015	6	752 248			777 633	-25 385	25 385	3,37%		
	2016	7	863 308			784 598	78 710	78 710	9,12%		
	2017	8	885 524			791 564	93 960	93 960	10,61%		
	2018	9	803 042			798 530	4 512	4 512	0,56%		
Forecast	2019	10			707 210	805 495	805 495			98 285	13,90%
	2020	11			598 808	812 461	812 461			213 653	35,68%
	2021	12			420 412	819 427	819 427			399 015	94,91%
	2022	13			280 164	826 392	826 392			546 228	194,97%

4.16 EMEA - FCA - Diesel Case – Trend Data

				K =		2				
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Abs. Squared Error <i>ABS(Error)_t²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1 148 873	1 148 994							
	2011	983 405								
	2012	909 116		1 066 139	-157 023	24 656 222 529	17,27%			
	2013	958 100		946 261	11 840	140 173 760	1,24%			
	2014	999 534		933 608	65 926	4 346 237 476	6,60%			
	2015	1 180 454		978 817	201 637	40 657 479 769	17,08%			
	2016	1 221 956		1 089 994	131 962	17 413 969 444	10,80%			
	2017	1 189 797		1 201 205	-11 408	130 142 464	0,96%			
	2018	1 148 994		1 205 877	-56 883	3 235 618 806	4,95%			
Forecast	2019		1 056 309	1 169 396	1 169 396				113 087	10,71%
	2020		1 023 160	1 159 195	1 159 195				136 035	13,30%
	2021		1 223 647	1 164 295	1 164 295				59 352	4,85%
	2022		1 290 835	1 161 745	1 161 745				129 090	10,00%

t = 9

$\sum (X_j - P_j)^2$	90 579 844 249
t-(k+1)	6
σ_k	122 868

4.17 EMEA - FCA - Gasoline Case – Moving Average Data_K=2

		K =		3					
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 148 873	1 148 994						
	2011	983 405							
	2012	909 116							
	2013	958 100		1 013 798	-55 698	3 102 267 204	5,49%		
	2014	999 534		950 207	49 327	2 433 152 929	5,19%		
	2015	1 180 454		955 583	224 871	50 566 816 727	23,53%		
	2016	1 221 956		1 046 029	175 927	30 950 192 044	16,82%		
	2017	1 189 797		1 133 981	55 816	3 115 388 645	4,92%		
Forecast	2018	1 148 994		1 197 402	-48 408	2 343 366 736	4,04%		
	2019		1 056 309	1 186 916				130 607	12,36%
	2020		1 023 160	1 175 236				152 076	14,86%
	2021		1 223 647	1 170 382				53 265	4,35%
	2022		1 290 835	1 177 511				113 324	8,78%

t = 9

$\sum (X_j - P_j)^2$	92 511 184 286
t-(k+1)	5
σ_k	136 023

4.17 EMEA - FCA - Gasoline Case – Moving Average Data_K=3

		K =		4					
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 148 873	1 148 994						
	2011	983 405							
	2012	909 116							
	2013	958 100							
	2014	999 534		999 874	-340	115 260	0,03%		
	2015	1 180 454		962 539	217 915	47 487 056 183	18,46%		
	2016	1 221 956		1 011 801	210 155	44 165 124 025	17,20%		
	2017	1 189 797		1 090 011	99 786	9 957 245 796	8,39%		
Forecast	2018	1 148 994		1 147 935	1 059	1 120 952	0,09%		
	2019		1 056 309	1 185 300				128 991	12,21%
	2020		1 023 160	1 186 512				163 352	15,97%
	2021		1 223 647	1 177 651				45 996	3,76%
	2022		1 290 835	1 174 614				116 221	9,00%

t = 9

$\sum (X_j - P_j)^2$	101 610 662 215
t-(k+1)	4
σ_k	159 382

4.17 EMEA - FCA - Gasoline Case – Moving Average Data_K=4

		K =		5					
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1 148 873							
	2011	983 405							
	2012	909 116							
	2013	958 100							
	2014	999 534							
	2015	1 180 454		999 806	180 648	32 633 844 423	15,30%		
	2016	1 221 956		1 006 122	215 834	46 584 401 890	17,66%		
	2017	1 189 797		1 053 832	135 965	18 486 481 225	11,43%		
Forecast	2018	1 148 994	1 148 994	1 109 968	39 026	1 523 013 066	3,40%		
	2019		1 056 309	1 148 147				91 838	8,69%
	2020		1 023 160	1 177 870				154 710	15,12%
	2021		1 223 647	1 177 353				46 294	3,78%
	2022		1 290 835	1 168 432				122 403	9,48%

t = 9

$\sum(X_j - P_j)^2$	99 227 740 603
t-(k+1)	3
σ_k	181 868

4.17 EMEA - FCA - Gasoline Case – Moving Average Data_K=5

		K =		2					
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecast Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	1 148 873							
	2011	983 405							
	2012	909 116		983 405	-74 289	5 518 854 070	8,17%		
	2013	958 100		909 116	48 984	2 399 433 141	5,11%		
	2014	999 534		958 100	41 434	1 716 777 145	4,15%		
	2015	1 180 454		999 534	180 920	32 732 049 993	15,33%		
	2016	1 221 956		1 180 454	41 502	1 722 416 977	3,40%		
	2017	1 189 797		1 221 956	-32 159	1 034 200 500	2,70%		
Forecast	2018	1 148 994	1 148 994	1 189 797	-40 803	1 664 883 844	3,55%		
	2019		1 056 309	1 148 994				92 685	8,77%
	2020		1 023 160	1 148 994				125 834	12,30%
	2021		1 223 647	1 148 994				74 653	6,10%
	2022		1 290 835	1 148 994				141 841	10,99%

t = 9

$\sum(X_j - P_j)^2$	46 788 615 670	Weight	
t-(k+1)	6	w(l-1)	1,00
σ_k	88 307	w(l-2)	0,00

4.18 EMEA - FCA - Gasoline Case – Weighted Moving Average Data_K=2

			K =		3					
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 148 873	1 148 994							
	2011	983 405								
	2012	909 116								
	2013	958 100		909 116	48 984	2 399 432 256	5,39%			
	2014	999 534		958 100	41 434	1 716 776 356	4,32%			
	2015	1 180 454		999 534	180 920	32 732 046 400	18,10%			
	2016	1 221 956		1 180 454	41 502	1 722 416 004	3,52%			
	2017	1 189 797		1 221 956	-32 159	1 034 201 281	2,63%			
	2018	1 148 994		1 189 797	-40 803	1 664 884 809	3,43%			
Forecast	2019		1 056 309	1 148 994	1 148 994				92 685	8,77%
	2020		1 023 160	1 148 994	1 148 994				125 834	12,30%
	2021		1 223 647	1 148 994	1 148 994				74 653	6,10%
	2022		1 290 835	1 148 994	1 148 994				141 841	10,99%

t = 9

$\sum (X_j - P_j)^2$	41 269 757 106	Weight	
t-(k+1)	5	w(l-1)	1,00
σ_k	90 851	w(l-2)	0,00
		w(l-3)	0,00

4.18 EMEA - FCA - Gasoline Case – Weighted Moving Average Data_K=3

				K =		4				
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	1 148 873	1 148 994							
	2011	983 405								
	2012	909 116								
	2013	958 100								
	2014	999 534			966 905	32 629	1 064 644 278	3,26%		
	2015	1 180 454			998 790	181 664	33 001 966 040	15,39%		
	2016	1 221 956			1 167 930	54 026	2 918 763 680	4,42%		
	2017	1 189 797			1 209 778	-19 981	399 230 273	1,68%		
	1 148 994				1 181 015	-32 021	1 025 371 762	2,79%		
Forecast	2019		1 056 309	1 150 446	1 150 446				94 137	8,91%
	2020		1 023 160	1 153 747	1 153 747				130 587	12,76%
	2021		1 223 647	1 155 410	1 155 410				68 237	5,58%
	2022		1 290 835	1 155 114	1 155 114				135 721	10,51%

t = 9

$\sum (X_j - P_j)^2$	38 409 976 033	Weight	
t-(k+1)	4	w(l-1)	0,95
σ_k	97 992	w(l-2)	0,00
		w(l-3)	0,00
		w(l-4)	0,05

4.18 EMEA - FCA - Gasoline Case – Weighted Moving Average Data_K=4

				K =		5					
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv	
Real Data Market	2010	1 148 873	1 148 994								
	2011	983 405									
	2012	909 116									
	2013	958 100									
	2014	999 534									
	2015	1 180 454			1 026 466	153 988	23 712 403 386	13,04%			
	2016	1 221 956			1 144 918	77 038	5 934 797 812	6,30%			
	2017	1 189 797			1 165 539	24 258	588 464 602	2,04%			
	2018	1 148 994			1 148 013	981	962 419	0,09%			
Forecast	2019			1 056 309	1 122 041				65 732	6,22%	
	2020			1 023 160	1 132 575				109 415	10,69%	
	2021			1 223 647	1 148 694				74 953	6,13%	
	2022			1 290 835	1 156 106				134 729	10,44%	

t = 9

$\Sigma(X_j - P_j)^2$	30 236 628 220	Weight	
$t - (k+1)$	3	w(l-1)	0,82
σ_k	100 394	w(l-2)	0,00
		w(l-3)	0,00
		w(l-4)	0,00
		w(l-5)	0,18

4.18 EMEA - FCA - Gasoline Case – Weighted Moving Average Data_K=5

				Alpha		1,00				
	Year	FCA EMEA Gasoline Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntvvs Qltv</i>
Real Data Market	2010	1 148 873	1 148 994							
	2011	983 405		1 148 873	-165 468	27 379 659 024	16,83%			
	2012	909 116		983 405	-74 289	5 518 855 521	8,17%			
	2013	958 100		909 116	48 984	2 399 432 256	5,11%			
	2014	999 534		958 100	41 434	1 716 776 356	4,15%			
	2015	1 180 454		999 534	180 920	32 732 046 400	15,33%			
	2016	1 221 956		1 180 454	41 502	1 722 416 004	3,40%			
	2017	1 189 797		1 221 956	-32 159	1 034 201 281	2,70%			
	2018	1 148 994		1 189 797	-40 803	1 664 884 809	3,55%			
Forecast	2019		1 056 309	1 148 994	1 148 994				92 685	8,77%
	2020		1 023 160	1 148 994	1 148 994				125 834	12,30%
	2021		1 223 647	1 148 994	1 148 994				74 653	6,10%
	2022		1 290 835	1 148 994	1 148 994				141 841	10,99%

t = 9
J = 2

$\Sigma(X_j - P_j)^2$	46 788 612 627
$t - (k+1)$	7
σ_k	81 756

4.19 EMEA - FCA - Gasoline Case – Single Exponential Smoothing Data

			K =		2,00				
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) $Real Data_t - Forecast_t$	Absolute Squared Error $ABS(Error)^2$	Error %	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	851 787	803 042						
	2011	884 084							
	2012	623 047		867 936	-244 889	59 970 377 432	39,30%		
	2013	624 170		753 566	-129 396	16 743 195 420	20,73%		
	2014	648 792		623 609	25 184	634 208 672	3,88%		
	2015	752 248		636 481	115 767	13 401 998 289	15,39%		
	2016	863 308		700 520	162 788	26 499 932 944	18,86%		
	2017	885 524		807 778	77 746	6 044 440 516	8,78%		
Forecast	2018	803 042		874 416	-71 374	5 094 247 876	8,89%		
	2019		707 210	844 283				137 073	19%
	2020		598 808	823 663				224 855	38%
	2021		420 412	833 973				413 561	98%
	2022		280 164	828 818				548 654	196%

t =	9	$\sum (X_j - P_j)^2$	128 388 401 150
		t-(k+1)	6
		σ_k	146 281

4.20 EMEA - FCA - Diesel Case – Moving Average Data_K=2

			K =		3				
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) $Real Data_t - Forecast_t$	Abs. Squared Error $ABS(Error_t)^2$	Error % $Forecast$ vs Real Data	Abs. Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	851 787	803 042						
	2011	884 084							
	2012	623 047							
	2013	624 170		786 306	-162 136	26 288 082 496	20,62%		
	2014	648 792		710 434	-61 642	3 799 695 069	8,68%		
	2015	752 248		632 003	120 245	14 458 860 025	19,03%		
	2016	863 308		675 070	188 238	35 433 544 644	27,88%		
	2017	885 524		754 783	130 741	17 093 296 242	17,32%		
Forecast	2018	803 042		833 693	-30 651	939 504 235	3,68%		
	2019		707 210	850 625				143 415	20,28%
	2020		598 808	846 397				247 589	41,35%
	2021		420 412	833 355				412 943	98,22%
	2022		280 164	843 459				563 295	201,06%

t =	9	$\sum (X_j - P_j)^2$	98 012 982 711
		t-(k+1)	5
		σ_k	140 009

4.20 EMEA - FCA - Diesel Case – Moving Average Data_K=3

				K =		4				
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	851 787								
	2011	884 084								
	2012	623 047								
	2013	624 170								
	2014	648 792			745 772	-96 980	9 405 120 400	14,95%		
	2015	752 248			695 023	57 225	3 274 672 013	7,61%		
	2016	863 308			662 064	201 244	40 499 046 914	23,31%		
	2017	885 524			722 130	163 395	26 697 762 630	18,45%		
	2018	803 042	803 042		787 468	15 574	242 549 476	1,94%		
Forecast	2019		707 210	826 031	826 031				118 821	16,80%
	2020		598 808	844 476	844 476				245 668	41,03%
	2021		420 412	839 768	839 768				419 356	99,75%
	2022		280 164	828 329	828 329				548 165	195,66%

t = 9

$\sum(X_j - P_j)^2$	80 119 151 433
t-(k+1)	4
σ_k	141 527

4.20 EMEA - FCA - Diesel Case – Moving Average Data_K=4

				K =		5				
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	851 787								
	2011	884 084								
	2012	623 047								
	2013	624 170								
	2014	648 792								
	2015	752 248			726 376	25 872,0	669 360 384	3,44%		
	2016	863 308			706 468	156 839,8	24 598 722 864	18,17%		
	2017	885 524			702 313	183 211,0	33 566 270 521	20,69%		
	2018	803 042	803 042		754 808	48 233,6	2 326 480 169	6,01%		
Forecast	2019			707 210	790 583				83 373	11,79%
	2020			598 808	818 941				220 133	36,76%
	2021			420 412	832 280				411 868	97,97%
	2022			280 164	826 074				545 910	194,85%

t = 9

$\sum(X_j - P_j)^2$	61 160 833 938
t-(k+1)	3
σ_k	142 783

4.20 EMEA - FCA - Diesel Case – Moving Average Data_K=5

			K =		2					
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	851 787	803 042							
	2011	884 084								
	2012	623 047		884 084	-261 037	68 140 321 995	41,90%			
	2013	624 170		623 047	1 123	1 261 099	0,18%			
	2014	648 792		624 170	24 622	606 242 427	3,80%			
	2015	752 248		648 792	103 456	10 703 142 012	13,75%			
	2016	863 308		752 248	111 060	12 334 321 453	12,86%			
	2017	885 524		863 308	22 216	493 550 158	2,51%			
	2018	803 042		885 524	-82 482	6 803 282 446	10,27%			
Forecast	2019		707 210	803 042	803 042				95 832	14%
	2020		598 808	803 042	803 042				204 234	34%
	2021		420 412	803 042	803 042				382 630	91%
	2022		280 164	803 042	803 042				522 878	187%

t = 9

$\sum (X_j - P_j)^2$	99 082 121 590	Weight	
t-(k+1)	6	w(l-1)	1,00
σ_k	128 506	w(l-2)	0,00

4.21 EMEA - FCA - Diesel Case – Weighted Moving Average Data_K=2

				K =		3					
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecast Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv	
Real Data Market	2010	851 787	803 042								
	2011	884 084									
	2012	623 047									
	2013	624 170			623 047	1 123	1 261 129	0,18%			
	2014	648 792			624 170	24 622	606 242 884	3,94%			
	2015	752 248			648 792	103 456	10 703 143 936	15,95%			
	2016	863 308			752 248	111 060	12 334 323 600	14,76%			
	2017	885 524			863 308	22 216	493 550 656	2,57%			
	2018	803 042			885 524	-82 482	6 803 280 324	9,31%			
Forecast	2019			707 210	803 042	803 042			95 832	13,55%	
	2020			598 808	803 042	803 042			204 234	34,11%	
	2021			420 412	803 042	803 042			382 630	91,01%	
	2022			280 164	803 042	803 042			522 878	186,63%	

t = 9

$\sum (X_j - P_j)^2$	30 941 802 529	Weight	
t-(k+1)	5	w(l-1)	1,00
σ_k	78 666	w(l-2)	0,00
		w(l-3)	0,00

4.21 EMEA - FCA - Diesel Case – Weighted Moving Average Data_K=3

			K =		4					
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Absolute Squared Error <i>ABS(Error)²</i>	Error % <i>Forecast vs Real Data</i>	Absolute Forecat Error Qntv vs Qltv (Unit Engines)	Error % Qntv vs Qltv
Real Data Market	2010	851 787	803 042							
	2011	884 084								
	2012	623 047								
	2013	624 170								
	2014	648 792			652 790	-3 998	15 980 541	0,62%		
	2015	752 248			678 377	73 871	5 456 985 527	9,82%		
	2016	863 308			736 003	127 305	16 206 606 363	14,75%		
	2017	885 524			833 240	52 284	2 733 634 284	5,90%		
	2018	803 042		855 758	-52 716	2 779 013 698	6,56%			
Forecast	2019			707 210	796 655				89 445	12,65%
	2020			598 808	805 036	805 036			206 228	34,44%
	2021			420 412	815 156	815 156			394 744	93,89%
	2022			280 164	813 633	813 633			533 469	190,41%

$$t = 9$$

$\Sigma(X_j - P_j)^2$	27 192 220 413	Weight	
$t - (k+1)$	4	W(I-1)	0,87
σ_k	82 450	W(I-2)	0,00
		W(I-3)	0,00
		W(I-4)	0,13

4.21 EMEA - FCA - Diesel Case – Weighted Moving Average Data_K=4

			K =		5					
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)		Error (Unit Engines) <i>Real Data_t – Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	851 787								
	2011	884 084								
	2012	623 047								
	2013	624 170								
	2014	648 792								
	2015	752 248			705 816	46 432	2 155 910 728	6,17%		
	2016	863 308			789 283	74 025	5 479 756 104	8,57%		
	2017	885 524			795 815	89 709	8 047 665 729	10,13%		
	2018	803 042		812 106	-9 064	82 154 091	1,13%			
Forecast	2019		707 210	759 711	759 711				52 501	7,42%
	2020		598 808	757 614	757 614				158 806	26,52%
	2021		420 412	787 305	787 305				366 893	87,27%
	2022		280 164	814 896	814 896				534 732	190,86%

$\Sigma(X_j - P_j)^2$	15 765 486 653	Weight	
$t - (k+1)$	3	W(I-1)	0,72
σ_k	72 492	W(I-2)	0,00
		W(I-3)	0,00
		W(I-4)	0,00
		W(I-5)	0,28

4.21 EMEA - FCA - Diesel Case – Weighted Moving Average Data_K=5

			Alpha	0,52					
	Year	FCA EMEA Diesel Market	Qualitative Forecast	Quantitative Forecast (Unit Engines)	Error (Unit Engines) <i>Real Data_t - Forecast_t</i>	Abs. Squared Error <i>ABS(Error_t)²</i>	Error % <i>Forecast vs Real Data</i>	Abs. Forecat Error <i>Qntv vs Qltv (Unit Engines)</i>	Error % <i>Qntv vs Qltv</i>
Real Data Market	2010	851 787	803 042,00						
	2011	884 084		444 998	439 086	192 796 269 795	49,67%		
	2012	623 047		674 389	-51 342	2 636 043 962	8,24%		
	2013	624 170		647 567	-23 397	547 403 105	3,75%		
	2014	648 792		635 344	13 448	180 860 568	2,07%		
	2015	752 248		642 369	109 879	12 073 303 373	14,61%		
	2016	863 308		699 773	163 535	26 743 637 104	18,94%		
	2017	885 524		785 209	100 315	10 063 195 075	11,33%		
	2018	803 042		837 616	-34 574	1 195 378 262	4,31%		
Forecast	2019		707 210,00	819 554	819 554			112 344	15,89%
	2020		598 808,00	819 554	819 554			220 746	36,86%
	2021		420 412,00	819 554	819 554			399 142	94,94%
	2022		280 164,00	819 554	819 554			539 390	192,53%

t =	9
J =	2

$\sum (X_j - P_j)^2$	53 439 821 450
t-(k+1)	7
σ_k	87 374

4.22 EMEA - FCA - Diesel Case – Single Exponential Smoothing Data

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