POLITECNICO DI TORINO

Master of Science in Aerospace Engineering

Master Thesis

# Computer Aided Design for Manufacturing and Tolerancing for a Turbine Stage

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# Abstract

The design process has always been a complicated, long and iterative procedure and this is particularly true in the case of an aviation jet engine, therefore this thesis, which was carried out inside Avio Aero who is a world leader in the design of low pressure turbine, has the aim of finding and implementing new and innovative methods to achieve a smarter, more cost efficient and more optimized design process, in this case for the low pressure turbine component.

But first of all, a brief introduction of an aviation jet engine is given, going over all the main principles, the significant factors that influence the performances of an aviation engine, as well as describing the most common configurations and architectures of an aviation engine, especially the most used one that is the turbofan architecture. Going deeper, a description of the main components present inside the engine is also made, with particular focus on the low pressure turbine.
This introduction is needed in order to understand the principles behind the design process of an aviation engine and only then it will be clear why certain components have certain shapes and materials.

Afterwards, a small digression is made to describe the project of Great2020, its birth, its objectives set for the year 2020 compatible with those set by ACARE, and the main actors and laboratories participating in this project. This is important to understand the current state of the art and the context on which this thesis finds itself in as well as to understand the possible future trends and scenarios reserved for the aviation engine of future generation.

Then we will shift our attention back to the low pressure turbine, in particular the description of some of the analysis needed to verify this complex component, such as the modal analysis and the static analysis. A particular focus will be given to the overall procedure to carry out these analyses and showing the possible results that can be obtained.

Next we will move to a different topic but still significant in the design of a component, and that is manufacturing, and therefore a brief introduction is given of the main manufacturing operations, especially those relevant to the manufacture of a turbine blade, such as casting and machining. In addition, possible future trends regarding this field is also presented.
This will be necessary to understand the next part, which is the main topic of

this thesis and it concerns the concept of design for manufacturability (DFM). A brief description of its principles and how it works is present, with particular focus to its benefits compared to the traditional design process.
Then we will move on with the implementation of a possible DFM code for the low pressure turbine blade and the results that can be achieved.

Aside from the analysis of manufacturability, another analysis has also been carried out concerning the assembly operation, and that is tolerancing stack up analysis. That is why after a brief introduction to the stackup methods, a deep analysis is shown of a new and innovative method, through a commercial software, to carry out a 3D variational analysis based on Monte Carlo simulations, which could be used to substitute the old traditional methods.

Finally, this thesis will end with the description of future proposals to improve and optimize both the code for DFM and the 3D variational analysis as well as possible trends and scenarios reserved for the field of design for manufacturing of a turbine blade.

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# An introduction to aircraft engine

The aircraft engine is a very complex component of an aircraft which has evolved in many forms over the years starting from the very first engine, a reciprocating engine, also known as piston engine, of the Wright Brothers aircraft to the modern jet propulsion engine integrated in different forms to provide the necessary thrust for the most common seen aircraft.
The jet propulsion is based on the principle of reaction, in other words, a force is applied by the engine on a certain mass of fluid, usually air, and the reaction force thus generated represent the thrust given to the aircraft enabling it to accelerate and take off.

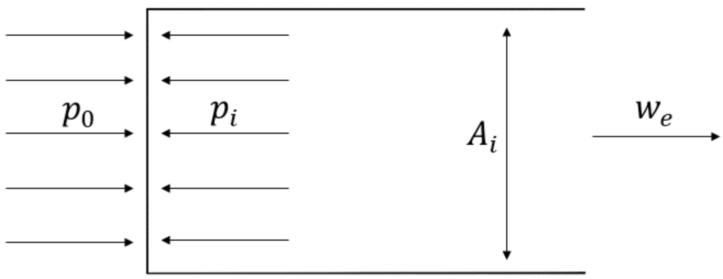


Figura 1.1: Simplified model of a jet propulsion engine.

In order to understand this phenomenon, we can consider an overly simplified model of the engine, that is an open ended tube as seen from the figure 1.1, in which air is supplied, in some way, to the engine and a steady state thrust can be then generated as long as

$$p_i > p_0$$

the thrust can also be easily evaluated by applying the balance law of linear momentum

$$\mathscr{T} = A_i(p_i - p_0)$$

this equation actually represent the ideal thrust generated by the distribution of internal and external pressures of an engine with the drag term due to shear

stresses neglected.
It is needless to say that such a simplified model does not correctly represent the real turbine engine due to the presence of much more complex geometries present inside the engine as well as complex phenomena happening inside, but nonetheless this relation can still give us an idea of what are the main parameter that influences the generated thrust.
Thanks to the introduction of the jet principle into the aircraft engines, thus bringing forth the turbojet concept substituting the old reciprocating engine, it was then possible to:

- increase the flight speed, reaching eventually the supersonic realm.
- reduce the air travel cost, by providing a higher thrust to weight ratio as well as reduce the maintenance cost.

From a thermodynamics point of view, however, the working cycle of the jet propulsion engine is actually very simple, in fact the engine can be seen as a heat machine in which heat energy is given to the airflow, previously compressed in order to have a good enough efficiency, and afterwards it converts this energy, through a series of expansion, into kinetic energy, in other words velocity. Using this process, it is then possible to accelerate the incoming flow and generate the required thrust.
Then all the thermodynamic processes happening inside the engine can be seen as part of a thermodynamic cycle, in particular the engine adopts a Joule-Brayton cycle, as represented in figure 1.2, which is, in a ideal scenario, characterized by the following four phases:

- adiabatic compression;
- isobaric heat addition;
- adiabatic expansion;
- isobaric heat removal.

In a real scenario, the last phase does not actually take place, as the exhaust gas, after the expansion, is not recovered and a supply of new fresh air is instead used to return to the initial condition.

## 1.1 The jet engine performances

Before going into detail about the engine configurations and its components, a brief description about the analysis of the performances is required in order to fully comprehend the meaning behind each configuration.
In particular, we shall focus our attention on two main parameters that can define more or less the goodness of a certain engine design and they are:

- engine thrust;
- engine efficiency.

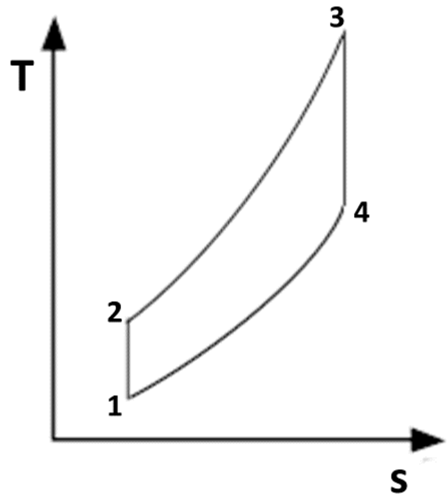


Figura 1.2: Joule - Brayton cycle.

Unfortunately generally it is hard to find an optimal solution that satisfy both parameters and a trade-off has to be often considered.

### 1.1.1 Thrust equation

The main purpose of an engine is certainly to provide the required thrust for the aircraft to operate and by definition its value can be defined as the integral of the pressure forces, both internal and external, projected into the longitudinal axis of the engine.
However, this approach is not feasible as it is hard to have a good estimate of the pressure forces, especially inside the engine, and that is why it is common to evaluate the thrust by applying the balance law of linear momentum to a certain control surface, as represented in figure 1.3.
Then the sum of all the applied forces in the longitudinal direction is equal to

$$\sum F_x = \int_S \rho u(u \cdot n) dS$$

assuming a reversible flow, then the only applied forces are the thrust and the net pressure force

$$\sum F_x = \mathscr{T} + (p_0 - p_e)A_e$$

whereas the integral part can be evaluated by considering the continuity equation, bringing forth the following relation

$$\int_S \rho u(u \cdot n) dS = \dot{m}_e w_e - \dot{m}_0 u$$

the thrust can then be evaluated as

$$\mathscr{T} = \dot{m}_0[(1+f)w_e - u] + (p_e - p_0)A_e$$

in case of air-breathing engines where the equivalence ratio is usually

$$f << 1$$

then the thrust equation can be furthermore simplified as

$$\mathscr{T} = \dot{m}_0(w_e - u) + (p_e - p_0)A_e$$

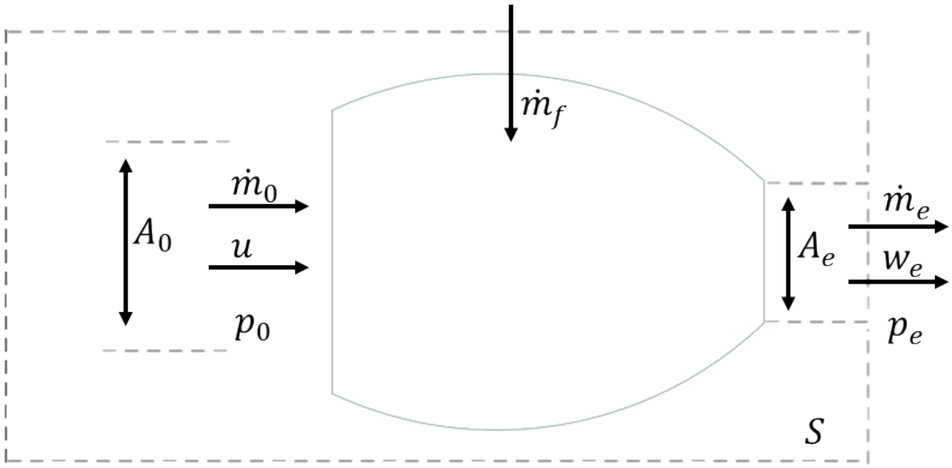


Figura 1.3: Control surface for thrust evaluation.

### 1.1.2 Engine efficiency

Another important parameter which defines the overall performance of an engine is the efficiency, however different definitions of efficiency do exist and each of them focuses on different contributions.

- propulsive efficiency $\eta_p$:
  the propulsive efficiency is defined as the ratio of thrust power to the rate of production of propellant kinetic energy, and in case of a single stream it can be written as

  $$\eta_p = \frac{\mathscr{T} u}{\dot{m}_0 \left[(1+f)\frac{w_e^2}{2} - \frac{u^2}{2}\right]}$$

  and by this definition, it can be considered as a measure of the performance of the propulsive system. Analogously, this relation can be furthermore simplified by neglecting the equivalence ratio term and the pressure term present in the thrust equation, reaching the following relation

  $$\eta_p = \frac{(w_e - u)u}{\frac{w_e^2}{2} - \frac{u^2}{2}}$$

from this equation it can be easily deduced that for thrust to happen, the exhaust gas must be of a higher velocity with respect to the velocity of the aircraft, however the higher the difference between these two velocity the lower is actually the propulsive efficiency.

- thermal efficiency $\eta_p$:
  the thermal efficiency is defined as the ratio of the rate of kinetic energy obtained by the propellant to the total energy consumption rate

$$\eta_{th} = \frac{\left[(1+f)\frac{w_e^2}{2} - \frac{u^2}{2}\right]}{fq_r}$$

  with $q_r$ the heat of reaction obtained from the fuel.

In any case, it is straightforward how both efficiencies are not sufficient, if considered alone, to measure the performance of an engine. That is why, it is common to define an overall efficiency $\eta_g$ simply defined as

$$\eta_g = \eta_p \eta_{th} = 2\eta_{th} \left( \frac{u}{w_e \left(1 + \frac{u}{w_e}\right)} \right)$$

in other words, the overall efficiency of an engine depend on the thermal efficiency and the velocity ratio.

## 1.2 The jet engine configuration

As mentioned previously, from a thermodynamics point of view, the engine can be modeled as a heat machine which realize the Joule-Brayton cycle, sometimes called gas turbine cycle, and to achieve this a certain number of components are present inside. Now it should be clear why an engine usually present the following components:

- intake;
- fan and compressor;
- combustion chamber;
- turbine;
- nozzle.

nonetheless, even with the same working cycle, different configurations of a gas turbine engine are still possible and are still being used. Moreover, even though the role of each component is pretty straightforward, the same function can be still carried out by different arrangement of the same component with each of them having its own benefit and disadvantages.
That is why a certain trade-off is still needed and why different configurations have been developed in order to satisfy different flight conditions and customer requirements.
Nonetheless, since the basic working cycle is often not heavily modified, it

is still possible to define a basic configuration, called gas generator which is represented in figure 1.4, and to define conventional configurations of a civil jet engine according to the variation of a certain component or function.
Lastly, it must also be noted that the design of a single of each of the component is surely not an easy task and it usually takes a long time to develop from the conceptual design of the component to its actual production due to the complexity and the multidisciplinary nature of this field.
However, for this thesis, we shall focus mainly on one particular component which is the turbine stage, in particular the Low Pressure Turbine(LPT) stage, commonly found in the multi-spool engine configuration, for example a dual spool turbofan, which is the case for most of the modern aeroengines.

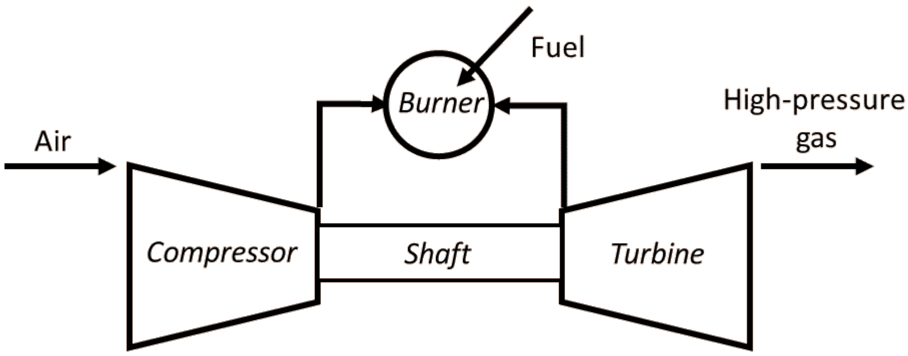


Figura 1.4: Basic gas generator scheme.

### 1.2.1 Turbojet

The first and straightforward engine configuration developed which embodies the jet propulsion principle is certainly the turbojet configuration which consists of mainly a basic gas generator group placed in-between an intake and a nozzle. Through this gas generator group, it is possible to generate high energy exhaust gas which is sent straightly to a nozzle where a further expansion takes place and all the kinetic energy is then converted into velocity, in other words thrust.
The turbojet configuration was popular in the past but nowadays it has been mostly substituted by the turbofan configuration which, even though it usually provides lower thrust, it has however a higher overall efficiency, which is becoming more and more important nowadays, if not the most important, performance parameter in the evaluation of a jet engine.
This is especially true for civil aircraft, as companies tend to prefer a higher efficiency engine which can help in reducing costs, mainly fuel, as well as satisfy pollution requirements which are getting more and more strict.

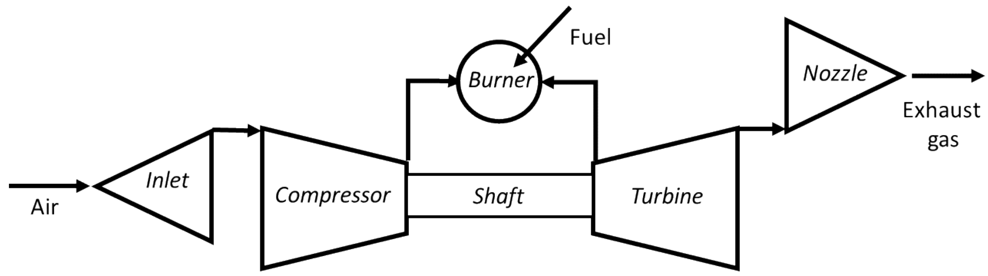


Figura 1.5: Basic scheme of a turbojet engine.

Figura 1.6: A turbojet engine model.

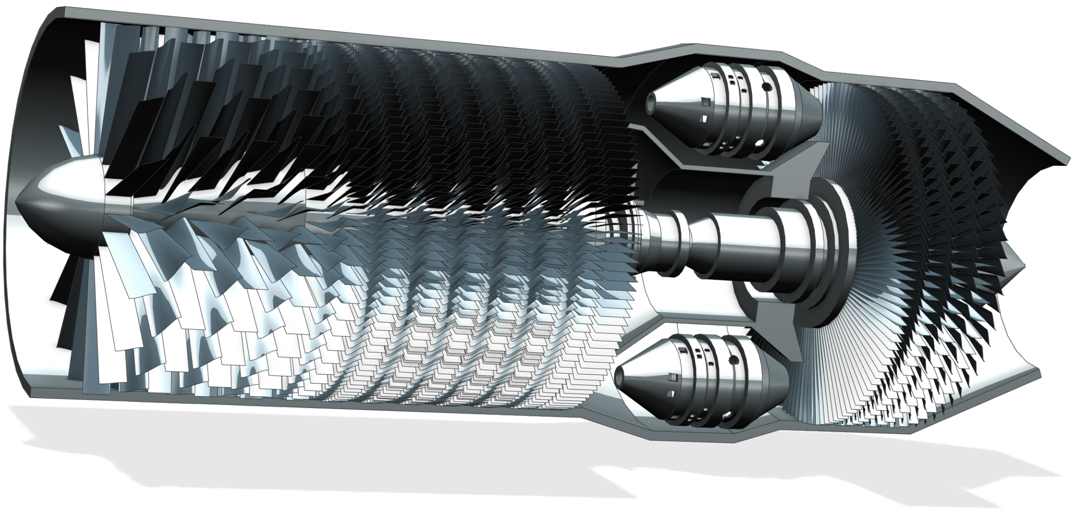

Figura 1.7: A turbojet engine section view.

### 1.2.2 Turbofan

The turbofan configuration can be considered as an evolution from the basic turbojet configuration, in fact it is very similar to the turbojet configuration as the turbofan also has a main gas generator system at its core but it presents two main differences:

- the presence of a fan which functionally is similar to a compressor stage however its design criteria is still very different and due to its position and functionality, it usually presents very few stages compared to the compressor.
- the presence of a bypass duct, in fact the inlet air flow, after being compressed by the fan, is actually divided into two different flow. One of which goes inside the core and that is why it is usually called the hot flow, the other one also called cold flow bypasses the core and goes directly into a mixer or nozzle.

Furthermore, the presence of a fan brings another complication since its optimal rotational speed is often very different compared to the optimal rotational speed of a compressor stage, and that is why a common choice is to use a multi-shaft configuration in order to have different rotational speed, increasing furthermore the efficiency and off-design performances.
Generally, two shafts or three shafts configuration have been used and considering a two shaft configuration, it is then possible to define a high pressure group and a low pressure group of compressor and turbine stages.
Lastly, depending on whether the cold and hot airflow are mixed or not before the expansion in the nozzle, two different versions of a turbofan can then be defined.
By and large, the mixed version usually has a higher performance in terms of

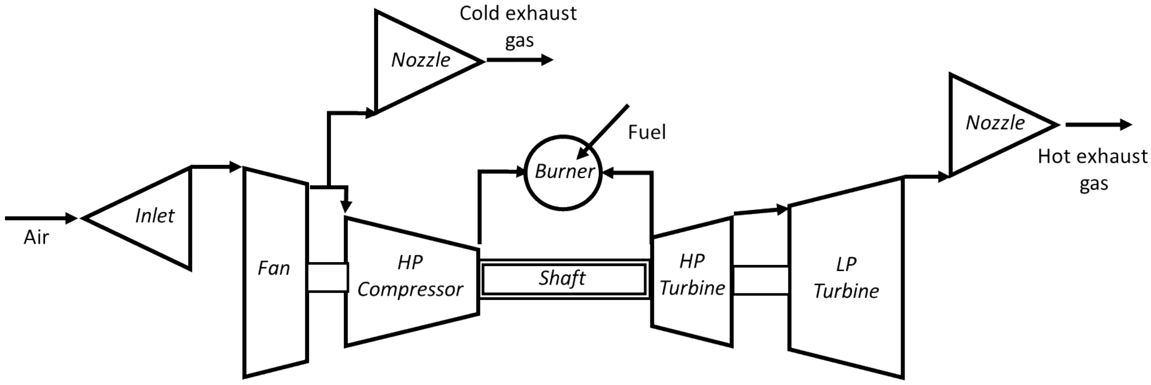


Figura 1.8: Basic scheme of a turbofan engine.

specific thrust and consumption, nonetheless the additional weight cannot be neglected either.
Researches have shown that the separated version is more suitable in case of subsonic flight, and thus for civil aircraft, whereas the mixed version is slightly better for the supersonic flight.

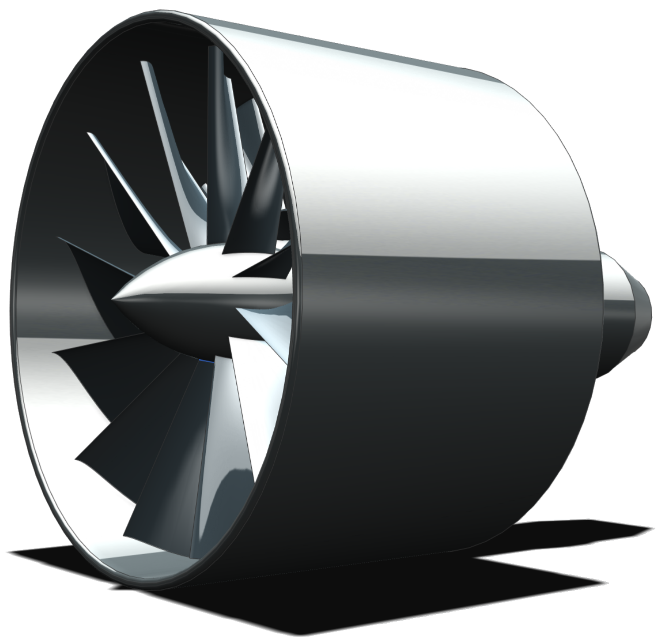

Figura 1.9: A turbofan engine model.

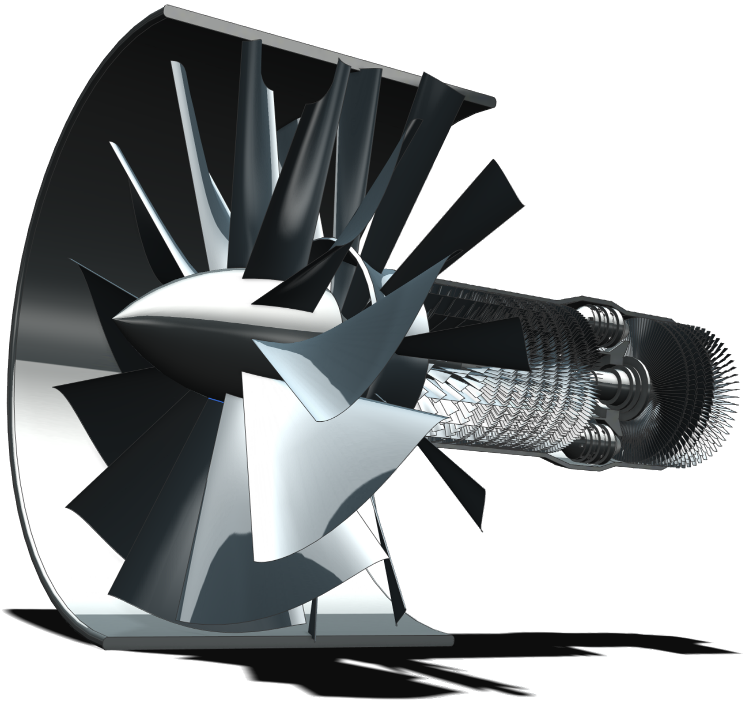

Figura 1.10: A turbofan engine section view.

### 1.2.3 Turboprop

Even though the turbofan configuration has already been widely used, smaller aircraft for instance a regional jet most often present a different engine, that is a turboprop.
The main reason behind this choice is that it is more convenient and efficient, in fact it can be proved that the thrust generated by the use of propellers is generally more efficient compared to the thrust generated by a jet stream, since the propellers with its huge diameter can move a higher mass of air.
This configuration is still based on a main gas generator group, and compared to the previous configurations, a main difference can be found in the turbine section as a portion of the generated power goes into a gearbox to move the propeller, and the gearbox is needed in order for the core engine and propeller to move at optimum rotational speed. At the same time, the exhaust gas is also expanded through a nozzle then, in this way, it is possible to obtain thrust both from the propeller and exhaust gas and obviously the thrust generated by the propeller is much more significant compared to the jet stream.
However, the presence of a propeller, without any surrounding duct, can be also considered to be a limit of this configuration as its presence inevitably mean that the aircraft cannot reach high flight speed and that is why this configuration is suitable for small aircraft.

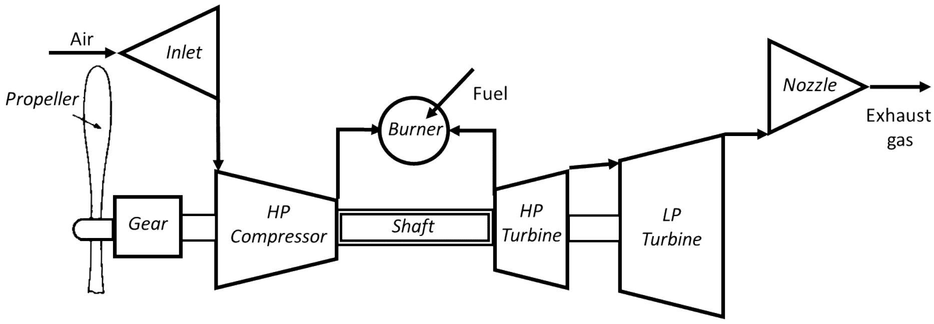


Figura 1.11: Basic scheme of a turboprop engine.

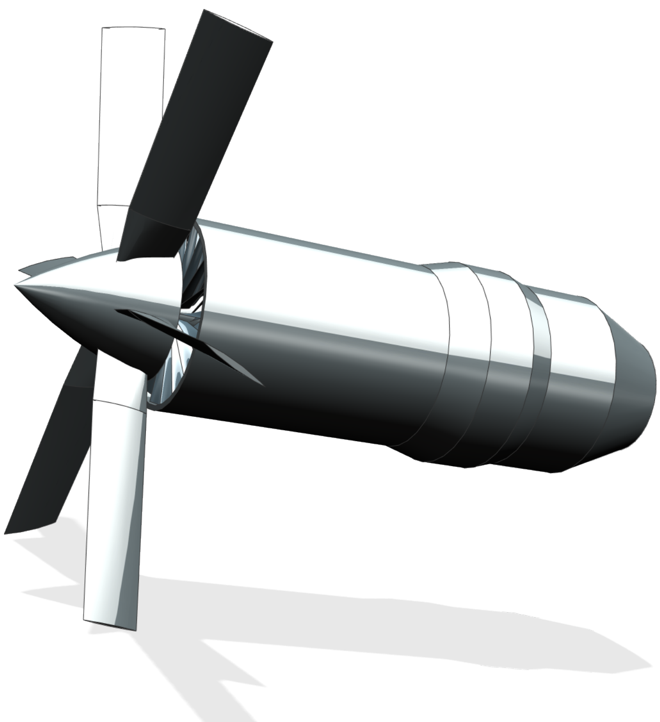

Figura 1.12: A turboprop engine model.

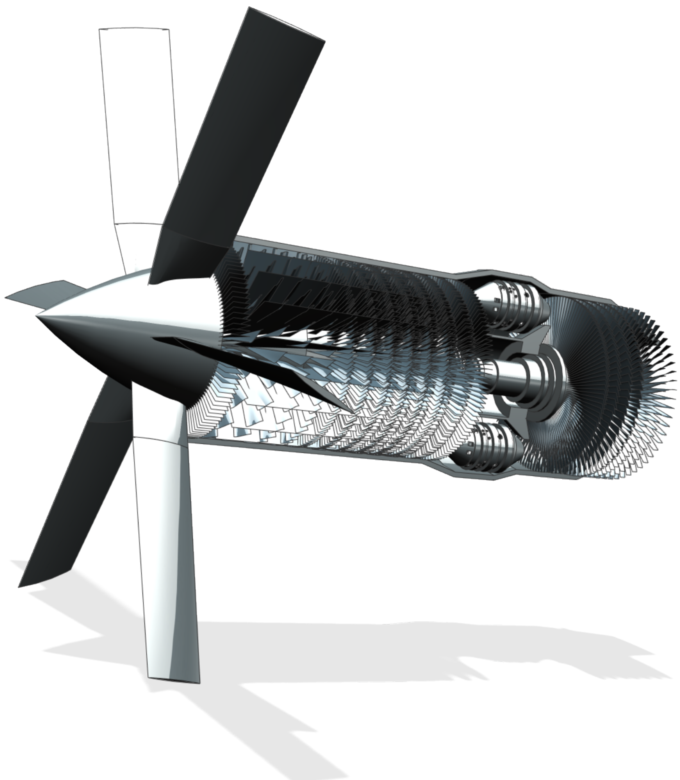

Figura 1.13: A turboprop engine section view.

## 1.3 The design process of a jet engine

As mentioned previously, the design of a jet engine is a long and complex process but nonetheless, it is still possible to identify a flow chart containing the critical phases and elements of the complete design of a new engine.
In fact, the first step of a new design is to identify the requirements or needs, as the design process is started by and as well as constrained by the need, which can come from a market analysis, statistical analysis or new limiting regulations that translate into a need of a new aircraft, or sometimes new engine only.
It must be noted that the engine has a significant impact on the performances of an aircraft and that is why the design of an engine must also take into account the parameters of a specified aircraft, or a specified application, and these are used as input for the design process.
However, from a defined set of inputs there is not a unique design process, in fact the order for the next steps to be made depend mostly on the experience of the company and its application, for instance a completely new configuration of an engine will definitely require deeper analysis compared to a modification of an existing engine.
In spite of that, a possible flow chart of a design process can still represented as in figure 1.14 containing all the critical steps for the complete design of a new engine, being aware of the fact that some order may be changed, and from this chart it is clear that different field of expertise are present, which means a different team for each field is required to deal with its analysis. It is also interesting to note that the process is inherently iterative and so it is often required to return to a previous step.
In conclusion, it must be added that the design rarely will reach an unique and optimum solution, instead it will often lead to multiple feasible solutions and that's why a systematic approach must also be adopted to identify a final optimum solution and taking into account the consequent trade-offs.

## 1.4 The turbine stage

One of the most critical component influencing the performances of an aircraft engine is surely the turbine who is responsible for the extraction of energy from the hot exhaust gas coming from the combustion chamber in order to provide the power to drive the compressor, shaft power for propeller or rotor, if present.
Two main types of turbines have been developed over the years and are currently used:

- axial turbine;
- radial turbine.

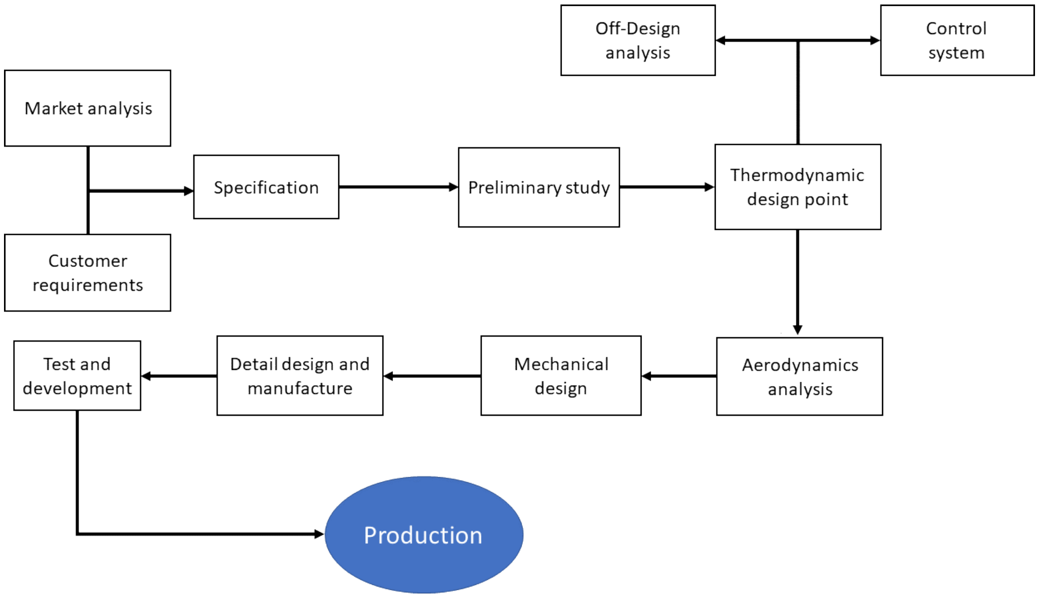


Figura 1.14: The design process flow chart.

where in the first one the fluid flows in the axial direction, whereas in the second one the fluid flows essentially in the radial direction instead. But that is not the only difference, in fact with the same overall diameter the axial turbine can handle a greater mass flow, however for small mass flow the radial turbine is actually more efficient than the axial configuration and it can provide a higher pressure ratio per stage. That is why an unique choice does not exist and it mostly depends on its application.
For most common civil aircraft engines, the choice lands on the axial turbine in order to minimize the nacelle drag and engine weight and it will be the only configuration considered for this thesis.
Compared to the compressor, the turbine usually present a higher efficiency and from a fluid dynamic point of view its design is also easier since the pressure drop in the turbine can reduce the risk of flow separation problems which is a big constraint for the compressor and therefore it is possible to achieve a much higher work and thus pressure ratio per stage.
On the other side, the turbine faces a much more critical stress problem related to the very high temperature of the exhaust gas coming from the combustion chamber and so the design process revolves more about stress and cooling considerations.
As mentioned previously, the main goal of the turbine is to extract energy however in order to provide the necessary energy, a single stage of a turbine is usually not enough and therefore several stages are present, where each stage is composed by one row of stationary nozzles, called stator, and one row of rotating blades, called rotor, such that:

- in the stator we have a conversion of pressure into kinetic energy.

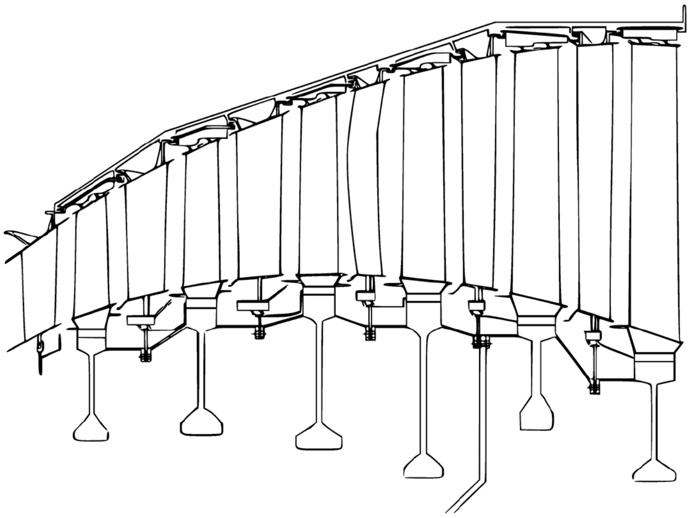

Figura 1.15: A low pressure turbine section.

- in the rotor we have a conversion of kinetic energy into mechanical energy.

Due to the high pressure ratio, the nozzle and blade are of increasing length as the gas expands along the stages and the number of stages required is also lower compared to the compressor.
Considering a section of a turbine stage and an appropriate velocity diagram as shown in figure 1.16, in this case the change in tangential velocity produces torque on the rotor and so the fluid does work on the rotor and the power output is given as

$$P_t = \dot{m}\,(U_2 C_{\theta 2} - U_1 C_{\theta 1})$$

and as for the work per unit of mass

$$W_t = c_p\,(T_1^\circ - T_3^\circ) = U\,(C_{\theta 2} - C_{\theta 3})$$

it can be clearly seen that the higher the change in tangential velocity and blade speed the higher is the work and power output, however a certain number of factors exist that limit these parameters, in fact:

- blade speed is limited by the rotational stresses, especially at the tip.
- tangential velocity change is limited in order to have a high enough efficiency.

Regarding the stage efficiency, it is deeply related with the aerodynamic losses in each stage and these losses influence the choice of blade type, which may be impulse or reaction type, aspect ratio, flow incidence angle, spacing Reynolds number and Mach number. Different expressions can be written for the turbine efficiency, such as total-to-static or total-to-total

$$\eta_{ts} = \frac{T_1^\circ - T_3^\circ}{T_1^\circ - T_3} = \frac{1 - \frac{T_3^\circ}{T_1^\circ}}{1 - \left(\frac{p_3}{p_1^\circ}\right)^{\frac{\gamma-1}{\gamma}}}$$

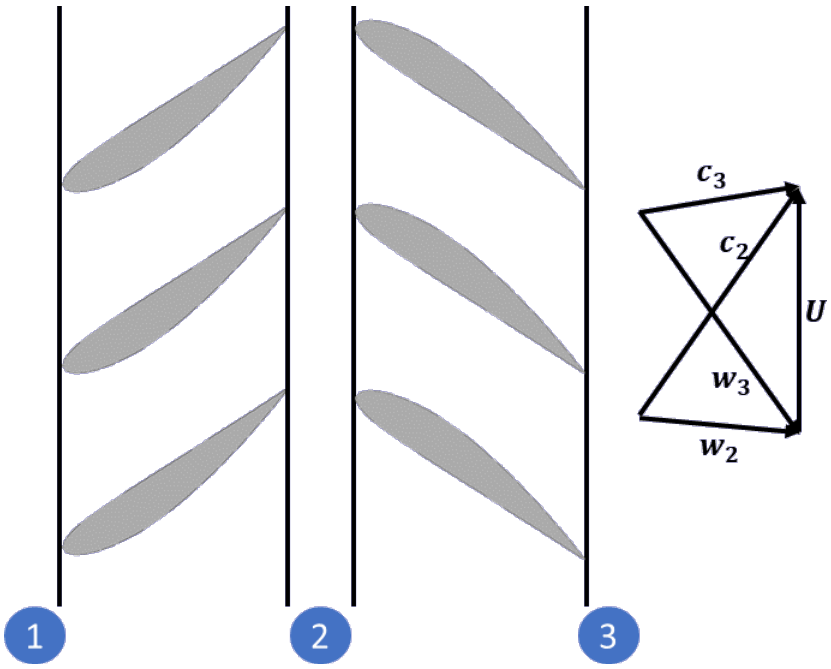


Figura 1.16: Typical turbine stage section and velocity diagram.

$$\eta_{tt} = \frac{T_1^\circ - T_3^\circ}{T_1^\circ - T_3^\circ} = \frac{1 - \frac{T_3^\circ}{T_1^\circ}}{1 - \left(\frac{p_3^\circ}{p_1^\circ}\right)^{\frac{\gamma-1}{\gamma}}} > \eta_{ts}$$

generally speaking the desired output is shaft power whereas the exhaust kinetic energy is often considered to be a loss, or at least the kinetic energy associated with the tangential velocity component as it cannot be converted into thrust when expanding in the nozzle.
An important parameter that significantly influence the turbine stage efficiency is the nozzle and blade spacing, it is known that a close spacing implies higher viscous dissipation both in the boundary layer and in the downstream wake, whereas for a wider spacing the number of boundary layers and wake is lower and thus less dissipation however a larger blade force is needed in order to have the same given total torque and therefore a larger pressure difference between the pressure and suction side of the airfoil which could lead to large boundary layer growth and larger viscous losses. That is why various trade-offs must be made in order to identify an optimum nozzle and blade spacing.
As mentioned earlier, a critical factor in the design of a turbine stage is the high stress due to the critical environment the rotor turbine blade as well as disc resides. One of the possible consequence of the high temperature environment is the risk of creep of the material and for high performance gas turbines, it is important to avoid it, especially for extended period, and this is partly caused by the grain boundaries present in the material by which a turbine blade is composed and this can be considered to be one of the main source of weakness for the blade at high stress.

There are different ways to raise the blade strength and some of them are:

- directional solidification: the turbine blade and nozzle is cast so that the solidification process goes along the stress direction of the blade and therefore columnar crystals can be formed aligned in that direction. In this way it is possible to obtain a turbine blade which can operate at higher stress as well as temperature.
- single crystal: an even better solution is to cast the turbine blade as a single crystal and this provides the possibility to operate at even higher stress and temperature.

Three main types of stress can be identified for the turbine blade and they are

- centrifugal stress: this represents a significant stress due to the high rotational velocity of the turbine and it depends mainly on the blade alloy density, hub to tip ratio and blade tip speed. Moreover it increases in presence of a blade twist but it can be decreased by tapering the blade the blade cross section.
- bending stress: this stress is originated by the aerodynamic forces that act on the blade, both steady and unsteady. It is especially critical at the blade root and, through a control volume analysis, it can found that it is of the same order of the centrifugal stress.
- thermal stress: this stress is generated due to the substantial temperature gradients within the blade as it is internally cooled through cold air coming from the compressor while being at contact with hot gas externally.

### 1.4.1 Turbine cooling

Due to the operating condition and the necessity of high efficiency, the turbine finds himself in contact with extremely high temperature exhaust gas coming right from the combustion chamber and it is known that a significant decrease of strength, as the temperature rises, occurs which leads to the implementation of solutions to face this problem and one of which is the turbine cooling through cold air coming from the compressor. Usually less than 10% of cooling air is needed to cool both the turbine nozzles and blades and the temperature of the extracted cooling air depends on several factors such as the pressure ratio of the compressor and the flight Mach number.
Since air flow has been extracted from the compressor, this obviously leads to a loss of work, but nonetheless the benefits coming from a higher turbine inlet temperature, for example a higher engine efficiency, overwhelm the losses and that is why this solution is commonly used.
Different ways are possible to cool the turbine blade and typical choices include an internal passage of cooling air, and for this a hollow turbine blade is needed, and if this is not enough small holes can be drilled onto the blade surface so that the cooling air can generate a film on the airfoil surface and an appropriate

choice of the injection rate as well as the hole diameter and spacing must be chosen.
In case from the preliminary estimate, a higher percent of air flow extraction is needed to obtain the desired cooling then the losses start to become significant and its effect related to efficiency must be seriously investigated.

## 1.5 The Low Pressure Turbine(LPT) design

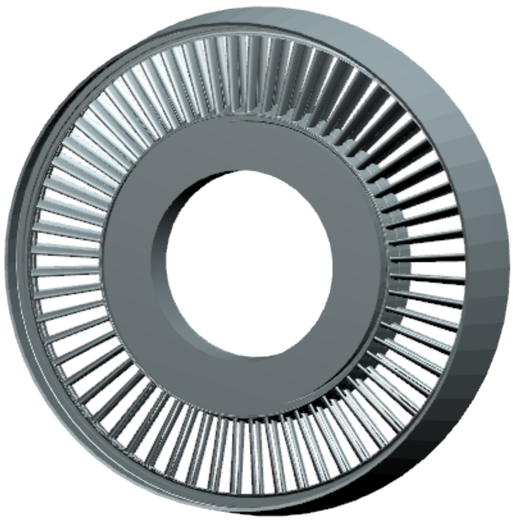

Figura 1.17: Low Pressure turbine stage.

Considering all the characteristics, properties and functions of a turbine stage as described earlier, it is clear that the design of a turbine is by no way an easy task, but not only, in fact due to the complexity of the field it works in and all the different phenomena that can arise, a multidisciplinary team must be involved to deal with this task.
Nonetheless, it is still possible to identify two main fields that need to be studied and analyzed bringing forth the following two distinct groups:

- aerodynamic design;
- structural design.

The aerodynamic design is aimed at finding the optimal airfoil that can maximize the energy extracted from the fluid, in other words to maximize the conversion of high pressure and high temperature from the exhaust gas into mechanical energy while at the same time reducing as much as possible the losses that come with it, especially the aerodynamic losses.
The structural design, instead, is focused on the behavior of the blade under static and dynamic loads and verify that all the deformations are reasonable and under acceptable values so that the turbine can operate without incurring

into any failure. Another aspect of the structural design is the choice of the material to be adapted which must be able to satisfy all the structural requirements without increasing too much the engine weight.
Obviously, these two designs cannot be carried out separately, in fact the interaction between fluid and structure must also be taken into account as possible unstable phenomenon due to vibrations can arise due to their interaction and its effects need to be studied so that the turbine can work safely. In figure 1.18 the nomenclature used for a turbine blade is shown.

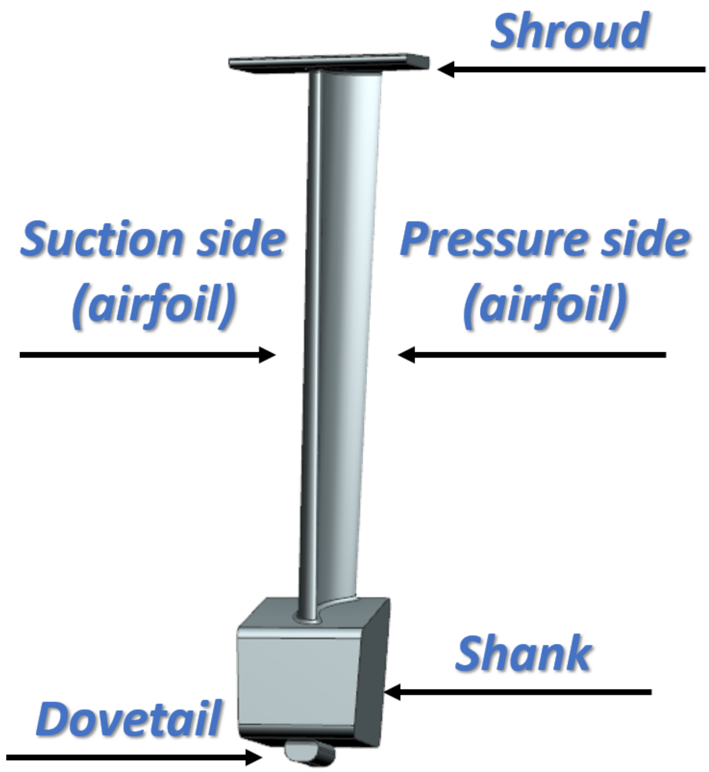


Figura 1.18: Turbine blade nomenclature.

### 1.5.1 Aerodynamic design

The aerodynamic design, as already mentioned, is tasked with finding the optimal airfoil for each of the turbine stage and to do that, the turbine flowpath must be defined first, which simply is the channel traveled by the coming flow for a certain fixed operating condition.
The overall process can be divided into three main steps:

- 1D Analysis:
  after having defined all the necessary inputs in terms of requirements to meet and as well as goals to reach, to start with the aerodynamic design a preliminary design has to be made by carrying out a 1D analysis with

various hypothesis to reduce the complexity of the problem.
With this analysis, the main goal is to define the velocity triangles, as shown in figure 1.16, and therefore some features of the turbine blade airfoil such as the leading edge and trailing edge orientation for various radial location, going from the hub to the tip. A typical design criteria, in this phase, is to set equal work along the radial direction then the airfoil of the blade cannot remain unchanged and in fact different values for the stagger angle need to be considered.
Furthermore, it must be noted that this process is inherently iterative until an optimum has been identified to be used as an input for the next analysis.

- 3D Analysis:
  after having identified an optimum solution for the airfoil from the 1D analysis, the next step is to transform this basic solution, through various empiric correlation based on the company's experience, into a complete 3D turbine blade shape with all the necessary details.
  Once the blade shape is defined then a deeper analysis needs to be carried out through a computational fluid dynamics analysis for all the various stages of the LPT in order to determine its response.
  The outcome from this analysis is also needed for the definition of boundary conditions as well as loads which will be used for the next and final step.
- Optimization analysis:
  the next and final step in the aerodynamic design of a low pressure turbine blade involve the optimization process in which, through various iterations, the best shape for the turbine blade is selected such that it holds both the boundary conditions and the performance requirements set at the beginning.
  Once this is defined, this configuration is then frozen and a deeper and specific analysis is carried out to define with more details some features of the blade such as the leading and trailing edge, chordal length and thickness distribution.
  At last, this final configuration will be then used for the structural design in order to be verified from a structural point of view.

### 1.5.2 Structural design

Through the aerodynamic design, the shape of the airfoil has been defined and after adding the appropriate inner and outer elements which are essentially:

- inner blade vane(IBV) and outer blade vane(OBV) for the nozzle.
- dovetail, shank and shroud for the blade.

then it is possible to proceed with the generation of a CAD model.
However, the turbine is designed for a specific operating condition which is

the "hot" condition but the turbine is assembled in cold condition then an appropriate correction is needed for everything to work as designed. This is done through a hot to cold scaling which can be divided into three main steps:

- scaling:
  the first step is to carry out a scale reduction to take into account the thermal expansion that comes due to the high temperature.
- twisting:
  the scaled blade is then twisted along the radial direction to obtain the blade for the cold condition.
- positioning:
  the last step is to define the position of the inner and outer elements so that the airfoil is fixed.

Afterwards, from the CAD model it is possible to start generating a finite element model which will be used as the basis for the coming analysis such as the modal analysis and the static analysis.
The modal analysis is aimed at studying the dynamic effects of the turbine blade and its vibratory behavior which should be under acceptable ranges, whereas the static analysis is focused on analyzing the static behavior under the various loads that occur during its operating condition.
During this analysis a negative outcome, in other words an unacceptable value of deformation, may arise and therefore changes in the geometry of the turbine blade need to be implemented without changing too much its aerodynamics. it is then clear that this process is also iterative until a good compromise is reached and good results are obtained.
After that, more detailed analysis is performed such as the campbell diagram, to study the turbine response in different conditions, the forced response, for resonance phenomena, and flutter analysis, to study the effects coming from the interaction between fluid and structure. But that is not all, in fact further analysis, such as from the fatigue point of view, also needs to be carried out but at this point the turbine blade's geometry has already been consolidated and sent for the manufacturing team to be produced, it is then clear the importance of choosing appropriate safety factors in order to pass all the necessary tests.
In figure 1.19 an example of turbine blade and nozzle is shown and the whole row is shown instead in figure 1.20 and 1.21.

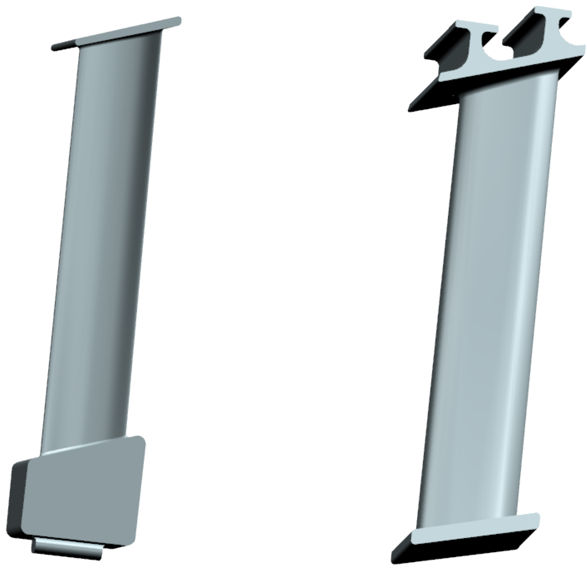

Figura 1.19: Examples of Low Pressure Turbine blade and nozzle.

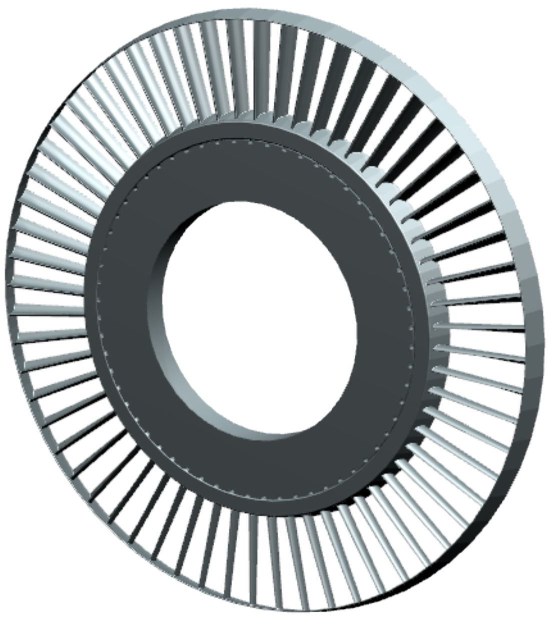

Figura 1.20: Row of turbine bladed disk.

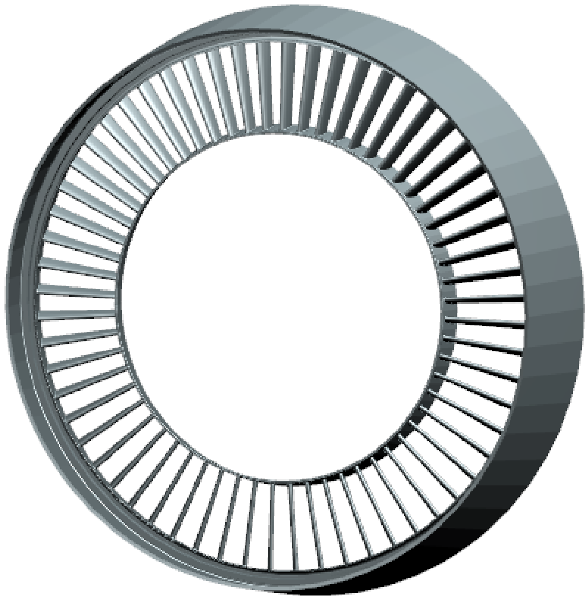

Figura 1.21: Row of turbine nozzle with casing.

# The Great 2020

Figura 2.1: The Great 2020.

The GReen Engine for Air Traffic 2020 (GREAT 2020) is a project born in 2009 in order to reach the goals set for the year 2020 by the Advisory Council for Aeronautical Research in Europe(ACARE) for the aerospace industry due to the ever increasing needs, both commercial and political, to reduce the fuel consumption and as well as the $CO_2$ produced by the aircraft engines.
The environmental issue has significant impact on the aerospace industry and represent one of the biggest constraint to consider, that is why many research programs, such as the GREAT2020, have been founded and started in order to develop a new engine that is both safe, efficient and as well as environmentally sustainable, in particular the goals for short and long terms for these research programs are to meet the requirements set by Vision and Strategic Research and Innovation Agenda (SRIA) by 2020 and Flightpath by 2050.

## 2.1 Toward ACARE 2020

The objectives set for 2020 by ACARE, published in Vision2020 and SRIA, concern the engine, the aircraft and as well the operations related, in other words the objectives fixed for 2020 regard the overall air transportation system. The main constraints, especially for the engine, are essentially:

- to reduce the emissions of $CO_2$;
- to reduce the emissions of $NO_x$ by 80%;

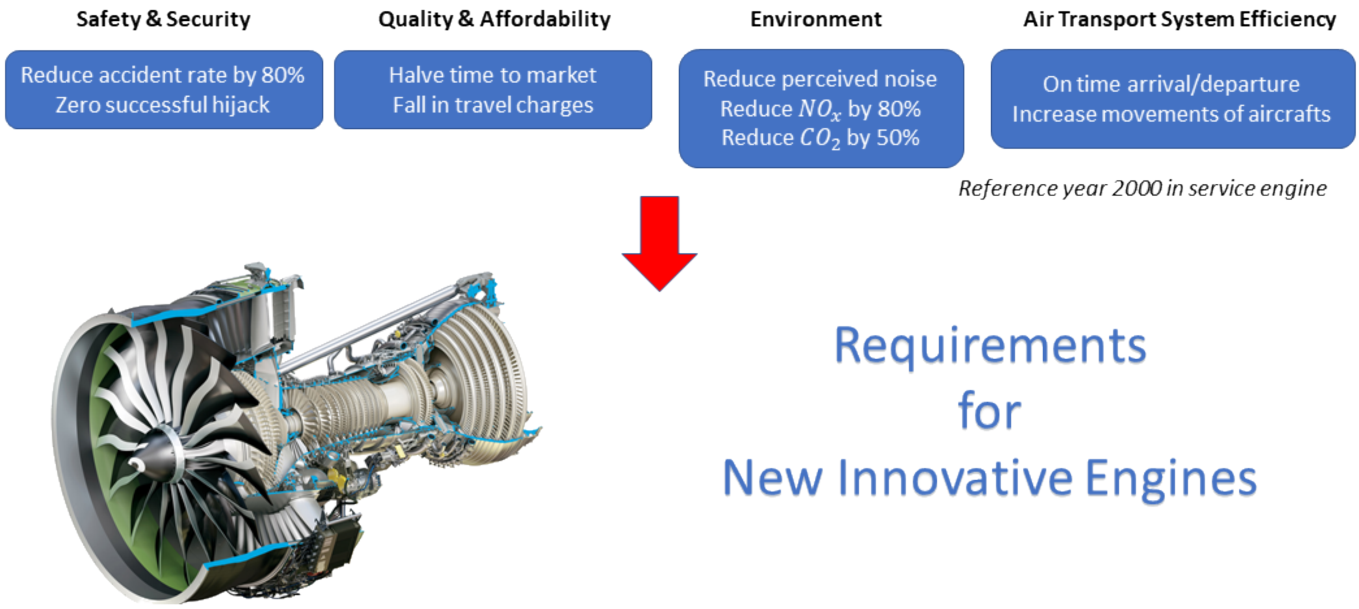


Figura 2.2: ACARE objectives for 2020.

- to reduce the perceived the noise by $10dB$;
- to reduce specific fuel consumption by 15% to 20%.

where the values are referred to the year 2000.
The choice of adopting these values as targets for 2020 are not obviously random, but it is a result obtained by taking into account different factors, first and foremost the volatility of fuel price has to be considered for the design of new engines, then the environmental issue, especially the climate change, is also a major concern for the future and it is deeply related to the emissions of carbon dioxide.
Inevitably, the aerospace industry has a significant contribution to the emission of carbon dioxide and considering also the traffic growth expected in the next years then the emission also represents a substantial requirement to be met for the new engines and new technological improvements are needed to address this issue.
In order to reach this goal, a drastic change in the design process has to be made, as in the past the main focal point was to reduce the specific fuel consumption by increasing the thermal efficiency and propulsive efficiency. To improve the first one, an increase in the component efficiency as well as the inlet temperature is needed, whereas for the latter the main choice was to increase the bypass ratio (BPR), which could also reduce the engine noise, however, an increase of bypass ratio has also the effect of increasing the engine weight and installation drags.
This means that the value of bypass ratio is limited by these constraints and

it cannot be increased indefinitely, that's why, nowadays, the focus has been shifted in order to find new technological solutions to satisfy the ever increasing restrictive requirements, such as those ones set by ACARE for the year 2020, into the development of new engine architectures.

### 2.1.1 Partners

Before diving into the results achieved by the research program GREAT2020, it is interesting, first of all, to look at the main participants in this research program.
This research program was conceived in the Piedmont region due to the presence of different companies, research center and university, the Piedmont region actually represents the most suitable place for this research program to be born and to face the challenges set for the next generation engines.
In particular, the GREAT2020 research program is a natural outcome reached by the collaboration of Avio Aero, Polytechnic University of Turin and many other companies and businesses who are based in the aerospace industry and thanks to the GREAT2020 it is possible to have a structured approach to reach a common goal.
The main participants in this research program are:

- **Piedmont region**:
  The Piedmont region is one of the core participant in the research program GREAT2020 and it has supported the birth of a collaboration between the various companies, both small and big, of the aerospace industry present here.
- **Avio Aero**:
  Avio Aero is a GE Aviation business involved in the design, manufacturing and maintenance of components and systems for the aviation industry, mainly in the field of mechanical transmissions, turbines and combustors.
  Its main aim is to develop new technologies that can help reduce the energy consumption and provide better performance as well as reduce the engine weight. That's why Avio Aero has always invested in research which is clearly seen by the already consolidated network of relationships with major universities and international research centers.
  Nowadays, thanks also to its long history, Avio Aero has now several offices in different countries, with the headquarter in Rivalta di Torino, counting thousands of employees.
- **Polytechnic University of Turin**:
  The Polytechnic University of Turin is an engineering university based in Turin who offers several courses in the fields of engineering, architecture and industrial design, but even more importantly it has invested a lot in education and research activities, such as the research program GREAT2020.

The main departments involved in this research program are the department of mechanical and aerospace engineering(DIMEAS), the department of applied science and technology(DISAT), the department of energy(DENERG) and the department of management and production engineering(DIGEP).
Moreover, from the partnership between the university and Avio Aero a specific laboratory was constituted to work on research projects and thesis projects.

- **ISTEC-CRN**:
  The Institute of Science and Technology for Ceramics is a structure which belongs to the National Research Council of Italy providing long term activity programs concerning a wide range of ceramic materials.
  Their main focus is about developing, studying and manufacturing of materials and as for the research program GREAT2020 their activities regard the analysis study of sustainable manufacturing system of alloys used in the aerospace industry.
- **Innovative firms**:
  Many other companies present in the Piedmont region actually participate in the GREAT2020 research program with fields of expertise complementary to the aforementioned participants, going as far as materials science to the field of mechatronics.

### 2.1.2 Laboratories

Under the GREAT2020, six main laboratories have been constituted in different locations across the Piedmont region to face the challenges set for the future of the aviation industry, in particular the goals set by ACARE for 2020. Each laboratory has a specific objective to achieve and a specific field of expertise, moreover in each laboratory a team composed by people of Avio Aero, research centers and other firms is present to carry out the research activities and to develop strategies to realize the next generation of aviation engines.
The laboratories are as follows:

- **Lift Lab**:
  The Lift laboratory is focused on developing new materials with low density but high endurance and to use them for new innovative design of engine architectures as well as the its manufacturing process and certification.
  Going into more detail, this laboratory is focused on:
  - the development of components to be used for the turbine and combustor produced with the additive manufacturing technology using alloys of titanium-aluminium and chromium-cobalt as well as the development of high endurance steel to be used in the gearbox;

  - the design of advanced systems for the control of dynamic effect present in the turbine with high reliability;
  - the simulation and optimization of transformation concerning new materials.

- **Aeronflux Lab**:
  The Aeronflux laboratory is focused on developing innovative technologies to maximize the efficiency of the low pressure turbine and at the same time reduce the noise emitted by the same component.
  In particular, this laboratory is focused on:

  - the effects of unsteady aerodynamics for an advanced design of blade airfoil with optimum efficiency;
  - the management of thermal effects of the turbine and its control system to minimize the losses during its flight.
  - the design of turbine configuration with low noise impact.

- **Ageades Lab**:
  The Ageades laboratory is focused on developing innovative technologies for a robust design of mechanical transmissions to be used in the innovative engine architectures, that is geared turbofan and geared open rotor.
  In particular, this laboratory is focused on:

  - the design of planetary gearbox, in particular its dynamic effects;
  - the design of advanced tools to be used to predict the behavior of key components so that the reliability of mechanical transmissions can be improved and to meet the requirements given by the transfer of high density power, typical of the next generation systems.

- **Zec Lab**:
  The Zec laboratory is focused on developing the next generation of combustion systems with reduced emissions, compatible with alternative fuels and even more importantly that can satisfy the requirements set by sustainability, cost and reliability.
  In particular, this laboratory is focused on:

  - the study of alternative fuels and their impacts on the emission produced as well as the overall performance of the combustor;
  - the development of new combustion models to be adapted for alternative fuels with injector "Low-$NO_x$";
  - the development of design process and advanced manufacturing for the next generation of combustion systems.

- **MC Lab**:
  The MC laboratory is focused on, first of all, integrating smart and intelligent technologies into the engine system by which it will be possible to track the overall health of the engine, and secondly it is developing systems for a high efficiency generation of electric power to be used for

a next generation aircraft of type "more electric".
In particular, this laboratory is focused on:

  - the development of algorithms and advanced electronic systems to be used for the prediction of possible failures during its flight;
  - the development of electric power generator integrated in the engine system to be used to power on board.

- **ECOPRO Lab**:
  The ECOPRO laboratory is focused on improving and consolidating the engineering knowledge concerning the transformation process and manufacturing process of components used in the aerospace industry and to make them eco-sustainable as well as to reduce the manufacturing time.
  In particular, this laboratory is focused on:

  - the development of new systems to help the sustainable manufacturing processes, without reducing the structural integrity of the final products;
  - the study of innovative production system that is flexible and intelligent to be used under different conditions with reduced need of human inputs;
  - the integration of advanced methods for the inspection of titanium-aluminium alloy used in the engine components.

## 2.2 The engine

The key component of an aircraft to work on in order to achieve the goals set by ACARE for the year 2020 is obviously the engine which is needed to provide the necessary thrust.
Nonetheless, it is also the engine who is the main responsible for the emission of pollutants, such as $CO_2$ and $NO_x$, which come out from the combustion process along with water $H_2O$. Generally speaking, their amount is directly proportional to the amount of fuels burned but not only, in fact in case of $NO_x$, the temperature inside the combustion chamber also comes into play.
Aside from the emission of pollutants, another important factor not to be neglected is the noise generated which is important especially near the airports and that's why it also faces strict requirements for the future.

### 2.2.1 Emissions

By and large, the main environmental challenge is due to the climate change, specifically the global warming, which is caused by the emissions produced on earth.
One of the main pollutant is the carbon dioxide and a big part comes from the combustion of petroleum, methane and carbon which is increasing more and

more over the years due to the increasing industrial activities.
The impact of the emissions, generated by the aviation industry, can be classified into three different levels:

- on a global level, the emissions at a high altitude have a bigger impact compared to the emissions at sea level;
- on a regional level, the emissions at a higher altitude increase the thickness of the tropospheric ozone;
- on a local level, the emissions directly influence the atmospheric pollution, especially nearby the airports.

Nonetheless, according to the Intergovernmental Panel on Climate Change(IPCC) the emissions of carbon dioxide generated by the aviation industry is only around 2% of the global carbon dioxide emissions, and around 4% of the overall emissions, however a growth in the air traffic is to be expected for the next years and therefore the overall impact of the emission on the environment cannot be neglected.

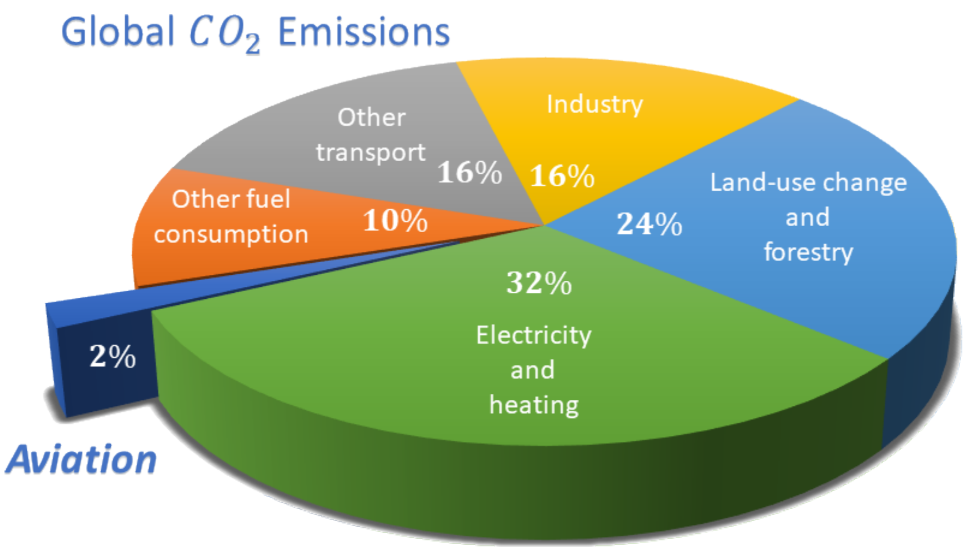


Figura 2.3: Global $CO_2$ emissions.

The next natural question would be what are the main parameters that influence the emissions of the main pollutants, carbon dioxide and nitrogen oxides, and what changes are needed to reduce these emissions.
With regard to the carbon dioxide $CO_2$ emission, it is directly proportional to the amount of fuel burned and possible solutions to reduce the fuel consumption are to either increase the overall engine efficiency of the engine or to reduce the weight of other sub systems.
The efficiency of the engine, as mentioned previously, depends first of all on the configuration adopted which is commonly the turbofan architecture characterized by the presence of two distinct flow, hot and cold, and the higher their ratio the less how flow is needed to generate the same thrust, in other words less

fuel is needed. That is why the first step toward reduced emission is to search for new configurations that allow a high bypass ratio, nonetheless this value cannot be increased indefinitely as it presents the following disadvantages:

- increase of aerodynamic drag due to the larger engine;
- increase of the weight, especially of the low pressure components;
- reduced efficiency of low pressure turbine, due to the limiting constraint of rotational velocity of the larger fan.

Regarding the nitrogen oxides $NO_x$ emissions, they do not depend only on the amount of fuel burned but it is mainly caused by the high temperature present in the combustion chamber. That's why new technologies are needed that allow a high temperature, which is beneficial for the efficiency of the engine and thus carbon dioxide emissions, as well as a reduced emission of nitrogen oxides.
A possible solution is reached with the Lean-Burn configuration which can reduce the peak temperature and the residence time and so limiting the nitrogen oxides emissions, moreover this configuration can also help to maintain the flame stability and it permits the re-ignition if needed during the flight.

### 2.2.2 Innovative engine architectures

As mentioned previously, to address these challenges and to meet the future requirements and constraints, the classical configuration of the turbofan may not be enough and that is the reason why new configuration of the engine have been researched that permit the implementation of new technologies, especially to overcome the bypass ratio limit, without however changing too much the core of the turbofan concept.
In this regard, the following possible configurations, characterized by a high value of bypass ratio, are being analyzed and researched:

- Advanced Turbofan Engine: this configuration maximizes the efficiency of the currently used configuration by increasing the bypass ratio, and so the efficiency of the engine, which is obtained by the implementation of new light material for the low pressure group as well as an increase of the thermal efficiency of the core;
- Open Rotor/Propfan: this configuration maximizes the propulsive efficiency through the implementation of counter-rotating propellers outside the casing, however it brings forth a big challenge regarding the reduction of acoustic emissions;
- Geared Turbofan: this configuration presents a gearbox to free the low pressure turbine from the fan so that both can rotate at optimum speed.

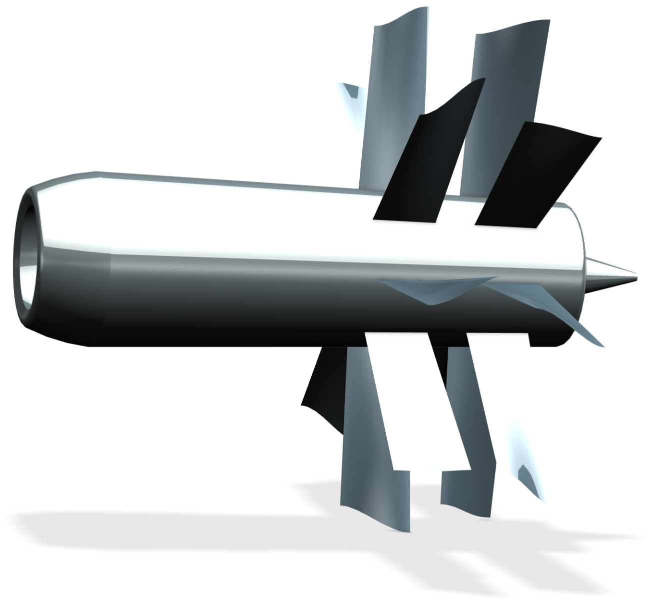

Figura 2.4: Open Rotor concept model.

## 2.3 Results

Over the years, many results have been reached by the GREAT2020 research program with hundreds people taking part in this project. The main results can be summarized as:

- 8 decibel reduction;
- 18% reduction of carbon dioxide emission;
- 60% reduction of nitrogen oxides emission, thanks to the implementation of Lean-Burn technologies;
- 90% reduction of greenhouse gas emissions for the manufacturing of a turbine component in titanium-aluminium alloy through additive manufacturing.

all the values are referred to the year 2000.
To reach this result, hundreds and hundreds of test have been made through dedicated laboratories and the achievement is proven by the large numbers of technical reports, thesis and scientific papers published as well as events organized.

Thanks to GREAT2020, the Piedmont region emerged as a center of excellence in the aviation jet engine industry and an increase both in terms of economic growth as well as level of employment are expected for the next years. That is why the Piedmont region will continue to research and innovate on the next generation engine.

# Modal analysis of a turbine bladed disc

After having thoroughly introduced the turbine stage, its purpose and main features, along with its main components, such as the turbine bladed disc and nozzles, it is then clear that during its operating condition, many sources of loads are present and they are mostly dynamic loads by nature which causes mechanical vibrations along with aeroelastic effects and this needs to be carefully studied in order to avoid failures during its operation.
From this description then it is clear that the turbine stage operates in an interdisciplinary realm involving several fields of expertise. This interdisciplinary nature can be clearly described through the scheme represented in figure 3.1 which was conceived by the Professor A.R. Collar and it depicts the three main discipline:

- Fluid mechanics is the field focused on the evaluation of the aerodynamic forces acting on a body of a specified shape.
- Structural mechanics is the field focused on the evaluation of the deformation along with the elastic forces generated for a specified load.
- Dynamics is the field focused on the evaluation of the inertial forces

Obviously, most real applications actually involve an interaction between these various fields giving birth to new interdisciplinary fields such as static aeroelasticity, structural mechanics and dynamic stability, but even more important the interaction among all three is the dynamic aeroelasticity and the turbine stage resides in this field.
One of the dynamic phenomena generated is the mechanical vibrations that provoke a loss of efficiency in the conversion of pressure into shaft power and stress that may generate cracks in the structure, and thus failure. In fact the turbine blade suffer fatigue damage typical of high cycle fatigue, since the induced stress is below the maximum allowed, and a high number of cycles can be reached before failure, moreover it is characterized by a large range of frequency response due to the different rotational speed that are present for each possible throttle setting, however these oscillating frequencies can be near resonance conditions and this must be accurately investigated in order to avoid them during all the operating conditions of the turbine.

Two typical dynamic analysis that need to be carried out are:

- forced response: the forced response consists of analyzing the effects of perturbations in the air flow provoked by the adjacent stages and they can then be considered as harmonic exciting forces on the turbine blade and their frequency depends on the rotational velocity and number of blades of the adjacent stages.
- flutter: the flutter analysis investigates the effects due to the interaction between fluid and structure and the most important phenomenon to be analyzed is the flutter instability which is an auto-excited oscillation, and that is the deformation of the structure generates the aerodynamic forces needed to sustain and magnify this oscillation.

Both of them are dynamic by nature and their effects must be thoroughly studied and analyzed for a safe operation of the turbine.

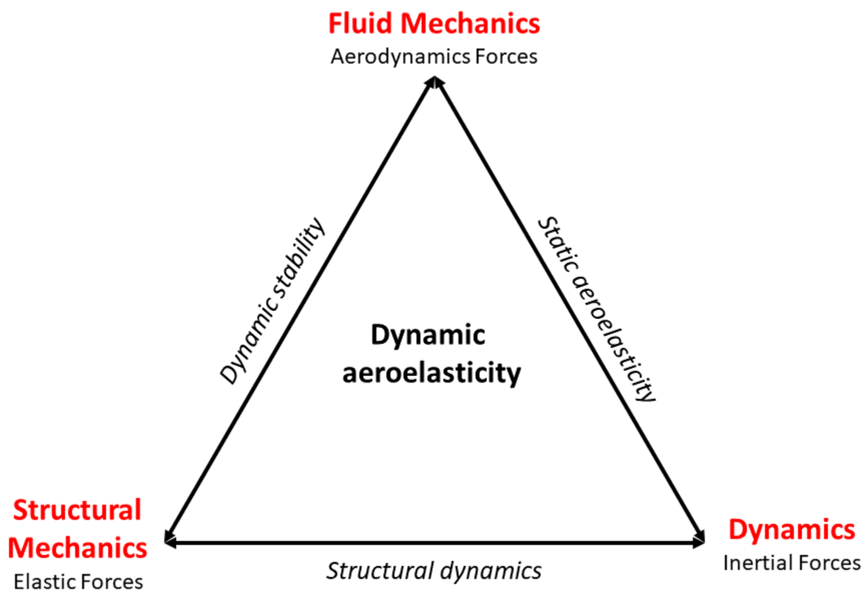


Figura 3.1: Fields scheme.

## 3.1 Modal analysis

One of the preliminary dynamic analysis that needs to be performed in order to study the vibrations is the modal analysis from which it is possible to evaluate the natural frequencies and mode shapes of a turbine rotor, which will be then used in the forced response and flutter analysis. The mode shapes actually depend mainly on the material properties such as the mass, stiffness and damping, in conjunction with the boundary conditions and thus it is an intrinsic property of the structure under analysis.
The most common approach, nowadays, to modal analysis is to conduct it over

a finite element model of the structure and evaluate the natural frequencies and mode shapes of this meshed structure. That's why a critical step in this analysis is surely the meshing procedure and an accurate mesh is needed to obtain reasonably valid solutions.
A finite element model is characterized by the fact that the structure is not analyzed as a single element but it is in fact divided into numerous small elements and to each of them a certain number of nodes is associated and the mesh will greatly differ depending on the choice of the shape as well as the order of the chosen element to be used for the discretization of the structure.
Once the meshing of the structure is defined, it is then possible to write the equation of motion which is valid for a structural problem

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{x}}(t)\} + [\boldsymbol{C}]\{\dot{\boldsymbol{x}}(t)\} + [\boldsymbol{K}]\{\boldsymbol{x}(t)\} = \{\boldsymbol{f}(t)\}$$

where $[\boldsymbol{M}]$, $[\boldsymbol{C}]$ and $[\boldsymbol{K}]$ are the mass matrix, the gyroscopic or damping matrix and the stiffness matrix respectively, $\{\dot{\boldsymbol{x}}(t)\}$ is a vector containing the degree of freedoms associated with each node whereas $\{\boldsymbol{f}(t)\}$ is the vector containing the external force applied on the structure. The dimension of the matrices and vectors are obviously related to the number of nodes implemented during the meshing procedure.
For the modal analysis, the system considered is undamped and without external forces applied then the equation of motion can be rewritten as an eigenvalue problem

$$\left([\boldsymbol{K}] - \boldsymbol{\omega}_i^2[\boldsymbol{M}]\right)\{\boldsymbol{u_i}\} = \{\boldsymbol{0}\}$$

where $\boldsymbol{\omega_i}$ is the $i$th eigenvector and $\{\boldsymbol{u_i}\}$ is the $i$th eigenvalue of this eigenproblem. With the conditions appropriately specified, it is then possible to evaluate the eigenvalues, which corresponds to the natural frequencies, and the eigenvectors, which corresponds to the mode shapes of the structures.
Since a finite element model has been adopted then the natural frequencies, and the mode shapes as well, are limited, in fact the maximum number of eigenvalues is equal to the maximum number of degrees of freedom of the structure, and once again it emphasizes the importance of the meshing procedure. However in reality the structure is continuous and so the number of degree of freedoms is infinite, which means there are an infinite number of natural frequencies and mode shapes in a real application.
Nonetheless the complete structure of a single stage of a turbine rotor is still considerable and a finite element analysis of the complete structure is not really feasible as the time needed for the computation is exorbitant due to the substantial computational cost.
Luckily, a simplification for this analysis can be adopted as the turbine rotor has a rotational periodicity, also known as cyclic symmetry, and it is possible to drastically reduce the computational cost by considering the turbine rotor as composed by a finite number of substructures or substructure and carry out the analysis only for a single substructure. It can be shown that a condition found for a specific position at an angle $\theta$ is identical for any point positioned

at $\theta + n\phi_0$, given $\phi_0 = 2\frac{\pi}{N}$, where $N$ is the number of blades and $n$ is any integer lower than the number of blades. So the complete structure can be obtained by repeated rotation of this substructure.
As an example, a turbine rotor with 36 substructure has been represented in figure 3.2 where the simplest case for the substructure has been adopted consisting of a disc portion and one blade attached to it.

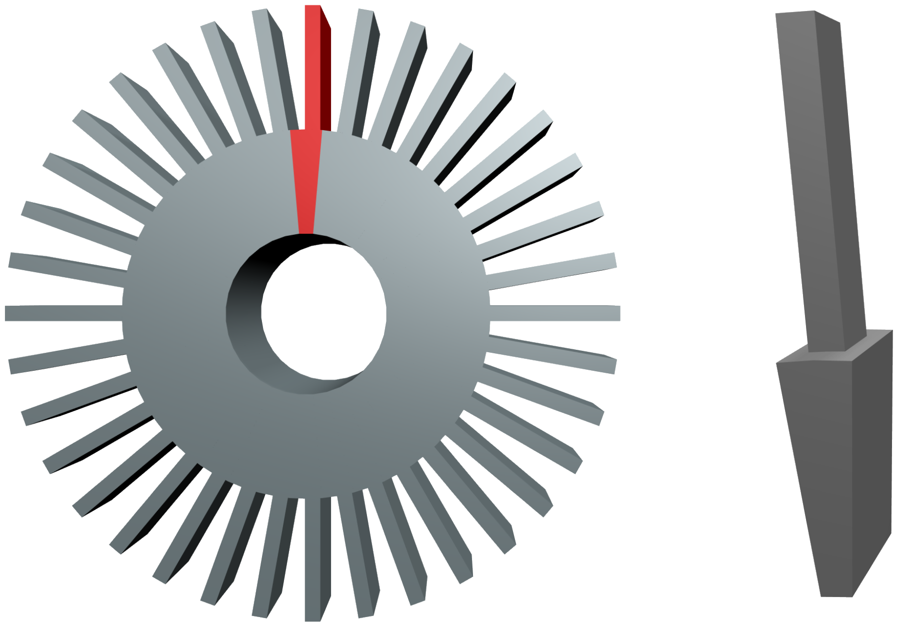

Figura 3.2: Dummy turbine rotor model highlighting a substructure of it.

### 3.1.1 Mode shapes

It is then clear that from the modal analysis it is possible to obtain the natural frequencies and the mode shapes of the considered structure however since a finite element model has been adopted, their numbers are limited. Before going on with the description of the various mode shapes that characterize a turbine blade, it is useful to define the concepts of nodal diameters and nodal circumferences.
The nodal diameter is defined as a line passing through the rotor disc center characterized by a zero modal displacement, analogously the nodal circumferences is defined as a circumference concentric with the disc characterized by zero modal displacement.
Moreover, it must be added that multiple number of nodal diameters may be present and this can be clearly seen from the figure 3.3 with no nodal diameter, figure 3.4 characterized by one nodal diameter, figure 3.5 characterized by two nodal diameter and figure 3.6 characterized by six nodal diameter. For sake of completeness, other mode shapes have also been represented such as a torsional mode, represented in figure 3.7, characterized by different nodal diameters

and a bending mode, represented in figure 3.8, where the nodal circumference can be clearly seen.

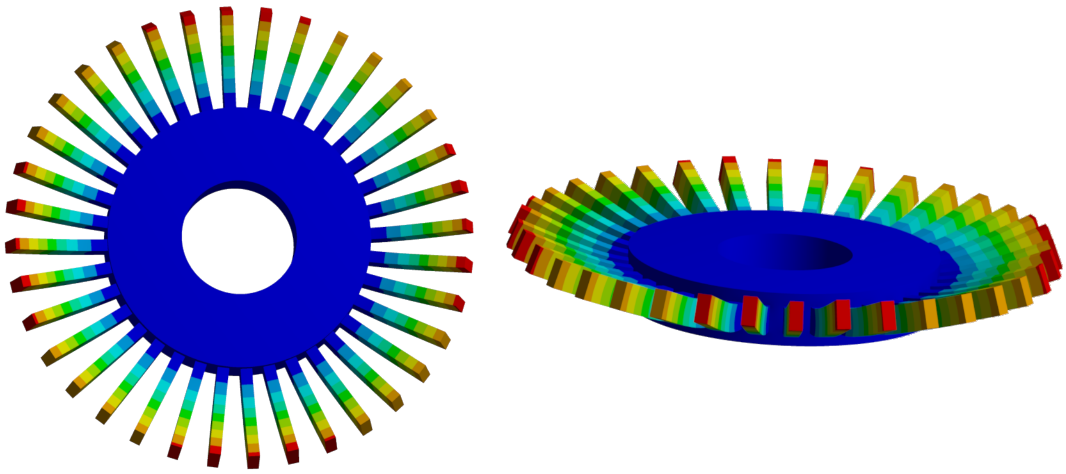

Figura 3.3: Turbine rotor with bending mode shape characterized by no nodal diameter and no nodal circumference.

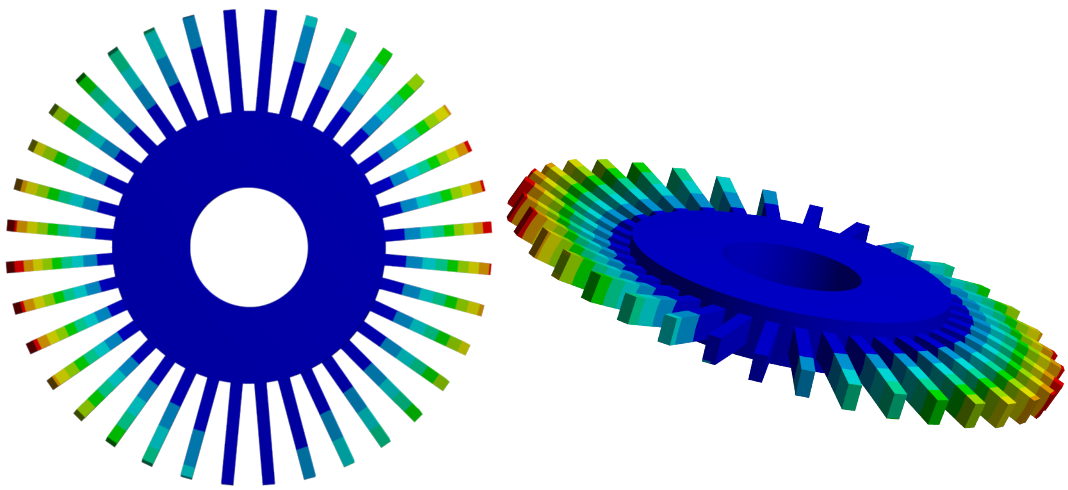

Figura 3.4: Turbine rotor with bending mode shape characterized by one nodal diameter and no nodal circumference.

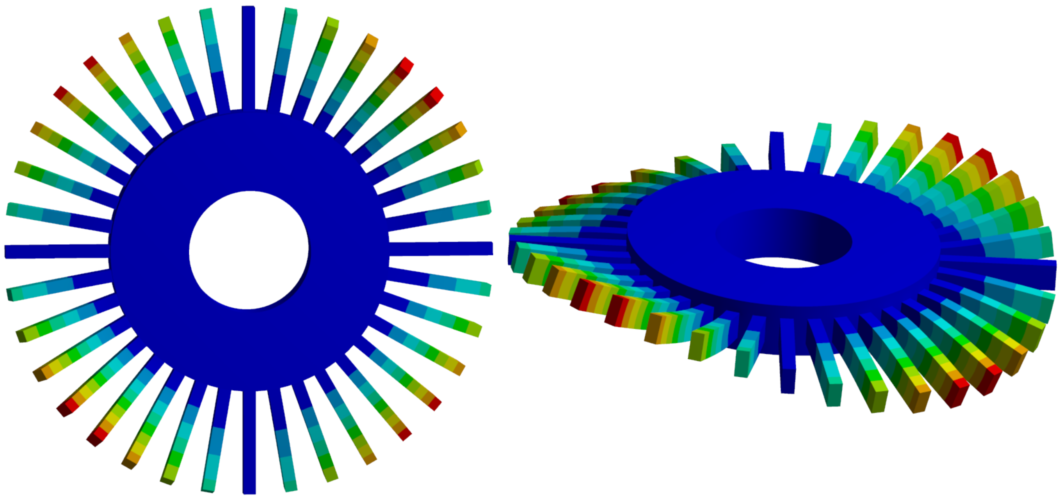

Figura 3.5: Turbine rotor with bending mode shape characterized by two nodal diameter and no nodal circumference.

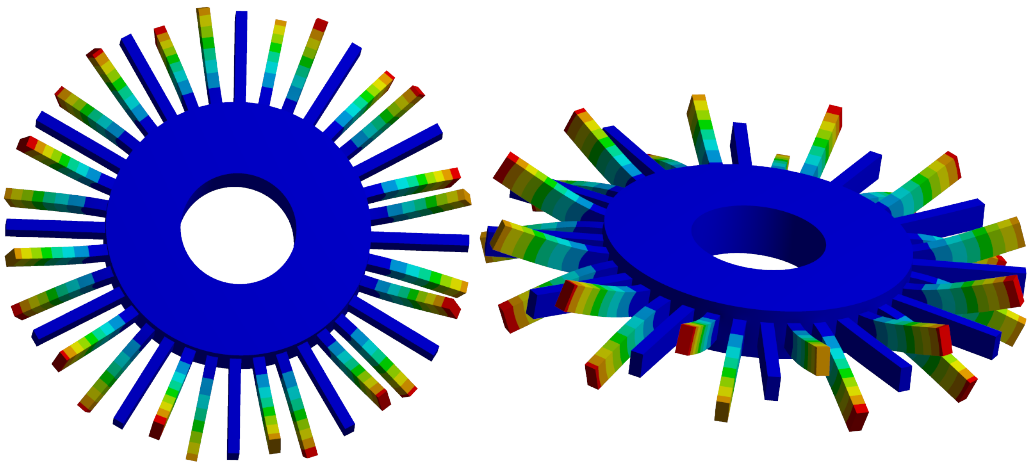

Figura 3.6: Turbine rotor with bending mode shape characterized by six nodal diameter and no nodal circumference.

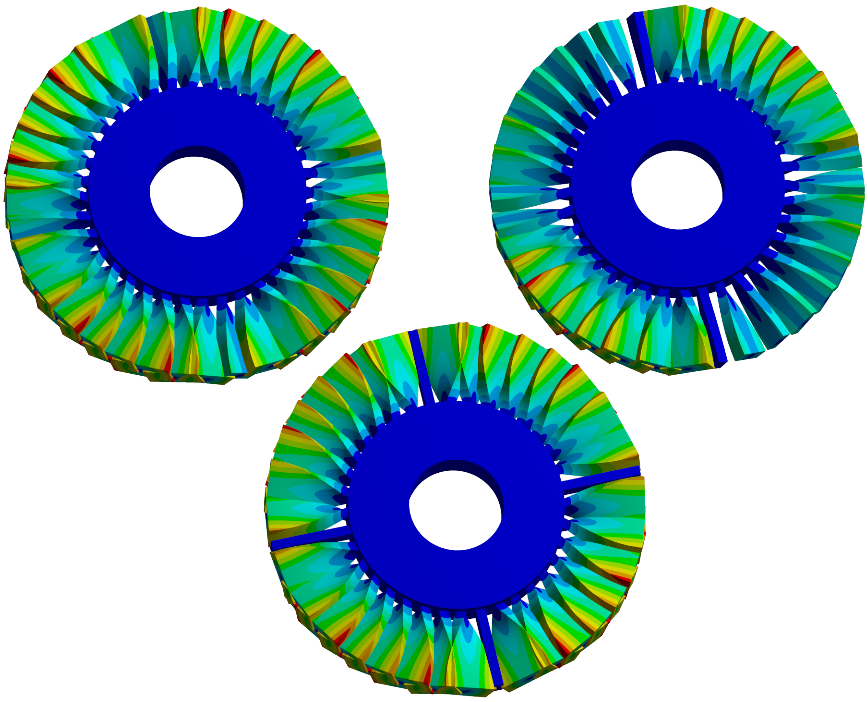

Figura 3.7: Turbine rotor with torsional mode characterized by various nodal diameter but no nodal circumference.

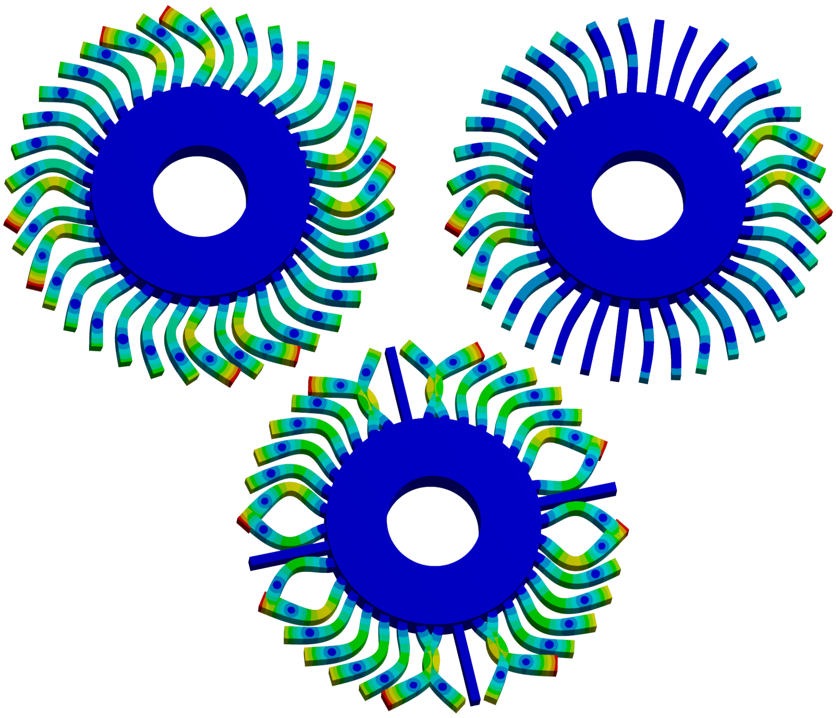

Figura 3.8: Turbine rotor with bending mode characterized by one nodal circumference but no nodal diameter.

As mentioned previously, the mode shapes can be obtained by solving the eigenvalue problem

$$\left([\boldsymbol{K}] - \boldsymbol{\omega}_{\boldsymbol{i}}^2[\boldsymbol{M}]\right)\{\boldsymbol{u_i}\} = \{\boldsymbol{0}\}$$

it is however convenient to reorder the eigenvalues, which in turn means the degrees of freedom, so that the first $m$ elements are the eigenvalues of the first substructure, supposing each substructure has $m$ degrees of freedom, followed by $m$ elements of the next substructure and so on

$$\{\boldsymbol{u}\} = \{\boldsymbol{u}^1, \boldsymbol{u}^2, \boldsymbol{u}^3, \dots \boldsymbol{u}^{N-1}, \boldsymbol{u}^N\}^T$$

where $\{\boldsymbol{u}^i\}$ is a vector containing the real $m$ eigenvalues of the $i$th substructure and thus the total length is given by the $N$ substructures times the $m$ degrees of freedom.
The rotational periodic structure is similar in some way with an axisymmetric structures, in fact for axisymmetric structures most modes of vibration occur in degenerate orthogonal pairs. Similarly, for rotational periodic structures, the mode shapes obtained for each substructure can be divided into one of the following classes according to the mode shapes of the neighboring substructure:

- Case 1: all the substructures present the same mode shapes as the neighboring substructures, then

  $$\boldsymbol{u}^i = \boldsymbol{u}^{i+1}$$

  in this case it is possible to rewrite the eigenvalues as follows

  $$\{\boldsymbol{u}\} = \{\boldsymbol{u}^1, \boldsymbol{u}^1, \boldsymbol{u}^1, \dots \boldsymbol{u}^1\}^T$$

  it is clear from this expression that when rotating, the mode shape remains the same and so this mode shape can already describe the complete structure without any further additional mode shapes.
  For this class of mode shapes, then, no kind of degeneracy is present.
- Case 2: all the substructures present the same mode shapes as the neighboring substructure, however the substructure is vibrating in anti-phase with the neighboring substructures, then

  $$\boldsymbol{u}^i = -\boldsymbol{u}^{i+1}$$

  this class of mode shape can happen only if the number of blades of the turbine rotor is even and the eigenvalues present themselves as follows

  $$\{\boldsymbol{u}\} = \{\boldsymbol{u}^1, -\boldsymbol{u}^1, \boldsymbol{u}^1, -\boldsymbol{u}^1, \dots \boldsymbol{u}^1, -\boldsymbol{u}^1\}^T$$

  from this expression, it can be seen that if the rotation happens through even numbers of substructures then the mode shape remains the same, as it happens for the first case, however if the rotation goes through an odd number of substructures then the mode shape is still the same but it presents in anti-phase.
  For this class of mode shapes as well, no kind of degeneracy is present.

- Case 3: all the other possible mode shapes and they may exhibit degeneracy, in particular they show a double degeneracy.

To describe the third class of mode shapes, which present the condition of

$$\{\boldsymbol{u}^i\} \neq \{\boldsymbol{u}^{i+1}\} \quad \{\boldsymbol{u}^i\} \neq -\{\boldsymbol{u}^{i+1}\}^T$$

assuming that it is normalized so that

$$\boldsymbol{u}^T\boldsymbol{u} = 1$$

By definition, all the substructures chosen are identical to each other and with this condition, the deflected shape $\{\boldsymbol{u}'\}$ obtained from rotating the mode shape around a substructure is still an eigenvector although different from the original mode shape $\{\boldsymbol{u}\}$. In addition, this deflected shape is not orthogonal to the original mode shape $\{\boldsymbol{u}\}$ and this means that there exists another eigenvector $\{\bar{\boldsymbol{u}}\}$ which is orthogonal to the original mode shape $\{\boldsymbol{u}\}$ in order for $\{\boldsymbol{u}'\}$ to exists and together with $\{\boldsymbol{u}\}$ it forms a vector space.
The rotated mode shape is of the form

$$\{\boldsymbol{u}'\} = \{\boldsymbol{u}^N, \boldsymbol{u}^1, \boldsymbol{u}^2, \ldots \boldsymbol{u}^{N-1}\}^T$$

and this eigenvector is also normalized so that

$$\{\boldsymbol{u}'\}^T\{\boldsymbol{u}'\} = 1$$

however the rotated eigenvector can also be expressed as a linear combination of the basis of this vector space

$$\{\boldsymbol{u}'\} = c\{\boldsymbol{u}\} + s\{\bar{\boldsymbol{u}}\}$$

with $c$ and $s$ as constants. Analogously, it is possible to define an eigenvector $\{\bar{\boldsymbol{u}}'\}$ orthogonal to the previous eigenvector $\{\boldsymbol{u}'\}$ and express it as a linear combination of the basis of the aforementioned vector space. It is then in the form of

$$\{\bar{\boldsymbol{u}}'\} = -s\{\boldsymbol{u}\} + c\{\bar{\boldsymbol{u}}\}$$

$$\begin{Bmatrix} \boldsymbol{u}' \\ \bar{\boldsymbol{u}}' \end{Bmatrix} = \begin{bmatrix} c\mathcal{I}_{NJ} & s\mathcal{I}_{NJ} \\ -s\mathcal{I}_{NJ} & c\mathcal{I}_{NJ} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ \bar{\boldsymbol{u}} \end{Bmatrix} = \begin{bmatrix} \mathcal{R} \end{bmatrix} \begin{Bmatrix} x\boldsymbol{u} \\ \bar{\boldsymbol{u}} \end{Bmatrix}$$

where $\mathcal{I}_{NJ}$ denotes the unit matrix of order $NJ$ and $\begin{bmatrix} \mathcal{R} \end{bmatrix}$ is a transformation matrix through which it is possible to rotate the eigenvectors around $J$ substructures. From the assumption of normalized eigenvectors then the next relation holds true

$$c^2 + s^2 = 1$$

which can be proved by the assumption of normalized rotated eigenvector $\{\boldsymbol{u}'\}$. Therefore, by analogy, it is possible to define the two coefficients as

$$c = \cos\psi \quad s = -\sin\psi$$

then from the previous expression, it is possible to relate deflections on any substructure as a function of the deflections present on the first substructure.

Previously, it has been mentioned that the pair of eigenvectors considered are all real, however the eigenvalue problem

$$\Big([\boldsymbol{K}] - \boldsymbol{\omega}_i^2[\boldsymbol{M}]\Big)\{\boldsymbol{u_i}\} = \{\boldsymbol{0}\}$$

can also be solved if we consider a pair of complex eigenvectors written as

$$\{\boldsymbol{z}\} = \{\boldsymbol{u}\} + i\{\bar{\boldsymbol{u}}\}$$

and analogously the rotated eigenvector expression can be rewritten for the complex eigenvector case and therefore

$$\{\boldsymbol{z}'\} = e^{-i\psi}\{\boldsymbol{z}\}$$

however it is known that by applying the transformation equation above as many times as the number of substructures does not change the complex eigenvector, and this means that the value of $\psi$ is not independent but it must assume one of the values given by

$$\psi = \pm\frac{2\pi}{N}ND$$

for the case of bladed discs the angle $\psi$ denotes the Inter Blade Phase Angle (IBPA) and it represents the difference of phase between two neighboring substructures, whereas $ND$ represents the nodal diameter which is a measure of the periodicity of a mode shape over the entire structure.
By analogy, the mode shapes of the neighboring substructures can be expressed as

$$\{\boldsymbol{z}^{j-1}\} = e^{-i\psi}\{\boldsymbol{z}^{j}\} \quad \text{or} \quad \{\boldsymbol{z}^{j}\} = e^{-i\psi}\{\boldsymbol{z}^{j-1}\}$$

it is then clear that both the real pair of normalized eigenvector and the complex eigenvector can satisfy the aforementioned eigenvalue problem, but the difference between them resides within the time evolution of the deflected mode shape.
In fact for a real eigenvector $\{\boldsymbol{u}\}$ the deflected shape is given by the real part of $\{\boldsymbol{u}\}e^{i\omega t}$ which is simply

$$\mathcal{R}\left[\{\boldsymbol{u}\}e^{i\omega t}\right] = \{\boldsymbol{u}\}\cos\omega t$$

and the peaks always occur at the same points during the rotation, however for a complex eigenvector $\{z\}e^{i\omega t}$, the instantaneous deflected shape is given by the real part of the complex eigenvalue

$$\mathcal{R}\left[\{\boldsymbol{z}\}e^{i\omega t}\right] = \{\boldsymbol{u}\}\cos\omega t - \{\bar{\boldsymbol{u}}\}\sin\omega t$$

when

$$t = \frac{\psi}{\omega}$$

the deflected shape is the same as the deflected shape as that at $t = 0$ with the difference that it has rotated round one substructure, this can be described through figures 3.9 and 3.10.
Therefore it is possible to consider the mode shape given by real eigenvector as a standing wave mode shape, whereas the mode shape given by a complex eigenvector can be considered as a rotating wave mode shape.

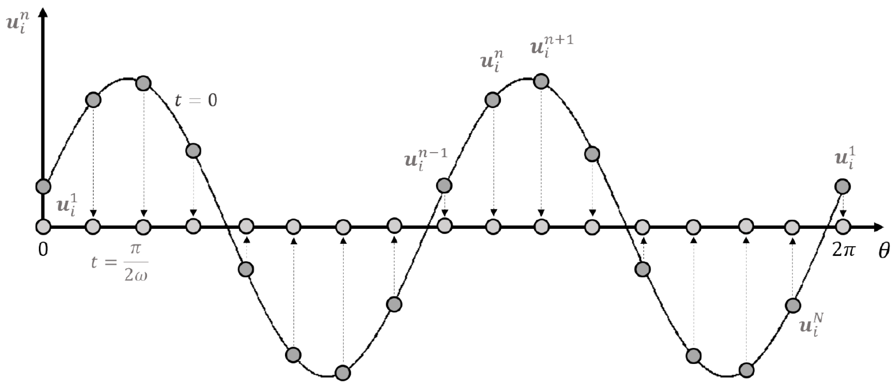


Figura 3.9: Standing wave mode shapes with fixed peaks.[7]

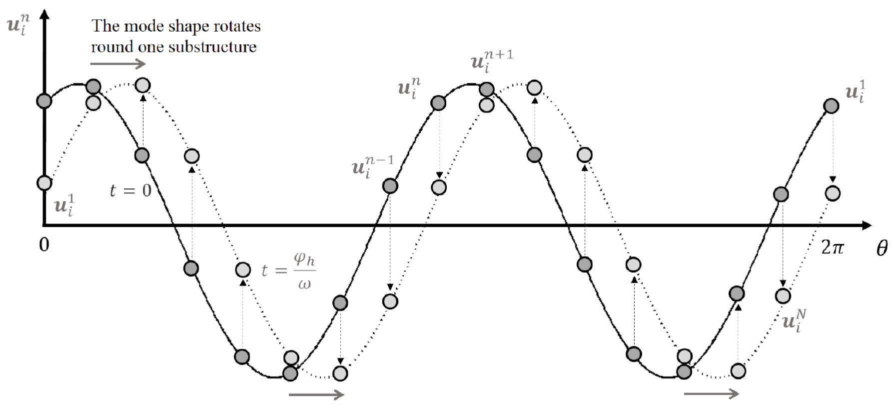


Figura 3.10: Rotating wave mode shapes with moving peaks.[7]

Furthermore, it has been found that the instantaneous mode shapes can always be expressed as a combination of sines and cosines of the argument $ND\theta$, where $\theta$ is the circumferential coordinate and $ND$ is simply the nodal diameter, whose maximum number is not independent but it is a function of the number of blades

$$\begin{cases} 0 \leq ND \leq \frac{N}{2} & \text{if } N \text{ is even} \\ 0 \leq ND \leq \frac{N-1}{2} & \text{if } N \text{ is odd} \end{cases}$$

Moreover, it is possible to define the periodicity of the mode shapes, once the number of blades has been defined and this is done by considering the Inter Blade Phase Angle (IBPA) $\psi$ and, generally, a positive value indicate a clockwise rotating mode shapes, whereas a negative value indicate a counter-clockwise rotating mode shape.
The values assumed by $\psi$ depends on the number of blades and generally, it holds:

$$\begin{cases} \psi \in \left[-\pi, -\frac{2\pi\left(\frac{N}{2}-1\right)}{N}, ..., -\frac{4\pi}{N}, -\frac{2\pi}{N}, 0, \frac{2\pi}{N}, \frac{4\pi}{N}, ..., -\frac{2\pi\left(\frac{N}{2}-1\right)}{N}, \pi\right] & \text{if } N \text{ is even} \\ \psi \in \left[-\frac{2\pi(N-1)}{N}, ..., -\frac{4\pi}{N}, -\frac{2\pi}{N}, 0, \frac{2\pi}{N}, \frac{4\pi}{N}, ..., -\frac{2\pi(N-1)}{N}\right] & \text{if } N \text{ is odd} \end{cases}$$

It is now possible to redefine the three classes of mode shapes introduced previously:

- Case 1: in this case, all the substructures are in-phase between them and that is why these mode shapes can be described as a standing wave mode shapes, which brings to

$$\psi = 0$$

$$ND = 0$$

this mode shape presents a single real eigenvector and it is present in any rotational periodic structure.
- Case 2: in this case, all the substructures are in anti-phase between them and so these mode shapes can be also described as standing wave mode shapes, which is possible only for a structure with even number of blades and then

$$\psi = \pm\pi$$

$$ND = \frac{N}{2}$$

also in this case, the mode shape presents a single real eigenvector.
- Case 3: in this case, all the mode shapes are represented by rotating wave mode shapes and so each substructure has a different $\psi$ compared to the neighboring substructure. The number of nodal diameters can be calculated by the aforementioned expression

$$\begin{cases} 0 \leq ND \leq \frac{N}{2} & \text{if } N \text{ is even} \\ 0 \leq ND \leq \frac{N-1}{2} & \text{if } N \text{ is odd} \end{cases}$$

Considering now the specific case of a turbine rotor, which falls into the category of rotational periodic structure, it is possible to define a further classification

of the mode shapes.
As mentioned previously, the number of mode shapes are infinite in reality, and it is still very large for a finite element model, however not all of them are actually considered when a modal analysis is carried out, in fact generally only the first eigenvalues are considered, then inside this range of frequencies the mode shapes can be classified into one of the following mode of vibration:

- bending mode (B);
- torsion mode (T);
- edgewise mode (EW);
- flapwise mode (FW).

however, mode shapes of the same class does not necessarily mean they are the same, in fact another aspect to be considered is the order of the mode shape. To deal with the modal analysis, a typical choice consist in sorting the mode shapes into groups, called Modal Families, and the sorting method can be either based on the frequencies of the modes or on the type of the modes, in addition the Modal Families technique is needed when a FREND diagram is to be plotted.
The FREND diagram is used to represent the modal families characterized by nodal diameters on the $x$ axis and frequency on the $y$ axis. Inside this diagram each line represent a modal family and an example of FREND diagram is represented in figure 3.11. Finally, some examples of the mode shapes of a turbine blade are also shown in figures 3.12, 3.13, 3.14, 3.15.

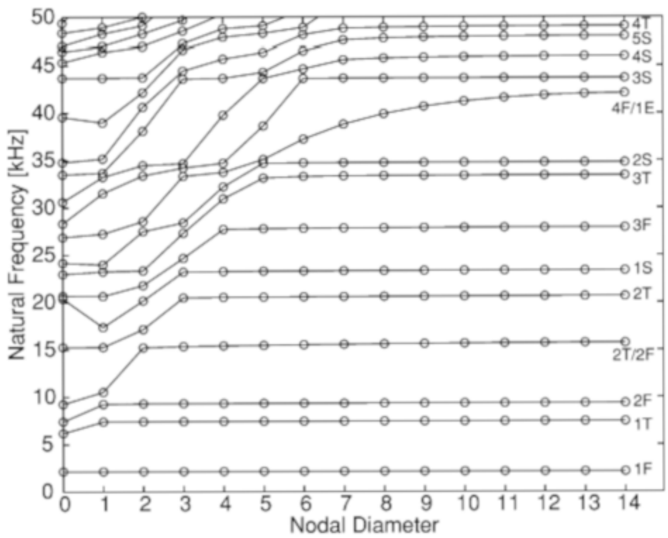


Figura 3.11: FREND diagram.[20]

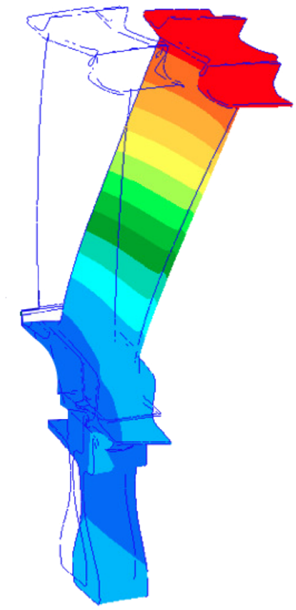

Figura 3.12: Flapwise mode.[18]

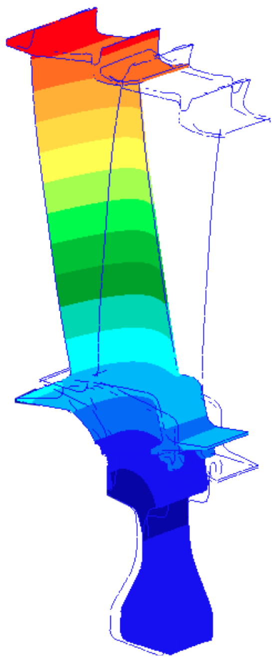

Figura 3.13: Edgewise mode.[18]

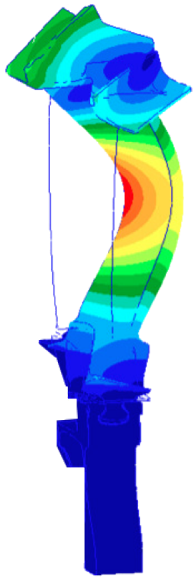

Figura 3.14: Bending mode.[18]

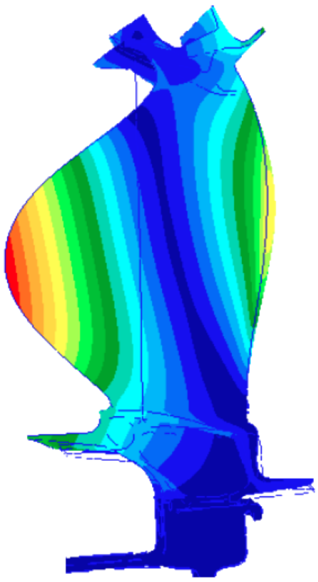

Figura 3.15: Torsion mode.[18]

## 3.2 Modal analysis with cyclic symmetry condition

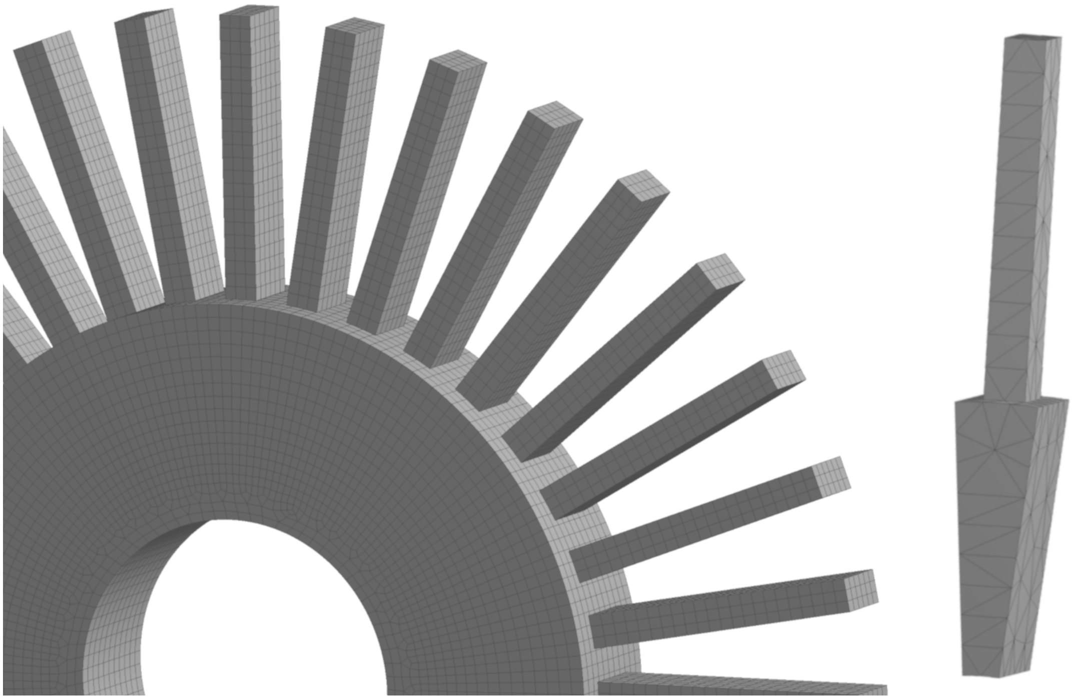

Figura 3.16: Meshed model of a dummy turbine rotor.

As mentioned previously, a common technique to deal with a modal analysis of a structure is the implementation of a finite element model of the structure, through an appropriate meshing procedure, and the analysis is then conducted over this structure characterized by a finite number of elements and degrees of freedoms.
In the case of turbine rotor, this structure has the benefit of being a rotational periodic structure and so it is possible to exploit the cyclic symmetric condition to furthermore reduce the number of nodes to be used as we can then simply analyze a single substructure and rotate it to obtain the complete structure by using the relations mentioned previously.
By considering now the equation of motion, the mass and stiffness of the structure are matrices of limited elements since a finite element model has been implemented and they are characterized by a circulant symmetric structure and block structure, with each block representing a substructure. The circulant symmetry is characterized by the fact that each row of the block has the same elements of the previous matrix except that it is shifted to the right by one

element. This is clearly shown by the following matrices expressions

$$[\boldsymbol{M}] = \begin{bmatrix} [\boldsymbol{M}_0] & [\boldsymbol{M}_1] & [\boldsymbol{M}_2] & ... & [\boldsymbol{M}_2] & [\boldsymbol{M}_1] \\ [\boldsymbol{M}_1] & [\boldsymbol{M}_0] & [\boldsymbol{M}_1] & ... & [\boldsymbol{M}_3] & [\boldsymbol{M}_2] \\ [\boldsymbol{M}_2] & [\boldsymbol{M}_1] & [\boldsymbol{M}_0] & ... & [\boldsymbol{M}_4] & [\boldsymbol{M}_3] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ [\boldsymbol{M}_1] & [\boldsymbol{M}_2] & [\boldsymbol{M}_3] & ... & [\boldsymbol{M}_0] & [\boldsymbol{M}_1] \\ [\boldsymbol{M}_0] & [\boldsymbol{M}_1] & [\boldsymbol{M}_2] & ... & [\boldsymbol{M}_1] & [\boldsymbol{M}_0] \end{bmatrix}$$

$$[\boldsymbol{K}] = \begin{bmatrix} [\boldsymbol{K}_0] & [\boldsymbol{K}_1] & [\boldsymbol{K}_2] & ... & [\boldsymbol{K}_2] & [\boldsymbol{K}_1] \\ [\boldsymbol{K}_1] & [\boldsymbol{K}_0] & [\boldsymbol{K}_1] & ... & [\boldsymbol{K}_3] & [\boldsymbol{K}_2] \\ [\boldsymbol{K}_2] & [\boldsymbol{K}_1] & [\boldsymbol{K}_0] & ... & [\boldsymbol{K}_4] & [\boldsymbol{K}_3] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ [\boldsymbol{K}_1] & [\boldsymbol{K}_2] & [\boldsymbol{K}_3] & ... & [\boldsymbol{K}_0] & [\boldsymbol{K}_1] \\ [\boldsymbol{K}_0] & [\boldsymbol{K}_1] & [\boldsymbol{K}_2] & ... & [\boldsymbol{K}_1] & [\boldsymbol{K}_0] \end{bmatrix}$$

$$\{\boldsymbol{x}\} = \begin{Bmatrix} \{\boldsymbol{x}_0\} \\ \{\boldsymbol{x}_1\} \\ \{\boldsymbol{x}_2\} \\ \vdots \\ \{\boldsymbol{x}_{N-1}\} \\ \{\boldsymbol{x}_N\} \end{Bmatrix} \qquad \{\boldsymbol{f}\} = \begin{Bmatrix} \{\boldsymbol{f}_0\} \\ \{\boldsymbol{f}_1\} \\ \{\boldsymbol{f}_2\} \\ \vdots \\ \{\boldsymbol{f}_{N-1}\} \\ \{\boldsymbol{f}_N\} \end{Bmatrix}$$

The presence of a circulant symmetry has the benefit of being diagonalizable through a Fourier transform and for a generic nodal displacement of a substructure $j$, it can be expressed as the following Fourier series

$$\{\mathbf{x}_j\} = \frac{1}{\sqrt{N}}\left\{\mathbf{u}^0\right\} + \frac{2}{\sqrt{N}}\sum_{k=1}^{N-1}\left[\left\{\mathbf{u}_c^k\right\}\cos\left((j-1)\psi_k\right) + \left\{\mathbf{u}_s^k\right\}\sin\left((j-1)\psi_k\right)\right] \\ + \frac{1}{\sqrt{N}}\left(-1\right)^{j-1}\left\{\mathbf{u}^N\right\}$$

where $\{\mathbf{u}\}$ represents the Fourier coefficients and the subscripts, $c$ and $s$, denotes respectively the cosine and sine components. It is interesting to note from this expression that the nodal displacement is influenced significantly by the contribution given for each of the nodal diameter, as it can be seen from the summation operator.

The eigenvalue problem can be rewritten through a transformation of the coordinate system in order to obtain a block diagonal form of the stiffness and mass matrices

$$[\boldsymbol{M}] = \begin{bmatrix} [M_0^*] & 0 & 0 & ... & 0 & 0 \\ 0 & \left[M_{1,c}^*\right] & 0 & ... & 0 & 0 \\ 0 & 0 & \left[M_{2,c}^*\right] & ... & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & ... & \left[M_{2,s}^*\right] & 0 \\ 0 & 0 & 0 & ... & 0 & \left[M_{1,s}^*\right] \end{bmatrix}$$

$$[\boldsymbol{K}] = \begin{bmatrix} [K_0^*] & 0 & 0 & ... & 0 & 0 \\ 0 & \left[K_{1,c}^*\right] & 0 & ... & 0 & 0 \\ 0 & 0 & \left[M_{2,c}^*\right] & ... & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & ... & \left[K_{2,s}^*\right] & 0 \\ 0 & 0 & 0 & ... & 0 & \left[K_{1,s}^*\right] \end{bmatrix}$$

by defining $\{\boldsymbol{x}^*\}$ as the transformed coordinates, the eigenvalue problem then becomes as

$$\Big([\boldsymbol{K}] - \boldsymbol{\omega}_i^2 [\boldsymbol{M}]\Big) \begin{Bmatrix} \{x_0^*\} \\ \left\{x_{1,c}^*\right\} \\ \left\{x_{2,c}^*\right\} \\ \vdots \\ \left\{x_{2,s}^*\right\} \\ \left\{x_{1,s}^*\right\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ \vdots \\ \{0\} \\ \{0\} \end{Bmatrix}$$

And the benefit of this diagonalization is the possibility to solve each of the $N$ single block as a separate eigenvalue problem.
Considering now the individual $j$th substructure, the nodal displacements vector can be divided into superficial nodes and internal nodes, moreover the superficial nodes can be divided into left side and right side of the substructure

$$\{\boldsymbol{x}_j\} = \begin{Bmatrix} \{x_{L,j}\} \\ \{x_{I,j}\} \\ \{x_{R,j}\} \end{Bmatrix}$$

this process is useful since it is known that the regions in cyclic symmetry condition are the faces of the disk and the shroud, which are also in common with adjacent substructures.
Analogously, we can express the vector of exciting forces in a similar way

$$\{\boldsymbol{F}_j\} = \begin{Bmatrix} \{F_{L,j}\} \\ \{F_{I,j}\} \\ \{F_{R,j}\} \end{Bmatrix}$$

Once the mode shape has been specified, a relation exists between the nodes of the left region and the nodes of the right region, in fact they are bound by a fixed phase difference equal to the $IBPA$

$$\{x_R\} = e^{i\psi}\{x_L\}$$

and therefore

$$\{\boldsymbol{x}_j\} = \begin{Bmatrix} \{x_{L,j}\} \\ \{x_{I,j}\} \\ e^{i\psi}\{x_L\} \end{Bmatrix}$$

Similarly, the stiffness and mass matrices can be reduced by exploiting this relation

$$[\boldsymbol{K}_j] = [T]^T [K] [T] = \begin{Bmatrix} [I] \\ [I] \\ \left[e^{i\psi} I\right] \end{Bmatrix}^T [K] \begin{Bmatrix} [I] \\ [I] \\ \left[e^{i\psi} I\right] \end{Bmatrix}$$

$$[\boldsymbol{M}_j] = [T]^T [M] [T] = \begin{Bmatrix} [I] \\ [I] \\ \left[e^{i\psi} I\right] \end{Bmatrix}^T [M] \begin{Bmatrix} [I] \\ [I] \\ \left[e^{i\psi} I\right] \end{Bmatrix}$$

and the eigenproblem relative to this substructure becomes

$$\left([\boldsymbol{K}_j] - \boldsymbol{\omega}_i^2 [\boldsymbol{M}_j]\right) \begin{Bmatrix} \{x_{L,j}\} \\ \{x_{I,j}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix}$$

It is then clear that the solution to this eigenproblem depends on the phase shift, that is $IBPA$, but that is related to specific mode shapes and nodal diameters. In other words, if we hypothetically solve the eigenproblem for all possible values of $IBPA$ then it is possible to obtain all the possible mode shapes of the complete structure.
As for the exciting forces, the main contributions comes from the pressure

generated on the airfoil and disk, and this region is an unsteady region as the flow is highly perturbed by the presence of stages upstream and downstream, and this perturbation provokes further stress to the structure. In addition mechanical vibrations are also present due to the presence of asymmetry.
Since the blades are rotating, the exciting forces are also rotational periodic and this may give rise to resonance phenomena with destructive consequences, and many factors influence the exciting frequency such as the number of airfoils and rotational velocity.
It is then useful to define a parameter, Engine Order (EO), as the ratio between the frequency of the exciting force and the rotational velocity of the structure

$$\text{EO} = \frac{\omega_F}{\omega}$$

this parameter will be useful to analyze resonance phenomena, and as this happens when the frequency of exciting forces is equal or multiple of the rotational velocity, then the Engine Order is always an integer number.

### 3.2.1 Campbell diagram

To study resonance phenomena, it is useful to use the campbell diagram in which the $x$ axis is made of the rotational velocity and $y$ axis is the frequency. Then it is possible to plot the natural frequencies, as constant line, and the frequencies of the exciting forces which are oblique line of slope equal to the Engine Order. The intersections between these curves then represent the potential resonance conditions which occur at those rotational velocity. Those rotational velocity should be avoided during the operative range of the engine, if that is not possible then a deeper analysis of the energy of the resonance must be carried out to check whether the resonance phenomena are strong enough to bring the structure to failure. Physically, the dependance from the rotational velocity can be explained from the fact that the centrifugal load and temperature distribution change the material properties either stiffening or softening.

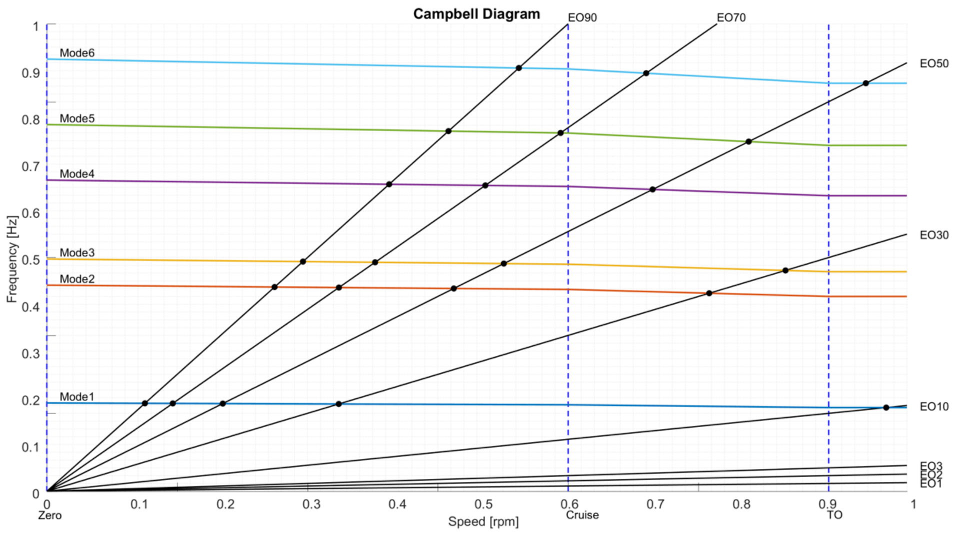


Figura 3.17: Example of Campbell diagram.[17]

# Static analysis of a turbine bladed disc

From the modal analysis, as already mentioned, it is possible to obtain the main mode shapes and natural frequencies of the turbine rotor, but surely this analysis alone is not enough to determine whether the structure can operate safely through all the operating conditions.
Another typical analysis that is carried out in the design of a turbine bladed disc is the static analysis where all the forces acting on the structure are time independent which means that they do not vary in magnitude and direction over time, or at least they are assumed to be that way.
Considering the functionality of the turbine stage and its position inside a jet engine, it is clear that the main forces acting on the turbine is surely the pressure field generated around the surface of the airfoil, which can be divided into pressure side and suction side, according to how the gas flow is deflected around the blade, moreover the pressure difference also generate a bending moment contribution. In addition, for the rotor component, the whole structure is rotating at a very high speed which means there is a contribution of inertial load that cannot be neglected.
Furthermore, the surrounding gas flow has very high temperature which, once it comes into contact with the surface of the blade, provides an additional stress due to the thermal gradient that is generated on the blade airfoil.
Finally, having described the main loads, there is an additional contribution that cannot be neglected, in fact, the turbine stage is generally mounted with a pretwist around the radial direction and this contribution must also be taken into account.
These loads due to their nature may not seem constant and a static analysis may not seem appropriate at first, nonetheless their evolution over time is still very small, in fact they can be considered to be quasi-static loads, which is valid when its frequency is significantly lower than the natural frequency of the structure. And this is true for these loads so all of them can be assumed to be constant and a static analysis can already provide a good approximation of what are the stresses generated, the most critical regions and how it deforms.

## 4.1 Static loads

The first step to take in order to carry out a static analysis is similar to the modal analysis as nowadays the most common approach to this kind of problem is the implementation of a finite element model, as it provides the most accurate results albeit it requires significant computational cost, and thus an appropriate mesh must be first generated.
Aside from the mesh procedure, the main difference resides in the static loads acting on the structure. A brief description of them has already been given, however it is still possible to dive deeper into their characteristics:

- inertial load:
  as mentioned previously, the main function of the turbine stage is to provide shaft power for the compressor and this power is proportional to the rotational velocity of the rotor and that is why the rotor rotates at a very high velocity which in turn means a non-negligible inertial load.
  It is then clear that the centrifugal force thus generated is surely proportional to the material density and rotational velocity, a simplified formulation can be expressed as follows
  $$F_C = \rho\omega^2 \int_r^t A r dr$$
  from this definition, it is easy to see that, due to the contribution of the cross section, the maximum stress lies around the root of the blade.
- pressure load:
  in order to fulfill its objective, different pressure fields are generated around the blade airfoil, which in turn is converted into mechanical shaft power, moreover, since the blade is clamped at its root and/or tip, the blade is subjected to an additional stress due to bending moment, however this last effect can be reduced through camber so that a new bending moment can be generated from the centrifugal force and it goes against the previous bending moment.
- thermal load:
  since the turbine blade comes into contact with highly energized hot gas, inevitably this prolonged contact puts the turbine blade into significant stress, and this is caused not by the rising temperature per se of the structure itself but due to the fact that the structure is constrained which does not allow it to expand freely as a consequence of the rising temperature. The following expression is a simple equation that can be used to understand the rate of expansion related to the temperature difference, through the thermal expansion
  $$\epsilon_T = \alpha \Delta T$$
  This effect can be reduced by using cooling system, as already mentioned previously, but an accurate analysis of its effect become quite complicated.

On a side note, the prolonged exposure to high temperature has another critical impact as a creep deformation may occur, in other words, the structure under stress for a long duration may deform permanently with dangerous consequences even for stresses under its yield strength.

- pretwist load:
  in order to ensure an optimal positioning of the turbine blade during the operating condition, a pretwist is generally needed during the assembly procedure and this needs to be accurately calculated by taking into account the adjacent blades since interference may occur in the shroud, and this depends mainly on temperature gradient and centrifugal load.

After having correctly defined all the loads and the constraints, it is possible to execute a finite element analysis and the accuracy of the results will be highly influenced by the quality of the mesh generated during the meshing procedure. During this analysis it must be guaranteed that these deformations do not produce interference with other parts of the final assembly.

# Manufacturing overview

From the etymology of the word manufacturing which comes from the combination of the latin word "manu", that is hand, and "factura", which means working, and therefore it involves a process of hand working which was typical for that period when the word first originated.
However, its meaning has evolved a lot from the original definition going through many important historical events, such as industrial revolution, and nowadays the term manufacturing refer to the process of material working through automated machinery.
Now considering a modern context, two proper definitions can be given for the word, or concept, manufacturing which are equally valid, depending on the context:

- from a technological point of view, manufacturing can be defined as a set of individual processes, both physical and chemical, to change the properties and appearance of an initial product into a desired one. In addition, it also includes processes needed for the assembly operation. Each of these processes are carried out through the use of highly accurate and precise machinery tools.
- from an economical point of view, manufacturing can still be defined as a set of individual processes used, instead, to add value, economically, to an initial product by modifying its properties and appearance, and thus the final product is more valuable than the initial product.

On a side note, it is interesting to discuss firstly the difference between the word, and the concept, of manufacture and production, as they are commonly used as synonyms between each other. Nonetheless a difference, however small, is still present between them in fact the term manufacturing is usually used to define the production process in which raw materials are transformed into a tangible product, especially in the context of industrial sector, and the raw materials are often procured from outside. The term production, instead, is more general and the input used does not only consist of raw materials but it may also consist of intangible materials, and as such the output can also be intangible.
For this thesis, the two terms will be used interchangeably as this thesis is restricted to the aerospace industry and the final product is the aeroengine

turbine blade. This is the reason why the engineering materials considered will be only metal materials, as this is the common material used for the low pressure turbine blade, and all the following considerations will take this into account. Finally, for this next part, most of the definition and concepts presented here will be referred to the work by Mikell[14].

## 5.1 Manufacturing processes

From the aforementioned definitions, the manufacturing processes may be wrongly interpreted as a simple deed, obviously that is not the case as it is a set of many operations, and each of them refers to an individual operation that carries out a small or large alteration to the initial product and bringing it closer to the final product.
In any case, these operations can be divided into two basic categories:

- processing operation;
- assembly operation.

the first one is the class of manufacturing processes that involves a transformation of its physical and chemical properties, and thus the initial product becomes more valuable by changing its size, weight and appearance. The processing operations can also be divided into three subcategories:

- shaping process;
- surface process;
- property enhancing process.

The assembly operation, instead, is the class of manufacturing processes that are needed to connect different components in order to obtain a mid or final assembly. Analogously, these operations can be divided into the following subcategories:

- permanent joining process;
- mechanical fastening.

### 5.1.1 Processing operations

The processing operations are those concerned with changing the physical, geometrical and chemical properties of an initial product through specific machine tools and therefore the product becomes more valuable as energy is spent to bring these changes, and by energy not only mechanical energy is considered but human energy as well is needed to control and oversee the machine tools. As outcome along with the final product, scraps and waste are also present which may occur either naturally as part of a manufacturing process or occasionally in case one of the product or part is considered to be defective as it does not satisfy the design requirement. Either way, these waste must be

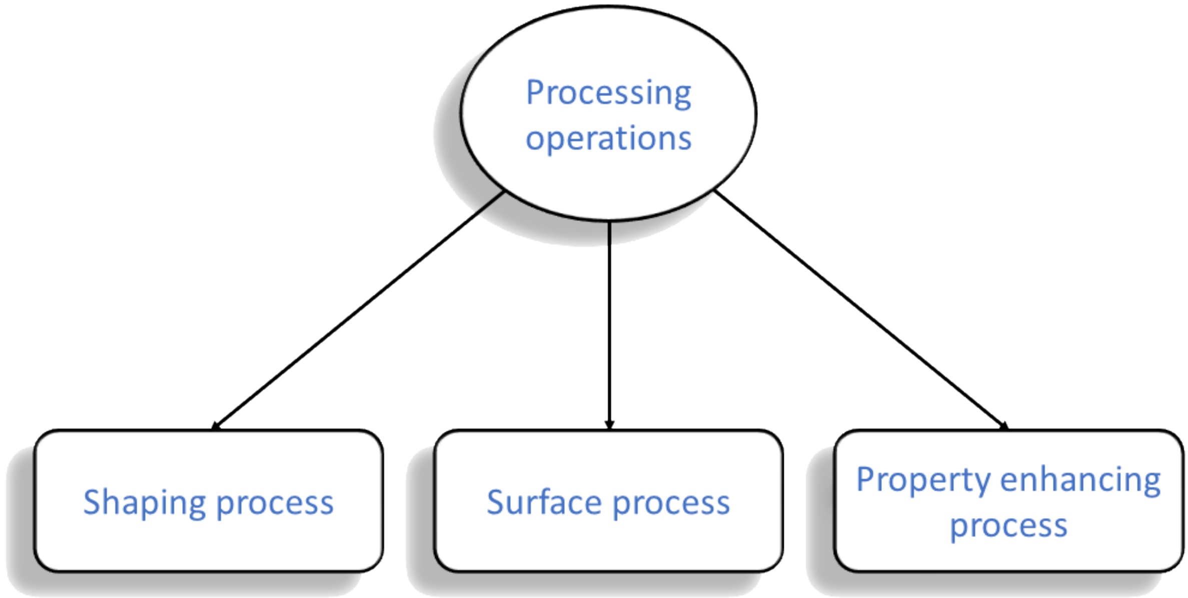


Figura 5.1: Processing operations scheme.

contained and possibly reduced.
As mentioned previously, this class of operations can be divided into into three additional categories: shaping process, surface process, property enhancing process.
The shaping process, as the name implies, refer to those processes that can alter the geometry and size of an initial product, however this can be achieved through different ways. The first is through a solidification process, or casting for metal material, where the initial material is melted by heating and then this liquid is poured into designated molds to solidify. Once the material is extracted, it will present the shape of the desired product.
A second method is particulate processing in which the initial material is in the form of powders, and then they are pressed and heated to obtain the desired shape.
A third method is the deformation process in which the shape is altered by applying a force such that the tension generated is higher than the yield strength of the material. This process is typically used for metal materials as they are ductile by nature, which is an essential requirement for this process, and to achieve a higher ductility the material is heated before the deformation operation. The main operations of this method are the forging and extrusion process in the case of metalworking
A fourth and last method is the material removal process in which, as the name implies, material is slowly removed from the initial material untill the desired shape is achieved. However it is clear from this description that, during its operation, a lot of waste and scraps are inevitably generated and this represent the negative side. In case of metals, this method is also called machining and

it includes operations such as milling, drilling and turning.
The surface process refer to those operations concerning the external region of the product and they can be divided into:

- cleaning: this process refer to those operations, chemical and mechanical, to remove surface contamination such as dirt and oil.
- surface treatment: this process refer to those operations, mechanical and physical, aimed at improving its mechanical and physical properties.
- coating: this process refer to the operation of coating the surface of the product with a specific material.
- thin film deposition: this process refer to the deposition of thin films on the exterior surface of the product.

The property enhancing process refer to those operations in which the mechanical and physical properties are enhanced without changing, at least not intentionally, the overall geometry of the initial product. One of the most used of these kinds of processes, especially for metals, is the heat treatment through which the metal is first heated then cooled and the rate of cooling heavily impacts the mechanical and physical properties of the final product, such as strength, hardness, ductility and also corrosion resistance. Some typical heat treatment techniques are:

- annealing: this technique makes the metal more workable and thus higher ductility as the metal is brought closer to its equilibrium state and this is achieved by heating the metal above its upper critical temperature, then it is slowly cooled.
- quenching: this technique hardens the metal and just like annealing it also involve heating metal above its upper critical temperature, however it is quickly cooled to room temperature so that the microstructure is not altered.
- precipitation hardening: this technique strengthens the metal by uniforming its grain structure and it is achieved by heating and fast cooling process which takes place in an inert atmosphere. The time required however depends on the thickness of the metal.
- tempering: this technique gives the metal a higher hardness, toughness and reduces its brittleness, in other words the metal becomes more ductile.

### 5.1.2 Assembly operations

The assembly operations are those concerned with joining and connecting two or more components into a new component, that is an assembly, however the connection can be of two types either permanent connection or disassemblable. A typical example of permanent assembly operation is welding, in fact the components, once joined, cannot be separated, whereas through mechanical assembly, in particular with threaded fasteners, it is still possible to easily

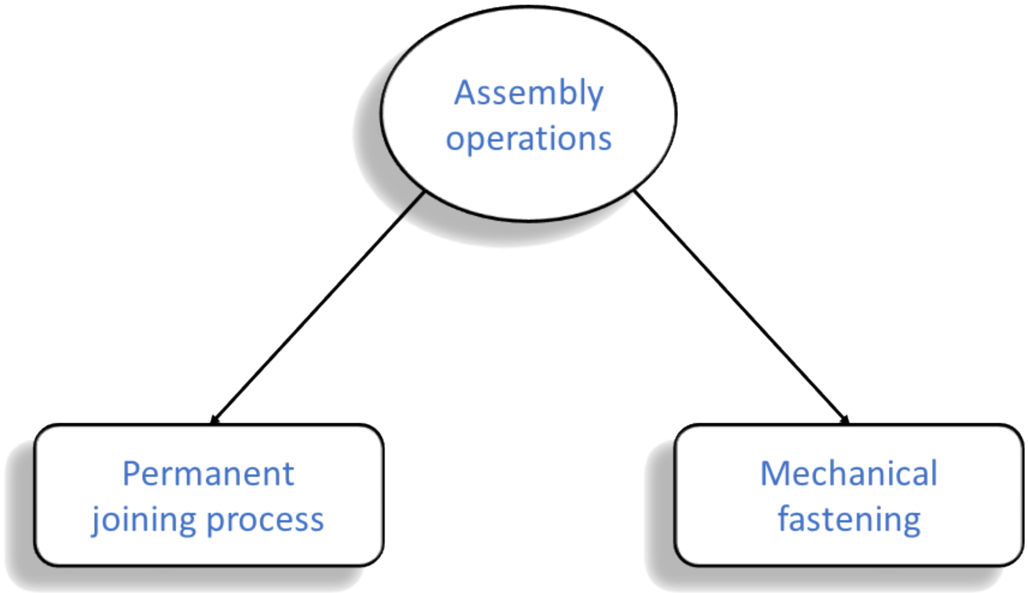


Figura 5.2: Assembly operations scheme.

disassemble the two original components.
Some of the most common assembly operations are:

- welding: the welding process is an operation through which two or more components are joined at their contact surface and this is achieved through application of heat or pressure. In some cases, a filler material can be added between the two materials to facilitate the welding and if this filler material has higher strength properties then the welding joint will also present higher properties than the initial materials, but the welded point may suffer from defects which are hard to detect and this may reduce its strength. In addition, this operation is economic in terms of material usage and fabrication cost, nonetheless, since this process is usually manual, it has higher labor cost. Finally, two main types of welding are commonly used and they are fusion welding, which use heat to join the components, and solid state welding. which is based on pressure and heat to join the components.
- brazing: the brazing process is the process through which no melting is required of the base metals to be connected, in fact a filler metal, called brazing material, is instead melted and distributed along the surfaces of the base material and generally the strength of the joint is superior to the brazing material. This process has the benefit, compared to welding, of the possibility to use any metals and it can be performed quickly, however the strength of the joined entity is usually less compared to the welded joint.
- soldering: the soldering process is similar to the brazing process in fact no melting of the base metals occurs and a filler metal, called solder,

is melted instead, and analogously it is distributed along the surfaces of the base metals to be connected. The difference consist in the fact that the filler metal, once melted and distributed, forms a metallurgical bond with the base metals and for this to occur the surfaces must be preemptively cleaned and to be free of oils and oxides. However, this process usually produce low joint strength.

- adhesive bonding: the adhesive bonding process is a joining process through which a filler material, a structural adhesive which is nonmetallic and so it is not melted compared to the previously mentioned methods, is used to join the components. The structural adhesive is able of forming strong enough permanent bonds and the physical properties achieved depends on the curing process, in other words it depends on what chemical reaction occurs to pass from the liquid state into the solid state which is needed to form the bonding with the surface of the base metals. The chemical reactions generally used are polymerization, condensation or vulcanization and usually heat and pressure are also applied to aid the joining procedure. One of the downside of the adhesive bonding is the time necessary for the curing process to take place, called setting time.
- mechanical assembly: the mechanical assembly refer to a group of operations that join two components mechanically and they can be generally divided into operations that allow for disassembly and those that do not permit so. This category of operation is usually preferred for its ease of assembly and disassembly, moreover the necessary tooling is also simple. Some of the most commonly used techniques are threaded fasteners, which are components that make use of external or internal threads to connect the different parts which permit the disassembly, rivets and eyelets, which can perform a permanent joint between two parts, interference fit, in other words two parts are connected through mechanical interference which occurs during assembly, stitching, stapling, sewing and many others.

### 5.1.3 Manufacturing capability

A particular concept that will be significant for this thesis is the manufacturing capability which refer to the presence of limitations due to physical, geometrical and as well as technical limitations of the manufacturing plant and this represent a big constraint for the design, especially of a new component.
To describe the manufacturing capability, it is useful to define the following concepts:

- technological processing capability:
  the technological processing capability refers to the availability of the various manufacturing processes, as a specific machinery can carry out a specific process for a limited variety of material, and therefore the more tools a manufacturing plant has, the more manufacturing processes it can

carry out. Generally speaking, a specific process is related to a specific material and so the more manufacturing processes may also indicate that more materials can be worked on. However, these are still not enough to determine the technological process capability, in fact not only physical processes are considered but the proficiency of the plant personnel is also taken into account.
Overall, it is important for a company to know its own technological processing capability in order to be able to design and manufacture a new product.

- physical product limitations:
the physical product limitations refers to the physical limitations of a product in other words size and weight, in fact the manufacturing plant must be equipped with the right tools to manage the product. For example, large and heavy products may require an appropriate crane to move, whereas small and light products may just need a conveyor.
In addition, the physical properties also influence the manufacturing equipment, in fact in order to manufacture a large product, an appropriate machine of the right size is also required.
Analogously, a company must know its physical product limitations in order to be able to design and actually manufacture a product compatible with its manufacturing plant.
- production capacity:
the production capacity refers to the maximum allowed production quantity of a manufacturing plant for a certain period of time, under a specific operating condition. In other words, given a certain operating condition, which depends on factors such as number of shifts and hours per shift, we are interested in knowing what is the production quantity related to that condition.
Moreover, the result is usually shown in terms of number of units for a certain period of time, if the outputs can be regarded homogeneous, else other measures may need to be taken into consideration.

These matters are of crucial importance in the design of a new product, since it may not be manufacturable with the company's own manufacturing plant, that is why more and more companies are implementing a design for manufacturing approach. This will be described in more and deeper details in the next chapters.

## 5.2 Metal casting

Casting is a manufacturing process in which the initial product is a material in liquid form which is then poured into specific molds and left to solidify, and once this happens the material has taken the shape of the mold, which represents the desired shape. In addition, the casting process is actually not an

Figura 5.3: Casting operation.

single individual process but it is actually a group of various processes that is based on this method and these processes can be obviously used with different materials but in this thesis we shall focus only on processes that are inherent to metal casting.
The main features of the casting process are:

- both small and large components can be produced through this process.
- it is possible to produce components with complex internal and external shapes.
- it is possible to produce final components without additional manufacturing processes, although in most cases further manufacturing operations, such as machining, is needed to achieve a good surface finish and tolerances.
- this process can be used for any metal as long as it can be melted into liquid state.
- the final product may be characterized by low mechanical property, poor accuracy and surface finish.

The mold is an essential part of casting as it contains the cavity which has the shape of the desired geometry of the final product, although it is slightly over-sized in order to take into account the shrinkage during cooling. After having adequately heating the liquid metal is then poured in this cavity and once it solidifies through cooling, it has reached the desired geometry. The mold used can be divided into open and closed, where in the first type the liquid metal is poured directly into the cavity whereas in the second type a

passageway is needed to allow the liquid, from outside, to enter the cavity and this passageway is also called gating system.
Once the liquid is inside the cavity, it will start to cool naturally and the solidification process starts in which a change of phase takes place from liquid to solid which has the shape of the internal cavity of the mold, however this process requires a significant amount of time to release the necessary heat. Additional steps may still be needed to remove excessive parts, surface cleaning and heat treatment, inspection of the final product, or in some cases a machining process may still be required to obtain the desired product, especially to reach the necessary tolerances on some features.
As for the mold, they can be either expendable or permanent:

- expendable mold: a casting process which uses this mold is characterized by the destruction of the mold in order to extract the cast part and they are usually made of sand with appropriate binders.
- permanent mold: this mold instead is characterized by the possibility of re-usability and thus the same mold can be used many times to produce the cast part, with the same geometry. In this case it is usually made of metal or ceramic refractory material so that it can withstand the high temperature of the liquid metal and it usually presents an opening mechanisms to permit the extraction of cast part.

The casting operation can be divided into the following steps:

- heating phase:
  through this phase, heat is provided to liquefy the metal, initially solid, and the required heat is given by the sum of the heat needed to reach the melting point of the metal, heat of fusion needed for phase change and the required heat to reach the needed temperature for pouring.
- pouring phase:
  after the metal is heated, the pouring phase starts which needs to be correctly analyzed as the liquid should first of all fill all of the cavities before solidifying and this depends on the pouring temperature, pouring rate and turbulence as well. The turbulence refer to the rapid variation of velocity in both magnitude and directions which produces agitated and irregular fluid flow instead of a smooth flow. This should be avoided as it has negative consequences such as the acceleration of the formation of metal oxides and mold erosion which may affect the geometry of cast part.
- solidification phase:
  this is the last phase and it takes place once the liquid metal has cooled enough inside the mold cavity and this may vary depending on the metal composition, however for most alloys, the liquid will freeze over a range of temperatures instead of a single point as it happens with pure metals and this causes the cast part to be characterized by high variation in chemical composition. Another important factor is the time needed for

complete solidification and it depends on the type of metal, volume to surface area ratio and shape as well of the casting.

A major problem of the casting operation is the shrinkage which happens due to the lowering temperature during the cooling process and this can be divided into three main steps:

- contraction during liquid phase;
- contraction during phase change, also called solidification shrinkage;
- contraction during solid state.

the second phase is particularly important as it is the cause of the shrinkage cavity which happens as the remaining liquid is limited and a void in the metal is then formed. To counter this effect the mold cavity is usually slightly over-sized and this margin is called pattern shrinkage allowance, however other form of compensation to this effect exist such as the use of risers, for sand casting, and use of pressure, for die casting.

### 5.2.1 Sand casting

Sand casting is one of the most popular process and it is based on the use of a mold made by sand and binders. The sand mold is made of two separate parts called cope, the top part, and drag, the bottom part, and they are inside a box called flask.
To form the internal cavity of the mold, it is frequent to use patterns made of wood, metal or plastic and then sand is packed around this pattern to form the cope and drag, afterwards the pattern is removed and the shape of the cast part is generated. In case internal surfaces are needed for the cast part, then cores made of sand can be inserted inside the mold. In addition, the size of the pattern must also take into account shrinkage as well as further machining allowances.
The liquid metal is poured inside a pouring cup, to minimize splash, which is connected to a downsprue and then to the runner before reaching the main cavity or riser, if present, and both of them make the gating system of a casting process. In addition as the metal liquid flows in, the air entrapped needs to be released, but thanks to the porosity of the sand mold, it can permit the expulsion of air and gas.
The riser is used in sand casting to provide liquid metal for casting and it can be used to provide a directional solidification in order to avoid the formation of shrinkage cavities inside the cast part, moreover it must be appropriately designed, in particular the sectional area of the passageway between the riser and the main cavity, as a low volume is required to lower the metal waste but a high sectional area is still needed to avoid premature freezing in the passageway. Once finished the riser is removed and remelted for the next casting.

In order to make the sand mold, different factors must be considered such as grain size, shape and its distribution in the mixture, in fact small grain size will produce better surface finish, whereas larger ones tend to be more permeable, moreover irregular grains will provide higher strength compared to regular rounded grains due to interlocking between themselves. Nonetheless, these properties can also be enhanced by changing the mixture and additives. Once the mold has been formed, several factors are available to determine the quality of the sand mold:

- strength: which denotes its ability to both keep its shape and resist erosion during the casting process and it mainly depends on grain shape and binder;
- permeability: which denotes the ability to release air and gas during the pouring operation;
- thermal stability: which denotes the ability to resist cracking and buckling during the contact with the hot liquid metal;
- collapsibility: which denotes the ability to allow the shrinkage of the cast part without collapsing and along with this definition, it also refers to the possibility of sand removal after the process;
- reusability: which denote the possibility of being reused for more casting processes.

Finally, it must also be noted that not all of these qualities are compatible so trade-offs will be necessary when choosing a mold.

### 5.2.2 Die casting

Die casting is a a type of casting process characterized by the use of permanent mold, called die, which is made of metal for both sections, and the cavity must be accurately machined to take into account all the factors mentioned previously. Analogously the cores can also be made of metal however they must permit the possibility to be removed after the casting or at least collapsible in order to be mechanically removed later. If both of these cases are not possible, then sand cores can be used and this process is referred as semi-permanent mold casting.
Through a permanent mold casting, the mold must be preheated to facilitate the metal flow inside the gating system and into the cavity after having been appropriately coated to aid heat dissipation, however, differently from expendable castings, the metal mold is opened as soon as the metal start solidifying to avoid the formation of cracks inside the structure due to cooling contractions.
Generally the permanent mold has the advantages of being able to provide good surface finish and finer grain structure, nonetheless the mold is much more expensive and it is not advised for complex geometries.

Specifically for die castings, the liquid metal is injected inside the cavity under high pressure and this pressure is maintained during solidification until the mold is opened. The die castings operation can be carried out in two configurations:

- hot-chamber machines: this configuration is characterized by the fact that the molten metal is inside a container near the main machine and it is injected, under pressure, through a piston. It is then clear that the injection system is particularly stressed and this configuration can only be used for metals with low melting point.
- cold-chamber machines: contrary to the previous one, the liquid metal is poured from an external container and a piston drives the liquid inside the cavity under pressure.

Finally, since metal molds have no porosity, venting holes and passageways must be appropriately built for the expulsion of gas and hot air.
From this description it is clear that die casting provides the possibility for high production rates and it is an economical solution for large quantity production.

### 5.2.3 Investment casting

An alternative to the previous casting methods is the investment casting, also called lost-wax process, characterized by the fact that the pattern is made of wax which is subsequently coated with refractory material to form the mold. Afterwards, the wax can be melted leaving only the refractory material mold into which the liquid metal is then poured to solidify.
This method has the advantages of being able to produce cast parts of high accuracy and complex shapes, nonetheless since the wax pattern is melted then the pattern must be created every time and generally this is carried out by pouring hot wax inside a master die, otherwise for very complicated geometry it is possible to use separate wax parts joined together.
In order to coat the wax mold with refractory materials, the wax mold is dipped repeatedly inside the refractory material in powder form and its small fine grain provide good quality of surface finish. In summary, with this technique it is possible to cast parts with great complexity albeit it is an expensive process. Moreover this technology is limited by the weight of the component and it is generally suitable for small and light parts.

## 5.3 Metal machining

Another important manufacturing technology is the material removal process and this can be achieved with different techniques, both traditional and non conventional. The group of traditional techniques, also called traditional machining, is characterized by the use of a sharp cutting tool to remove the

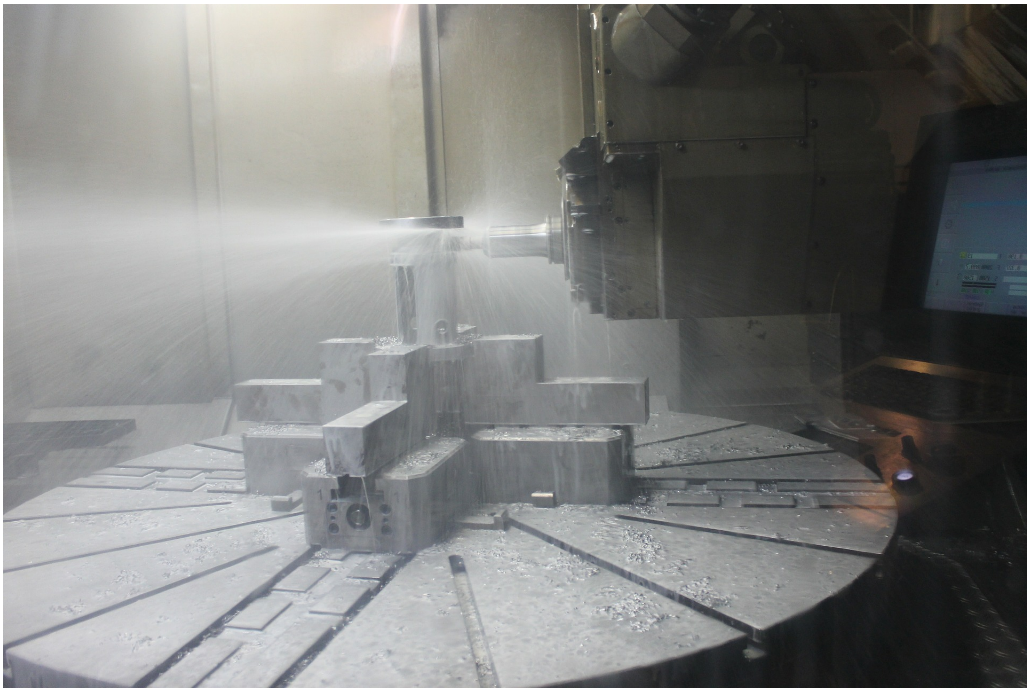

Figura 5.4: Machining operation.

material until the desired geometry is achieved. The material removal is achieved through shear deformation of the material through cutting tool to form a chip which is removed as the process progresses and for this to happen relative motion between tool and work material is needed which are called cutting speed, the primary motion, and feed, the secondary motion.
This family of processes present the following features:

- variety of materials;
- variety of shape and geometry;
- dimensional accuracy;
- good surface finish;
- high material waste;
- time consuming.

generally, machining is carried out to improve the features of a part produced through other manufacturing processes.
As for the cutting tool, it presents a cutting edge made of a material stronger than the work material in order for the material removal to happen. Near the cutting edge are the rake face, which directs the flow of the chip, at an angle called rake angle and the flank, which protects the new surface from abrasion, at an angle called relief angle.
The machining operations can be divided into two main categories according to its purpose:

- roughing cuts: this represents the case in which large amount of materials

need to be removed and this is carried out rapidly. However through this cut, subsequent processes are needed to achieve the final product.
- finishing cuts: this represents the case in which the product is machined to achieve the final dimensions and tolerances.

In any case, a cutting fluid is needed during the machining operation to cool as well as lubricate the cutting tool and the choice of which cutting fluid to use depends on the cutting condition and tool.
This family consists of:

- turning;
- drilling;
- milling.

In addition, the machine operation can divided into two categories depending on how the final shape is created:

- generating: through this case, the geometry of the part is created according to the trajectory of the cutting tool.
- forming: through this case, the geometry is created from the shape of the cutting tool which present the reverse of the shape to be produced.

Finally, an important concept regarding manufacturing often neglected is the machinability of a certain manufacturing process which depends on several factors, such as tool life, forces and power, surface finish and chips. It is clear then that it depends both on the chosen material, the tool and the process itself and in order to evaluate it, machinability testings can be carried out through which an index, called machinability rating, is calculated which is related to a standard base material.

### 5.3.1 Turning

Turning is a machining operation characterized by the fact that the material is removed while the part is rotating through a single point tool which is fed by a machine tool called lathe, which also rotates the part to be machined. This tool then moves in the same direction of the axis of rotation to generate a cylindrical shape.
The main operations carried out with a turning operation are:

- facing: through which a flat end is created.
- taper turning: through which a tapered cylinder or conical shape is generated by providing an angle to the tool while feeding.
- contour turning: through which a contoured form in the turned part is generated by providing a contour to the tool while feeding.
- form turning: through which the shape present on the tool is formed on the turned part by pressing radially.

- chamfering: through which a chamfer is generated on the part.
- cutoff: through which the end of the part can be cut off by feeding radially.
- threading: through which threads are created on a cylinder by feeding linearly and parallel to the axis of rotation.
- boring: through which a single point tool is used to feed linearly on the inside diameter of a hole present in a part.

### 5.3.2 Drilling

Drilling is a machining operation used mainly to create a hole in a part which is achieved through a tool called drill bit, that is a rotating cylindrical tool with two cutting edges, and this is carried out on a drill press.
In a drilling operation, the drill bit is fed into a fixed part to form a hole on it whose diameter is equal to the diameter of the drill.
The main operations carried out with a drilling operation are:

- reaming: through which a hole can be slightly enlarged.
- tapping: through which internal threads can be created on a hole.
- counterboring: through which counterbored hole can be created on a part.
- countersinking: through which a cone-shaped hole is created.
- centering: through which a starting hole is created to establish the location for other drilling processes.

### 5.3.3 Milling

Milling is a machining operation characterized by a rotating cylinder with multiple cutting edges, but differently from drilling, the axis of rotation of the cutting tool is perpendicular to the feed direction. The cutting tool for milling is called milling cutter and it is carried out through a milling machine.
The main operations carried out with a milling operation can be divided into two main categories called peripheral milling and face milling.
The peripheral milling consist of:

- slab milling: through which the width of the cutting tool is higher than the part.
- slot milling: through which the width of the cutting tool is smaller than the part, creating a slot.
- side milling: through which the side of the part is machined.
- straddle milling: through which both sides of the part is machined simultaneously.
- form milling: through which a specific form is shaped on the part according to the shape of the milling teeth.

The face milling consist of:

- conventional face milling: through which the diameter of the cutting tool is higher than the part.
- partial face milling: through which a portion of the cutting tool is outside the part.
- end milling: through which the diameter of the cutting tool is smaller than the part.
- profile milling: through which the outside surface of a part is machined.
- pocket milling: through which a pocket is machined into a surface.
- surface contouring: through which a countoured surface is generated by moving the cutting tool along a curvilinear path.

## 5.4 Trends in manufacturing

The realm of manufacturing is surely not a static field but a continuous evolving industry to find smarter and more optimized solutions for the overall manufacturing process. Some of the newest trends in improving and changing the traditional manufacturing are:

- lean production:
  this concept was derived from Toyota Production System which is based on the reduction of waste which refer to defective parts, production of more than needed parts, excessive inventories, unneeded processes, unnecessary movements and workers waiting. It is easy to see that through lean production not only material use is optimized but all the other resources are used more efficiently.
- six sigma:
  this concept was derived from the Motorola Corporation and it is based on reducing the variability in the processes and products and this is achieved by setting measurable targets to obtain near perfection in the processes. This concept is based on six steps which are:
  - definition of the problem;
  - measurement of the process and performances;
  - analysis of the process;
  - improvements recommendation;
  - development of control plan for future improvements.
- globalization:
  nowadays, the whole world is closely connected and this contributed in the creation of an international economy characterized by a freer flow of products. That is why a new tendency is to buy resources from different countries over the world by evaluating all the advantages and disadvantages. Then it is possible to differentiate between offshore outsourcing, that is from companies based oversea, and nearshore outsourcing, that is from companies in the same country.

- environmentally conscious manufacturing:
  the manufacturing processes, as described previously, inevitably produce waste and the energy needed to feed these systems is not negligible either. That is why, the society is more and more interested in finding more environmentally sustainable solutions for the field of manufacturing which aim at determining operations that use materials more efficiently and minimize the consequence on environment. This tendency is closely related to green manufacturing, cleaner production and sustainable manufacturing and all of them can be considered to be a part of the new approach design for environment.

# Design for manufacturing

In the previous chapter, a brief description of the main manufacturing processes has been carried out, especially those related to the aeroengine industry, and their main features as well as limits have also been highlighted.
However, in a traditional design process, these constraints are not taken into account and their possible benefits are not fully exploited as well. In fact, the traditional design process can be described roughly as follows:

- problem definition:
  this phase represents the first step of a design process in which the problem, to which the component represent the solution, must be defined. In other words, from the definition of the problem, it is possible to deduce the main functionality of the component to be designed, although in this phase it is defined only at an abstract level and it will evolve going further on.
- conceptual design:
  the next step is the conceptualization of the starting abstract idea and by conceptualization it is meant the phase of project planning in which all the various ideas with their relative costs and risks are evaluated. This process can be more efficiently carried out by implementing various tools and methods that can accurately highlight their relative strengths and weaknesses.
- preliminary design:
  once the best idea has been chosen and it has roughly started to take form, the preliminary design phase takes place, in which a first assessment of its feasibility is performed by modeling a first prototype which can represent roughly the researched product. This step is necessary and critical as the complete design of a new product is expensive and time consuming and therefore through a preliminary design it is already possible to determine whether this to be designed product can satisfy all the previously defined requirements.
- detailed design:
  when the preliminary product is validated, the component's main features are then fixed and the detailed design begins in which the more specific features, needed for the effective production and assembly, are

defined. In this phase then, the procurement of materials, drawings and specifications are also carried out.

- production planning and tool design:
  after all the high level details have been carefully designed, the new component model is then frozen and its design is basically complete. What comes next is the phase concerning the planning and selection of manufacturing processes and production, if not already available, of the tools necessary to manufacture this part.
- production:
  finally, once everything has been correctly designed, it is possible to start the production phase.

These steps, represented in figure 6.1, more or less describe the workflow of a traditional design process, which is still practiced nowadays, however with this order of sequence, a big flaw is present as no consideration about the manufacturability is made before the production planning and this is problematic because during this phase manufacturability problems may arise related to the fact that the designed component is not manufacturable in some parts, or even if the product is manufacturable it may be too expensive to produce it. These information are received only during the last few phases and this forces, if the manufacturability problem is not too big, the design process only a few steps back, or if the manufacturability problem is significant to the design of a whole new component.
This problem was accepted in the past as satisfying its main design requirements, without taking into account the manufacturing requirements, was the utmost importance, and so there were little to no collaboration between the design department and manufacturing department, but it is clear that it is not an optimized and smart process.
Indeed, it has been noticed, as shown in figure 6.2 and 6.3, that this has a significant impact on the overall cost of the product, in fact from various studies it has been shown that the actual cost of design is relatively low, around $\sim 5\%$, however its influence on the cost can be as much as $\sim 70\%$, and this is one of the reasons why the design for manufacturing concept along with concurrence engineering are becoming more and more popular.

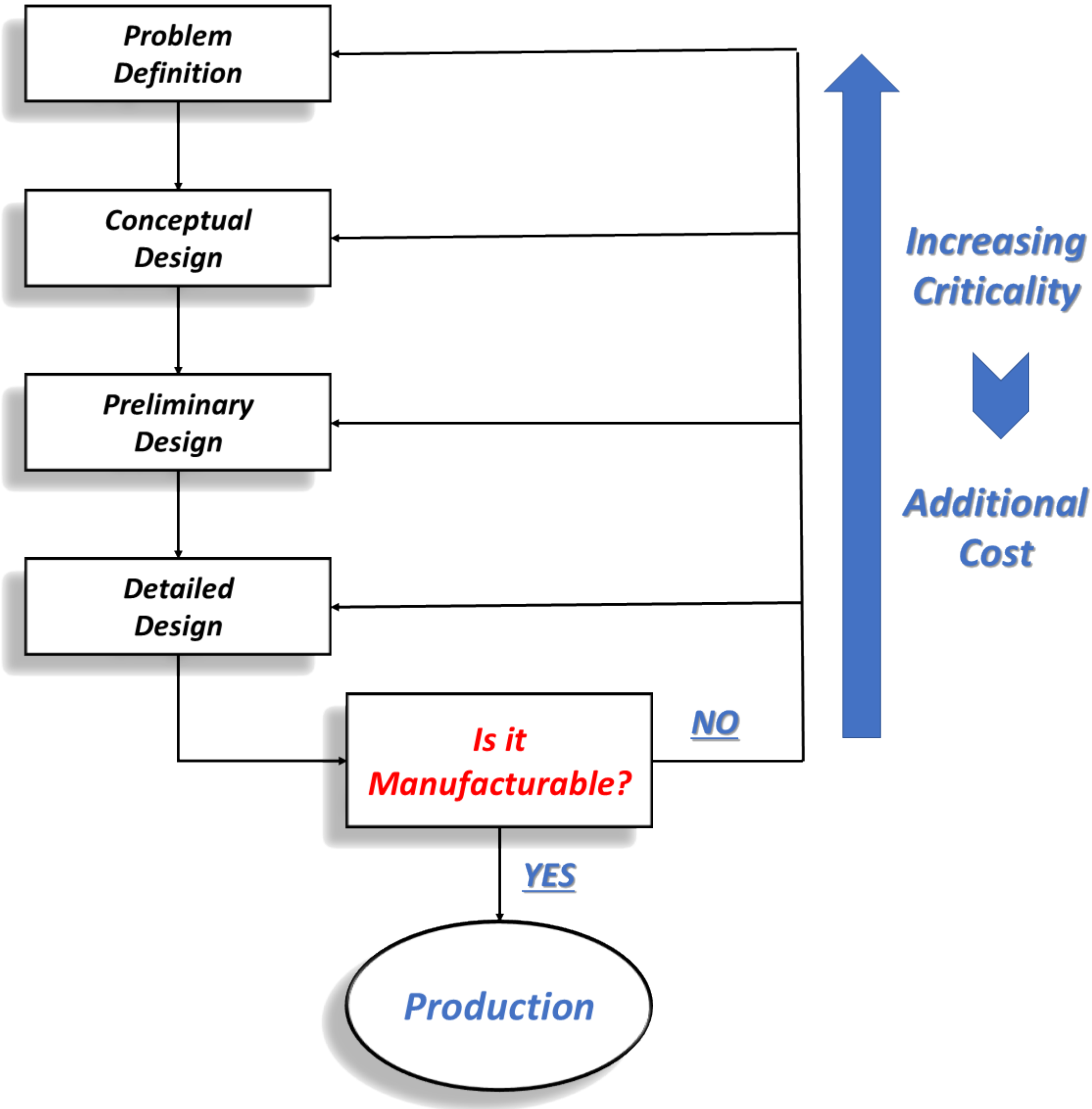


Figura 6.1: Traditional design process.

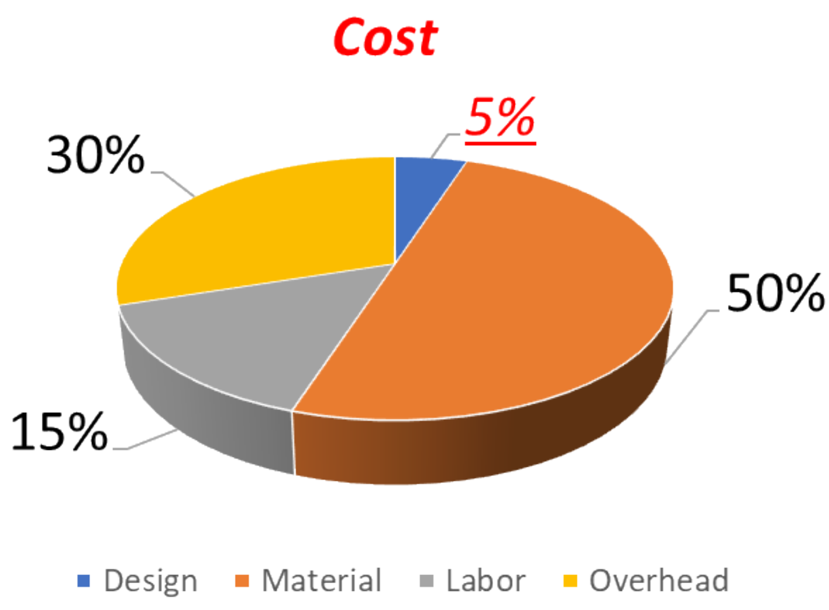


Figura 6.2: Cost contributions.

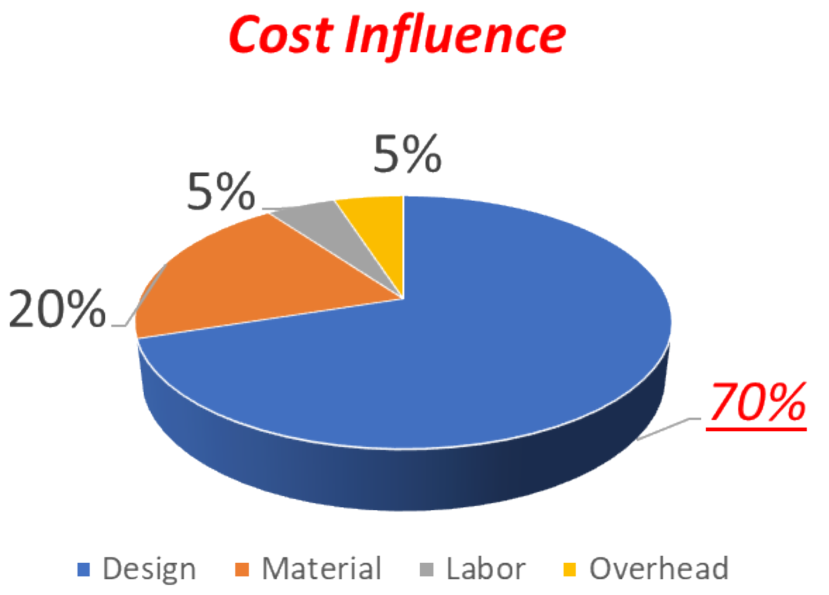


Figura 6.3: Cost influence.

## 6.1 DFM Concept

Poorly designed component for manufacturability raises several problems, as mentioned earlier, such as:

- difficult features to manufacture;
- inefficient assembly;
- need for specifically made tools;
- launch difficulties;
- numerous changes;
- parts proliferation.

all of these lead to increased cost and delayed time to market, and in addition these problems will also drain resources from the team both in terms of people and money. All of these problems can be avoided or at least dealt with when it is still not too late, for example during the preliminary design phase, by implementing a design for manufacturing approach. The design process that implements a DFM concept is represented in figure 6.4, and compared to the traditional design process, the key difference is that both the design team and manufacturing team will start early on in order to provide the necessary inputs for the preliminary design phase, and thus fully embracing the concept of concurrence engineering.
In substance, implementing a design process that embraces a design for manufacturing philosophy enables:

- optimization of all the manufacturing processes and assembly;
- insurance of best cost, quality, reliability, regulatory compliance, time to market;
- insurance of the fact that functionality, styling and product delivery are not compromised.

Some of the reasons of why this concept is not commonly implemented yet despite its benefits reside mainly on misconceptions that the problems related to manufacturability can be actually dealt with at a later time or that by adding the manufacturability constraints along with design requirements the problem may become too complicated. However, what must be highlighted is that the longer these conditions are neglected the more difficult it will become to solve without incurring in big delays and additional significant costs.
From a different point of view, it is possible to interpret the goal of this concept as the necessity to achieve a single correct design already at the preliminary design phase in order to avoid the need to implement any time-consuming and expensive changes later on in the design process. In order to achieve this, surely the experience and feedback received from past projects related to similar component represent an essential source not to be neglected and they represent a good starting point.
On the other hand, through a correct implementation of this concept it is

possible to achieve the following benefits:

- lower production cost: this is achieved by reducing parts, thus lower assembly cost, and smoother product development to avoid expensive and time consuming change orders.
- higher quality: this is obtained by adopting a more robust and optimal design along with appropriate manufacturing processes and tools.
- faster time to market: through DFM, it is possible to have a more optimized part by prioritizing te use of tested processes and standardized parts and thus reducing the need of special tools for manufacturing.
- lower equipment costs: since standardized parts are preferred, special tools can be avoided and overall less tools will be needed.
- fewer design changes: by introducing all the constraints, both design and manufacturability, then a product that satisfy all these requirements is less likely to incur in changes later on.
- factory availability: through DFM concept, it is possible to have a more efficient production and this lead to higher availability for other products.

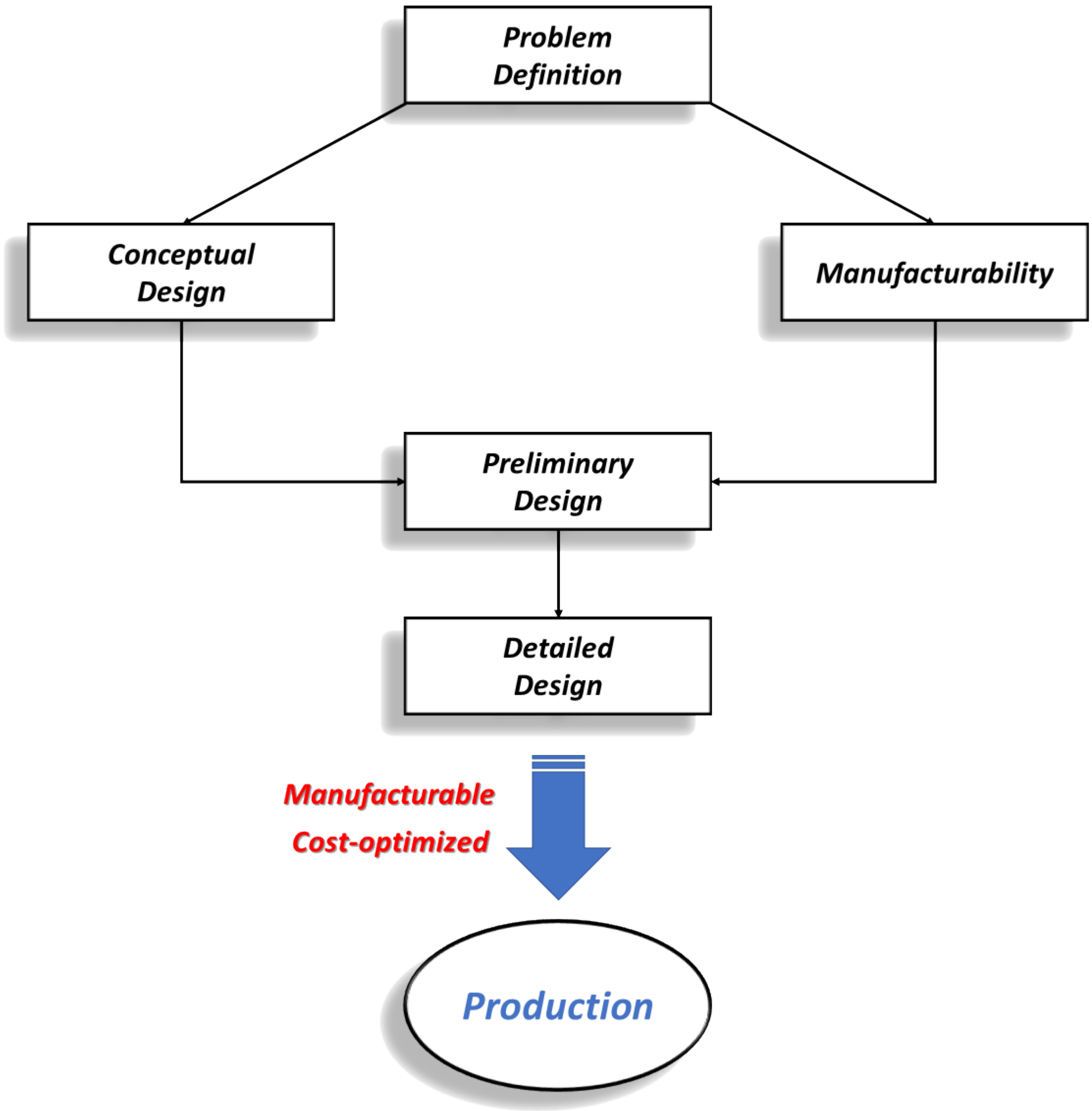


Figura 6.4: DFM-based design process.

## 6.2 DFM Guidelines

In this next part, a deeper description of the main guidelines suggested by the design for manufacturing concept is given, emphasizing the reason behind a certain choice and decision and to do this, we shall refer to the work by Anderson[5].

### 6.2.1 Design Strategy

In order to implement a design for manufacturing concept, an optimal design strategy must be established first at a very early stage of the design process, as it is critical that certain practices must be maximized and some minimized, if not avoided at all, in order to achieve an optimized design process and all of these must be known by the design team before the actual design begins to achieve a smooth and clean design.
Some of the most common practices that should be prioritized are:

- standard parts: the design should be carried out around standard parts to improve the availability and avoid expensive parts. In order to achieve this a list of standard parts should be provided in the early stage of preliminary design.
- off-the-shelf parts: similarly to the previous guideline, the design should always prioritize the off-the-shelf parts, and this will also simplify the design itself, as otherwise the designers have to make arbitrary decisions which may bring complications later on.
- proven processing and design: processing used for past product should not be neglected and new processes should be used only when the former cannot be used. This is due to the fact that new process need to be concurrently developed with the possibility of delays and risks.
  Analogously, when designing a part, it is advised to take into consideration past similar parts as they represent an invaluable source of feedback.
- tolerances: the tolerances should be chosen to be the widest whenever possible without compromising its functionality and it is important to identify the presence of tight tolerances and try to eliminate them if possible.

Nonetheless, over-constraining must still be avoided as they can provoke additional cost, compromise functionality and quality as well.

### 6.2.2 Design for everything

A fundamental difference between a DFM approach and a traditional design process is that through a traditional design process the focus is mainly and only on the functionality of the component, whereas with a DFM approach

many other parameters are taken into account during the design.
Some of the most important parameters are:

- cost: this is one of the core parameter behind a DFM-based design process and as shown previously, the design has a significant influence on cost. However achieving the lowest cost does not always represent a good solution, as quality may degrade, so a trade-off is still necessary.
- delivery: another important parameter, especially for new product is the delivery and it depends highly on the design, in particular its complexity to build, to assemble, to procure the various parts. A standardization of parts will help this aspect.
- quality and reliability: this part is also influenced in great part by the design, in fact the quality is specified along with the part and it is the quality of each part that determine the overall quality of the product.
- ease of assembly: during the design process, the assembly operation must also be taken into account and optimized whenever possible.
- time to market: aside from functionality and cost, the time to market is also an important factor, due to the competitiveness that characterize the modern market.
- future design: the design should be carried out so that the future designs can be based on the current design. Through this, it is possible to reduce significantly future design cost and time.

### 6.2.3 Part design

In order to be able to correctly apply a DFM approach, the designer must have a clear idea, before the actual design, of how the part is going to fit in the final product and its relation with other components.
Some general guidelines ro improve the manufacturability are:

- design for fixturing: designing and dimensioning parts appropriately by taking into account fixturing for each manufacturing processes will help on improving cost, time and quality and to achieve this, the designer must be familiar with the manufacturing processes.
- prioritize symmetry: since symmetrical part do not need to be oriented when assembled, this can help in avoiding quality problem during assembly. If this is not a feasible solution, then it is better to design the part to be very asymmetrical which is less likely to be oriented in the wrong way, which may happen for slightly asymmetrical component.
- reduce tooling complexity: engage in concurrent engineering of parts and tooling to minimize tooling complexity and this can help a lot in terms of cost and delivery time.
- optimal tolerance: the tolerances, especially tight tolerances, have significant influence on cost and therefore an optimal tolerance must be looked for, not only for the part itself but also as an assembly. For the

later, a stackup analysis can help in providing feedback in the choice of a good tolerance, which will be described later.

- optimal manufacturing process: when designing a part, the optimal process must be appropriately chosen at an early stage and the design should take that factor into account. This can also be achieved by concurrence engineering.
- design for work holdings: during the design, the work holdings are also needed to be taken into consideration as the designed part need to be rigid enough to withstand the work holding force.
- one setup for machining: whenever possible, the designed part must try to minimize the number of reposition during machining, as more setups is equivalent to higher time and extra cost.
- cutting tools: analogously the number of cutting tools must also be minimized.
- optimal selection of material: an optimal selection of material may help on reducing the post-processing steps to increase the mechanical properties, therefore when choosing a material its raw cost alone is not enough to determine the optimal material.

Obviously there are many more guidelines to be followed in order to fully embrace a DFM approach, however for complex products, such as a turbine blade, more specific guidelines are needed which are mostly based on past products and experience.

## 6.3 Implementation in the design process

After having described the main ideas behind the design for manufacturing concept, a description of a possible implementation of a design for manufacturing concept is now given. Obviously, a complete implementation of all the possible guidelines is very complicated and it is a long process as it concerns different teams which will need specific training in order to change the traditional mindset to the design of a new product and this is beyond the scope of this thesis. Since it is a long process, it is reasonable to implement it step by step and in this next part the first step to the implementation of a design for manufacturing is carried out.
The first step is actually the development of a code that can automatically carry out a series of checks aimed at validating some aspects of manufacturability of a certain product and considering the previous flowchart, it inserts itself in the preliminary design part. In particular, it can be used both during, and right after as well, the modeling process in which a 3D model of the product is generated so that the user can receive an almost instant feedback on the manufacturability aspects and best practices of the product to be designed.
As of now, the code is able to perform the following checks:

- material: the first check to be run regards the material assignment, although this check is not directly related to the manufacturability of a component, it is however important that the correct material has been assigned for the analysis that need to be carried out later.
- fillet radius and facial radius: the second check verifies that the value of a fillet radius and facial radius are inside a certain range. This is particularly critical as a very small radius is difficult to manufacture, special tools may need to be produced to manufacture this part and overall it is one of the main parameters that directly influence the cost of a product. Through this check it is then possible to verify that all the fillet radius are above a minimum radius value and below a maximum value as well, the latter is not critical and as such it produces a warning instead of a failure.
- thickness of the parts: the next check verifies that the thickness of some features are inside a definite range, in particular, through this rule, it is possible to check locally various wall thickness inside the shank, dovetail and shroud of the turbine blade. This checks aims at verifying that the thickness chosen are in accordance with the limits related to manufacturability, or more accurately related to a specific manufacturing technology, for example this check is highly relevant for a cast part as thickness may arise castability problems. In addition, this is particularly important during the preliminary design, aside from the manufacturability aspects, as it is possible to check whether the thickness chosen are similar to those suggested by best practices which are based on past designs.
- dimensional tolerances: the fourth check is aimed at verifying that the chosen tolerances are inside a specified range and it is obvious that a too small value of tolerance may need special tools and additional processes to achieve it, which in turn means additional cost. However, too large tolerances may generate problems related to interference with other parts of the assembly.
- geometrical tolerances: analogously, the geometrical tolerances values are also checked and they can be compared with best practices based on past designs.
- mass properties: finally some general properties such as total mass and volume of the product is compared to limiting values, this check is significant as some casting processes is limited by the total mass of the product to be cast.

In order to execute these checks, the limiting values need to be appropriately chosen and given as inputs to this code and they need to come from the manufacturing team who must be familiar with the manufacturing plants, especially their limits and constraints.
Once the inputs have been given, it is possible to run the code and the results are presented in two forms. The first result is a visual one, meaning that all

the failed features are highlighted, the second one is a report file that summarizes all the checks with their relative results along with a general comment regarding this particular check and the reason why it is needed.
In figure 6.5, an example of the output is shown applied to a dummy turbine blade and the feature that did not pass the checks can be clearly seen.

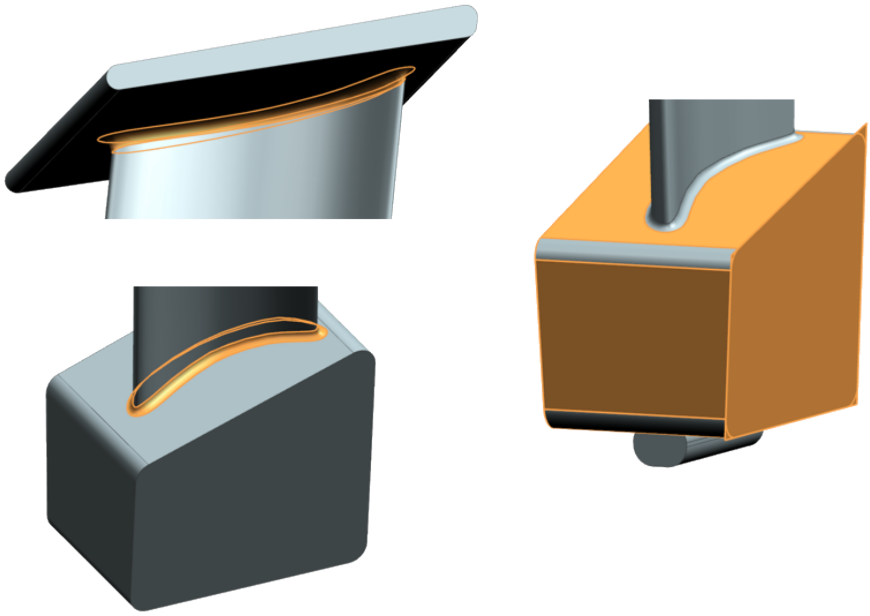

Figura 6.5: Output example applied to a dummy turbine blade.

The results can be generated almost instantly and basing on this result, it is possible to carry out all the necessary changes and the code can then be run again to verify the results.
In summary, although this code is still at its basic stage, this code can potentially act as an interface between the design team and manufacturing team and thus substitute the concurrence between them.
Finally, this code is compiled so that it can also be inserted in an automatic procedure such as the tool PRIME.

### 6.3.1 PRIME

The process of DFM code just shown can be used either as a standalone as well as introduced in an automatic process.
One of the possible choices is the PRIME tool which is a tool who is able to carry out a basic preliminary design of a turbine blade and vane starting from, simply, aerodynamic inputs which need to be provided by the aerodynamic department.
As shown in the work by Prino[17], this unique tool is characterized by the following steps:

- CAD generation:
  by giving the aerodynamics data, it is possible to generate, first of all, a 3D airfoil and then the appropriate inner and outer elements are automatically attached to the airfoil with customizable size and shape.
- FEM model generation:
  once the CAD model has been generated, it is used as an input for the finite element analysis which will be conducted later. So in this part, the meshing procedure is executed then the loading and boundary conditions are appropriately applied.
- static analysis:
  since the finite element model has already been generated, it is possible to carry out a static analysis. Through this analysis it is then possible to check the stresses as well as its behavior under these loads, in addition this is important to verify that no interference occur with neighboring blades under deformation.
- modal analysis:
  afterwards, a modal analysis can be carried out starting from a pre-stress condition and it is possible to obtain the modeshapes and natural frequencies related to this model.
- flutter analysis:
  after the modal analysis, it is possible to execute the flutter analysis to check its stability behavior, in particular the stability of the flutter phenomena. Of particular importance is the aerodynamic damping parameter which, at some moment, may go below zero and this indicates an instability.
- forced response:
  finally by combining the aerodynamic and free response, it is possible to evaluate the work done by the aerodynamic exciting force which will be used to evaluate the modal force.

This tool, as of now, is mainly based on functional requirements, as it is common for the preliminary design, and it lacks any feedback for the manufacturability aspects. That is why, the previous DFM code could be thought of being added in this automated procedure to be executed as a parallel task right after the CAD generation and provide a final feedback on the manufacturability aspects, which is in some way related to costs, and perform changes based on all the reports generated.

# Tolerance stackup analysis

Tolerance stackup analysis is a part of a broader analysis, that is tolerance analysis, and it is used to evaluate the effect of tolerance accumulation in an assembly and this is needed due to the fact that the distance between two features is generally not dimensioned directly and a stackup analysis can provide the right answer for this kind of problem.
In other words, through a stackup analysis it is possible to obtain information regarding the variation of a certain gap, usually in terms of a maximum and minimum value, and this information can be used to determine whether changes in tolerance for some parts should be applied or not, however changing the tolerances for a part means that the part must be redesigned which is time-consuming and expensive but unavoidable in case interference is highly likely to happen between two or more parts.
Overall, the interference check is not the only reason behind a stackup analysis, other reasons include:

- optimization of tolerances for a new design:
- determination of the required tolerances to satisfy all the requirements;
- determination of whether a specified tolerance yield an acceptable amount of variation;
- determination of the allowable tolerances for a part if the assembly tolerance is already determined;
- determination of possible malfunctioning;
- determination of the effect of a tolerance on the overall assembly tolerance;
- possibility to test design alternatives.

all of them can contribute in achieving an optimal design and reduce number of defective products, and therefore the overall cost can be optimized.

## 7.1 Description of variation

It is obvious that products having all dimensions coincident with nominal values are impossible to achieve and this is why variation happens and stackup analysis are needed. However, the exact reason why variation happens for a

specific part or product is not easy to identify as many factors exist which contribute to variation and they are mainly related to tolerances present in the drawing, inspection process and assembly process.
Some of the main factors are:

- process capability: this factor refers to the limitation of the manufacturing process as they have limited accuracy and precision, and the tighter the tolerances the harder it becomes to achieve it, in order words it becomes more expensive as more defective products will be manufactured and all of these effects are described through the process capability,
- tool wear: all the tools used in the manufacturing processes inevitably wear with continuous usage, mainly due to friction during contact with the work piece, and this causes the tool to become smaller then the manufactured part will also be affected.
- error and bias: during the manufacturing process improper operations and human factors may occur to contribute to higher variation. This effect can be reduced with appropriate training and organization of personnel.
- material: the raw material itself may already present variations and this will inevitably produce variation in the final product.
- ambient conditions: all the ambient factors also affect the variation of the final product and this is can be clearly seen in the cooling temperature which depends on the temperature of the environment.
- difference in process and equipment: if different machines and processes are used then this will also contribute in variation.
- inspection process variation: ironically, one of the main sources of variation may actually reside in the inspection process as it may be carried out through different methods and tool which inevitably produce different reports.
- assembly process variation: the assembly process is also a contributor to variation as this may be carried out with different tools and methods.

## 7.2 Traditional methods for stackup analysis

The stackup analysis due to its criticality must always be analyzed, however an unique valid method to adopt to carry out this analysis as an unique solution does not exist due to its unpredictability nature.
As shown in the work by Fischer[10], traditionally two main methods have been developed in the past and used widely to deal with a stackup analysis of an assembly:

- Worst case tolerance analysis: through this method it is assumed that the tolerances will happen at their extreme values, and by adding each contribution it is possible to determine the minimum and maximum tolerances allowed and thus the minimum and maximum dimensions possible of an assembly as well. However, it is clear that this method is too conservative and it is appropriate only for very critical measurement.

- Statistical tolerance analysis: through this method, a statistical approach is taken instead which is based on the assumption that all the tolerances have a normal distribution and that they are all independent between them. Since it is a statistical approach, the output values are to be interpreted as the minimum and maximum most likely to occur for a certain measurement. This method is mainly limited by the fact that it cannot take into account the relationships of the various tolerances as they are assumed to be independent and this will inevitably affect the accuracy of the final result.

Both of these two methods are carried out manually or through the use of spreadsheet and they are mostly limited to one-dimensional, or two-dimensional at most, as a three-dimensional analysis would be too complicated for a manual analysis. Therefore linearization will also be needed to simplify the assembly function which also needs to be explicit and manually identified before the stackup analysis is carried out.
In order to have a clearer understanding of all these methods a test case is presented, represented in figure 7.1, consisting of a dummy assembly composed of by casing, nozzle and disc where the tolerances which contributes in the measurement have also been represented. In this case the measurement of interest is the gap between the nozzle and disc and no assembly shifts will be considered for simplicity.

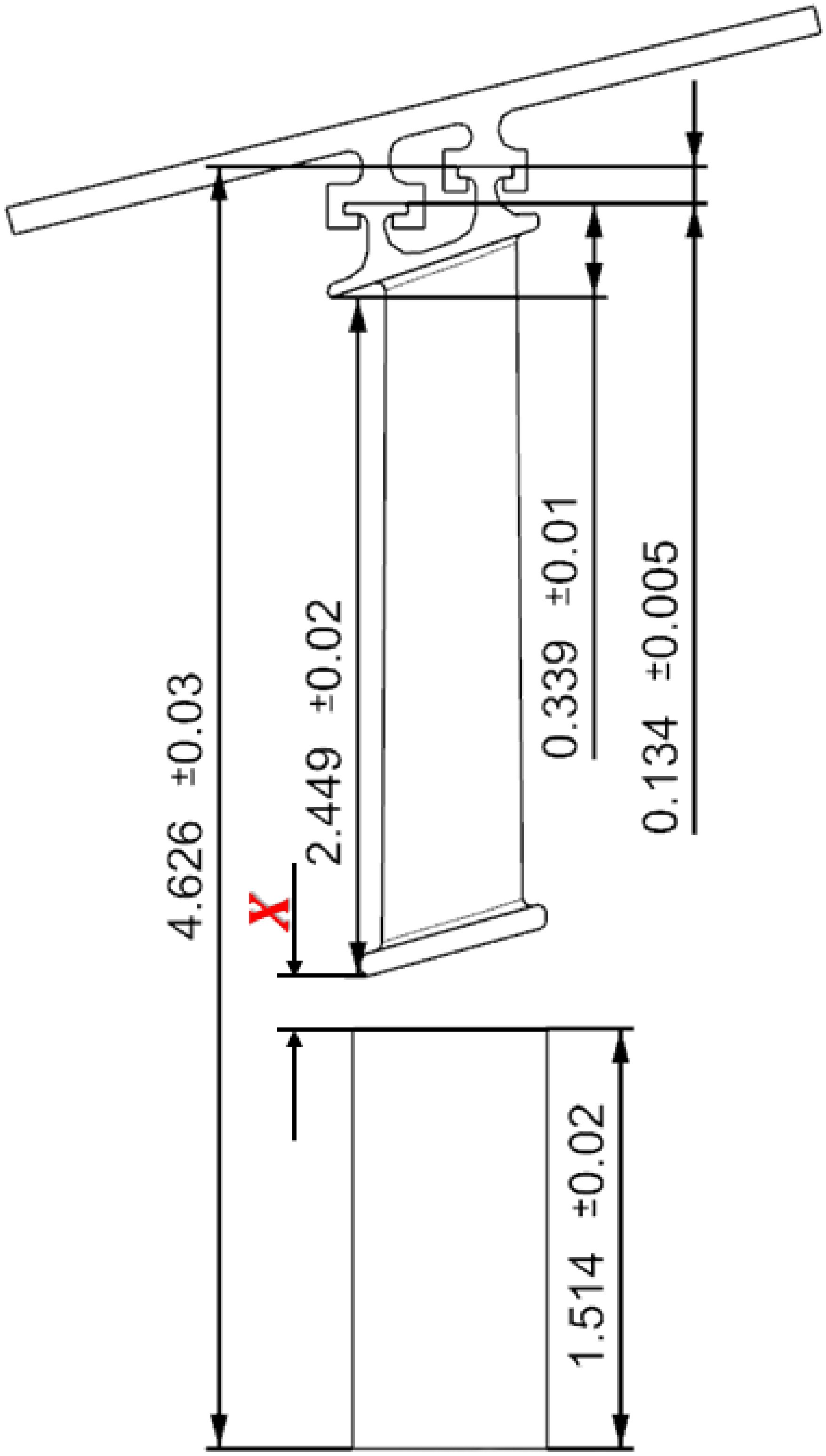


Figura 7.1: Example for tolerance stackup analysis.

### 7.2.1 Worst case tolerance analysis

The worst case method represents the most traditional way of carrying out a stackup analysis as it is the most straightforward, in fact, as mentioned earlier, through this method each individual tolerance is assumed to be at their extreme values, and hence the name worst case, then the stackup measurement is simply given by the sum of these tolerances. The calculated measurements are then the lower extreme value, if the lower limit of tolerances are taken, and the upper extreme value, if the upper limit of tolerances are taken, and the real measurement will never exceed these limits. It is clear then that this method is useful whenever critical stackup measurement are present and its limits should always be guaranteed.
Nonetheless, the assumption of extreme values for tolerances may be too conservative, moreover if a certain measurement has only a strict margin available then the design tolerances for each part is affected by very tight tolerances, and these tight tolerances translate into expensive manufacturing processes and inspection processes as finer tools will be required but high scrap rates will happen as well. In addition, this assumption is clearly not realistic and only for highly rare cases this situation, where each part is affected either by maximum or minimum tolerances, happen in reality and therefore the worst case scenario does not represent a very accurate method.
So this method should be used only when the criticality of a certain measurement strictly requires it, in all other cases a different approach should be instead adopted.
Going back to the test case previously described, it is possible to apply the worst case method, as shown in the work by Fischer[10], and obtain the extreme limits.

| | + | − | Tolerances |
|---|---|---|---|
| | 4.626 | | ±0.02 |
| | | 1.514 | ±0.02 |
| | | 0.134 | ±0.005 |
| | | 0.339 | ±0.01 |
| | | 2.449 | ±0.02 |
| Total | 4.626 | 4.436 | ±0.075 |

$$x_{nom} = 4.626 - 4.436 = 0.19$$

$$x_{max} = 0.19 + 0.075 = 0.265$$

$$x_{min} = 0.19 - 0.075 = 0.115$$

### 7.2.2 Statistical tolerance analysis

The worst case tolerance analysis, due to being too conservative, is used only for critical measurements, otherwise it is more common to adopt a statistical approach to evaluate a stackup measurement whose result will be more realistic compared to worst case. One of the statistical approach is the Root Sum Squared method, which is based on the assumption that each dimension will mostly fall in the vicinity of the center of its tolerance range and very few parts will fall far from the center of the tolerance range. Therefore the Root Sum Squared method assumes a normal distribution for each tolerance which needs two parameters to be defined:

- mean value;
- standard deviation.

and by adding the root sum squared of the standard deviations to the means, it is possible to obtain an estimate of the tolerance stack. The overall standard deviation, in other words the standard deviation of the assembly, can be calculated as

$$\sigma_{ass} = \sqrt{\sum_{i=1}^{n} \sigma_i^2}$$

where $\sigma_i$ is the standard deviation of the $i$th component, $n$ is the total number of components of the assembly and $\sigma_{ass}$ is the standard deviation of the final assembly.
The normal distribution is characterized by the fact that around 68.2% of values occur within one standard deviation of the mean, analogously around 95.4% will occur within two standard deviation and around 99.7% within three standard deviation.
To get the values of the mean and standard deviation of each tolerance, a gathering of measurements must be carried out before performing a stackup analysis, however the calculated standard deviation is valid only if a certain amount of measurement is achieved. If the number of measurement is lacking then the sample standard deviation formula should be used instead

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (\bar{x} - x_i)^2}{N - 1}}$$

If information about measurements cannot be obtained, as in the case of preliminary design phase where no parts have been produced yet and we are interested only in a rough assessment of the assembly's variation, then it is possible to assume all dimensions centered in the tolerance range and three standard deviations to describe the tolerance range. Obviously, this makes sense only if the manufacturing plant is actually able to produce parts with tolerance equal to three standard deviations, in other words process capability equal to 1.33.

Applying this method, as shown in the work by Fischer[10], to our test case, it is clear that the range is lower than the limits provided with the worst case method which is too conservative.

| | + | − | Tolerances | Squared tolerances |
|---|---|---|---|---|
| | 4.626 | | $\pm 0.02$ | $\pm 0,00004444$ |
| | | 1.514 | $\pm 0.02$ | $\pm 0,00004444$ |
| | | 0.134 | $\pm 0.005$ | $\pm 0,00000278$ |
| | | 0.339 | $\pm 0.01$ | $\pm 0,00001111$ |
| | | 2.449 | $\pm 0.02$ | $\pm 0,00004444$ |
| Total | 4.626 | 4.436 | | $\pm 0,0364$ |

$$x_{nom} = 4.626 - 4.436 = 0.19$$

$$x_{max} = 0.19 + \sqrt{0,0364} = 0.2264$$

$$x_{min} = 0.19 - \sqrt{0,0364} = 0.1536$$

## 7.3 Computer Aided Tolerancing

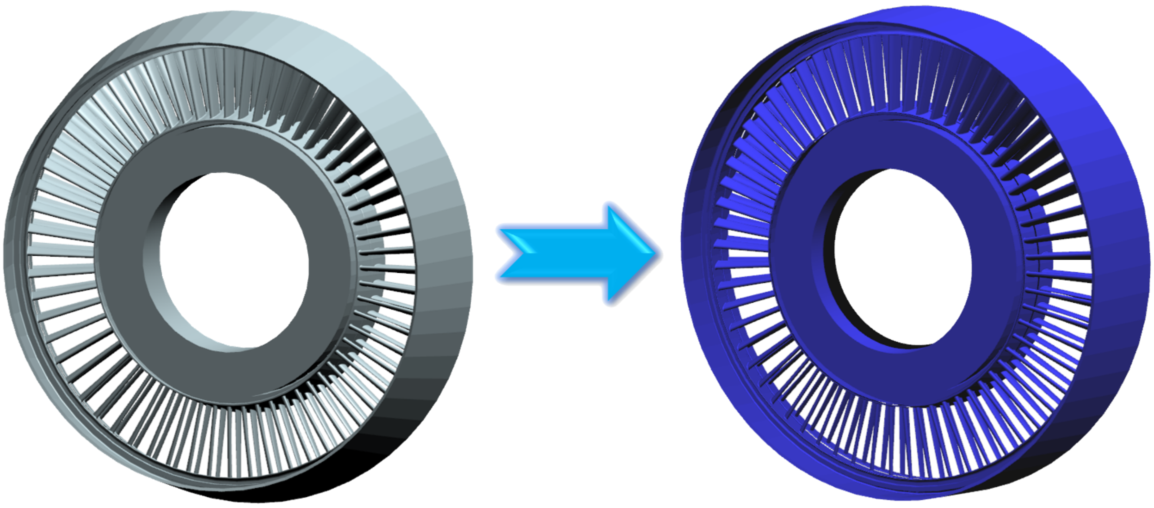

Figura 7.2: Computer Aided Tolerancing.

All the previously described methods are pretty straightforward to implement and easy to use, however they are characterized by assumptions that are not truly realistic and the real tolerances of the assembly may still differ from calculated values. In addition, these methods are also very time consuming as they require the manual identification of the assembly function and of each significant tolerances for each stackup analysis. That is why new and innovative solutions are being developed over the years in order to substitute the old traditional approach to a stackup analysis by overcoming the shortcomings typical of the traditional methods.
One of these possible solutions is to carry out a simulation based variational analysis through a computer aided tolerancing software, and since this is an automated process it is more time-efficient while at the same time the results obtained tend to be closer to reality, in other words it is possible to optimize the tolerances assigned and reduce the number of defective products. Therefore the overall benefit is a cost reduction for the entire design, manufacturing and assembly processes.

### 7.3.1 Monte Carlo based simulation

As previously mentioned, the variational analysis implemented on a computer aided tolerancing software, is simulation based, in particular the sampling is obtained by running a certain number of Monte Carlo simulations, which surely is not the only solution but it is the most popular all the while.
The Monte Carlo simulation is a very popular and powerful tool used in different application, also outside of the field of engineering, and this is due to the fact that it can be used to deal with complex problems characterized by high

number of random variable, distributions and nonlinear models.
The Monte Carlo simulation can be easily summarized in three fundamental steps:

- random generation of the input variable according to their own distributions
$$\boldsymbol{X} = \{X_1, X_2, X_3, ..., X_N\}$$
- computation of output from the input through the assembly function
$$\boldsymbol{Y} = F(\boldsymbol{X})$$
- extraction of statistical information from the output variable

this method works both for dependent and independent variables.
In the first step, samples are generated randomly according to their distributions and this is achieved by initially generating a set of random variables uniformly distributed between a range of 0 and 1

$$\boldsymbol{Z} = \{Z_1, Z_2, Z_3, ..., Z_N\}$$

through pseudo random number generators present on computers.
Once these numbers are obtained, these uniform variables are then transformed into the random variables that comply with a specified distribution $D_{X_i}(X_i)$. There are several ways to obtain these random variables, one of them is the inverse transformation method

$$X_i = D_{X_i}^{-1}(Z_i)$$

in which $D_{X_i}^{-1}$ is the inverse of the cumulative distribution function of the random variable $X_i$. These two steps is done for a certain number of times, called number of simulations $N_s$, and therefore a set of input variable is generated.
In the next step, this set of input variable is given to the assembly function

$$\boldsymbol{Y_i} = F(\boldsymbol{X_i})$$

and this is solved $N_s$ times to find the output variable for each simulation.
Finally, the last step is to carry out a statistical analysis from the output obtained in the previous step and thus the mean and variance are calculated as follows

$$\bar{Y} = \frac{1}{N_s}\sum_{i=1}^{N_s} Y_i$$

$$\sigma_Y^2 = \frac{1}{N_s - 1}\sum_{i=1}^{N_s}\left(Y_i - \bar{Y}\right)^2$$

afterwards, the probability of failure can be evaluated as

$$p_f = \frac{N_f}{N_s} = \frac{1}{N_s}\sum_{i=1}^{N_s} I(x_i) = \bar{I}(x)$$

where $N_f$ is the number of samples that do not satisfy the limit condition and $I$ is the indicator function which can be either 1 or 0 depending if the value satisfies the limit or not

$$I(x) = \begin{cases} 1 & \text{if } g(x) \leq L, g(x) \geq U \\ 0 & \text{otherwise} \end{cases}$$

then the reliability is simply

$$R = 1 - p_f = \frac{N_s - N_f}{N_s}$$

the cumulative distribution function is determined instead as

$$cdf(y) = \frac{1}{N_s} \sum_{i=1}^{N_s} I(y_i)$$

where the indicator now has is given by

$$I(x) = \begin{cases} 1 & \text{if } g(x) \leq y \\ 0 & \text{otherwise} \end{cases}$$

and the probability distribution function *pdf* can be easily derived from the numerical differentiation of *cdf*.
An important consideration not to be neglected consist in how much error is being committed through this method and it is clear that it depends mainly on the number of simulations $N_s$, in theory the solution will converge to the true probability if the number of simulations approaches infinity. Since infinite simulation is clearly not possible, then the question becomes how many simulations are needed to reach the specified accuracy.
Assuming a 95% confidence level, the error percentage can be given by

$$\text{error} = 200\sqrt{\frac{(1 - p_f^T)}{N_s p_f^T}}$$

where $p_f^T$ is the true probability of failure, which is unknown, and must be first estimated in some way.
To summarize, the Monte Carlo simulation present the following features:

- easy to use without the need for deep knowledge of statistical analysis;
- feasible for any distribution and assembly function;
- robust and it will always converge for most of the cases;
- suitable to deal with problems characterized very high number of variables;
- high computational cost.

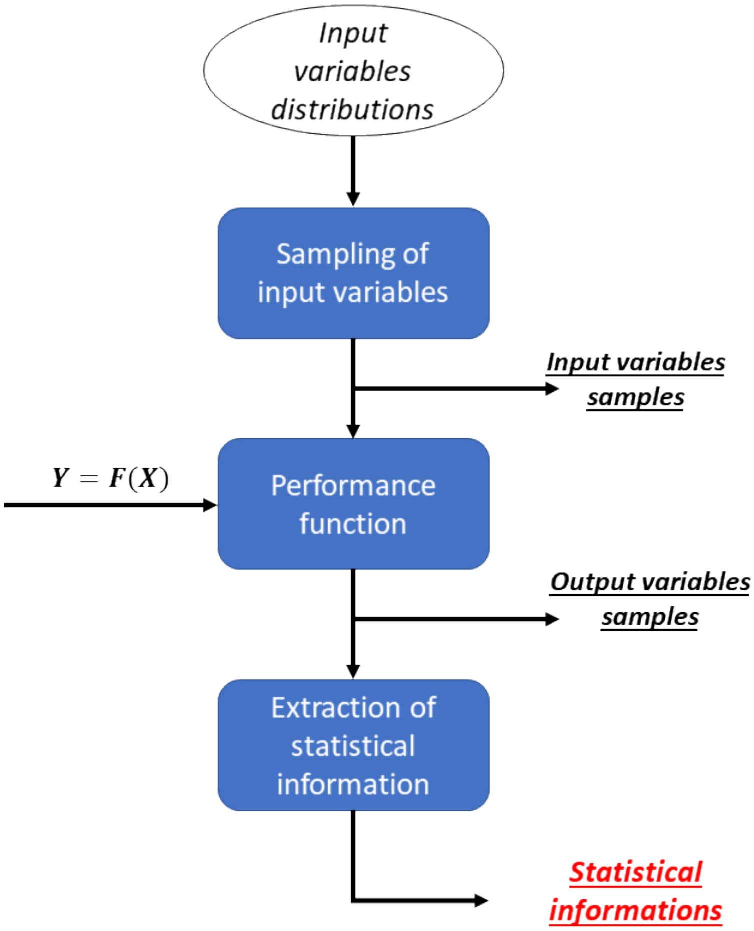


Figura 7.3: Monte Carlo simulation chart.

### 7.3.2 3D Variational analysis

The variational analysis represents a possible alternative solution to deal with a stackup analysis characterized by a whole new and different approach as this method is based on a computer aided tolerancing software which implements parametric geometric modeling where the geometry itself is modeled through mathematical equations as a function of a certain number of parameters and its dimensions can be represented as a vector so the assigned tolerances can then be considered as a small variation of these vectors.
Since this is done on a computer, then it is also possible to use the Monte Carlo simulation to exploit its benefits for a certain number of simulations, where each of them consist of a case with randomly generated set of tolerances, following still a specified distribution, and as well as a certain value of output. After all the simulations have been carried out, it is then possible to build a distribution of the assembly variations which is our main output of interest.
Several tools are available to perform this kind of analysis, and in this thesis we

shall focus primarily on Teamcenter Visualization MockUp(Siemens Software Inc.) which is able to simulate the manufacturing and assembly processes and then provide an estimate of the amount and causes of assembly variation as well as the main contributors to this variation.
Its first benefit is that it does not need any dimensional approximations, either in 1D or 2D, as it is fully capable of analyzing and performing a 3D analysis, in fact it receives, as input, the CAD file of the whole assembly.
To carry out a stackup analysis through this tool, the main steps to take are the followings:

- import of CAD assembly;
- modeling;
- Product and Manufacturing Information definition;
- measurement definition;
- VSA settings.

After the CAD has been correctly imported, the modeling procedure begins in which the assembly geometry is remodeled inside the software as it does not read directly the CAD assembly file imported but considers, for the stackup analysis, only the features reconstructed inside. And these features are used not only to describe the shape of the parts of the assembly but also to be used for the assignment of tolerances. What must be noted is that the assignment of tolerances is dependent on the features on which they are assigned and so this step represent a critical part for the stackup analysis. Together with the assignment of tolerances, the distribution related to each tolerance must also be specified and the possible choices are:

- normal distribution: this distribution, also known as bell curve due to its shape, is a symmetrical curve where half of the values falls to the left and right from the mean. Additional characteristics of this distribution are that the mean, mode and median are all equal and that the total area below the curve is unitary.
- uniform distribution: this distribution, also known as rectangular distribution, is characterized by constant probability, in other words all the values inside a specified range have equal probability of occurring. To describe this distribution, two parameters are enough which represent the minimum and maximum. Additionally, the area below the curve is also unitary.
- extreme value distribution: this distribution models how large or small the values will probably get and thus it is useful to check the limits of a set of data. This distribution is then characterized by the fact that the minimum and maximum tolerance values have an equal probability of occurring.
- Pearson distribution: this distribution represent a family of distributions

satisfying the following equation

$$f'(x) = (x - d)\frac{f(x)}{(ax^2 + bx + c)}$$

it can be used to describe a wide scale of distributions characterized by different skewness and kurtosis. By choosing an appropriate pair of these two parameters, it is then possible to obtain the distribution that describe the original set of data.

- trapezoidal distribution: this distribution takes the form of a trapezoid which is suitable for data that present a rapid linear growth and rapid linear decay. This distribution is expressed as a function of two main parameters that are shift and width.

Once all the geometry of interest has been remodeled and the tolerances has been correctly assigned, it is now possible to define all the measurements, as is or calculated, we are interested in analyzing.
Finally, the number of Monte Carlo simulations must be appropriately chosen in order to have a good accuracy of the results

Considering once again our test case, it is interesting to check the results that can be achieved through this method. After carrying out all the steps as mentioned above, in figure 7.4 the modeled structure is represented with all the tolerances already defined. It is then possible to run the simulation by choosing a high enough number of simulations, in this case 1000, and an example of the output is shown in figure 7.5 where it is easy to see that the output is in the form of a distribution and the following parameters are computed as follows:

- process capability index

$$C_p = \frac{0.5(U - L)}{3\sigma}$$

- bias Factor

$$k = \frac{\mu - 0.5(U + L)}{0.5(U - L)}$$

- performance Index

$$C_{pk} = C_p(1 - k)$$

In order to have a direct comparison, it is possible to impose the limits given by the worst case tolerance analysis and statistical tolerance analysis on the distribution.
Compared to worst case tolerance analysis, represented in figure 7.6, it can be verified that the entire distribution is inside the range given by the limits of worst case, and this is to be expected as worst case provide only the extreme limits. In addition, it can be clearly seen that a significant margin is present especially toward the lower limit due to the conservativeness of the worst case method.

Compared to statistical tolerance analysis, represented in figure 7.7, the overall variation is larger than the limits provided by the statistical approach, roughly $\sim 5\%$ above the upper limit, and this is caused by the limits mentioned previously of the RSS method, such as the assumption that all the tolerances are independent between themselves.
Finally, it is also possible to investigate the contributions given by each of the tolerances, represented in figure 7.8, which is an important feedback in case changes in tolerances are to be made and it is possible then to find an optimal set of tolerances for the entire assembly, minimizing the overall cost.

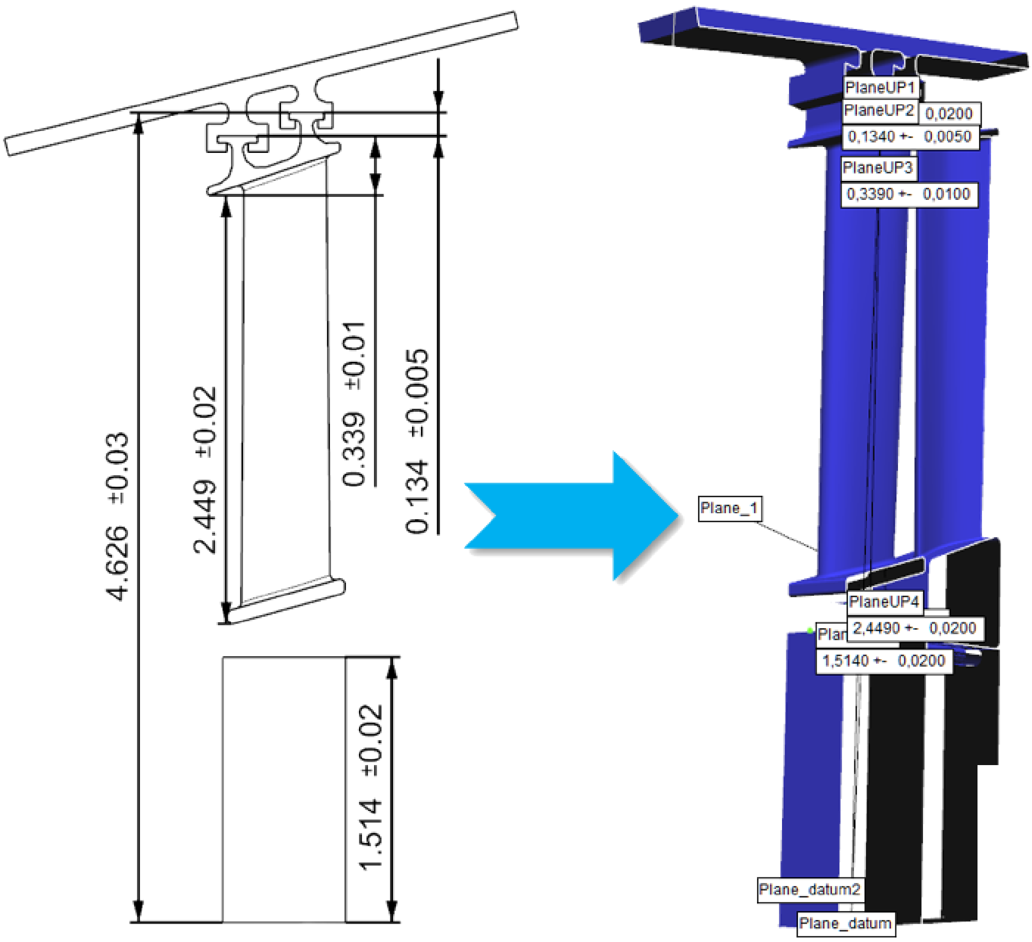


Figura 7.4: Modeling and PMI definition of the test case.

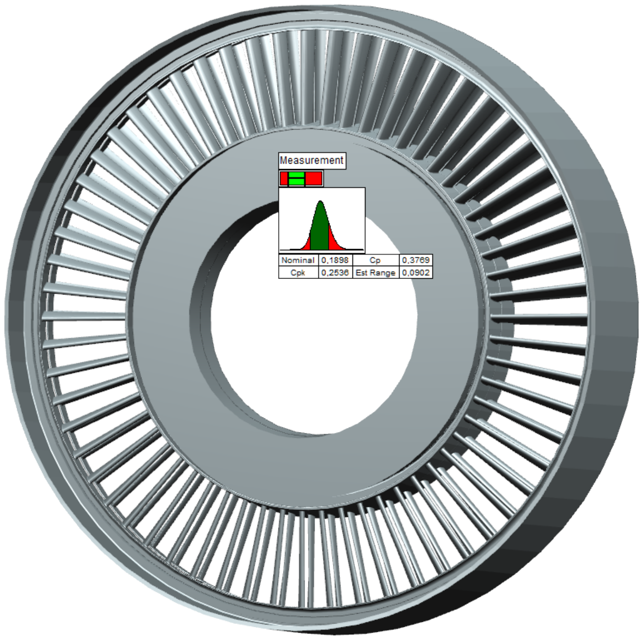


Figura 7.5: Test case output.

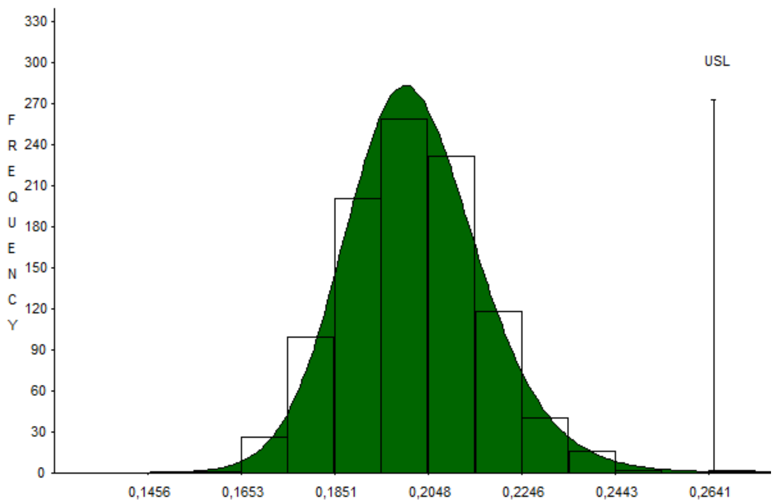


Figura 7.6: Results comparison with worst case tolerance analysis.

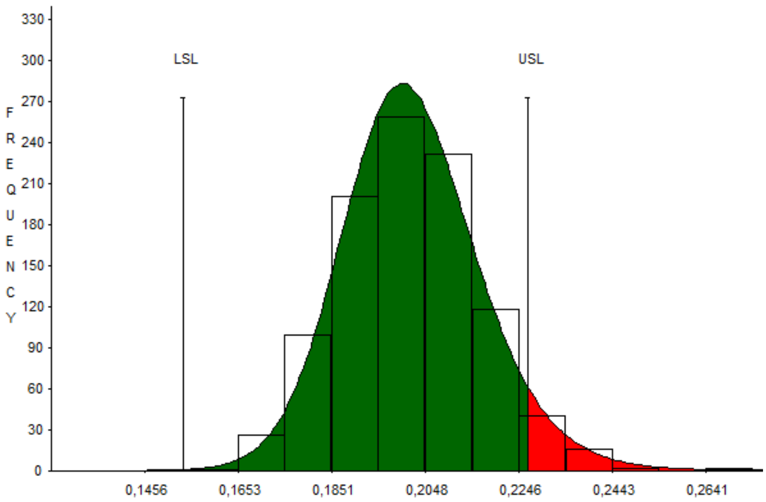


Figura 7.7: Results comparison with statistical tolerance analysis.

| Contributors | Effective Tole... | Sensitivity | Effect |
|---|---|---|---|
| 1. Dummy_casing_c - PlaneUP1 -> 4,6258 +- 0,020 | 0,0471 | 1,1779 | 37,04% |
| 2. Dummy_casing_c - PlaneUP4 -> 2,4490 +- 0,020 | 0,0400 | 1,0000 | 26,69% |
| 3. Dummy_casing_c - Plane_disk -> 1,5140 +- 0,020 | 0,0400 | 1,0000 | 26,69% |
| 4. Dummy_casing_c - PlaneUP3 -> 0,3390 +- 0,010 | 0,0217 | 1,0866 | 7,88% |
| 5. Dummy_casing_c - PlaneUP2 -> 0,1340 +- 0,005 | 0,0101 | 1,0091 | 1,70% |

Figura 7.8: Tolerance contributions.

# Conclusion

By and large, the aerospace industry has always been a unique industry characterized by the presence of many challenges set for the future, and this is particularly true for the aeroengine where there is an ever present pursue for higher and higher performance for it to remain competitive as well as more and more demanding requirements to satisfy.
This is the main framework for the activities carried out in this thesis dissertation, although it only focuses on the low pressure turbine stage, and so after a brief description of various aspects concerning this component, a research is carried out for new solutions to improve and substitute the traditional design process in order to overcome its limitations and constraints and this is possible by completely changing the traditional mindset and approach to the design of a new product by starting to take into account more parameters that are related directly to the function requirements, such as manufacturability and cost. In order to achieve this, the first solution proposed here is the implementation of design for manufacturing inside the design process which could greatly improve both in terms of cost and time to delivery, however a complete implementation is by no means an easy task as it is a long procedure and it would also require the involvement of different teams which is beyond the scope of this thesis. Therefore, a possible first step to design for manufacturing is instead described which consists of the development of a basic code to be used as a tool to verify the requirements of manufacturability and so it could potentially act as an interface between the manufacturing team and design team, in other words it could act as a substitute to concurrence.
The second solution proposed here to improve the design of a product is the description of a new and innovative analysis, that is a simulation based 3D variational analysis, which could be used to substitute the old and manual stackup analysis in order to achieve a faster analysis, as it can avoid the manual identification of contributing tolerances typical of traditional stackups since they can be automatically detected, as well as more accurate results as less assumptions are needed, in fact a traditional stackup is usually carried out in 1D or 2D whereas here a 3D analysis is possible. In addition, the higher accuracy and the possibility to identify the key contributors to each stackup measurement enables the possibility to find, through iteration, an optimal set of tolerances for each part and thus reducing the overall cost.

Obviously, deeper investigation should still be carried out and the solutions presented here can still be greatly improved. In fact, the design for manufacturing code could be improved by implementing more checks that are specific to each manufacturing process, and so different codes could be developed for this purpose based on the feedback from the manufacturing team. Analogously, a further investigation can be carried out on the 3D variational analysis to estimate the possible benefit, cost-wise, of using this tool iteratively to find an optimal set of tolerances.
In summary, the main goal of this thesis dissertation is to provide a new and innovative approach to design that can overall improve the traditional design process and overcome its main limitations and thus achieving a smarter and more optimized design process that can be time-efficient and cost optimized at the same time.

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