POLITECNICO DI TORINO

Department of Mechanical and Aerospace Engineering

Master Degree in Aerospace Engineering

Master Degree Thesis

# Study of the effects of tooth geometry on characteristics of contact

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# Abstract

In the aeronautic field even the smallest aspect can results determining in term of performance and competitiveness. There is a continuous request for lighter product to minimize weights and costs.
In this context vibrations have a very important role. They are source of noise and could bring at unexpected resonances, therefore represents a critical aspect to analyze paying extreme attention.
Even if this kind of investigations would be referred to a system, it it not possible yet to neglect the sub-systems analysis. For this reason, this thesis is focused on static and dynamic aspects of gears, in particular way on spur gears.
One of the main causes of vibrations in these elements is that gears are not infinitely rigid, but they have teeth whose deformations varies in time. The reason of this variation is that the number of theet varies during a mesh cycle. This variable stiffness is called Mesh Stiffness and it is the cause of Transmission Error, a discrepancy between the pinion motion and the wheel motion.
The aim of this thesis is to understand how geometric characteristics of the gears could affect on the variation of transmission error and, then, on the mesh stiffness. To realize that, after a brief description of of the main kind of gears and of their relative dynamic, the following step will be a detailed description of the two main modeling software used. Then will be analyzed from a static approach a model a gears pair varying the profile modifications and the torque applied, calculating the mesh stiffness with two traditional method and one slightly different to check the precious two.
Once again form the static approach, form tooth geometry it will pass on the gear geometry, calculating the mesh stiffness studying what happens analyzing rigid and flexible gears varying the rim thickness.
In the end, it will be presented a modeling solution in Transmission 3D, an Ansol software, which allows a fats and reliable dynamic analysis.
In particular, thanks to this solution the time analysis has been reduced by 80%.

# Sommario

Nel campo aeronautico anche il piú piccolo degli aspetti puó risultare determinante in termini di prestazioni e competitivitá. Alle aziende vengono richiesti prodotti piú leggeri per ridurre al minimo costi e pesi.
In questo contesto le vibrazioni hanno un ruolo determinante. Esse sono fonte di rumore e possono portare a risonanze inaspettate, pertanto rappresentano un aspetto cruciale da analizzare con estrema attenzione.
Anche se questo tipo di investigazioni andrebbero riferite a livello di sistema, non é ancora possibile trascurare l'analisi dei sotto componenti. Per questo motivo, questa tesi é incentrata su aspetti statici e dinamici di ingranaggi, in particolare a denti dritti.
Una delle cause principali di vibrazioni in tali elementi é che le ruote non sono infinitamente rigide, ma hanno denti che si deformano in modo diverso nel tempo per via del fatto che il numero di coppie in presa varia. Questa rigidezza variabile é detta Rigidezza di mesh e causa un errore di trasmissione, ovvero una discrepanza tra il moto della ruota trainante (pignone) e il moto della ruota condotta.
Lo scopo di questa tesi é capire come le caratteristiche gerometriche della ruota dentata influiscono sulla variazione dell'errore di transmissione e, quindi, della rigidezza di mesh.
Per fare ció, dopo una breve descrizione delle principali tipologie di ingranaggi e della relativa dinamica si passerá a descrivere nel dettaglio i software di modellazione che sono stati utilizzati.
In seguito verrá analizzato staticamente un modello di accoppiamento al variare delle modifiche di profilo, della coppia applicata da un punto di vista statico, calcolando la rigidezza di mesh attraverso due approcci tradizionali e uno leggermente diverso a verifica dei primi due.
Ancora dal punto di vista statico, dalla geometria del dente ci si sposterá sulla geometria della ruota, esaminando ció che accade analizzando ruote flessibili e infinitamente rigide al variare dello spessore del rim.
Infine, verrá presentata una soluzione di modellazione con Transmission 3D, un software della Ansol, che permette una rapida e affidabile analisi dinamica.
In particolare, grazie a quest'ultima é stato possibile ridurre il tempo di analisi circa dell'80%.

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# Chapter 1

# Introduction

In the field of aerospace engineering structural analysis has a primary role. His target is to realize techniques that permit to validate components and systems, ensuring that failures doesn't occur.
In this sense, analyses are carried out with two approach, **static** and **dynamic**.
In static it is assumed that the system does not move, analyzing every instant of time like a series of steady state. For example, in a study like that does not make sense to set a speed because its effects it is not considered in the calculation. Obviously in the reality this is not true, but for some structures results can be reliable and, because of its simpler formulation, a static analysis is preferable.
On the other hand, some structures can incur to failure even if their static requirements have been fulfilled. In these cases a dynamic analysis is necessary. The aim of this thesis is to understand how the gear geometry could affect the static of a system and how it is possible to carry out a reliable dynamic analysis in reasonable time.

Before that, will be presented a brief introduction on the gears world and its dynamic.

## 1.1 Gears

Before to well understand the problems related to use of gears and to the hertzian contact between teeth, it is useful to give an overview of the gears world. A gear is a toothed wheel that often present itself round. The purpose of gears is to transmit motion or power between two shaft. Obviously this transmission is not uniform, but can show changes in speed, direction and shaft-torque. Parallel axis gear transmit power with greater efficiency than any other type or form of gearing.

### 1.1.1 Spur gears

The spur gears are the most common and most used toothed wheel. Teeth are on the outside of the cylinder and they are parallel to its axis. Their purpose is to exchange power between parallel shaft. The tooth shape is that of an involute form, in order to guarantee constant transmission ratio and to avoid impulsive behaviours, that can compromise their correct operation. There are some exceptions like cycloidal form, used for situations that require more precision. The advantages of these gears are:

- High stiffness shapes, making them suitable for high power transmission
- Straight teeth implies minimum axial forces, permitting an easier design of shaft and bearings.

The negative side is that the contact is not very smooth (usually due to low contact ratio), so the meshing generates vibrations.

Figure 1.1: Spur Gear

### 1.1.2 Helical gears

When tooth shape in the axial direction is no longer parallel to the axis, gears are designed as **helical**. In this case there is a bigger surface in contact during the mesh process. Therefore the contact is smoother and more gradual, allowing big power transmission with relatively small noise phenomena. In addiction, the load transmitted can be larger, or the life of gears can be longer with the same loading, than with an equivalent pair of spur gears. On the other hand, more surface means more frictions, so for this kind of gears will be necessary the use of particular lubrificants. Generally, the meshing is carried out with two gears with opposite helix angle, in order to balance the axial forces that are introduced by their particular shape.

Figure 1.2: Helical Gear

### 1.1.3 Bevel gears

Differently than the previous two cases, bevel gears are not generated from cylinders, but their primitive geometry is a cone. They are used to connect two non-parallel axis. These gears can have both straight and helical teeth, according to load conditions. In aeronautical gearboxes, the great majority of bevel gears have helical teeth, given the strict operating conditions which these transmission are subject to. In fact, as described above, helical teeth guarantee smoother contact and better distribution of forces. [1]

Figure 1.3: Bevel Gear

### 1.1.4 Profile modifications

There are many factors that prevent the attainment of true involute contact in gear meshes, as errors of manufacture or deflections of mountings under load. As a result, theory does not find confirmations in applications. Excessive contact pressure at the ends of the teeth or premautere contact at the tip could give rise to noise and gear failure.

**Profile modifications** is a usual practise to reduce thins unwanted effects.

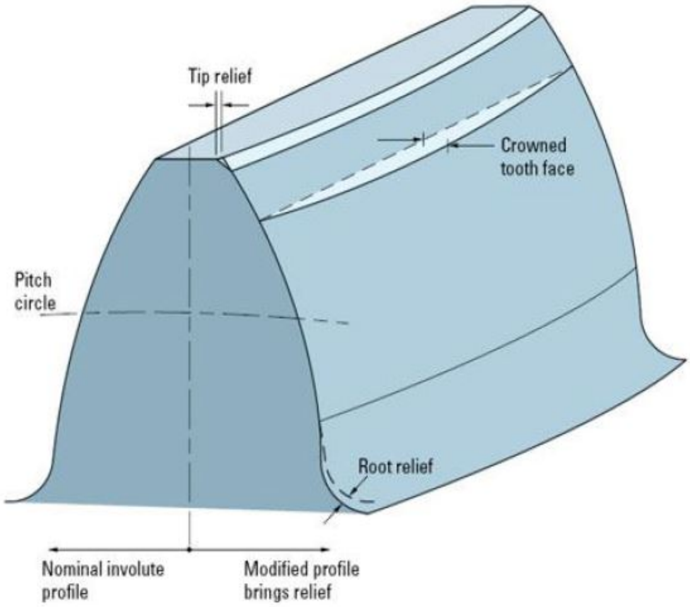


Figure 1.4: Tip relief and crowning modifications

Even tough there us a wide range of possible solutions, the main modifications carried out in the aeronautical fields are:

- **Tip/root relief**: this modification consists in removing material near the tip/root of the tooth. The idea is to ensure a smoother contact between the two teeth.
- **Crowning**: this modification consists in reshaping the face width curvature in order to enforce the contact pressure to be limited in a known area.

In the design process these modifications play a very important role, and a lot of time is spent analyzing their effect. This is usually done with static analyses, since a characterization in dynamics is more demanding and of difficult interpretation.
A large part of this thesis will focus on the effects that modifications have on the aspects analyzed.

## 1.2 Basics of dynamics

In this chapter, it will be briefly described the basic and general concepts of dynamics. A general configurations in space of a mechanical system is described using independent parameters (i.e. translation and rotation coordinates). The number of this parameters defines the total amount of degrees of freedom needed to describe accurately the dynamic behaviour of a mechanical system. Because of real systems have an infinite number of degrees of freedom, approximate solutions have been developed to permit simulations. In particular, finite element modeling techniques are the most widely used, reducing the system to a finite set of elements and nodes.
To each node is assigned a certain number of dof and an ideal mass, whereas the interaction between nodes is modeled using springs and dumpers as it can be seen in the figure

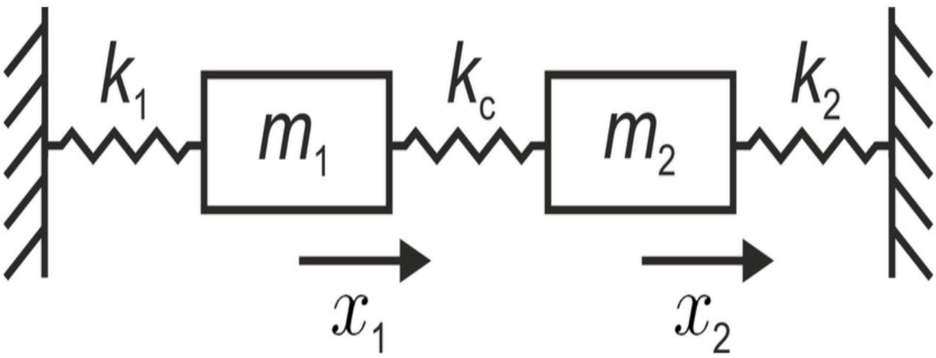


Figure 1.5: Two d.o.f. system

The equilibrium equqtions of motion for the two masses are

$$\begin{cases} m_1\ddot{x}_1 + k_1x_1 - k_c(x_2 - x_1) = F_1 \\ m_2\ddot{x}_2 + k_2x_2 - k_c(x_2 - x_1) = F_2 \end{cases} \tag{1.1}$$

from wich is obtained the sistem of motion in matrix form:

$$M\ddot{x} + Kx = F \tag{1.2}$$

where

$$\begin{cases} M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \\ \\ K = \begin{bmatrix} k_1 + k_c & -k_c \\ -k_c & k_2 + k_c \end{bmatrix} \\ \\ F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \end{cases}$$

If interactions account also for damping it becomes

$$M\ddot{x} + B\dot{x} + Kx = F \tag{1.3}$$

where B is the damping matrix which has the same structure of the stiffness matrix. Considering F=0 means finding the free evolution of the system, that is how it vibrates naturally. If the system does not start from an equilibrium condition, the displacement vector assumes the following form

$$x = A\cos(\omega t - \phi) \tag{1.4}$$

where

- A is a constant
- $\omega$ is frequency of vibration
- $\phi$ is the phase

Now, substituting this form of solution in the equation 5.1 with F=0, the solutions of the system exists when

$$det(K - \omega^2 M) = 0$$

The values of $\omega$ which satisfy this equation are called natural frequencies and represents how the systems tends to vibrate naturally.
Knowing natural frequencies allows to calculate mode shapes, that are the deformations shapes which occur during vibration, and above all, to solve the equation of motion with F≠0 obtaining displacements, velocities and accelerations at each time instant. One of the most important phenomenon is **resonance**, which may occur when the excitation frequency is comparable with natural frequency. This is very dangerous because resonance implies an increase of vibration amplitude over time.[2]

## 1.3 Gear dynamics

All mechanical properties of a system can be summarized in

- **Mass**: Indicates the distribution of the weights in the system
- **Stiffness**: Indicates the interactions among the every parts of the system
- **Damping**: Accounts for the friction within the system

these elements give birth to the equation of motion

$$M\ddot{u} + B\dot{u} + Ku = F$$

where u is the displacement vector and F is the external force vector.
In static is not very important how weights are distributed, while the dumping **B** is negligible, so the equation can be simplified

$$Ku = F$$

with $F = cost$.
For this reason, static is only conditioned by the intensity of loads acting on the system, not considering the frequencies of this forces, that are fundamental in dynamics. As a conseuqnce, the main goal of dynamic studies is to design systems in order to eliminate resonances in operating conditions.
In a simple case like in the figure 1.6, each gear has a natural frequencies that can be triggered by excitation.

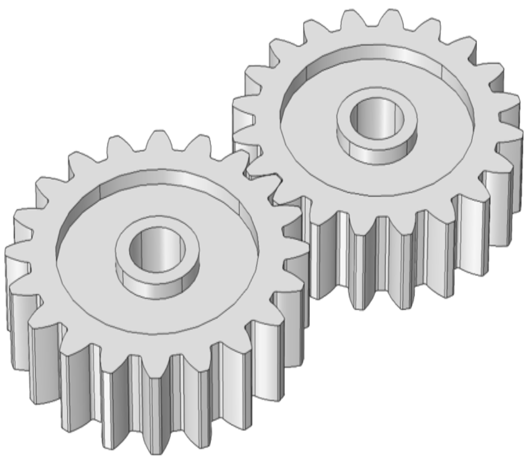

Figure 1.6: Simple gear pair

More generally, when toothed wheels start to vibrate, some areas remain fixed in the mode shapes. The number of these areas depends on the frequency at which vibration occurs, and in particular increases the frequency. Because of the symmetry of this particular system, they occur in couples and these couples are called nodal diameter. A nodal diameter represents a symmetry plane of modal shape. [3] In the figure 1.7, 1.8, 1.8 there are some examples.

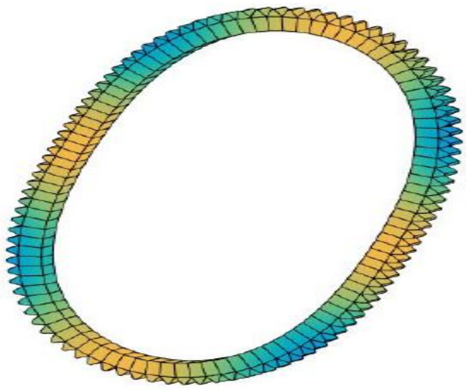

Figure 1.7: Two nodal diameters

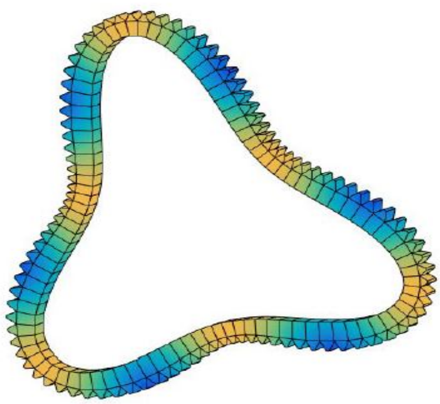

Figure 1.8: Three nodal diameters

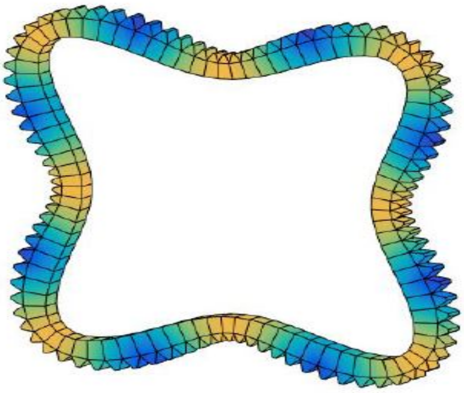

Figure 1.9: Four nodal diameters

Therefore, it can be said that nodal diameters establish the mode shape of a toothed wheel at a certain frequency.
The excitation in gears is caused by contact between teeth. The problem is that the number of teeth in contact changes over time, making also the stiffness varying over time. That is the main form of excitation which increases the vibrations generation. An other factor that increases vibrations is the fact that the contact is not perfectly smooth.
**Mesh frequency** is the frequency at which the contact occurs and depend on the speed and the number of teeth.
One of the most important diagram in gears dynamic is the **Campbell diagram**, represented in the figure 1.10

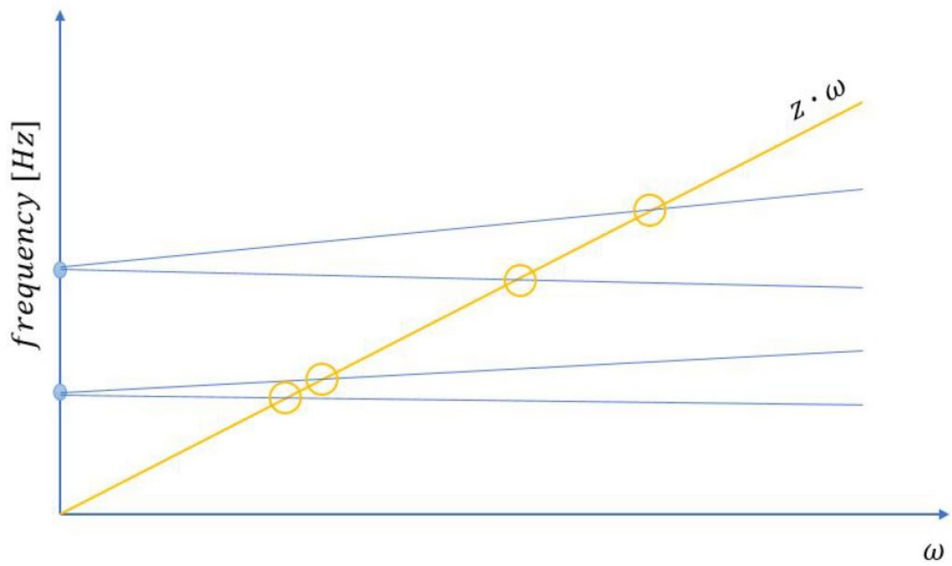


Figure 1.10: Campbell diagram of a gear

The blue dots in the vertical axis represent the **natural frequencies** of a gear, that is

the frequencies with rotational speed $\omega = 0$.
Since gears vibrate while rotating, two harmonic waves contribute to dynamics. This can be seen in the Campbell diagram, where there are two lines starting from natural frequencies, whose aperture , theoretically, depends on the number of nodal diameters of that specific mode shape following the law

$$f = f_0 \pm N_D \Omega$$

valid only if the gyroscopic effects are neglected.

- $f$ is the system frequency at a certain speed
- $f_0$ is the natural frequency
- $N_D$ is number of nodal diameter
- $\Omega$ is the rotational speed of the gear

The yellow circle indicate the possible resonances, that could be of two kind:

- Forward resonance
- Backward resonance

The first occurs when the two harmonics have the same sign and it represents the highest frequency of the mode, the latter represents the lowest frequency of the mode and it happens when the two harmonics have opposite sign. [4]

# Chapter 2

# Gear geometry

I this thesis it has been used mainly the following model, whose geometry is summarized in the table 2.1. In the figure 2.1 there is this pair molded in the editor of Transmission 3D.

| | Symbol | Pinion | Gear | Unit |
|---|---|---|---|---|
| Module | m | 2.54 | 2.54 | mm |
| Number of teeth | z | 43 | 102 | - |
| Pressure angle | $\phi$ | 20 | 20 | deg |
| Internal radius | $r_i$ | 45.7200 | 114.9350 | mm |
| Tip radius | $r_{tip}$ | 57.1500 | 132.0800 | mm |
| Root radius | $r_{root}$ | 51.4350 | 126.3650 | mm |
| Face width | b | 11.4300 | 11.43 | mm |

Table 2.1: First gears pair geometry

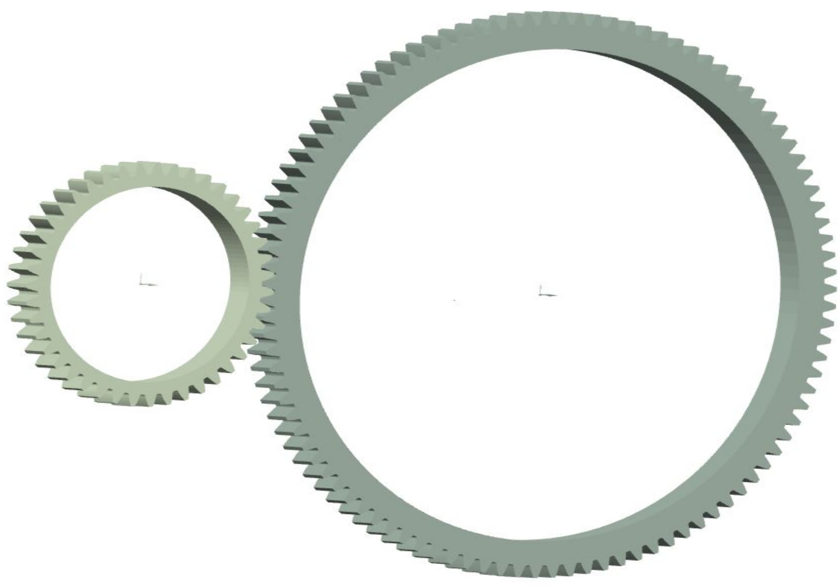

Figure 2.1: First gears model

Below are presented three more pairs, that have been used occasionally at confirmation of results acquired with the first model.

| | Symbol | Pinion | Gear | Unit |
|---|---|---|---|---|
| Module | m | 2.5 | 2.5 | mm |
| Number of teeth | z | 40 | 132 | - |
| Pressure angle | $\phi$ | 20 | 20 | deg |
| Internal radius | $r_i$ | 40.6250 | 153.1250 | mm |
| Tip radius | $r_{tip}$ | 52.5000 | 167.5000 | mm |
| Root radius | $r_{root}$ | 46.8750 | 161.8750 | mm |
| Face width | b | 5.6250 | 5.6250 | mm |

Table 2.2: Second gears pair geometry

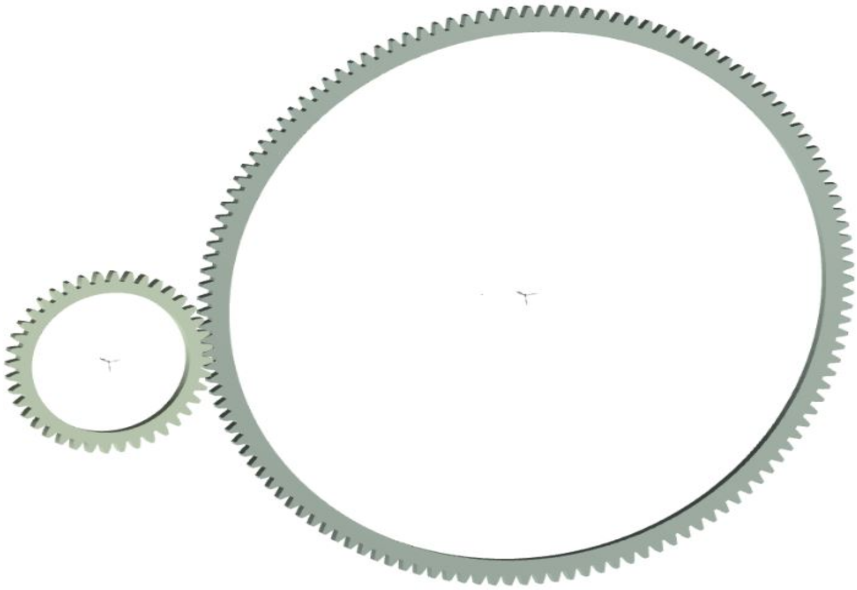

Figure 2.2: Second gears model

This is a particularly thin model, characterized by a small face width.

| | Symbol | Pinion | Gear | Unit |
|---|---|---|---|---|
| Module | m | 2.54 | 2.54 | mm |
| Number of teeth | z | 20 | 60 | - |
| Pressure angle | $\phi$ | 20 | 20 | deg |
| Internal radius | $r_i$ | 16.000 | 64.0000 | mm |
| Tip radius | $r_{tip}$ | 27.9400 | 78.7400 | mm |
| Root radius | $r_{root}$ | 21.8400 | 72.6440 | mm |
| Face width | b | 25.4000 | 25.4000 | mm |

Table 2.3: Third gears pair geometry

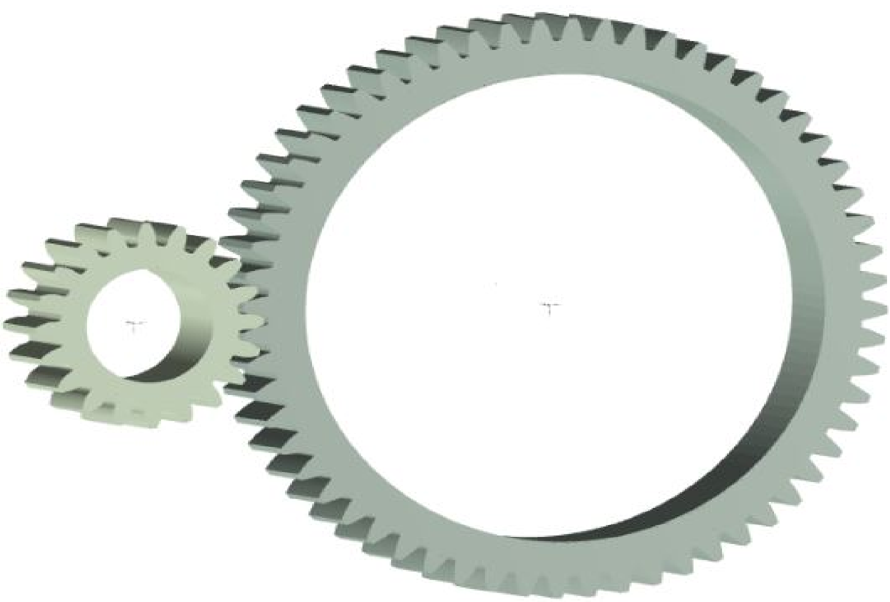

Figure 2.3: Third gears model

This model presents a thicker gears pair. It belongs to Windows LDP samples. It has been chosen because is more massif than the others.

| | Symbol | Pinion | Gear | Unit |
|---|---|---|---|---|
| Module | m | 3 | 3 | mm |
| Number of teeth | z | 20 | 60 | - |
| Pressure angle | $\phi$ | 21.88 | 21.88 | deg |
| Internal radius | $r_i$ | 21.000 | 83.5000 | mm |
| Tip radius | $r_{tip}$ | 31.5000 | 94.0000 | mm |
| Root radius | $r_{root}$ | 24.7500 | 87.4300 | mm |
| Face width | b | 40.0000 | 40.0000 | mm |

Table 2.4: Fourth gears pair geometry

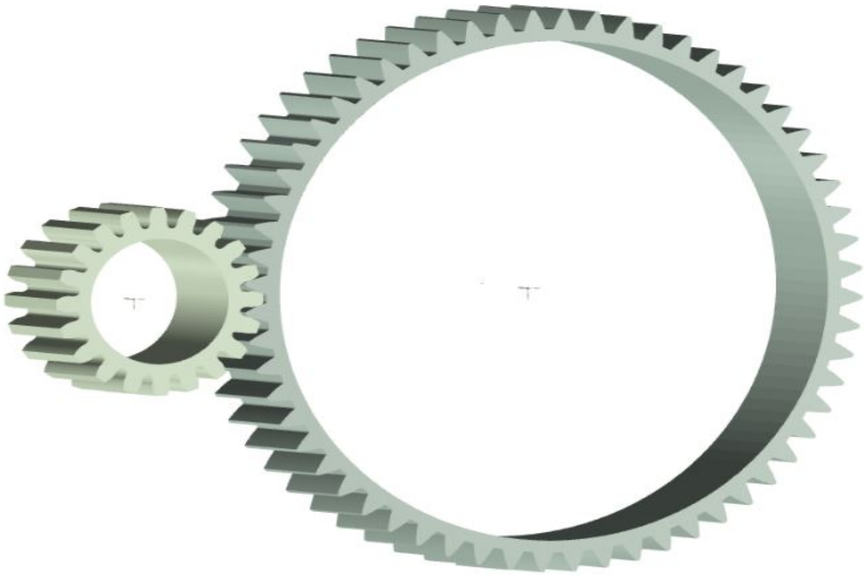

Figure 2.4: Fourth gears model

This gears pair is very similar to the previous one, indeed it has a face width even thicker. But the rim thickness is very small.

# Chapter 3

# Transmission 3D - Winsows LDP

The first phase of this thesis was aimed at understanding the two main software used: Transmission 3D and Windows LDP. Both are software of contact simulation, and give information like load distribution, displacement and transmission error.

## 3.1 Transmission 3D

Transmission 3D is a software capable of modeling complex gear system. It includes helical, straight bevel, spiral bevel, hypoids, beveloids and worms. it has a big variety of data output such as displacements, bending stress, contact patterns and, the most important for this thesis, transmission error.
The software presents two interfaces

- **Guide**
- **iSys**

Guide is the file creator/editor interface. It can be used to create the model, to run the analysis and to to run access postprocessing menus to obtain results. It is not convenient to create the model on guide because it has not a graphic interface, so it's not easy to build a model without looking at.
iSys is an improved editor that has been recently released. The interface is more intuitive and that improve the ease of data entry. Moreover, it is possible to see the 3D model during the creation, making more easy the detection of any error. Transmission 3D is a software exclusively developed for power transmission systems. One of the big problems of this kind of analysis is the non-linearity of the contact response. This is the reason why this is called semi-analytic software. It uses a finite element to analyze what happens in the meshing teeth far form the contact.

With this method is possible to have reliable results form the contact in term of displacements, force and contact pressure.

### 3.1.1 Model

This model, has been developed in transmission through a series of very detailed input.
Transmission 3D systems are built based on a combination of shaft, gears and bearings. The ensemble of these elements make a rotor. The rotor can't exist without a gear and a shaft, but it is possible to create a rotor without the bearing. Indeed, the model analyzed includes two rotor formed by a gear and a shaft.
For this thesis aim, the effects of the shaft won't be considered, so in the model has been design a very thin shaft, which is hardly visible in the figure **??**. It is connected to the gears by welding.

#### Contact grid

Before to set the geometric characteristic there are some important parameters that defines the **contact grid**

- **SEPTOL**: Is the SEParation TOLerance. Surfaces separated by more than this distance will not considered in the contact analysis

- **NPROFDIVS**: This variable controls the number of contact patches or cells that will be used to cover the contact zone between gear teeth.

- **NFACEDIVS**: This variable controls the number of contact patches or cells that will be used along the length of the contact zone

- **DSPROF**: This variable controls the width of the contact zone in physical length units. It is the most important parameters.

These parameter are very important because thanks to them it is possible to set the accuracy of the analysis, but also the duration of it. For this reason it is essential that the level of accuracy is compatible with the time available.

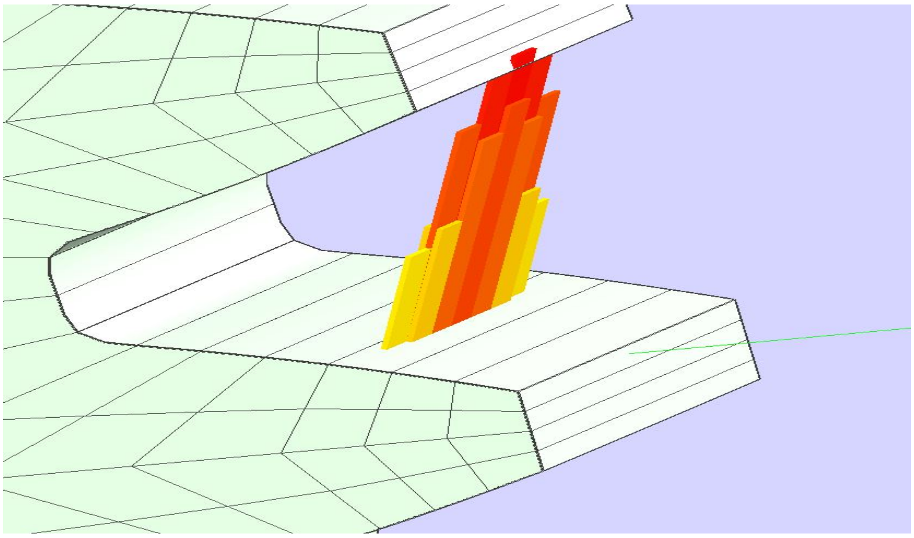

Figure 3.1: Distribution of the contact pressure

In the figure 3.1 it is possible to see the distribution of the contact pressure. Based on the type of analysis or on the desired results the grid can be more or less dense. For example if it is required the contact pressure on a certain point of the tooth profile is better a very dense grid, so NPROFDIVS and NFACEDIVS must be high while DSPROF must be smaller. Obviously in this situation the analysis could be longer.

**Torque**

it is important to spend some words also on the approach that Transmission 3D uses to determine the torque and the speed that characterize the gears pair. It is possible to set every rotor as INPUT or OUTPUT. In the first case the program will ask for the speed of the rotor, in the second case the request will be the resistant torque. The convention is to set the pinion as the INPUT rotor, setting the speed that will be important only in a dynamic analysis while the wheel will be OUTPUT .

**Mesh template**

The next important step is to set the mesh template. The finite element meshes in the Transmission3D package are created with very little input from the user. The user does not need to provide any of the node numbering and element connectivity information to the model generator. This information is read by the program from pre-existing files called "template" files.
While the contact grid have the purpose to regulate the accuracy of features such as contact force, contact pressure and displacements on the face width along the profile, this mesh is focused on stresses far form the contact.

There are three different templates

- MEDIUM
- FINEROOT
- FINEST

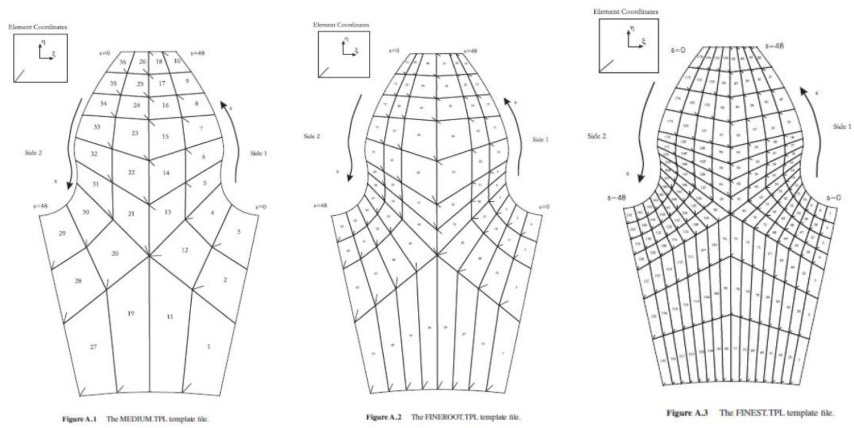


Figure 3.2: Tooth mesh templates

The differences among them are shown in the figure 3.2. It is evident that a more dense mesh template permit to have better results in the zone of the tooth root.
If the goal of the analysis is to calculate the displacements at the tooth tip, then MEDIUM or FINEROOT could give reliable results with short times. If it is important to know, for example, the root stresses then FINEST is required, even if the duration of the analysis is longer.
Chosen the template, also the number of elements trough the face width have to be set. According to some tests it seems that one face width element is sufficient for reliable results, since the contact parameters can be chosen separately, permitting to have a precise modeling of the contact using a coarse mesh. [5]

### Profile modification

In the following figures there are two diagram showing the two main profile modifications that will be considered in this thesis.

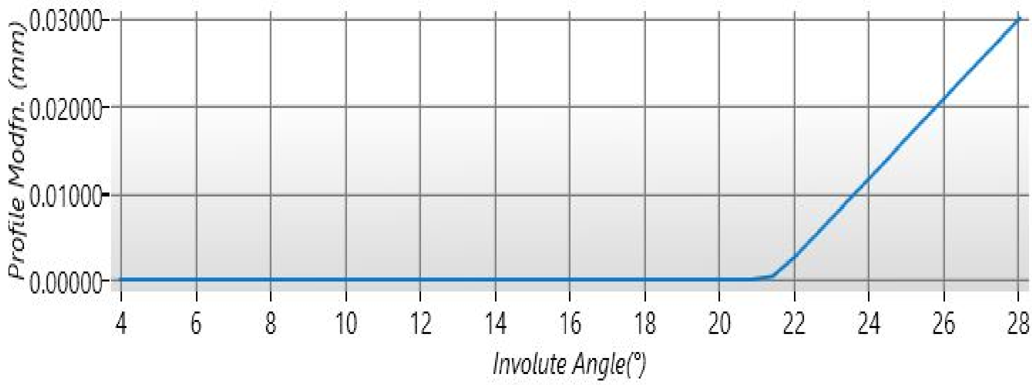


Figure 3.3: Linear tip modification

The linear tip relief start form the HPSTC, the highest point single tooth contact in both gears.

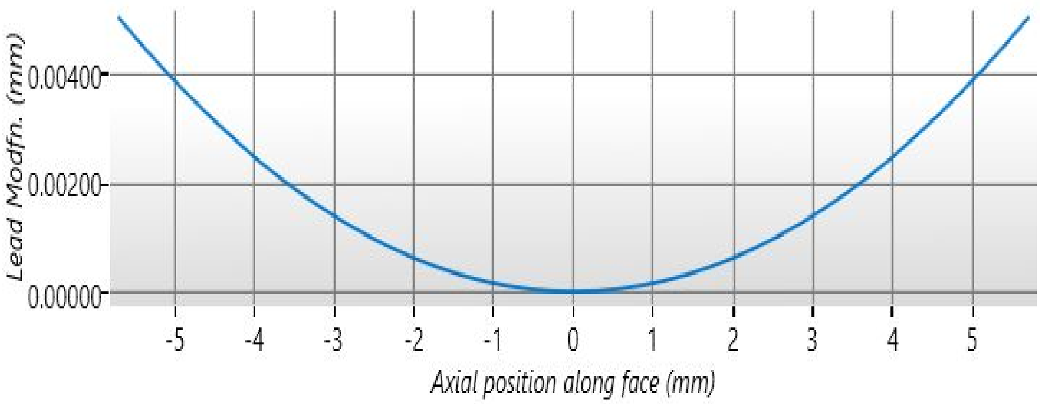


Figure 3.4: Lead crown modification

#### Analysis

The setting of the analysis is a critical moments in Transmission 3D. This is can be carried out from SETUP on the structural tree. The main commands are summarized below

- **MEMORY_USAGE_LEVEL**: This variable controls how aggressive the solver Calyx will be in consuming system memory. The higher the setting, the more data it will retain in RAM in order to speed up execution.
- **POSTPROCWRITE**: If this flag is set, a finite element post-processing file will be created containing displacement and stress information. This file can be very large, it takes up a huge amount of disk space and make the duration of the analyses very longer. On the other hand, this is the only way to create IGLASS file, that permit to observe every post-processing information in graphic interface (Figure 3.5).

- **NTIMESTEPS**: This variable represents the number of equally spaced position to analyze in the mesh cycle. For a static analysis it must be set at least 21, where the first and the last position are the same. 20 is the minimum of these positions to find a reliable transmission error. For a dynamic analysis the number of these steps increase. The more the steps, the more accurate will be the analysis, paying attention on the duration.

- **NSTEPSWRITE**: This variable is the number of time steps after which to write post-processing information. This is very useful for dynamic analysis where there could be thousand of steps because it permits to preserve the memory of the work-station and it allows faster analyses.

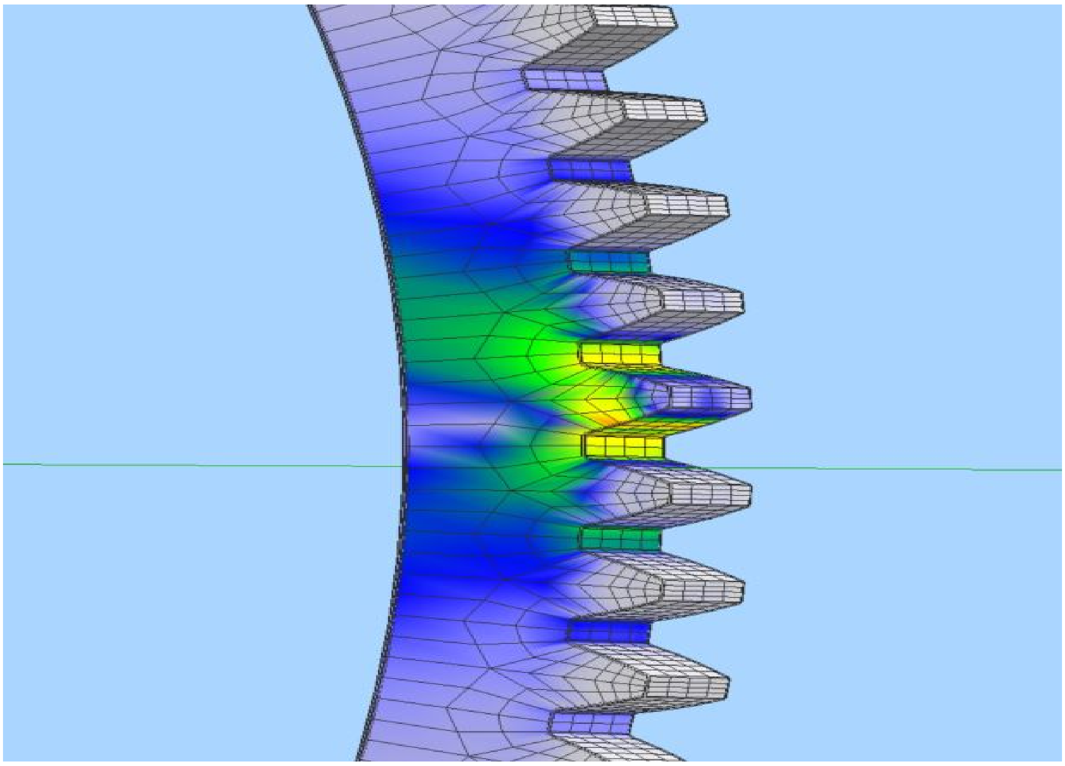

Figure 3.5: Vom Mises stress on the wheel on iGlass interface

## 3.2 Window LDP

Windows LDP (Load Distribution Program) is a computer program for predicting the load distribution across the zone of contact for a single pair of spur or helical gears. The gears may have an internal or external mesh and may be mounted in shafts between centers or overhung. The model assumes the load distribution to be a function of the elasticity of the gear system and errors or modifications on the gear teeth.
[6] For elastic deformations calculations the program makes some assumptions:

- The total elastic deformation is the sum of the individual elastic deformations.
- The elastic deformations are small, thus tooth contact is assumed to remain on the line of contact.
- The gear bodies and supporting shaft behave as solid cylinders for the purpose of determining the bending and torsional deformations.
- The deflections of any given tooth pair are not influenced by the loads on other tooth pairs.

### 3.2.1 Model

To build a new model it needed to use the input module, that include a series of data input divided in some tab as it possible to see in the following figure.

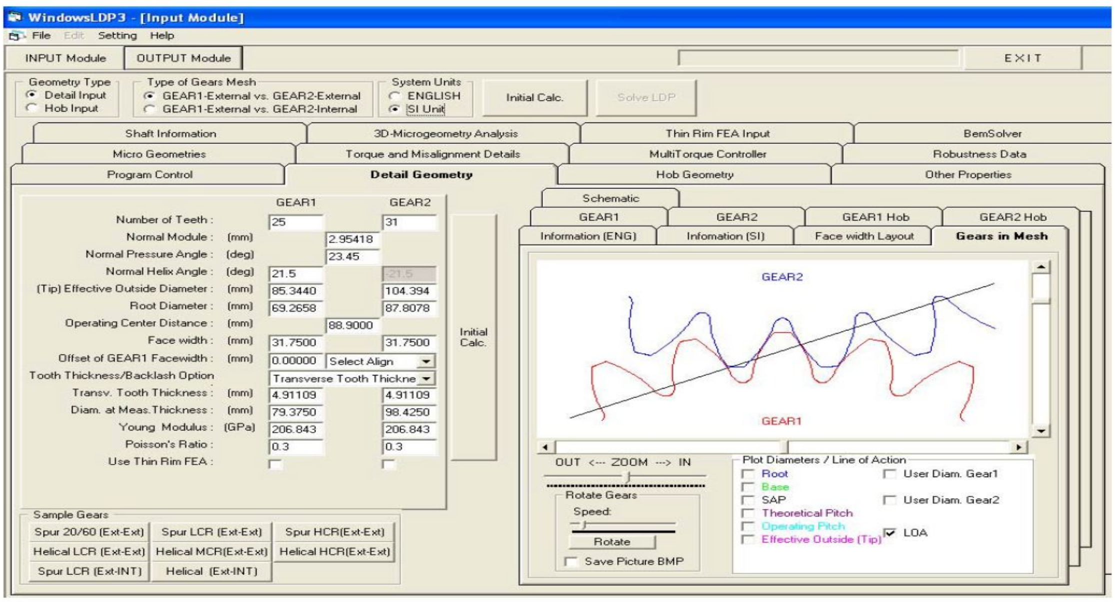


Figure 3.6: Geometry data input WinLDP

On of the most important differences with Transmission 3D is the lack of the gear rim. WinLDP is a software which consider only the teeth for the tip to the root.
In the figure 3.6, is possible to see the tab relative to the Detailed Geometry. Once every data are correctly inserted it is possible to click Initial Calculation to observe the model in a graphic interface. There are also some sample gears in the bottom of the interface. These are public geometries that cover a broad range of contact ratios of spur and helical gearing. The 25 tooth/31 tooth gear pairs are family of gears designed by Boeing Helicopters that have been tested extensively at NASA and OSU. The default run of each set is a single torque run using a perfect involute geometry.

### Profile modifications

Profile modifications can be set from Micro Geometries tab.

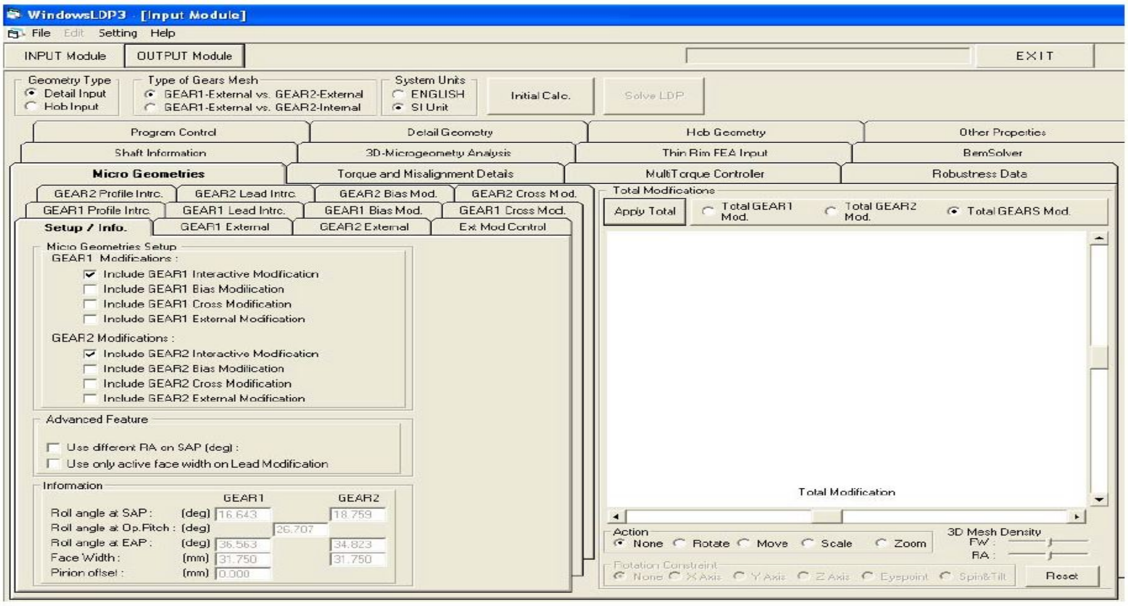


Figure 3.7: Micro Geometries tab

This menu is where one enters modifications or errors to the involute tooth shape. Entries may be from the interactive entry of simple shapes and/or from external files. Thee are several different options that may be included in the interactive inputs, namely tip and root relief, bias modifications, cross modifications and external modifications. Any or all of the modifications may be used and all are additive to one another. The graphic to the right side of this window allows one to graph the modification that is applied.
On each "macro-tab" there are lots of additional tabs and in this case every sub-tab is related on a different modification. On Setup/Info it is possible to select the forms of modifications that are to be applied. One can apply modifications to either gear or to

both of them.
Above there are some examples of the 3D interface which show the different modifications

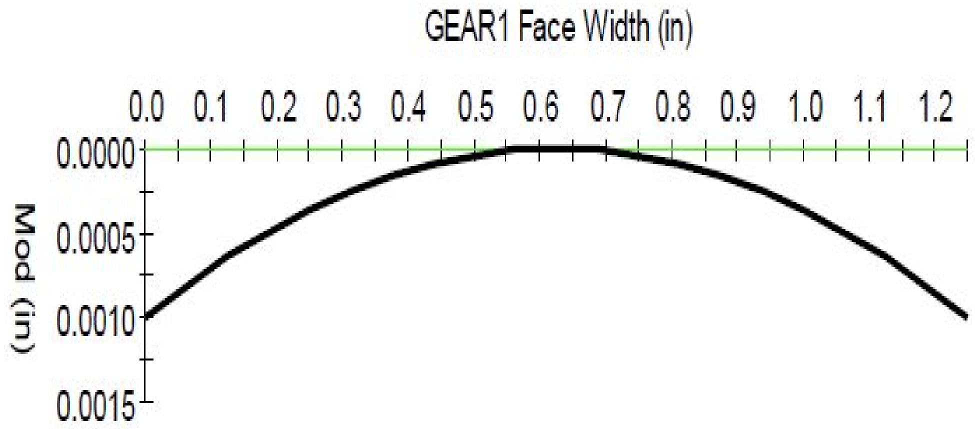


Figure 3.8: Total modification along face width (2D)

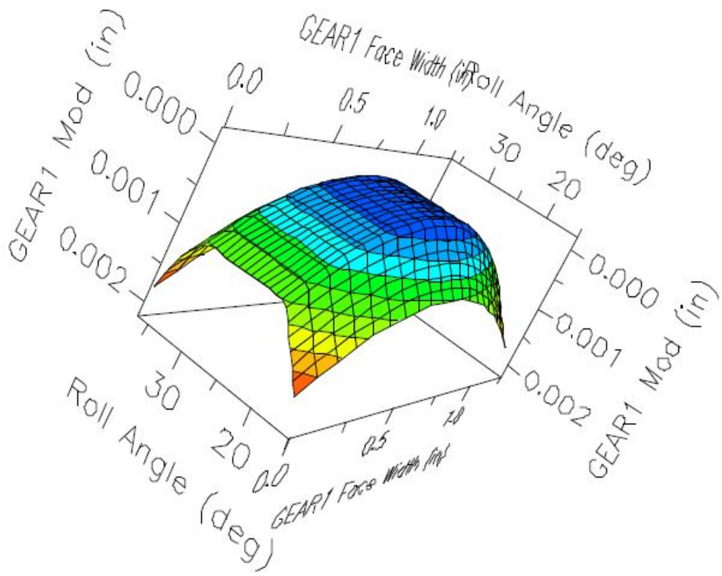


Figure 3.9: Total modification (3D)

**Torque**

The torque has to be set in the Torque and Misalignment Tab. This tab allows one to control the torques that the program will be run at. Also, misalignments may be added individually for each torque value, but it won't be done in this work of thesis.

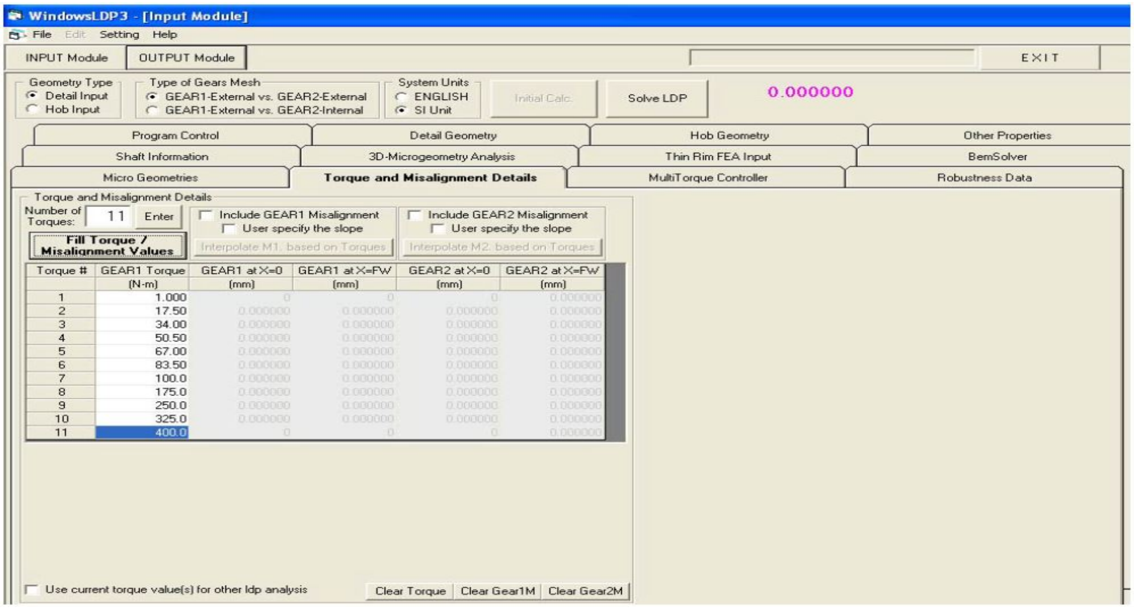


Figure 3.10: Torque and Misalignment tab

The basic LDP run assumes that the input torque is held constant and the output torque is also constant at the input torque multiplied by the gear ratio. This is technically incorrect since friction losses in the drive will cause output toques to be slightly smaller the calculated value. .The number of torque command allows LDP to run repeat runs at a progression of torques. For spur gears with low contact ratio, 6-10 torques are appropriate, but for higher contact ratio gears, it may be necessary to run 30 or 40 torques in order to properly see the torque variation.

A second variation of torque specification is to vary the torque for each position. For instance, if there is a severe torque variation at twice per shaft revolution, this could be entered from an external file.

### Analysis

To set the analysis there is the Program Control tab. It is possible to set run parameters such as number of discrete points used along the lines of action and the number of positions of rotation to be analyzed.

It could be useful to set the beginning of position control in the meshing cycle. For example, when zero is selected as the beginning of the contact, this is the position where one tooth pair has just left contact. At the same way, it is possible to set the ending of position control. When 1.0 is selected as the end of contact, this is the position just prior to a pair of teeth leaving contact. 0.0 and 1.0 are, virtually, the same point, but different numbers of teeth are in theoretical contact. It is possible to perform analyses over more than one base pitch, but in most analyses it would mean the repetition of the analysis from 0.0 to 1.0.

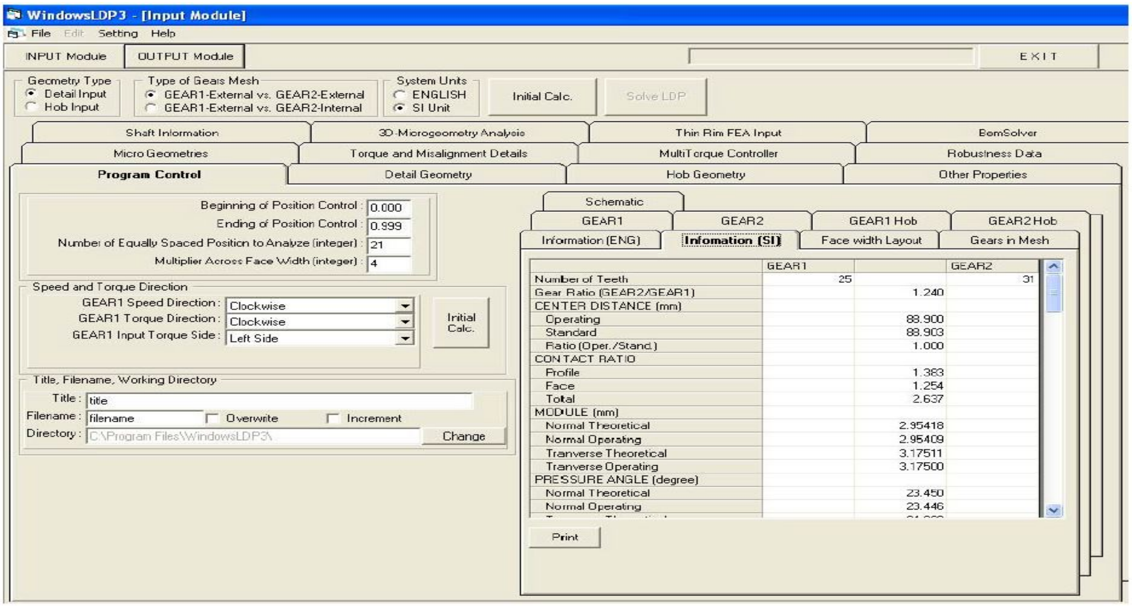


Figure 3.11: Program Control tab

The most important run parameter is the **Number of equally spaced positions to analyze**. The default is to have 11 positions in a mesh cycle. This essentially gives 10 equally space positions since 0 and 1.0 are virtually the same position. If one wants more position detail, a larger number of positions may be used. As it was already said, for this thesis will be set 21 position.

In the end through the Speed and Torque Direction section it is possible to define which gear will be the INPUT and which one will be the out put. By maintaining the same convention of Transmission 3D "Gear1 Speed Direction" must be set opposite to "Gear1 Torque Direction".

## 3.3 Software differences

The most important difference between Transmission 3D and Windows LDP is the duration of the analysis. While T3D requires some seconds per step, Win LDP gives instantly every results.
The problem is the discrepancy of results in the two software. It is known that Transmission 3D is the most complete software of gears mesh simulation, then their results are more reliable than the Win LDP ones. The next step is to understand if it is possible to discover the features that make Windows LDP less precise and transform them in a "corrective factor". This would lead to fast and precise analyzes.
In particular the output analyzed has been the **Transmission Error**, that is bigger with Winodws LDP.

### 3.3.1 Rim

The first feature analyzed is the fact that Windows LDP does not consider the rim in his model, there is just the teeth from the tip to the root. Then a possible source of error could be the fact that in T3D the rim is considered.
The first step was to eliminate the rim in T3D specifying an Inner diameter equal to the difference between the Outer diameter and the sum of Addendum (the module m) and Dedendum

$$D_i = D_o - (m + 1.25m)$$

The problem is that it is not possible to create a model like this in Transmissio 3D because of the impossibility to create a personalized mesh template as said before. Then, the QUAD element of the template, indifferently MEDIUM, FINEROOT or FINEST, collapses on themselves making the calculations on the analysis impossible.
Since that, the only way has been to create a very thin rim to get closer to the LDP model, but the discrepancy remain the same.
Then, it has been possible to insert a rim on the Win LDP model, but the Transmission error, instead of decrease to get closer to the T3D one, increased. The conclusion is that the presence of the rim causes a constrain on the tooth, inducing the growing of the transmission error.
From this evidence, the presence of the rim could not be the responsible of this discrepancy.

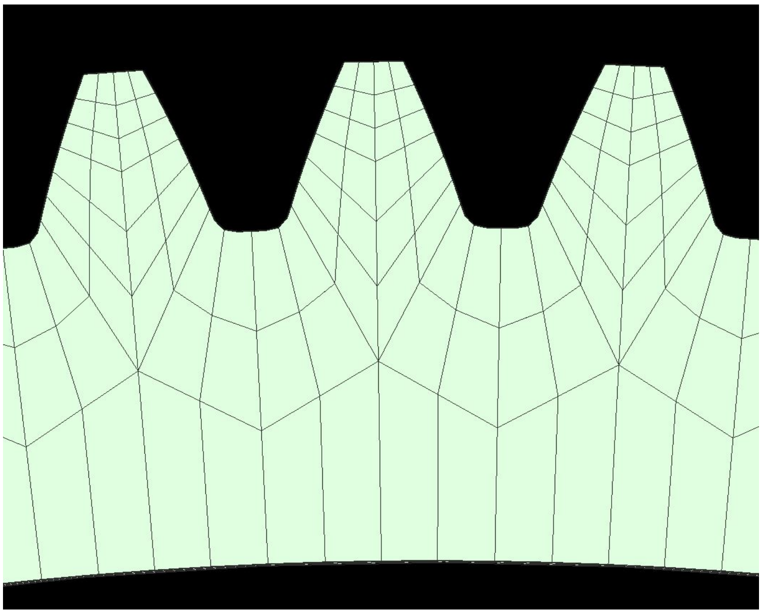

Figure 3.12: Medium mesh template on gear rim

### 3.3.2 Mesh template

The second difference in terms of model is that the mesh template medium is coarser than the one on Win LDP.
Using FINEST the average transmission error is closer but not enough to consider this as the real source of error. Moreover, the duration of calculation increase very much, so it does not make sense to lose so much time for such a small decrease of discrepancy.

### 3.3.3 Shaft presence

It is impossible to create a model without a shaft in Transmission 3D, a rotor for exist mast be composed by a shaft and a gear.

These attempts has been carried out also changing the geometry, trying the models described in the tables of the previous chapter.

### 3.3.4 Conclusion

In conclusion it has not been possible to detect an error factor that permit to use indifferently the two software obtaining the same results. The reason is that they are too different in terms of input. In particular Transmission 3D ask for input like

- Raleigh coefficient of mass matrix $\alpha$
- Raleigh coefficient of stiffness matrix $\beta$

- Thermal expansion coefficient

While Windows LDP ask for parameters like

- Conducivity
- Specific heat
- Oil type
- Inlet bulk temperature
- Friction coefficient

Nevertheless, from the analyses carried out, it can be said that Transmission 3D is a more reliable software and it must be used when is important to know the exact value of stress, displacement or any other output even if the duration of calculation is longer than LDP; on the other hand, if the aim of the study is just to know the trend a particular gear characteristic, Windows LDP is a very reliable program that permits fast results.

# Chapter 4

# Mesh stiffness

Specific tooth load per unit tooth deflection in a meshed gear system is called **Mesh Stiffness**. While gears in operation, the points of contact are moving continuously and the load shared at these points vary in magnitude and direction. Due to this dynamic loading, the gear teeth were subjected to bending fatigue which also effect on stiffness and vibrations character of the meshed gear tooth. In addiction to dynamic loading effect, transmission error and sliding friction is also the major sources of noise and vibration in meshing gears. In civil aviation one of the most important challenge is to reduce the noise in order to make the flight more comfortable. The difference between the effective and the ideal position of the output shaft with respect to the input shaft is called the transmission error. Dynamic loading, transmission error and sliding friction are sources of noise and vibration in meshing gears due to non-uniform motion in gear tooth mesh. This occurs due to adjacent pitch error, profile error, misalignment and lead errors.
One of the most important purpose of this thesis is to better understand how to calculate mesh stiffness in the most correct way.

## 4.1 Transmission Error

Theoretically, for two gears with perfect involutes and an infinite stiffness, the rotation of the output gear would be a function of the input rotation and the gear ratio. A constant rotation of the input shaft would therefore result in a constant rotation of output shaft. Due to both intended shape modifications and unintended modifications, such as manufacturing errors, gears will be a motion error of the output gear relative to the input gear. The transmission error and mesh stiffness variation is often considered to be the primary excitation of gear noise and a minimization of the transmission error is believed to minimise noise. The definition of transmission error is "the difference between the actual position of the output gear and the position it occupy if the gear drive were perfectly conjugate".

### 4.1.1 Static transmission error

The transmission error features depend on the instantaneous moments of the meshing tooth pairs. Obviously it will be senseless to set a speed for this kind of analysis.
Under operating conditions, the variation o the mesh stiffness generate dynamic mesh force which is transmitted to the housing through the shaft. So the noise is related to the vibrations of the housings. To reduce noise is very important to predict the static transmission error.

### 4.1.2 Dynamic transmission error

While in the static transmission error we can just consider the gear pair, for dynamic transmission error the gears should be in their gearbox, because the dynamical properties of the system (casing, bearing, shaft and gears) are very important.
That is why, in the last years, many studies were been carried out to predict this kind of error working both on characteristics of the tooth such as profile modifications and backlash, and gear characteristics such as rim thickness and face width.

## 4.2 Calculation methods

Mesh stiffness is the slope of the Transmission error-Force diagram. Currently, we use mainly two method: average slope method and local slope method. The first finds the stiffness by dividing the nominal mesh force by the mesh deflection (transmission error)

$$K_{mesh} = \frac{NominalForce[N]}{Averageranmissionerror[mm]} \tag{4.1}$$

the latter is based on the concept of the finite differences, so two analysis are needed, the first with the nominal torque and the second one with the nominal torque increased of a little percentage. The stiffness will be calculated as

$$K_{mesh} = \frac{\Delta T}{\Delta TE}$$

In the following paragraph these two methods will be described.

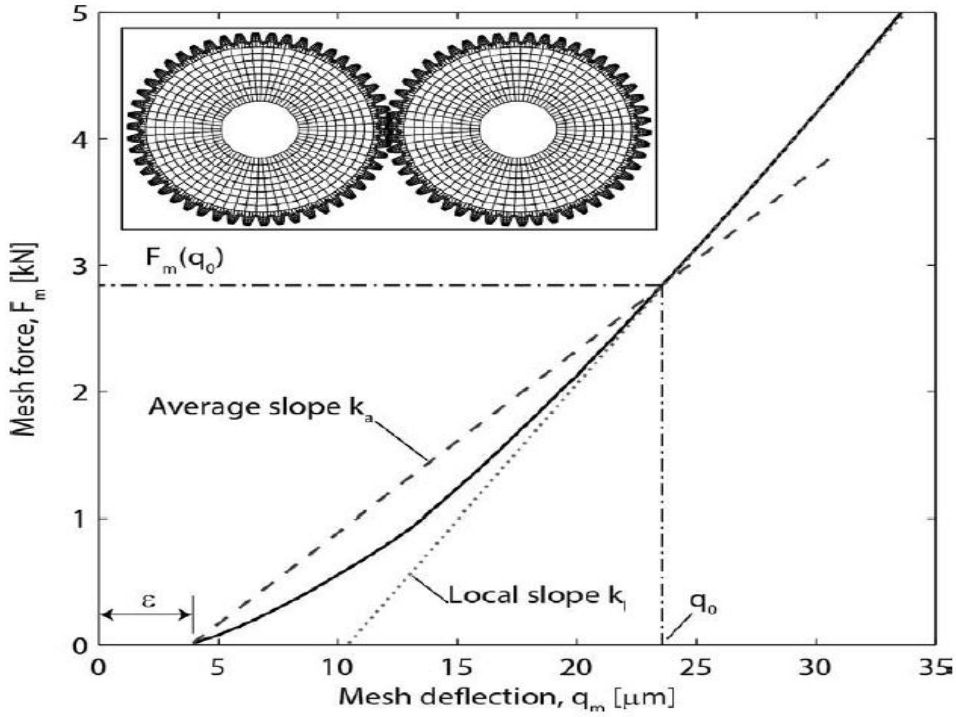


Figure 4.1: Transmission error - Force diagram

### 4.2.1 Average slope method

The gears are analyzed statically for a number of different over one mesh cycle to capture the effects caused by changing contact conditions. As already said, the mesh stiffness at each point in the mesh cycle is

$$K_{mesh} = \frac{F_m}{TE_m} \tag{4.2}$$

where the transmission errore is calculated as

$$TE = r_b\theta \tag{4.3}$$

$r_b$ is the base radius of the wheel, while $\theta$ is the absolute gear rotational deflections measured relative to perfectly conjugate gear motion. For the considerations of this thesis, pinion will be considered infinitely rigid, so deflections like transmission error will be calculated only on the wheel.
The tooth mesh force is

$$F_m = \frac{T_2}{r_b} \tag{4.4}$$

The rotational deflection $\theta$ is calculated with the two software presented in the previous chapter, Transmission 3D and Windows LDP.
Now is important to do a correction.

The correct form of the equation 4.3 is the following

$$TE = r_b\theta - \epsilon \tag{4.5}$$

Where $\epsilon$ is the **unloaded factor** and it depends on tooth surface modifications. For unmodified tooth $\epsilon$ is equal to zero. For vanishing applied load the static transmission error becomes the unloaded transmission error.
To find $\epsilon$ it is sufficient to set a very small torque on software (zero is not accepted neither in T3D nor in WinLDP).
As it is possible to see in the figure 4.1, at each point of the mesh cycle stiffness from the average slope approach is the slope of a line extending from the deflection $\epsilon$ at zero mesh force to the point on the curve corresponding to the force and deflection values for the given torque; this is the average stiffness over the deflection range beginning from zero torque to the final deflection for the given torque.

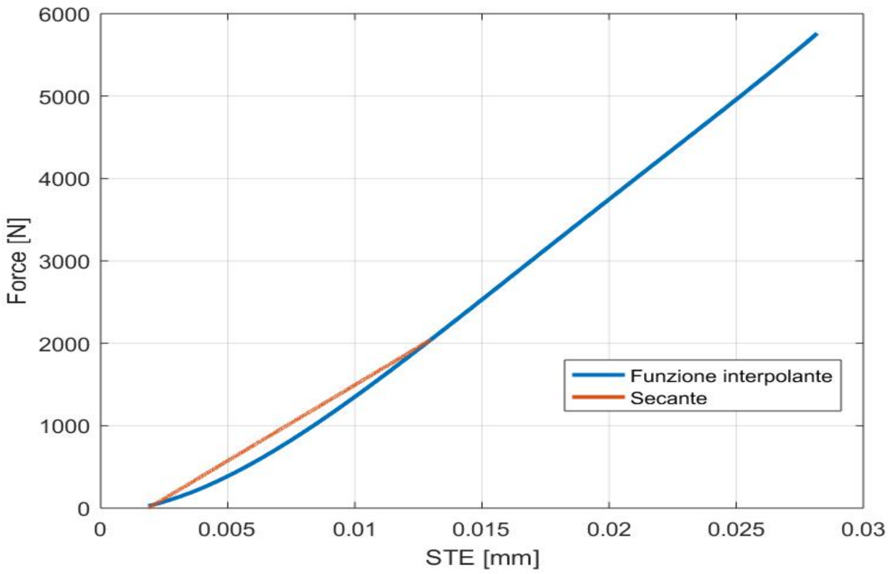


Figure 4.2: Average method on Transmission error - Force diagram

In figure 4.2 there is the Transmission error - Force diagram drawn interpolating average static transmission error, obtained with several simulations on Windows LDP, on Matlab tool. Mesh stiffness is the slope of the red line that represents the average slope method.

### 4.2.2 Local slope method

In this approach the tooth In this approach the tooth stiffness is the local slope of the force-deflection curve (Figure 4.1) at some nominal deflection $q_m$. The local slope of this curve using first-order finite difference approximation gives the mesh stiffness

$$k_{mesh} = \frac{F_m(q_m + \Delta q_m) - F_(q_m)}{\Delta q_m} \tag{4.6}$$

where the parentheses indicate the deflection values where the force is calculated, $\Delta q_m$ is a specified small change in mesh deflection and $F_m$ is calculated from the model.
In the reality, how has been said in the two software the torque is the input, while the output is the rotational deflections. Then, it was used a Matlab tool to invert input and output obtaining diagrams similar to the one in figure 4.1.
The displacement $\Delta q_m$ or mesh force $\Delta F_m$ step size for the finite difference calculation must be carefully chosen. Excellent convergence was obtained using a step size of 1% of the nominal torque for the gear pair analyzed in this study. Higher-order finite difference expressions yield no additional accuracy for stiffness calculations using the finite element/contact mechanics method.
To calculate the mesh stiffness using the average slope approach for unmodified gears one simulation at the operating torque is necessary. For teeth with modifications an additional simulation at very low torque is necessary to calculate the unloaded transmission error Îţ. Calculation of mesh stiffness using the local slope approach requires two simulations: one above the nominal deflection or load, and another below it. In terms of computation time, the local slope approach is no different than the average slope approach when the gear teeth have modifications. [7]

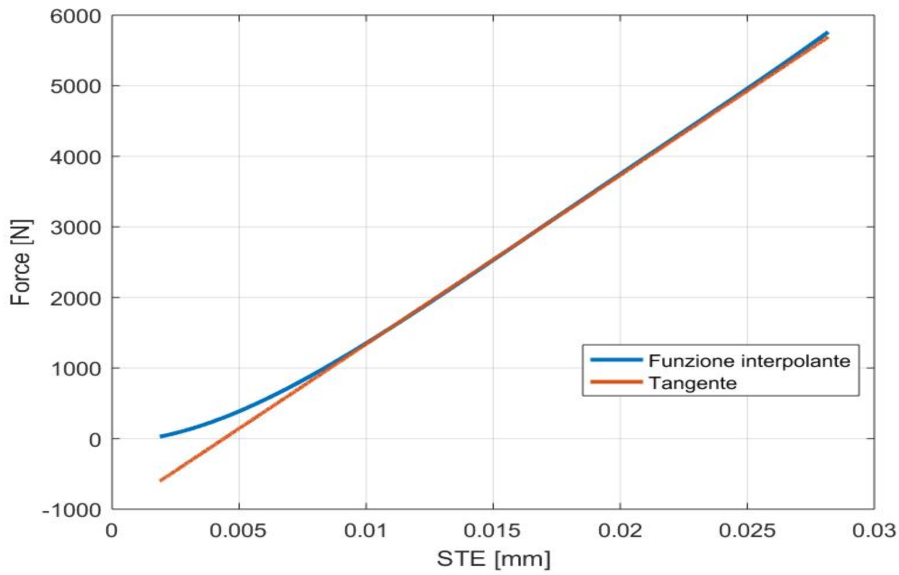


Figure 4.3: Local method on Transmission error - Force diagram

In figure 4.3 there is the Transmission error - Force diagram drawn interpolating average static transmission error, obtained with several simulations on Windows LDP, on Matlab tool. Mesh stiffness is the slope of the red line that represents the local slope method.

### 4.2.3 Derivative method

How to understand which one is precise? The mesh stiffness is the slope of the diagram on a certain point. Then, the idea is to use the mathematical definition of slope to calculate this stiffness.
The slope of a function at a certain abscissa is the derivative of this function evaluated at that abscissa. To do this, several analysis have been made using Windows LDP varying the value of the torque from 0 to 500 Nm with a pitch of 10. The purpose of these analysis was to calculate the average transmission error on a mesh cycle. The next steps has been the implementation of a Matlab tool (already mentioned previously) that take in input the text file containing every values of torque with the relative transmission error, interpolate this data finding a function and deriving a the torque requested.

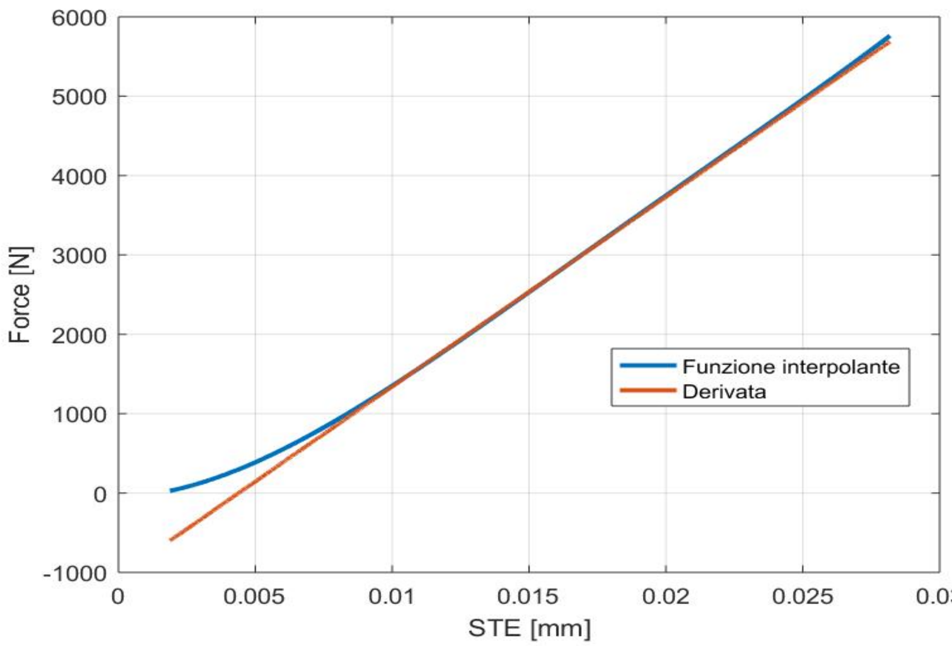


Figure 4.4: Derivative method on Transmission error - Force diagram

As it possible to see from the figure 4.4, this method is very similar to the local slope method. This analogy is all the more accentuated as the curvature of the function is reduced.
Obviously this method requires a huge computational cost, as it is necessary to run enough simulation to create the static transmission error - torque diagram. The advantage is that once the diagram is drawn, it is possible to know the mesh stiffness on every point instantaneously.

### 4.2.4 Profile modification and Unloaded factor

Below will be presented these methods applied to four cases of gears pairs

- Gears pair with no modifications
- Gears pair with linear tip relief
- Gears pair with lead crown
- Gears pair with linear tip relief and lead crown

**Gears pair with no modifications**

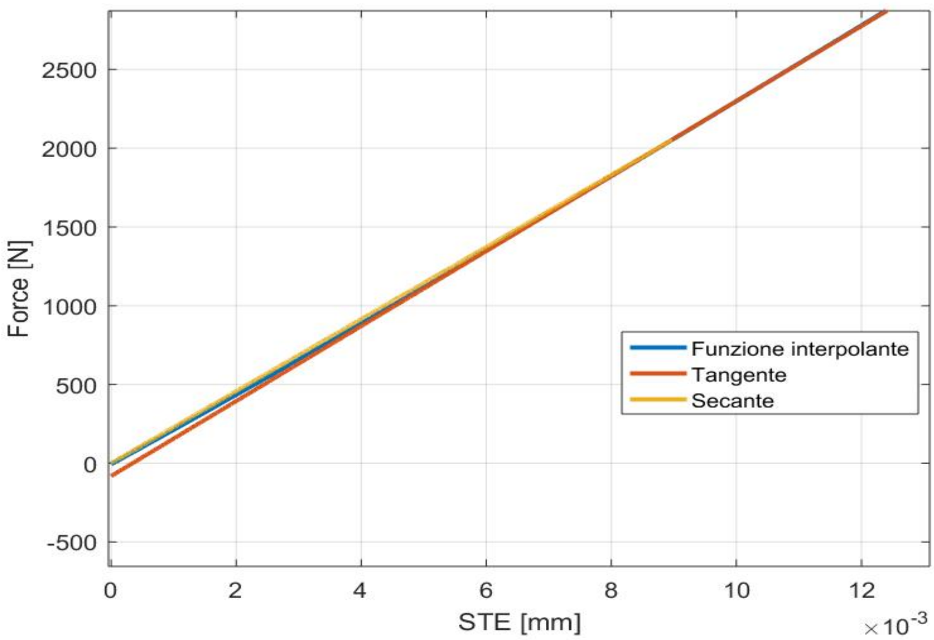


Figure 4.5: Transmission error - Force diagram for gears pair with no profile modifications

| | Average slope | Local slope | Derivative |
|---|---|---|---|
| Mesh Stiffness [N/mm] | 228890.3391 | 238068.6724 | 238016.9311 |

Table 4.1: Mesh stiffness values for gears pair with no modifications

The absence of modification means first that if the torque is negligible there will not be an unloaded factor. Moreover the trend of the curve is quite linear. In such a situation the differences among the methods are quite canceled. The line of the average and local methods have slope that are very similar, and mesh stiffness values reported in the table 4.1 confirm that statement.

### Gears pair with linear tip relief

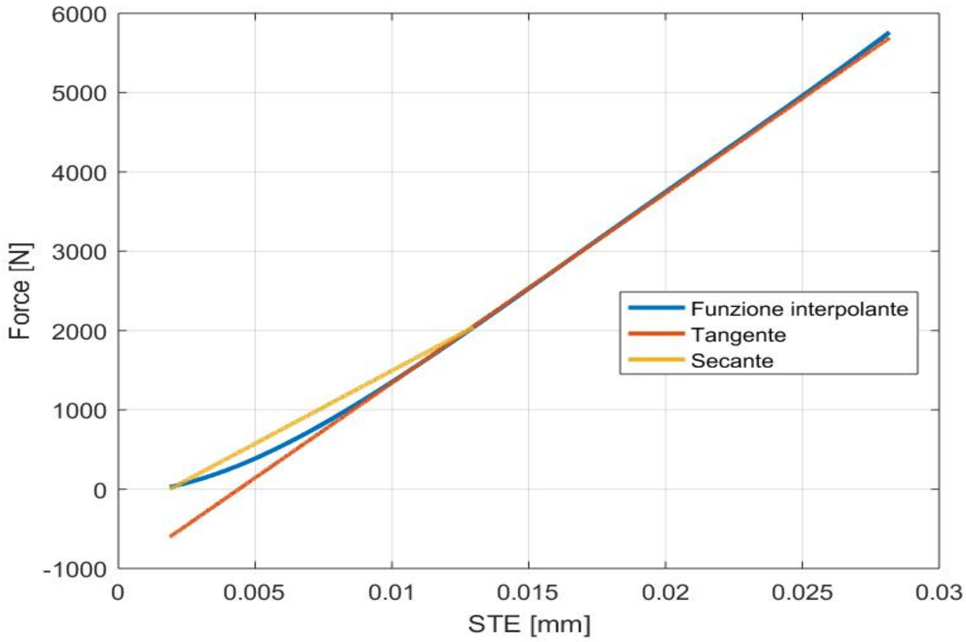


Figure 4.6: Transmission error - Force diagram for gears pair with tip relief

| | Average slope | Local slope | Derivative |
|---|---|---|---|
| Mesh Stiffness [N/mm] | 183930.7768 | 239241.8368 | 239030.5836 |

Table 4.2: Mesh stiffness values for gears pair with tip relief

Adding a linear tip relief modifies the trend of the curve transmission error-torque. In particular this curve does not start from the origin of the axis and that means that the tooth presents a deformation even unloaded. This deformation is the Unloaded factor. An other curve characteristic is that at low torque the trend is not linear but increase more slowly. This is a positive effect because it means that at low torque transmission error is limited and so it is the noise (and obviously the vibrations).
In this situation, to calculate mesh stiffness with average slope method, it is necessary to know the unloaded factor, then at least two simulations are required. The advantage is that this factor does not depends on the torque, then once calculated, to know transmission error relative to an other torque would not required an other simulation for the unloaded factor. The disadvantage is that, as it possible to see in the table 4.2 mesh stiffness values presents a value less similar to other methods than the model with no modification. On the other hand, local slope method is more precise as the table assets, but two simulation are always required.

### Gears pair with lead crown

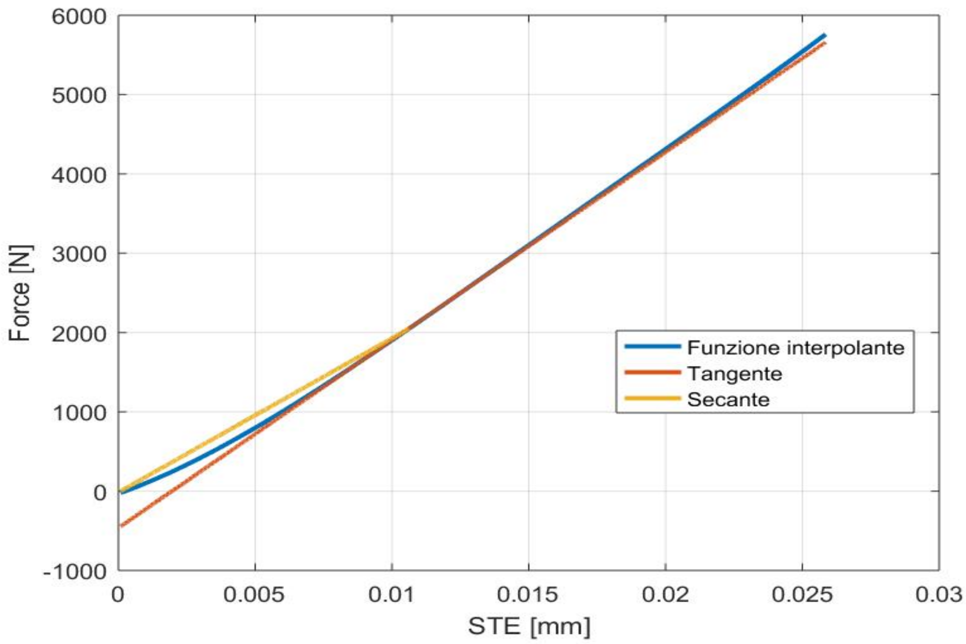


Figure 4.7: Transmission error - Force diagram for gears pair with lead crown

| | Average slope | Local slope | Derivative |
|---|---|---|---|
| Mesh Stiffness [N/mm] | 194371.0615 | 236873.6404 | 236723.7658 |

Table 4.3: Mesh stiffness values for gears pair with lead crown

In this situation, unloaded factor is very small so lead crown modification affects almost only the trend of the curve. Since in this situation is possible to neglect the unloaded factor, average slope approach seems the best from a computational point of view. Clearly, remains the problem of precision at low torques.

### Gears pair with linear tip relief and lead crown

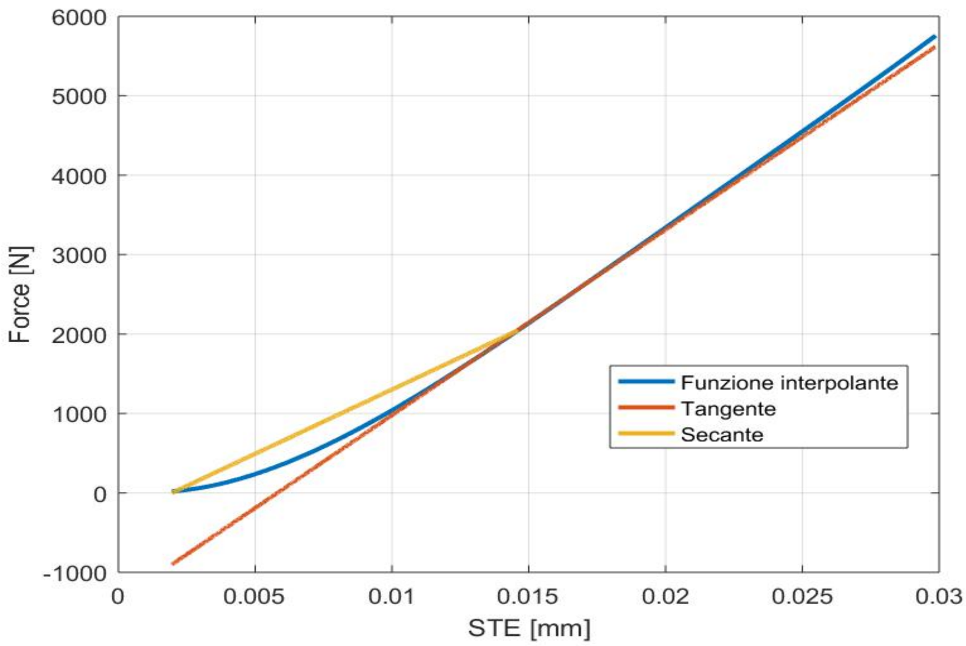


Figure 4.8: Transmission error - Force diagram for gears pair with tip relief and lead crown

| | Average slope | Local slope | Derivative |
|---|---|---|---|
| Mesh Stiffness [N/mm] | 161786.5639 | 233501.6076 | 233205.9048 |

Table 4.4: Mesh stiffness values for gears pair with tip relief and lead crown

In this situation returns the unloaded factor and the curvature of the trend at low torque is more accentuated. So the considerations already done remain valid.
In conclusion it is possible to say that average slope method is computationally convenient, but to be precise it must be used only at high torque. For low torque, where there is the non-linearity, local slope method is necessary.

### 4.2.5 Load effect

Now it will be presented the effect of the variation of the torque on the mesh stiffness along a mesh cycle.

#### Gears pair with no modifications

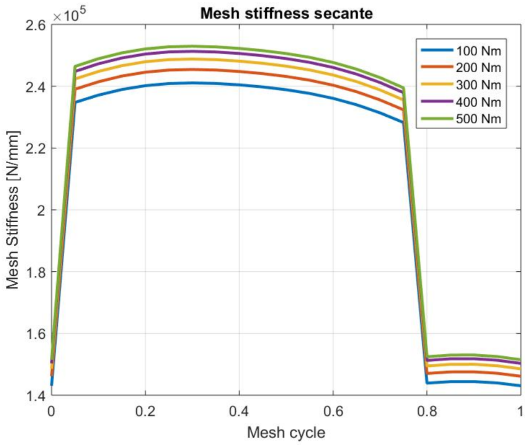


Figure 4.9: Average slope method

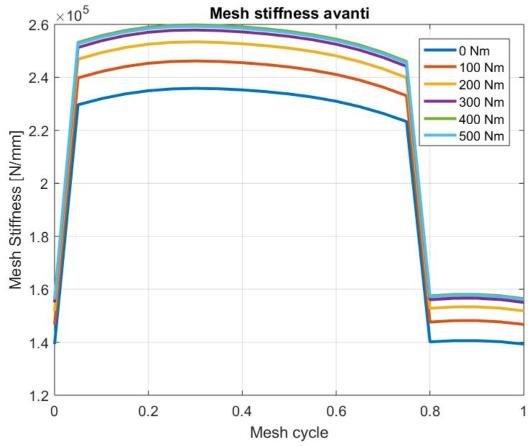


Figure 4.10: Local slope method

Here there are not modification, so increasing the load, mesh stiffness become higher, but the trend remains the same. The reason of this behavior is that the displacement does not increase linearly with the torque. Then increasing the torque the displacement will increase always a bit less until the failure.

#### Gears pair with linear tip relief

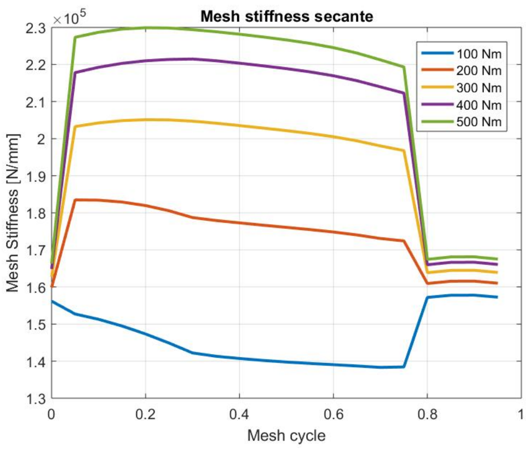


Figure 4.11: Average slope method

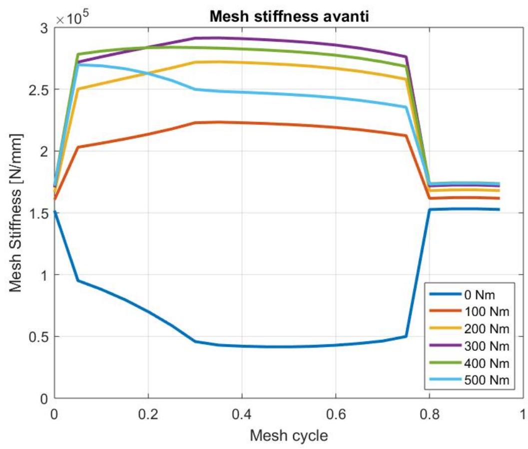


Figure 4.12: Local slope method

In this situation the first feature that can be seen is the trend for the negligible torque. The torque can be considered equal to zero while the unloaded factor is higher than zero, then is the only case where torque is lower than the displacement, so the trend is the opposite of the others.

### Gears pair with lead crown

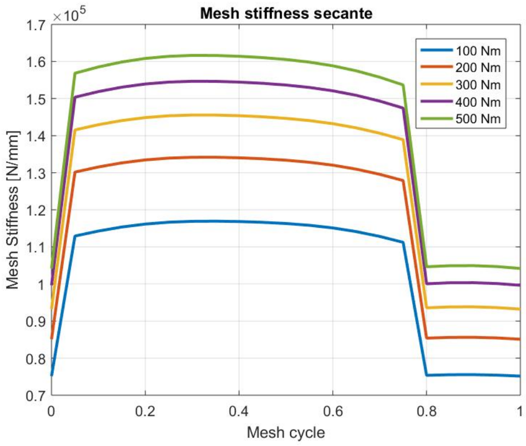


Figure 4.13: Average slope method

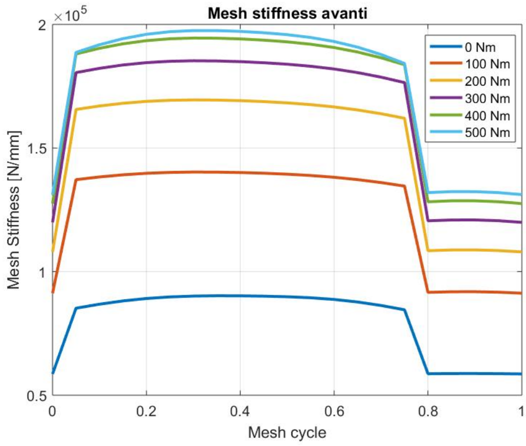


Figure 4.14: Local slope method

In figure 4.7, it has been already noticed that unloaded factor is almost inexistent, then the trends are more similar to the case with no modifications.

In every situations the increase of the load leads to an increase of the mesh stiffness since a bigger torque causes more important displacements.

# Chapter 5

# Body deflection

Until now every simulations has been carried out assuming a rigid connection between gear and shaft. This means that the deformations of the ring body has not been considered in the mesh stiffness calculations.
How body deflection affects the mesh stiffness? How it was already said, mesh stiffness is a tooth characteristic, then, to analyze it, it has been considered only the tooth features.

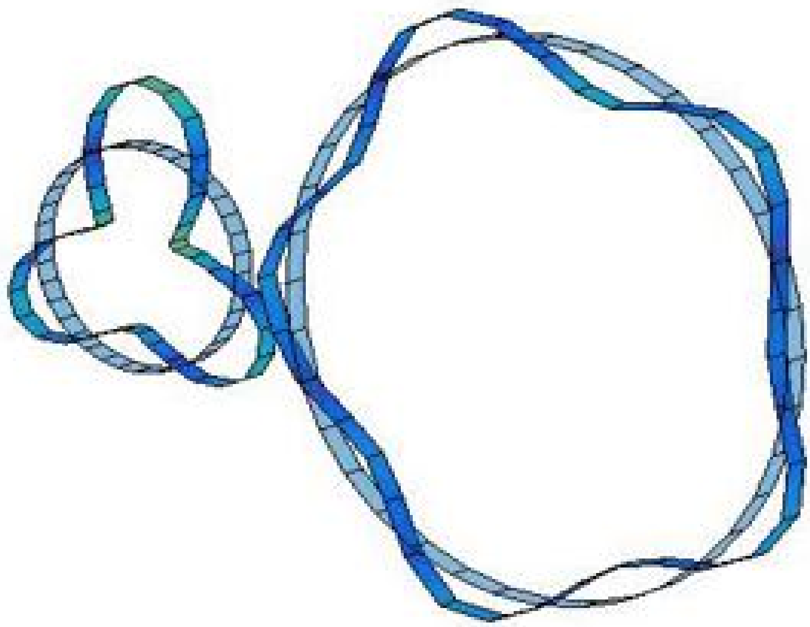

Figure 5.1: Gear body deflection

In the reality, the body is not a rigid ring, the it could suffer some deformations, in particular in this section will be considered radial deformations.
From the analyses the it has been observe that rigid rim leads to a bigger mesh stiffness, but not as much as it was expected.

At first sight, it seems not very intuitive because something rigid could lead to a smaller deflection. That is exact, but it is not the tooth to be rigid but just the rim.
That is obtained with a Transmission 3D feature. When is the moment to choose the connection between the thin shaft and the gear it has to chose IDCONSTRAINED that make the inner diameter of the shaft constrained, and so not free to move. After that the option ODRACE connect the outer diameter of the shaft with the inner diameter of the gear welding them together. At this point is possible to chose the constrain of the inner diameter oh the shaft. It is possible to chose RIGID or FLEXIBLE.

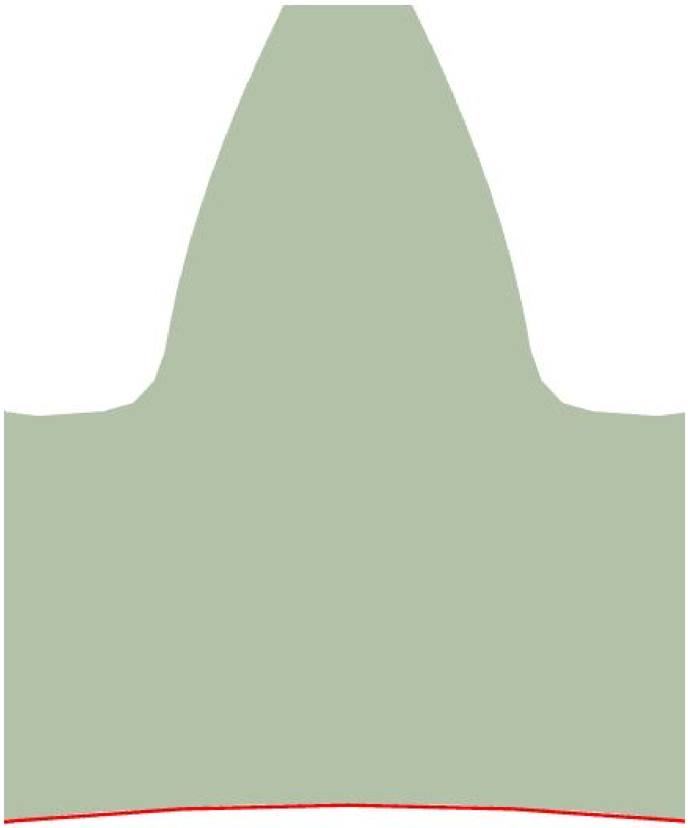

Figure 5.2: Transmission 3D welding between shaft (red) and gear (green)

In the reality is has been demonstrated that the tooth has a dual behavior that depends on the rim thickness. For thin rim, the system is free to move, then the mesh stiffness is very low. When the rim is very thick it acts like a cantilevered beam (fig 5.3) which makes the system less free, favoring tooth deformation. What in the figure 5.3 is V, in the gears is the transmission error.

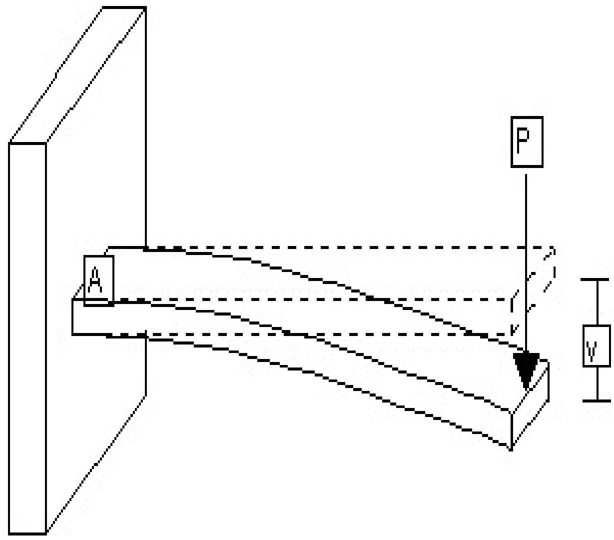


Figure 5.3: Displacement of a cantilevered beam

| | RIGID | FLEXIBLE |
|---|---|---|
| Mesh Stiffness thin rim [N/mm] | 378990 | 18636 |
| Mesh Stiffness thick rim [N/mm] | 208838 | 192801 |

Table 5.1: Mesh stiffness values for rigid and flexible rim

The mesh stiffness values reported in the table 5.1 for thin rim show that the difference between the two values is not negligible. On the other hand, if the thickness is big enough, the values tend to converge.
Then, to make this section complete, it is important to speak about the role of the gears thickness.

## 5.1 Gears thickness

To analyze the influence of the gear thickness it has been used the first model described in chapter 2, whose geometry is summarized in the table 2.1.
The rim thickness has been modified in term of reduction of inner diameter of the gear (and, of course, with a coherent reduction of outer and inner diameter of the shaft) in the Transmission 3D model.
Three cases has been analyzed

- Thicker wheel
- Thicker pinion
- Both gears thicker

Below it is possible to see the T3D model increasing wheel rim thickness until 40%.

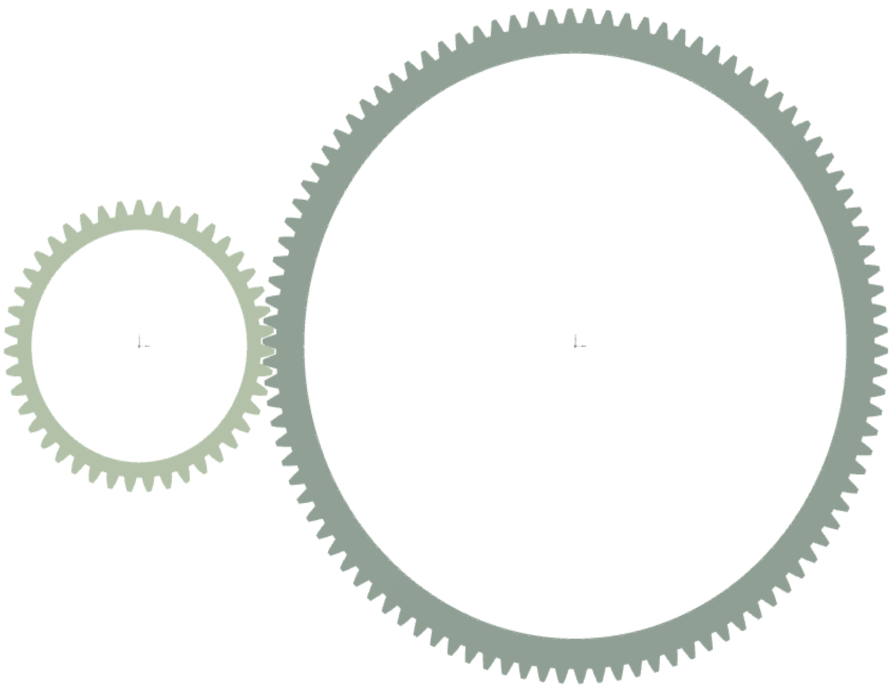

Figure 5.4: Original gears model

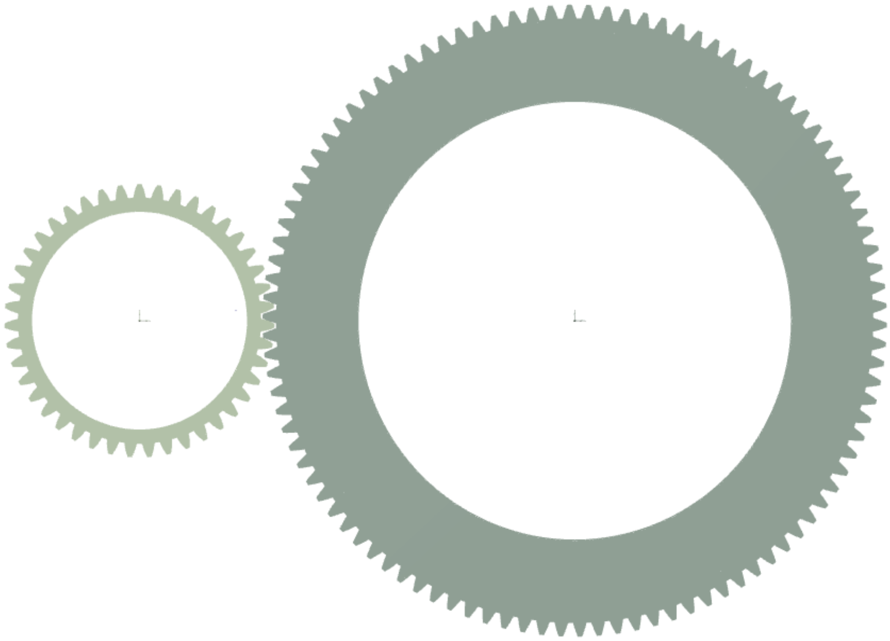

Figure 5.5: Gears model with a wheel inner diameter reduction of 20%

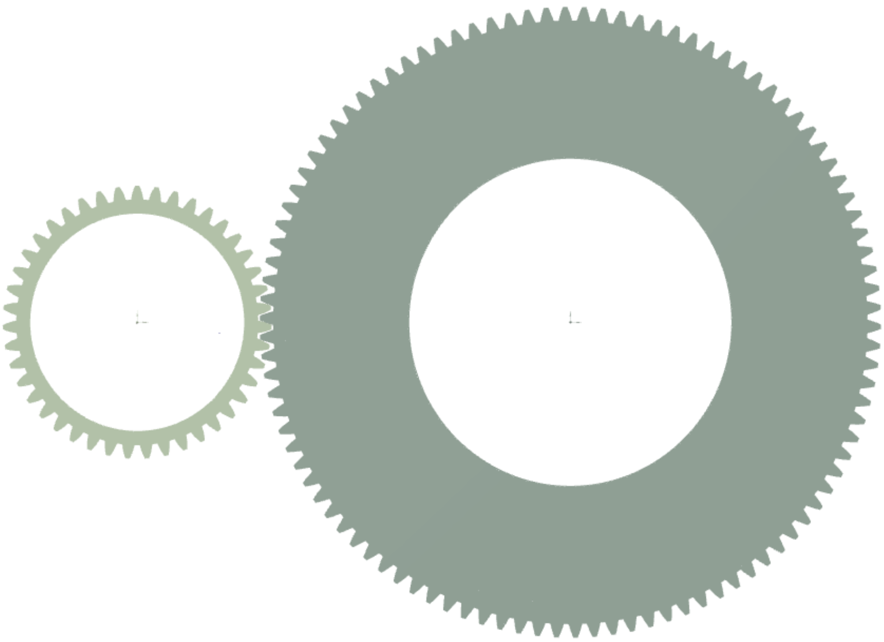

Figure 5.6: Gears model with a wheel inner diameter reduction of 40%

### 5.1.1 Thicker wheel

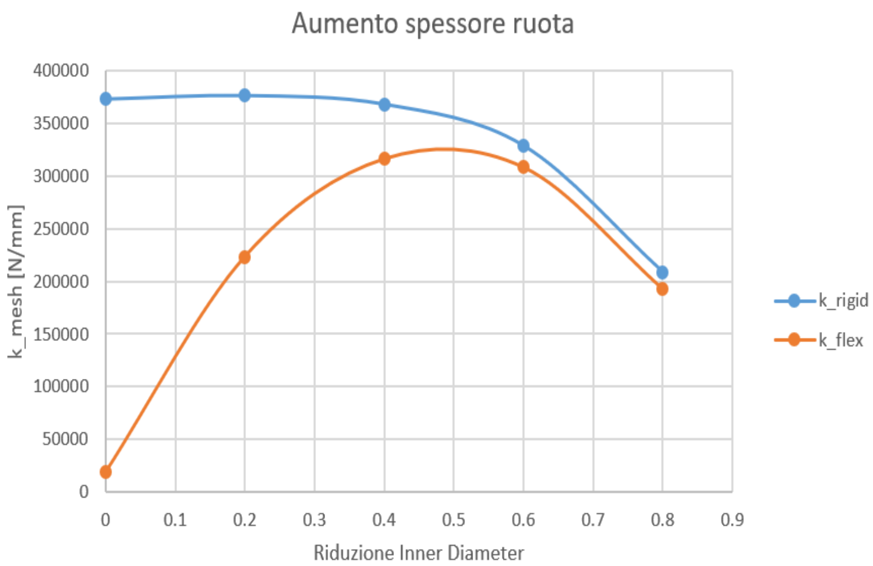


Figure 5.7: Rigid and flexible mesh stiffness for a thicker wheel

As it possible to see in the figure 5.7 the rigid mesh stiffness remains constant until the 40% of reduction, then collapses following a linear decrease. The flexible mesh stiffness, instead, start from a very low value of stiffness because the rim is very thin and the tooth is more free to move. Obviously, increasing the rim thickness also the flexible stiffness increase, but not until the infinite. Around a reduction of 50% flexible stiffness reaches its maximum. After that, the two curves has the same linear trend.
To explain that is necessary to remember how it works the beam theory. From an engineering point of view, a beam is a structure whose longitudinal length is prevailing on the other dimensions. Imaging to break the gear and to stretch along the circumference it would obtain a sort of toothed beam like in the figure 5.8

Figure 5.8: Straight gear

The stiffness of a beam is

$$K = \frac{EA}{L} \tag{5.1}$$

where

- E is the Young modulus
- A is the cross section
- L is the length of the straight gear corresponding to the circumference of the original gear

Now the Young modulus is a constant since increasing the rim thickness does not change the material. Also the length of the gear remains the same, because the pitch diameter does not change. The only characteristic that change is the cross section. Then, from the equation 5.1, it is possible to say that stiffness varies linearly with the cross section. So when the thickness, and then the cross section, is thick enough, the stiffness is so big that the rim could be considered as a very strong constrain for the tooth. It is here that the tooth start is behavior like a beam as explained previously, and that is the reason why the mesh stiffness starts to decrease.

### 5.1.2 Thicker pinion

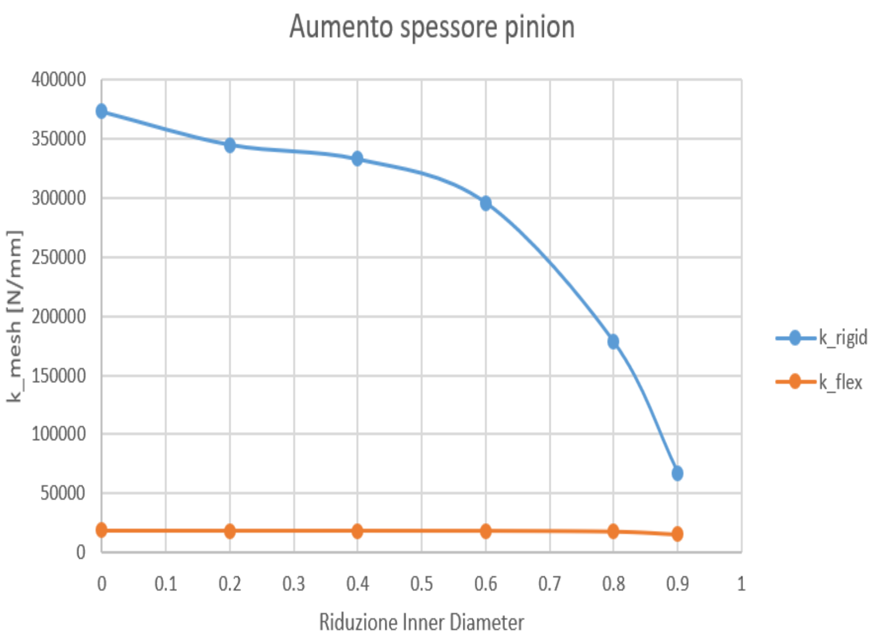


Figure 5.9: Rigid and flexible mesh stiffness for a thicker pinion

How it has been already said, every analysis has been carried out, focusing on the wheel. Then it has been considered the pinion as a infinitely rigid element, and the transmission error has been calculated only for the wheel. Therefore, an analysis where the pinion rim thickness varies could seem senseless. In the reality the reason of this analysis is that a bigger pinion could lead to a bigger tooth wheel displacement.
As it possible to see in the figure 5.9, values of rigid stiffness are the same of the previous, while the flexible stiffness remain constant on low values because the wheel rim remain thin.

### 5.1.3 Both pinion and wheel thicker

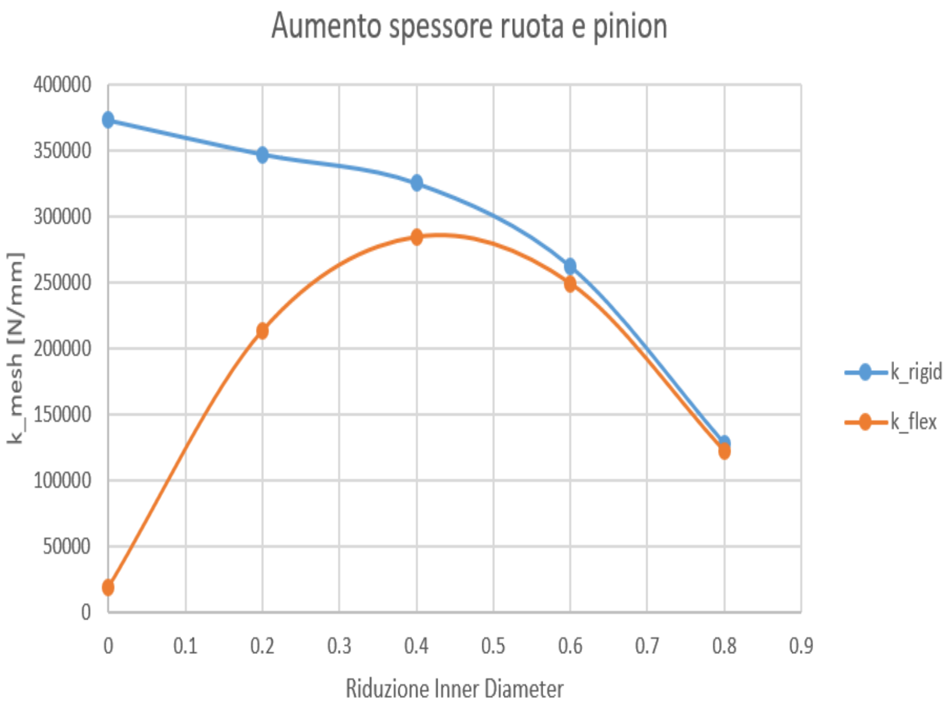


Figure 5.10: Rigid and flexible mesh stiffness for thicker gears

From the evidence of the two cases just mentioned, to increase the rim thickness of both gears leads to a similar situation than the first one. Indeed, figure 5.7 and figure 5.10 are quite the same.

## 5.2 Rigid and flexible analyses on Transmission 3D

As already extensively explained in chapter 3, one of the most critical aspects of Transmission 3D is the duration of the simulations.
One of the aspects that affects this duration is the choice between rigid and flexible analyses. Clearly the second one, is more realistic, but causes an increase of the time that is not negligible. So it is of primary importance to understand when it is really important to have a realistic simulation and when using a rigid model could represents a good approximation of the reality.

| | Pinion rim [mm] | Wheel rim [mm] | Thickness/Outer diameter wheel [%] |
|---|---|---|---|
| 0 | 5.715 | 11.43 | 4.3269 |
| 0.2 | 14.859 | 34.417 | 13.0288 |
| 0.4 | 24.003 | 57.404 | 21.7308 |
| 0.6 | 33.147 | 80.391 | 30.4327 |
| 0.8 | 42.291 | 103.378 | 39.1346 |

Table 5.2: Geometric correspondence in gears model

From the diagrams in the figures 5.7 and 5.10 is evident that around the 40% of the inner diameter reduction flexible and rigid mesh stiffness tend to converge. Obviously, having a gears model, there is not a reduction as reference. Then the idea has been to link this reduction to the outer diameter of the wheel. In the table 5.2 there is the wheel thickness-outer diameter ratio linked to the reduction done in these analyses, so the 40% of inner diameter reduction corresponds to a rim thickness that is the 20% of the outer diameter.
It can be concluded that a flexible analysis is necessary only when the rim thickness is under the 20% of the wheel outer diameter. This could be applied both in static and in dynamic simulations.

# Chapter 6

# Dynamic model

Until now, every analyses has been carried out from a static point of view. As already explained above, a static analysis need very few time steps of calculation to divide a mesh cycle. Since a good number of these steps is 21, is not necessary to work on gears model geometry in order to obtain very fast analyses. This is a positive thing because it is possible to obtain realistic static results. On the other hand, even if static is widely used in the field of aeronautic gearboxes, it is very important also to understand the behavior of the gears from a dynamic point of view.
The problem is that a dynamic analysis needs lots of time of simulation due to several reasons. Two of the most important are

- Mesh cycle must be divided in lots of time steps because of the vibration of the system.
  Imagine to have an harmonic with low frequency like in figure 6.1.

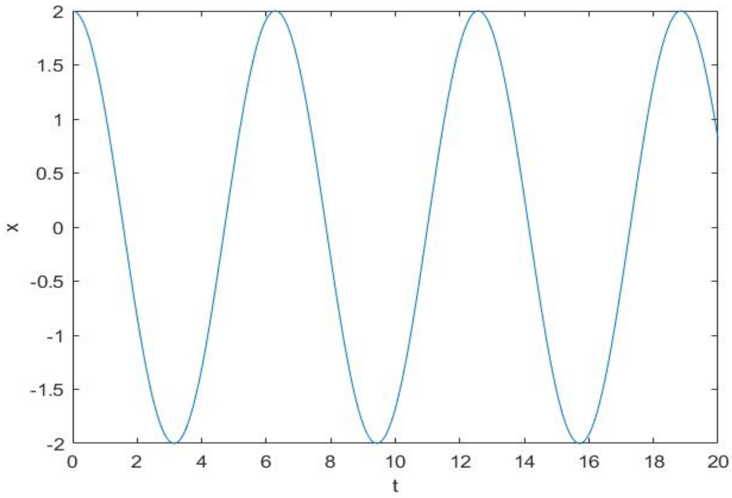


Figure 6.1: Low frequency harmonic

It is possible to discretize it with few time steps without loosing important information. That is an advantage because few time steps leads to small duration of simulation. In transmission 3D simulations have an average duration of six second per step. It is immediately to understand that in static, where there is no need of lots of steps, simulations are quite rapid.
On the other hand in dynamic the situation is more similar to the figure 6.2.

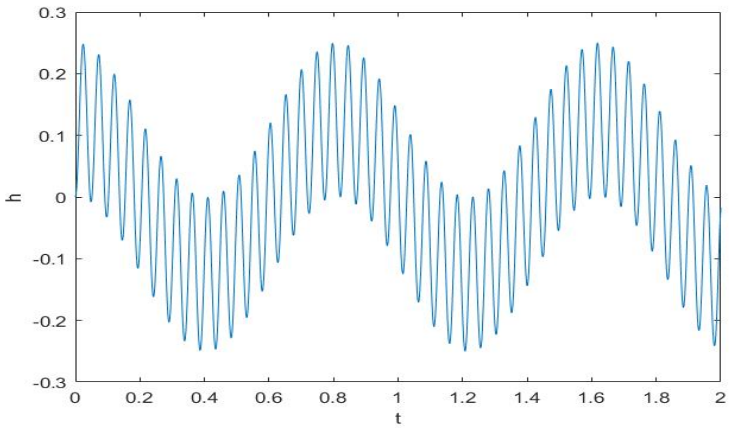


Figure 6.2: High frequency harmonic

Frequency is very high and a small number of time steps could lead to the lost of important fluctuations. Generally, for a single mesh cycle, neglecting the initial transient, at least 1000 are required.

- Before to analyze the real behavior, the initial transient must be overcome.
  Even if in the previous point it has been neglected the initial transient that is not possible.

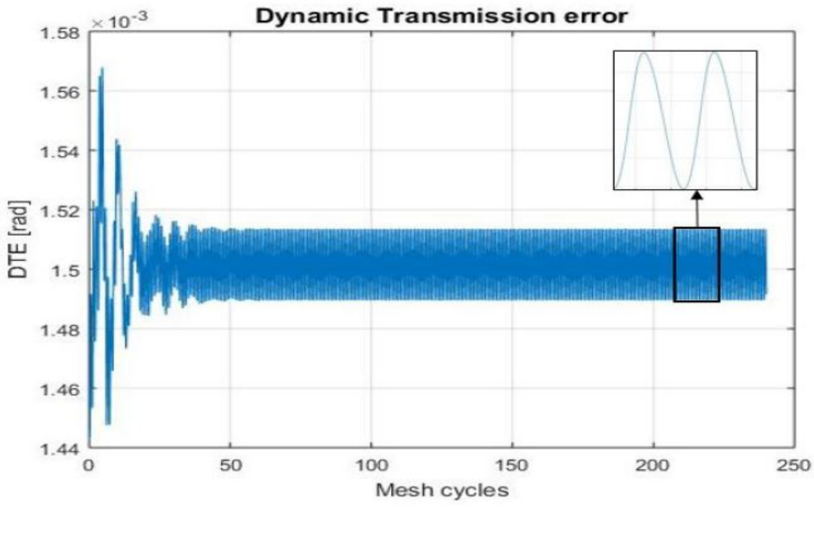


Figure 6.3: High frequency harmonic with initial transient

As it is possible to see in the figure 6.3, for the first 50 mesh cycles the fluctuations are very unsteady. Then, to analyze the first mesh cycle will never return reliable results. It could be captured an high peak in displacements, and this would lead to a very low mesh stiffness. In this case the gear would be designed excessively robust, causing an increase of weight and an increase of costs.

Considering that, it seems that Transmission 3D is not the perfect software for dynamic analyses because of the long duration of simulation.
In absence of any other faster software, the only way is the optimization of the model in order to speed up the simulation.

## 6.1 Transmission 3D optimized model

The idea has been to simulate a 2D software, creating a model with a very low face width, as it is possible to see in the figure 6.4.

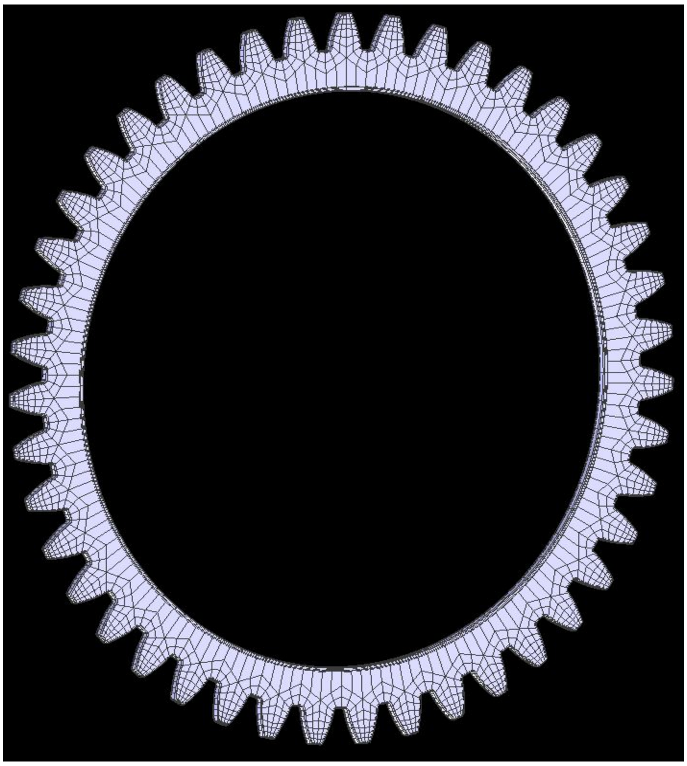

Figure 6.4: Thin model for dynamic analyses

It has been used the mesh template MEDIUM, and the contact grid has the following characteristics

| | |
|---|---|
| ADAPTIVE GRID | OFF |
| DSPROF | 0.08 |
| NPROFDIVS | 5 |
| NFACEDIVS | 1 |
| SEPTOL | 0.1 mm |

Table 6.1: Contact grid parameters for check 1

With these geometric correction and the support of a performing workstation it has been possible to pass form a simulation of six seconds per step to an analysis of one second per step, with a reduction of the 80%.
Clearly, this model presents some limitations. The main one is the impossibility to consider some kind of modification like lead crown.
In conclusion, this model could be used for dynamic analyses of simple gears pair like spur gears with at the most, a linear or parabolic tip relief.

# Chapter 7

# Conclusion

In this thesis work have been presented some methods to optimize gears design.

It has been presented how to use two different software for the same reason maximizing their qualities and minimizing defects.

Always in order to minimize the lost of time, two different methods, with different computational time, for mesh stiffness calculation have been analyzed in order to chose the most suitable for every situations.

It has been developed a practice that permits to chose the most rapid kind of analysis just knowing the geometry of the gears.

In the end has been presented a gears model that permits to reduce time simulation for dynamic in Transmission 3D.

## 7.1 Future developments

In the future could be useful to extend analyses done maintaining constant modification varying them. For example

- To use a parabolic tip relief instead of the linear one.
- To evaluate how mesh stiffness varies moving tip relief under the pitch diameter.

Moreover, every simulations has been carried out using spur gears models so it could be interesting to create models with helical or bevel gears.
The most important limitation of this work has been the impossibility to analyze real gears models from a dynamic point of view. Then it would be very important to understand

how to calculate mesh stiffness with a dynamic approach with short times. Average slope method is not appropriate because of the big fluctuations that characterize dynamic, while local slope method require very long simulations at least twice.
In the end a big challenge could be the possibility to understand the behavior of the transmission error in such a way to calculate without the support of commercial software in order to reduce costs and to be more efficient. About this, extensive research on the gear geometry could lead to new discoveries related to transmission error and, then, to mesh stiffness.

# Appendix A

# Transmission 3D contact grid

This appendix has the purpose to prove how the grid parameter in Transmission 3D has been set. As explained, Transmission 3D do not use finite element model to calculate the contact pressure. In fact, it uses an independent grid, enabling to use also a coarse mesh but with very fine grid parameters, even tough fine grid is not always the best option. First, just remind the parameters definition:

**SEPTOL**

Is the separation tolerance. Surfaces separated by more than this distance will not considered in the contact analysis.

**NPROFDIVS**

This variable controls the number of contact patches or cels that will be used to cover the contact zone between gear teeth.

**NFACEDIVS**

This variable controls the number of contact patches or cell that will be used along length of the contact zone

**DSPROF**

This variable controls the width of the contact zone in physical length units. It is the most important parameter.
This last parameter is the most important one because it defines the shape of the contact pressures. The software offers the ADAPTIVEGRID solution, that automatically sets the profile element's width. In dynamic analysis it has been proven that it is not a reliable solution, since in some cases it reduces to much this size letting the pressure to diverge.

As an example, a test varying DSPROF has been done at constant speed and checking the contact pressures pattern at the same time instant in all cases. From the figures it is possible to notice how the ADAPTIVEGRID can be dangerous and how the shape of the contact pattern changes with the parameters.

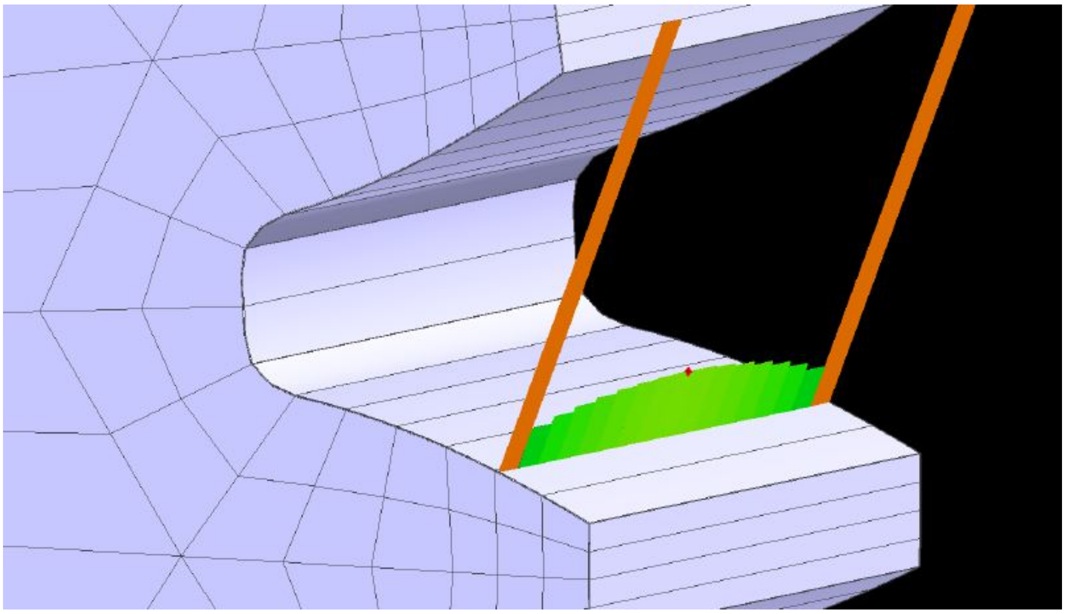

Figure A.1: Contact pressure check 1

| | |
|---|---|
| ADAPTIVE GRID | ON |
| NPROFDIVS | 8 |
| NFACEDIVS | 8 |
| SEPTOL | 0.1 mm |

Table A.1: Contact grid parameters for check 1

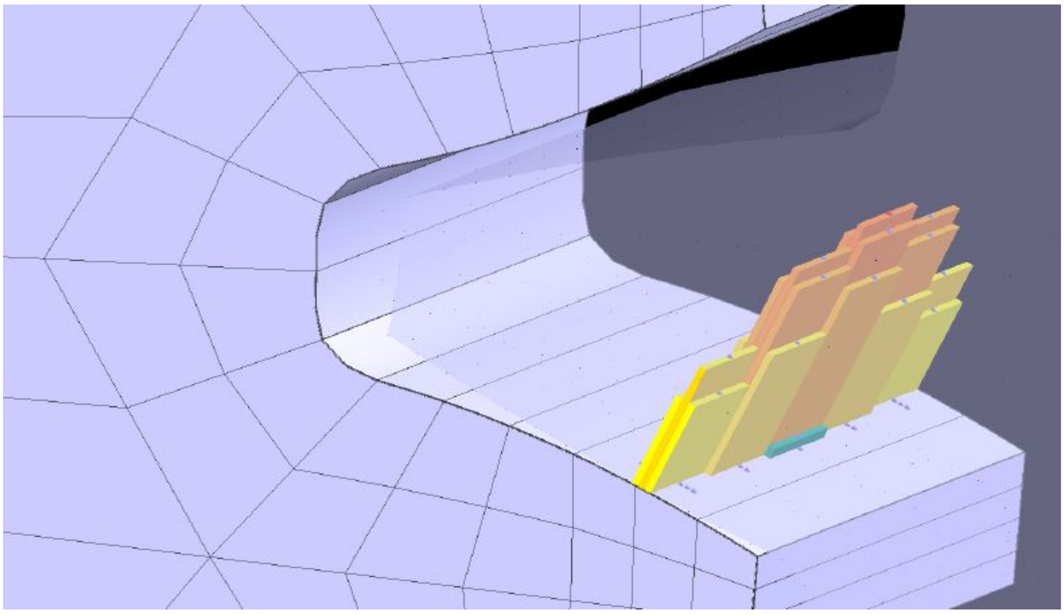

Figure A.2: Contact pressure check 2

| | |
|---|---|
| ADAPTIVE GRID | OFF |
| DSPROF | 0.09 mm |
| NPROFDIVS | 4 |
| NFACEDIVS | 4 |
| SEPTOL | 0.1 mm |

Table A.2: Contact grid parameters for check 2

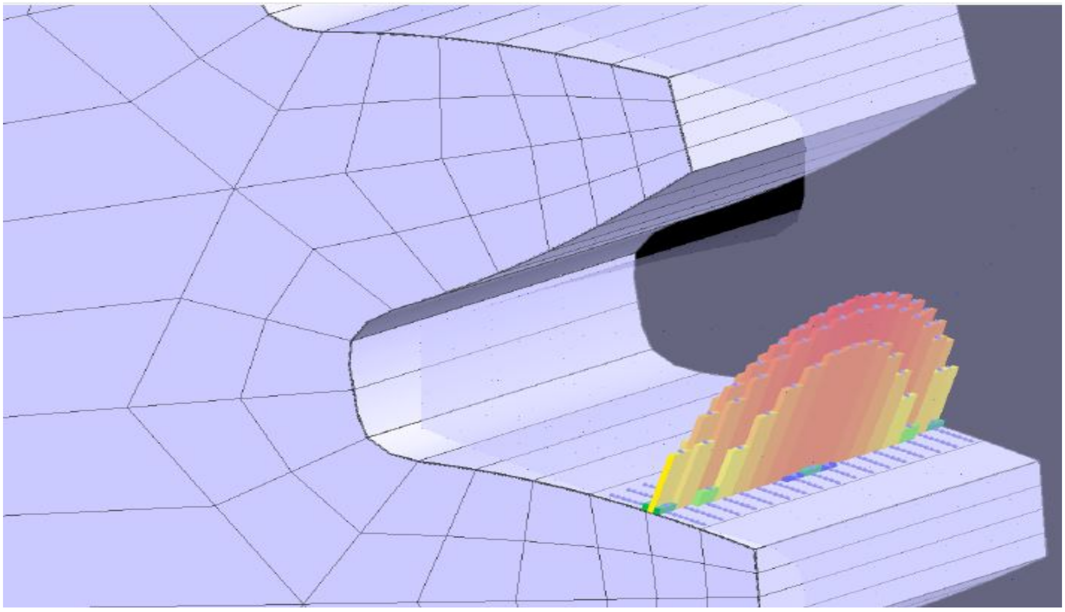

Figure A.3: Contact pressure check 3

| | |
|---|---|
| ADAPTIVE GRID | OFF |
| DSPROF | 0.09 mm |
| NPROFDIVS | 8 |
| NFACEDIVS | 8 |
| SEPTOL | 0.1 mm |

Table A.3: Contact grid parameters for check 3

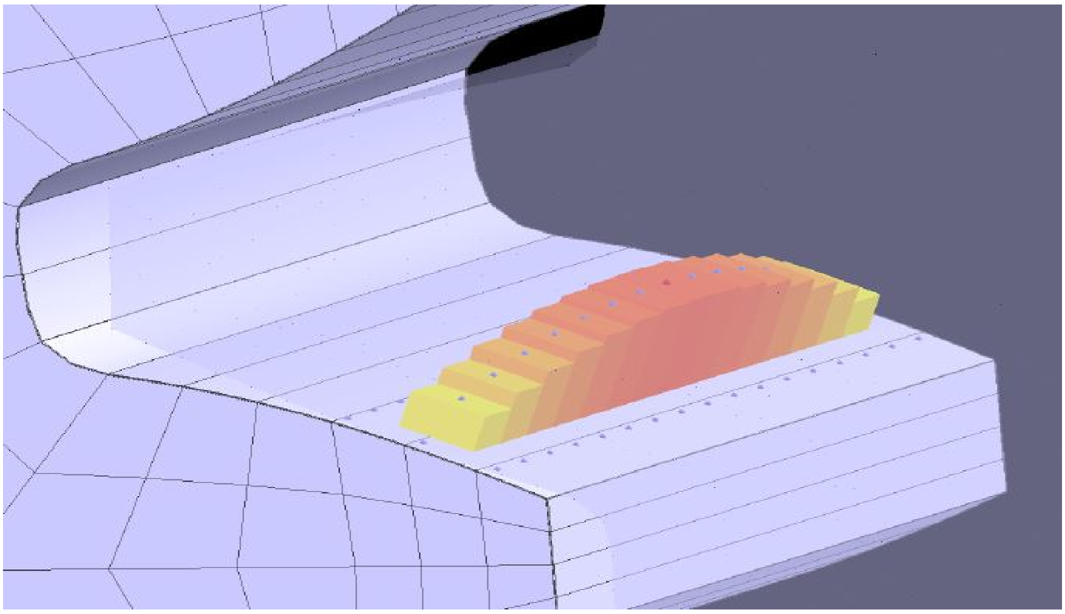

Figure A.4: Contact pressure check 4

| | |
|---|---|
| ADAPTIVE GRID | OFF |
| DSPROF | 0.9 mm |
| NPROFDIVS | 8 |
| NFACEDIVS | 8 |
| SEPTOL | 0.1 mm |

Table A.4: Contact grid parameters for check 4

# Appendix B

# Matlab tool

Below is presented the Matlab tool developed to interpolate the average transmission error and to implement every methods of mesh stiffness calculation.

---

```

close all
clear all
clc
format long
%----------Input--------------------------

rb_rt = 121.728;                % [mm] Raggio ruota
Nom_Torque = 250e3 / rb_rt;     % [N] Forza alla quale voglio stiffness
m = 4;                          % Ordine del polinomio interpolante

%----------Importazione valori------------

% txt = importdata('no_mod.txt');
% txt = importdata('te_relief.txt');
txt = importdata('te_crown.txt');
% txt = importdata('relief_crown.txt');

x = txt(:,2) * 10^(-3);
% [mm] Nel vettore x metto i valori di errore medio con modifiche
y = txt(:,1) * 10^3 / rb_rt;
% [N] Nel vettore y metto le forze

%----------Interpolazione----------------

n = length(x);
```

```
a = x(1);                                   % Coppia di partenza
b = x(n);                                   % Coppia finale

p = polyfit(x,y,m);             % Interpolazione
xx = linspace(a,b,500);         % Passaggio per plottare la f interpolante
yy = polyval(p,xx);             % Passaggio per plottare la f interpolante
der = polyder(p);               % Derivo il polinomio interpolante

%----------Generazione grafici------------

figure
plot(xx,yy,'linewidth',2)     % Plotto la funzione interpolante
ylabel('Force␣[N]')
xlabel('STE␣[mm]')
grid on
hold on

i=1:10:length(xx);
xx_p = xx(i);


i=1:10:length(yy);
yy_p = yy(i);

% plot(xx_p,yy_p,'d','linewidth',2)    %Plotto la funzione interpolante
                                        % per punti

% plot(x,y,'linewidth',2)     %Plotto la funzione con i punti di input
% hold on

c = Nom_Torque;           % Coppia corrispondente all'STE per cui voglio
                          % sapere la derivata [Nm]

k = 0;
for i=1:1:length(x)       % Questo ciclo for serve a individurare l'STE
    if y(i) <= c          % corrispondente alla Nom_Torque
        k=k+1;
    end
end

t = x(k);               % [mm] STE corrispondente alla Nom_Torque
```

```
espn = zeros(1,length(der)); % Questo ciclo for serve a implementare
for f=1:1:length(der)-1      % automaticamente il vettore degli
    espn(f+1) = espn(f) + 1; % esponenti della derivata del polinomio
end                          % a seconda dell'ordine m che scelgo
esp = fliplr(espn);

der_val= 0;
% Questo ciclo for serve a valutare la derivata
for j=1:1:m
% del polinomio alla coppia desiderata
    der_hand =  der(j) * t^(esp(j));
    der_val = der_val + der_hand;
end

u = linspace(a, b, 10000);          % Discretizzo l'asse delle coppie
y_q = y(k); % Il ciclo for serve a trovare il te relativo alla nom_torque
q = y_q - der_val * t;        % Trovo il termine noto della retta
y_retta = der_val * u + q;  % Equazione della retta tangente nel punto

% plot(u,y_retta,'linewidth',2)
% hold on
%
disp(['Mesh␣stiffness␣con␣derivata␣=␣' num2str(der_val) '␣N/mm']);

%% Calcolo con metodo parker (avanti e centrato)

% Questo script e' affetto da un piccolo errore dovuto al fatto che la
% funzione che interpola le y (cioe' le forze) non da un valore nullo
% per il valore di STE che invece da valore nullo di coppia. E'
% comunque trascurabile

c = Nom_Torque;           % Coppia per cui voglio sapere la derivata [Nm]
t = x(k);                 % Errore corrispondente alla coppia [mm]

espn1 = zeros(1,length(p)-1);    % Questo ciclo for serve a implementare
for f=1:1:length(p)-1            % automaticamente il vettore degli
    espn1(f+1) = espn1(f) + 1;   % esponenti del polinomio a seconda
end                              % dell'ordine m che scelgo
esp1 = fliplr(espn1);
```

```
poly_val= 0;
% Questo ciclo for serve a valutare il polinomio
for j=1:1:m+1
% alla forza desiderata. E' fatto in modo che scegliendo
    poly =  p(j) * t^(esp1(j));
% La coppia, automatico trova l'STE nelle x. In pratica
    poly_val = poly_val + poly;
% aggira il fatto di dare in input la y invece che la x
end

poly_val_1= 0;
% Questo ciclo for serve a valutare il polinomio
for j=1:1:m+1
% all'errore maggiorato dell'1%, quindi mi dice F(STE+0.01STE)
    poly =  p(j) * (t + 0.01 * t)^(esp1(j));
    poly_val_1 = poly_val_1 + poly;
end

poly_val_2= 0;
% Questo ciclo for serve a valutare il polinomio
for j=1:1:m+1
% all'errore minorato dell'1%, quindi mi dice F(STE-0.01STE)
    poly =  p(j) * (t - 0.01 * t)^(esp1(j));
    poly_val_2 = poly_val_2 + poly;
end

for i=1:1:length(x)
 if exist('x(i)<=0')
  s = 0;
% Questo ciclo for serve ad individurare l'indice per cui
  for j=1:1:length(y)
% la funzione interpolante si annulla, quindi per trovare
     if yy(j) <= 0
% l'ascissa corrispondente,ovvero xx(s)
         s=s+1;
     end
  end
 else
   s = 1;
 end
end
```

```
k_mesh_avanti = (poly_val_1 - poly_val) / (0.01 * t );
% Calcolo la k_mesh con diff in avanti
k_mesh_centrata = (poly_val_1 - poly_val_2) / (2 * 0.01 * t );
% Calcolo la k_mesh con diff centrata
k_mesh_secante = poly_val / (t - xx(s));
% Calcolo la k_mesh con metodo secante

q_av = y_q - k_mesh_avanti * t;
% Trovo il termine noto della retta
y_av = k_mesh_avanti * u + q_av;
% Retta differenze in avanti

q_cent = y_q - k_mesh_centrata * t;
% Trovo il termine noto della retta
y_cent = k_mesh_centrata * u + q_cent;
% Retta differenze centrate

u_sec = linspace(a, t, 10000);
% Discretizzo l'asse delle coppie
q_sec = -k_mesh_secante * xx(s);                % Retta secante
y_sec = k_mesh_secante * u_sec + q_sec;

plot(u,y_av,'linewidth',2)
hold on
% plot(u,y_cent,'linewidth',2)
% hold on
plot(u_sec,y_sec,'linewidth',2)
hold on

legend('Funzione␣Interpolante','Punti␣interpolati', 'Tangente␣esatta',...
'Differenza␣in␣avanti','Differenza␣centrata','Secante')


disp(['Mesh␣stiffness␣Parker␣avanti='num2str(k_mesh_avanti)'N/mm']);
disp(['Mesh␣stiffness␣Parker␣centrata='num2str(k_mesh_centrata)'N/mm']);
disp(['Mesh␣stiffness␣Parker␣secante='num2str(k_mesh_secante)'N/mm']);
```

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