POLITECNICO DI TORINO

Master's Degree in Mechanical Engineering

Master's Degree Thesis

Combustion engine crankshaft balancing analysis

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April 2019

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Abstract

This thesis is focused on the study of the crankshaft balancing. The aim of the work is to analyze different engine configurations in terms of cylinders number and layout and, therefore, different crankshaft arrangement. Both *In-Line engines* and *V engines* are treated.

All of the above is done by developing a Matlab script that allows to quickly define and set up a first approximation crankshaft design in terms of engine type (In-line or V-engine), number of cylinders, crank throws disposition, geometrical properties and some other key factors in order to scan rapidly between several possibilities of crankshaft layouts to check which forces and moments are "naturally" balanced and which are not. The Matlab script also allows to implement a balancing strategy to balance the first order rotating forces and moments.

Once the crankshaft arrangement and the balancing strategy are defined, the reactions on the main journals are derived.

All the results are shown through representative plots, depending on the crank angle.

This script can be useful to have, as already said to a first approximation, the orders of magnitude of the forces acting on the system in order to have a first talk with the bearing supplier.

Acknowledgments

A chiusura di questi cinque lunghi anni di Università che mi hanno portato, per forza di cose, ad allontanarmi dall'unico posto che ritengo possa essere chiamato Casa, sento la necessità di ringraziare innanzitutto chi, questa Casa, ha cercato di farmela sentire meno distante.

Grazie a tutta la mia famiglia.

Grazie ai miei Genitori per avermi supportato e sopportato in qualunque situazione mi sia trovato davanti, nei momenti difficili ed anche in quelli "facili", cercando di non farmi sentire il peso di un sacrificio che non tutti sono disposti a compiere. Senza di loro tutto ciò non sarebbe stato possibile, e se sono riuscito a raggiungere questo traguardo il merito va anche a loro.

Grazie ai miei Nonni, per i loro sempre utili consigli da maestri di vita.

Grazie ai miei Zii, perchè nonostante le mie assenze più o meno lunghe sanno sempre farmi sentire il loro affetto.

Grazie a Caterina, perchè malgrado le distanze, malgrado le incomprensioni e le differenze di vedute, sei sempre stata un porto sicuro in cui tornare e trovare tranquillità e amore.

Grazie ai miei pochi ma buoni amici, a quelli che hanno condiviso "da Casa" questo mio percorso ed a quelli che ho trovato qui a Torino, per avermi alleggerito le giornate strappandomi un sorriso nei momenti più difficili, e per aver diviso con me un carico spesso troppo pesante da portare da soli.

Un grazie va anche ai miei Relatori di Tesi, al Prof. Massimo Sorli ed al Prof. Stefano Mauro, per il loro supporto e per avermi dato la possibilità di cimentarmi in quest'ultima sfida universitaria.

Thanks to all the guys and girls of the McLaren Woking Team for their help, support and knowledge.

Un grazie va infine ad Angelo, persona da ammirare sia sotto l'aspetto professionale che umano. Grazie per aver condiviso con me la tua conoscenza e per avermi dato la possibilità di confrontarmi con un mondo a me nuovo.

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Chapter 1

Introduction

The engine balance is a subject that covers many areas in the design, production and tuning. An engine as a whole can be considered balanced when it produces an acceptable level of vibrations and stress on the engine supports and on the main bearings.

The reasons why an engine needs to be balanced are fundamentally two:

- To have as much as possible constant reactions on the engine supports.
- To reduce the main bearings loads.

The first one can be considered as an *"external reason"* because is referred to the relationship between the engine and the external environment, the second one instead is an *"internal reason"* because it concerns on what happen inside the crankcase and in particular on the crankshaft, and this thesis work is focused on it.

The engine vibrations can be broadly divided into two categories [1]:

1. Vibrations of the engine and its rigid component as a whole, in which no elastic yielding of the various components is considered. These vibrations are caused by the imbalance of rotating and reciprocating components.
2. Vibrations of engine parts due to the elastic deformation of the materials under the influence of periodic combustion impulses that causes torsional and lateral oscillations of the crankshaft and camshaft.

In this thesis work, only the first category of vibrations listed is treated. These vibrations, due to the forces and moments of the rotating and reciprocating masses, are eliminated doing the *crankshaft equilibrium*, where the latter is considered as a rigid straight shaft with these masses opportunely located along the crankshaft length at a distance equal to the crank radius from the axis of rotation.

In order that the crankshaft is completely balanced, both the statical and the dynamical balance must be achieved.

A crankshaft is statically balanced when the resultant of the centrifugal forces is equal to zero, that is when its centre of gravity is on the axis of rotation.

A crankshaft is dynamically balanced when the resultant of the moments due to the centrifugal forces is equal to zero for any reference point considered.

The aim of this thesis work is to create a Matlab script that allows to study and analyze different engines, and therefore crankshaft, configurations in terms of engine type (In-line engine of V-engine), number of cylinders, crank throws arrangement and so on, in order to examine the engine state-of-balance, apply a balancing strategy and finally calculate the reactions on the main bearings.

Based on the assumptions made the crankshaft is considered ad a monodimensional hyperstatic beam, therefore to find the reactions on the main bearing it was necessary to use the *Three-moment equation of Clapeyron* which allows to solve hyperstatic beams.

In order to give the reader a better view of the logic with which the work was developed, the contents of each chapter are reported briefly below.

In the Chapter 2 the slider-crank mechanism is analyzed. The latter is the simplest system and all the following reasonings are based on that. In particular, in the chapter 2 are shown all the mathematical relations that regulate this system both in the configurations with or without pin offset. In the final part the analysis of the forces that acting on this system is shown.

In the Chapter 3 the fundamental theory of the engine balancing is treated. The involved forces and the method of analisys is shown. Moreover, the parameter *Balancing Factor* is introduced and the relationship between the BF and the crankshaft weight is shown.

Chapter 4 is dedicated to the analysis of the Inline engines and for each of them at least one configuration is shown. For each layout the first and second order states-of-balance of the crankshaft are examined and the bearings loads calculus procedure is reported.

The Chapter 5 is similar to the chapter 4, but it concerns about V-engines.

The Chapter 6 shows how the Matlab code works. The figures produced by Matlab are also reported and analyzed giving the conclusions.

Chapter 2

The Slider-Crank Mechanisms

The slider-crank mechanism is widely applied in gasoline and diesel engines, where the gas force acts on the slider and the motion is transmitted through the connecting rod, then this type of mechamism converts the reciprocating motion of the piston in rotating motion of the crankshaft. The system can be *In-line* type or *Offset* type.

2.1 The In-line Slider-Crank Mechanisms

The In-line slider-crank mechanism has its slider (or piston) positioned so that the slider axis crosses the crankshaft axis of rotation (Figure 2.1) [2].

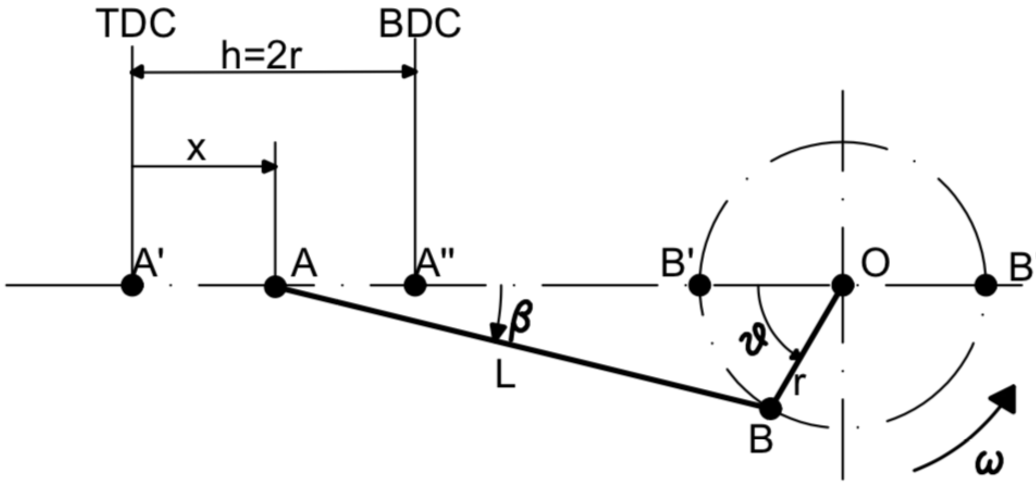

Figure 2.1: In-line slider-crank mechanism.

In the Figure 2.1 it can be seen:

- L = Conrod lenght.
- r = Crank radius.
- h = Stroke.
- θ = Crank rotation respect the TDC.
- β = Angle between conrod and piston axis.
- x = Piston motion, referred to the TDC.

In order to find the velocity and the acceleration of the piston, the kinematics relation between the terms θ and x is needed [3].

Considering the Figure 2.1, it can be written:

$$r \sin\theta = L \sin\beta \rightarrow \sin\beta = \frac{r}{L}\sin\theta = \lambda \sin\theta \tag{2.1}$$

$$\Rightarrow \cos\beta = \sqrt{1 - \lambda^2 \sin^2\theta}; \tag{2.2}$$

with $\lambda = \frac{r}{L}$ ratio.

The stroke h can be expressed as:

$$h = rcos\theta + Lcos\beta = r\left[\cos\theta + \frac{1}{\lambda}\cos\beta\right]; \tag{2.3}$$

and the piston motion x is:

$$x = r(1 - cos\theta) + L(1 - cos\beta) = r(1 - cos\theta) + L(1 - \sqrt{1 - \lambda^2 \sin^2\theta}); \tag{2.4}$$

Differentiating the term x and assuming $\sqrt{1 - \lambda^2 sin^2\theta} \cong 1$, the piston velocity expression is:

$$v = \dot{x} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \ldots = \omega r\left(\sin\theta + \frac{1}{\lambda}\frac{2\lambda\sin\theta\cos\theta}{2\sqrt{1 - \lambda^2\sin^2\theta}}\right) \cong \omega r\left(\sin\theta + \frac{\lambda}{2}\sin 2\theta\right); \tag{2.5}$$

and the piston acceleration expression is:

$$a = \ddot{x} = \frac{d^2x}{d\theta^2}\frac{d^2\theta}{dt^2} = \omega^2\frac{d^2x}{d\theta^2} = \ldots \cong \omega^2 r(\cos\theta + \lambda\cos 2\theta); \tag{2.6}$$

Typical trends of motion, velocity and acceleration of an *In-line Slider-Crank mechanism* are shown in the figures below (2.2, 2.3, 2.4):

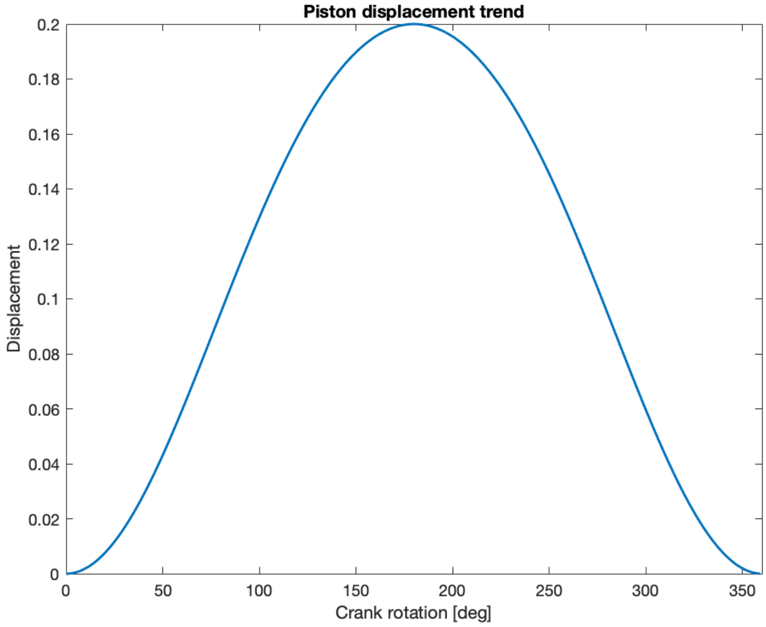

Figure 2.2: Piston displacement of an inline mechanism.

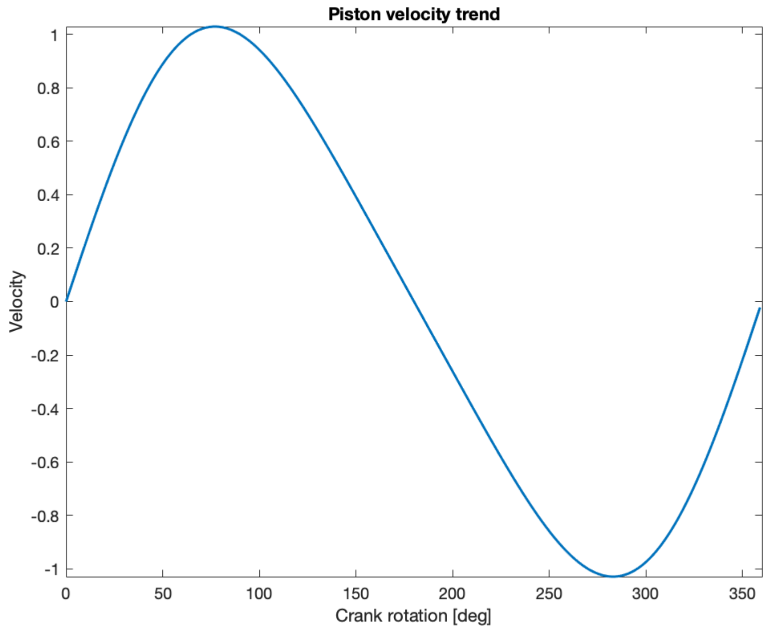

Figure 2.3: Piston velocity of an inline mechanism.

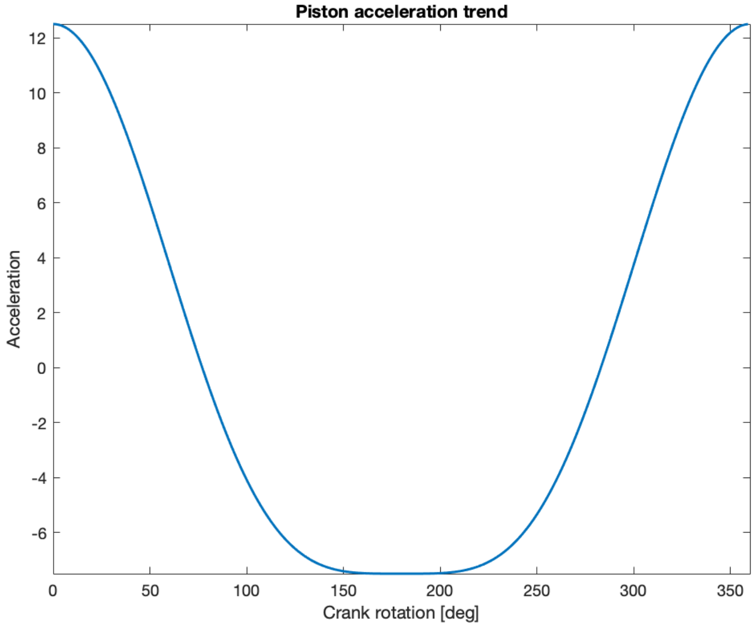

Figure 2.4: Piston acceleration of an inline mechanism.

2.2 The Offset Slider-Crank Mechanisms

As can be seen in the Figure 2.5, in this case the cylinder axis does not cross the axis of rotation of the crankshaft, but there is an offset concordant to the crankshaft rotation verse when the piston is at the TDC.

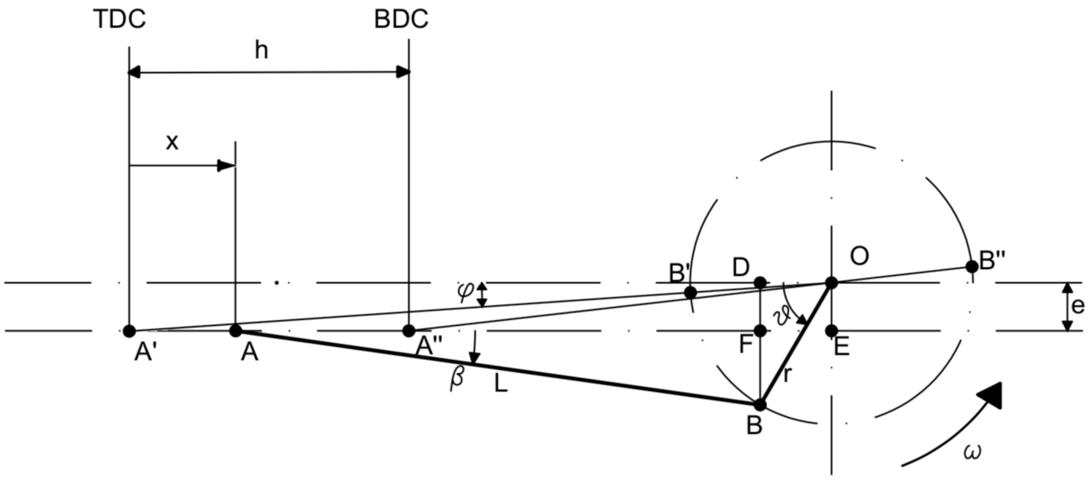

Figure 2.5: Offset slider-crank mechanism.

The term e in the Figure 2.5 represents the offset, typical offset values are $0.8 \div 1$ *mm*.

In this way the conrod has a lower inclitation angle during the *Intake* and *Combustion* phases and an higher inclination angle during the *Compression* and *Exhaust* phases. This is useful, in particular in Diesel engines, because allows to reduce the lateral thrust between the piston and the cylinder chamber during the expansion phase. Because of the offset, the piston dead centres do not coincide with the crank dead centres and the *Intake* and *Combustion* have an angle slightly higher than 180°, while the *Compression* and *Exhaust* phases have an angle slightly lower than 180°.

Analyzing the system in Figure 2.5, the piston motion x can be written as:

$$x = A'E - AF - FE = (L + r)\cos\phi - L\cos\beta - r\cos\theta; \tag{2.7}$$

The conrod tilt angle can be derived considering that:

$$DB = r\sin\theta = L\sin\beta + \phi \rightarrow \sin\beta = \frac{r}{L}\sin\theta - \frac{e}{L}; \tag{2.8}$$

and assuming $\lambda = \frac{r}{l}$ and $\delta = \frac{e}{L}$:

$$\Rightarrow \sin\beta = \lambda\sin\theta - \delta; \tag{2.9}$$

This relation allows to obtain, for each value of the crank angle θ, the conrod angle β. Considering that:

$$\cos\beta = \sqrt{1 - sin^2\beta} = \sqrt{1 - (\lambda\sin\theta - \delta)^2}; \tag{2.10}$$

$$\cos\phi = \sqrt{1 - sin^2\phi} = \sqrt{1 - (\delta/(1 + \lambda))^2}; \tag{2.11}$$

by replacing in the expression of the piston motion x:

$$x = r\left[(1 + \frac{1}{\lambda})\sqrt{1 - \left(\frac{\delta}{1 + \lambda}\right)^2} - \cos\theta - \frac{1}{\lambda}\sqrt{1 - (\lambda\sin\theta - \delta)^2}\right]; \tag{2.12}$$

Differentiating the term x the expression of the velocity of the piston can be obtained:

$$v = \dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \omega r\left[\sin\theta + \frac{\cos\theta(\lambda\sin\theta - \delta)}{\sqrt{1 - (\lambda\sin\theta - \delta)^2}}\right]; \tag{2.13}$$

and the piston acceleration epression is:

$$a = \ddot{x} = \frac{d^2x}{d\theta^2}\frac{d^2\theta}{dt^2} = \omega^2 r\left[\cos\theta + \frac{\lambda\cos^2\theta - \sin\theta(\lambda\sin\theta - \delta)}{\sqrt{1 - (\lambda\sin\theta - \delta)^2}} + \frac{\lambda\cos^2\theta(\lambda\sin\theta - \delta)^2}{(1 - (\lambda\sin\theta - \delta)^2)^{3/2}}\right]; \tag{2.14}$$

The parameter δ is generally in the range $0 \div 0.045$.

The Figures 2.6, 2.7 and 2.8 below show a comparison between trends of an *Inline Slider-Crank* and an *Offset Slider-Crank* with the same values of $r, \lambda, omega$ and with $\delta = 0.03$.

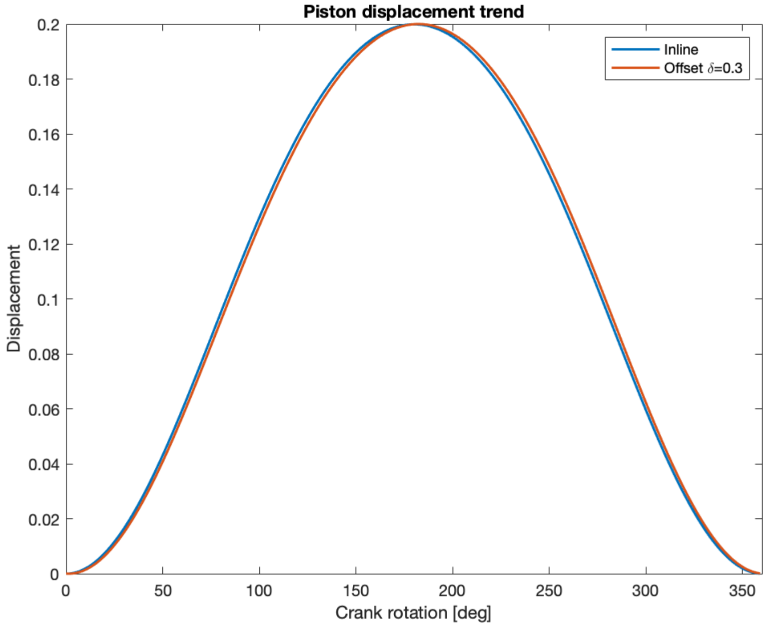

Figure 2.6: Inline vs. Offset displacement comparison.

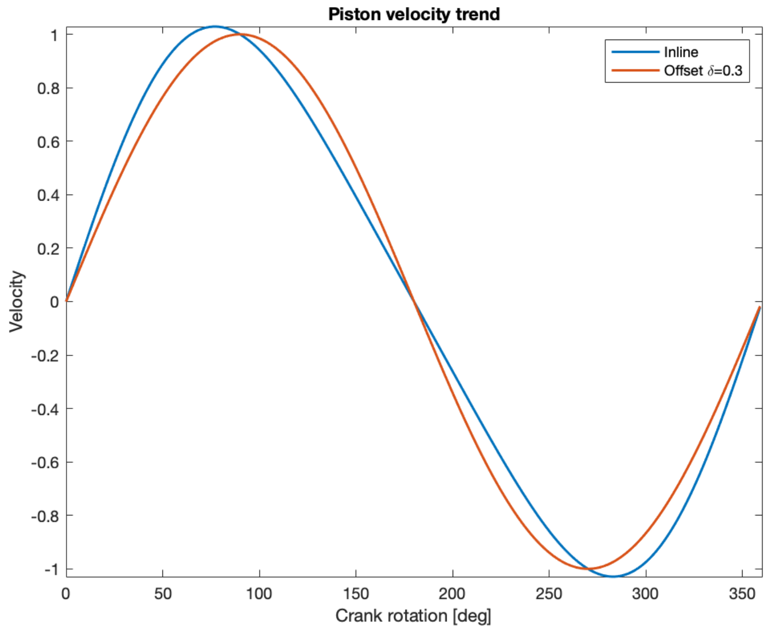

Figure 2.7: Inline vs. Offset velocity comparison.

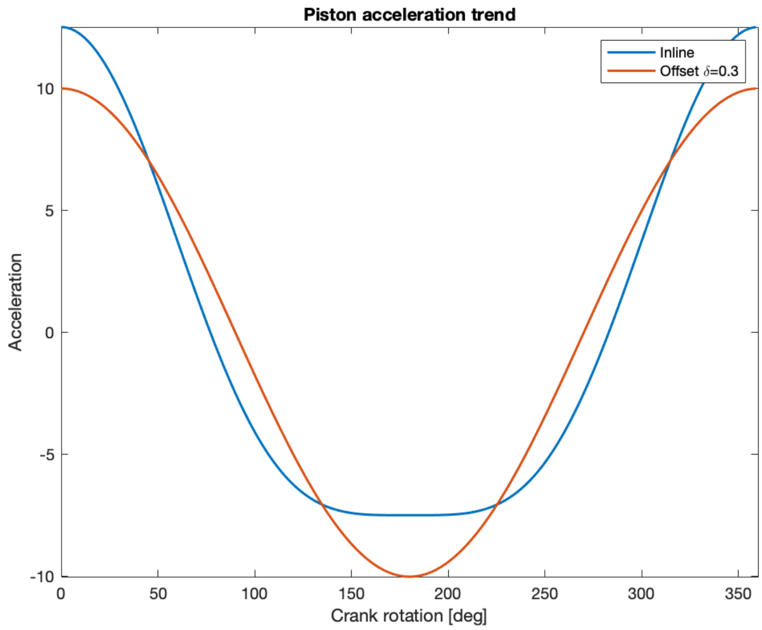

Figure 2.8: Inline vs. Offset acceleration comparison.

2.3 Slider-Crank System Force Analysis.

It can be considered the Figure 2.9 below, under the assumption of *no friction*.

Considering an *In-Line Slider-Crank Mechanism*, the fundamental parameters that must be known are:

- The bore (D)
- The piston stroke (TDC, BDC, h)
- The crank length (r)
- The ratio ($\lambda = r/L$)
- The crankshaft angular velocity (ω)
- The alternating (or reciprocating) masses (m_{ALT})
- The rotating masses (m_{ROT})
- The $p_{(\theta)}$ law

The forces acting on the system can be divided in forces due to the gas pressure in the cylinder chamber, and inertia forces acting on the moving parts [4].

The *Gas Force* can be written as:

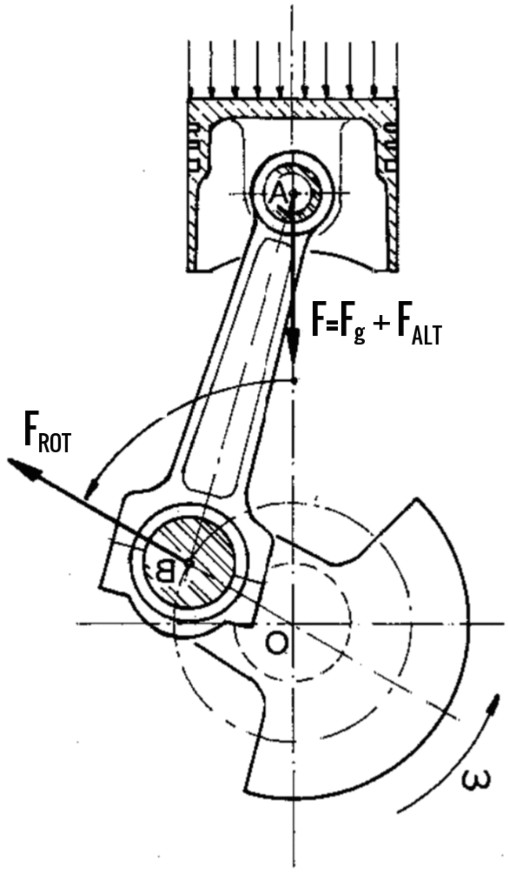

Figure 2.9: Forces acting on the crank (non considering counterweight)

$$F_g = (p_{(\theta)} - p_0) \cdot \frac{\pi D^2}{4}; \tag{2.15}$$

where the term p_θ represents the gas pressure in the cylinder chamber, function of the crank angle θ, and the termo p_0 is the ambient pressure (with ambient considered as the interior part of the crankcase).

The inertia forces acting on the moving parts of the system can be divided in inertia forces of the *reciprocating* parts of the system, and inertia forces of the *rotating* parts of the system, that is the *centrifugal forces.*

It is possible to obtain, once the masses and the laws of motion of the system are known, the forces acting on the system.

2.3.1 Alternating Inertia Forces and Centrifugal Forces.

In each engine there are masses that move reciprocating motion and masses that move rotating motion, and it is useful to clarify what type of motion have the different parts of the system.

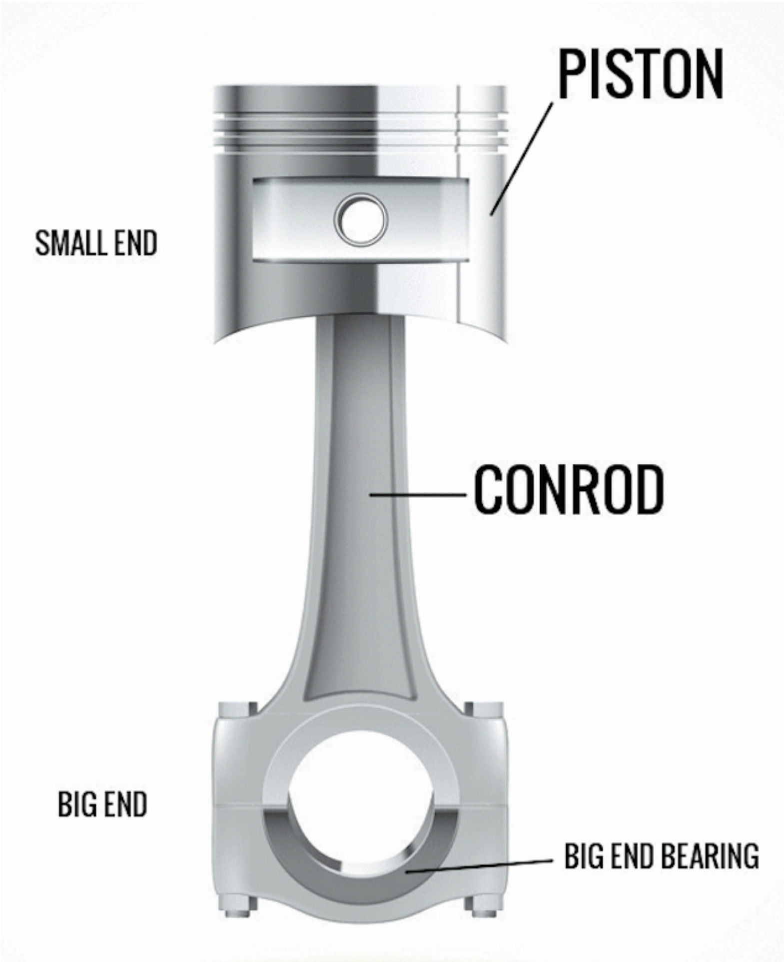

Figure 2.10: Piston-conrod system.

In the Figure 2.10 is represented a *Piston-conrod system*. The piston and the parts directly applied to it (i.e. piston rings) moves with reciprocating motion. However, as regards the conrod it is necessary to divide its mass in a rotating component and in an alternating component.

Since the *small end* of the conrod moves with reciprocating motion and the *big end* instead moves with rotating motion, for the conrod body it can be assumed that 1/3 of its mass moves as alternating, and the remaining 2/3 moves rotating.

Therefore, the masses considered concentrated on the piston pin (or on the conrod small end) that move with alternating motion are:

- The piston, the piston pin and the piston rings.
- The 1/3 of the conrod mass (including small end bearing).

The masses that moves with rotating motion and that are considered concentrated on the crank pin are:

- The crank pin mass.
- The 2/3 of the conrod mass.
- The big end bearing.

Moreover, the crank and the possible counterweights are to be considered as rotating masses that generate centrifugal forces, concentrated, for hypothesis, on the crank pin.

The alternating and the centrifugal forces are very important for the engine balancing, the first one is directed only along the cilynder axis, therefore they act on the system similarly to the gas pressure and they modifying the engine torque. The second one instead do not affect the engine torque value, because their direction is constantly passing through the center of rotation.

Considering the general expression of the inertial forces:

$$\overrightarrow{F_{ALT}} = -m_{ALT}\,\vec{\ddot{a}}\,; \tag{2.16}$$

replacing the term $\vec{\ddot{a}}$ with the acceleration of the piston, the expression of the *Alternating Inertia Force* can be obtained:

$$F_{ALT} = -m_{ALT}\omega^2 r(\cos\theta + \lambda\cos 2\theta); \tag{2.17}$$

this formula (2.17) is made by two components:

$$F'_{ALT} = -m_{ALT}\omega^2 r\cos\theta; \tag{2.18}$$

that represents the *First Order Alternating Force*, and:

$$F''_{ALT} = -m_{ALT}\omega^2 r\lambda\cos 2\theta; \tag{2.19}$$

that expresses the *Second Order Alternating Force.*

In the Figure 2.11 below there is represented a typical trend of first and second order alternating forces. The second one has twice frequency than the other.

The inertia forces are one of the most important causes of vibrations of engines, and they can be balanced completely or only partially, depending on the system configuration.

The rotating parts of the engine are subjected to the *Rotating (or Centrifugal) Forces*, that can be expressed as:

$$F_{ROT} = m_{ROT}\omega^2 r; \tag{2.20}$$

that represents a rotating vector, with constant magnitude, passing through the axis on rotation of the crank.

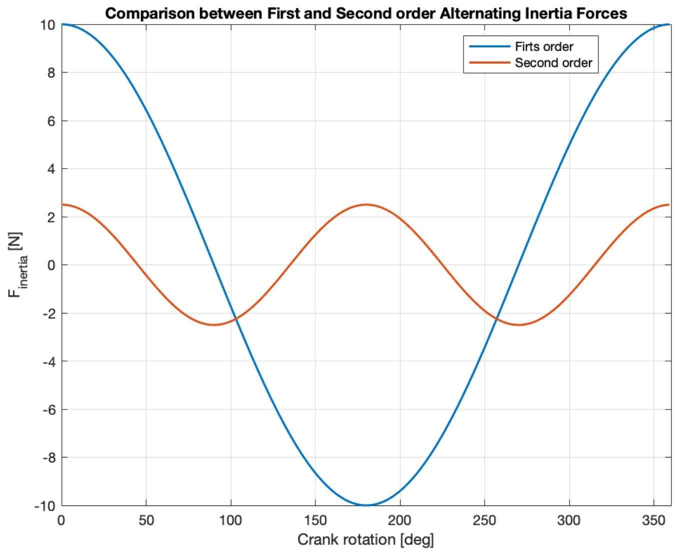

Figure 2.11: Example of first and second order alternating inertia forces.

2.3.2 Crankcase internal forces.

The gases inside the combustion chamber of the cylinders and the moving parts such as pistons and rods exerts forces on the engine, that can be easily calculated considering a single cylinder engine case, as shown in the Figure 2.12.

The gases pressure gives rise to the force F_g that is transmitted directly to the engine head and, equal and opposite, to the piston head.

Moreover, the piston exerts on the cylinder a force with direction perpendicular to its axis. The *thrust on the cylinder chamber* is equal to:

$$F_L = F_b \cdot \sin\beta = F \cdot \tan\beta; \tag{2.21}$$

Furthermore, there is a force exerted on the conrod with direction coincident with the conrod axis and directed towards the crank pin:

$$F_b \cdot \cos\beta = F \rightarrow F_b = \frac{F}{\cos\beta}; \tag{2.22}$$

The force F, that can be called *Piston effort*, is the composition of the *Gas Force* F_g and of the *Alternating Inertia Force* F_{ALT}, which will be explained later on.

$$F = F_g + F_{ALT}; \tag{2.23}$$

The component of F_b along the crank (in radial direction) produces a thrust on the crankshaft bearings.

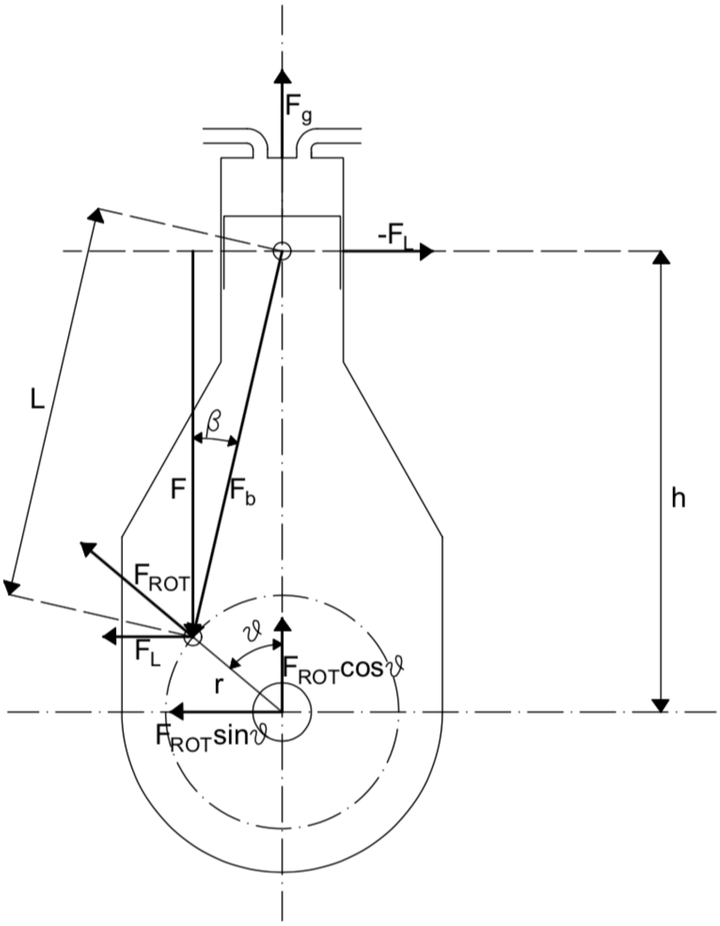

Figure 2.12: Crankcase internal forces.

$$F_r = F_b \cdot \cos(\theta + \beta) = \frac{F}{\cos\beta} \cdot \cos(\theta + \beta); \tag{2.24}$$

The *Crank Effort* F_t is the net force applied to the crank pin, perpendicular to the crank, which gives the required turning moment on the crankshaft.

$$F_t = F_b \cdot \sin(\theta + \beta) = \frac{F}{\cos\beta} \cdot \sin(\theta + \beta); \tag{2.25}$$

Finally, also the *Centrifugal Force* F_{ROT}, applied on the crank pin, in transferred to the bearings.

This *Centrifugal Force* exerts on the crankcase a vertical force equal to:

$$F_{ROT} \cdot \cos\theta \tag{2.26}$$

and an horizontal force equal to:

$$F_{ROT} \cdot \sin\theta \tag{2.27}$$

Looking at the Figure 2.12, the resultant forces in vertical and horizontal directions acting on the crankcase can be calculated as:

$$F_g - F + F_{ROT}\cos\theta = F_{ALT} + F_{ROT}\cos\theta; \tag{2.28}$$

$$-F_L + F_L + F_{ROT}\sin\alpha = F_{ROT}\sin\alpha; \tag{2.29}$$

As can be seen in these formulas, the vertical resultant (along the cylinder axis) of all the forces acting inside the crankcase does not depend on the *Gas Force* F_g, as it might appear to be at first sight, but is only dependant on the *Alternating Inertia Forces* F_{ALT} and on the *Rotating Forces* F_{ROT}. This is because the *Gas Force* F_g is thrusting both on the piston head and on the engine head, and these two forces balance each other.

The horizontal resultant (perpendicular to che cylinder axis) force instead is only a function of the *Rotating Forces* F_{ROT}.

The expressions of the *Engine torque* M and of the *Reaction torque* M_R can be written as:

$$\begin{aligned} M &= F_t \cdot r = F_b \cdot b = F \cdot \overline{OD} = \\ &= \frac{Fr}{\cos\beta}\sin(\theta+\beta) = \frac{Fr}{\cos\beta}(\sin\theta\cos\beta + \cos\theta\sin\beta) = \\ &= Fr\left(\sin\theta + \cos\theta\frac{\sin\beta}{\cos\beta}\right) = Fr\left(\sin\theta + \cos\theta\sin\theta\frac{1}{\lambda}\frac{2\lambda^2}{2\sqrt{1-\lambda^2\sin^2\theta}}\right) \cong \\ &\cong Fr\left(\sin\theta + \frac{\lambda}{2}\sin 2\theta\right); \end{aligned} \tag{2.30}$$

$$\begin{aligned} M_R &= F_L \cdot h = [(F_g + F_{i_{ALT}})\tan\beta]r\left(\cos\theta + \frac{1}{\lambda}\cos\beta\right) \cong \\ &\cong (F_g + F_{i_{ALT}})r\left(\sin\theta + \frac{\lambda}{2}\sin 2\theta\right) = Fr\left(\sin\theta + \frac{\lambda}{2}\sin 2\theta\right); \end{aligned} \tag{2.31}$$

with the aproximation $\sqrt{1-\lambda^2\sin^2\theta} \cong 1$.

As it can be seen, the *Reaction Torque* is equal to the *Engine Torque*, but it has opposite verse.

In the light of the above, the actions transmitted by the engine to the engine supports changes periodically in modulus and direction; therefore, the engine supports must react with variable reactions. On account of this, the engine and the engine frame can take a vibratory motion.

In order to cancel the vibrations, the actions transmitted by the engine to the environment must be as much as possible constant. In other words, the engine must be balanced.

All the considerations done for a *single-cylinder system* can be extended to engines with more cylinders.

2.3.3 Few considerations about the parameter λ

The importance of the $\lambda = \frac{r}{L} = \frac{h}{2L}$ have only a mechanical character, because it do not affect the thermodynamic properties of the system.

The lower the λ the lower the lateral thrust on the cylinder chamber F_L. This could be an advantage because, with a lower thrust, the piston skirt can be made shorter and then the piston mass is reduced. This is an advantage in terms of alternating inertia forces. But the lower the λ the higher the L (conrod length) and the higher the conrod mass. If the conrod mass is higher, it will be also raise the part of conrod mass which moves of alternating motion and then the alternating inertia force increases.

Typical values of λ are in the range $0.2 \div 0.3$, to which they correspond values of conrod length L equal to $(2.5 \div 1.7) \cdot h$ (where h = *stroke*).

Chapter 3

Engine Balancing

As mentioned in the Paragraph 2.3.2, the main causes of imbalance in engine are due to the vertical and horizontal resultat forces, that are generated by the rotating and alternaring inertia forces and by the reaction torque. These forces have a periodical variation, that is fuction of the crank angle θ, which is the cause of the system vibrations.

In order to have a complete balancing, the crankshaft must be balanced both statically and dynamically. The dynamic balance can only be achieved if the crankshaft is statically balanced [5][2].

A crankshaft is statically balanced if the resultant of the centrifugal forces is equal to zero, that is when its COG is located on the axis of rotation. For engines with more than one cylinder is usually to choose the proper arrangement of cylinders, and therefore of the crank throws, in order to have a configuration that is as much as possible self-balanced. Generally, the crank throws are arranged to obtain a uniform phase displecement of the duty cycles and achieve an engine torque as constant as possible.

In many cases the arrangement is such that the static balance is automatically satisfied because there is a plane of symmetry passing through the axis of rotation, in other cases it is necessary to add the counterweights on the crankshaft [4] [2].

A crankshaft is dynamically balanced when the resuntant of the moments of the centrifugal forces, calculated from a reference point assumed arbitrarily, is zero. In this condition, when the crankshaft rotates the reactions generated on the supports are only due to its own mass. In other words, the crankshafts with more than two crank throws are dynamically balanced when they are statically balanced, and when there is a symmetry plane perpedicular to the axis of rotation of the shaft against which the crank throws are symmetrical in number, form and arrangement. All crankshafts that do not have this feature are not naturally balanced, but the balancing can be achieved adding the counterweights. It is easy to deduce that for all the crankshafts with odd cylinders number and for *Two-Strokes Engines* the complete balance is reachable only adding the counterweights [4] [2].

It is important to stress that, while the static balance concerns only the whole crankshaft, the dynamic balance can be considered even on each crankshaft bay.

Often the dynamic balance is achieved when the resultant of various moments is zero: it means that bending moments can exist in each bay, and the inflection is prevented by the reactions of the bearings. For this reason, bearings are subjected even to centrifugal stresses. To cancel these stresses, especially in high speed engines, is usually to balance the single bays with counterweights.

Now, let us see how the forces that acting on the system can be balanced, considering a case with only one cylinder. All the concepts obtained can be applied to multi-cylinder engines.

3.1 Balancing of Centrifugal Forces

In the *Single-Cylinder Engines* the only way to balance the centrifugal forces is the addition of counterweights on the crankstaft. In the Figure 3.1 a simple example can be observed of one of the possible configurations, obtained adding two counterweights opposite to the rotating mass.

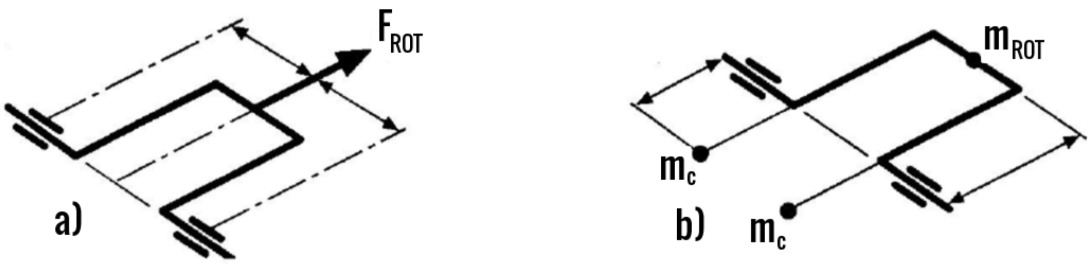

Figure 3.1: *a*)Single-cylinder crankshaft. *b*)Single-cylinder crankshaft with counterweights.

3.2 Balancing of Alternating Inertia Forces

In terms of *Alternating Inertia Forces*, these are characterised by the fact that their direction is constantly along the cylinder axis, and they have variable towards and magnitude. The expression of these forces is:

$$F_{ALT} = F'_{ALT} + F''_{ALT} = m_{ALT}\omega^2 r(\cos\theta + \lambda\cos 2\theta); \tag{3.1}$$

The two components are now analyzed separately.

3.2.1 Balancing of First Order Alternating Forces

The firts order alternating force can be written as:

$$F'_{ALT} = m_{ALT}\omega^2 r\cos\theta; \tag{3.2}$$

Considering the Figure 3.2 below:

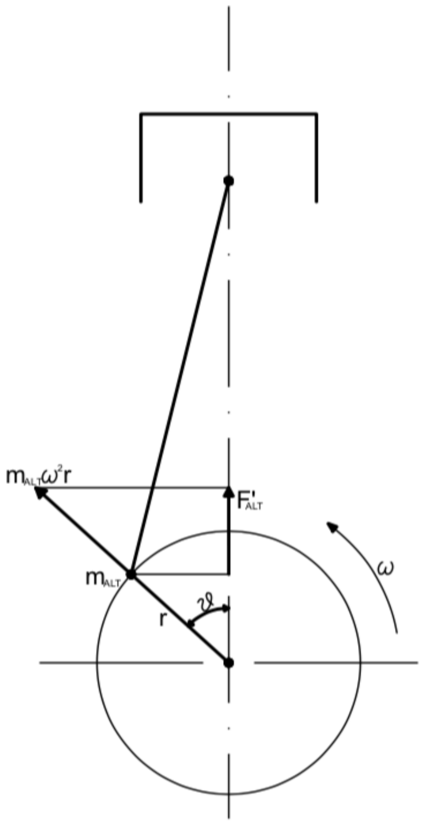

Figure 3.2: First order alternating force.

The first order alternating force (F'_{ALT}) can be considered as the projection on the y axis of a fictious centrifugal force, equal to $m_{ALT}\omega^2 r$. This force is generated considering the alternating mass m_{ALT} concentrated on the crank pin. Therefore even this alternating force can be balanced as done for the rotating force.

Observing the Figure 3.3:

F_{ALT} can be balanced by the vertical component of the centrifugal force $-m_{ALT}\omega^2 r$ that is produced by another mass (counterweight). The counterweight is in opposition to the crank pin and gives a static torque equal to $m_{ALT}r$.

Moreover, as shown in the Figure 3.3, there is a "new" force $F_0 = -m_{ALT}\omega^2 rsin\theta$ that has x direction and the same amplitude and pulsation of the alternating force. Therefore, in this case, the result is just the rotation by $\pi/2$ of the alternating force.

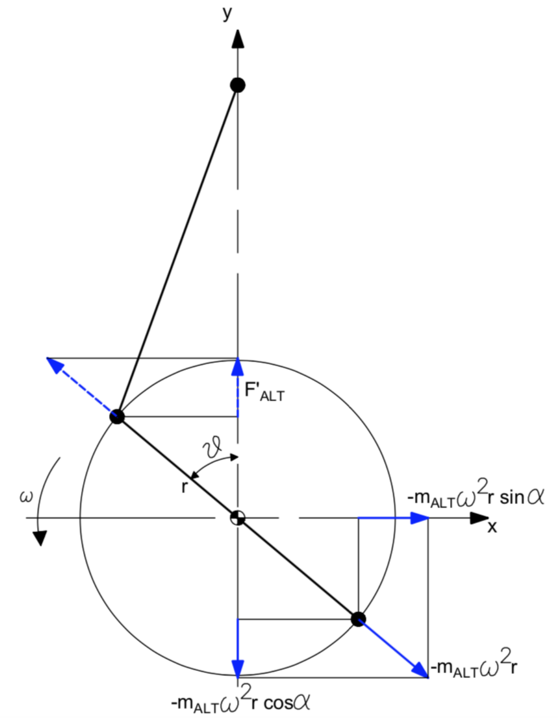

Figure 3.3: Balancing of first order alternating force.

However, if the added mass is equal to half of m_{ALT}, as can be seen in Figure 3.4, the result is the balance of an half of the alternating force, and there is a force perpendicular to the cylinder axis equal to $F_0/2$. Composing these two perpendicular forces, the resultant is a rotating force characterized by $-\omega$ angular velocity and $\frac{1}{2}m_{ALT}\omega^2$ intensity that can not be balanced.

This is the maximum grade of balance for first order alternating forces that can be reached up for single-cylinder engines.

To obtain a system completely balanced it is necessary to add some auxiliary shafts, in order to balance both the rotating component and the counter-rotating component of the first order alternating force. In the Figure 3.5 below it is shown an example:

3.2.2 Balancing of Second Order Alternating Forces

This force is equal to:

$$F''_{i_{ALT}} = m_{ALT}\omega^2 r\lambda\cos 2\theta; \tag{3.3}$$

The second order alternating force can be seen as the projection on the cylinder axis of a centrifugal force equal to $m_{ALT}\omega^2 r\lambda$, whose frequency is twice that of the first order force. The angle between the force and the cylinder axis is twice the crank angle.

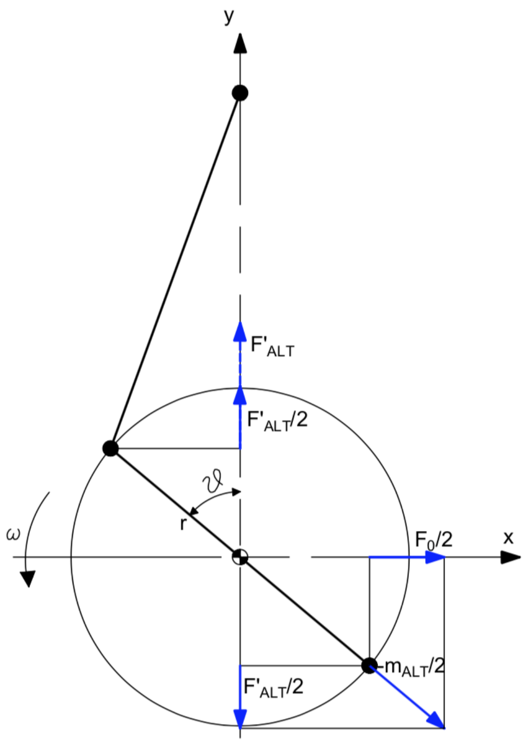

Figure 3.4: Transformation of a first order alternating force in a rotating force.

It is important to stress that the second order torques and forces can not be balanced (not even partially) with the addition of counterweights on the crankshaft, because any added mass should rotate with twice as fast as the crankshaft.

Therefore, there is no relationship between crankshaft balancing and balancing of the second order forces and torques, differently to what occurs for the first order alternating forces. Anyway, second order alternating forces are less important in term of vibrations than first order forces because the values are in λ relationship (with $\lambda = 0.2 \div 0.3$ generally).

A first order alternating force $m_{ALT}\omega^2 r\cos\theta$ can be considered as the resultant of two forces F_1 and F_2 equal to $1/2 m_{ALT}\omega^2 r$ rotating with the crank, one with ω speed and one with $-\omega$ speed (counter-rotating) as shown in the Figure 3.5. The same observation applies to second order alternating forces, but in this case the rotating forces must have a modulus equal to $1/2 m_{ALT}\omega^2 r\lambda$ and an angular velocity equal to 2ω and -2ω.

Finally, it is possible to balance the first order alternating force using two auxiliary shafts, each one having half the mass of m_{ALT}, and the second order alternating force adding another two auxiliary shafts whose mass should be equal to $1/8 m_{ALT}\lambda$, rotating at 2ω and -2ω angular speed, as shown in the Figure 3.6:

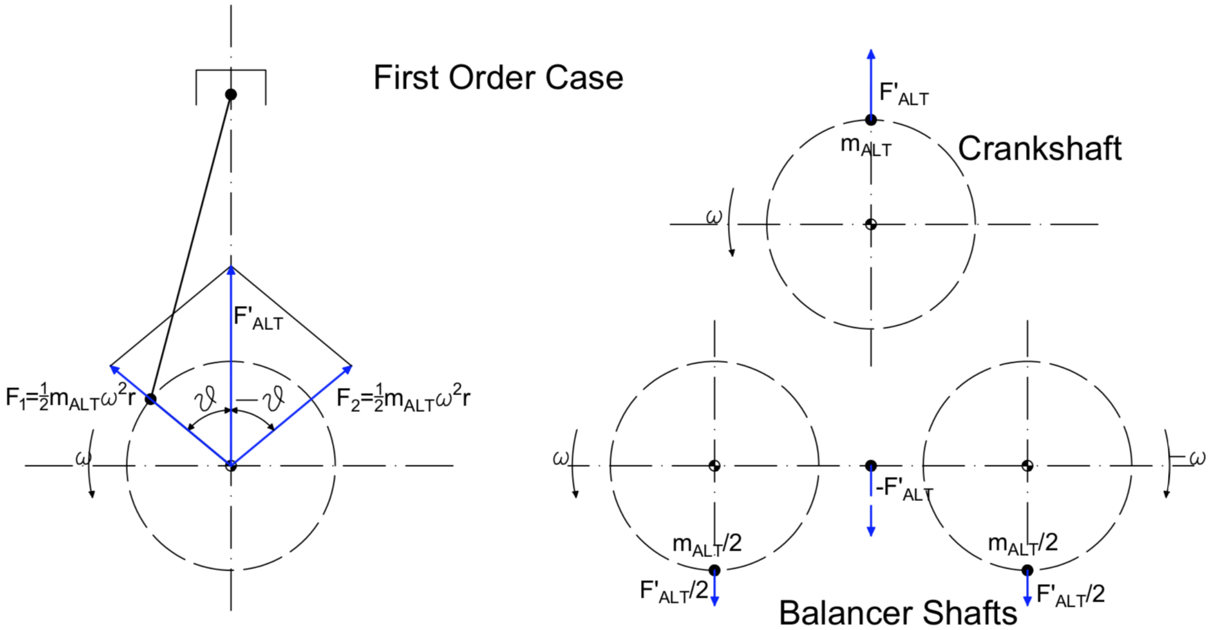

Figure 3.5: Balancing of a first order alternating force with two rotating forces.

This solution is anyway never used in practical applications because the system would be excessively complex and expensive to produce. Usually in single-cylinder engines only the centrifugal forces and the rotating part of first order alternating forces are balanced, even if not completely.

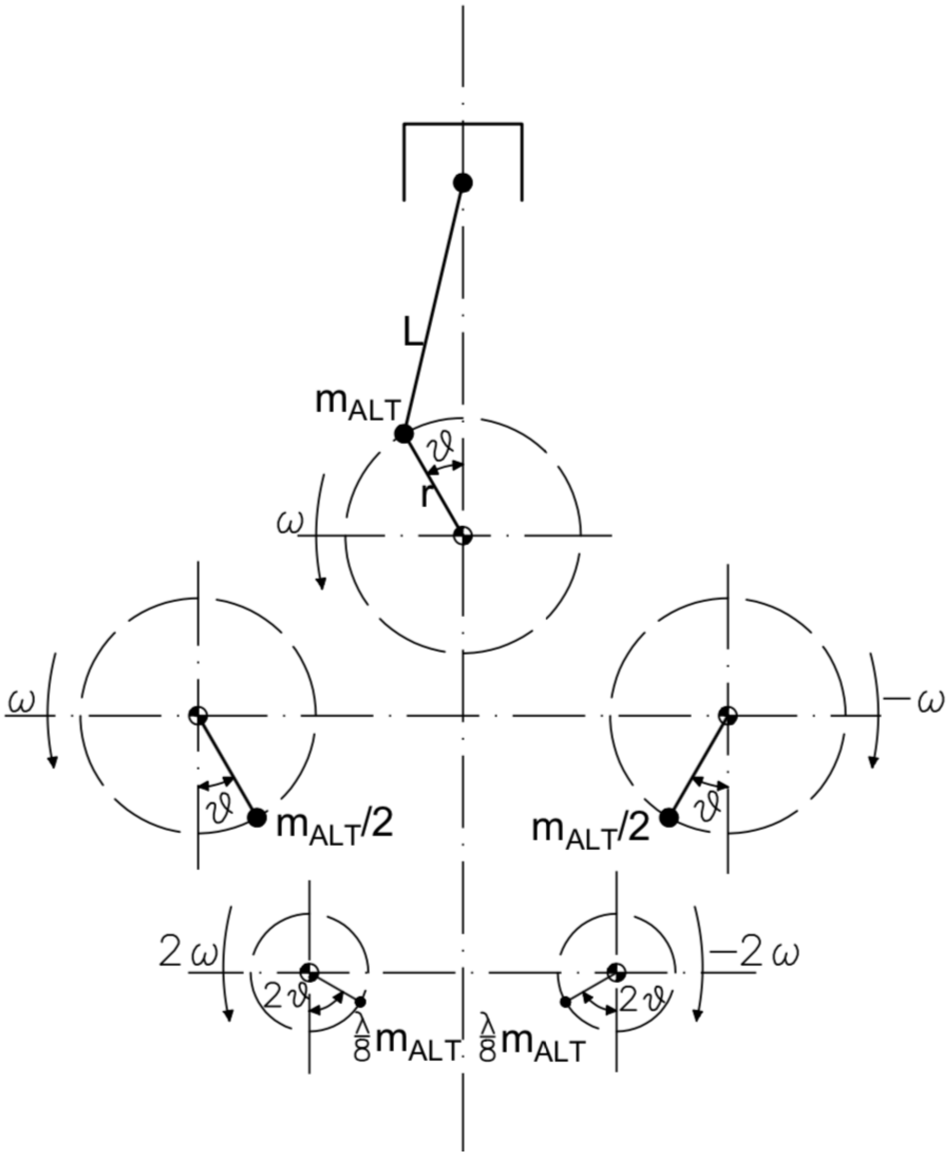

Figure 3.6: Complete balancing of a single-cylinder engine.

3.3 The Balancing Factor

The *Balancing Factor (BF)* can be described as a ratio between forces, in particular between centrifugal forces acting on the engine and forces whose balance them.

For engine with more than one cylider, a different balance factor can be defined for every single crankshaft bay, in order to find a system configuration with the lightest possible crankshaft and the lowest possible bearings loads, without compromising the structural shaft integrity.

Once the balancing factor in chosen, bearing loads and shaft weight can be determined.

3.3.1 Rotating masses

Let us consider the conrod: every conrod has a part of its mass that can be considered as rotating. Observing a generic conrod, as shown in Figure 3.7:

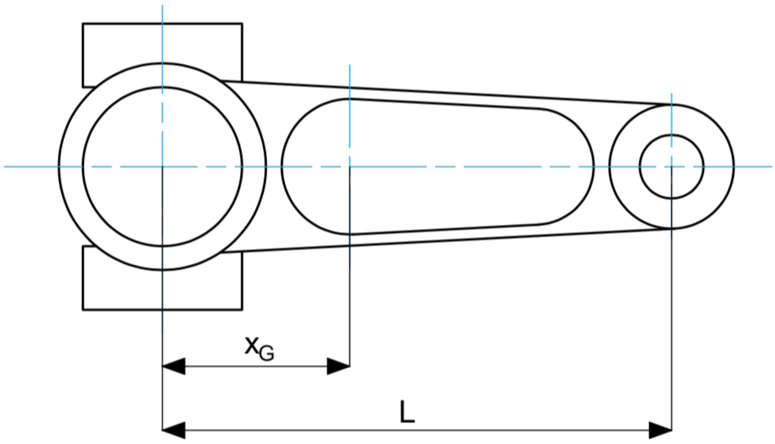

Figure 3.7: Conrod scheme.

the main parameters that needs to be taken in consideration are: the *conrod length* L_{ROD}, the *conrod mass* m_{ROD} and the distance between conrod COG and the centre on the big end bearing x_G. With these parameters, the rotating part of the conrod mass che be calculated, as shown in the equation (3.4).

$$m_{ROT_{ROD}} = m_{ROD}\frac{L_{ROD} - x_G}{L_{ROD}}; \tag{3.4}$$

this mass is assumed concentrated in the crank pin, at the crank radius $r = \frac{stroke}{2}$.

Considering the crank, the rotating masses, as shown in the Figure 3.8, are pin mass m_{PIN}, the masses of the two webs m_{WEB_A} and m_{WEB_B} and the masses of the counterweights, if they are (in the case of the Figure 3.8 m_{CW_A} and m_{CW_B}).

The balancing factor can be written as:

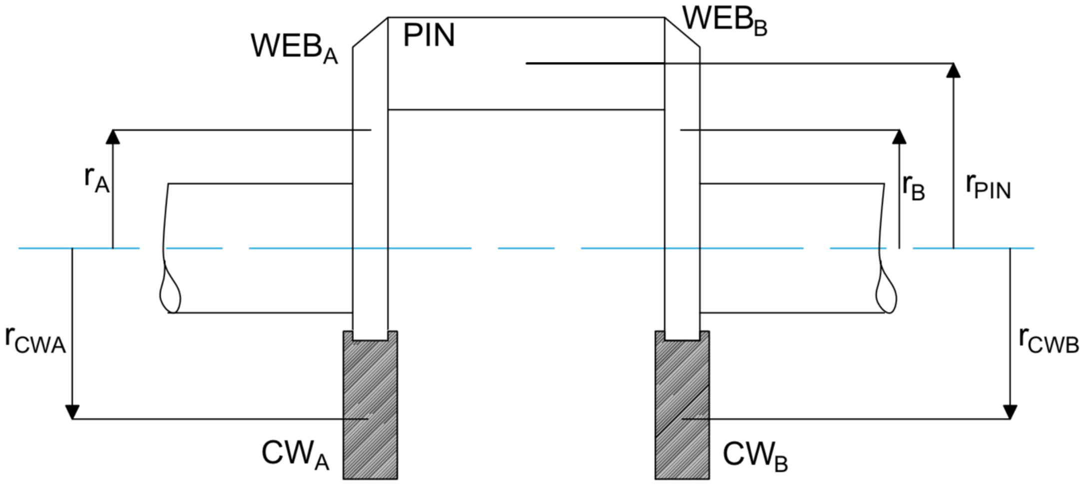

Figure 3.8: Crank scheme with rotating masses.

$$BF_{ROT} = \frac{m_{CW_A} r_{CW_A} \omega^2 + m_{CW_B} r_{CW_B} \omega^2}{m_{PIN} r_{PIN} \omega^2 + m_{ROT_{ROD}} \frac{s}{2} \omega^2 + m_{WEB_A} r_A \omega^2 + m_{WEB_B} r_B \omega^2} = \frac{m_{CW_A} r_{CW_A} + m_{CW_B} r_{CW_B}}{m_{PIN} r_{PIN} + m_{ROT_{ROD}} \frac{s}{2} + m_{WEB_A} r_A + m_{WEB_B} r_B}; \quad (3.5)$$

3.3.2 Reciprocating masses

The same reasoning done for the rotating masses applies considering the alternating inertia forces.

In the Figure 3.9 are represented both the rotating and alternating masses. As can be seen, instead of considering the alternating mass concentrated on the piston pin, it can be assumed concentrated on the crank pin. Moreover this mass is conceptually "divided" in rotating and counter-rotating part: for the first one the definition of the *BF* is exactly the same that for the rotating masses, and it can be balanced at least for 50% adding coutnerweights.

The counter-rotating part instead can be "naturally" balanced by the geometrical arrangement of the pistons (for engines with number of cylinder higher than one) or by a balancer shaft.

The general expression of the *Balancing Factor BF* for a single bay with two counterweights is:

$$BF = \frac{m_{CW_A} r_{CW_A} + m_{CW_B} r_{CW_B}}{m_{PIN} r_{PIN} + m_{ROT_{ROD}} \frac{s}{2} + m_{WEB_A} r_A + m_{WEB_B} r_B + \frac{1}{2} m_{ALT} \frac{s}{2}}; \quad (3.6)$$

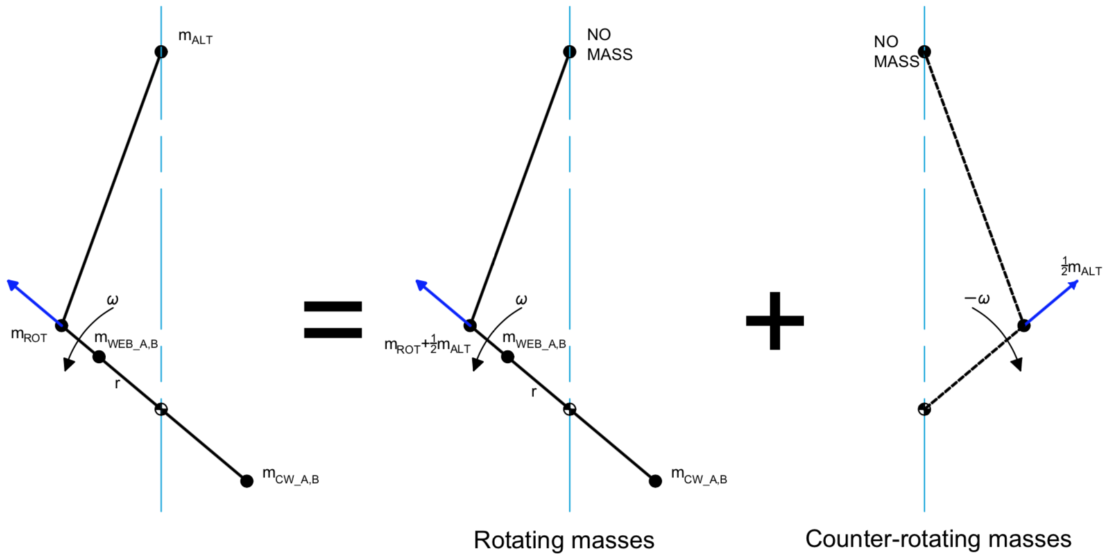

Figure 3.9: Alternating masses.

Often, some assumptions can be maked:

- $r_{CW_A} = r_{CW_B}$ → Counterweights are placed at the same radius.
- $r_{PIN} = \frac{stroke}{2}$ → This assumption is valid only if the oil drillings are neglected.
- $r_A = r_B$ → The crank throws have the concentrated mass at the same radius. It is not exactly true all the time, for example for webs with thrust washer, but it can still assumed as a valid aproximation.

The *Balancing Factor* may assume different values:

- $BF > 1$: is never uses, because it would be an overbalancing of the system.
- $BF = 1$: the rotating forces are completely balanced, which means 100% of pure rotating and 50% of alternatig considering as rotating.
- $BF < 1$: the rotating forces are not completely balanced.

The *Balancing Factor choice* is a complex topic, and it would require to study in deep the problem and analyze the bearings loads, the shaft natural frequencies, the shaft bending behaviour and other important phenomenon.

A low *BF* gives a crankshaft with higher natural frequencies, but higher shaft bending as speed increase. An high *BF* gives an heavier crankshaft, but lower loads on the bearing ad high speeds.

The equation 3.6 applies for *In-Line Engines* that have only one conrod on each throw. For *V-Engines* the definition it is similar, but it must pay attention to the fact that, in this case, there are two conrods on each throw. Therefore in the *BF* formula appears the mass of a second cordod (that usually is equal to the other, and hence a factor 2 on the conrod mass) and a second alternating mass of the second piston (again the same to the other piston, hence a factor 2 here too).

$$BF = \frac{m_{CW_A} r_{CW_A} + m_{CW_B} r_{CW_B}}{m_{PIN} r_{PIN} + 2m_{ROT_{ROD}} \frac{s}{2} + m_{WEB_A} r_A + m_{WEB_B} r_B + m_{ALT} \frac{s}{2}}; \tag{3.7}$$

The Figure 3.10 explains what is already written.

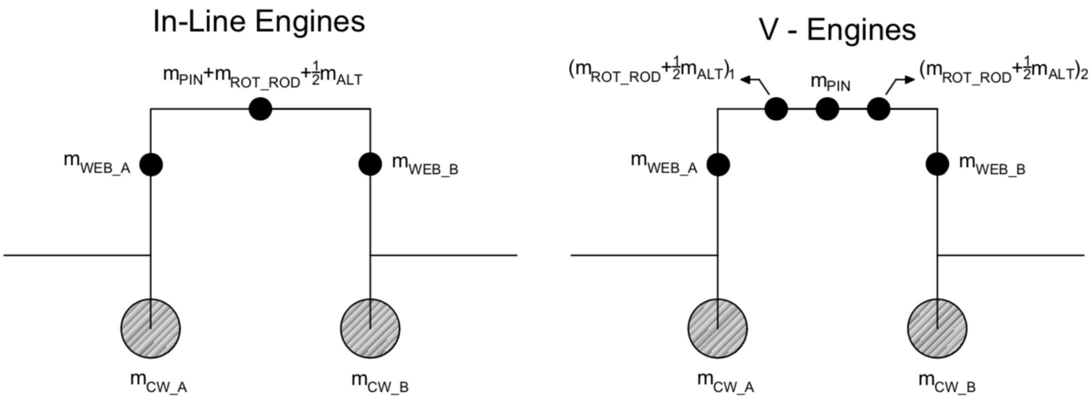

Figure 3.10: Masses on Inline engines and V-engines.

In conclusion, there are engines with different *Balancing Factors* on the same crankshaft in order to reduce the loads on the main bearings, expecially at high speeds. Therefore, the *BF* can assume a different value *bay-by-bay*. Common *BF* values used are in the range $0.65 \div 0.7$, but for very high speed engines it may be equal to 1, or approximately 1.

It is clear that, for a rigid shaft (as in this thesis work is considered when balancing it), the bearings loads are a function of the *BF* chosen for each bay. In particular, considering the approximation of give all the load of a throw only to the adjacent bearings, it is understandable that the local reactions depends only on local *BF*.

In the following paragraphs, a series of different engines with different configurations will be analyzed in detail, starting with the simplest one: the *Single-Cylinder Engine*. All the key concepts obtained in the single-cylinder engine case can be applied to engines with higher number of cylinders.

3.3.3 Influence of the Balancing Factor on the crankshaft weight

One of the parameters that must be kept under control during the design is the crankshaft weight.

A lighter crankshaft allows to reach higher engine revs without compromising its integrity.

The *Balancing Factor* has an influence on the weight and, to have a better view of the relation between this two parameters, let us consider a simple crankshaft case.

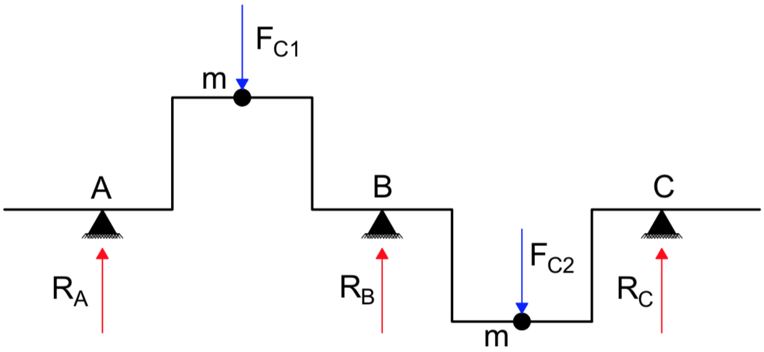

Figure 3.11: Two-cylinder crankshaft

In the Figure 3.11 is reported a two-cylinder crankshaft supported by three main bearings. It is assumed that all the masses are concentrated ad the crank throw radius, indicated with m and since there are no counterweights, the BF is equal to zero. Therefore, in this layout the total crankshaft mass is equal to $2m$ and the reactions on the main bearings R_A, R_B and R_C are the highest possible.

Considering now to set the *BF=1* and assuming to consider, just to simplify, that the counterweights radius is exactly the same as the crank throw radius. Looing at the Figure 3.12, the BF formula 3.6 becomes:

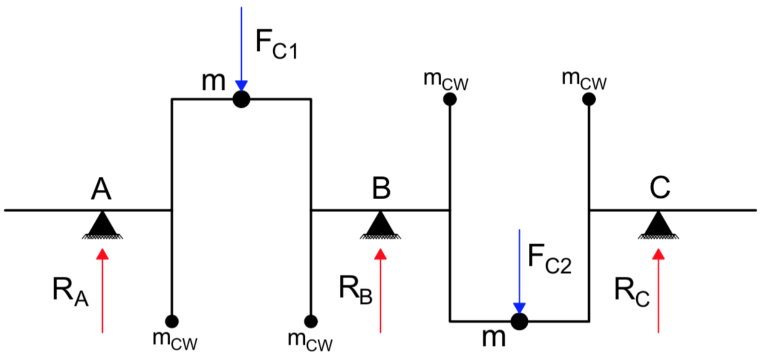

Figure 3.12: Two-cylinder crankshaft with counterweights

$$BF = \frac{m_{CW}r + m_{CW}r}{mr} = 1 \quad \Rightarrow \quad 2m_{CW} = m \quad \Rightarrow \quad m_{CW} = \frac{m}{2}; \tag{3.8}$$

This formula 3.8 is referred to only one of the two bays, but is the same for the two bays.

Therefore, in this way the crankshaft is fully balanced to first order rotating forces and moment and the bearing reactions R_A, R_B and R_C are zero. This is also the heaviest configuration because the total mass is equal to $4m$. Actually, if *BF>1* the system would be heavier than $4m$, but this solution would not make sense because the crankshaft would be over-balanced and heavier.

Considering the Figure 3.12, if the *BF* is equal, for example, to 0.7:

$$BF = \frac{m_{CW}r + m_{CW}r}{mr} = 0.7 \quad \Rightarrow \quad 2m_{CW} = 0.7m \quad \Rightarrow \quad m_{CW} = 0.35m; \qquad (3.9)$$

This is an "intermediate configuration", where the crankshaft weight is equal to $2m + 4 \cdot 0.35m = 3.4m$ and it is lower than the case with *BF=1*, but the reactions on the main journals are higher because the moment is not completely balanced.

Anyway, the configuration shown in the Figure 3.12 is not the only possible way to place the counterweights on the shaft. For example, assuming to add only one counterweight for each bay, there are two main configurations that the system can assume that are shown in the Figures 3.13 and 3.14.

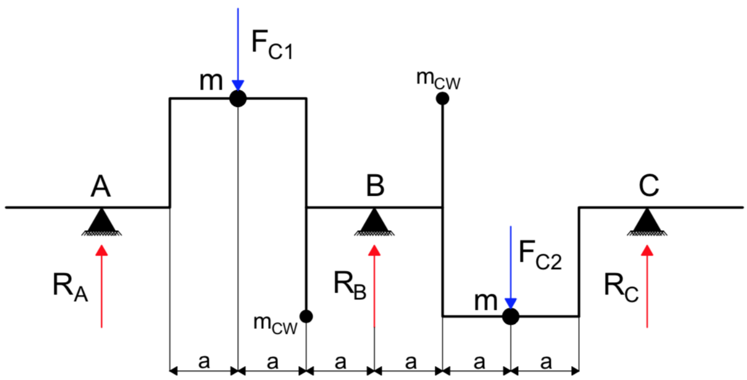

Figure 3.13: Two-cylinder crankshaft with internal balancing

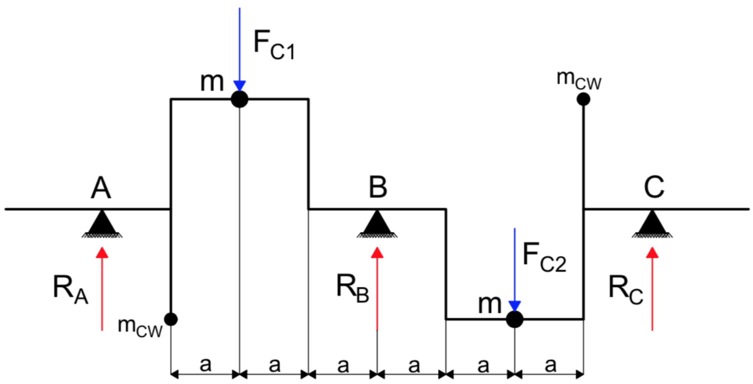

Figure 3.14: Two-cylinder crankshaft with external balancing

Considering the case *"Internal Balancing"*, in order to have a system completely balanced (*BF=1*) to the first order moment, an equilibrium moment equation can be written respect to the central point (reaction B):

$$4a(m\omega^2 r) = 2a(m_{CW}\omega^2 r) \quad \Rightarrow \quad m_{CW} = 2m; \tag{3.10}$$

Therefore, the total mass of the crankshaft is in this case equal to $6m$.

Considering instead the case *"External Balancing"*, to have a full-balanced system the balancing mass must be equal to:

$$4a(m\omega^2 r) = 6a(m_{CW}\omega^2 r) \quad \Rightarrow \quad m_{CW} = \frac{2}{3}m; \tag{3.11}$$

And therefore the whole crankshaft mass is equal to $\frac{10}{3}m$ that is less than $6m$ of the internal balancing case.

Obviously, this method can be applied in a similar way to other crankshaft configurations even with higher number of cylinders.

It is possible to understand how, depending on the system layout, there are several possibilities to balance the system and each of them leads to different results in terms of balancing, weight and geometry.

There is no correct or wrong configuration, but there are better and worse configurations and in each single case the designer must find the layout that fits best with the goal to have the lightest system with the lowest bearing reactions.

3.4 Single-Cylinder Engine

In order to have a clearer view of the problem, in term of which forces are acting on the system, these forces can be represented through the *vectors star*. This type of representation allows to easily see which forces are balanced and which not.

Considering a generic configuration of the system, as shown in Figure 3.15, the vectors star can be derived.

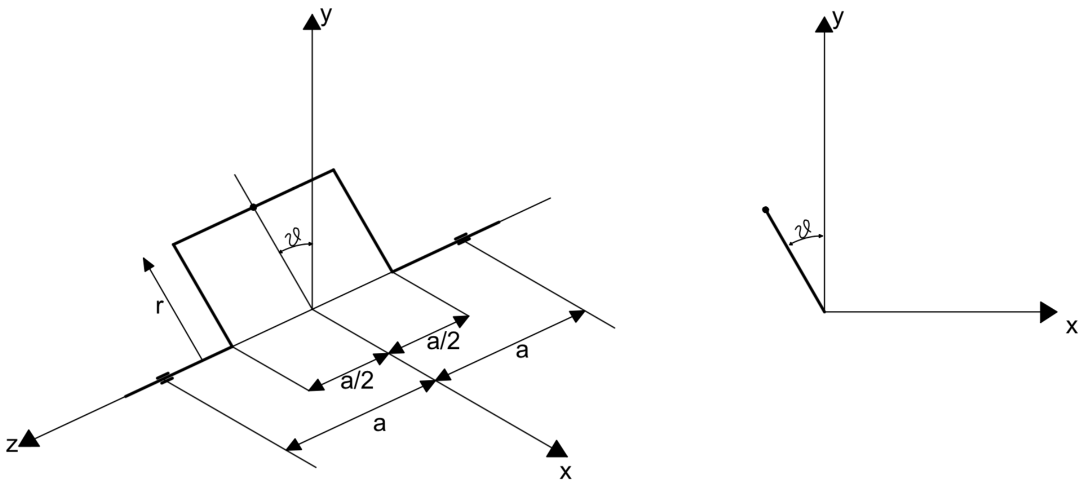

Figure 3.15: Single crank.

In the following Figures 3.16 and 3.17 are reported the *First order vectors star* and the *Second order vectors star*.

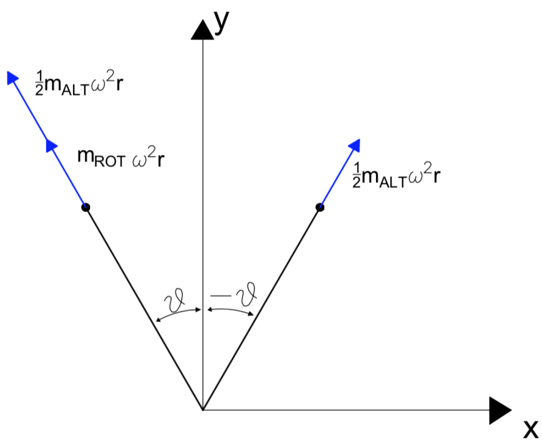

Figure 3.16: First order vectors star

As can be seen in the Figure 3.16:

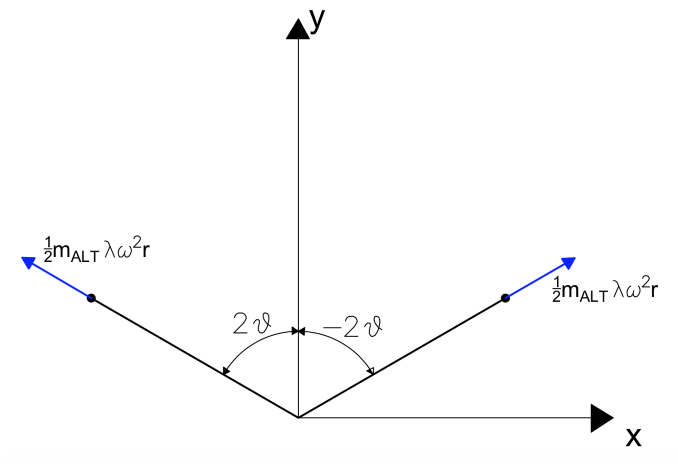

Figure 3.17: Second order vectors star

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) $\Rightarrow$ Not balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Not balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Not balanced.

and in the Figure 3.17:

- Rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) $\Rightarrow$ Not balanced.
- Counter-rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) $\Rightarrow$ Not balanced.

In this type of engine there are not unbalanced moments both of first and second order.
The equilibrium of forces can be written also in an analytical form.
The fudamental equations are:

$$\begin{cases} x) & \sum_i m_i\,\omega^2\,x_i = \sum_i m_i x_i = m_1 r\sin\theta = 0; \Rightarrow \text{Not balanced} \\ y) & \sum_i m_i\,\omega^2\,y_i = \sum_i m_i y_i = m_1 r\cos\theta = 0; \Rightarrow \text{Not balanced} \\ x \circlearrowleft & \sum_i (m_i\,\omega^2\,x_i) z_i = \sum_i m_i x_i z_i = m_1 r\sin\theta\cdot 0 = 0; \Rightarrow \text{Balanced} \\ y \circlearrowleft & \sum_i (m_i\,\omega^2\,y_i) z_i = \sum_i m_i y_i z_i = m_1 r\cos\theta\cdot 0 = 0; \Rightarrow \text{Balanced} \end{cases} \tag{3.12}$$

with $m_1 = m_{ROT} + \frac{1}{2}m_{ALT}$.

Therefore, as already explained, in order to have a balanced engine it is necessary to add counterweights on the crankshaft.

3.4.1 Balancing strategy

In order to balance centrifugal forces and the rotating part of the first order alternating forces, it is necessary to add counterweights on the crankshaft. The system can be represented as in Figure 3.18, having taken the assumption to add two counterweights.

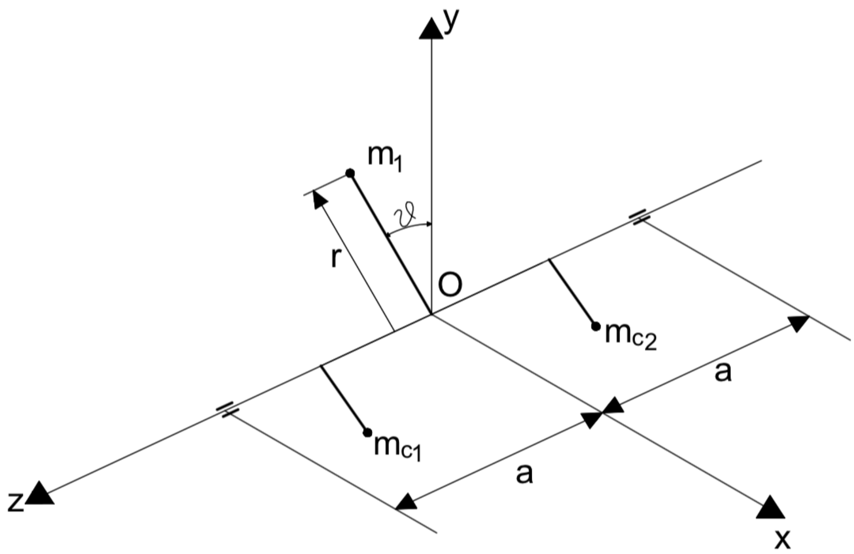

Figure 3.18: Single cylinder crankshaft with two counterweights

$$m_1 \Rightarrow \begin{pmatrix} r\sin\theta \\ r\cos\theta \\ 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \qquad m_{c_1} \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}; \qquad m_{c_2} \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix};$$

The system of equilibrium equation can be written as:

$$\begin{cases} x)\ m_{c_1}\omega^2 x_1 + m_{c_2}\omega^2 x_2 + m_1\omega^2 x = 0 \\ y)\ m_{c_1}\omega^2 y_1 + m_{c_2}\omega^2 y_2 + m_1\omega^2 y = 0 \\ O_x \circlearrowleft (m_{c_1}\omega^2 x_1)z_1 + (m_{c_2}\omega^2 x_2)z_2 = 0 \\ O_y \circlearrowleft (m_{c_1}\omega^2 y_1)z_1 + (m_{c_2}\omega^2 y_2)z_2 = 0 \end{cases} \tag{3.13}$$

There are four equations and four unknowns, so we have the freedom of choise on where to place the masses and which mass should be placed in those positions $(m_{c_1}x_1, m_{c_2}x_2, m_{c_1}y_1, m_{c_2}y_2)$.

The solution is:

$$x_1 m_{c_1} = \frac{x z_2}{z_1 - z_2} m_1; \tag{3.14}$$

$$x_2 m_{c_2} = \frac{x z_1}{z_2 - z_1} m_1; \tag{3.15}$$

$$y_1 m_{c_1} = \frac{y z_2}{z_1 - z_2} m_1; \tag{3.16}$$

$$y_2 m_{c_2} = \frac{y z_1}{z_2 - z_1} m_1; \tag{3.17}$$

where x, y and m_1 are kwown, and z_1, z_2 chosen.

The different parameters can be assumed as shown:

1. $m_{c_1}, m_{c_2} \rightarrow$ I get x_1, y_1, x_2, y_2 (masses positions).
2. x_1, x_2 (or y_1, y_2) $\rightarrow$ I get $m_{c_1}, m_{c_2}, y_1, y_2$ (or x_1, x_2).

If $z_1 = -z_2$: (symmetrical planes)

$$x_1 m_{c_1} = x_2 m_{c_2} = -\frac{x}{2} m_1; \tag{3.18}$$

$$y_1 m_{c_1} = y_2 m_{c_2} = -\frac{y}{2} m_1; \tag{3.19}$$

If:

$$x_1 = x_2 \Rightarrow m_{c_1} = m_{c_2} \quad \text{and } y_1 = y_2; \tag{3.20}$$

$$y_1 = y_2 \Rightarrow m_{c_1} = m_{c_2} \quad \text{and } x_1 = x_2; \tag{3.21}$$

The parameters assumpitons are maked based on different projectual restrictions, ad for example the encumbrances and the available empty spaces.

By way of example, in the Figure 3.19 is reported a possible configuration of a *Single-Cylinder Engine* in which all the first order forces are balanced. It can be seen the balancer shaft that rotates with the same ω speed than the crankshaft, but with opposite verse. This balancer shaft has a specific mass that allows to the system to be fully first order forces balanced.

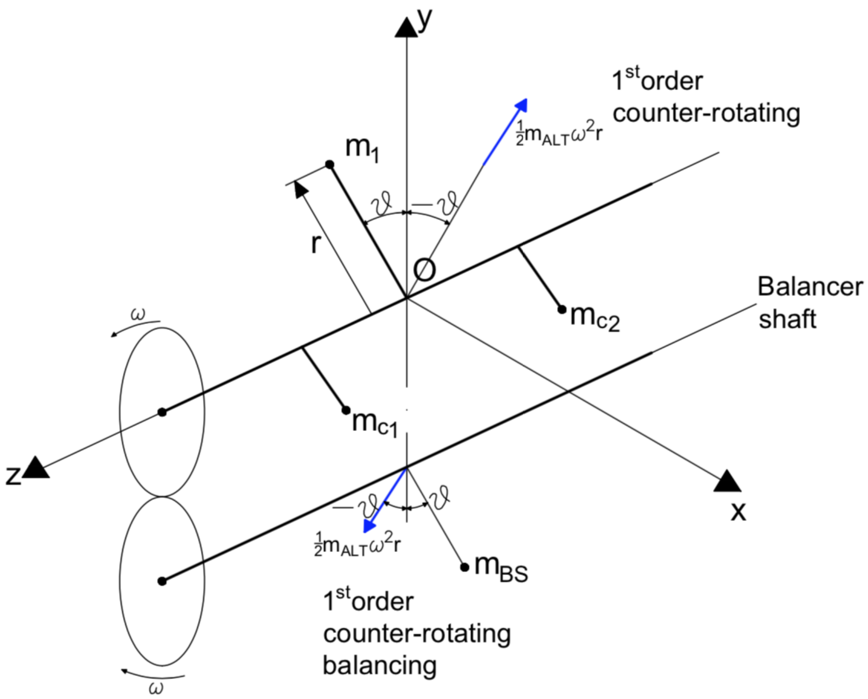

Figure 3.19: Single cylinder engine with a balancer shaft

Chapter 4

Inline Engines

4.1 Two-Cylinder Inline Engines

There are three possible configurations of *Two-Cylinder Inline Engine*:

- β = 180°
- β = 360°
- β = 270° (or 90°)

The term β is the angle bewteen crank throws.

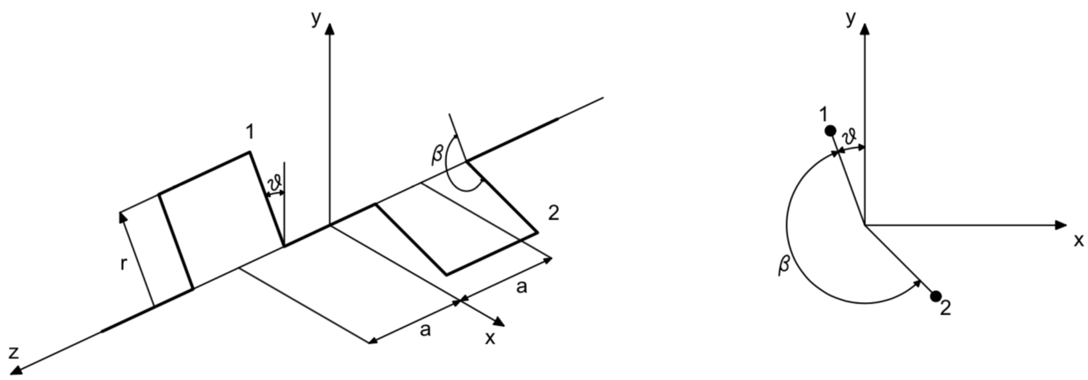

Figure 4.1: Generic configuration of a two-cylinder inline engine

Considering the configuration shown in the Figure 4.1 with a generic β angle the fudamental equations system, that shows which forces are balanced or not, can be written as:

$$\text{Cylinder 1} \Rightarrow \begin{pmatrix} r\sin\theta \\ r\cos\theta \\ a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \qquad \text{Cylinder 2} \Rightarrow \begin{pmatrix} r\sin(\theta+\beta) \\ r\cos(\theta+\beta) \\ -a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix};$$

$$\begin{cases} x) \quad \sum_i m_i\,\omega^2\,x_i = \sum_i m_i x_i = m_1 r\sin\theta + m_2 r\sin(\theta+\beta) = 0; \\ y) \quad \sum_i m_i\,\omega^2\,y_i = \sum_i m_i y_i = m_1 r\cos\theta + m_2 r\cos(\theta+\beta) = 0; \\ x \circlearrowleft \quad \sum_i (m_i\,\omega^2\,x_i) z_i = \sum_i m_i x_i z_i = m_1 r\sin\theta(a) + m_2 r\sin(\theta+\beta)(-a) = 0; \\ y \circlearrowleft \quad \sum_i (m_i\,\omega^2\,y_i) z_i = \sum_i m_i y_i z_i = m_1 r\cos\theta(a) + m_2 r\cos(\theta+\beta)(-a) = 0; \end{cases} \tag{4.1}$$

Let us consider now each single case separately and, as already done for the single cylinder case, derive the equations and relationships that regulate the systems.

4.1.1 Two-Cylinder Inline Engines with β=180°

In the 4-strokes two-cylinder engines with β=180° the gap between the duty cycles is not uniform, because of the useful phases succeed each other with different intervals (of 180° and 540°).

Therefore, in terms of torque variations, this configuration is worse than the configuration with β=360°.

The system can be represented as shown in the Figure 4.2:

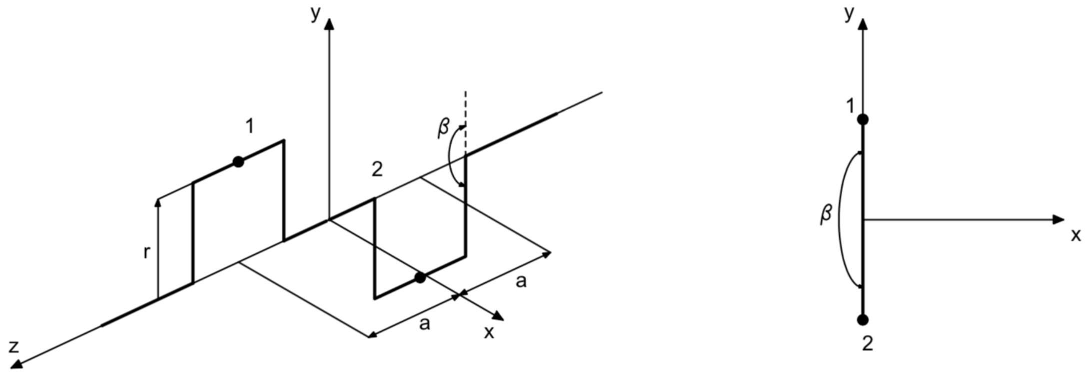

Figure 4.2: Two-cylinder inline engine with β=180°

The forces acting on the system can be divided in first and second order, and they can be represented in the first order and in the second order vectors stars.

First order vectors star

As can be seen in the Figure 4.3:

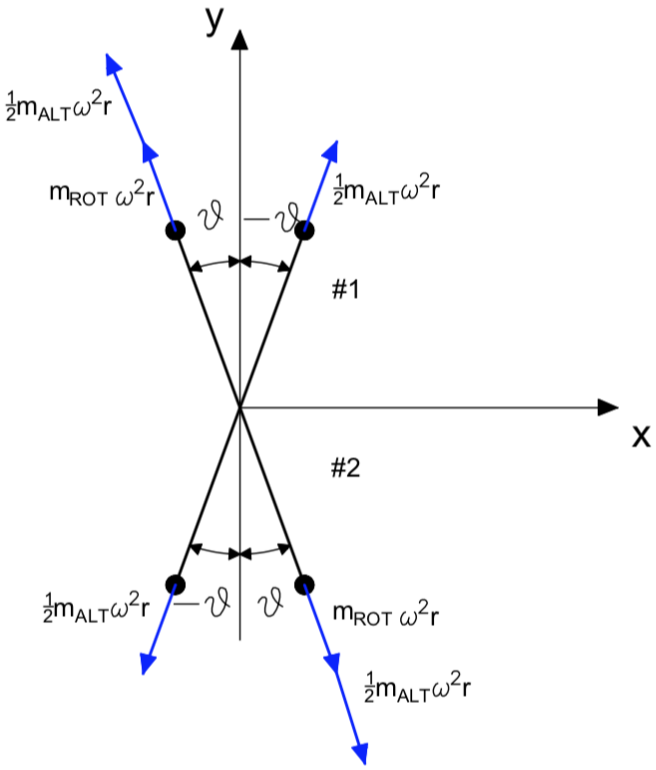

Figure 4.3: Firts order vectors star with β=180°

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) ⇒ Balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

Second order vectors star

As can be seen in the Figure 4.4:

- Rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) ⇒ Not balanced.
- Counter-rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) ⇒ Not balanced.

As shown in the Figure 4.4, the second order forces, that rotate with 2ω speed, due to the cylinder #1 and the cylinder #2 are overlapped, as can be derived in this quick demonstration:

- Cyl. #1 $\rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. #2 $\rightarrow (\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;

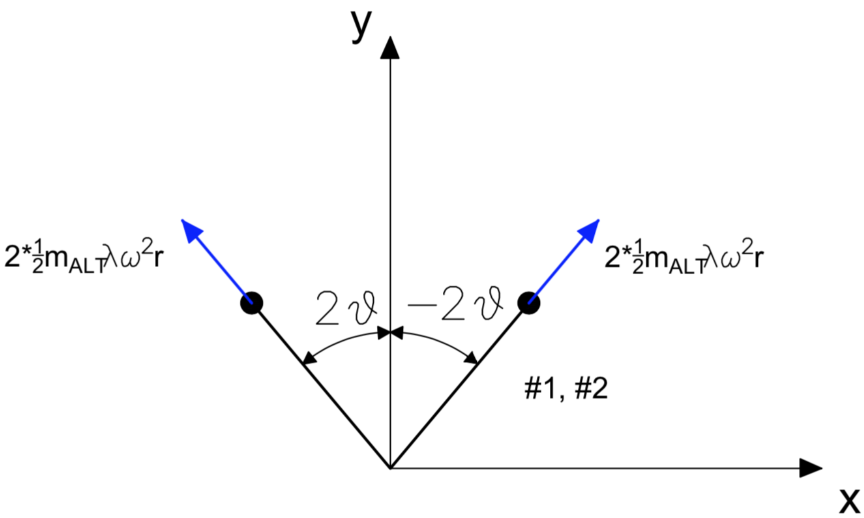

Figure 4.4: Second order vectors star with β=180°

In this configuration of the system the first order moments are not balanced, instead the second order moments are balanced.

Analytical form and fundamental equations

The considered system is shown in the Figure 4.5:

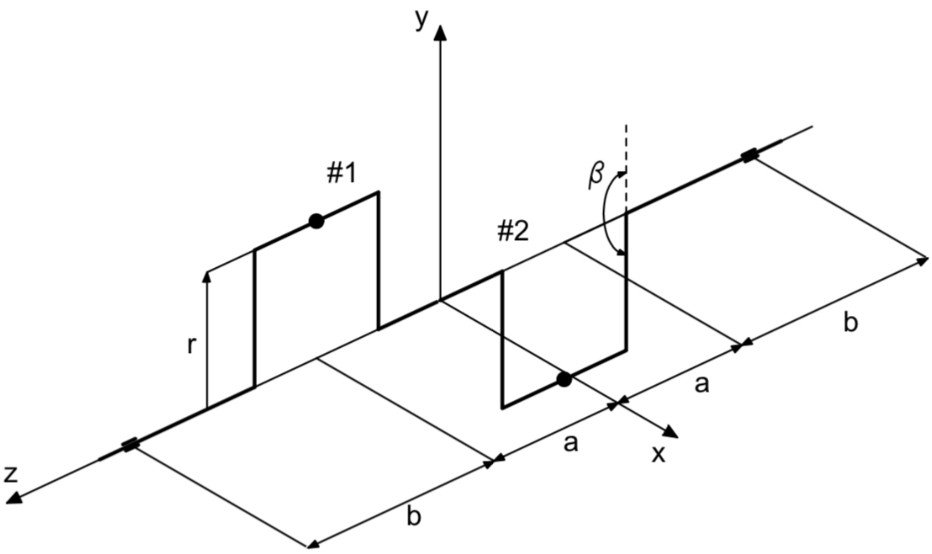

Figure 4.5: Two-cylinder inline engine with β=180°

$$\text{Cylinder } \#1 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = a \end{pmatrix} \qquad \text{Cylinder } \#2 \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = -a \end{pmatrix}$$

$$\begin{cases} x) & \sum_i m_i\, \omega^2\, x_i = \sum_i m_i x_i = m_1 x_1 + m_2 x_2 = 0; \Rightarrow \text{Balanced} \\ y) & \sum_i m_i\, \omega^2\, y_i = \sum_i m_i y_i = m_1 y_1 + m_2 y_2 = 0; \Rightarrow \text{Balanced} \\ x \circlearrowleft & \sum_i (m_i\, \omega^2\, x_i) z_i = \sum_i m_i x_i z_i = m_1 x_1 a + m_2 x_2 (-a) = 0; \Rightarrow \text{Must be balanced} \\ y \circlearrowleft & \sum_i (m_i\, \omega^2\, y_i) z_i = \sum_i m_i y_i z_i = m_1 y_1 a + m_2 y_2 (-a) = 0; \Rightarrow \text{Must be balanced} \end{cases} \tag{4.2}$$

The resultant force in x direction is balanced in each system position because $x_1 = -x_2$ for any θ angle considered. For the resultant in y direction it can be followed the same reasoning considering that $y_1 = -y_2$ for any θ angle considered.

Balancing Strategy

It has to be chosen to consider a configuration with the addition of tho counterweights to the crankshaft, one for each cylinder. It is important to stress that this is only one of the many possible layouts. Let us consider the Figure 4.6, where the system are schematically reported:

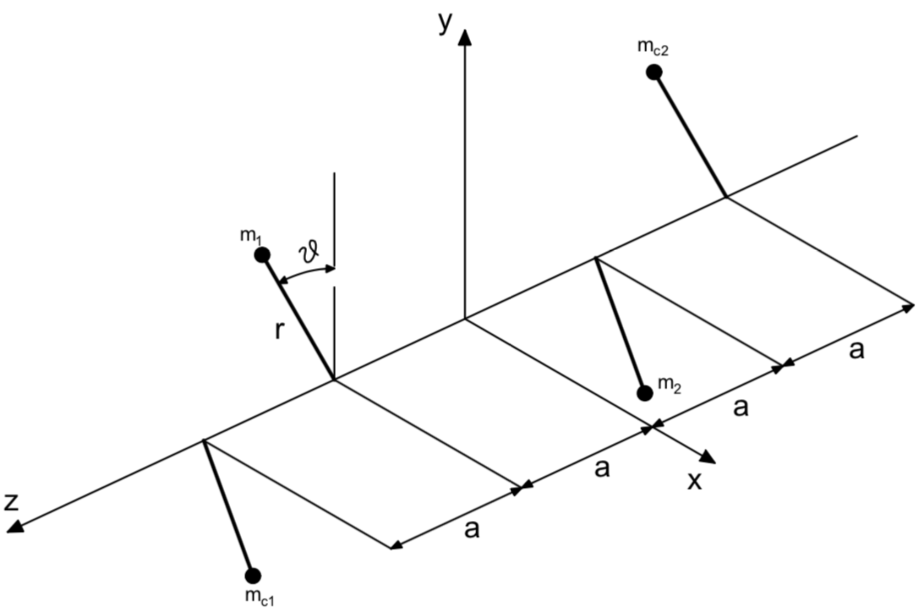

Figure 4.6: Two-cylinder inline engine with β=180° with counterweights

$$m_1 \Rightarrow \begin{pmatrix} x_1 = -r\sin\theta \\ y_1 = r\cos\theta \\ z_1 = a \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 = r\sin\theta \\ y_2 = -r\cos\theta \\ z_2 = -a \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = 2a \end{pmatrix}; \qquad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = -2a; \end{pmatrix};$$

(considering $m_1 = m_{ROT_1} + \frac{1}{2} m_{ALT_1}$ and $m_1 = m_2$).
The system can be written as:

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_1\,\omega^2\,(-r\sin\theta) + m_2\,\omega^2\,r\sin\theta = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_1\,\omega^2\,r\cos\theta + m_2\,\omega^2\,(-r\cos\theta) = 0 \\ x \circlearrowleft \ (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_1\,\omega^2\,(-r\sin\theta))z_1 + (m_2\,\omega^2\,r\sin\theta)z_2 = 0 \\ y \circlearrowleft \ (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_1\,\omega^2\,r\cos\theta)z_1 + (m_2\,\omega^2\,(-r\cos\theta))z_2 = 0 \end{cases} \tag{4.3}$$

with $z_{c_1} = 2a$, $z_{c_2} = -2a$, $z_1 = a$ and $z_2 = -a$ taken as assumption.

If $m_1 = m_2 = m$:

$$\begin{cases} x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} = 0 \\ y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} = 0 \\ x \circlearrowleft \ m_{c_1}x_{c_1} - m_{c_2}x_{c_2} - mr\sin\theta = 0 \\ y \circlearrowleft \ m_{c_1}y_{c_1} - m_{c_2}y_{c_2} + mr\cos\theta = 0 \end{cases} \tag{4.4}$$

The solutions are:

$$m_{c_1}x_{c_1} = \frac{mr\sin\theta}{2}; \tag{4.5}$$

$$m_{c_2}x_{c_2} = -\frac{mr\sin\theta}{2}; \tag{4.6}$$

$$m_{c_1}y_{c_1} = -\frac{mr\cos\theta}{2}; \tag{4.7}$$

$$m_{c_2}y_{c_2} = \frac{mr\cos\theta}{2}; \tag{4.8}$$

If $x_{c_1} = -x_{c_2} = x$ and $y_{c_1} = -y_{c_2} = y$ the equations system can be rewritten as:

$$\begin{cases} x) \quad (m_{c_1} - m_{c_2})x = 0 \\ y) \quad (m_{c_1} - m_{c_2})y = 0 \\ x \circlearrowleft \ (m_{c_1} + m_{c_2})x = mr\sin\theta \\ y \circlearrowleft \ (m_{c_1} + m_{c_2})y = -mr\cos\theta \end{cases} \tag{4.9}$$

and the solutions are:

$$m_{c_1} = m_{c_1} = m_c; \tag{4.10}$$

$$m_c x = \frac{mr\sin\theta}{2}; \tag{4.11}$$

$$m_c y = -\frac{mr\cos\theta}{2}; \tag{4.12}$$

Fixing the masses m_{c_1} and m_{c_2} the parameters $x_{c_1}, x_{c_2}, y_{c_2}, y_{c_2}$ can be obtained ; fixing x_{c_2} and x_{c_2} (or y_{c_2} and y_{c_2}) it is possible to derive $m_{c_1}, m_{c_2}, y_{c_1}, y_{c_2}$ (or x_{c_1}, x_{c_2}).

To balance the first order counter-rotating moment it is necessary to place a balancer shaft that rotates at $-\omega$ speed. In the Figure 4.7 it can be seen a possible system configuration, with a balancer shaft equipped with two eccentric masses that provide to balance the first order counter-rotating forces. The second order moments are balanced instead.

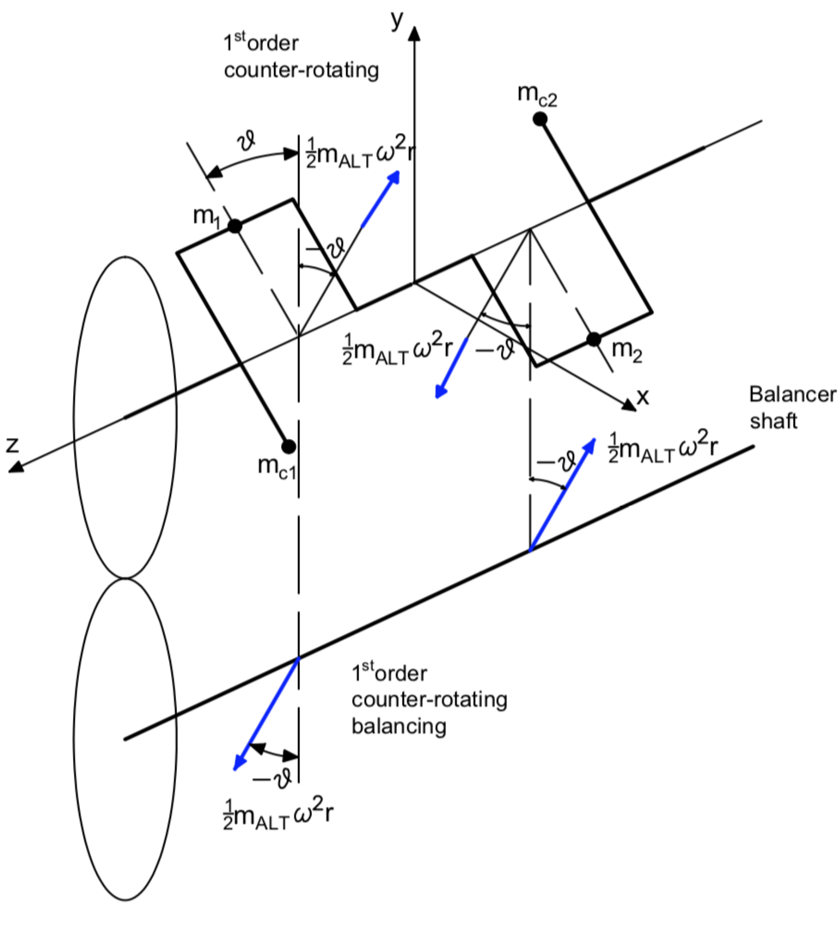

Figure 4.7: Two-cylinder inline engine with β=180° with balancer shaft

4.1.2 Two-Cylinder Inline Engines with β=360°

In this configuration, with β=360°, the gap duty cycles is uniform, therefore the useful phases succeed each other always with the same interval. Then, this is the best layout in terms of torque variations for a two-cylinder engine. The system can be represented as shown in the Figure 4.8:

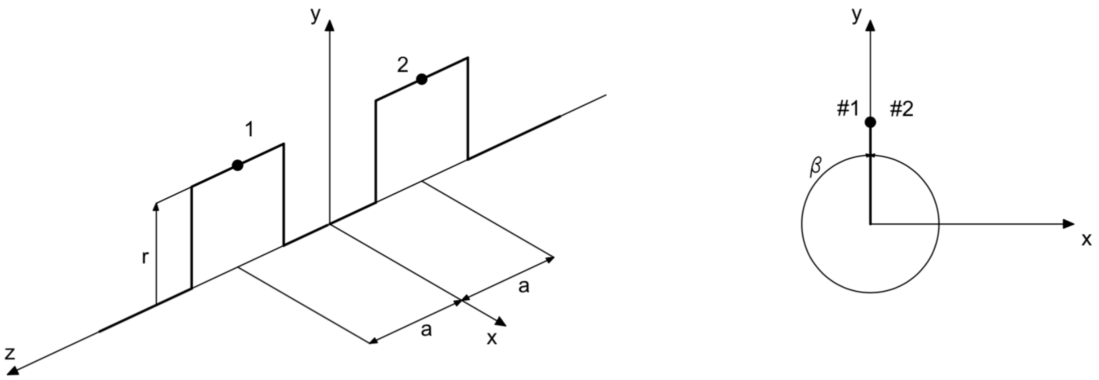

Figure 4.8: Two-cylinder inline engine with β=360°

The forces acting on the system can be divided in first and second order, and they can be represented in the first order and in the second order vectors stars.

First order vectors star

As can be seen in the Figure 4.9:

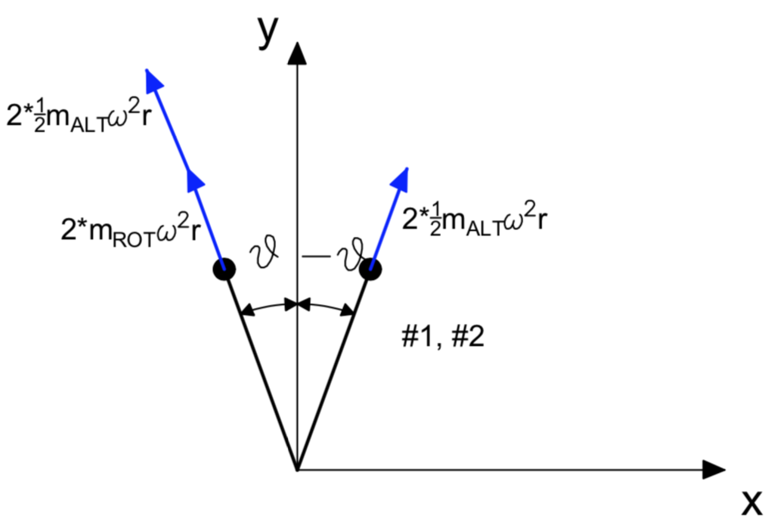

Figure 4.9: Firts order vectors star with β=360°

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) $\Rightarrow$ Not balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Not balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Not balanced.

Second order vectors star

As can be seen in the Figure 4.10:

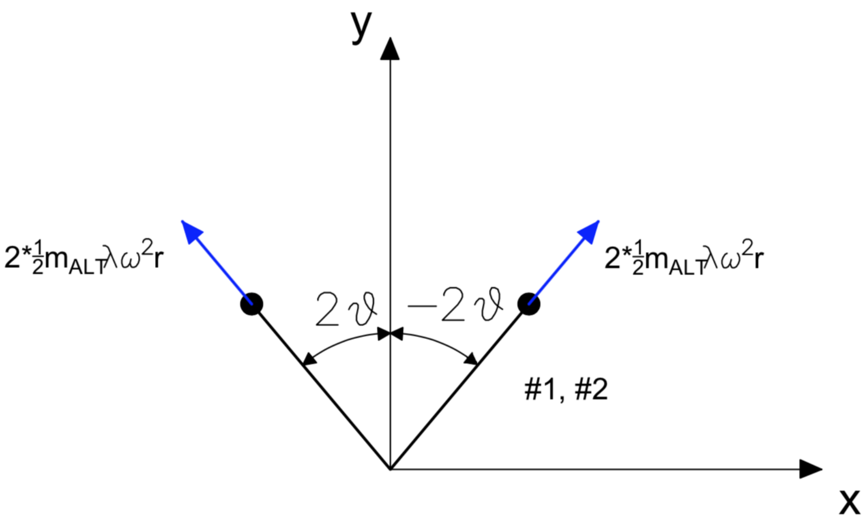

Figure 4.10: Second order vectors star with β=360°

- Rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) $\Rightarrow$ Not balanced.
- Counter-rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) $\Rightarrow$ Not balanced.

In this system configuration, first order and second order moments due to rotating and counter-rotating first order and second order forces are balanced.

Analytical form and fundamental equations

The considered system in shown in the Figure 4.11:

$$\text{Cylinder \#1} \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = a \end{pmatrix} \qquad \text{Cylinder \#2} \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = -a \end{pmatrix}$$

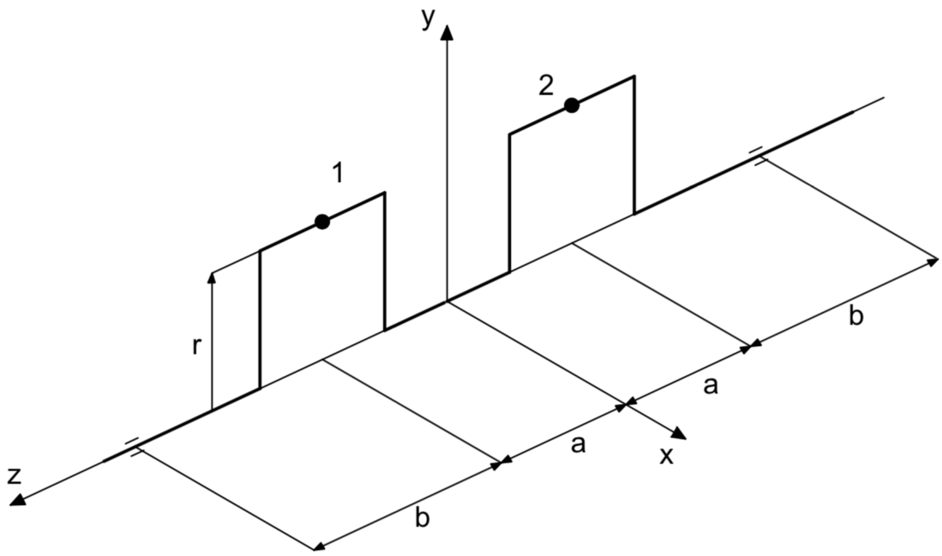

Figure 4.11: Two-cylinder inline engine with β=360°

$$\begin{cases} x) & \sum_i m_i\,\omega^2\,x_i = \sum_i m_i x_i = m_1 x_1 + m_2 x_2 = 0; \Rightarrow \text{Must be balanced} \\ y) & \sum_i m_i\,\omega^2\,y_i = \sum_i m_i y_i = m_1 y_1 + m_2 y_2 = 0; \Rightarrow \text{Must be balanced} \\ x \circlearrowleft & \sum_i (m_i\,\omega^2\,x_i) z_i = \sum_i m_i x_i z_i = m_1 x_1 a + m_2 x_2 (-a) = 0; \Rightarrow \text{Balanced} \\ y \circlearrowleft & \sum_i (m_i\,\omega^2\,y_i) z_i = \sum_i m_i y_i z_i = m_1 y_1 a + m_2 y_2 (-a) = 0; \Rightarrow \text{Balanced} \end{cases} \tag{4.13}$$

The resultant force in x direction is not balanced because $x_1 = x_2$ for each θ angle considered. For the resultant in y direction it can be followed the same reasoning considering that $y_1 = y_2$ for any θ angle considered.

Both the moments x and y are instead balanced for all the θ angles.

Balancing Strategy

To balance this engine configuration, it is necessary to add counterweights on the crankshaft. Considering a configuration with the addition of 4 balancing masses, two for each cylinder, as shown in Figure 4.12:

$$m_1 \Rightarrow \begin{pmatrix} x_1 = -r\sin\theta \\ y_1 = r\cos\theta \\ z_1 = 2a \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 = -r\sin\theta \\ y_2 = r\cos\theta \\ z_2 = -2a \end{pmatrix};$$

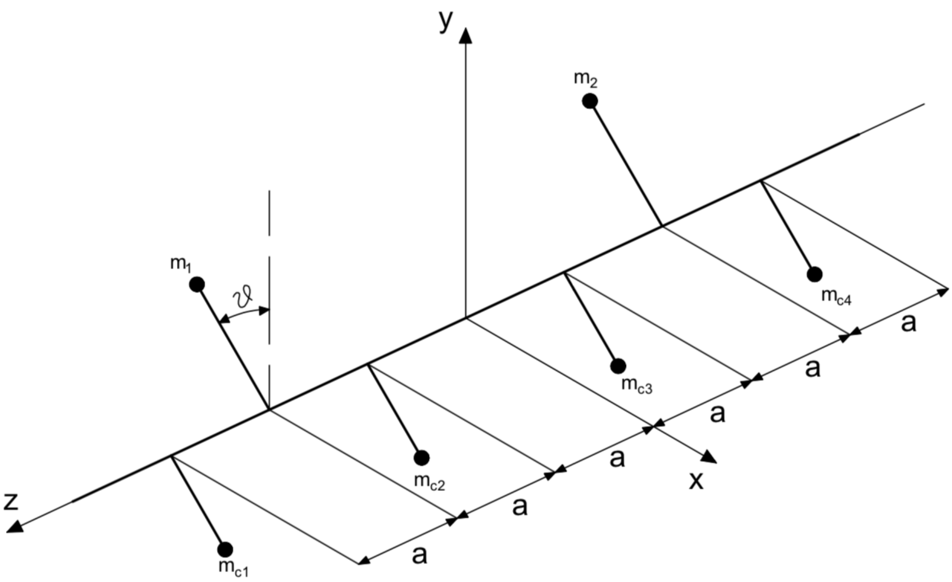

Figure 4.12: Two-cylinder inline engine with β=360° with counterweights

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = 3a \end{pmatrix}; \qquad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = a; \end{pmatrix};$$

$$m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = -a \end{pmatrix}; \qquad m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = -3a; \end{pmatrix};$$

This type of balancing is commonly called *Bay-by-bay*.

The system can be written as:

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + m_1\,\omega^2\,(-r\sin\theta) + m_2\,\omega^2\,(-r\sin\theta) = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + m_1\,\omega^2\,r\cos\theta + m_2\,\omega^2\,r\cos\theta = 0 \\ x \circlearrowleft \; (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + \\ \qquad +(m_1\,\omega^2\,(-r\sin\theta))z_1 + (m_2\,\omega^2\,(-r\sin\theta))z_2 = 0 \\ y \circlearrowleft \; (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} + \\ \qquad +(m_1\,\omega^2\,r\cos\theta)z_1 + (m_2\,\omega^2\,r\cos\theta)z_2 = 0 \end{cases} \tag{4.14}$$

If $m_1 = m_2 = m$ and $x_1 = x_2 = x$ and $y_1 = y_2 = y$:

$$\begin{cases} x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_4} + m_{c_4}x_{c_4} + 2mx = 0 \\ y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_4} + m_{c_4}y_{c_4} + 2my = 0 \\ x \circlearrowleft \; 3m_{c_1}x_{c_1} + m_{c_2}x_{c_2} - m_{c_3}x_{c_3} - 3m_{c_4}x_{c_4} = 0 \\ y \circlearrowleft \; 3m_{c_1}y_{c_1} + m_{c_2}y_{c_2} - m_{c_3}y_{c_3} - 3m_{c_4}y_{c_4} = 0 \end{cases} \tag{4.15}$$

In order to make ad example, assuming that the counterweights are placed in opposition to the cylinders (therefore with an angle of 180° respect of the cylinders) and that the four counterweights masses are the same ($m_{c_1} = m_{c_2} = m_{c_3} = m_{c_4} = m_c$):

The solution is:

$$m_c x_c = -\frac{mx}{2}; \tag{4.16}$$

$$m_c y_c = -\frac{my}{2}; \tag{4.17}$$

The values counterweights masses and positions are a function of many parameters, as the balancing factos chosen for each bay, the empty spaces in the crankcase, the maximum acceptable loads on the bearing, the maximum acceptable shaft weight, etc.

In the Figure 4.13 is shown a possible balancing configuration of the first order counter-rotatiung forces, using a balancer shaft that rotates with the same and opposite speed of the crankshaft.

It this way the system is fully balanced to the first order forces.

4.1.3 Two-Cylinder Inline Engines with β=270°

In the Figure 4.14 it is represented the system considered:

As already done for the cases with β=180° and β=360°, the forces are divided in firts order and second order and representer in the vectors stars.

First order vectors star

As can be seen in the Figure 4.15:

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) ⇒ Not balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Not balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Not balanced.

Second order vectors star

As can be seen in the Figure 4.16:

- Rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) ⇒ Balanced.
- Counter-rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r\lambda$) ⇒ Balanced.

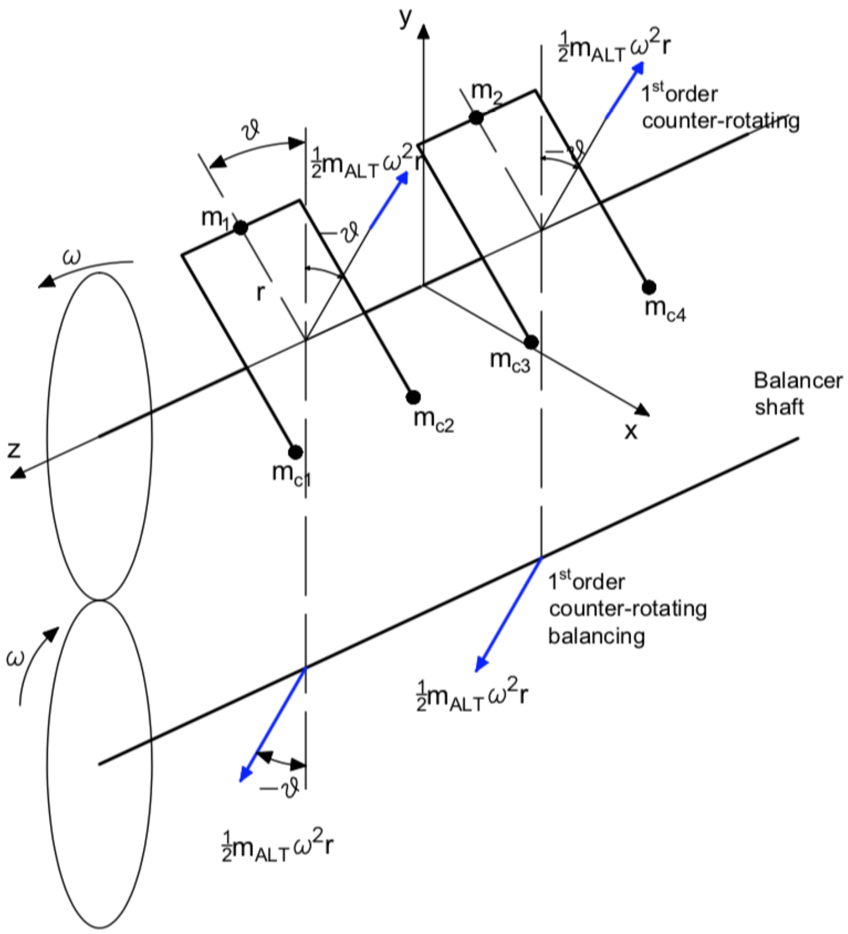

Figure 4.13: Two-cylinder inline engine with β=360° with balancer shaft

As shown in the Figure 4.16, the second order forces, that rotate with 2ω speed, due to the cylinder #1 and the cylinder #2 are overlapped, as can be derived in this quick demonstration:

- Cyl. #1 $\rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. #2 $\rightarrow (\theta + 270°) \Rightarrow 2(\theta + 270°) \Rightarrow \cos(2\theta + 540°)$;

In this configuration the *first order moments* are not balanced, instead the *second order moments* are balanced.

Analytical form and fundamental equations

The considered system in shown in the Figure 4.17:

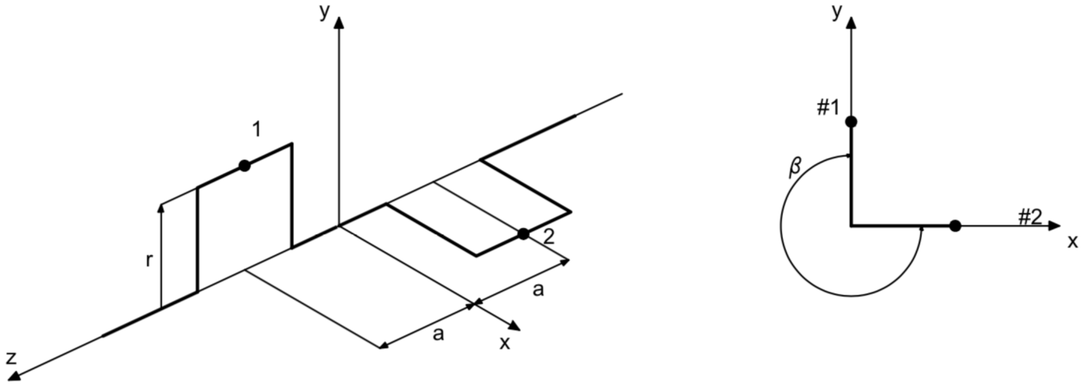

Figure 4.14: Two-cylinder inline engine with β=270°

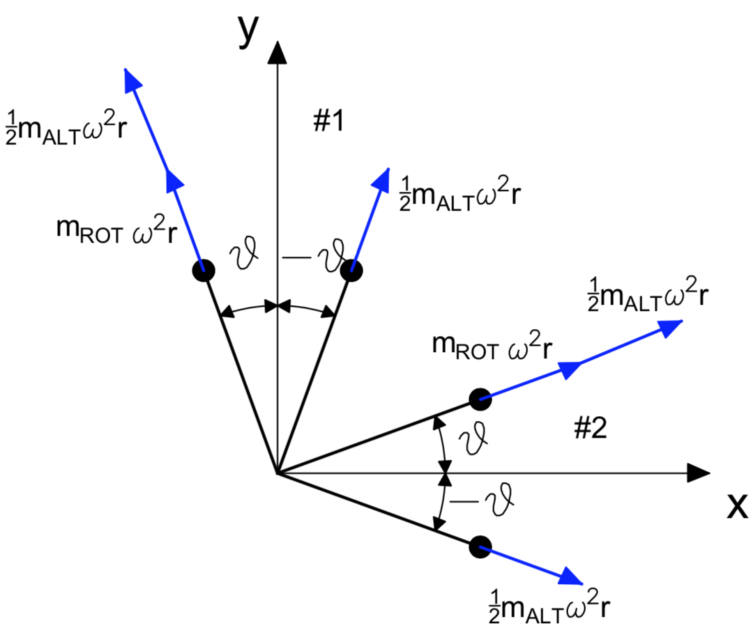

Figure 4.15: Firts order vectors star with β=270°

$$\text{Cylinder \#1} \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = a \end{pmatrix} \qquad \text{Cylinder \#2} \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = -a \end{pmatrix}$$

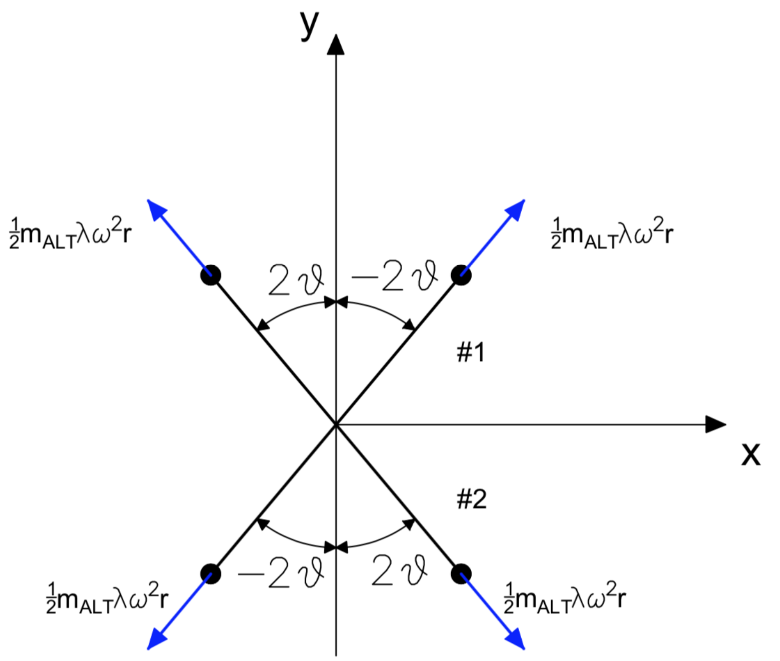

Figure 4.16: Second order vectors star with β=270°

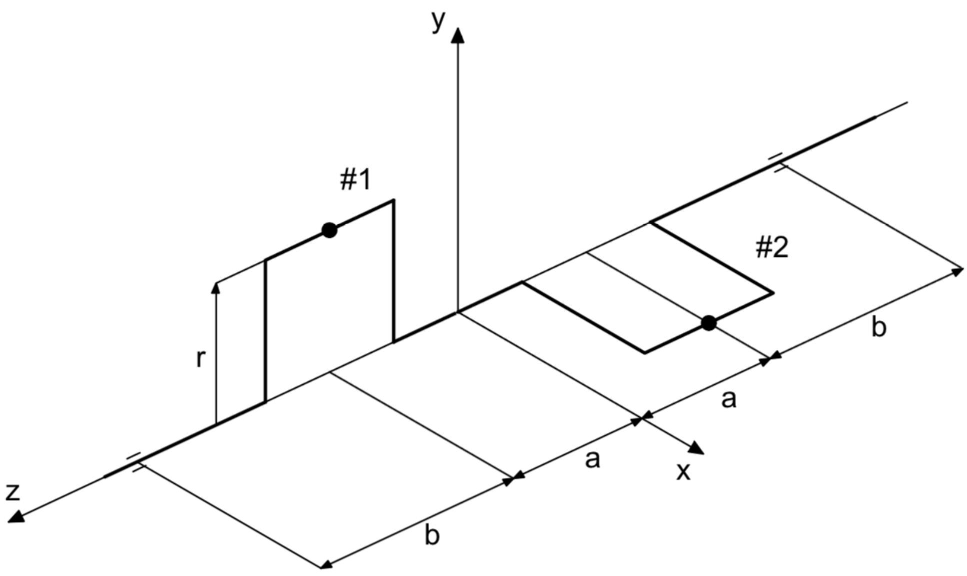

Figure 4.17: Two-cylinder inline engine with β=270°

$$\begin{cases} x) \quad \sum_i m_i\, \omega^2\, x_i = \sum_i m_i x_i = m_1 x_1 + m_2 x_2 = 0; \Rightarrow \text{Must be balanced} \\ y) \quad \sum_i m_i\, \omega^2\, y_i = \sum_i m_i y_i = m_1 y_1 + m_2 y_2 = 0; \Rightarrow \text{Must be balanced} \\ x \circlearrowleft \quad \sum_i (m_i\, \omega^2\, x_i) z_i = \sum_i m_i x_i z_i = m_1 x_1 a + m_2 x_2 (-a) = 0; \Rightarrow \text{Must be balanced} \\ y \circlearrowleft \quad \sum_i (m_i\, \omega^2\, y_i) z_i = \sum_i m_i y_i z_i = m_1 y_1 a + m_2 y_2 (-a) = 0; \Rightarrow \text{Must be balanced} \end{cases} \tag{4.18}$$

In this engine configuration, first order forces and moments are not balanced for every θ angle considered.

The expressions of the moments around x and y axis can be derived as showh below in 4.19 and 4.20:

$$M_x = aF\cos\theta - aF\sin\theta = aF(\cos\theta - \sin\theta); \quad (4.19)$$

$$M_y = aF\sin\theta + aF\cos\theta = aF(\sin\theta + \cos\theta); \quad (4.20)$$

considering the trigonometric relations $\sin\theta = -\cos(\frac{3\pi}{2} - \theta)$ and $\cos\theta = -\sin(\frac{3\pi}{2} - \theta)$ and using the prosthaphaeresis formulas, the moments expressions will be:

$$M_x = -\sqrt{2}aF\cos\left(\theta - \frac{3\pi}{4}\right); \quad (4.21)$$

$$M_y = -\sqrt{2}aF\sin\left(\theta - \frac{3\pi}{4}\right); \quad (4.22)$$

The moments M_x and M_y are on a plane that follows by $3\pi/4 = 135°$ the plane of the crank of the cylinder #1, in each position considered.

Balancing Strategy

Because of the relations 4.21 and 4.22, in this engine layout it is necessary to add two (or more) counterweights on the plane where the moments are located in order to balance them.

Considering a configuration with $\theta = 45°$ as shown in the Figure 4.18:

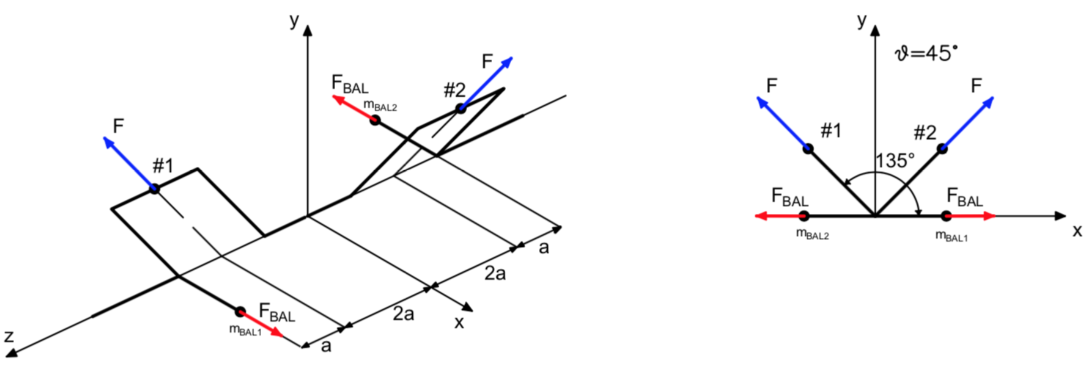

Figure 4.18: Two-cylinder inline engine with β=270° and θ=45°

With $\theta = 45°$ the moment M_x is self-balanced. Doing a moment balancing, the forces due to the counterweights masses can be derived as shown below:

$$4aF\sin 45 = 2\sqrt{2}aF = 6aF_{BAL} \quad (4.23)$$

$$F_{BAL} = m_{BAL}\omega^2 r_{BAL} = \frac{\sqrt{2}}{3}F = \frac{\sqrt{2}}{3}\omega^2 r(m_{ROT} + \frac{1}{2}m_{ALT}); \tag{4.24}$$

In this way it can be balanced both the pure rotating forces and the part of the firts order alternating forces that can be reconduced to rotating and, moreover, the first order rotating moment due to these forces.

In order to balance the counter-rotating forces it is necessary to have a balancer shaft, as shown in the Figure 4.19:

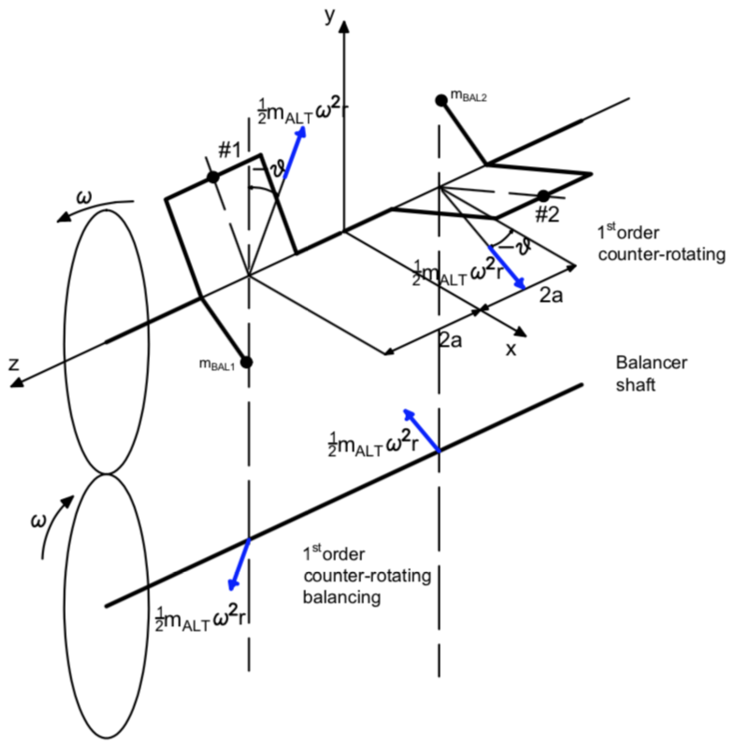

Figure 4.19: Two-cylinder inline engine with β=270° and balancer shaft

The plane where the forces due to the eccentric masses of the balancer shaft are placed preceed by 135° the plane of the cyl. #1 in each position considered.

This way of balancing produces a system that is globally balanced to the first order forces and moments, but it is not locally balanced. In order to have a system that is also locally balanced it is possible to apply the *Bay-by-bay balancing strategy.*

Considering to add two counterweights for each cylinder, the system can be represented as shown in the Figure 4.20:

$$m_1 \Rightarrow \begin{pmatrix} x_1 = -r\sin\theta \\ y_1 = r\cos\theta \\ z_1 = 2a \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 = r\cos\theta \\ y_2 = r\sin\theta \\ z_2 = -2a \end{pmatrix};$$

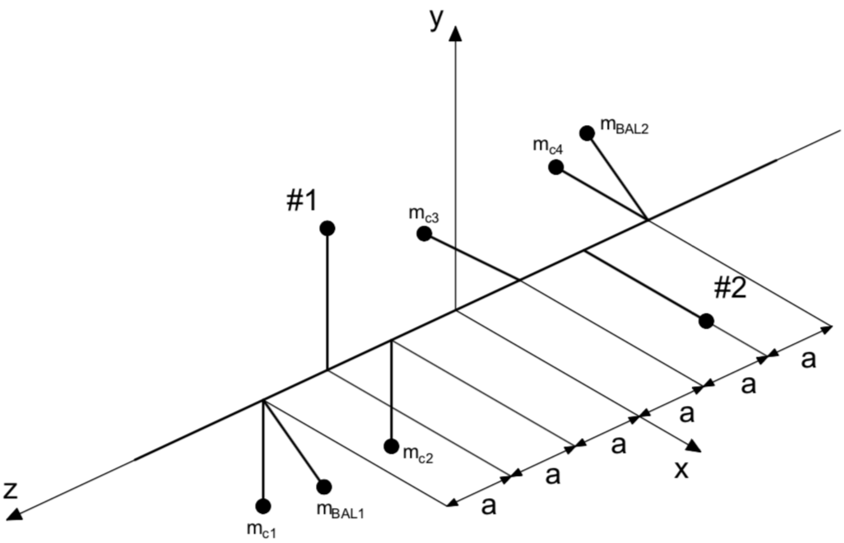

Figure 4.20: Two-cylinder inline engine with β=270° with bay-by-bay balancing

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = 3a \end{pmatrix} \quad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = a \end{pmatrix} \quad m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = -a \end{pmatrix} \quad m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = -3a \end{pmatrix}$$

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + \\ \qquad +m_1\,\omega^2\,(-r\sin\theta) + m_2\,\omega^2\,(r\cos\theta) = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + \\ \qquad +m_1\,\omega^2\,r\cos\theta + m_2\,\omega^2\,r\sin\theta = 0 \\ x \circlearrowleft \quad (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + \\ \qquad +(m_1\,\omega^2\,(-r\sin\theta))z_1 + (m_2\,\omega^2\,r\cos\theta)z_2 = 0 \\ y \circlearrowleft \quad (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} + \\ \qquad +(m_1\,\omega^2\,r\cos\theta)z_1 + (m_2\,\omega^2\,r\sin\theta)z_2 = 0 \end{cases} \tag{4.25}$$

that can be rewritten as:

$$\begin{cases} x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_1(-r\sin\theta) + m_2(r\cos\theta) = 0 \\ y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_1 r\cos\theta + m_2 r\sin\theta = 0 \\ x \circlearrowleft \quad (m_{c_1}x_{c_1})3a + (m_{c_2}x_{c_2})a + (m_{c_3}x_{c_3})(-a) + (m_{c_4}x_{c_4})(-3a) + \\ \qquad +(m_1(-r\sin\theta))2a + (m_2 r\cos\theta)(-2a) = 0 \\ y \circlearrowleft \quad (m_{c_1}y_{c_1})3a + (m_{c_2}y_{c_2})a + (m_{c_3}y_{c_3})(-a) + (m_{c_4}y_{c_4})(-3a) + \\ \qquad +(m_1 r\cos\theta)2a + (m_2 r\sin\theta)(-2a) = 0 \end{cases} \tag{4.26}$$

Assuming that:

- $m_1 = m_2 = m$;
- $x_{c_1} = x_{c_2} = x_{c_{12}}$ and $x_{c_3} = x_{c_4} = x_{c_{34}}$
- $y_{c_1} = y_{c_2} = y_{c_{12}}$ and $y_{c_3} = y_{c_4} = y_{c_{34}}$
- $m_{c_1} = m_{c_2} = m_{c_{12}}$ and $m_{c_3} = m_{c_4} = m_{c_{34}}$

the system becomes:

$$\begin{cases} x) \quad mr(\cos\theta - \sin\theta) + 2m_{c_12}x_{c_12} + 2m_{c_34}x_{c_34} = 0 \\ y) \quad mr(\cos\theta + \sin\theta) + 2m_{c_12}y_{c_12} + 2m_{c_34}y_{c_34} = 0 \\ x \circlearrowleft \; 2amr(-\cos\theta - \sin\theta) + 4am_{c_12}x_{c_12} - 4am_{c_34}x_{c_34} = 0 \\ y \circlearrowleft \; 2amr(\cos\theta - \sin\theta) + 4am_{c_12}y_{c_12} - 4am_{c_34}y_{c_34} = 0 \end{cases} \tag{4.27}$$

Solving this system it is possible to find a solution with all the parameters that balance the engine. Obviously, this solution is a function of the assumptions maked.

4.1.4 Bearings loads

The way used to find the bearing loads is based on the *Three Moment Equation of Clapeyron* [6] [7]. The crankshaft is hyperstatically constrained and can be considered as a beam, in this case with two span.

The three moment equation method require to use ad additional equation for each excess constrain. In the equations shown below it is assumed that the Young's modulus E and the moment of inertia of the crankshaft section J are constant along the crankshaft, therefore they cancel each other in all the terms.

The procedure is the same for all the I2 crankshaft configurations analyzed.

The three moment equation is derived considering the system in the Figure 4.21.

the three moment equation is equal to:

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = -\frac{F_{c1}}{l_1}a(l_1^2 - a^2) - \frac{F_{c2}}{l_2}d(l_2^2 - d^2); \tag{4.28}$$

The moments M_A and M_C are zero and the unknown moment M_B is equal to:

$$M_B = \frac{1}{2(l_1 + l_2)}\left[-\frac{F_{c1}}{l_1}a(l_1^2 - a^2) - \frac{F_{c2}}{l_2}d(l_2^2 - d^2)\right]; \tag{4.29}$$

Considering now the each single span, the shear forces and the reaction on the supports can be derived as shown below:

- System "AB" (Figure 4.22):

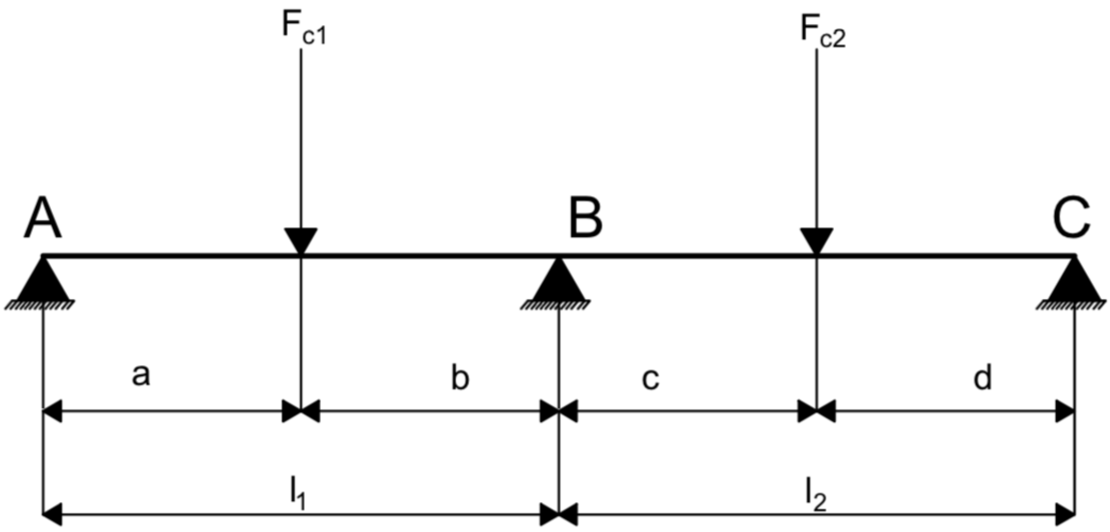

Figure 4.21: I2 crankshaft seen as a two-span beam

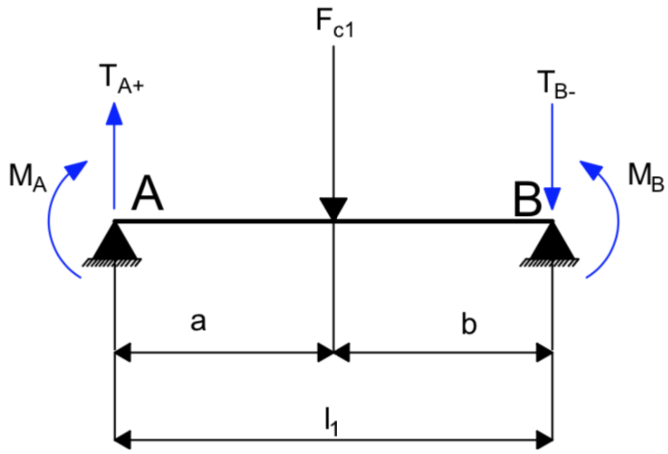

Figure 4.22: System AB

$$\begin{cases} B \circlearrowright \;\; M_B - M_A + F_{c1}b - T_{A^+}l_1 = 0 \\ \qquad T_{A^+} = \frac{1}{l_1}(M_B + F_{c1}b); \\ \uparrow) \; T_{A^+} - F_{c1} - T_{B^-} = 0 \\ \qquad T_{B^-} = T_{A^+} - F_{c1}; \end{cases} \tag{4.30}$$

- System "BC" (Figure 4.23):

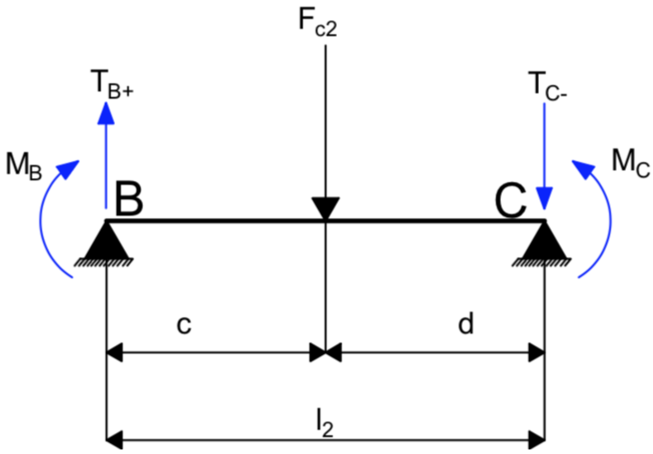

Figure 4.23: System BC

$$\begin{cases} C \circlearrowleft \;\; M_C - M_B + F_{c2}d - T_{B^+}l_2 = 0 \\ \qquad T_{B^+} = \frac{1}{l_2}(-M_B + F_{c2}d; \\ \uparrow) \; T_{B^+} - F_{c2} - T_{C^-} = 0 \\ \qquad T_{C^-} = T_{B^+} - F_{c2}; \end{cases} \tag{4.31}$$

The reactions can be obtaiden considering the convention shown in Figure 4.24.

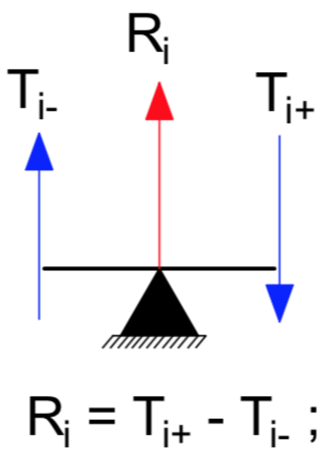

Figure 4.24: Shear forces convention

$$\begin{cases} R_A = T_{A^+}; \\ R_B = T_{B^+} - T_{B^-}; \\ R_C = -T_{C^-}; \\ \end{cases} \tag{4.32}$$

All the procedure shown above is valid for the case of a non-balanced crankshaft as no forces due do the counterweights are were considered. Anyway, the same method can be used for a balanced crankshaft just adding the forces due to the counterweights, as shown in the Figure 4.25.

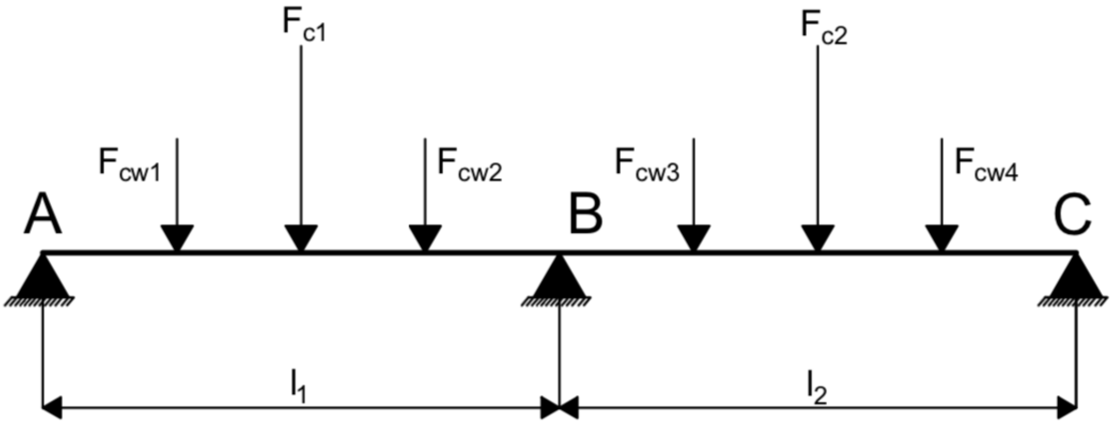

Figure 4.25: Model of forces in a I2 balanced crankshaft

4.2 Three-Cylinder Inline Engines

In this paragraph equations and relationships that regulate three cylinders inline engines will be obtained.

For these engines the angle between each crank is equal to $\frac{720°}{3} = 240°$ and there are two possibilities of crank throws layout:

- $\beta_2 = 120°$, $\beta_3 = 240°$ (F.O. 1-3-2)
- $\beta_2 = 240°$, $\beta_3 = 120°$ (F.O. 1-2-3)

In the Figure 4.26 is shown what β_2 and β_3 mean.

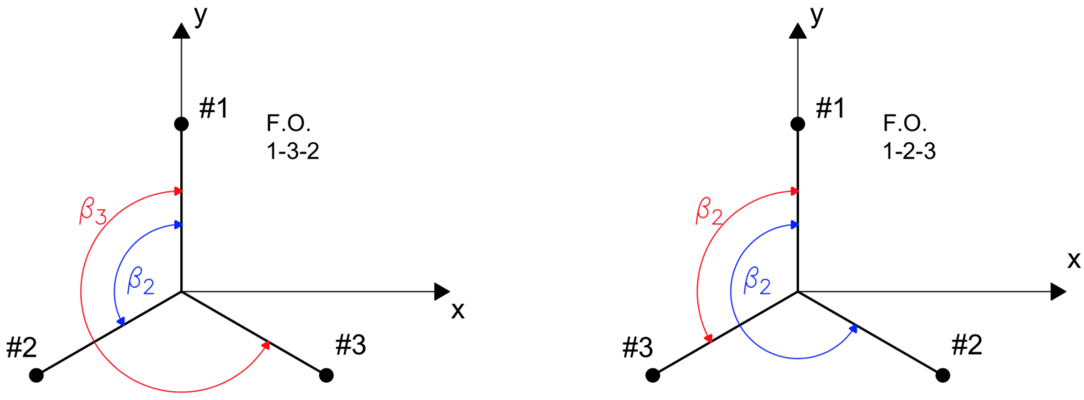

Figure 4.26: Three-cylinder Firing Orders

Only one of the two cases listed is analyzed, because, with the same reasoning, it is easy to come to similar results.

4.2.1 Three-cylinder Inline engine with $\beta_2 = 120°$ and $\beta_3 = 240°$

This case corresponding to a *F.O.* equal to 1-3-2, as shown in the Figure 4.27

The first and the second order vectors stars are now reported, in order to show the forces acting on the system.

First order vectors star

As can be seen in the Figure 4.28:

As can be seen:

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) ⇒ Balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

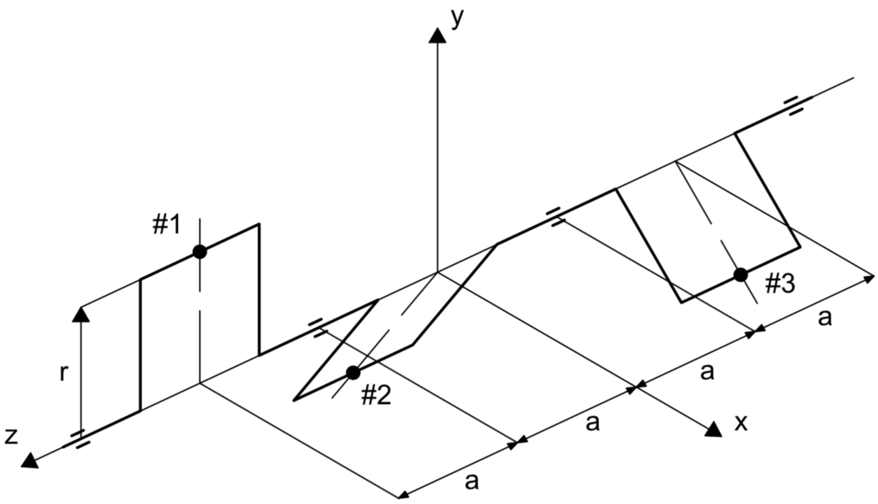

Figure 4.27: Three-cylinder inline engine with F.O. 1-3-2.

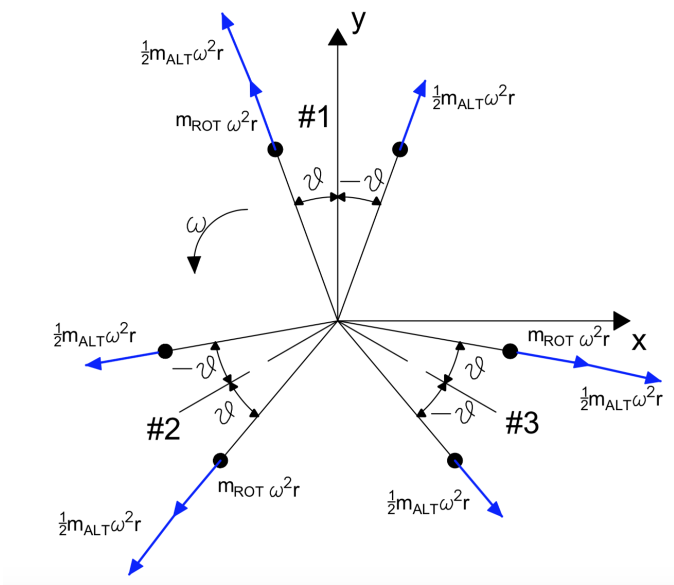

Figure 4.28: Firts order vectors star

Second order vectors star

In Figure 4.29 the second order forces, that rotate with 2ω speed, due to the cylinder #2 and #3 exchange their position, as quickly derived below:

- Cyl. $\#1 \rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. $\#2 \rightarrow (\theta + 120°) \Rightarrow 2(\theta + 120°) \Rightarrow \cos(2\theta + 240°)$;
- Cyl. $\#3 \rightarrow (\theta + 240°) \Rightarrow 2(\theta + 240°) \Rightarrow \cos(2\theta + 480°) = \cos(2\theta + 120°)$;

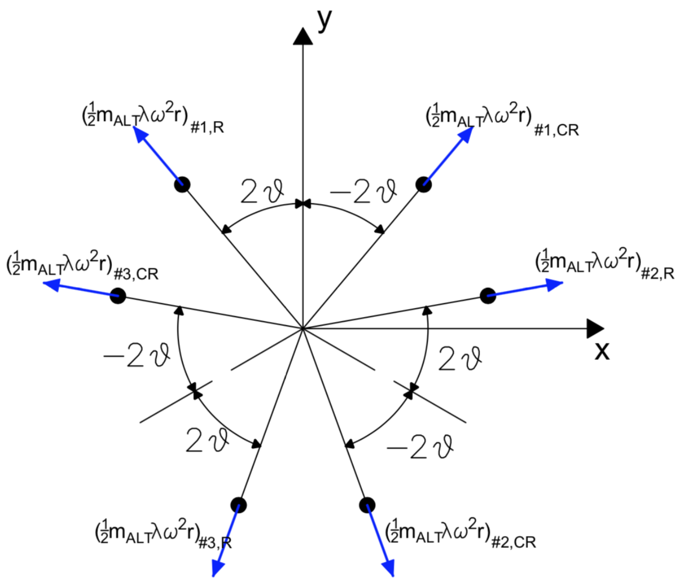

Figure 4.29: Second order vectors star

As can be seen in the Figure 4.29:

- Rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.
- Counter-rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.

In these engines, all first and second order forces are balanced, but the moments are not balanced (this phenomenon is typical of systems with odd cylinders number, because there is no symmetry plane).

Analytical form and fundamental equations

$$\text{Cyl. } \#1 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = 2a \end{pmatrix}; \qquad \text{Cyl. } \#2 \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = 0 \end{pmatrix} \qquad \text{Cyl. } \#3 \Rightarrow \begin{pmatrix} x_3 \\ y_3 \\ z_3 = -2a \end{pmatrix}$$

The system of equations can be written as:

$$\begin{cases} x) \quad \sum_i m_i\,\omega^2\,x_i = \sum_i m_i x_i = m_1x_1 + m_2x_2 + m_3x_3 = 0; \Rightarrow \text{Balanced} \\ y) \quad \sum_i m_i\,\omega^2\,y_i = \sum_i m_i y_i = m_1y_1 + m_2y_2 + m_3y_3 = 0; \Rightarrow \text{Balanced} \\ x \circlearrowleft \quad \sum_i (m_i\,\omega^2\,x_i) z_i = \sum_i m_i x_i z_i = m_1x_1 2a + m_2x_2 \cdot 0 + m_3x_3(-2a) = 0; \Rightarrow \text{Must be balanced} \\ y \circlearrowleft \quad \sum_i (m_i\,\omega^2\,y_i) z_i = \sum_i m_i y_i z_i = m_1y_1 2a + m_2y_2 \cdot 0 + m_3y_3(-2a) = 0; \Rightarrow \text{Must be balanced} \end{cases} \tag{4.33}$$

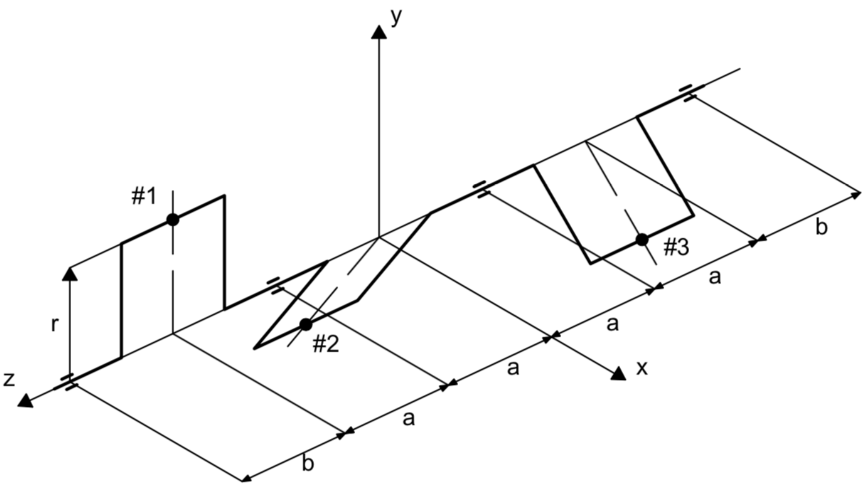

Figure 4.30: Three-cylinder inline crankshaft

Considering a case with $\theta = 0°$ and the assuptions that $m_1 = m_2 = m_3 = m$ and $r_1 = r_2 = r_3$, the system becomes:

$$\text{Cyl. } \#1 \Rightarrow \begin{pmatrix} x_1 = 0 \\ y_1 = r \\ z_1 = 2a \end{pmatrix}; \qquad \text{Cyl. } \#2 \Rightarrow \begin{pmatrix} x_2 = -r\cos 30° = -\frac{\sqrt{3}}{2}r \\ y_2 = -r\sin 30° = -\frac{1}{2}r \\ z_2 = 0 \end{pmatrix}$$

$$\text{Cyl. } \#3 \Rightarrow \begin{pmatrix} x_3 = r\cos 30° = \frac{\sqrt{3}}{2}r \\ y_3 = -r\sin 30° = -\frac{1}{2}r \\ z_3 = -2a \end{pmatrix}$$

$$\begin{cases} x) \quad \sum_i m_i\,\omega^2\,x_i = \sum_i m_i x_i = m\cdot 0 + m(-\frac{\sqrt{3}}{2}) + m\frac{\sqrt{3}}{2} = 0; \Rightarrow \text{Balanced} \\ y) \quad \sum_i m_i\,\omega^2\,y_i = \sum_i m_i y_i = mr + m(-\frac{r}{2}) + m(-\frac{r}{2}) = 0; \Rightarrow \text{Balanced} \\ x \circlearrowleft \quad \sum_i (m_i\,\omega^2\,x_i)z_i = \sum_i m_i x_i z_i = m\cdot 0\cdot 2a + m(-\frac{\sqrt{3}}{2}r\cdot 0) + m(\frac{\sqrt{3}}{2}r\cdot(-2a)) = \\ \qquad = -\sqrt{3}arm = 0; \Rightarrow \text{Must be balanced} \\ y \circlearrowleft \quad \sum_i (m_i\,\omega^2\,y_i)z_i = \sum_i m_i y_i z_i = mr2a + m(-\frac{r}{2}\cdot 0) + m(-\frac{r}{2}(-2a)) = \\ \qquad = 3arm = 0; \Rightarrow \text{Must be balanced} \end{cases} \tag{4.34}$$

That system confirms what as already written, in terms of which forces are balanced and which not.

Considering the Figure 4.31, the expressions of the moments around x and y axis can be derived as showh below in 4.35 and 4.36:

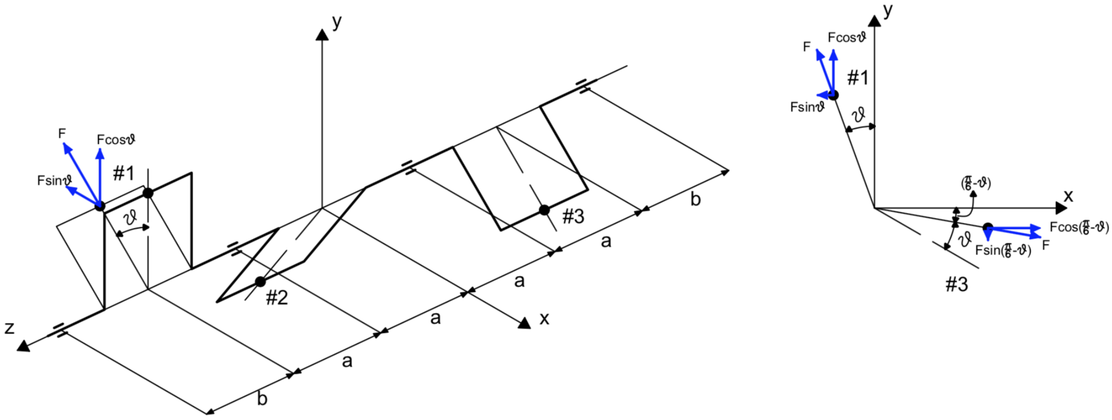

Figure 4.31: Three-cylinder inline crankshaft

$$M_x = F\cos\theta 2a + F\sin\left(\theta - \frac{\pi}{6}\right)2a = 2aF\left[\cos\theta + \sin\left(\theta - \frac{\pi}{6}\right)\right] = 2aF\left[\cos\theta - \cos\left(\frac{4\pi}{3} + \theta\right)\right]; \tag{4.35}$$

considering the trigonometric relation: $\sin\left(\theta - \frac{\pi}{6}\right) = -\cos\left(\frac{4\pi}{3} + \theta\right)$.

$$M_y = F\sin\theta 2a + F\cos\left(\frac{\pi}{6} - \theta\right)2a = 2aF\left[\sin\theta + \cos\left(\frac{\pi}{6} - \theta\right)\right] = 2aF\left[\sin\theta - \sin\left(\frac{4\pi}{3} + \theta\right)\right]; \tag{4.36}$$

considering the trigonometric relation: $\cos\left(\frac{\pi}{6} - \theta\right) = -\sin\left(\frac{4\pi}{3} + \theta\right)$.

Continuing to develop the moments expressions through trigonometric relations and prosthaphaeresis formulas, the results will be:

$$M_x = 2\sqrt{3}aF\cos\left(\frac{\pi}{6} + \theta\right); \tag{4.37}$$

$$M_y = 2\sqrt{3}aF\sin\left(\frac{\pi}{6} + \theta\right); \tag{4.38}$$

The term M_x is the *Pitching Moment*, M_y is the *Yawing Moment*.

These moments M_x and M_y, considering as θ the angle of the mass 1, preceed 30° (= $\pi/6$) the position of this cylinder, in each reference position considered (as shown in the Figure 4.32).

Balancing Strategy

In order to study the engine balancing, to have a better view of the physical meaning it is useful to put the cylinder number 2 at TDC (on shafts with odd cylinders number it is usual to put the "central" cylinder at the TDC), as shown in the Figure 4.33:

In this configuration:

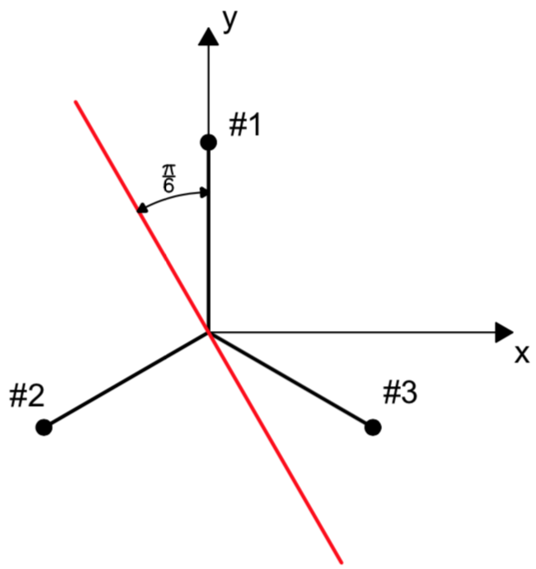

Figure 4.32: Plane where the moments are located on.

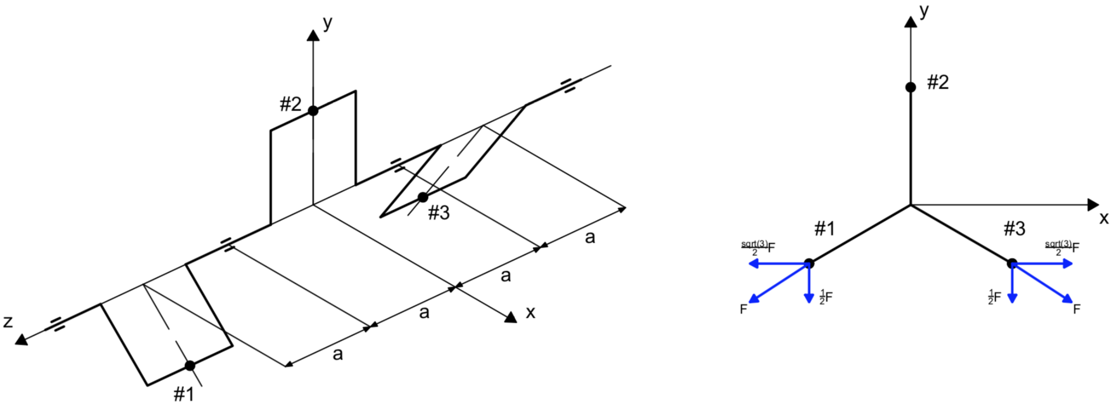

Figure 4.33: Configuration with the cyl.#2 at TDC

$$M_x = 0; \tag{4.39}$$

$$M_y = F\frac{\sqrt{3}}{2}4a = 2\sqrt{3}aF; \tag{4.40}$$

with $F = \omega^2 r(m_{ROT} + \frac{1}{2}m_{ALT})$.

Because of the relations 4.37 and 4.38, in this engine layout it is necessary to add two (or more) counterweights on the plane where the moments are located in order to balance them, as it can be seen in the Figure 4.34.

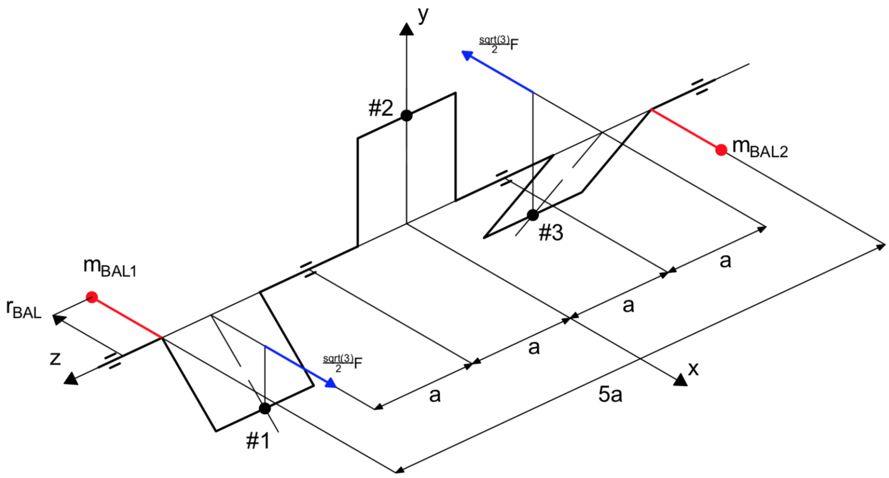

Figure 4.34: Balanced configuration of the system

Doing a moment balancing, the forces due to the counterweights masses can be derived as shown below:

$$\frac{\sqrt{3}}{2}F4a = F_{BAL}5a \tag{4.41}$$

then:

$$F_{BAL} = m_{BAL}\omega^2 r_{BAL} = \frac{4}{5}\frac{\sqrt{3}}{2}F = \frac{2\sqrt{3}}{5}F = \frac{2\sqrt{3}}{5}\omega^2 r\left(m_{ROT} + \frac{1}{2}m_{ALT}\right); \tag{4.42}$$

where m_{BAL} is equal to the counterweight mass and r_{BAL} is equal to the radius where the counterweight mass is placed.

In this way it can be balanced both the pure rotating and the part of first order alternating force that can be recoduced to rotating.

In order to balance the counter-rotating moment it is necessary to use balancer shafts. In the Figure 4.35 is shown an example with a counter-shaft that balances the firts order counter-rotating moments.

The direction of the balancing moment is in a plane trailing by 30° the cylinder #1. It is possible to demonstrate this statement as shown below.

Assuming to consider the cylinder #1 at the TDC, as shown in the Figure 4.36:

The first order counter-rotating moments can be written as:

$$M_{x_{CR}} = F\cos\theta 2a + F\sin\left(\frac{\pi}{6}+\theta\right)2a = 2aF\left[\cos\theta - \cos\left(\frac{4\pi}{3}-\theta\right)\right]; \tag{4.43}$$

$$M_{y_{CR}} = F\sin\theta 2a + F\cos\left(\frac{\pi}{6}+\theta\right)2a = 2aF\left[\sin\theta - \sin\left(\frac{4\pi}{3}-\theta\right)\right]; \tag{4.44}$$

Using the same trigonometric and prosthaphaeresis formulas used to find the plane of the first order rotating moment, the expressions become:

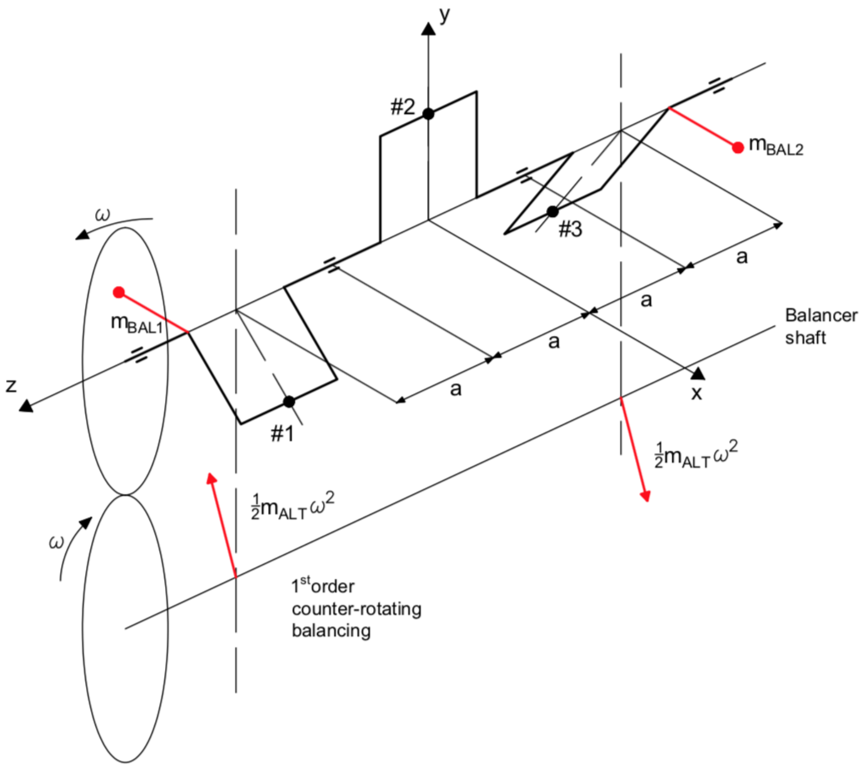

Figure 4.35: Configuration with a balancer shaft

$$M_{x_{CR}} = 2\sqrt{3}aF\cos\left(\frac{\pi}{6} - \theta\right); \tag{4.45}$$

$$M_{y_{CR}} = 2\sqrt{3}aF\sin\left(\frac{\pi}{6} - \theta\right); \tag{4.46}$$

The term F is, in this case, only due to the counter-rotating part of the first order alternating forces equal to $\frac{1}{2}m_{ALT}\omega^2 r$.

The same reasoning can be applied to balance the second order moments adding other two balancer shafts that rotate with double speed than the crankshaft, one of them rotating with the same verse than the crankshaft that balance the second order rotating moments, and the other rotating with opposite verse that balance the counter-rotating moments. Usually these second order moments are not balanced in these engines, because their balance introduces complexity and weight to the system and, moreover, costs.

This way of balancing produces a system that is globally balanced to the first order forces and moments, but it is not locally balanced. In order to have a system that is also locally balanced it is possible to apply many strategies, one of the most common methods is the *Bay-by-bay balancing strategy.* Considering to add two counterweights for

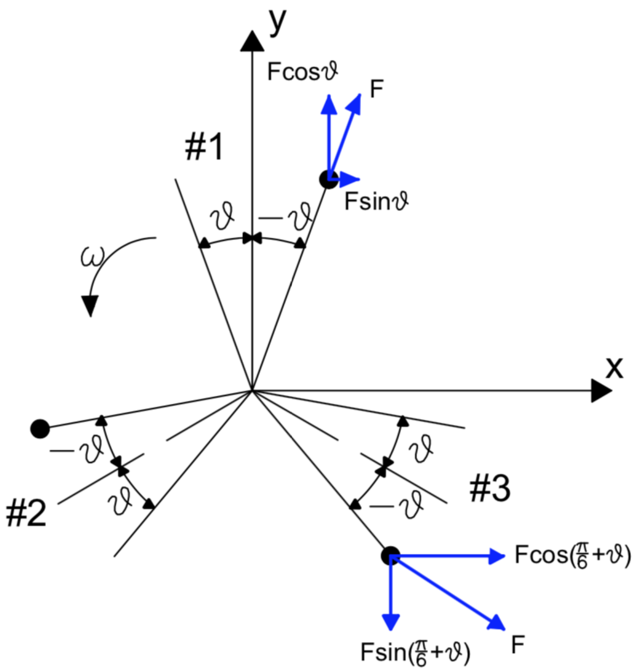

Figure 4.36: Counter-rotating forces that produce moments.

each cylinder, the system can be represented as shown in Figure 4.37:

$$m_1 \Rightarrow \begin{pmatrix} x_1 = -r\sin\theta \\ y_1 = r\cos\theta \\ z_1 = 3a \end{pmatrix}; \quad m_2 \Rightarrow \begin{pmatrix} x_2 = -r\sin(\theta+120) \\ y_2 = r\cos(\theta+120) \\ z_2 = 0 \end{pmatrix}; \quad m_3 \Rightarrow \begin{pmatrix} x_3 = -r\sin(\theta+240) \\ y_3 = r\cos(\theta+240) \\ z_3 = -3a \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = 4a \end{pmatrix}; \quad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = 2a; \end{pmatrix}; \quad m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = a \end{pmatrix};$$

$$m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = -a; \end{pmatrix}; \quad m_{c_5} \Rightarrow \begin{pmatrix} x_{c_5} \\ y_{c_5} \\ z_{c_5} = -2a; \end{pmatrix}; \quad m_{c_6} \Rightarrow \begin{pmatrix} x_{c_6} \\ y_{c_6} \\ z_{c_6} = -4a \end{pmatrix};$$

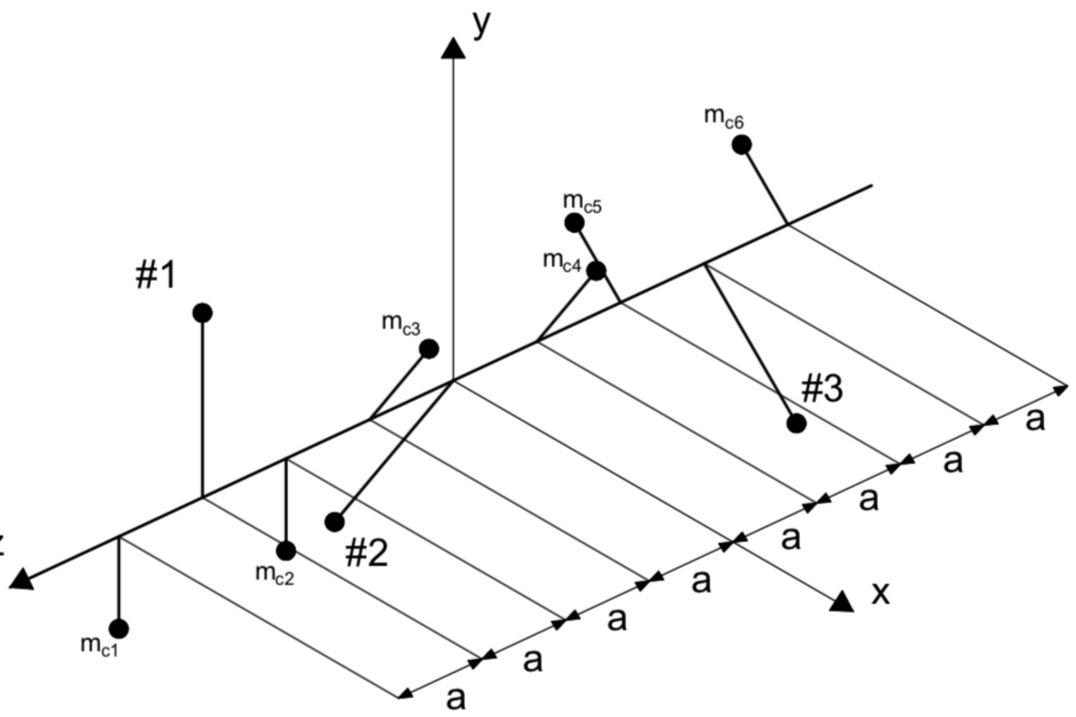

Figure 4.37: Configuration globally balanced

The system of equations can be written as:

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + m_{c_5}\,\omega^2\,x_{c_5} + m_{c_6}\,\omega^2\,x_{c_6} + \\ \qquad + m_1\,\omega^2\,x_1 + m_2\,\omega^2\,x_2 + m_3\,\omega^2\,x_3 = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + m_{c_5}\,\omega^2\,y_{c_5} + m_{c_6}\,\omega^2\,y_{c_6} + \\ \qquad + m_1\,\omega^2\,y_1 + m_2\,\omega^2\,y_2 + m_3\,\omega^2\,y_3 = 0 \\ x \circlearrowleft \; (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + (m_{c_5}\,\omega^2\,x_{c_5})z_{c_5} + \\ \qquad + (m_{c_6}\,\omega^2\,x_{c_6})z_{c_6} + (m_1\,\omega^2\,x_1)z_1 + (m_2\,\omega^2\,x_2)z_2 + (m_3\,\omega^2\,x_3)z_3 = 0 \\ y \circlearrowleft \; (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} + (m_{c_5}\,\omega^2\,y_{c_5})z_{c_5} + \\ \qquad + (m_{c_6}\,\omega^2\,y_{c_6})z_{c_6} + (m_1\,\omega^2\,y_1)z_1 + (m_2\,\omega^2\,y_2)z_2 + (m_3\,\omega^2\,y_3)z_3 = 0 \end{cases} \tag{4.47}$$

that can be rewritten as:

$$\begin{cases} x) & m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + \\ & \quad + m_1(-r\sin\theta) + m_2(-r\sin(\theta+120)) + m_3(-r\sin(\theta+240)) = 0 \\ y) & m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + \\ & \quad + m_1 r\cos\theta + m_2 r\cos(\theta+120) + m_3 r\cos(\theta+240) = 0 \\ x \circlearrowleft & (m_{c_1}x_{c_1})4a + (m_{c_2}x_{c_2})2a + (m_{c_3}x_{c_3})(a) + (m_{c_4}x_{c_4})(-a) + (m_{c_5}x_{c_5})(-2a) + (m_{c_6}x_{c_6})(-4a) + \\ & \quad + (m_1(-r\sin\theta))3a + (m_2(-r\sin(\theta+120)))\cdot 0 + (m_3(-r\sin(\theta+240)))(-3a) = 0 \\ y \circlearrowleft & (m_{c_1}y_{c_1})4a + (m_{c_2}y_{c_2})2a + (m_{c_3}y_{c_3})(a) + (m_{c_4}y_{c_4})(-a) + (m_{c_5}y_{c_5})(-2a) + (m_{c_6}y_{c_6})(-4a) + \\ & \quad + (m_1 r\cos\theta)3a + (m_2 r\cos(\theta+120))\cdot 0 + (m_3 r\cos(\theta+230))(-3a) = 0 \end{cases} \tag{4.48}$$

Assuming that:

- $m_1 = m_2 = m_3 = m$;
- $x_{c_1} = x_{c_2} = x_{c_{12}}$ and $x_{c_3} = x_{c_4} = x_{c_{34}}$ and $x_{c_5} = x_{c_6} = x_{c_{56}}$
- $y_{c_1} = y_{c_2} = y_{c_{12}}$ and $y_{c_3} = y_{c_4} = y_{c_{34}}$ and $y_{c_5} = y_{c_6} = y_{c_{56}}$
- $m_{c_1} = m_{c_2} = m_{c_{12}}$ and $m_{c_3} = m_{c_4} = m_{c_{34}}$ and $m_{c_5} = m_{c_5} = m_{c_{56}}$

the system becomes:

$$\begin{cases} x) & 2m_{c_{12}}x_{c_{12}} + 2m_{c_{34}}x_{c_{34}} + 2m_{c_{56}}x_{c_{56}} - mr(\sin\theta + \sin(\theta+120) + \sin(\theta+240)) = 0 \\ y) & 2m_{c_{12}}y_{c_{12}} + 2m_{c_{34}}y_{c_{34}} + 2m_{c_{56}}y_{c_{56}} + mr(\cos\theta + \cos(\theta+120) + \cos(\theta+240)) = 0 \\ x \circlearrowleft & m_{c_{12}}x_{c_{12}}6a - m_{c_{56}}x_{c_{56}}6a + 3amr(\sin(\theta+240) - \sin\theta) = 0 \\ y \circlearrowleft & m_{c_{12}}y_{c_{12}}6a - m_{c_{56}}y_{c_{56}}6a + 3amr(\cos(\theta+120) + \cos\theta) = 0 \end{cases} \tag{4.49}$$

Solving this system it is possible to find a solution with all the parameters that balance the engine. Obviously, this solution is a function of the assumptions made.

4.2.2 Bearings loads

The way used to find the bearing loads is based on the *Three Moment Equation of Clapeyron*. The crankshaft is hyperstatically constrained and can be considered as a beam, in this case with three span.

The three moment equation method requires use of an ad additional equation for each excess constrain. In the equations shown below it is assumed that the Young's modulus E and the moment of inertia of the crankshaft section J are constant along the crankshaft, therefore they cancel each other in all the terms.

In this case the system can be divided in two subsystems, the ABC and the BCD. For each subsystem a three moment equation can be derived.

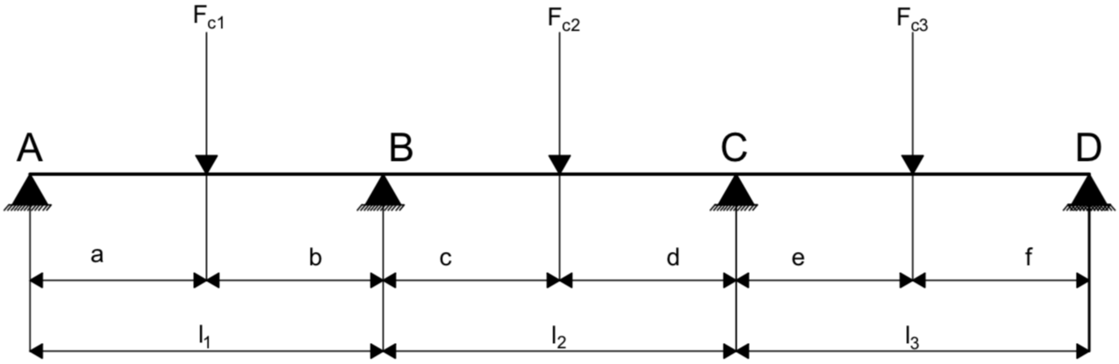

Figure 4.38: I3 crankshaft seen as a three-span beam

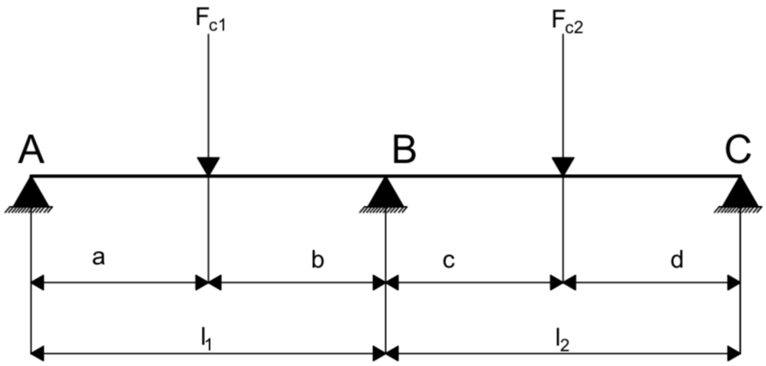

Figure 4.39: System ABC

Figure 4.38 represents the whole system considered.
Considering now the system ABC shown in Figure :
the three moment equation is equal to:

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = -\frac{F_{c1}}{l_1} a(l_1^2 - a^2) - \frac{F_{c2}}{l_2} d(l_2^2 - d^2); \tag{4.50}$$

for the system BCD shown in the Figure 4.40 the three moment equation is equal to:

$$M_B l_2 + 2M_C(l_2 + l_3) + M_D l_3 = -\frac{F_{c2}}{l_2} c(l_2^2 - c^2) - \frac{F_{c3}}{l_3} f(l_3^2 - f^2); \tag{4.51}$$

The moments M_A and M_D are zero, and M_B and M_C can be calculated considering the system in matrix form like $Ax = B$ with:

$$A = \begin{bmatrix} 2(l_1 + l_2) & l_2 \\ l_2 & 2(l_2 + l_3) \end{bmatrix}$$

;

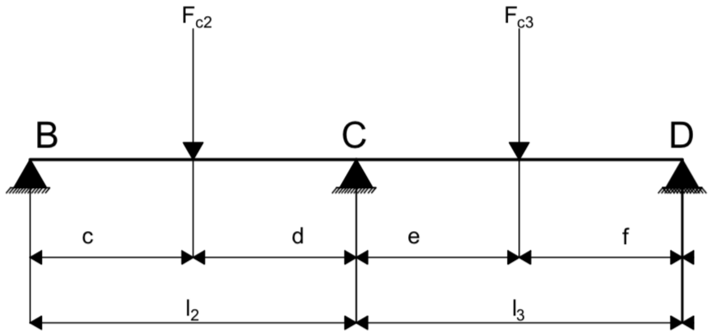

Figure 4.40: System BCD

$$x = \begin{pmatrix} M_B \\ M_C \end{pmatrix}$$

and the vector B equal to the known term:

$$B = \begin{pmatrix} -\frac{F_{c1}}{l_1} a(l_1^2 - a^2) - \frac{F_{c2}}{l_2} d(l_2^2 - d^2) \\ -\frac{F_{c2}}{l_2} c(l_2^2 - c^2) - \frac{F_{c3}}{l_3} f(l_3^2 - f^2) \end{pmatrix}$$

and then:

$$x = \begin{pmatrix} M_B \\ M_C \end{pmatrix} = A^{-1} * B$$

Considering now each single span, the shear forces and the reaction on the supports can be derived as shown below:

- System "AB" (Figure 4.41):

$$\begin{cases} B \circlearrowleft \; M_B - M_A + F_{c1} b - T_{A^+} l_1 = 0 \\ \quad T_{A^+} = \frac{1}{l_1}(M_B + F_{c1} b); \\ \uparrow) \; T_{A^+} - F_{c1} - T_{B^-} = 0 \\ \quad T_{B^-} = T_{A^+} - F_{c1}; \end{cases} \tag{4.52}$$

- System "BC" (Figure 4.42):

$$\begin{cases} C \circlearrowleft \; M_C - M_B + F_{c2} d - T_{B^+} l_2 = 0 \\ \quad T_{B^+} = \frac{1}{l_2}(M_C - M_B + F_{c2} d); \\ \uparrow) \; T_{B^+} - F_{c2} - T_{C^-} = 0 \\ \quad T_{C^-} = T_{B^+} - F_{c2}; \end{cases} \tag{4.53}$$

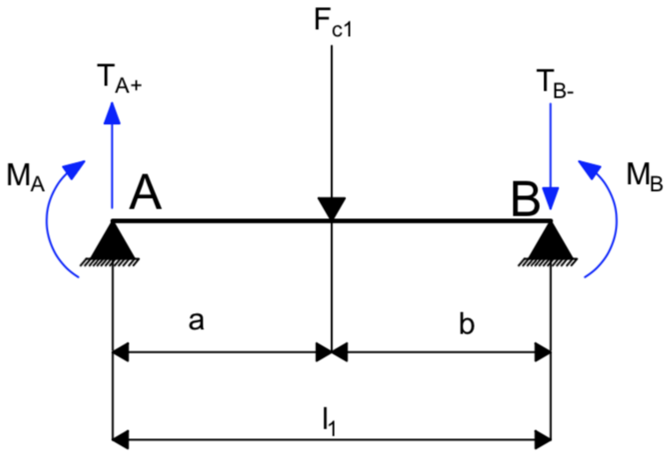

Figure 4.41: System AB

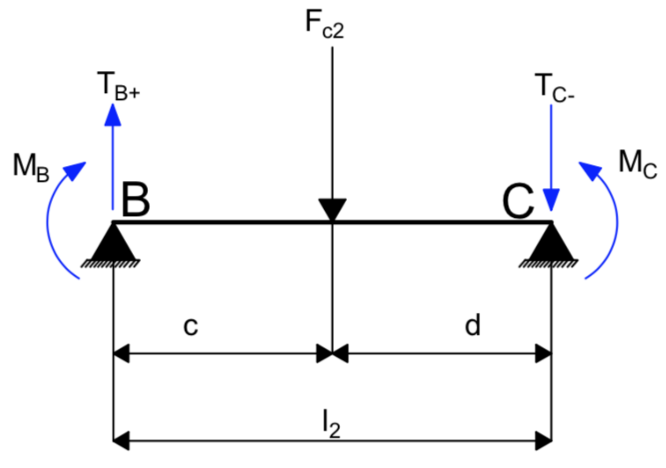

Figure 4.42: System BC

- System "CD" (Figure 4.43):

$$\begin{cases} D \circlearrowright \;\; M_D - M_C + F_{c3} f - T_{C^+} l_3 = 0 \\ \qquad T_{C^+} = \frac{1}{l_3}(-M_C + F_{c3} f); \\ \uparrow) \; T_{C^+} - F_{c3} - T_{D^-} = 0 \\ \qquad T_{D^-} = T_{C^+} - F_{c3}; \end{cases} \tag{4.54}$$

The reactions can be obtained considering the convention shown in Figure 4.44.

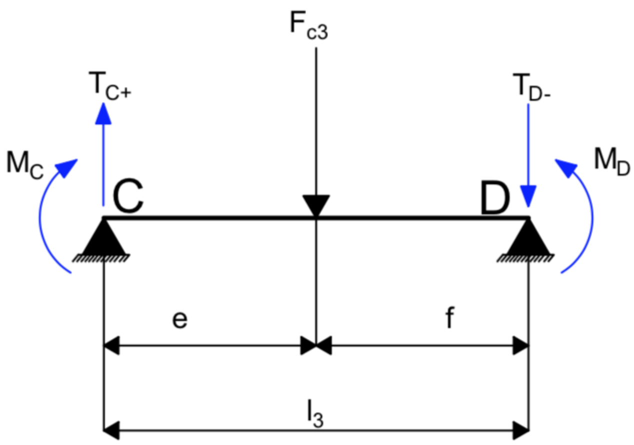

Figure 4.43: System CD

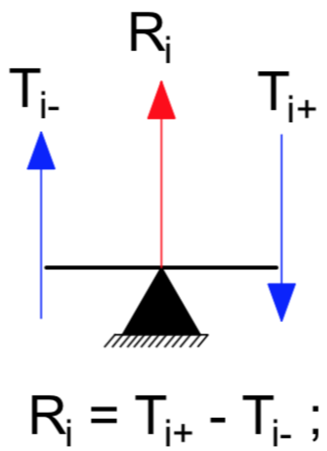

Figure 4.44: Shear forces convention

$$\begin{cases} R_A = T_{A^+}; \\ R_B = T_{B^+} - T_{B^-}; \\ R_C = T_{C^+} - T_{C^-}; \\ R_D = -T_{D^-}; \end{cases} \tag{4.55}$$

All the procedure shown above is valid for the case of a non-balanced crankshaft as no forces due do the counterweights are were considered. Anyway, the same method can be used for a balanced crankshaft just adding the forces due to the counterweights, as shown in the Figure 4.45.

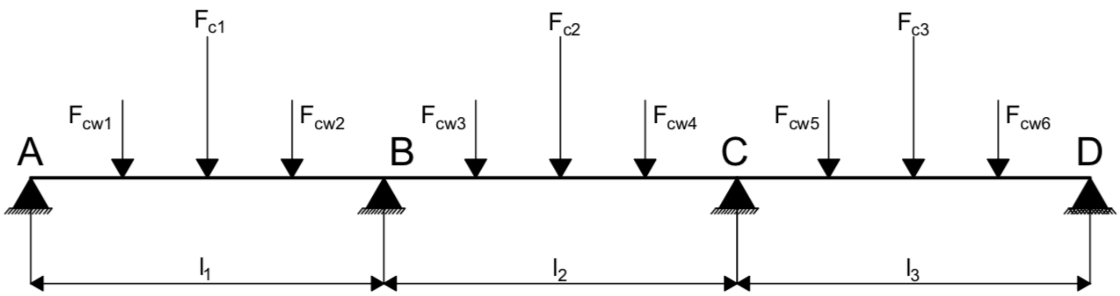

Figure 4.45: Model of forces in a I3 balanced crankshaft

4.3 Four-Cylinder Inline Engines

In this paragraph the relationships that regulate the four cylinders inline engines will be analyzed. The angle between each crank is in this case equal to $\frac{720}{4}$ = 180° and this type of engines are called *Flat-Plane*. Moreover, it is possible to have an angle equal to 90° between the throws, and in these cases the engines are called *Cross-Plane*.

There are different possible firing orders, for example:

- 1-3-4-2 (probably the most common F.O. used)
- 1-2-4-3 (some British Ford and Riley engines)
- 1-3-2-4 (Subaru 4-cylinder engine, Yahama R1 crossplane)
- 1-4-3-2 (Volkswagen air-cooled engines)

Let us now analyze in details two of these configurations. In the following discussion only the F.O. equal to 1-3-4-2 will be dealt, because the procedure is similar between the different cases.

4.3.1 Four-cylinder Inline Engine Flat-Plane with F.O. 1-3-4-2

This is a *Flat-Plane* configuration and the crank throws are arranged as shown in the Figure 4.46:

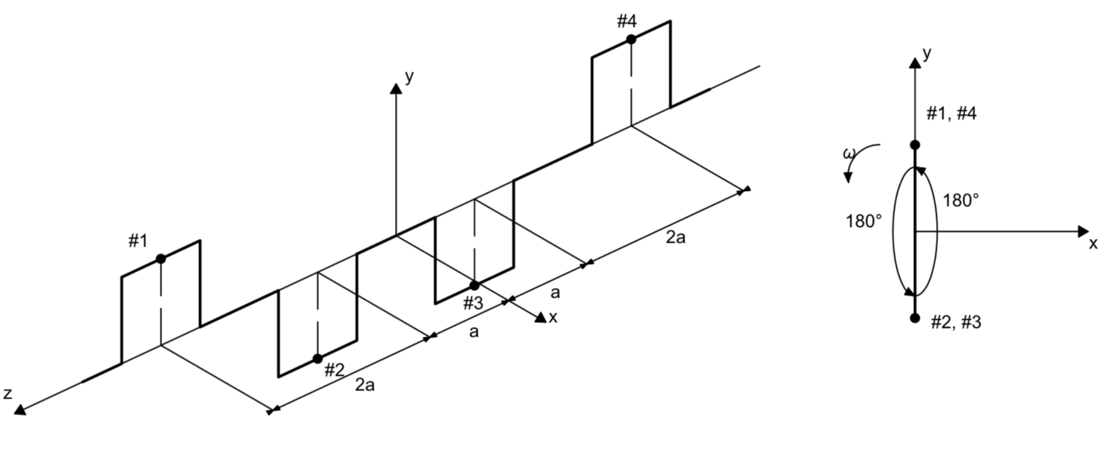

Figure 4.46: Four-cylinder flat-plane inline engine

The crank angles β compared to the crank #1, are equal to:

- β_2 = 180°, β_3 = 180°, β_4 = 360° (F.O. 1-3-4-2)

The first and the second order vectors stars are now reported, in order to show the forces acting on the system.

First order vectors star

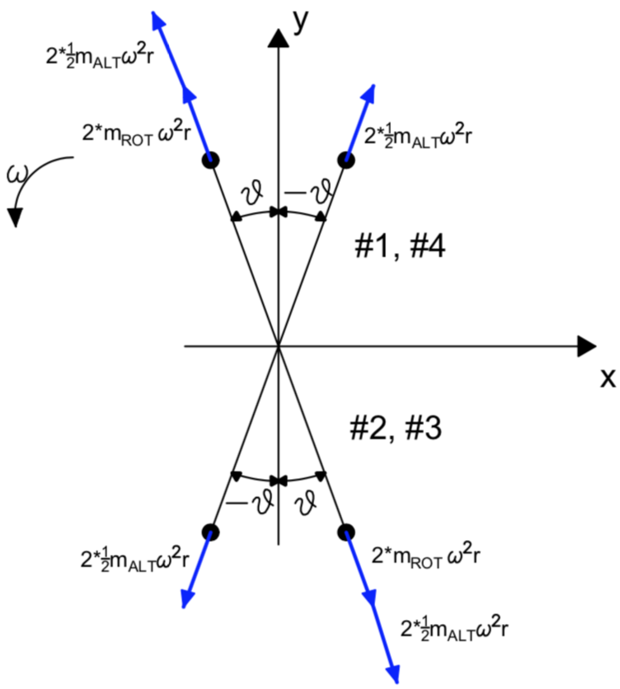

Figure 4.47: Firts order vectors star

As can be seen in the Figure 4.47:
As can be seen:

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) $\Rightarrow$ Balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Balanced.

Second order vectors star

As can be seen in the Figure 4.48:

- Rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Not balanced.
- Counter-rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Not balanced.

As shown in the Figure 4.48, the second order forces, that rotate with 2ω speed, due to the cylinder $\#1, \#2, \#3$ and $\#4$ are overlapped, as can be derived in this quick demonstration:

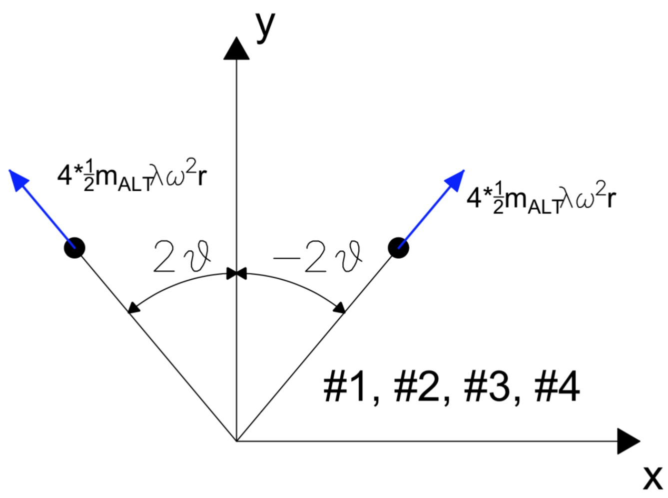

Figure 4.48: Second order vectors star

- Cyl. #1 $\rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. #2 $\rightarrow (\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;
- Cyl. #3 $\rightarrow (\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;
- Cyl. #4 $\rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;

In this system layout, both the first and the second order moments are balanced.

Analytical form and fundamental equations

In the Figure 4.49 is shown the system considered:

$$\text{Cyl. \#1} \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = 3a \end{pmatrix}; \qquad \text{Cyl. \#2} \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = a \end{pmatrix}$$

$$\text{Cyl. \#3} \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = -a \end{pmatrix}; \qquad \text{Cyl. \#4} \Rightarrow \begin{pmatrix} x_3 \\ y_3 \\ z_3 = -3a \end{pmatrix}$$

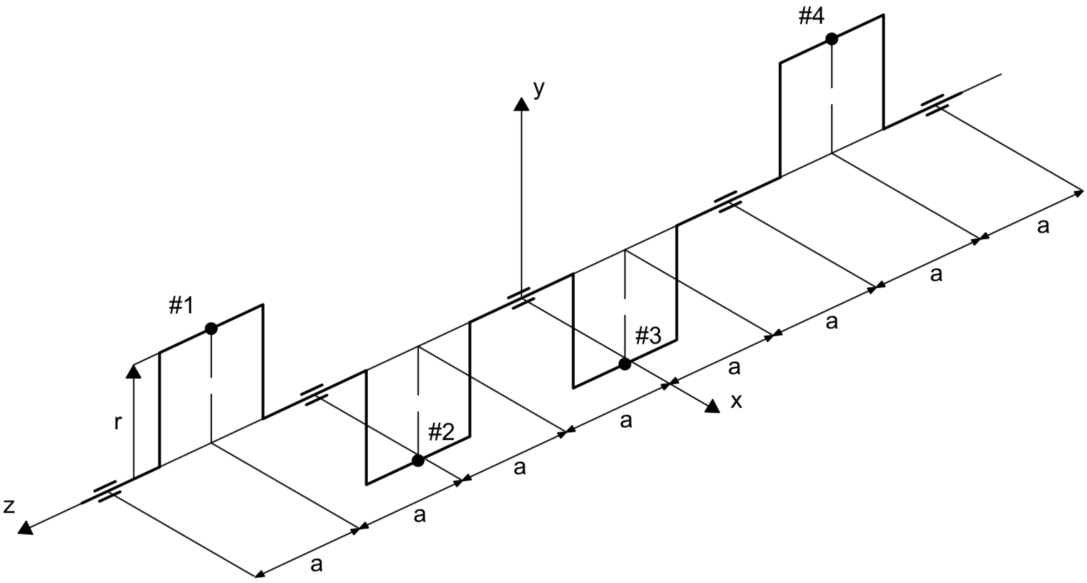

Figure 4.49: Four-cylinder flat-plane inline engine

The system of equations can be written as:

$$
\begin{cases}
x) \quad \sum_i m_i\, \omega^2\, x_i = \sum_i m_i x_i = m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 = 0; \\
y) \quad \sum_i m_i\, \omega^2\, y_i = \sum_i m_i y_i = m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 = 0; \\
x \circlearrowleft \quad \sum_i (m_i\, \omega^2\, x_i) z_i = \sum_i m_i x_i z_i = m_1 x_1 3a + m_2 x_2 a + m_3 x_3 (-a) + m_4 x_4 (-3a) = 0; \\
y \circlearrowleft \quad \sum_i (m_i\, \omega^2\, y_i) z_i = \sum_i m_i y_i z_i = m_1 y_1 3a + m_2 y_2 a + m_3 y_3 (-a) + m_4 y_4 (-3a) = 0;
\end{cases}
\tag{4.56}
$$

considering a configuration with $\theta = 0°$ and assuming $m_1 = m_2 = m_3 = m_4$ the system becomes:

$$
\begin{cases}
x) \quad \sum_i m_i\, \omega^2\, x_i = \sum_i m_i x_i = m_1 \cdot 0 + m_2 \cdot 0 + m_3 \cdot 0 + m_4 \cdot 0 = 0; \Rightarrow \text{Balanced} \\
y) \quad \sum_i m_i\, \omega^2\, y_i = \sum_i m_i y_i = m_1 r + m_2 (-r) + m_3 r + m_4 (-r) = 0; \Rightarrow \text{Balanced} \\
x \circlearrowleft \quad \sum_i (m_i\, \omega^2\, x_i) z_i = \sum_i m_i x_i z_i = m_1 \cdot 0 \cdot 3a + m_2 \cdot 0 \cdot a + m_3 \cdot 0 \cdot (-a) + \\
\qquad + m_4 \cdot 0 \cdot (-3a) = 0; \Rightarrow \text{Balanced} \\
y \circlearrowleft \quad \sum_i (m_i\, \omega^2\, y_i) z_i = \sum_i m_i y_i z_i = m_1 r 3a + m_2 r a + m_3 r (-a) + \\
\qquad + m_4 r (-3a) = 0; \Rightarrow \text{Balanced}
\end{cases}
\tag{4.57}
$$

Balancing Strategy

As show in the equations 4.56, 4.57 the system looks balanced, but it is important to stress that this is only a global balance, not local.

In order to have a system that is also locally balanced it is possible to apply the *Bay-by-bay balancing strategy*. Assuming to add two counterweights for each crank throw, the crankshaft can be schematically represented as shown in the Figure 4.50:

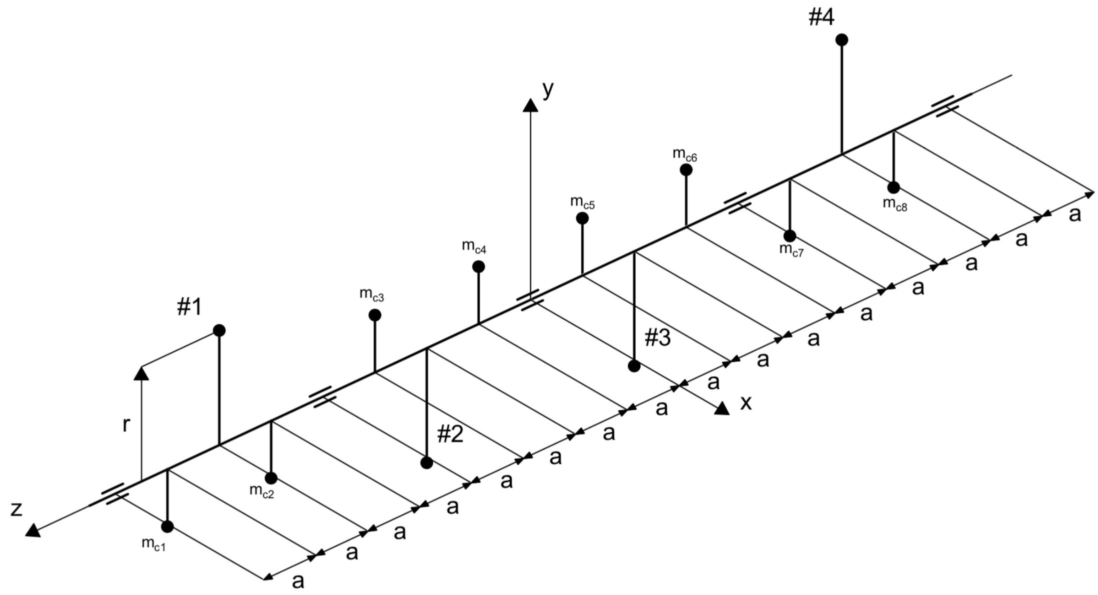

Figure 4.50: Four-cylinder inline flat-plane engine with bay-by-bay balancing

$$m_1 \Rightarrow \begin{pmatrix} x_1 = -r\sin\theta \\ y_1 = r\cos\theta \\ z_1 = 6a \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 = -r\sin(\theta+180) \\ y_2 = r\cos(\theta+180) \\ z_2 = 2a \end{pmatrix};$$

$$m_3 \Rightarrow \begin{pmatrix} x_3 = -r\sin(\theta+180) \\ y_3 = r\cos(\theta+180) \\ z_3 = -2a \end{pmatrix}; \qquad m_4 \Rightarrow \begin{pmatrix} x_4 = -r\sin\theta \\ y_4 = r\cos\theta \\ z_4 = -6a \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = 7a \end{pmatrix}; \quad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = 5a; \end{pmatrix}; \quad m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = 3a \end{pmatrix}; \quad m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = a \end{pmatrix};$$

$$m_{c_5} \Rightarrow \begin{pmatrix} x_{c_5} \\ y_{c_5} \\ z_{c_5} = -a \end{pmatrix}; \quad m_{c_6} \Rightarrow \begin{pmatrix} x_{c_6} \\ y_{c_6} \\ z_{c_6} = -3a \end{pmatrix}; \quad m_{c_7} \Rightarrow \begin{pmatrix} x_{c_7} \\ y_{c_7} \\ z_{c_7} = -5a \end{pmatrix}; \quad m_{c_8} \Rightarrow \begin{pmatrix} x_{c_8} \\ y_{c_8} \\ z_{c_8} = -7a \end{pmatrix};$$

The system of equations can be written as:

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + m_{c_5}\,\omega^2\,x_{c_5} + m_{c_6}\,\omega^2\,x_{c_6} + \\ \qquad + m_{c_7}\,\omega^2\,x_{c_7} + m_{c_8}\,\omega^2\,x_{c_8} + m_1\,\omega^2\,x_1 + m_2\,\omega^2\,x_2 + m_3\,\omega^2\,x_3 + m_4\,\omega^2\,x_4 = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + m_{c_5}\,\omega^2\,y_{c_5} + m_{c_6}\,\omega^2\,y_{c_6} + \\ \qquad + m_{c_7}\,\omega^2\,y_{c_7} + m_{c_8}\,\omega^2\,y_{c_8} + m_1\,\omega^2\,y_1 + m_2\,\omega^2\,y_2 + m_3\,\omega^2\,y_3 + m_4\,\omega^2\,y_4 = 0 \\ x \circlearrowleft \; (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + \\ \qquad + (m_{c_5}\,\omega^2\,x_{c_5})z_{c_5} + (m_{c_6}\,\omega^2\,x_{c_6})z_{c_6} + (m_{c_7}\,\omega^2\,x_{c_7})z_{c_7} + (m_{c_8}\,\omega^2\,x_{c_8})z_{c_8} + \\ \qquad + (m_1\,\omega^2\,x_1)z_1 + (m_2\,\omega^2\,x_2)z_2 + (m_3\,\omega^2\,x_3)z_3 + (m_4\,\omega^2\,x_4)z_4 = 0 \\ y \circlearrowleft \; (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} + (m_{c_5}\,\omega^2\,y_{c_5})z_{c_5} + \\ \qquad + (m_{c_6}\,\omega^2\,y_{c_6})z_{c_6} + (m_{c_7}\,\omega^2\,y_{c_7})z_{c_7} + (m_{c_8}\,\omega^2\,y_{c_8})z_{c_8} + \\ \qquad + (m_1\,\omega^2\,y_1)z_1 + (m_2\,\omega^2\,y_2)z_2 + (m_3\,\omega^2\,y_3)z_3 + (m_4\,\omega^2\,y_4)z_4 = 0 \end{cases} \tag{4.58}$$

that can be rewritten as:

$$\begin{cases} x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + m_{c_7}x_{c_7} + m_{c_8}x_{c_8} + \\ \qquad - r\sin\theta(m_1 + m_4) - r\sin(\theta + 180)(m_2 + m_3) = 0 \\ y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + m_{c_7}y_{c_7} + m_{c_8}y_{c_8} + \\ \qquad + r\cos\theta(m_1 + m_4) + r\cos(\theta + 180)(m_2 + m_3) = 0 \\ x \circlearrowleft \; 7a(m_{c_1}x_{c_1} - m_{c_8}x_{c_8}) + 5a(m_{c_2}x_{c_2} - m_{c_7}x_{c_7}) + 3a(m_{c_3}x_{c_3} - m_{c_6}x_{c_6}) + \\ \qquad + a(m_{c_4}x_{c_4} - m_{c_5}x_{c_5}) + 2ar\sin(\theta + 180)(m_3 - m_2) + 6ar\sin\theta(m_4 - m_1) = 0 \\ y \circlearrowleft \; 7a(m_{c_1}y_{c_1} - m_{c_8}y_{c_8}) + 5a(m_{c_2}y_{c_2} - m_{c_7}y_{c_7}) + 3a(m_{c_3}y_{c_3} - m_{c_6}y_{c_6}) + \\ \qquad + a(m_{c_4}y_{c_4} - m_{c_5}y_{c_5}) + 2ar\cos(\theta + 180)(m_2 - m_3) + 6ar\cos\theta(m_1 - m_4) = 0 \end{cases} \tag{4.59}$$

Making the assumption that:

- $m_1 = m_2 = m_3 = m_4 = m$;
- $x_{c_1} = x_{c_2} = x_{c_7} = x_{c_8} = x_{c_D}$ and $x_{c_3} = x_{c_4} = x_{c_5} = x_{c_6} = x_{c_H}$
- $x_{c_D} = -x_{c_H}$
- $y_{c_1} = y_{c_2} = y_{c_7} = y_{c_8} = y_{c_D}$ and $y_{c_3} = y_{c_4} = y_{c_5} = y_{c_6} = y_{c_H}$
- $y_{c_D} = -y_{c_H}$

the system becomes:

$$\begin{cases} x) \quad x_{c_D}(m_{c_1} + m_{c_2} + m_{c_7} + m_{c_8}) + x_{c_H}(m_{c_3} + m_{c_4} + m_{c_5} + m_{c_6}) = 0 \\ y) \quad y_{c_D}(m_{c_1} + m_{c_2} + m_{c_7} + m_{c_8}) + y_{c_H}(m_{c_3} + m_{c_4} + m_{c_5} + m_{c_6}) = 0 \\ x \circlearrowleft \; x_{c_D}[7(m_{c_1} - m_{c_8}) + 5(m_{c_2} - m_{c_7})] + x_{c_H}[3(m_{c_3} - m_{c_6}) + (m_{c_4} - m_{c_5})] = 0 \\ y \circlearrowleft \; y_{c_D}[7(m_{c_1} - m_{c_8}) + 5(m_{c_2} - m_{c_7})] + y_{c_H}[3(m_{c_3} - m_{c_6}) + (m_{c_4} - m_{c_5})] = 0 \end{cases} \tag{4.60}$$

and then:

$$\begin{cases} x) \quad (m_{c_1} + m_{c_2} + m_{c_7} + m_{c_8}) = (m_{c_3} + m_{c_4} + m_{c_5} + m_{c_6}) \\ y) \quad (m_{c_1} + m_{c_2} + m_{c_7} + m_{c_8}) = (m_{c_3} + m_{c_4} + m_{c_5} + m_{c_6}) \\ x \circlearrowright \; [7(m_{c_1} - m_{c_8}) + 5(m_{c_2} - m_{c_7})] = [3(m_{c_3} - m_{c_6}) + (m_{c_4} - m_{c_5})] \\ y \circlearrowright \; [7(m_{c_1} - m_{c_8}) + 5(m_{c_2} - m_{c_7})] = [3(m_{c_3} - m_{c_6}) + (m_{c_4} - m_{c_5})] \end{cases} \tag{4.61}$$

In this way it is possible to find a configuration that balances the system locally, this solution is a function of the assumptions made.

Another possible solution could be to add only one counterweight for each crank, appropriately positioned.

In order to balance the second order rotating and counter-rotating forces, two balancer shafts must be properly placed. In the Figure 4.51 is shown an example of possible configuration.

4.3.2 Four-cylinder Inline Engine Cross-Plane with F.O. 1-3-2-4

The *Crossplane Design* means that each of the crank throws are at an angle of 90° compared to the next. Therefore, the crankpins are in two plane crossed at 90°, thus the name *Cross-Plane Crankshaft*. The *F.O.* results in a new sequence of combustion equal to *1-3-2-4* with the following intervals: 270°, 180°, 90°, 180°. Therefore this is an "irregular" firing engine compared to the classic four-cylinder flat-plane wich constant firing intervals equal to 180°.

In the Figure 4.52 is reported the considered system configuration.

When the piston #1 is at TDC, the piston #4 is at BDC and vice versa and when the piston #2 is at TDC, the piston #3 is at BDC and vice versa.

The first and the second order vectors stars are now reported, in order to show the forces acting on the system.

First order vectors star

As can be seen in the Figure 4.53:

As can be seen:

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) ⇒ Balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

Second order vectors star

As can be seen in the Figure 4.54:

- Rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

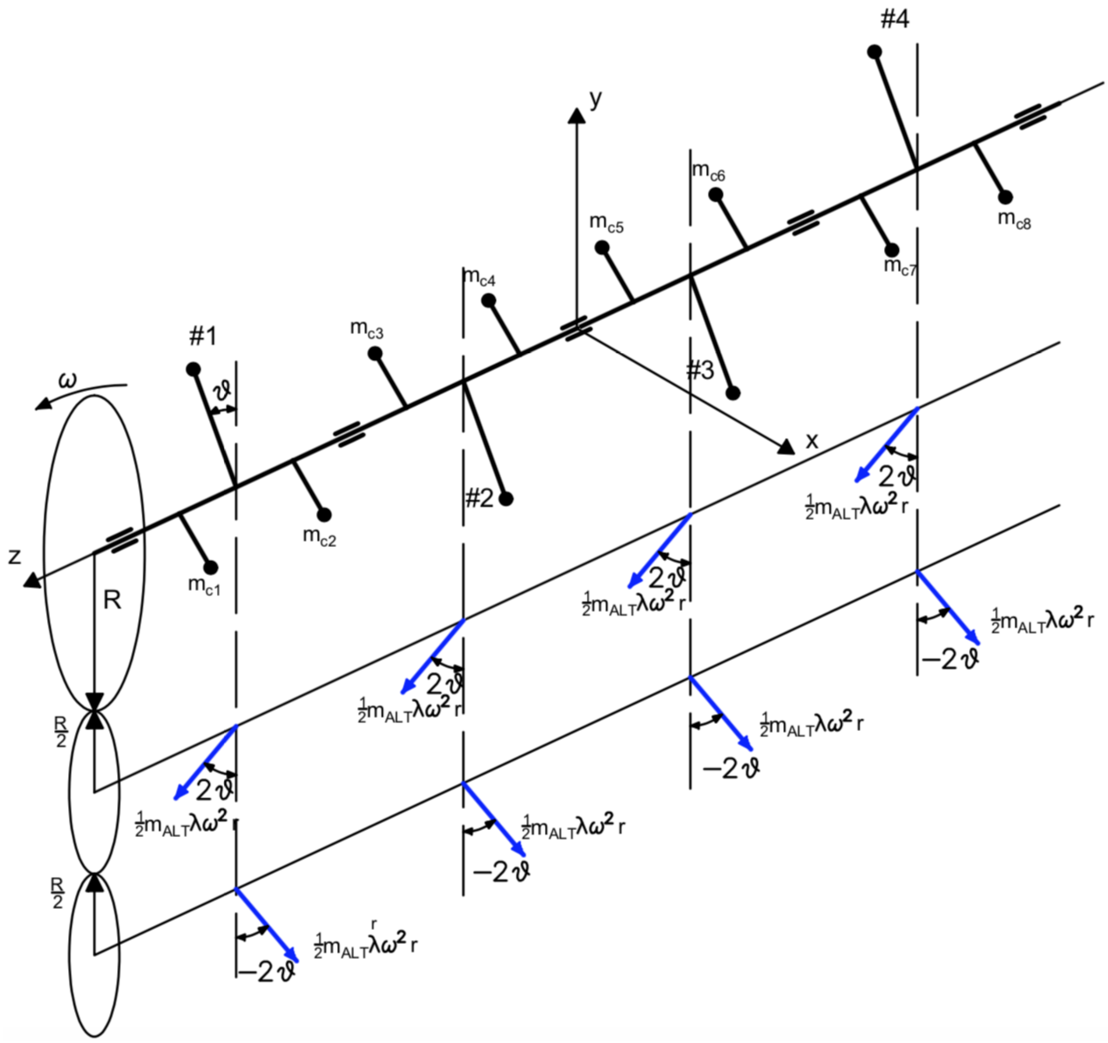

Figure 4.51: Four-cylinder inline flatplane completely balanced

- Counter-rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

As shown in the Figure 4.54, the second order forces, that rotate with 2ω speed, due to the cylinder #1, #2, #3 and #4 are overlapped, as can be derived in this quick demonstration:

- Cyl. $\#1 \rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. $\#2 \rightarrow (\theta + 270°) \Rightarrow 2(\theta + 270°) \Rightarrow \cos(2\theta + 540°)$;
- Cyl. $\#3 \rightarrow (\theta + 90°) \Rightarrow 2(\theta + 90°) \Rightarrow \cos(2\theta + 180°)$;
- Cyl. $\#4 \rightarrow (\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;

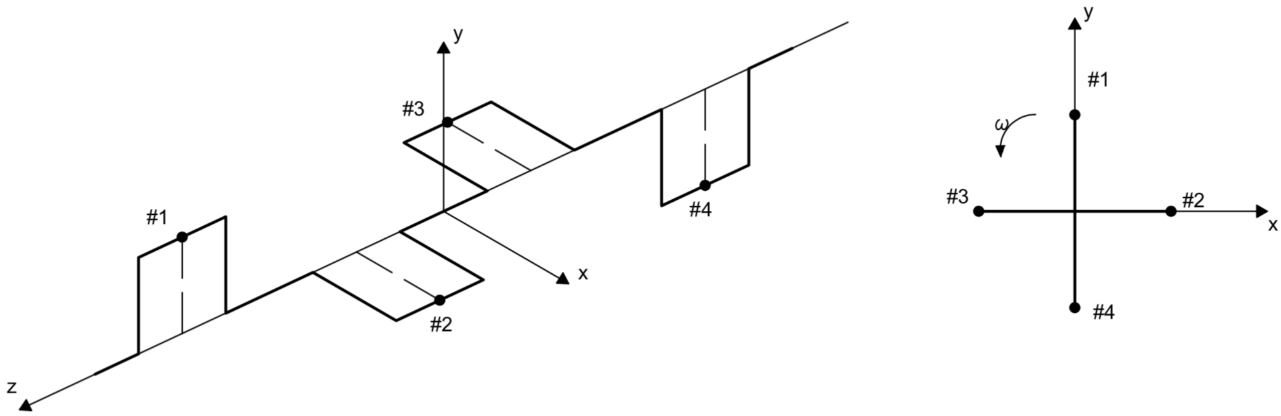

Figure 4.52: Four-cylinder inline crossplane configuration

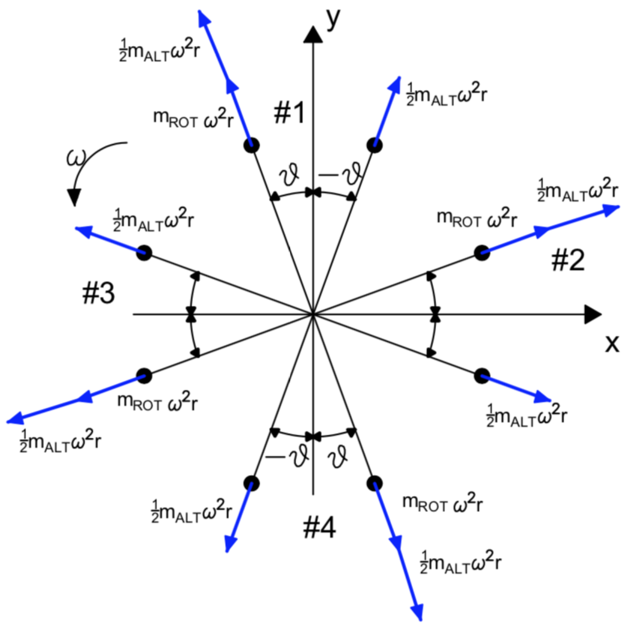

Figure 4.53: Firts order vectors star

Obviously: $\cos(2\theta + 540°) = \cos(2\theta + 180°)$.

In this engine layout the moments due to the first order forces are not balanced, the moments due to the second order forces are balanced instead.

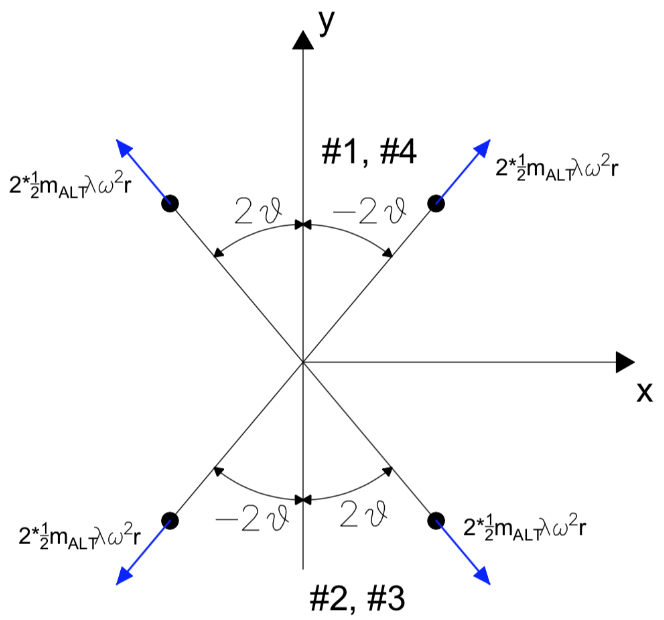

Figure 4.54: Second order vectors star

Analytical form and fundamental equations

The system considered is shown in the Figure 4.55

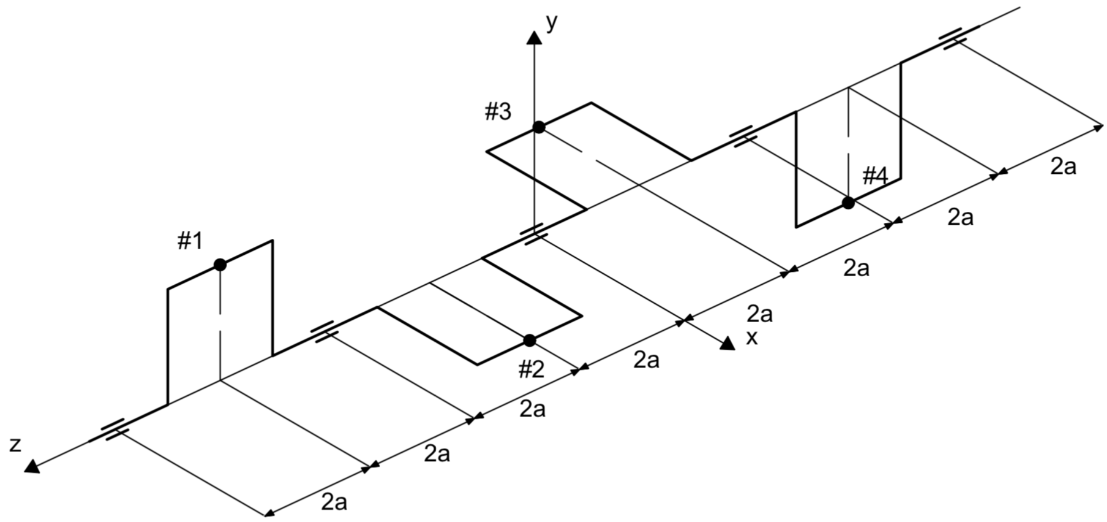

Figure 4.55: Four-cylinder inline crossplane configuration

$$\text{Cyl. } \#1 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = 3a \end{pmatrix}; \qquad \text{Cyl. } \#2 \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = a \end{pmatrix}$$

$$\text{Cyl. } \#3 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = -a \end{pmatrix}; \qquad \text{Cyl. } \#4 \Rightarrow \begin{pmatrix} x_3 \\ y_3 \\ z_3 = -3a \end{pmatrix}$$

The system of equations can be written as:

$$\begin{cases} x) & \sum_i m_i\, \omega^2\, x_i = \sum_i m_i x_i = m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 = 0; \\ y) & \sum_i m_i\, \omega^2\, y_i = \sum_i m_i y_i = m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 = 0; \\ x \circlearrowleft & \sum_i (m_i\, \omega^2\, x_i) z_i = \sum_i m_i x_i z_i = m_1 x_1 6a + m_2 x_2 2a + m_3 x_3(-2a) + m_4 x_4(-6a) = 0; \\ y \circlearrowleft & \sum_i (m_i\, \omega^2\, y_i) z_i = \sum_i m_i y_i z_i = m_1 y_1 6a + m_2 y_2 2a + m_3 y_3(-2a) + m_4 y_4(-6a) = 0; \end{cases} \tag{4.62}$$

considering a configuration with $\theta = 0°$ and assuming $m_1 = m_2 = m_3 = m_4$ and $r_1 = r_2 = r_3 = r_4 = r$, the system becomes:

$$\begin{cases} x) & \sum_i m_i\, \omega^2\, x_i = \sum_i m_i x_i = m_1 \cdot 0 + m_2 r + m_3(-r) + m_4 \cdot 0 = 0; \Rightarrow \text{Balanced} \\ y) & \sum_i m_i\, \omega^2\, y_i = \sum_i m_i y_i = m_1 r + m_2 \cdot 0 + m_3 \cdot 0 + m_4(-r) = 0; \Rightarrow \text{Balanced} \\ x \circlearrowleft & \sum_i (m_i\, \omega^2\, x_i) z_i = \sum_i m_i x_i z_i = m_1 \cdot 0 \cdot 6a + m_2 r 2a + m_3(-r)(-2a) + \\ & \quad + m_4 \cdot 0 \cdot (-6a) = 4mra = 0; \Rightarrow \text{Must be balanced} \\ y \circlearrowleft & \sum_i (m_i\, \omega^2\, y_i) z_i = \sum_i m_i y_i z_i = m_1 r 6a + m_2 \cdot 0 \cdot 2a + m_3 \cdot 0(-2a) + \\ & \quad + m_4(-r)(-6a) = 12mra = 0; \Rightarrow \text{Must be balanced} \end{cases} \tag{4.63}$$

Balancing Strategy

The cenfrifugal and the first order alternating forces generate two moments acting on two planes mutually perpendicular. In order to balance these moments it is necessary to balance the forces through counterweights, in other words adding a proportionate mass for each crank throw that gives a force equal and opposite to the sum of the rotating and the first order alternating. This strategy is the *Bay-by-bay balancing*.

Let us now consider a *Bay-by-bay balancing* with two counterweights for each crank, as shown in the Figure 4.56:

$$m_1 \Rightarrow \begin{pmatrix} x_1 = -r\sin\theta \\ y_1 = r\cos\theta \\ z_1 = 6a \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 = r\cos\theta \\ y_2 = r\sin\theta \\ z_2 = 2a \end{pmatrix};$$

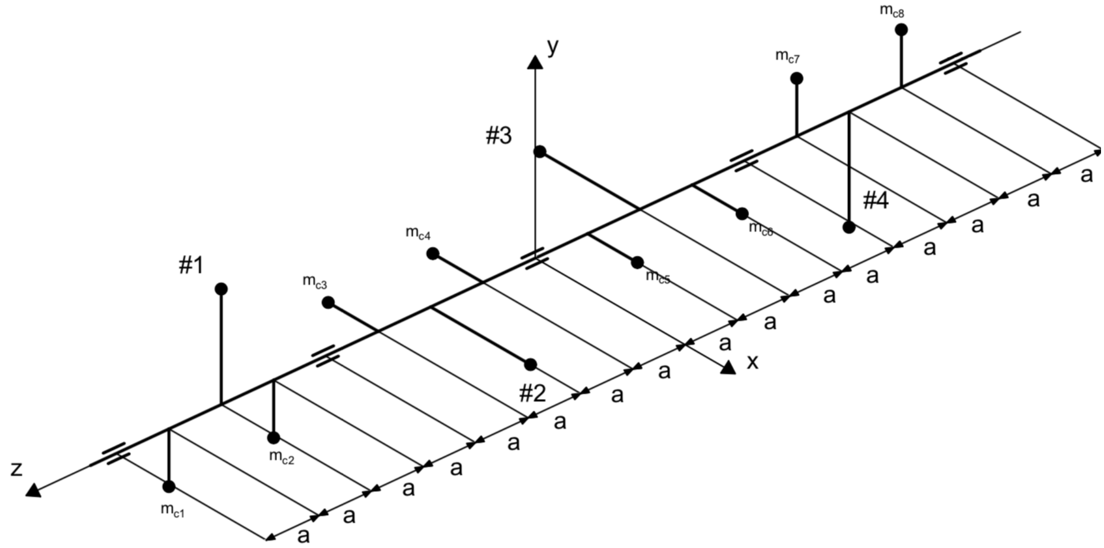

Figure 4.56: Four-cylinder inline cross-plane engine with bay-by-bay balancing

$$m_3 \Rightarrow \begin{pmatrix} x_3 = -r\cos\theta \\ y_3 = -r\sin\theta \\ z_3 = -2a \end{pmatrix}; \qquad m_4 \Rightarrow \begin{pmatrix} x_4 = r\sin\theta \\ y_4 = -r\cos\theta \\ z_4 = -6a \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = 7a \end{pmatrix}; \quad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = 5a; \end{pmatrix}; \quad m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = 3a \end{pmatrix}; \quad m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = a \end{pmatrix};$$

$$m_{c_5} \Rightarrow \begin{pmatrix} x_{c_5} \\ y_{c_5} \\ z_{c_5} = -a; \end{pmatrix} \quad m_{c_6} \Rightarrow \begin{pmatrix} x_{c_6} \\ y_{c_6} \\ z_{c_6} = -3a \end{pmatrix} \quad m_{c_7} \Rightarrow \begin{pmatrix} x_{c_7} \\ y_{c_7} \\ z_{c_7} = -5a; \end{pmatrix} \quad m_{c_8} \Rightarrow \begin{pmatrix} x_{c_8} \\ y_{c_8} \\ z_{c_8} = -7a \end{pmatrix};$$

The system of equations can be written as:

$$
\begin{cases}
x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + m_{c_5}\,\omega^2\,x_{c_5} + m_{c_6}\,\omega^2\,x_{c_6} + \\
\qquad + m_{c_7}\,\omega^2\,x_{c_7} + m_{c_8}\,\omega^2\,x_{c_8} + m_1\,\omega^2\,x_1 + m_2\,\omega^2\,x_2 + m_3\,\omega^2\,x_3 + m_4\,\omega^2\,x_4 = 0 \\
y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + m_{c_5}\,\omega^2\,y_{c_5} + m_{c_6}\,\omega^2\,y_{c_6} + \\
\qquad + m_{c_7}\,\omega^2\,y_{c_7} + m_{c_8}\,\omega^2\,y_{c_8} + m_1\,\omega^2\,y_1 + m_2\,\omega^2\,y_2 + m_3\,\omega^2\,y_3 + m_4\,\omega^2\,y_4 = 0 \\
x \circlearrowleft \quad (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + \\
\qquad + (m_{c_5}\,\omega^2\,x_{c_5})z_{c_5} + (m_{c_6}\,\omega^2\,x_{c_6})z_{c_6} + (m_{c_7}\,\omega^2\,x_{c_7})z_{c_7} + (m_{c_8}\,\omega^2\,x_{c_8})z_{c_8} + \\
\qquad + (m_1\,\omega^2\,x_1)z_1 + (m_2\,\omega^2\,x_2)z_2 + (m_3\,\omega^2\,x_3)z_3 + (m_4\,\omega^2\,x_4)z_4 = 0 \\
y \circlearrowleft \quad (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} + (m_{c_5}\,\omega^2\,y_{c_5})z_{c_5} + \\
\qquad + (m_{c_6}\,\omega^2\,y_{c_6})z_{c_6} + (m_{c_7}\,\omega^2\,y_{c_7})z_{c_7} + (m_{c_8}\,\omega^2\,y_{c_8})z_{c_8} + \\
\qquad + (m_1\,\omega^2\,y_1)z_1 + (m_2\,\omega^2\,y_2)z_2 + (m_3\,\omega^2\,y_3)z_3 + (m_4\,\omega^2\,y_4)z_4 = 0
\end{cases}
\tag{4.64}
$$

that can be rewritten as:

$$
\begin{cases}
x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + m_{c_7}x_{c_7} + m_{c_8}x_{c_8} + \\
\qquad + r\sin\theta(m_4 - m_1) + r\cos\theta(m_2 - m_3) = 0 \\
y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + m_{c_7}y_{c_7} + m_{c_8}y_{c_8} + \\
\qquad + r\cos\theta(m_1 - m_4) + r\sin\theta(m_2 - m_3) = 0 \\
x \circlearrowleft \quad 7a(m_{c_1}x_{c_1} - m_{c_8}x_{c_8}) + 5a(m_{c_2}x_{c_2} - m_{c_7}x_{c_7}) + 3a(m_{c_3}x_{c_3} - m_{c_6}x_{c_6}) + \\
\qquad + a(m_{c_4}x_{c_4} - m_{c_5}x_{c_5}) + 2ar\cos\theta(m_2 + m_3) - 6ar\sin\theta(m_1 + m_4) = 0 \\
y \circlearrowleft \quad 7a(m_{c_1}y_{c_1} - m_{c_8}y_{c_8}) + 5a(m_{c_2}y_{c_2} - m_{c_7}y_{c_7}) + 3a(m_{c_3}y_{c_3} - m_{c_6}y_{c_6}) + \\
\qquad + a(m_{c_4}y_{c_4} - m_{c_5}y_{c_5}) + 2ar\sin\theta(m_2 + m_3) + 6ar\cos\theta(m_1 + m_4) = 0
\end{cases}
\tag{4.65}
$$

Making the assumpition that:

- $m_1 = m_2 = m_3 = m_4 = m$;
- $x_{c_1} = x_{c_2} = x_{c_{12}}$, $x_{c_3} = x_{c_4} = x_{c_{34}}$, $x_{c_5} = x_{c_6} = x_{c_{56}}$,and $x_{c_7} = x_{c_8} = x_{c_{78}}$
- $x_{c_{12}} = -x_{c_{78}}$ and $x_{c_{34}} = -x_{c_{56}}$
- $y_{c_1} = y_{c_2} = y_{c_{12}}$, $y_{c_3} = y_{c_4} = y_{c_{34}}$, $y_{c_5} = y_{c_6} = y_{c_{56}}$,and $y_{c_7} = y_{c_8} = y_{c_{78}}$
- $y_{c_{12}} = -y_{c_{78}}$ and $y_{c_{34}} = -y_{c_{56}}$

the system becomes:

$$
\begin{cases}
x) \quad x_{c_{12}}(m_{c_1} + m_{c_2}) + x_{c_{34}}(m_{c_3} + m_{c_4}) + x_{c_{56}}(m_{c_5} + m_{c_6}) + x_{c_{78}}(m_{c_7} + m_{c_8}) = 0 \\
y) \quad y_{c_{12}}(m_{c_1} + m_{c_2}) + y_{c_{34}}(m_{c_3} + m_{c_4}) + y_{c_{56}}(m_{c_5} + m_{c_6}) + y_{c_{78}}(m_{c_7} + m_{c_8}) = 0 \\
x \circlearrowleft \quad 7ax_{c_{12}}(m_{c_1} + m_{c_8}) + 5ax_{c_{12}}(m_{c_2} + m_{c_7}) + 3ax_{c_{34}}(m_{c_3} + m_{c_6}) + \\
\qquad + ax_{c_{34}}(m_{c_4} + m_{c_5}) + 4mar\cos\theta - 12mar\sin\theta = 0 \\
y \circlearrowleft \quad 7ay_{c_{12}}(m_{c_1} + m_{c_8}) + 5ay_{c_{12}}(m_{c_2} + m_{c_7}) + 3ay_{c_{34}}(m_{c_3} + m_{c_6}) + \\
\qquad + ay_{c_{34}}(m_{c_4} + m_{c_5}) + 4mar\sin\theta + 12mar\cos\theta = 0
\end{cases}
\tag{4.66}
$$

and, then:

$$\begin{cases} x) \quad x_{c_{12}}(m_{c_1} + m_{c_2} - m_{c_7} - m_{c_8}) + x_{c_{34}}(m_{c_3} + m_{c_4} - m_{c_5} - m_{c_6}) = 0 \\ y) \quad y_{c_{12}}(m_{c_1} + m_{c_2} - m_{c_7} - m_{c_8}) + y_{c_{34}}(m_{c_3} + m_{c_4} - m_{c_5} - m_{c_6}) = 0 \\ x \circlearrowleft \; ax_{c_{12}}(7(m_{c_1} + m_{c_8}) + 5(m_{c_2} + m_{c_7})) + ax_{c_{34}}(3(m_{c_3} + m_{c_6}) + (m_{c_4} + m_{c_5}))+ \\ \qquad +mar(4\cos\theta - 12\sin\theta) = 0 \\ y \circlearrowleft \; ay_{c_{12}}(7(m_{c_1} + m_{c_8}) + 5(m_{c_2} + m_{c_7})) + ay_{c_{34}}(3(m_{c_3} + m_{c_6}) + (m_{c_4} + m_{c_5}))+ \\ \qquad +mar(4\sin\theta + 12\cos\theta) = 0 \end{cases} \tag{4.67}$$

and if $m_{c_1} = m_{c_2} = m_{c_3} = m_{c_4} = m_{c_5} = m_{c_6} = m_{c_7} = m_{c_8} = m_c$:

$$\begin{cases} x) \quad x_{c_{12}} \cdot 0 + x_{c_{34}} \cdot 0 = 0 \\ y) \quad y_{c_{12}} \cdot 0 + y_{c_{34}} \cdot 0 = 0 \\ x \circlearrowleft \; x_{c_{12}}(24m_c) + x_{c_{34}}(8m_c) + mr(4\cos\theta - 12\sin\theta) = 0 \\ y \circlearrowleft \; y_{c_{12}}(24m_c) + y_{c_{34}}(8m_c) + mr(4\sin\theta + 12\cos\theta) = 0 \end{cases} \tag{4.68}$$

Solving the system 4.68, a balanced configuration of this engines layout can be reached. Oblsiously the solution is a fuction of the assumptions made.

However it is also possible to balance the resultant moment directly, in this way the crankshaft weight decreases because of the smaller addes mass. In order to find the resultant moment value, the two components are projected on their action planes, as shown in the Figure 4.57.

The component on the plane yy is equal to:

$$M_y = F \cdot 12a = (F_{ROT} + F'_{ALT})12a = \omega^2 r(m_{ROT} + m_{ALT})12a; \tag{4.69}$$

The component on the plane xx is equal to:

$$M_x = F \cdot 4a = (F_{ROT} + F'_{ALT})4a = \omega^2 r(m_{ROT} + m_{ALT})4a; \tag{4.70}$$

The value of the resultant moment can be obtained making a vector sum:

$$M_{res} = \sqrt{(12aF)^2 + (4aF)^2} = \sqrt{160a^2F^2} \cong 12{,}65aF; \tag{4.71}$$

The angle γ can be obtained as:

$$\tan\gamma = \frac{M_x}{M_y} = -\frac{1}{3} \Rightarrow \gamma = -18.43°; \tag{4.72}$$

therefore, the plane where the resultant moment acts (indicated with the red line in the Figure 4.57) is trailing by γ the cylinder #1. The minus signs is due to the fact that the two moments M_x and M_y have different verse, one of them is clockwise and the other counter-clockwise.

The balancing can be obtained through two or four counterweights properly positioned.

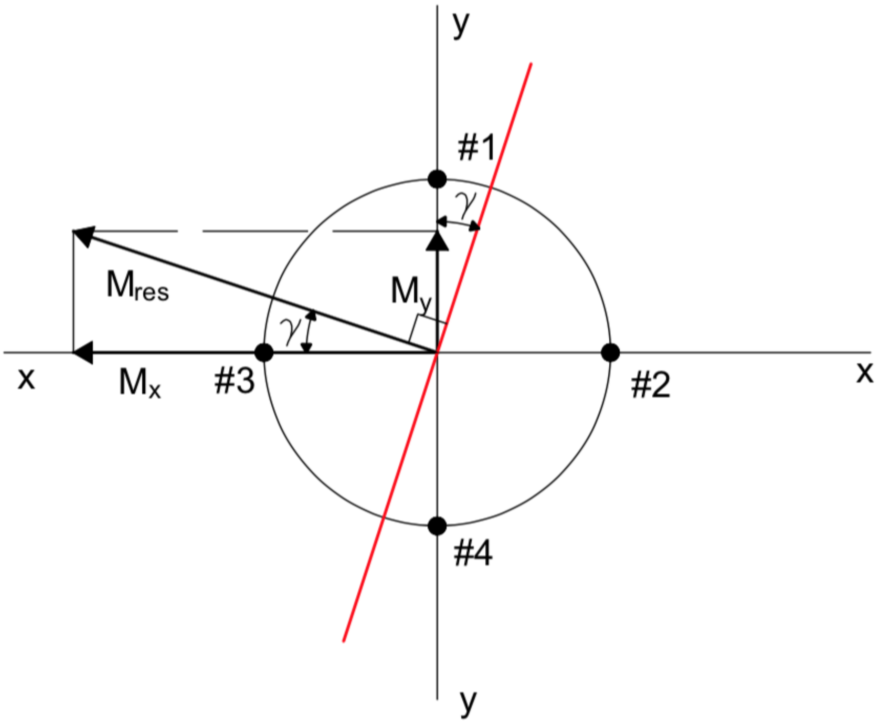

Figure 4.57: Resultant moment

4.3.3 Bearings loads

The way used to find the bearing loads is based on the *Three Moment Equation of Clapeyron*. The crankshaft is hyperstatically constrained and can be considered as a beam, in this case with four span.

The three moment equation method require to use ad additional equation for each excess constrain. In the equations shown below it is assumed that the Young's modulus E and the moment of inertia of the crankshaft section J are constant along the crankshaft, therefore they cancel each other in all the terms.

In this case the system can be divided in three subsystems, the ABC, the BCD and the CDE. For each subsystem a three moment equation can be derived.

The equations are written in a general form, so the results are valid both for flat-plane and cross-plane V8 crankshaft.

In the Figure 4.58 is represented the whole system considered.

Considering now the system ABC shown in Figure :

the three moment equation is equal to:

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = -\frac{F_{c1}}{l_1}a(l_1^2 - a^2) - \frac{F_{c2}}{l_2}d(l_2^2 - d^2); \tag{4.73}$$

for the system BCD shown in the Figure 4.60 the three moment equation is equal to:

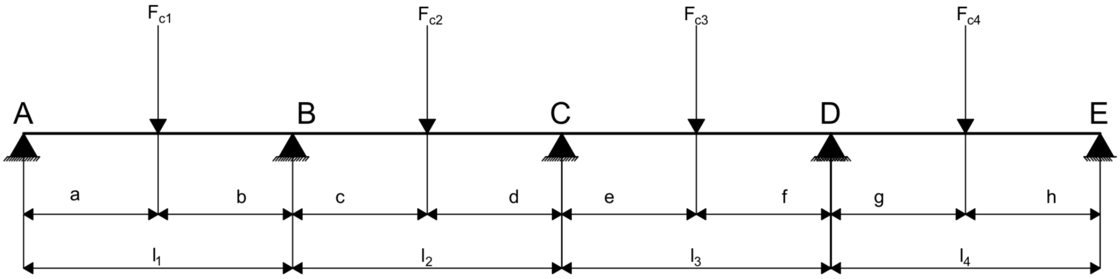

Figure 4.58: I4 crankshaft seen as a four-span beam

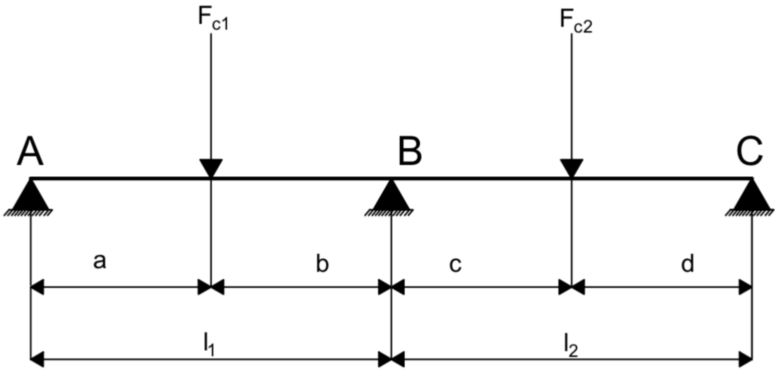

Figure 4.59: System ABC

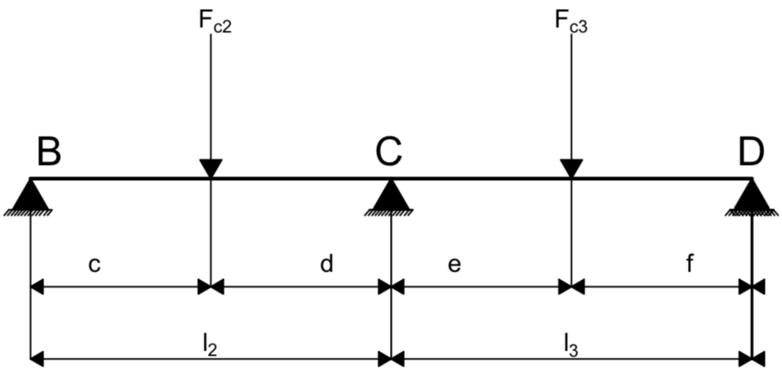

Figure 4.60: System BCD

$$M_B l_2 + 2M_C(l_2 + l_3) + M_D l_3 = -\frac{F_{c2}}{l_2} c(l_2^2 - c^2) - \frac{F_{c3}}{l_3} f(l_3^2 - f^2); \tag{4.74}$$

for the system *CDE* shown in the Figure 4.61 the three moment equation is equal to:

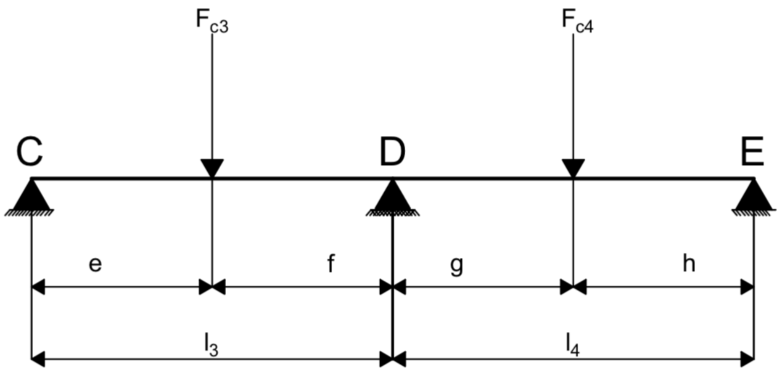

Figure 4.61: System BCD

$$M_C l_3 + 2M_D(l_3 + l_4) + M_E l_4 = -\frac{F_{c3}}{l_3} e(l_3^2 - e^2) - \frac{F_{c4}}{l_4} h(l_3^2 - h^2); \tag{4.75}$$

The moments M_A and M_E are zero, and M_B, M_C and M_D can be calculated considering the system in matrix form like $Ax = B$ with:

$$A = \begin{bmatrix} 2(l_1 + l_2) & l_2 & 0 \\ l_2 & 2(l_2 + l_3) & l3 \\ 0 & l_3 & 2(l3 + l4) \end{bmatrix}$$

;

$$x = \begin{pmatrix} M_B \\ M_C \\ M_D \end{pmatrix}$$

and the vector B equal to the known term:

$$B = \begin{pmatrix} -\frac{F_{c1}}{l_1} a(l_1^2 - a^2) - \frac{F_{c2}}{l_2} d(l_2^2 - d^2) \\ -\frac{F_{c2}}{l_2} c(l_2^2 - c^2) - \frac{F_{c3}}{l_3} f(l_3^2 - f^2) \\ -\frac{F_{c3}}{l_3} e(l_3^2 - e^2) - \frac{F_{c4}}{l_4} h(l_4^2 - h^2) \end{pmatrix}$$

and then:

$$x = \begin{pmatrix} M_B \\ M_C \\ M_D \end{pmatrix} = A^{-1} * B$$

Considering now the each single span, the shear forces and the reaction on the supports can be derived as shown below:

- System "AB" (Figure 4.62):

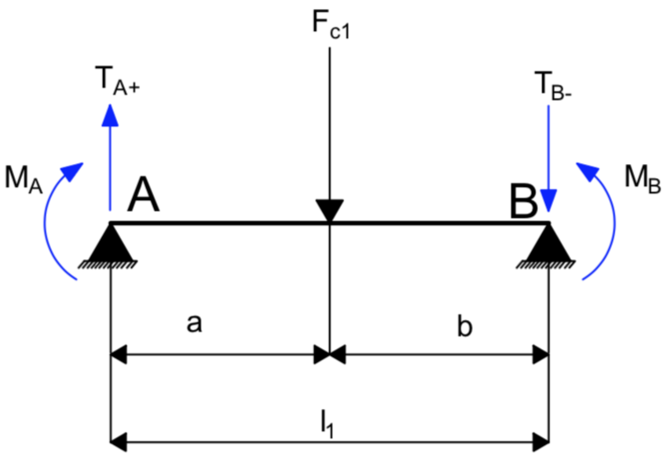

Figure 4.62: System AB

$$
\begin{cases}
B \circlearrowright \; M_B - M_A + F_{c1} b - T_{A^+} l_1 = 0 \\
\qquad T_{A^+} = \frac{1}{l_1}(M_B + F_{c1} b); \\
\uparrow) \; T_{A^+} - F_{c1} - T_{B^-} = 0 \\
\qquad T_{B^-} = T_{A^+} - F_{c1};
\end{cases}
\tag{4.76}
$$

- System "BC" (Figure 4.63):

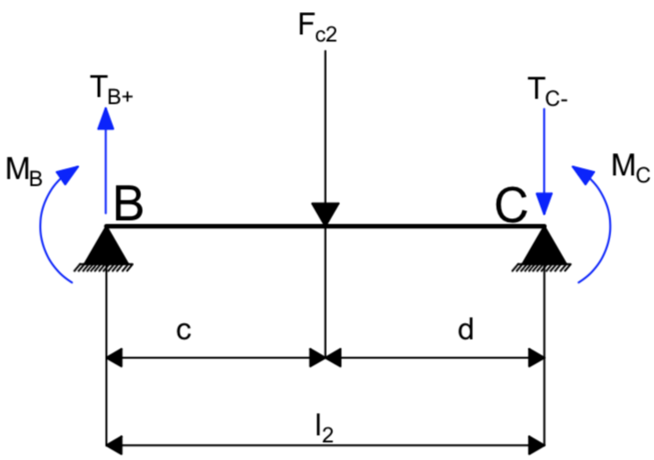

Figure 4.63: System BC

$$\begin{cases} C \circlearrowleft \quad M_C - M_B + F_{c2}d - T_{B^+}l_2 = 0 \\ \qquad T_{B^+} = \frac{1}{l_2}(M_C - M_B + F_{c2}d); \\ \uparrow) \; T_{B^+} - F_{c2} - T_{C^-} = 0 \\ \qquad T_{C^-} = T_{B^+} - F_{c2}; \end{cases} \tag{4.77}$$

- System "CD" (Figure 4.64):

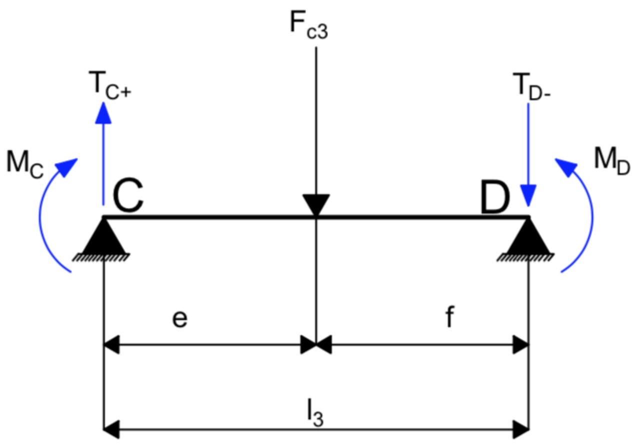

Figure 4.64: System CD

$$\begin{cases} D \circlearrowleft \quad M_D - M_C + F_{c3}f - T_{C^+}l_3 = 0 \\ \qquad T_{C^+} = \frac{1}{l_3}(M_D - M_C + F_{c3}f); \\ \uparrow) \; T_{C^+} - F_{c3} - T_{D^-} = 0 \\ \qquad T_{D^-} = T_{C^+} - F_{c3}; \end{cases} \tag{4.78}$$

- System "DE" (Figure 4.65):

$$\begin{cases} E \circlearrowleft \quad M_E - M_D + F_{c4}h - T_{D^+}l_4 = 0 \\ \qquad T_{D^+} = \frac{1}{l_4}(-M_D + F_{c4}h); \\ \uparrow) \; T_{D^+} - F_{c4} - T_{E^-} = 0 \\ \qquad T_{E^-} = T_{D^+} - F_{c4}; \end{cases} \tag{4.79}$$

The reactions can be obtaiden considering the convention shown in Figure 4.66.

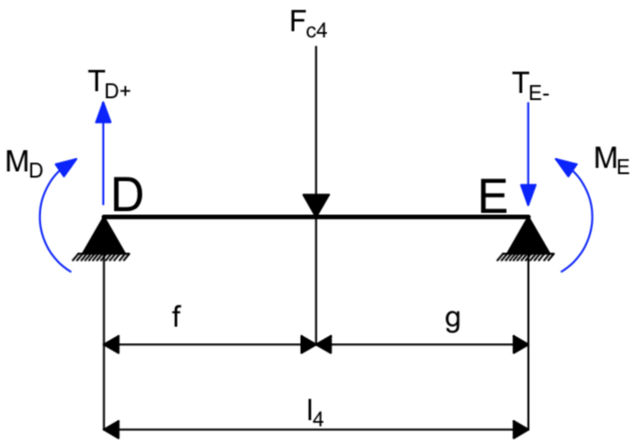

Figure 4.65: System DE

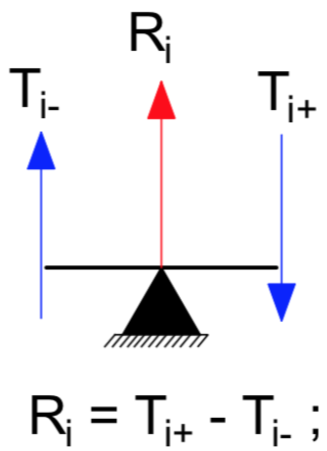

Figure 4.66: Shear forces convention

$$\begin{cases} R_A = T_{A^+}; \\ R_B = T_{B^+} - T_{B^-}; \\ R_C = T_{C^+} - T_{C^-}; \\ R_D = T_{D^+} - T_{D^-}; \\ R_E = -T_{E^-}; \end{cases} \tag{4.80}$$

All the procedure shown above is valid for the case of a non-balanced crankshaft as no forces due do the counterweights were considered. Anyway, the same method can be used for a balanced crankshaft just adding the forces due to the counterweights, as shown in the Figure 4.67.

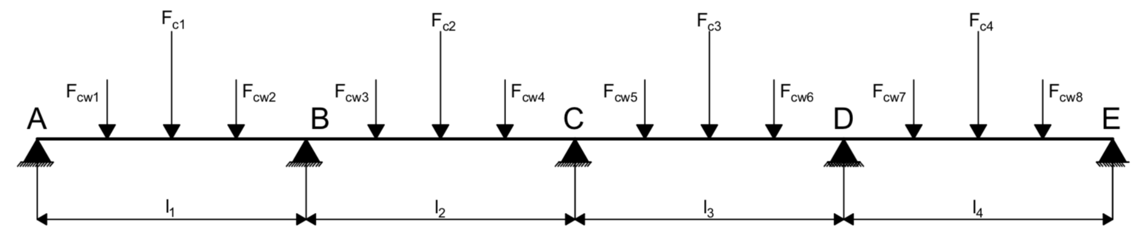

Figure 4.67: Model of forces in a I4 balanced crankshaft

4.4 Five-cylinder Inline Engines

In this paragraph the relationships that regulate the five cylinders inline engines will be analyzed. The angle between crank throws is in this case equal to $\frac{720}{5} = 144°$. There many firing orders used in practical applications, two of the most common are the *F.O. 1-5-2-3-4* and the *F.O. 1-2-4-5-3*; the first one will be analyzed below.

4.4.1 Five-cylinder Inline Engine with F.O. 1-5-2-3-4

The crankshaft is arranged as shown in the Figure 4.68:

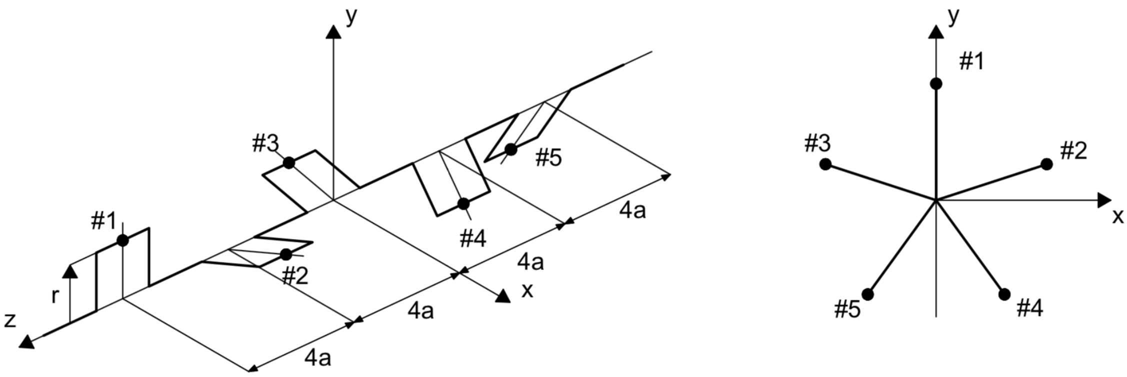

Figure 4.68: Five-cylinder inline crankshaft

The first and the second order vectors stars are now reported, in order to show the forces acting on the system.

First order vectors star

As can be seen in the Figure 4.69:

As can be seen:

- Pure centrifugal forces $(m_{ROT}\omega^2 r)$ ⇒ Balanced.
- Rotating part of first order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r)$ ⇒ Balanced.
- Counter-rotating part of first order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r)$ ⇒ Balanced.

Second order vectors star

As can be seen in the Figure 4.70:

- Rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r)$ ⇒ Not balanced.
- Counter-rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r)$ ⇒ Not balanced.

For the second order forces, in this case:

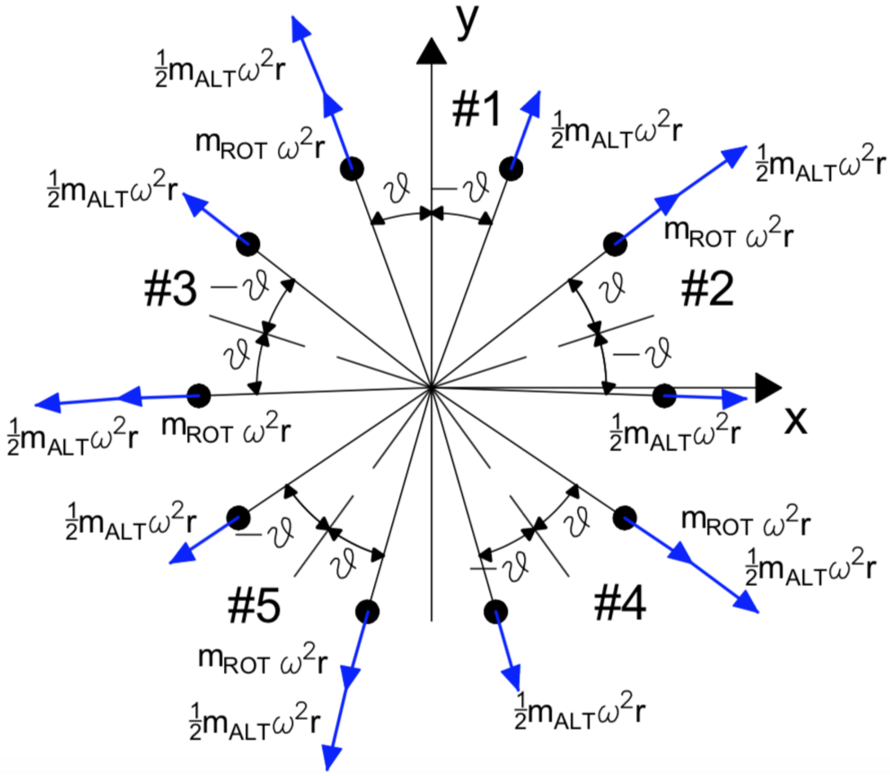

Figure 4.69: Firts order vectors star

- Cyl. #1 → $\theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. #2 → $(\theta + 288°) \Rightarrow 2(\theta + 288°) \Rightarrow \cos(2\theta + 576°) = \cos(2\theta + 216°)$;
- Cyl. #3 → $(\theta + 72°) \Rightarrow 2(\theta + 72°) \Rightarrow \cos(2\theta + 144°)$;
- Cyl. #4 → $(\theta + 216°) \Rightarrow 2(\theta + 216°) \Rightarrow \cos(2\theta + 432°) = \cos(2\theta + 72°)$;
- Cyl. #5 → $(\theta + 144°) \Rightarrow 2(\theta + 144°) \Rightarrow \cos(2\theta + 288°)$;

In this crankshaft layout, both the first and the second order moments are not balanced.

For this engine layout the study was not completed because the focus of this work is on V-engines from 4 to 8 cylinders, therefore since the "basic configurations" of the latter have already been discussed, for the I5 and the I6 cases is shown only the state-of-balancing. This engine is however totally covered in the Matlab code.

Anyway, it can be summarized that for an Inline five-cylinder configuration al least the first order moments must be balanced, therefore it is necessary to find the plane of the resultant moment. Developing the calculations this plan is following by 18° the plane of the crank throw #1 (this value is dependant on the firing order chosen, in fact if the firing order is 1-2-4-5-3 the resultant moment plane follows the crank throw #1 by 54° degrees).

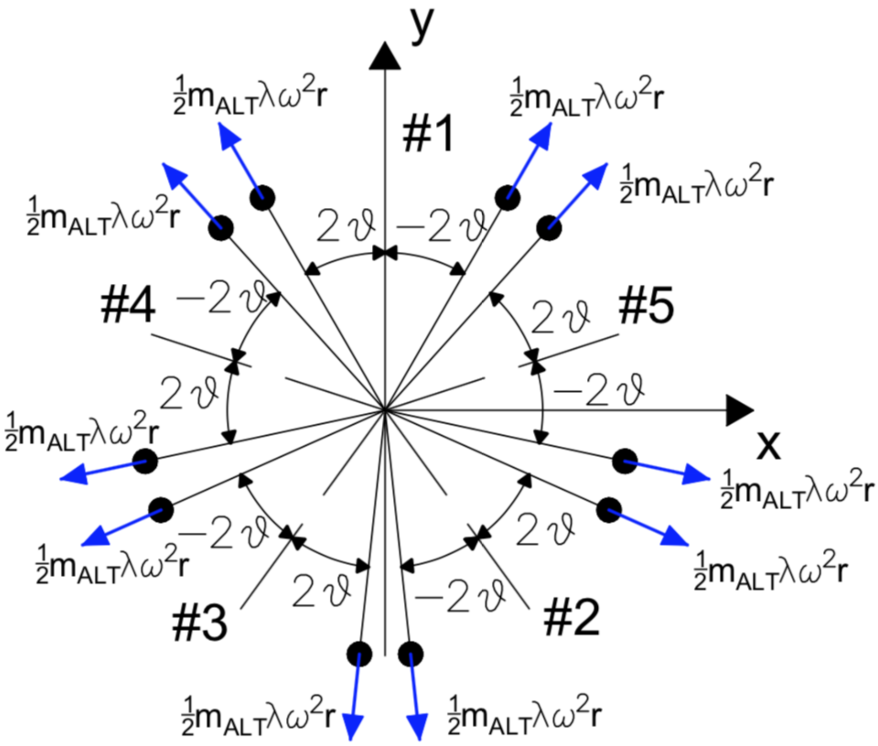

Figure 4.70: Second order vectors star

In order to have a globally balanced system it is possible to place two counterweights on the external crank arms opportunely skewed of the angle of the resultant moment. If instead the goal is to reduce the bearings loads the crankshaft must be balanced both globally and locally, so a bay-by-bay balancing strategy can be used with better results.

4.5 Six-cylinder Inline Engines

In this paragraph the relationships that regulate the six cylinders inline engines will be analyzed. The angle between crank throws is in this case equal to $\frac{720}{6} = 120°$. There many firing orders used in practical applications, one of the most common is the *F.O. 1-5-3-6-2-4* that will be analyzed below.

As already said for the I5 case even in this case only the system state-of-balancing is shown, but it is fully developed on Matlab.

4.5.1 Six-cylinder Inline Engine with F.O. 1-5-3-6-2-4

The crankshaft is arranged as shown in the Figure 4.71:

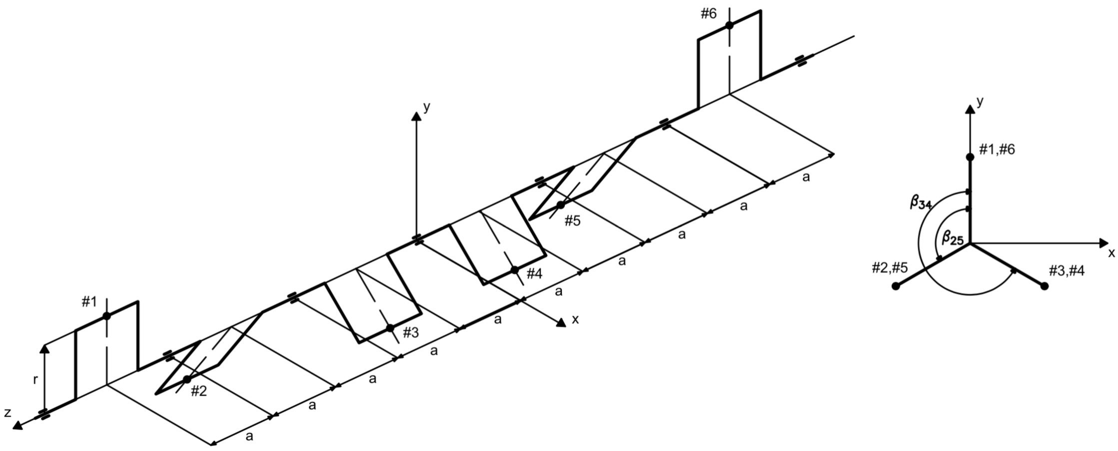

Figure 4.71: Six-cylinder inline crankshaft

The first and the second order vectors stars are now reported, in order to show the forces acting on the system.

First order vectors star

As can be seen in the Figure 4.72:

As can be seen:

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) ⇒ Balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

Second order vectors star

As can be seen in the Figure 4.73:

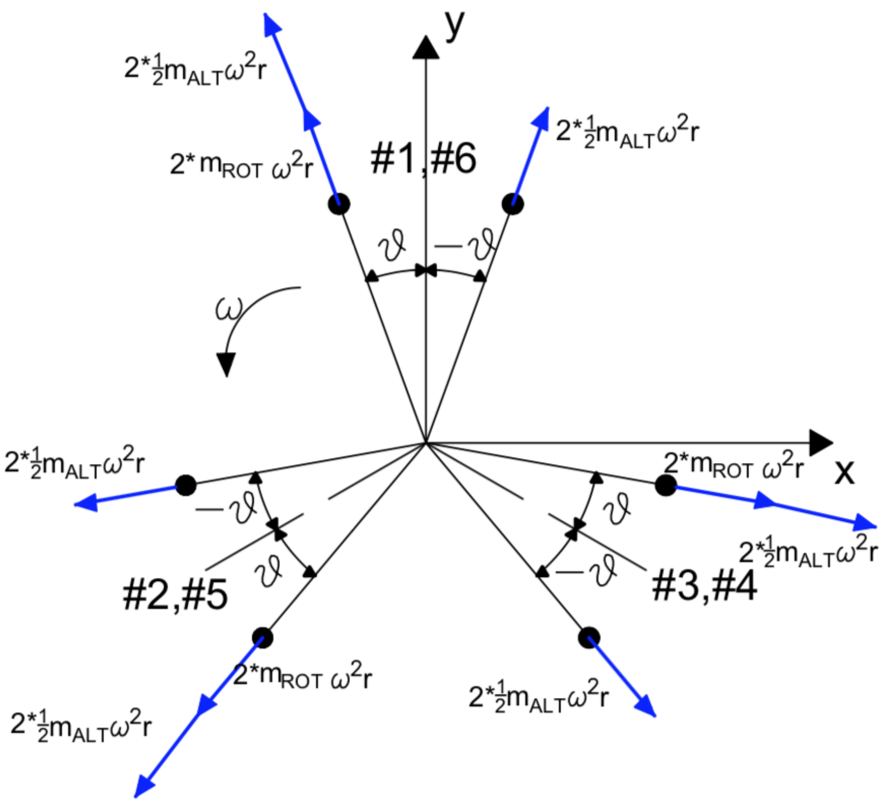

Figure 4.72: Firts order vectors star

- Rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.
- Counter-rotating part of second order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

For the second order forces, in this case:

- Cyl. #1 → $\theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. #2 → $(\theta + 120°) \Rightarrow 2(\theta + 120°) \Rightarrow \cos(2\theta + 240°)$;
- Cyl. #3 → $(\theta + 240°) \Rightarrow 2(\theta + 240°) \Rightarrow \cos(2\theta + 480°) = \cos(2\theta + 120°)$;
- Cyl. #4 → $(\theta + 240°) \Rightarrow 2(\theta + 240°) \Rightarrow \cos(2\theta + 480°) = \cos(2\theta + 120°)$;
- Cyl. #5 → $(\theta + 120°) \Rightarrow 2(\theta + 120°) \Rightarrow \cos(2\theta + 240°)$;
- Cyl. #6 → $\theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;

In this crankshaft layout, both the first and the second order moments are balanced. As can be seen, all the forces and moments are balanced in this configuration, in fact the six cylinder inline engine is the most self-balanced engine existing.

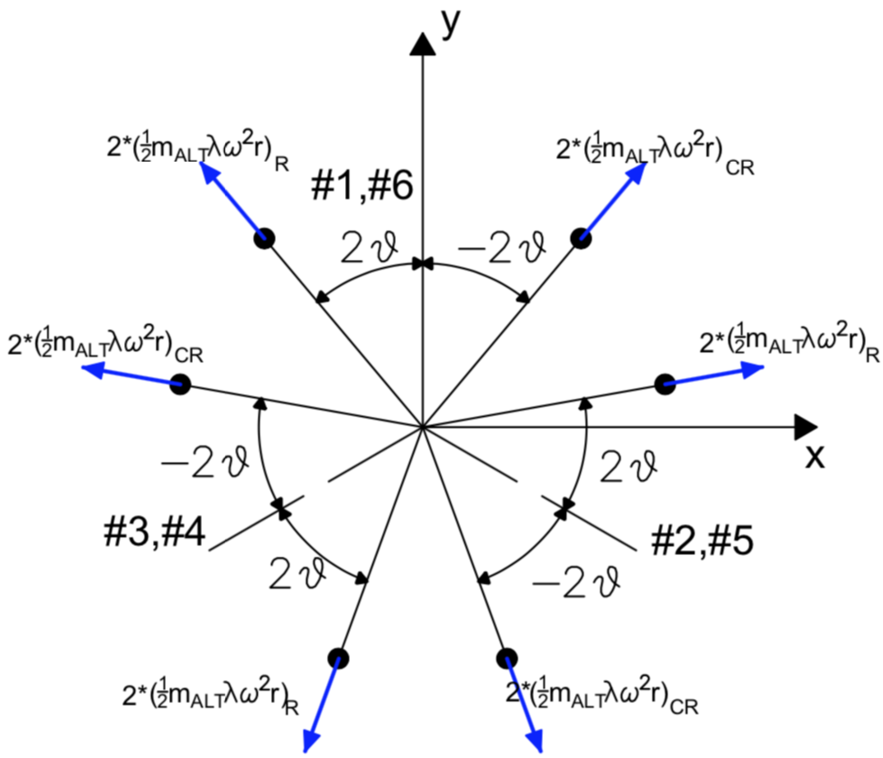

Figure 4.73: Second order vectors star

Anyway, if the system has a good global balance it is not the same in terms of local balance and therefore if the designer wants to reduce the bearings reactions even this layout must be locally balanced through the addition of counterweights opportunely placed.

Chapter 5

V-Engines

This is a common layout for internal combustion engines. In a *V-Engine* (or *Vee Engine*) cylinders and pistons are aligned but separated in two different planes, called *banks*. Observing the system along the crankshaft axis, these two planes form an angle between each other and they appear to be in a V. Both the banks are considered as two inline engines separated as *V-angle*.

The primary reason to use a V-engine is packaging. They generally allows to reduce the overall engine length, heigth and weight compared to an inline configuration.

Usually, a common crankpin is shared between a pair of corresponding pistons from each bank. There are several conrods which can be used to connect a pair of pistons:

- Two ordinary connecting rods placed side-by-side.
- Master and Slave connecting rod.
- Fork and Blade connecting rod.

The most common solution is to use a pair of ordinary connecting rods placed side-by-side. However, with this configuration, the two axis of the couple of cylinders considered are not on the same plane and there is an offset between them. It means that the left bank and the right bank of the engine are staggered in order to accomodate the two conrods for each pair of cylinders. With *Master and Slave conrod* or *Fork and Blade conrod* solutions this offset is obviously canceled.

The *V-angle* can be chosen from 1° to 180° and depending upon the number of cylinders. There may be some that work better than other in term of engine stability.

Also the firing order is relevant for the engine balancing.

5.1 Generic configuration of a V-Twin Engine

Figure 5.1 illustrates a generic system formed by two pistons connected through two conrods at a common crank throw.

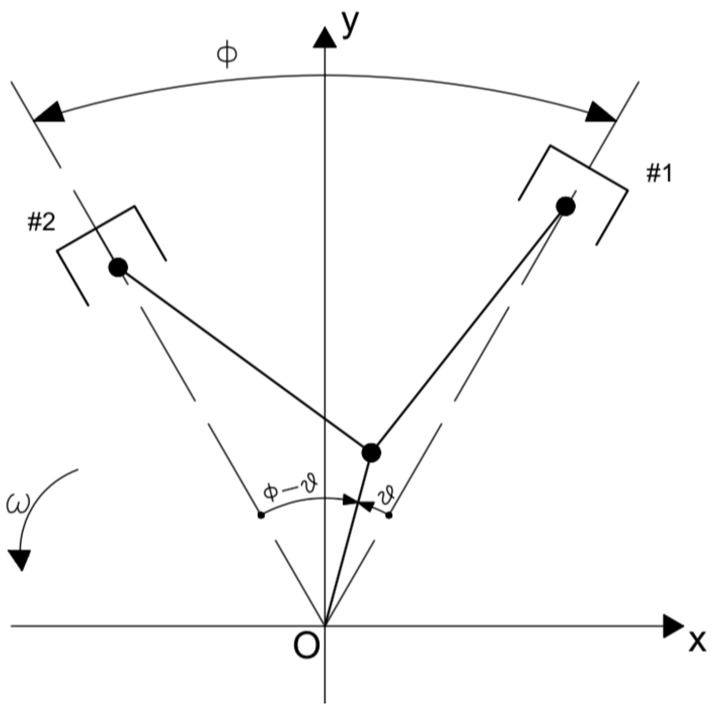

Figure 5.1: Generical configuration of a V-Twin Engine

The rotating and counter-rotating first and second order forces of the considered configuration are shown in the Figure 5.2

Observing the Figure 5.2 it can be noticed that, depending on the V-angle ϕ, some forces can be self-balanced. It can be seen that:

- Centrifugal forces and the rotating part of the first order reciprocating forces always add up.
- The second order reciprocating rotating forces form an angle equal to ϕ between them.
- The first order reciprocating counter-rotating forces form ad angle equal to 2ϕ between them.
- The second order reciprocating counter-rotating forces form ad angle equal to 3ϕ between them.

These statements are valid in any case.

Therefore, as already said depending on the ϕ angle some forces can balance each other. For example, if ϕ = 90° the first order reciprocating counter-rotating forces are self balanced, because the angle between them is equal to 180°, so they are equal and opposite (if the masses considered for cylinders 1 and 2 are equal).

The magnitude of the forces along the main directions can be calculated considering using trigonometric relations through the angles θ and ϕ. The terms *main directions* mean

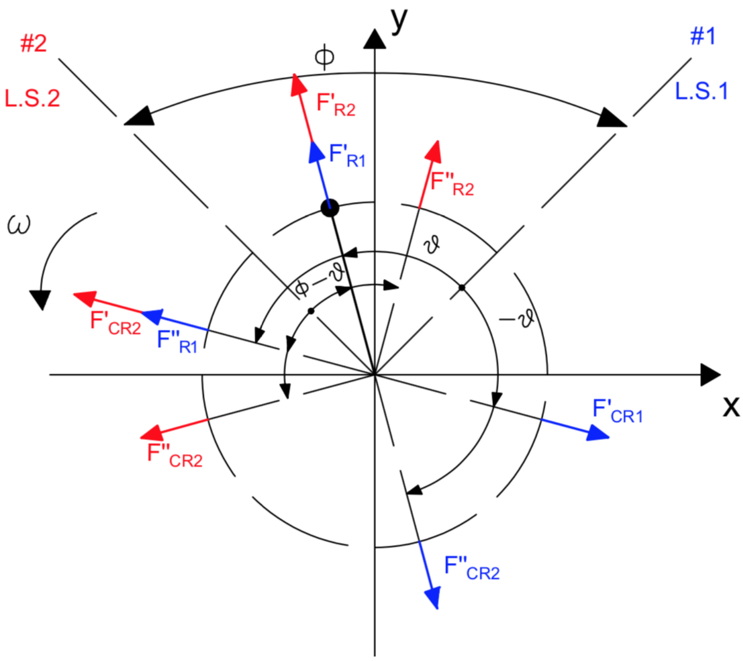

Figure 5.2: Generical configuration of a V-Twin Engine with main angles and directions

the x-axis direction, y-axis direction and the lines of stroke of the two cylinders considered.

It is important to stress that the V-angle ϕ also influences the moment values. Considering the Figure 5.3 it can be observed that:

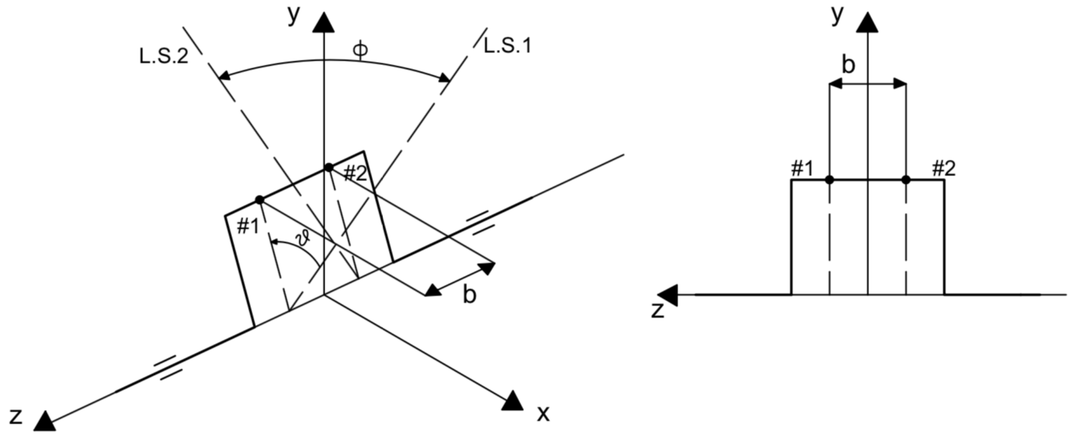

Figure 5.3: V-Twin Engine with a single crank throw

- The moment due to the first order forces (pure centrifugal and reciprocating) is zero if the forces due to the two piston have the same magnitude.

- The moment due to the first order counter-rotating forces depend on the value of the ϕ angle.
- Also the moments due to second order rotating and counter-rotating forces are dependant by the ϕ angle value.

For example, if $\phi = 180°$ the angle between second order rotating forces is equal to $2\phi = 360°$. It means that the forces are on the same plane, with the same verse. If the masses are equal the forces magnitude are equal and the moment is zero.

Considering now a system with more than one crank throw as in the Figure 5.4, some important relations can be derived.

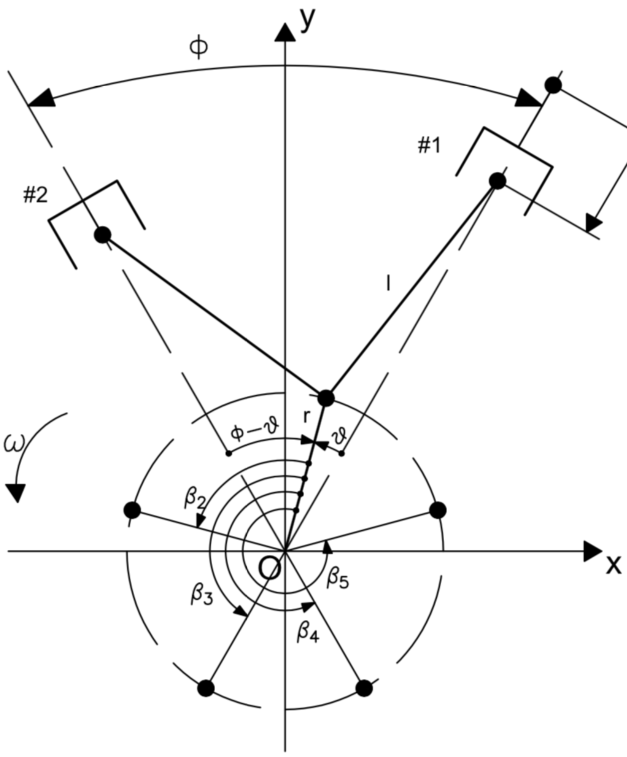

Figure 5.4: V-Engine with five crank throws

where:

- r = crank radius = $\frac{1}{2}stroke$;
- l = conrod length;
- $\lambda = \frac{r}{l}$;

The piston displacement can be written like:

$$s = r(1 - cos\theta_1) + l(1 - \sqrt{1 - \lambda^2 \sin^2 \theta_1}); \tag{5.1}$$

and considering the series expansion of the term $\sqrt{1-\lambda^2 \sin^2\theta_1}$:

$$\sqrt{1-\lambda^2 sin^2\theta_1} = 1 - \frac{1}{2}\lambda^2 \sin^2\theta_1 - \frac{1}{8}\lambda^4 \sin^4\theta_1 - \frac{1}{16}\lambda^6 \sin^6\theta_1 - ... \tag{5.2}$$

the 5.1 can be written as:

$$s = r(1 - cos\theta_1) + \frac{r}{4}\lambda(1 - \cos 2\theta_1); \tag{5.3}$$

The first and second order reciprocating forces can be written as:

$$F'_{ALT} = m_{ALT}\omega^2 r \cos\theta_1; \tag{5.4}$$

$$F''_{ALT} = m_{ALT}\omega^2 r\lambda \cos 2\theta_1; \tag{5.5}$$

Considering the first bank, the summation of the forces is equal to:

- First order forces:

$$\begin{aligned}\sum_{i=1}^{n} F'_{ALT_1} = m_{ALT}\omega^2 r \sum_{i=1}^{n} \cos(\theta_1 + \beta_i) = \\ = m_{ALT}\omega^2 r \sum_{i=1}^{n} [\cos\theta_1 \cos\beta_i - \sin\theta_1 \sin\beta_i] = \\ = m_{ALT}\omega^2 r [\cos\theta_1 \sum_{i=1}^{n} \cos\beta_i - \sin\theta_1 \sum_{i=1}^{n} \sin\beta_i];\end{aligned} \tag{5.6}$$

- Second order forces:

$$\sum_{i=1}^{n} F''_{ALT_1} = m_{ALT}\omega^2 r\lambda [\cos 2\theta_1 \sum_{i=1}^{n} \cos 2\beta_i - \sin 2\theta_1 \sum_{i=1}^{n} \sin 2\beta_i]; \tag{5.7}$$

where n is the number of crank throws considered, in this case equal to 5.
For the second bank:

- First order forces:

$$\begin{aligned}\sum_{i=1}^{n} F'_{ALT_2} = m_{ALT}\omega^2 r \sum_{i=1}^{n} \cos(-\phi + \theta_1 + \beta_i) = \\ = m_{ALT}\omega^2 r \sum_{i=1}^{n} [\cos(-\phi + \theta_1) \cos\beta_i - \sin(-\phi + \theta_1) \sin\beta_i] = \\ = m_{ALT}\omega^2 r [\cos(-\phi + \theta_1) \sum_{i=1}^{n} \cos\beta_i - \sin(-\phi + \theta_1) \sum_{i=1}^{n} \sin\beta_i];\end{aligned} \tag{5.8}$$

- Second order forces:

$$\sum_{i=1}^{n} F''_{ALT_2} = m_{ALT}\omega^2 r\lambda[\cos 2(-\phi+\theta_1)\sum_{i=1}^{n}\cos 2\beta_i - \sin 2(-\phi+\theta_1)\sum_{i=1}^{n}\sin 2\beta_i]; \quad (5.9)$$

To calculate the shaking couples, let us consider the system in the Figure 5.5:

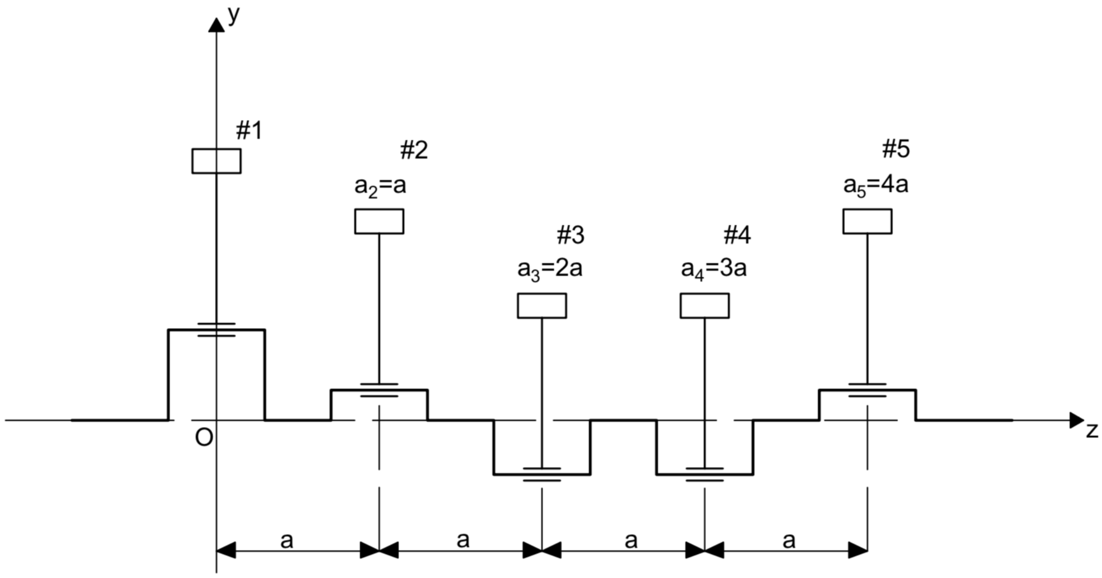

Figure 5.5: V-Engine with five crank throws

For the first bank:

- First order couples:

$$C'_1 = \sum_{i=2}^{5} F'_{ALT_i} a_i = m_{ALT}\omega^2 r \sum_{i=2}^{5}(a_i \cos(\theta_1+\beta_i)) =$$
$$= m_{ALT}\omega^2 r[\cos\theta_1 \sum_{i=1}^{n}(a_i\cos\beta_i) - \sin\theta_1\sum_{i=1}^{n}(a_i\sin\beta_i)]; \quad (5.10)$$

- Second order couples:

$$C''_1 = \sum_{i=2}^{5} F''_{ALT_i} a_i = m_{ALT}\omega^2 r\lambda \sum_{i=2}^{5}(a_i \cos(2(\theta_1+\beta_i))) =$$
$$= m_{ALT}\omega^2 r\lambda[\cos 2\theta_1 \sum_{i=2}^{5}(a_i\cos 2\beta_i) - \sin 2\theta_1\sum_{i=2}^{5}(a_i\sin 2\beta_i)]; \quad (5.11)$$

For the second bank:

- First order couples:

$$C'_2 = \sum_{i=2}^{5} F'_{ALT_i} a_i = m_{ALT}\omega^2 r \sum_{i=2}^{5} (a_i \cos(-\phi + \theta_1 + \beta_i)) =$$
$$= m_{ALT}\omega^2 r[\cos(-\phi + \theta_1) \sum_{i=1}^{n} (a_i \cos\beta_i) - \sin(-\phi + \theta_1) \sum_{i=1}^{n} (a_i \sin\beta_i)]; \quad (5.12)$$

- Second order couples:

$$C''_2 = \sum_{i=2}^{5} F''_{ALT_i} a_i = m_{ALT}\omega^2 r\lambda \sum_{i=2}^{5} (a_i \cos(2(-\phi + \theta_1 + \beta_i))) =$$
$$= m_{ALT}\omega^2 r\lambda[\cos 2(-\phi + \theta_1) \sum_{i=2}^{5} (a_i \cos 2\beta_i) - \sin 2(-\phi + \theta_1) \sum_{i=2}^{5} (a_i \sin 2\beta_i)]; \quad (5.13)$$

It is possible to represent the forces like:

$$\sum_{i=1}^{n} F'_{ALT} = m_{ALT}\omega^2 r[\cos\theta_1 \sum_{i=1}^{5} \cos\beta_i - \sin\theta_1 \sum_{i=1}^{5} \sin\beta_i] = m_{ALT}\omega^2 r[C_1 \cos\theta_1 - C_2 \sin\theta_1]; \quad (5.14)$$

$$\sum_{i=1}^{n} F''_{ALT} = m_{ALT}\omega^2 r\lambda[\cos 2\theta_1 \sum_{i=1}^{5} \cos 2\beta_i - \sin 2\theta_1 \sum_{i=1}^{5} \sin 2\beta_i] = m_{ALT}\omega^2 r[C_3 \cos 2\theta_1 - C_4 \sin 2\theta_1]; \quad (5.15)$$

for a 5 cylinder engine, the terms C_1, C_2, C_3 and C_4 are equal to:

$$C_1 = \sum_{i=1}^{5} \cos\beta_i = \cos\beta_1 + \cos\beta_2 + \cos\beta_3 + \cos\beta_4 + \cos\beta_5; \quad (5.16)$$

$$C_2 = \sum_{i=1}^{5} \sin\beta_i = \sin\beta_1 + \sin\beta_2 + \sin\beta_3 + \sin\beta_4 + \sin\beta_5; \quad (5.17)$$

$$C_3 = \sum_{i=1}^{5} \cos 2\beta_i = \cos 2\beta_1 + \cos 2\beta_2 + \cos 2\beta_3 + \cos 2\beta_4 + \cos 2\beta_5; \quad (5.18)$$

$$C_4 = \sum_{i=1}^{5} \sin 2\beta_i = \sin 2\beta_1 + \sin 2\beta_2 + \sin 2\beta_3 + \sin 2\beta_4 + \sin 2\beta_5; \quad (5.19)$$

The shaking couples can be expressed as:

$$\sum_{i=1}^{n} C' = m_{ALT}\omega^2 r[\cos\theta_1 \sum_{i=1}^{5} (a_i \cos\beta_i) - \sin\theta_1 \sum_{i=1}^{5} (a_1 \sin\beta_i)] =$$
$$= m_{ALT}\omega^2 r[C_5 \cos\theta_1 - C_6 \sin\theta_1]; \quad (5.20)$$

$$\sum_{i=1}^{n} C'' = m_{ALT}\omega^2 r\lambda[\cos 2\theta_1 \sum_{i=1}^{5}(a_i \cos 2\beta_i) - \sin 2\theta_1 \sum_{i=1}^{5}(a_1 \sin 2\beta_i)] = \\ = m_{ALT}\omega^2 r[C_7 \cos 2\theta_1 - C_8 \sin 2\theta_1]; \quad (5.21)$$

for a 5 cylinder engine, the terms C_5, C_6, C_7 and C_8 are equal to:

$$C_5 = \sum_{i=1}^{5} a_i \cos\beta_i = a_1\cos\beta_1 + a_2\cos\beta_2 + a_3\cos\beta_3 + a_4\cos\beta_4 + a_5\cos\beta_5; \quad (5.22)$$

$$C_6 = \sum_{i=1}^{5} a_i \sin\beta_i = a_1\sin\beta_1 + a_2\sin\beta_2 + a_3\sin\beta_3 + a_4\sin\beta_4 + a_5\sin\beta_5; \quad (5.23)$$

$$C_7 = \sum_{i=1}^{5} a_i \cos 2\beta_i = a_1\cos 2\beta_1 + a_2\cos 2\beta_2 + a_3\cos 2\beta_3 + a_4\cos 2\beta_4 + a_5\cos 2\beta_5; \quad (5.24)$$

$$C_8 = \sum_{i=1}^{5} a_i \sin 2\beta_i = a_1\sin 2\beta_1 + a_2\sin 2\beta_2 + a_3\sin 2\beta_3 + a_4\sin 2\beta_4 + a_5\sin 2\beta_5; \quad (5.25)$$

These equations are valid for only one bank.
For the second bank the crank angle is equal to $(-\phi + \theta_1)$, therefore it can be written:

$$\sum_{i=1}^{n} F'_{ALT} = m_{ALT}\omega^2 r[C_1\cos(\theta_1 - \phi) - C_2\sin(\theta_1 - \phi)]; \quad (5.26)$$

$$\sum_{i=1}^{n} F''_{ALT} = m_{ALT}\omega^2 r\lambda[C_3\cos 2(\theta_1 - \phi) - C_4\sin 2(\theta_1 - \phi)]; \quad (5.27)$$

$$\sum_{i=1}^{n} C' = m_{ALT}\omega^2 r[C_5\cos(\theta_1 - \phi) - C_6\sin(\theta_1 - \phi)]; \quad (5.28)$$

$$\sum_{i=1}^{n} C'' = m_{ALT}\omega^2 r\lambda[C_7\cos 2(\theta_1 - \phi) - C_8\sin 2(\theta_1 - \phi)]; \quad (5.29)$$

Making a system of the equations above, it is possible fo find a solution that balance the shaking forces and couples. In some cases it is possible to have a full balanced configuration, in other cases instead it is possible to balance only some of the forces and couples that stressing the system. If it is possible to make a choice, in generally is more important to balance the first order shaking forces because their magnitude is bigger compared with the second order forces, because of the factor $\lambda = \frac{r}{l}$ that will always be less than 1.

As already mentioned, in most cases conrods and pistons will be offset along the *z-axis* (axis of the crankshaft). Therefore, the shaking forces of the two pistons connected to the same crank throw will not be the same plane and there is a possibility of having a shaking moment.

Generally, it is better to have a crankshaft carefully designed in order to have the maximum possible internal balance. In the cases in which it is not possible to balance completely the alternating forces, some additional counterweights can be placed, but with a careful design the size of these counterweights can be smaller.

Let us consider now some important cases of engine configurations.

5.2 V4 Engines

A *V4 Engine* is a four-cylinder engines with its cylinders arrangend in two banks V configuration. V4 engines are used both in automobiles and motorcycles and industrial applications.

A V4 engine can be seen as a pair of V-Twin engines mounted end-to-end, or a pair of Two-cylinder inline engines separated as a V-angle and sharing the same crankshaft. Most part of V4 engines support the crankshaft with three main bearings, and have two crankpinks. Each of these crankpins are shared by opposing cylinders from different banks.

Compared with a classic Four-cylinder inline engine, a V4 engine is more compact because of the crankshaft length is smaller. A V4 produces less rocking couples than an Inline-4, and since the crankshaft is shorter its stiffness is higher compared to an Inline-4 crankshaft. A disadvantage is that usually a V4 is more difficult and expensive to design and produce than ad Inline-4.

5.2.1 90° V4 Engine with crank throws at 180°

Considering the system show in the Figure 5.6:

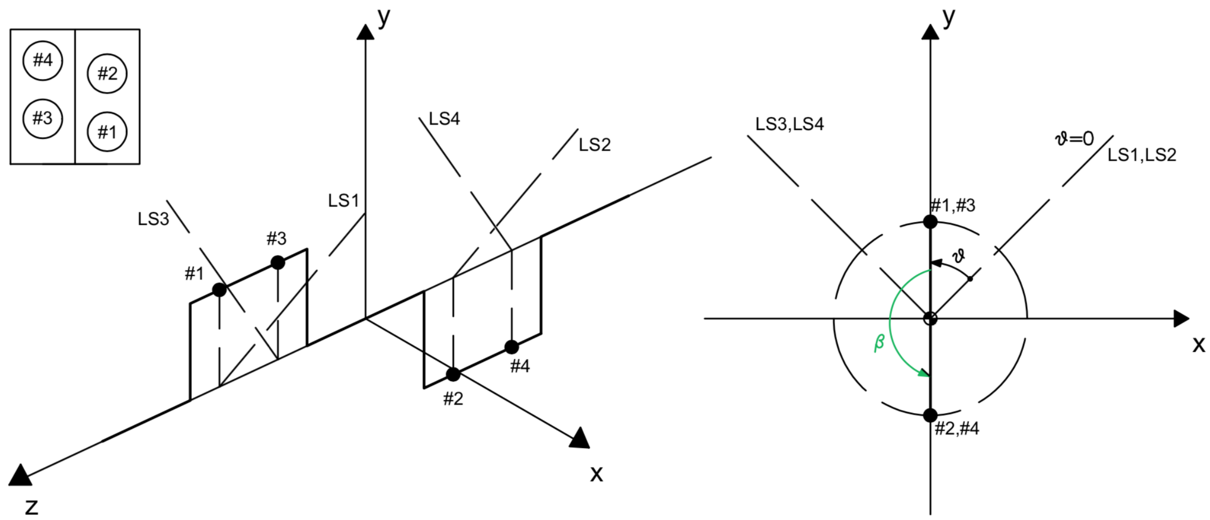

Figure 5.6: 90° V4 Engine with crank throws at 180°

On the right bank there are cylinders #1 and #2, on the left #3 and #4. β is the angle between the two crank throws, in this case equal to 180°.

It is assumed that the position θ = 0° is coincident with the *line of stroke* of the cylinders #1 and #2.

The V-angle ϕ is equal to 90°.

The forces acting on the system can be represented through the vectors stars.

First order vectors star

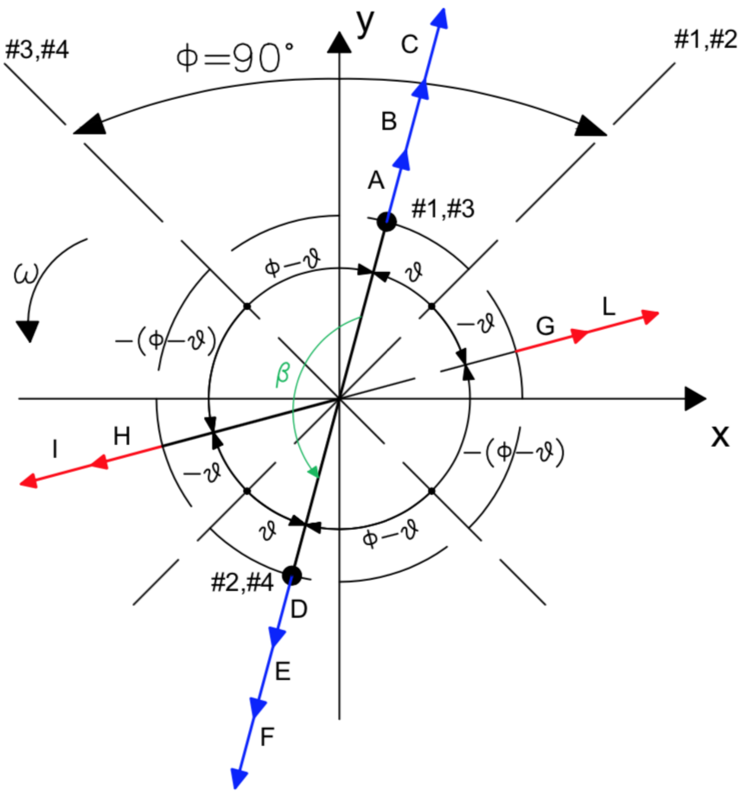

Figure 5.7: 90° V4 Engine first order vectors star

The rotating forces are indicated in blue, in red the counter-rotating.
The forces reported in the Figure 5.7 are equal to:

$$A = m_{ROT_{13}}\omega^2 r; \quad B = (\tfrac{1}{2}m_{ALT_1}\omega^2 r)_R; \quad C = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_R;$$
$$D = m_{ROT_{24}}\omega^2 r; \quad E = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_R; \quad F = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_R;$$
$$G = (\tfrac{1}{2}m_{ALT_1}\omega^2 r)_{CR}; \quad H = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_{CR};$$
$$I = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_{CR}; \quad L = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_{CR};$$

with:

- $m_{ROT_{13}} = m_{crank_{13}} + m_{ROT_{rod_1}} + m_{ROT_{rod_3}}$;
- $m_{ROT_{24}} = m_{crank_{24}} + m_{ROT_{rod_2}} + m_{ROT_{rod_4}}$;

As can be seen in the Figure 5.7:

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) $\Rightarrow$ Balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) $\Rightarrow$ Balanced.

Second order vectors star

The angle β is equal to 180°, therefore, in order to find the second order forces directions, it can be written:

- Cyl. #1 $\rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. #2 $\rightarrow (\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;
- Cyl. #3 $\rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cyl. #4 $\rightarrow (\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;

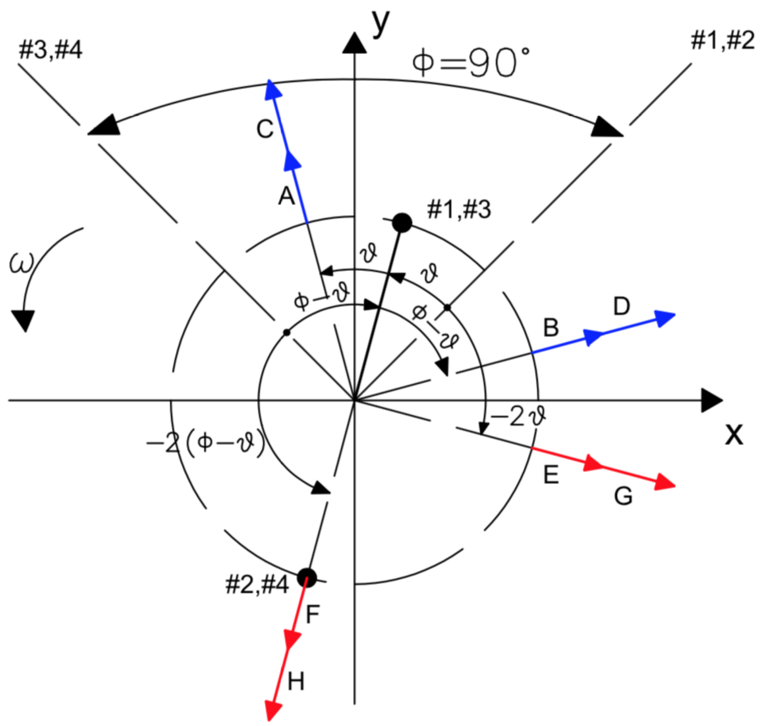

Figure 5.8: Second order vectors star

The rotating forces are indicated in blue, in red the counter-rotating.
The forces reported in the Figure 5.8 are equal to:

$$A = (\tfrac{1}{2}m_{ALT_1}\omega^2 r\lambda)_R; \qquad B = (\tfrac{1}{2}m_{ALT_2}\omega^2 r\lambda)_R;$$

$$C = (\tfrac{1}{2} m_{ALT_3} \omega^2 r \lambda)_R; \qquad D = (\tfrac{1}{2} m_{ALT_4} \omega^2 r \lambda)_R;$$

$$E = (\tfrac{1}{2} m_{ALT_1} \omega^2 r \lambda)_{CR}; \qquad F = (\tfrac{1}{2} m_{ALT_2} \omega^2 r \lambda)_{CR};$$

$$G = (\tfrac{1}{2} m_{ALT_3} \omega^2 r \lambda)_{CR}; \qquad H = (\tfrac{1}{2} m_{ALT_4} \omega^2 r \lambda)_{CR};$$

As can be seen in the Figure 5.8:

- Rotating part of second order alternating forces $(\frac{1}{2} m_{ALT} \omega^2 r) \Rightarrow$ Not balanced.

- Counter-rotating part of second order alternating forces $(\frac{1}{2} m_{ALT} \omega^2 r) \Rightarrow$ Not balanced.

Observing the 5.8, it is important to stress that in this particular configuration ($\phi = 90°, \beta = 180°$) the resultant of the sum of second order rotating and counter-rotating forces along the y-axis is equal to zero, for each theta angle considered. Therefore, since both rotating and counter-rotating first order forces are balanced, this engine configuration is globally balanced to all the forces in y direction.

In terms of couples, the second order moments are not balanced, the first order rotating moment are not balanced too but the first order counter-rotating moment are balanced instead. This is due to this particular configuration of the engine with the V-angle equal to 90° because, as already said in Paragraph 5.1, the first order counter-rotating forces form an angle equal to 2ϕ between them, therefore, since the moment arms are equal to each other, the moment is canceled.

The resultant of first order rotating moment is on the same plane of the two crank throws. Considering the Figure 5.9 this fact can be demonstrated as shown below.

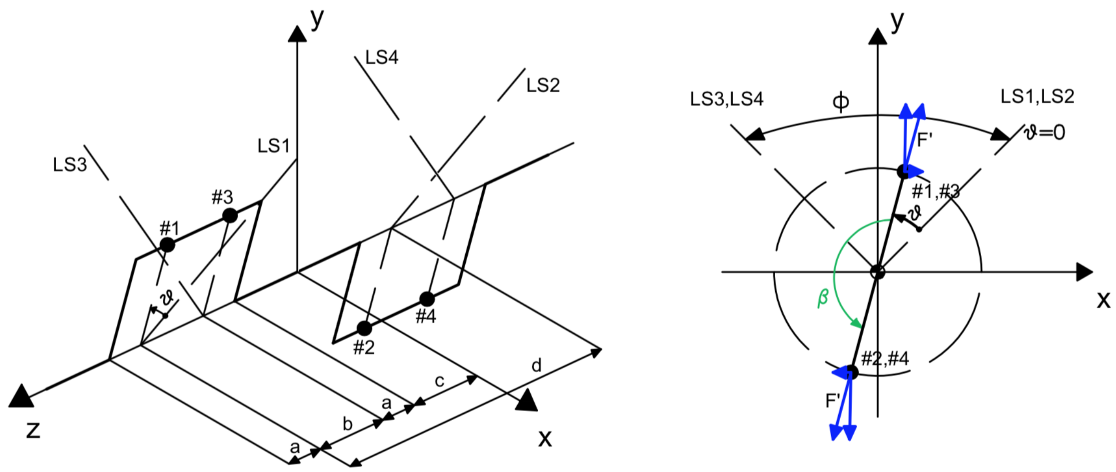

Figure 5.9: V4 engine with crank throws at 180°

The moment around x-axis can be written as:

$$M'_x = F'_{\#1}\cos\left(-\frac{\phi}{2}+\theta\right)(a+b+c) + F'_{\#3}\cos\left(-\frac{\phi}{2}+\theta\right)(a+c)+ \\ - F'_{\#2}\cos\left(-\frac{\phi}{2}+\theta+\beta\right)(a+c) - F'_{\#4}\cos\left(-\frac{\phi}{2}+\theta+\beta\right)(a+b+c); \quad (5.30)$$

considering that:

- $F'_{\#1} = F'_{\#2} = F'_{\#3} = F'_{\#4} = F'$
- $\frac{\phi}{2} = 45°$ and $\beta = 180°$

$$M'_x = F'[\cos(-45° + \theta)(2a + b + 2c) - \cos(-45° + \theta + 180°)(2a + b + 2c)] = \\ = F'(2a + b + 2c)[\cos(-45° + \theta) - \cos(-45° + \theta + 180°)]; \quad (5.31)$$

The moment around y-axis is equal to:

$$M'_y = F'_{\#1}\sin\left(-\frac{\phi}{2}+\theta\right)(a+b+c) + F'_{\#3}\sin\left(-\frac{\phi}{2}+\theta\right)(a+c)+ \\ - F'_{\#2}\sin\left(-\frac{\phi}{2}+\theta+\beta\right)(a+c) - F'_{\#4}\sin\left(-\frac{\phi}{2}+\theta+\beta\right)(a+b+c) = \\ = F'[\sin(-45° + \theta)(2a + b + 2c) - \sin(-45° + \theta + 180°)(2a + b + 2c)] = \\ = F'(2a + b + 2c)[\sin(-45° + \theta) - \sin(-45° + \theta + 180°)]; \quad (5.32)$$

Considering now the Figure 5.10 the angle δ between the crank throws and the plane where the resultant first order moment is placed can be obtained.

$$\tan\alpha = \frac{M'_y}{M'_x} = \frac{\sin(-45° + \theta) - \sin(-45° + \theta + 180°)}{\cos(-45° + \theta) - \cos(-45° + \theta + 180°)}; \quad (5.33)$$

therefore:

$$\alpha = \tan^{-1} = \frac{\sin(-45° + \theta) - \sin(-45° + \theta + 180°)}{\cos(-45° + \theta) - \cos(-45° + \theta + 180°)}; \quad (5.34)$$

and finally:

$$\delta = \alpha + \frac{\phi}{2} - \theta; \quad (5.35)$$

Considering for example:

- $\theta = 0° \Rightarrow \alpha = \tan^{-1}\left(\frac{-\sqrt{2}}{\sqrt{2}}\right) = -45° \Rightarrow \delta = 0°;$
- $\theta = 60° \Rightarrow \alpha = \tan^{-1}\left(\frac{\frac{\sqrt{6}-\sqrt{2}}{2}}{\frac{\sqrt{6}+\sqrt{2}}{2}}\right) = \tan^{-1}(-\sqrt{3}+2) = 15° \Rightarrow \delta = 0°;$

The plane of the resultant moment is therefore the same as for the crank throws, for each θ angle.

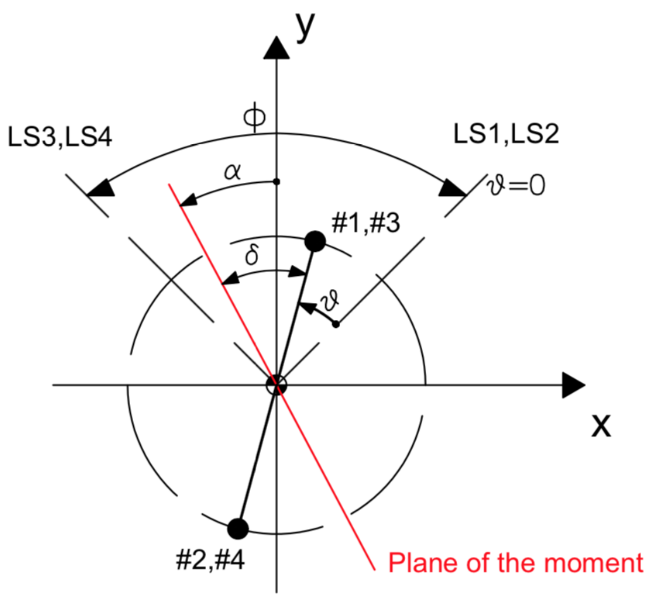

Figure 5.10: Angle between crank throws and resultant first order moment plane

Balancing Strategy

In terms of global balance, the considered system is balanced only to the first order forces and first order counter-rotating moments. All the rest of forces and couples is not balanced.

If the purpose is to balance first order actions, adding two counterweights in proper positions it is possible to achieve to a system globally balanced. Considering the Figure 5.11:

these counterweights are placed in the same plane of the crank throws, opposite to them.

Doing a moment balancing as shown in 5.36 and considering $F_{BAL} = m_{BAL}\omega^2 r_{BAL}$ it is possible to find the required counterweights mass and the radius which they are positioned.

$$2(2a + b + c)F_{BAL} = 2(a + b + c)F_{\#1,\#4} + 2(a + c)F_{\#2,\#3}; \tag{5.36}$$

considering:

- $F_{\#1,\#4} = F_{\#2,\#3} = F'$

$$(4a + 2b + 2c)F_{BAL} = (2a + 2b + 2c)F_{\#1,\#4} + (2a + 2c)F_{\#2,\#3} = (4a + 2b + 4c)F'; \tag{5.37}$$

and then:

$$F_{BAL} = \frac{(4a + 2b + 4c)}{(4a + 2b + 2c)}F'; \tag{5.38}$$

In this way the system results globally balanced to the first order rotating moments.

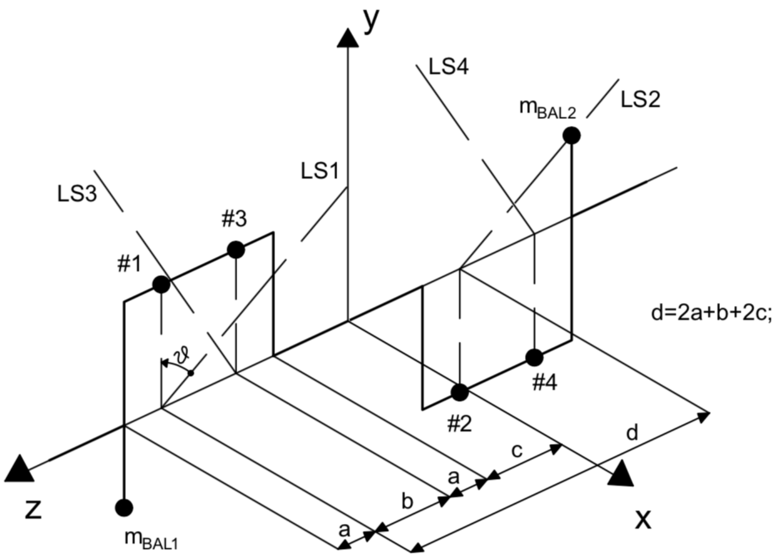

Figure 5.11: V4 engine global balanced

Bay-By-Bay Balancing

It is useful to have a system both globally and locally balanced because the reactions on the main journals are lower, and often there is a better mass distribution. In this case a bay-by-bay balancing can be achieved adding two counterweights per crank throw.

The system considered is shown in the Figure 5.12:

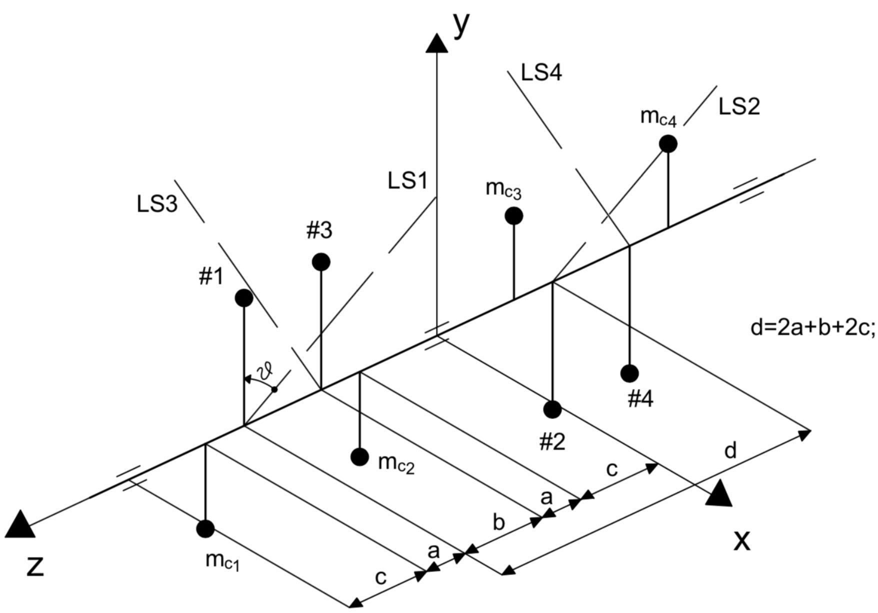

Figure 5.12: Bay-by-bay balancing on a V4 crankshaft

$$m_1 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = (a+b+c) \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = -(a+c) \end{pmatrix};$$

$$m_3 \Rightarrow \begin{pmatrix} x_3 \\ y_3 \\ z_3 = (a+c) \end{pmatrix}; \qquad m_4 \Rightarrow \begin{pmatrix} x_4 \\ y_4 \\ z_4 = -(a+b+c) \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = (2a+b+2c) \end{pmatrix}; \qquad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = c; \end{pmatrix};$$

$$m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = -c \end{pmatrix}; \qquad m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = -(2a+b+2c); \end{pmatrix};$$

In order to simplify the notation, the hypotesys that $m_1 = m_{ROT_{ROD_1}} + \frac{1}{2} m_{crank_{13}}$ was made; the remaining part of $m_{crank_{13}}$ is considered in the m_3. Idem for the $m_{crank_{24}}$ with m_2 and m_4.

The system of equations can be written as:

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + \\ \qquad + m_1\,\omega^2\,x_1 + m_2\,\omega^2\,x_2 + m_3\,\omega^2\,x_3 + m_4\,\omega^2\,x_4 = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + \\ \qquad + m_1\,\omega^2\,y_1 + m_2\,\omega^2\,y_2 + m_3\,\omega^2\,y_3 + m_4\,\omega^2\,y_4 = 0 \\ x \circlearrowleft \; (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + \\ \qquad + (m_1\,\omega^2\,x_1)z_1 + (m_2\,\omega^2\,x_2)z_2 + (m_3\,\omega^2\,x_3)z_3 + (m_4\,\omega^2\,x_4)z_4 = 0 \\ y \circlearrowleft \; (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} + \\ \qquad + (m_1\,\omega^2\,y_1)z_1 + (m_2\,\omega^2\,y_2)z_2 + (m_3\,\omega^2\,y_3)z_3 + (m_4\,\omega^2\,y_4)z_4 = 0 \end{cases} \tag{5.39}$$

Assuming:

- $m_1 = m_2 = m_3 = m_4 = m$;
- $x_1 = x_3 = x_{13}$ and $x_2 = x_4 = x_{24}$;
- $x_{13} = -x_{24}$;
- $y_1 = y_3 = y_{13}$ and $y_2 = y_4 = y_{24}$;
- $y_{13} = -y_{24}$;

$$
\begin{cases}
x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + 2mx_{13} + 2mx_{24} = 0 \\
y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + 2my_{13} + 2my_{24} = 0 \\
x \circlearrowleft \ m_{c_1}x_{c_1}z_{c_1} + m_{c_2}x_{c_2}z_{c_2} + m_{c_3}x_{c_3}z_{c_3} + m_{c_4}x_{c_4}z_{c_4} + \\
\qquad +mx_{13}(a+b+c) + mx_{24}(-(a+c)) + mx_{13}(a+c) + mx_{24}(-(a+b+c)) = 0 \\
y \circlearrowleft \ m_{c_1}y_{c_1}z_{c_1} + m_{c_2}y_{c_2}z_{c_2} + m_{c_3}y_{c_3}z_{c_3} + m_{c_4}y_{c_4}z_{c_4} + \\
\qquad +my_{13}(a+b+c) + my_{24}(-(a+c)) + my_{13}(a+c) + my_{24}(-(a+b+c)) = 0
\end{cases}
\tag{5.40}
$$

and then:

$$
\begin{cases}
x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} = 0 \\
y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} = 0 \\
x \circlearrowleft \ m_{c_1}x_{c_1}z_{c_1} + m_{c_2}x_{c_2}z_{c_2} + m_{c_3}x_{c_3}z_{c_3} + m_{c_4}x_{c_4}z_{c_4} + 2mx_{13}(2a+b+2c) = 0 \\
y \circlearrowleft \ m_{c_1}y_{c_1}z_{c_1} + m_{c_2}y_{c_2}z_{c_2} + m_{c_3}y_{c_3}z_{c_3} + m_{c_4}y_{c_4}z_{c_4} + 2my_{13}(2a+b+2c) = 0
\end{cases}
\tag{5.41}
$$

Fixing the counterweights masses $m_{c_1}, m_{c_2}, m_{c_3}$ and m_{c_4} the values x_c and y_c can be obtained. On the contrary, the massed can be derived fixing the positions of the four counterweights.

5.2.2 Bearing loads

The way used to find the bearing loads is based on the *Three Moment Equation of Clapeyron*. The crankshaft is hyperstatically constrained and can be considered as a beam, in this case with two span.

The three moment equation method require to use ad additional equation for each excess constraint. In the equations shown below it is assumed that the Young's modulus E and the moment of inertia of the crankshaft section J are constant along the crankshaft, therefore they cancel each other in all the terms.

Considering the Figure 5.13, the three moment equation can be written as in the 5.42:

$$
M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\frac{F_{c1}}{l_1}a(l_1^2-a^2) - \frac{F_{crank_{13}}}{l_1}(a+b)(l_1^2-(a+b)^2) - \frac{F_{c3}}{l_1}(a+b+c)(l_1^2-(a+b+c)^2) +
$$
$$
-\frac{F_{c2}}{l_2}(f+g+h)(l_2^2-(f+g+h)^2) - \frac{F_{crank_{24}}}{l_2}(g+h)(l_2^2-(g+h)^2) - \frac{F_{c4}}{l_2}h(l_1^2-h^2); \tag{5.42}
$$

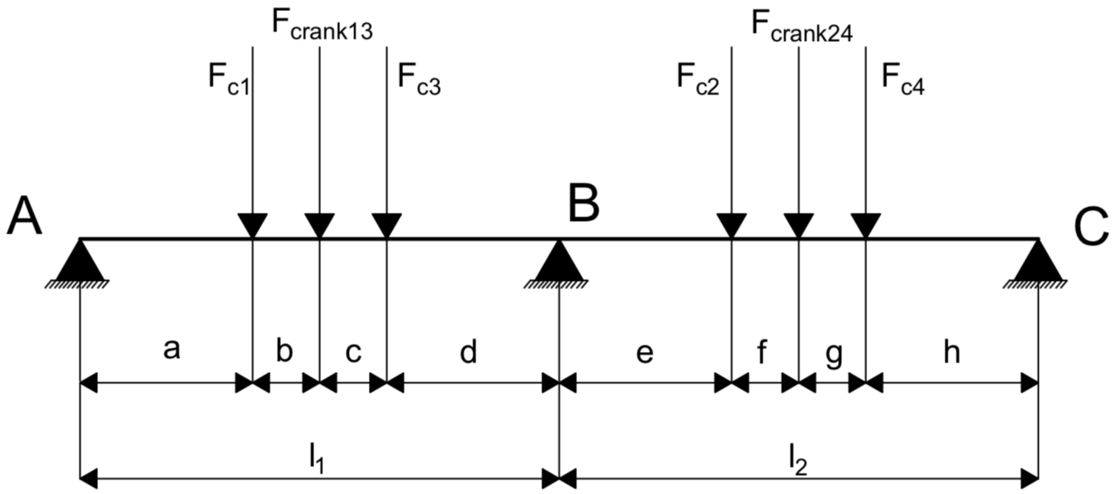

Figure 5.13: V4 crankshaft seen as a two-span beam

Considering that $M_A = 0$ and $M_C = 0$, the unknown moment M_B is equal to:

$$M_B = \frac{1}{2(l_1+l_2)}\left[-\frac{F_{c1}}{l_1}a(l_1^2-a^2)-\frac{F_{crank_{13}}}{l_1}(a+b)(l_1^2-(a+b)^2)-\frac{F_{c3}}{l_1}(a+b+c)(l_1^2-(a+b+c)^2)+\right.$$
$$\left.-\frac{F_{c2}}{l_2}(f+g+h)(l_2^2-(f+g+h)^2)-\frac{F_{crank_{24}}}{l_2}(g+h)(l_2^2-(g+h)^2)-\frac{F_{c4}}{l_2}h(l_1^2-h^2)\right]; \tag{5.43}$$

The shear forces can be expressed considering each single bay of the beam.

- System "AB" (Figure 5.14):

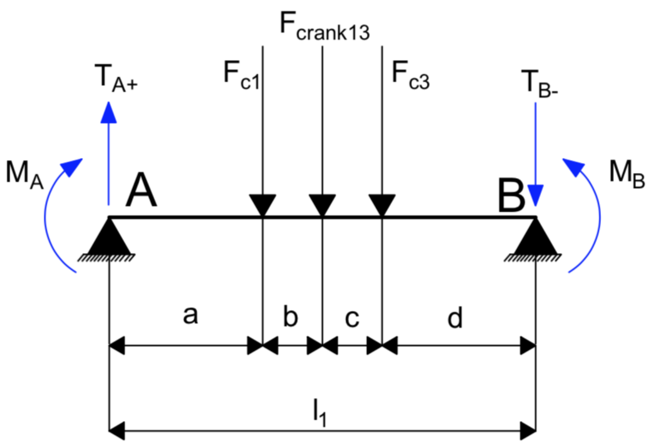

Figure 5.14: System AB

$$\begin{cases} B \circlearrowleft \;\; M_B - M_A + F_{c1}(b+c+d) + F_{c3}d + F_{crank_{13}}(c+d) - T_{A^+}l_1 = 0 \\ \qquad T_{A^+} = \frac{1}{l_1}(M_B - M_A + F_{c1}(b+c+d) + F_{c3}d + F_{crank_{13}}(c+d)); \\ \uparrow) \; T_{A^+} - F_{c1} - F_{c3} - F_{crank_{13}} - T_{B^-} = 0 \\ \qquad T_{B^-} = T_{A^+} - F_{c1} - F_{c3} - F_{crank_{13}}; \end{cases} \tag{5.44}$$

- System "BC" (Figure 5.15):

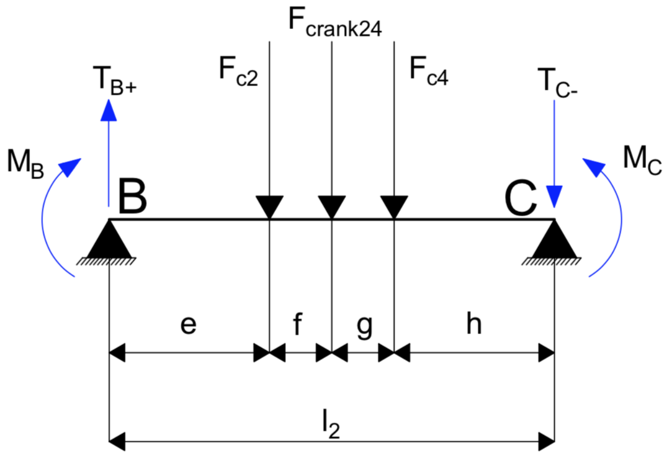

Figure 5.15: System BC

$$\begin{cases} C \circlearrowleft \;\; M_C - M_B + F_{c2}(f+g+h) + F_{c4}h + F_{crank_{24}}(g+h) - T_{B^+}l_2 = 0 \\ \qquad T_{B^+} = \frac{1}{l_2}(-M_B + F_{c2}(f+g+h) + F_{c4}h + F_{crank_{24}}(g+h)); \\ \uparrow) \; T_{B^+} - F_{c2} - F_{c4} - F_{crank_{24}} - T_{C^-} = 0 \\ \qquad T_{C^-} = T_{B^+} - F_{c2} - F_{c4} - F_{crank_{24}}; \end{cases} \tag{5.45}$$

The reactions can be obtained considering the convention shown in Figure 5.16.

$$\begin{cases} R_A = T_{A^+}; \\ R_B = T_{B^+} - T_{B^-}; \\ R_C = -T_{C^-}; \end{cases} \tag{5.46}$$

All the procedure shown above is valid for the case of a non-balanced crankshaft as no forces due do the counterweights are were considered. Anyway, the same method can be used for a balanced crankshaft just adding the forces due to the counterweights, as shown in the Figure 5.54.

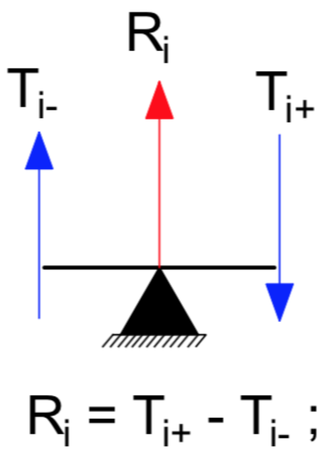

Figure 5.16: Shear forces convention

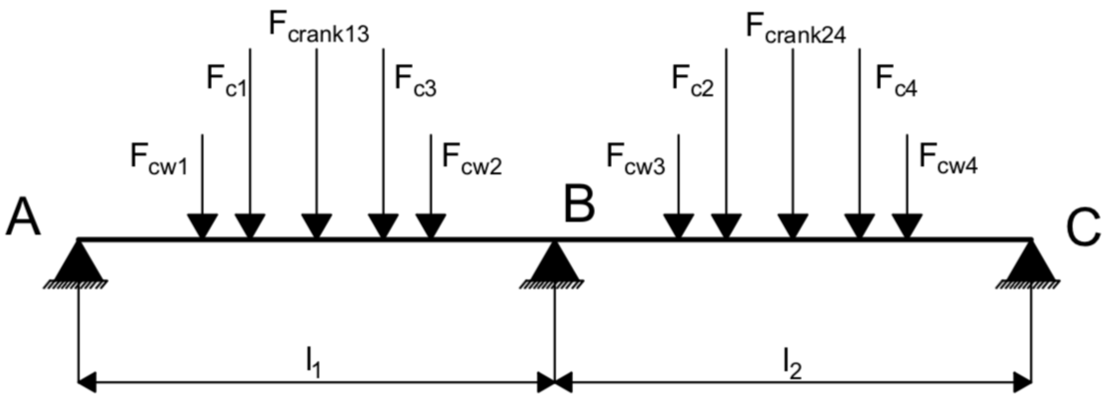

Figure 5.17: Forces in a V4 balanced crankshaft

5.3 V6 Engines

A *V6 Engine* is a six-cylinder engines with the cylinders arrangend in two banks V configuration, usually the V-angle can be equal to 60, 90 or 120 degrees. V6 Engines are used mainly in automobiles applications.

Similarly to as already said for the V4, a V6 engine can be seen as a pair of Three-cylinder inline engines separated as a V-angle and sharing the same crankshaft. In most part those engines the crankshaft is supported by four main bearings.

A V6 engine is more compact compared to an inline six-cylinders especially in terms of lengthiness, because the crankshaft length in smaller. Often those engines are shorter than an inline four-cylinders too. In terms of balancing and vibrations, this engine configuration is not as well as balanced as the inline six-cylinders, in fact counterweights on the crankshaft and counter-rotating balancer shafts are required to compensate the first order couples.

In this thesis work is considered that there are only three crank pins, and in each crank pin are connected two conrods, one for each bank. However, it is important to stress that with V-angles equal to 60 or 90 degrees, having two conrods of different banks attached on the same crank pin produces an uneven firing order. For this reason, in modern V6 engines the designers usually split crank pins in order to connect each conrod to a single crank pin. The crank pins are opportunely spaced to have an even firing order.

5.3.1 60° V6 Engine.

Considering the system show in the Figure 5.18:

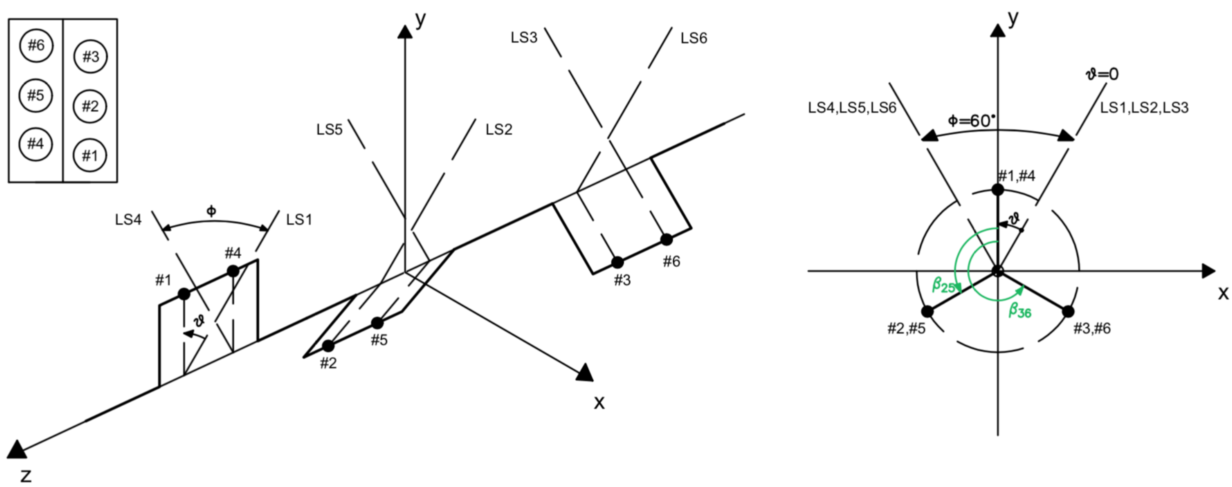

Figure 5.18: 90° V4 Engine with crank throws at 180°

On the right bank there are cylinders #1, #2 and#3, on the left #4, #5 and #6. β is the angle between the crank throws, in this case $\beta_{25} = 120°$ and $\beta_{25} = 240°$.

It is assumed that the position $\theta = 0°$ is coincident with the *line of stroke* of the cylinders #1, #2 and #3.

The V-angle ϕ is equal to 60°.

The forces acting on the system can be represented through the vectors stars.

First order vectors star

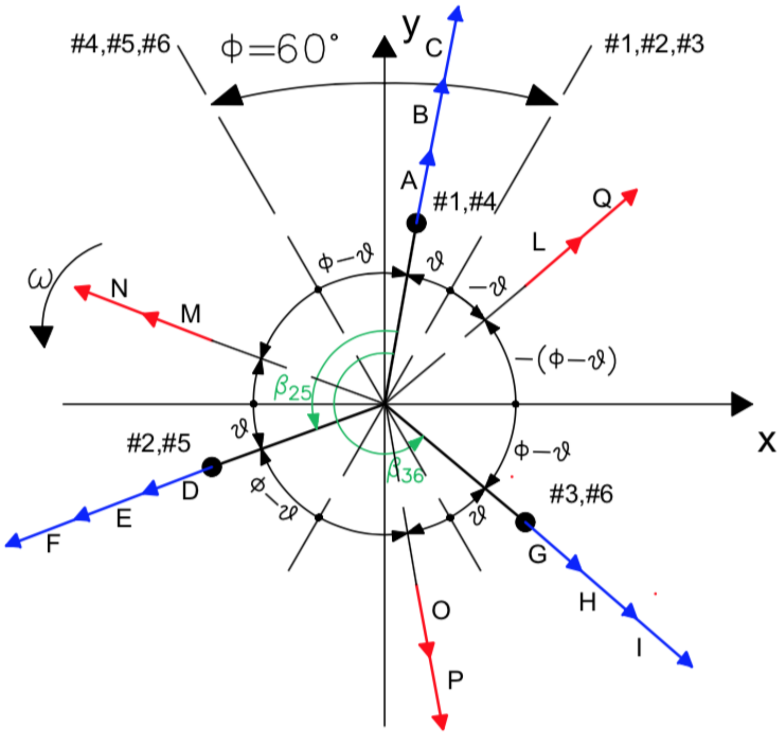

Figure 5.19: 60° V6 Engine first order vectors star

In blue are indicated the rotating forces, in red the counter-rotating.

The forces reported in the Figure 5.19 are equal to:

$$A = m_{ROT_{14}}\omega^2 r; \quad B = (\tfrac{1}{2}m_{ALT_1}\omega^2 r)_R; \quad C = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_R;$$

$$D = m_{ROT_{25}}\omega^2 r; \quad E = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_R; \quad F = (\tfrac{1}{2}m_{ALT_5}\omega^2 r)_R;$$

$$G = m_{ROT_{36}}\omega^2 r; \quad H = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_R; \quad I = (\tfrac{1}{2}m_{ALT_6}\omega^2 r)_R;$$

$$L = (\tfrac{1}{2}m_{ALT_1}\omega^2 r)_{CR}; \quad M = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_{CR}; \quad N = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_{CR};$$

$$O = (\tfrac{1}{2}m_{ALT_5}\omega^2 r)_{CR}; \quad P = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_{CR}; \quad Q = (\tfrac{1}{2}m_{ALT_6}\omega^2 r)_{CR};$$

with:

- $m_{ROT_{14}} = m_{crank_{14}} + m_{ROT_{rod_1}} + m_{ROT_{rod_4}}$;
- $m_{ROT_{25}} = m_{crank_{25}} + m_{ROT_{rod_2}} + m_{ROT_{rod_5}}$;
- $m_{ROT_{36}} = m_{crank_{36}} + m_{ROT_{rod_3}} + m_{ROT_{rod_6}}$;

As can be seen in the Figure 5.19:

- Pure centrifugal forces $(m_{ROT}\omega^2 r) \Rightarrow$ Balanced.
- Rotating part of first order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.
- Counter-rotating part of first order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.

Second order vectors star

The angle β_{25} is equal to 120° and β_{36} to 240°, therefore, in order to find the second order forces directions, it can be written:

- Cylinders #1 and #4 $\rightarrow \theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cylinders #2 and #5 $\rightarrow (\theta + 120°) \Rightarrow 2(\theta + 120°) \Rightarrow \cos(2\theta + 240°)$;
- Cylinders #3 and #6 $\rightarrow (\theta + 240°) \Rightarrow 2(\theta + 240°) \Rightarrow \cos(2\theta + 480°) = \cos(2\theta + 120°)$;

In blue are indicated the rotating forces, in red the counter-rotating.
The forces reported in the Figure 5.20 are equal to:

$$
\begin{aligned}
A &= (\tfrac{1}{2}m_{ALT_1}\omega^2 r\lambda)_R; \quad & B &= (\tfrac{1}{2}m_{ALT_4}\omega^2 r\lambda)_R; \quad & C &= (\tfrac{1}{2}m_{ALT_2}\omega^2 r\lambda)_R;\\
D &= (\tfrac{1}{2}m_{ALT_5}\omega^2 r\lambda)_R; \quad & E &= (\tfrac{1}{2}m_{ALT_3}\omega^2 r\lambda)_R; \quad & F &= (\tfrac{1}{2}m_{ALT_5}\omega^2 r\lambda)_R;\\
G &= (\tfrac{1}{2}m_{ALT_1}\omega^2 r\lambda)_{CR}; \quad & H &= (\tfrac{1}{2}m_{ALT_4}\omega^2 r\lambda)_{CR}; \quad & I &= (\tfrac{1}{2}m_{ALT_2}\omega^2 r\lambda)_{CR};\\
L &= (\tfrac{1}{2}m_{ALT_5}\omega^2 r\lambda)_{CR}; \quad & M &= (\tfrac{1}{2}m_{ALT_3}\omega^2 r\lambda)_{CR}; \quad & N &= (\tfrac{1}{2}m_{ALT_6}\omega^2 r\lambda)_{CR};
\end{aligned}
$$

As can be seen in the Figure 5.20:

- Rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.
- Counter-rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.

The first and second order moments are, in this configuration, not balanced.
The plane where the first order resultant moment is placed can be derived as shown below.

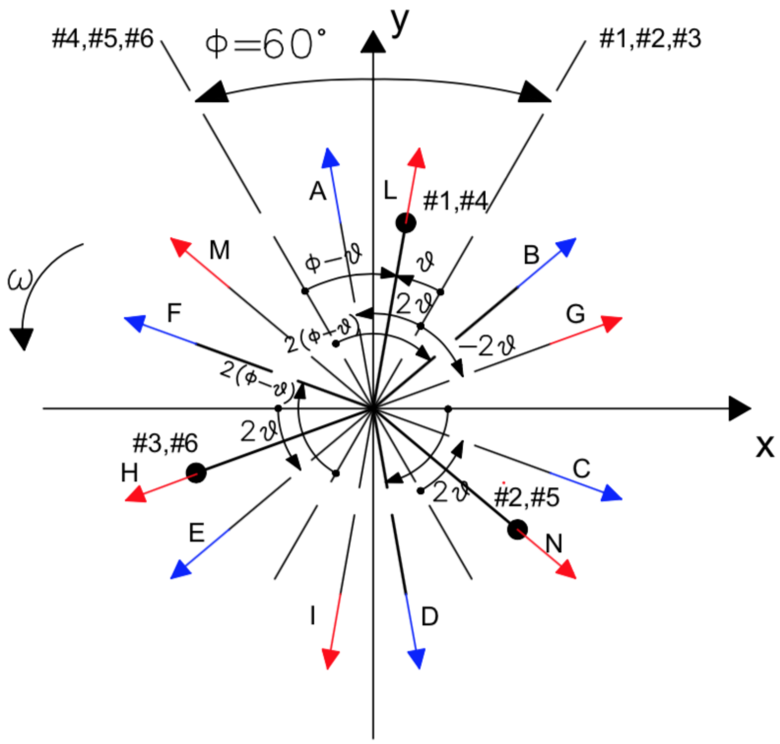

Figure 5.20: 60° V6 Engine second order vectors star

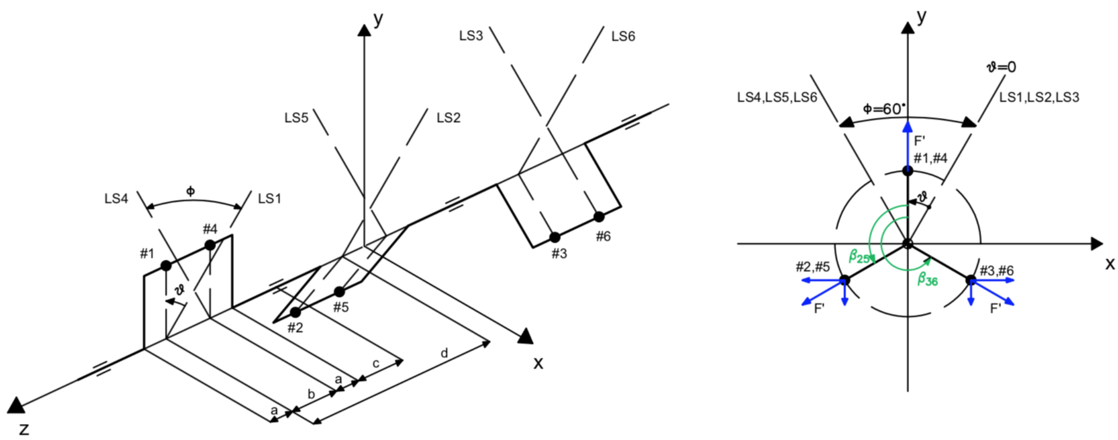

Figure 5.21: V6 engine layout

Considering the Figure 5.21, the moment around x-axis can be written as:

$$M'_x = F'_{\#1}\cos\left(-\frac{\phi}{2}+\theta\right)\left(d+\frac{b}{2}\right)+F'_{\#4}\cos\left(-\frac{\phi}{2}+\theta\right)(d-\frac{b}{2})-F'_{\#2}\sin\theta\left(\frac{b}{2}\right)+F'_{\#5}\sin\theta\left(\frac{b}{2}\right)+$$

$$+F'_{\#3}[-\sin(-\phi+\theta)]\left(d-\frac{b}{2}\right)+F'_{\#5}[-\sin(\phi+\theta)]\left(d+\frac{b}{2}\right); \quad (5.47)$$

considering that:

- The components due to cylinders #2 and #5 cancel each other.
- $F'_{\#1} = F'_{\#2} = F'_{\#3} = F'_{\#4} = F'$;
- $\frac{\phi}{2} = 30°$;

$$M'_x = F'[2d\cos(-30° + \theta) - 2d\sin(-60° + \theta)] = 2dF'[\cos(-30° + \theta) - \sin(-60° + \theta)]; \quad (5.48)$$

The moment around y-axis is equal to:

$$M'_y = F'_{\#1}\sin\left(-\frac{\phi}{2}+\theta\right)\left(d+\frac{b}{2}\right) + F'_{\#4}\sin\left(-\frac{\phi}{2}+\theta\right)\left(d-\frac{b}{2}\right) - F'_{\#2}\cos\theta\left(\frac{b}{2}\right) + F'_{\#5}\cos\theta\left(\frac{b}{2}\right) + + F'_{\#3}\cos(-\phi+\theta)\left(d-\frac{b}{2}\right) + F'_{\#6}\cos(-\phi+\theta)\left(d+\frac{b}{2}\right); \quad (5.49)$$

and then, making simplifications:

$$M'_y = 2dF'[\sin(-30° + \theta) - \cos(-60° + \theta)]; \quad (5.50)$$

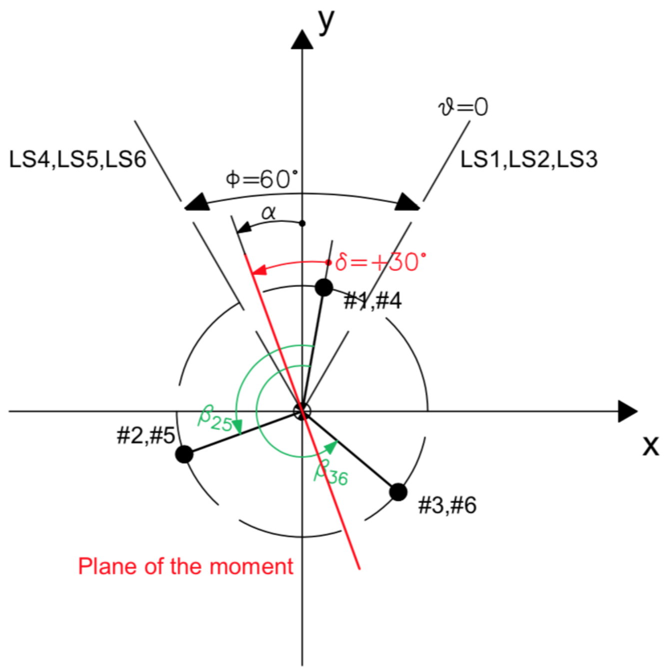

Figure 5.22: Angle between crank throw of cylinders #1 and #4 and resultant first order moment plane

Considering now the Figure 5.22 the angle δ between the crank throws and the plane where the resultant first order moment is placed can be obtained.

$$\tan\alpha = \frac{M'_y}{M'_x} = \frac{[\sin(-30° + \theta) - \cos(-60° + \theta)]}{[\cos(-30° + \theta) - \sin(-60° + \theta)]}; \tag{5.51}$$

therefore:

$$\alpha = \tan^{-1}\left(\frac{M'_y}{M'_x}\right) = \frac{\sin(-30° + \theta) - \cos(-60° + \theta)}{\cos(-30° + \theta) - \sin(-60° + \theta)}; \tag{5.52}$$

and finally:

$$\delta = \alpha + \frac{\phi}{2} - \theta; \tag{5.53}$$

Considering for example:

- $\theta = 0° \Rightarrow \alpha = \tan^{-1}\left[\frac{-\frac{1}{2}+\frac{1}{2}}{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}\right] = 0; \Rightarrow \delta = 30°;$
- $\theta = 75° \Rightarrow \alpha = \tan^{-1}\left[\frac{\sin(-30°+75°)-\cos(-60°+75°)}{\cos(-30°+75°)-\sin(-60°+75°)}\right] = \tan^{-1}(\sqrt{3}+2) = 75° \Rightarrow \delta = 30°;$

The plane of the resultant moment is therefore 30 degrees preceding the plane of the crank throw of the cylinders #1 and #4 for each θ angle considered.

As already done in the Paragraph 4.2.1 for the Inline three-cylinder, in order to have globally balanced system to first order moment it is possible to place two counterweights on a plane skewed of 30° degrees than the plane of the crank throw of the cylinders #1 and #4 that balance that moment. Considering the Figure 5.23:

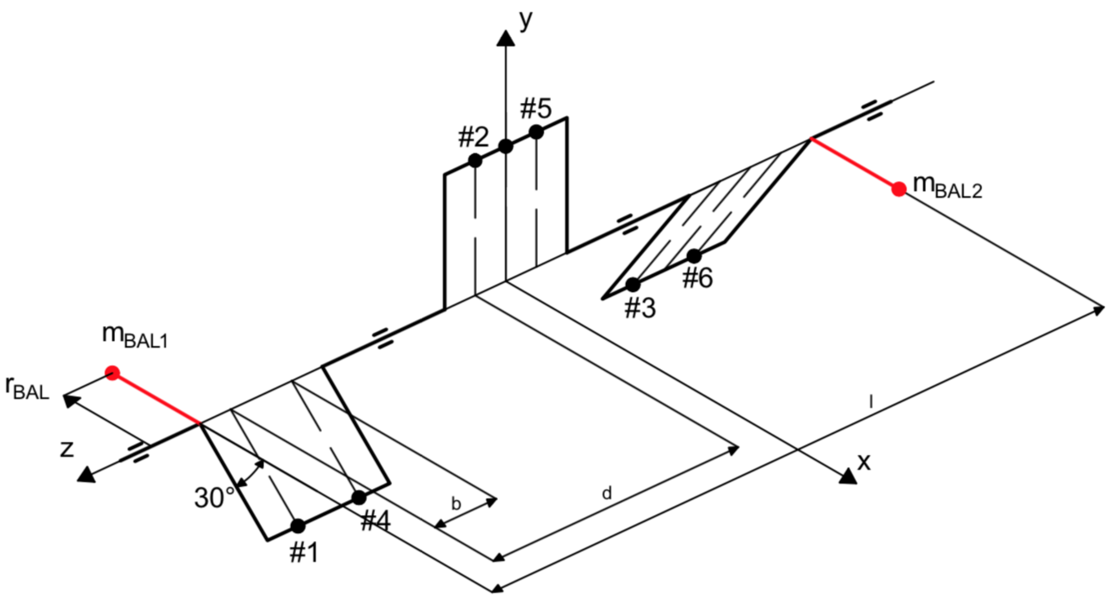

Figure 5.23: V6 engine global balanced

in the same way as the inline case, doing a moment balancing in the plane of the resultant balancing moment it can be written:

$$2F' \cos(30°)\left(d+\frac{b}{2}\right)+2F' \cos(30°)\left(d-\frac{b}{2}\right)=4dF' \cos(30°)=F_{BAL}l; \tag{5.54}$$

it means that:

$$F_{BAL}=\frac{4dF' \cos 30°}{l}; \tag{5.55}$$

with $F_{BAL}=m_{BAL}\omega^2 r_{BAL}$.

This solution, as already mentioned, gives only a global balanced system. In order to have a system balanced both globally and locally one of the possible solutions is the bay-by-bay balancing.

Bay-By-Bay Balancing

Considering the system shown in the Figure 5.24:

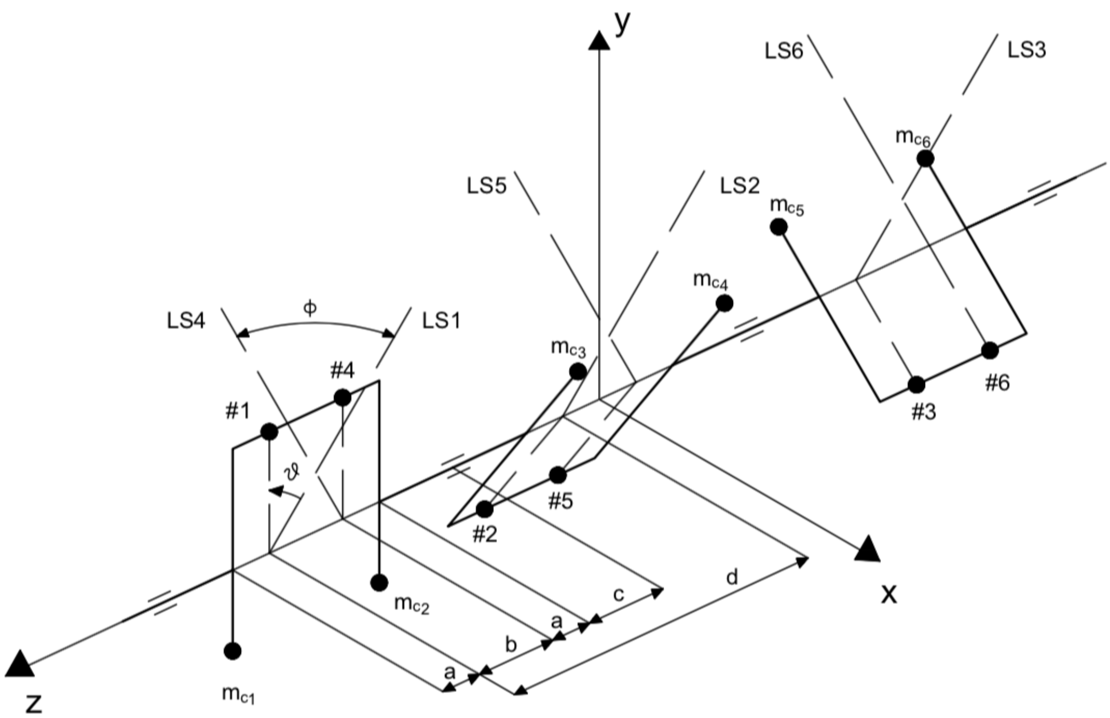

Figure 5.24: V6 engine with bay-by-bay balancing

$$m_1 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = d+\frac{b}{2} \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = \frac{b}{2} \end{pmatrix}; \qquad m_3 \Rightarrow \begin{pmatrix} x_3 \\ y_3 \\ z_3 = -d+\left(\frac{b}{2}\right) \end{pmatrix};$$

$$m_4 \Rightarrow \begin{pmatrix} x_4 \\ y_4 \\ z_4 = d - \frac{b}{2} \end{pmatrix}; \qquad m_5 \Rightarrow \begin{pmatrix} x_5 \\ y_5 \\ z_5 = -\frac{b}{2} \end{pmatrix}; \qquad m_6 \Rightarrow \begin{pmatrix} x_6 \\ y_6 \\ z_6 = -(d + \frac{b}{2}) \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = \left(d + \frac{b}{2} + a\right) \end{pmatrix}; \quad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = \left(d - \frac{b}{2} - a\right) \end{pmatrix}; \quad m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = \left(\frac{b}{2} + a\right) \end{pmatrix};$$

$$m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = -\left(\frac{b}{2} + a\right) \end{pmatrix}; \quad m_{c_5} \Rightarrow \begin{pmatrix} x_{c_5} \\ y_{c_5} \\ z_{c_5} = -\left(d - \frac{b}{2} - a\right) \end{pmatrix}; \quad m_{c_6} \Rightarrow \begin{pmatrix} x_{c_6} \\ y_{c_6} \\ z_{c_6} = -\left(d + \frac{b}{2} + a\right) \end{pmatrix};$$

The system of equilibrium equations can be written as:

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + m_{c_5}\,\omega^2\,x_{c_5} + m_{c_6}\,\omega^2\,x_{c_6} + \\ \qquad + m_1\,\omega^2\,x_1 + m_2\,\omega^2\,x_2 + m_3\,\omega^2\,x_3 + m_4\,\omega^2\,x_4 + m_5\,\omega^2\,x_5 + m_6\,\omega^2\,x_6 = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + m_{c_5}\,\omega^2\,y_{c_5} + m_{c_6}\,\omega^2\,y_{c_6} + \\ \qquad + m_1\,\omega^2\,y_1 + m_2\,\omega^2\,y_2 + m_3\,\omega^2\,y_3 + m_4\,\omega^2\,y_4 + m_5\,\omega^2\,y_5 + m_6\,\omega^2\,y_6 = 0 \\ x \circlearrowleft \quad (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + \\ \qquad + (m_{c_5}\,\omega^2\,x_{c_5})z_{c_5} + (m_{c_6}\,\omega^2\,x_{c_6})z_{c_6} + \\ \qquad + (m_1\,\omega^2\,x_1)z_1 + (m_2\,\omega^2\,x_2)z_2 + (m_3\,\omega^2\,x_3)z_3 + (m_4\,\omega^2\,x_4)z_4 + \\ \qquad + (m_5\,\omega^2\,x_5)z_5 + (m_6\,\omega^2\,x_6)z_6 = 0 \\ y \circlearrowleft \quad (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} \\ \qquad + (m_{c_5}\,\omega^2\,y_{c_5})z_{c_5} + (m_{c_6}\,\omega^2\,y_{c_6})z_{c_6} + \\ \qquad + (m_1\,\omega^2\,y_1)z_1 + (m_2\,\omega^2\,y_2)z_2 + (m_3\,\omega^2\,y_3)z_3 + (m_4\,\omega^2\,y_4)z_4 + \\ \qquad + (m_5\,\omega^2\,y_5)z_5 + (m_6\,\omega^2\,y_6)z_6 = 0 \end{cases} \tag{5.56}$$

Considering and assuming that:

- $m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m$;
- $x_1 = x_4 = x_{14}$, $x_2 = x_5 = x_{25}$ and $x_3 = x_6 = x_{36}$;
- $y_1 = y_4 = y_{14}$, $y_2 = y_5 = y_{25}$ and $y_3 = y_6 = y_{36}$;

the system becomes:

$$
\begin{cases}
x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + \\
\qquad +2mx_{14} + 2mx_{25} + 2mx_{36} = 0 \\
y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + \\
\qquad +2my_{13} + 2my_{25} + 2my_{36} = 0 \\
x \circlearrowright \quad m_{c_1}x_{c_1}z_{c_1} + m_{c_2}x_{c_2}z_{c_2} + m_{c_3}x_{c_3}z_{c_3} + m_{c_4}x_{c_4}z_{c_4} + \\
\qquad +m_{c_5}x_{c_5}z_{c_5} + m_{c_6}x_{c_6}z_{c_6} + 2dmx_{14} - 2dmx_{36} = 0 \\
y \circlearrowright \quad m_{c_1}y_{c_1}z_{c_1} + m_{c_2}y_{c_2}z_{c_2} + m_{c_3}y_{c_3}z_{c_3} + m_{c_4}y_{c_4}z_{c_4} + \\
\qquad +m_{c_5}y_{c_5}z_{c_5} + m_{c_6}y_{c_6}z_{c_6} + 2dmy_{14} - 2dmy_{36} = 0
\end{cases}
\tag{5.57}
$$

Solving this system, the position and masses of the counterweights than balace the crankshaft can be derived.

5.3.2 Bearing loads

The way used to find the bearing loads is based on the *Three Moment Equation of Clapeyron*. The crankshaft is hyperstatically constrained and can be considered as a beam, in this case with three span.

The three moment equation method require to use ad additional equation for each excess constraint. In the equations shown below it is assumed that the Young's modulus E and the moment of inertia of the crankshaft section J are constant along the crankshaft, therefore they cancel each other in all the terms.

In this case the system can be divided in two subsystems, one the ABC and the other BCD. For each subsystem a three moment equation can be derived. In the Figure 5.25 is represented the whole system considered.

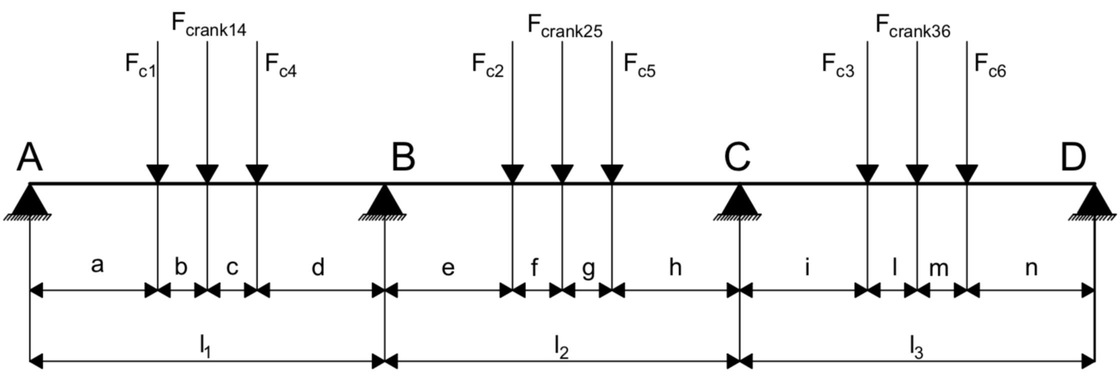

Figure 5.25: V6 crankshaft seen as a three-span beam

Considering now the system ABC shown in Figure :

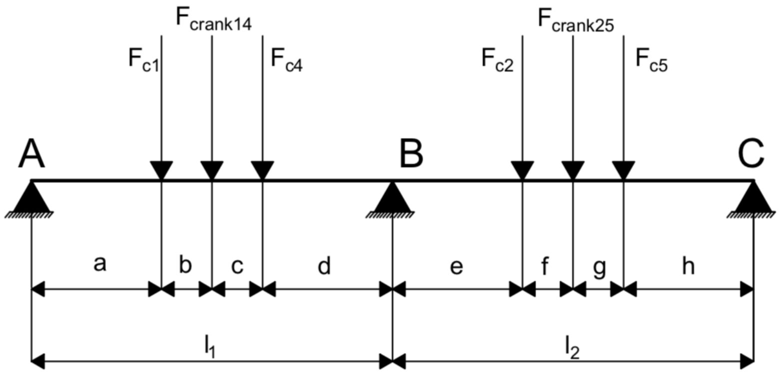

Figure 5.26: System ABC

the three moment equation is equal to:

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\frac{F_{c1}}{l_1} a(l_1^2 - a^2) - \frac{F_{crank_{14}}}{l_1}(a+b)(l_1^2-(a+b)^2) - \frac{F_{c4}}{l_1}(a+b+c)(l_1^2-(a+b+c)^2) + \\ - \frac{F_{c2}}{l_2}(f+g+h)(l_2^2-(f+g+h)^2) - \frac{F_{crank_{25}}}{l_2}(g+h)(l_2^2-(g+h)^2) - \frac{F_{c5}}{l_2} h(l_1^2 - h^2); \tag{5.58}$$

for the system *BCD* shown in the Figure 5.27 the three moment equation is equal to:

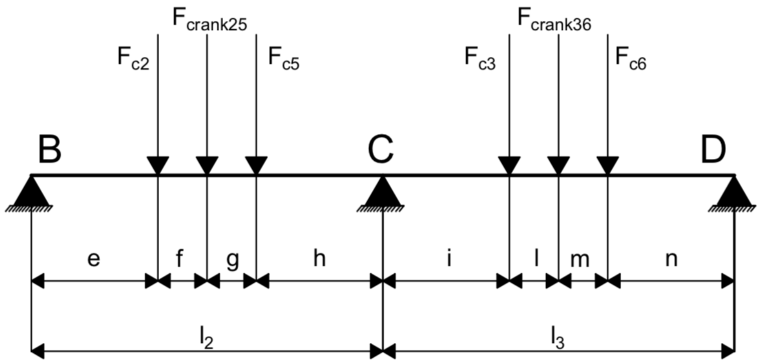

Figure 5.27: System BCD

$$M_B l_2 + 2M_C(l_2+l_3) + M_D l_3 = -\frac{F_{c2}}{l_2} e(l_2^2 - e^2) - \frac{F_{crank_{25}}}{l_2}(e+f)(l_2^2-(e+f)^2) - \frac{F_{c5}}{l_2}(e+f+g)(l_2^2-(e+f+g)^2) + \\ - \frac{F_{c3}}{l_3}(n+m+l)(l_3^2-(n+m+l)^2) - \frac{F_{crank_{36}}}{l_3}(n+m)(l_3^2-(n+m)^2) - \frac{F_{c6}}{l_3} n(l_1^2 - n^2); \tag{5.59}$$

The moments M_A and M_D are zero, and M_B and M_C can be calculated considering the system in matrix form like $Ax = B$ with:

$$A = \begin{bmatrix} 2(l_1 + l_2) & l_2 \\ l_2 & 2(l_2 + l_3) \end{bmatrix}$$

;

$$x = \begin{pmatrix} M_B \\ M_C \end{pmatrix}$$

and the vector B equal to the known term:

$$B = \begin{pmatrix} -\frac{F_{c1}}{l_1} a(l_1^2 - a^2) + \ldots - \frac{F_{c5}}{l_2} h(l_1^2 - h^2) \\ -\frac{F_{c2}}{l_2} e(l_1^2 - e^2) + \ldots - \frac{F_{c6}}{l_3} n(l_1^2 - n^2) \end{pmatrix}$$

and then:

$$x = \begin{pmatrix} M_B \\ M_C \end{pmatrix} = A^{-1} * B$$

Considering now the each single span, the shear forces and the reaction on the supports can be derived as shown below:

- System "AB" (Figure 5.28):

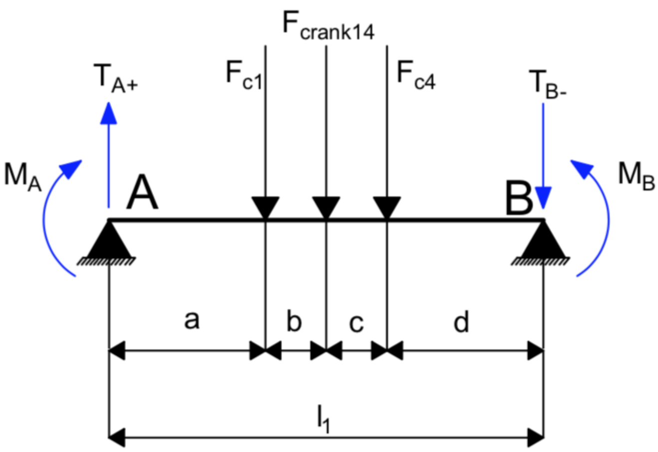

Figure 5.28: System AB

$$\begin{cases} B \circlearrowright \; M_B - M_A + F_{c1}(b + c + d) + F_{c4}d + F_{crank_{14}}(c + d) - T_{A^+}l_1 = 0 \\ \qquad T_{A^+} = \frac{1}{l_1}(M_B + F_{c1}(b + c + d) + F_{c4}d + F_{crank_{14}}(c + d)); \\ \uparrow) \; T_{A^+} - F_{c1} - F_{c4} - F_{crank_{14}} - T_{B^-} = 0 \\ \qquad T_{B^-} = T_{A^+} - F_{c1} - F_{c4} - F_{crank_{14}}; \end{cases} \tag{5.60}$$

- System "BC" (Figure 5.29):

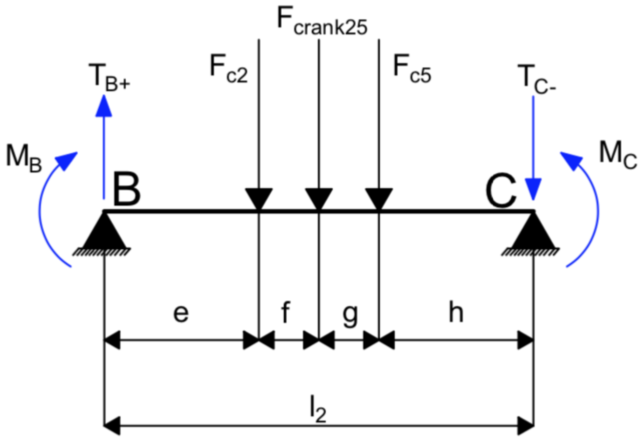

Figure 5.29: System BC

$$
\begin{cases}
C \circlearrowleft \; M_C - M_B + F_{c2}(f+g+h) + F_{c5}h + F_{crank_{25}}(g+h) - T_{B^+}l_2 = 0 \\
\qquad T_{B^+} = \frac{1}{l_2}(M_C - M_B + F_{c2}(f+g+h) + F_{c5}h + F_{crank_{25}}(g+h)); \\
\uparrow) \; T_{B^+} - F_{c2} - F_{c5} - F_{crank_{25}} - T_{C^-} = 0 \\
\qquad T_{C^-} = T_{B^+} - F_{c2} - F_{c5} - F_{crank_{25}};
\end{cases}
\tag{5.61}
$$

- System "CD" (Figure 5.30):

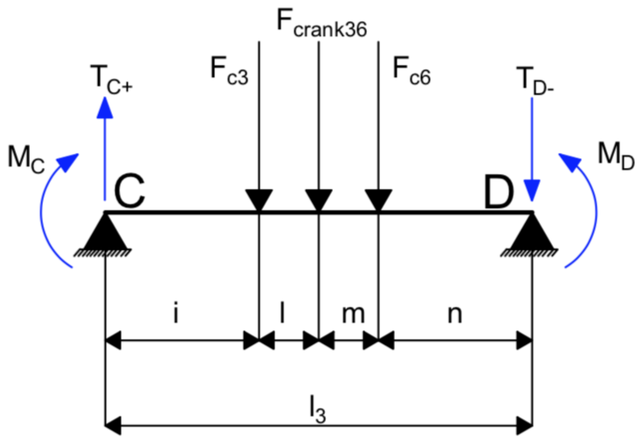

Figure 5.30: System CD

$$
\begin{cases}
E \circlearrowleft \; M_D - M_C + F_{c3}(l+m+n) + F_{c6}m + F_{crank_{36}}(m+n) - T_{C^+}l_3 = 0 \\
\qquad T_{C^+} = \frac{1}{l_3}(-M_C + F_{c3}(l+m+n) + F_{c6}m + F_{crank_{36}}(m+n)); \\
\uparrow) \; T_{C^+} - F_{c3} - F_{c6} - F_{crank_{36}} - T_{D^-} = 0 \\
\qquad T_{D^-} = T_{C^+} - F_{c3} - F_{c6} - F_{crank_{36}};
\end{cases}
\tag{5.62}
$$

The reactions can be obtained considering the convention shown in Figure 5.31.

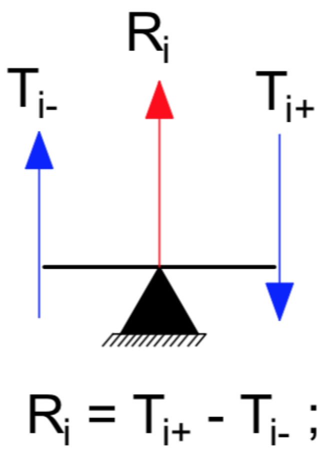

Figure 5.31: Shear forces convention

$$
\begin{cases}
R_A = T_{A^+}; \\
R_B = T_{B^+} - T_{B^-}; \\
R_C = T_{C^+} - T_{C^-}; \\
R_D = -T_{D^-};
\end{cases}
\tag{5.63}
$$

All the procedure shown above is valid for the case of a non-balanced crankshaft as no forces due do the counterweights are were considered. Anyway, the same method can be used for a balanced crankshaft just adding the forces due to the counterweights, as shown in the Figure 5.32.

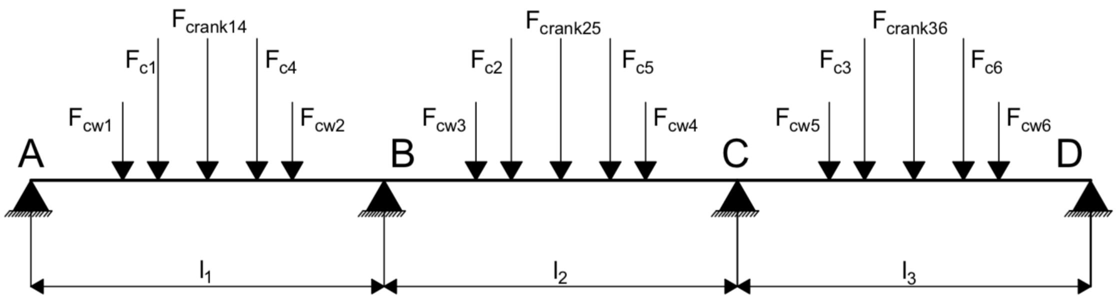

Figure 5.32: Model of forces in a V6 balanced crankshaft

5.4 V8 Engines

5.4.1 90° V8 Flat-Plane Engine

In this V8 configuration, shown in the Figure 5.33, the angle between crank throws is equal to 180°.

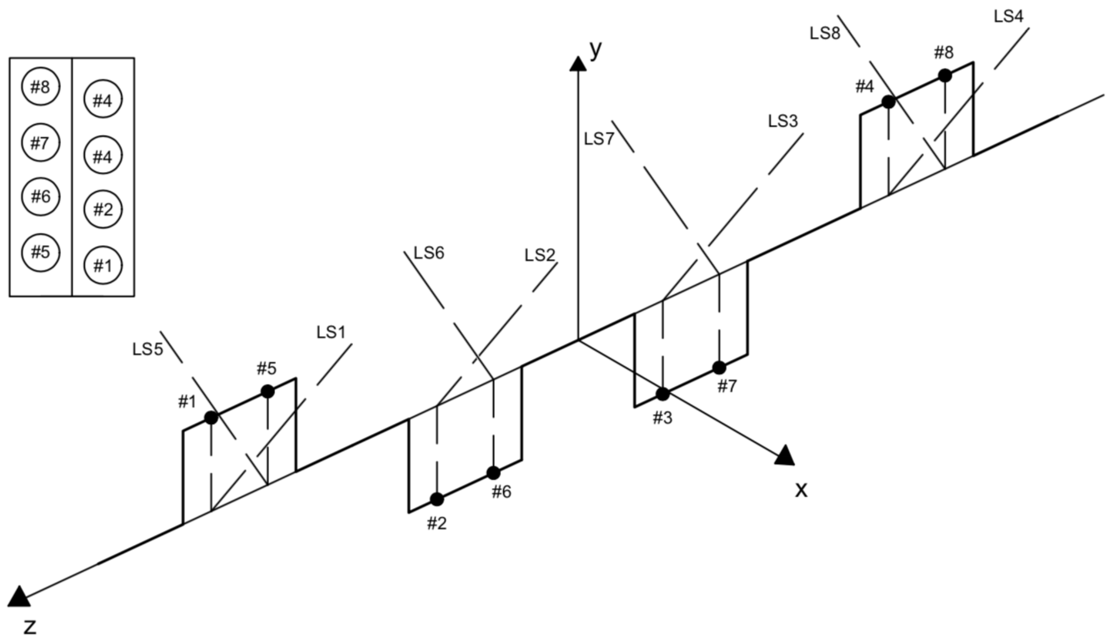

Figure 5.33: Flat-plane V8

On the right bank there are the cylinders #1, #2, #3 and #4, on the left #5, #6, #7, #8. β is the angle between crank throws, in this case $\beta_{26} = \beta_{37} = 180°$. It is assumed that the position $\theta = 0°$ is coincident with the *line of stroke* of the cylinders #1, #2, #3 and #4 (right bank). The V-angle in equal to 90°.

This engine layout carries more vibrations and it is much louder that a cross-plane configuration, therefore is no longer used in most mass production cars, but remains a useful configurations for racing cars, where the level of allowed vibrations is higher.

The forces acting on the system can be represented through the vectors stars.

First order vectors star

The rotating forces are shown in blue, in red the counter-rotating.

The forces reported in the Figure 5.35 are equal to:

$$A = m_{ROT_{15}}\omega^2 r; \qquad B = \left(\tfrac{1}{2}m_{ALT_1}\omega^2 r\right)_R; \qquad C = \left(\tfrac{1}{2}m_{ALT_5}\omega^2 r\right)_R;$$

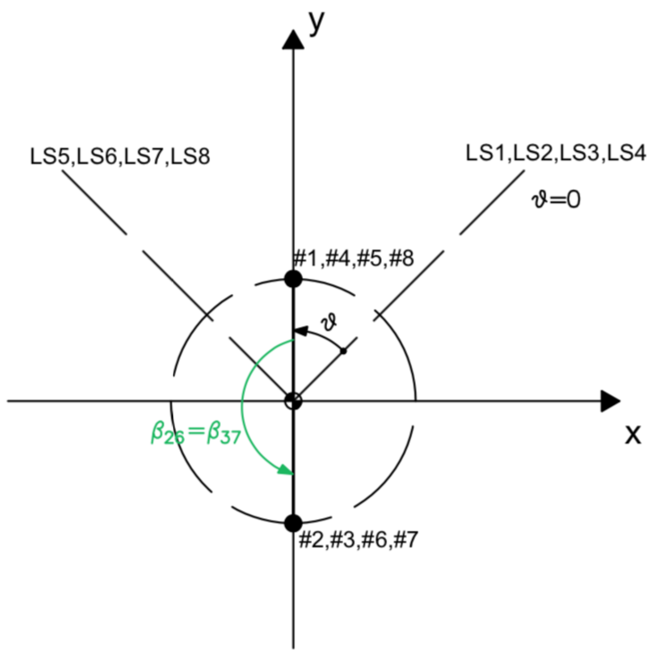

Figure 5.34: Flat-plane V8

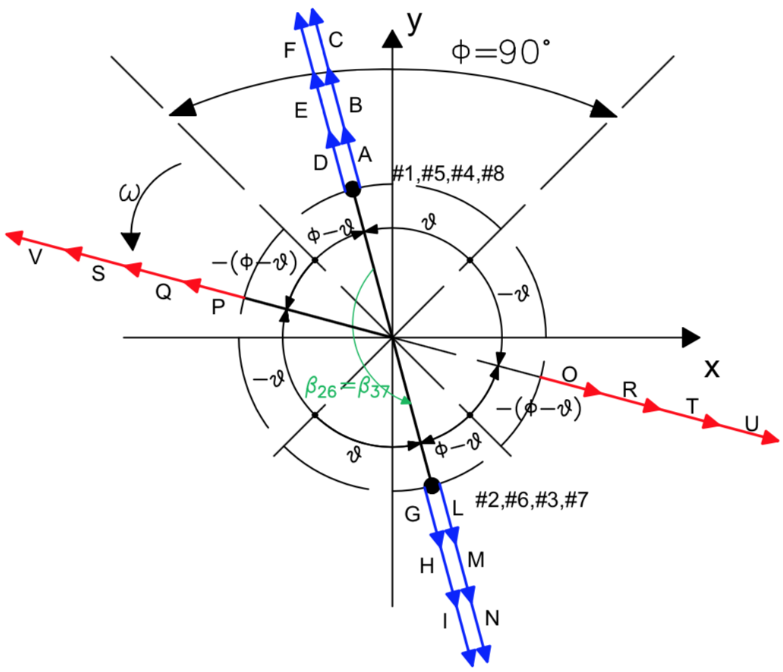

Figure 5.35: 90° V8 flat-plane engine first order vectors star

$$D = m_{ROT_{48}}\omega^2 r; \qquad E = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_R; \qquad F = (\tfrac{1}{2}m_{ALT_8}\omega^2 r)_R;$$

$$G = m_{ROT_{26}}\omega^2 r; \quad H = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_R; \quad I = (\tfrac{1}{2}m_{ALT_6}\omega^2 r)_R;$$
$$L = m_{ROT_{37}}\omega^2 r; \quad M = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_R; \quad N = (\tfrac{1}{2}m_{ALT_7}\omega^2 r)_R;$$
$$O = (\tfrac{1}{2}m_{ALT_1}\omega^2 r)_{CR}; \quad P = (\tfrac{1}{2}m_{ALT_5}\omega^2 r)_{CR}; \quad Q = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_{CR};$$
$$R = (\tfrac{1}{2}m_{ALT_6}\omega^2 r)_{CR}; \quad S = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_{CR}; \quad T = (\tfrac{1}{2}m_{ALT_7}\omega^2 r)_{CR};$$
$$U = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_{CR}; \quad V = (\tfrac{1}{2}m_{ALT_8}\omega^2 r)_{CR};$$

with:

- $m_{ROT_{15}} = m_{crank_{15}} + m_{ROT_{rod_1}} + m_{ROT_{rod_5}}$;
- $m_{ROT_{26}} = m_{crank_{26}} + m_{ROT_{rod_2}} + m_{ROT_{rod_6}}$;
- $m_{ROT_{37}} = m_{crank_{37}} + m_{ROT_{rod_3}} + m_{ROT_{rod_7}}$;
- $m_{ROT_{48}} = m_{crank_{48}} + m_{ROT_{rod_4}} + m_{ROT_{rod_8}}$;

As can be seen in the Figure 5.35:

- Pure centrifugal forces ($m_{ROT}\omega^2 r$) ⇒ Balanced.
- Rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.
- Counter-rotating part of first order alternating forces ($\frac{1}{2}m_{ALT}\omega^2 r$) ⇒ Balanced.

Second order vectors star

The angles β_{26} and β_{37} are equal to 180°,instead the angle β_{48} is equal to 0°. Therefore, in order to find the second order forces directions, it can be written:

- Cylinders #1 and #5 → $\theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cylinders #2 and #6 → $(\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;
- Cylinders #3 and #7 → $(\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;
- Cylinders #4 and #8 → $\theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;

In blue are indicated the rotating forces, in red the counter-rotating.
The forces reported in the Figure 5.36 are equal to:

$$A = (\tfrac{1}{2}m_{ALT_1}\omega^2 r\lambda)_R; \quad B = (\tfrac{1}{2}m_{ALT_5}\omega^2 r\lambda)_R; \quad C = (\tfrac{1}{2}m_{ALT_4}\omega^2 r\lambda)_R;$$
$$D = (\tfrac{1}{2}m_{ALT_8}\omega^2 r\lambda)_R; \quad E = (\tfrac{1}{2}m_{ALT_2}\omega^2 r\lambda)_R; \quad F = (\tfrac{1}{2}m_{ALT_6}\omega^2 r\lambda)_R;$$
$$G = (\tfrac{1}{2}m_{ALT_3}\omega^2 r\lambda)_R; \quad H = (\tfrac{1}{2}m_{ALT_7}\omega^2 r\lambda)_R;$$
$$I = (\tfrac{1}{2}m_{ALT_1}\omega^2 r\lambda)_{CR}; \quad L = (\tfrac{1}{2}m_{ALT_5}\omega^2 r\lambda)_{CR}; \quad M = (\tfrac{1}{2}m_{ALT_4}\omega^2 r\lambda)_{CR};$$

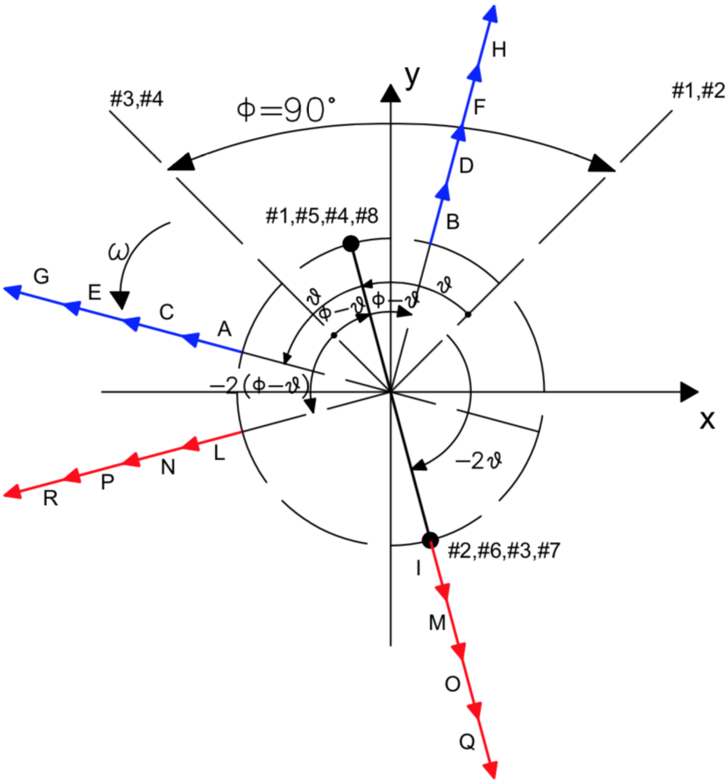

Figure 5.36: 90° V8 flat-plane engine second order vectors star

$$N = (\tfrac{1}{2} m_{ALT_8} \omega^2 r\lambda)_{CR}; \quad O = (\tfrac{1}{2} m_{ALT_2} \omega^2 r\lambda)_{CR}; \quad P = (\tfrac{1}{2} m_{ALT_6} \omega^2 r\lambda)_{CR};$$
$$Q = (\tfrac{1}{2} m_{ALT_3} \omega^2 r\lambda)_{CR}; \quad R = (\tfrac{1}{2} m_{ALT_7} \omega^2 r\lambda)_{CR};$$

As can be seen in the Figure 5.20:

- Rotating part of second order alternating forces $(\frac{1}{2} m_{ALT} \omega^2 r)$ ⇒ Not balanced.
- Counter-rotating part of second order alternating forces $(\frac{1}{2} m_{ALT} \omega^2 r)$ ⇒ Not balanced.

In this crankshaft configuration the first order moments are balanced, and the second order moments are balanced too. To be precise, the second order moments are balanced if the crankshaft is assumed symmetrical and the forces are considered applicated in a "medium point" of each crank throw. If these assumpitions are not satisfied there are small resultant moments.

Balancing Strategy

Since there are no unbalanced first order moments, in crankshafts with this layout the counterweights are placed with the aim to reduce the local stress in each bay. Considering the Figure 5.37 where is reported a classic bay-by-bay strategy:

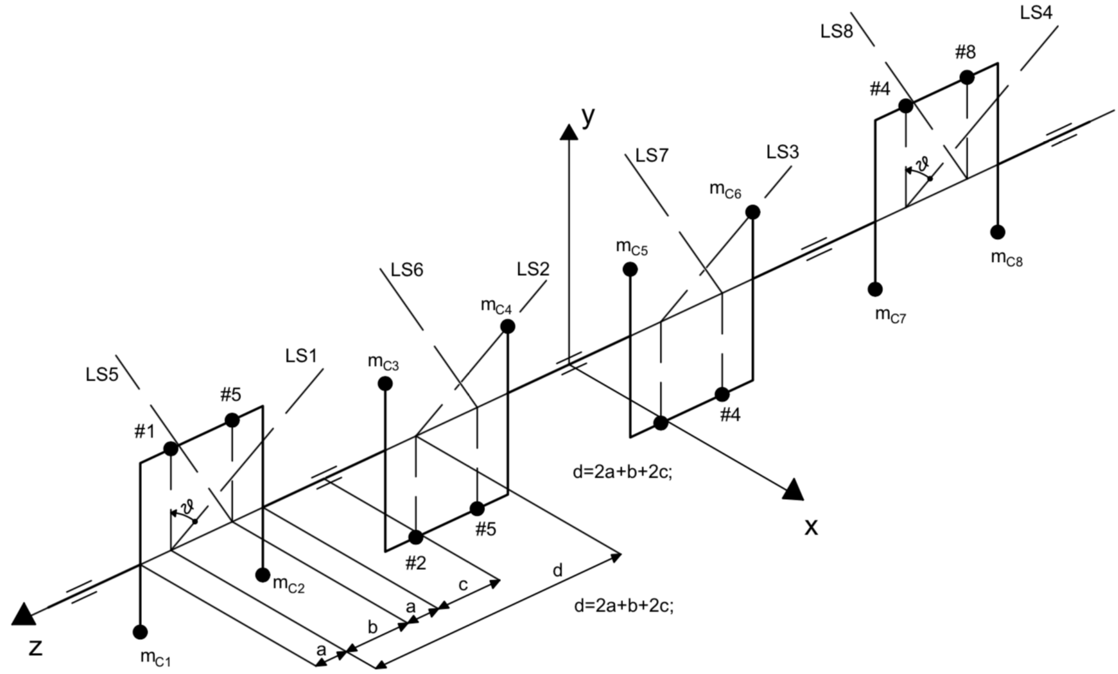

Figure 5.37: 90° V8 flat-plane crankshaft bay-bay-by balanced

it is assumed, for simplification purposes, that the bore spacing d is equal to $d = 2a{+}b{+}2b$, where a is the distance between the crank arm and the conrod, b is the distance between the two conrods equal to the offset between banks and c is the distance between the crank arm and the adjacent main journal.

$$m_1 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = (a+b+c+d) \end{pmatrix}; \qquad m_2 \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = (a+b+c) \end{pmatrix}; \qquad m_3 \Rightarrow \begin{pmatrix} x_3 \\ y_3 \\ z_3 = -(a+c) \end{pmatrix};$$

$$m_4 \Rightarrow \begin{pmatrix} x_4 \\ y_4 \\ z_4 = -(a+c+d) \end{pmatrix}; \qquad m_5 \Rightarrow \begin{pmatrix} x_5 \\ y_5 \\ z_5 = (a+c+d) \end{pmatrix}; \qquad m_6 \Rightarrow \begin{pmatrix} x_6 \\ y_6 \\ z_6 = (a+c) \end{pmatrix};$$

$$m_7 \Rightarrow \begin{pmatrix} x_7 \\ y_7 \\ z_7 = -(a+b+c) \end{pmatrix}; \qquad m_8 \Rightarrow \begin{pmatrix} x_8 \\ y_8 \\ z_8 = -(a+b+c+d) \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = (2a+b+c+d) \end{pmatrix} \quad m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = (2a+b+3c) \end{pmatrix} \quad m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = (2a+b+c) \end{pmatrix}$$

$$m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = c \end{pmatrix}; \quad m_{c_5} \Rightarrow \begin{pmatrix} x_{c_5} \\ y_{c_5} \\ z_{c_5} = -c; \end{pmatrix}; \quad m_{c_6} \Rightarrow \begin{pmatrix} x_{c_6} \\ y_{c_6} \\ z_{c_6} = -(2a+b+c) \end{pmatrix};$$

$$m_{c_7} \Rightarrow \begin{pmatrix} x_{c_7} \\ y_{c_7} \\ z_{c_7} = -(2a+b+3c); \end{pmatrix}; \quad m_{c_8} \Rightarrow \begin{pmatrix} x_{c_8} \\ y_{c_8} \\ z_{c_8} = -(2a+b+c+d); \end{pmatrix};$$

The system of equations can be written as:

$$\begin{cases} x) \quad m_{c_1}\,\omega^2\,x_{c_1} + m_{c_2}\,\omega^2\,x_{c_2} + m_{c_3}\,\omega^2\,x_{c_3} + m_{c_4}\,\omega^2\,x_{c_4} + m_{c_5}\,\omega^2\,x_{c_5} + m_{c_6}\,\omega^2\,x_{c_6} + \\ \quad +m_{c_7}\,\omega^2\,x_{c_7} + m_{c_8}\,\omega^2\,x_{c_8} + m_1\,\omega^2\,x_1 + m_2\,\omega^2\,x_2 + m_3\,\omega^2\,x_3 + m_4\,\omega^2\,x_4 + \\ \quad +m_5\,\omega^2\,x_5 + m_6\,\omega^2\,x_6 + m_7\,\omega^2\,x_7 + m_8\,\omega^2\,x_8 = 0 \\ y) \quad m_{c_1}\,\omega^2\,y_{c_1} + m_{c_2}\,\omega^2\,y_{c_2} + m_{c_3}\,\omega^2\,y_{c_3} + m_{c_4}\,\omega^2\,y_{c_4} + m_{c_5}\,\omega^2\,y_{c_5} + m_{c_6}\,\omega^2\,y_{c_6} + \\ \quad +m_{c_7}\,\omega^2\,y_{c_7} + m_{c_8}\,\omega^2\,y_{c_8} + m_1\,\omega^2\,y_1 + m_2\,\omega^2\,y_2 + m_3\,\omega^2\,y_3 + m_4\,\omega^2\,y_4 + \\ \quad +m_5\,\omega^2\,y_5 + m_6\,\omega^2\,y_6 + m_7\,\omega^2\,y_7 + m_8\,\omega^2\,y_8 = 0 \\ x \circlearrowleft \quad (m_{c_1}\,\omega^2\,x_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,x_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,x_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,x_{c_4})z_{c_4} + \\ \quad +(m_{c_5}\,\omega^2\,x_{c_5})z_{c_5} + (m_{c_6}\,\omega^2\,x_{c_6})z_{c_6} + (m_{c_7}\,\omega^2\,x_{c_7})z_{c_7} + (m_{c_8}\,\omega^2\,x_{c_8})z_{c_8} + \\ \quad +(m_1\,\omega^2\,x_1)z_1 + (m_2\,\omega^2\,x_2)z_2 + (m_3\,\omega^2\,x_3)z_3 + (m_4\,\omega^2\,x_4)z_4 + \\ \quad +(m_5\,\omega^2\,x_5)z_5 + (m_6\,\omega^2\,x_6)z_6 + (m_7\,\omega^2\,x_7)z_7 + (m_8\,\omega^2\,x_8)z_8 = 0 \\ y \circlearrowleft \quad (m_{c_1}\,\omega^2\,y_{c_1})z_{c_1} + (m_{c_2}\,\omega^2\,y_{c_2})z_{c_2} + (m_{c_3}\,\omega^2\,y_{c_3})z_{c_3} + (m_{c_4}\,\omega^2\,y_{c_4})z_{c_4} + (m_{c_5}\,\omega^2\,y_{c_5})z_{c_5} + \\ \quad +(m_{c_6}\,\omega^2\,y_{c_6})z_{c_6} + (m_{c_7}\,\omega^2\,y_{c_7})z_{c_7} + (m_{c_8}\,\omega^2\,y_{c_8})z_{c_8} + \\ \quad +(m_1\,\omega^2\,y_1)z_1 + (m_2\,\omega^2\,y_2)z_2 + (m_3\,\omega^2\,y_3)z_3 + (m_4\,\omega^2\,y_4)z_4 + \\ \quad +(m_5\,\omega^2\,y_5)z_5 + (m_6\,\omega^2\,y_6)z_6 + (m_7\,\omega^2\,y_7)z_7 + (m_8\,\omega^2\,y_8)z_8 = 0 \end{cases} \tag{5.64}$$

Considering and assuming that:

- $m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m$;
- $x_1 = x_5 = x_{15}$, $x_2 = x_6 = x_{26}$, $x_3 = x_7 = x_{37}$,and $x_4 = x_8 = x_{48}$
- $x_{15} = x_{48} = x_{1548}$ and $x_{26} = x_{37} = x_{2637}$
- $x_{1548} = -x_{2637}$
- $y_1 = y_5 = y_{15}$, $y_2 = y_6 = y_{26}$, $y_3 = y_7 = y_{37}$,and $y_4 = y_8 = y_{48}$
- $y_{15} = y_{48} = y_{1548}$ and $y_{26} = y_{37} = y_{2637}$
- $y_{1548} = -y_{2637}$

the system of equation becomes:

$$\begin{cases} x) & m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + m_{c_7}x_{c_7} + m_{c_8}x_{c_8} + \\ & \quad +4m(x_{1548}) + 4m(x_{2637}) = 0 \\ y) & m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + m_{c_7}y_{c_7} + m_{c_8}y_{c_8} + \\ & \quad +4m(y_{1548}) + 4m(y_{2637}) = 0 \\ x \circlearrowright & (2a+b+c+d)(m_{c_1}x_{c_1} - m_{c_8}x_{c_8}) + (2a+b+3c)(m_{c_2}x_{c_2} - m_{c_7}x_{c_7}) + \\ & \quad +(2a+b+c)(m_{c_3}x_{c_3} - m_{c_6}x_{c_6}) + c(m_{c_4}x_{c_4} - m_{c_5}x_{c_5}) + \\ & \quad +mx_{1548}(2a+b+2c+2d-2a-b-2c-2d) + mx_{2637}(2a+b+2c-2a-b-2c) = 0 \\ y \circlearrowright & (2a+d+c+d)(m_{c_1}y_{c_1} - m_{c_8}y_{c_8}) + (2a+b+3c)(m_{c_2}y_{c_2} - m_{c_7}y_{c_7}) + \\ & \quad +(2a+b+c)(m_{c_3}y_{c_3} - m_{c_6}y_{c_6}) + c(m_{c_4}y_{c_4} - m_{c_5}y_{c_5}) + \\ & \quad +my_{1548}(2a+b+2c+2d-2a-b-2c-2d) + my_{2637}(2a+b+2c-2a-b-2c) = 0 \end{cases} \tag{5.65}$$

and:

$$\begin{cases} x) & m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + m_{c_7}x_{c_7} + m_{c_8}x_{c_8} = 0 \\ y) & m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + m_{c_7}y_{c_7} + m_{c_8}y_{c_8} = 0 \\ x \circlearrowright & (2a+b+c+d)(m_{c_1}x_{c_1} - m_{c_8}x_{c_8}) + (2a+b+3c)(m_{c_2}x_{c_2} - m_{c_7}x_{c_7}) + \\ & \quad +(2a+b+c)(m_{c_3}x_{c_3} - m_{c_6}x_{c_6}) + c(m_{c_4}x_{c_4} - m_{c_5}x_{c_5}) = 0 \\ y \circlearrowright & (2a+b+c+d)(m_{c_1}y_{c_1} - m_{c_8}y_{c_8}) + (2a+b+3c)(m_{c_2}y_{c_2} - m_{c_7}y_{c_7}) + \\ & \quad +(2a+b+c)(m_{c_3}y_{c_3} - m_{c_6}y_{c_6}) + c(m_{c_4}y_{c_4} - m_{c_5}y_{c_5}) = 0 \end{cases} \tag{5.66}$$

Solving this system, a balanced configuration can be reached.

5.4.2 90° V8 Cross-Plane Engine

In this V8 configuration, shown in the Figure 5.38, the angle between crank throws is equal to 90°.

On the right bank there are the cylinders #1, #2, #3 and #4, on the left #5, #6, #7, #8. β is the angle between the crank throws, in this case $\beta_{26} = 270°, \beta_{37} = 90°$, and $\beta_{48} = 180°$ It is assumed that the position $\theta = 0°$ is coincident with the *line of stroke* of the cylinders #1, #2, #3 and #4 (right bank). The V-angle is equal to 90°.

The cross-plane crankshaft is probably the most popular crankshaft layout used in V8 road cars.

The most common V8 cross-plane crankshaft for a 90° V8 engine has four crankpins with two adjacent conrods for each crankpin. The crankpins are in two planes crossed ad 90°, hence the name cross-plane crankshaft. The number of main bearing is usually five, but in can be reach nine if the crankshaft has eight crank throws.

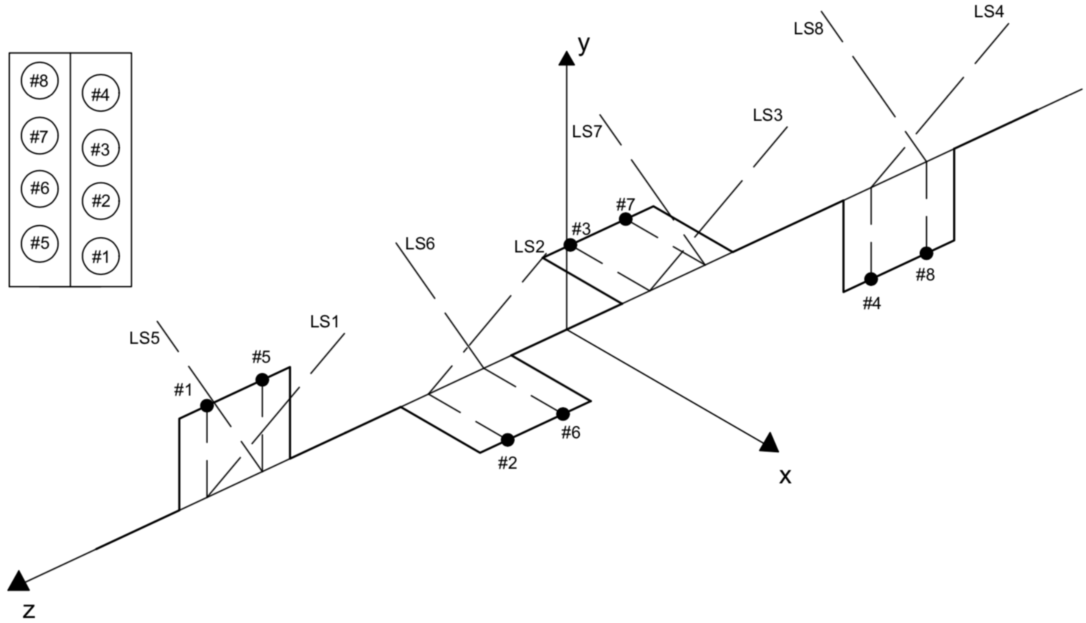

Figure 5.38: Cross-plane V8

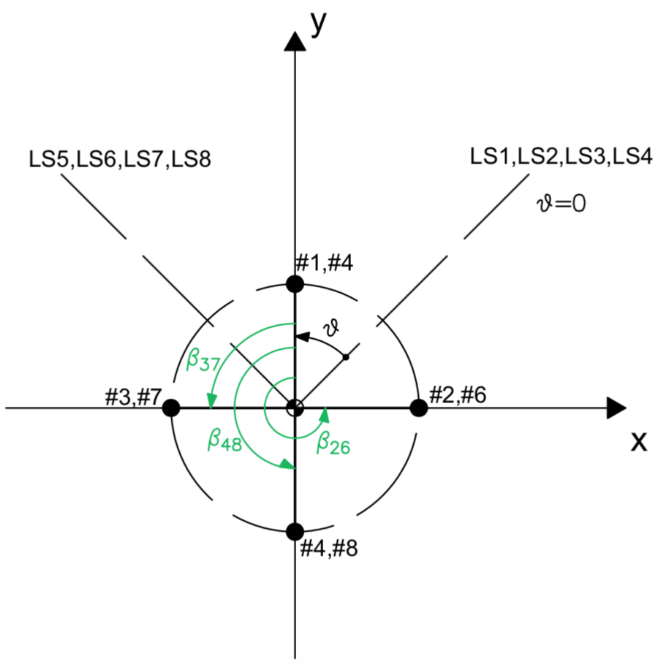

Figure 5.39: Cross-plane V8

One of the reasons the cross plane crank was developed, is that the non-balanced second order forces of a flat-plane V8 gives a conspicuous contribution, especially in large displacement engines. In a cross-plane crankshaft instead the particular geometry allows to have, in each bank of the engine, four distinct piston phases that are able to cancel the second order contribution entirely.

However, compared to a flat-plane crankshaft, in this case the crank throws layout provide a resultant first order couple that can be anyway easily balanced by adding counterweights opportunely on the crankshaft. Those counterweigths can be, depending on the circumstances, very heavy; it means that, in general, a cross-plane crankshaft can reach lower rotating speed compared to a flat-plane configuration.

The forces acting on the system can be represented through the vectors stars.

First order vectors star

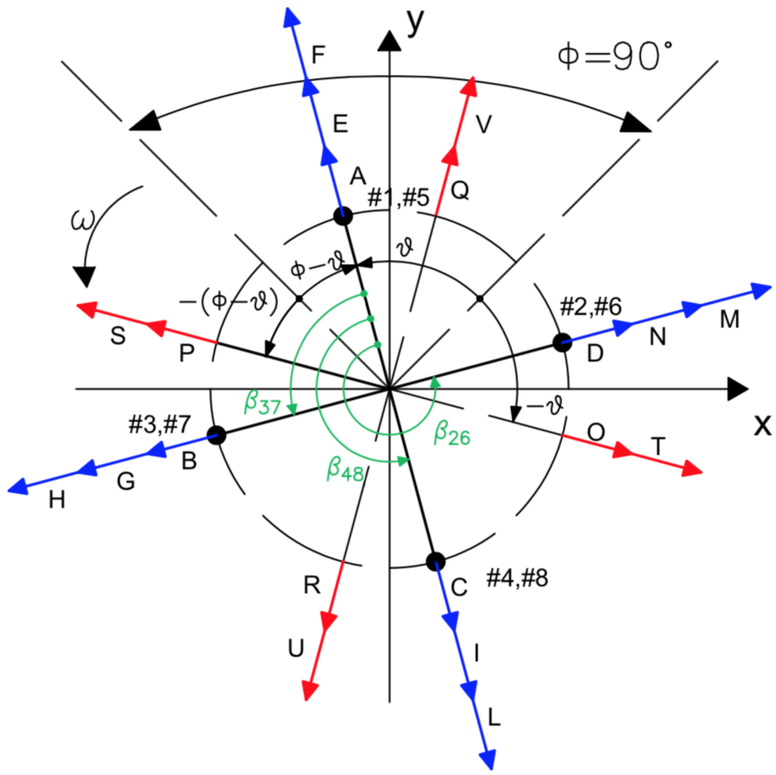

Figure 5.40: 90° V8 cross-plane engine first order vectors star

In blue are indicated the rotating forces, in red the counter-rotating.

The forces reported in the Figure 5.40 are equal to:

$$A = m_{ROT_{15}}\omega^2 r; \qquad B = m_{ROT_{37}}\omega^2 r; \qquad C = m_{ROT_{48}}\omega^2 r;;$$

$$D = m_{ROT_{26}}\omega^2 r; \qquad E = (\tfrac{1}{2}m_{ALT_1}\omega^2 r)_R; \qquad F = (\tfrac{1}{2}m_{ALT_5}\omega^2 r)_R;$$

$$G = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_R; \qquad H = (\tfrac{1}{2}m_{ALT_7}\omega^2 r)_R; \qquad I = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_R;$$

$$L = (\tfrac{1}{2}m_{ALT_8}\omega^2 r)_R; \quad M = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_R; \quad N = (\tfrac{1}{2}m_{ALT_6}\omega^2 r)_R;$$
$$O = (\tfrac{1}{2}m_{ALT_1}\omega^2 r)_{CR}; \quad P = (\tfrac{1}{2}m_{ALT_5}\omega^2 r)_{CR}; \quad Q = (\tfrac{1}{2}m_{ALT_3}\omega^2 r)_{CR};$$
$$R = (\tfrac{1}{2}m_{ALT_7}\omega^2 r)_{CR}; \quad S = (\tfrac{1}{2}m_{ALT_4}\omega^2 r)_{CR}; \quad T = (\tfrac{1}{2}m_{ALT_8}\omega^2 r)_{CR};$$
$$U = (\tfrac{1}{2}m_{ALT_2}\omega^2 r)_{CR}; \quad V = (\tfrac{1}{2}m_{ALT_6}\omega^2 r)_{CR};$$

with:

- $m_{ROT_{15}} = m_{crank_{15}} + m_{ROT_{rod_1}} + m_{ROT_{rod_5}}$;
- $m_{ROT_{26}} = m_{crank_{26}} + m_{ROT_{rod_2}} + m_{ROT_{rod_6}}$;
- $m_{ROT_{37}} = m_{crank_{37}} + m_{ROT_{rod_3}} + m_{ROT_{rod_7}}$;
- $m_{ROT_{48}} = m_{crank_{48}} + m_{ROT_{rod_4}} + m_{ROT_{rod_8}}$;

As can be seen in the Figure 5.40:

- Pure centrifugal forces $(m_{ROT}\omega^2 r)$ ⇒ Balanced.
- Rotating part of first order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r)$ ⇒ Balanced.
- Counter-rotating part of first order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r)$ ⇒ Balanced.

Second order vectors star

The angle β_{26} is equal to 270°, $\beta_{37} = 90°$ and β_{48} is equal to 180°. Therefore, in order to find the second order forces directions, it can be written:

- Cylinders #1 and #5 → $\theta \Rightarrow 2\theta \Rightarrow \cos(2\theta)$;
- Cylinders #2 and #6 → $(\theta + 270°) \Rightarrow 2(\theta + 540°) \Rightarrow \cos(2\theta + 540°) = \cos(2\theta + 180°)$
- Cylinders #3 and #7 → $(\theta + 90°) \Rightarrow 2(\theta + 90°) \Rightarrow \cos(2\theta + 180°)$;
- Cylinders #4 and #8 → $(\theta + 180°) \Rightarrow 2(\theta + 180°) \Rightarrow \cos(2\theta + 360°) = \cos(2\theta)$;

In blue are indicated the rotating forces, in red the counter-rotating.
The forces reported in the Figure 5.41 are equal to:

$$A = (\tfrac{1}{2}m_{ALT_1}\omega^2 r\lambda)_R; \quad B = (\tfrac{1}{2}m_{ALT_5}\omega^2 r\lambda)_R; \quad C = (\tfrac{1}{2}m_{ALT_4}\omega^2 r\lambda)_R;$$
$$D = (\tfrac{1}{2}m_{ALT_8}\omega^2 r\lambda)_R; \quad E = (\tfrac{1}{2}m_{ALT_3}\omega^2 r\lambda)_R; \quad F = (\tfrac{1}{2}m_{ALT_7}\omega^2 r\lambda)_R;$$
$$G = (\tfrac{1}{2}m_{ALT_2}\omega^2 r\lambda)_R; \quad H = (\tfrac{1}{2}m_{ALT_6}\omega^2 r\lambda)_R;$$
$$I = (\tfrac{1}{2}m_{ALT_1}\omega^2 r\lambda)_{CR}; \quad L = (\tfrac{1}{2}m_{ALT_5}\omega^2 r\lambda)_{CR}; \quad M = (\tfrac{1}{2}m_{ALT_4}\omega^2 r\lambda)_{CR};$$
$$N = (\tfrac{1}{2}m_{ALT_8}\omega^2 r\lambda)_{CR}; \quad O = (\tfrac{1}{2}m_{ALT_3}\omega^2 r\lambda)_{CR}; \quad P = (\tfrac{1}{2}m_{ALT_7}\omega^2 r\lambda)_{CR};$$

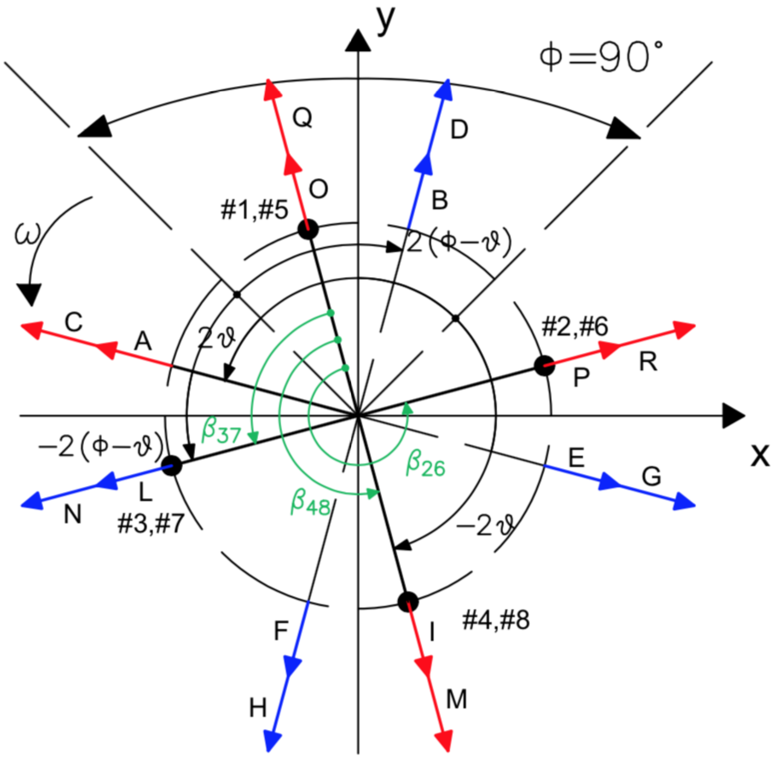

Figure 5.41: 90° V8 cross-plane engine second order vectors star

$$Q = (\tfrac{1}{2}m_{ALT_2}\omega^2 r\lambda)_{CR}; \qquad R = (\tfrac{1}{2}m_{ALT_6}\omega^2 r\lambda)_{CR};$$

As can be seen in the Figure 5.20:

- Rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.
- Counter-rotating part of second order alternating forces $(\frac{1}{2}m_{ALT}\omega^2 r) \Rightarrow$ Balanced.

In general, in this crankshaft configuration (V8 cross-plane) the first order moments are not balanced and the second order moments are balanced instead.

But in this particular case, with the *V angle* equal to 90°, the first order counter-rotating moment is balanced because, as already mentioned in the Paragraph 5.1, the angle between the first order counter-rotating forces is equal to $2\phi = 2 \cdot 90° = 180°$ therefore there are four pair of forces skewed at 180° between themselves and the resultant moment is balanced.

Balancing Strategy

It is possible to find the rotating first order moment resultant and the plane where this moment is placed. Considering the Figure 5.42:

the moments around the directions x and y can be expressed as:

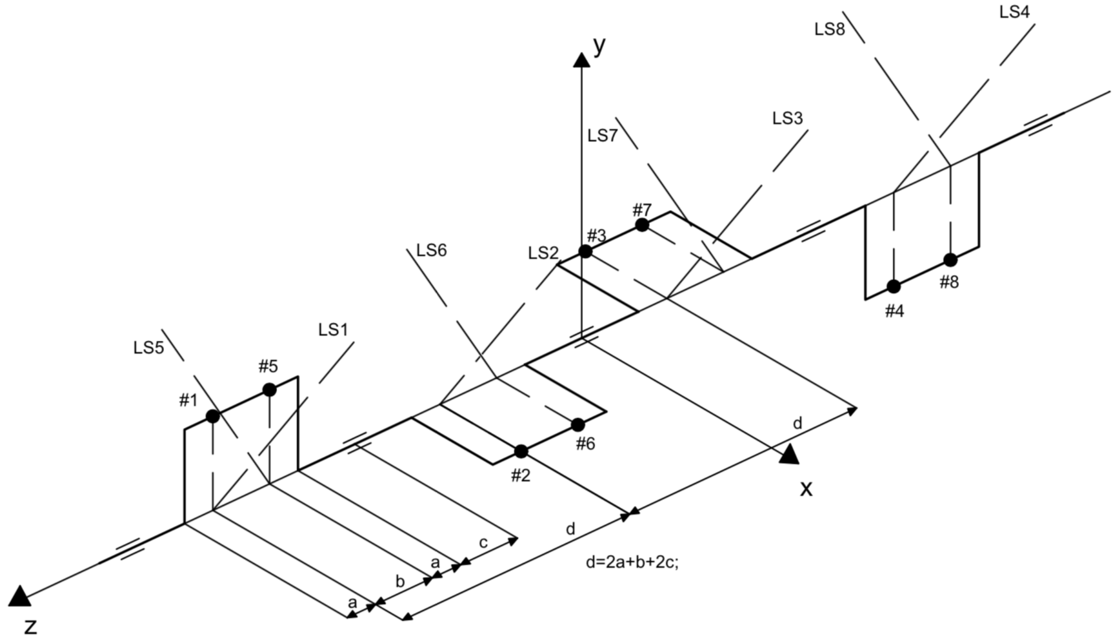

Figure 5.42: 90° V8 cross-plane crankshaft

$$
\begin{aligned}
M'_x = {} & F'_{\#1}\cos\left(-\frac{\phi}{2}+\theta\right)(3a+2b+3c) + F'_{\#5}\cos\left(-\frac{\phi}{2}+\theta\right)(3a+b+3c)+ \\
& + F'_{\#2}\cos\left(-\frac{\phi}{2}+\theta+\beta_2\right)(a+b+c) - F'_{\#6}\cos\left(-\frac{\phi}{2}+\theta+\beta_2\right)(a+c)+ \\
& - F'_{\#3}\cos\left(-\frac{\phi}{2}+\theta+\beta_3\right)(a+c) - F'_{\#7}\cos\left(-\frac{\phi}{2}+\theta+\beta_3\right)(a+b+c)+ \\
& - F'_{\#4}\cos\left(-\frac{\phi}{2}+\theta+\beta_4\right)(3a+b+3c) - F'_{\#8}\cos\left(-\frac{\phi}{2}+\theta+\beta_4\right)(3a+2b+3c); \quad (5.67)
\end{aligned}
$$

$$
\begin{aligned}
M'_y = {} & F'_{\#1}\sin\left(-\frac{\phi}{2}+\theta\right)(3a+2b+3c) + F'_{\#5}\sin\left(-\frac{\phi}{2}+\theta\right)(3a+b+3c)+ \\
& + F'_{\#2}\sin\left(-\frac{\phi}{2}+\theta+\beta_2\right)(a+b+c) - F'_{\#6}\sin\left(-\frac{\phi}{2}+\theta+\beta_2\right)(a+c)+ \\
& - F'_{\#3}\sin\left(-\frac{\phi}{2}+\theta+\beta_3\right)(a+c) - F'_{\#7}\sin\left(-\frac{\phi}{2}+\theta+\beta_3\right)(a+b+c)+ \\
& - F'_{\#4}\sin\left(-\frac{\phi}{2}+\theta+\beta_4\right)(3a+b+3c) - F'_{\#8}\sin\left(-\frac{\phi}{2}+\theta+\beta_4\right)(3a+2b+3c); \quad (5.68)
\end{aligned}
$$

making the assumptions that:

- $F'_{\#1} = F'_{\#2} = F'_{\#3} = F'_{\#4} = F'_{\#5} = F'_{\#6} = F'_{\#7} = F'_{\#8} = F'$
- $\phi = 90°$

the expressions become:

$$\begin{aligned} M'_x = {} & F'\cos(-45+\theta)(6a+3b+6c) + F'\cos(-45+\theta+\beta_2)(2a+b+2c)+ \\ & - F'\cos(-45+\theta+\beta_3)(2a+b+2c) - F'\cos(-45+\theta+\beta_4)(6a+3b+6c) = \\ & = F'(6a+3b+6c)[\cos(-45+\theta) - \cos(-45+\theta+\beta_4)]+ \\ & + F'(2a+b+2c)[\cos(-45+\theta+\beta_2) - \cos(-45+\theta+\beta_3)]; \end{aligned} \tag{5.69}$$

$$\begin{aligned} M'_y = {} & F'\sin(-45+\theta)(6a+3b+6c) + F'\sin(-45+\theta+\beta_2)(2a+b+2c)+ \\ & - F'\sin(-45+\theta+\beta_3)(2a+b+2c) - F'\sin(-45+\theta+\beta_4)(6a+3b+6c) = \\ & = F'(6a+3b+6c)[\sin(-45+\theta) - \sin(-45+\theta+\beta_4)]+ \\ & + F'(2a+b+2c)[\sin(-45+\theta+\beta_2) - \sin(-45+\theta+\beta_3)]; \end{aligned} \tag{5.70}$$

Therefore, the angle of the resultant moment plane can be derived as:

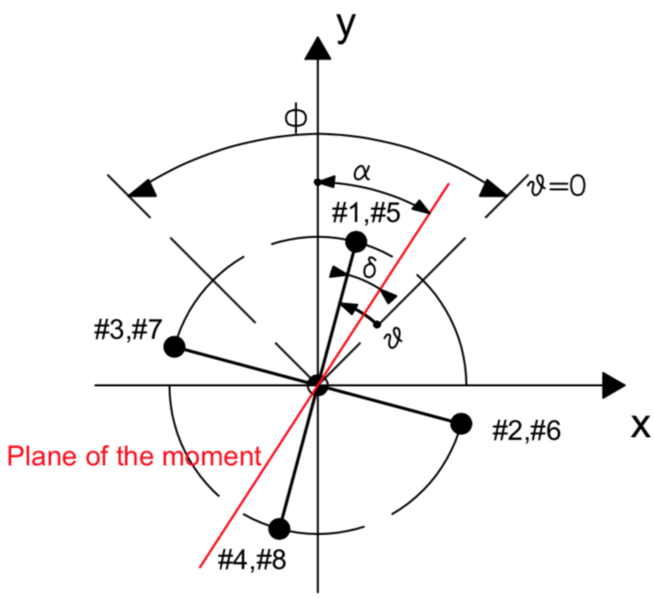

Figure 5.43: Angle between crank throw of cylinders #1 and #5 and resultant first order moment plane

Considering now the Figure 5.43 the angle δ between the crank throws and the plane where the resultant first order moment is placed can be obtained.

$$\tan\alpha = \frac{M'_y}{M'_x} \Rightarrow \alpha = \tan^{-1}\left(\frac{M'_y}{M'_x}\right); \tag{5.71}$$

and finally:

$$\delta = \alpha + \frac{\phi}{2} - \theta = -18.43°; \tag{5.72}$$

It can be noticed that the angle in exactly the same that in the I4 cross-plane.

In order to have the first order rotating moment, it is possible to place two counterweights on the external crankshaft webs and skewed by the angle just derived. However, in this way only a globally balance can be reached.

If the goal is to reduce forces and moments on each crankshaft bay, the *bay-by-bay balancing strategy* can be used with better results. Considering the Figure 5.44:

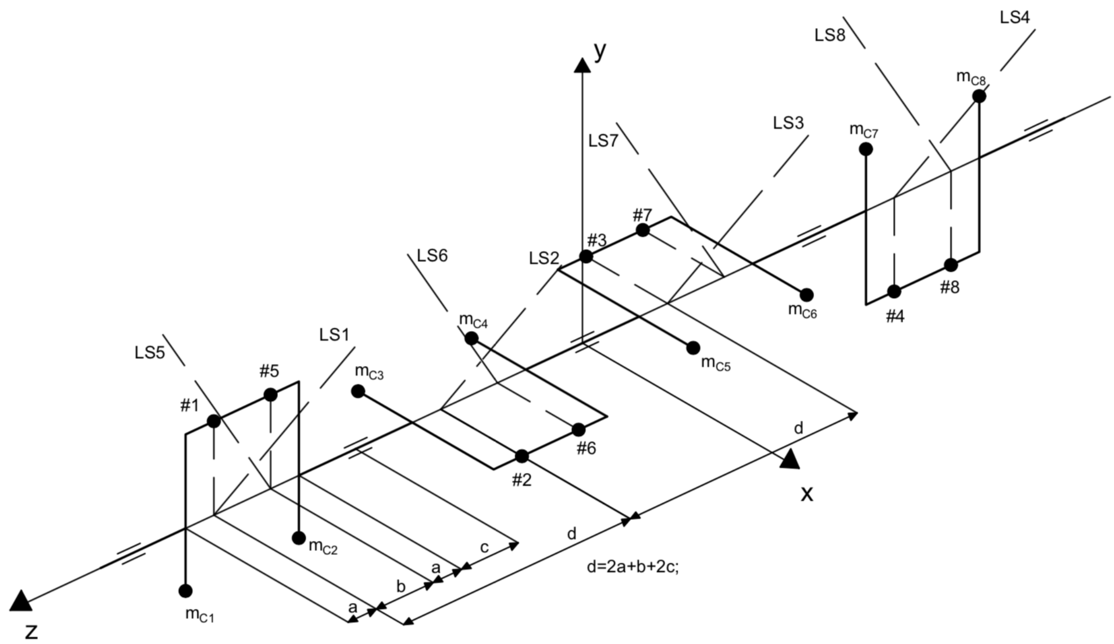

Figure 5.44: 90° V8 cross-plane crankshaft with bay-by-bay balancing

$$m_1 \Rightarrow \begin{pmatrix} x_1 \\ y_1 \\ z_1 = (a+b+c+d) \end{pmatrix}; \quad m_2 \Rightarrow \begin{pmatrix} x_2 \\ y_2 \\ z_2 = (a+b+c) \end{pmatrix}; \quad m_3 \Rightarrow \begin{pmatrix} x_3 \\ y_3 \\ z_3 = -(a+c) \end{pmatrix};$$

$$m_4 \Rightarrow \begin{pmatrix} x_4 \\ y_4 \\ z_4 = -(a+c+d) \end{pmatrix}; \quad m_5 \Rightarrow \begin{pmatrix} x_5 \\ y_5 \\ z_5 = (a+c+d) \end{pmatrix}; \quad m_6 \Rightarrow \begin{pmatrix} x_6 \\ y_6 \\ z_6 = (a+c) \end{pmatrix};$$

$$m_7 \Rightarrow \begin{pmatrix} x_7 \\ y_7 \\ z_7 = -(a+b+c) \end{pmatrix}; \qquad m_8 \Rightarrow \begin{pmatrix} x_8 \\ y_8 \\ z_8 = -(a+b+c+d) \end{pmatrix};$$

$$m_{c_1} \Rightarrow \begin{pmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} = (2a+b+c+d) \end{pmatrix} \; m_{c_2} \Rightarrow \begin{pmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} = (2a+b+3c) \end{pmatrix} \; m_{c_3} \Rightarrow \begin{pmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} = (2a+b+c) \end{pmatrix}$$

$$m_{c_4} \Rightarrow \begin{pmatrix} x_{c_4} \\ y_{c_4} \\ z_{c_4} = c \end{pmatrix}; \quad m_{c_5} \Rightarrow \begin{pmatrix} x_{c_5} \\ y_{c_5} \\ z_{c_5} = -c; \end{pmatrix}; \quad m_{c_6} \Rightarrow \begin{pmatrix} x_{c_6} \\ y_{c_6} \\ z_{c_6} = -(2a+b+c) \end{pmatrix};$$

$$m_{c_7} \Rightarrow \begin{pmatrix} x_{c_7} \\ y_{c_7} \\ z_{c_7} = -(2a+b+3c); \end{pmatrix}; \quad m_{c_8} \Rightarrow \begin{pmatrix} x_{c_8} \\ y_{c_8} \\ z_{c_8} = -(2a+b+c+d); \end{pmatrix};$$

The general system of equilibrium equations can be written in the same way that in 5.51.

Considering and assuming that, in this case:

- $m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m$;

- $x_1 = x_5 = x_{15}$, $x_2 = x_6 = x_{26}$, $x_3 = x_7 = x_{37}$,and $x_4 = x_8 = x_{48}$

- $x_{15} = -x_{48} =$ and $x_{26} = -x_{37}$

- $y_1 = y_5 = y_{15}$, $y_2 = y_6 = y_{26}$, $y_3 = y_7 = y_{37}$,and $y_4 = y_8 = y_{48}$

- $y_{15} = -y_{48}$ and $y_{26} = -y_{37}$

the system becomes:

$$
\begin{cases}
x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + m_{c_7}x_{c_7} + m_{c_8}x_{c_8} + \\
\qquad +2m(x_{15}) + 2m(x_{48}) + 2m(x_{26}) + 2m(x_{37}) = 0 \\
y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + m_{c_7}y_{c_7} + m_{c_8}y_{c_8} + \\
\qquad +2m(y_{15}) + 2m(y_{48}) + 2m(y_{26}) + 2m(y_{37}) = 0 \\
x \circlearrowleft \quad (2a+b+c+d)(m_{c_1}x_{c_1} - m_{c_8}x_{c_8}) + (2a+b+3c)(m_{c_2}x_{c_2} - m_{c_7}x_{c_7}) + \\
\qquad +(2a+b+c)(m_{c_3}x_{c_3} - m_{c_6}x_{c_6}) + c(m_{c_4}x_{c_4} - m_{c_5}x_{c_5}) + \\
\qquad +mx_{15}(2a+b+2c+2d) + mx_{48}[-(2a+b+2c+2d)] + \\
\qquad +mx_{26}(2a+b+2c) + mx_{37}[-(2a+b+2c)] = 0 \\
y \circlearrowleft \quad (2a+d+c+d)(m_{c_1}y_{c_1} - m_{c_8}y_{c_8}) + (2a+b+3c)(m_{c_2}y_{c_2} - m_{c_7}y_{c_7}) + \\
\qquad +(2a+b+c)(m_{c_3}y_{c_3} - m_{c_6}y_{c_6}) + c(m_{c_4}y_{c_4} - m_{c_5}y_{c_5}) + \\
\qquad +my_{15}(2a+b+2c+2d) + my_{48}[-(2a+b+2c+2d)] + \\
\qquad +my_{26}(2a+b+2c) + my_{37}[-(2a+b+2c)] = 0
\end{cases}
\tag{5.73}
$$

and then:

$$
\begin{cases}
x) \quad m_{c_1}x_{c_1} + m_{c_2}x_{c_2} + m_{c_3}x_{c_3} + m_{c_4}x_{c_4} + m_{c_5}x_{c_5} + m_{c_6}x_{c_6} + m_{c_7}x_{c_7} + m_{c_8}x_{c_8} = 0 \\
y) \quad m_{c_1}y_{c_1} + m_{c_2}y_{c_2} + m_{c_3}y_{c_3} + m_{c_4}y_{c_4} + m_{c_5}y_{c_5} + m_{c_6}y_{c_6} + m_{c_7}y_{c_7} + m_{c_8}y_{c_8} = 0 \\
x \circlearrowleft \quad (2a+b+c+d)(m_{c_1}x_{c_1} - m_{c_8}x_{c_8}) + (2a+b+3c)(m_{c_2}x_{c_2} - m_{c_7}x_{c_7}) + \\
\qquad +(2a+b+c)(m_{c_3}x_{c_3} - m_{c_6}x_{c_6}) + c(m_{c_4}x_{c_4} - m_{c_5}x_{c_5}) + \\
\qquad +mx_{15}(4a+2b+4c+4d) + mx_{26}(4a+2b+4c) = 0 \\
y \circlearrowleft \quad (2a+d+c+d)(m_{c_1}y_{c_1} - m_{c_8}y_{c_8}) + (2a+b+3c)(m_{c_2}y_{c_2} - m_{c_7}y_{c_7}) + \\
\qquad +(2a+b+c)(m_{c_3}y_{c_3} - m_{c_6}y_{c_6}) + c(m_{c_4}y_{c_4} - m_{c_5}y_{c_5}) + \\
\qquad +my_{15}(4a+2b+4c+4d) + my_{26}(4a+2b+4c) = 0
\end{cases}
\tag{5.74}
$$

Solving this system, a balanced configuration can be reached.

5.4.3 Bearing Loads

The way used to find the bearing loads is based on the *Three Moment Equation of Clapeyron*. The crankshaft is hyperstatically constrained and can be considered as a beam, in this case with four span.

The three moment equation method require to use ad additional equation for each excess constraint. In the equations shown below it is assumed that the Young's modulus E and the moment of inertia of the crankshaft section J are constant along the crankshaft, therefore they cancel each other in all the terms.

In this case the system can be divided in three subsystems, the *ABC*, the *BCD* and the *CDE*. For each subsystem a three moment equation can be derived. In the Figure 5.45 is represented the whole system considered.

The equations are written in a general form, so the results are valid both for flat-plane and cross-plane V8 crankshaft.

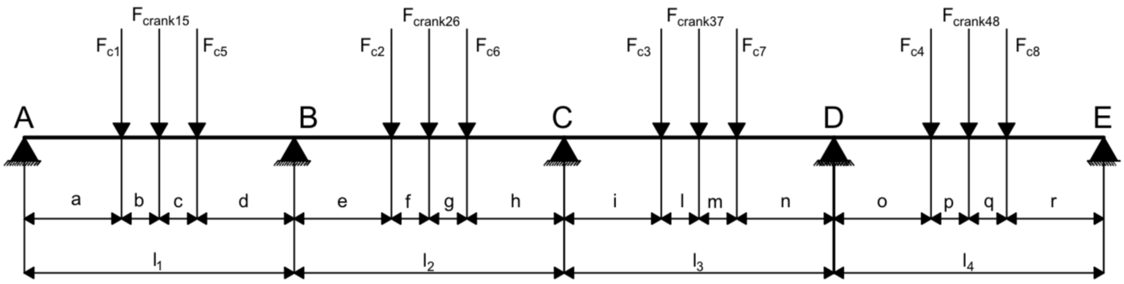

Figure 5.45: V8 crankshaft seen as a four-span beam

Considering now the system ABC shown in Figure :

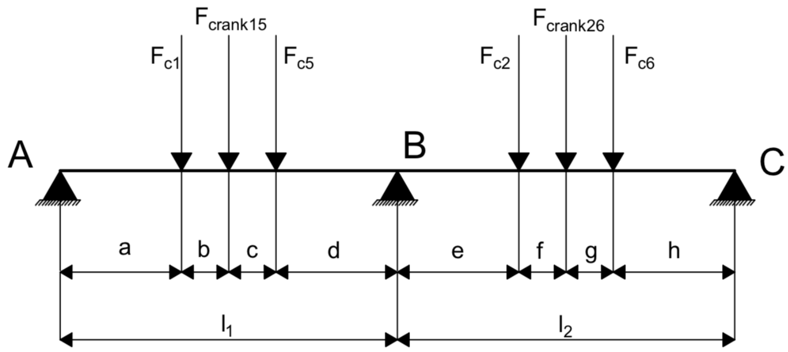

Figure 5.46: System ABC

the three moment equation is equal to:

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\frac{F_{c1}}{l_1} a(l_1^2 - a^2) - \frac{F_{crank_{15}}}{l_1}(a+b)(l_1^2 - (a+b)^2) - \frac{F_{c5}}{l_1}(a+b+c)(l_1^2 - (a+b+c)^2) + \\ - \frac{F_{c2}}{l_2}(f+g+h)(l_2^2 - (f+g+h)^2) - \frac{F_{crank_{26}}}{l_2}(g+h)(l_2^2 - (g+h)^2) - \frac{F_{c6}}{l_2} h(l_1^2 - h^2); \tag{5.75}$$

for the system BCD shown in the Figure 5.47 the three moment equation is equal to:

$$M_B l_2 + 2M_C(l_2+l_3) + M_D l_3 = -\frac{F_{c2}}{l_2} e(l_2^2 - e^2) - \frac{F_{crank_{26}}}{l_2}(e+f)(l_2^2 - (e+f)^2) - \frac{F_{c6}}{l_2}(e+f+g)(l_2^2 - (e+f+g)^2) + \\ - \frac{F_{c3}}{l_3}(n+m+l)(l_3^2 - (n+m+l)^2) - \frac{F_{crank_{37}}}{l_3}(n+m)(l_3^2 - (n+m)^2) - \frac{F_{c7}}{l_3} n(l_1^2 - n^2); \tag{5.76}$$

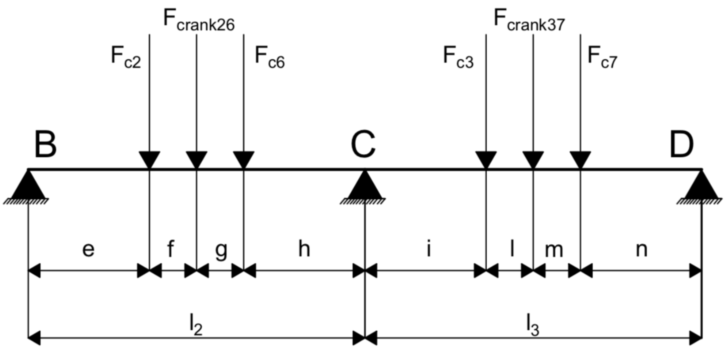

Figure 5.47: System BCD

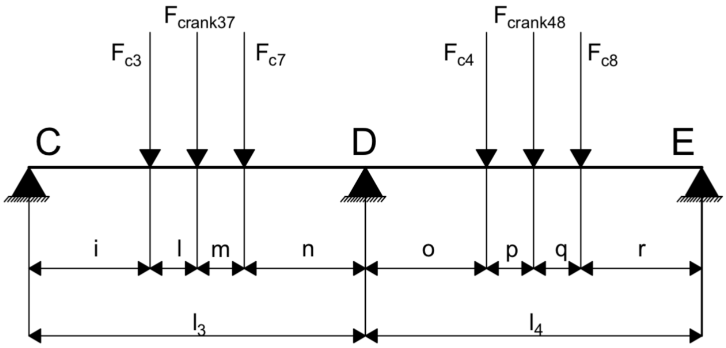

Figure 5.48: System CDE

for the system CDE shown in the Figure 5.48 the three moment equation is equal to:

$$M_C l_3 + 2M_D(l_3+l_4) + M_E l_4 = -\frac{F_{c3}}{l_3} i(l_3^2 - i^2) - \frac{F_{crank_{37}}}{l_3}(i+l)(l_3^2-(i+l)^2) - \frac{F_{c7}}{l_3}(i+l+m)(l_3^2-(i+l+m)^2) +$$
$$-\frac{F_{c4}}{l_4}(r+q+p)(l_4^2-(r+q+p)^2) - \frac{F_{crank_{48}}}{l_4}(r+q)(l_4^2-(r+q)^2) - \frac{F_{c8}}{l_4} r(l_4^2 - r^2); \quad (5.77)$$

The moments M_A and M_E are zero, and M_B, M_C and M_D can be calculated considering the system in matrix form like $Ax = B$ with:

$$A = \begin{bmatrix} 2(l_1+l_2) & l_2 & 0 \\ l_2 & 2(l_2+l_3) & l3 \\ 0 & l_3 & 2(l3+l4) \end{bmatrix}$$

;

$$x = \begin{pmatrix} M_B \\ M_C \\ M_D \end{pmatrix}$$

and the vector B equal to the known term:

$$B = \begin{pmatrix} -\frac{F_{c1}}{l_1}a(l_1^2 - a^2) + \ldots - \frac{F_{c6}}{l_2}h(l_2^2 - h^2) \\ -\frac{F_{c2}}{l_2}e(l_2^2 - e^2) + \ldots - \frac{F_{c7}}{l_3}n(l_3^2 - n^2) \\ -\frac{F_{c3}}{l_3}i(l_3^2 - i^2) + \ldots - \frac{F_{c8}}{l_4}r(l_4^2 - r^2) \end{pmatrix}$$

and then:

$$x = \begin{pmatrix} M_B \\ M_C \\ M_D \end{pmatrix} = A^{-1} * B$$

Considering now the each single span, the shear forces and the reaction on the supports can be derived as shown below:

- System "AB" (Figure 5.49):

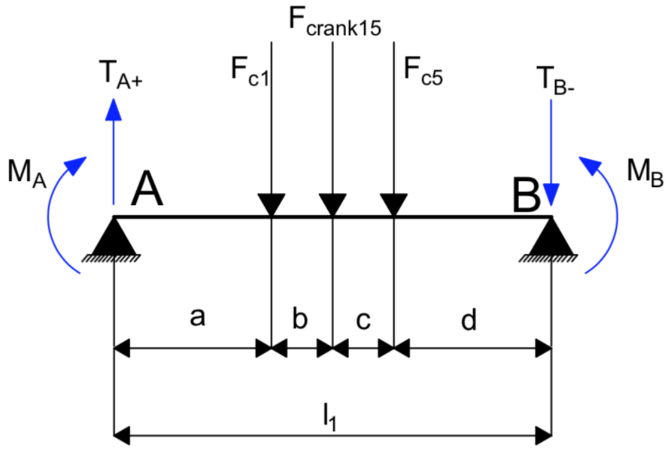

Figure 5.49: System AB

$$\begin{cases} B \circlearrowright \;\; M_B - M_A + F_{c1}(b+c+d) + F_{c5}d + F_{crank_{15}}(c+d) - T_{A^+}l_1 = 0 \\ \qquad T_{A^+} = \frac{1}{l_1}(M_B + F_{c1}(b+c+d) + F_{c5}d + F_{crank_{15}}(c+d)); \\ \uparrow)\; T_{A^+} - F_{c1} - F_{c5} - F_{crank_{15}} - T_{B^-} = 0 \\ \qquad T_{B^-} = T_{A^+} - F_{c1} - F_{c5} - F_{crank_{15}}; \end{cases} \tag{5.78}$$

- System "BC" (Figure 5.50):

$$\begin{cases} C \circlearrowright \;\; M_C - M_B + F_{c2}(f+g+h) + F_{c6}h + F_{crank_{26}}(g+h) - T_{B^+}l_2 = 0 \\ \qquad T_{B^+} = \frac{1}{l_2}(M_C - M_B + F_{c2}(f+g+h) + F_{c6}h + F_{crank_{26}}(g+h)); \\ \uparrow)\; T_{B^+} - F_{c2} - F_{c6} - F_{crank_{26}} - T_{C^-} = 0 \\ \qquad T_{C^-} = T_{B^+} - F_{c2} - F_{c6} - F_{crank_{26}}; \end{cases} \tag{5.79}$$

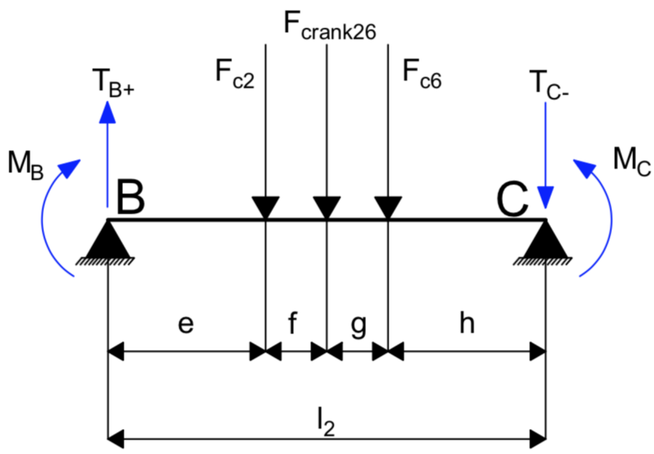

Figure 5.50: System BC

- System "CD" (Figure 5.51):

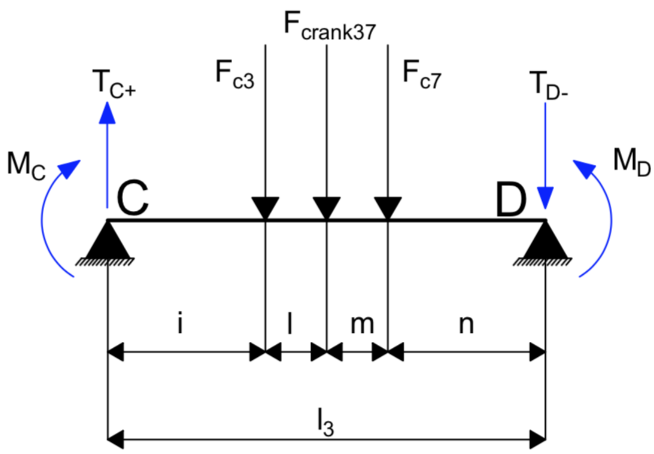

Figure 5.51: System CD

$$\begin{cases} D \circlearrowleft \; M_D - M_C + F_{c3}(l+m+n) + F_{c7}m + F_{crank_{37}}(m+n) - T_{C^+}l_3 = 0 \\ \quad T_{C^+} = \frac{1}{l_3}(M_C - M_B + F_{c3}(l+m+n) + F_{c7}m + F_{crank_{37}}(m+n)); \\ \uparrow) \; T_{C^+} - F_{c3} - F_{c7} - F_{crank_{37}} - T_{D^-} = 0 \\ \quad T_{D^-} = T_{C^+} - F_{c3} - F_{c7} - F_{crank_{37}}; \end{cases} \tag{5.80}$$

- System "DE" (Figure 5.52):

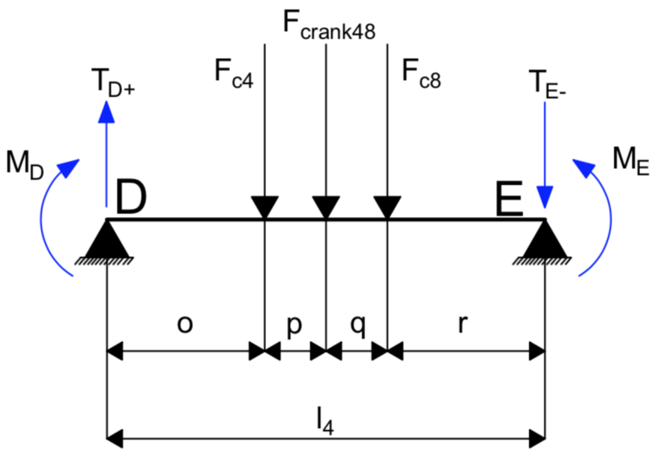

Figure 5.52: System DE

$$\begin{cases} E \circlearrowleft \; M_E - M_D + F_{c4}(l+m+n) + F_{c8}m + F_{crank_{48}}(m+n) - T_{D^+}l_4 = 0 \\ \qquad T_{D^+} = \frac{1}{l_4}(-M_D + F_{c4}(l+m+n) + F_{c8}m + F_{crank_{48}}(m+n)); \\ \uparrow) \; T_{D^+} - F_{c4} - F_{c8} - F_{crank_{48}} - T_{E^-} = 0 \\ \qquad T_{E^-} = T_{D^+} - F_{c4} - F_{c8} - F_{crank_{48}}; \end{cases} \tag{5.81}$$

The reactions can be obtained considering the convention shown in Figure 5.53.

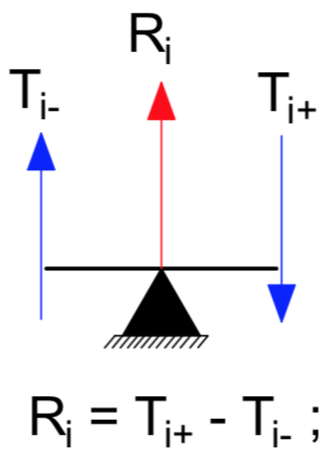

Figure 5.53: Shear forces convention

$$\begin{cases} R_A = T_{A^+}; \\ R_B = T_{B^+} - T_{B^-}; \\ R_C = T_{C^+} - T_{C^-}; \\ R_D = T_{D^+} - T_{D^-}; \\ R_E = -T_{E^-}; \end{cases} \tag{5.82}$$

All the procedure shown above is valid for the case of a non-balanced crankshaft as no forces due to the counterweights were considered. Anyway, the same method can be used for a balanced crankshaft just adding the forces due to the counterweights, as shown in the Figure 5.54.

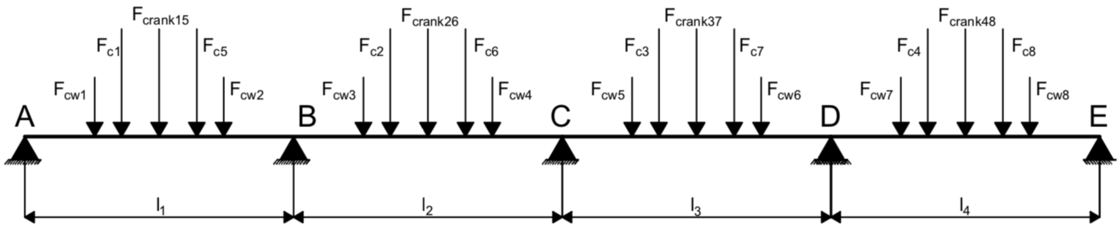

Figure 5.54: Model of forces in a V8 balanced crankshaft

Chapter 6

Matlab Scripts and Results

The aim of this thesis work is to create a Matlab script that allows the user to quickly set up a crankshaft layout and find the crankshaft state of balancing and the reactions on the main bearings derived with the methods shown in the previous paragraphs.

The crankshafts that can be analyzed in this Matlab code are from Inline engines from 2 to 6 cylinders, and V-engines with 4, 6 or 8 cylinders.

I chose to divide the whole code in two main parts and, therefore, in two main scripts. The following figures show some screenshots of the required choices and parameters during the running of the code.

6.1 Matlab script without balancing

The first part of the code governs the forces and moments analysis considering a non-balanced crankshaft; this script is useful to let the user know about which forces and moments are naturally balanced in the shaft configuration considered in order to think about a possible balancing strategy that can be applied in that particular case.

First of all, the script requires to set if the user wants to consider an Inline or V crankshaft layout as shown in the Figure 6.1:

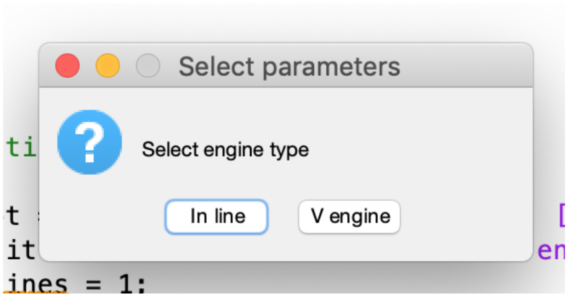

Figure 6.1: Inline or V engine selection

then the number of cylinders are required, as shown in the Figures 6.2 and 6.3.

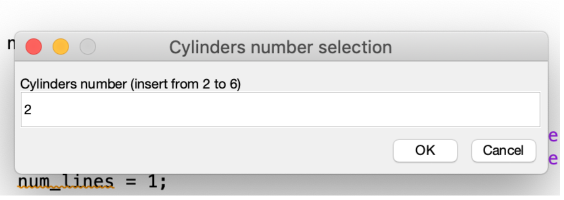

Figure 6.2: Inline cylinders number selection

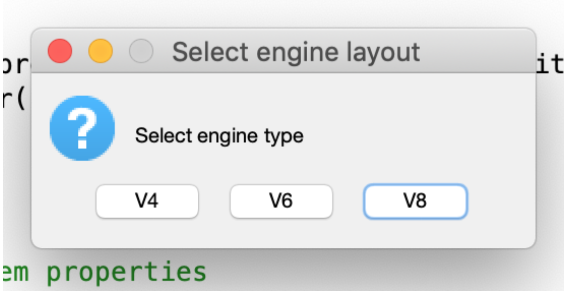

Figure 6.3: V-engine cylinders number selection

After that, the user have to insert some physical parameters of the system, as reported in Figure 6.4:

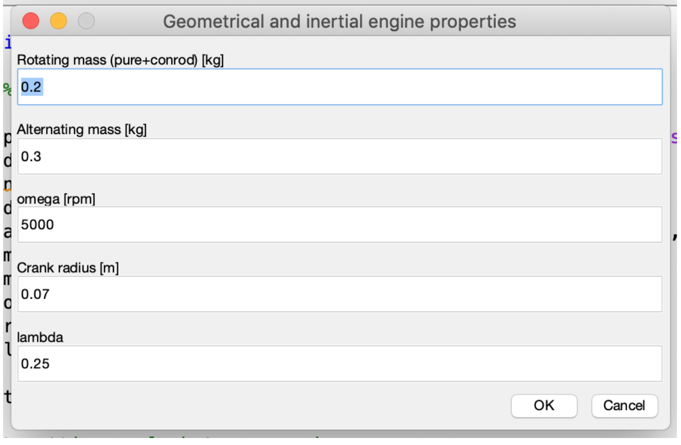

Figure 6.4: Parameters setting

The parameter λ is the ratio between the crank throw radius and the conrod length. Only in V-engine cases is now required to set up the V-angle (Figure 6.5).

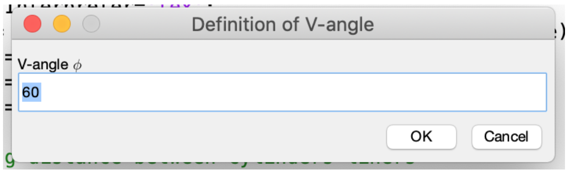

Figure 6.5: V-angle setting

The next step is to set the crank throws disposition in terms of angles between themselves. Considering for example a I3 engine, the values of the angle between the crank throws #1 and #2 and between #1 and #3 as shown in the Figure 6.6.

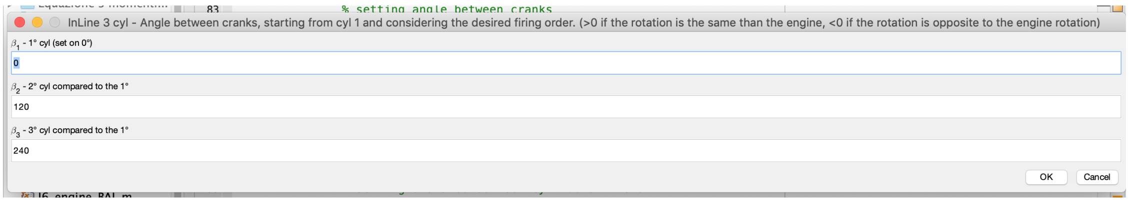

Figure 6.6: Angles between crank throws

The last parameter to set is the bore spacing (Figure 6.7), that is useful in the moments and bearing reactions calculation.

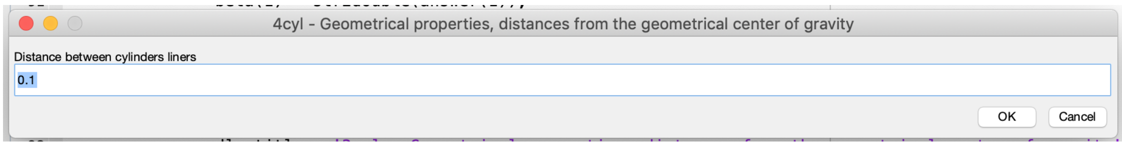

Figure 6.7: Bore spacing for Inline engines

In V-engine cases only, besides the bore spacing also some other parameters are required (Figure 6.8), as the offset between banks, the distance from the crank arm to the adjacent main bearing and the distance between the crank arm and the adjacent conrod (assuming that the two conrods are placed symmetrically respect to the centreline plane of the crank throw).

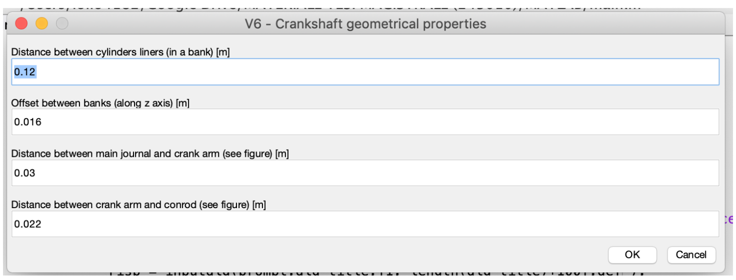

Figure 6.8: Required parameters for V-engines

Once the necessary parameters have been set, the script recalls a function that manage all the calculation required to find the first and second order forces and moments and the reactions on the supports. A specific function was written for each case that can be analyzed, therefore there are four funcions that manage Inline crankshafts from 2 to 6 cylinders, and three fuctions that manage V4, V6 and V8 crankshaft layouts.

Those functions give back to the user the values of forces, moments and reactions acting on the system considering the crankshaft without any type of balancing and a parameter that represents the shaft weight, obviously according to the assumptions and simplifications made in this study. Moreover also the angle of the plane of the resultant moment (if it exists) is returned to the user. There are also some plots in output that allow to graphically observe the trends of the parameters returned by the functions.

6.2 Matlab script with balancing

The second main script does the crankshaft balancing.

In the initial part the user has to set up the same parameters set in the case without the balancing, so the engine type, the number of cylinders, the masses, the crank throw length, the ratio λ, the engine rotation speed ω, geometrical parameters how the angle between crank throws, the bore spacing, the distance between crank throws and main bearings and the offset between banks in case of a V-engine.

After that the script requires the key parameter for the crankshaft balance: the *Balancing Factor*.

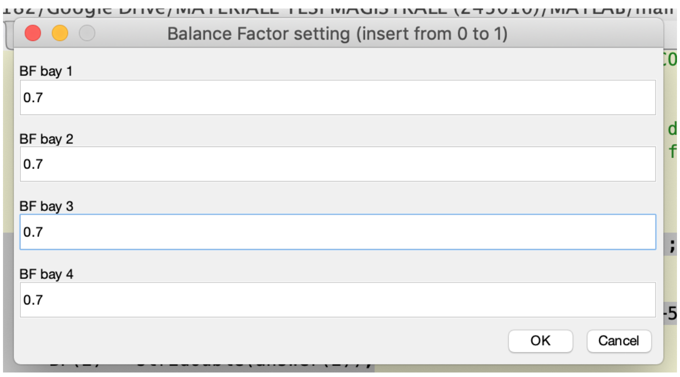

Figure 6.9: BF setting for a V8 engine

As can be seen in the Figure 6.9 referred to a V8 engine (with four bays), each single bay has its balancing factor.

Once the BF values are inserted, the script requires a definition of the masses and the disposition of the counterweights. First of all the position along the crankshaft axis must be set (Figure 6.10) and then the angle between each crank throw and the couterweights placed in each of them, as shown in the Figure 6.11:

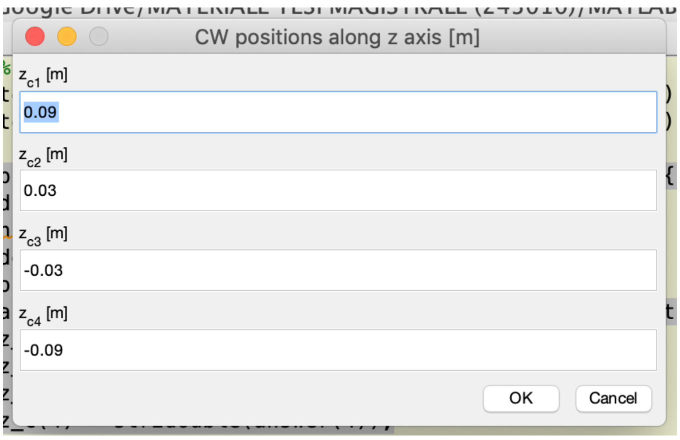

Figure 6.10: Counterweight position along crankshaft axis for a V4 engine

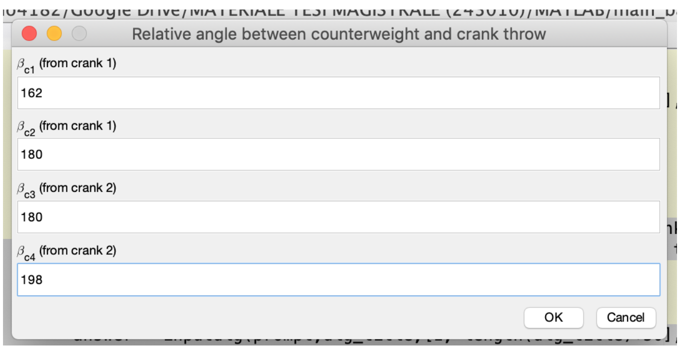

Figure 6.11: Counterweight angle from the relative crank for a V4 engine

The angle that must be set in the Figure 6.11 is a relative angle referred to each crank and it allows to skew the counterweights of the desired angle. This skew angle can be very useful when there is a rocking couple to counteract. For example in the Figure 6.11 is reported the setting window for a V4 engine (with two bays) where a maximum of 4 counterweights can be placed, two for each crank throw.

After that the script gives to the user the possibility to choose (Figure 6.12) to follow an "easy way" to do the balance, that is to have equal countwerweights in each bay in terms of mass and radius, or a "more complex way" that is to set the properties for each counterweight.

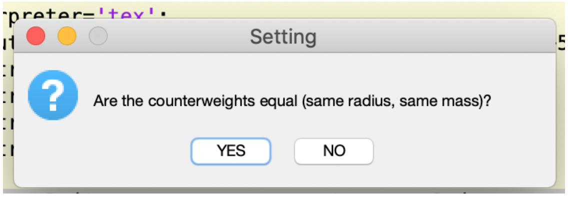

Figure 6.12: Counterweight type selection

If the "easy way" is chosen the code asks which parameter the designer want to set between the counterweights mass or the radius (Figure 6.13), in order to find the parameter unknown. Usually the radius can be assumed (Figure 6.14), because there is the constraint of the available space inside the crankcase and the maximum counterweights radius can be set equal to the crank throw radius.

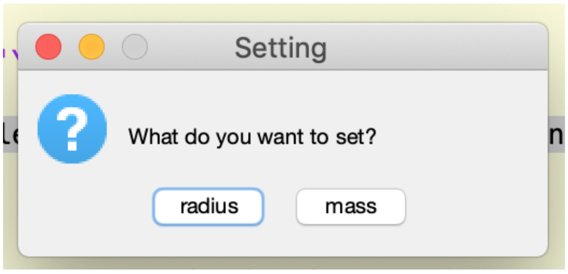

Figure 6.13: Input

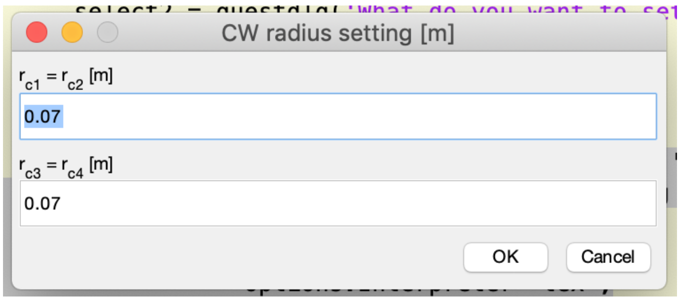

Figure 6.14: Counterweights radius input

Instead, if the "more complex way" is chosen the designer have to set the radius and the mass of each counterweight of each bay, according to the BF chosen initially (Figure 6.15).

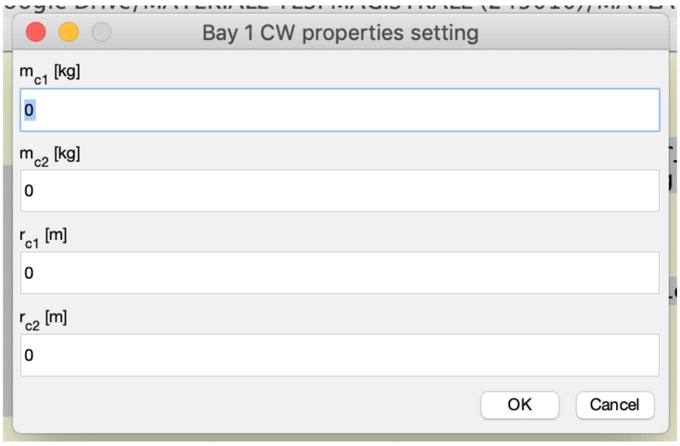

Figure 6.15: Counterweights settings for a single crankshaft bay

In the same way as done for the script without the balancing part, for each possible engine configuration in terms of type (Inline or V) and nymber of cylinders, a specific fuction was written.

Those functions give back the values of forces, moments and reactions acting on the system considering the crankshaft balancing through counterweights and a parameter that represents the shaft weight, obviously according to the assumptions and simplifications made in this study. Moreover also the angle of the plane of the resultant moment (if it exists) is returned to the user. There are also some plots in output that allow to graphically observe the trends of the parameters returned by the functions.

6.3 Output plots

In this Paragraph are reported the output plots of the Matlab scripts created to analyze the crankshaft state of balancing.

In particular, in the Pararagraph 6.3.1 are shown the Figures referred to the Matlab script that carries out the analysis of the crankshaft without any type of balacing strategy, in order to verify which forces and moments are balanced, to find the plane of the resultant moment (if it is existing) and to calculate the whole crankshaft weight based on the hypotheses made. The Paragraph 6.3.2 instead shows figures about balanced crankshaft configurations in order to check graphically the results produced by the balancing strategy applied in terms of main bearings reactions and shaft weight.

For the sake of brevity are reported just few of the possible cases that the Matlab script is able to analyze, in particular are illustrated the figures referred to the Inline three-cylinder engine (that can be easily extended to a V6 configuration), the 90° V8 flat-plane engine and the 90° V8 cross-plane engine.

6.3.1 Output plots of the Matlab script without balancing

Inline three-cylinder engine plots

The input data assumed for the analysys are:

m_{ROT}	m_{ALT}	r	λ	ω	Bore spacing
[kg]	[kg]	[m]	-	[rpm]	[m]
0.5	0.65	0.045	0.25	6000	0.1

Table 6.1: Input data I3 crankshaft

Where m_{ROT} is considered ad the sum of the pure rotating mass and the two-thirds of the conrod mass. The m_{ALT} is the sum of the one-third of the conrod mass and the piston mass.

The output figures are:

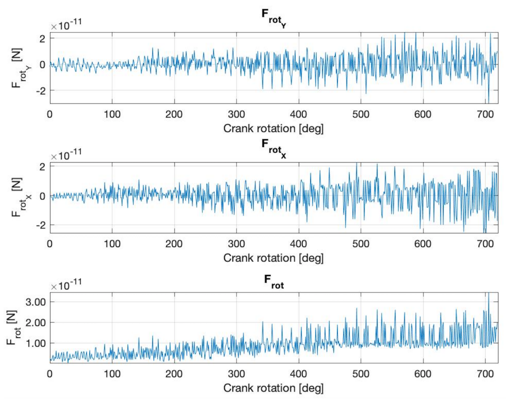

Figure 6.16: Pure rotating forces

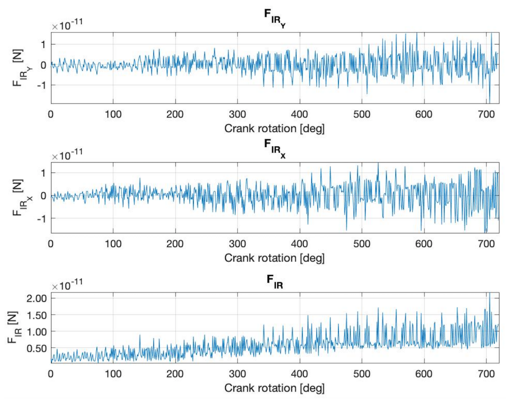

Figure 6.17: First order rotating forces

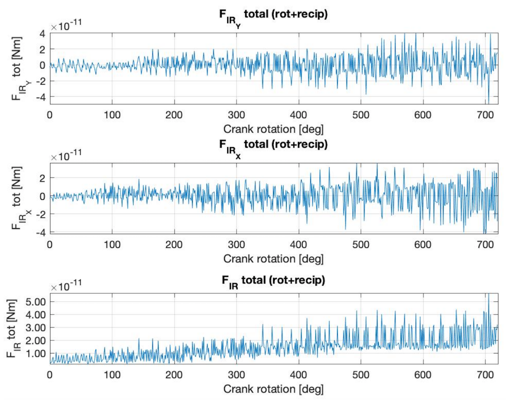

Figure 6.18: Total first order rotating forces

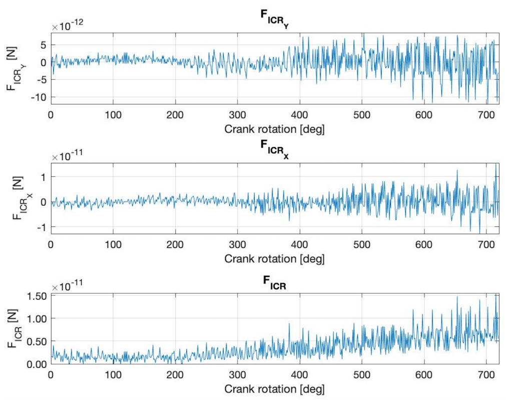

Figure 6.19: First order counter-rotating forces

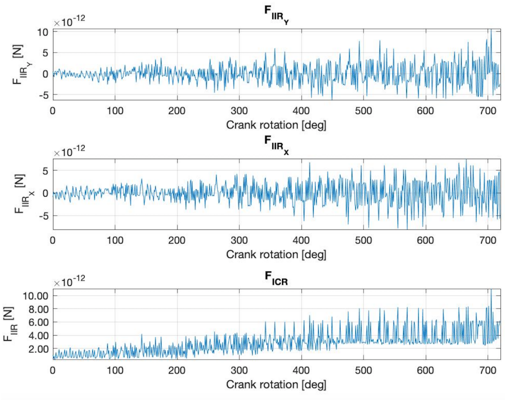

Figure 6.20: Second order rotating forces

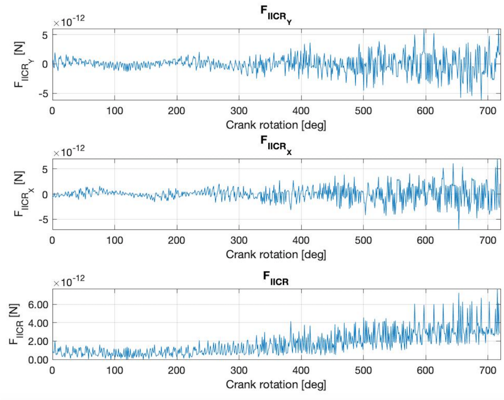

Figure 6.21: Second order counter-rotating forces

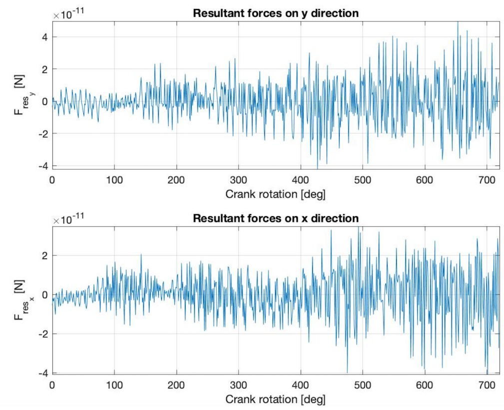

Figure 6.22: Resultant forces

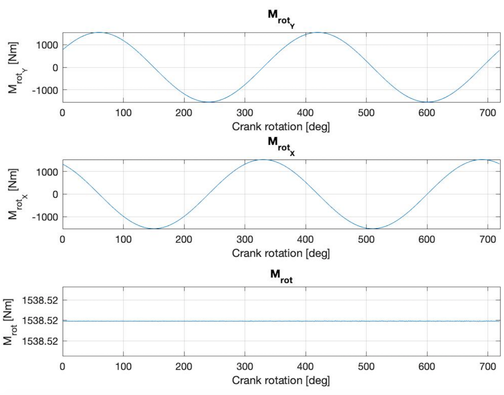

Figure 6.23: Pure rotating moments

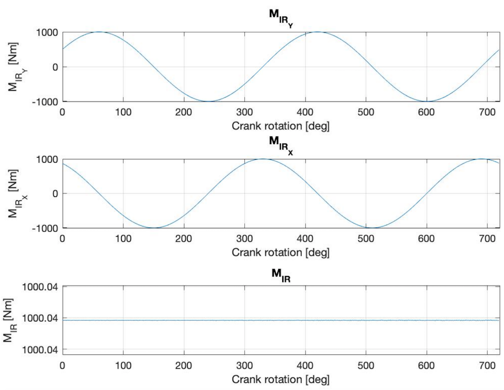

Figure 6.24: First order rotating moments

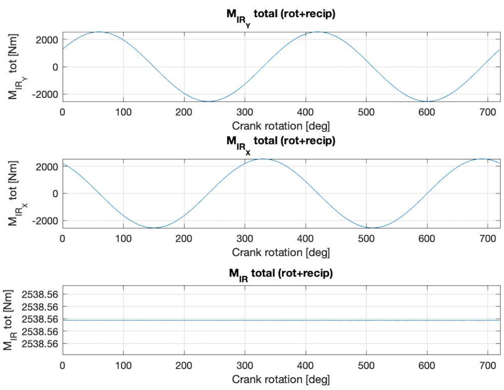

Figure 6.25: Total first order rotating moments

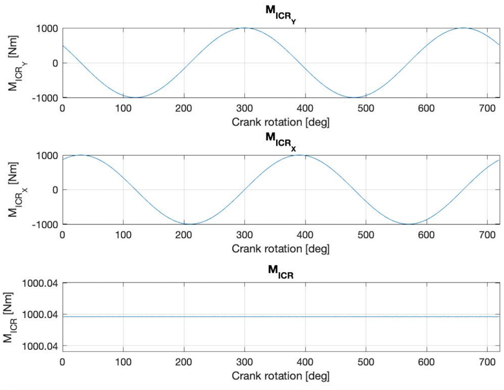

Figure 6.26: First order counter-rotating moments

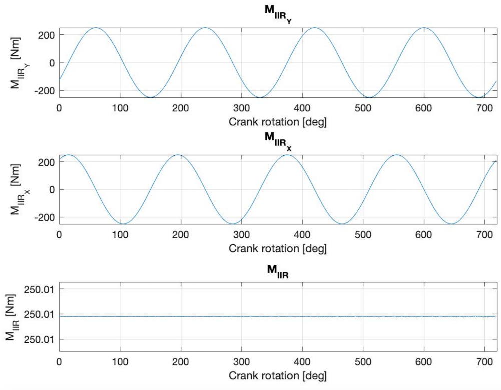

Figure 6.27: Second order rotating moments

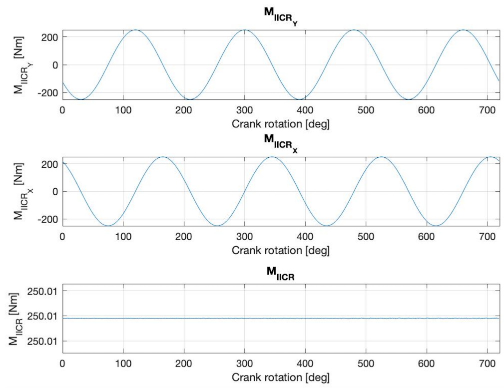

Figure 6.28: Second order counter-rotating moments

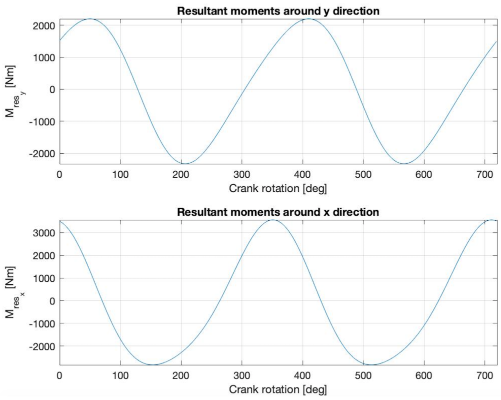

Figure 6.29: Resultant moments

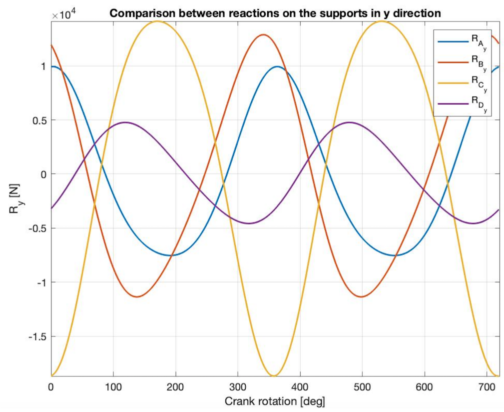

Figure 6.30: Bearings reactions in y

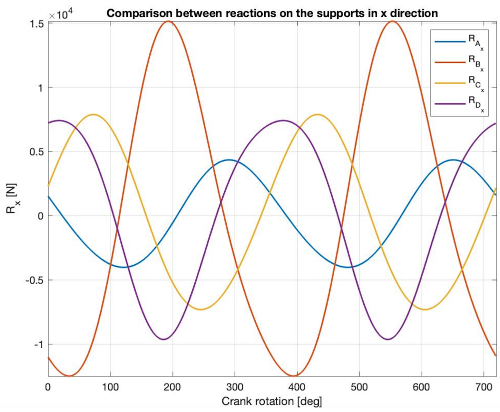

Figure 6.31: Bearings reactions in x

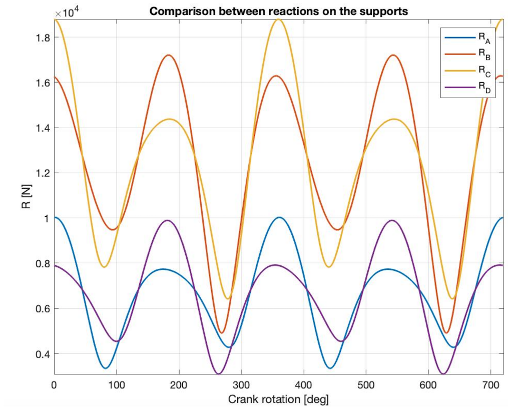

Figure 6.32: Resultant bearings reactions

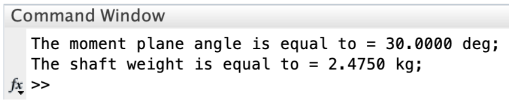

Figure 6.33: Output parameters

The figure shown above represent all the main stress to which the crankshaft is subjected. The results obtained are consistent with the theory reported in the Paragraph 4.2 because:

- The first order forces, both rotating and counter-rotating, reported in the Figures from 6.16 to 6.19 are balanced.
- The second order forces, both rotating and counter-rotating, shown in the Figures 6.20 and 6.21 are balanced.
- The first order moments, both rotating and counter-rotating, reported in the Figures from 6.23 to 6.26 are not balanced.
- The second order moments, both rotating and counter-rotating, shown in the Figures from 6.27 to 6.28 are not balanced.

The Figures from 6.30 to 6.32 shown the bearings reactions trends, and it can be observed that without any type of balancing the most stressed are the central ones (B and C). The last Figure 6.33 reports the values of the calculated shaft weight and of the angle between the plane of the resultant first order moment and the plane of the crank throw #1 that is, according to the theory, equal to 30°.

90° V8 flat-plane engine plots

The input data assumed for the analysys are:

$m_{ROT_{pure}}$	$m_{ROT_{ROD}}$	m_{ALT}	r	λ	ω	V angle
[kg]	[kg]	[kg]	[m]	-	[rpm]	[deg]
0.2	0.3	0.65	0.045	0.25	6000	90

Bore spacing	a	b	c
[m]	[m]	[m]	[m]
0.12	0.022	0.016	0.03

Table 6.2: Input data V8 flat-plane crankshaft

Where m_{ALT} is the sum of the one-third of the conrod mass and of the piston mass. For the distances a, b and c the reader can look at the Figure 5.42.

The output figures are:

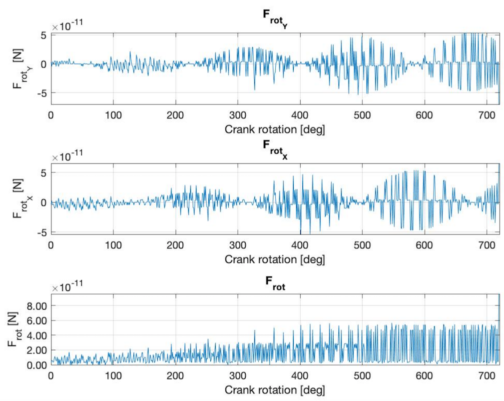

Figure 6.34: Pure rotating forces

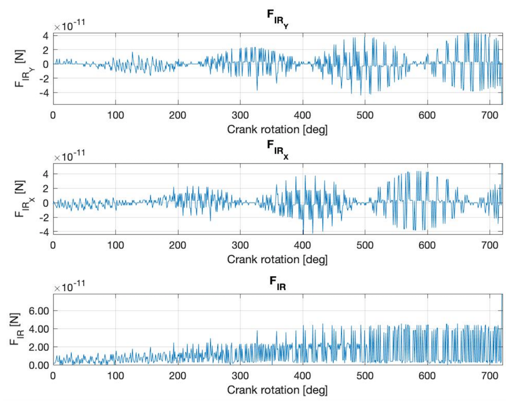

Figure 6.35: First order rotating forces

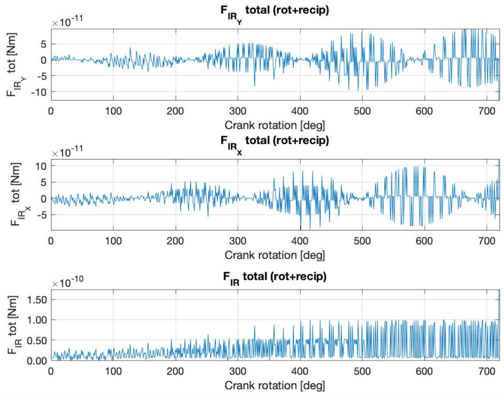

Figure 6.36: Total first order rotating forces

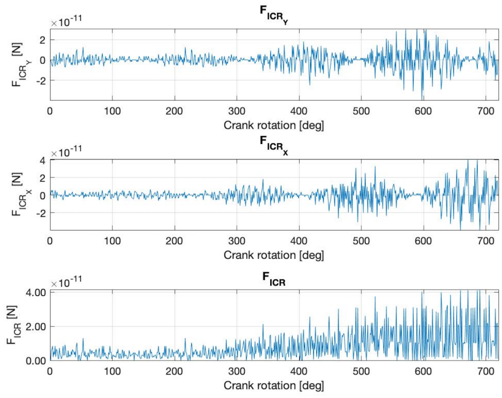

Figure 6.37: First order counter-rotating forces

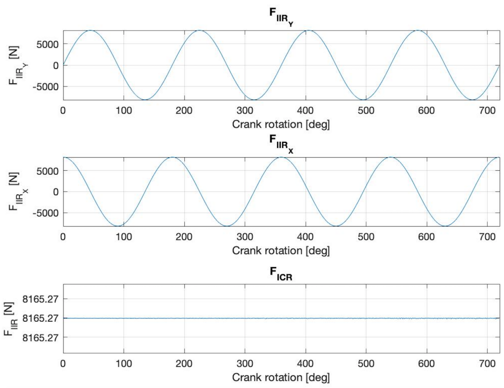

Figure 6.38: Second order rotating forces

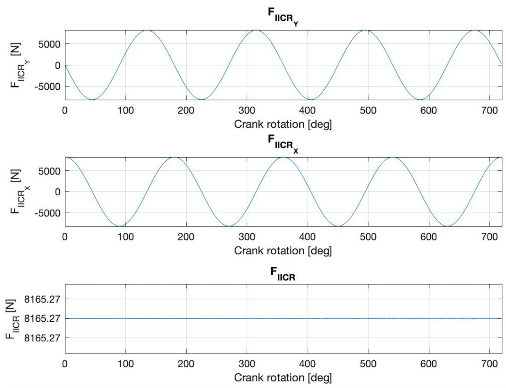

Figure 6.39: Second order counter-rotating forces

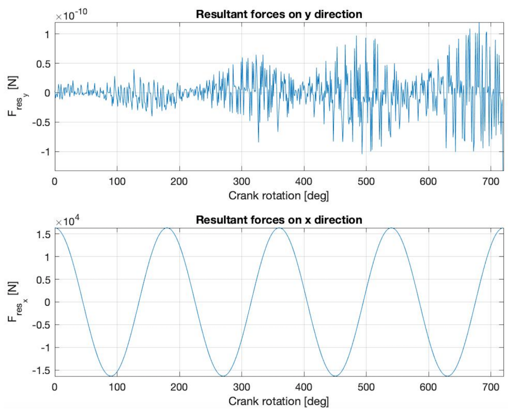

Figure 6.40: Resultant forces

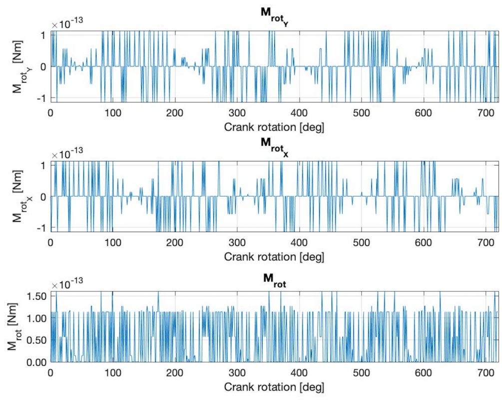

Figure 6.41: Pure rotating moments

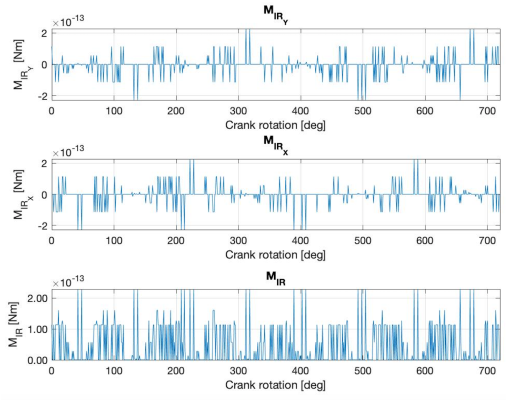

Figure 6.42: First order rotating moments

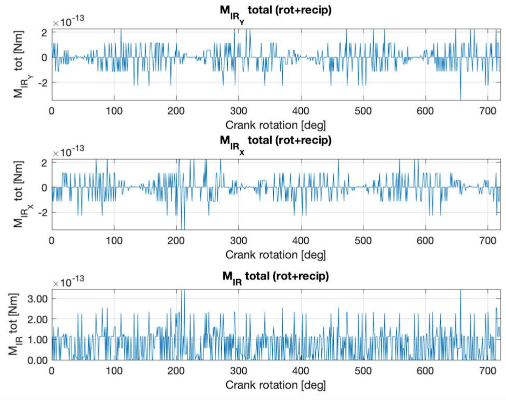

Figure 6.43: Total first order rotating moments

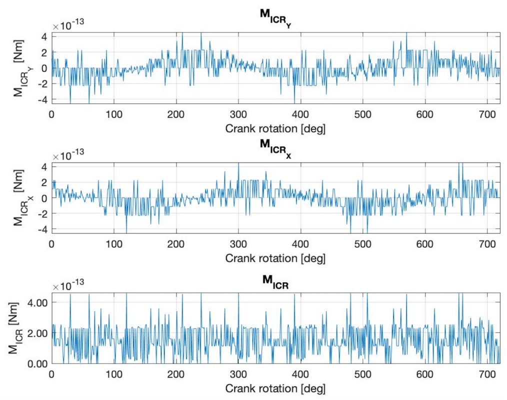

Figure 6.44: First order counter-rotating moments

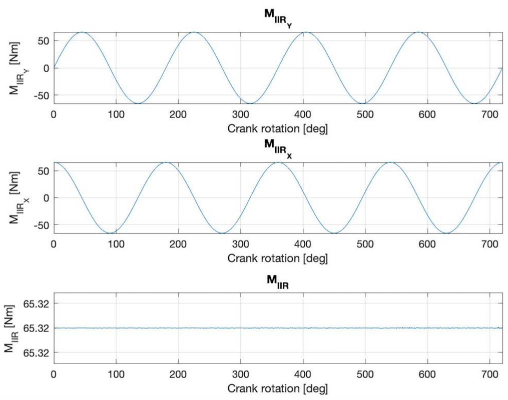

Figure 6.45: Second order rotating moments

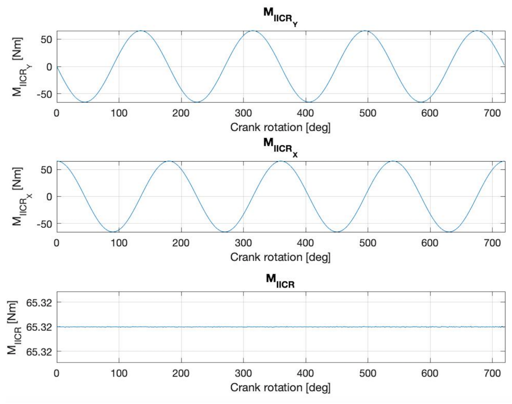

Figure 6.46: Second order counter-rotating moments

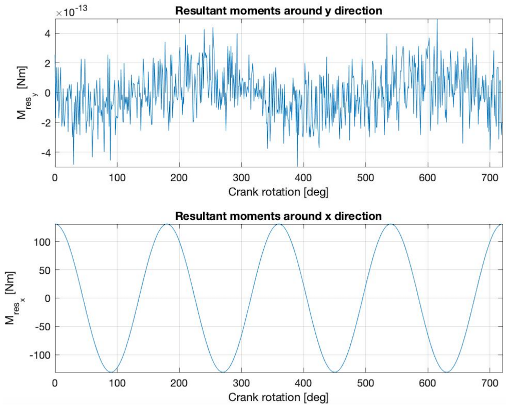

Figure 6.47: Resultant moments

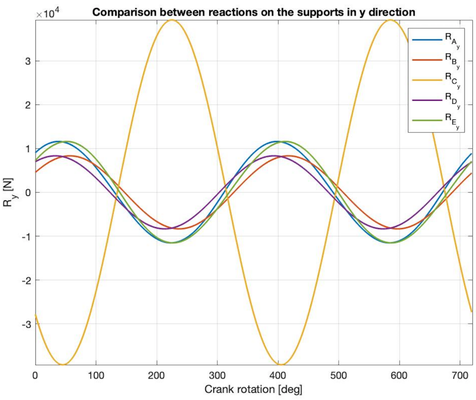

Figure 6.48: Bearings reactions in y

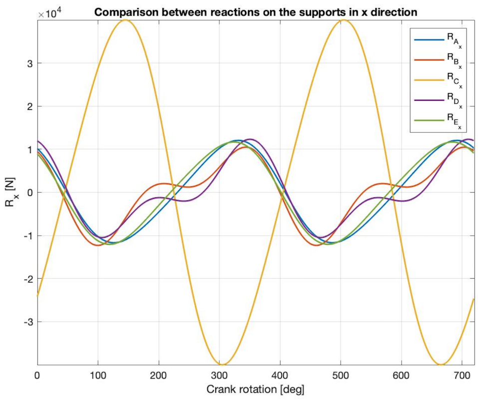

Figure 6.49: Bearings reactions in x

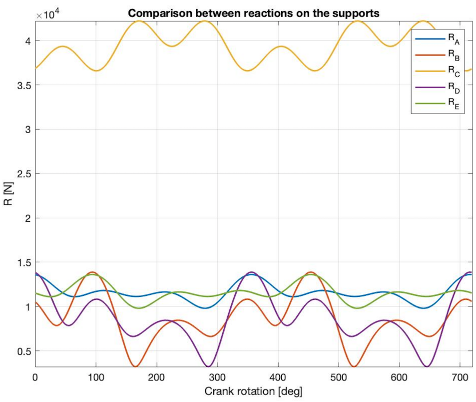

Figure 6.50: Resultant bearings reactions

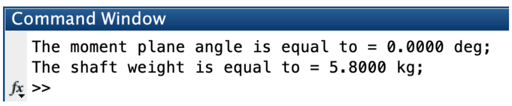

Figure 6.51: Output parameters

The figures shown above represent all the main stress to which a 90° flat-plane V8 crankshaft is subjected. The results obtained are consistent with the theory reported in the Paragraph 5.4.1 because:

- All the first order forces, both rotating and counter-rotating, illustrated in the Figures from 6.34 to 6.37 are balanced.
- The second order forces, both rotating and counter-rotating, shown in the Figures 6.38 and 6.39 are not balanced.
- The first order moments, both rotating and counter-rotating, illustrated in the Figures from 6.41 to 6.44 are balanced.
- The second order moments, both rotating and counter-rotating, shown in the Figures from 6.45 to 6.46 are not balanced.

It can be noticed that in the Figure 6.40 the resultant forces (sum of rotating and counter-rotating contributions) are balanced in y direction, but are not balanced in x direction. This fact can be easily understood looking at the Figure 5.36 in which it is clear that the sum of rotating and counter-rotating second order forces is equal to zero in y direction, but is different from zero along x direction.

The Figures from 6.48 to 6.50 shown the bearings reactions trends and it can be observed that for this crankshaft layout the most stressed bearing is the central one. In the last Figure 6.51 are reported the values of the calculated shaft weight and of the angle between the plane of the resultant first order moment and the plane of the crank throw #1 that is in this case equal to 0° because there are no first order moments.

90° V8 cross-plane engine plots

The input data assumed for the analysys are the same reported on the Table 6.2. Obiously in this is different than the flat-plane case because of the angles between the crank throws.

The output figures are:

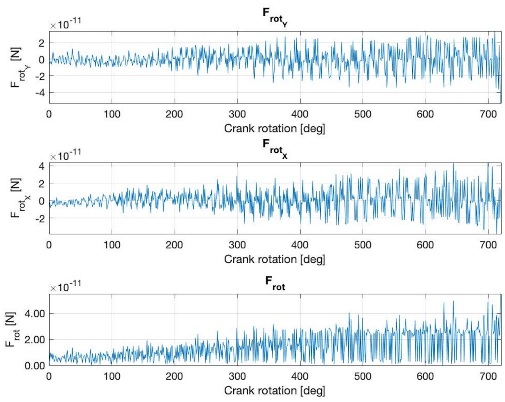

Figure 6.52: Pure rotating forces

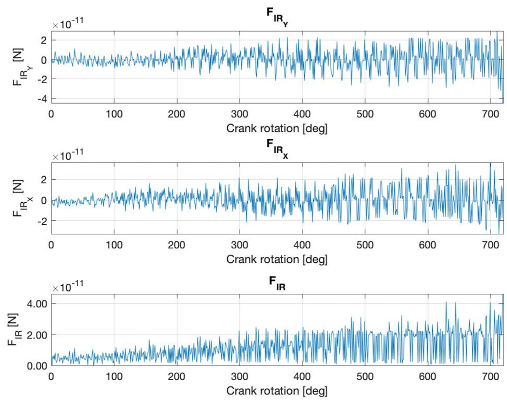

Figure 6.53: First order rotating forces

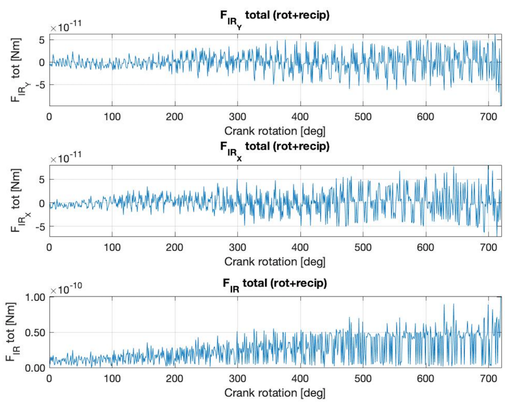

Figure 6.54: Total first order rotating forces

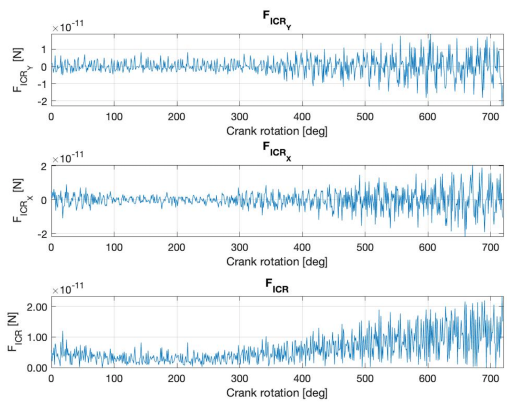

Figure 6.55: First order counter-rotating forces

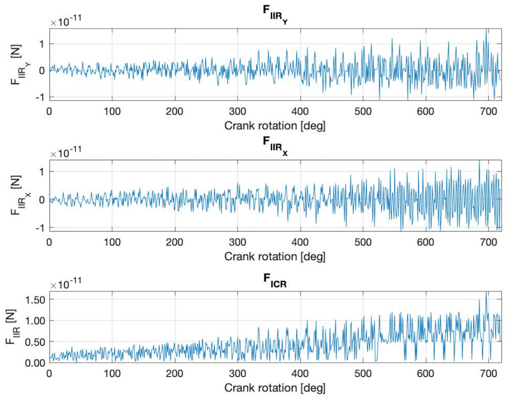

Figure 6.56: Second order rotating forces

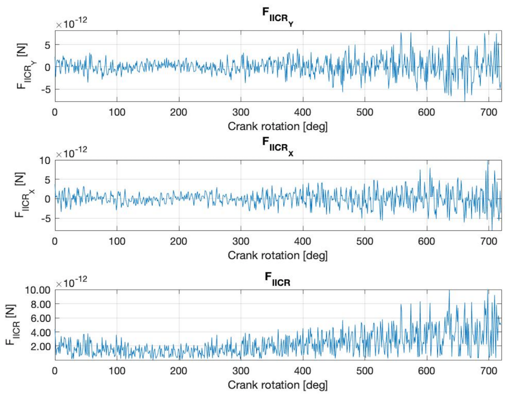

Figure 6.57: Second order counter-rotating forces

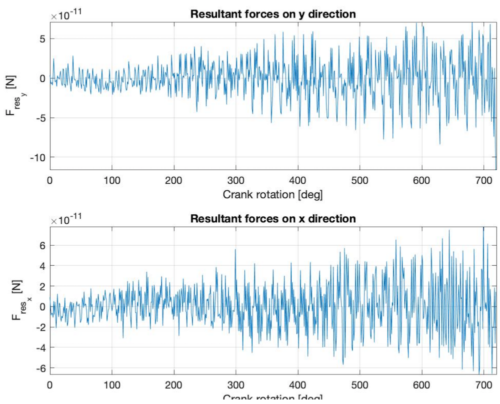

Figure 6.58: Resultant forces

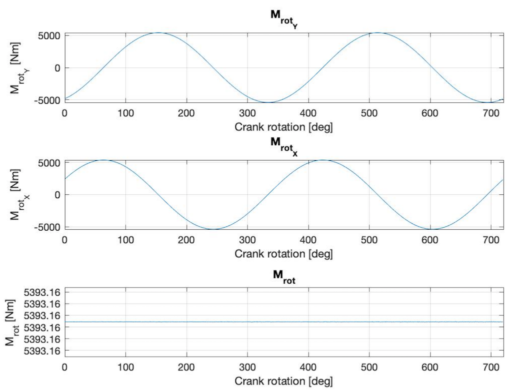

Figure 6.59: Pure rotating moments

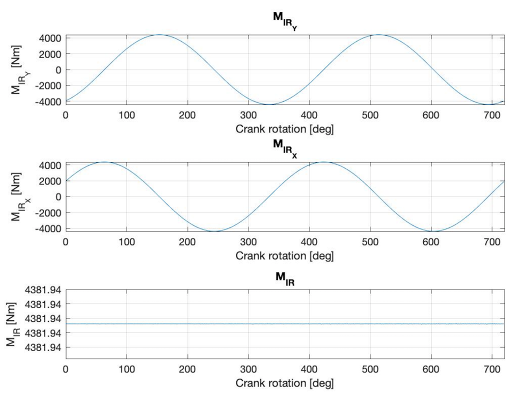

Figure 6.60: First order rotating moments

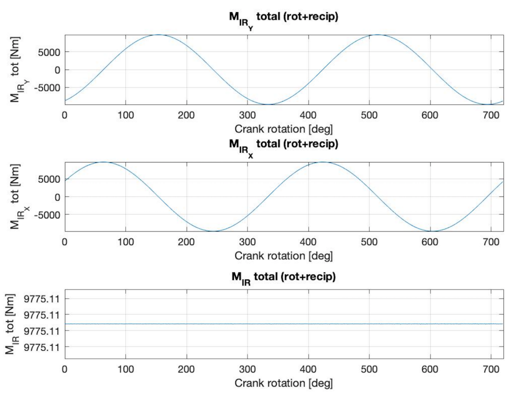

Figure 6.61: Total first order rotating moments

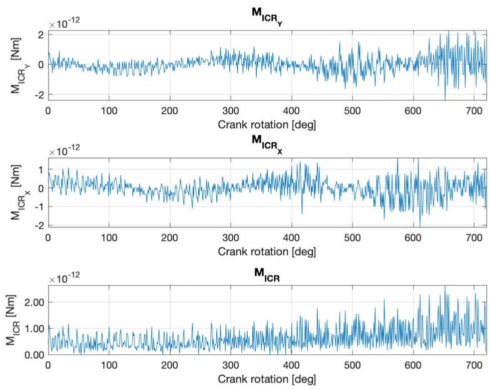

Figure 6.62: First order counter-rotating moments

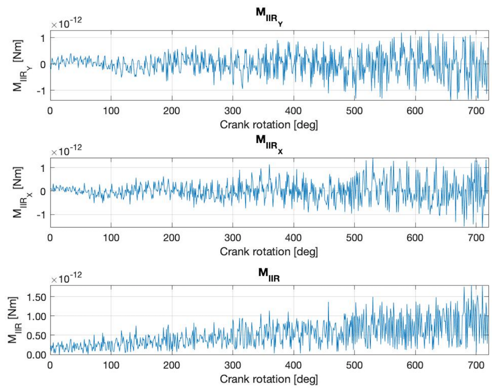

Figure 6.63: Second order rotating moments

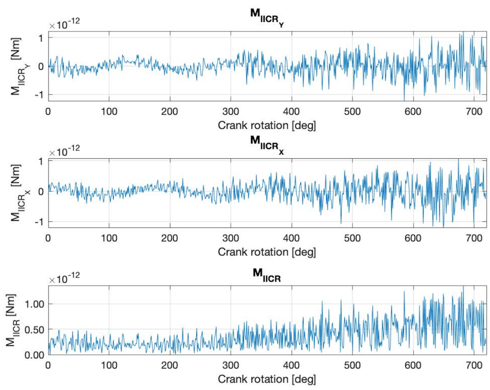

Figure 6.64: Second order counter-rotating moments

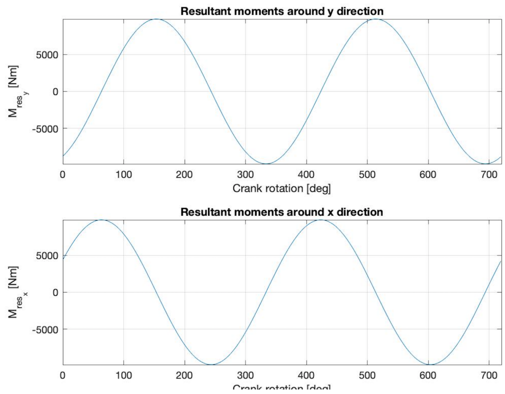

Figure 6.65: Resultant moments

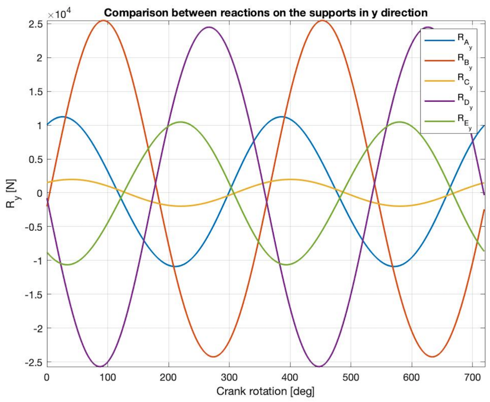

Figure 6.66: Bearings reactions in y

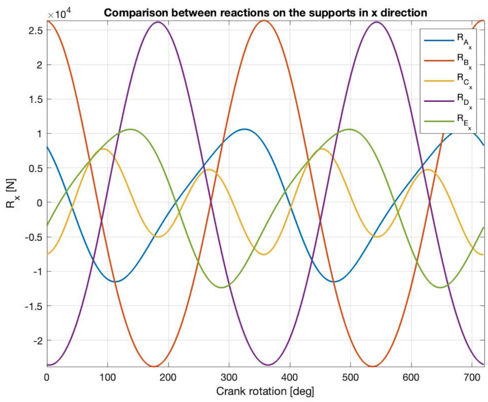

Figure 6.67: Bearings reactions in x

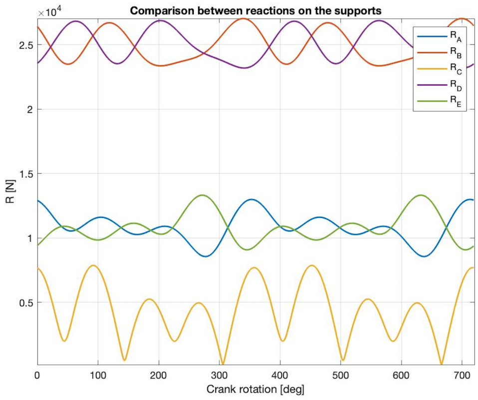

Figure 6.68: Resultant bearings reactions

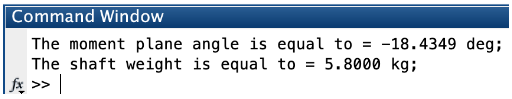

Figure 6.69: Output parameters

The figures shown above represent all the main stress to which a 90° cross-plane V8 crankshaft is subjected. The results obtained are consistent with the theory reported in the Paragraph 5.4.2 because:

- All the first order forces, both rotating and counter-rotating, reported in the Figures from 6.52 to 6.55 are balanced.
- The second order forces, both rotating and counter-rotating, shown in the Figures 6.56 and 6.57 are balanced.
- The first order rotating moment reported in the Figure 6.59 is balanced.
- The second order moments, both rotating and counter-rotating, shown in the Figures from 6.63 to 6.64 are not balanced.

In this case, the first order counter-rotating moment is balanced due to the fact to have a V-angle equal to 90°. As already written in the Paragraph 5.1, since the angle between the first order reciprocating counter-rotating forces is equal to 2ϕ (with $\phi = Vangle$), this moment results balanced.

The Figures from 6.66 to 6.68 illustrate the bearings reactions trends and it can be observed that for this crankshaft layout the most stressed bearings are the "internal" ones. In the last Figure 6.69 are reported the values of the calculated shaft weight and of the angle between the plane of the resultant first order moment and the plane of the crank throw #1 that is in this case equal to −18.43°, according to the value reported in the Paragraph 5.4.2 in the Equation 5.72.

In order to make a comparison between the flat-plane and cross-plane configurations, looking at the figures reported it can be concluded that:

- With the same input parameters considered, the bearing reactions of the flat-plane case are greater than the cross-plane.

- The bearing reactions in the flat-plane case are greater because of the second order forces make a conspicuous contribution on the whole crankshaft stress. In a cross-plane configuration instead the second order forces are balanced, and this is one of the main reasons why the latter was developed.

These two statements confirm what is reported in the Paragraphs 5.4.1 and 5.4.2.

6.3.2 Output plots of the Matlab script without balancing

In this paragraph a balancing strategy is applied to the same engine layouts illustrated in the Paragraph 6.3.1. For each of them, several configurations with different input parameters are reported in order to compare the output graphs.

Plots concenring the balanced Inline three-cylinder engine

For an Inline three-cylinder engine, in order to have a completely balanced crankshaft both the first and second order moments should be counteracted, with particular attention to the first order one because it gives the most powerful stress.

However, using the *bay-by-bay balancing strategy* only the rotating first order stress can be balanced, and this thesis work is focused on it.

Let us now show some different balanced configurations of this crankshaft:

- $BF_1 = BF_2 = BF_3 = 0.65$

  The same input parameters reported in the Table 6.1 are used. However the code requires to set up other parameters to locate the counterweights, reported in the tables below.

a	c
[m]	[m]
0.02	0.03

Table 6.3: Input data I3 crankshaft balancing (1)

Where a is equal to the distance between the crank arm and the center line of the conrod, and c is the distance bewteen the crank arm and the adjacent bearing (these distances are assumed to be constant in all the crankshaft bays).

z_{c1}	z_{c2}	z_{c3}	z_{c4}	z_{c5}	z_{c6}
[m]	[m]	[m]	[m]	[m]	[m]
0.12	0.08	0.02	-0.02	-0.08	-0.12

Table 6.4: Input data I3 crankshaft balancing (2)

Table 6.4 shows the counterweights positions along the crankshaft axis.

β_{c1}	β_{c2}	β_{c3}	β_{c4}	β_{c5}	β_{c6}
[deg]	[deg]	[deg]	[deg]	[deg]	[deg]
180°	180°	180°	180°	180°	180°

Table 6.5: Input data I3 crankshaft balancing (3)

Table 6.5 shows relative angles between each counterweight and the crank throw in which is located.

For reasons of simplification, it is chosen to have equal conterweights on each bay and to set their radius equal to the crank radius (Figures 6.12, 6.13, 6.14 and Table 6.6).

$r_{c1} = r_{c2}$	$r_{c3} = r_{c4}$	$r_{c5} = r_{c6}$
[m]	[m]	[m]
0.045	0.045	0.045

Table 6.6: Input data I3 crankshaft balancing (4)

For the sake of brevity, only the figures that are modified by the balancing operations are shown.

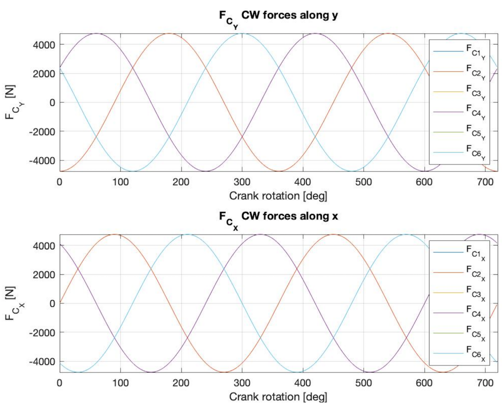

Figure 6.70: CW forces

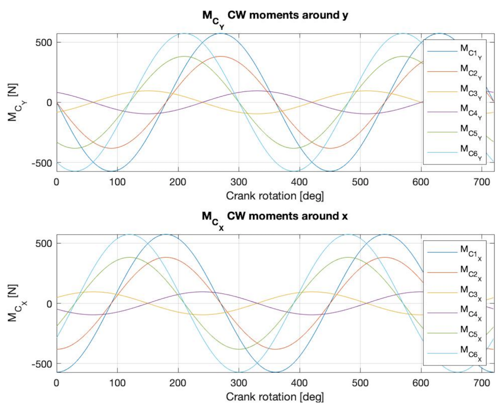

Figure 6.71: CW moments

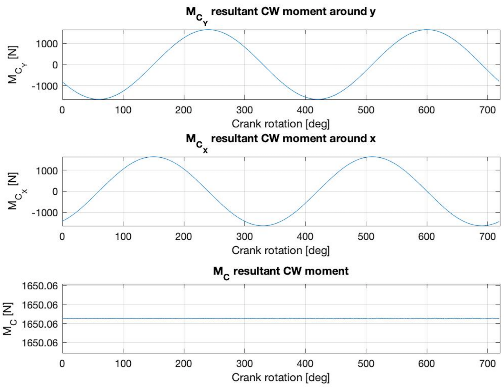

Figure 6.72: CW resultant moments

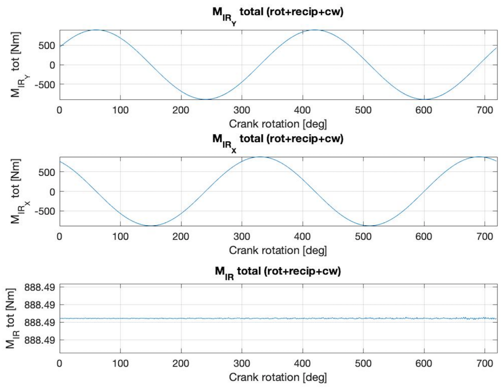

Figure 6.73: Total first order moments

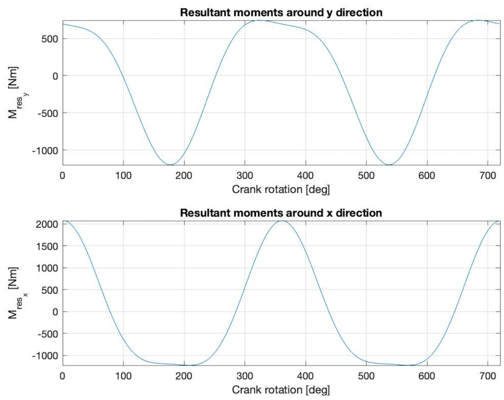

Figure 6.74: Resultant moments

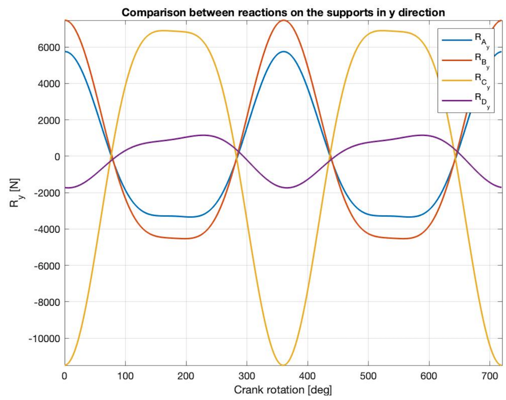

Figure 6.75: Bearings reactions in y

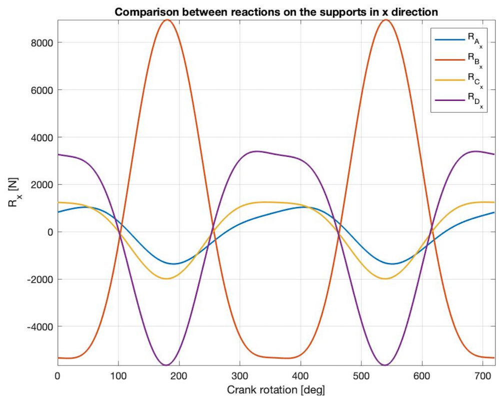

Figure 6.76: Bearings reactions in x

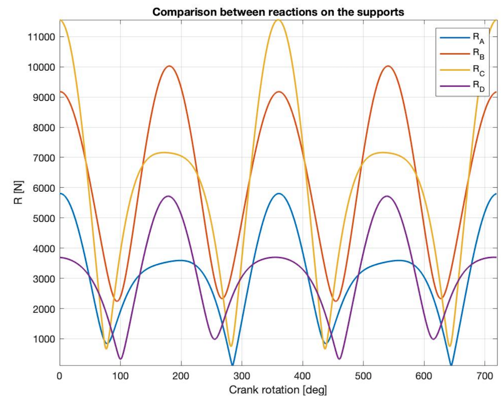

Figure 6.77: Resultant bearings reactions

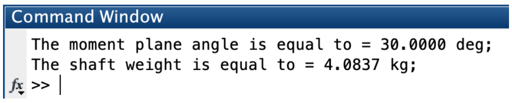

Figure 6.78: Output parameters

It can be observed that:

- Comparing the Figures 6.73 and 6.25 there is a reduction of the total first order rotating moment, due to the counteweights addition.
- The reduction of the first order rotating moment is obviously also reflected on the resultant moments trend (Figure 6.74).
- The positive and negative maximums of the bearings reactions are lower, it means that the local balancing in each bay is better than the case without counterweights.
- The crankshaft is heavier and, since the BF values are equal to each other, the resultant first order moment plane is still 30° ahead to the plane of the crank throw #1.

- $BF_1 = BF_2 = BF_3 = 1$

  Setting all the BFs equal to 1, the configuration with the maximum possible balance is reached.

  All the assumptions and parameters set in the previous case (BF=0.65) are also used in this case (Tables 6.3, 6.4, 6.5, 6.6).

  The produced graphs are:

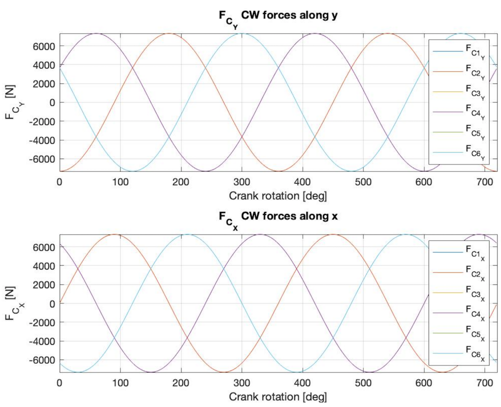

Figure 6.79: CW forces

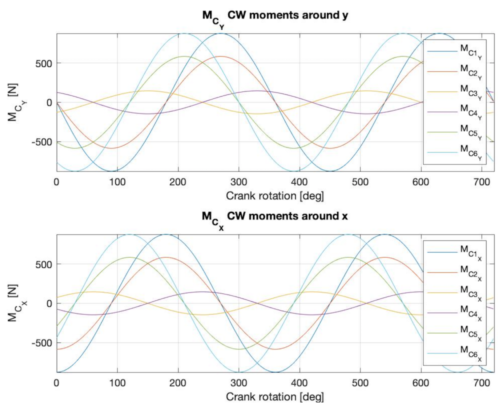

Figure 6.80: CW moments

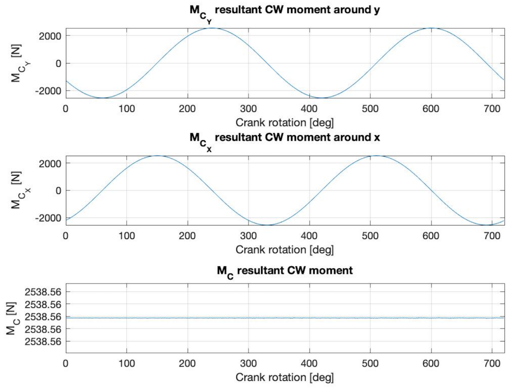

Figure 6.81: CW resultant moments

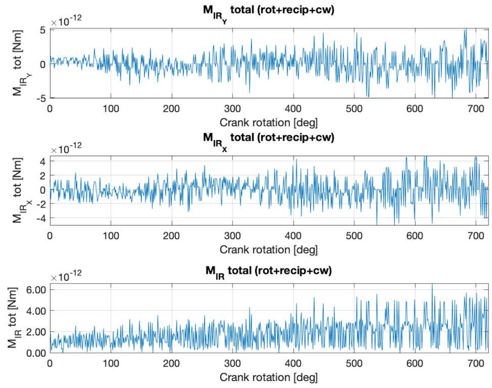

Figure 6.82: Total first order moments

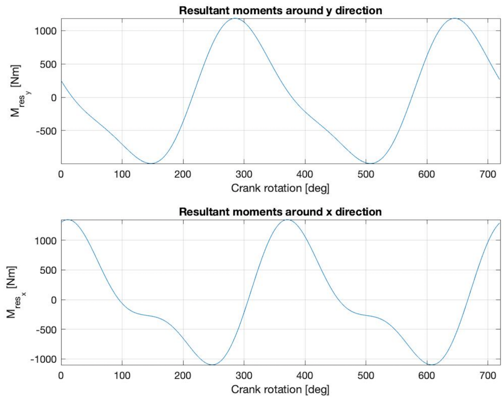

Figure 6.83: Resultant moments

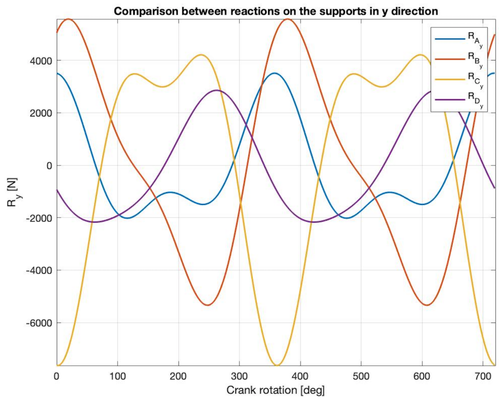

Figure 6.84: Bearings reactions in y

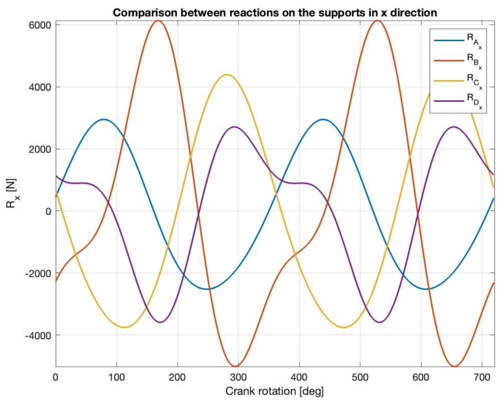

Figure 6.85: Bearings reactions in x

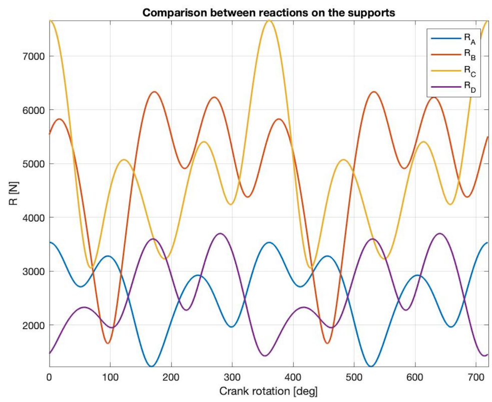

Figure 6.86: Resultant bearings reactions

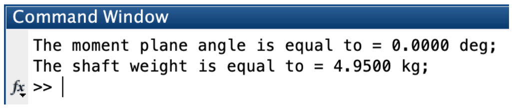

Figure 6.87: Output parameters

It can be observed that:

– Figure 6.82 shows that the first order moment are, in this case, completely balanced.

– The reduction of the first order rotating moment is obviously also reflected on the resultant moments trend (Figure 6.83).

– The positive and negative maximums of the bearings reactions are the lowest, it means that the local balancing in each bay is the best that can be achieved.

– On the contrary, this is the heaviest possible crankshaft configuration.

It is important to notice that the external counterweights (#1 and #6) can be skewed of an angle equal to the resultant first order moment plane angle, in order to counteract the first order rotating moment.

Considering to skewing the external counterweights as reported in the Table 6.7 and to keep all the BFs equal to 1:

β_{c1}	β_{c2}	β_{c3}	β_{c4}	β_{c5}	β_{c6}
[deg]	[deg]	[deg]	[deg]	[deg]	[deg]
210°	180°	180°	180°	180°	150°

Table 6.7: Input data I3 crankshaft balancing (3)

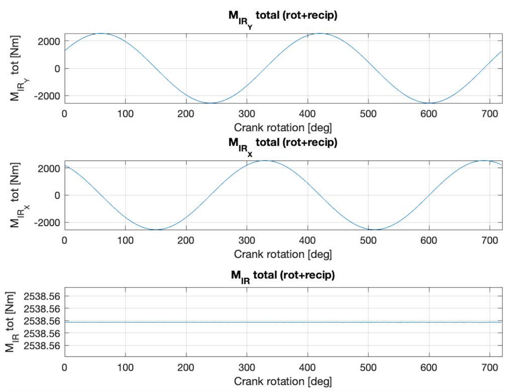

Figure 6.88: Total first order moment

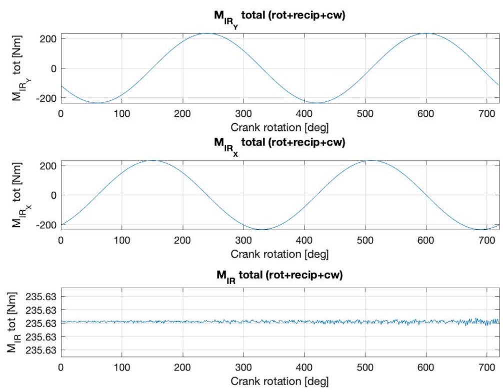

Figure 6.89: Total first order moment with CW

It can be observed that in this case the system is over-balanced, it means that skewing the external counterweights it is possible to reduce their mass continuing to have a completely balanced system to rotating first order moment.

This is an advantage because the reduction of the counterweights mass determines also the reduction of the whole crankshaft mass, therefore the shaft can rotate at higher speed.

On the contrary, because of the external counterweights #1 and #6 skewing, the first order forces are no longer balanced.

Finally, the above are just some of the possible balancing configurations that can be realized and the best possible configuration must be founded based on the specific case analized.

Plots concenring the balanced 90° V8 flat-plane engine

The main reason to add counterweights in a V8 flat-plane crankshaft is to reduce the local stress on the bearings in each bay.

Let us now show a balanced configuration:

- Considering: $BF_1 = BF_2 = BF_3 = BF_4 = 0.65$

  The same input parameters reported in the Table 6.2 are used. However the code requires to set up other parameters to locate the counterweights, reported in the tables below.

z_{c1}	z_{c2}	z_{c3}	z_{c4}	z_{c5}	z_{c6}	z_{c7}	z_{c8}
[m]	[m]	[m]	[m]	[m]	[m]	[m]	[m]
0.21	0.15	0.09	0.03	-0.03	-0.09	-0.15	-0.21

Table 6.8: Input data V8 flat-plane crankshaft balancing (1)

Table 6.8 shows the counterweights positions along the crankshaft axis.

β_{c1}	β_{c2}	β_{c3}	β_{c4}	β_{c5}	β_{c6}	β_{c7}	β_{c8}
[deg]	[deg]	[deg]	[deg]	[deg]	[deg]	[deg]	[deg]
180°	180°	180°	180°	180°	180°	180°	180°

Table 6.9: Input data V8 flat-plane crankshaft balancing (2)

Table 6.9 shows relative angles between each counterweight and the crank throw in which is located.

For reasons of simplification, it is chosen to have equal conterweights on each bay and to set their radius equal to the crank radius (Figures 6.12, 6.13, 6.14 and Table 6.10).

$r_{c1} = r_{c2}$	$r_{c3} = r_{c4}$	$r_{c5} = r_{c6}$	$r_{c7} = r_{c8}$
[m]	[m]	[m]	[m]
0.045	0.045	0.045	0.045

Table 6.10: Input data V8 flat-plane crankshaft balancing (3)

For the sake of brevity, only the figures that are modified by the balancing operations are shown.

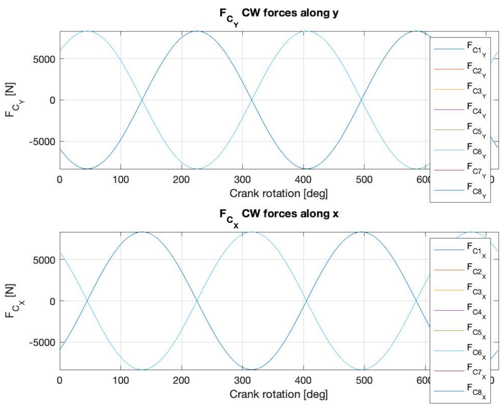

Figure 6.90: CW forces

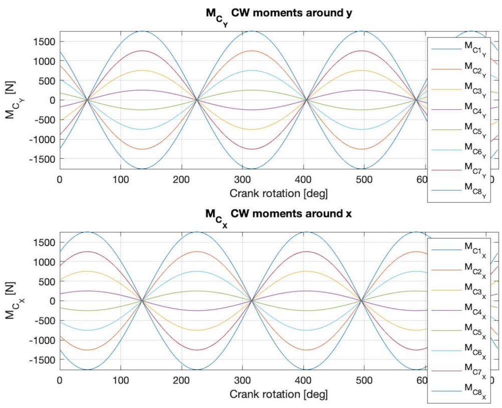

Figure 6.91: CW moments

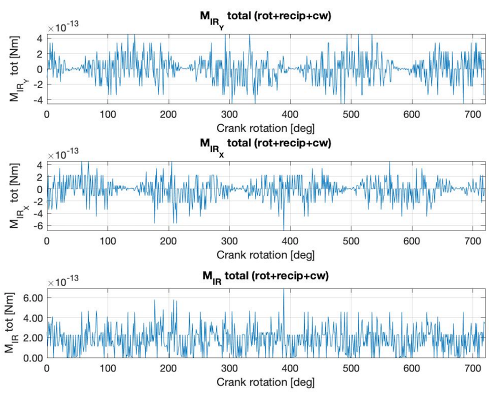

Figure 6.92: Total first order moment

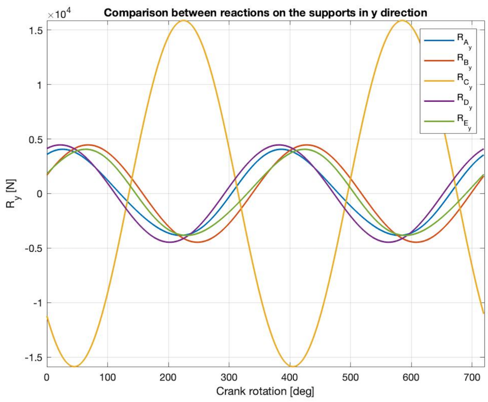

Figure 6.93: Bearings reactions in y

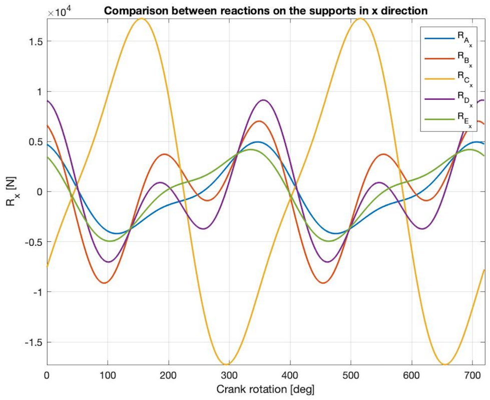

Figure 6.94: Bearings reactions in x

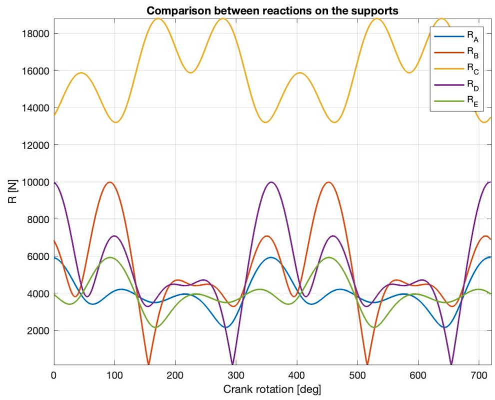

Figure 6.95: Resultant bearings reactions

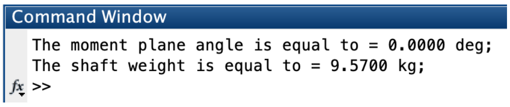

Figure 6.96: Output parameters

It can be said that:

- Balancing with the same BF in all the shaft bays with the counterweights placed opposite to the relative crank (at 180°) does not cause any change in terms of global balancing. In fact, the only unbalanced forces are the second order rotating and counter-rotating ones.

- The addition of counterweights does not make any variations in terms of resultant forces or moments.

- As can be seen in the Figures 6.93, 6.94 and 6.95 the bearings reactions are decreased; this is ad advantage for the bearings duration. On the contrary, the shaft weight (Figure 6.96) is increased.

- Increasing the BF the bearings reactions decrease and the weight increases.

  The reactions trends in a case where all balancing factors values are set equal to 1 are reported below (all the other parameters are considered the same as the case above).

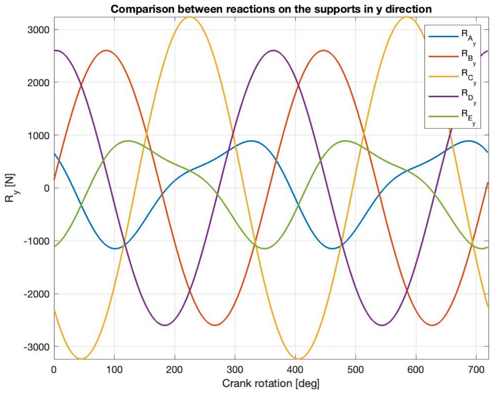

Figure 6.97: Bearings reactions in y

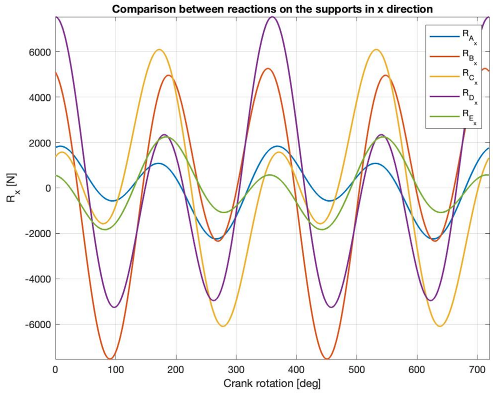

Figure 6.98: Bearings reactions in x

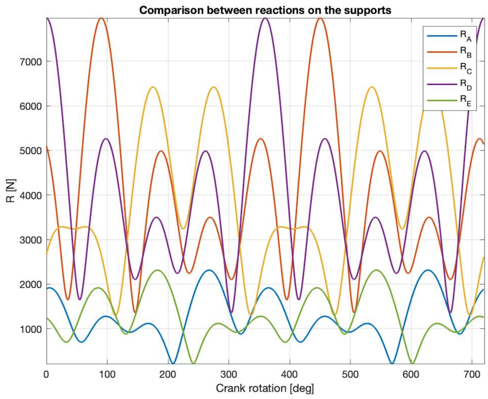

Figure 6.99: Resultant bearings reactions

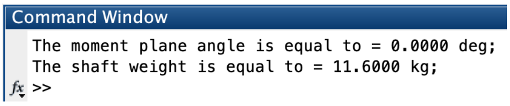

Figure 6.100: Output parameters

As can be observed this is the configuration with the lowest reactions but the heaviest crankshaft.

Moreover, it can be understood that in this configuration it is not particularly useful to skew the counterweight respect to their referred crank throw, unless exceptional cases.

Plots concenring the balanced 90° V8 cross-plane engine

The main reasons to add counterweights in a V8 cross-plane crankshaft are to counteract the first order rotating moment and to decrease the bearings reactions.

Let us now illustrate a balanced configuration:

- Considering: $BF_1 = BF_2 = BF_3 = BF_4 = 0.65$

  The same input parameters reported in the Tables 6.2, 6.8, 6.9 and 6.10 are used.

  Even in this case for the sake of brevity, only the plots affected by the balancing are reported.

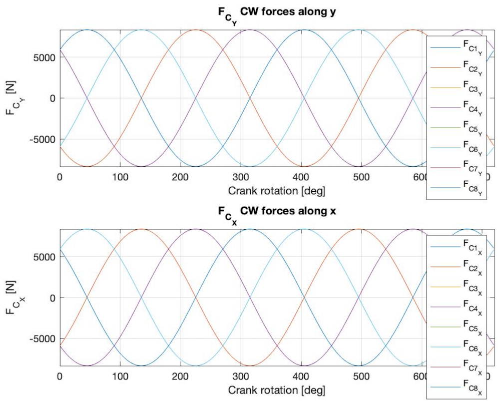

Figure 6.101: CW forces

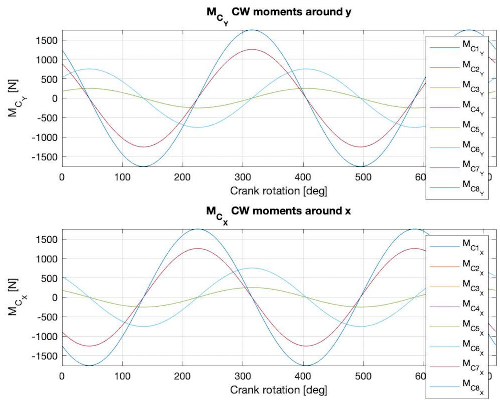

Figure 6.102: CW moments

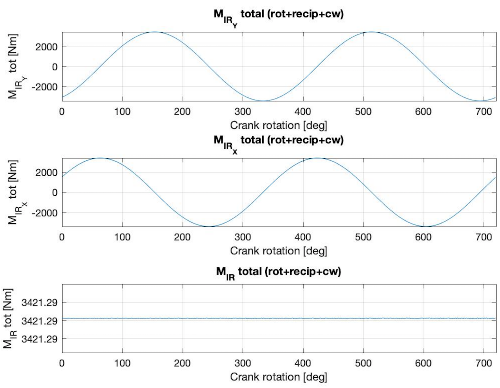

Figure 6.103: Total first order moments

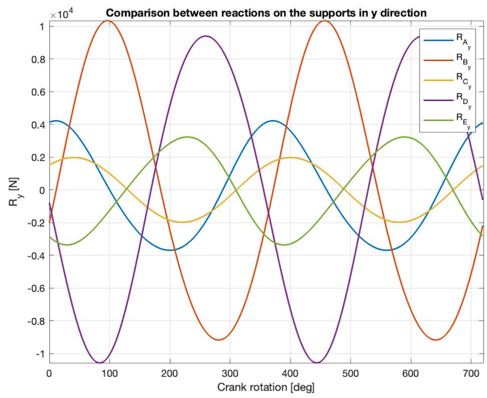

Figure 6.104: Bearings reactions in y

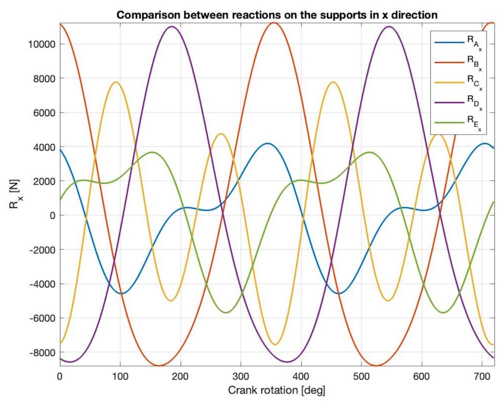

Figure 6.105: Bearings reactions in x

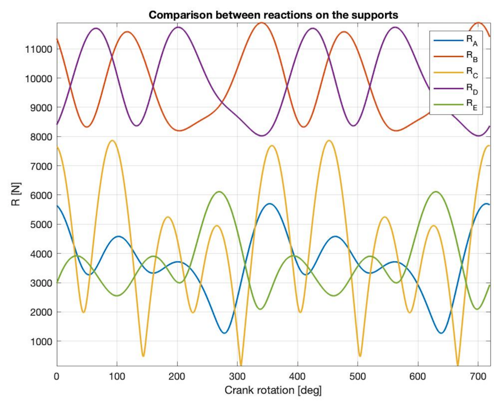

Figure 6.106: Resultant bearings reactions

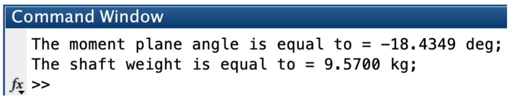

Figure 6.107: Output parameters

It can be observed that:

- Making a comparison between 6.103 and 6.61 it can be seen that the resultant first order moment is decreased, because of the addition of counterweights.
- The most stressed bearings are the same as the non-balanced configuration (B and D) but the positive and negative maximum are lower.
- The crankshaft is heavier than the non-balanced configuration and since the BF values are equal to each other the moment plane is still −18.43° behind the plane of the crank throw of the cylinders #1 and #5.

- Considering now $BF_1 = BF_2 = BF_3 = BF_4 = 1$:

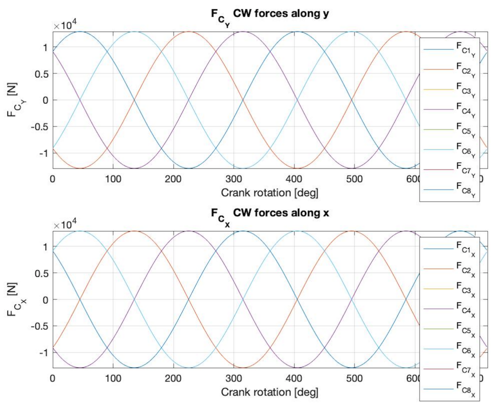

Figure 6.108: CW forces

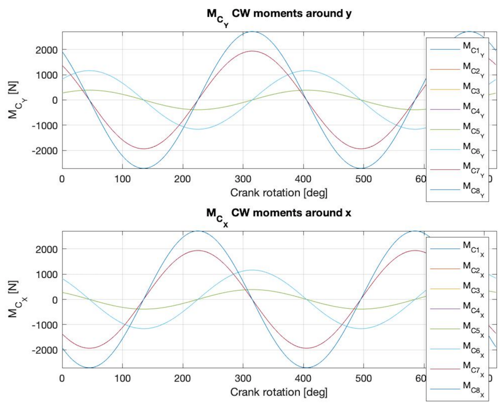

Figure 6.109: CW moments

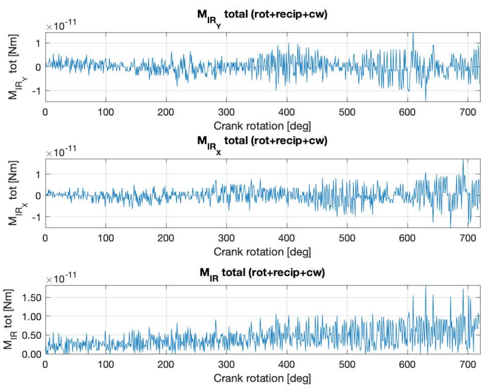

Figure 6.110: Total first order moments

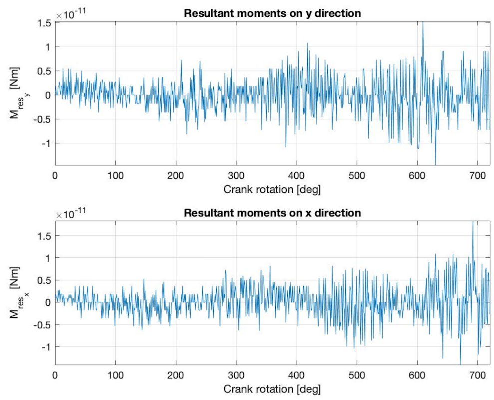

Figure 6.111: Resultant moments

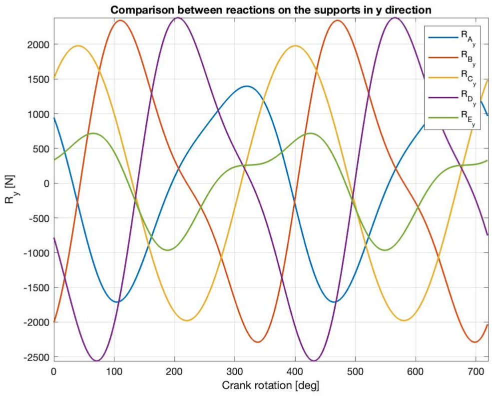

Figure 6.112: Bearings reactions in y

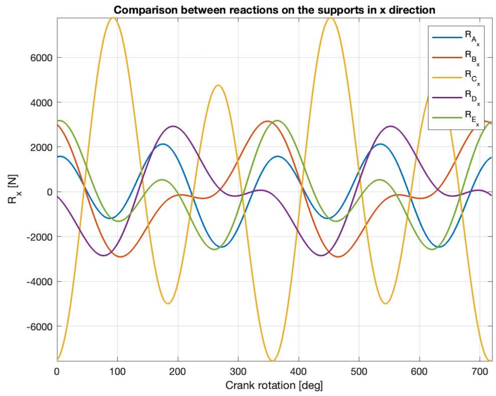

Figure 6.113: Bearings reactions in x

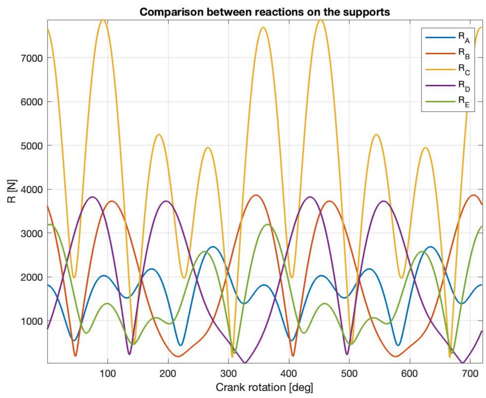

Figure 6.114: Resultant bearings reactions

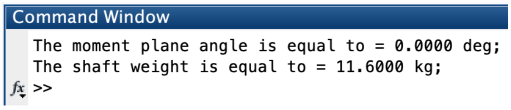

Figure 6.115: Output parameters

As can be seen:

- The first order rotating moment is, in this case, completely balanced (Figure 6.110). For the layout considered, this means that even the resultant moments are completely balanced (Figure 6.111).
- The addition of counterweights does not make any variation in terms of global first and second order forces balance.
- The BF values are the highest possible, it means that this is the configuration with the lowest bearings reaction but also the heaviest.

As already said for the balancing of the I3-cylinder, it is important to notice that skewing the external counterweights of an angle equal to the resultant first order moment angle plane (of the non-balanced configuration), it is possible to reach a system in which the first order rotating moment is completely balanced, but with a lighter crankshaft.

In order to summarize, the crankshaft balancing is a complex topic with a lot of variables to consider during the study.

Starting from the basic theory of the slider-crank system, in this thesis work attempts were made to give to the reader an overview to what does it mean to balance a crankshaft, which are the forces at work and what role they play in the crankshaft state-of-balancing. To do all this, several engines, and therefore crankshafts, configurations have been illustrated, and for each of them mathematical relationships that governs the "balancing process" have been written.

However, the main purpose of the thesis was to write a Matlab script capable of manage and analyze in an "easy way" any layout that the user deems useful to simulate.

It is obvious that this script is only a first approximation, because a series of simplifying hypotheses have been made in order to make the system easy to study via Matlab.

Nonetheless, looking at the figures illustrated in the Chapter 6 it can be concluded that the Matlab code, according to the hypoteses made, provides a good approximation of the stresses involved both in the case of a non-balanced system and in the case with the balancing strategy applied.

The next step of this work could to be find a way to optimize the parameters at play. To be clear: to design a strategy that takes into account all the key variables, with their relative weight in the balancing procedure, that allows to find the best set of parameters (masses, radii, balance factors) for the desired crankshaft configuration which grant to have the most possible crankshaft balancing, to minimize the bearing reactions and to have the lightest possible shaft.

Bibliography

[1] H. Heisler, *Advanced vehicle technology*. Elsevier, 2002.

[2] D. Giacosa, *Motori endotermici*. HOEPLI EDITORE, 2000.

[3] R. L. Norton, "Design of Machinery Edition", 1999.

[4] G. Bocchi, *Motori a quattro tempi*. HOEPLI EDITORE, 1987, vol. 4040.

[5] Wikipedia. (2019). Engine balance, [Online]. Available: https://en.wikipedia.org/wiki/Engine_balance.

[6] Zanghi. (2018). Travi continue, [Online]. Available: https://profzanghi.weebly.com/uploads/9/0/0/4/9004706/travi_continue.pdf.

[7] A. Carpinteri, *Scienza delle costruzioni*. Pitagora, 1993.