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# Abstract

Magnetic 3D tracking of human body motion is an important technology in many biomedical applications such as human motion analysis, human-guided surgical instrumentation and human training simulator systems. Most of these tracking technologies are low cost, low power and small sized, features that make these devices wearable and so allow to monitor the patient's activities outside the clinical environs. This thesis presents a novel algorithm for monitoring the movement of a human body part in a space by computing the position and orientation using magnetic 3D tracking technology. The algorithm takes as input a set of real-time data collected by a magnetic angular rate and gravity (MARG) sensor which is part of a wireless and low-cost embedded six degrees of freedom (6DOF) tracking system. Generation and sensing of a magnetic-dipole field at a known frequency provides enough information to determine the position of the sensor relatively to the source, while Earth's magnetic field, gravitational field and angular rate sensing provides the orientation estimation of the sensor. In this work, several orientation algorithms are investigated and compared such as TRIAD and QUEST algorithms which use only accelerometer and magnetometer measurements, and Madgwick and the Kalman Filter which use also the gyroscope measurement. All the presented algorithms use the quaternion representation which avoids the problem of singularities associated with orientation estimation. Performances in terms of accuracy and computational time have been evaluated to determine the best algorithm to perform human body motion tracking.



# Chapter 1

## Introduction

Accurate measurement of position and orientation of an object in the space is a key technology in many fields such as aerospace [5], robotics [1], [4] and navigation [11]. In the last years this type of problem became interesting in many biomedical applications based on the human motion tracking. Minimally invasive surgery is realized with the use of medical robotic systems in [3], hand motion analyses is performed with an electromagnetic tracking system in [2]. In rehabilitation, motion tracking is an enabling technology, specially to monitor the patient's activities outside the clinical environs and then correct him. To this purpose is needed a wearable system able to record and store data for a long period of time (days or an entire week). The technology used for the motion tracking varies in accordance with the performance requested for the specific application. Optical and magnetic tracking offer the better solution in terms of accuracy. Optical tracking system can determine the position reaching sub-millimeter accuracy as the VICON [12] and NDI optical systems [8]. The limitation of optical systems is the dependence to the line-of-sight (LOS) between an unobstructed marker and a stereo camera, which is a crucial aspect in image-guided surgical instrumentation and applications where visually-impaired people could obstruct the device to be tracked, such as TAMO3 systems [7]. All Polhemus tracking systems [9] are commercial devices and use electromagnetic technology to track motion of different part of the body as head, hand and fingers. There are also many studies on the human motion tracking using inertial sensors, as the ambulatory monitoring system developed in [10] for the estimation of walking parameters and the measurement system of inclination of body segments and activity of daily living (ADL) developed by [6]. An IMU (Inertial Measurement Unit) is made of gyroscope and accelerometer allowing the tracking of rotational and translational movements. A drawback of IMU sensor is that they aren't able to measure the rotation around the gravity axis, so the attitude estimation is incomplete. A MARG (Magnetic, Angular Rate, and Gravity) sensor is a IMU which incorporates a tri-axis magnetometer and is able to provide a complete measurement of orientation relative to the direction of gravity and the earth's magnetic field.

### 1.1 Tracking 3D

### 1.2 Application fields

### 1.3 State of the art



## Chapter 2

# Attitude determination

### 2.1 Mathematical representation of the orientation

To describe the orientation of a body two different coordinate systems are used: one is attached to the body itself and it's referenced to the second, which is fixed. If the coordinate system is the Cartesian, we use the frame that is a set of three orthonormal vectors. In this work we consider the body frame, which moves, and the Earth fixed frame with three axes  $x, y, z$ . There are multiple ways to represent the rotation of an object in the space, the commonly used are rotation matrices, euler angles and quaternions.

### 2.2 Quaternion representation of the orientation

A rotation in the space can be completely characterized by quaternion. A quaternion is a 4 dimensional vector, it belongs to the  $\mathfrak{R}^4$  space. Their form is:

$$q = ( q_1 , q_2 , q_3 , q_4 ) \quad (2.1)$$

where  $q_1$  is the scalar part and  $(q_2, q_3, q_4)$  is the vectorial part. It is possible use the notation  $q = (q_1, w)$  where  $w = (q_2, q_3, q_4)$ .

The most intuitive quaternion representation is:

$$q = \left[ \cos \left( \frac{\theta}{2} \right), \hat{e} \sin \left( \frac{\theta}{2} \right) \right] \quad (2.2)$$

where  $\hat{e} = (e_x, e_y, e_z)$  is the axis vector and  $\theta \in [0, 2\pi]$  is the the angle of rotation around  $\hat{e}$ .

### 2.3 Magnetic field and gravitational field

The Earth's magnetic field is like the one produced by a magnet bar-field dipole. The geographical North pole is not coincident with the north pole of the magnet because the magnet axis is tilted 11,5 degrees from the Earth axis . By convention invisible magnetic field lines generated by the Earth are directed from south to north pole of the magnet. The Earth's magnetic field is approximately stationary, because it's caused by the flow of liquid iron in Earth's core, which creates electric currents that produces at finally the magnetic field. The unit of magnetic field is the Gauss (G) or Tesla (T) which is the SI unit. The conversion is  $1\text{G} = 100 \mu\text{T}$ . The Earth's field intensity is in the interval between 0.25 G and 0.65 G (25-65  $\mu\text{T}$ ).

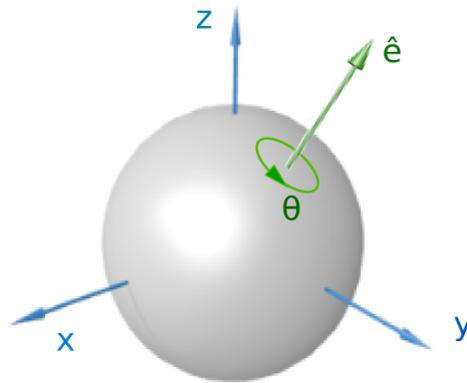


Figure 2.1: quaternion representation

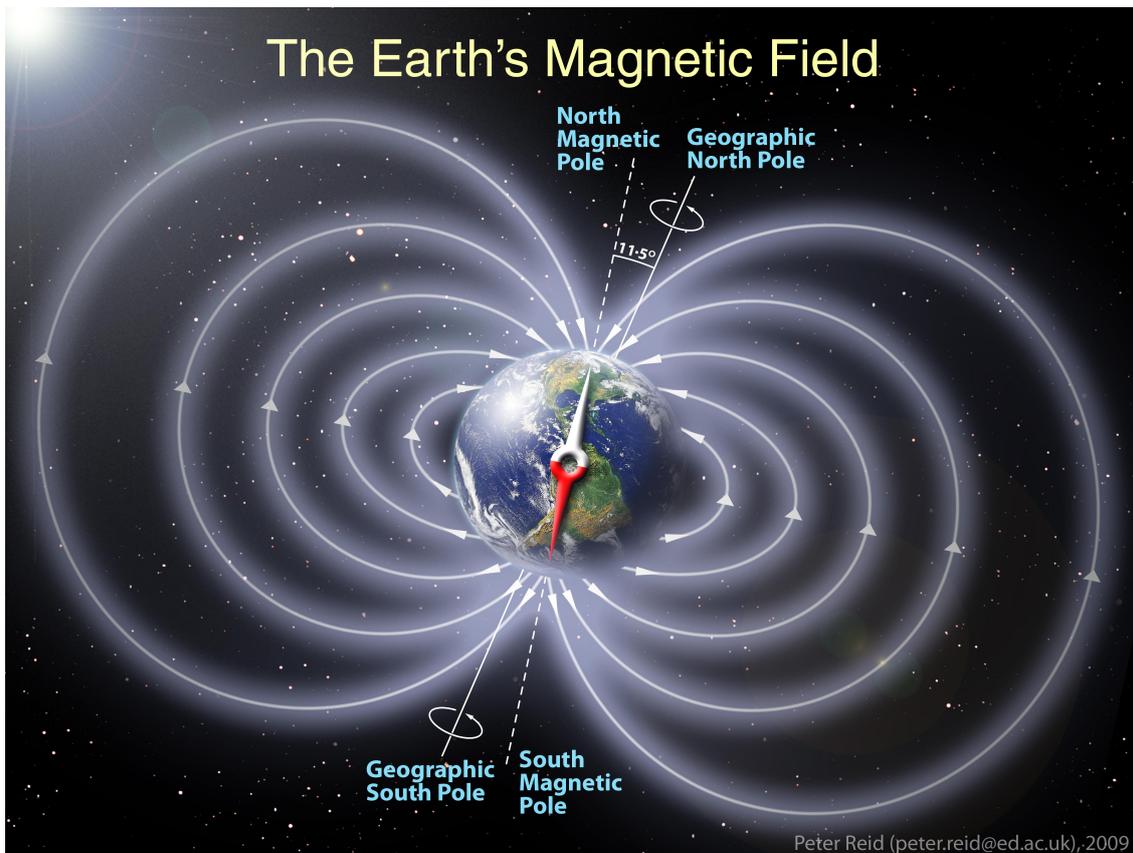


Figure 2.2: The Earth's magnetic field

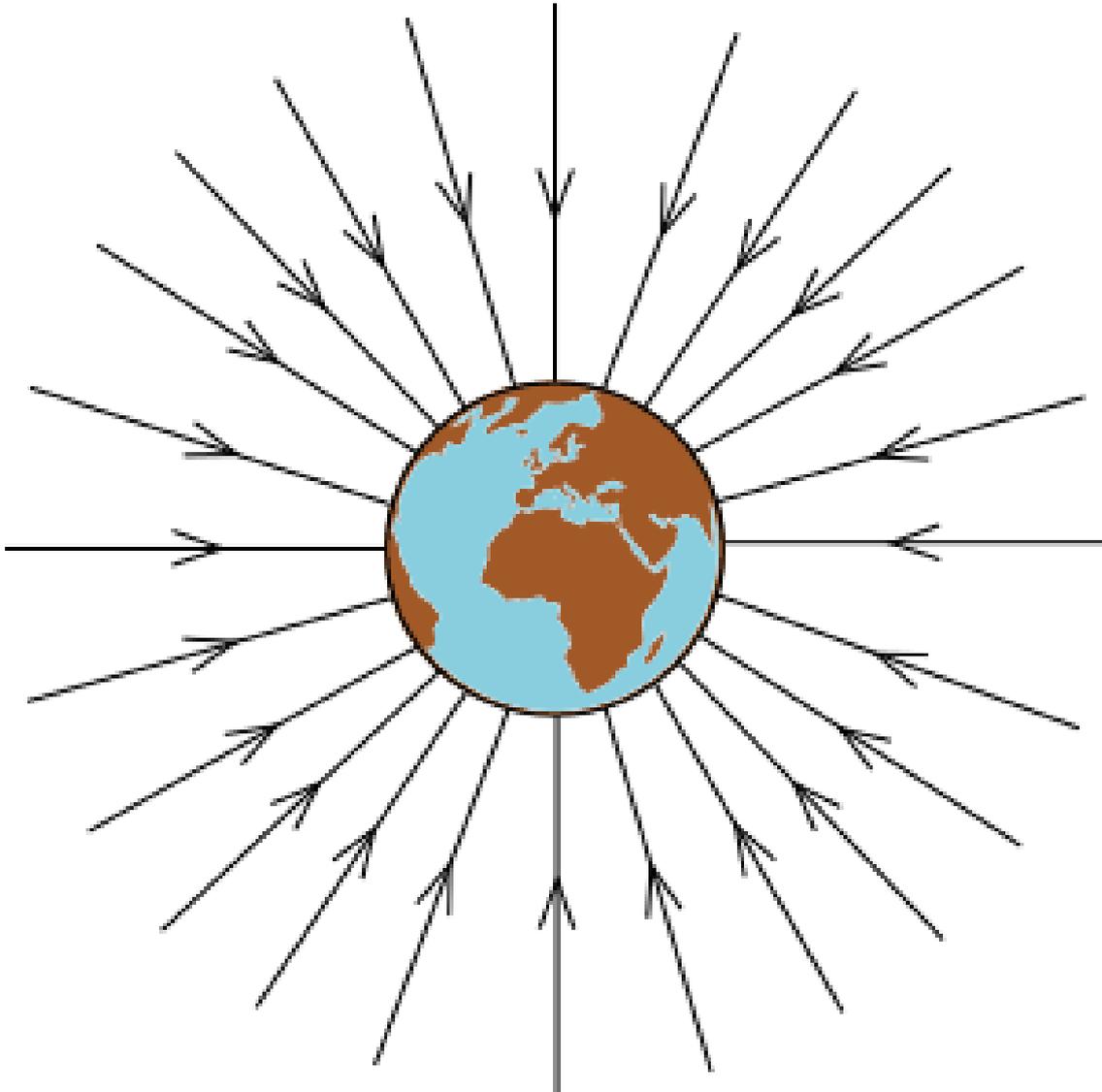


Figure 2.3: The Earth's gravitational field

The Earth's mass exercises attraction on the other bodies. The gravitational field can be seen as multiple vectors directed to the Earth core and whose amplitude decreasing by leaving the Earth. The acceleration due to Earth's gravity varies depending on the distance from the Earth core. At poles where the distance is minimal (the terrestrial ray at Poles is about 6356,988 km) the gravity measures  $g=9,823 \text{ m/s}^2$ , while at Equator that measure the maximum distance (The terrestrial ray at Equator is about 6378,388 km) gravity is  $g=9,789 \text{ m/s}^2$ . So over Earth's surface the gravity is mainly constant and can be computed as a mean of this two values, approximately of  $g \approx 9.81 \text{ ms}^2$ . To measure it the sensor needs to be stationary, otherwise it will sense linear acceleration due to the body motion. To avoid this, it's possible to implement a low-pass filter, to register only the gravitational bias.

## 2.4 Algorithms which use accelerometer and magnetometer

### 2.4.1 AQUA

### 2.4.2 QUEST

### 2.4.3 TRIAD

The starting point of this algorithm are the two nonparallel reference unit vectors  $V_1$  and  $V_2$  and their correspond

$$W_1 = \frac{a}{\sqrt{a_x^2 + a_y^2 + a_z^2}} \quad (2.3)$$

$$W_2 = \frac{m}{\sqrt{m_x^2 + m_y^2 + m_z^2}} \quad (2.4)$$

The orthogonal matrix  $A$  we want to find satisfy the following equations:

$$\begin{cases} AV_1 = W_1 \\ AV_2 = W_2 \end{cases} \quad (2.5)$$

By these equations,  $A$  is overdetermined, so it's possible to construct two triads of orthonormal reference and observation vectors:

$$r = (r_1, r_2, r_3) \quad (2.6)$$

$$s = (s_1, s_2, s_3) \quad (2.7)$$

Now the relation is:

$$Ar_i = s_i, \quad i = 1, 2, 3 \quad (2.8)$$

$A$  is unique and determined by these two equivalent expressions, which define the TRIAD solution:

$$A = \sum_{i=1}^3 s_i r_i^T \quad (2.9)$$

By solving equations (2.5) the attitude matrix also has to satisfy a necessary and sufficient condition expressed by:

$$V_1 \cdot V_2 = W_1 \cdot W_2 \quad (2.10)$$

Once obtained the direct cosine matrix, we transform this with the following equation to compute the corresponding quaternion:

$$q_1 = \frac{1}{2} \sqrt{1 + A_{11} + A_{22} + A_{33}} \quad (2.11)$$

$$q_2 = \frac{1}{4q_1} (A_{23} - A_{32}) \quad (2.12)$$

$$q_3 = \frac{1}{4q_1} (A_{31} - A_{13}) \quad (2.13)$$

$$q_4 = \frac{1}{4q_1} (A_{12} - A_{21}) \quad (2.14)$$

The final quaternion, the one used to rotate the frame, is the conjugate:

$$q = (q_1, -q_2, -q_3, -q_4) \quad (2.15)$$

## 2.5 Algorithms which use accelerometer, magnetometer and gyroscope

### 2.5.1 Madgwick algorithm

The Madgwick algorithm uses three set of three vectors measurement. The assumption of the algorithm is that accelerometer measures only gravity and magnetometer measures only the earth's magnetic field. So data needs to be preprocessed with a low pass filtering, to remove linear acceleration and the magnetic interference from the magnetometer data. Initial condition: the reference frame is considered the first measurement of the sensor, so the initial data recorded by the sensor will represent the orientation of the reference frame. The algorithm will calculate variation from this first measurement. Madgwick use as reference frame known vectors of gravity and magnetic field and start to align the sensor to these referenced vector. This is less practiced, we prefer choose the reference direction as first measurement of the sensor with respect to a predefined reference direction. DESCRIZIONE E RAGIONAMENTO PER IL LOW PASS FILTER Campionamento da 1 a 515 Hz: da 10 Hz in poi sul paper va. Errore statico  $\pm 2^\circ$ , dinamico  $\pm 7^\circ$  campionando a 10 Hz. Un livello di accuratezza adatto a human motion.

### Orientation from angular rate

The gyroscope measurements in rad/s are collected in a vector called  ${}^S\boldsymbol{\omega}$  and from this the quaternion is obtained as:

$${}^S\boldsymbol{\omega} = [0 \ \omega_1 \ \omega_2 \ \omega_3] \quad (2.16)$$

The quaternion derivative calculated from the gyroscope measurements as in eq. 2.17 and represents the rate of change of the earth frame relative to the sensor frame.

$${}^S_E\dot{\mathbf{q}} = \frac{1}{2} {}^S_E\hat{\mathbf{q}} \otimes {}^S\boldsymbol{\omega} \quad (2.17)$$

So it is possible to obtain the orientation of the earth frame relative to the sensor frame at instant  $t$  (iteration  $k$ ),  ${}^S_E\mathbf{q}_{\omega,t}$  can be computed by numerically integrating the quaternion derivative, if the initial conditions are known: eq. 2.18 and 2.19, where  $\Delta t$  is the sampling period ( $1/f_s$ ),  ${}^S\boldsymbol{\omega}_t$  is the angular rate measured at time  $t$  and  ${}^S_E\hat{\mathbf{q}}_{est,t-1}$  is the orientation estimation at the previous instant.

$${}^S_E\dot{\mathbf{q}}_{\omega,t} = \frac{1}{2} {}^S_E\hat{\mathbf{q}}_{est,t-1} \otimes {}^S\boldsymbol{\omega}_t \quad (2.18)$$

$${}^S_E\mathbf{q}_{\omega,t} = {}^S_E\hat{\mathbf{q}}_{est,t-1} + {}^S_E\dot{\mathbf{q}}_{\omega,t}\Delta t \quad (2.19)$$

### Orientation from observation vectors

The quaternion that rotates the earth reference system to the sensor reference system is found as the minimum of an error function. It is a typical optimization problem 2.20 where the objective function or error function is defined as in 2.21.

$$\min_{{}^S_E\hat{\mathbf{q}} \in \mathbb{R}^4} \mathbf{f} \left( {}^S_E\hat{\mathbf{q}}, {}^E\hat{\mathbf{d}}, {}^S\hat{\mathbf{s}} \right) \quad (2.20)$$

$$\begin{aligned}
\mathbf{f}\left(\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}, \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}, \begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}}\right) &= \begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}^* \otimes \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}} \otimes \begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}} - \begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}} = \\
&= \begin{bmatrix} 2d_x \left(\frac{1}{2} - q_3^2 - q_4^2\right) + 2d_y (q_1q_4 + q_2q_3) + 2d_z (q_2q_4 - q_1q_3) - s_x \\ 2d_x (q_2q_3 - q_1q_4) + 2d_y \left(\frac{1}{2} - q_2^2 - q_4^2\right) + 2d_z (q_1q_2 + q_3q_4) - s_y \\ 2d_x (q_1q_3 + q_2q_4) + 2d_y (q_3q_4 - q_1q_2) + 2d_y \left(\frac{1}{2} - q_2^2 - q_3^2\right) - s_z \end{bmatrix} \quad (2.21)
\end{aligned}$$

In 2.21  $\begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}$  is the quaternion representing the earth frame field,  $\begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}}$  is the quaternion representing the sensor measurement frame of magnetic or gravitational frame and  $\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}$  is the quaternion computed from the optimization problem.

So the minimum of this function corresponds to the minimal distance between the reference direction of the field in the earth frame  $\begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}$  and the measured field in the sensor frame  $\begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}}$ .

The simplest solution used by Madgwick is the gradient descent algorithm, an iterative solution which finds the orientation estimation is 2.22.

$$\begin{smallmatrix} S \\ E \end{smallmatrix} \mathbf{q}_{k+1} = \begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_k - \mu \frac{\nabla \mathbf{f}\left(\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_k, \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}, \begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}}\right)}{\|\nabla \mathbf{f}\left(\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_k, \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}, \begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}}\right)\|} \quad k = 0, 1, 2, \dots, n \quad (2.22)$$

Where  $\mu$  is a variable step size and the iteration for  $k=0$  is  $\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_0$  is the "initial guess" orientation and the error direction of the solution is computed by the objective function  $\mathbf{f}$  and its jacobian  $\mathbf{J}$  as 2.23.

$$\nabla \mathbf{f}\left(\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_k, \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}, \begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}}\right) = \mathbf{J}^T\left(\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_k, \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}\right) \mathbf{f}\left(\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_k, \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}, \begin{smallmatrix} S \\ \end{smallmatrix} \hat{\mathbf{s}}\right) \quad (2.23)$$

The Jacobian matrix is obtained by orderfully differentiating the components of the objective function for the quaternion components reaching 9

$$\mathbf{J}^T\left(\begin{smallmatrix} S \\ E \end{smallmatrix} \hat{\mathbf{q}}_k, \begin{smallmatrix} E \\ \end{smallmatrix} \hat{\mathbf{d}}\right) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \quad (2.24)$$

Where:

$$\begin{aligned}
a_{11} &= 2d_yq_4 - 2d_zq_3 \\
a_{12} &= 2d_yq_3 + 2d_zq_4 \\
a_{13} &= -4d_xq_3 + 2d_yq_2 - 2d_zq_1 \\
a_{14} &= -4d_xq_4 + 2d_yq_1 + 2d_zq_2 \\
a_{21} &= -2d_xq_4 + 2d_zq_2 \\
a_{22} &= 2d_xq_3 - 4d_yq_2 + 2d_zq_1 \\
a_{23} &= 2d_xq_2 + 2d_zq_4 \\
a_{24} &= -2d_xq_1 - 4d_yq_4 + 2d_zq_3 \\
a_{31} &= 2d_xq_3 - 2d_yq_2 \\
a_{32} &= 2d_xq_4 - 2d_yq_1 - 4d_zq_2 \\
a_{33} &= 2d_xq_1 + 2d_yq_4 - 4d_zq_3 \\
a_{34} &= 2d_xq_2 + 2d_yq_3
\end{aligned}$$

These are the general formulations of the algorithm applied to a field predefined in any direction. The Madgwick solution simplifies these equations because gravitational earth field has only z component  $\mathbf{g} = (0, 0, 1)$  (normalized vector) and magnetic field has null y component  $\mathbf{B} = (b_x, 0, b_z)$ . So substituting  ${}^E\hat{\mathbf{d}}$  with the quaternion deriving from  $\mathbf{g}$  and  $\mathbf{B}$  it is possible to obtain 2.25 and 2.26. The correspondent measurement quaternion vectors  ${}^S\hat{\mathbf{s}}$  in the sensor frame of gravitational and magnetic fields are 2.27 and 2.28.

$${}^E\hat{\mathbf{g}} = [0 \ 0 \ 0 \ 1] \quad (2.25)$$

$${}^E\hat{\mathbf{b}} = [0 \ b_x \ 0 \ b_z] \quad (2.26)$$

$${}^S\hat{\mathbf{a}} = [0 \ a_x \ a_y \ a_z] \quad (2.27)$$

$${}^S\hat{\mathbf{m}} = [0 \ m_x \ m_y \ m_z] \quad (2.28)$$

Jacobian and objective function are simplified for the magnetic measurements:

$$\mathbf{f}_b \left( {}^S_E\hat{\mathbf{q}}, {}^E\hat{\mathbf{b}}, {}^S\hat{\mathbf{m}} \right) = \begin{bmatrix} 2b_x(0.5 - q_3^2 - q_4^2) + 2b_z(q_2q_4 - q_1q_3) - m_x \\ 2b_x(q_2q_3 - q_1q_4) + 2b_z(q_1q_2 + q_3q_4) - m_y \\ 2b_x(q_1q_3 + q_2q_4) + 2b_z(0.5 - q_2^2 - q_3^2) - m_z \end{bmatrix} \quad (2.29)$$

$$\mathbf{J}_b \left( {}^S_E\hat{\mathbf{q}}, {}^E\hat{\mathbf{b}} \right) = \begin{bmatrix} -2b_zq_3 & 2b_zq_4 & -4b_xq_3 - 2b_zq_1 & -4b_xq_4 + 2b_zq_2 \\ -2b_xq_4 + 2b_zq_2 & 2b_xq_3 + 2b_zq_1 & 2b_xq_2 + 2b_zq_4 & -2b_xq_1 + 2b_zq_3 \\ 2b_xq_3 & 2b_xq_4 - 4b_zq_2 & 2b_xq_1 - 4b_zq_3 & 2b_xq_2 \end{bmatrix} \quad (2.30)$$

And for the gravitational field as:

$$\mathbf{f}_g \left( {}^S_E\hat{\mathbf{q}}, {}^S\hat{\mathbf{a}} \right) = \begin{bmatrix} 2(q_2q_4 - q_1q_3) - a_x \\ 2(q_1q_2 + q_3q_4) - a_y \\ 2\left(\frac{1}{2} - q_2^2 - q_3^2\right) - a_z \end{bmatrix} \quad (2.31)$$

$$\mathbf{J}_g \left( {}^S_E\hat{\mathbf{q}} \right) = \begin{bmatrix} -2q_3 & 2q_4 & -2q_1 & 2q_2 \\ 2q_2 & 2q_1 & 2q_4 & 2q_3 \\ 0 & -4q_2 & -4q_3 & 0 \end{bmatrix} \quad (2.32)$$

Equations ((??????)) and ((??????)) have a global minimum defined by a line, while the objective function are defined as eq. 2.33 and 2.34 and have a single point solution, representing a unique orientation of the sensor.

$$\mathbf{f}_{g,b} \left( {}^S_E\hat{\mathbf{q}}, {}^S\hat{\mathbf{a}}, {}^E\hat{\mathbf{b}}, {}^S\hat{\mathbf{m}} \right) = \begin{bmatrix} \mathbf{f}_g \left( {}^S_E\hat{\mathbf{q}}, {}^S\hat{\mathbf{a}} \right) \\ \mathbf{f}_b \left( {}^S_E\hat{\mathbf{q}}, {}^E\hat{\mathbf{b}}, {}^S\hat{\mathbf{m}} \right) \end{bmatrix} \quad (2.33)$$

$$\mathbf{J}_{g,b} \left( {}^S_E\hat{\mathbf{q}}, {}^E\hat{\mathbf{b}} \right) = \begin{bmatrix} \mathbf{J}_g^T \left( {}^S_E\hat{\mathbf{q}} \right) \\ \mathbf{J}_b^T \left( {}^S_E\hat{\mathbf{q}}, {}^E\hat{\mathbf{b}} \right) \end{bmatrix} \quad (2.34)$$

Standard approach of optimization requires more iterations of equation 2.22 to find the optimal quaternion, but Madgwick's algorithm computes one iteration per sample time  $t$ . It's possible rewrite equation 2.22 in this way:

$${}^S_E \hat{\mathbf{q}}_{\nabla,t} = {}^S_E \hat{\mathbf{q}}_{est,t-1} - \mu_t \frac{\nabla \mathbf{f}}{\|\nabla \mathbf{f}\|} \quad (2.35)$$

$$\nabla \mathbf{f} = \begin{cases} \mathbf{J}_g^T ({}^S_E \hat{\mathbf{q}}_{est,t-1}) \mathbf{f}_g ({}^S_E \hat{\mathbf{q}}_{est,t-1}, {}^S \hat{\mathbf{a}}_t) \\ \mathbf{J}_{g,b}^T ({}^S_E \hat{\mathbf{q}}_{est,t-1}, {}^E \hat{\mathbf{b}}) \mathbf{f}_{g,b} ({}^S_E \hat{\mathbf{q}}_{est,t-1}, {}^S \hat{\mathbf{a}}_t, {}^E \hat{\mathbf{b}}, {}^S \hat{\mathbf{m}}_t) \end{cases} \quad (2.36)$$

$${}^S_E \hat{\mathbf{q}}_{\nabla,t} = {}^S_E \hat{\mathbf{q}}_{est,t-1} - \mu \frac{\nabla ({}^S_E \hat{\mathbf{q}}_{est,t-1}, {}^E \hat{\mathbf{g}}, {}^S \hat{\mathbf{a}}, {}^E \hat{\mathbf{b}}, {}^S \hat{\mathbf{m}})}{\|\nabla ({}^S_E \hat{\mathbf{q}}_{est,t-1}, {}^E \hat{\mathbf{g}}, {}^S \hat{\mathbf{a}}, {}^E \hat{\mathbf{b}}, {}^S \hat{\mathbf{m}})\|} \quad (2.37)$$

$\mu_t$  is a step-size which represents the convergence rate of the estimated orientation and it is equal or greater than the rate of change of the sensor orientation. Avoiding multiple iteration to compute the quaternion (k<sub>l</sub>) the result is a real-time output with a greater speed. However this remains an approximation so second order errors are introduced with this simplified formulation. Errors irrelevant in case of quasi-static movement, while for a dynamic sensor behavior the gradient the algorithm will give not at all reliable output. The value of  $\mu_t$  is recomputed for each time sample to avoid overshooting as eq. 2.38.

$$\mu_t = \alpha \|{}^S_E \hat{\mathbf{q}}_{\omega,t}\| \Delta t, \quad a > 1 \quad (2.38)$$

${}^S_E \hat{\mathbf{q}}_{\omega,t}$  is calculated with equation 2.18 and  $\alpha$  is an augmentation coefficient to take into account noise in the accelerometer and magnetometer measurements.

Fusion process The two estimated quaternions  ${}^S_E \hat{\mathbf{q}}_{\omega,t}$  and  ${}^S_E \hat{\mathbf{q}}_{\nabla,t}$  have to be fused to obtain the optimal quaternion that represents the orientation of sensor. Gyroscope measurements are affected from noise and if accelerometer is not stationary or magnetometer is exposed to interference, measurements from them are incorrect. The goal of the fusion algorithm is to filter out high frequency errors so introduces weights applied to each orientation calculation:

$${}^S_E \hat{\mathbf{q}}_{est,t} = \gamma_t {}^S_E \hat{\mathbf{q}}_{\nabla,t} + (1 - \gamma_t) {}^S_E \hat{\mathbf{q}}_{\omega,t}, \quad 0 \leq \gamma_t \leq 1 \quad (2.39)$$

Two extremes cases are:

- $\gamma = 0$  the sensor orientation is determined only by magnetometer and accelerometer measurements, so sensor is stationary cause no angular acceleration are detected from gyroscope.
- $\gamma = 1$  the sensor attitude is determined only by gyroscope measurements.

Gamma is chosen as the value

## 2.5.2 Integrazione Filtro Kalman + (AQUA,QUEST,TRIAD)

## Chapter 3

# Position estimation

### 3.1 Generation of a magnetic-dipole field

The 3D space target, in which the object we want to track moves, is permeated by a magnetic field generated by a solenoid. The expression for dipole model to describe the field generated by a bobbin is the following:

### 3.2 Magnetic field tracking



## Chapter 4

# The algorithm

### 4.1 Flowchart

### 4.2 Steps



# Chapter 5

## Test

### 5.1 Instrumentation and setting

A MPU 92/65 and a microcontroller are used to collect data at a given frequency of 100 Hz, than transmitted to the computer where are processed in Matlab.

### 5.2 Data set acquisition

All algorithms are tested with actual measurement data collected from a MARG sensor. During the period when data are collected, the sensor was rotated 90 degrees, followed by a -180 degrees and then other 90 degrees about each axis. Two test with the same rotation were performed: a static test with an angular rate of 110 DPS and a dynamic test at 190 DPS. These angular velocity are chosen in the range of human motion.

### 5.3 Static and dynamic test



## Chapter 6

# Results



## Chapter 7

# Conclusions



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