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Market Efficiency and Portfolio Optimization Using Markowitz Model: A Study of African Stock Exchanges

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Abstract

In the recent decades the African continent has achieved a relative economic development due to the political stability across most of the continent's countries, this could be directly revealed in form of increasing in the importance of the capital markets; in general all over Africa but indeed, in a broader sense to the countries which have been witnessing greater economic flourish. As a result of this, a great deal of investments opportunities can be anticipated through these up-steps in some African stock exchanges, especially considering the increasing level of competition in the Western stock markets would drive investors to other alternative markets.

In this study the "Investability" across Africa is examined through studying four major capital markets of different countries in the content. There are two main aspects considered to carry out that; the first is testing the weak-form of market efficiency and investigating whether these stock exchanges follow the random walk hypothesis; however the second aspect is building optimal portfolios out of these African markets using Markowitz model and making comparisons worldwide by including well-known capital markets throughout the world.

In the first part, basic tests -parametric and non-parametric- of the weak-form of the random walk hypothesis have been conducted on the data-sets of indices closing prices that represent four African Stock Exchanges; Johannesburg Stock Exchange, Nigerian Stock Exchange, Casablanca Stock Exchange, and Egyptian Stock Exchange, data from Nov. 2013 to Nov. 2018. In addition to the statistical description, Runs test, Ljung-Box Statistic (White Noise Test) and Arch Effect Test are employed empirically test the random walk characteristics of stock returns and examine the weak-form efficiency of the African stock markets.The results reject the null of random walk for the chosen indices, which is an evidence to the inefficiency of the chosen four African markets.

In the second part, the Mean-variance(Markowitz) model is applied in three distinctive cases; the first case is building optimal portfolios only from the African indices for different risk-aversion levels to explain the trade-off investors make between minimizing risks and simultaneously maximizing returns; the second case considers the risk of exchange rates that practically investors are faced with, since it can tell something about the political and economic stability across the continent;and the last case includes DAJA, NEKKEI and DAX which compares the risk-return values these African hold worldwide. Results of optimal portfolios and efficient frontiers reveal in general relatively higher levels of return-risk values for the African indices compared to the Western ones. However, exchange rates risk in Africa is substantial and it would radically change the investors' choices, that could be assigned to different ways, for instance the monetary policies counties in question undertake, but its implications on African financial markets are remarkable and worth further long-horizons studies.

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Chapter 1

Introduction

1.1 Overview of Market Efficiency and Portfolio Selection Problems

In the recent times some African countries started to boom economically due to the relative stability in political systems across the continent, these booms could substantially be reflected in so many aspects; this study in concerned with the aspect that shades the financial flourish and the capital markets developments that could be clearly noticed to the close observer. This study undertakes in ample of details the investability across the continent by taking four countries as instances to highlight the potential markets, this study, indefinitely, does not have the capability of concluding the subject matter for the entire continent; instead, it gives investments opportunities and markets' indications to these four African stock exchanges that might be unforeseen to the financial community in general.

The selection process of these four African stock exchanges is based on the performance of these capital markets in general, as well as the economic stability of the countries embrace them. It is also considered to choose the stock exchanges from different countries, though there would be a broader image about the continent. In addition to that, the economic level of impact plays an important role in the selection; for example, the Nigerian Stock Exchange (NSE) is basically taken because the country has the biggest economy in Africa.

This study takes this investability through considering two general objectives that even the thesis is road-mapped accordingly; the investigation of market efficiency of these African exchanges and the application of Markowitz's portfolio selection model to build an optimal portfolio out of these selected capital markets.

The first objective of this study is the market efficiency which could be defined in terms of how prices can reflect all the available information according to the Efficient Market Hypothesis [1]. If the stock market is efficient, the available information that is relevant to estimate or anticipate the future performance of a company should be reflected in the prices of the company's shares, which in different words the market price of a share should be equal to its intrinsic value. Also, the predictability and the market efficiency could be linked in a conceptual way; the more random the price of the future thus the less able to foresee the price change, so the efficient markets tend to be completely random and unpredictable. Regarding this efficiency the study carries out the weak-form of the random walk tests, since it's the form with the most practical experiments among the other forms. There will be more detailing discussions in the further parts of this study about the market efficiency and why the weak-form of the random walk choice.

The second aspect of the investability is building optimal portfolios out of these African markets using Markowitz model; though that would provide the capability of comparing these African portfolios, in terms of return-risk measures, with other portfolios investors make of the well-known stock exchanges around the world. The Mean-Variance model, as Markowirz model is named sometimes, is quite appropriate to use it such situations since there is analysis to historical data and then based on deciding about the future. So, there are three distinctive cases discussed in this study; the first one is building optimal portfolios out of African assets only without paying attention to the exchange rates risk of these African countries; then there is a case of taking the exchange rates risk into account while building this optimal portfolio; and the last case is building optimal portfolio out of African assets and well-known indices from across the world like DJIA, DAX, and NIKKEI, this gives a holistic view to the positions of the African capital markets to the rest of the world

The study is not concerned with investigating or criticizing the theories of market efficiency and portfolio selection model, instead, it applies them to give some lights to the investments in the mother continent.

1.2 Objective and Contribution of the Study

The objective of this study in general is giving some lights to investments in African capital markets sine there are very few studies published regarding these markets; in more details the main aims of this study could be summarized as follow:

- It investigates the weak-form efficiency of these African markets; and that through testing the weak-form of the random walk hypothesis, whether each exchange, represented by its selected index, follows the random walk hypothesis or not and the economic justifications behind that.

- Building optimal portfolios out of African markets only based on Markowitz model for different levels of risk-aversions; showing the investments opportunities that might be provided by this continent in its stock exchanges and enabling the financial community to see some unforeseen potentials there.

- Highlighting the effect of the exchange rates risk on the optimal portfolios and then of the investments choices in the continent since the political stability can be shown through the fluctuations in the exchange rates.

- Building optimal global portfolios out of these African markets and other global famous stock exchanges such as NYSE and TSE; and that for the sake of showing the position of the African investment opportunities and the comparing them to the ones provided by stock exchanges across the world.

1.3 Limitations of the Study

There are lots of limitations could be stated in this study weather from the perspective of the data taken to represent specific markets or from the methodological point of views to the theories applied in this study; though the main limitations could be briefed as follows:

- The time horizon of the date is just five years; since this could be insufficient for answering some questions like the predictability and the efficiency of the market.
- The criticisms for the random walk theory are still valid; for instance, linking the price predictability to investigating the historical data, which indicates that the economic factors for the future will be the same as the past.
- There are still other tests that could be carried out by the study in the market efficiency part; however, for the simplicity the most important tests are considered to answer the randomness question.
- Markowitz approach is quite significant in portfolio theory, but still the criticisms of building a portfolio for the future based on a belief that the average of historical risk and return will continue are still realistic. Taking withstand that the determinants of the future are completely independent from the past.
- Outlining investment opportunities in Africa based only on four indices is kind of defective, clearly the vision would be clearer if more assets were considered which could highlight more African stock exchanges.

1.4 Thesis Organization

The thesis is decomposed as follows:

- **Chapter one (Introduction)** is divided to several parts; the first part is an overview to the main segments of this study and the emerging significance of the African capital markets; then there is the part that illustrates the aims of the thesis and the potential contributions this study may yield to this specific field of science; the third part is dedicated to the limitations of this study either in its unrealistic assumptions or the selected data in a way of representing the real objects of the study; then the last part of the first chapter reveals the structure of the thesis and what each chapter contains.

- **Chapter two** shows the subject of this study; the African stock exchanges taken in this study and their main indices, it also gives information about the history and the market capitalization each market has, though then the reason that is behind choosing them to represent the capital market in the mother continent.

- **Chapter three** gives an overview to the definition of the market efficiency and price predictability, it goes into details about the random walk and its forms. Mainly its literature reviews the market efficiency.

- **Chapter four** shows the application of the market efficiency theory on the African markets; it starts by illustrating the used data and the cleaning process carried out to it, then it discusses the weak-form of random walk tests used it this study and their methodologies, lastly it describes the outlined results and discusses them.

- **Chapter five** introduces the portfolio selection problem according to Markowitz model; what is trying to achieve, and the main parameters used as valuable measures in this quadratic optimization problem. It also tells about the basic model and the risk-aversion model in details since is the basic theory used in this part of the study.

- **Chapter six** illustrates building optimal portfolios from African indices as well as the case if world-wide indices are included; it shows the data that used in this part and the cleaning process used to prepare the data as well, then it reveals the methodologies of the Markowitz model and its cases, lastly it describes the outlined results and discusses them.

- **Chapter seven** shows the conclusions of this study and summarizes the answers this study is trying to provide. It also gives potential future works that may be carried based on this study.

Chapter 2

Main African Stock Exchanges and Their Indices

This chapter basically gives an overview to the African stock exchanges to which the indices, considered in this study, belong. It is very demanding to highlight the capital markets in the continent, since there are worthy investments opportunities after the relative political and economic stability that recently brought to some African countries.

Based on the political stability and the economic prosperity across the continent, four stock exchanges are chosen that belong to different countries. Also, the market capitalizations attained by capital markets play an essential role in the selection process.

Accordingly, this part provides a brief description to the four selected stock exchanges as well as their indices. Sinking in the details is being avoided as possible, since the history of the capital is quite important for investments decision, there is a short brief for each selected stock exchange. Some financial information is also presented to give a general intuition about these markets.

2.1 Johannesburg Stocks Exchange

The JSE is currently ranked the 19th largest stock exchange in the world by its market capitalization which estimated over one billion US dollar and the largest exchange in the African continent. It strives to offer efficient, secure primary and secondary capital markets across a diverse range of instruments, enhanced by post-trade and regulatory services. The JSE was founded in 1887 motivated by the first South African gold strike. Following the first legislation wide-spreading the worlds' financial markets in 1947, the JSE stepped into the World Federation of Exchanges in 1963 and upgraded to an electronic trading system in the early 1990s. The bourse demutualized and listed on its own exchange in 2005.

In 2003, JSE started an alternative exchange, AltX, for small and medium-sized listings, followed by the interest rate and currency instruments yield X. There are nearly 400 companies in the Exchange in the Main Board and in AltX. In 2001, the JSE acquired the South African Futures Exchange (SAFEX) and in 2009 the South African Bond Exchange (BESA).

It offers five financial instruments today, namely equities and bonds and derivatives of financial commodities and interest rates. The JSE is the front-line regulator for the exchange, establishment and enforcement of listing requirements and trade rules. The Financial Services

Board (FSB) oversees JSE 's regulatory functions of its regulatory duties.

Over the past few years, the JSE has undertaken major technological upgrades to a consistent drive to improve trade, clearing and settlement, which continues. In July 2013, the JSE implemented a new Millennium Exchange trading platform in the equity market while moving the trading system from London to Johannesburg. Following this successful transition, trades can now be executed up to 400 times faster than under the previous Trad-Elect system. The change allows for increased liquidity and more algorithmic traders.

The regulatory framework is set to be considerably flexible for future developments, as South Africa seeks to implement a twin peak supervisory model. Under the new system, prudential supervision is transferred to the South African Reserve Bank (SARB) and the regulation of market conduct is guided by a strengthened FSB. South Africa is currently ranked 1st in the world in terms of securities exchange regulation in the 2013- 2014 Global Competitiveness Survey of the World Economic Forum. This is a commendation for the JSE and its regulators.

Another regulatory change that could have widespread consequences is the decision in 2011 to change the internal listing rules of South Africa, allowing foreign domestic companies to be treated as domestic listings. Although foreign companies were permitted to list in the JSE since 2004, they were previously subject to foreign exchange rules, which limited the amount of these shares held by local investors. The removal of these restrictions was an important regulatory change for the Exchange and makes the JSE more attractive. In the following we would like to introduce the indices available for trading in JSE with briefings about any one, directly taken from the JSE website[2]:

2.1.1 FTSE/JSE Africa Index Series

The FTSE/JSE Africa Index Series resulted from a joint venture between the JSE Limited (JSE) and the FTSE Group (FTSE), a world leader in the creation and management of indices. The series brought with it a change in the philosophy and methodology for calculating indices and classifying sectors. The FTSE/JSE Africa Index Series replaced the JSE Actuaries indices on 24 June 2002.

The FTSE/JSE Africa Index Series complies with the recommendations made by the International Organization of Securities Commissions (IOSCO), as laid out in the Principles for Financial Benchmarks published in July 2013 (the IOSCO Principles).

2.1.2 JSE Fixed Income Index Series

The Fixed Income Indices measure the total return of representative bond portfolios and provide a benchmark for historical performance. The indices provide a standard against which investment performance in the bond market can be measured. Specifically, they provide:

- A barometer for daily movements in the Bond Market.

- A history of market levels and movements for technical analysis of the Bond Market.
- A benchmark for measuring portfolio performance.
- A means to analyze sub-sectors of the market.

2.1.3 Rand Index (RAIN)

The Rand Index (RAIN) is a currency index that tracks the performance of the rand against a basket of five other currencies, representing South Africa's most prominent international trading partners. The five currencies and their weights in the index is determined based on the value of goods imported and exported between South Africa and its trading partners, as reported by the South African Revenue Service (SARS). The five currencies included in the index are the euro, US dollar, Chinese yuan, UK pound and Japanese yen.

2.1.4 RFTSE/JSE Responsible Investment index series

The JSE, the first emerging market and first stock exchange to form a Socially Responsible Investment Index (SRI Index) in 2004, announced on 3 June 2015 that it is partnering with FTSE Russell, the global index provider, in progressing the JSE's work around promoting corporate sustainability practices over the last decade. The JSE has adopted the FTSE Russell ESG Ratings process to create the following two indices, launched on 12 October 2015.

2.1.5 JSE Responsible Investment Top 30 Index

A market-cap weighted index calculated on an end of day basis – benchmark. It comprises all eligible companies who achieve the required minimum FTSE Russell ESG rating as set out in the Ground Rules from time to time.

2.1.6 The JSE Socially Responsible Investment (SRI) Index promotes sustainable and transparent business practices

The SRI index series evolved considerably since it was launched in May 2004. The advent of sustainability initiatives internationally and the King code locally, also saw the index created to foster good corporate citizenship and promote sustainable development. Listed companies in the FTSE/JSE All Share index were reviewed annually against a set of Environmental, social and governance (ESG) concerns.

2.2 Nigerian Stock Exchange (NSE)

NSE serves the largest economy in Africa and promotes the development of the financial markets of Africa. The market capitalization of NSE is estimated over 0.8 billion US dollar. The NSE, a limited by guarantee registered company, was established in 1960 and is licensed under the Investments and Securities Act (ISA) and regulated by the Nigerian Securities and Exchange Commission (SEC). The Exchange offers listing and trading services, licensing services, market data solutions, ancillary technology services and more.

The NSE is committed to the implementation of the highest international standards. NSE is a member of several international and regional organizations that promote the development and integration of global best practices throughout its operations to support this commitment. It is a member of the International Organization of Securities Commissions (IOSCO), the World Federation of Exchanges (WFE), Sustainable Stock Exchanges (SSE) Initiative, the SIIA's Financial Information Services Division (FISD) and the Intermarket Surveillance Group (ISG). The Exchange is a founding member and executive committee member of the African Securities Exchanges Association (ASEA).

The NSE continues to evolve in order to meet the needs of its valued customers and to achieve the highest level of competitiveness. It is an open, professional and vibrant exchange, connecting Nigeria, Africa and the world.

The NSE was set up as the Lagos Stock Exchange in 1960. It changed its name from the Lagos stock exchange to the Nigerian stock exchange in 1977. As of 31 May 2018, 169 companies have been listed with a total market capitalization of more than 13 billion Euros. All listings are included in the index of all shares of the Nigerian Exchange. The Nigerian stock exchange is the third largest stock exchange in Africa as far as market capitalization is concerned.

The Nigerian Stock Exchange was established in 1960 as the Lagos Stock Exchange, and the Stock Exchange Council was inaugurated on 15 September 1960. Operations officially began on 25 August 1961 with 19 trading securities, but informal operations had begun earlier in June 1961. Initially, operations were carried out in the Central Bank building with the exchange of four companies as market dealers: Inlaks, John Holt, C.T. Bowring and ICON (Investment Company of Nigeria). The volume for August 1961 was approximately 80,500 pounds and in September of the same year rose to approximately 250,000 pounds with the bulk of government securities investments. In December 1977, the Nigerian stock exchange was established with branches in some of the country's major commercial cities. In December 1977, the Nigerian stock exchange was established with branches in some of the country's major commercial cities.

The NSE is regulated by the Securities and Exchange Commission, which has the exchange monitoring mandate to prevent market violations and to detect and detect unfair manipulation and trade practices. The exchange has an automated system of trading. The performance data of listed companies are published every day, weekly, monthly, quarterly and yearly.

Since 27 April 1999, the Nigerian Stock Exchange has operated an Automated Trading

System (ATS) with dealers trading on a server-connected computer network. The ATS has a remote trading and monitoring facility. As a result, many trading members trade online from their Lagos offices and from all 13 branches throughout the country. The Exchange is in the process of establishing more branches for online real time trading. Trading on The Exchange starts at 9.30 a.m. every business day and closes at 2.30 p.m.

The government has abolished legislation to prevent the flow of foreign capital into the country in order to encourage foreign investment in Nigeria. This has enabled foreign brokers to become traders on the Nigerian stock exchange and investors of all nationalities can invest freely. Multiple and cross-border listings in foreign markets are also allowed to Nigerian companies. The Nigerian Stock Exchange suspended 17 companies on 7 July 2017 for failure to comply with the regulatory provisions of the Corporate Governance Act and the existing post-listing guidelines.

The Nigerian Capital Market was deregulated in 1993. Consequently, prices of new issues are determined by issuing houses and stockbrokers, while on the secondary market prices are made by stockbrokers only. The market/quote prices, along with the All-Share Index plus NSE 30 and Sector Indices, are published daily in The Stock Exchange Daily Official List, The Nigerian Stock Exchange CAPNET (an intranet facility), newspapers, and on the stock market page of the Reuters Electronic Contributor System.

In the following we would like to introduce the most important indices available for trading in JSE with a briefing about any one, directly taken from the NSE website[3]:

2.2.1 The All-Share Index

The All-Share Index tracks the general market movement of all listed equities on the Exchange, including those listed on the Alternative Securities Market (ASeM), regardless of capitalization.

2.2.2 Premium Index

Tracks the Premium Board companies in terms of market capitalization and liquidity. It is a price index and is weighted by adjusted market capitalization—the number of a company's listed shares, multiplied by the closing price of that company, multiplied by a capping factor. Only fully paid-up common shares are included in the index.

2.2.3 Pension Index

The NSE pension tracks the top 40 companies in terms of market capitalization and liquidity. It is a total return index and is weighted by adjusted market capitalization. Adjusted market capitalization of a company is the number of its listed shares multiplied by the closing price of the company multiplied by its capping factor. Index is also adjusted for a free float factor.

2.2.4 Banking Index

Designed to provide an investable benchmark to capture the performance of the banking sector, this index comprises the most capitalized and liquid companies in banking. The index is based on the market capitalization methodology.

2.2.5 NSE 30 Index

Tracks the top 30 companies in terms of market capitalization and liquidity. It is a price index and is weighted by adjusted market capitalization—the number of a company's listed shares, multiplied by the closing price of that company, multiplied by a capping factor. Only fully paid-up common shares are included in the index.

2.2.6 ASEM

This Index tracks price movements of all equities listed on the Alternative Securities Market. It is a value-based index.

2.3 Egyptian Stocks Exchange

A company was founded by some wealthy investors and brokers for this purpose, however the brokers formed a union and formulated a law to regulate their affairs like the union established by their colleagues in Alexandria. Then, a stock exchange was then instituted in Cairo in 1904 based on an agreement between the union and the company and has limited its membership to securities brokers only. The Egyptian Stock Exchange (EGX) is one of the oldest stock markets established in Middle East, who traced its origin from the merger of two stock exchanges in Egypt namely, Alexandria Stock Exchange which was founded in 1883 and Cairo Stock Exchange in 1903, now known as Egyptian Stock Exchange which was formerly known as Cairo and Alexandria Stock Exchange (CASE). Both exchanges were very well traded in 1940s, and both the exchanges combined were ranked fifth in the world.

The main performance indication or measure of trading of Egyptian Stock Exchange (EGX) is EGX30, which was previously known as CASE30 index. It includes the top 30 companies in terms of liquidity or 30 most active companies in the Egyptian Stock Exchange. The CASE 30 was started with base value of 1000 points on January 2, 1998 and was re-termed to EGX 30 on March 1st, 2009. Any company in order to be listed or included in EGX30 should have a 15 percent free float.

The Egyptian Exchange (EGX) is one of the Middle East 's oldest stock markets from 1883. The exchange covers the main market, an over - the-counter market and the Nilex market, which is a relatively new market for SMEs. As of December 2015, there were 221 companies listed on the main market, with a total market capitalization of LE430bn (equivalent to 22.8 billion US dollar as of December 2016); and 31 listed on the Nilex, with a total market capitalization of LE1bn (53m US dollar).

The two most widely followed market indices are the EGX30 Index, a free-float market capitalization-weighted index of the 30 most actively traded companies, and the EGX70 Index, a price index that tracks the performance of the next 70 most active stocks on the main market. Both indexes are rebalanced in February and August every year.

The Egyptian stock market is one of the oldest stocks in the world since 1883. It includes the stock exchange of Alexandria in 1883 and the exchange of Cairo in the 1940s. During the 1950s, however, Egypt adopted a socialist regime that led to a wave of nationalization that, as these companies became state property, considered all the outstanding stocks of previously traded companies to be obsolete.

The Egyptian government implemented economic reform programs throughout the 1990s, including the privatization of state-owned enterprises; the liberalization of the financial market; and the adoption of a market-based economy. This led to the recovery of the Egyptian stock market (capital market law 95/1992). This led to the recovery of the Egyptian stock market (capital market law 95/1992). In addition, Presidential Decree No. 51 redefined the legal structure of the exchanges in 1997, resulting in the merger of the Cairo and Alexandria stock exchanges into one entity (Cairo and Alexandria stock exchange or CASE) with one President and one Board of Directors and two locations: Cairo and Alexandria respectively.

As the Egyptian stock market grew, investors from inside and outside Egypt began to invest in the stock market in order to maximize the value of their investments and increase their portfolios. The current increase in the Egyptian stock market began in 2003 and peaked in 2005.

In the following we would like to introduce the most important indices available for trading in JSE with a briefing about any one, directly taken from the NSE website[4].

2.3.1 EGX 30 Index

EGX 30 index, previously named CASE 30 Index, is designed and calculated by EGX. EGX started disseminating its index on 2 February 2003 via data vendors, its publications, web site, newspapers etc. The start date of the index was on 4/1/1998 with a base value of 1000 points. EGX 30 index value is calculated in local currency terms and denominated in US dollars since 1998. EGX started publishing its dollar denominated index on 1st of March 2009. EGX 30 index includes the top 30 companies in terms of liquidity and activity. EGX 30 Index is weighted by market capitalization and adjusted by the free float. Adjusted Market capitalization of a listed company is the number of its listed shares multiplied by the closing price of that company multiplied by the percent of freely floated shares.

For a company to be included in EGX 30 index, it must have at least 15 percent free float or if its adjusted market capitalization in not less than the median of adjusted market capitalization for all traded companies during the review period, and its free float is compatible with the Listing Rules. This ensures market participants that the index constituents truly represent actively traded companies and that the index is a good and reputable barometer for the Egyptian market.

2.3.2 EGX50 EWI Index

The Egyptian Exchange has developed and launched equally weighted index EGX50 EWI. The index includes top 50 companies in terms of liquidity and activity. The index is designed to balance the impact of price changes among the constituents of the index as they will have a fixed weight of 2 percent at each quarterly review. EGX50 EWI developed in consistent with the international best practices. EGX50 EWI avoids concentration on one industry and therefore has a good representation of various industries/sectors in the economy. The starting date of the index is 2nd of July 2013 with a value of 1000 points.

2.3.3 EGX 70 Index

The Egyptian Exchange has launched EGX70 price index on 1st of March 2009 with a base date of 02/01/2008 and a base value of 1000 points. EGX introduced EGX70 index to diversify its family of indices after introducing the benchmark index EGX30 in 2003. The Price Index (EGX70) aims at providing wider tools for investors to monitor market performance. It is worth mentioning that one of the most important examples of price indices is Dow Jones Industrial Average Index.

2.3.4 EGX 100 Index

EGX introduced a new price index; EGX 100 Price Index, on 2nd of August 2009, which tracks the performance of the 100 active companies, including both the 30 constituent-companies of EGX 30 Index and the 70 constituent-companies of EGX 70 Index. EGX 100 index measures the change in the companies' closing prices, without being weighted by the market capitalization, and was retroactively computed as of 1 January 2006.

2.3.5 EGX 30 Capped Index

The Egyptian Exchange has launched the EGX30 Capped Index on February 3, 2019. The index is designed to measure the performance of the most traded companies in accordance with the rules set for mutual funds, as is stated in Article 174 of the executive regulations of the Capital Market Law (95) of 1992. The law stipulates that "any fund shall not invest more than 15 percent in the securities of a single company, nor exceed 20 percent of the securities issued by such company".

EGX30 Capped Index sets a maximum percentage on the weight of the constituents that is determined by its adjusted market capitalization. Constituents are capped quarterly so that the weight of each constituent will not exceed 15 percent of the total weight. This would minimize the control of a limited number of constituents on the overall performance of the index.

2.3.6 S&P-EGX&ESG Index

The Environment, Social and Governance (ESG) Index for Egypt has been created by the Egyptian Institute of Directors, Standard & Poor's and Crisil. EIoD conducts the ESG research for scoring, under the guidance of Standard and Poor's and Crisil, and with the assistance of the Egyptian Stock Exchange (EGX), the Egyptian Exchange also tested the historical data for consistence. The purpose of S&P/EGX ESG index is to raise the profile of those companies that perform well along the three parameters of environmental, social and corporate governance responsibility when compared to their market peers. Linking stock market performance to ESG is, perhaps, the most effective way to highlight the concept of corporate-level Environment, Social and Governance responsibilities. More and more indices are being used to create derivative products, exchange traded funds (ETFs), OTC products and structured products, all of which provide liquidity and investability to specific market segments. Investors, in turn, have access to an investable tool which matches their investment preferences. As investment in ESG products increases, it will become imperative for companies to delve into their business practices and strive to improve them.

2.3.7 Nile Index

The Egyptian Exchange (EGX) has launched Nile Index on 2 February 2014. It comes within the Exchange strategic framework and its interest to increase the Nilex market liquidity as one of the promising markets in the Egyptian market. In 2011, The Egyptian Exchange changed the trading mechanisms of Nilex Exchange from auction trading to continuous trading system, followed by increasing the number of trading hours from one hour to four hours at October 2013. The Index is weighted by market capitalization and adjusted by free float. Nile Index avoids concentration on one industry and therefore has a good representation of various industries/sectors in the economy. The Egyptian Exchange started publishing Nile Index, which has a base date of 02/07/2012 and a base value of 1000 points.

2.4 Casablanca Stocks Exchange

The Casablanca Stock Exchange is a stock exchange in Casablanca, Morocco. The Casablanca Stock Exchange (CSE) is the third largest stock exchange in Africa after Johannesburg Stock Exchange (South Africa) and Nigerian Stock Exchange in Lagos. It achieves one of the best performances in the region of the Middle East and North Africa (MENA). It has 19 members and 81 listed securities with a total market capitalization of $71.1 billion in 2018.

The Casablanca Stock Exchange was established in 1929 as one of the oldest exchanges in Africa. There were some factors that prompted the authorities at that time to found and regulate organizations and operations of the Stock Exchange, such as, the growing importance of the securities market and the introduction of foreign exchange controls. There were also market's shortcomings highlighted, however, its attractiveness at a time when domestic investors revealed a dramatic interest in stock market investments, to jump over these shortcomings, legislative reforms were introduced in 1967, providing Morocco with financial markets with a robust legal and technical framework. In 1993 reforms were made to refine investors protection, stimulate transparency and electronic trading was introduced. Those reforms were enhanced in 1997 by the launch of "MAROCLEAR", the clearing house of Casablanca Stock Exchange. An outcome of the success of amended reforms can be reflected in the market capitalization rate through the jump in the ratio of stock market capitalization to GDP from 23.8 percent in 2002 to 86.1 percent in 2006. As of 2012, CSA was Africa's stock market number four when it comes to value traded in USD and market capitalization. Also, a recent study by the International Monetary Fund (IMF) outlined that economic growth since 1980 till 2010 has been mainly driven by capital accumulation on the supply side, that indefinitely enhances the importance of the stock market as a major source for capital availability which promotes the country's economy growth. According to this study, capital accumulation was close to 45 percent regarding the mentioned period; labor has been the second largest contributor to growth at 40 percent. Human capital and total factor productivity accounted together for less than 15percent.

The CSE lists 75 companies in total till 2017. These companies can be categorized in 19 industries such as Banking, Mining, Oil, Real Estate, Pharmaceutical and others. At the end of 2005, the stock-market's capitalization valued Morocco Dinars (MAD) 252.3 billion, registering 22 percent increase compared to 2004. For the year ended 31st December 2006, the stock market's capitalization got MAD 417. 1 billion, an increase in 65.30 percent compared to 2005.

The Casablanca stock market computes two general indices to track the performance of the listed companies as taken from it's website [5]: MASI (Morocco All Share Index) and MADEX (Most Active Shares Index).

2.4.1 MASI Index

The Morocco Casablanca Stock Exchange MASI index (Moroccan All Shares Index) is a major stock market index created January 2, 2002 that tracks the performance of all companies listed in the Casablanca Stock Exchange and allows to follow up all listed values and to have a long-term visibility. The MASI is a free-float, capitalization-weighted index and is considered the major stock market index in the Maghreb region. The MASI has a base value of 1000 as of December 31, 1991. Data was sourced from Casablanca stock market.

2.4.2 MADEX Index

It is the other one of the two general indexes at the stock exchange, MADEX is an abbreviation to (Moroccan Most Active Shares Index). We can refer to MASI as a proxy of Casablanca stock market because it represents the performance of all listed shared, whereas the MADEX is limited to the most actively traded companies with the top 62 highly liquid stocks and 16 individual stocks all known to be family business companies.

Chapter 3

Short Notes on Market Efficiency

3.1 Introduction to Market Efficiency and Asset Predictability

One of the most debatable questions in financial econometrics throughout decades is the question of assets predictability, can financial securities be forecasted or no. Because it gains its importance from the fact that, it is so much fundamental to the investors' decisions, mathematical models of asset prices based on historical data have been developed over many years and been engaged in by lots prominent financial mathematicians and scientists. Indefinitely, modern financial-econometricians are putting extensive efforts on the idea of "Beating the market" as a major concern of this field that has been discussed and still being.

The problem of asset predictability, namely, forecasting the changes in prices may happen in the future is considered using only past data of the asset's prices for building this predict. Whereas, we certainly know that considering only the past data as a determinant to the price forecast means giving up some other factors that have considerable influences on future prices. For instance, investors' decisions get affected significantly by the diversity of the information among the investors which yield different investors' behaviors to the future and consequently to the future prices. In any case, we must emphasize here to the fact that using the past prices data only to construct a model for forecasting future is not conceptually enough, but it gives a very good financial intuition for these assets' markets efficiency. We shall see an important concept in this discussion quickly, due to its importance in the historical researches applied to the efficiency of the markets, that is, the random walk hypothesis.

The analogy of the market efficiency and its predictability could be seen interestingly in a different aspect: the more efficient the market is, the more random the price of the future thus the less able to foresee the price change, so the efficient markets tend to be completely random and unpredictable.

3.2 The Random Walk Hypothesis and The Market Efficiency

The market efficiency could also be defined in terms of how prices can reflect all the available information according to the Efficient Market Hypothesis (Fama 1970). If the stock market is efficient, the available information that is relevant to estimate or anticipate the future performance of a company should be reflected in the prices of the company's shares, which in different words the market price of a share should be equal to its intrinsic value. Also, the change in the anticipated future performance of the company as expressed in new information should be immediately reflected in a change in share price, because any delay in that translation of information into price would seem completely irrational.

Following this simple logic, changes in prices should be a reaction only to new information. As much random the information is, much the prices change or fluctuate unpredictably.

Depending on the definition of the relevant information sub-sets, namely the weak, semi-strong and strong form, Fama (1970) classified the information in three sub-sets and suggested three forms (levels) of EMH. The weak form of market efficiency hypothesis is a special case of efficient market theory and is the lowest form of efficiency that defines the market as efficient if current prices fully reflect all information contained in previous prices. This form complies with the fact that past prices cannot be used as a tool for predicting future changes in stock prices.

So, therefore an agent cannot make abnormal returns using only the past prices history. It contradicts the Chartist and Technical School, which believes that current prices are the result of past trends and that all fluctuations and turbulence in prices are discounted on average and that average trends change in a predictable way as the history of trends are repeated over and over[6].

According to Fama (1970), the Efficient Market Hypothesis (EMH) poses three types or forms of the market efficiency: weak form, semi-strong form, and strong form. In the weak form, prices assumed to reflect all the historical information. Instead, prices additionally reflect the most available information to all market participants in the semi-strong form of market efficiency. Whereas in the strong form also the "internal" information known by any market participant should be reflected in the prices. As may be noticed from the definitions the semi-strong and the strong forms of the (EMH) are very difficult to be tested in real-life situations, not like the theoretical definitions for the all forms which tend to be quite clear and satisfactory[1].

So based on the former discussion, we shall elaborate in some details of each form, just highlighting the difference because it is quite important in this study to justify the logic behind selecting these specific tests used here:

- **Weak-form of Market Efficiency** Weak-form of Market Efficiency can be attained as we explained formerly when the historical price changes are entirely reflected in the price changes of the future. This directly leads to serial independence of returns yielded from the price changes over time horizons. Though in case of this serial independence, it would be impossible for traders to make unusual profits only by forecasting the future changes in the prices. It is quite important here to notice the statistical connections between this form of market efficiency and the random walk model through these serial correlations of the returns.

- **Semi strong-form of Market Efficiency** implies that not only the historical information but also all public information reflected in prices in this case. Consequently, traders cannot make excess returns, neither by studying the historical information nor by doing so with other public information. That means gaining abnormal profit, from a market that is efficient in a semi-strong form based on fundamental analysis to the most available information of this market, is not possible at all.

- **Strong-form of Market Efficiency** is not, practically, achievable in the real world that implies the maximum level of efficiency. In addition to the information reflected in the other two forms, prices also reflect the insider "internal" information. Attaining this strong-form means traders are not capable of making excess returns even if they know superior information since it is already reflected or expressed in prices.

Though practically, the efforts of the academic communities are mainly focused only on the weak-form of the market efficiency hypothesis. As long as the tests for the weak-form pass then it would be quite sufficient to claim the efficiency of the market which implies that it is not possible for anyone to predict the prices' changes of the future based on the historical performance.

Since of this practical significance of the weak-form emerges, we shall be highlighting that with ample of details showing the practicality of this form and well as some statistical technicalities for carrying out this form in real-world data. In the following, we shall reveal some statistical concepts that are necessary for investigating the weak-form of the market efficiency, the most popular hypotheses in the academic community of the financial econometrics: Rational Expectations, Martingale Process, and Random Walk Hypothesis.

1. **Rational Expectations (RE):** it is explained clearly in the paper if John F. Muth published in 1961 who one of the leading researchers in this approach, the prices changes will be random in the future since the at time t no one knows what is going to happen at time t+1. However, more importantly, he formed that differently, that is, because of the "expected forecasted error" will be 0 as described in the model:

$$P_{t+1} = E_t[P_{t+1}] + \epsilon_{t+1} \tag{3.1}$$

   where:
   $E_t[P_{t+1}]$:the expected value of the price at $t+1$ given the information till time t.
   ϵ_{t+1}:the forecat error.

   then:

$$E_t[\epsilon_{t+1}] = E_t[P_{t+1} - E_t[P_{t+1}]] = 0 \tag{3.2}$$

Cleary, the last formula shows that the conditional expectation of the price at time t+1 is "unbiased" and the forecast error should be correlated with the information available in the market. So the investors at time t are not able to identify what is going to happen at t+1, which means the process of the error should be serially correlated for all lags or leads[7].

2. **Martingale Model:** It is almost the earliest model of financial asset prices, which its bases formed on the probability theory and game of chance. The "fair game" as defined by Girolamo Cardano the Italian mathematician, a game for which none of the players has any privilege over the other, became the building block of the martingale. We can describe a stochastic process Pt as martingale if it satisfies the following condition:

$$E\left[P_{t+1}|P_t, P_{t-1}, ...\right] = p_t \tag{3.3}$$

That simply states the price at t+1 is expected to be equal to the price at t given the entire information history till time t. Alternatively, the expected price difference in the future, given the price history, is equal to zero, though the likelihood of the price to increase is equal to the likelihood of the price to decrease. Another implication of the martingale hypothesis is that nonoverlapping price changes are uncorrelated at all lags and leads [8].

3. **Random Walk Hypothesis (RWH):** It is the most important model among all the modern ones that describe the dynamics of the change in general. Because it incorporates the basic assumptions of the martingale hypothesis, it has become now extremely significant for virtually every scientific field concerned with dynamics. We shall go briefly over the three different forms of the random walk hypothesis since it is the essence of this study:

   - **Random Walk 1 (RW1)** is the simplest form of the random walk. It assumes price increments that are independently and identically distributed (IDD). As explained here:

$$P_t = \mu + P_{t-1} + \epsilon_t \tag{3.4}$$

   and:

$$\epsilon_t \sim IDD(0, \sigma^2) \tag{3.5}$$

   where p is the price, μ is the drift and ϵ_t in the increment or the error that is independently and identically distributed with a mean 0 and variance σ^2.

The IDD increment assumption indicates that the definition of the martingale is included in the random walk, so it is also a fair game. Additionally, it implies that any nonlinear functions of the increments are also uncorrelated [8].

- **Random Walk 2 (RW2):** Besides the popularity of the RW1 due to its theoretical simplicity, it is unrealistic to assume that the distributions of price changes are identical over some time because a look to the data of this study demonstrates that point. Though in RW2 keeps the assumption of the independent increment, whereas it relaxes the identicality where the increments are still independent but not identically distributed (INID). RW2 is more general form compared to RW1 that enables wider and more realistic characteristics to describe stock's dynamics, for instance, allowing for unconditional heteroskedasticity is very useful given the timely change of volatility of the financial assets in general. However, it maintains the independence of the increments which keeps the unpredictability of the future price changes based on the price increments history.
- **Random Walk 3 (RW3):** This is the most general forms of the random walk hypothesis that is obtained by also relaxing the assumption of the in independent increments of RW2 to include processes with uncorrelated increments but might be dependent. So, RW3 is the weakest form of the random walk hypothesis that contains RW1 and RW2 as special cases. Well, we must necessarily state that this weak form of the random walk (RW3) is the one most often tested in the empirical studies of the financial data.

According to the following discussions and complying with previous similar studies, this study is taking the most general version, of the random walk, the weak form, RW3. Assuming the prices of our indices follow RW3.

Chapter 4

The Efficiency of African Markets

4.1 Data

The data used in this study primarily consist of daily closing prices series for 5 years; from the 1st November 2013 to the 31th of October 2018; of four general indices of different African Stock Exchange, namely, FTSE/JSE Top 40 index from Johannesburg Stock Exchange, NSE 30 index from Nigerian Stock Exchange, MASI index from Casablanca Stock Exchange, and EGX 30 index from Egyptian Stock Exchange. The data were taken from their websites.

Well, it's interesting here to mention that because we were interested in finding the covariances between the indices, we needed the data to be timely matching, whereas the countries in question have different calendar holidays for instance the weekend in Egypt is Friday and Saturday instead of Saturday and Monday in South Africa also the Christmas holidays in South Africa are corresponded by working days in Morocco. To go over this we had to come up with that, making the prices during the holidays equal to the prices of the daily directly before the holidays which means for example the prices in the weekend in Egypt are Thursday's (directly before the weekend) prices. This way we could make match of the data so we can get an acceptable covariances values between the indices.

Then, a natural logarithmic transformation is performed so the data is converted to generate a time series of continuously compounded returns, daily returns of the indices are computed as follows:

$$r_t = logp_t - logp_{t-1} = log(p_t/p_{t_-1}) \tag{4.1}$$

Where p_t and p_{t-1} are the indices prices at time t and $t-1$ respectively.

So accordingly, we got then 1825 logarithmic returns for each index. FTSE/JSE Top 40, NSE 30, MASI and EGX 30 logarithmic returns are respectively illustrated in figure 41, figure 4.2, figure 4.3 and figure 4.4.

4.2 Methodology

Therefore, to investigate the weak-form efficiency of African markets reflected in the selected indices, the following tests are curries out knowing that there are all implemented regarding the main following hypotheses: Ho: prices of the selected index follows a random walk, then

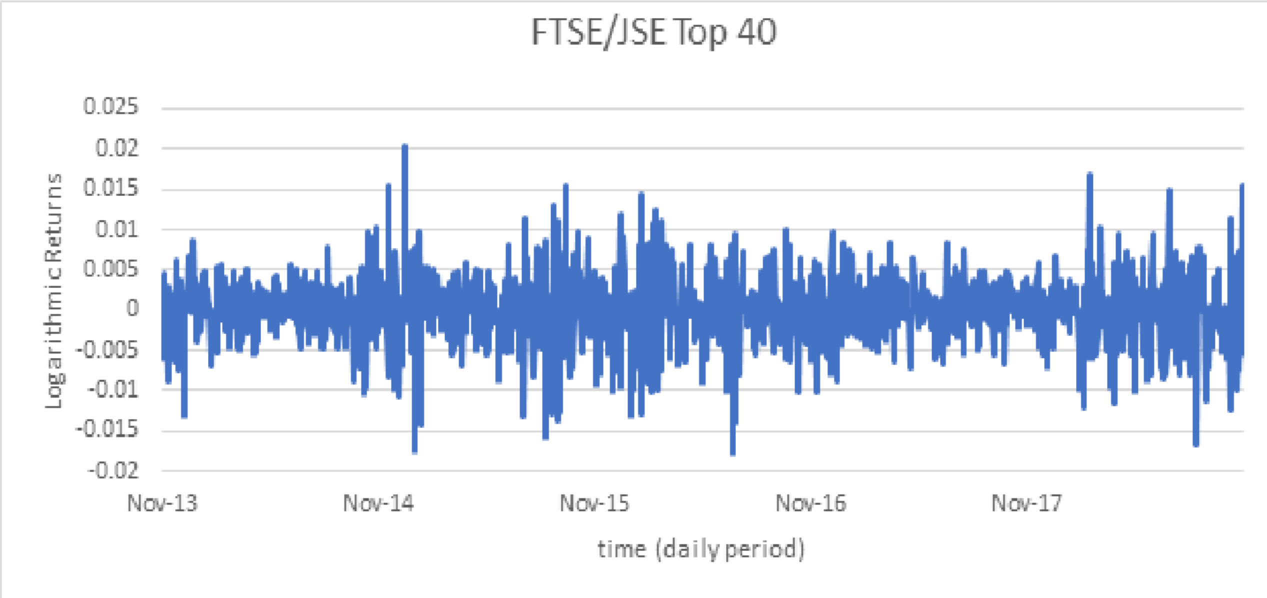

Figure 4.1: Daily logarithmic returns changes of FTSE/JSE Top 40 from Nov. the 1st 2013 to Oct. 31th 2018.

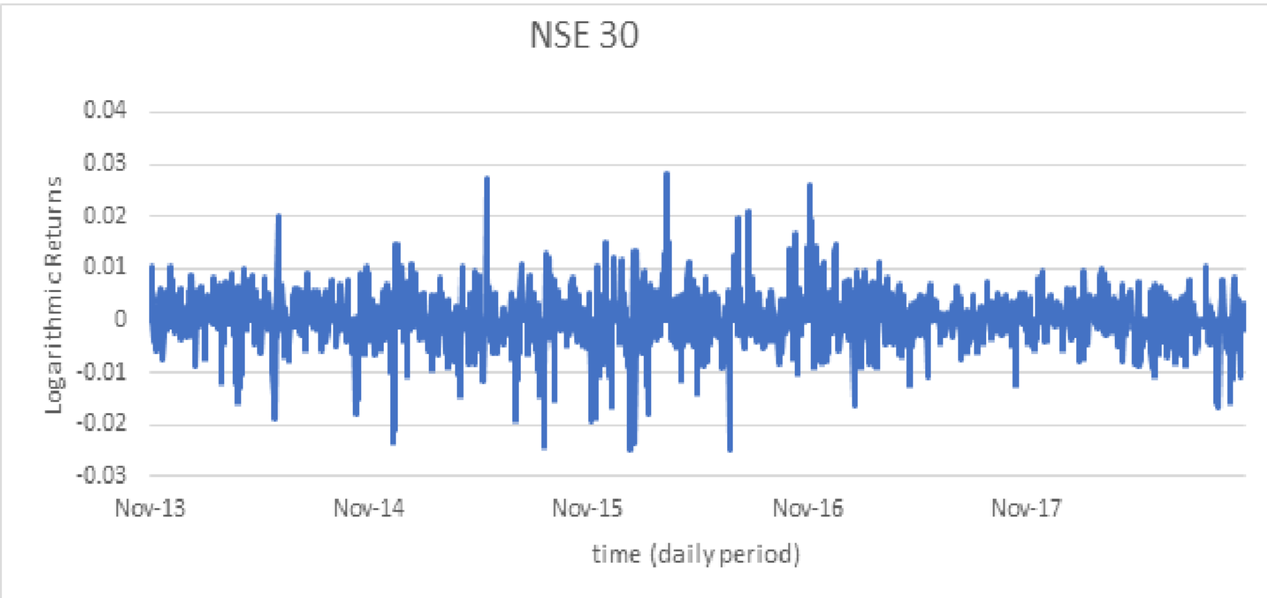

Figure 4.2: Daily logarithmic returns changes of NSE 30 from Nov. the 1st 2013 to Oct. 31th 2018.

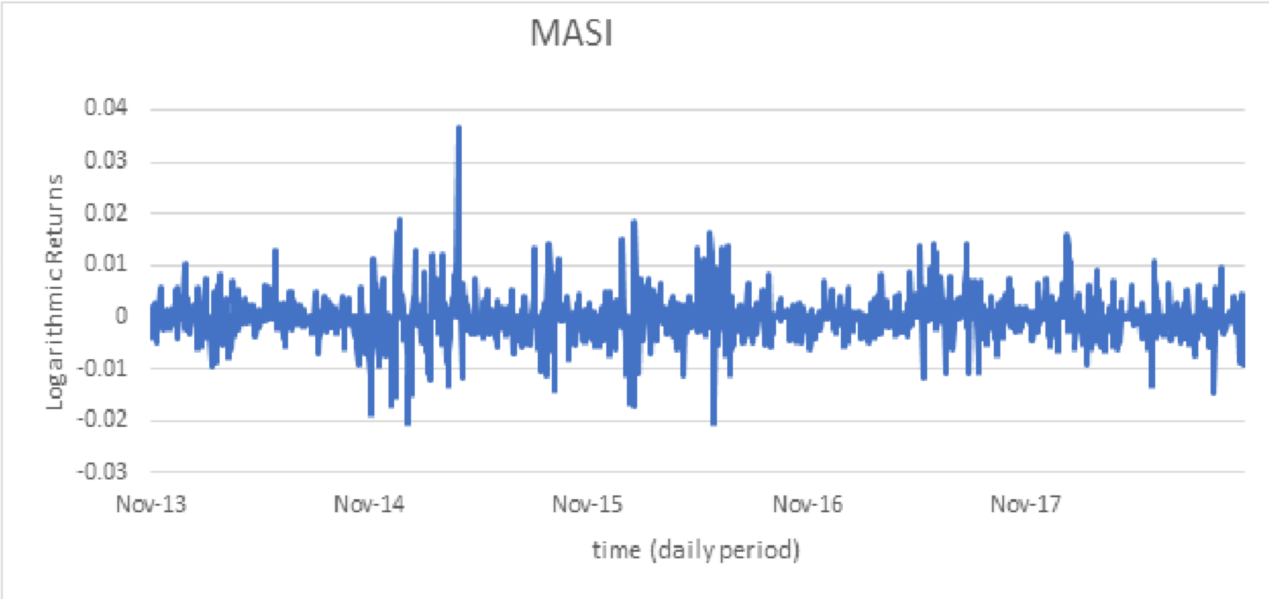

Figure 4.3: Daily logarithmic returns changes of MASI from Nov. the 1st 2013 to Oct. 31th 2018.

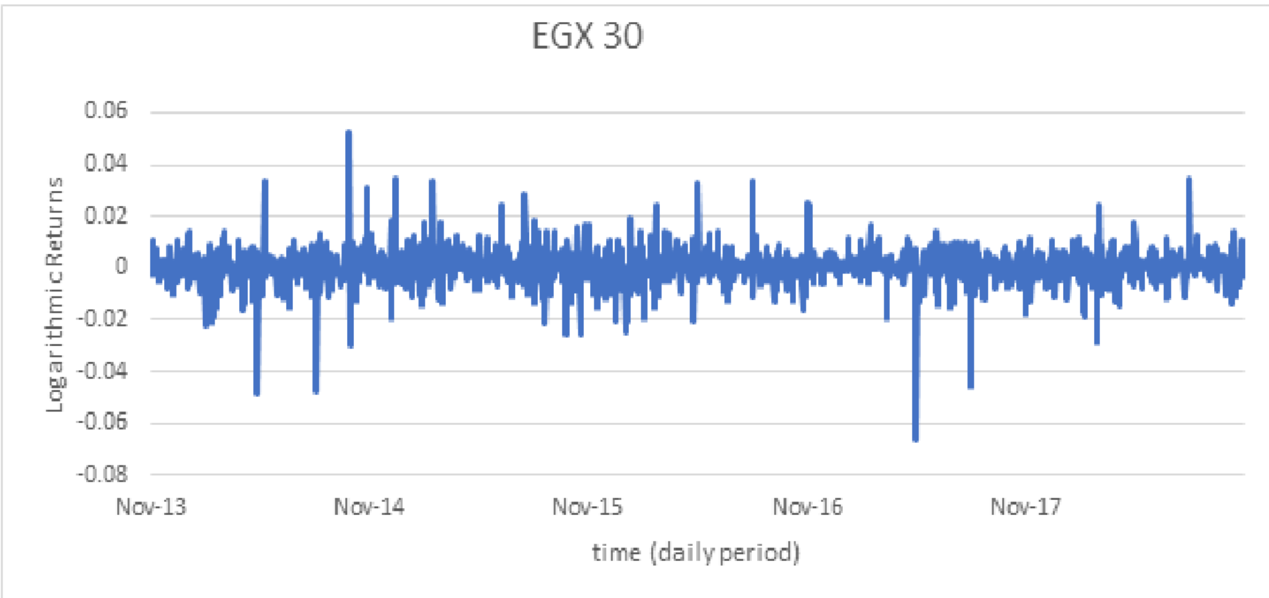

Figure 4.4: Daily logarithmic returns changes of EGX 30 from Nov. the 1st 2013 to Oct. 31th 2018.

the market is efficient. H1: prices on the index does not follow a random walk, then the market is not efficient.

4.2.1 Runs Test

We use the runs test as a non-parametric test for testing the randomness or serial independence for each of our four African indices. A run occurs every time the log price series changes its sign from increasing to decreasing for example or from a decrease to being unchanged. There are three possible changes: positive (+), negative (-), and zero changes (0). The run's test is basically a comparison between the observed (Actual) number of runs that directly taken from the observing the change in the logarithmic returns of each index with the expected number of runs, which is calculated under the assumption that prices fluctuate randomly and independently (IDD) due to the hypothesis of the weak-form of the random walk[9]. If there is a significant difference between the actual number of runs and the expected number of runs, the null hypothesis of randomness in successive price changes is rejected. We estimate the expected number of runs as:

$$ExpectedRuns(m) = \frac{\left[N(N+1) - \sum_{i=1}^{3} n_i^2 \right]}{N} \tag{4.2}$$

Where N is the total number of return observations, and n_i is the number of price changes of each sign.

The standard error of the series S_M of runs can be illustrated to be (Fama, 1965):

$$S_M = \left\langle \sum_{i=1}^{3} n_i^2 \left[\sum_{i=1}^{3} N_i^2 + N(N+1) \right] - 2N \sum_{i=1}^{3} n_i^3 - N^3 \right\rangle^{1/2} / N^2(N-1) \tag{4.3}$$

To test the significance of the difference between the actual R and the expected m number of runs, we can get use of the abundance of the used data (large sample) considering that the distribution is approximately normal, and then, measuring the difference by converting into the usual standardized normal variable Z:

$$Z = \frac{[R \pm 0.5] - m}{S_M} \tag{4.4}$$

where the half in the numerator is a discontinuity adjustment which may have positive or negative value depending on the difference is it positive or negative.

When the difference is found to be significant then rejecting the null hypothesis, it simply reflects that daily logarithmic returns are not random and that there is an opportunity to make abnormal returns which is, in different words, the market is not efficient. If the actual number of runs are significantly less than the expected value, it could be assigned to a market's overreaction to information, whereas a higher number of runs reveals a lagged response to information (Poshokwale, 1996).

4.2.2 Ljung-Box Statistic (White Noise Test)

This test examines the processes generating the observed returns of each index is a series of IID (identical independent distribution) random variables. To test the null hypothesis of the random walk since it implies that the joint hypothesis which says that all autocorrelations coefficient are zero, we use a statistic test that is very powerful when it compares to other statistics, Ljung and Box's (1978) portmanteau $Q'_m - statistic$, the modified form of $Q'_m - statistic$ due to Box and Pierce (1970), is used under our null hypothesis.

$$Q'_m = T(T+2)\sum_{k=1}^{m}\frac{\rho^2(k)}{T-k} \tag{4.5}$$

where:
T:the sample size;
k:the order of the autocorrelations (Lags)
$\rho(k)$:autocorrelation coefficient of order k

$$\rho(k) = \frac{cov(r_t, r_{t+k})}{\sqrt{Var(r_t)}\sqrt{Var(r_{t+k})}} \tag{4.6}$$

$$Q'_m \sim X_i^2 \tag{4.7}$$

Because Q'_m is asymptotically distributed as X_i^2 and it gives a better fit to the X_i^2, we measure the statistic against X_i^2. As know X_i^2 is a one-tail statistical test, if the p-value is greater than the significance level (α) leads us to that we can't reject the null hypothesis.

it is very useful to estimate how successive values

It is very useful to estimate how successive values of serial correlation (Autocorrelation) between the logarithmic returns are significantly away from zero. Taking into our consideration that choosing the number of autocorrelations m is a bit tricky, if it is chosen too few, we will be missing any higher-order autocorrelations that might be present, but also if it is too much, the power of the test would decrease due to the insignificance of higher-order autocorrelations. So, in real life cases practitioners used to choose the upper lag limit m as $logT$, and we followed that in our study[10].

4.2.3 Arch Effect Test

In the tests of the weak-form of the random walk we take for granted that the volatility of the returns in a stock is constant over long time horizons, but this is simply not true and statistically inconsistent we can notice that from having a look at the graphs of our African indices formerly plotted in this study which you may see the presence of what's called volatility clustering, that is, not directly a sign of market inefficiency but may affect it or in particular the validity of the tests that conducted for the market efficiency, and the leave conclusions on market efficiency. This volatility clustering is quite common when studying the random walk hypothesis of financial data for long time horizons, to avoid falling in such a mistake we consider the serial correlation of the volatility which is estimating the serial correlation coefficients for squared excess of logarithmic returns or absolute excess of logarithmic returns. In order to capture this, Engle proposed ARCH test, the class of Autoregressive Conditionally Heteroskedastic. So, we included this ARCH test in our study. Basically, the ARCH effect test is a white-noise test, but for the squared time series. Alternatively, we are estimating a higher order (non-linear) of autocorrelation.

$$y_t = X_t^2 \tag{4.8}$$

In this case the process x_t is represented by the logarithmic returns:

$$R_t = X_t \tag{4.9}$$

The ARCH effect test is about time varying volatility so:

$$\sigma_t^2 = E[x_t - \bar{x}_t{}^2] = E[x_t^2] - \bar{x}_t^2 \tag{4.10}$$

Where:
σ_t^2: conditional variance
$\bar{x}_t$: conditional mean

In practice, financial logarithmic means are close to zero, so it would be more convenient (but not very precise) to neglect the conditional mean so:

$$\sigma_t^2 = E[x_t^2] \approxeq x_t^2 \tag{4.11}$$

Obviously, the conditional volatility varies over time, with the assumption of serial correlation of the squared time series of the logarithmic returns, also may show a clustering effect for instance periods of swings followed by periods of relative calm. This ARCH test helps to investigate this time varying effect in the conditional volatility.

4.3 Discussions and Analysis of Results

In this section there will be discussions of the results of the random walk hypothesis tests that are carried out in this study as well as the general statistical descriptions for the selected indices.

The descriptive statistics for the four African indices are shown in Table 4.1. The highest average daily return is attained by MASI, the Moroccan index, registering 0.0355 percent,

Table 4.1: The statistical description of the daily returns of the four indices

Index	Mean	St. Dev	Minimum	Maximum	Skewness	Kurtosis
FTSE/JSE Top 40	2.97E-05	0.0036611	-0.0175858	0.020318703	-0.121393383	3.5870454
EGX 30	0.0001812	0.0047422	-0.0250308	0.028179512	-0.20504971	5.8635561
NSE 30	-4.66E-05	0.0039447	-0.0201125	0.036584219	0.439461736	9.0566038
MASI	0.0003549	0.0064845	-0.0666881	0.052601359	-0.622049776	16.668007

then with a relatively big difference we find the Egyptian index EGX 30 with 0.018 percent. The South African index FTSE/JSE Top 40 makes close to zero daily return which is 0.003 percent, whereas the Nigerian index NSE 30 presents a minus daily returns which does not make anything except reflecting substantial problem the companies included in that index have been through. Well, it's very imprecise to use this indices' average returns as bases to compare between the countries' economies because they give really limited information about that, not just because of the small roles played by these indices in their economies, but also the economic insignificance that the African stock exchange markets hold compared to ones existed in Europe and the United States. Along with the mean returns, we find the volatility measured by standard deviations keeps the same order with an exception of the Nigerian index which reflects a higher standard deviation than FTSE/JSE Top 40.

The distributions of the returns clearly are not normal because they are negatively skewed revealing that the returns are flatter to the left with respect to the normal distribution, except the Nigerian index which is positively skewed, flatter to the right compared with the normal distribution.

The rejection of the normal distribution is also emphasized by the high values of the kurtosis of the returns which might also indicate the distributions of returns are leptokurtic, i.e. revealing sharp peaks compared to the normal distribution.

Table 4.2: Runs tests results for the four African indices

Index	Actual Runs	Expected Runs	Standard Error	Z-Statistic
FTSE/JSE Top 40	1405	1135.3	19.882	13.542
EGX 30	1344	1147.7	19.887	9.8452
NSE 30	1315	1144.4	19.877	8.5603
MASI	1440	1136.5	19.885	15.239

Note Under null of random walk, actual runs should be equal to expected runs. Asterisked values indicate rejection of null of random walk at 1% level of significance.

The study also carried out a non-parametric test for the RWH for the indices which is the runs test.

As it is shown in table 4.2 the results of the runs test, the actual runs are the number of changes in the returns all the positive or negative ones counted in the returns' series of each index. The expected number of runs are the required changes in case the data set (series of each index) follows the random walk hypothesis and then generates a random process. From the results appeared in table 4.2 there are differences between the actual and the expected numbers of runs but how significant the difference so we cannot accept it, it's a very important question to consider that why here we take an advantage of the big size of the sample and assume the distribution is normal, then we can measure using Z-Statistic how significant this difference could be accepted according to our 1% significance level.

As clear in the results table, the hypothesis of random walk has been rejected for all the indices according to the runs test, though the random walk hypothesis does not explain the behavior these African indices.

Table 4.3: Autocorrelations coefficients and Ljung-Box Q statistic for FTSE/JSE Top 40 index

Lag	**LB Q'_m Statistic**	**LB Q'_m' significance**
1	1.13	28.70%
2	4.07	13.10%
3	4.42	22.00%
4	5.03	28.40%
5	5.8	32.60%
6	6.36	38.40%
7	7.54	37.50%
8	8.29	40.60%
9	10.15	33.90%
10	10.28	41.60%

Table 4.4: Autocorrelations coefficients and Ljung-Box Q statistic for EGX 30 index

Lag	**LB Q'_m Statistic**	**LB Q'_m' significance**
1	72.16	0.00%
2	73.67	0.00%
3	78.05	0.00%
4	80.9	0.00%
5	81.02	0.00%
6	83.76	0.00%
7	83.78	0.00%
8	84.51	0.00%
9	85.74	0.00%
10	85.85	0.00%

The Ljung-Box Q statistics for the indices are given in tables 4.3, 4.4, 4.5, and 4.6 for different orders of autocorrelations up to the 10th. Asterisked value rejects the null hypothesis at 1% significance level. As clear in the tables, the South African index FTSE/JSE Top 40 is

Table 4.5: Autocorrelations coefficients and Ljung-Box Q statistic for NSE 30 index

Lag	**LB Q'_m Statistic**	**LB Q'_m' significance**
1	197.78	0.00%
2	203.36	0.00%
3	204.26	0.00%
4	208.8	0.00%
5	213.53	0.00%
6	215.17	0.00%
7	221.7	0.00%
8	225.54	0.00%
9	226.41	0.00%
10	226.92	0.00%

Table 4.6: Autocorrelations coefficients and Ljung-Box Q statistic for MASI index

Lag	**LB Q'_m Statistic**	**LB Q'_m' significance**
1	0	0
2	0	0
3	0	0
4	5.03	28.40%
5	5.8	32.60%
6	6.36	38.40%
7	7.54	37.50%
8	8.29	40.60%
9	10.15	33.90%
10	10.28	41.60%

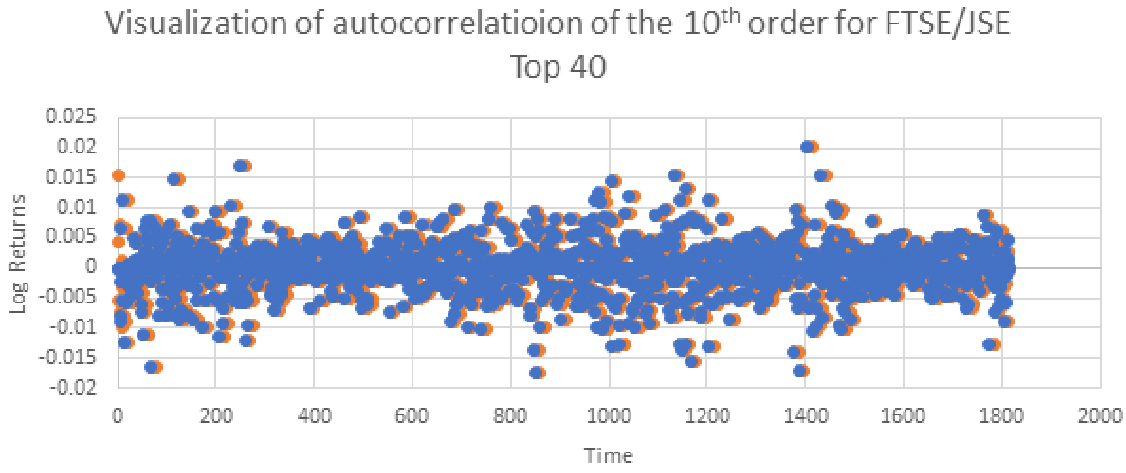

Figure 4.5: The 10th order of autocorrelations for FTSE/JSE Top 40.

following the random walk hypothesis because we cannot reject the null hypothesis given our significance level which means there in no serial correlations in this index'. Whereas for the other three indices we reject the null hypothesis and though these indices do not follow the random walk hypothesis, or in other words there is serial correlations in each of these indices' datasets.

In the following figures; figure 4.5 , figure 4.6, figure 4.7, and figure 7.8; autocorrelations of tenth order are visualized for the four indices; the tenth order because it is very difficult to visualize the first order autocorrelation with respect to the timeframe of the data (one day out of five years).

Table 4.7: Tests for the ARCH effect for the four African indices

Index	**P-value**
FTSE/JSE Top 40	0.00%
EGX 30	0.00%
NSE 30	0.00%
MASI	0.00%

From results of the ARCH effect test appeared in table 4.7, according to a significant level of 99We reject the null hypothesis of no ARCH effect for the all indices, which is confirming the existence of heteroscedasticity in the distributions of the results. That is directly against the hypothesis of the independent distributions of the daily asset returns over the time horizon (Random Walk Assumption). That explains also, as we discussed formerly, the presence of serial correlation for squared excess of returns, that is confirming the presence of the volatility clustering as well for each of the studies indices. This discussion will be quite interesting in case of the Random Walk Hypothesis tests claim efficiency of the markets in question, because it would question the assumptions of the hypothesis it's self not the outcomes through the

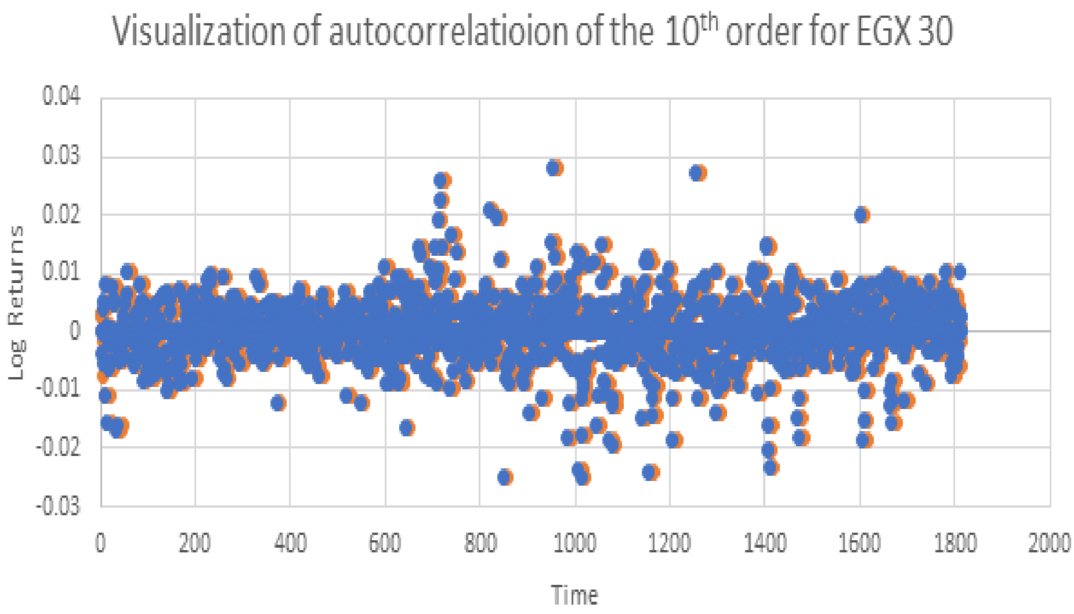

Figure 4.6: The 10th order of autocorrelations for EGX 30.

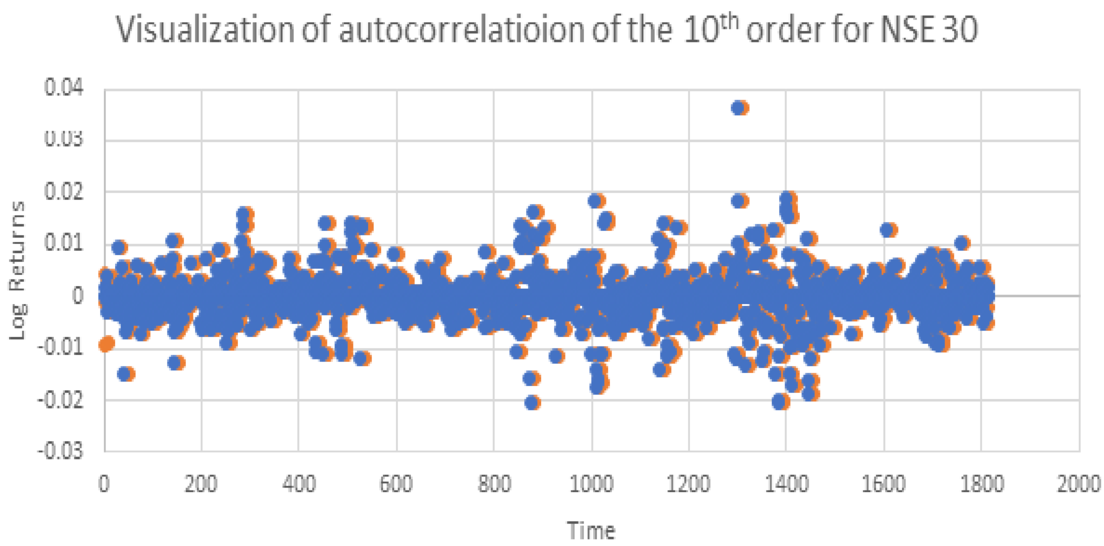

Figure 4.7: The 10th order of autocorrelations for NSE 30.

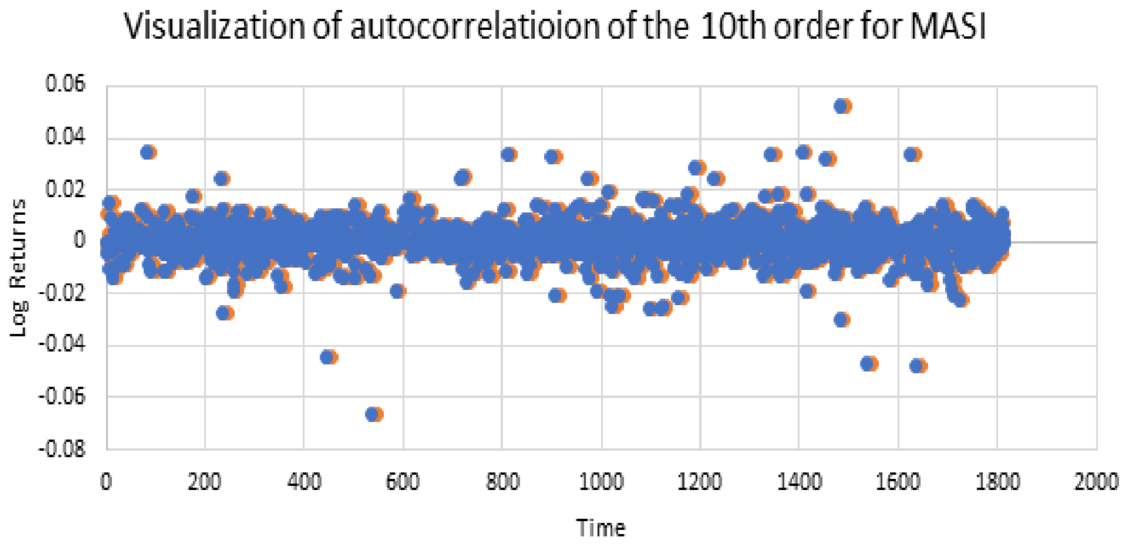

Figure 4.8: The 10th order of autocorrelations for MASI.

presence of heteroscedasticity in the distributions.

Chapter 5

Notes on Markowitz Model

Before publishing the famous article, "Portfolio Selection", in 1952 by Harry M. Markowitz, investors were just picking the assets through estimating their returns and risk individually, and then building portfolios. The process of investments decisions was based on selecting the securities with the higher returns and the lower risks, and accordingly forming the portfolio with weights consistent with the estimated returns and risks. Instead, Markowitz deals with that based on the returns and the risks of the portfolio as whole, that is called diversification of the portfolio, whereas in this approach the portfolio decision is made after estimation of the overall portfolio risk and return, that means in different words, selecting the portfolio in aggregate, instead of selecting each security separately based on the distinctive evaluation of its risk and the return. That triggers the question of the relationships between the securities for instance, the covariances between the securities rather than the emphasis on the characteristics of the securities themselves.

Decades from that time, Markowitz was awarded the Nobel Prize in Economics for this breakthrough on portfolio theory. The static model, in terms of expressing the returns of each security for a time period, assumes that these returns of the portfolio's securities are random variables so the statistical measures as the expected return and standard deviation of these variables could be obtained to estimate the risk of the investment. In details, the expected return of the portfolio $E[r_p]$ could be considered as a measure the reward of this investment, similarly the standard deviation of the portfolio σ_p could be obtained to measure the risk of the investment. The significance of this theory comes from the idea of diversification came up by Markowitz, because it gives attention to the fact that considering the covariance between the portfolio's assets could be a critical factor in estimating the overall risk of the portfolio, and then the decision of the selection itself [11].

5.1 Methodologies

It is quite important to introduce the parameters used in Markowitz model and other specific considerations taken in this study.

5.1.1 Rate of Return

As known, it is the ration an investor gets from putting amount of money in specific investment or security or any general project. The money invested is assigned to as the principal, so the rate of return (r) is usually taken as a ratio of the principal. Well, there is a substantial controversy behind estimating this rate of return, even the article of Markowitz does consider this, as quoted "The R (and consequently R_p) are considered to be random variables, i.e., we assume that the investor does (and should) act as if he had probability beliefs concerning these variables. I n general we would expect that the investor could tell us, for any two events (A and B), whether he personally considered A more likely than B, B more likely than A, or both equally likely. If the investor were consistent in his opinions on such matters, he would possess a system of probability beliefs. We cannot expect the investor to be consistent in every detail. We can, however, expect his probability beliefs to be roughly consistent on important matters that have been carefully considered. We should also expect that he will base his actions upon these probability beliefs even though they be in part subjective. This paper does not consider the difficult question of how investors do (or should) form their probability beliefs"[11]. In this study this rate of return for our African indices is obtained using historical prices as expectation means for the future, although this approach has its valid criticism, but still this is not the focal point of this study. Furthermore, the rate of return in this study, regardless of the timeframe (daily or annually), it's always takes as consciously compounded as follows:

$$r_t = logp_t - logp_{t-1} = log(p_t/p_{t-1}) \tag{5.1}$$

Where
p_t and p_{t-1} are the indices prices at time t and $t-1$ respectively.

Based on this historical approach, the expected rate of return for historical dataset of rates of return for an asset i could be calculated as the average:

$$\mu_i = E[r^i] = \frac{\sum_{i=1}^{m} r_t^i}{m} \tag{5.2}$$

where:
r_t^i : the rate of return on asset i between periods $t-1$ and t,for $t = 1, \ldots, m$;and
m:the number of periods.

5.1.2 Variance and Standard Deviation

Following the same approach of using historical data, the variance of an asset i is calculated as follows:

$$Var(r^i) = \sigma_i^2 = \frac{\sum_{i=1}^{m} (r_t^i - \mu_i)^2}{m-1} \tag{5.3}$$

Then using the variance, the standard deviation (volatility) is obtained which is the variable that is assumed to measure the risk of the assets according to Markowitz. It is very common the use this statistical measure as an indication of risk in studying the financial data-set, and obviously, investors are seeking lees risk, i.e., lower values of standard deviation are preferred.

Though it is calculated as follows:

$$\sigma_i = \sqrt{Var(r^i)} = \sqrt{\frac{\sum_{i=1}^{m}(r_t^i - \mu_i)^2}{m-1}} \tag{5.4}$$

5.1.3 Covariance

Since the level of diversification of the portfolio basically depends of the covariances between the assets that composing the portfolio, as with the variance and standard deviation, historical data of prices is used to estimate the covariances matrix $\Omega_{(m*n)}$ as follows:

$$\Omega_{(m*n)} = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & . & . & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & . & . & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & . & . & \sigma_{1,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ \sigma_{m,1} & \sigma_m^2 & . & . & \sigma_m^2 \end{bmatrix}$$

where:

$$\sigma_{ij} = cov(r^i, r^j) = \frac{\sum_{i=1}^{m}(r_t^i - \mu_i)(r_t^j - \mu_j)}{m} \tag{5.5}$$

As shown, the diagonal of the covariances matrix represents the variances of the assets.

5.1.4 The Basic Markowitz Model

The first model proposed by the article represents a mathematical representation for quadratic optimization problem to either maximize the expected return of the portfolio given a specific degree of risk estimated by the variance of the portfolio, or to minimize the variance of the portfolio given a specific expected return. Let's first explain the mathematical representations according to Markowitz for the portfolio expected return and the standard deviation. Obviously, the expected portfolio return is the weighted average of the returns of the assets making the portfolio. As follows:

$$\mu_p = E[R_p] = \sum_{i=1}^{n} w_i \mu_i = \mu^T W \tag{5.6}$$

Where:

w_i:the weight of asset i;the percentage of capital that will be invested in asset i;
R_p:the expected return of asset i;
μ_p:the expected return of the portfolio.

The variance of the portfolio is formulated as follows:

$$Var(R_p) = \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i \sigma_{ij} w_j = W^T \Sigma W \tag{5.7}$$

As shown, the representation is formulated also terms of matrix form which is quite useful practically; the weight matrix is W and Σ is the covariance matrix. Though, the following is the initial model published by Markowitz proposes that the investor tends to maximize the portfolio expected return and minimize the portfolio variance simultaneously. Markowitz excludes the short selling by making a constrains for the weights values to be not less than zero.

This leads to the following quadratic optimization problem:

$$max\ \mu_p = \sum_{i=1}^{n} w_i \mu_i \tag{5.8}$$

$$min\ \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i \sigma_{ij} w_j \tag{5.9}$$

$$s.t. \sum_{i=1}^{n} w_i = 1 \tag{5.10}$$

$$w_i \geq 0 \tag{5.11}$$

Intuitively, the sum of the assets' weights should be equal to zero.

5.1.5 The Modified Markowitz Model Used in This Study

In this study, there is a very important parameter $\lambda \in [0, 1]$ used to convey the investors risk aversion, that is, to which extent they are willing to rake risk to increase their returns or giving up returns to enjoy low levels of risk. The case with $\lambda = 1$ represents maximizing the portfolio mean return without paying attention to the variance and the optimal solution will be formed only by the asset with the greatest expected return. However, the case with $\lambda = 0$ represents minimizing the total variance associated to the portfolio regardless of the mean returns and the optimal solution will apparently include several assets. Any value of λ inside the interval (0,1) represents a trade-off between mean return and variance, generating a solution between the two extremes $\lambda = 0$ and $\lambda = 1$. It is also considered two cases, following the classical model in excluding the short selling which means considering the constraint of non-negative values of the weights, and a more general case that includes the short selling in the portfolio by taking that constraint away[12].

Accordingly, the optimization problem with the case of excluding the short selling as follows:

$$max\ \lambda\left[\sum_{i=1}^{n} w_i\mu_i\right] - (1-\lambda)\left[\sum_{i=1}^{n}\sum_{j=1}^{n} w_i\sigma_{ij}w_j\right] \tag{5.12}$$

$$s.t. \sum_{i=1}^{n} w_i = 1 \tag{5.13}$$

$$w_i \geq 0 \tag{5.14}$$

And, the optimization problem with the case of including the short selling is:

$$min\ (1-\lambda)\left[\sum_{i=1}^{n}\sum_{j=1}^{n} w_i\sigma_{ij}w_j\right] - \lambda\left[\sum_{i=1}^{n} w_i\mu_i\right] \tag{5.15}$$

$$s.t. \sum_{i=1}^{n} w_i = 1 \tag{5.16}$$

5.1.6 Portfolio Efficient Frontier

Graphically, to illustrate the above results may be outcome from the classical model of Markowitz, it sounds good to plot them in risk-return space; the x-axis represents the risk of the portfolio in form of the standard deviation and the y-axis represents the expected portfolio return. Then, all the possible portfolios could be plotted in this risk-return space. The efficient frontier could be defined as the line that the combinations along it represents the portfolios for which there is a highest expected return given a specific level of risk. Or conversely, given a specific level of return, the portfolios plotted along in the efficient frontier represent the one with the least possible level of risk[13].

The efficient frontier line, to sum up, only gives the optimal portfolios for varying risk aversions. Though it shows the increase of the expected returns while taking more risks through accepting higher values of standard deviations, reversely it also shows the investors' desire of having lower level of risk by giving up some level of return. Mathematically, the efficient frontier represents the portfolios could be obtained out of applying the optimization model of Markowitz defined formerly, given different levels of investors' risk aversions preferences. So, each point along the line gives an optimal solution to the quadratic problem.

Graphically, as shown in figure 5.1, the efficient frontier shows a curve with slope which is consistent with the financial fact of risk-reward principal. The figure also shows the entire investment opportunities for investors which represents all attainable combinations of risk-returns held by portfolios combined out of risky assets. The line represents only the optimal combinations of portfolios varying with the risk aversions[14]

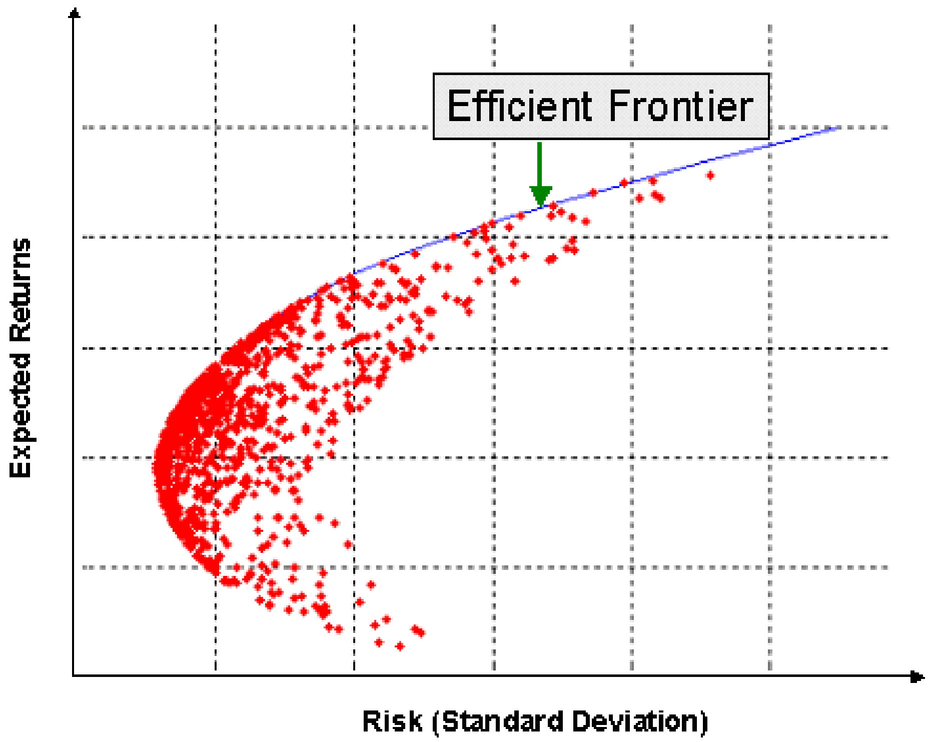

Figure 5.1: Efficient Frontier.

Chapter 6

Building Optimal Portfolios of African Markets

6.1 Data

The datasets used in this part of portfolio optimization are the same data that used for the market efficiency, which id the daily closing prices series for 5 years; from the 1st November 2013 to the 31th of October 2018; of four general indices of different African Stock Exchange, namely, FTSE/JSE Top 40 index from Johannesburg Stock Exchange, NSE 30 index from Nigerian Stock Exchange, MASI index from Casablanca Stock Exchange, and EGX 30 index from Egyptian Stock Exchange. The data were taken from their websites. In this part, for having more concrete interpretations of the results, the exchange rates risk of the countries, where these African indices traded, are considered. Though data-sets of the daily exchanges rates of the countries' currencies to the USD are gathered for the same period in questions; from the 1st November 2013 to the 31th of October 2018. Then, all the indices are converted into USD, putting the risk of the exchange rates on the shoulders of investors, since trading in those countries involves taking considerations of exchange rates changes.

Other datasets included in this study are closing prices of famous general indices across the world to make the thorough picture of the African stock exchanges and its position among the world. Though, closing prices of Dow Jones Industrial Index (DJI) from New York Stock Exchange, NIKKEI from Tokyo Stock Exchange and DAX Index from Frankfort Stock exchange are acquired for the same 5 years; from the 1st November 2013 to the 31th of October 2018. Since it quite significant in this study to compare the investment opportunities made by the African stock exchanges to the ones provided in the Western stock exchanges.

Well, the datasets of course needed a clearing process because lots of problems contained in that which would have affected the results without this data clearing process. For instance, it's interesting here to mention that because we were interested in finding the covariances between the indices, we needed the data to be timely matching, whereas the countries in question have different calendar holidays for instance the weekend in Egypt is Friday and Saturday instead of Saturday and Monday in South Africa also the Christmas holidays in South Africa are corresponded by working days in Morocco. To go over this we had to come up with that, making the prices during the holidays equal to the prices of the daily directly before the holidays which means for example the prices in the weekend in Egypt are Thursday's (directly before

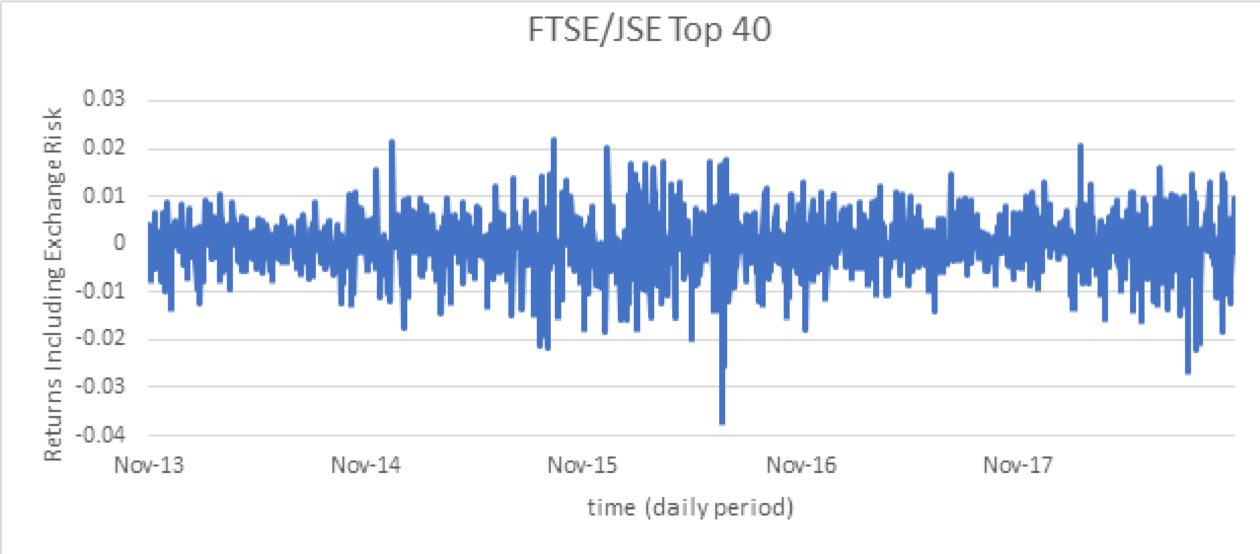

Figure 6.1: Daily returns changes of FTSE/JSE Top 40 from Nov. the 1st 2013 to Oct. 31th 2018 including the exchange rate risk (returns calculated for the indices in USD).

the weekend) prices. This way we could make match of the data so we can get an acceptable covariances values between the indices. Then, a natural logarithmic transformation is performed so the data is converted to generate a time series of continuously compounded returns, daily returns of the indices are computed as follows:

$$r_t = logp_t - logp_{t-1} = log(p_t/p_{t\_1}) \tag{6.1}$$

where:
p_t and p_{t-1} are the indices prices at time t and $t-1$ respectively.

Since the four African indices returns are illustrated in the part of market efficiency, and for avoiding repetition in the following there will be graphs of those indices including the risk of exchange rates. Or in other words, the continuously compounded returns are calculated after exchanging the daily closing prices into USD using the datasets defined formerly. Accordingly, figure 6.1, figure 6.2, figure 6.3 and figure 6.4 represent the continuously compounded returns, after exchanging into USD, of FTSE/JSE Top 40, NSE 30, MASI and EGX 30 respectively. However, this study is to highlight investments in African Stoch Exchanges, it is not a necessity to study the datasets of the closing prices of DJI, NIKKEI and DOX. Since they are included for the sake of making fruitful comparisons. Consequently, the following graphs draw attention only to the four African indices; the price fluctuations considering the exchange rate.

6.2 Methodologies

Formerly in the previous chapter the modified version of Markowitz model is presented in plenty of details, but here the entire process of producing the optimal portfolio is illustrated

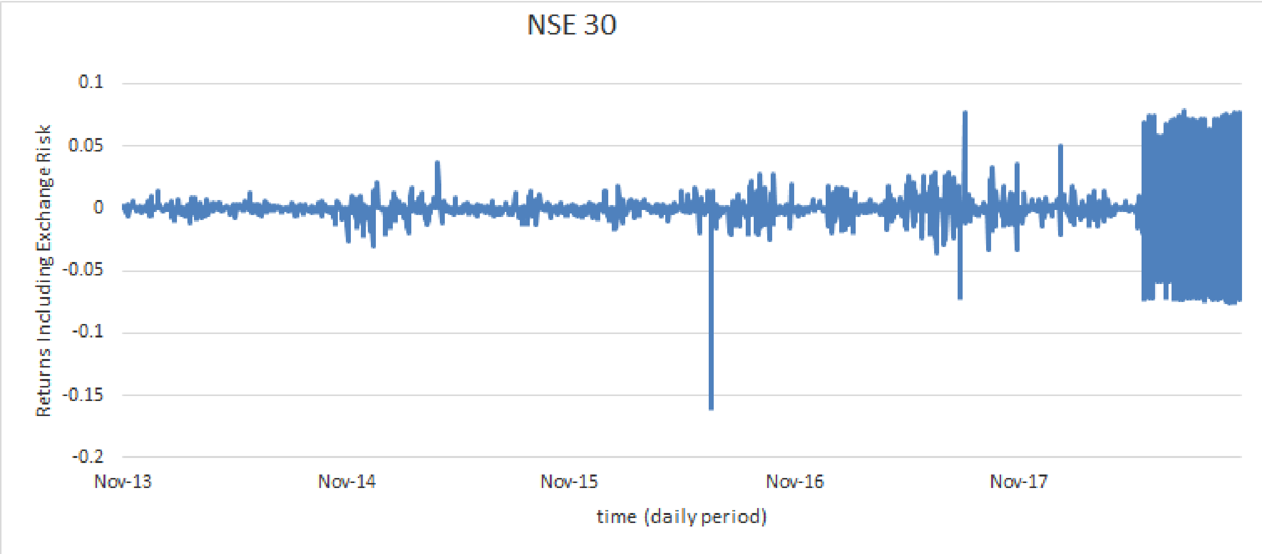

Figure 6.2: Daily returns changes of NSE 30 from Nov. the 1st 2013 to Oct. 31th 2018 including the exchange rate risk (returns calculated for the indices in USD).

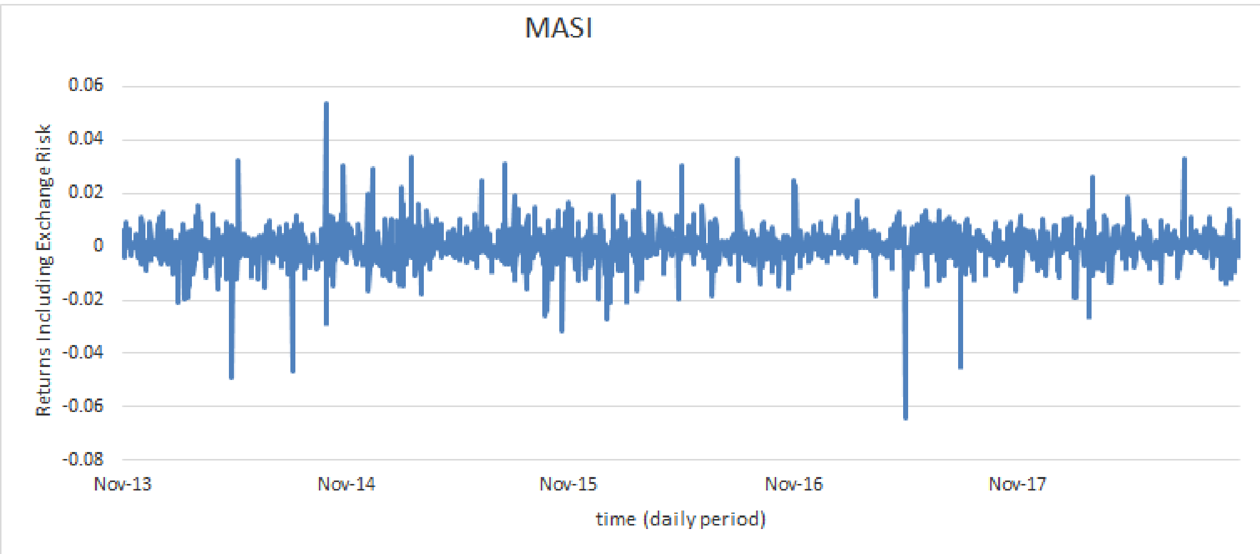

Figure 6.3: Daily returns changes of MASI from Nov. the 1st 2013 to Oct. 31th 2018 including the exchange rate risk (returns calculated for the indices in USD).

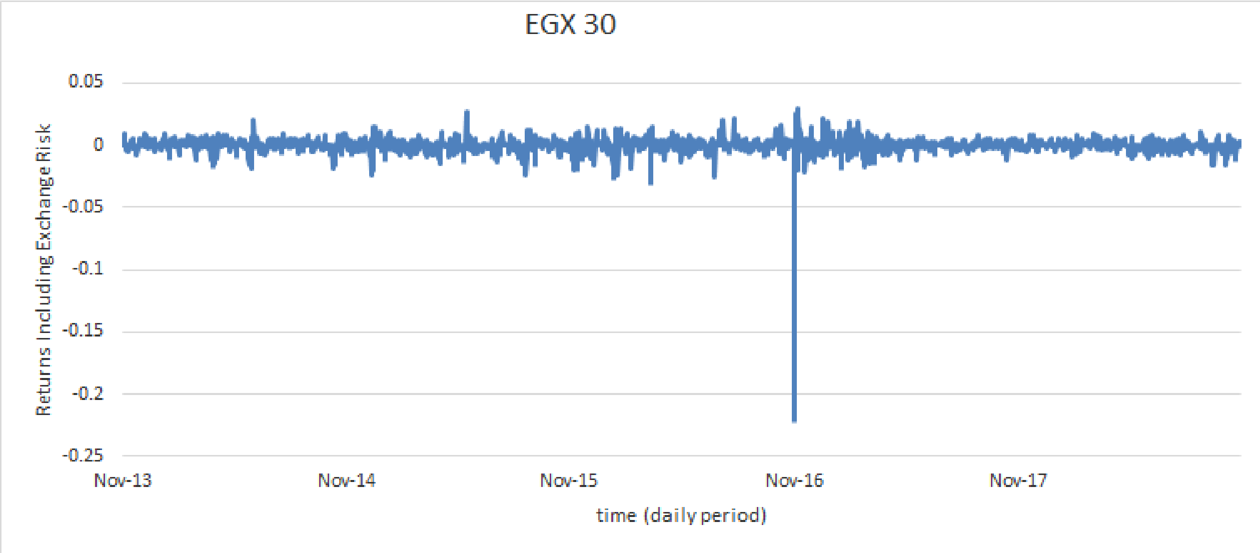

Figure 6.4: Daily returns changes of EGX 30 from Nov. the 1st 2013 to Oct. 31th 2018 including the exchange rate risk (returns calculated for the indices in USD).

in practical sense, however it is real-world problem based on market's data-sets. After clearing the data; the indices' prices are converted into USD in the case where the exchange risk in considered, as follows:

$$p_t(USD) = p_t * R_t \tag{6.2}$$

where:

p_t:the index price in the country's original currency at time t

R_t: the country's exchange rate from the original currency into USD at time t

p_t (USD): the index price in USD at time t

Again, the natural logarithmic transformation is performed to the price ratio, so the data is converted to generate a time series of continuously compounded returns, daily returns of the indices are computed as follows:

$$r_t = logp_t - logp_{t-1} = log(p_t/p_{t\_1}) \tag{6.3}$$

where:

p_t and p_{t-1} are the indices prices at time t and $t-1$ respectively.

Part of implementing the risk-aversion modified model of Markowitz is estimating the co-variances between the assets considered, the common practice realizes that in form of covariance matrix:

$$\Omega_{(m*n)} = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & . & . & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & . & . & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & . & . & \sigma_{1,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ \sigma_{m,1} & \sigma_m^2 & . & . & \sigma_m^2 \end{bmatrix}$$

where:

$$\sigma_{ij} = cov(r^i, r^j) = \frac{\sum_{i=1}^{m}(r_t^i - \mu_i)(r_t^j - \mu_j)}{m} \tag{6.4}$$

This is the basic formula of this study; the risk-aversion modified model of Markowitz in case of restricting the short-selling.

$$max\ \lambda \left[\sum_{i=1}^{n} w_i \mu_i\right] - (1-\lambda)\left[\sum_{i=1}^{n}\sum_{j=1}^{n} w_i \sigma_{ij} w_j\right] \tag{6.5}$$

$$s.t. \sum_{i=1}^{n} w_i = 1 \tag{6.6}$$

$$w_i \geq 0 \tag{6.7}$$

The risk-aversion modified model of Markowitz in case of including the short-selling.

$$max\ \lambda \left[\sum_{i=1}^{n} w_i \mu_i\right] - (1-\lambda)\left[\sum_{i=1}^{n}\sum_{j=1}^{n} w_i \sigma_{ij} w_j\right] \tag{6.8}$$

$$s.t. \sum_{i=1}^{n} w_i = 1 \tag{6.9}$$

6.3 Discussions of the Results

This is the most significant part of this study because it gives clear insights about investments opportunities in Africa, through taking the most flourished stock exchanges in the mother continent. Not just in terms expected returns and volatilities, but also the risk of exchange rates is considered in this study to draw a thorough intuition using the risk-aversion modified version of Markowitz model of portfolio optimization. This part is divided to two subparts; the first one reveals general statistical description to the collected data emphasizing on expected returns, the risk of volatility and the covariances between the indices because they are the basics of model of the portfolio selection; the second part discuss the optimal portfolios obtained in the three cases considered in this study.

6.3.1 Statistical descriptions of data

The general descriptive statistics for the all indices used in this study are shown in Table 6.1. The African stocks already described in the previous part. But regarding the case of including DJIA, NIKKEI and DAX, still MASI comes in the front in terms of returns; .0355 percent a day, followed by EGX 30. But DJIA and NIKKEI record higher returns than the South African index; 0.011 and 0.010 percent respectively. However, Dax has the smallest among the indices; 0.0057 percent, but not the smallest return considering that NSE30 makes loss; -.00466 percent.

Along with the mean returns, we find the volatility measured by standard deviations keeps the same order with an exception of the Nigerian index which reflects a higher standard deviation than FTSE/JSE Top 40 and DJIA and DAX which reflects a higher volatility as well with respect to FTSE/JSE Top 40and DJIA. The distributions of the returns clearly are not normal because they are negatively skewed revealing that the returns are flatter to the left with respect to the normal distribution, except the Nigerian index which is positively skewed, flatter to the right compared with the normal distribution. The rejection of the normal distribution is also emphasized by the high values of the kurtosis of the returns which might also indicate the distributions of returns are leptokurtic, i.e. revealing sharp peaks compared to the normal distribution.

Table 6.1: The statistical description of the daily returns of the indices without the risk of exchange rates.

Index	Mean	St. Dev	Minimum	Maximum	Skewness	Kurtosis
FTSE/JSE Top 40	2.97E-05	0.0036611	-0.0175858	0.0203187	-0.1213934	3.5870454
EGX 30	0.0001812	0.0047422	-0.0250308	0.0281795	-0.2050497	5.8635561
NSE 30	-4.66E-05	0.0039447	-0.0201125	0.0365842	0.4394617	9.0566038
MASI	0.0003549	0.0064845	-0.0666881	0.0526014	-0.6220498	16.668007
DJIA	0.0001108	0.0028586	-0.0204739	0.016831	-0.6899371	6.5462622
NIKKEI	0.0001033	0.0045471	-0.035842	0.0322514	-0.2889064	9.0632474
DAX	5.70E-05	0.0040655	-0.0306928	0.0210722	-0.4062592	4.7646822

Table 6.2 shows annual returns and risks for the sake of familiarity since the average returns and standard deviations are commonly considered annually, besides that in this study there are two categories in terms of time for optimal portfolios made; daily and annually.

It is very essential in the concept of portfolio optimization to consider the correlation between the securities, where Markowitz suggests the formation of portfolios of stocks that have less than perfect positive correlation yields a better level of diversification. due to that the calculated covariances of the daily returns appear in Table 6.3.

It is too way better to analyze the annual version of the covariances, more practically

Table 6.2: The means and the standard deviations of the annual returns of the indices without the risk of exchange rates.

	FTSE/JSE Top 40	EGX 30	NSE 30	MASI	DJIA	NIKKEI	DAX
Mean	0.007422384	0.0453079	-0.0116505	0.0887267	0.0276986	0.0258237	0.014259
St. Dev	0.057886797	0.074981	0.0623706	0.1025284	0.0451978	0.0718967	0.064281

Table 6.3: The covariances matrix of the daily returns of the indices without the risk of exchange rates.

	FTSE/JSE Top 40	EGX 30	NSE 30	MASI	DJI	NIKKEI	DAX
FTSE/JSE Top 40	1.34E-05						
EGX 30	2.29E-06	2.25E-05					
NSE 30	6.26E-07	7.71E-07	1.56E-05				
MASI	2.05E-06	8.87E-07	-1.84E-07	4.20E-05			
DJIA	3.82E-06	5.92E-07	1.20E-07	6.67E-06	8.17E-06		
NIKKEI	5.02E-06	3.04E-06	9.96E-07	3.24E-06	2.22E-06	2.07E-05	
DAX	8.27E-06	1.72E-06	6.32E-07	5.27E-06	6.09E-06	5.21E-06	1.65E-05

common and sensible especially when the compared subjects belong to real-world securities with real data. Though Table 6.4 shows the annual covariances for the studied indices; the table shows 21 covariances of each index with other indices. Well, clearly there is no serious correlations between the indices indicating to the different economic factors affect each stock exchange. That may reflect a single fact which is in Africa the due to the less economic cooperation agreements compared to the West there is less interaction then correlation between the African market, especially the stock exchange. What supports that is the highest annual covariance is assigned to FTSE/JSE Top 40 and DAX; 0.21%, however there is almost no correlation between MASI and DJIA; recording the smallest one among the group by 0.003% It is also quite remarkable to notice that all covariances are positive with an exception of the covariance between NSE 30 and MASI; which reflects minus correlations between the indices. This less correlations among indices sound a good factor for the diversification of the risk according to Markowitz, in fact it would be extremely pointless to build a portfolio out of perfectly correlated assets yielding a greater value to the portfolio variance which supposed to be minimized.

Table 6.4: The covariances matrix of the annual returns without the risk of exchange rates.

	FTSE/JSE Top 40	EGX 30	NSE 30	MASI	DJI	NIKKEI	DAX
FTSE/JSE Top 40	0.0033509						
EGX 30	0.0005728	0.005622					
NSE 30	0.0001565	0.000193	0.00389				
MASI	0.0005128	0.000222	-4.59E-05	0.0105121			
DJIA	0.0009545	0.000148	3.01E-05	0.0016668	0.002043		
NIKKEI	0.0012541	0.000759	0.000249	0.0008106	0.000556	0.005169	
DAX	0.0020684	0.00043	0.000158	0.0013185	0.001524	0.001303	0.00413

Another case in this study, as it is formerly explained, carries out the comparison between the indices considering the exchange rate risk. Table 6.5 summarizes the annual means and standard deviations of the indices including the risk of exchange rates. It is figured out that exchange rates risk has passive effect on returns in general sine three of turned having negative vales; FTSE/JSE Top 40, EGX 30, and NSE 30. Even MASI got remarkably less annual return compared to the case that does not take the exchange rates risk into considerations. As expected, the risk level now represented standard deviations attained much higher values relative to previous case reflecting instability and turbulence in the continent's economies expressed through the constant change in the exchange rates for the four African counties in question.

Table 6.6. shows the covariances between the daily returns of the indices including the risk of exchange rates. The covariances now attaining more negative values surprisingly which could

Table 6.5: The means and the standard deviations of the annual returns of the indices including the risk of exchange rates.

	FTSE/JSE Top 40	EGX 30	NSE 30	MASI
Mean	-0.014740371	-0.011539155	-0.050709882	0.080352296
St. Dev	0.084161671	0.114338096	0.216596562	0.10318158

be due to lots of possible justifications, for instance, the political instability any country may have which would leave a print on exchange rates. It is not one of the objectives of this study to investigate this, however describing correlations may be quite essential in building optimal portfolios.

Table 6.6: The covariances matrix of the daily returns of the indices including the risk of exchange rates.

	FTSE/JSE Top 40	EGX 30	NSE 30	MASI
FTSE/JSE Top 40	2.83E-05			
EGX 30	1.11E-06	5.23E-05		
NSE 30	-1.37E-07	-6.04E-08	0.000187656	
MASI	6.71E-06	-3.00E-06	-2.17E-06	4.26E-05

Table 6.7, as the previous case, shows the annual covariances of returns including the risk of exchange rates. Clearly, correlations are too way less considering the risk of exchange rates which is financially sensible. Uncorrelation between the African exchange rates could be rooted to uncorrelated political and economic circumstances for each country, especially due to the lack of active economic integration agreements (like the EU in Europe) in Africa.

Table 6.7: The covariances matrix of the annual returns including the risk of exchange rates.

	FTSE/JSE Top 40	EGX 30	NSE 30	MASI
FTSE/JSE Top 40	0.007083187			
EGX 30	0.000278495	0.0130732		
NSE 30	-3.43E-05	-1.51E-05	0.046914071	
MASI	0.001676266	-0.000751119	-0.000542661	0.010646439

6.3.2 Optimal Portfolios and Efficient Frontier

This is the core part of this study; since it analyses the outcomes from applying the modified model of building portfolio and it compares, through distinctive scenarios, between the yielded portfolios based on different categorization factors. There are three cases undertakes in this study for the application of portfolio selection model; in the first case the optimal portfolios are built only out of the four African indices in their local currencies which means without including the exchange risk; the second case presents building optimal portfolios also only from the four African indices but after exchanging all the values into USD, that is, including the exchange risk; in the third case it is purposed to highlight the investments opportunities in the African continent among the other stock exchanges across the world, accordingly the portfolios made up out of all the indices considered in this study without giving attention to the exchange risk. In each one of these three cases there are two factors used for categorization; the timeframe of the risk and return, which considered either daily or annually; and the condition of the short-selling, either allowing the short-selling or restricting it. To sum up, each case undertakes these two factors outlining four subcases; including short-selling annually, including short-selling daily, restricting short-selling annually, and restricting short-selling daily.

In addition to that, the values of the risk aversion factor λ range from 0, which represents paying attention only to minimize the portfolio risk without considering the return, to 1, that is, the willingness to take any level of risk to increase the portfolio reward. With a 10% increase increasing rate to λ, it would end up having 12 portfolios with different risk aversion scenarios; $\lambda = 0, \lambda = 0.1, \lambda = 0.2, \ldots, \lambda = 0.9, \lambda = 1$. However, the portfolios appeared in each table are optimal or graphically they could be represented by points along the efficient frontier varying with the level of risk aversion. The tables also show in detail, for the sake of the comparison, the portfolio returns, variances, and standard deviations.

6.3.2.1 Case 1: Optimal portfolios of African only Indices without Considering the Exchange Risk

This case concentrates only about comparing between the African markets in terms of returns and volatility, though the tables from Table 6.8 to Table 6.11 show the built optimal portfolios for this case and their reward and risk measures for various risk aversion levels. Each table represents a different subcase.

Table 6.8 and 8.9 show the subcase that includes the short selling, in the annual measures when considering only minimizing the variance ($\lambda = 0$), the portfolio allocation is only based on the indices' standard deviations and covariances, that is why the biggest share 35.1 % is assigned to the least risk index; FTSE/JSE Top 40 with 5.8% standard deviation. As the risk aversion level increases in form of increasing λ, the emphasis on the returns increases. Though at $\lambda = 0.5$ for instance, clearly NSE 30 is short-sold since its return is negative which is quite logical financially, FTSE/JSE Top 40 as well is short-sold, by less quantity compared to NSE 30, but differently because it is better used to minimize the portfolio risk than to maximize the return, however the returns if MASI and EGX3 30 are markedly bigger than FTSE/JSE Top 40. The lion share belongs to MASI due to having the biggest return among the indices followed by EGX3 30; recording 3.5 and 2.9 respectively in the annual measures as appeared in table 6.8. When investors only pay attention to increasing the portfolio return ($\lambda = 1$),

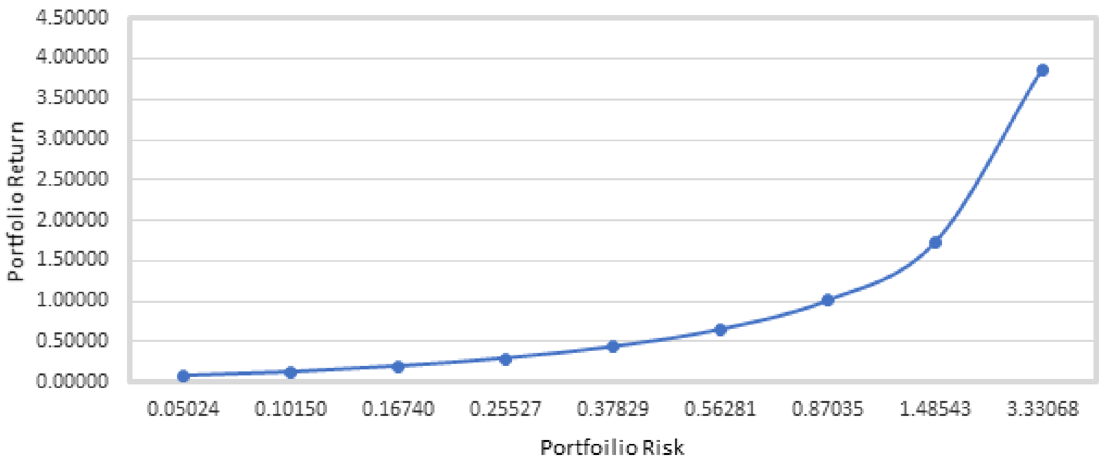

Figure 6.5: Efficient frontier minimizing risk of portfolio for case 1 including short-selling in annual measures.

and since short-selling is not restricted there are very big figures representing the portfolio while minimizing the variance is not a concern. Apparently, the weights of the portfolio are not bounded by minimum or maximum values, so the algorithm is free to give high values as showed in table 6.8 and table 6.9. Results are almost similar when a comparison is made between table 6.8 and table 6.9; allowing the short-selling annually and daily respectively, with slight differences due to the non-linear nature of Markowitz model otherwise the weights would be identical regardless of the timeframe of the measures.

Table 6.10 and 6.11 show the subcase that excludes the short-selling, the portfolio when $\lambda = 0$ is identical to the case of including short-selling. But as λ goes higher with restricting the short-selling NSE 30; since it has negative value, would be taken out of the portfolio from $\lambda = 0.1$ and higher which is financially sensible. Looking at portfolios at $\lambda = 0.3$ or higher, it is entirely made of MASI not just because if excluding the short-selling for the case of NSE 30 and FTSE/JSE Top 40, but also due to that, the difference in the volatility between MASI and EGX 30 is rewarded by a relatively bigger difference in the return. Results are almost similar when a comparison is made between table 6.10 and table 6.11; restricting the short-selling annually and daily respectively.

Figures 6.5 and 6.7 show the efficient frontiers for case 1 including and excluding the short-selling in the annual measures. Instead, figures 6.6 and 6.8 show the optimal portfolios calculated for the same conditions. .

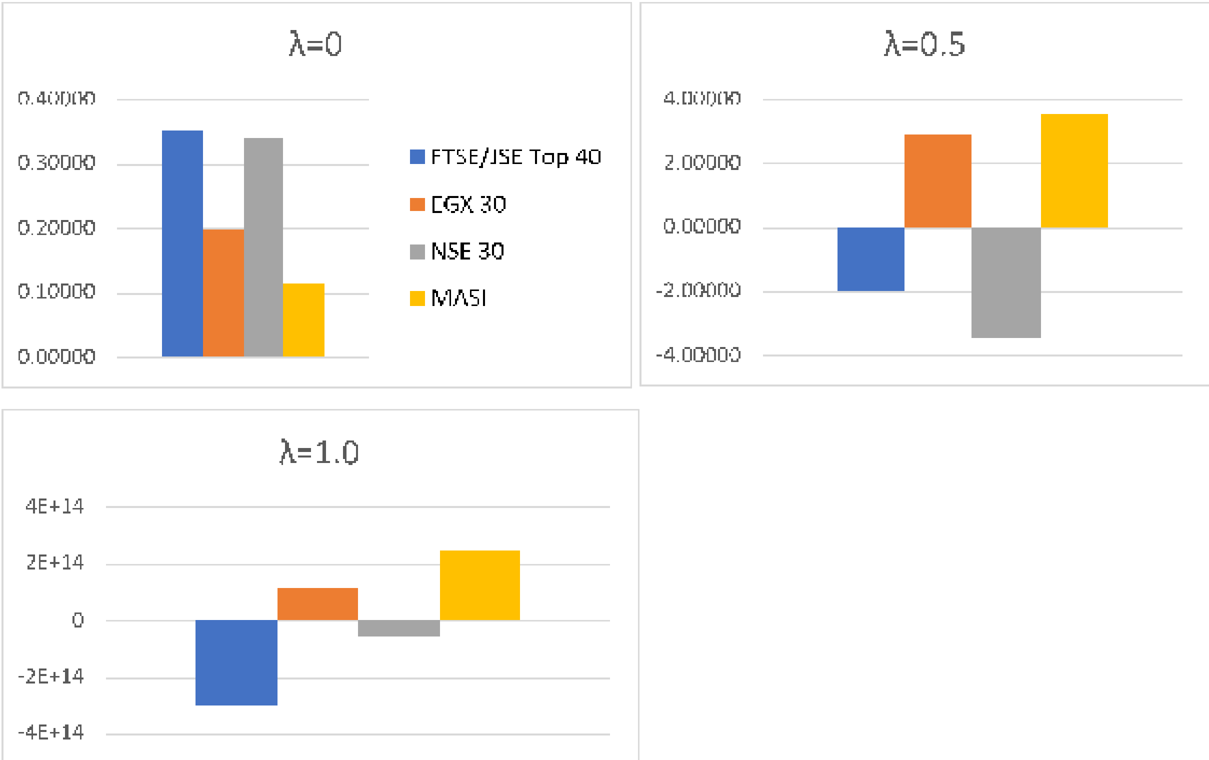

Figure 6.6: The optimal portfolios for different risk-aversion levels for case 1 including short-selling in annual measures.

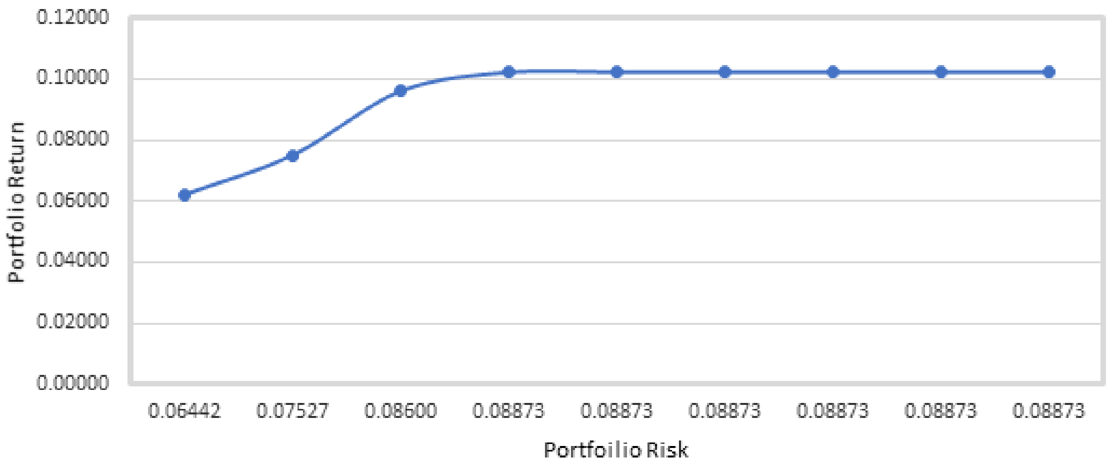

Figure 6.7: Efficient frontier minimizing risk of portfolio for case 1 excluding short-selling in annual measures.

Figure 6.8: The optimal portfolios for different risk-aversion levels for case 1 excluding short-selling in annual measures.

Table 6.8: Optimal portfolios (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of annual measures including short selling.

Lambda(λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.35137	0.09149	-0.23336	-0.65101	-1.20789	-1.98752	-3.15697	-5.10605	-9.0042	-20.69866	-3.01E+14
EGX 30	0.19743	0.4962	0.86967	1.34983	1.99006	2.88637	4.23084	6.47162	10.95318	24.39786	1.14E+14
NSE 30	0.33771	-0.08117	-0.60475	-1.27794	-2.17552	-3.43213	-5.31704	-8.45857	-14.74163	-33.59079	-5.73E+13
MASI	0.11349	0.49347	0.96844	1.57912	2.39335	3.53328	5.24318	8.093	13.79265	30.89159	2.44E+14
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Annual)	0.01769	0.06789	0.13064	0.21132	0.3189	0.46951	0.69541	1.07193	1.82496	4.08404	2.53E+13
Variance (Annual)	0.0014	0.00419	0.01552	0.04289	0.10181	0.22731	0.5097	1.23135	3.61594	18.29998	9.17E+26
Standard Deviation (Annual)	0.03744	0.06473	0.12458	0.20711	0.31907	0.47677	0.71393	1.10966	1.90156	4.27785	3.03E+13
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.0014	0.00302	0.01371	0.03337	0.06648	0.1211	0.21337	0.38095	0.73678	1.84564	2.53E+13

Table 6.9: Optimal portfolios (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of daily measures including short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.35137	0.07382	-0.27313	-0.71919	-1.31395	-2.1466	-3.39559	-5.47723	-9.64052	-22.13038	-1.40E+15
EGX 30	0.19743	0.37817	0.60408	0.89455	1.28184	1.82404	2.63734	3.99284	6.70385	14.83687	5.56E+14
NSE 30	0.33771	-0.01126	-0.44746	-1.00828	-1.75606	-2.80294	-4.37326	-6.99046	-12.22487	-27.92808	-2.80E+14
MASI	0.11349	0.55927	1.1165	1.83293	2.78817	4.1255	6.13151	9.47485	16.16153	36.22158	1.13E+15
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00008	0.00028	0.00054	0.00088	0.00132	0.00194	0.00287	0.00443	0.00753	0.01685	5.02E+11
Variance (Daily)	0.00001	0.00002	0.00006	0.00018	0.00042	0.00094	0.0021	0.00508	0.01492	0.07549	7.92E+25
Standard Deviation (Daily)	0.00237	0.00414	0.00799	0.0133	0.02049	0.03062	0.04585	0.07127	0.12213	0.27475	8.90E+12
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00001	0.00001	0.00006	0.00014	0.00028	0.0005	0.00088	0.00157	0.00304	0.00762	5.02E+11

Table 6.10: Optimal portfolios (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of annual measures without short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.35137	0.04751	0	0	0	0	0	0	0	0	0
EGX 30	0.19743	0.47075	0.30993	0.06286	0	0	0	0	0	0	0
NSE 30	0.33771	0	0	0	0	0	0	0	0	0	0
MASI	0.11349	0.48174	0.69007	0.93714	1	1	1	1	1	1	1
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.01769	0.06442	0.07527	0.086	0.08873	0.08873	0.08873	0.08873	0.08873	0.08873	0.08873
Variance (Daily)	0.0014	0.00384	0.00564	0.00928	0.01051	0.01051	0.01051	0.01051	0.01051	0.01051	0.01051
Standard Deviation (Daily)	0.03744	0.06199	0.0751	0.09633	0.10253	0.10253	0.10253	0.10253	0.10253	0.10253	0.10253
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.0014	0.00298	0.01054	0.0193	0.02918	0.03911	0.04903	0.05896	0.06888	0.0788	0.08873

Table 6.11: Optimal portfolios (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of daily measures without short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.35137	0.06772	0	0	0	0	0	0	0	0	0
EGX 30	0.19743	0.37464	0.12366	0	0	0	0	0	0	0	0
NSE 30	0.33771	0	0	0	0	0	0	0	0	0	0
MASI	0.11349	0.55765	0.87634	1	1	1	1	1	1	1	1
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00008	0.00028	0.00037	0.00041	0.00041	0.00041	0.00041	0.00041	0.00041	0.00041	0.00041
Variance (Daily)	0.00001	0.00002	0.00003	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004
Standard Deviation (Daily)	0.00237	0.00412	0.00573	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00001	0.00001	0.00005	0.00009	0.00014	0.00018	0.00023	0.00027	0.00032	0.00036	0.00041

6.3.2.2 Case 2: Optimal portfolios of African Indices only including the Exchange Risk

However, this case highlights the effect of the exchange rate risk on the optimal choices made in case 1, giving more practical perspective due to fact of the exposure to the exchange risk by deciding to invest in a foreign country. Tables from Table 6.12 to Table 6.15 show the built optimal portfolios for this case and their reward and risk measures for varying risk aversion levels. Each table represents a different subcase.

Table 6.12 and 6.13 show the subcase that includes the short selling, in the annual measures when considering only minimizing the variance ($\lambda = 0$), the portfolio allocation is only based on the indices' standard deviations and covariances, that is why the biggest share 39.4% is assigned to the least risk index; FTSE/JSE Top 40 with 5.8% standard deviation. As the risk aversion level increases in form of increasing λ, the emphasis on the returns increases. From $\lambda = 0.3$ and more, the weights of FTSE/JSE Top 40, EGX 30 and NSE 30 are negative indicating short selling, but the biggest portion of short selling does not belong to the least return's index; NSE 30 with -5%, instead in belongs to the one with the least volatility; FTSE/JSE Top 40. As before, there are very big values of weights when the focus is only on increasing the portfolio return $\lambda = 1$ with no restriction on short selling. Results are almost similar in terms of signs (buying or short-selling) when a comparison is made between table 6.12 and table 6.13; allowing the short-selling annually and daily respectively, with slight differences in quantities due to the non-linear nature of Markowitz model otherwise the weights would be identical regardless of the timeframe of the measures.

On the other side, Table 6.14 and 6.15 show the subcase that excludes the short-selling, the portfolio when $\lambda = 0$ is identical to the case of including short-selling. As λ goes up, the focus is getting on MASI, when $\lambda \geq 0.2$ the portfolio is solely composed by MASI since the other indices have negative returns with restricting the short-selling. Because buying those negative indices would not just increase the portfolio volatility, but also would decrease its return. On timely manner, results of the annual and the daily measures are identical except when $\lambda = 0.1$, that could be due to the same reason as before; the nonlinearity of the model.

Figures 6.9 and 6.11 show the efficient frontiers for case 1 including and excluding the short-selling in the annual measures. Instead, figures 6.10 and 6.12 show the optimal portfolios calculated for the same conditions.

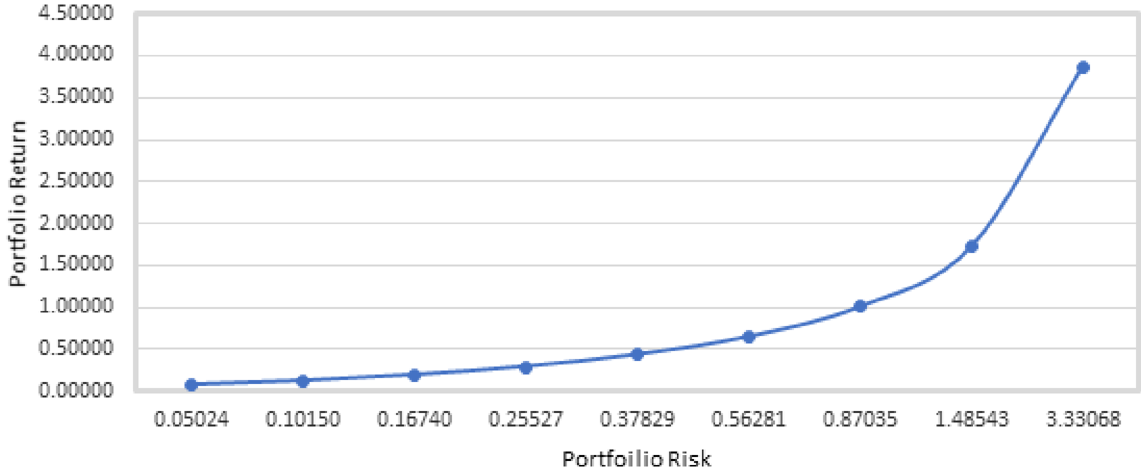

Figure 6.9: Efficient frontier minimizing risk of portfolio for case 2 including short-selling in annual measures.

Figure 6.10: The optimal portfolios for different risk-aversion levels for case 2 including short-selling in annual measures.

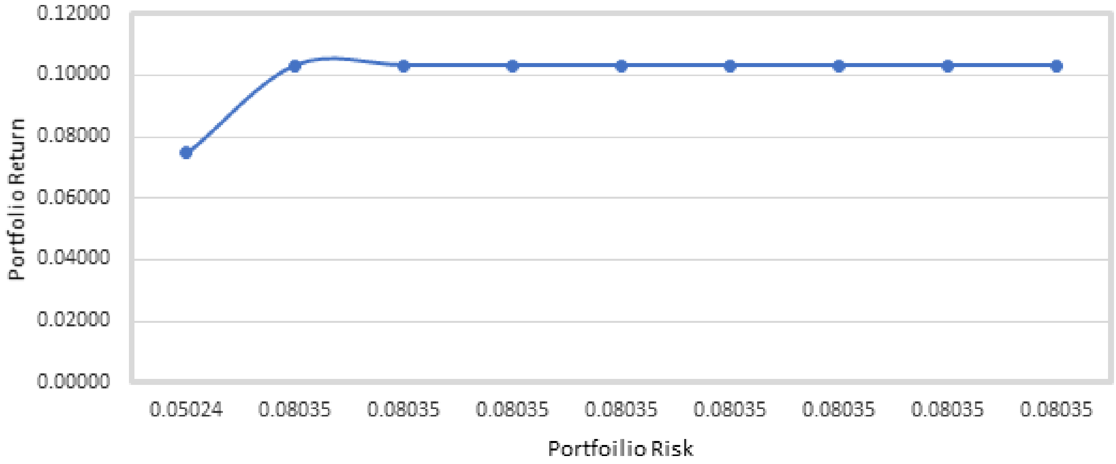

Figure 6.11: Efficient frontier minimizing risk of portfolio for case 2 excluding short-selling in annual measures.

Figure 6.12: The optimal portfolios for different risk-aversion levels for case 2 excluding short-selling in annual measures.

Table 6.12: Optimal portfolios including exchange risk (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of annual measures including short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.39368	0.11103	-0.24227	-0.69652	-1.30218	-2.15011	-3.422	-5.54183	-9.78147	-22.50041	-3.04E+14
EGX 30	0.26076	0.20185	0.12821	0.03354	-0.09269	-0.26941	-0.53449	-0.97629	-1.85989	-4.51069	1.14E+14
NSE 30	0.07414	0.00765	-0.07547	-0.18234	-0.32483	-0.52432	-0.82356	-1.32228	-2.31972	-5.31204	-5.90E+13
MASI	0.27143	0.67947	1.18953	1.84532	2.7197	3.94384	5.78005	8.84039	14.96108	33.32314	2.49E+14
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00924	0.05024	0.1015	0.1674	0.25527	0.37829	0.56281	0.87035	1.48543	3.33068	2.61E+13
Variance (Daily)	0.00331	0.00559	0.01485	0.03721	0.08532	0.18784	0.41849	1.00795	2.9557	14.94979	1.34E+27
Standard Deviation (Daily)	0.05756	0.07478	0.12185	0.19289	0.2921	0.4334	0.64691	1.00397	1.71922	3.8665	3.67E+13
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00331	-0.00001	0.00842	0.02418	0.05091	0.09522	0.17029	0.30686	0.59721	1.50263	2.61E+13

Table 6.13: Optimal portfolios including exchange risk (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of daily measures including short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.39368	0.09079	-0.28781	-0.77459	-1.42363	-2.33228	-3.69525	-5.96688	-10.51014	-24.13991	-1.40E+15
EGX 30	0.26076	0.09371	-0.11511	-0.38358	-0.74154	-1.24269	-1.99441	-3.24728	-5.75303	-13.27026	5.56E+14
NSE 30	0.07414	0.0178	-0.05262	-0.14317	-0.2639	-0.43292	-0.68645	-1.109	-1.95409	-4.48939	-2.80E+14
MASI	0.27143	0.7977	1.45554	2.30134	3.42907	5.00789	7.37611	11.32316	19.21726	42.89955	1.13E+15
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00004	0.00031	0.00065	0.00108	0.00166	0.00248	0.0037	0.00573	0.0098	0.02201	5.10E+11
Variance (Daily)	0.00001	0.00003	0.00009	0.00024	0.00056	0.00123	0.00276	0.00666	0.01955	0.09892	1.15E+26
Standard Deviation (Daily)	0.00364	0.00532	0.00946	0.01541	0.02358	0.03513	0.05254	0.08162	0.13982	0.31452	1.07E+13
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00001	0.00001	0.00006	0.00016	0.00033	0.00062	0.00111	0.00202	0.00393	0.00992	5.10E+11

Table 6.14: Optimal portfolios including exchange risk (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of annual measures without short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.39368	0.11103	0	0	0	0	0	0	0	0	0
EGX 30	0.26076	0.20185	0	0	0	0	0	0	0	0	0
NSE 30	0.07414	0.00765	0	0	0	0	0	0	0	0	0
MASI	0.27143	0.67947	1	1	1	1	1	1	1	1	1
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00924	0.05024	0.08035	0.08035	0.08035	0.08035	0.08035	0.08035	0.08035	0.08035	0.08035
Variance (Daily)	0.00331	0.00559	0.01065	0.01065	0.01065	0.01065	0.01065	0.01065	0.01065	0.01065	0.01065
Standard Deviation (Daily)	0.05756	0.07478	0.10318	0.10318	0.10318	0.10318	0.10318	0.10318	0.10318	0.10318	0.10318
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00331	-0.00001	0.00755	0.01665	0.02575	0.03485	0.04395	0.05305	0.06215	0.07125	0.08035

Table 6.15: Optimal portfolios including exchange risk (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of daily measures without short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.39368	0.09079	0	0	0	0	0	0	0	0	0
EGX 30	0.26076	0.09371	0	0	0	0	0	0	0	0	0
NSE 30	0.07414	0.0178	0	0	0	0	0	0	0	0	0
MASI	0.27143	0.7977	1	1	1	1	1	1	1	1	1
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00004	0.00031	0.00041	0.00041	0.00041	0.00041	0.00041	0.00041	0.00041	0.00041	0.00041
Variance (Daily)	0.00001	0.00003	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004
Standard Deviation (Daily)	0.00364	0.00532	0.00653	0.00653	0.00653	0.00653	0.00653	0.00653	0.00653	0.00653	0.00653
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00001	0.00001	0.00005	0.00009	0.00014	0.00018	0.00023	0.00027	0.00032	0.00036	0.00041

6.3.2.3 Case 3: Optimal portfolios of All the Indices without Considering the Exchange Risk

For having more general perspective, case 3 illustrates the position of investing in African markets among other markets locating across the world, this may highlight any unforeseen investing opportunities in African stock exchanges. Tables from Table 6.16 to Table 6.19 show the built optimal portfolios and their reward and risk measures for varying risk aversion levels. Each table represents a different subcase.

Table 6.16 and 6.17 show the subcase that includes the short selling for the annual and the daily measures respectively. What might seem surprising is that at $\lambda = 0$ there is a short selling of DAX as appeared for the first time when investors only pay attention to minimize the portfolio risk, that is obviously not due to negative value of DAX return which is positive but it could be due to the covariances of DAX to other indices compose the portfolio. Then is very clear that the weights are proportional to the indices' standard deviation, the lowest risk has the biggest share since increasing the portfolio return is not a concern at $\lambda = 0$. When risk aversion level is increasing, short selling includes, beside DAX, NSE 30 due to the negative return and FTSE/JSE Top 40 most likely because of the correlation with other better-return indices. The bigger shares still belong to the high-return indices, at $\lambda = 0.5$ MASI has the highest weight followed by EGX 30 then DJIA and lastly NIKKEI, following the same order of the returns. As before, there are very big values of weights when the focus is only on increasing the portfolio return $\lambda = 1$ with no restriction on short selling. Results of weights are identical in terms of signs and quantities (buying or short-selling) when a comparison is made between table 6.6 and table 6.17; allowing the short-selling annually and daily respectively, with slight differences in quantities due to the non-linear nature of Markowitz model otherwise the weights would be identical regardless of the timeframe of the measures.

However, table 6.18 and table 6.19 reveal subcase that restricts the short selling for the annual and the daily measures respectively. At $\lambda = 0$ the weights of the portfolio in identical to the previous subcase except for taking DAX out since it was previously short sold. As risk aversion levels goes up$\lambda = 0.1$ in addition to DAX, NSE 30 and FTSE/JSE Top 40 are also taken out for the same reason, the portfolio converges by the increase of λ to be completely composed by MASI (the index with the biggest return) when $\lambda \geq 0.4$. As before, there are very big values of weights when the focus is only on increasing the portfolio return $\lambda = 1$ keeping zero weights for FTSE/JSE Top 40 and NSE 30. On timely manner, results of the annual and the daily measures are identical.

Figures 6.13 and 6.15 show the efficient frontiers for case 1 including and excluding the short-selling in the annual measures. Instead, figures 6.14 and 6.16 show the optimal portfolios calculated for the same conditions.

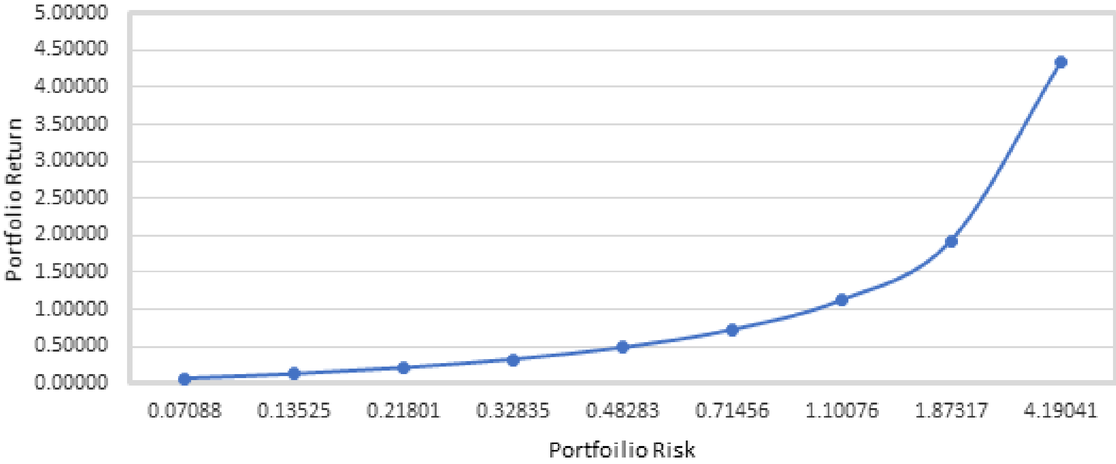

Figure 6.13: Efficient frontier minimizing risk of portfolio for case 3 including short-selling in annual measures.

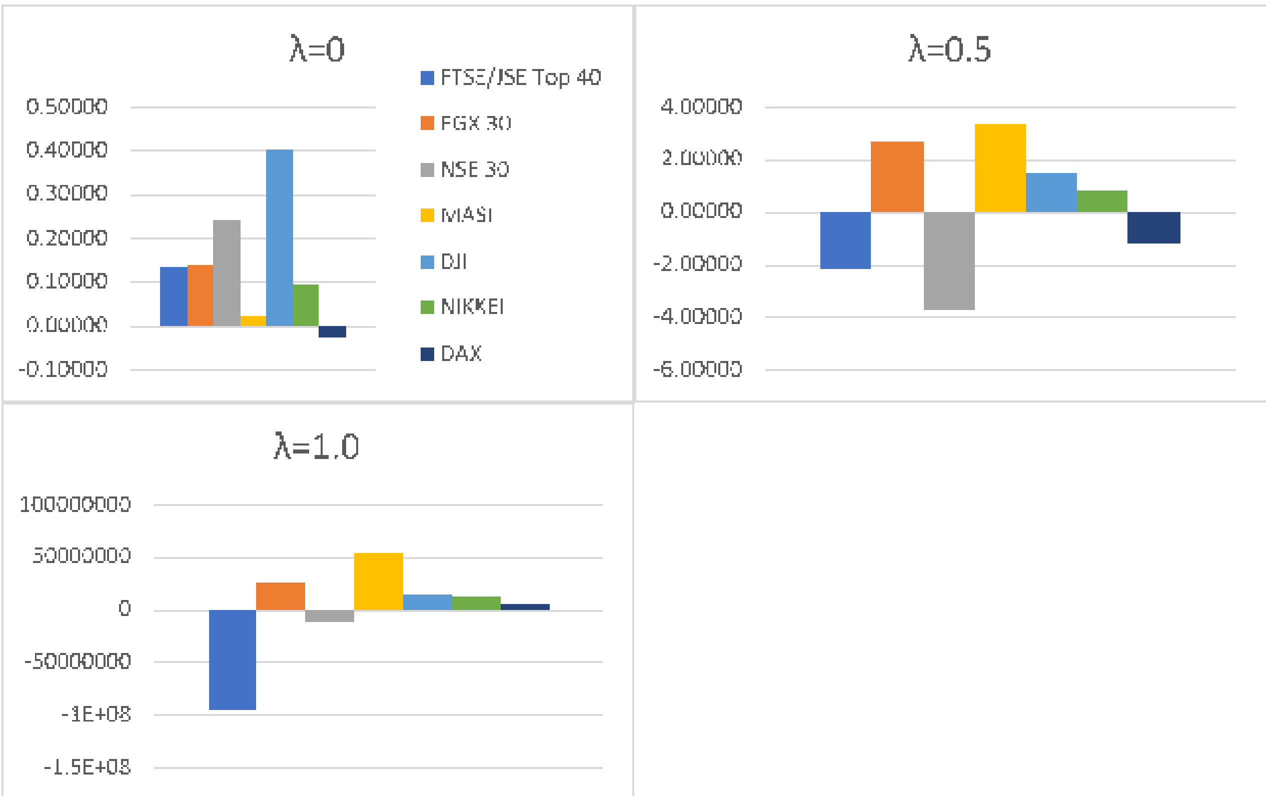

Figure 6.14: The optimal portfolios for different risk-aversion levels for case 3 including short-selling in annual measures.

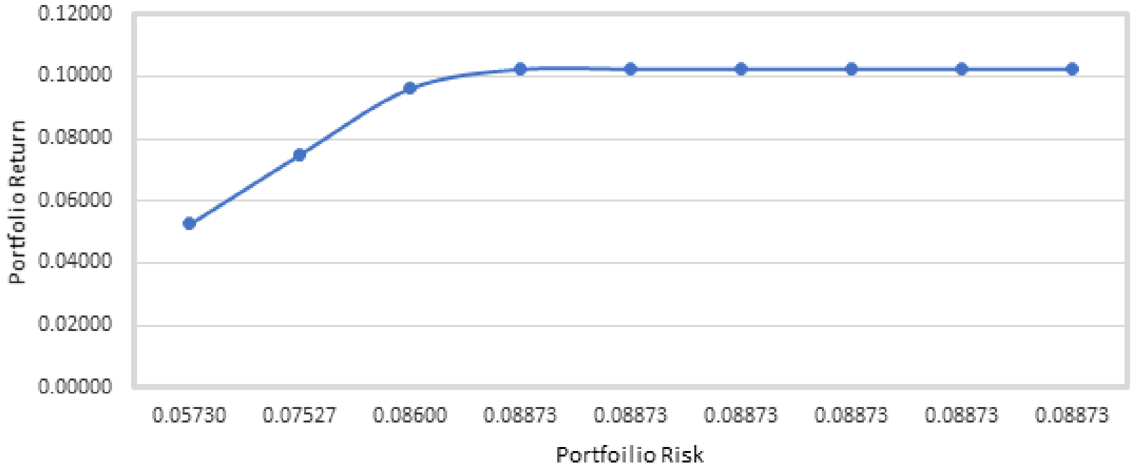

Figure 6.15: Efficient frontier minimizing risk of portfolio for case 3 excluding short-selling in annual measures.

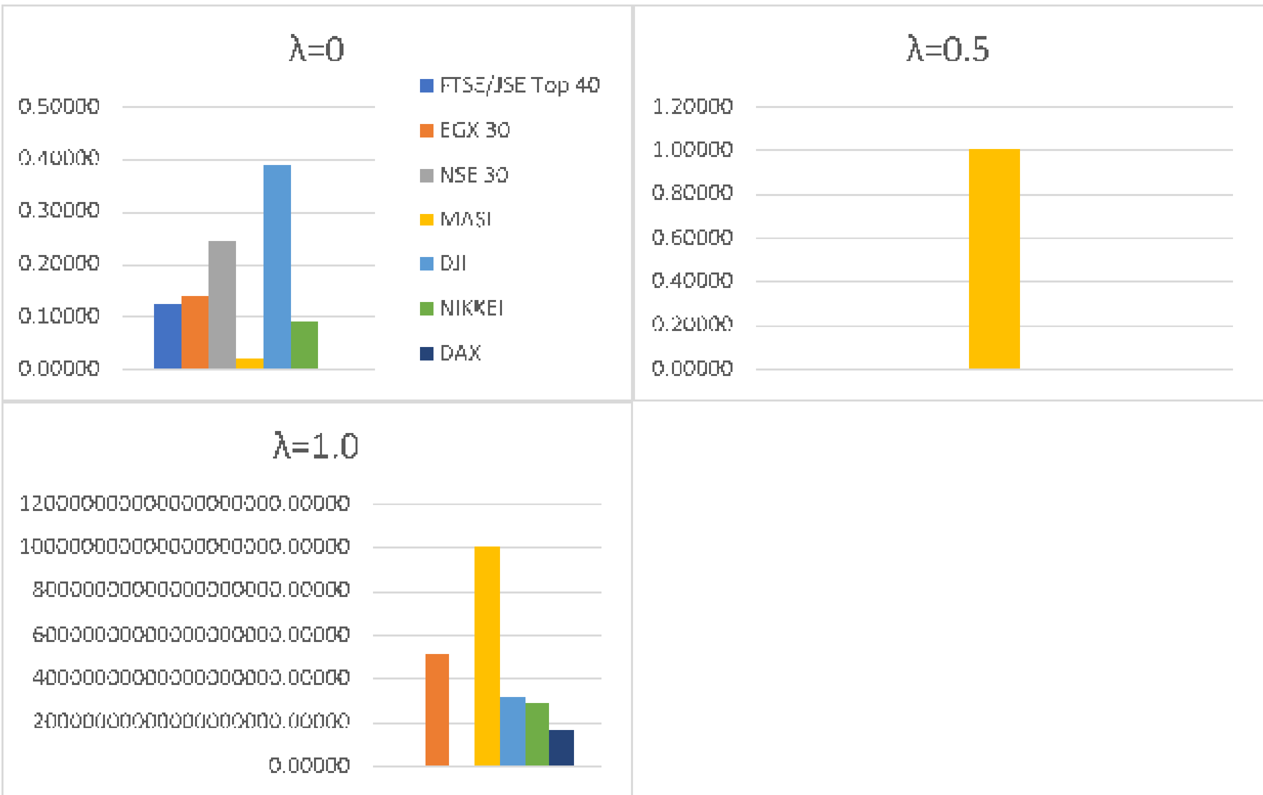

Figure 6.16: The optimal portfolios for different risk-aversion levels for case 3 excluding short-selling in annual measures.

Table 6.16: Optimal portfolios including DJIA, NIKKEI and DAX (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of annual measures including short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.1342	-0.12623	-0.45177	-0.87032	-1.42838	-2.20968	-3.38161	-5.33485	-9.24131	-20.96069	-96163550
EGX 30	0.13728	0.41761	0.76803	1.21856	1.81927	2.66027	3.92177	6.02427	10.22927	22.84426	25016679
NSE 30	0.24179	-0.20093	-0.75433	-1.46585	-2.41454	-3.74271	-5.73496	-9.05538	-15.69621	-35.61871	-12594274
MASI	0.02131	0.38509	0.83982	1.42446	2.204	3.29534	4.93235	7.66071	13.11743	29.48758	53687091
DJIA	0.40033	0.51648	0.66166	0.84831	1.09719	1.44562	1.96826	2.83933	4.58147	9.8079	13388829
NIKKEI	0.09328	0.17007	0.26605	0.38946	0.55401	0.78437	1.12991	1.70581	2.85762	6.31304	12150839
DAX	-0.02819	-0.16209	-0.32945	-0.54463	-0.83154	-1.23321	-1.83572	-2.83991	-4.84827	-10.87337	4514386.9
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.01939	0.07088	0.13525	0.21801	0.32835	0.48283	0.71456	1.10076	1.87317	4.19041	6078899.8
Variance (Daily)	0.00102	0.00388	0.0155	0.04358	0.10401	0.23274	0.5224	1.26262	3.70859	18.77061	5.75E+13
Standard Deviation (Daily)	0.0319	0.06228	0.1245	0.20876	0.3225	0.48243	0.72277	1.12366	1.92577	4.33251	7580810.7
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00102	0.0036	0.01465	0.0349	0.06894	0.12505	0.21977	0.39175	0.75682	1.8943	6078899.8

Table 6.17: Optimal portfolios including DJIA, NIKKEI and DAX (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of daily measures including short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.1342	-0.12623	-0.45177	-0.87032	-1.42838	-2.20968	-3.38161	-5.33485	-9.24131	-20.96069	-9.07E+10
EGX 30	0.13728	0.41761	0.76803	1.21856	1.81927	2.66027	3.92177	6.02427	10.22927	22.84426	2.36E+10
NSE 30	0.24179	-0.20093	-0.75433	-1.46585	-2.41454	-3.74271	-5.73496	-9.05538	-15.69621	-35.61871	-1.188E+10
MASI	0.02131	0.38509	0.83982	1.42446	2.204	3.29534	4.93235	7.66071	13.11743	29.48758	5.064E+10
DJIA	0.40033	0.51648	0.66166	0.84831	1.09719	1.44562	1.96826	2.83933	4.58147	9.8079	1.263E+10
NIKKEI	0.09328	0.17007	0.26605	0.38946	0.55401	0.78437	1.12991	1.70581	2.85762	6.31304	1.146E+10
DAX	-0.02819	-0.16209	-0.32945	-0.54463	-0.83154	-1.23321	-1.83572	-2.83991	-4.84827	-10.87337	4.258E+09
Sum	1	1	1	1	1	1	1	1	1	1	0.9999995
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00008	0.00028	0.00054	0.00087	0.00131	0.00193	0.00286	0.0044	0.00749	0.01676	22935202
Variance (Daily)	0	0.00002	0.00006	0.00017	0.00042	0.00093	0.00209	0.00505	0.01483	0.07508	2.05E+17
Standard Deviation (Daily)	0.00202	0.00394	0.00787	0.0132	0.0204	0.03051	0.04571	0.07107	0.1218	0.27401	452234042
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	0	0.00001	0.00006	0.00014	0.00028	0.0005	0.00088	0.00157	0.00303	0.00758	22935202

Table 6.18: Optimal portfolios including DJIA, NIKKEI and DAX (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ), portfolio returns, portfolios variances and portfolios standard deviations for the case of annual measures without short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.12208	0	0	0	0	0	0	0	0	0	0
EGX 30	0.13737	0.34711	0.30993	0.06286	0	0	0	0	0	0	5.11E+19
NSE 30	0.24187	0	0	0	0	0	0	0	0	0	0
MASI	0.02086	0.38691	0.69007	0.93714	1	1	1	1	1	1	1.00E+20
DJIA	0.38685	0.19868	0	0	0	0	0	0	0	0	3.12E+19
NIKKEI	0.09096	0.0673	0	0	0	0	0	0	0	0	2.91E+19
DAX	0	0	0	0	0	0	0	0	0	0	1.61E+19
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.01923	0.0573	0.07527	0.086	0.08873	0.08873	0.08873	0.08873	0.08873	0.08873	5.21E+16
Variance (Daily)	0.00102	0.00278	0.00564	0.00928	0.01051	0.01051	0.01051	0.01051	0.01051	0.01051	6.24E+35
Standard Deviation (Daily)	0.03193	0.05276	0.0751	0.09633	0.10253	0.10253	0.10253	0.10253	0.10253	0.10253	7.90E+17
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	-0.00102	0.00322	0.01054	0.0193	0.02918	0.03911	0.04903	0.05896	0.06888	0.0788	5.21E+16

Table 6.19: Optimal portfolios including DJIA, NIKKEI and DAX (each optimal portfolio represented by weights of indices that compose it) for different risk aversion levels (represented by λ, portfolio returns, portfolios variances and portfolios standard deviations for the case of daily measures without short selling.

Lambda (λ)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Portfolio's Weights										
FTSE/JSE Top 40	0.12214	0	0	0	0	0	0	0	0	0	0
EGX 30	0.13747	0.34711	0.30993	0.06286	0	0	0	0	0	0	5.11E+19
NSE 30	0.24179	0	0	0	0	0	0	0	0	0	0
MASI	0.02087	0.38691	0.69007	0.93714	1	1	1	1	1	1	1.00E+20
DJIA	0.38691	0.19868	0	0	0	0	0	0	0	0	3.12E+19
NIKKEI	0.09083	0.0673	0	0	0	0	0	0	0	0	2.91E+19
DAX	0	0	0	0	0	0	0	0	0	0	1.61E+19
Sum	1	1	1	1	1	1	1	1	1	1	1
	Portfolio Returns and Risks										
Expected Return (Daily)	0.00008	0.00023	0.0003	0.00034	0.00035	0.00035	0.00035	0.00035	0.00035	0.00035	5.21E+16
Variance (Daily)	0	0.00001	0.00002	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	6.24E+35
Standard Deviation (Daily)	0.00202	0.00334	0.00475	0.00609	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	7.90E+17
$max\ \lambda * \mu_p - (1-\lambda) * \sigma_p^2$	0	0.00001	0.00004	0.00008	0.00012	0.00016	0.0002	0.00024	0.00028	0.00032	5.21E+16

6.3.3 Outlines of comparisons between the three cases

This part considers comparing the three cases together for the same risk-aversion level; for the sake of the comparison lets fix λ to be 0.5 which represents investors with equal concerns to portfolio risk and return. Through watching the former histograms plotted above, it can be easily seen the following:

- When only African indices considered in case 1, MASI takes the biggest share since it has the highest return followed -not with a big difference- by EGX 30, the other two are short-sold with the dominance of NSE 30 which has a negative return.

- When the exchange rates risk is included in case 2, clearly, Moroccan exchange rates risk is the lowest among the indices, so MASI's share increases with respect to case 1. The high risk of exchange in Egypt changes the position of EGX 30 to short-selling because its return gets negative with the exchange risk. In South Africa the amount of short-selling even increases since the exchange risk makes it worse - it increases the return's negative value. However, in Nigeria the relatively low exchange risk decreases the amount of the short-selling.

- In general, the exchange risk effect is positive on MASI and NSE 30 due to relative economic stability of Morocco and Nigeria in the studied period, whereas it plays a passive role on EGX 30 and FTSE/JSE Top 40, since the fluctuations of the exchange rates could be interpreted due to deteriorations of the South African and the Egyptian economies.

- A remarkable point to state is that, when a global portfolio in formed in case 3 the returns of DJI, NIKKEI, and DAX are medium compared to the African indices; since they are in the middle because MASI and EGX 30 are higher than them and NSE 30 and FTSE/JSE Top 40 are lower than them.

- So accordingly, DJI, NIKKEI, and DAX compete the African indices relative to the return-risk value; that yield the inclusion of the Western indices but with lower shares in case of DJI and NIKKEI and with a lower amount of short-selling in the case of DAX.

- These medium risk-return values of Westerns stock exchanges reflect an economic stability of the indices' countries compared to the considered African counties. However, the inclusion of these Western indices is insignificant due to these medium risk-return values, following the simple logic, investors either afford risk to increase the return in this case MASI and EGX 30 are preferred (higher risk-return values), or they are willing to give up returns to decrease the portfolio's risk in this case they prefer NSE 30 and FTSE/JSE Top 40(lower risk-return values).

- When the short-selling option is constrained, the portfolio is entirely made of MASI in the three different cases; considering only African indices without exchange rates risk;

considering only African indices including exchange rates risk; and the global portfolio case that blend the African indices with the three Western indices. So, the restriction of short-selling makes investors put all their money in MASI since it has the best risk-return combination among the indices with the positive returns.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

In this section, a summary for the work completed throughout this thesis is presented, besides highlighting the general outcomes that drown out by this study.

In chapter (4), the weak-form of the random walk hypothesis has been tested for four indices taken from African stock exchanges; Johannesburg Stock Exchange, Egyptian Stock Exchange, Nigerian Stock Exchange and Casablanca Stock Exchange. Three basic tests have been carried out; Runs Test, Ljung-Box Statistic (White Noise Test) and Arch Effect Test. It is concluded that all the four African indices reject the null hypothesis which assumes following the random walk and accordingly all the considered African markets are not efficient. This is outlined through taking the results of all the tests together into considerations and ending up with an overall conclusion, it does not necessarily mean that each index fails in every individual test.

In chapter (6), risk-aversion modified model of Markowitz portfolio selection theory is applied on the same indices. Three different cases are applied; building optimal portfolios considering only African indices without exchange rates risk; building optimal portfolios considering only African indices including exchange rates risk; and the global portfolios case that blend the African indices with the three Western indices.

In each case there are two dividing factors considered; including and restricting short-selling; and the timeframe of the risk-return measure is either annual or daily. That yielded four subcases for each case of the three. The risk-aversion level represented by λ is varying from 0 to 1 by 0.1 rate of change for each subcase.

Case (1) figures out that portfolios are formed based on the risk-returns values of the indices and the risk-aversion level investors prefer. At $\lambda = 0$ investors only pay attention to risk minimizing though priorities given to the least-risk indices; FTSE/JSE Top 40 and NSE 30, however at $\lambda = 1.0$ investors care only about maximizing the portfolio return which yield a focus on the biggest returns' assets that are MASI and EGX 30 the rest are short-sold.

The effect of exchange rates risk, as considered by case (2), varies from country to another. Due to the economic stability Morocco and Nigeria in the period in question, the effect of having the lowest exchange risk increases the shares of the two indices of these countries. Whereas, Egyptian and South African currencies have been through instable fluctuations, which could impact negatively on their indices' positions, for instance the exchange risk effect turns EGX 30 into short-selling position.

Case (3) instead reveals that, the risk-return levels of Western indices are medium compared to the African ones which hold either higher returns with higher risks or lower risks with lower returns (even negative) to the Western ones. This point outlines insignificant positions of the Western indices in the portfolios as seen in the formerly plotted histograms; either included with lower shares in case of DJI and NIKKEI or with a lower amount of short-selling in the case of DAX.

When the short-selling option is constrained, the portfolio is entirely made of MASI in the three different cases. So, the restriction of short-selling makes investors put all their money in MASI since it has the best risk-return combination among the indices with the positive returns.

For all the subcases, it is plotted the efficient frontiers of portfolios; which represents the optimal portfolios according to Markowitz model of portfolio selction.

Results are almost similar when a comparison is made between the annual and daily measures, with slight differences due to the non-linear nature of Markowitz model otherwise the weights would be identical regardless of the timeframe of the measures.

7.2 Future Work

Starting from results achieved in this study, it would be better to propose a more general model of Markowitz that includes other risk factors; such as risk of default and the transaction cost.

This study takes only the effect of the exchange rates risk; however, due to the massive effect of the exchange risk effect on investors' decisions that this study figures out, there should be a study that investigates the financial and economic causes behind this change of exchange risk.

Finally, a similar study with long horizons risk-return measures (for instance 70 years) would be proposed, since it could yield better precision and representation to the African capital markets.

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