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Modeling and trajectories simulation for complex aerospace systems

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With Space Shuttle retirement in 2011, worldwide aerospace industry focused on new technologies aimed at develop of an innovative new line of reusable Space Vehicles. Furthermore, on the wake of the new NASA strategies, commercial companies start to heavily be involved, aiming at easier and cheaper access to space. In particular, in the perspective of a future space tourism era, the category of air launched flights is the most interesting to analyse. Indeed, missions using air-launch system are not addressed only to future space tourists but also to carry cargo, scientific payloads and crew to the ISS or, more in general, in LEO platforms. However these are still developing technologies, hence a mathematical modeling is fundamental to drive the project and the operational aspects. Moreover a mathematical approach can be a confirm to correlate experimental data to have a reliable and improvable model as the technology progresses.

Therefore the main focus of this work is the modeling and the trajectories simulation for air launch systems. After a general overview, a preliminary study is executed in order to stress the differences between ground launch and air launch. Then the attention shifts on the trajectory modeling approach related to two kinds of air launched missions: suborbital and orbital flights. For the mathematical model some assumptions are used as: point-like model, interest on longitudinal plane, classic dynamics equations etc. Finally the mathematical model is simulated using different and flexible graphical user interfaces (GUIs) on Matlab in which the user can insert input data, providing graphs and results useful for future and different applications.

Abstract

Con il ritiro dello Space Shuttle nel 2011, l'industria aerospaziale mondiale ha iniziato a focalizzarsi su nuove techologie atte allo sviluppo di una nuova innovativa linea di veicoli spaziali riutilizzabili. Inoltre, sull'onde delle nuove strategie della NASA, compagnie commerciali private hanno iniziato ad essere coinvolte in modo sostanziale, con l'obiettivo di permettere in futuro un piú facile e piú economico acceso allo spazio. In particolare, nella prospettiva di una futura imminente era del turismo spaziale, la categoria dei voli avio trasportati risulta la piú interessante da analizzare. Infatti le missioni che utilizzano spazioplani non sono indirizzate solo ai futuri turisti spaziali ma anche per il trasporto di cargo, payload scientifici ed equipaggio verso la Stazione Spaziale Internazionale o, piú in generale, su piattaforme LEO. Tuttavia queste sono tecnologie ancora in via di sviluppo, quindi un modello matematico é fondamentale per guidare il progetto e per gli aspetti operativi. Inoltre l'approccio matematico puó essere una conferma per correlare i dati sperimentali ed avere un sistema affidabile e migliorabile man mano che la tecnologia va avanti. Dunque, il tema fondamentale di questo lavoro é la modelizzazione e la simulazione di traiettoria per i sistemi avio trasportati. Dopo una breve overview, si procede con uno studio preliminare per mettere in luce le differenze tra l'avio lancio e il lancio da terra. Dopodiché l'attenzione si sposta sull'approccio matematico riferito a due tipi di missioni di avio lancio: suborbitale e orbitale. Per il modello matematico son state fatte alcune assunzioni: modello puntiforme, interesse sul piano longitudinale, equazioni della dinamica ecc. Infine il modello matematico viene simulato usando differenti interfacce grafiche di Matlab in modo che l'utente possa avere piú flessibilitá inserendo i dati di input e fornendo grafici e risultati utili per applicazioni future.

Chapter 1

Introduction

1.1 Background

The next generational leap in aviation will come with the integration of affordable commercial suborbital and orbital spaceflight. Reusable launch vehicles (RLVs) have critical site requirements to support their flight profiles. Aviation authorities, who plan future development with RLVs in mind, stand to reap the rewards of commercial spaceflight. Creation of a Spaceport Development Plan can serve to identify critical path infrastructure, siting, and facilities needed to support RLV requirements, and streamline the Spaceport licensing process.

Spaceport development has continued to evolve and is now a mix of approaches. An Aerospaceport, as considered in this paper, is a traditional airport that has also become licensed to support space launch operations. The current generation of Aerospaceports is typically a former military or general aviation airport that has obtained a FAA licensing to operate as a launch site in support of suborbital Reusable Launch Vehicles (RLVs). Aerospaceports are expected to evolve to support a wide range of missions including orbital space access for both passengers and cargo. Air-launch of RLVs fits in this context[1].

There many different typologies of horizontal air launch systems that have been evaluated during the past years and now they are more showing a comeback in commercial applications. This is due to the fact that horizontal air launch concepts offer multiple benefits. "In fact, one of the pros is that often the vehicle concept of operations can be simplified for faster launch turn-around times along with easier vehicle integration and increased payload delivery flexibility". Moreover, the 1st stage of the system, called carrier aircraft, can provide a very mobile and flexible launch platform which can avoid inclement weather, it can also offer a very large number of possible launch to orbits and "it can provide for covert launching capability"[2].

One of the main benefits of a rocket or spaceplane that is air launched by a carrier aircraft is that it could not fly through the lower and denser layers of atmosphere. Moreover, it is well known that the drag force leads to a very large amount of extra work and this requires inevitably propellant mass. In fact, in the atmosphere layers at lower altitudes the density is higher and this results in stronger drag forces acting on the vehicle. "In addition, thrust is lost due to over-expansion of the exhaust at high ambient pressure and under-expansion at low ambient pressure; a fixed nozzle geometry cannot provide optimal exhaust expansion over the full range of ambient pressure, and represents a compromise solution."[24] For this reason, there are optimization for lower ambient pressure for vehicles launched from high altitudes, achieving greater levels of thrust during the mission time. In addiction, the carrier vehicle lifts the rocket or spaceplane to altitude in a more efficient way, thus the propellant is conserved. On-board oxidizer storage is not required by the engines because they can use the surrounding air to produce thrust force, using for example a turbofan. In this way the launch system conserves a very large amount of mass that would otherwise be used for fuel. So, rocket mass can also include payload, reducing costs for launching it.

As said before, there are many commercial offers that air launch gives for aircraft-like operations and one of these is surely launching on demand, without too many weather constraints because the aircraft can avoid bad weather conditions flying to better launch sites and launching payloads into orbits with any inclination at any time. "Insurance costs are reduced as well, because launches occur well away from land, and there is no need for a launch pad or blockhouse". Another important feature of air launch systems is that they can reduce ΔV needed to achieve orbit. This leads to a larger payload and it reduces the cost for launching to orbit[3].

For this reason in this general framework of air launch, there is a need to create mathematical models to represent the physical nature of the air-launch physics and to develop software simulators to simulate the problem and to analyze the obtained results. This work aims to reach the abovementioned goal, developing a reliable mathematical tool which includes models and simulations to compare with experimental results.

Figure 1.1: An example of air launched flight: Virgin Orbit's LauncherOne [credits Virgin Orbit]

1.2 Objectives

The main purpose of this work is to develop mathematical and physical model to represent the nature of air-launch and to create a software for modeling and simulating trajectories of complex aerospace systems using a graphical user interface (GUI) developed with Matlab. Thanks to this software, the most relevant design drivers, operations and relevant ground segment will be identified.

First of all, an overview of the main air launched flights will be presented, showing the principal advantages of air launch with respect to vertical launch and showing a list of the different types of air launch. Then the attention will shift to two real case of air launch:

- Virgin Galactict's SpaceShip2
- Virgin Orbit's LauncherOne

Then the attention moves to the trajectory modeling approach, a chapter in which mathematical models will be presented, from the rigid body model to the simplification of point-like mass one, from parameters modeling, including aerodynamic forces, thrust curves, thermal flux, air density, to the numerical method implemented in the simulator. Chapter 4 provides the process to create the software simulator using Matlab GUIs, focusing on two study case: SpaceShip2 and LauncherOne for suborbital and orbital missions. In the final chapter results will be analyzed, comparing the output given by the simulator with experimental data. Then conclusions, future works and upgrades will be presented.

Chapter 2

An overview of air launched spaceflight transportation

In this chapter, after we explained a general outline of air launch, its advantages with respect to ground launch will be presented, showing some examples and focusing our attention on two different types of airlaunch:

- suborbital air launch by reference to Virgin Galactic's SpaceShip2
- orbital air launch by reference to Virgin Orbit's LauncherOne

2.1 What is air launch?

The air launch system is a complex aerospace system made up of at least two stages: the first one, called carrier, is an aircraft statically or dinamically supported (in this last case it could be subsonic or supersonic). The second stage is usually a rocket (sometimes also a spaceplane like the SS2), completely expandable and often multistage. The main advantage of air launch is its high flexibility e reactivity. This grounbracking system is catching on in the aerospace field during these last decades where at least 136 studies to air launch were conducted [6] and currently still a lot of concepts are in development showing that air launch is also seen as a promising concept in the scientific world.

Air launch spans a variety of concepts, from expendable to reusable launch vehicles, from rocket propulsion to airbreathing, from balloons to hypersonic waves-riders as carrier. Indeed, the emerging market for nano and microsatellites (1-50 kg) is seen by many people as the potential prime market for air launched vehicles. [4]. Microsatellites are often launched together with larger satellites because a dedicated launch would be too expensive. However, for the small satellite a major disadvantage is that the primary client dictates the final destination and launch date. This leads to various limitations such that the full potential of the small satellite's mission is not always exploited.

The second potential market is the market for operationally responsive space (ORS). ORS is

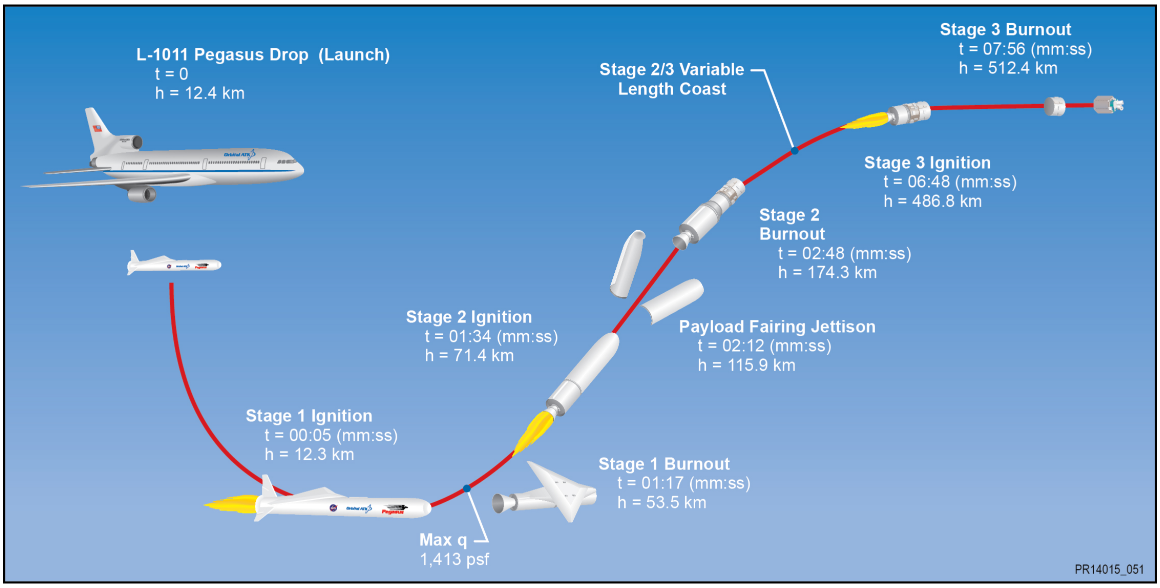


Figure 2.1: Pegasus air launched rocket [credits Northrop Grumman]

the capability to develop satellites within a couple of months and launch them almost instantaneously on demand [5]. Currently, the development time of a conventional satellite is 4-10 years and for a microsatellite (10-100 kg) 1-4 years [6]. Air launch removes the need for cumbersome launch facilities and, therefore, fits perfectly for ORS.

On the other hand air launch is emerging also in suborbital field, like SpaceShip2, both for letting "common" peolple experiencing a certain time of microgravity both for scientific experimentation.

Figure 2.2: SpaceShip2 spaceplane [credits Virgin Orbit]

2.1.1 ALTO vs GLTO

Which are the advantages and the drawbacks of air launch flights compared to ground launch? Why is there this interest and research? This section will answer to these questions.

Let's begin with the advantages:

- One of the main advantages of air-launch is that it can be set up according to the requirements of a single client who want to put into orbit a payload.

- All mission features, like orbital parameters (eccentricity e, semimajor axis a, inclination i, longitude of the ascending node Ω, argument of periapsis ω and true anomaly θ), can be chosen in according with client's needs. This cannot be satisfied by a conventional ground launch, where the above-mentioned parameters are fixed usually by the principal payload client. Indeed, the owners of minor payloads have to adapt their requirements to main client's ones, including also delays. For this reason, launching a payload with a conventional ground launch could take long time!

- A spaceport with launch platforms is not required because everything you need is a runway and a structure for the integration of all components. Rocket release could be carried out anywhere, in every direction, complying with regulations. Ground launch, on the other hand, is limited to operate in existing spaceports and within a certain direction range.

- Air launch is not influenced by adverse weather conditions which can be easily bypassed.
- Since the rocket is released by the carrier through less dense atmospheric layers, it has not to store the required propellant to contrast and win friction forces of lower altitudes. Indeed, the carrier aircraft only stores the needed propellant to reach the separation point, while the rocket or spaceplane uses its energy to climb in an environment where friction is much lower than sea level (at 10 km altitude, atmospheric density is 25% of that at sea level!). For this reason air launch choice is ”drag optimizing”.
- Choosing ALTO configuration, the required ΔV to reach the desired orbit is reduced because the rocket has an initial velocity given by the carrier and the gravity force acts on it for less time. This implies a minor cost of fuel.
- Many existing militar or civil aircrafts are no longer usable for their purpose, so they can perform the role of carrier. In this case we cannot spend our time and resources to design, to product and to certify a new launch platform.

Of course all that glitters is not gold, therefore there are also some drawbacks with respect to ground launch, for examples:

- Payload mass is limited by carrier size and by maximum takeoff weight.
- As said above, the studies, the research and the experiences on air launch are still not enough and this increases the complexity of this configuration. Indeed, the most critical phase is the release one where rocket or spaceplane separates from carrier aircraft.
- After the release phase, the danger of collision between carrier and rocket (or spaceplane) must be avoid. Mission profiles could help for this purpose.
- Since a pilot is needed, human factors have to take into account. Moreover launch operations must be done mandatorily on uninhabited areas.
- Rocket release could suddenly modify aerodynamics and flight quality.

2.1.2 Examples

Many organizations have proposed air launch Reusable Launch Vehicle (RLVs, like rocket or spaceplane) due to a renewed interest generated by NASA's second Generation Space Launch Initiative. Air launched RLVs are categorized as captive on top, captive on bottom, towed, aerial refueled and internally carried. In this section these different types of air launch will be presented, finding that many concepts are not possible with today's technology[7].

- CAPTIVE ON TOP

  In this concept the rocket or spaceplane is carried on the back or top of carrier aircraft. [7]One of the main benefits of captive on top configurations is that the aircraft carrier can carry a very large and heavy RLV on its top. However, there also many disadvantages that tell us why this system is not so used and one of the main cons is that the RLV and the carrier aircraft could collide at the release phase, so there is the need of having active controls.

Figure 2.3: RLV captive on top [7]

  There are three principal strategy to avoid collision between RLV and carrier during the release phase:

  - Winged RLV
  - Trapeze top launch
  - Barrel Roll Manouvre

- CAPTIVE ON BOTTOM

  This is the most common configuration for air launch. Here the RLV is attached to carrier belly or wing. It is also the only one commercially employed, looking at Pegasus XL launcher. There are many advantages of this configuration, such as proven and easier separation from carrier vehicle without collision between the RLV and the carrier, less thermal protection system concerns, and smaller wing for the RLV. Moreover, since gravity force pulls away the two vehicle, this air launch method turns out to be also the simplest and safest one. However, also this configuration is not devoid of disadvantages. In fact, since the RLV is carried on carrier aircraft belly, this leads to limiting the RLV size. "Furthermore the launcher increases aircraft drag and modifies aerodynamics"[7]. There are many examples of air launch vehicle in a captive on bottom configuration:

  1. Pegasus: an air launched rocket developed by Orbital Science Corporation which carries small payloads into LEO. It is the world's only operational air launch vehicle with over 30 launches to its credit. It consists of expendable 3-stage solid rocket booster with wings attached to the first stage. Only the carrier is reusable.
  2. SpaceShip1 and SpaceShip2: air launched suborbital spaceplane designed for space tourism developed by Virgin Galactic.
  3. LauncherOne: an air launched to orbit rocket designed to launch "smallsat" payloads into Sun-synchronous orbit. It has been developed by Virgin Orbit and launches are projected to begin in early 2019.

Figure 2.4: PegasusXL: captive on bottom vehicle [credits Northrop Grumman]

- TOWED

  The air launched rocket is hooked up to carrier through a cable. On of the main benefits of a towed configuration are easier separation from the towing aircraft without collision and lower costs to modify the towing aircraft. On the other hand, one of the main disadvantages are safety concerns including ”broken towlines and a towing aircraft take-off abort”.

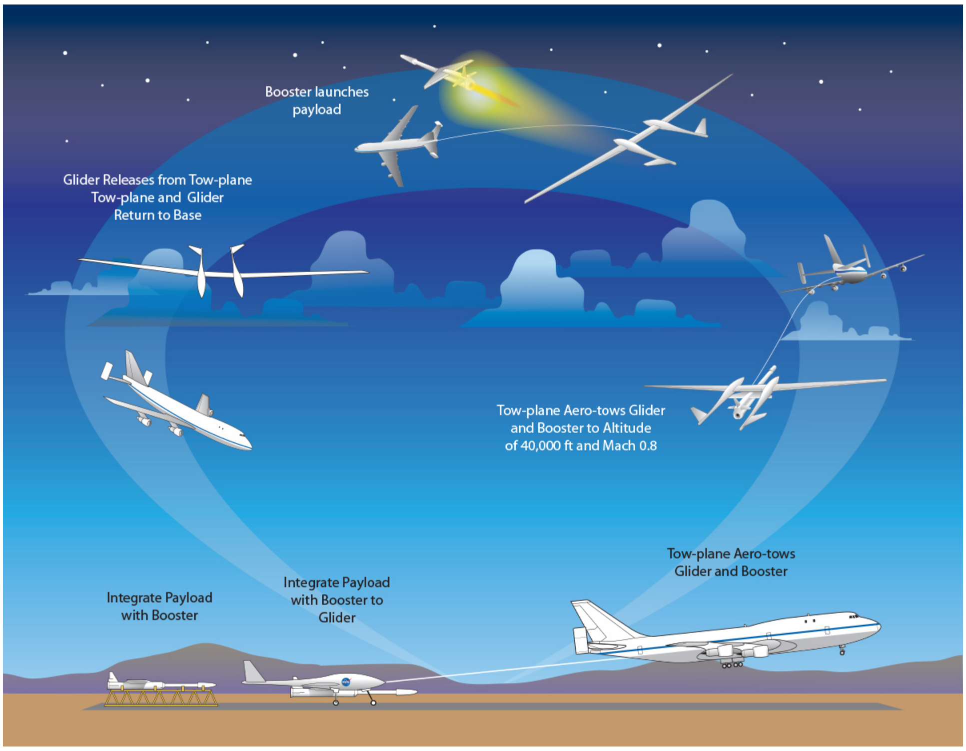


Figure 2.5: Typical towed air-launch profile mission [credits Utah State University]

- INTERNALLY CARRIED

  The RLV is housed inside the carrier aircraft on a longitudinal sliding platform. This technique is usually used for military missions to parachute vehicles. Since this configurations does not need frequent modifications, there are lower development and operations costs. There are also less troubles due to propellant boil-off since the RLV carried inside the carrier is not subject to sun radiation heating or airstream convective heating. Moreover crews get in the launch vehicle until just before the launch and this reduces the safety constraints of carrying a launch vehicle with a manned carrier aircraft. In addictions, the internally carried configuration eliminates weather's and with this system we are able to carry heavier RLVs and release them at higher altitudes with respect to externally carried systems[7]. However, the RLV must be fited inside the carrier aircraft and dangerous fuels cannot be carried internally.

- AERIAL REFUELED

  This strategy is the process of transferring aviation fuel from one military aircraft (the tanker) to another (the receiver) during flight. The principal advantages of aerial refueling is that it reduces the size of the carrier aircraft's wing and landing gear.

2.2 Simulation tools overview

In this section we will present an overview related to the existing trajectory simulation tools and we will explain why they cannot be utilized. Hence the need to create a new software simulator using Matlab GUI.

One of the main tool utilize in aerospace field for modeling trajectories is the U.S. "STK Astrogator". This tool is often used for orbit maneuver and spacecraft trajectory design. "It supports an unlimited series of events for modeling and targeting a spacecraft's trajectory, including impulsive and finite burns and high-fidelity orbit propagation, while providing the ability to target specified and optimized orbit states that reference customizable control and result parameters"[23]. With STK we can define the trajectory as a sequence of events called

MCS segments. Some of these segments are the initial state, the launch state, the follow segment, the maneuver, the propagate etc.

Moreover STK includes all elements needed to construct a trajectory such as the MCS segments, propagators, stopping conditions, central bodies, engine models, and more. Target sequences are used to calculate and subsequently define the required maneuver characteristics necessary to meet specified or optimal mission parameters. Once inserted all these parameters, the results produce a trajectory that reaches the goals needed. Here an example of STK simulation:

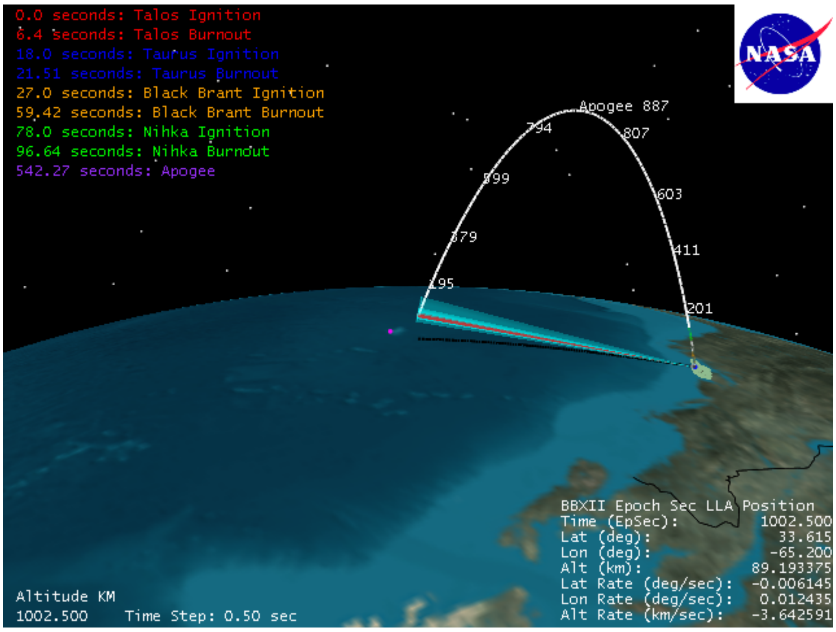


Figure 2.6: Trajectory simulated with STK software [credits NASA]

On the other hand there is "ASTOS". With this tool, the user can analyze all the aspects of the mission he desires, he can optimizes the trajectory or the vehicle design and he can simulate different mission scenarios such as the launch phase, the release phase, the re-entry phase, the orbit insertion etc. "ASTOS is being extensively used at ESA and aerospace industry community to calculate mission analysis, optimal launch and entry trajectories".[8] This tool has got a very large library of differential equations both 3-DOF and 6-DOF, it presents many different mathematical models related to propulsion, thrust curves, aerodynamics coefficients and the user can choose all these things by the usef of a friendly graphical user interface (GUI). ESA has used ASTOS for many years for project related to different kind of missions. This tool is very intuitive and the first steps to analyze a trajectory are performed in the so called "Model

Browser", the GUI that links all the models of the ASTOS library. To model the vehicle we need to have a "txt" or "Excel" file with data related to already avilable models, we need to know masses, thrust curves, aerodynamic models etc.

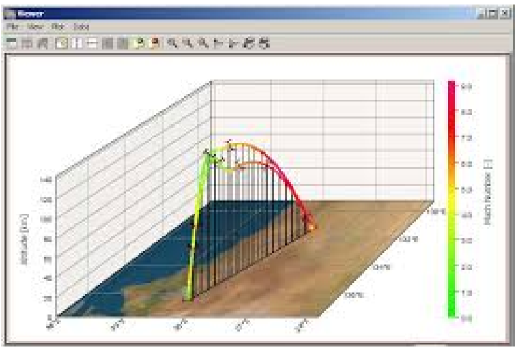


Figure 2.7: Trajectory simulated with ASTOS software [credits ASTOS]

The aforementioned tools are very strong and very employed in aerospace field. However, their "weak spot" is that the user must have a lot of information with regard to the mission he wants to do. Indeed, we know air launch is a very new research field and the companies are still running experimental tests so the data are limited and most of the time confidential. Hence, the need to create a new software simulator, using Matlab environment, which can handle and manage all available data related to air launch. Therefore, to implement a reliable simulation code a proper mathematical model should be developed, starting with the well-known motion equations. This part will be discussed extensively in chapter 3 while the software implementation will be discussed in chapter 4 followed by a comparison between the simulation results and the existing data and graphs.

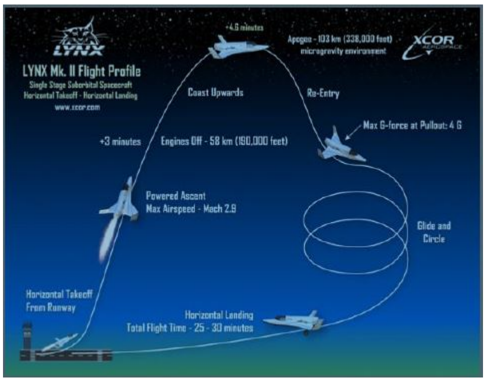
2.3 Suborbital air launch

What is meant by suborbital air launch or in general suborbital flight?

A suborbital spaceflight is a particular flight where the vehicle can reach the space outer from Earth's atmosphere but at the same time it will not complete one orbital revolution once launched from the carrier aircraft. For instance, the trajectory of a vehicle launched from

Earth that reaches the Karman line (at 100 km (62 mi) above sea level), and then falls back to Earth, is considered a suborbital spaceflight.[9]

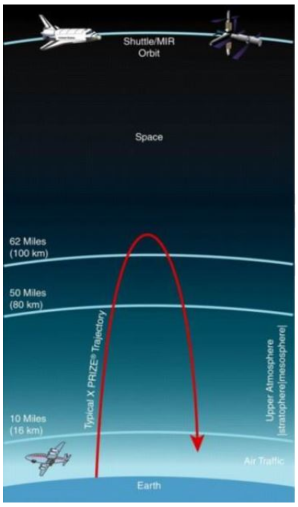
Figure 2.8: A suborbital mission profile: Lynx [credits XCOR]



Suborbital spaceflights could be divided into two main categories:

- Touristic mission: suborbital spaceflights are born also to carry passengers to experience the outer space. During these flights there will be a time interval where the crew and the passengers can experience weightlessness, the so called microgravity phase. One of the main problem is that passengers have to deal with vertical ascent and high levels of gs. Therefore they must have a good health state and perhaps they have to do some training before the mission. During the microgravity phase, which lasts few minutes, the passengers can unbuckle their seat belt and float in absence of gravity.
- Scientific mission: to conduct experiments in microgravity or above the atmosphere.

Figure 2.9: Suborbital path [credits HobbySpace]



One of the main goal of air launch is to carry out suborbital space flights. As said previously, the new millennium has simultaneously led to the birth and rise of the first private space launch companies. Moreover, the rising interest in the suborbital re-usable vehicles operations, mainly addressed at manned missions for touristic and scientific purposes, is totally modifying the concept of space business and preparing to make the space progressively more accessible to everyone.

In this framework, several private companies are developing the vehicles aimed at managing the new business of suborbital touristic flight along with the needed ensemble of infrastructures able to overcome the challenge of opening the shutters on the first solely-commercial operational spaceport of the human history. The current technological development trends for suborbital vehicles mainly considers RLVs to ensure a sufficient launch cost reduction aimed at offering the chance for a space adventure to potential passengers at competitive costs. According to the Federal Aviation Administration (FAA) Annual Compendium [10] released in early 2016, the currently realized suborbital RLVs are:

- Blue Origin's New Shepard, a single stage vertical take-off and landing vehicle aimed at commercial applications, designed to carry up to six passenger slightly over the Karman line and landing vertically. Its maiden flight in 2015 made it the first manned vehicle to take-off vertically, overcoming the Karman line1 and landing vertically.

- Virgin Galactic's SpaceShipTwo, an air launched spaceplane aimed at transporting up to

8 people (including 2 pilots) to the Karman line, at re-entering as a glider and at landing horizontally. The potentialities of this suborbital mission concept have been proven by the two consecutive SpaceShipOne flights in September and October 2004, becoming the winners of the Ansari X-Prize for suborbital flight vehicles.

- XCOR Aerospace's Lynx, a small suborbital spaceplane conceived for single passenger flights to the Karman line. The mission concept considers horizontal take-off and landing without any carrier vehicle, while the designed propulsion system is fully liquid propellant-based.

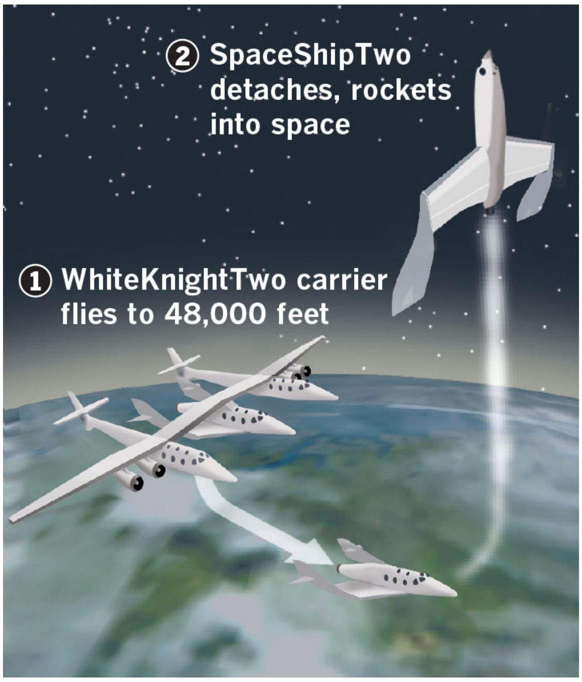
2.3.1 Virgin Galactic's SpaceShip2

Virgin Galactic is building the world's first commercial spaceline, making space accessible to everyone[11]. Their air launch system consists of two aircraft:

- WhiteKnightTwo (WK2) that is the carrier aircraft;
- SpaceShipTwo (SS2), that is the RLV air launched vehicle used for suborbital missions for reaching the outer space.

The SpaceShipTwo was thoughts as an evolution of SpaceShipOne and it was developed before 2010 similarly to the mission concept of SpaceShipOne. It can carry up two pilots and six passengers over the Karman line reaching the outer space. As said before all the people can experience during the flight periods of microgravity that last few minutes. The SpaceShipTwo mission lasts about 90 minutes from takeoff to landing. WhiteKnightTwo relases SpaceShipTwo at about 15200 m where the carried vehicles free falls for 5 seconds before igniting the rockets. Once ignited the rocket, the vehicle begins to reach an AoA of 90 degrees for its vertical ascent. Then after the burn-out, the coast phase begins where it reaches its peak apogee at around Mach 3.

Figure 2.10: WhiteKnight2 and SpaceShip2 [credits Virgin Galactic]



This spaceplane used an innovative mission concept: the space vehicle was lifted up to 15000 m of altitude by a carrier air-breathing propelled vehicle, named WhiteKnight, and then released. After a short burn, the vehicle coasted up to slightly overcoming the Karman line, and then it activated its high drag devices, often called ”feathers”, in order to perform a ballistic re-entry. Once the descent to atmospheric altitude and subsonic speeds was completed, it de-activated the drag devices in order to glide back to the departure airport.

What is feathering?

While coasting, SS2 will ’feather’ its wings and tail booms-activating a unique and highly reliable mechanism first proven by SpaceShipOne-in order to achieve an extremely reliable re-entry. In SpaceShipTwo’s feathered configuration, the entire tail structure is rotated upwards about 60 degrees, creating high drag as the spaceship enters the atmosphere. After reentry, the vehicle resumes its original configurationat approximately 70,000 feet and SpaceShipTwo glides safely back to a smooth runway landing at spaceport.

The SpaceShipTwo is a delta-winged aircraft designed for suborbital flight. The fuselage equips the two-pilots cockpit, the six seats cabin and the motor, composed by an oxidizer tank, a solid (rubber) propellant cartridge, and a supersonic nozzle. On the two edges of the delta wing, two tail fins are mounted, each provided with one horizontal elevon, in charge of controlling both the pitch and the roll angle, that mainly extends on the outer side. Each fin has a steerable rudder on the rear termination in order to gain directional controllability during the atmospheric phases of the suborbital flights. The vehicle is provided with a small micro-thrusters-based attitude control systems in order to assure the full attitude controllability in the space phase of its mission.

Figure 2.11: Activating feathers [credits Virgin Galactic]

The SpaceShipTwo propulsion system consists in a single hybrid rocket engine, named RocketMotorTwo. The engine burns a rubber-like solid propellant (based on Hydroxyl-Terminated Polybutadiene, HTPB) and uses Nitrous Oxide (N2O, also known simply as ”Nitrous”) as oxidizer. The selection of this typology of propellant relies in its safety properties: the rubber propellant needs a constant flow of oxidizer to keep ignited. If the injection of N2O is terminated, the propellant ignition is instantaneously stopped [12]. Moreover, the usage of N2O as oxidizer is usually considered ”safer”, in spite of its worse performances, than liquid oxygen or hydrogen, or kerosene. Nitrous oxide is indeed not flammable and not explosive.
As stated in the Environmental Assessment [13], the SpaceShipTwo operates by burning ap-

proximately 6800 kg of propellant (including oxidizer and solid rocket fuel) in 60 seconds. Its Maximum Take-Off Weight (MTOW) equals approximately 13000 kg. Therefore, 50% of the total vehicle mass is expelled within the first $60 - 70$ seconds after the spaceplane release. The engine thrust varies from 222 kN to 378 kN. The thrust level can depend on the oxidizer flow and on the HTPB grain geometry.

The SpaceShipTwo mission profile follows the design guidelines of the precursor vehicle, SpaceShipOne. The spaceplane is air launched by a carrier vehicle, called WhiteKnightTwo, whose four turbofan engines have been modified to maximize the ceiling altitude. The air launch is performed at 15200 m of altitude, while the WhiteKnightTwo is in horizontal flight conditions. The SpaceShipTwo performs a slight flare maneuver shortly after the release, in order to reduce the horizontal speed and the vertical speed modulus. A few seconds later, the engine is ignited and the spaceplane quickly starts its final nose-up maneuver, that is completed when the aircraft is pointing perfectly upwards. After 60 seconds from the engine ignition, when the spaceplane is over 40000 m of altitude, the engine burn-out occurs, assuming the propellant completely consumed. Therefore, the coast or ballistic phase of the spaceflight starts. The vehicle reaches an apogee over the Karman line. A few seconds before reaching the apogee, the SpaceShipTwo activates the feathers, in order to prepare and maintain the proper re-entry configuration. When starting its descent, the vehicle keeps its wings leveled to the horizon, in order to maximize the exposed wing surface to the ram air stream. At the beginning of the descent, the passengers seats are inclined and maintained parallel to the vehicle horizontal plane. The composition of the seats inclination angle and the vehicle attitude during the descent allows the passengers to minimize the effects of the re-entry high G-number, while ensuring that the acceleration is experienced in front-to-back direction [Guidance for Medical Screening of Commercial Aerospace Passengers]. Finally, when the vehicle has completed its re-entry, falling with a subsonic Mach number, the pilots command the "de-feathering" maneuver, by blocking the feathers again in glider position. The spaceplane then performs a final slight nose-down maneuver in order to begin to generate aerodynamic force with its wings. Finally, the SpaceShipTwo approaches the landing airport as a glider and it performs an unpowered landing.

In chapter 3 we will see the model for SpaceShip2 implemented in the simulator.

2.4 Orbital air launch

Air launch system is also used for orbital spaceflights. As seen in section 2.1.1, the major advantage of using this innovative system is that orbital parameters can be choosen in according with client's needs.

How to get into orbit and which are air launch benefits?
We know that the change of velocity, called Δv, that a launch object experiences, deals with several losses due to gravity force, aerodynamic forces or steering that is the mismatch of the thrust vector and the vehicle velocity vector. All these losses can be minimized with specific choices. Gravity loss is given by the following formula:

$$\int_{t_{ignition}}^{t_{burnout}} gsin\gamma(t)dt$$

Therefore we can clearly understand that a trajectory with a path angle γ, i.e.the angle between the vehicle velocity vector and the local horizontal, minimizes this loss. Typical value of losses due to gravity are about 1300 m/s. On the other hand the drag loss, due to interaction between vehicle and air layers, is given by this formula:

$$\int \frac{D(t)}{m(t)}dt$$

Typical values of drag losses are in the order of about 152 m/s. This loss can be minimized building a long slender body because we know that drag increases if the surface increases.
Steering losses are instead typically about 100 m/s. If we want to find a good choice to try to reduce all the losses, then a compromise trajectory must be chosen because we know that a vertical path that would minimize drag losses but at the same time increases gravity losses while an horizontal trajectory would decrease gravity losses but increases drag ones.
Moreover, Δv depends on launch site: obviously "the best place to launch is the equator in due east direction because the Earth's rotation helps with a free velocity increment of 463 m/s"[7].
One of the main advantage of air launch is that it can reduce the Δv required to reach the desired orbit because the carrier aircraft provides a positive speed of the order of 200 m/s.

Moreover, since the vehicle is released at certain altitude then gravity and drag losses are reduced. In addiction, as said in [7] ”to provide a performance benefit, the carrier aircraft must be capable of releasing the launch vehicle at a positive flight path angle above the local horizon. A subsonic release at $\gamma = 25$ provides about 488 m/s Δv benefit for a winged launch vehicle”. Further increases in γ provide little additional benefit for winged launch vehicles but does provide additional benefit for unwinged launch vehicles.

In the following section we will take a look at the main developed (or in development) orbital air launch system.

2.4.1 Northrop Grumman’s Pegasus

The Pegasus is an air-launched rocket developed by Orbital Sciences Corporation, now part of Northrop Grumman Innovation Systems[14]. This aircraft can carry small payloads of up to 443 kilograms into LEO. Its first flight was in 1990 and Pegasus has remained operative until 2018. The release phase from its carrier aircraft happens at about 12,000 m, and its first stage has a wing and a tail to generate lift force and to control vehicle attitude during the mission[14]. Pegasus is a mature and flight-proven launch system that has demonstrated consistent accuracy and dependable performance. The Pegasus launch system has achieved a high degree of reliability through its significant flight experience. It offers a variety of capabilities that are uniquely suited to small spacecraft.

The mission profile of Pegasus has explained in the figure above.

2.4.2 Virgin Orbit’s LauncherOne

Virgin Galactic, part of Richard Branson’s Virgin Group, began development of its LauncherOne system in mid-2012, after preliminary study of the idea beginning in 2007. The company’s initial goal was to be able to boost 120 kg to sun synchronous low earth orbit for less than $10 million. The company initially contemplated use of the White Knight Two aircraft that was built for the SpaceShipTwo program to drop-launch the LauncherOne rocket[15].

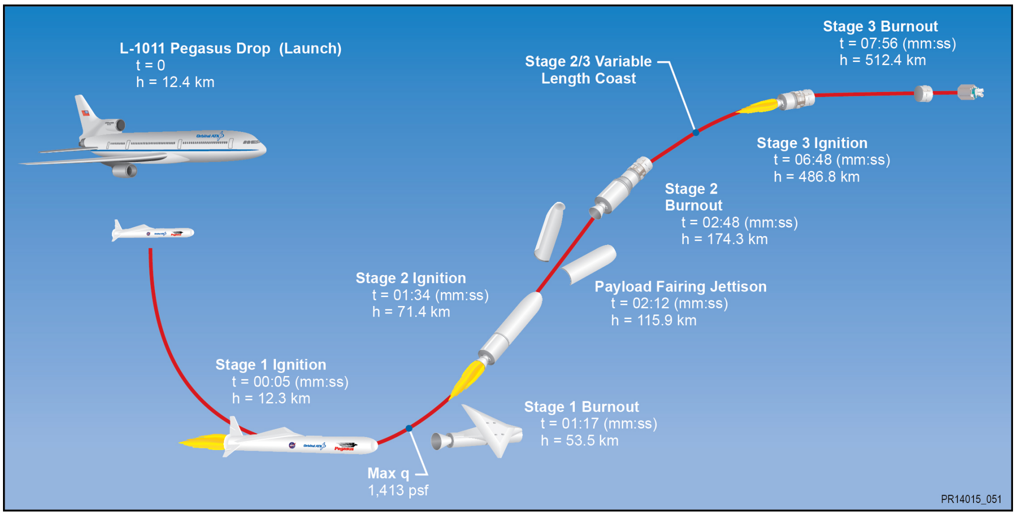


Figure 2.12: Profile mission of Pegasus rocket [credits Northrop Grumman]

Early development focused on rocket engines for the two stages and on composite propellant tanks. The group had successfully test fired its thrust "NewtonThree" first stage engine and had tested the gas generator for the "NewtonFour" second stage engine. Both engines were pump-fed LOX/RP-1 types. Earlier pressure-fed "NewtonOne" and "NewtonTwo" engines had been previously tested by the company, but abandoned in favor of the pump-fed designs. These engines had been developed under DARPA's ALASA (Airborne Launch Assist Space Access) program.

Figure 2.13: LauncherOne on wing bottom of CosmicGirl [credits Virgin Orbit]

Engine testing took place at Mojave, using Virgin Galactic's "Necker" test stands. During a typical flight, the first stage engine would fire for about three minutes. The second stage engine would perform multiple burns for a total of nearly six minutes.

The decision to go with pump-fed engines, combined with a decision to bypass White Knight in favor of a larger aircraft, increased payload to 200 kg by mid-2015.

On December 3, 2015, the company announced that a 747-400 aircraft named ”Cosmic Girl” would be used to drop-launch the LauncherOne rocket from 11000 m altitudes. The 747 had previously flown for Virgin Atlantic. Modifications of the 747 began to allow it to carry LauncherOne on a new pylon under its port wing, inboard of the engines, and to provide updated communications systems to allow it to serve as a ”flying launch site”.

In March, 2017, Virgin Galactic announced that it was forming a new company, Virgin Orbit, to handle the LauncherOne program. The company would be led by President Dan Hart, previously Vice President of Boeing Government Satellite Systems.

By early 2017, LauncherOne performance was listed at 300 kg to a 500 km sun synchronous orbit or 500 kg to a 200 km x 28.5 deg low earth orbit. Virgin Orbit was planning for initial operations to be based in Mojave, California.

Chapter 3

Trajectory modeling approach

3.1 Introduction

In this chapter the main objective is to present some mathematical models used for modeling our physical problems. We illustrate the equations that govern our phenomena, showing the differences between the profile phases. Then we dwell on the more critical phase, the re-entry phase, studying the variables that affect the most the aircraft, like dynamic pressure and thermal flux. Since there are at stake many models and input parameters, a graphical user interface GUI has been implemented with MatLab and the results has been compared with those available in literature.

3.2 Trajectory modeling approach

3.2.1 Rigid body model

In this chapter we will see the rigid body equations taking a cue from [22]. We need to reference frames for talking of displacement, velocity and acceleration related to the relative motion of two rigid bodies. The first frame we consider is the inertial frame that does not accelerate with

respect to any other frames. For example, if we consider flights at Mach number ≤ 3 and at altitude ≤ 3 km we can take the Earth as our inertial frame I even if its spinning. We can also assume for simplicity that the axis Z is aligned with the gravitational force[22].
The position of the aircraft center of mass in the inertial reference frame is given by:

$$r_c(t) = [x_c(t), y_c(t), z_c(t)]^T,\ t \geq 0$$

Since aircraft have several moving parts we have to consider an "orthonormal reference frame J fixed with the aircraft and the choice of the axes strongly depends on the problem considered". Once we have defined our two frames I and J,Tait-Bryan angles $\varphi, \theta, \psi : [0, +\infty) \rightarrow \mathbb{R}$ named roll, pitch, and yaw angles, can describe the vehicle attitude. The velocity of the aircraft center of mass with respect to the wind is denoted by $[u, v, w]^T : [0, +\infty) \rightarrow \mathbb{R}^3$ and the aircraft angular velocity with respect to the inertial reference frame I is denoted by $[p, q, r]^T : [0, +\infty) \rightarrow \mathbb{R}^3$. Both of them are expressed in the body reference frame J. So, we can define the aircraft state vector χ as[22]:

$$\chi := [x_c, y_c, z_c, \varphi, \theta, \psi, u, v, w, p, q, r]^T$$

Moreover we know that the aircraft attitude can change by the use of some controls. Therefore, we can define the vehicle control vector as:

$$\eta := [\delta_T, \delta_{E \cdot A}, \delta_R]^T$$

where δ_T is the throttle ratio, δ_E represents elevators deflection angle, δ_A is ailerons deflection angle and δ_R stands for rudder deflection angle.
The angle of attack α and the sideslip angle β play key roles in the study of aircraft dynamics. The first one is the angle between the axis x and the projection of the velocity of the aircraft center of mass with respect to the wind on the aircraft plane of symmetry. The second one at the aircraft center of mass is the angle between the vector velocity and the aircraft plane of symmetry[22]. In some applications it is convenient to express the aircraft state vector as:

$$\chi := [x_c, y_c, z_c, \varphi, \theta, \psi, u, \alpha, \beta, p, q, r]^T$$

The study of the aerodynamic, gravitational, and propulsive forces and moment of the forces plays a fundamental role both in the analysis of the aircraft dynamics and the synthesis of effective control actions. In general, the aerodynamic and propulsive forces acting on an aircraft and their moments explicitly depend on $\chi, \dot{\chi}, \eta$ but rarely depend on higher derivatives of the state vector.

Modeling an aircraft as a rigid body, the equations of motion in the reference frame I are:

$$\frac{d}{dt}\begin{pmatrix} x_c(t) \\ y_c(t) \\ z_c(t) \end{pmatrix} = \begin{pmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{pmatrix}\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix}$$

$$\begin{pmatrix} x_c(0) \\ y_c(0) \\ z_c(0) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}, \ t \geq 0$$

$$\frac{d}{dt}\begin{pmatrix} \varphi(t) \\ \theta(t) \\ \psi(t) \end{pmatrix} = \begin{pmatrix} 1 & sin\varphi\, tg\theta & cos\varphi\, tg\theta \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi\, sec\theta & cos\varphi\, sec\theta \end{pmatrix}\begin{pmatrix} p(t) \\ q(t) \\ r(t) \end{pmatrix}$$

$$\begin{pmatrix} \varphi(0) \\ \theta(0) \\ \psi(0) \end{pmatrix} = \begin{pmatrix} \varphi_0 \\ \theta_0 \\ \psi_0 \end{pmatrix}, \ t \geq 0$$

$$\frac{d}{dt}\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \frac{1}{m}\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} - \begin{pmatrix} p(t) \\ q(t) \\ r(t) \end{pmatrix} \times \begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix}$$

$$\begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix}, \ t \geq 0$$

$$\frac{d}{dt}\begin{pmatrix} p(t) \\ q(t) \\ r(t) \end{pmatrix} = \begin{pmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{pmatrix}^{-1} \begin{pmatrix} L \\ M \\ N \end{pmatrix} - \begin{pmatrix} p(t) \\ q(t) \\ r(t) \end{pmatrix} \times \begin{pmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{pmatrix} \begin{pmatrix} p(t) \\ q(t) \\ r(t) \end{pmatrix}$$

$$\begin{pmatrix} p(0) \\ q(0) \\ r(0) \end{pmatrix} = \begin{pmatrix} p_0 \\ q_0 \\ r_0 \end{pmatrix},\ t \geq 0$$

We can notice how these equations are strongly nonlinear. It is common practice in nonlinear systems theory to analyze a nonlinear system by observing the behavior of the linearized system in a neighborhood of a conveniently chosen point, such as an equilibrium point.

3.2.2 Point-like mass model

The point-like model simplifies the problem considering the aircraft as a material point. This model takes in account:

- aerodynamic forces, in terms of drag and lift;
- gravity force, considering a flat-Earth reference frame;
- thrust force, given by the propulsion system;
- mass loss, due to the aircraft fuel consumption.

The classical model equations are the following:

$$
\begin{aligned}
\frac{dV}{dt} &= \frac{1}{m}(T - D - Wsin\gamma) \\
\frac{d\psi}{dt} &= \frac{1}{mVcos\phi}Lsin\phi \\
\frac{d\gamma}{dt} &= \frac{1}{mV}(Lcos\phi - Wcos\gamma) \\
\frac{dx}{dt} &= Vcos\gamma cos\psi \\
\frac{dy}{dt} &= Vcos\gamma sin\psi \\
\frac{dz}{dt} &= Vsin\gamma \\
\frac{dm}{dt} &= -W_c
\end{aligned}
$$

where V indicates the aircraft speed, T is the engine thrust, D and L are respectively the drag and lift aerodynamic forces, W is the weight and m is the mass. γ represents the climb angle (between the velocity vector and the ground plane), ϕ is the roll angle (between the vehicle symmetry plane and the local vertical vector) and ψ is the directional angle (between the aircraft heading and a reference direction on the ground plane, often indicating the geographic North). It has already said that the mass changes with respect to time, so W_c represents the fuel consumption (kg/s). This model is very useful for calculating aircraft trajectories for what concerns problems of classical flight mechanics or for fixed-wings vehicles use. Indeed, the usual flight conditions are those whose AoA spans within small values centered on the zero. The AoA is defined as $\alpha = \theta - \gamma$, (where θ represents the pitch angle)and the aforementioned model works for very small values of α. Therefore there is always a match between the pitch and the climb angle. For our objectives the hypotheses of small values of α will be removed, since in airlaunching the aircraft attitude changes from horizontal to vertical and vice versa.

To a first approximation we can simplify our model supposing that the airlaunched aircraft profile mission does not consider any turn and any roll maneuver between the initial and final phases. Hence, ϕ remains zero and ψ remains constant. In this way we can operate a coordinate transformation aligning the x-axis to the trajectory direction ground component, deleting one dimension of the problem. So, the remaining equations are related to V, γ, x, z. Similarly we can consider the acceleration equations along the two directions of interest:

$$a_x = \frac{1}{m}(Tcos\gamma - Lsin\gamma + Dcos\gamma)$$
$$a_z = \frac{1}{m}(Tsin\gamma + Lcos\gamma - Dsin\gamma - W)$$

When considering flights in which the AoA does not span within small values centered on the zero, like Virgin Galactic SpaceShipTwo flight, the pitch and the climb angle cannot be considered equal. Therefore, the model shall now consider the pitch angle θ instead of the climb angle γ:

$$a_x = \frac{1}{m}(Tcos\theta - Lsin\theta + Dcos\theta)$$
$$a_z = \frac{1}{m}(Tsin\theta + Lcos\theta - Dsin\theta - W)$$

3.2.3 Parameters modeling

- THRUST

  The thrust is often modeled as:

  $$T = T_{max} \cdot \delta_T$$

  where T_{max} is the maximum thrust of the engine and δ_T, called "throttle ratio", represents the pilot control on the fuel valve. It expresses the percentage of thrust during the mission, so it varies from 0 to 1. The shape of thrust function mainly depends on the considered mission. Indeed, in aeronautic and aerospace applications there are three different types of fuel:

  – solid: as shown in Fig. 3.1 different grain geometries in solid-fuel rockets produce various thrust curves. Solid fuel is used because it's very simple to obtain, it's easily storable and usually fairly cheap compared to liquids. On the other hand it cannot be shut down or be throttled and it could ignite at any moment if not stored correctly.

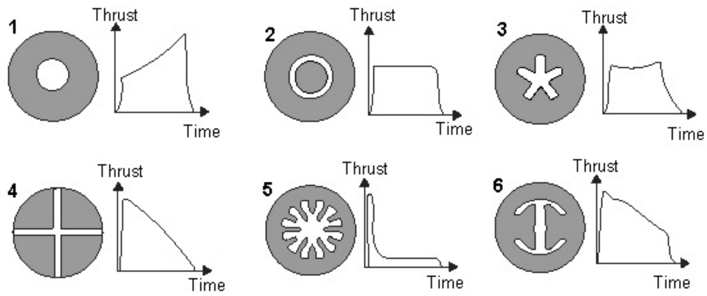


Figure 3.1: Different grain geometries and their relative thrust curves [credits http://www.aerospacengineering.net/?p=1255]

- liquid: pros and cons are specular compared to solid ones. Indeed, liquids can sometimes be throttled, shut down and they can be stored if using hypergolics. However, they are very expensive and complicated, requiring a lot of intricate machinery. Besides, you have to deal with fluid flow.
- hybrid: this model gathers the qualities of the previous ones. It represents a compromise between solid and liquid pros.

Usually the most common model applications consider thrust as a constant function. However, thrust curve can be modeled in many different ways in order to achieve different mission objectives. Indeed, the thrust trend can be optimized by considering a trade-off among mission factors.

- AERODYNAMIC FORCES

  The lift and drag forces are modeled considering the conventional form, as product of the dynamic pressure, the wing surface and the lift or drag coefficient:

$$L = \frac{1}{2}\rho S V^2 C_l$$

$$D = \frac{1}{2}\rho S V^2 C_d$$

where S represents the wing surface, ρ is the air density and C_l and C_d are the lift and drag coefficient. Defining the dynamic pressure as $q = \frac{1}{2}\rho V^2$ we can rewrite the aerodynamic forces as:

$$L = qSC_l$$

$$D = qSC_d$$

The hardest step is finding suitable model for C_l and C_d because they depend on many drivers.

- AIR DENSITY

  It is also necessary to investigate through a proper mathematical model the interaction of the vehicle itself with the atmosphere.

  The traditional assumption of considering the airflow as a continuous might no longer be consistent with the operational environment of high altitude vehicle, as their characteristic dimension may be of the same order of magnitude of the mean free path λ, namely the distance between two following interactions of a fluid particle with another one. In this sense, traditional concepts like the aeronautic stall cannot hold for this kind of vehicles, as well as the evaluation of the aerodynamic coefficient. Thus, it is necessary to introduce a model of rarefied gas to take into account the effective molecular mean free path and provide a successful strategy to assess with a better approximation the aerodynamic coefficients of high altitude flying vehicle during the transition between the aeronautic and the space domains.

  Usually in aircraft applications the air density ρ is modeled as an exponential function of altitude:

$$\rho = \rho_0 \cdot e^{-\beta z}$$

  where ρ_0 is the reference air density at mean sea level, β is the inverse of scale height and it is usually equal to 10^{-4} and z is the altitude. The exponential model provides a good approximation below 30km, so, as we will see, it does not fit our purposes related to trans-atmospheric vehicles. The model that will be selected in this work is the U.S. Standard Atmosphere model, which guarantees up to 150 km of altitude[17].

  Once basic thermodynamics are known, the key parameter of the present model can be evaluated: the mean free path λ. By definition, it denotes the mean distance between two consecutive interactions of a fluid particle with two other ones; it is easy to argue that such parameter grows along with the altitude. Through λ it is possible to obtain a criterion to distinguish between continuous and discrete air-flows, with significant impacts on the evaluation of the aerodynamic coefficients. Mean free path is a function of the absolute temperature T and pressure P according to the following equation:

$$\lambda = \frac{3e - T^2}{(T + 110.4)P}$$

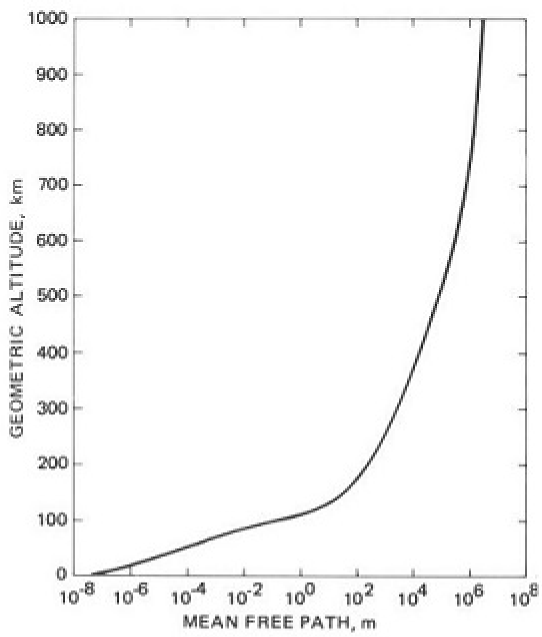


Figure 3.2: Mean free path vs altitude

From Fig. 3.2 we can notice that at almost 100 km of altitude, the mean free path is nearly equal to 0.1 m, so it is comparable to the dimension of an aircraft wing. Therefore from this point onwards we cannot consider the airflow as a continuum anymore.

The non-dimensional, key parameter permitting to distinguish between this two kinds of interactions is the Knudsen number Kn, defined as follows:

$$Kn = \frac{\lambda}{L}$$

where L represents the characteristics dimension of the vehicle. When this parameter equals 0.1, the assumption of continue airflow cannot be considered valid any more, and in this case it is necessary to examine the interaction with the vehicle from a statistic point of view.

3.3 Re-entry phase

[18]Returning from space is one of the most difficult challenge to deal with. While the objects plummet through Earth's atmosphere, they experience high levels of deceleration due to the interaction with more dense air layers. This is the most dangerous phase because if an object returning from space hit the atmosphere too steeply or too fast, it risks a fiery end. On the other hand, if its impact is too weak, it may come back into the space.

The re-entry phase is characterized by a set of requirements to reach such as all the mission phases. In this case we must balance three of these requirements that most of the time are competing:

- Deceleration
- Heating
- Accuracy of landing or impact

During the re-entry phase the vehicle and hence the crew experience high levels of "g's" that have to be limited to prevent damages. We have to keep in mind that also little deceleration have to be avoided because if the object come back very slowly it could back into space.

Another important factor during re-entry phase is the values of heating due to the friction between the body and the air. The vehicle design must be thought so as to withstand high temperatures. Information about these are given by the total heating and the peak heating rate.

The last mission requirement is accuracy of landing or impact that depends for example on what area the vehicle land on. Greater the area, less important the constraints of accuracy.

Therefore the vehicle returning from space must choose a good compromise among all these principal factors. This compromise is a three-dimensional re-entry corridor whose size depends on the constraints of the three competing factors.

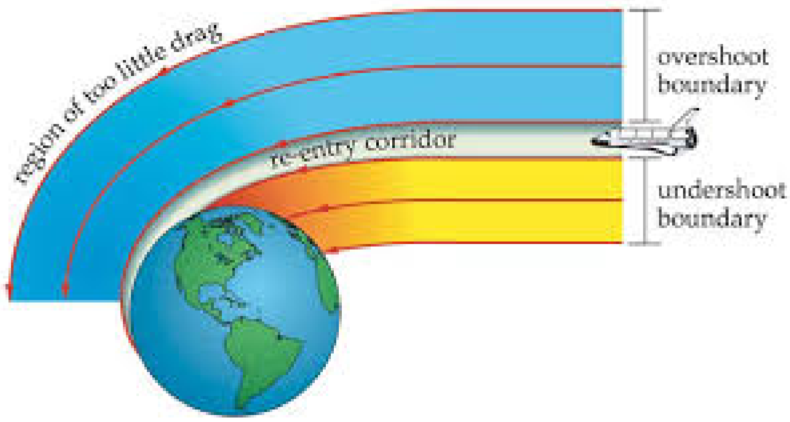


Figure 3.3: Re-entry corridor [18]

3.3.1 Re-entry Motion

[18]The main force acting on the vehicle during the re-entry phase is drag:

$$D = \frac{1}{2}\rho V^2 C_d A$$

To understand how streamlined or not streamlined an object is we can introduce a parameter called shape coefficient of drag, C_d, that determines how big the drag force is on our aircraft. This parameter is computed and validated by engineers through experiments in wind tunnels. For Newton's Second Law we have that:

$$\vec{a} = \frac{1}{2}\rho V^2 \frac{C_d A}{m}\vec{e}$$

where $\vec{e}$ represents the unit vector along drag direction. Moreover we can introduce a new quantity, $\frac{C_d A}{m}$, that has a special significance in describing how an object moves through the atmosphere. By convention, engineers invert this term and call it the ballistic coefficient, B:

$$B = \frac{m}{C_d A}$$

The ballistic coefficient gives information about how deceleration an object experiences returning from outer space.

As shown from Fig. 3.4, we can summarize the competing mission requirements during the re-entry phase are as follows[18]:

- ”Trajectory design, including changes to
  1. Re-entry velocity, $V_{re-entry}$
  2. Re-entry flight-path angle, γ
- Vehicle design, including changes to
  1. Vehicle size and shape (B)
  2. Thermal-protection systems (TPS)”[18]

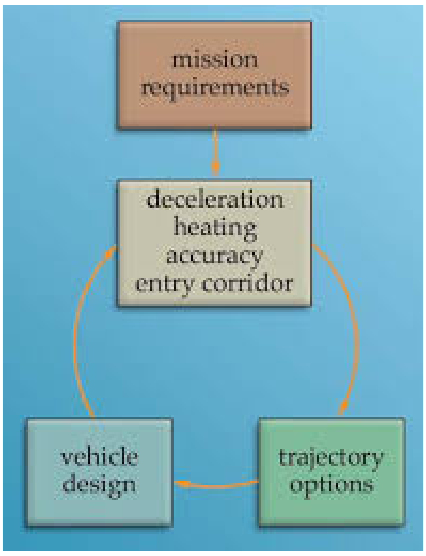


Figure 3.4: Mission requirements for the re-entry phase [18]

3.3.2 Trajectory and deceleration

During re-entry phase we have understood that the vehicle subject to high levels of deceleration and heating. Deceleration increases gradually until a maximum value, a_{max}, and then begins to decrease again. From literature [18], it can be prooven that the maximum deceleration and the altitude at which it occurs are the following:

$$a_{max} = \frac{V_{re-entry}^2 \beta sin\gamma}{2e}$$

$$z_{a_{max}} = \frac{1}{\beta} ln \left(\frac{\rho_0}{B \beta sin \gamma} \right)$$

where β is the atmospheric scale length used to describe the density profile of atmosphere (”for Earth $\beta = 0.000139\, m^{-1}$”[18]). We can notice that max depends on the re-entry velocity and flight-path angle, while the altitude z_{amax}, that is the altitude at which the max deceleration occurs, depends only on the flight-path angle. Fig. 3.5 shows us that higher the re-entry velocity, greater the maximum deceleration and also higher the re-entry flight-path angle then greater the maximum deceleration experienced.

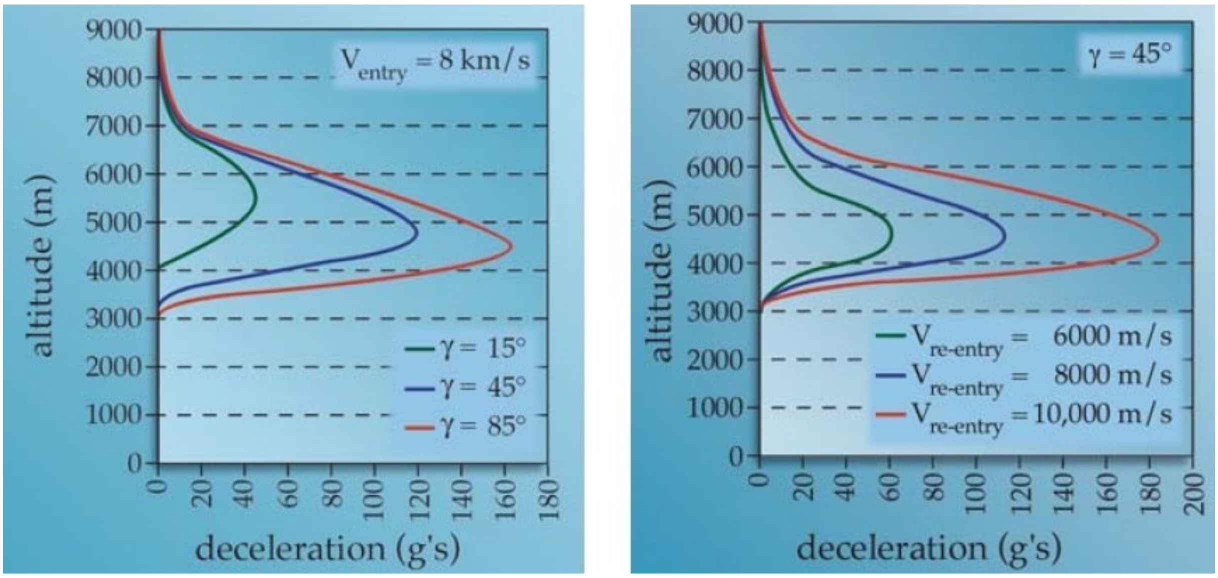


Figure 3.5: altitude vs deceleration [18]

3.3.3 Trajectory and heating

For what concerns heating we know that a vehicle con gets hot through radiation, conduction and convection. The first one is the transfer of heat from a zone to another region through electromagnetic waves; the second one through a physical medium, while convection happens when there is an energy transfer from a fluid and an object[18].

Moreover, since the objects that we will study have very high velocities during the re-entry phase, a shock wave forms along all the vehicle's body.

"We can quantify the heating rate, $\dot{q}$, a re-entry vehicle experiences, expressing it in watts per square meter, i.e. heat energy per unit area per unit time. It's a function of the vehicle's velocity and nose radius, and the density of the atmosphere"[18].

Empirically, for Earth's atmosphere, using Sutton and Graves equation, this becomes approximately:

$$\dot{q} = 1.83 \cdot 10^{-4} v^3 \sqrt{\frac{\rho}{r_{nose}}}$$

As done before for deceleration, from literature we can prove which are the altitude and velocity where the maximum heating rate occurs:

$$h_{\dot{q}_{max}} = \frac{1}{\beta} ln \left(\frac{\rho_0}{3B\beta \sin(\gamma)} \right)$$

and

$$v_{\dot{q}_{max}} \approx 0.846 v_{re-entry}$$

3.4 Suborbital mission study case: Spaceship 2

In this section the model for suborbital spaceflight simulation will be presented.

This model is used when the user choices are:

- mission → suborbital
- number of stages → 1

then the software simulator takes a cue from the study case of Virgin Galactic's Spaceship 2. The code has been divided into three main parts each one linked to the three different mission phases:

1. RELEASE PHASE

   The spaceplane is released horizontally by the carrier at altitude h_0 and with a certain velocity v_{carr}, then:

   $$state_{in} = [x_0, y_0, v_{x0}, v_{y0}] = [0, h_0, v_{carr}, 0]$$

   The release phase lasts $t_{release}$ seconds and in this time interval the attitude of our vehicle behaves in two different ways:

   - if $0 \leq t \leq \frac{t_{release}}{2}$, then the attitude remains constant, i.e. C_d, C_l and α don't vary

   - if $\frac{t_{release}}{2} \leq t \leq t_{release}$, we suppose a linear increment for all abovementioned parameters because the spaceplane makes a flare maneuver which increments the AoA from 0 to 10 degrees:

   $$\begin{aligned} C_d(t) &= C_{d0} + (C_{df} - C_{d0})\,\frac{t - 0.5t_{rel}}{0.5t_{rel}} \\ C_l(t) &= C_{l0} + (C_{lf} - C_{l0})\,\frac{t - 0.5t_{rel}}{0.5t_{rel}} \\ \alpha(t) &= \alpha_0 + (\alpha_f - \alpha_0)\,\frac{t - 0.5t_{rel}}{0.5t_{rel}} \end{aligned}$$

   This assumption is based on the direct proportionality between AoA α with C_d and C_l. For the release phase the motion equations are the following:

   $$\begin{cases} ma_x = -L\sin\alpha - D\cos\alpha \\ ma_y = -mg + L\cos\alpha - D\sin\alpha \end{cases}$$

   where $L = \frac{1}{2}\rho v^2 S C_l = qSC_l$ and $D = qSC_d$.

2. THRUST PHASE

During the burn phase we have to consider three different time intervals:

- if $0 \leq t \leq t_0$ the spaceplane keeps its attitude like the final one of the release phase and the thrust is at its minimum value;
- if $t_0 \leq t \leq t_1$ the angle γ increases until the vehicle gets perpendicular with an angle of 90 degrees. We assume these following behaviors:

$$\begin{aligned}
\gamma(t) &= \gamma_0 + (\gamma_f - \gamma_0)\sin\left[\frac{\pi}{2}\left(\frac{t-(t_{rel}+t_0)}{t_1}\right)\right] \\
C_l(t) &= C_{l_0} + (C_{l_f} - C_{l_0})\frac{t-(t_{rel}+t_0)}{t_1} \\
C_d(t) &= C_{d_0} + (C_{d_f} - C_{d_0})\frac{t-(t_{rel}+t_0)}{t_1}
\end{aligned}$$

- if $t_1 \leq t \leq t_{thrust}$ the attitude remains constant in its final set.

It should be mentioned how the thrust has been modelled. Using the simulator, the user has to insert the minimum and the maximum value of thrust and, of course, its duration. For this purpose, the simulator provides a thrust curve implemented following the literature and video of Spaceship2. These are the equations:

$$T(t) = \begin{cases} T_{min}, & 0 \leq t \leq t_0 \\ T_{min} + (\delta T_{max} - T_{min})\left(\frac{(t-(t_{rel}+t_0+t_1))}{t_{thrust}-t_1-t_0}\right)^3, & t_0 \leq t \leq t_1 \\ \delta T_{max} + (T_{min} - \delta T_{max})\left(\frac{(t-(t_{rel}+t_0+t_1))}{(t_{thrust}-t_1-t_0)}\right)^3, & t_1 \leq t \leq t_{thrust} \end{cases}$$

As said in section 2.3 the suborbital mission can be divided into two different categories:

- scientific mission
- touristic mission

This difference is modelled introducing a parameter called δ which represents the percentage of maximum thrust that is used during all the burn phase (90% and 80% respectively). The thrust modulation trend was optimized by considering a trade-off among the following simulation drivers:

- the RocketMotorTwo shall begin the burn with a low thrust level, in order not to cause an excessive increase of the horizontal speed, which would enlarge the ground track of the mission profile. Moreover, a higher horizontal speed would affect the AoA during the burn nose-up maneuver;
- the RocketMotorTwo shall increase the thrust while the nose-up maneuver is performed, until reaching the maximum allowed thrust value when the vehicle is in vertical position;
- the RocketMotorTwo shall rapidly decrease the thrust level while maintaining the vertical attitude, in order to optimally contrast the divergent rise of the g-number experienced by the on-board crew.

The optimal thrust profile was therefore obtained by superposing two curves:

- an increasing, diverging, third order curve, leading the thrust from the minimum to the maximum value during the nose-up maneuver time;
- a decreasing, converging, third order curve, forcing the thrust to diminish from the maximum to the minimum value and starting once the SpaceShipTwo has reached a perfectly vertical attitude.

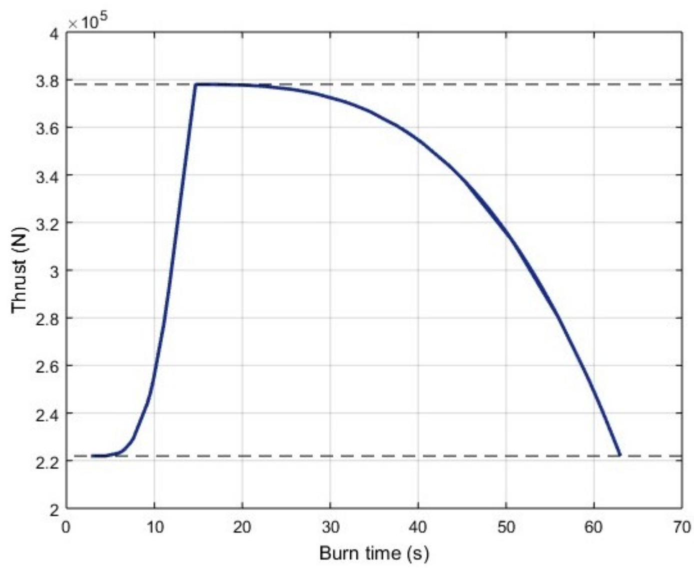


Figure 3.6: Thrust curve of RocketMotorTwo

Moreover we have to remember that in this phase mass changes following this relation:

$$\begin{cases} \dot{m}(t) = -\frac{m_{fuel}}{t_{thrust}} \\ m(t_{release}) = m_0 \end{cases}$$

i.e.

$$m(t) = m_0 - \frac{m_{fuel}}{t_{thrust}}(t - t_{release})$$

Since the spaceplane aims to reach a vertical position, there will be a phase in which the vehicle will not produce lift. Then we impose a constraint on lift:

$$\text{if } \; v_x \lll v_y \; \text{ OR } \; v_x \leq v_{xlim} \longrightarrow L = 0$$

Finally the motion equations are on the same page of the release phase, i.e.:

$$\begin{cases} ma_x = T\cos\gamma - L\sin\gamma - D\cos\gamma \\ ma_y = T\sin\gamma - mg + L\cos\gamma - D\sin\gamma \\ \dot{m} = -\frac{m_{fuel}}{t_{thrust}} \end{cases}$$

3. COAST PHASE

Once the burn phase has ended, the vehicle experiences a ballistic flight, entering in the coast phase. Here we suppose that the fuel has been all exhausted, i.e. $m = m_0 - m_{fuel}$. During this phase, in particular during the re-entry phase, the SpaceShipTwo is going to activate the feathers. In this way we can split the coast phase into two pieces:

(a) if $v_y \geq 0$, then the feathers are still not activated and we can use C_l and C_d as the previous ones.

(b) Otherwise if $v_y \leq 0$, i.e during the re-entry phase, then the feathering system will be activated and the vehicle will behave like a re-entering capsule. This behavior can be modelled supposing $C_l = 0$ and an increase of the reference area S.

The motion equations are the same of the release phase.

3.5 Orbital mission study case: PegasusXL-LauncherOne

In this section the model for orbital spaceflight simulation will be presented.

This model is used when the user choices are:

- mission $\rightarrow$ orbital
- number of stages $\rightarrow \geq 2$ (from now on let's suppose a 2 stages vehicle)

then the software simulator takes a cue from the study case of Northrop Grumman's PegasusXL or Virgin Orbit's LauncherOne. Also this code has been divided in more parts related to:

1. RELEASE PHASE

   Here one of the main difference between the suborbital case is that the vehicle is released with a higher inclination due to the orbital height and inclination to be reached. Hence, if we call θ_{rel} the release angle, the initial state can be expressed as:

   $$state_{in} = [x_0, y_0, v_{x0}, v_{y0}] = [0, h_0, v_{carr} \cos \theta_{rel}, v_{carr} \sin \theta_{rel}]$$

   Usually, in literature we know that the release angle has to be greater than or equal to 25 degrees and, according to the mass center and the pressure center, three different scenarios can be considered:

   - the attitude remains constant for all the release duration;
   - linear increment of the AoA and therefore of aerodynamic parameters;
   - linear decrease of the AoA and therefore of aerodynamic parameters.

   The motion equations remains the same of the suborbital case. Then the attention shifts to the first of the thrust phases, because we have to keep in mind that usually in the orbital case there are more than one stage, so more thrust phases have to be considered and modeled.

2. 1st THRUST PHASE

Since the input parameters given by the user for this phase are T_{max}, T_{min}, t_{thrust} and $t_{T_{max}}$ (the time to reach the maximum value of thrust), we model the thrust curve as a third degree polynomial curve similar to the suborbital case's one. This assumption opens a large field of research of the optimal thrust curve for the specific mission. Indeed, this type of information are still confidential, hence we can only take a cue from similar past cases and from the information that we can know from the companies. Moreover, since orbital vehicles usually have not lifting surfaces, in this phase we can consider the lift negligible with respect to thrust and drag forces.

So the equations are the following:

$$\begin{cases} ma_x = T\cos\gamma - D\cos\gamma \\ ma_y = T\sin\gamma - mg - D\sin\gamma \\ \dot{m} = -\frac{m1_{fuel}}{t_{thrust}} \end{cases}$$

where γ represents the AoA.

However, using this model and running the simulation, the results show us the rocket cannot increase its height and loses its altitude. This fact lets us thinking that we are not considering something in the model. One of the main reason for which a rocket cannot increase its altitude is that vertical component thrust is lower than vertical component drag and weight force. In fact we are considering an AoA lower than 45 degrees (usually 25 or 30), so we give more importance to horizontal components with respect to the vertical ones. So, in this way, the vehicle cannot increase its altitude. A possible solution is considering gimbaled thrust even if this is another assumption that cannot be demostraded as real or false for lack of data, but it will lead to good and reliable results. Gimbaled thrust is the system of thrust vectoring used in most rockets, including the Space Shuttle, the Saturn V lunar rockets, and the Falcon 9. In a gimbaled thrust system, the exhaust nozzle of the rocket can be swiveled from side to side. As the nozzle is moved, the direction of the thrust is changed relative to the center of gravity of the rocket. If we consider ψ_{gimb} as the gimbal angle of thrust, the equations of our system become the following:

$$\begin{cases} ma_x = T\cos\psi_{gimb} - D\cos\gamma \\ ma_y = T\sin\psi_{gimb} - mg - D\sin\gamma \\ \dot{m} = -\frac{m1_{fuel}}{t_{thrust}} \end{cases}$$

Figure 3.7: Gimbaled thrust

Introducing the concept of this angle, we can give more weight to vertical thrust component so that it can contrast the weight force and the drag force to increase vehicle altitude.

In this way, in addition to the AoA γ, the gimbal angle ψ comes into play and it becomes a new parameter to be modelled. Indeed, we can consider different scenarios related to its behaviour, for example, given its initial value ψ_{0gimb}, we can assume a constant trend or a linear decrease or polynomial decrease etc. It would be interesting to understand the differences between a choice and another one.

3. COAST PHASE BETWEEN THRUST PHASES Once the first ignition has been completed there might be a coast phase before second stage thrust starting, let's call it t_{1-2}. This parameter is confidential for LauncherOne's case, while it is known for PegasusXL's one. What's the advantage to carry out a coast phase?

Keeping a burnt-out stage attached doesn't hurt until it's time to start the next stage. In fact, keeping it attached until shortly before it's time to ignite the next stage can improve the total launch Δv.

Another parameter that can be used is the time in which the 1st stage is released from the principal body.

For what concerns the equations, they are still the same of suborbital case.

4. 2nd THRUST PHASE

   The reasoning is similar to the 1st thrust phase with different input values. The gimbal approach is used here too.

5. FINAL COAST PHASE AND ORBIT INSERTION

   The ending phase, when 2nd burn-out occurs, is characterized by a coasting motion or ballistic flight. Once the apogee has been reached, the 2nd stage is released and the payload can insert into orbit. This last phase is not of interest to us, because the most important thing is to reach the required altitude and velocity to orbit.

3.6 Numerical methods

For what concerns numerical method, the fourth order Runge-Kutta method (RK4) is implemented to integrate our physical equations. The Runge-Kutta methods are a family of explicit or implicit iterative methods used for approximating solutions of ordinary differential equations. Let us consider the following problem:

$$
\begin{aligned}
y'(t) &= f(t, y(t)) \\
y(t_0) &= y_0
\end{aligned}
$$

where y is the unknown scalar or vector function of variable t (often representing time), which we would like to approximate; $\dot{y}$ is the changing rate of y, which is a function of t and also of y itself; y_0 represents the value of y at initial time t_0. Let's consider a evenly spaced time interval, defining Δt to be the time step size and $t_i = t_0 + i\Delta t$ with $i = 0, ..., n-1$. We define:

$$
\begin{aligned}
y_{n+1} &= y_n + \frac{1}{6}\Delta t(k_1 + 2k_2 + 2k_3 + k_4) \\
t_{n+1} &= t_n + \Delta t
\end{aligned}
$$

where y_{n+1} is the approximation of $y(t_{n+1})$ and:

$$
\begin{aligned}
k_1 &= f(t_n, y_n) \\
k_2 &= f(t_n + \frac{\Delta t}{2}, y_n + \Delta t\frac{k_1}{2}) \\
k_3 &= f(t_n + \frac{\Delta t}{2}, y_n + \Delta t\frac{k_2}{2}) \\
k_4 &= f(t_n + \Delta t, y_n + \Delta t k_3)
\end{aligned}
$$

So we can notice as the approximation y_{n+1} is determined by the value at present time y_n plus the weighted average of four increments, where each increment is the product of the size of the interval, Δt, and an estimated slope specified by function f on the right-hand side of the differential equation.

The RK4 method is a fourth-order method, meaning that the local truncation error is on the order of $\mathcal{O}(h^5)$, while the total accumulated error is on the order of $\mathcal{O}(h^4)$.

Now we want to understand the stability of the numerical method that has been used. Let's take a look at our system and, for simplicity, we consider only the release phase of the suborbital case:

$$\begin{cases} ma_x = -L\sin\alpha - D\cos\alpha \\ ma_y = -mg + L\cos\alpha - D\sin\alpha \end{cases}$$

This is a second order ODE system which can be transformed into a first order system as follows: let $\vec{z}$ be the status vector, i.e.:

$$\vec{z} = [z_1,\, z_2,\, z_3,\, z_4] = [x,\, y,\, v_x,\, v_y]$$

then we have:

$$\begin{cases} \dot{z_1} = z_3 \\ \dot{z_2} = z_4 \\ m\dot{z_3} = -L\sin\alpha - D\cos\alpha \\ m\dot{z_4} = -mg + L\cos\alpha - D\sin\alpha \end{cases}$$

and expliciting all the terms:

$$\begin{cases} \dot{z_1} = z_3 \\ \dot{z_2} = z_4 \\ \dot{z_3} = -\frac{1}{2m}S\rho(z_2)(z_3^2 + z_4^2)[C_l(t)\sin(\alpha(t)) + C_d(t)\cos(\alpha(t))] \\ \dot{z_4} = \frac{1}{2m}S\rho(z_2)(z_3^2 + z_4^2)[C_l(t)\cos(\alpha(t)) - C_d(t)\sin(\alpha(t))] - g(z_2) \end{cases}$$

From that it is easy to notice how the system is highly non-linear; therefore a non-linear stability theory is needed. Before doing that, we can streamline the system introducing some functions: let $h(z_2, z_3, z_4) = \frac{1}{2m}S\rho(z_2)(z_3^2 + z_4^2)$, $c_1(t) = [C_l(t)\sin(\alpha(t)) + C_d(t)\cos(\alpha(t))]$ and

$c_2(t) = [C_l(t)\cos(\alpha(t)) - C_d(t)\sin(\alpha(t))]$, then our system turns into:

$$\begin{cases} \dot{z}_1 = z_3 \\ \dot{z}_2 = z_4 \\ \dot{z}_3 = -h(z_2, z_3, z_4)c_1(t) \\ \dot{z}_4 = h(z_2, z_3, z_4)c_2(t) - g(z_2) \end{cases}$$

whose Jacobian matrix is:

$$J = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{2m}S\frac{d\rho}{dz_2}(z_3^2 + z_4^2)g_1(t) & -\frac{1}{m}S\rho(z_2)z_3g_1(t) & -\frac{1}{m}S\rho(z_2)z_4g_1(t) \\ 0 & \frac{1}{2m}S\frac{d\rho}{dz_2}(z_3^2 + z_4^2)g_2(t) - \frac{dg}{dz_2} & \frac{1}{m}S\rho(z_2)z_3g_2(t) & \frac{1}{m}S\rho(z_2)z_4g_2(t) \end{pmatrix}$$

Since ρ is an exponential function of the type $\rho = \rho_0 e^{\beta(z_2 - z_{20})}$, then $\frac{d\rho}{dz_2} = \beta\rho$. Moreover we can consider gravity as a constant because we are in the release phase, so we have:

$$J = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{2m}S\beta\rho(z_3^2 + z_4^2)g_1(t) & -\frac{1}{m}S\rho z_3g_1(t) & -\frac{1}{m}S\rho z_4g_1(t) \\ 0 & \frac{1}{2m}S\beta\rho(z_3^2 + z_4^2)g_2(t) & \frac{1}{m}S\rho z_3g_2(t) & \frac{1}{m}S\rho z_4g_2(t) \end{pmatrix}$$

Jacobian matrix will be useful for understanding the stability of our system.

3.6.1 Non-linear stability theory

Very often linear stability theory is inadequate when applied to non-linear or even linear variable coefficient systems. We know that for a linear constant coefficient system $y' = Ay$, negativity of the real parts of the eigenvalues of A implies that $\|y(t)\|$ decreases, but also implies that neighbouring solution curves get closer together as t increases. It turns out to be much more fruitful to seek generalizations of this second property. We are thus motivated to make the following definition[19]:

Definition 3.1 *CONTRACTIVITY*
Let $y(t)$ and $\bar{y}(t)$ be any two solutions of the system $y' = f(t, y)$ satisfying initial conditions $y(t_0) = y_0$, $\bar{y}(0) = \bar{y}_0$, $y_0 \neq \bar{y}_0$. Then if:

$$\|y(t_2) - \bar{y}(t_2)\| \leq \|y(t_2) - \bar{y}(t_2)\|$$

for all t_2, t_1 such that $t_0 \leq t_1 \leq t_2 \leq t_{end}$, the solutions of the system are said to be contractive in $[t_0, t_{end}]$.

There is also an analogous definition for numerical solutions. Here, the inevitable introduction of discretization errors in a numerical solution can be thought of as being equivalent to jumping on to a neighbouring solution curve; if we demand that the numerical solutions be contractive whenever the exact solutions are, then we are ensuring that the numerical solution cannot wander away from the exact solution. Numerical evidence shows that the problem fails to be contractive, therefore special care has to be devoted to numerical integration.

3.6.2 Comparison between numerical methods

A comparison between numerical methods has been carried out for understanding the reliability and the accuracy of the numerical method implemented in the simulator. We have tested different routines that are already present in Matlab:

- ode45: it solves nonstiff differential equations with a medium-high order;
- ode15s: it can solve problems with a mass matrix that is singular, known as differential-algebraic equations (DAEs), hence it also solves stiff equations;
- ode23: it solves nonstiff differential equations with a low order.

for simplicity, these differential equations solvers have been used to find the solution for the release phase. Here the graphs that shows the comparison between the numerical methods:

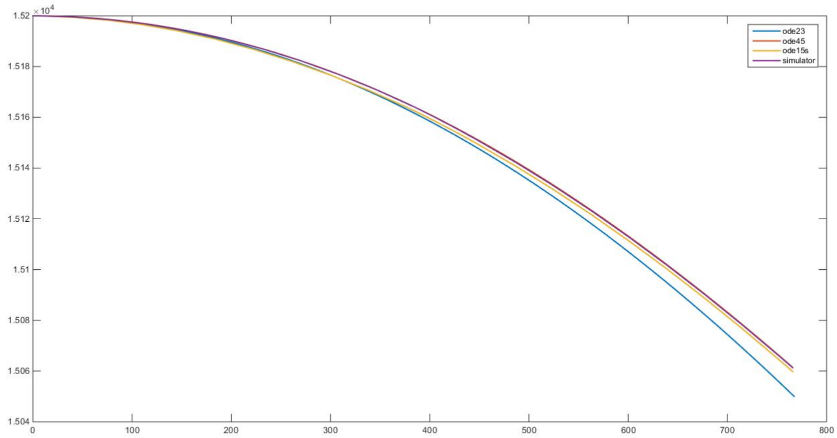


Figure 3.8: Comparison between methods

From the figure above we can clearly notice that there is a curve that departs from the others. In fact, this is the solution of ode23 that has a lower order with respect to other numerical methods. On the other hand we ascertain how our solution, called "simulator", is near enough to ode45 and ode15s solutions and this could be a confirm for our method and for the choice of integration time step.

3.7 Flight Domains

It is a generally accepted opinion that every flight occurring beyond 100/110 km above sea level must be considered out of the atmosphere, or, equivalently, that the re-entry phase of a capsule/vehicle starts somewhere between 100/110 km of altitude. However, there is not a univocal approach to determine the transition from the atmosphere to the outer space. According to a physical approach, atmosphere ends where the airflow cannot support the machine anymore and outer space begins where an object can briefly maintain an orbit. This opinion was for first brought up by Theodore von Karman, which is why that line between air space and outer space is sometimes named Karman line. As the altitude raises, in order to compensate from the diminution of the atmosphere density, the vehicle velocity should increase so that the lift can balance the vehicle's weight. At a certain point the required velocity so that lift equals weight becomes equal to that of a circular orbit where the centrifuge force equals weight: this altitude is defined as the Karman line. Calculations of that line differ, since they depend on the vehicle geometric features.
This consideration shows the intrinsic difficulty of establishing a conceptual division in a continuous medium. Such a difficulty can be circumvented by considering that the division must descend from the flying qualities of the flying vehicles. This means that different vehicles may have different transition zones between consecutive atmosphere layers[20].
There are three main zones in which the atmosphere can be divided, corresponding to different physical behaviour of the vehicles flying in it:

- The aircraft flight domain;
- the High Altitude Flight (HAF) domain;
- the Space domain.

So, the HAF layer can be simply defined as the transition zone between the layer where a vehicle behaves like an airplane and where it behaves like a satellite. Therefore, we could define the HAF as the flight occurring between two reference altitudes, a lower and an upper limit, in a range within a vehicle must show hybrid features between an airplane and a space satellite. Although reasonable, such altitudes are still arbitrary and chosen on the bases of the common sense; in other terms, the lower and the upper limit do not descend from a physical/mathematical criterion. As it will be shown later, the division cannot rely only on the solely value of altitude, but it must be based at least on a combination of altitude, flight speed and attitude.

In the aeronautical domain, control actions ruling the stability of a given equilibrium position, in terms of trajectory and attitude, or the transition between two of them, depend on the dynamic pressure and the vehicle's aerodynamic properties through the interaction with the atmosphere.

In the space domain, control actions ruling the stability of a given equilibrium position, in terms of orbital parameters and attitude, or the transition between two of them, depend only on the given propulsion system used for the active control, regardless to the satellite's altitude, velocity and geometrical properties.

The main physical quantities that must be considered are the lift L, the weight W, thrust T, the drag D and the centrifuge force F_c. For an airplane, the weight W of the airplane is fully balanced by the lift L, and the centrifuge force is negligible:

$$W = L$$

For a spacecraft the situation is opposite, since the weight W is balanced by the centrifuge force F_c, whereas the aerodynamic effect of lift is negligible:

$$W = F_c$$

It is meaningful therefore to consider a complete force environment, when dealing with vehicle with an hybrid behaviour:

$$W = L + F_c$$

where:

$$L = \frac{1}{2}\rho S V^2 C_l$$

$$F_c = m\frac{V^2}{R_e + z}$$

How can we quantify the transition between aeronautical domain and the HAF domain and between the HAF domain and the space domain?

Transition from aeronautical domain to HAF domain

It is reasonable to consider the transition from aeronautical domain to HAF domain starting when the effect of the centrifuge force F_c begins to be comparable to that of the lift L. Therefore, in this case, we might consider the weight W will be balanced at 90% by the lift, and at 10% by the centrifuge force:

$$F_c = \frac{1}{10}W$$

i.e.

$$m\frac{V^2}{R_e + z} = \frac{1}{10}mg$$

i.e.

$$V_{t_1} = \sqrt{\frac{1}{10}g(R_e + z)}$$

Since $z \ll R_e$ and $g/10 \simeq 1$, we obtain that:

$$V_{t1} \simeq \sqrt{R_e} = 2524\, m/s$$

On the other hand:

$$L = \frac{9}{10}W$$

i.e.

$$\frac{1}{2}\rho S V^2 C_l = \frac{9}{10}mg$$

i.e.

$$\rho(z) = \frac{9}{5C_l}\frac{mg}{SV^2}$$

Recalling that $\rho(z) = \rho_0 e^{-\frac{z}{H}}$, we obtain the following result:

$$z = H \cdot ln \left[\frac{5}{9} \frac{\rho_0 S V^2 C_l}{mg} \right]$$

Therefore:

- The transition from the aeronautic to the atmospheric domain depends not only on the value of altitude, but also on the value of the velocity, and that such values are linked one to the other.

- While the velocity corresponding to the transition does not depend on the vehicle aerodynamic features, the transition altitude do. This is an extremely important consideration, since it reveals that the velocity behaviour transcend the vehicle's geometry.

Transition from HAF domain to space domain

The transition from the space domain to the HAF one starts when the effect of the lift force, thanks to the increase of the dynamic pressure, starts to be comparable to that of the centrifuge force. Therefore, in a circular Earth orbit reasonably close to the Earth surface, the weight W of the orbiting vehicle will be balanced at 90% by the centrifuge force, and at 10% by the lift:

$$F_c = \frac{9}{10} W$$

i.e.

$$m \frac{V^2}{R_e + z} = \frac{9}{10} mg$$

i.e.

$$V_{t_2} = \sqrt{\frac{9}{10} g (R_e + z)}$$

Taking in account the same previous considerations, we obtain that:

$$V_{t2} \simeq 3\sqrt{R_e} = 7572\, m/s$$

On the other hand:

$$L = \frac{1}{10}W$$

i.e.

$$\frac{1}{2}\rho SV^2 C_l = \frac{1}{10}mg$$

i.e.

$$\rho(z) = \frac{1}{5C_l}\frac{mg}{SV^2}$$

Recalling that $\rho(z) = \rho_0 e^{-\frac{z}{H}}$, we obtain the following result:

$$z = H \cdot ln\left[5\frac{\rho_0 SV^2 C_l}{mg}\right]$$

Even in this case, one should note that:

- The transition from the HAF to the space domain depends not only on the value of altitude, but also on the value of the velocity, and that such values are linked one to the other.
- While the velocity corresponding to the transition does not depend on the vehicle aerodynamic features, the transition altitude do. This is an extremely important observation, since it reveals that the velocity behaviour transcend the vehicle's geometry.

Which are the limiting factors to take in account within these transition zones?

- If the vehicle executes a transition from the aeronautical to the HAF domain, the first limiting factor will be the propulsion system. Indeed in this zone the aerodynamic drag is still meaningful and the vehicle's velocity is not so high to permit an inertial motion like that of satellites.
- If the vehicle approaches the transition to the HAF domain from the space domain, the limiting factors to be taken into account are thermal and mechanical stresses. In this case, the vehicle owns a high kinetic energy and, for this reason, it is subjected to a high dynamic pressure and thermal flux, proportional to ρV^2 and ρV^3 respectively

Chapter 4

Software Simulator

4.1 GUI

To create an appropriate software simulator it has been decided to utilize Matlab GUIs. This is due to the need of an extremely user-friendly graphical interface.
Why use a GUI in MATLAB?
The main reason GUIs are used is because it makes things simple for the end-users of the program. If GUIs were not used, people would have to work from the command line interface, which can be extremely difficult and frustrating. In Matlab there is an interactive tool, called ”Guide”, for designing and building Graphical User Interfaces[21]. GUI building process involves:

- designing of the user interface and layout
- programming of the GUI and its components
- testing, debugging and finally running it

Guide can be simply opened typing ”guide” on the command window. This will open the Quickstart GUI template selection window. The user interface will usually be made up of:

- toolbars and menus;
- input control components such as
    1. push buttons, radio buttons, check boxes;
    2. pop-up menus, list boxes;
    3. sliders;
    4. edit text;
- graphical objects, like axes objects;
- text objects, like static text.

The initial stages of the GUI design will involve selecting the control components we wish to use from the palette and dragging and dropping them to a location of our choice on the layout area. Once dropped into the layout area,these components can be moved around and resized at will until a pleasant interface layout is achieved. In the following chapter all the steps to create the software simulator using Matlab GUI will be shown.

4.2 Implementation

The software simulator has been divided into four main windows:

- the main window;
- the specs window;
- the input window;
- the results window.

Main window

In the main window the user can choose the type of mission among:

- orbital;
- suborbital;
- point to point.

Then he can choose the number of stages, selecting a number between 1 and 3, the mathematical model, choosing or the rigid body model or the point like mass model, and the unit system deciding from US or SI system. Here the screen of the main window selection:

Figure 4.1: Main window

If, for example, the user want a mission similar to Spaceship2's one, i.e. a suborbital mission, then he has to choose:

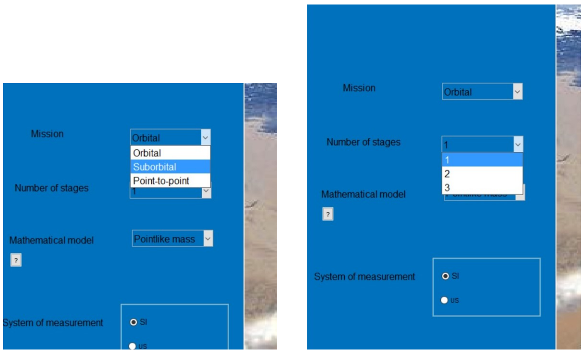


Figure 4.2: Surborbital choice

Once the user has done all the choices occurring on the main window, he can switch to the specs window, where specs means technical specifications. Here the specs window:

Figure 4.3: Specs window

This has been divided into:

- propulsion which can be chosen from solid, liquid or hybrid;

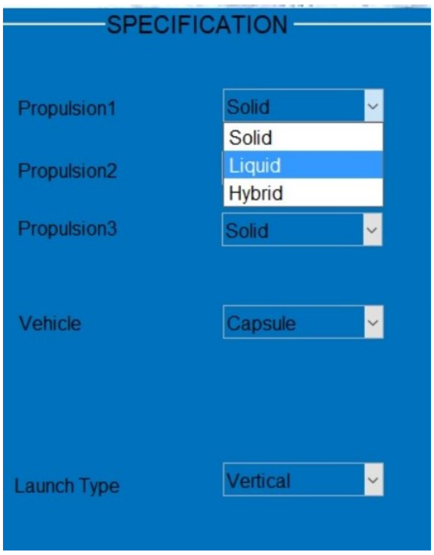
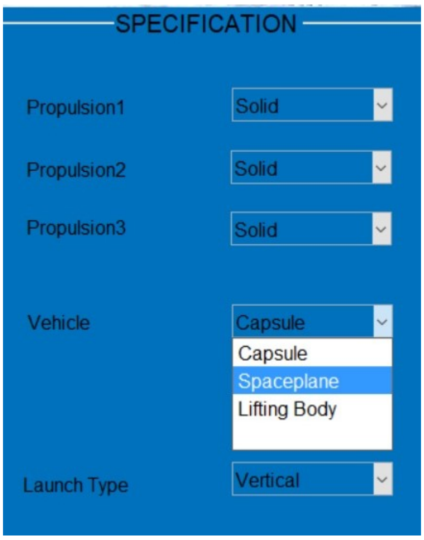


Figure 4.4: Propulsion choice

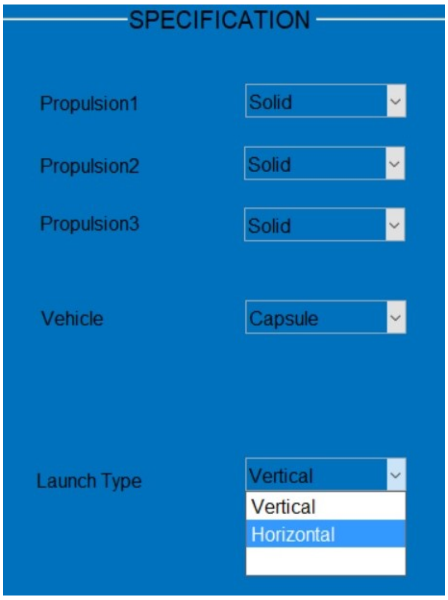
- vehicle which can be chosen from capsule, spaceplane or lifting body;

Figure 4.5: Vehicle choice



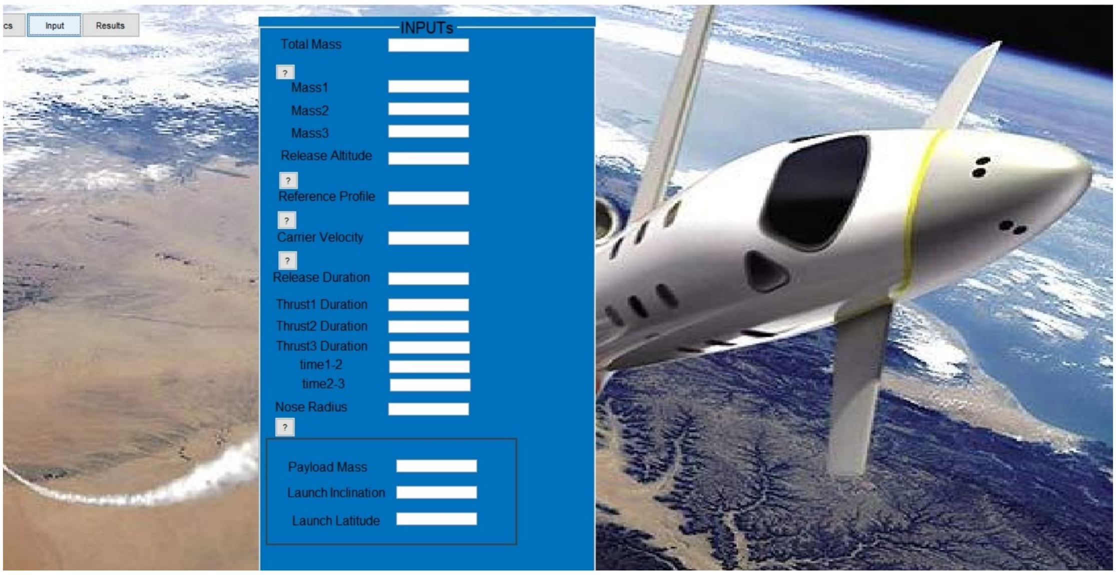
- launch type which can be chosen from vertical or horizontal.

Figure 4.6: Launch choice



The last window the user can managed is the input one where he can insert all the data required for that specific mission.

Figure 4.7: Input window



When the user enters the thrust duration, another window appears: that is thrust's input parameters window. This object is different if the chosen mission is suborbital or if it is orbital. In fact:

Once all these input parameters have been inserted, the last thing the user has it to click

Figure 4.8: Thrust parameters windows: left-suborbital right-orbital

the results button. In this way, a new window appears on the screen in which the user, with according to his needs, can choose different plots divided into:

- trajectory
- ground track
- altitude
- velocity
- horizontal velocity
- vertical velocity
- load factor
- dynamic pressure
- thermal flux

Here the figure of the results window:

In the next section we will see the results related to suborbital and orbital cases respectively.

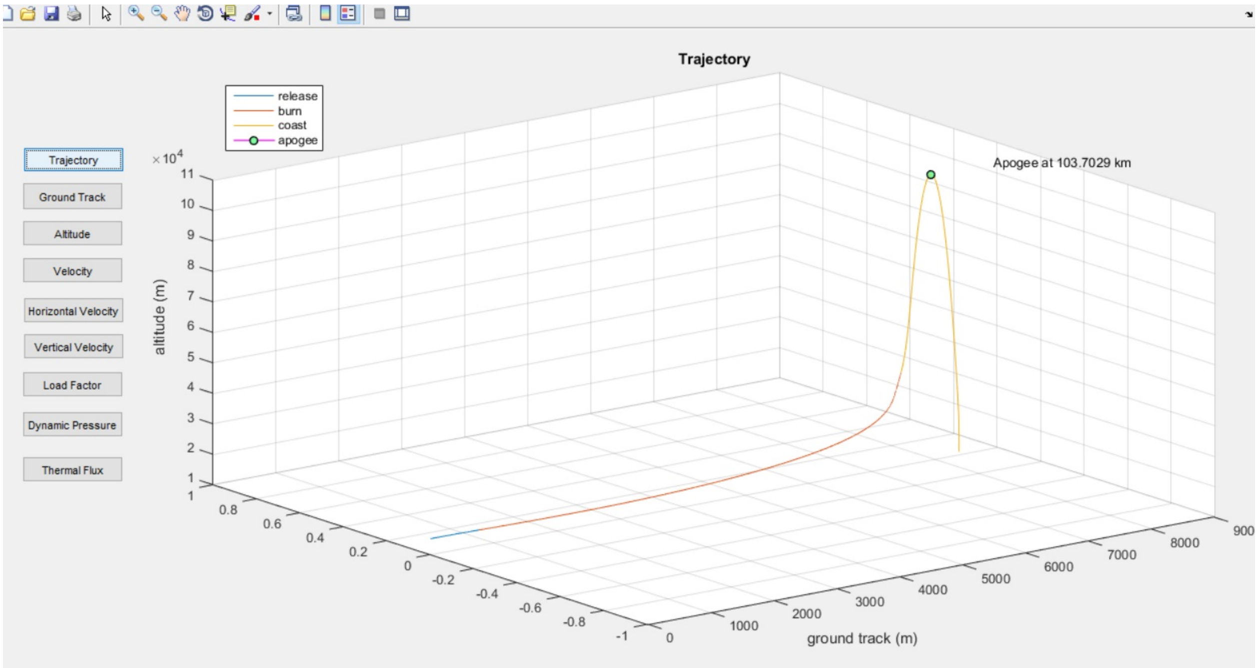


Figure 4.9: Results window

4.3 Results and the study case of Italian spaceport of Grottaglie

4.3.1 Suborbital case

For the suborbital case the choices are the following:

- mission → suborbital
- number of stages → 1
- propulsion → hybrid
- vehicle → spaceplane
- launch → horizontal

while the input parameters are:

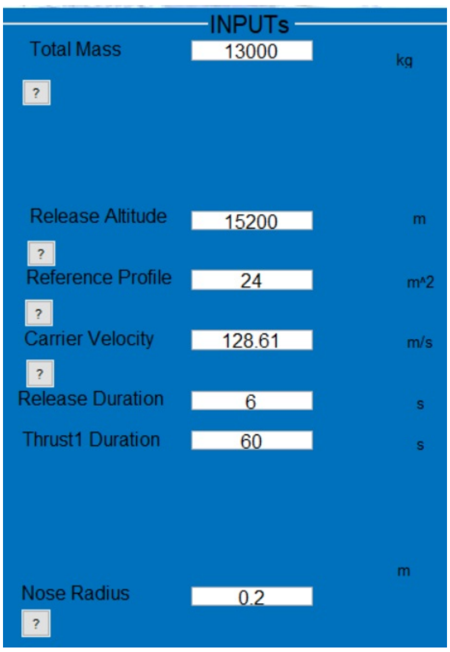


Figure 4.10: Input parameters for suborbital mission

Once these inputs have been entered, the user will click on the results button for taking a look at the plots. In the next page some results are shown.

TRAJECTORY

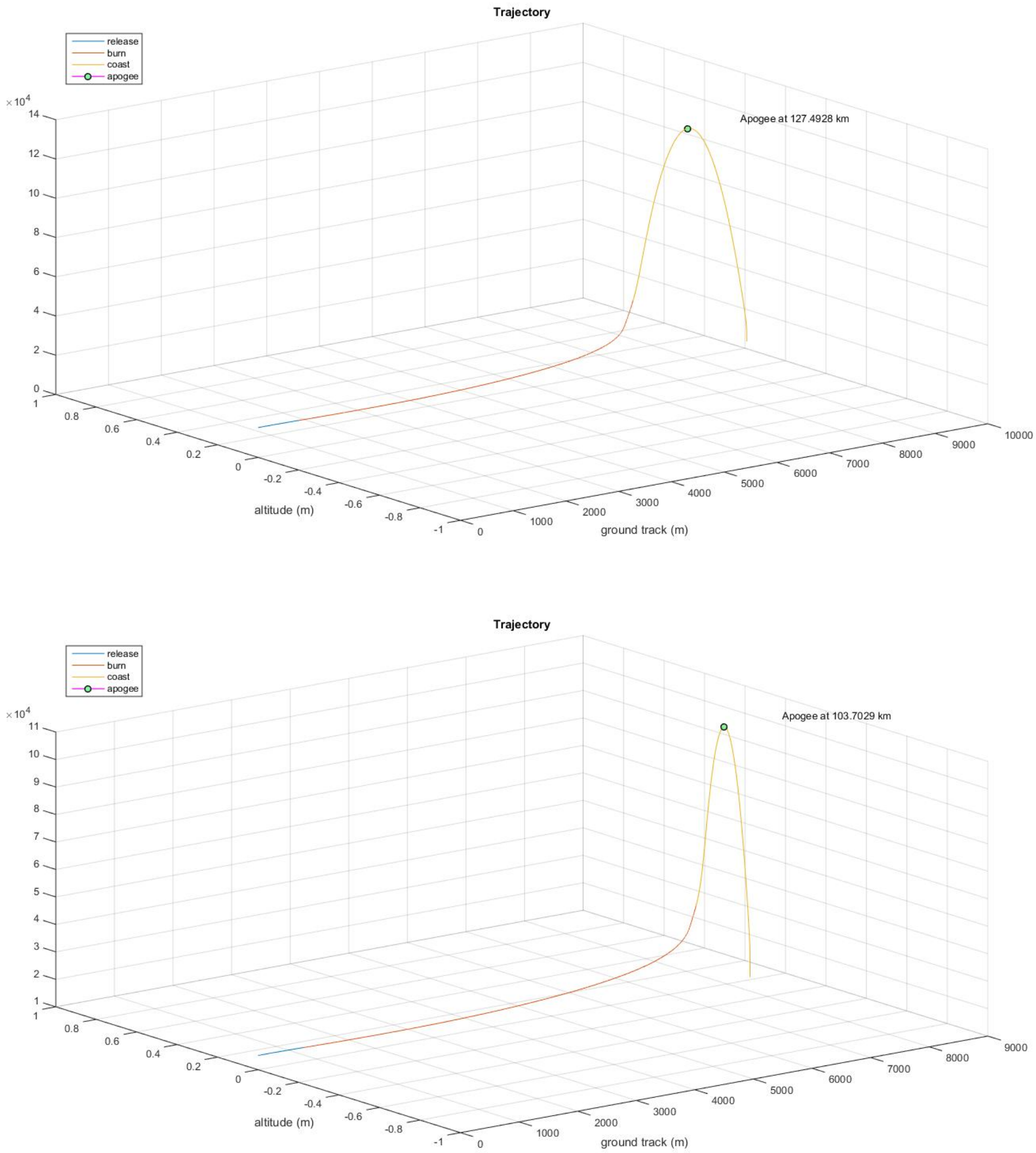


Figure 4.11: Top-Scientific trajectory Bottom-Touristic trajectory

We notice that, as we might expect, the scientific mission apogee is higher than the touristic one and this is due to the higher percentage of maximum thrust. Moreover, for what concerns touristic mission, the apogee is reached at about 104 km. This is in accordance with Virgin Galactic's goals on the basis of which the presumed apogee is between 100 and 110 km.

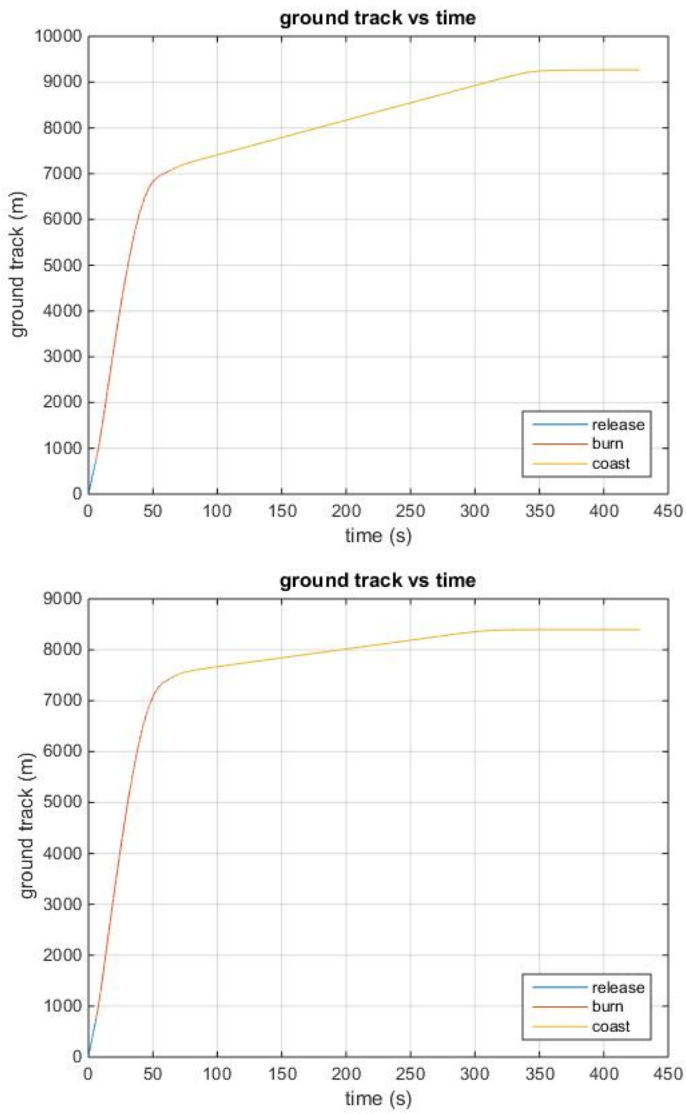


Figure 4.12: Top-Scientific ground track Bottom-Touristic ground track

GROUND TRACK

Information about ground tracks are useful for the planning of the ground segment that consists of all infrastructures (like ground stations, ground networks, terminals, mission control centers, operations centers, facilities etc.) and applications that allow the monitoring of the mission. For our purpose the ground track is fundamental to define a specific zone in the air space and the ground space during all the mission as we will see for Grottaglie's spaceport.

VELOCITY

The graph highlights the great difference in the maximum reached Mach number, that equals 3.99 for the scientific profile (and 3.38 for the touristic profile). Moreover, it is clearly evident how the scientific profile final re-entry part is again superposed to the touristic one. This is caused by the combination of the higher maximum mechanical energy combination with the higher energy dissipation (resulting in higher g-values). A perfect equalization between the two effects makes possible the superposition to a lower-energy trajectory, represented by the touristic profile. The re-entry sonic boom is generated at 24930 m, circa 400 m below the sonic boom altitude of the touristic profile, in perfect compliance with the Environmental Assessment requirement.

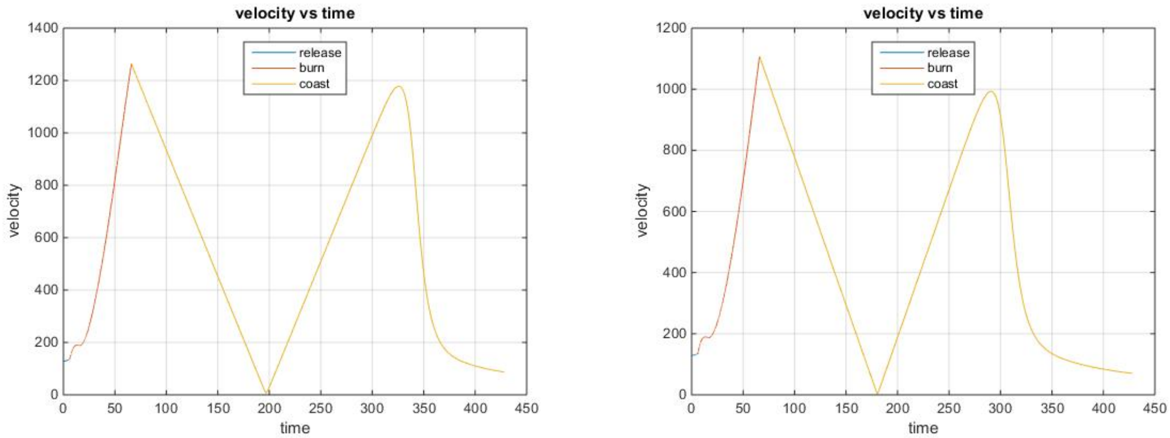


Figure 4.13: Left-Scientific velocity Right-Touristic velocity

LOAD FACTOR

The load factor is a crucial parameter to evaluate the quality of the obtained mission profile, since the SpaceShipTwo is mainly aimed at allowing people (passengers or scientists with experiments) to experience milli- or micro-gravity. The main mission requirement related to the minimum acceleration values is the milli-gravity phase duration.

Moreover, the commercial aim of the Virgin Galactic flights imposes precise requirements on the maximum g-number that passengers can experience during the spaceflight opportunity. The

usual requirements on acrobatic or orbital flights suggest to reduce the maximum g-number to 3 (continuous) and 4 (peak), in order to ensure a complete survivability to all the passengers. However, if higher g-force values are needed, preliminary, gradual tests can be performed with the candidate passengers and astronauts by means of Human Training Centrifuges (HTCs), test facilities aimed at testing the effect of high g-force values on astronauts.
Virgin Galactic includes a HTC test among the basic training sessions provided to the space tourists, in order to prove their capability to tolerate higher g-force values. In particular, preliminary reference information provided by ALTEC during the development of this work indicates that the maximum SpaceShipTwo profile g-numbers are 4 for the burn phase and 6 (peak, with 4 continuous) for the re-entry phase. The possibility to incline the spaceplane seats will reduce the effects of such g-force values on the passengers' bodies, by allowing to orient the acceleration in front-to-back direction during both the burn and the re-entry phases.

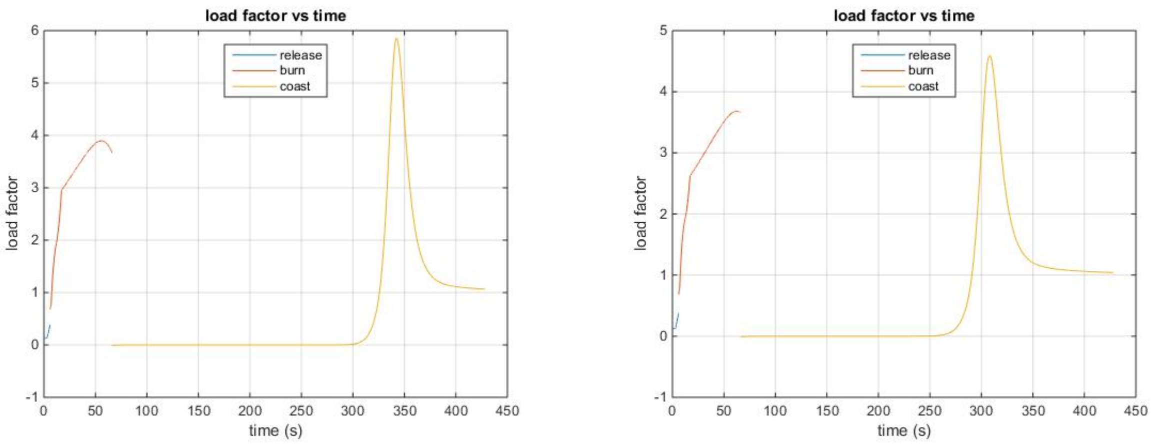


Figure 4.14: Left-Scientific load factor Right-Touristic load factor

As we notice in the figures, in the scientific mission the higher throttle ratio (percentage of max thrust), imposed as control parameter, forces the g-force values to increase, due to the higher allowed thrust during the burn phase, and due to the dissipation of a greater mechanical energy during the re-entry. However, the higher apogee altitude will permit to remain in milli-gravity for a greater time interval (171s vs 237s).

DYNAMIC PRESSURE and THERMAL FLUX

Information about dynamic pressure and thermal flux are necessary for the aircraft design, including the thermal protection system. It is important to understand when and at which altitudes the maximum peaks occur to comprehend, for example, which materials to be used to secure the passengers.

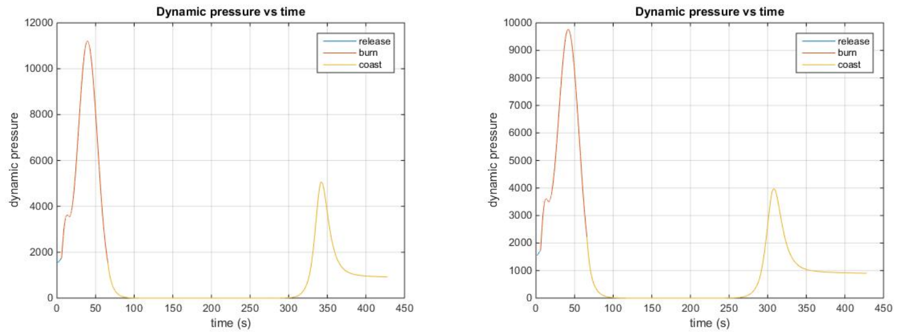


Figure 4.15: Left-Scientific pressure Right-Touristic pressure

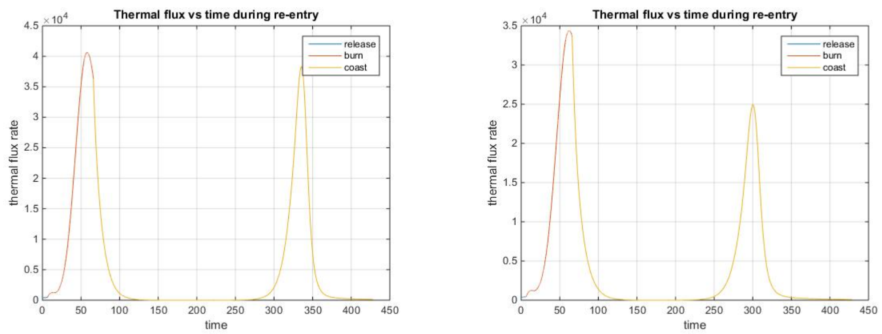


Figure 4.16: Left-Scientific thermal flux Right-Touristic thermal flux

4.3.2 Abort case and horizontal thrust case

In this section we will see two interesting cases related to the suborbital mission. This kind of studies have been made to support evaluations related to aerospace segregation in case of contingency scenarios.

The first one has been called abort case, i.e. when, during the burn phase, the aircraft engine ceases to work for some reason. We have simulated three different cases according to when the abort occurs:

- $t_{abort} = 10$
- $t_{abort} = 20$
- $t_{abort} = 40$

This study case is very useful for defining the ground segment and for securing it since, when an abort occurs, the propellant mass is not exhausted completely. From the figures in the next pages we can notice that only in the case of abort at 40 s the spaceplane exceeds its glider altitude while in the other cases it can behave immediately like a glider.
On the other hand the second case represents the situation where for some reason during the burn phase the vehicle keeps its AoA constant, equal to the final AoA of the release phase. Hence, in this case the spaceplane walks away almost horizontally until the propellant has exhausted or until the aircraft hits the ground. Even here we have tested two different cases on the basis of thrust duration: 60 and 80 respectively.

4.3.3 Orbital case

For the orbital case the choices are the following:

- mission → orbital
- number of stages → 2
- propulsion → liquid or solid
- vehicle → capsule
- launch → horizontal

while the input parameters have been got from PegasusXL data sheet:

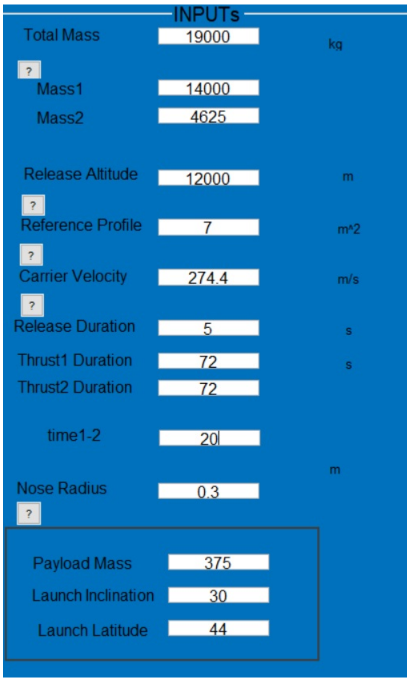


Figure 4.17: Input parameters for orbital case

Even for the orbital case the user can obtain all the graphs that are present in the results window.

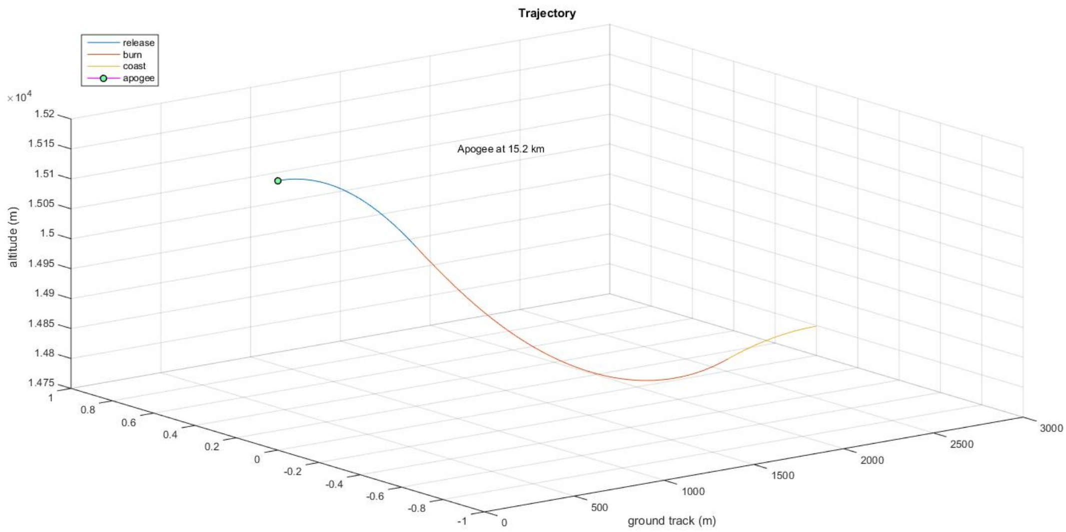


Figure 4.18: Abort at 10s

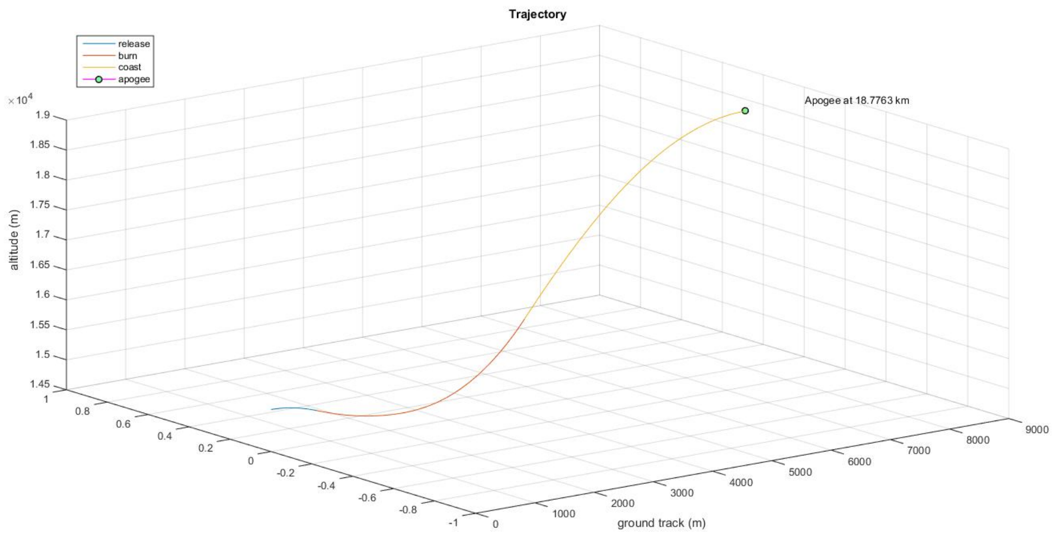


Figure 4.19: Abort at 20s

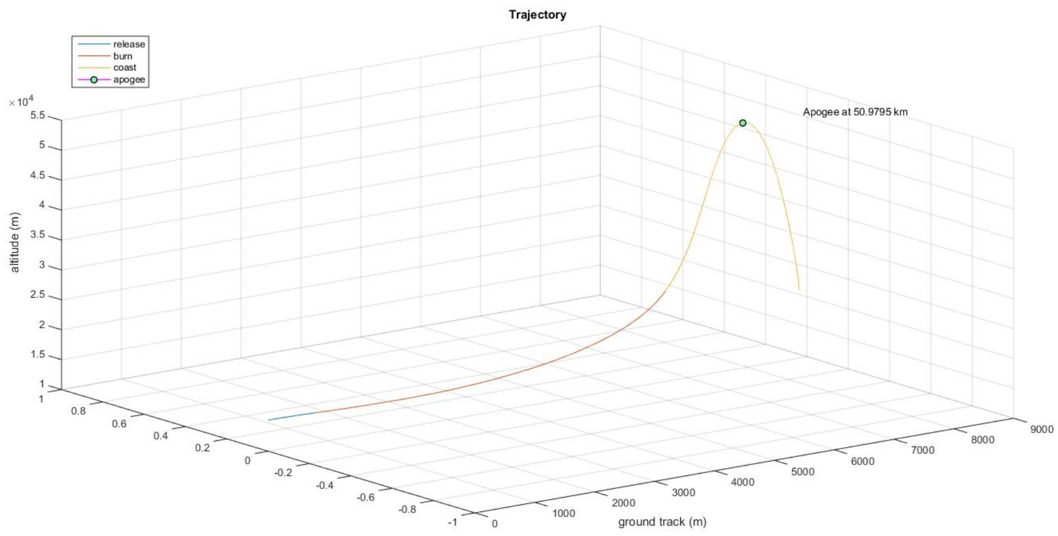


Figure 4.20: Abort at 40s

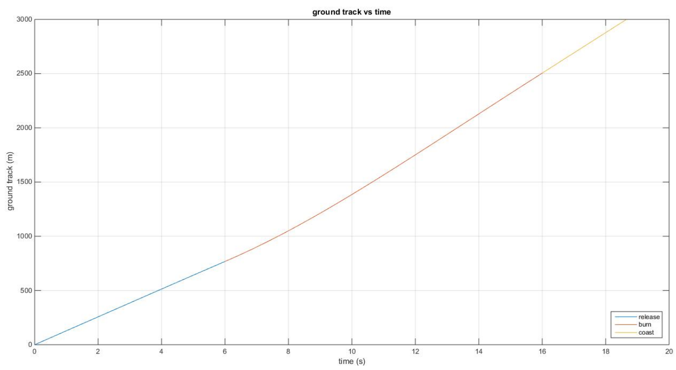


Figure 4.21: Ground track with abort at 10s

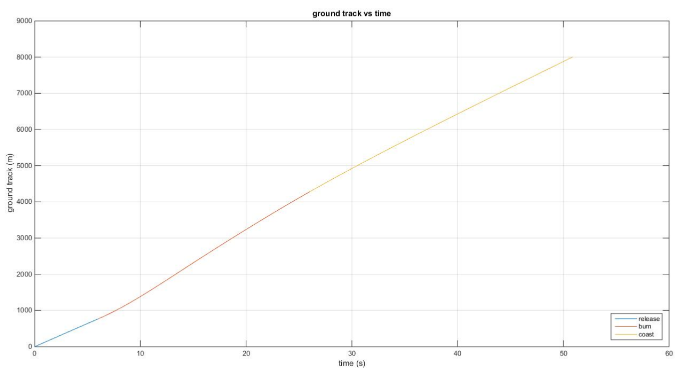


Figure 4.22: Ground track with abort at 20s

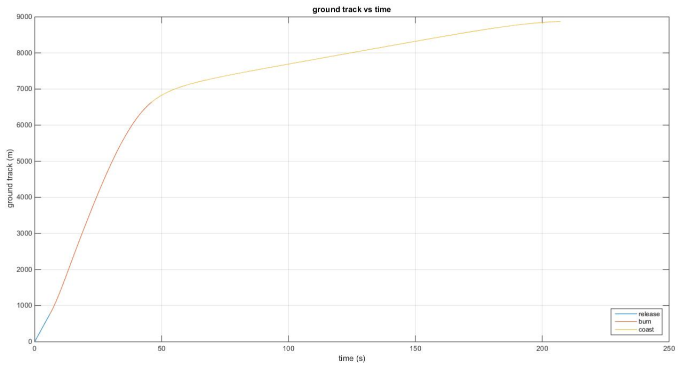


Figure 4.23: Ground track with abort at 40s

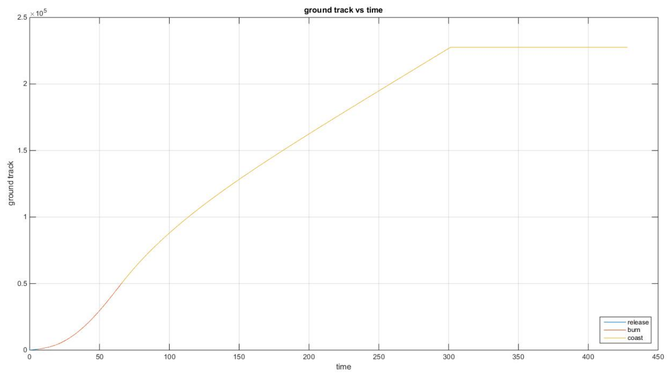


Figure 4.24: Ground track, thrust duration=60s

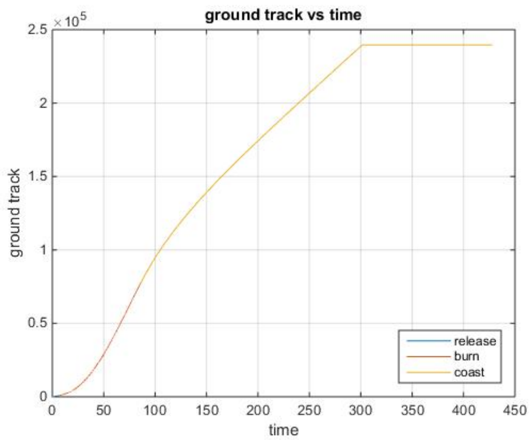
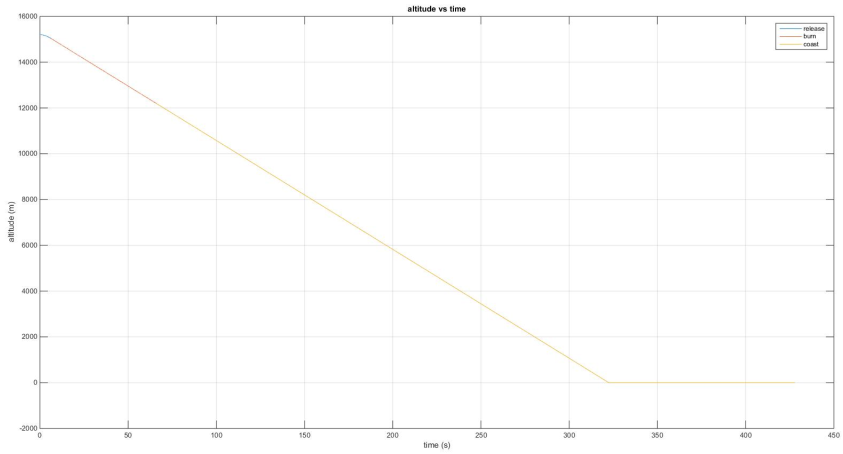


Figure 4.25: Ground track, thrust duration=80s



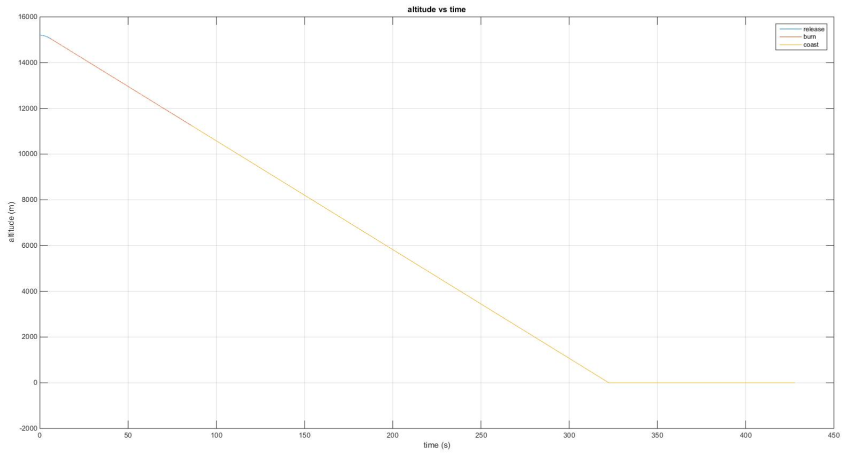
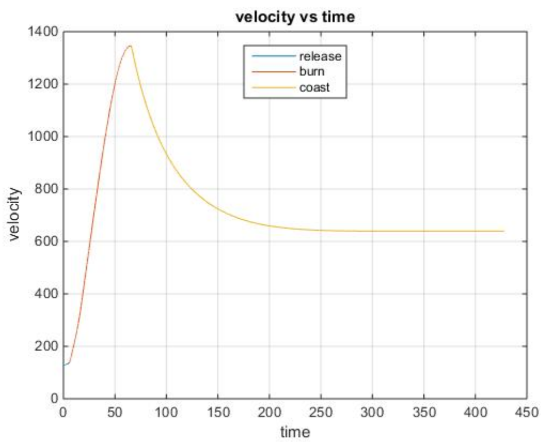


Figure 4.26: Altitude, Top: thrust duration=60s - Bottom: thrust duration=80s



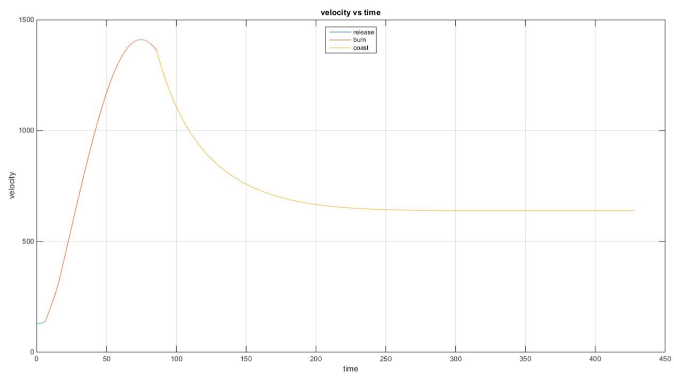


Figure 4.27: Velocity, left: thrust duration=60s - right: thrust duration=80s

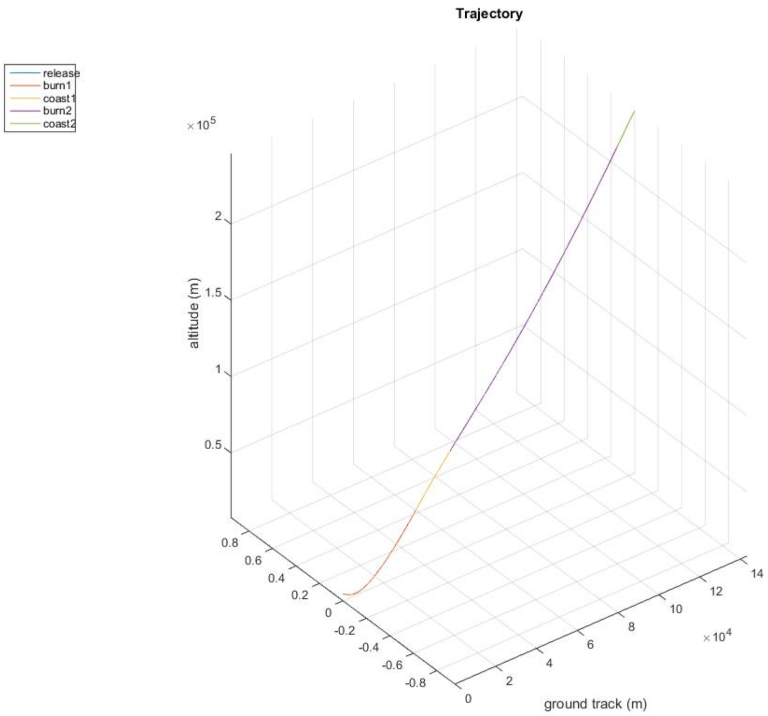


Figure 4.28: Trajectory for orbital case

Chapter 5

Conclusions and future works

5.1 Conclusions

In this study we have achieved a suitable mathematical model for air launch.

Based on Virgin's test cases, we have found some differences in the mathematical approach between suborbital and orbital missions. Some assumptions have been made to simplify the problem and they have been tested obtaining the mission goals requested by the companies.

Moreover this study has shown as the simulator implemented with Matlab is really flexible for running different missions: the user can choose whatever kind of mission he wants, he can insert his available input data and he can choose whatever result or graph he desires. The trajectory simulator has given very useful results and graphs for aerospace segregation, for operational aspects and different mission scenarios. Finally, an important application has been found in Grottaglie's spaceport, designated as national spaceport for suborbital needs. The most of considerations is surely theoretical and cannot be fully validated until the number of flights and tests grows: the effort for the ground segment development will be greater at the beginning but it will get easier over time by acquiring a knowledge base and a fixed commercial activity.

5.2 Future works

As a prosecution of this work of great interest may be more detailed studies and developments for what concerns the mathematical model. Many assumptions could be refined, finding a more accurate methodology. For example, aerodynamics coefficients could be upgraded, the model could be extended also to lateral-directional plane and many simplifications could be relaxed. Other study and test cases could be proven using the trajectory simulator for validating them and another possibility may be extending this kind of work to a point-to-point mission.

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