Analysis of flow channel insert deformations influence on the transport phenomena in liquid metal channels under magnetic fields

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Relatori:
Laura Savoldi
Daniel Suarez i Cambra
alla mia famiglia che c'è sempre stata,
alla mia famiglia che non c'è più …
I was lucky enough to conduct this thesis by interfacing with a foreign university: the UPC. Here I had the opportunity to meet Daniel Suarez, my tutor, who personally followed me, supported, taught to use OpenFOAM but above all he supported me in the most difficult moments of stress, especially towards the end, always having a constructive comment on my job and never destructive. It gave me the strength not to stop and in the end I obtained satisfactory results.

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Abstract

The dual coolant lithium lead (DCLL) concept is a candidate to be an effective breeding blanket (BB) for nuclear fusion technologies. One critical point of this concept is the magneto-hydrodynamic (MHD) effects involving Lorentz damping force. Flow Channel Insert (FCI) seems to be the best solution to electrically de-coupling the liquid PbLi from the EUROFER walls. The impact of the FCI on the velocity profile is analyzed here. First the velocity for a channel without flow channel insert is computed, then a parametric study (varying Re or Ha, keeping the other constant) is conducted. MHD effects under fusion relevant condition ($Ha = 7570$ and $Re = 22700$) are investigated and a further temperature calculation under the assumption of non-buoyant fully developed channel, is made. At the end the critical zone for FCI deformations is detected and a suitable geometries is built up. The effect of the deformation and possible rupture of the flow channel insert on the velocity profile, and the corresponding variation of the pressure drop, are then investigated. In presence of deformation, the relative pressure drop is lower, presenting the D-wave in the velocity profile. If the alumina should be subjected to too much stress, it could break and open a passage to the current density leading to an increase in pressure drop term.
TABLE OF CONTENTS

List of Tables ix
List of Figures xi

1 Introduction 1
  1.1 The Breeding Blanket ........................................ 1
  1.2 The Dual Coolant Lithium Lead ............................... 3
  1.3 The Flow Channel Insert .................................... 4
  1.4 Scope of the thesis ......................................... 6

2 Magnetohydrodynamics 9
  2.1 Mathematics: governing equations and models .......... 9
    2.1.1 Hypotheses ............................................... 10
    2.1.2 Pure MHD .................................................. 11
    2.1.3 Low $R_m$ approximation ................................ 12
    2.1.4 $\phi$-formulation ...................................... 12
  2.2 Dimensionless numbers ...................................... 13
    2.2.1 Fusion Relevant Condition .............................. 15
  2.3 Physics: MHD effects ........................................ 15
  2.4 Parametric study ............................................. 20

3 The implementation 23
  3.1 The algorithm ............................................... 23
  3.2 Time step ..................................................... 24
  3.3 Density current conservation ................................ 25
  3.4 The source term .............................................. 26
  3.5 Schemes ........................................................ 26
  3.6 Validation ........................................................ 26

4 Geometry and physical properties 29
  4.1 Physical properties .......................................... 30
# TABLE OF CONTENTS

5 Results

5.1 FCI impact ................................................................. 32
5.2 Heat analysis ............................................................ 37
5.3 FCI deformation ......................................................... 38
  5.3.1 Strategy .............................................................. 39
  5.3.2 Geometry and expectations ....................................... 40
  5.3.3 Results .............................................................. 41

6 Conclusions ................................................................. 47

6.1 Improvements ............................................................ 48

A Mesh construction ......................................................... 49
  A.0.1 MHD grading ........................................................ 50
  A.0.2 Thermo MHD grading .............................................. 50
  A.0.3 Grading in flow rate direction .................................... 51

B Numerical Accuracy ....................................................... 55

C Thermal coupling: φT-formulation ...................................... 61

D FCI settings for sandwich like .......................................... 63

Bibliography ................................................................. 69
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Relevant Dimensionless numbers for a front channel of the breeding zone present in the equatorial module of the OB segment</td>
<td>15</td>
</tr>
<tr>
<td>2.2 Summary table for the parametric study, $dP/dx$ in [Pa/m]. $C_w = 0.25$</td>
<td>21</td>
</tr>
<tr>
<td>3.1 $\Delta t$ with respect to different $Ha$.</td>
<td>25</td>
</tr>
<tr>
<td>3.2 Results of calculation [Pa/m]</td>
<td>27</td>
</tr>
<tr>
<td>3.3 FCI results comparison with Urgorri et al. [25] $Ha=7570$, $Re=22700$</td>
<td>28</td>
</tr>
<tr>
<td>4.1 Input geometry parameters</td>
<td>30</td>
</tr>
<tr>
<td>4.2 Physical properties of the materials</td>
<td>30</td>
</tr>
<tr>
<td>5.1 Resume table of pressure drop [Pa/m]</td>
<td>37</td>
</tr>
<tr>
<td>5.2 Input parameter for heat analysis</td>
<td>38</td>
</tr>
<tr>
<td>B.1 Grid nodes</td>
<td>55</td>
</tr>
<tr>
<td>B.2 Results of GCI. $Ha=2500$, $Re=355$</td>
<td>56</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Sketch of tokamak: overall view</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Representation of BB sector with its segment and module. In the segment overview (yellow) is clear the modular nature of BB. (taken from Boccaccini el al. [2])</td>
<td>2</td>
</tr>
<tr>
<td>1.3 DCLL BB segment overview (a) and representation of OB and IB (b) (taken from Urgorri et al. [25])</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Final FCI fabricated in CIEMAT (taken from Fernandez [9]) (a) and sketch of sandwich-like FCI (b) with different materials clearly distinguished</td>
<td>5</td>
</tr>
<tr>
<td>1.5 Graphical representation of the steps made to conduct this master thesis work</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Channel’s sketch</td>
<td>16</td>
</tr>
<tr>
<td>2.2 Vectorial illustration of current density, Lorentz force and velocity in the channel</td>
<td>16</td>
</tr>
<tr>
<td>2.3 Layer’s thickness with respect to Ha number($a = 80.8mm$)</td>
<td>17</td>
</tr>
<tr>
<td>2.4 Velocity field inside the channel Ha=3000, Re=355, $C_w = 0$</td>
<td>18</td>
</tr>
<tr>
<td>2.5 Velocity profile along transverse direction Ha=3000, Re=355, $C_w = 0$</td>
<td>18</td>
</tr>
<tr>
<td>2.6 Velocity field inside the channel Ha=3000, Re=355, $C_w = 0.25$</td>
<td>19</td>
</tr>
<tr>
<td>2.7 Velocity profile along transverse direction Ha=3000, Re=355, $C_w = 0.25$</td>
<td>19</td>
</tr>
<tr>
<td>2.8 Pressure drop comparison. Shercliff vs Hunt</td>
<td>20</td>
</tr>
<tr>
<td>2.9 $dP/dx$ to three different Re numbers</td>
<td>21</td>
</tr>
<tr>
<td>2.10 $dP/dx$ with respect to three different Ha numbers</td>
<td>21</td>
</tr>
<tr>
<td>3.1 $\phi-PISO$ algorithm flow chart</td>
<td>24</td>
</tr>
<tr>
<td>3.2 Validation for three different Ha numbers, Re=355</td>
<td>28</td>
</tr>
<tr>
<td>3.3 Validation for three different Ha numbers, Re=750</td>
<td>28</td>
</tr>
<tr>
<td>4.1 Geometry description of the channel studied in this thesis</td>
<td>30</td>
</tr>
<tr>
<td>5.1 $Ha = 7570, Re = 22700, C_w = 0$</td>
<td>32</td>
</tr>
<tr>
<td>5.2 Conducting wall results under fusion relevant conditions:$C_{FW} = 0.25$, $Ha = 7570, Re = 22700$. Overall view (a) and near-wall zoom (b)</td>
<td>33</td>
</tr>
<tr>
<td>5.3 Comparison between $C_w = 0$ and $C_w = 0.25$. Overall view (a) and near First Wall zoom (b).</td>
<td>33</td>
</tr>
</tbody>
</table>
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4</td>
<td>Mesh used for the present calculations. ( Ha = 7570 )</td>
<td>34</td>
</tr>
<tr>
<td>5.5</td>
<td>Graph of velocity (a) and overall view of the channel (b)</td>
<td>35</td>
</tr>
<tr>
<td>5.6</td>
<td>Comparison between three cases. Total duct (a) and near wall zoom (b)</td>
<td>36</td>
</tr>
<tr>
<td>5.7</td>
<td>Pressure drop comparison, ( Ha = 7570, Re = 22700 )</td>
<td>36</td>
</tr>
<tr>
<td>5.8</td>
<td>Sketch of channel's section underlining the most critical part for the deformation of the FCI</td>
<td>39</td>
</tr>
<tr>
<td>5.9</td>
<td>Temperature profile on outlet cross-section (FW on the left)</td>
<td>39</td>
</tr>
<tr>
<td>5.10</td>
<td>Deformation overview</td>
<td>41</td>
</tr>
<tr>
<td>5.11</td>
<td>( D - Wave ): ( Ha = 3000, Re = 355 )</td>
<td>42</td>
</tr>
<tr>
<td>5.12</td>
<td>Pressure drop comparison, ( Ha = 3000 Re = 355 )</td>
<td>43</td>
</tr>
<tr>
<td>5.13</td>
<td>( V - shape ). ( Ha = 300, Re = 35 )</td>
<td>43</td>
</tr>
<tr>
<td>5.14</td>
<td>Streamline path and velocity field over the section.</td>
<td>44</td>
</tr>
<tr>
<td>5.15</td>
<td>Alumina crack visualization. The entity of the crack was chosen in order to guarantee at least three in the crack itself. ( Ha = 300 )</td>
<td>44</td>
</tr>
<tr>
<td>5.16</td>
<td>Pressure drop comparison between normal FCI, deformed and with rupture of Alumina. ( Ha = 300, Re = 32 )</td>
<td>45</td>
</tr>
<tr>
<td>A.1</td>
<td>Channel cross section. Frontal view with Wall (red), GAP (blue), FCI (green) and bulk (grey).</td>
<td>49</td>
</tr>
<tr>
<td>A.2</td>
<td>Mesh overview, pure MHD, ( Ha = 2500 )</td>
<td>51</td>
</tr>
<tr>
<td>A.3</td>
<td>Mesh zoom, blockMesh issue</td>
<td>51</td>
</tr>
<tr>
<td>A.4</td>
<td>Mesh overview, thermo MHD, ( Ha = 300 )</td>
<td>52</td>
</tr>
<tr>
<td>A.5</td>
<td>( x-y ) plane. Three cells in ( x )-direction needed</td>
<td>53</td>
</tr>
<tr>
<td>B.1</td>
<td>Coarse mesh. Overall view (a) and corner zoom (b)</td>
<td>57</td>
</tr>
<tr>
<td>B.2</td>
<td>Intermediate mesh. Overall view (a) and corner zoom (b)</td>
<td>58</td>
</tr>
<tr>
<td>B.3</td>
<td>Fine mesh. Overall view (a) and corner zoom (b)</td>
<td>59</td>
</tr>
<tr>
<td>D.1</td>
<td>Electrical conductivity ([S/m]) for different material.</td>
<td>64</td>
</tr>
<tr>
<td>D.2</td>
<td>Streamline current</td>
<td>64</td>
</tr>
<tr>
<td>D.3</td>
<td>Streamline current-particular</td>
<td>65</td>
</tr>
<tr>
<td>D.4</td>
<td>Specific heat for three different material of FCI</td>
<td>65</td>
</tr>
<tr>
<td>D.5</td>
<td>density for three different material of FCI</td>
<td>66</td>
</tr>
<tr>
<td>D.6</td>
<td>thermal conductivity for three different material of FCI</td>
<td>66</td>
</tr>
<tr>
<td>D.7</td>
<td>Thermal diffusivity for three different material of FCI</td>
<td>67</td>
</tr>
</tbody>
</table>
This thesis work was done at the Polytechnic University of Catalunya, in Barcelona, under the direct tutoring of Daniel Suarez and Elisabeth mas de les Valls.

The EU-DEMOnstration power plant (DEMO) is an EUROfusion project [8], whose scope is to demonstrate the possibility of producing electricity through fusion reaction. The confinement of the plasma and the controlled nuclear fusion reaction should be reached using a tokamak system.

When heated to fusion temperatures, the electrons in atoms disassociate, resulting in a fluid of nuclei and electrons known as a plasma. Unlike electrically neutral atoms, a plasma is electrically conductive, and can, therefore, be manipulated by electrical or magnetic fields [3].

Tokamak is a machine that, taking advantage of this, is able to confine the plasma by mean of magnetic fields.

In this chapter one of the main component of future DEMO will be discussed\(^1\) : the Breeding Blanket.

### 1.1 The Breeding Blanket

The breeding blanket (BB), in its life, is called to accomplish a triple task:

1. cooling: this task is carried out by the coolant (which can change according to the design), through which the conversion from fusion energy to electrical energy is made possible;

2. breeding: the key task of the breeding blanket is to ensure the self-sustainability of the fusion reaction. In other words, the tritium necessary for the fusion reaction is produced

\(^1\)To learn more about the tokamak and other components, all the informations can be found on the EUROfusion/DEMO website. ([8])
here, in the BB, and subsequently extracted through an external system and re-introduced into the plasma in the form of small pellets;

3. shielding: the BB protects the superconducting magnets and the whole external environment from the radiation coming from the plasma, to guarantee a perfect functioning of the first ones and to prevent possible releases of external contamination.

The BB consists of a series of sectors distributed toroidally around the central solenoid (see 1.1). In the EU-DEMO design currently under investigation, each of these toroidal sectors consists of 3 outboard segments (OB) and 2 inboard segments (IB).
1.2 THE DUAL COOLANT LITHIUM LEAD

Four BB concepts are being designed for the EU-DEMO over the years, and they are:

1. Helium Cooled Pebble Bed (HCPB): It uses solid Li-ceramics as breeder and beryllium as neutron multiplier material;

2. Helium Cooled Lithium Lead (HCLL): It uses helium as coolant and the eutectic PbLi as breeder material and multiplier;

3. Water Cooled Lithium Lead (WCLL): It uses water as coolant and the eutectic PbLi as breeder material;

4. Dual Coolant Lithium Lead (DCLL).

1.2 The Dual Coolant Lithium Lead

This concept is the one of main interest for this thesis, and will therefore be dealt with separately in this section.
The Dual Coolant Lithium Lead (DCLL) is an advanced BB concept that uses eutetic PbLi as breeder and it uses two different coolants: the PbLi itself and the He. The first one is the main coolant and it flows into the module channels at about $\approx 1 \text{ cm/s}$, and the second one has to cool the EUROFER supporting structure.

Like the other BBs, the sectors are placed along the toroidal plan and each sector is composed of three OBs and two IBs. Each segment consists of 8 modules which are connected to the same back supporting structure (BSS). This BSS acts as a support but also as a manifold: this is why there are two channels of PbLi (BSS cold and hot in figure 1.3 (b)) and four of He in it (see figure 1.3 (b)). By its nature the PbLi conducts electricity and, therefore, once invested by the magnetic field coming from the external magnets, will give rise to the typical magnetohydrodynamic (see next chapter 2) effects developing a Lorentz force that will damp the flow of the same coolant and will cause a huge pressure drop.

As we will see in the last chapter (5), the magnitude of these induced forces is considerable and the associated pressure drop very high. This because if the surrounding walls are not isolated, the density current became higher implying a greater Lorentz force and, thus, a greater pressure drop. The only way to solve this problem is to isolate, in some way, the channel walls of the DCLL modules.
1.3 The Flow Channel Insert

The Flow Channel Insert (FCI) is the best solution for the reduction of pressure drop resulting from the induction of Lorentz force. In its latest design, the FCI is of the sandwich-like type: it is composed of three layers, which inside corresponds to the actual insulator.
Norajitra et al. (2017, [17]) studied different insulator composition, and from the experiments came out that only ceramic tiles (Alumina or SiC) in sandwich form between two thin steel layers could be considered candidates for insulating the flowing PbLi. So, following this suggestion, the research is moving towards this type of insulation. The European design deals with FCI alumina based, the Americans with SiC. In this master thesis, it was decided to study the first one. In CIEMAT (Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas, Madrid) prototypes of FCI are being manufactured, we can see one in this picture (figure 1.4):

![FCI Prototype](image)

Figure 1.4: Final FCI fabricated in CIEMAT (taken from Fernandez [9]) (a) and sketch of sandwich-like FCI (b) with different materials clearly distinguished

A detailed investigation was made by Urgorri et al.[25] where three configuration of FCI were compared:

1. Naked FCI: only 5mm of naked alumina where considered;
2. Thin sandwich FCI: sandwich like FCI 0.5mm – 1mm – 0.5mm of steel-alumina-steel;
3. Thick sandwich FCI: sandwich like FCI 1mm – 5mm – 1mm of steel-alumina-steel.

From MHD results, the naked FCI mitigates the MHD effect more than the others, considerably reducing the pressure drop (being present only a ceramic insulator). Regarding the other two configurations, the thick FCI has a greater pressure drop of a factor of about 2.

The FCI has a double function: if on the one hand it acts as an electrical insulator between the liquid metal and the steel wall, on the other it limits the flow of heat from the PbLi to the He, which has the task of cooling the structures in EUROFER. In addition, PbLi is the primary coolant and is responsible for converting thermal energy into electrical energy, transporting heat. A loss to Helium channels would result in a loss of efficiency for the entire BB.

A thermal analysis (Urgorri et al. [25]) revealed that the naked FCI, is not suitable for thermally isolating the PbLi, leaving too much heat from the bulk flow to the Helium channels.
CHAPTER 1. INTRODUCTION

For this reason we rejected the naked FCI configuration. In CIEMAT, and throughout Europe, there is no real choice regarding this topic: the scientific community is divided between thin and thick. The former relies on the minor drop in pressure and the latter by appealing to structural issues related to the relative resistance between a FCI of 2 mm and one of 7 mm.

We, at this juncture, have decided to move in the second way, choosing to study the thick FCI. So from now on we will talk no more about thick FCI, but only of FCI.

1.4 Scope of the thesis

Under neutron irradiation, both PbLi and the solid structures heat up. The FCI, like any solid under heat flow, can undergo thermal expansion and, therefore, deformation. The aim of this thesis is to study the effect of this deformation on the MHD phenomena.

In order to do that, after the geometry definition and mesh construction, a pure MHD analysis was performed. The velocity profile was taken from this calculus and a further thermal analysis was conducted in order to detect the most critical zone for FCI deformation. Subsequently, new geometry was designed and a new pure MHD simulation was performed.

The work can be subdivided into the following steps:

- Definition of initial geometry: based on the latest EU-DEMO DCLL BB design geometric values have been taken into account and the geometry has been designed including the presence of FCI;
- Condition flow and physical properties of materials and PbLi was taken into account and conditions flow posed;
- Building up of suitable mesh to study the problem;
- Performing of pure MHD analysis;
- Heat deposition and thermal properties were defined;
- New suitable mesh was built up;
- Thermal analysis considering the temperature as a passive scalar was made;
- Looking at thermal results, critical zone of deformation was detected and a new geometry was designed;
- New suitable mesh was built up;
- Pure MHD analysis was made with new deformated geometry.
Figure 1.5: Graphical representation of the steps made to conduct this master thesis work
Magnetohydrodynamics (MHD, called also magneto-fluid dynamics or hydromagnetics) is a branch of fluid dynamics that studies the behavior of electrically conducting fluid under the influence of external magnetic fields.

The first studies concerning this field lead us to name Hannes Alfvén, a swedish physicist and winner of the 1970 Nobel Prize in Physics for his work on MHD [1]. Over the years more and more studies have been conducted on magnetohydrodynamics, and their fields of application concern the most different subjects: from astrophysics to geophysics.

From an engineering point of view, MHD is related to the study of plasma confinement stability [10] and liquid metal refrigeration.

This last field is the study of interest of this dissertation, focusing the efforts on the eutectic PbLi.

2.1 Mathematics: governing equations and models

Before entering the depth of the physics behind the magnetohydrodynamic, a small presentation is given about the mathematical model used (which will be the basis of the numerical codes used here and throughout the world). A first list concerning the hypotheses that lie at the base of the models used will be given, and then present the equations that govern the purely magnetohydrodynamic problem and then, finally, thermomagnetohydrodynamic.

It is a duty to specify that the hypotheses considered, as well as the mathematical models, are in agreement with the doctoral thesis of Prof. Mas de les Valls [6] and the Davidson book [5].
CHAPTER 2. MAGNETOHYDRODYNAMICS

2.1.1 Hypotheses

First a list of hypotheses will be given which, soon after, will be explained.

1. *Continuum media*: considering the fluid in a macroscopic way we can apply an infinitesimal calculus;

2. *Incompressible fluid*: this strong simplification allow us to simplify the Navier-Stokes equations;

3. *Boussinesq hypothesis*: if a thermal gradient is present, in the momentum equation the variation of density is expected only in buoyant term where the gravity acceleration is present (see appendix C);

4. *Newtonian fluid with the Stokes condition for the bulk viscosity*: this hypothesis help us to further simplify the Navier-Stokes equations;

5. *Homogeneity and isotropy of materials*: the properties of the material are constant in all domain and they do not vary with the direction;

6. *Electrically conducting fluid*: for definition PbLi is a liquid metal and so it conducts electricity;

7. *Relativistic terms neglected in the constitutive relations for the electric displacement and the magnetic induction*: the relativistic effects in the Maxwell equation are not taken into account;

8. *No magnetic monopolies*: magnetic monopolies are not considered in Maxwell equations. In this way can be demonstrated the solenoidal nature of the magnetic field;

9. *Symmetric Maxwell stress tensor*;

10. *Low magnetic Reynolds number approximation*: this approximation, in fusion technology conditions, is verified (see section 2.1.3);

11. *Negligible expansion/contraction work, viscous dissipation and Joule generation*: this hypothesis allow us to simplify the energy equation in the thermal analysis.

In this thesis, the thermal analysis was made considering the temperature as a passive scalar. Hypotheses 3 and 11 are related to appendix C.
2.1.2 Pure MHD

Considering all of the hypotheses above mentioned we can, finally, present the governing equations of the problem considered.

As suggested by the name itself, the magnetohydrodynamics involves, at the same time, equations of classical hydrodynamics, such as Navier-Stokes, and electromagnetism equations, ie those of Maxwell.

Taking into account the hypotheses made in section 2.1.1, the Navier-Stokes equations will be (Davidson’s [5]):

Navier-Stokes equations

\[
\begin{align*}
\nabla \cdot v &= 0 \quad (2.1a) \\
\frac{\partial v}{\partial t} + (v \cdot \nabla)v &= -\frac{\nabla p}{\rho} + \nu \nabla^2 v + b_f \quad (2.1b)
\end{align*}
\]

where \(v\), \(p\), \(\rho\), \(\nu\) and \(b_f\) are the velocity, pressure, fluid density, kinematics viscosity and body forces, respectively.

A simplified set of Maxwell equations is represented by set of equations 2.2:

Maxwell equations

\[
\begin{align*}
\nabla \cdot B &= 0 \quad (2.2a) \\
\nabla \times E &= -\frac{\partial B}{\partial t} \quad (2.2b) \\
\nabla \times B &= \mu_m j \quad (2.2c) \\
\n\nabla \cdot j &= 0 \quad (2.2d) \\
\n\ j &= \sigma_m(E + v \times B) \quad (2.2e) \\
\n\ F &= j \times B \quad (2.2f)
\end{align*}
\]

where \(B\), \(E\), \(j\), \(\mu_m\), \(\sigma_m\) are the magnetic field, electric field, current density, magnetic permeability and electric conductivity, respectively.

The 2.2a represents the Solenoidal nature of \(B\); 2.2b represents the Faraday’s law of induction; 2.2c is the Ampere’s law equation; 2.2d is the charge conservation; 2.2e Ohm’s law; 2.2f represents the Lorentz force. In particular, in our problem, this Force corresponds to the last term of 2.1b.\footnote{This is true for pure MHD problem, when thermal gradient are not involved}

Through algebraic manipulations, which are beyond the scope of this work, we can arrive at the formulation of a complete set of equations that describes the entire MHD phenomenon. This set is 2.3:
CHAPTER 2. MAGNETOHYDRODYNAMICS

Set of complete equations: B-formulation

\[ \nabla \cdot v = 0 \]  \hspace{0.5cm} (2.3a)

\[ \frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{\nabla p}{\rho} + v \nabla^2 v + \frac{j \times B}{\rho} \]  \hspace{0.5cm} (2.3b)

\[ \frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta \nabla^2 B \]  \hspace{0.5cm} (2.3c)

\[ \nabla \cdot B = 0 \]  \hspace{0.5cm} (2.3d)

where \( \eta = \frac{1}{(\sigma_m \mu_m)} \).

In Davidson [5], the phenomenon of MHD is explained in three steps:

1. The movement of a conducting fluid inside a magnetic field, for the Faraday’s law of induction, induce a electromagnetic force. This last, for the Ohm’s law, produce an electric current;

2. For the Ampere’s law 2.2c from this induced current, a further magnetic field is generated;

3. The combination of the two magnetic fields, with the interaction of the induced electrical current, generate a Lorentz force 2.2f. Generally the direction of the latter is opposite to the flow of the flow, so as to inhibit it.

The 2\textsuperscript{nd} point suggests us that \( B \) in 2.3 is the total magnetic field: \( B_{\text{tot}} = B_{\text{external}} + B_{\text{induced}} \).

2.1.3 Low \( R_m \) approximation

In equation 2.3c, the ratio between the 2\textsuperscript{nd} and the 1\textsuperscript{st} terms of the r.h. side, allows us to define an important dimensionless number: the Reynolds magnetic number, \( R_m = \frac{vL}{\eta} \).

If \( R_m << 1 \) this mean that we can do a further approximation called inductionless approximation (or Low \( R_m \) approximation). This mean that the fluid does not influence the magnetic field, which remains constant and equal to the external one. In pp. 65 of Muller (2001) [14], we can find more detail about this issue.

If this condition is reached \( B_{\text{tot}} = B_{\text{external}} + B_{\text{induced}} = B_{\text{external}} \) and in momentum equation the Lorentz force depends only on the external magnetic field. With the fluid here considered and with our geometry, \( Rm \approx 10^{-3} \).

2.1.4 \( \phi \)-formulation

From the point of view of computational cost, the calculation of a vector rather than a scalar involves a considerable increase in the difficulty. For this reason a formulation has been developed, called \( \phi \– \text{formulation} \). This formulation uses \( \phi \) (electric potential) as a main driver and is the same used in the algorithm for the numerical calculus. Such an algorithm will be explained in chapter 3.

The electric potential is related to the electric field through the Poisson equation \( E = -\nabla \phi \). In
2.2. DIMENSIONLESS NUMBERS

this way the Ohm’s law take this form: \( j = \sigma_m (-\nabla \phi + v \times B) \). For the charge conservation law and by applying the divergence operator to 2.2e, we can find a further formulation for the electric potential: \( \nabla^2 \phi = \nabla \cdot (v \times B) \).

Now, finally, we can define the decisive set of equations: 2.4

Final set of equation

\[
\begin{align*}
(2.4a) & \quad \nabla \cdot v = 0 \\
(2.4b) & \quad \frac{\partial v}{\partial t} + (v \cdot \nabla) v = -\frac{\nabla p}{\rho} + v \nabla^2 v + \frac{j \times B_{ext}}{\rho} \\
(2.4c) & \quad \nabla^2 \phi = \nabla \cdot (v \times B) \\
(2.4d) & \quad j = \sigma_m (-\nabla \phi + v \times B)
\end{align*}
\]

Unlike 2.3 here is present only the external magnetic field \( B_{ext} \). Moreover we should underline that in this set of equations, now is present 2.4c instead of 2.3c. Here we are calculating a scalar instead of a vector and this strongly simplify the calculation.

2.2 Dimensionless numbers

As a fluid dynamics study, the magnetohydrodynamics is characterised by a group of dimensionless numbers. These give us informations on the fluid flow condition for a given problem. Mainly they are three:

- the Hartmann number, \( Ha \);
- the Reynolds number, \( Re \);
- the Grashof number, \( Gr \).

The first one, which square corresponds to the ratio between electromagnetic forces and viscous forces, is found by mean of this simple expression:

\[
(2.5) \quad Ha = BL \sqrt{\frac{\sigma}{\mu}}
\]

where \( B, L, \sigma, \mu \) are the external magnetic applied field, the characteristic length, the fluid electrical conductivity and the fluid dynamic viscosity, respectively.

The \( Re \) and \( Gr \) numbers are well known and they relate the inertia forces and the bouyancy forces, respectively, to the viscous ones.

\[
(2.6) \quad Re = \frac{\rho v L}{\mu}
\]

\[
(2.7) \quad Gr = \frac{g \beta \Delta T L^3}{\nu^2}
\]
where \( \rho, v, L, g, \beta, \Delta T \) and \( \nu \) are the fluid density, fluid velocity, characteristic length, acceleration gravity, thermal expansion coefficient, characteristic thermal gradient and kinematic viscosity, respectively. Note that the three \( L \) are not the same for the three numbers. Each number has a certain characteristic length, and it changes among the different problems. Their combinations can be useful in characterizing convective flow in the blanket [24].

The Stuart number (or Interaction Parameter)

(2.8) \[ N = \frac{Ha^2}{Re} \]

tells us the relative importance of the magnetic field in a certain configuration. It stands for the ratio between magnetic forces and inertia forces and if it is sufficiently high, the problem could be considered inertialess.

In fluid dynamics the regime move from a laminar flow to a turbulent one as \( Re \) increase. In MHD flow the question is much more complicated. The dimensionless number that plays fundamental role in the transition are the ratio \( Re/Ha \) and the wall conductivity ratio \( C_w = \sigma_w t_w / \sigma L \).

\( Re/Ha \) is a sort of "Reynolds number built through the thickness of the Hartmann layer" [24] and as stated from the authors the critical value for the transition is \( (Re/Ha)_{cr} = 300 \) [23]. For fusion relevant condition in the blanket, this ratio is expected to be well below such limit. In MHD flows, turbulence can be found in quasi two-dimensional (Q2D) flows. For more details about Q2D turbulence the reader is referred to Davidson book (2001) [5].

When we deal with the thermo-magnetohydrodynamics, we should consider also the Prandtl number, \( Pr \), which is an intrinsic dimensionless number, proper to the considered fluid.

(2.9) \[ Pr = \frac{\mu c_p}{k} \]

where \( c_p \) and \( k \) are the specific heat and thermal conductivity, respectively.

The ratio between advective and conductive heat transfer in a fluid is well know, and it is the Péclet number, \( Pe = Pr Re \). Under transversal magnetic field Pe number can be written as :

(2.10) \[ Pe = \frac{Pr Re}{N} = \frac{Pr Gr}{Ha^2} \]

as can be found by the dimensionless manipulation of the energy balance equation C.4 (see chapter 12, [14]).

An equivalent term to the Interaction Parameter, \( N \), is the so called Lykoudis number.

(2.11) \[ Ly^2 = \frac{Ha^4}{Gr} \]

As 2.8, if it is big enough, the inertia term can be neglected in momentum equation. According to 2.11, the equivalent number would be \( Re = Gr/Ha^2 \).
2.2.1 Fusion Relevant Condition

To have an idea about the characteristic numbers for an EU-DCLL breeding blanket, a table 2.1 is reported below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Ha}$</td>
<td>$7.57 \cdot 10^3$</td>
</tr>
<tr>
<td>$\text{Re}$</td>
<td>$2.27 \cdot 10^4$</td>
</tr>
<tr>
<td>$\text{Gr}$</td>
<td>$5.98 \cdot 10^{11}$</td>
</tr>
<tr>
<td>$\text{Pr}$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\text{N}$</td>
<td>$2.52 \cdot 10^3$</td>
</tr>
<tr>
<td>$\text{Pe}$</td>
<td>195</td>
</tr>
<tr>
<td>$L_y^2$</td>
<td>$5.5 \cdot 10^3$</td>
</tr>
<tr>
<td>$\text{Re}/\text{Ha}$</td>
<td>3</td>
</tr>
<tr>
<td>$R_m$</td>
<td>$1.92 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 2.1: Relevant Dimensionless numbers for a front channel of the breeding zone present in the equatorial module of the OB segment

This values refer to the front channel of the breeding zone present in the equatorial module of the OB segment. [25]

The Stuart number $N \approx 10^3$ is of the order $\approx 10^3$ this mean that the problem can be considered as inertialess. The same can be stated for TMHD problem, being $L_y \approx 10^5$.

The ratio $\text{Re}/\text{Ha}$ is well below the critical value of 300, this mean that the turbulence is founded in the form of Q2D. The Peclet number $\text{Pe} = \text{PrGr}/\text{Ha}^2 = 195$ this mean that in the energy equation the advective term has more effectiveness on the problem more than the diffusive one. This last, in any case, is not negligible.

To confirm the low $R_m$ (section 2.1.3) approximation, we see that $R_m \approx 10^{-3}$.

To complete this section: the Hartmann number 2.5 was constructed by using $a$; Reynolds 2.6 and Grashof 2.7, with using $b$. This is the standard procedure in MHD word to evaluate these numbers.

2.3 Physics: MHD effects

As predicted at the end of 2.1.2, the motion of fluid electrically conducting in a magnetic field develops a Lorentz force that acts as a damping force for the motion itself.

To simplify the understanding of the current, Lorentz force and magnetic field vector directions, a sketch of a rectangular channel of side $2a \times 2b$ is proposed.

The half-length $a$ lies along the magnetic field line and the other half-length, $b$, remains perpendicular to them.

In this simple example an external, constant and uniform, magnetic field goes in $y$-direction. So $\mathbf{B} = (0, B_y, 0)$. The fluid velocity vector enters in the sheet, along the positive $x$-direction:
\( \mathbf{v} = (v_x, 0, 0) \). The walls perpendicular to magnetic field are called Hartmann walls, the other two side walls (see figure 2.1).

From the interaction of these two fields, an electric current, \( \mathbf{j} \), is induced in the positive \( z \)-

direction and, for the conservation of charge 2.2d, it travels along the walls to close its path. Let’s assume that the walls are perfect insulators so that, following this assumption, the electric current is not able to cross them and it closes its path in the bulk flow. Now, the interaction between \( \mathbf{j} \) and \( \mathbf{B} \) generates a Lorentz force, by mean of 2.2f.

In figure 2.2 we can appreciate the current’s path and understand where is present the Lorentz force and in which direction. This case, is the one which can be found in [22]. The Lorentz force is not present near side walls (\( \mathbf{j} \not\parallel \mathbf{B} \)), and promotes bulk flow in Hartmann layers. In the center of fluid domain, this force acts as a damping force, opposing the motion. Always present at walls,
2.3. PHYSICS: MHD EFFECTS

Figure 2.3: Layer’s thickness with respect to Ha number ($a = 80.8 \text{mm}$)

The viscous forces are in the opposite sense of velocity. In this framework, the so-called side and Hartmann layers are developed at the side walls and Hartmann walls, respectively. Their depth can be calculated as $\delta_{\text{side}} = a/\sqrt{Ha}$ and $\delta_{Ha} = a/Ha$. As the Hartmann number increase, $\delta_{Ha}$ is thinner and thinner respect to the side layer (see figure 2.3).

Results for $Ha = 3000$ and $Re = 355$ is shown in figure 2.4 and 2.5. The flat profile in bulk core is founded. Due to the High Hartmann number $\delta_{Ha}$ is not appreciable in this figure.

Let’s define an important parameter: the wall conductivity ratio, $C_w$. This parameter found a definition in equation 2.12.

\[
C_w = \frac{\sigma_w t_w}{\sigma L}
\]

where $\sigma_w$, $t_w$ and $\sigma$ are the solid wall electrical conductivity, the wall thickness and the $PbLi$ electrical conductivity. $L$ is the characteristic length. Basically it tells us how the electrical conductivity of the walls is relevant respect to the one of the flow.

For the case above mentioned, $C_w = 0$ (Shercliff 1953 [22]).

In reality, $C_w \neq 0$. This mean that the electric current, now, can cross the Hartmann walls and close its path through them. As a result, the Lorentz force in $\delta_{Ha}$ that was previously acting as a
promoter of motion now takes on the role of inhibitor. Now, almost all of the core flow is opposed by the Lorentz force, which remains almost absent at the side walls. This helps us understand why almost all the fluid flow is carried out in $\delta_{side}$, where the velocity is greater than the average by a factor of $\sqrt{Ha/2}$ [14] (see figures 2.6 and 2.7). A consequence of this new velocity field is a noticeable increase in pressure drop per unit length (about 400 times), from 0.25Pa/m up to 106Pa/m, as can be noticed in figure 2.8.
We have finally shown the need to isolate solid walls. To do that, a sandwich-like Flow Channel Insert (FCI) was designed.
CHAPTER 2. MAGNETOHYDRODYNAMICS

2.4 Parametric study

The Reynolds number and the Hartmann number are two fundamental parameters for studying the effects of magnetohydrodynamics. The first gives an indication of the magnitude of the velocity of the liquid metal, while the second gives us information about the relative intensity of the magnetic field in our geometry. Looking at Maxwell equations, 2.2e and 2.2f, results easy to understand what we could expect from this study: at the same density of electric current, if Ha increases (at equal Re), the magnetic field increases, B. The latter, increasing, makes the impact of the Lorentz force more and more effective. Instead, with the same Ha, if the speed increases, the intensity of the electric current increases and, consecutively, the magnitude of the Lorentz force. These conditions would lead to a greater "damping" of the flow, which results in a greater pressure drop.

If \( Re_1 > Re_2 \), it follows that \( \nabla P_1 > \nabla P_2 \). A linear increase in pressure drop was found with increasing Re. So that \( \nabla P_1/\nabla P_2 \sim Re_1/Re_2 \), as can be seen in figure 2.9.

On the other hand, a parabolic dependence was found relating to magnetic field, so that \( \nabla P_1/\nabla P_2 \sim Ha^2_1/Ha^2_2 \) (being \( Ha \sim B \)). In figure 2.10 we have also three point so is difficult to understand but, for instance, if we consider that (taking Re=355) \( dP/dx \) for \( Ha_1 = 2500 \), we can easily find that \( dP/dx \) for \( Ha_2 = 3500 \) is \( \sim 74 \cdot 3500^2/2500^2 \sim 145 \) [Pa/m].

A resume table with all these values is given (2.2): All these results are in agreement with the correlations provided by Kirillov et al., which was used as a validation method (Section 3.6).
2.4. PARAMETRIC STUDY

Figure 2.9: $dP/dx$ to three different Re numbers

Figure 2.10: $dP/dx$ with respect to three different Ha numbers

<table>
<thead>
<tr>
<th></th>
<th>$Ha = 2500$</th>
<th>$Ha = 3000$</th>
<th>$Ha = 3500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re = 355$</td>
<td>74.4</td>
<td>106.5</td>
<td>145.8</td>
</tr>
<tr>
<td>$Re = 750$</td>
<td>157.1</td>
<td>224.7</td>
<td>307.6</td>
</tr>
<tr>
<td>$Re = 1065$</td>
<td>222.6</td>
<td>318.3</td>
<td>435.7</td>
</tr>
</tbody>
</table>

Table 2.2: Summary table for the parametric study, $dP/dx$ in [Pa/m]. $C_w = 0.25$
After talking about the mathematical models and the physical effects related to MHD, we will now discuss the implementation of them on OpenFOAM. Regarding the construction of the grid used and its numerical accuracy, please refer to the appendix B and A.

3.1 The algorithm

In chapter 2, section 2.1.4, the full set of complete equation, considering $\phi$–formulation is given. Scope of this section, is explain how, in OpenFOAM, we can solve equations 2.4. The name of the algorithm used is $\phi$–PISO, because it uses the $\phi$–formulation set of equation to solve the MHD phenomena and the Pressure-Implicit with Splitting of Operators (PISO) algorithm to treat the pressure-velocity coupling, intrinsic aspect of the Navier-Stokes equations 2.1. A deep explanation about the methodology, and implementation can be found at chapter 5, section 5.2.1 of [6].

So let's try to understand the logic behind this implementation, focusing in the sequential steps, which are typical of an iterative process.

1. An initial map of $v$, $p$ and $\phi$ is given. The external magnetic field, $B$ is constant and is another input. With this data, is possible to calculate the density current, and the corresponding Lorentz force;

2. Now the momentum equation 2.4b, can be solved, with a fixed number of iterations with PISO algorithm obtaining $p$ and $v$ correctors;

3. Solve the continuity equation to estimate the error;
4. Correct the velocity field obtaining new $v$;

5. Now, with new $v$ and $p$, we can calculate, again, $j$, $\phi$ and the Lorentz force;

6. Next time step, return to 2.

![Flow Chart](Image)

Figure 3.1: $\phi$ – PISO algorithm flow chart

The same algorithm is used if we would deal with temperature. At step 1, we give as an input also an initial map Temperature. Then, in step 2, the momentum equation is solver considering the thermal coupling. This algorithm is called $\phi T$ – PISO.

### 3.2 Time step

In the discretization of the momentum equation each term is discretized with an implicit scheme, except for Lorentz force. The latter is expressed explicitly: this mean that the time step chosen is crucial for the stability of the solution (Patankar [20]).

In openFoam the input parameters that we have to give are two: *maxCo* and *maxDeltaT*, that correspond to maximum magnetic Courant number and maximum time step, respectively. The relation between the two parameters is simple, and it involves the external applied magnetic field and the natural physical properties of the considered fluid.

$$\tag{3.1} Co = \frac{\sigma B^2 \Delta t}{\rho}$$

24
3.3 Density current conservation

In [6] it is suggested to use $Co \leq 0.2$ for pure MHD simulations. This because this conservative value allow us to have a stable and reliable solution. Nevertheless, different calculations have been conducted to find the "minimum $Co$" for which a stable or non-stable condition can be obtained with a $c2c = 1.25$ (see appendix A). From the results it turns out that under $Co \leq 0.3$ it is possible to obtain stable solutions with reasonable residuals. On the contrary, if $Co \geq 0.3$, divergent or highly unstable solutions can be obtained. With Ha progressively increasing, the time steps to be used become smaller and smaller, requiring more calculation precision and computational time. For this reason we chose to use $Co = 0.2$ for $Ha = 7570$ and $Co = 0.3$ for $Ha \leq 7570$. An idea is given in table 3.1.

<table>
<thead>
<tr>
<th>$\Delta t$ [s]</th>
<th>$Co = 0.1$</th>
<th>$Co = 0.3$</th>
<th>$Co = 0.5$</th>
<th>$Co = 0.7$</th>
<th>$Co = 0.9$</th>
<th>$Co = 1$</th>
<th>$Co = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ha = 300$</td>
<td>$4.71 \cdot 10^{-2}$</td>
<td>$1.41 \cdot 10^{-1}$</td>
<td>$2.36 \cdot 10^{-1}$</td>
<td>$3.3 \cdot 10^{-1}$</td>
<td>$4.24 \cdot 10^{-1}$</td>
<td>$4.71 \cdot 10^{-1}$</td>
<td>$9.42 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>$Ha = 3000$</td>
<td>$4.71 \cdot 10^{-4}$</td>
<td>$1.41 \cdot 10^{-3}$</td>
<td>$2.36 \cdot 10^{-3}$</td>
<td>$3.3 \cdot 10^{-3}$</td>
<td>$4.24 \cdot 10^{-3}$</td>
<td>$4.71 \cdot 10^{-3}$</td>
<td>$9.42 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$Ha = 7570$</td>
<td>$7.4 \cdot 10^{-5}$</td>
<td>$2.22 \cdot 10^{-4}$</td>
<td>$3.7 \cdot 10^{-4}$</td>
<td>$5.18 \cdot 10^{-4}$</td>
<td>$6.66 \cdot 10^{-4}$</td>
<td>$7.4 \cdot 10^{-4}$</td>
<td>$1.48 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 3.1: $\Delta t$ with respect to different Ha.

3.3 Density current conservation

Let’s re-write the set of equations of $\phi – formulation$:

\[(3.2a) \quad \nabla \cdot v = 0\]
\[(3.2b) \quad \frac{\partial v}{\partial t} + (v \cdot \nabla)v = - \frac{\nabla p}{\rho} + \nu \nabla^2 v + \frac{j \times B_{ext}}{\rho}\]
\[(3.2c) \quad \nabla^2 \phi = \nabla \cdot (v \times B)\]
\[(3.2d) \quad j = \sigma m (-\nabla \phi + v \times B)\]

From the second and third equations, we can calculate the velocity and the pressure and the electric potential, respectively. The current density equation is a consequence of the previous two, and is outside the PISO loop.

Even if the calculus of $\phi$ is made through a current density conservation (Ni et al. [16]), the aspect just mentioned is a fact.

The result is that the current error estimated does not converge and, rather, could diverge. Nevertheless, the code has been validated and the results demonstrate coherence with the analytical and experimental results and comparative results to other studies conducted in this field.

Thus, we will consider as an index of convergence only the velocity field.

25
CHAPTER 3. THE IMPLEMENTATION

3.4 The source term

In section A.0.3, we have talked about the cyclic boundary condition. This condition is achieved by imposition at the InletOutlet boundary the "cyclic" type boundary condition. Moreover, as an input is given an average velocity, labeled $U_{bar}$, which must be maintained throughout the calculation. This is made possible through the imposition, in the equation of the momentum of a "Source Term" (El termine fuente). This term, in a certain sense, 'pushes' the fluid from the outlet to the outlet with a certain speed and guarantee the condition mentioned above.

3.5 Schemes

In this short paragraph will be briefly discussed the numerical schemes used for time, gradient terms, divergence terms, Laplacian terms and interpolation.

Time scheme
As a time scheme, type "Euler" was used. This is implicit and is of the first order.

$$\frac{\partial \phi}{\partial t} = \frac{\phi - \phi_0}{\delta t}$$

where $\phi$ is a generic variable.

Interpolation scheme
As interpolation scheme was chosen the "linear" one. Basically, it is an arithmetic average between two values.

Gradient and Divergence scheme
These two type of terms where solved with the type "Gauss <Interpolation scheme>", so "Gauss Linear", using the famous Gauss theorem:

$$\int_V \delta u dV = \oint (nu) dS$$

Laplacian scheme
This scheme use the Gauss method, as before: "Gauss <interpolationScheme> <snGrad-Scheme>". Since a correction for non-orthogonal mesh is yet implemented in the code, the scheme is "Gauss linear corrected".

3.6 Validation

The validation of the pure MHD code (without wall or FCI) was made by Prof. Mas de les Valls in her Ph.D. thesis ([6]) with respect to three different case: Hartmann flow...
3.6. VALIDATION

(Hartmann 1937), Shercliff case (Shercliff 1953 [22]) and Hunt’s case (Hunt 1966 [11]). Here will be shown a validation of results obtained considering the coupling fluid/solid and so considering the presence of the solid conducting wall.

For this study were considered three different Ha numbers and two different Re numbers. The present results were compared with correlations found in the bibliography (Kirillov et al. [12]).

\[
\frac{dP}{dx} = k_p \sigma U_0 B_0^2
\]

Where \( k_p, U_0, \sigma, B_0 \) are pressure drop coefficient, mean velocity, PbLi electrical conductivity and external magnetic field, respectively. Since the nature of our geometry, (three different wall thickness, see 1.4, \( k_p \) can be calculated as:

\[
k_p = \frac{1}{1 + \frac{1}{c + \frac{\sigma}{\sigma a} c_1 + c_2}}
\]

where

\[
c = \frac{\sigma_w t_{SW}}{\sigma a}
\]

\[
c_1 = \frac{\sigma_w t_{FW}}{\sigma a}
\]

\[
c_2 = \frac{\sigma_w t_{RW}}{\sigma a}
\]

where, according to table 4.1, \( t_{SW}, t_{RW} \) and \( t_{FW} \) stands for thickness of Radial wall, Rear wall and First wall respectively.

<table>
<thead>
<tr>
<th></th>
<th>Ha=2500</th>
<th>Ha=3000</th>
<th>Ha=3500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re=355</td>
<td>75</td>
<td>106.5</td>
<td>145.8</td>
</tr>
<tr>
<td>Re=750</td>
<td>157</td>
<td>224.75</td>
<td>307.65</td>
</tr>
</tbody>
</table>

Table 3.2: Results of calculation [Pa/m]

In table 3.2 are shown the results of present calculations and hereafter will be presented a comparison with respect to the Kirrilov et al. ([12]) one. From figures 3.2 and 3.3 we can notice that the code underestimates the results obtained from correlations found in the literature. However, the error we have is around \( 1 \pm 2\% \).

Regarding the results obtained considering the FCI, there are no correlations or experiments to validate our results. despite this, the result obtained for Ha and Re high was compared to that obtained by CIEMAT (Madrid) present in Urgorri et al. [25]. The two results are almost identical. The relative percentage of error is around \( 6\% \).
CHAPTER 3. THE IMPLEMENTATION

Figure 3.2: Validation for three different Ha numbers, Re=355

Figure 3.3: Validation for three different Ha numbers, Re=750

<table>
<thead>
<tr>
<th>-</th>
<th>Pressure Drop [Pa/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urgorri et al.</td>
<td>3 861</td>
</tr>
<tr>
<td>Present calculation</td>
<td>4 112</td>
</tr>
</tbody>
</table>

Table 3.3: FCI results comparison with Urgorri et al [25] Ha=7570, Re=22700
In this chapter we will define the geometric dimensions used for our study and the physical properties of the materials we consider. We will focus our attention on the frontal channel of the equatorial module of the DCLL OB segment (see figure 1.3). We will refer to the EU-DEMO DCLL BB, using the same configuration that was used in latest paper of Urgorri et al. [25], for CIEMAT research center. The physical properties were taken from literature (see [7] and [25]). A summary table 4.1 and a figure 4.1 (obtained through a post-processing tool called paraView) are presented below. The latter, moreover, will help us to understand the spatial arrangement of the channel with respect to the external magnetic field (coming from the magnetic arrangement of the plasma) and with respect to the thermal flow (coming from neutron irradiation) and will allow us to deny, once and for all, the dimensionless numbers essential for our study (see 2.2).

In table 4.1 the symbols $a$, $b$, $L$, $t_{FCI}$, $t_{gap}$, $t_{FW}$, $t_{SW}$ and $t_{RW}$ stand for half-length of the channel parallel to the external magnetic field, half-length of the channel perpendicular to magnetic field, total length of the duct, Flow Channel Insert thickness, gap thickness, first wall thickness, radial wall thickness and rear wall thickness, respectively.

In figure 4.1, $B$ and *heat* stand for external magnetic field and external heat flux, respectively.

---

1. In figure 4.1 the FCI is represented as a single component, but in reality is the kind of sandwich-like, see Appendix D.
### Chapter 4. Geometry and Physical Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2a$ (mm)</td>
<td>161.6</td>
</tr>
<tr>
<td>$2b$ (mm)</td>
<td>282</td>
</tr>
<tr>
<td>$L$ (m)</td>
<td>2</td>
</tr>
<tr>
<td>$t_{FCI}$ (mm)</td>
<td>7</td>
</tr>
<tr>
<td>$t_{gap}$ (mm)</td>
<td>2</td>
</tr>
<tr>
<td>$t_{FW}$ (mm)</td>
<td>18.64</td>
</tr>
<tr>
<td>$t_{SW}$ (mm)</td>
<td>13.48</td>
</tr>
<tr>
<td>$t_{RW}$ (mm)</td>
<td>16.50</td>
</tr>
</tbody>
</table>

**Table 4.1: Input geometry parameters**

![Figure 4.1: Geometry description of the channel studied in this thesis](image)

### 4.1 Physical properties

Now will be presented the values of the physical properties used during this work. In table 4.2 are put together the properties of PbLi, Eurofer and Alumina.

<table>
<thead>
<tr>
<th>Material</th>
<th>PbLi</th>
<th>EUROFER</th>
<th>Alumina</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kgm$^{-3}$)</td>
<td>9720</td>
<td>7680</td>
<td>3950</td>
</tr>
<tr>
<td>$\sigma$ (Sm$^{-1}$)</td>
<td>7.63·$10^5$</td>
<td>8.33·$10^5$</td>
<td>10$^{-8}$</td>
</tr>
<tr>
<td>$\mu$ (Pa s)</td>
<td>1,497·$10^{-3}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$c_p$ (JkgK$^{-1}$)</td>
<td>189</td>
<td>730</td>
<td>880</td>
</tr>
<tr>
<td>$\kappa$ (WmK$^{-1}$)</td>
<td>15,14</td>
<td>30,35</td>
<td>28</td>
</tr>
</tbody>
</table>

**Table 4.2: Physical properties of the materials**

With this data we can evaluate each single parameter of interest, like, for instance, $C_w = 0.25$. Note that, in the calculus of wall conductivity ratio, $C_w$ (eq. 2.12), was used the thickness of the first wall. Nevertheless as input in numerical simulation is not given $C_w$, but directly $\sigma_w$. In this way, we could consider the value $C_w$ as an indicative value.
This part is the heart of the present thesis. We will focus our attention on the Magnetohydrodynamics (MHD) effects related to Breeding Blanket Fusion relevant conditions. A first pure MHD study was performed, showing the necessity of the Flow Channel Insert (FCI) as insulator of the rectangular duct. Then, a thermal analysis is conducted, in order to detect the critical zone for deformation. Once this is done, a new geometry is designed by using blockMesh tool. A new MHD study with deformed geometry was made, and a comparison is presented to the reader.

These effects were studied using the OpenFOAM toolbox, with the yet implemented solver made by Elisabet Mas de les Valls in her doctoral dissertation [6]. These solvers have been kindly provided to the author of this thesis, and through them it has been possible to conduct an accurate and validated study.

OpenFOAM (Field Operation And Manipulation) is a free, open source toolbox. It is a finite volume CFD tool, which has more developer houses. Here we used foam-extended3.2 because within it there is the possibility of dealing with multi-region cases and coupling fluid and solid domains (see Chapter 5, page 73 [6]).

More information about OpenFOAM codes can be found in manual user or in the official OpenFOAM website (https://www.openfoam.com).
5.1 FCI impact

In chapter 2, discussing the effects of magnetohydrodynamics, it was mentioned the need for the Flow Channel Insert to electrically de-couple the current conducting fluid and the surrounding walls. Now we will try to take a deeper look at the subject, which deserves due attention.

We will present here the results obtained from calculations conducted under fusion relevant condition: \( Ha = 7570, Re = 2.27 \cdot 10^5 \).

A fully 2D developed calculation was made and velocity profile taken over the line from one side wall to the other is given.

As a first result, a velocity profile for perfectly conducting wall is shown. The flat profile in core bulk region is retrieved. Compared to the high \( Ha \) and high \( Re \), the pressure drop is quite small, being \( \approx 42.8 \text{ Pa/m} \).

Then a “normal electrical conductivity” was considered for EUROFER surrounding wall, \( \sigma_w = 8.33 \cdot 10^5 \text{ S/m} \). Now the current density is able to close its path inside the conductive wall and the Lorentz force begin more effective. A damping force is generated, with an increasing of pressure drop and with the typical near-wall high-velocity jets are founded.

![Figure 5.1: \( Ha = 7570, Re = 22700, C_w = 0 \)]
5.1. FCI IMPACT

Figure 5.2: Conducting wall results under fusion relevant conditions: $C_{FW} = 0.25$, $Ha = 7570, Re = 22700$. Overall view (a) and near-wall zoom (b).

The increase in pressure drop is very considerable, being greater than a factor of 1000. As a matter of fact, the pressure drop that results from this calculation is $4.3 \cdot 10^4 \, \text{Pa/m}$. The plateau magnitude found with $C_w = 0$ is greater than the constant velocity in the core bulk flow with $C_w \neq 0$, a symptom that Lorentz force prevails in the second case. A more explaining figure is 5.3, where the two velocity profile are reported in the same graph.

Figure 5.3: Comparison between $C_w = 0$ and $C_w = 0.25$. Overall view (a) and near First Wall zoom (b).
Once again, we have proven the need to find a way to electrically decouple the liquid metal from the solid wall.

The Flow Channel Insert (FCI) seems to be the best candidate to overcome the problem of fluid/solid electric coupling. A 7mm FCI sandwich-like steel-Al-steel (1−5−1mm) was here considered. With the presence of the FCI, we will deal with two new region: FCI region and the GAP region. The GAP is a thin space, filled of liquid metal, leaved between the FCI and steel wall. This is because the FCI could undergo thermal expansions and, if the GAP were not present, the mechanical constriction of the steel walls could lead to the breaking of the same FCI.

Consistent with the scientific community, having two regions where fluid is present, we will use the terms bulk flow and GAP flow respectively, depending on whether we are referring to the BULK region or the GAP region.

What we expected from this analysis is a consistent reduction of the pressure drop and a consistent reduction of the near wall jets.

These two aspects are actually found, but there is another very interesting aspect from the analysis of the FCI. In the GAP region, the gap flow has larger jets than those found in the BULK region near the walls. The reason is the following: by design nature, the FCI is a component made up of two materials, such as steel and alumina. These two materials are translated into three layers, which central layer is constituted by alumina. This means that the gap flow "touches" two steel walls, with a consequent increase in the density of electric current and a more predominant Lorentz force. As a result, the MHD effect is more pronounced here than in the BULK region (see fig 5.5).

For the mesh construction and numerical accuracy issue we will refer to appendix A and B, respectively. Here a \( c_2c = 1.25 \) was used to determine the grading. A sketch of the mesh is provided in figure.

Nevertheless, from a comparison between the three cases just shown, the average velocity
in the bulk core, in the case of the conducting wall, is lower than in the other two, this because the more effectiveness of the MHD effects. The entity of the GAP jets is similar to the one of the near-wall jets for the conducting wall for the FW, while for the RW a greater jet for conducting wall is showed. On the other hand, even if the gap flow is "wrapped" by steel, in the FCI there are 5mm of alumina that "dampen" the increase in current density, translating this phenomenon into a lower MHD effect than in the case without FCI. In figure 5.6, an explanatory graph will help us to better understand the phenomena described above.
The pressure drop reduction with the FCI insertion is of 10 times the pressure drop of the case with conducting wall, and, moreover, for completeness, the increase of pressure with respect of the case insulating wall is only 96 times (against 1000 times of the pressure drop increase in the case of conducting wall).

In figure 5.7 an histogram is given. The magnitude of the bars helps us to get an idea about what was said above.

To complete the report, a table 5.1 that represents these values will be reported where $dP/dx_0$ indicates the pressure drop of perfecting insulating wall.
### 5.2 Heat analysis

In order to detect the critical part for FCI deformation, a thermal analysis was done. In appendix C is presented the theoretical coupling between temperature and velocity, while in our calculation we have considered the temperature as a passive scalar. In this way only the energy equation 5.1 was solved starting from velocity field obtained from the previous pure MHD calculation with FCI.

\[
\frac{\partial T}{\partial t} + \nabla (vT) = \alpha \nabla^2 T + S_{\text{thermal}}
\]

Let’s focus on this \( S_{\text{thermal}} \). It is an external source, an precisely, is related to the deposition of the neutrons coming from the nuclear reaction. The fusion reaction that takes place into the plasma is the \( D - T \) reaction. This reaction gives rise to alpha and neutron particles \((D + T \rightarrow \alpha(3.5MeV) + n(14MeV))\). The former are engaged in heating the plasma itself and the latter, investing the blanket, to produce tritium and deposit energy, which will then be converted into electricity.

Such a heat source can be expressed by an exponential law:

\[
S_{\text{thermal}} = q_0 e^{-mz}
\]

where \( q_0 \) and \( m \) are two parameters of interest for this formula. This first is the peak of the thermal load, corresponds to the value of \( S_{\text{thermal}} \) at the FW. The second is a parameter that "shapes" the exponential. In [25] was made a best fitting of more complete studies made by Iole Palermo et al 2017 Nucl. [19] from which came out that \( m = 6.3 \).

Regarding \( q_0 \) we can extrapolate it starting from the Grashof number. From the Grashof number, 2.7, we can know the characteristic temperature difference, and from that, the average volumetric heating in the channel: \( \bar{q} = k \Delta T/b^2 \) and, for definition, \( \bar{q} = \)
\[ \frac{1}{V} \int_V q_0 e^{-mz} dV. \] This is the standard way to calculate \( q_0 \). This means that this parameter is only related to the deposition in the \( PbLi \) and not to the solid structure, like FCI and solid wall. In the calculation, thus, was assumed that the neutron are deposited only in the \( PbLi \) and not in the other two solid structure. Then the heat is exchanged by conduction between \( PbLi \) and FCI like between \( PbLi \) and wall, and by convection between He cooling channel and EUROFER wall.

If on the one hand the present neutron flux heats the liquid metal, on the other hand the helium channels that surround the main channel have the role of removing the excess heat from the steel structures. The governing equation is the heat exchange equation by convection. So that:

\[ q = A h_{He} (T_{wall} - T_{He}) \]

where \( A, h_{He}, T_{wall} \) and \( T_{He} \) stands for surface area, coefficient exchange, wall temperature and helium temperature, respectively. Just to be precise: in openFOAM we do not have a proper boundary condition for convective problem. Since we know the effective heat flow that from the \( PbLi \) goes the He channels and we know the average heat transfer coefficient for the FW and RW, [25], 5.2, we fix a \( \nabla T \) as a boundary condition of our simulation, being \( q = A h_{He} (T_{wall} - T_{He}) = k_w A \nabla T \).

From the calculation we expect that, looking at the inlet, the part of Ha gap more exposed to the neutron irradiation of more heat. This is because the presence of wall jets in the side-gap and in the side wall promotes the removal of heat by convection. Instead, in the Hartmann layer and in the Ha GAP, the velocity is almost stagnant, removing a minimum amount of heat.

What we expected was founded effectively and in figure 5.9, is clear what we just have said.

### 5.3 FCI deformation

Finally, we arrive at the final section: the FCI deformation. The paragraph will be divided into three more sub-sections:

<table>
<thead>
<tr>
<th>input</th>
<th>First wall</th>
<th>Radial wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_{PbLi-He} , [kW] )</td>
<td>10.675</td>
<td>9.185</td>
</tr>
<tr>
<td>( h_{He} , [kW/m^2K])</td>
<td>3.084</td>
<td>3.366</td>
</tr>
</tbody>
</table>

Table 5.2: Input parameter for heat analysis
5.3. FCI DEFORMATION

Figure 5.8: Sketch of channel’s section underlining the most critical part for the deformation of the FCI

Figure 5.9: Temperature profile on outlet cross-section (FW on the left)

1. strategy, where will be explained the logic and assumptions of the deformation’s geometry and the assumptions;

2. geometries and expectations, where the new geometries will be presented;

3. results, where the MHD results come from our calculus are shown.

5.3.1 Strategy

This is a crucial point: finding a way to model geometry that is consistent with physics, with structured and logical aspects.

From the thermal analysis it has come out that the temperature peak, considered as the average on the section, is located at the outlet of the channel. This means that the most critical section will be right at the outlet. In addition, a more careful analysis of the
CHAPTER 5. RESULTS

section of interest suggests that the most critical point is located in the Hartmann gap, as anticipated in the previous section,(5.2). From [9] we learn that the outer steel layer detached from the Alumina one, and the internal one "push" the Alumina from inner region. Since the deformation occurs near the outlet we can assume that no PbLi will be present in the gap between steel and Alumina.

Then, let’s consider the formulation of Young modulus, \( E \):

\[
E = \frac{\sigma}{\varepsilon} = \frac{F}{A\varepsilon}
\]

where \( \sigma = F/A \) is the stress and \( \varepsilon \) is the relative deformation. The area \( (A) \) on which the forces act is practically the same (if we consider the surface area of steel layers and Al layer the differences is really small). The force \( (F) \) acting on steel and Alumina is basically the same (if we consider that all force is acting on FCI). So the problem it reduce on the Young’s modulus:

\[
\frac{E_{\text{EUROFER}}}{E_{\text{Al}}} = \frac{\varepsilon_{\text{Al}}}{\varepsilon_{\text{EUROFER}}} \implies \varepsilon_{\text{Al}} = \varepsilon_{\text{EUROFER}} \frac{E_{\text{EUROFER}}}{E_{\text{Al}}}
\]

considering that \( E_{\text{EUROFER}} = 300 \text{MPa} \) [9] and \( E_{\text{Al}} = 300 \text{GPa} \) [21] comes out that \( \varepsilon_{\text{Al}} = 10^{-3} \varepsilon_{\text{EUROFER}} \). This mean that we can assume, without making a big mistake, that the deformation occurs only on the steel layer.

For reasons of clarity and order, we summarize the assumptions:

- the deformation occurs near the outlet of out channel;
- only the steel layers deform, being the Young’s modulus of Alumina really bigger than EUROFER’s one;
- the critical point is detected after a thermal analysis, made on the near outlet section;
- the PbLi do not fill the gap between Al and EUROFER, and we will consider it EUROFER too.

Having all this instruments, we can now proceed to the design of the geometry.

5.3.2 Geometry and expectations

Geometry definition was made through the use of blockMesh, using arc. As input parameter, the only deformation “\( d \)” (mm) is needed to create the all grid.
5.3. FCI DEFORMATION

As a result, we can see a deformation, in the upper right side (seen from the inlet) of our channel. In figure 5.10 it is possible to appreciate the deviation of the superior part of the FCI, which moves towards the wall in EUROFER. Note that the FCI do not touch the wall, consistent with [9], and for reasons of simplicity of blockMesh and code itself\(^1\).

Let’s forget for a moment the engineering aspect of deformations and information technology and try to focus on the physics of the problem: Lorentz's strength by its very nature is a vectorial force. Corresponds to the vector product between the electric current vector (electrical current density) and the magnetic field vector. Therefore \( F_{\text{Lorentz}} = j \times B = jB \sin \theta \), where \( \theta \) is the angle in between the two vectors. As was said in the chapter 2 this force acts as a damping force and its magnitude is maximum when \( j \perp B \), or \( \theta = 90^\circ \). Until now this was verified, but if FCI deforms the new \( \theta \) is no more a right angle. This leads us to think that the first damping force was highest throughout the BULK now, where deformation is present, it is no longer. In that area we expect a higher speed, and in general a lower pressure drop.

5.3.3 Results

What was anticipated in the previous section is actually found: a minor Lorentz force acts on the liquid metal around the deformation. This brings to a new phenomena:

---

\(^1\)If the FCI were to touch the wall in EUROFER, the region of the GAP would be "squeezed" in one point, causing the collapse of several cells and the impossibility of performing any simulation. Moreover, due to the nature of the code used, the contact between FCI and solid wall is not foreseen. Last but not least, [9] performed a structural study consistent with the expected temperature gradients, and it is estimated that at most the FCI would "approach" the Wall for 1.5 mm, shorter length to the thickness of the GAP (2 mm)
near the deformations, the curvilinear dumps the Lorentz force because the angle in between \( j \) and \( B \) is no more 90°. Near the side walls the jets are present. Now, near the deformation, a new phenomena is present: the spatial profile of velocity reminds us of a wave, presenting a "swelling" near the deformation. Precisely for this reason, the author of this thesis has decided to call it near-deformation wave (D-Wave).

What has just been said lets us understand how the Lorentz force, now, is less effective. From the hydraulic point of view, the force that first dampened the flow and led to large drops of pressure, is now lower. What we expect is therefore a lower pressure drop in our channel.

Indeed, this happens. In figure 5.12 we can have a visual explanation of the difference in the pressure drop.

The pressure drop is slightly less and the magnitude of this decrease is a factor of 1.4. This, paradoxically, could lead us to think that a deformation could be "desired".

However, if the EUROFER were to "push" too much Alumina, the Alumina could break, thus creating a 'passage' for the current density that could lead to new configurations of speed and new phenomena.

Near the breaking of alumina, a new passage for current lines is created. This protests in the manifestation of a new Lorentz force that dampens the motion in a circumscribed space in the presence of the "crack" in alumina. In two points, contiguous to the crack, the current lines are parallel to the magnetic field. In these points the Lorentz force is almost absent and leads to the formation of two new "jets". The shape, remind us a "V". For this reason, we call it "V-shape" (see 5.13).

In order to justify the \( V - shape \), we have to look at the density current path.
Near the crack, the density current close its path. This means that there are two points in which the current line are parallel to the magnetic field (see figure 5.14). In those points, the Lorentz force is almost not present, and this justify the two "jets" which constitute the tips of V-shape. The pressure drop in this case, is greater. In figure 5.16 the greater pressure drop is related to the rupture of Alumina. The relative percentage with respect to the non deformed FCI, is about 3%. By the way we should say that with increasing Hartmann numbers, MHD regimes are changing.

What we expect from bigger Hartmann is a velocity profile similar to this with $Ha = 300$,
CHAPTER 5. RESULTS

Figure 5.14: Streamline path and velocity field over the section.

Figure 5.15: Alumina crack visualization. The entity of the crack was chosen in order to guarantee at least three in the crack itself. Ha=300
with more pronounced MHD effects and higher pressure drops. However, this does not mean that results that differ from expectations may come out of the calculations. In any case, this image helps us to understand that the breaking of the Alumina interrupts the decoupling in one point and leads to greater pressure drops.
The Breeding Blanket is one of the most studied component of a future fusion reactor. In this thesis we focused our attention on the Dual Coolant Lead Lithium type. The main coolant in this concept is a liquid metal (LM): \( PbLi \). By its nature, it conducts electricity and interacting with the external magnetic field, damping Lorentz force is produced leading to strong pressure drops in the channel. Electrically decoupling the liquid metal and the solid wall, the pressure drop is strongly reduced. The Flow Channel Insert (FCI) seems to be the best solution to ensure this task. The FCI here considered is the kind of sandwich like steel-alumina-steel insulator, and it is inserted in the flow channel leaving a thin GAP of LM between the wall and itself. The goal, in this thesis, was to study the MHD effects on a geometry where the FCI is deformed. The FCI deformation studied is thought to be the result of thermal expansion of the FCI itself due to heat exchange with the LM. This goal has been achieved and a further study was performed considering a "crack" of alumina and showing its effects. The conductive walls lead to a huge pressure drop under significant fusion conditions, giving life to the typical "near-wall jets". The FCI mitigates these effects, drastically reducing the pressure drop and being itself an excellent solution to solve the problem. Typically, in this configuration, high side gap jets are present and their magnitude is bigger than the bulk jets one. The deformation of FCI leads to a change in the spatial profile of the velocity. In fact, near the deformation, the Lorentz force is less effective and leads to a paradoxical "gain"
in terms of pressure drop. Here, the D-Wave is formed, further promoting the flow of LM. According to our hypothesis, the alumina does not deform and remains in its initial position, leaving the steel to be deformed. The result is that the steel layer in the Hartmann gap curves, approaching the wall and "squeezing" the GAP itself, while the inner layer pushes the alumina, bringing it to bend. If the magnitude of this stress is significant, the alumina may break, thus opening a passage to the current lines. This brings us to a new configuration: the "V-shape". Where a real "hole" is formed at the breaking point of alumina. In this case a greater pressure drop is generated, because the MHD effects are more effective.

6.1 Improvements

In our calculations we used blockMesh, using simpleGrading. This forced us to construct and use a varied number of blocks, required for cell grading. To facilitate access to the blockMeshDict files, it would be appropriate (but not mandatory) to construct a geometry using blockMesh + multiGrading. In this way it will be possible to reduce the number of blocks and to easily "draw" new geometries more simply. As for the computational cost, looking at the figure A.3) the use of the snappyHexMesh tool could solve the problem. However, for the calculations we conducted and for the purpose of this thesis, we chose to use blockMesh with simpleGrading (already created by [6]) and to continue with the latter.

Moreover a more detailed structural analysis should be carried out in order to study a more realistic deformation. In our study, we considered a deformation consistent with our temperature study and their conclusions, assuming further that alumina does not deform.

Regarding the thermal analysis, the standard procedure described in paragraph 5.2 refers to the deposition of heat only in the liquid metal, and therefore the solid structures heat up by conduction. A more detailed procedure should be developed to take into account heat deposition even in solid structures, considering the different probabilities of neutron interaction.

Furthermore, we calculate the temperature as a passive scalar, i.e. with no bouyancy. In reality the temperature is coupled with velocity and magnetic field, leading to particular effects (see 4.2 of [6]). A more detailed analysis should be done in this sense.
For our problem, we used blockMesh, following a step-by-step procedure that can be found on webpage of openFOAM Wiki [18] the geometry was designed and then using the simpleGrading method, the mesh was created. The number of cells and grading coefficients have been calculated with a simple formulas.

In $\delta_{Ha}$ and $\delta_{side}$, we need (at least) 8 and 25 cells, respectively [13].

Be N the number of cells on which we would like to apply a certain grading. These cells should be present in a certain length. Let’s call $t$ this length (thickness). Define the cell-to-cell ratio, $c2c$ as the ratio between two adjacent cells, more precisely, between the greater cell and the smaller cell, $G = \frac{c_{big}}{c_{small}}$. Said this, we can express the total length as:

![Figure A.1: Channel cross section. Frontal view with Wall (red), GAP (blue), FCI (green) and bulk (grey).]
The previous equivalence is demonstrable through the use of a simple geometric sequence. The grading coefficient can be expressed as:

\[ G = \frac{C_{\text{max}}}{C_{\text{min}}} = c 2c^{N-1} \]

So, considering \( N = N_{Ha} \) or \( N = N_{side} \) we are able to find the grading coefficients in two layers.

Regarding the grading applied to the BULK, FCI, GAP and Wall regions, two different grades were applied depending on whether they were in magnetohydrodynamic or thermo-magnetohydrodynamics.

**A.0.1 MHD grading**

As we learned in chapter 2, whether we are in the presence of perfectly insulated walls, perfectly conductive, or an intermediate case, we note that the relevant velocity gradients are present only within the two layers (\( \delta_{Ha} \) and \( \delta_{side} \)).

In the bulk core, velocity gradient is not present, so that we can apply a grading (more or less generous) in this region. Equation A.1 and A.2 have been applied. Now the \( C_{\text{min,bulk}} = C_{\text{max,layer}} \). A bi-dimensional grading is applicable thanks to the simpleGrading tool implemented in blockMesh. Similar treatment was reserved for the remaining regions. In figure A.2 results of our method.

The most expert and critical eye will not have missed a detail: the angle, in this mesh, presents a "waste" of cells, thus increasing the computational cost. This is certainly true, however, and an intrinsic "waste" that manifests itself using blockMesh. The need to create a consistent mesh and to make the blocks fit together correctly, leads to the creation of zone with more cells than is necessary.

**A.0.2 Thermo MHD grading**

Now the situation is much more complicated. The temperature gradients are present in each region and they are quite strong (\( Gr \approx 10^9 \div 10^{11} \)). This mean that we are not more allowed to apply a full gradient in bulk, FCI, GAP and Wall regions. Considering that heat flux is directed along the z axis, (ie perpendicular to magnetic field), the very strong variation of temperature is in the same direction. What we do is apply a small grading to y-axis and no-grading to z-axis.
In figure A.4 is shown a case with $Ha=300$. This was done only for a certain visual comfort, not to "crush" the cells too much and not to make the image "too blue".

### A.0.3 Grading in flow rate direction

As each CFD analysis, also here an optimization process, from the computational cost point of view, was made. In the flow rate direction, that for us stands for $x$–direction, only a three cell, uniform-grading was applied. This type of strategy is adopted when we are interested in a fully developed solution. This aspect found an answer in a simple keyword: "cyclic boundary condition". To understand this particular type of boundary condition (BC) we can resume the entire
process in three steps:

**Step 1**
*Impose an initial map for the velocity field, that in this work is considered uniform, and impose an average velocity to maintain throughout the calculation.*

**Step 2**
*Make a calculation with the algorithm of interest and obtain the outlet values.*

**Step 3**
*Take this outlet values and put them at the inlet. Repeat loop.*

These kind of BC, is widely used in CFD analysis, and its setting depends on the program used. OpenFOAM, in particular, require, at least, three cells. Just think of the steps subdivision just made: at the first iteration, in the first cell, a velocity map is given; then, in the second cell, the values are processed and manipulated according to the equation implemented in the code; at the end, in the third cell, the values are taken and 'moved' to the first cell. Now the second iteration starts with the first cell that has a new maps of values and the loop process can proceed.

In figure A.5 a visual explanation of the process is given.
Figure A.5: x-y plane. Three cells in x-direction needed
In this thesis was used a method recommended by American Society of Mechanical Engineers (ASME) [4]. This method is called Grid Convergence Index (GCI). To conduct this study a representative case was taken, consisting of a PbLi channel surrounded by an electrically conductive wall. A fully developed simulation was performed, considering the presence of conducting wall with $Ha = 2500$, $Re = 355$ and $C_w = 0.25$ The grid has been built in such a way that the cell to cell ratio ($c2c$) has a certain value $A$. Here three different values of $c2c$ were considered:

- $c2c = 1.25$;
- $c2c = 1.1$;
- $c2c = 1.05$.

In this way we will find three different type of grids: fine, intermediate and coarse. B.1. Basically the procedure consists in five steps, that can be found in [4].

<table>
<thead>
<tr>
<th>type</th>
<th>$c2c$</th>
<th>N° cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>fine</td>
<td>1.05</td>
<td>137'120</td>
</tr>
<tr>
<td>intermediate</td>
<td>1.1</td>
<td>63'608</td>
</tr>
<tr>
<td>coarse</td>
<td>1.25</td>
<td>28'984</td>
</tr>
</tbody>
</table>

Table B.1: Grid nodes

Only the salient passages will be described here.
First of all we define a representative grid size \( h \), where

\[
    h = \left( \frac{1}{N} \sum_{i=1}^{i} \Delta V_i \right)^{1/3}
\]

Where \( N \) and \( \Delta V \) are the total number of cells and the volume of the \( i \) - \text{th} cell, respectively.

Now we have three values of \( h \) and we can run three different simulation and proceed to step 2.

From the previous three different simulation we can select a certain variable of interest (here a pressure gradient) and define \( r_{21} = h_2/h_1 \) and \( r_{32} = h_3/h_2 \). A recommendation, based on experience, tells us that this two values should be greater than 1.3. Then we can calculate the apparent order of the method, the extrapolate value of our variable of interest and, finally, the GCI.

In table B.2 the results of GCI are shown.

A sketch of the three different grids are below attached.

It is worth highlighting the three different "densities" of the grids in the reader’s eyes. Using the coarser allows us to save computational time and obtain a reasonably accurate result.
Figure B.1: Coarse mesh. Overall view (a) and corner zoom (b)
Figure B.2: Intermediate mesh. Overall view (a) and corner zoom (b)
Figure B.3: Fine mesh. Overall view (a) and corner zoom (b)
In the section 2.1.1 the hypotheses n° 3, introduce the so-called Boussinesq hypothesis. According to this one, the density variation due to a thermal gradient is present only in the buoyancy term, while in the other is considered constant. For this reason the fluid is considered incompressible and the eq. 2.1a is still applied.

In eq. 2.1b in the body force term, \( b_f \), now we have a new component: the buoyancy term. This one is added to the, yet mentioned, Lorentz’s force. For simplicity, to treat the density variation we use a linear Taylor expansion:

\[
\rho = \rho_o - \left( \frac{\partial \rho}{\partial T} \right)_o (T - T_o) = \rho_o (1 - \beta (T - T_o))
\]

where the subscript \( (_o) \) indicates the reference state and, since its definition of thermal expansion coefficient, 
\[
\beta = - \left( \frac{\partial \rho}{\partial T} \right)_o \frac{1}{\rho_o}.
\]

The force term is \( \rho g = \rho_o (1 - \beta (T - T_o)) g \), where \( g \) is the gravity vector. The relation between this term and \( -\nabla p \) is explicitly stated below:

\[
-\nabla p + \rho g = -\nabla (p - p_{\text{hydrostatic}}) - \rho_o \beta o g (T - T_o) = -\nabla p_d - \rho_o \beta o g (T - T_o)
\]

where \( p_d = p - p_{\text{hydrostatic}} \) is the dynamic pressure. This one is the increase of pressure due to motion of fluid and is the total pressure minus the hydrostatic one [15].

So now, finally, we can state the total equation of momentum that include also the buoyancy term:

\[
\frac{\partial v}{\partial t} + (v \cdot \nabla) v = -\frac{\nabla p_d}{\rho_o} + \nu \nabla^2 v + \frac{j \times B_{\text{ext}}}{\rho_o} - \beta o g (T - T_o)
\]
APPENDIX C. THERMAL COUPLING: $\phi T$-FORMULATION

Before defining the total set of equations used to solve the thermal MHD, we should introduce one last equation: the energy equation. Basically it is an energy balance made between the various energy terms: thermal gain, expansion / contraction, ohmic, viscous, and considering external heat sources. This equation has a name: the first law of thermodynamics.

Manipulating this equation and applying the simplification made in 2.1.1, we will deal with a compact, simple, and elegant formulation of transport equation, having only four terms:

\[
\frac{\partial T}{\partial t} + \nabla \cdot (vT) = \alpha \nabla^2 T + S_{\text{thermal}}
\]

where $\alpha$ is the thermal diffusivity.

In the end we have all the tools necessary to implement the thermomagnetohydrodynamic study: electromagnetism, hydrodynamics, thermofluiddynamics. In other words: we are going to solve this set of equations C.5:

\begin{align*}
(C.5a) & \quad \nabla \cdot v = 0 \\
(C.5b) & \quad \frac{\partial v}{\partial t} + (v \nabla)v = -\frac{\nabla p_d}{\rho_o} + \nu \nabla^2 v + \frac{j \times B_{\text{ext}}}{\rho_o} - \beta_0 g(T - T_o) \\
(C.5c) & \quad \nabla^2 \phi = \nabla \cdot (v \times B) \\
(C.5d) & \quad \frac{\partial T}{\partial t} + \nabla (vT) = \alpha \nabla^2 T + S_{\text{thermal}} \\
(C.5e) & \quad j = \sigma_m (-\nabla \phi + v \times B)
\end{align*}

Note that in C.5 the current density is calculated a posteriori. Is a sort of post-processing of other coupled variables. We will come back and we will better explain this topic in the Algorithm chapter (see 3.3).
Here we will present the manner in which the flow channel insert was treated in such a way that it could have been studied as "sandwich-like". By itself it is, on openFOAM, it is designed as a single component but later, through the use of setFields (already present on openFOAM but modified specifically so that it can operate in multiregion) it was possible to give it the appearance they want and make sure that inside there are three distinct layers.

Starting from the pure MHD study, in figure D.1 a distinction between the various electrical conductivity is clearly visible.

Starting from the left, we can see the orange (EUROFER, rear wall) then red (PbLi in the GAP), orange (EUROFER FCI, GAP interface), blue (Alumina), orange (EUROFER FCI, bulk interface) and again red (PbLi, bulk).

With this panorama, almost no current is able to cross the FCI, so that the Lorentz force is damped and its effects less effective. A sketch is given in figure D.2.

The same strategy was adopted in the calculation of the temperature: the code takes into account the different nature of the three layers and is able to understand where the EUROFER starts and where the Alumina starts.
Figure D.1: Electrical conductivity [S/m] for different material.

Figure D.2: Streamline current
Figure D.3: Streamline current-particular

Figure D.4: Specific heat for three different material of FCI
APPENDIX D. FCI SETTINGS FOR SANDWICH LIKE

Figure D.5: density for three different material of FCI

Figure D.6: thermal conductivity for three different material of FCI
Figure D.7: Thermal diffusivity for three different material of FCI
BIBLIOGRAPHY


[18] openFOAM wiki, *blockmeshdict*.


