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Provision of Ancillary Services with Uncertain Generation

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List of Abbreviations

AS	**A**ncillary **S**ervices
BRP	**B**alance **R**esponsible **P**arty
CDF	**C**umulative **D**ensity **F**unction
CHP	**C**ombined **H**eat and **P**ower
DER	**D**istributed **E**nergy **R**esources
ESS	**E**nergy **S**torage **S**ystems
GHG	**G**reenhouse **G**as
MG	**M**icro **G**rid
MPP	**M**ulti of **P**arametric **P**roblem
MPT	**M**ulti **P**arametric **T**oolbox
NMG	**N**etwork of **M**icro **G**rids
RED	**R**enewables **E**nergy **D**irectives
RES	**R**enewables **E**nergy **S**ources
ULG	**U**pper **L**evel **G**rid

Chapter 1

Introduction

The rising share of Renewable Energy Sources (RES), e.g wind, solar and geothermal, push the limits in the old electric grid. RES constitute important energy resources, but to cope with the volatility of RES it is necessary to increment the flexibility and ability to adapt to sudden changes in the conventional system.
The wide spread of distributed generation and RES introduces elements like bi-directionality, uncertainty and intermittency of the energy flow. These properties are not well reconciled with the passive structure of the old grid, having a negative impact in a feasible, reliable and efficient management of the grid.
Furthermore, the rapid spread of RES requires a growth in the ramping capability of the system enabling the compensation of the variability and uncertainty of these green sources. Moreover, the fast growth in load demand requires increased generation. Conventional large-scale plants able to provide fast response and more power need time to be built and they are expensive. Consequently, the large number of conventional units necessary to compensate the RES spreading can lead to network congestions (Majzoobi, Mahoor, and Khodaei, 2017).
Two possible solutions to tackle the limits of the actual grid are: starting long and expensive reinforcement of the grid, or exploiting controllability of distributed generation. The latter alternative has been intensively discussed as a prominent solution and it is associated with the transition to more efficient grid and smart grid (ENTSO-E, 2016).

The concept of Smart Grid arises because of increasing decentralized energy production based on Distributed Generation. Small production plants are spread over the whole territory, close to the consumers and often micro-generation is based on renewable energies. One of the main advantages of local generation is the reduction of transmission and distribution distances, which reduces in turn line losses and network congestion. Micro-generation systems have increased specially at the residential level. Consequently, the users evolve in "prosumers" producing electricity contributing to their own energy requirements and to support the grid.
A promising method to integrate the new energy distribution system based on distributed generation and to exploit the potential of micro-generation is to collect the local generation and the associated loads in sub-systems or Micro Grids (MGs).
A MG is a small local distribution system, consisting of distributed energy resources, loads and storage devices, that can work in connected-mode (connected to the Upper Level Grid (ULG)) and isolated-mode (disconnected from the ULG). In the connected-mode, the MG is able to interact with the grid operator and to provide support to

the ULG.

Into a MG, the presence of intermittent and unpredictable RES into a MG can cause the inability of self-providing the load demand at some time instants. This would require the purchase of expensive power from the ULG. Furthermore, a single MG might not be able to participate in the ancillary services and energy market, because of the limited internal capacity and the unreliability of intermittent RES.

A prominent solution to the mentioned problems is the cooperation between a new Active Distribution Network (ADN) and a Network of Micro Grids (NMG). The latter would address the issues related to a single MG enabling the usage of several various sources to satisfy the total load demand. At the same time the provision of reliable power to the ULG can be achieved in order to support the grid operations. From the ULG point of view, the ADN is able to exploit the local RES enhancing its flexibility and reducing the necessary investments.

The main scope of this thesis is to compute probabilistic boundaries for the dispatch of active power to an ULG. To that end, algorithm from multi parametric programming are deployed. This programming allows to divide the space of parameters in regions and to obtain an optimal solution in function of the parameters (Pistikopoulos, Georgiadis, and Dua, 2007; Borrelli, Bemporad, and Morari, 2017). The latter will be the output power of the uncertain sources and the power drawn to the ULG. The obtained regions will be analysed coupled to probabilistic forecast of the uncontrollable sources to obtain a result with probabilistic guarantees. The system analysed is composed of a NMG operating connected to an ULG. The control approach focuses on the cooperation among several MGs in order to meet the load demand, while respecting the network constraints, and on providing a reliable output power to the ULG. The main challenges tackled in this control formulation is in the intermittence of the output power of RES from different areas at the same time and the consideration of network constraints. The chosen approach allow an integration of the spatial correlation among uncertainty sources. Additionally, the output powers of RES are not considered as deterministic entities but they are random elements.

The next chapters are structured as follows. Chapter 2 shows the actual situation of the power system. Moreover, the main characteristics of RES as well as the advantages and disadvantages of their integration in the grid are highlighted. The chapter also introduces the concept of MG, of cooperation among MGs and the active role of MGs in supporting the ULG. Finally, a review of the solution proposed in the literature is outlined. Chapter 3 provides some mathematical definitions and theoretical background. Furthermore, it touches on several programming techniques and the properties of the chosen multi parametric programming. Chapter 4 describes the setup considered in this thesis of a network of cooperating MGs. Additionally, the chapter points out some limitations of a deterministic approach in scheduling with uncertainty sources and in providing support to the ULG. Chapter 5 describes the implementation of an optimization problem based on multi parametric programming aiming at an offer of variability range to the ULG. Chapter 6 shows the results of several simulations run to validate the method. Therein, the analysis of a network of two MGs is showed to ease the understanding of the implemented algorithm followed by the simulation of a real network. Chapter 7 sums up the main conclusions about the results and proposes further extensions of this work.

Chapter 2

Motivation

In recent years, an increasing amount of intermittent renewable energy generators has caused many power system problems, especially in the field of operation and system control. Conventional, large power production units, which have been providing stability and reliability of power system, are being replaced with intermittent generators. Maintaining the quality and stability of the power system has been shown as a challenge that requires more flexibility. One of the possible solutions is introduced through provision of support to the Upper Level Grid (ULG) from a Network of MicroGrids (NMG). Part of the scope of this thesis is to investigate the impact of uncontrollable power generated by Renewable Energy Sources (RES) on the operations of power system. Further on, to improve the stability of the ULG proposing a strategy for the participation of a NMG in provision of Ancillary Services (AS).

2.1 Transition from Fossil Fuels to RES

In last decades, global warming has been highlighted as one of the most critical problems that the world has been facing with. Meanwhile, the overall energy consumption has shown dramatic rise due to increasing world population, developing industry and higher standard of living. Most of the energy produced world-wide comes from fossil fuels and in order to deal with global warming, many different directives have been enacted. Among various measures, reduction of Greenhouse Gas (GHG) emissions is one of the central environmental policies. RES represent a prominent solution for fossil fuels and GHG reduction (Alexopoulos, Thomakos, and Tzavara, 2012). As it can be seen in Figure 2.1, it is expected that the share of low-carbon power generation in the world will grow significantly in the future allowing GHG emission to remain constant.

In the transition process towards clean energy, conventional power production is progressively replaced with intermittent generation. The intermittent power generation due to high amounts of RES has technical and economic impact on the entire power system. Consequently, with increased installed capacity of RES, new problems and challenges in terms of stability and flexibility of the electricity grid have been created (Nilsson, Söder, and Ericsson, 2016). In order to maintain, or even improve, reliability of power supply and its quality due to increased presence of

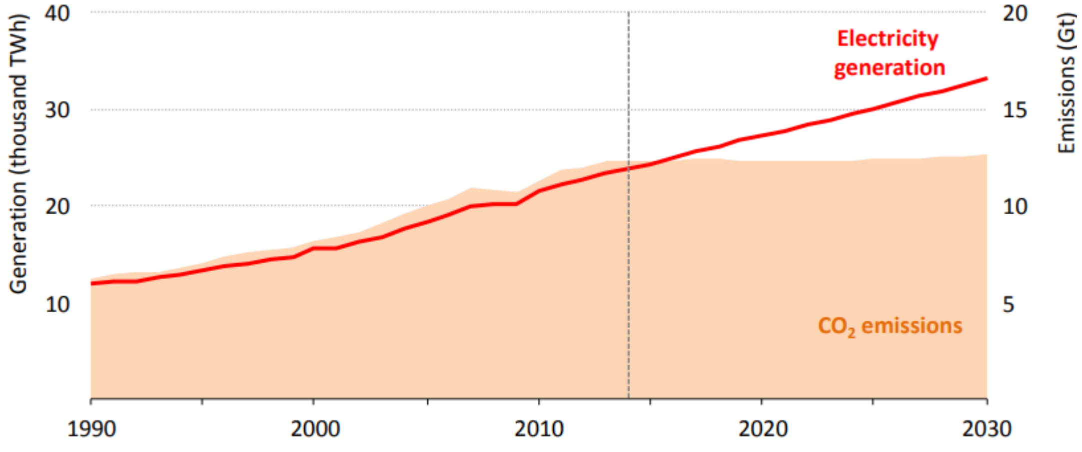

FIGURE 2.1: World electricity generation and related CO_2 emissions (*World energy outlook*).

RES, new strategies are required such as integration and optimal utilization of storage systems into the system and better connection and smart interaction of different Distributed Energy Resources (DER) (Carrasco et al., 2006).

2.1.1 European Directives on Renewable Energy

The Renewable Energy Directive (RED) is a set of legislations which establishes an overall policy for the renewable energy production in the European Union (EU). As for the rest of the world, dealing with climate changes and air pollution is EU key priority in the field of climate and energy. The three main directives enacted in the EU legislation are ”The Europe 2020 Strategy”, the ”2030 climate and energy framework” and the ”Energy Roadmap 2050” (*Climate strategies & targets*). The main targets of The Europe 2020 Strategy in the field of climate change and energy include reduction of GHG emissions by at least 20% compared to 1990 levels, 20% of energy production in the EU based on RES, improvement in energy efficiency by 20% and reaching 10% share of RE in transport sector. When it comes to the 2030 climate and energy framework, the main targets comprise the reduction of GHG emissions by at least 40% compared to 1990 levels, 27% of energy production in the EU based on RES and improvement in energy efficiency by at least 27-30%. One of the main EU long-term goals is the reduction of GHG by 80-95% when compared to 1990 levels by 2050 and this goal is a part of the Energy Roadmap 2050. Achieving the EU targets in the previously stated directives implies a significant reduction of the share of fossil fuels import which will consequently lead to more affordable energy in the EU (*2020 Energy Strategy*). In Figure 2.2, the share of RES till 2015 is shown for 28 European states. Additionally, the targets to accomplish the 2020 RED are presented.

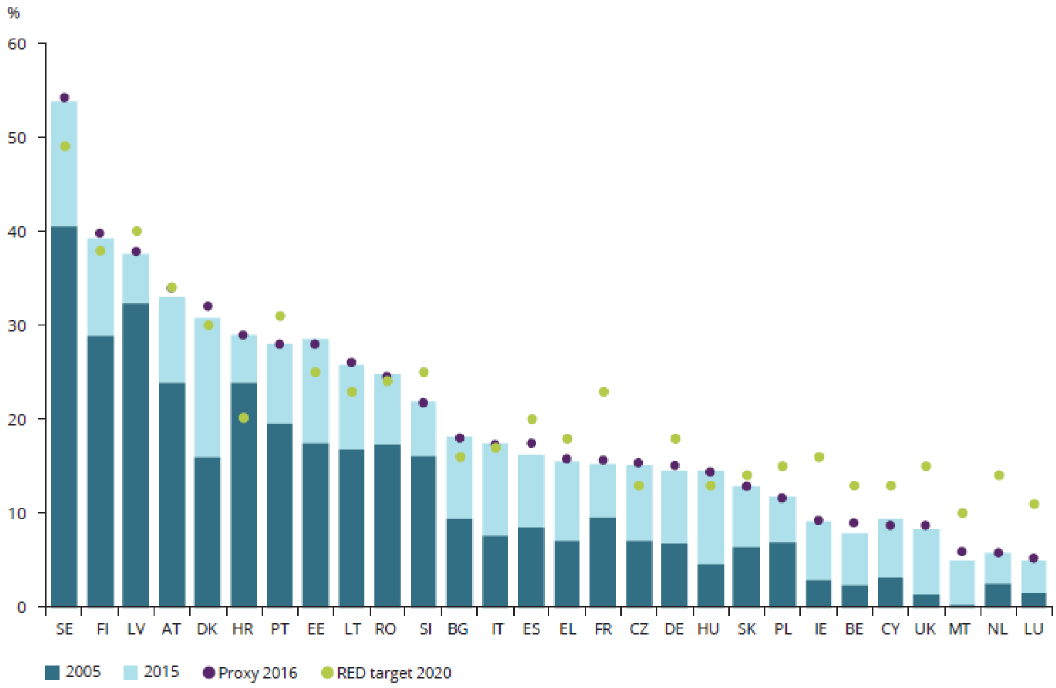

FIGURE 2.2: Recent and predicted RES share in 28 European states (*Renewable energy in Europe – 2017 Update Recent growth and knock-on effects*)

2.2 Renewable energy sources and impact on the grid

2.2.1 Renewable Energy Sources

As stated above, the energy policies aim to replace the fossil fuel generation with renewable generation. RES can be described by means of some main properties:

- Variability of the output power. In fact, RES depend on natural phenomena , like the wind speed and the solar irradiance, that influences the available range of power.
- Uncertainty related to weather phenomena that cannot be completely predicted.
- Interfacing the grid through power electronics devices that lack of inertia.
- Locally produced in specific areas where those phenomena are stronger.
- Providing energy almost to a null cost.
- Varying in a wide range of sizes from very small power rate to medium one.

Controlling the active power output of RES requires advanced forecast techniques. Furthermore, it is necessary the knowledge of the power availability, the system reliability and the actual capacity to control active power.
The predictability is highly dependent on the time scale considered during forecasts

and on the number of uncertainty sources examined (specially if from different areas).
The availability depends on the area, the weather conditions, the system set-up and the season while the reliability is associated to the capability of a RES to operate when and how required (Agency, 2014).

2.2.2 Impact on the Grid

The high rise of RES introduces several issues on distribution and transmission grids, leading to difficulties in operating the network in a reliable and secure way.
There are three fundamental requirements that must be considered in an operating power system. The first one is meeting the demand at all times. The system should constantly balance the generation and the demand during every time scale. Frequency regulation is an AS that is performed by automatic generation control and it consists of a rapid balancing between load and generation (seconds to minutes). Load following consists of varying a considerable amount of power in order to compensate net load deviations from set points (minutes to hours). Unit commitment is the process aimed to meet the predicted load, including possible fluctuations and ramping, when a hours to days timescale is considered (Majzoobi and Khodaei, 2017a).
Secondly, energy should be supplied at minimum cost and ecological impact.
Lastly, power quality must meet certain standards on frequency, voltage and reliability level. Control of power system aims at reliable production and delivery of power while maintaining voltage and frequency within permissible boundaries.

Before the introduction of RES, the variability was mainly imputable to the load demand. The volatility and intermittence of the output power of RES introduced new uncertainties that can endanger regular operations creating instabilities and, in the worst case, blackouts. Conventional generators, such as synchronous machines, can provide rotating inertia and frequency regulation to the system. The same cannot be expected from wind turbines and solar PV modules as they are connected to the grid through power electronics. Consequently, increasing installed capacity of wind turbine and PV as replacement for traditional generation reduces the system inertia. In addition, the usage of converters introduces harmonics affecting the power quality. The bidirectional power flow introduced by RES can causes voltage rises over the limits, congestions when there is low demand and enhancement of grid losses.

As an example, network load deviation at different percentage of wind generation is shown in Figure 2.3. The unpredictable nature of these fluctuations forces the system operator to constantly modify and update the balancing operations (economic dispatch and unit commitment) to keep the network reliable. With higher wind power generation, deviation of system power is larger. This means that more ramping systems would be required to compensate them.

Additionally, both the electrification of the transportation system and the production of the industry increase the demand and consequently the risks for the grid. Thus, this leads to an enhancement in the demand that can overcome the limit of the actual grid for a safe system. Looking, for example, at the distribution network,

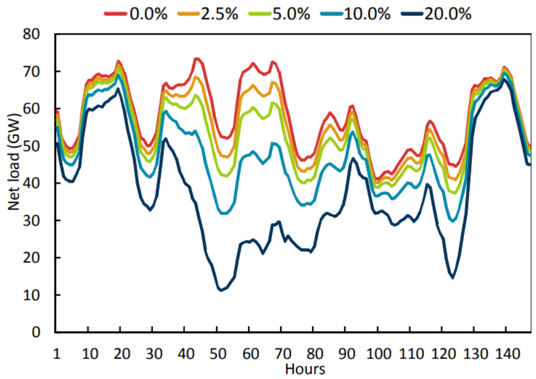

FIGURE 2.3: Impact of wind power generation on system (Agency, 2014)

this was built to afford the uncertainty pattern of classical consumers demand in any circumstances without any issues. With the high penetration of RES and the enhanced generation and uncertainty the grid is not able to ensure safe operations in any conditions. Thus, the situation should be efficiently addressed to improve the actual conditions.

2.3 Hierarchical Control

From the necessity of a paradigm shift in power system with DER and RES a new distribution network with an active role arises. An Active Distribution Network (ADN) is characterized by the capacity to exploit local DER to keep a stable and safe grid.
ADN is an alternative to an expansion and reinforcement of the distribution grid to avoid congestions or other issues (Gemine, Ernst, and Cornélusse, 2014). This investment requires time and may not be the most cost-efficient solution. Thus, the ADN aims to manage the distribution system in an efficient and cost-effective way. To this end are implemented control strategies, based on short-term (up to daily time scale) operations, to adjust the power generation and the load demand to prevent congestions or other limit violations.

In order to regulate the grid operations an ADN can make use of AS provided by grid-connected MGs. Communication between an ADN and MGs consists of three hierarchical levels.

- At the bottom there are the local controllers acquiring information from loads and sources of each MG.
- The second level is occupied by the MG Central Controller (MGCC) which interfaces all the local controllers.
- At the top there is the distribution management system (DMS) that enables to manage several MGs at the same time.

According to this hierarchy the control process can vary from a completely decentralized approach to a totally centralized one.
When each MG acts as an independent entity the local controllers are operating and it is a decentralized control. On the contrary, when a centralized approach is operated the MGCC acquires the information from local controllers and sends them back economical and technical instructions. At the DMS level several MGs are collected and interfaced to the DN.
Using a centralized control allows an efficient coordination because of the full knowledge about information, like power consumption/generation, from the MGs. In order to provide AS to an ADN and to participate in energy market a controllable distributed energy unit is required. In the MG system here described this unit is represented by the flexible device that is able to control its active power and consequently allows the participation in the provision of services (Braun, 2009; Saraiva and Gomes, 2010).

2.4 Ancillary Services

AS are necessary for the regulation of the operations of the grid. In particular, two complementary standard definitions from EURELECTRIC are:

> "Ancillary services are all services required by the transmission or distribution system operator to enable them to maintain the integrity and stability of the transmission or distribution system as well as the power quality (Group, 2004)."

> "Ancillary services are those services provided by generation, transmission and control equipment which are necessary to support the transmission of electric power from producer to purchaser (EURELECTRIC, 2000)."

A main basic clarification about the terms system services and ancillary services is useful.
The former, from the grid operator point of view, consists of all the services that the system operator provides to users connected to the system. AS are procured by the system operator from connected users in order to efficiently provide system services. In Figure 2.4 this difference is highlighted and a list, containing the commonest ancillary services, is provided.

The main scope of the electric grid is to match, with power generation, instantaneously and continuously the load demand. Therefore, the grid system operator ensures the power demand dispatch maintaining a sufficient active power reserve, in order to prevent the worst plausible contingency (loss of the largest generation or transmission facility).
In power systems, the power balancing reserves are used to decrease the frequency deviation and stabilize the frequency. Conventionally, both manual and automatic, operating frequency reserves, called primary regulation (PR), secondary regulation (SR) and tertiary regulation (TR), are used. In particular:

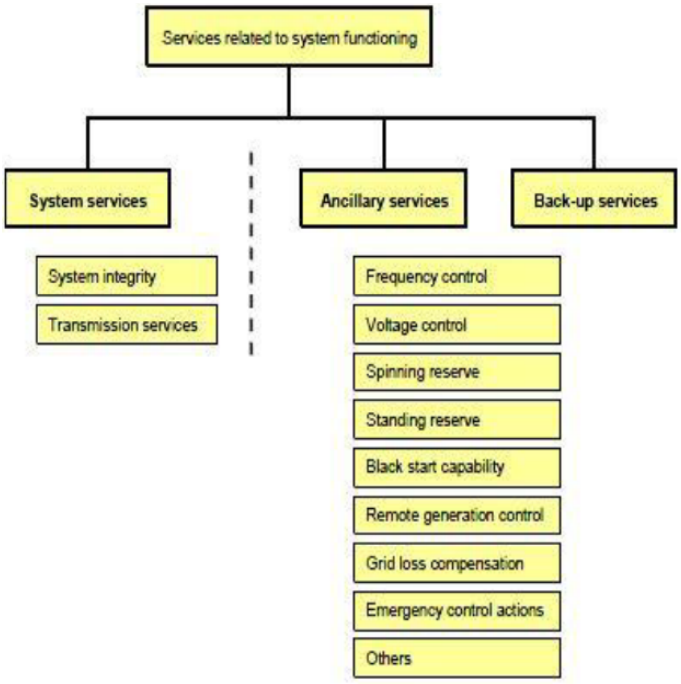

FIGURE 2.4: Services related to system functioning (Pirbazari, 2010)

- PR (frequency containment reserves) are provided by the governors of the generators. These are automatic reserves activated within seconds after the disturbance with complete deployment within 30 seconds after the disturbance.
- SR (frequency restoration reserves) are activated after the frequency is maintained by PR, namely, within 30 seconds to 15 minutes after the disturbance. Purpose of SR are to restore and maintain the frequency to its nominal value and replenish used PR. SR are automatic reserves which are provided by automatic generation control or load frequency control (LFC).
- TR (replacement frequency reserves) are provided manually with tertiary control which is slower than the previous two mentioned. Purposes of TR are to absolve the activated SR. The time scale of TR is from minutes to hours. Figure 2.5 shows different types of reserves activated by different controls as explained above (Das et al., 2015a; Das et al., 2015b).

The reserves need to be replenished fast enough, so the system does not get into an emergency state. Thus, special strategies are needed to prevent instabilities and blackouts. One of those strategies is Underfrequency Load Shedding. Underfrequency Load Shedding is the last option before economic losses and discomfort to the consumers happen. According to ENTSO-E recommendations, Underfrequency Load Shedding starts at 49 Hz and all frequency controls must act before the mentioned limit (*Frequency Stability Evaluation - Criteria for the Synchronous Zone - Requirements and impacting factors*).

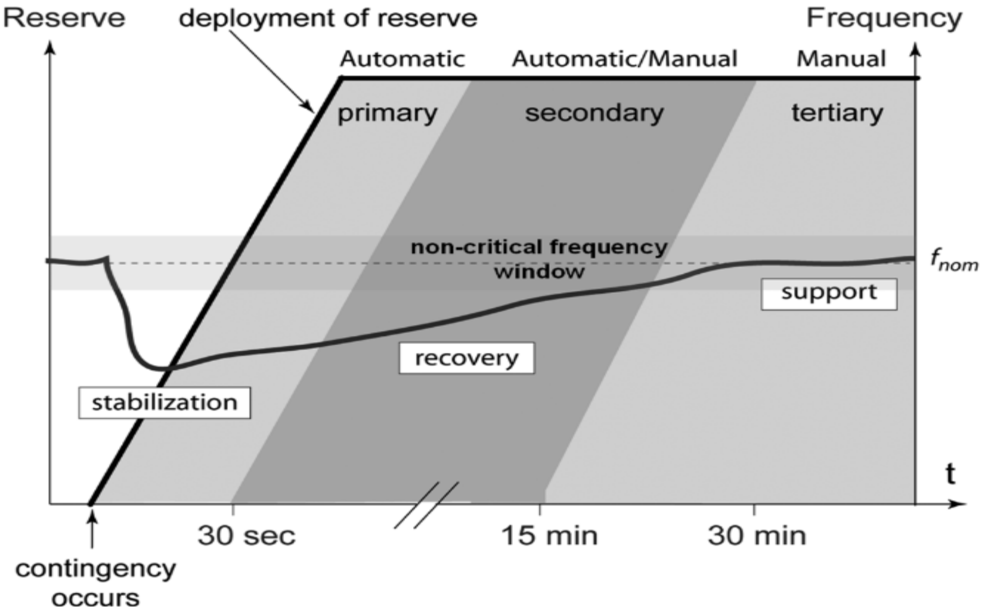

FIGURE 2.5: Activation of different types of reserves (Oudalov, Chartouni, and Ohler, 2007).

Spinning reserve is also included in contingency services, i.e., it is activated when there is an unpredicted outage of generators and tie-lines. It is a reliable reserve source and quickly available (no start-up delay) and gets the full capacity within 10 minutes.

Frequency regulation, together with spinning reserve, forms part of the services with almost zero energy. Both of them are exploited to compensate random and unrelated power deviations of the grid. They are both quickly and randomly activated, that is, the power request will promptly increase or decrease and the long term energy consumption is roughly zero (Pirbazari, 2010; Lin, Leung, and Li, 2014).

2.5 Distributed Flexibility Sources

As previously stated, the ADN represents a solution that controls DER to avoid, for example, the congestion of the distribution network. However, DER can be exploited in different ways. In fact, one possibility is curtailing the generation of RES. Undoubtedly this is an undesired operation because it consists in wasting "free" power. As an alternative Energy Storage Systems (ESS) are able to provide power (discharge) when the RES production is low, and absorb power (charge) when the production is high. Several drawbacks, like the maintenance cost, the limited capacity and the efficiency, make ESS a partial solution. However, a more efficient operation could be to use the flexibility provided by distributed generation in such a way to postpone and limit the necessary investments on reinforcement of the grid.

Furthermore, these flexibility products can support the operations of the system operators providing AS.

According to EURELECTRIC:

> "On an individual level, flexibility is the modification of generation injection and/or consumption patterns in reaction to an external signal (price signal or activation) in order to provide a service within the energy system. The parameters used to characterize flexibility in electricity include: the amount of power modulation, the duration, the rate of change, the response time, the location etc."(EURELECTRIC, 2014)

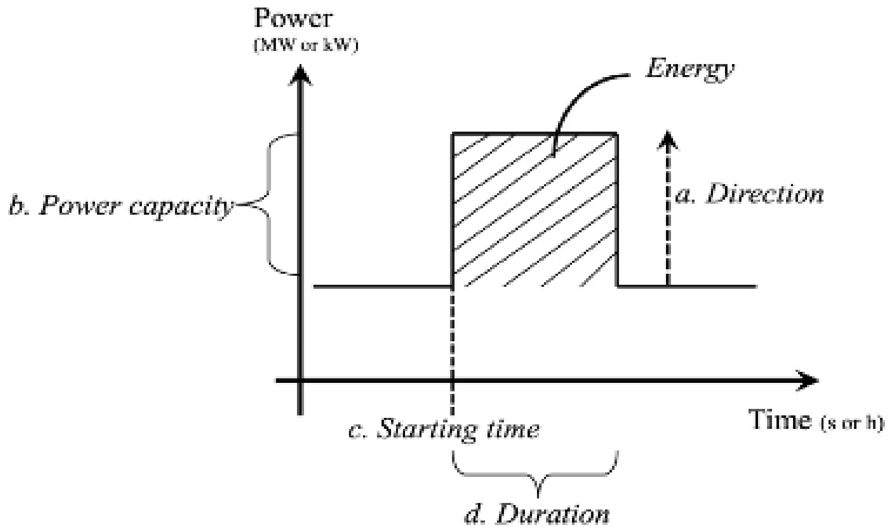

FIGURE 2.6: Main properties of a flexibility service (Eid et al., 2016).

The conventional passive consumer has now the capability to produce electricity becoming a prosumer; the consequence is a bidirectional power flow from the prosumer to the grid and vice versa. This player could provide flexibility to the grid along with other controllable sources, as storages, or by scheduling techniques able to predict in advance the production and adjust the power in real time.
The desired flexibility could derive from domestic or industrial prosumers. The stakeholders of these services could be the grid operator (TSO,DSO) or other suppliers. Individual users could interact directly with the interested part or through an aggregator that collects the flexibility bids from the prosumers and interface the stakeholders. In (EURELECTRIC, 2014; Force, 2015; Amicarelli, Tran, and Bacha, 2018; Roald et al., 2017) are addressed the main challenges to break down the barriers to enable the employing of flexibility services from distributed prosumers and the participation in energy and ancillary services markets.
The Balance Responsible Party (BRP) is designated to provide balancing between generation and demand to a group of users. The system operator has to maintain the balance over an assigned area and prevent violations of constraints in the area. These are examples of stakeholders interested in purchasing the flexibility services. Both were used to buy flexibility from conventional power plants but the introduction of RES has modified this trend. Both also purchase flexibility from MGs or

virtual power plants in specific markets and with short term transactions, differing from the long term ones required in energy markets. The main scope of these players is ensure enough reliable generation capacity to satisfy the load demand at all times.

Deviations from the balance can cause frequency drop if the demand is higher than the generation and frequency rise in opposite conditions. To restore the frequency and secondary/tertiary reserves flexibility services from distributed sources could be used. Reactive power control, congestion management and grid losses are other operations that can be addressed exploiting the mentioned services. The providers of flexibility are remunerated based on the prices/rules of the market and on the service that they can contribute to (e.g. demand adjustments, frequency regulation and so on). In Figure 2.7 a list of the services, the providers and the stakeholders is illustrated.

Service that can be provided	System user that can offer this service	Function that this product can fulfil	Flexibility user requiring this service
(E) Peak shifting (i.e. shifting the peak demand) (G) Peak shifting	Aggregated (or individual) industrial and commercial users Aggregated domestic customers	Long term congestion management. Portfolio optimization Generation capacity adequacy	DSO BRP TSO
(E&G) Demand adjustments – manually/automatically	Aggregated (or individual) industrial and commercial users Aggregated domestic customers	Short term congestion management Portfolio optimization Generation capacity adequacy	DSO BRP TSO
(E) FC/FRR/RR balancing services	Aggregated or individual industrial and commercial users Aggregated distributed generation	Frequency control	TSO
Generation adjustments	(Aggregated) distributed generation	Short term congestion management Grid losses reduction	DSO TSO
(G) Biogas injections	Distributed generation	Long term congestion management Portfolio optimization	DSO TSO BRP
Curtailment products	(Aggregated) distributed generation (Aggregated) industrial and commercial users Aggregated domestic customers	Short term congestion management	DSO TSO
Reactive power (mandatory)	(Aggregated) distributed generation	Voltage control	DSO TSO

FIGURE 2.7: Correlation among services that prosumers could provide and the interested parties, where E=Electricity and G=Gas systems (Force, 2015).

The aggregation of several distributed sources or prosumers enable the small entity to participate in energy, ancillary and flexibility markets. These users are usually characterized by a limited capacity. The uncertainty in their production is too high to make them reliable prosumers. Even if they could compensate these issues, the cost for transaction to join the markets is usually not affordable by the single user. Thus, because of the previous problems single users are not able to sell their flexibility products in the most remunerative markets. Aggregating the prosumers could reduce the total uncertainty of the aggregated system. The volatility of one user could be compensated by the other users of the system or by flexible devices into

the system. This could lead to a reliable aggregated structure able to provide certain amount of power with an high probability. The aggregation is an optimal way to overcome the limited capacity of one single low/medium voltage resource enabling the participation in provision of flexibility and other services.
In order to join the flexibility markets, the cooperation among several controllable (e.g. storage systems, electric vehicles) and uncontrollable devices (e.g. loads) should be exploited. A MG is a prominent solution to aggregate several sources and to actively manage their production.

2.6 Microgrid and its Components

According to (Lasseter, 2002) a MG is defined as a local distribution system, composed of distributed energy generators, storage devices and loads, that can work in two modes: either isolated mode, i.e., disconnected from the main distribution system (ULG), or connected mode, that means with a direct connection to the ULG.

The concept of MG has been introduced as a promising mean to exploit the intermittent nature of RES.
A MG is composed of loads and energy sources operating as a single controllable system aiming to provide electricity and heat in a local area. The main advantages of a MG are an improvement of the economic efficiency and the optimization of the usage of the resources. In fact, since the energy is consumed where is produced, the cost for the energy transportation is reduced. Furthermore, the improvement in controlling generators and loads leads to a better quality and continuity of service. The management of the bidirectional energy flow enables to complement conventional energy flow from producers to users, a parallel flow from the same users towards other users.
Other advantages are provided when considering several MGs; for example, the number of blackouts could decrease as MGs are able to operate disconnected from the ULG. In addition, since several generation units constitute the MGs, the possibility of losing great amount of power at a time is reduced.
A network of cooperating MGs could overcome the limits of one single unit. The volatility of one MG could be compensated by the flexible devices of the other MGs. Regarding to a large number of RES could reduce the whole uncertainty. In fact, the forecast over several uncertainties sources will be more accurate and the intermittences of one source can be mitigated by opposite deviations of the uncertainty sources in other areas. A minimum capacity could be required to provide services to the system operators; a NMG, in contrast to one unit, could meet this threshold of capacity.
In the following sections the main elements composing a MG, that is generation, storage and loads, are briefly illustrated.

2.6.1 Generation

Plants for distributed generation can be classified, according to the primary energy source, in plants from conventional sources (fossil fuels) and from renewable sources. The process of production is another element that allows a twofold classification in two categories: on the one hand, plants using thermodynamic processes, based primarily on fossil fuels usage for electric energy production; on the other hand plants where thermodynamic cycles are not necessary, as the ones using renewable sources or fuel cells. The power output of RES is controllable in a limited way because of the intermittent nature of their sources. The curtailment of this power is not recommended and is unwanted, since the environmental impact and the operation costs are low. Although curtailing is realizable, the output power from RES should be considered as a lower bound on energy generation in supply/demand balancing. On the contrary, the output power from fuel cells or from microturbines (dispatchable generators) is strongly controllable, with limits only due to the technologies used.

2.6.2 Storage

Energy storage systems (ESS) play a key role in improving the flexibility and efficiency of energetic systems and the usability of different energy sources. In fact, electrical energy is an highly versatile source, that can be converted in other forms of energy (e.g. mechanical, thermal, lighting) with high efficiency, and can be utilized far from the production centre. The main drawback is, with rare exceptions, the difficulty in accumulating it. Energy storage technologies differ based on the scope to which they are destined: overcome within fractions of second the fluctuations or the interruptions in the provision of electrical energy (power quality); ensuring continuity of service (system stability); adjust the electricity provision to the demand from users (energy management). ESS play a main role in exploiting widely and in large scale renewable sources, especially for wind and solar systems characterized by unpredictability of the weather conditions. A common solution is complementing the production from RES by an electrochemical ESS, called Battery Energy Storage Systems (BESS).

2.6.3 Loads

Loads can be classified in uncontrollable loads and controllable ones. The first are characterized by a time varying demand with a fixed amount at any instant that has to be met. On the contrary, controllable loads are flexible, i.e., the request can be modified by curtailing them (load shedding) or shifting it in time (load shifting). Since the capacity to adjust the output power of RES is limited, controllable loads could compensate this limitation enabling the so-called Demand Side Management. The main concept correlated to the latter is the demand response, that means varying the amount of energy exchanged from the load based on the availability (or price) of energy. Consequently, the energy pattern of the conventional final-user mutates according to the availability of energy. As stated before, load shifting and load shedding are the main techniques to control the flexible loads. The first consists in moving the consumption of high wattage loads to different times within a period of time. It does not lead to reduction in the net amount of electricity consumed. In load shedding instead, loads are prioritized, and if the total load starts increasing, a lower priority load, such as the uncontrollable part of a load, can be temporarily shut down for a certain period to reduce the total load.

2.7 Microgrid Energy Management and Market Operations

It has been stated that MGs can communicate with the ADN to provide AS and the hierarchy in the communication process has been described. This section briefly explains the operations of a NMG to participate in markets.
In particular, energy management denotes all the operations necessary to manage energy production and energy consumption units.
Two of the most important control operations of the energy management are unit commitment and economic dispatch. The latter consists of determining the output power of sources aiming to minimize the overall cost to meet the total demand. Economic dispatch refers its optimal scheduling to one particular load, but this load is varying from one instant to another and from one day to another. In order to meet these variations, a different number of sources have to be brought in or shut down. Unit commitment is a binary decision process that establishes the order with which the units are to be connected and the order in which the units are to be disconnected over a period of time (e.g. one day) in order to minimize the total operating cost (Saraiva and Gomes, 2010).

If it refers to a MG, energy management is the optimal scheduling of the different components (loads, sources and storages). The latter are programmed to satisfy the following goals: meeting load demand, minimizing the total cost, improve the usage of RES to replace conventional sources, and making the system cost-efficient by selling electricity to the ULG. MG energy management is usually operated interfacing to the MG system a MGCC coupled to local controllers for loads and sources (ESS and RES). MGCC communicates to either the upstream distributed market system (DMS) and MG local controllers.
In market operations, MGCC receive generation and load information from MG controllers. If there is a centralized control, the aggregator (e.g. DMS) calculates the flexibility of the connected MGCCs and sends the bid to the AS market. The system operator performs the day ahead market collecting all the bids and establishing the market clearing price. Thus, it sends the economic dispatch with all the accepted bids, with the right amount of power assigned to each player, to the aggregators. The market operator also verifies that the system constraints are never violated. In order to prevent violations, a secondary real time market is activated. The aggregators or the MG agents provide generation and load bids that are accepted according to the active losses and reactive power balancing necessity, or if a load interruption is requested. The purchased bids are divided by the aggregator over the connected units (Saraiva and Gomes, 2010; Braun, 2009).
Concluding, the intermittent nature of RES, the load demand uncertainty and the fluctuations in market prices lead to new issues in the economic management of a MG with respect to the energy management of the conventional grid.

In particular, it is challenging to determine the total power flexibility of a system made up of units from different zones.

2.8 Literature Review

In the literature it is possible to find several approaches to actively manage the distribution network exploiting the flexibility of DER. The document (EURELECTRIC, 2014) gives a clear definition of a flexibility service. Moreover, this work points out the main challenges about market rules with respect to the participation of small prosumers in balancing operations and in maintaining the constraints of the grid. Along the same lines, the extensive work in (Force, 2015) shows the advantages for the stakeholders (e.g. BRP, grid operators) in purchasing power from different prosumers. In parallel, it proposes the rights that should be guaranteed to the providers of flexibility services. Additionally, both works highlight the importance of aggregation and cooperation among several prosumers to access the market of AS. That is the reason why most of the successive studies review groups of prosumers cooperating with each other.

In order to implement the guidelines stated in the previous reports, (Amicarelli, Tran, and Bacha, 2018) proposes a structure for a new flexibility market to include the flexible products provided by prosumers, MGs and virtual power plants. This strategy is considered an alternative solution to the grid reinforcement. Furthermore, the method could increase the incomes of the prosumers and arise the competition of players that will sell their services to lower prices.
Similarly, (Majzoobi, Mahoor, and Khodaei, 2017) proposes to consider the distribution market operator as an instrument to collect the ramping capability of MGs. Additionally, the market operator interfaces the MGs with the ULG in order to support the ramping issues. The scheduling algorithm formulated for the market operator aims to maximize the profit of the MG. However, the ramping bid of each MG is assumed as assigned and it is not specified how to deal with the uncertainties in this calculation.

Even though the mentioned papers are proposing solutions to practically introduce flexibility services from DER in the conventional markets, the question of how to compute these products is not investigated in these works. Nevertheless, this is a well treated research question, several methods and different providers are presented in the literature.

(Sortomme and El-Sharkawi, 2012) and (Janjic and Velimirovic, 2015) formulate a bidding strategy to provide AS using a fleet of electric vehicles connected to the grid. The algorithms implemented aim to maximize the profit while supporting the operations of the grid. However, the beneficiary of that maximization in (Sortomme and El-Sharkawi, 2012) is the aggregator of electric vehicles while in (Janjic and Velimirovic, 2015) is the owner of the fleet. Differently, (Wang et al., 2016) look at the electric vehicles as flexible devices of a multi MG system. The method proposed there tries to avoid spikes of power exchange with the main grid considering the cooperation among MGs. To this end, a two-stage scenario-based method is implemented.

The provision of AS from a MG could be more remunerative than the only participation in the energy market (Qin, 2015). The techniques to include these services in the scheduling algorithm could be different.
For example, (Majzoobi and Khodaei, 2017b) regards the single MG as an instrument to compensate the fluctuations in the load/generation profile of other prosumers and consumers. This approach is proposed as well as an alternative to expensive reinforcement of the grid. The key aspect is to model the variability of the load pattern of prosumers and consumers as constraints in the scheduling of the MG. The focus is in providing ramping capability, but the uncertainties are not examined.
With a similar idea, (Majzoobi and Khodaei, 2017a) tackles the support to the utility grid through AS introducing specific constraints in the optimization problem of a MG. Thus, deviations in the net load of other customers are captured to set the flexibility constraint for the MG and providing frequency regulation up to 1 min. On the contrary to the mentioned works, the network constraints are here taken into account. However, the uncertain generation is still modelled through scenarios.

Most of the reported works consider the market as the starting point to calculate the flexibility services of the MGs. In fact, the power scheduling of the MG is computed based on unbalances or forecasting errors during the market operations. However, a market could be not available especially in the context of flexibility services (Majzoobi and Khodaei, 2017b). In fact, the market of flexibility products is still developing and limited to some regions. This thesis introduces a different point of view, where the main scope is the efficient management of the resources of several MGs providing at the same time the necessary or desired services to the ULG.
It is worth mentioning that there are plenty of works in the literature that look only at the analysis of a system without considering its participation in supporting the ULG. These works mainly focus on economic dispatch, optimal power flow and energy management.

One example is (Fathi and Bevrani, 2013), where the authors address the cooperation of MGs aiming to minimize the operational cost through a load demand management. The stochastic nature of demands and generation is considered but the problem is solved only through a real-time algorithm given unveiled uncertainties.
Similarly, (Rahbar, Chai, and Zhang, 2016) investigate the cooperation among two MGs as an appealing solution to the volatility of RES. The approach consists of an off-line optimization problem including network constraints followed by an online algorithm for real time management. Nonetheless, in the first step the uncertainties are considered known while in real time is introduced a deterministic noise to simulate the error in the realization of the energy profile. A clustering approach is considered to extend the solution to more than two MGs where the MGs are analysed two at a time.
In (Nikmehr and Ravadanegh, 2015), the authors perform the economic dispatch of a

system of MGs that can exchange power with each other. In particular, an optimization problem is implemented where the uncertainty sources are modelled relying on assumptions on their distribution. A particle swarm optimization is adopted to minimize the operational cost. The results show a reduction of cost when several MGs are cooperating.

Another characteristic of the previously reviewed papers is the choice of modelling the uncertainty variables by assuming a fixed set of scenarios. However, this assumption could restrict the solution. In fact, if a condition different from the scenarios formulated verifies the response of the system it is undetermined (Vrakopoulou and Hiskens, 2017; Liu et al., 2016). Moreover, employing a worst-case method to assess the uncertainties could lead to conservative results because the solution should be verified for every case (Liu et al., 2016). The usage of a specific distribution for the uncertainties has been validated as an inaccurate method that can result in insufficient solutions.

The multi parametric programming has been employed in several works (Pistikopoulos, Georgiadis, and Dua, 2007; Borrelli, Bemporad, and Morari, 2017) even if in different fields with respect to the one of this thesis. The novel ideas introduced and developed in this thesis move from the method presented in (Vrakopoulou and Hiskens, 2017). Therein, the authors employ multi parametric programming to calculate the stochastic optimal power flow of a network including line constraints and the uncertainty of wind power without assumptions on the distribution. In particular, a specific control strategy is formulated that allow to consider the uncertainty from different sources at the same time. In addition, a spatial correlation among scattered sources can be included.

2.9 Objective

This thesis is about the investigation of the capability of a system of MGs to provide flexibility to the ULG.
The goal is to compute reliable dispatchability boundaries of the active power output to the ULG while satisfying power balance and network constraints. The main scope of this work is the computation of optimal power dispatch schedule and the control of a NMG, grid connected, where each MG is constituted of inflexible generation/demand coupled to a flexible element.

With the term ”dispatchability” it refers to the capacity of a power-facility to provide required amounts of power (at or below the facility’s nameplate rating) on demand of the grid operator regardless of the time of day or weather conditions. RES power plants are not dispatchable because they depend on natural phenomena and are not always available.
”Reliability” refers to the probability of a device or system to perform its operations adequately, for the planned period of time, under the planned operating conditions (Prada, 1999).

The flexible element of a MG combined to the possibility of a cooperation and exchange of power among MGs should made RES power plants dispatchable and the net output power from the NMG to the ULG reliable.

In conclusion, even if representing a good starting point, the papers proposed in Section 2.8 leave several aspects unsolved. As stated in most of these works, the uncertainties are not considered or modelled by scenarios or assuming specific distributions. Moreover, the spatial correlation of random units is neglected. Thus, this thesis proposes a method where no assumptions are made over the distribution, to improve the accuracy of the results, and implements a method that allows the consideration of spatial correlation among the uncertainties in the NMG. In addition, the network constraints are included in this thesis while disregarded in most of the previous works. On the contrary to the mentioned papers that only consider the provision of AS in function of the market fluctuations or overlook the support to the ULG, this thesis proposes a method to compute a variability range independently of the presence of a market, meaning that the main focus is on efficiently distributing the active power among several MGs to obtain the desired power output.

Chapter 3

Mathematical background

In this chapter, most relevant mathematical definitions are presented. In addition, a description of the main categories of optimization problems is outlined. Thus, the chosen multi parametric programming is described with the main properties and advantages with respect to sensitivity analysis. The final part of the chapter shows the structure of a general multi parametric optimization problem.

3.1 Basic terminology and definitions

In this section basic definitions are given. Even though these definitions are standard, they are collected here in order to ease the reading and understanding of the thesis.

Definition: A set $C \subseteq R^n$ is said to be *affine* if the line through any two distinct points in C lies in C

$$\forall \mathbf{x}, \mathbf{y} \in C, \quad \lambda \in \mathbb{R} \quad \Rightarrow \quad \lambda \mathbf{x} + (1-\lambda)\mathbf{y} \in C \tag{3.1}$$

Definition: A set is said to be an *affine hull* of a set $C \subseteq \mathbb{R}^n$ if it is the smallest affine set that contains C

$$\textit{aff}(C) := \left\{ \sum_{i=1}^{k} \lambda_i x_i | x_i \in C, \lambda_i \in \mathbb{R}, i = 1, \ldots, k, \sum_{i=1}^{k} \lambda_i = 1, k \in N \right\} \tag{3.2}$$

An affine set in $\mathbb{R}^n$ containing the origin is a subspace of $\mathbb{R}^n$.

Definition: A function f: $\mathbb{R}^n \rightarrow \mathbb{R}$ is said to be *affine* if it can be expressed in this form

$$f(\mathbf{x}) = A\mathbf{x} + b \tag{3.3}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$.

Definition: A set $C \subseteq \mathbb{R}^n$ is said to be *convex* if the line segment connecting any pair of points of C lies entirely in C

$$\mathbf{x}, \mathbf{y} \in \mathbb{R}^n, 0 \leq \lambda \leq 1 \Rightarrow \lambda \mathbf{x} + (1-\lambda)\mathbf{y} \in C \tag{3.4}$$

Definition: A set is said to be a *convex hull* of a set $C \subseteq \mathbb{R}^n$ if it is the smallest convex set that contains C

$$co(C) := \{\sum_{i=1}^{k} \lambda_i x_i | x_i \in C, \lambda \geq 0, i = 1, \ldots, k, \sum_{i=1}^{k} \lambda_i = 1, k \in N\} \tag{3.5}$$

Definition: A function f: $C \rightarrow \mathbb{R}$ is said to be convex on C if its domain $C \in \mathbb{R}^n$ is a convex set and if

$$\forall \mathbf{x}, \mathbf{y} \in C, 0 \leq \lambda \leq 1 \Rightarrow f(\lambda \mathbf{x} + (1-\lambda)\mathbf{y}) \leq \lambda f(\mathbf{x}) + (1-\lambda) f(\mathbf{y}) \tag{3.6}$$

Definition: A set is called a *hyperplane* if it can be expressed in this form

$$\{\mathbf{x} \in \mathbb{R}^n | a\mathbf{x} = b\} \tag{3.7}$$

where $a \in \mathbb{R}^n$, $a \neq 0$, $b \in \mathbb{R}$.

Definition: A set is called a *half-space* in $\mathbb{R}^n$ if it can be defined in this form

$$H = \{\mathbf{x} \in \mathbb{R}^n | a\mathbf{x} \leq b\} \tag{3.8}$$

where $a \in \mathbb{R}^n$, $a \neq 0$, $b \in \mathbb{R}$.

Definition: A convex set $H \subset \mathbb{R}^d$ is called a convex *polyhedron* (polyhedron) when it is the intersection of a finite number of closed half-spaces

$$H = \{\mathbf{x} \in \mathbb{R}^d | A\mathbf{x} \leq b\} \tag{3.9}$$

where $A \in \mathbb{R}^{n \times d}$, $b \in \mathbb{R}^d$, $d < \infty$. The inequalities are considered component-wise.

Definition: a bounded polyhedron $P \subset \mathbb{R}^n$ is called *polytope* and can be represented as

$$H = \{\mathbf{x} \in \mathbb{R}^d | A\mathbf{x} \leq b\} \tag{3.10}$$

where $A \in \mathbb{R}^{n \times d}$, $b \in \mathbb{R}^n$, $d < \infty$. The inequalities are considered component-wise. A 2-dimensional polytope is called *polygon*.

A polyhedron can be represented in two different forms called half-space representation (H-rep) and vertex representation (V-rep). The first is the description based on inequalities as in the definition. The V-rep is as

$$H_v = \{x = \sum_{i=1}^{v} p\alpha_i V_i^P, 0 \leq \alpha_i \leq 1, \sum_{i=1}^{v} p\alpha_i = 1\} \tag{3.11}$$

where $V_i^P \in \mathbb{R}^n$ denotes the i-th vertex of P, and the total number of vertices of P is v_p. Figure 3.1 shows a 2-dimensional polyhedron in H-rep and a Polytope in V-rep. According to the Minkowski and Weyel theorem, it is possible to convert from one representation to the other one since they are equivalent. Either are necessary because of the different definitions. In fact, with the H-rep it is easier to verify the belonging of a point to a polytope, while with V-rep it is easier to sample points inside P (Baotic, 2005; Borrelli, Bemporad, and Morari, 2017).

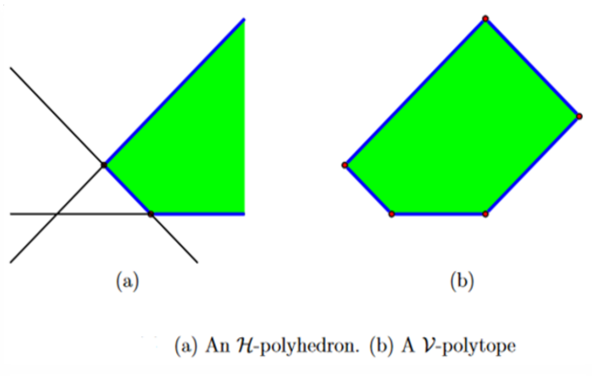

FIGURE 3.1: H-rep of a Polyhedron and V-rep of a Polytope (Borrelli, Bemporad, and Morari, 2017).

3.2 Mathematical Programming

Mathematical programming, or mathematical optimization, is the general context, in which stochastic and parameter programming are included, that consists of all the techniques to select the best (with respect to specific criteria) element from some available alternatives. A large number of problems, regarding the identification and the determination of decisions subjected to different constraints, can be modelled and solved with methods of mathematical programming. It includes all the minimization or maximization problems of a real function of real or integer variables, which are subject to constraints. To solve these kinds of problems using mathematical optimization, three steps can be highlighted: firstly modelling the problem, followed by a mathematical formulation of the previous model, and, as final step, the solution of this model by proper algorithms for the problem analysed. It is possible to include data or condition of uncertainty in a model, generalizing then deterministic models: among these there are sensitivity analysis, simulations, scenarios analysis, stochastic programming and parametric programming (Baotic, 2005).

Definition: A general *mathematical program* (MP) is an optimization problem with the following form:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \leq 0, \\ & h(\mathbf{x}) = 0, \\ & \mathbf{x} \in X \end{aligned} \tag{3.12}$$

where X is a subset of $\mathbb{R}^{n_x}$, $\mathbf{x}$ called *optimization variable*, is a vector of n_x components $x_1, \ldots, x_{n_x}$, and $f : X \to \mathbb{R}$, $g : X \to \mathbb{R}^{n_g}$ and $h : X \to \mathbb{R}^{n_h}$ are defined on X. The function f is usually called the *objective function* (criterion function or cost

function). Each of the constraints $g_i(\mathbf{x}) \leq 0$, $i = 1, \ldots, n_g$, is called an *inequality constraint*, and each of the constraints $h_i(\mathbf{x}) = 0$, $i = 1, \ldots, n_h$, is called an equality constraint. Usually X, called *set constraint*, includes lower and upper bounds on the variables; they are expressed separately from the inequality because of the possibility of some algorithms to handle them in a specific way. The MP problem *domain* is defined as $dom(MP) = dom(f) \cap dom(g) \cap dom(h)$. A vector $\mathbf{x} \in X$ satisfying all the constraints is defined as a *feasible solution* to the problem. Collecting all such point enables to describe the *feasible region (or feasible set)*. The MP problem is feasible if there is at least one feasible point, and infeasible if no feasible point exists. The MP problem (minimization problem in this case), then, is to find a feasible point $\mathbf{x}^*$ (*optimal point*) such that $f(\mathbf{x}) \geq f(\mathbf{x}^*)$ for each feasible point $\mathbf{x}$. The MP problem is said to be *unconstrained* if $n_g = 0$, $n_h = 0$ and $X = \mathbb{R}^{n_x}$. If there are feasible points x_k with $f(x_k) \to -\infty$ as $k \to \infty$ the problem is said *unbounded (below)* (Chachuat, 2007).
Definition: A constraint g_i is said to be inactive at $\mathbf{x}^*$, if $g_i(\mathbf{x}^*) < 0$. A constraint g_i is said to be active at $\mathbf{x}^*$, if $g_i(\mathbf{x}^*) = 0$. An equality constraint is always active.

3.3 Classification of Optimization Problems

Usually when trying to solve a problem that it is a special case, or a specific case of a more general problem, more difficulties in the solution may be encountered. MP is the most general description of an optimization problem. Actually, the majority of the optimization problems, especially the one considered in this thesis, can be traced back to particular categories that are sub-sets of the general MP. Different criteria can be used to classify optimization problems. A brief (not exhaustive) classification is done here based on the forms of the objective function, constraint functions and resolution approach.

Linear Programming

The objective function is linear, and constraints can be expressed by means of linear equalities and inequalities. Even though it is the simplest case among all the categories presented, most of the algorithms connect to this category of programming to solve optimization problems.

Quadratic Programming

The objective function is a second order polynomial (in one or two variables), while constraints are expressed with linear functions. Linear programming problems are a subcategory of quadratic ones.

Convex Programming

In this case the objective function and constraints are convex. This is a wide class of problems, that involves linear programming problems and part of the quadratic ones. These are problems widely and deeply analysed, then algorithms and well known theory have been developed for them. The problem treated in this thesis has convexity properties.

Nonlinear Programming

In this case the objective function and constraints are expressed by means of nonlinear functions. Being a pretty general category, including the previous categories, dealing with it is difficult and sometimes is treatable only using approximated solutions.

Stochastic Programming

Objective function or constraints depend on random variables. This category cannot be included in the previous ones because here the space of alternatives from which the optimum element is taken is different because it includes random variables.
The main scope of stochastic programming is to take optimal decisions about situation that present uncertainties. Stochastic programming is able to adapt to studied phenomena, with precision due to the solid probabilistic theory and stochastic mathematical basis. Stochastic is the opposite of deterministic, i.e. known, and implies that some parameters are undetermined. One element which distinguishes this programming is the hypothesis that some uncertain factors are properly representable by random variables, whose effect is either not or partially controllable and alterable. Stochastic programming is applied in several sectors, among which economy, industrial engineering and in mathematical branches as operative research and statistics.

Parametric Programming

Objective function or constraints depend on (random) parameters. This category mainly differs from the previous one because the probability distribution function is not necessary to solve the problems. This category is going to be used and treated in this thesis.

Integer Programming

In this case the random variables are bounded to assume integer values. Even though this limitation can appear as an advantage, inter programming cannot be included in the conventional categories (linear, quadratic, convex) in terms of used techniques, and its problems present non-trivial solution. The main reason behind this difficulty is that differential techniques are not applicable to integer numbers.

Dynamic Programming

Here the main difference is the approach to the solution. In fact, in this category, problems are addressed by means of an optimization strategy, involving the division of the problem in sub-problems easier to solve.

3.3.1 Parametric programming

Any process system is characterized by uncertainty and variability, usually represented by varying parameters. Modelling a process is the necessary step to translate a process-related phenomena to some descriptive form (quantitative or qualitative) and basically involves elements of uncertainty.
Technical characteristics, fluctuations in resources, market requirements and prices, which can affect the feasibility and economics of a project, are all possible representations of varying parameters. While the description of the uncertainty is itself an important modelling question, the potential effect of variability on process decisions regarding process design and operations constitutes another challenging problem. The two problem cannot be considered uncorrelated: in fact, if an optimal decision is totally insensitive to the presence of uncertainty, building a model to represent the uncertainty is superfluous.

One possible approach to analyse the effect of variations and uncertainty in process-systems engineering problems is parametric programming.
When an optimization problem structure is defined, where the aim is the minimization or maximization of a performance criterion subject to a set of constraints, and where some of the parameters are bounded between an upper and lower limit, parametric programming can be exploited. The latter is a technique for obtaining the objective function and the optimization variables as a function of the parameters, and the regions (called *Critical Regions*) in the space of the parameters where these functions are valid.
The space of parameters is mapped and the optimal solution is defined as an explicit function of the parameters. Once the latter is obtained, the optimization is solved, for a certain parameter value, evaluating the same explicit solution and is not necessary to solve the optimization. In other words, the exact mapping of optimal solution obtained from parametric programming takes the place of the optimization (Pistikopoulos, Georgiadis, and Dua, 2007).

Parametric programming has been developed to overcome the limitations of a sensitivity analysis. In fact, the latter requires changing one parameter at a time in the original model in order to evaluate the effect on the solution. On the contrary, linear parametric programming (or parametric programming) refers to the systematic study of changes in the optimal solution when the value of several parameters is modified at the same time, in a precise interval. Sensitivity analysis stops when it is known what happens if the the process conditions deviate from the nominal values to some value in its neighbourhood, parametric programming is concerned with the whole range of parametric variability. The former associates with the uncertainty and the latter to the variability of the process. This programming is a useful extension of sensitivity analysis and it is possible, for example, to verify the effect of simultaneous changes of correlated parameters.

Multi Parametric Problem Structure

The generic form for a Multi-Parametric Linear Problem (MPLP) or a Multi-Parametric Quadratic Problem (MPQP) is as follows:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \frac{1}{2}\mathbf{x}^T H\mathbf{x} + F\boldsymbol{\theta} + \mathbf{f}\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} + B\boldsymbol{\theta}, \\ & A_e\mathbf{x} = \mathbf{b}_e + E\boldsymbol{\theta}, \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}, \\ & A_\theta\boldsymbol{\theta} \leq \mathbf{b}_\theta \end{aligned} \tag{3.13}$$

where the matrices H, F, A, A_e, A_θ, B, E, and the vectors $\mathbf{f}$, $\mathbf{b}$, $\mathbf{b}_e$, $\mathbf{b}_\theta$, $\mathbf{l}$, $\mathbf{u}$ are the problem data. The vector $\mathbf{x}$ represents the decision variables and $\boldsymbol{\theta}$ is the vector of parameters. The Parametric linear complementarity problem is given to the Multi Parametric Toolbox (MPT) as:

$$\begin{array}{rcl} \mathbf{w} - My & = & \mathbf{q} + Q \\ \mathbf{w} & \geq & 0 \\ \mathbf{z} & \geq & 0 \\ \mathbf{w}(\boldsymbol{\theta})^T\mathbf{z}(\boldsymbol{\theta}) & = & 0 \\ A_\theta\boldsymbol{\theta} & \leq & \mathbf{b}_\theta \end{array} \tag{3.14}$$

where the matrices M, Q, A_θ, and vectors $\mathbf{q}$, $\mathbf{b}_\theta$ are the problem data, then $\mathbf{z}$, $\mathbf{w}$ are the decision variables, and the vector $\boldsymbol{\theta}$ contains the parameters.

Chapter 4

Problem Description

As stated the goal of this thesis is to compute reliable dispatchability ranges of the active power output to the ULG while satisfying power balance and network constraints.
This chapter describes the system setup exploited in subsequent analysis. The main drawbacks of a single MG with respect to a NMG are explained as well as the inaccuracy of a deterministic study.

4.1 Single MG Setup

A single MG adopted in this project is presented in Figure 4.1. It consists of inflexible generation and demand coupled to a flexible system. The generation provided from RES is uncertain and unpredictable, while the demand is represented by an uncontrollable load. The algebraic sum of the output power of RES and the power load is the net output power of the inflexible device. The inflexible element is usually governed by internal settings, this means that the output power is fixed or adjusted according to specific rules. This category include wind turbines, PV generators and residential loads. On the contrary, the flexible element, that can be represented by an aggregation of ESS or generators, have an output power that can be adjusted in such a way to obtain the desired control. The inflexible output power, denoted with $p_l \in \mathbb{R}$, can be composed of a large number of devices of different nature. This aggregation includes all the uncontrollable elements downstream. The output power of the flexible device is denoted with $p_s \in \mathbb{R}$, with positive power flow directed according to Figure 4.1 like p_l. The power balance of a MG , $p_g \in \mathbb{R}$, is given by the algebraic sum

$$p_{gi} = p_{si} + p_{li}, \qquad i \in \mathcal{N}_\mathrm{m}, \tag{4.1}$$

where $\mathcal{N}_\mathrm{m} = \{1, \dots, N_\mathrm{m}\}$.
Throughout this thesis the controllable element is considered as a device able to inject/absorb power within specific limits.

The output powers of controllable and uncontrollable devices are bounded as follows

$$\underline{p}_{si} \leq p_{si} \leq \overline{p}_{si}, \tag{4.2}$$

$$\underline{p}_{li} \leq p_{li} \leq \overline{p}_{li}, \tag{4.3}$$

with $i \in \mathcal{N}_\mathrm{m}$.

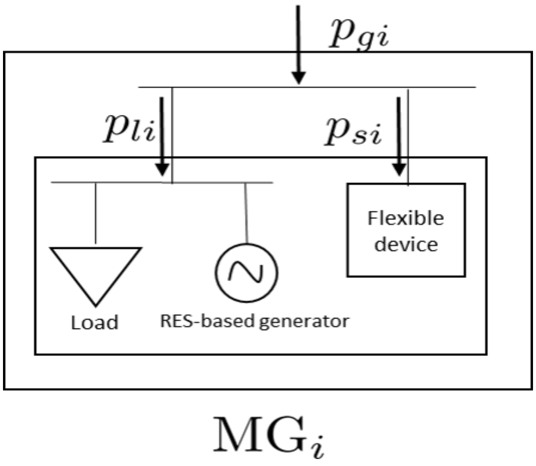

FIGURE 4.1: MG setup with all the main components

4.2 Network of MGs

Consider a NMG as in Figure 4.2 where every MG, described before, cooperates with each other exchanging active power. The network is characterized by nodes and lines interconnecting them. Note that each node could be the connection point of several MGs. In this project the MGs are operated in grid connected mode, meaning that there is always a direct or indirect connection to the ULG. In addition, a radial network is adopted where the ending node of any branch is unique and consequently the number of nodes match the number of branches.
Assume a power network comprising $N_\mathrm{m} \in \mathbb{N}$ MGs and $N_\mathrm{N} \in \mathbb{N}$ nodes.

The main focus of this work is on the analysis of generation and exchanges of *active power* (for the sake of brevity denoted as *power*). Therefore, the connections between nodes are assumed as ideal topological connections with limits on the amount of power that can flow through them. This modelling choice corresponds to a linear representation of the network. Notice that the transmission of reactive power over the lines is not directly modelled in this thesis as well as transmission losses. However, worst case conditions of transmission of reactive power are considered in limiting the flow of active power along the lines (see the following Section 6.2.3).
The usual extension of the NMG allows to assume a lossless system keeping the linearity of the structure. However, it is worth noting that the NMG cannot be treated as a unique element, i.e., considering the entire network as a single MG with generation/demand aggregation. In fact, the distance among them is not neglectable and line constraints have to be taken into account.

The active power injected in the two edges of a line is equal in magnitude and with opposite sign.
The power injection at one node is equal to the sum of the net output powers of

every MG connected to that node. The equivalent expression is

$$p_{ni} = \sum_{j=1}^{N_\mathrm{m}} C_{ij} p_{gj}, \qquad i \in \mathcal{N}_\mathrm{N}, j \in \mathcal{N}_\mathrm{m} \tag{4.4}$$

where $\mathcal{N}_\mathrm{N} = \{1, \dots, N_\mathrm{N}\}$ and matrix $C \in \mathbb{R}^{N_\mathrm{N} \times N_\mathrm{m}}$ is with element $C_{ij} = 1$ if the j-th MG is connected to node i-th, and zero otherwise. The power through one generic line is formulated as

$$p_{bi} = p_{ni} + \sum_{j=1}^{N_\mathrm{N}} B_{ij} p_{bj}, \qquad i, j \in \mathcal{N}_\mathrm{N}, \tag{4.5}$$

where the matrix $B \in \mathbb{R}^{N_\mathrm{N} \times N_\mathrm{N}}$ is with element $B_{ij} = 1$ if the j-th branch is directly connected to node i-th, and zero otherwise. The line rating is taken into account and it bounds the power flowing through the lines.

$$\underline{p}_{bi} \le p_{bi} \le \overline{p}_{bi}, \qquad i \in \mathcal{N}_\mathrm{N}. \tag{4.6}$$

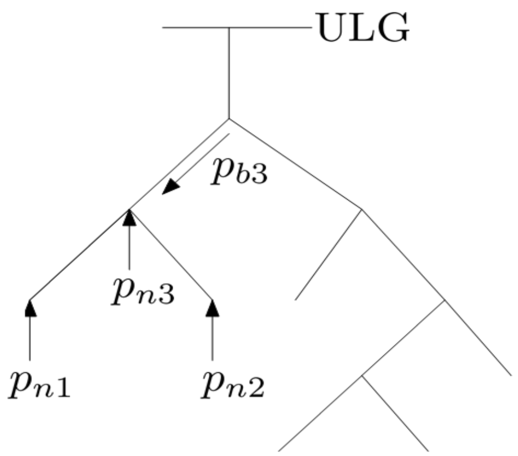

FIGURE 4.2: General topology of a radial grid

4.3 Control Actions

Throughout the thesis all controllable elements are represented by the output power of the flexible devices. An implicit feedback strategy for these powers can be generally written as

$$p_\mathrm{si} = f(\mathbf{p}_\mathrm{l}, p_{g0}), \qquad i \in \mathcal{N}_\mathrm{m} \tag{4.7}$$

where $\mathbf{p}_\mathrm{l} = [p_{l1}, \dots, p_{lN_\mathrm{m}}] \in \mathbb{R}^{N_\mathrm{m}}$, while p_{g0} represents the power flowing to/from the ULG. The expression refers to one flexible device and can be extended to all the controllable elements into the NMG. The exogenous signals of this implicit feedback strategy are: the generation/demand, dependent on uncertain weather conditions and on time of the day, of all the inflexible elements of the NMG and the power exchanged with the ULG. The latter can vary based on the demand of the ULG for a

certain amount of power to accomplish, for example, to a certain balancing service or to tackle a congestion problem.
Notice that the implicit feedback strategy could require other exogenous signals in input, in order to guarantee efficient system stability. For example, if the voltage regulation is addressed the reactive power is needed. However, only the terms of active power are expressed in (4.7), and the reactive power will be implicitly considered by testing the branch constraints.

The controllable power is adjusted according to the implicit feedback strategy $f(\cdot)$. This implicit feedback strategy describes in advance how the power of the flexible elements reacts when new data are attained. Meaning that once the updated informations are acquired the new value of the output power of the flexible devices is updated in function of the new data.
The implicit feedback strategy $f(\cdot)$ could differ for each controllable device. In particular, the function $f(\cdot)$ can assume several forms more or less elaborated. One simple possible selection could be an affine feedback strategy, where the control function $f(\cdot)$ is an affine function as in (3.3).
Choosing an affine structure for this implicit feedback strategy could lead to higher final cost and could be limiting with respect to, e.g., an arbitrary, more complex, function, or if the fixed points are optimally defined according to uncertainty scenarios. At the same time, it should be considered that the definition of arbitrary functions or the selection of an appropriate set of uncertainty scenarios for an optimization problem is non-trivial and could lead to scalability issues when large systems are evaluated. Even though initially the choice of a linear control function could appear a drawback because of the restricted control reaction, actually it presents several advantages.
This feedback control law can be easily implemented and it can be optimized without issues of computational tractability. To improve this control law maintaining these properties it could be adopted a Piecewise Affine (PWA) function as in (Roald et al., 2017; Vrakopoulou and Hiskens, 2017).
In this thesis it is assumed the existence of an optimal implicit feedback strategy capable to optimally allocate and adjust the controllable power as function of the inputs stated in (4.7).

4.4 Deterministic MGs Network

Consider a radial grid connected to the ULG like in Figure 4.3, with N_{N} nodes. Assume that only one MG is connected to each node and every MG has the same property as in Section 4.1.
As stated, the objective of this thesis is the provision of a reliable variability range to the ULG while satisfying the constraints stated in ((4.1)-(4.3)) and (4.6). In a conservative case it can be assumed that the variability range of each MG is calculated as

$$p_{gi}^{-} \leq p_{gi} \leq p_{gi}^{+} \tag{4.8}$$

$$p_{gi}^{+} = \underline{p}_{li} + \overline{p}_{si} \tag{4.9}$$

$$p_{gi}^{-} = \overline{p}_{li} + \underline{p}_{si} \tag{4.10}$$

where $\overline{p}_{li}$ and $\underline{p}_{li}$ are the boundaries of the inflexible device while $\overline{p}_{si}$ and $\underline{p_{si}}$ are the upper and lower limit of the controllable element. The upper bound (4.9) is considered in such a way to have the maximum capacity from the controllable element and the minimum power of the uncontrollable device. On the contrary, the lower bound is formulated in such a way to have the minimum capacity from the controllable element and the maximum power of the uncontrollable device. In this conservative formulation, an inconsistent case, where $p_{gi}^{-} > p_{gi}^{+}$, means that the variability range that it can be provided to the ULG is null.

The power flowing through one line has to account for line ratings, i.e., the power injected in one node is obtained comparing the limits of the output power of one MG (4.8), to the line limits (4.6).
Generalizing to the all nodes, the bounds of the conservative variability range for each node can be computed as

$$\overline{p}_{gi} = \quad \{\textstyle\sum_{j=0}^{N_{\mathrm{N}}} L_{ij}\mathrm{min}[\overline{p}_{bj}/\overline{p}_{gj}] \quad +p_{gi}^{+}\} \quad j \neq i \quad i \in \mathcal{N}_{\mathrm{N}}, \tag{4.11}$$

$$\underline{p}_{gi} = \quad \{\textstyle\sum_{j=0}^{N_{\mathrm{N}}} L_{ij}\mathrm{max}[\underline{p}_{bj}/\underline{p}_{gj}] \quad +p_{gi}^{-}\} \quad j \neq i \quad i \in \mathcal{N}_{\mathrm{N}}, \tag{4.12}$$

$$\underline{p}_{gi} \leq p_{gi} \leq \overline{p}_{gi}, \tag{4.13}$$

where L_{ij} is the element of the node-to-branch incidence matrix L[1] corresponding to the grid structure; subscript i indicates the columns of the matrix while j the rows.
The upper bound (4.11) of the i-th node is computed considering the upper bound (4.9) of the i-th MG. This term is added to the sum of the minimums between the upper rating of the j-th line $\overline{p}_{bj}$, connected to the i-th node, and the upper bound of the j-th node $\overline{p}_{gj}$, connected to the j-th line. The lower bound (4.12) is calculated with opposite considerations. The power range provided, dependent on the value of the uncertainty source, is the minimum range and the one with the higher reliability.

[1]The incidence matrix characterizes the relation between the nodes and the branches connecting theme. In a radial system the ending node of any brunch is unique. The incidence matrix L has general term $\{l_{ij}\}$ and dimension ($N_{\mathrm{N}} \times N_{\mathrm{N}}$). $l_{ij} = 1$ if branch i connects a sending node j, $l_{ij} = -1$ if branch i connects an ending node j, zero otherwise.

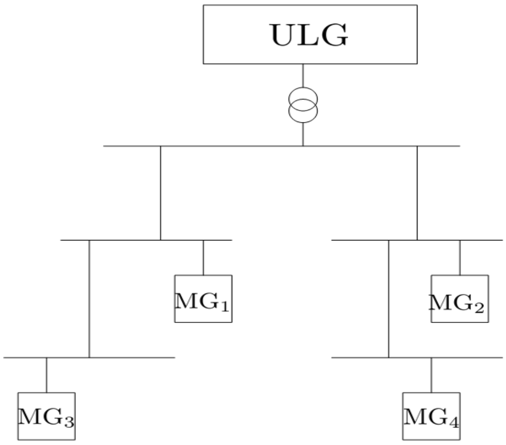

FIGURE 4.3: Radial grid with one MG at each node (except for the node of the ULG)

4.4.1 Deterministic Analysis

One MG Case

Assume a single MG, as described in Section 4.1, connected through a line to the ULG as in the Figure 4.4. Notice that in this figure the power through the line and the output power of one MG coincide, while they differ if more than one MG is connected to that line.

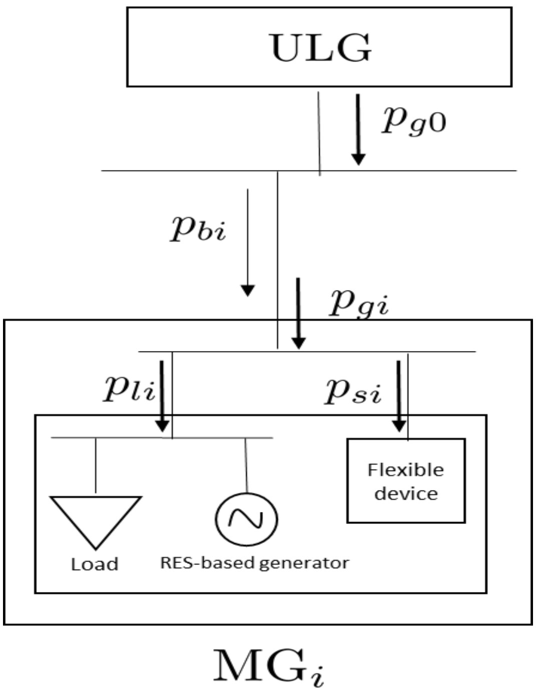

FIGURE 4.4: One MG connected to the ULG

Given worst case forecasts and the parameters of the system, the output power of the flexible device and the inflexible one and the limits for the power through the line are

$$
\begin{aligned}
-7.5 \leq p_{b1} \leq 7.5, \\
-3 \leq p_{s1} \leq 3, \\
3 \leq p_{l1} \leq 10.
\end{aligned}
\tag{4.14}
$$

where all the values are in kW. The output power of the flexible device can be controlled within the limits. The output power of the inflexible element is uncontrollable so it can randomly acquire any value within the defined interval.

Consider a target of power exchanged with the ULG, denoted as p_{g0}, into the following variability range

$$
7 \leq p_{g0} \leq 10, \tag{4.15}
$$

When $p_{l1} = 3$ kW the target cannot be satisfied. When $p_{l1} = 4$ kW instead the system is able to provide the target power to the ULG without violations of the constraints. Summing up, a target power exchange within the interval (4.15) could or could not be achieved depending on the value of p_{l1}.

Assume now that the variability range of power flowing to/from the ULG has to be provided with high reliability. To satisfy this condition the conservative case and the relations (4.11) and (4.12) are adopted.
However, this leads to the inconsistent interval

$$
7 \leq p_{g0} \leq 6. \tag{4.16}
$$

Here, the upper bound is given by $p_{s1} = 3$ kW plus $p_{l1} = 3$ kW, considering the line constraint the final value is $\overline{p}_{gi} = 6$ kW. The lower bound is obtained by $p_{s1} = -3$ kW and $p_{l1} = 10$ kW, after constraint evaluation results in $\underline{p}_{gi} = 7$ kW. This range is obviously inconsistent since the upper limit is smaller than the lower. This means that there is not any reliable range for the power drawn to the ULG with the given constraints.

A second MG with a different power output of the inflexible element is connected to the same ULG and present the following constraints

$$
\begin{aligned}
-7.5 \leq p_{b2} \leq 7.5, \\
-3 \leq p_{s2} \leq 3, \\
-8 \leq p_{l2} \leq -1.
\end{aligned}
\tag{4.17}
$$

The power to the ULG is

$$
-4 \leq p_{g0} \leq -5. \tag{4.18}
$$

The same considerations of the previous MG are valid and even this range is inconsistent since the upper limit is smaller than the lower. While MG2 is able to generate power to the ULG, the MG1 is constituted by a large power demand.

Two MGs Case

Consider a system composed of the two MGs described before with a direct connection to the ULG as shown in Figure 4.5. The two MGs are completely identical and they have the same property of the MG described in Section 4.1. In addition, they cooperate exchanging power with each other.

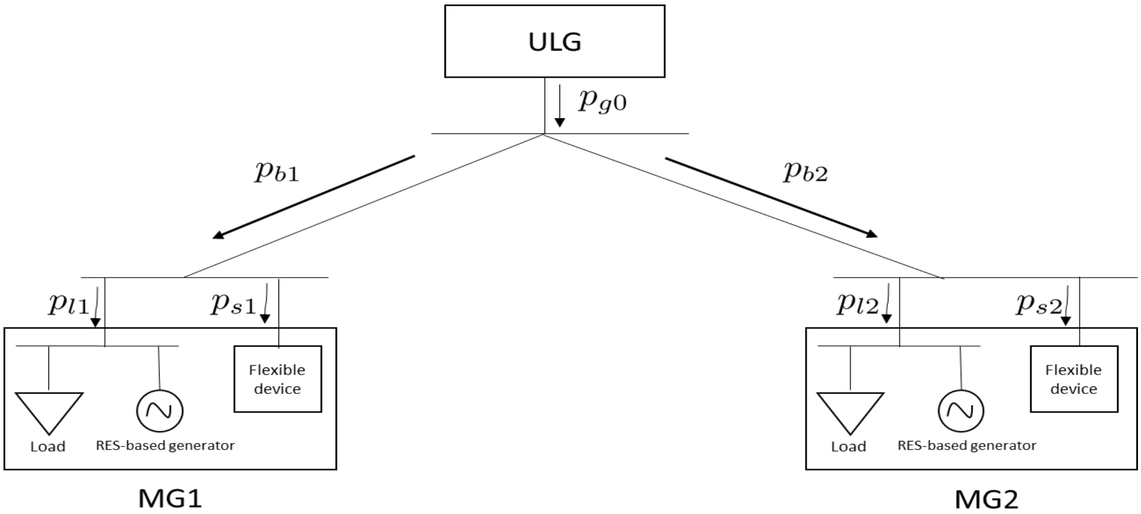

FIGURE 4.5: Two MG connected to the ULG

Consider the case where the network of 2 MGs provides the most reliable variability range.
According to (4.12), (4.11) and the considerations about the conservative range stated before, the variability range from the two MGs and the variability range provided to the ULG are

$$\begin{aligned} 7 \leq p_{g1} &\leq 6, \\ -4 \leq p_{g2} &\leq -5, \\ 3 \leq p_{g0} &\leq 1. \end{aligned} \tag{4.19}$$

The output power of the two MGs remain unchanged and with inconsistent intervals. The generation of MG2 compensate for part of the demand of MG1 and reduce the amount of power that is required to the MG1 from the ULG. However, the interval for p_{g0} is still inconsistent.
Consider now the new range for p_{l2}

$$-5 \leq p_{l2} \leq -1. \tag{4.20}$$

With a slight reduction of the interval of p_{l2}, e.g between -5 kW and -1 kW, it is possible to attain a different result. In fact, with these new values the obtained ranges are

$$\begin{aligned} 7 \leq p_{g1} &\leq 6, \\ -4 \leq p_{g2} &\leq -2, \\ 3 \leq p_{g0} &\leq 4. \end{aligned} \tag{4.21}$$

This variability range for p_{g0} is consistent because the upper bound is bigger than the lower one.
Changing the RES generation, for example curtailing the production, can bring to a consistent result. However, these are arbitrary examples. The power drop of p_{l2} is from -8 kW to -5 kW but an acceptable result could be obtained with also -5.5 kW, curtailing less power.
In another case, the capability to reduce the RES power, to obtain a consistent solution, could come from two different sources because of RES in MG1 and MG2. It can be dropped the power in MG1 and in MG2. How can we select a cost-effective range of curtailable power or the amount of power needed to obtain a new consistent range?

Comments

Some important considerations can be highlighted from these 2 analysed cases.

The first is about the uncertainties in the system.
The capability to satisfy the grid operator demand strongly depends on the realization of the uncertainties sources. This means that the ULG could not rely in this system on its operations because of RES intermittent nature. How should the system be analysed to take into account the volatility of RES?

The second aspect is related to the cooperation of MGs.
As it is shown in the two MGs case (4.19), the exchange of power among MGs could lead to improve the condition of the single MG. When MG1 operates independently it needs a certain amount of power from the ULG to satisfy the demand. When it cooperates with MG2 the power requested from the ULG decreases because of the support in load balancing from the generation of MG2. The uncertainty of each MG is mitigated by the controllable element of all the MGs in the system. Furthermore, the uncertainty of the single MG could be compensated by the uncertainty of the other MGs. The cooperation could lead to satisfy a request from the ULG that instead one grid could not meet because of the limited maximum power capacity.

Another consideration is about the reduction of p_l.
When the last consistent dispatch interval (4.21) for p_{g0} is computed, it is assumed an arbitrary drop in power production from MG2. However, as stated, when several MGs are composing the NMG it is not trivial to establish what is the MG that is causing that inconsistency. In addition, deciding what is the MG with a curtailable power is challenging. Choosing the amount of power to drop can play a key role in an economic environment and should not be done arbitrarily. In fact, when an assumption is completely arbitrary, it could lead to enhanced costs, so how to decide what is the more appropriate management approach?

The last consideration is about the probability of realization of some values of p_l. In real cases different values of output power of the inflexible devices could occur with different probability. Consider the solution in (4.15) where if the power p_{l1} is smaller than 4 kW the request cannot be satisfied. Analysing the probabilities of some values of the power p_{l1} it could turn out that the probability of realization of output powers less than four is very low. In this way the MG could avoid to deny the request of the ULG and it could accept with an associated probability to satisfy it. Consequently, it could be more accurate and cost-efficient a probabilistic analysis. The same consideration holds for the solution (4.21) where the power generation is curtailed from -8 kW to -5 kW. It could occur that the generation in that interval is highly improbable. This means that a probabilistic approach could evidence that the curtailment would be necessary only with a low probability.

The examples presented in this section point out that a stochastic analysis of the power system could be more efficient, cost-effective and accurate. Moreover, the sources of uncertainty could be modelled considering the spatial correlation and the possible cooperation among MGs. Thus, an optimization problem could be performed, including the uncertainty, in order to control the NMG with a precise objective (e.g cost minimization) and in such a way all the decisions are optimized. Furthermore, this control approach could be able to enhance the MGs reliability, enabling NMG to support the operations of the ULG.

4.4.2 Chapter Conclusions

The proposed examples points out the need for an adequate strategy able to provide a reliable power variability range to the ULG and capable to establish the necessary reduction of power in a NMG for a consistent solution. Furthermore, it could be more accurate an alternative analysis to the deterministic one because of the unpredictable behaviour of the RES. The uncertainties from different sources could compensate with each other. Therefore, a spatial correlation among them could be included in the analysis to improve the accuracy of the results. Further on, the MGs cooperate with each other. Meaning that the control strategy should be able to manage the response of the system considering the power from several scattered sources at the same time.

Chapter 5

Solution Methodology

As stated in the previous chapter the objective is to compute reliable variability ranges of active power to send to the ULG, while satisfying network constraints. It is a goal that should consider the intermittent and unpredictable nature of RES from several MGs at the same time. Moreover, it could be formulated an optimization problem in such a way to attain an optimal management of all the resources in a NMG. In this chapter such an optimization problem will be formulated and the potential of the chosen approach will be presented and explained.

5.1 Formulation as Optimization Problem

The system model has been introduced in Chapter 4 and the NMG will be exploited in this formulation. As the interest is mainly in calculating reliable ranges for p_{g0} an appropriate formulation is convenient to highlight this exogenous signal. To this end

$$p_{g0} = \hat{p}_{g0} + \Delta p_{g0}, \tag{5.1}$$

where $\hat{p}_{g0}$ is the target of power (denoted as target for the sake of brevity), while Δp_{g0} is the deviation from this target. Notice that with this formulation the target $\hat{p}_{g0}$ is now the exogenous signal in (4.7) together with the output power of all the inflexible elements $\mathbf{p}_{\mathrm{I}}$. The goal is the minimization of the whole operating cost of the NMG, based on power deviations from the target $\hat{p}_{g0}$, while respecting system constraints. The problem can be formulated as follows:

$$\underset{\Delta p_{g0}, \mathbf{p}_s, \mathbf{p}_b}{\text{minimize}} \quad \frac{1}{2} c \Delta p_{g0}^2 \tag{5.2a}$$

$$\text{subject to} \quad \hat{p}_{g0} + \Delta p_{g0} = \sum_{i=1}^{N_\text{m}} b_i p_{gi} + \sum_{i=1}^{N_\text{N}} c_i p_{bi}, \tag{5.2b}$$

$$p_{gi} = \sum_{j=1}^{N_\text{m}} C_{ij} (p_{sj} + p_{lj}), \tag{5.2c}$$

$$p_{bi} = p_{ni} + \sum_{j=1}^{N_\text{N}} B_{ij} p_{bj}, \tag{5.2d}$$

$$\underline{\mathbf{p}}_s \leq \mathbf{p}_s \leq \overline{\mathbf{p}}_s, \tag{5.2e}$$

$$\underline{\mathbf{p}}_b \leq \mathbf{p}_b \leq \overline{\mathbf{p}}_b, \tag{5.2f}$$

$$\underline{\mathbf{p}}_l \leq \mathbf{p}_\text{l} \leq \overline{\mathbf{p}}_l, \tag{5.2g}$$

$$\underline{\hat{p}}_{g0} \leq \hat{p}_{g0} \leq \overline{\hat{p}}_{g0} \tag{5.2h}$$

where b_i and c_i are unitary coefficients if the i-th branch or MG are respectively connected to the ULG (node 0), and zero otherwise.

Objective Function

Equation (5.2a) is a quadratic cost function based on cost of power deviations from the target exchanged with the ULG $\hat{p}_{g0}$. It is worth noting that the cost has been chosen quadratic to obtain symmetry with respect to zero. As stated, since the goal is to calculate reliable boundaries for $\hat{p}_{g0}$, any deviations from this parameter is undesired. Thus, $\Delta p_{g0} \in \mathbb{R}$ is introduced representing the deviation from the selected target exchanged with the ULG $\hat{p}_{g0}$. The solution of (5.2) is such that Δp_{g0} is not-null only if some constraint are active with the given set of the exogenous signals $\mathbf{p}_\text{l}$ and $\hat{p}_{g0}$. In other words, this term is a loophole to ensure that the selected target $\hat{p}_{g0}$ can be achieved.
This deviation is penalized with a large coefficient $c > 0$. Penalizing the fluctuations will result in optimal conditions where the power transferred to/from the ULG is as close as possible to the target $\hat{p}_{g0}$.

Equality Constraints

Equation (5.2c) refers to the power balance to each node. The expression is related to (4.4) but with p_{gj} decomposed in the MG balance $p_{gj} = p_{sj} + p_{lj}$.
The power balance of the ULG is formulated in (5.2b). This expression highlight the target $\hat{p}_{g0}$ and the two deviations from this value.

Inequality Constraints

The inequalities (5.2e-5.2h) express respectively the boundaries for all the controllable and uncontrollable devices, all the line ratings and for the target that can be exchanged with the ULG. These limits define the ranges of values in which these elements can vary.

The optimization problem (5.2) that has been formulated aims to minimize the deviation from the target $\hat{p}_{g0}$, given the exogenous signals $\hat{p}_{g0}, \mathbf{p}_l$, such that all the system constraints are satisfied. The approximations that has been mentioned about the structure of the NMG lead to a quadratic convex problem.

5.2 Multi Parametric Programming

The exogenous signals are the uncertain values of the optimization problem that has been formulated. As stated in 3.3.1 the uncertainty variables can be described as parameters varying within a selected interval. The optimization problem described presents parameters in the constraints. Consequently, the problem is addressed with multi parametric programming that it is supported by a well established theory (Pistikopoulos, Georgiadis, and Dua, 2007).
The formulation of the problem (5.2) can be expressed as a standard MPQP as in (3.13) and here reminded

$$\begin{aligned}
\underset{\{\mathbf{x}\}}{\text{minimize}} \quad & \frac{1}{2}\mathbf{x}^T H\mathbf{x} + F\boldsymbol{\theta} + \mathbf{f}\mathbf{x} \\
\text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} + B\boldsymbol{\theta}, \\
& A_e\mathbf{x} = b_e + E\boldsymbol{\theta}, \\
& \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}, \\
& A_\theta\boldsymbol{\theta} \leq \mathbf{b}_\theta
\end{aligned} \tag{5.3}$$

Objective Function and Constraints

The decision variables vector is $\mathbf{x} = [\Delta p_{g0}, \mathbf{p}_s, \mathbf{p}_b] \in \mathbb{R}^V$, V is the number of variables, while the parameters vector is $\boldsymbol{\theta} = [\mathbf{p}_l, \hat{p}_{g0}] \in \mathbb{R}^P$, P is the number of parameters. H is a diagonal matrix with the positive quadratic cost for the deviations, F is a null matrix and $\mathbf{f}$ the vector containing the null linear cost. It is worth noting that within the matrix H the costs associated to the variables different from the deviations are null. Meaning that the Hessian matrix of the objective function is positive semidefinite, therefore the function may or may not be strictly convex and consequently the solution may or may not be unique (Chachuat, 2007). Furthermore, notice that the degrees of freedom are lower than the number of variables expressed in the vector $\mathbf{x}$ due to the formulation of the equality constraints in the optimization problem (5.2).

The constraints of (5.2) are adapted to the standard form (5.3).

5.3 Parametric Solution

In (Pistikopoulos, Georgiadis, and Dua, 2007) a MPQP is solved giving a solution for a selected full range of parameter values. This is the approach applied in this thesis. The parameter space is optimally partitioned in regions. The latter are *Polyhedra* that, as stated in (3.10), if bounded are called *Polytopes*. The MPQP solution ensures the convexity and the existence of only bounded polyhedra. Calling M the number of polytopes $H_j \subseteq \Delta \in \mathbb{R}^P$ it holds $\bigcup_{i=1}^{M} H_j = \Delta$. The optimal value of each decision variable with respect to the MPQP can be computed via a Piece Wise Affine (PWA) function of the parameters $\boldsymbol{\theta}$ for each variable

$$x_z^*(\boldsymbol{\theta}) = \sum_{j=1}^{M} (G_j\boldsymbol{\theta} + k_j)\beta_j(\boldsymbol{\theta}), \qquad z \in \mathcal{V}, \tag{5.4}$$

where $\mathcal{V} = \{1, \ldots, V\}$, $\beta_j(\boldsymbol{\theta}) = 1$ if $\boldsymbol{\theta} \in R_j$ and $\beta_j(\boldsymbol{\theta}) = 0$ if $\boldsymbol{\theta} \notin H_j$. G_j is the j-th row, corresponding to the j-th region, of the matrix $G \in \mathbb{R}^{M \times P}$. The latter with the vector k are the coefficients, for the z-th variable, to denote the PWA function. The totality of the PWA functions is defined over the parametric space divided in Polytopes.
The solution is considered robust and feasible over the parameter space Δ. This means that for every point $\boldsymbol{\theta} \in \Delta$ the optimal solution $x_z^*(\boldsymbol{\theta})$ is defined (Vrakopoulou and Hiskens, 2017).

5.4 Probabilistic Forecast

As stated in 5.2, the interval within which the parameters vary is selected in a deterministic way. However, the analysis performed in Chapter 4 point out that a general probabilistic approach could be more efficient. In particular, the use of worst case scenarios to determine the ranges of values for the output power of a flexible device p_{li} could be conservative leading to results that are less accurate. To improve the accuracy, probabilistic forecasts of the output power of the inflexible devices can be used. In particular, probabilistic forecasts provide information about the probability of each of a number of different realizations of the random variable $\mathsf{P}_{li} \in \mathsf{L}^2(\Omega, \mu, \mathbb{R})$ (Perwass, 2009). Thanks to these informations it can be determined an interval of the random variable P_{li}, for a defined forecast horizon, and a probability that its realizations will lay into that interval.The methods adopted to calculate these intervals are out of scope. It is only mentioned that these intervals can be computed using pairs of quantile regression as described in (Ordiano et al., 2017), as this is considered in this thesis.
An example follows for one generic uncontrollable device

$$\mathrm{P}[\mathsf{P}_{li} \in \mathrm{I}_i] = \epsilon_i, \qquad i \in \mathcal{N}_{\mathrm{m}} \tag{5.5}$$

where $\mathrm{I}_i = [\underline{p}_{li}, \overline{p}_{li}]$. The probability that the random variable P_{li} will lay into the defined range, over a given interval of time, is equal to ϵ_i.
The set[1] containing all the possible realizations of all the parameters P_{li} is formulated

[1]Notice that a vector space has to include the zero vector. As the element that are considered in this thesis could not include the zero vector, the word *set* it is used.

as follows:

$$\Phi = \mathrm{I}_1 \times \ldots \times \mathrm{I}_{N_\mathrm{m}} \times \hat{p}_{g0}, \tag{5.6}$$

where $\Phi \subset \mathbb{R}^{\mathrm{P}2}$. Assuming that all the events $\mathcal{E}_i = \{P_{li} \in \mathrm{I}_i\}$ for $i \in \mathcal{N}_\mathrm{m}$ are independent, it follows that

$$\mathrm{P}[(\mathcal{E}_1) \cap \ldots \cap (\mathcal{E}_{N_\mathrm{m}})] = \pi = \prod_{i=1}^{N_\mathrm{m}} \epsilon_i. \tag{5.7}$$

The result is a set Φ containing all the possible realization of all the P_{li} with a probability π formulated as

$$\mathrm{P}[\mathbf{P}_l \in \Phi] = \pi, \tag{5.8}$$

where $\mathbf{P}_l = [\mathsf{P}_{l1}, \ldots, \mathsf{P}_{lN_\mathrm{m}}]$. Consider a unit vector $\overrightarrow{\hat{p}_{g0}}$ with direction given by the P-th dimension of the parametric space.
One property of the set Φ is the orthogonality to the unit vector $\overrightarrow{\hat{p}_{g0}}$, i.e. $\Phi \perp \overrightarrow{\hat{p}_{g0}}$ [2]. Thanks to defined probabilistic forecasts it can be determined a probability π given a set Φ, and vice versa, i.e.

$$\Phi(\pi) := \{\Phi \mid \mathrm{P}[\mathbf{P}_l \in \Phi] = \pi\} \tag{5.9a}$$

$$\pi(\Phi) = \mathrm{P}[\mathbf{P}_l \in \Phi]. \tag{5.9b}$$

5.5 Probabilistic Analysis

By referring to the optimization problem (5.2), according to the MPQP solution the parameter space is divided into optimal regions; an optimal PWA function is associated to this space for each variable. If one PWA function turns out to be null, $x_z^*(\boldsymbol{\theta}) = 0 \quad \forall \boldsymbol{\theta} \in \Delta$, that means the optimal value of the corresponding decision variable is independent of the parameter values and remains unchanged and constantly equals zero.

The connected[3] set $\Gamma \subseteq \Delta$ containing all the regions with a null solution for the variable Δp_{g0} is formulated as

$$\Gamma := \{H_j \in \mathcal{H} \mid x_1 = 0\} \tag{5.10}$$

where $\mathcal{H} = \{H_1, \ldots, H_M\}$, $\Gamma \in \mathbb{R}^P$. In this set the deviations from the target parameter are null, i.e. $\Delta p_{g0} = 0$. The set Γ is denoted as an *admissible* set, i.e., that the combination of parameters describing the set Γ is such that even if some constraints are activated there is no deviation from the value of the target $\hat{p}_{g0}$. Thus, target parameter $\hat{p}_{g0}$ defines the power drawn to the ULG, $p_{g0} = \hat{p}_{g0}$.

[2] It is worth mentioning that this set is a P-dimensional set with the P-th dimension fixed by a certain value of θ_P. In this Section 5.4 the fixed value of the P-th dimension it is implicitly considered and the focus is in the (P-1)-dimensional set.

[3] This property has been assumed based on several simulations that has been conducted. The result of the latter, in fact, always show a connection among the polytopes constituting the space Γ.

Consider the interval containing all the values of the target parameter in Γ defined as

$$\mathrm{I}_{g0} := \{\hat{p}_{g0} \in \mathbb{R} \mid \exists \boldsymbol{\theta} \in \Gamma, \quad \theta_P = \hat{p}_{g0}\}. \tag{5.11}$$

Similarly, consider the set containing all the values of the output power of the uncontrollable devices in Γ defined as

$$\Phi := \{\mathbf{p}_l \in \mathbb{R}^{P-1} \mid \exists \boldsymbol{\theta} \in \Gamma, \quad \boldsymbol{\theta}_{P-1} = \mathbf{p}_l\}, \tag{5.12}$$

where $\boldsymbol{\theta}_{P-1} = [\theta_1, \ldots, \theta_{P-1}]$ and the properties of this set have been described in 5.4. Assume a hypercuboid $\Psi \subseteq \Gamma$ whit the edges perpendicular to the Cartesian axes such that

$$\Psi = \Phi' \times \mathrm{I'}_{g0}. \tag{5.13}$$

where the set $\Phi' \subseteq \Phi$ and with the same properties of Φ while the interval $\mathrm{I'}_{g0} \subseteq \mathrm{I}_{g0}$. The direction for this range is defined by the unit vector $\overrightarrow{\hat{p}_{g0}}$, thus $\mathrm{I'}_{g0} \perp \Phi$. Figure 5.1 shows the main sets and intervals described. Notice that in the figure the bases Φ and Φ' coincide.

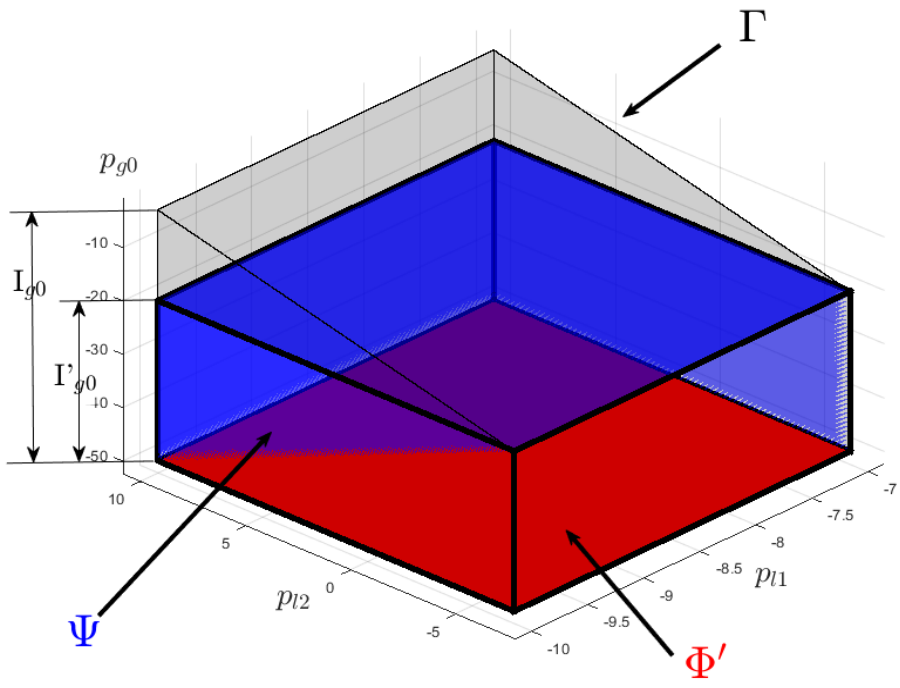

FIGURE 5.1: Parametric set Γ with the hypercuboid inscribed Ψ(blue) and the base Φ'(red).

The following cases are addressed employing the optimization problem described in (Boyd and Vandenberghe, 2004). The problem is formulated as follows:

$$\underset{u,l}{\text{minimize}} \quad -\sqrt[P]{\prod_{i=1}^{P}(u_i - l_i)} \tag{5.14a}$$

$$\text{subject to} \quad \sum_{j=1}^{P}(a_{ij}^{+}u_j - a_{ij}^{-}l_j) \leq b_i \quad i = 1, \ldots, S, \tag{5.14b}$$

$$l \leq b \tag{5.14c}$$

where

$$B := \{\mathbf{x} \in \mathbb{R}^P | l \leq \mathbf{x} \leq u\} \quad l, u \in \mathbb{R}^P \tag{5.15a}$$

$$H := \{\mathbf{x} \in \mathbb{R}^P | A\mathbf{x} \leq b\} \quad A \in \mathbb{R}^{S \times P}, b \in \mathbb{R}^S \tag{5.15b}$$

$$a_{ij}^{+} = \max\{a_{ij}, 0\} \qquad a_{ij}^{-} = \max\{-a_{ij}, 0\} \tag{5.15c}$$

$$B \subseteq H \Leftrightarrow \sum_{j=1}^{n}(a_{ij}^{+}u_j - a_{ij}^{-}l_j) \leq b_i \tag{5.15d}$$

The (5.15b) is the H-rep, introduced in (3.10), of the union of polytopes Γ. It is worth noticing that the set Γ is assumed to be convex. The vectors $\mathbf{l}, \mathbf{u}$ represent the upper and lower bound of edges of the hypercuboid Ψ.

The objective function aims to maximize the volume of a hypercuboid. The usage of the geometric mean it is only a loophole to ensure the convexity of the optimization problem. Adapting the general formulation to this specific analysis, the optimization problem aims to maximize the volume of the hypercuboid Ψ. The equation (5.13) can be formulated as

$$\Psi = \prod_{i=1}^{\mathrm{P}}(u_i - l_i), \tag{5.16}$$

$$\Phi' = \prod_{i=1}^{P-1}(u_i - l_i), \tag{5.17}$$

$$\mathrm{I'}_{g0} = (u_P - l_P). \tag{5.18}$$

5.5.1 Consistent Case : from Φ' to $\mathrm{I'}_{g0}$

Consider a set Φ' with a probability π' as in Section 5.4 and the problem of computing the maximum interval $\mathrm{I'}_{g0}$. The problem can be solved applying (5.14). The optimization problem aims to maximize the volume of the hypercuboid Ψ inscribed in Γ with a given hyperbase (or base) Φ'. Namely, the goal is the maximization of the interval $\mathrm{I'}_{g0}$ given the hyperbase Φ'.

The general optimization problem is adapted to the case of the maximization of $\mathrm{I'}_{g0}$. Consequently, the values of $\mathbf{u}_b = \{u_1, \ldots, u_{P-1}\}$ and $\mathbf{l}_b = \{l_1, \ldots, l_{P-1}\}$ are fixed based on the vertex of the hyperbase Φ'. The obtained optimal results u_P, l_P represent the maximum interval $\mathrm{I'}_{g0} = u_P - l_P$. The latter is a feasible and reliable interval of parameters target with an associated probability that the range $\mathrm{I'}_{g0}$ will be realizable in practice, i.e.

$$\mathrm{P}[\Delta \mathsf{P}_{g0} = 0 \mid \hat{p}_{g0} \in \mathrm{I'}_{g0}] \geq \mathrm{P}[\mathbf{P}_l \in \Phi'] = \pi'. \tag{5.19}$$

where $\Delta \mathsf{P}_{g0} \in \mathsf{L}^2(\Omega, \mu, \mathbb{R})$. Summarising, consider an assigned set of the parameters P_l defined by Φ', with associated probability π' as in 5.4. Solving the optimization problem (5.14) adapted to this case gives a range of power $\mathrm{I'}_{g0}$ with a probability that target $\hat{p}_{g0}$ will lay into this interval is at least equal to π'. In this way the dispatchability boundaries of power output to the ULG are reliable and calculated in advance, before the uncertainties are unveiled.

5.5.2 Consistent Case: from $\mathbf{I'}_{g0}$ to Φ'

Consider $\mathrm{I'}_{g0}$. In this case the optimization problem (5.14) aims to maximize the volume of the hypercuboid Ψ inscribed in Γ with a given interval $\mathrm{I'}_{g0}$. Namely, the goal is the maximization of the hyperbase Φ'. The general formulation is adapted to the case of the maximization of the hyperbase Φ'. Consequently, the values u_p, l_p are fixed based on the extreme limits of the interval $\mathrm{I'}_{g0}$. The optimal results $\mathbf{u}_b = \{u_1, \ldots, u_{P-1}\}$ and $\mathbf{l}_b = \{l_1, \ldots, l_{P-1}\}$ define the maximum hyperbase Φ' of the hypercuboid Ψ. As stated in 5.4 to this set Φ' can be associated a certain probability as in (5.8). Consequently the result is equal to (5.9).

Even though the final result is the same, the procedure is the opposite. Initially an interval $\mathrm{I'}_{g0}$ is assigned, from the ULG for example. Once the set Φ' is computed, the minimum probability with which the interval $\mathrm{I'}_{g0}$ will be realizable in practice is determined according to (5.9).

5.5.3 Inconsistent Case

The inconsistent case can occur when given a set Φ the result of the optimization problem (5.14) is an empty interval, $\mathrm{I'}_{g0} = \emptyset$. Another inconsistent case happens when assigned a certain interval $\mathrm{I'}_{g0}$ the maximization problem gives an empty set, $\Phi' = \emptyset$. When one of these two inconsistent cases arises, it can be outlined a twofold approach. Notice that neither the interval $\mathrm{I'}_{g0}$ nor the set Φ' are fixed.

The first approach consists of setting the optimization problem in order to find the maximum hyperarea Φ' inscribed in Γ. The target parameter is bounded within the maximum possible interval I_{g0}. The result is the set Φ' and a range of $\hat{p}_{g0}$. A probability π' is associated, according to (5.9), to this set Φ'. Consequently, it is determined the range of the target $\hat{p}_{g0}$ with the maximum probability to be realizable, i.e.

$$\max\{\mathrm{P}[\Delta \mathsf{P}_{g0} = 0 \mid \hat{p}_{g0} \in \mathrm{I'}_{g0}]\}. \tag{5.20}$$

The second approach consists of the maximization of the volume rectangle (or hypercuboid) Ψ inscribed in Γ. The results are the maximum instances of $\mathbf{l}, \mathbf{u}$ in such a way to define the hypercuboid with the maximum volume inscribed in Γ. The hyperbase of this hypercuboid is the set Φ' while the "hyperheight" is the interval $\mathrm{I'}_{g0}$.

The consideration about the probability are the same performed in all the previous cases.

5.5.4 Non Convex Cases

In all the analysed previous cases the *union* of polytopes Γ has been considered as convex. When a non-convex Γ occurs, for example as in Figure 5.2, the optimization problem (5.14) cannot be applied. In fact, the H-rep of Γ is not computable and the constraints of (5.14) cannot be formulated. An alternative approach can be applied instead in this case.

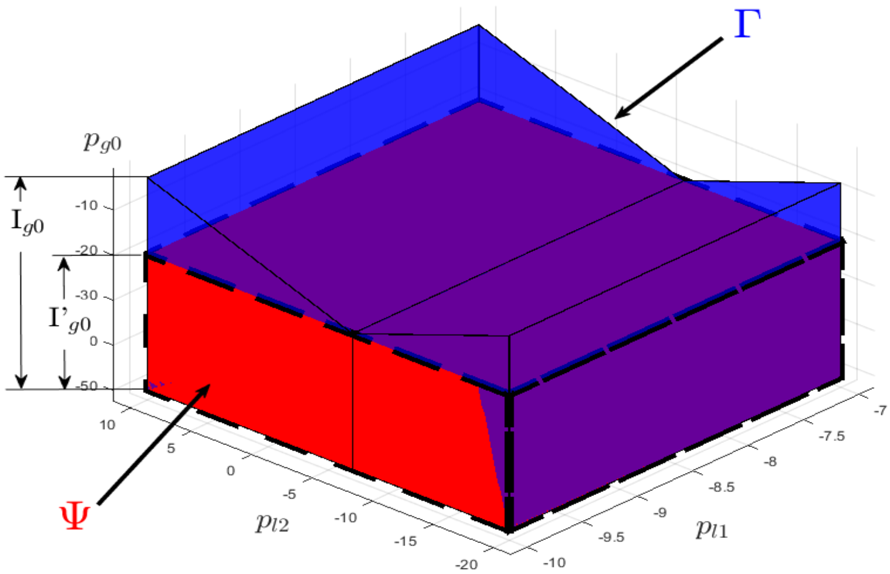

FIGURE 5.2: Non convex set Γ(blue) with a hypercuboid inscribed Ψ(red).

5.5.5 Non convex Case: from Φ' to I'_{g0}

Consider a set Φ' and a certain probability π' according to (5.9). The intersection between Γ and $\Psi = \Phi' \times I_{g0}$ is extracted, giving a new set Γ' (notice the difference between I'_{g0} and I_{g0}). Consequently, a twofold solution can occur: the new set Γ' is convex or non-convex. In the first case the optimization problem (5.14) can be implemented and all the analysis are unchanged with respect to the convex cases 5.5.1-5.5.3. The non-convexity prevents the use of (5.14). When this is the case, an alternative procedure is performed.

Consider the set

$$\Lambda(\hat{p}_{g0}) := \{\boldsymbol{\theta} \in \mathbb{R}^P \mid \theta_P = \hat{p}_{g0}\}, \tag{5.21}$$

where $\Lambda(\hat{p}_{g0}) \in \mathbb{R}^{P-1}$. Only the values of the target parameter corresponding to vertices of Γ' are practically evaluated. The comparison among the sets $\Phi', \Lambda(\hat{p}_{g0})$ is performed. If there is at least one match $\Phi' = \Lambda(\hat{p}_{g0})$ an interval can be formulated $\mathrm{F} = [\underline{\hat{p}}_{g0}(\Phi'), \overline{\hat{p}}_{g0}(\Phi')]$. The latter is formulated taking the preimage of all the sets $\Lambda(\hat{p}_{g0})$ meeting the comparison. Consequently, a probability is associated as in 5.5.1

$$\mathrm{P}[\Delta \mathrm{P}_{g0} = 0 \mid \hat{p}_{g0} \in \mathrm{F}] \geq \mathrm{P}[\mathbf{P}_l \in \Phi'] = \pi'. \tag{5.22}$$

The interval F is a reliable range of the target $\hat{p}_{g0}$ drawn to the ULG.

5.5.6 Non-Convex Case: from $\mathbf{I'}_{g0}$ to Φ'

The method used in this case is basically the same of 5.5.5. Assume a defined interval $\mathrm{I'}_{g0}$. The intersection between Γ and $\Psi = \Phi \times \mathrm{I'}_{g0}$ is extracted, giving a new set Γ' (notice the difference between Φ and Φ'). A possible representation of this case is shown in Figure 5.2 where the new set Γ' coincides with the hypercuboid Ψ. When Γ' is convex the optimization problem (5.14) is implemented and solved as in the previous convex cases 5.5.1-5.5.3. Within the non-convex case, the set (5.21) is defined exactly as in 5.5.5. The set with the smaller area rectangle (hyperarea) among the $\Lambda_{(\hat{p}_{g0})}$ is the new set Φ'. As in (5.9) a probability π' is associated to Φ'. The probability π' is the larger probability that the range $\mathrm{I'}_{g0}$ will be realizable in practice.

5.5.7 Non-Convex Case: Inconsistent Case

The last case that should be considered is when there are inconsistent results, as in 5.5.3, with non-convex Γ and also non-convex Γ', after the intersection as in 5.5.5-5.5.6. In fact, both the approaches performed in this thesis are not valid in these cases. The optimization problem cannot be formulated due to the lack of convexity. The approach based on (5.21) is not able to optimally calculate the maximum area rectangle or the maximum volume rectangle inscribed in a non convex Γ'. One possible solution could be reformulating the problem (5.14) as a convex mixed integer program that is non-trivial. However, this is out of the scope of this thesis.

5.6 Summary

This chapter shows that starting from the structure of a NMG it is possible to formulate a MPQP as in (3.13). Further on, it has been shown how the solution of this problem leads to an optimal reliable variability range calculated before the uncertainties are realized.
First of all, a reduction of the total parametric space Δ to the admissible set Γ, has been performed. The set Γ is defined in (5.10) and consists of all the regions with a null solution for the variable Δp_{g0}. The second step consists of evaluating the convexity of the set Γ. If Γ is convex the optimization problem (5.14) is implemented, otherwise alternative approaches should be employed. One possible solution has been presented in 5.5.4 and 5.5.5. Only the inconsistent case cannot be efficiently addressed with that alternative method.

5.7 Comments and Observations

The procedure illustrated in Chapter 5 proposes an efficient solution to deal with the challenges and questions raised in Chapter 4. An optimal solution of (5.2) satisfying all the system constraints has been calculated and reliable ranges of power dispatched to the ULG are optimally defined. This solution considers the response of the NMG to all the uncertainty sources from the various MGs.

Concluding, the described procedure allows to manage several cases, firstly the case in which a set Φ of values of the output power of all the inflexible devices P_l is given. Furthermore, it is assigned a certain probability that the P_l will fall into the set Φ by probabilistic forecasts. Thus, assumed the consistency of the solution, the two presented method can compute a reliable power variability range to the ULG.
Secondly, the case in which is given a certain power variability range I'_{g0}, e.g. from the ULG . Assuming the consistency of the solution, the method can establish if the power variability range can be satisfied and with what probability the deviation form the target $\Delta\mathsf{P}_{g0}$ will be null.
Finally, the cases in which is verified the convexity of Γ or Γ'. In this situation if an inconsistent solution is attained, as in 4.4.1, it can be efficiently and accurately determined: the necessary minimum amount of power and which output power of the uncontrollable devices to curtail to achieve a new consistent range. The latter will be provided with a certain probability to be realizable.
The capacity to provide these information is a key aspect in joining the AS market and in making the NMG a reliable system able to enhance the flexibility of the ULG.

Chapter 6

Simulation and Results

In this chapter the control strategy previously illustrated and the described optimization algorithm are applied to different test cases. A code has been implemented in MATLAB and includes Multi Parametric Toolbox (MPT), open-source optimization tool, to solve the MPQP (MATLAB, 2010; Herceg et al., 2013). Furthermore, YALMIP coupled to the solver SeDuMi has been exploited to solve geometrical optimal problems (Sturm, 1999; Lofberg, 2004). Finally, some simulations performed on a real test case use the software PowerFactory DigSILENT and a package of MATLAB called MATPOWER (Zimmerman, Murillo-Sánchez, and Thomas, 2011; Murillo-Sánchez et al., 2013).

Two main examples has been chosen to test the algorithm: the first it is a tutorial to show the steps outlined in Chapter 5 in a graphical way. The second consists of simulations on a real network to verify the accuracy of the results when a real case is considered.

6.1 Tutorial Example : 3-Nodes Network

First, a three nodes example is considered as shown in Figure 6.1.

The two MGs are both connected to the ULG drawing power to/from the ULG. Furthermore, the two MGs are able to exchange power with each other in order to compensate the fluctuations of RES.

This simple simulation is performed considering two household with rooftop PV generators and a battery.

The data for the uncontrollable devices are arbitrary chosen as well as the line constraints in such a way to highlight the main aspects of the algorithm. However, these values are not unrealistic, in fact, the powers output of the uncontrollable devices can come from the freely available dataset provided by Ausgrid, as in (Appino et al., 2018), incrementing the values by a scaling factor (Ratnam et al., 2017). The technical specifications of the battery are retrieved from the catalog of a commercial producer[1]. The target exchanged with the ULG can vary in a large interval because it is supposed the absence of limitations from the ULG. Table 6.1 shows the constraints of (6.1).

[1]www.tesla.com/powerwall

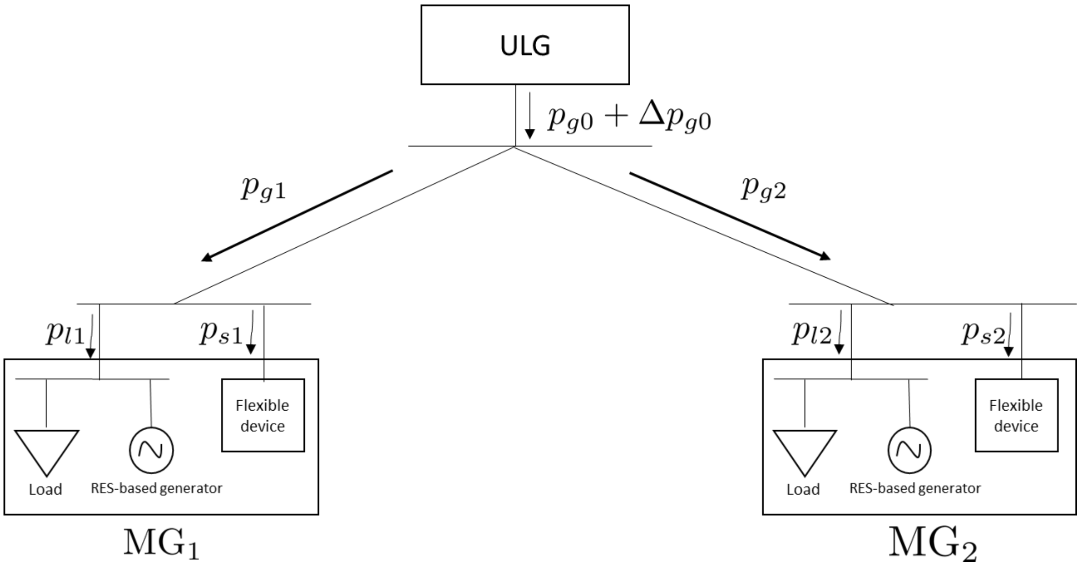

FIGURE 6.1: Three nodes test case

TABLE 6.1: Constraints for the 3-nodes network.

MG_1 (kW)	MG_2 (kW)
$-10 \leq p_{g1} \leq 10$	$-10 \leq p_{g2} \leq 10$
$-5 \leq p_{s1} \leq 5$	$-5 \leq p_{s2} \leq 5$
$-5 \leq p_{l1} \leq 15$	$-15 \leq p_{l2} \leq 5$
ULG (kW)	
$-50 \leq \hat{p}_{g0} \leq 50$	

The goal is to compute reliable variability ranges of the power output to the ULG. As in Chapter 5 a MPQP is formulated in order to minimize the whole operating cost, based on deviation from the target $\hat{p}_{g0}$ while satisfying the network constraints. The structure is the following

$$\begin{aligned}
\underset{\{\Delta p_{g0}, p_{s1}, p_{s2}, p_{g1}, p_{g2}\}}{\text{minimize}} \quad & \frac{1}{2} c \Delta p_{g0}^2 && (6.1a)\\
\text{subject to} \quad & \hat{p}_{g0} + \Delta p_{g0} = p_{g1} + p_{g2}, && (6.1b)\\
& p_{g1} = p_{s1} + p_{l1}, && (6.1c)\\
& p_{g2} = p_{s2} + p_{l2}, && (6.1d)\\
& \underline{p}_{s1} \leq p_{s1} \leq \overline{p}_{s1}, && (6.1e)\\
& \underline{p}_{s2} \leq p_{s2} \leq \overline{p}_{s2}, && (6.1f)\\
& \underline{p}_{g1} \leq p_{g1} \leq \overline{p}_{g1}, && (6.1g)\\
& \underline{p}_{g2} \leq p_{g2} \leq \overline{p}_{g2}, && (6.1h)\\
& \underline{p}_{l1} \leq p_{l1} \leq \overline{p}_{l1}, && (6.1i)\\
& \underline{p}_{l2} \leq p_{l2} \leq \overline{p}_{l2}, && (6.1j)\\
& \underline{\hat{p}}_{g0} \leq \hat{p}_{g0} \leq \overline{\hat{p}}_{g0}. && (6.1k)
\end{aligned}$$

where the large cost coefficient $c = 50$ is chosen to penalize the deviations. It is worth noticing that in this simple case the power through the lines p_{b1} (p_{b2}) is equal to the power balance of one MG, hence it is directly called p_{g1} (p_{g2}).
The decision variables vector is $\mathbf{x} = [\Delta p_{g0}, p_{s1}, p_{s2}, p_{g1}, p_{g2}] \in \mathbb{R}^5$ while the parameters vector is $\boldsymbol{\theta} = [p_{l1}, p_{l2}, \hat{p}_{g0}] \in \mathbb{R}^3$.

Using MPT, the parametric space is partitioned as in Figure 6.2 and from a different prospective in Figure 6.3

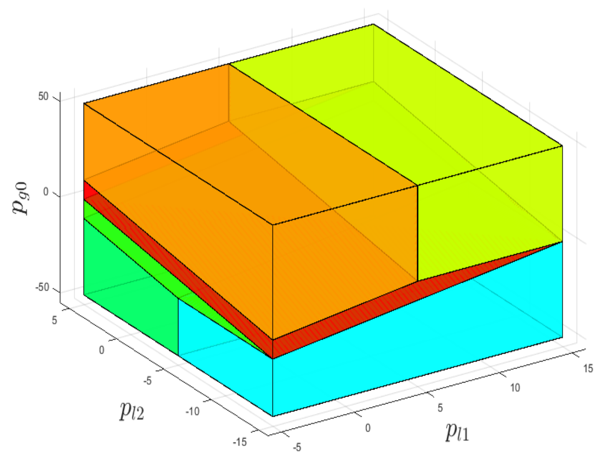

FIGURE 6.2: Partition of the parametric space (Δ) in polytopes H_j

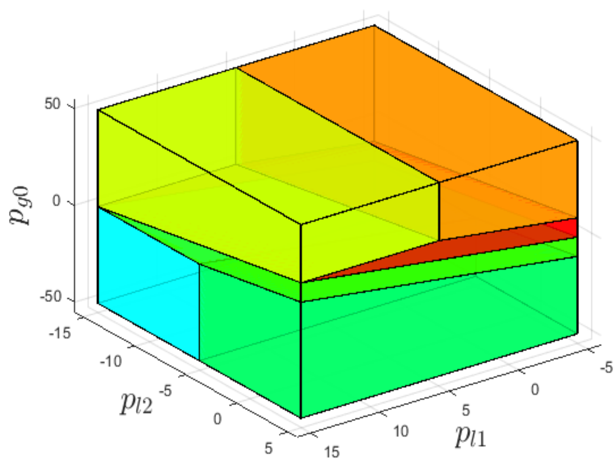

FIGURE 6.3: Division of the parametric space in regions (back of the previous Figure 6.2)

The regions assembling the parameter space are 6 and they are all convex polytopes denoted as $H_j, j = \{1, \ldots, 6\}$. The whole space of parameters has been denoted as Δ. In this space, to each region is associated an affine function of the parameters for each of the variables. For example, the solutions for the power deviation Δp_{g0} is formulated as

$$\Delta p_{g0} = a p_{l1} + b p_{l2} + c p_{g0} + d, \tag{6.2}$$

The coefficients of these solutions varies based on the considered region. The interest is on the regions where the power exchanged with the ULG is determined by the target $\hat{p}_{g0}$ without deviations. The regions characterized by a null solution for the power deviations are extrapolated from the whole parametric space. They assemble the set denoted as Γ as shown in Figure 6.4.

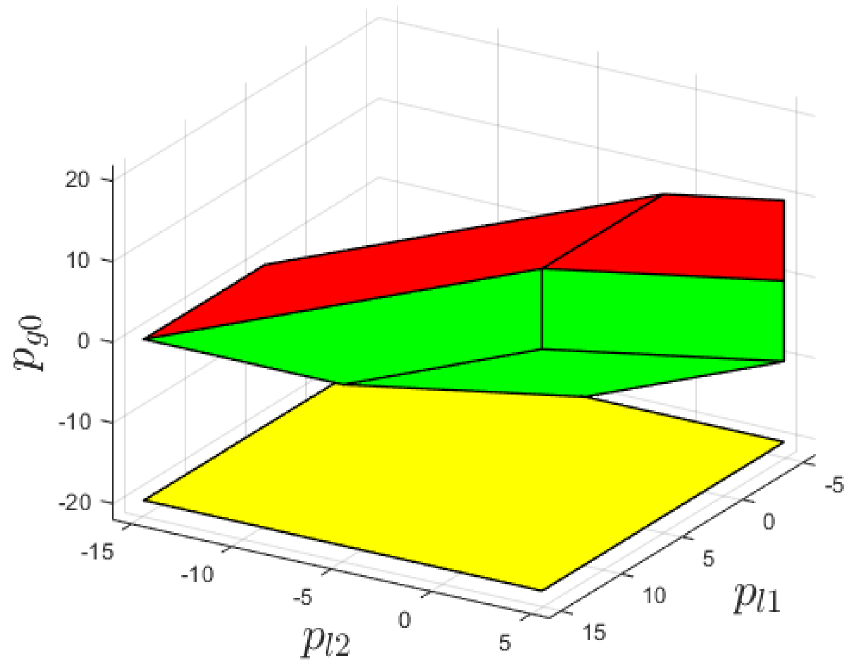

FIGURE 6.4: Regions with null solution for Δp_{g0} and the set Φ (in yellow)

The $\hat{p}_{g0}$-axes in Figure 6.4 shows all the values of the target $\hat{p}_{g0}$ in the interval defined as I_{g0}. As stated in Section 5.5, the powers within this interval can be drawn to the ULG without any deviation. The projection of Γ over the flat surface composed of p_{l1}, p_{l2} represents the set denoted as Φ and is represented in yellow in Figure 6.4. Since the set Γ is convex, all the considerations stated in 5.5.1-5.5.3 can be implemented in this case. Using a function of MPT the convex hull, defined in (3.5), of Γ is calculated and the matrix A and the vector $\mathbf{b}$ of the convex hull are obtained. Notice that being the set Γ convex, the convex hull is composed of the union of the two polytopes composing Γ as represented in Figure 6.5.

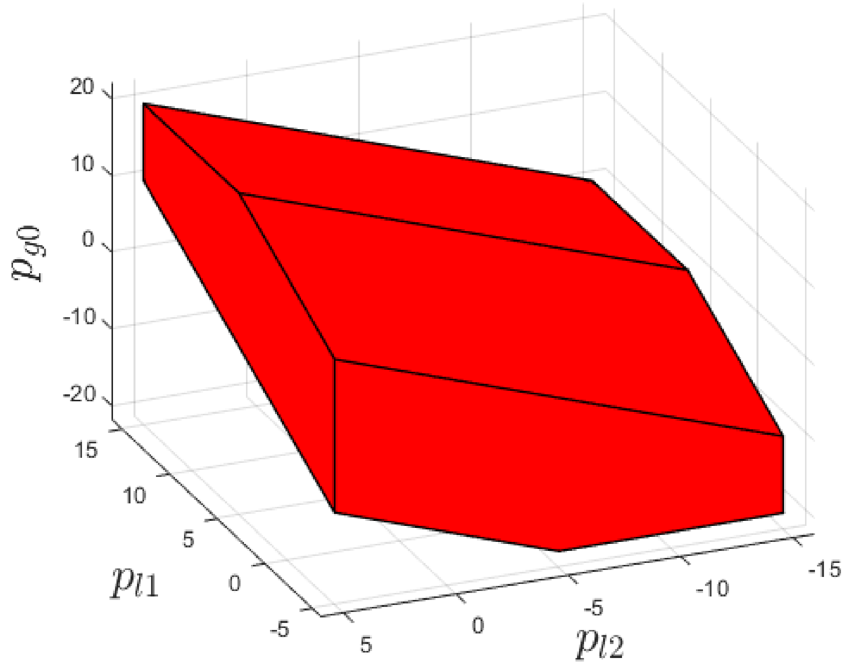

FIGURE 6.5: Convex hull of Γ.

$$A = \begin{bmatrix} -0.0000 & 0.1961 & -0.0000 \\ -0.1961 & 0.0000 & -0.0000 \\ -0.0985 & -0.0985 & 0.0985 \\ 0.0000 & -0.0664 & 0.0664 \\ -0.0000 & -0.0665 & -0.0000 \\ 0.0985 & 0.0985 & -0.0985 \\ 0.0665 & -0.0000 & 0.0000 \\ 0.0664 & -0.0000 & -0.0664 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 0.9806; \ 0.9806; \ 0.9853; \ 0.9956; \ 0.9978; \ 0.9853; \ 0.9978; \ 0.9956 \end{bmatrix}$$

From matrix A and vector $\mathbf{b}$ the constraints of the optimization problem (5.14) can be defined. Once this maximization problem it has been set, all the convex cases can be described.

6.1.1 Consistent Case : from Φ' to $\mathbf{I'}_{g0}$

Assume Cumulative Density Functions (CDFs)s for the random variables $\mathsf{P}_{l1}, \mathsf{P}_{l2}$ as shown in Figure 6.6 and in Figure 6.7. These CDFs are similar to the one that can be attained using probabilistic forecasts (Ordiano et al., 2017).

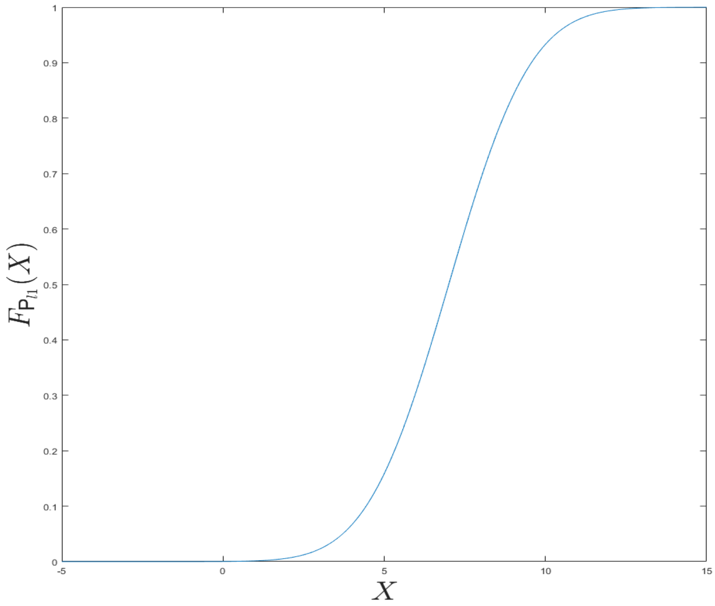

FIGURE 6.6: Normal CDF of P_{l1} with $\mu = 7$ and $\sigma = 4$.

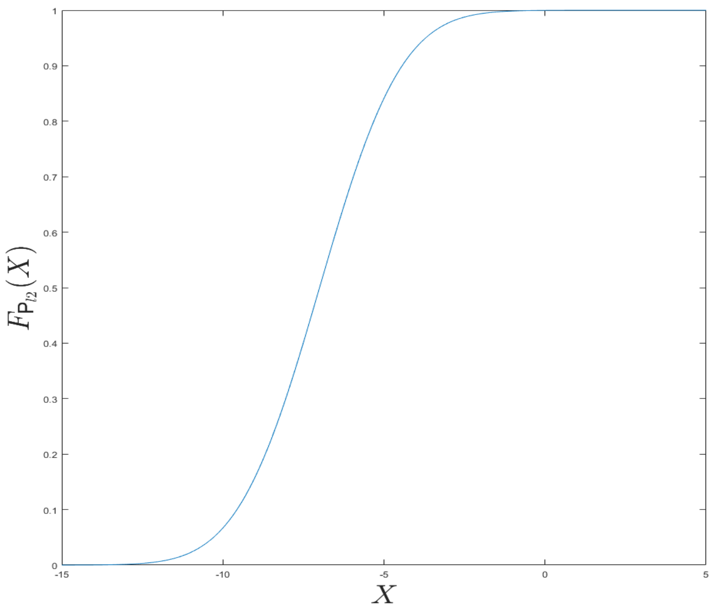

FIGURE 6.7: Normal CDF of P_{l2} with $\mu = -7$ and $\sigma = 4$.

By arbitrary assuming two ranges for p_{l1} and p_{l2}, from the two CDFs can be calculated the probability associated to these ranges as shown in Table 6.2.

TABLE 6.2: Given ranges and probabilities of P_{l1} and P_{l2}.

Ranges (kW)	$0 \leq p_{l1} \leq 8$	$-8 \leq p_{l2} \leq 0$
Probability	69.1%	69.1%

Consequently, from the informations in Table 6.2 the set Φ' and the probability π' are calculated. The first one is shown in Figure 6.8, while the probability is obtained as

$$\pi' = (69.1\%)(69.1\%) = 47.8\% \tag{6.3}$$

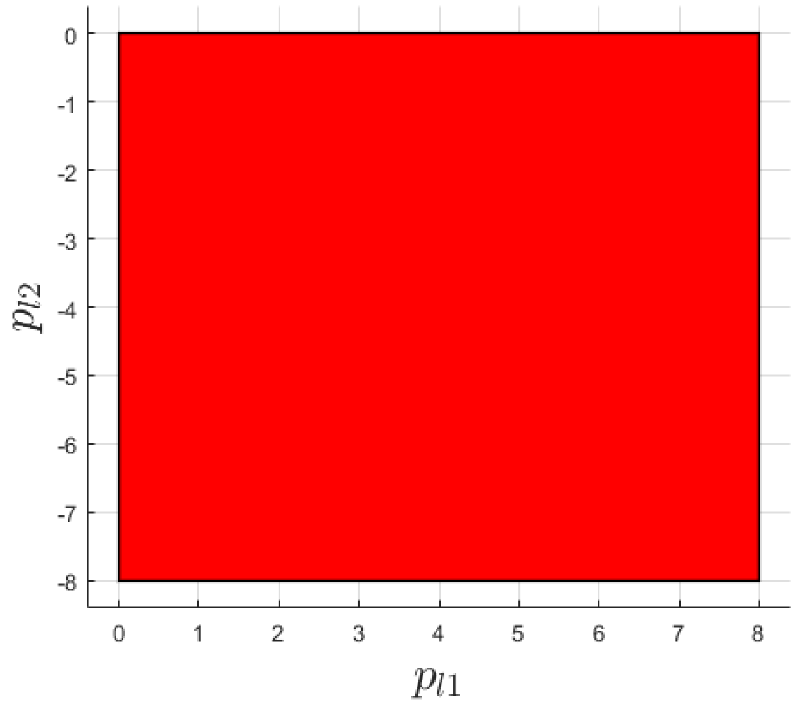

FIGURE 6.8: Set Φ' with probability $\pi' = 47,8\%$

As it can be observed, it is a 3-dimensional set, with the third dimension fixed, perpendicular to the dimension corresponding to the target $\hat{p}_{g0}$. Using the method outlined in 5.5.1, the maximum volume of a cuboid Ψ with base Φ' inscribed in Γ is computed and shown in Figure 6.9. The height of this cuboid represents the interval $\mathrm{I}'_{g0} = [-2; 2]$ with a probability to be realizable at least equal to $\pi' = 47.8\%$ according to (5.19).

6.1.2 Consistent Case: from $\mathbf{I}'_{g0}$ to Φ'

Consider that a certain interval $\mathrm{I}'_{g0} = [-4; 4]$ is given. The method presented in 5.5.2 is applied and the maximum volume of a cuboid Ψ with fixed height and inscribed in Γ is computed and shown in Figure 6.10. From the edges of the base of the cuboid the ranges for p_{l1} and p_{l2} can be defined. Then, it can be determined the probabilities associated to these intervals from the CDFs of P_{l1} and P_{l2} as presented in Table 6.3.

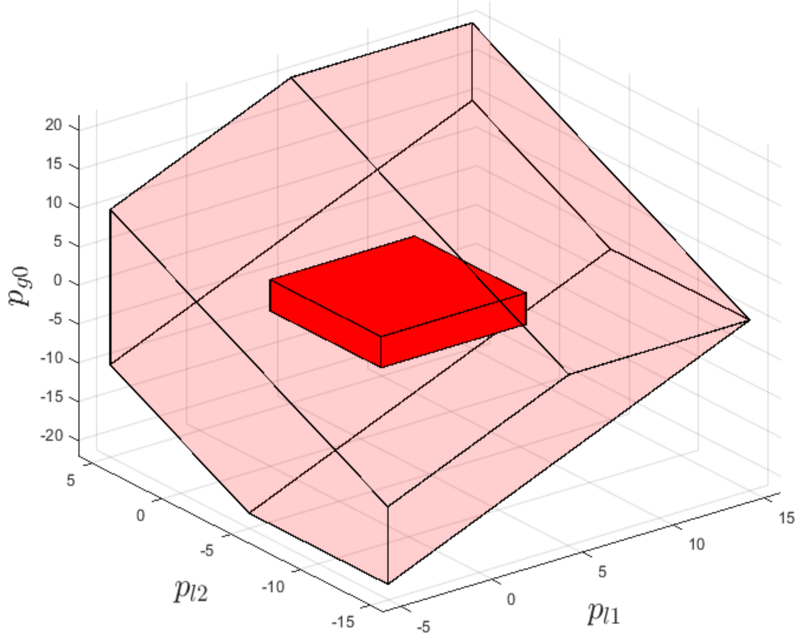

FIGURE 6.9: Cuboid with maximum volume inscribed in Γ with given base Φ'.

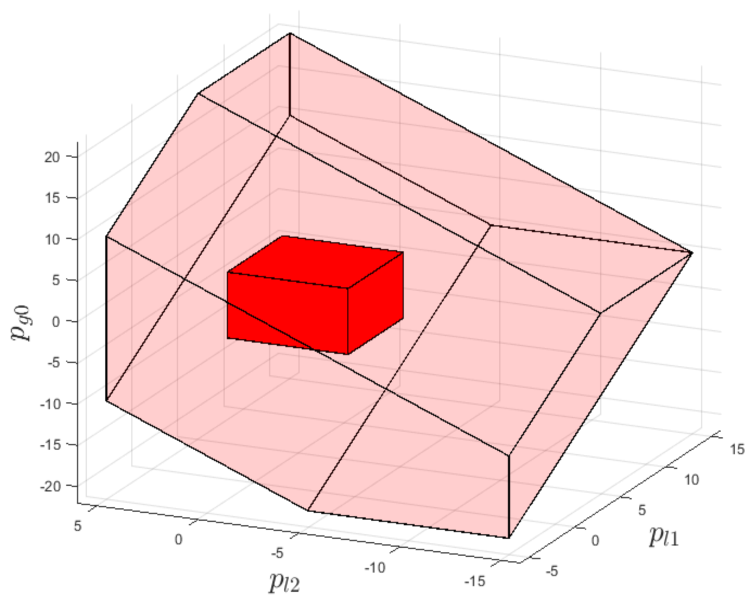

FIGURE 6.10: Cuboid with maximum volume inscribed in Γ with given height Γ'_{g0}.

TABLE 6.3: Given ranges and associated probabilities of P_{l1} and P_{l2}.

Ranges (kW)	$0.3409 \leq p_{l1} \leq 6.4309$	$-6.3409 \leq p_{l2} \leq -0.3409$
Probability	37.04%	37.04%

Consequently, from the information in 6.2 the set Φ' and the probability π' are calculated. The first is shown in Figure 6.11, while the probability is obtained as

$$\pi' = (37.04\%)(37.04\%) = 13.7\%. \tag{6.4}$$

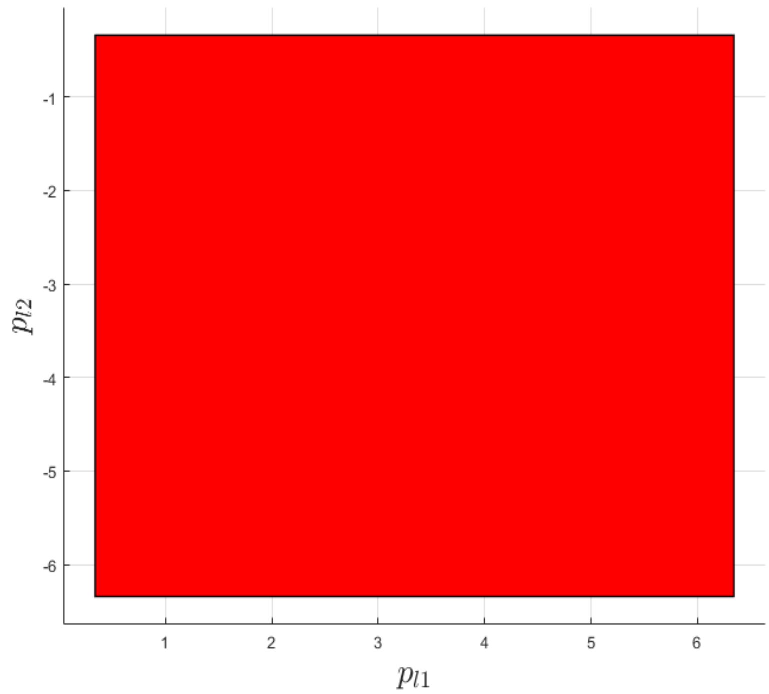

FIGURE 6.11: Set Φ' with probability $\pi' = 13.7\%$.

6.2 Real Test Case

6.2.1 Network Description

To provide a real case study, the developed strategy has been tested on a medium voltage distribution network simulated in DigSILENT PowerFactory. In Figure 6.12 is shown the grid, that is a three phase symmetrical and balanced system.

The rated voltage of the grid is $V_r = 20$ kW and is connected to the transmission grid at $V_t = 110$ kW, while the rated current of the lines is $I_r = 419$ A. Notice that the switch between node 3 and node 4 is open.

Before to carry out a complete analysis on this grid, some changes has been made. This choice is due to the absence in the grid of renewable generation, except for one node. Furthermore, the network is robust and can support high delivery of power in almost any condition, i.e. the actual loads cannot push the grid to the limits. Thus,

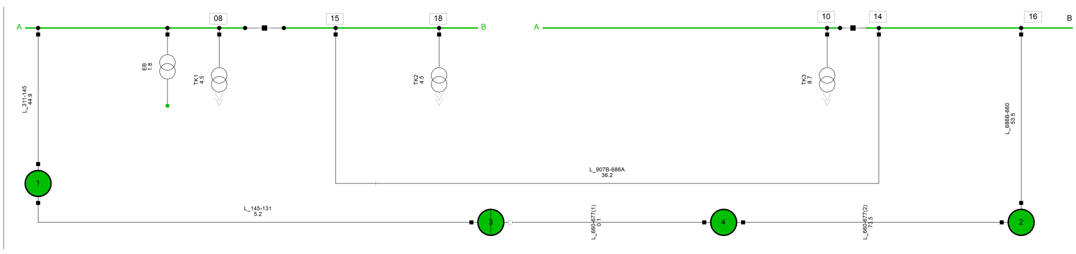

FIGURE 6.12: Real case grid.

it has been considered to insert uncontrollable generation/demand in each node simulated as a negative/positive load. Moreover, a flexible device is added to each node and simulated as a synchronous generator. In particular, a Combined Heat and Power (CHP) plant has been added in nodes 1 and 4 and a storage in nodes 3 and 2 with the rated power in MW shown in Table 6.4.

TABLE 6.4: Rated power of the flexible devices in each node.

	CHP (MW)	ESS (MW)
Node 1	$0 \leq p_{s1} \leq 4$	-
Node 3	-	$-1.5 \leq p_{s3} \leq 1.5$
Node 4	$0 \leq p_{s4} \leq 4$	-
Node 2	-	$-1.5 \leq p_{s2} \leq 1.5$

6.2.2 Network of Micro Grids

The described network with the considered modifications is a NMG with four MGs connected to the transmission grid (ULG) as displayed in Figure 6.13. A single MG consisting of a synchronous generator and a load is illustrated in Figure 6.14. The ULG is simulated with a synchronous generator for the deviation Δp_{g0} from the target $\hat{p}_{g0}$ represented as a negative/positive load as shown in Figure 6.15.

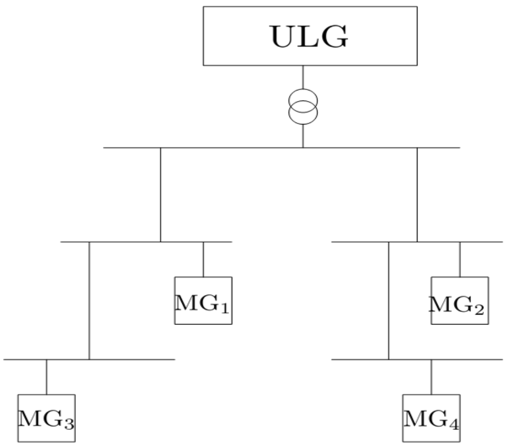

FIGURE 6.13: The real NMG with five nodes, single phase scheme.

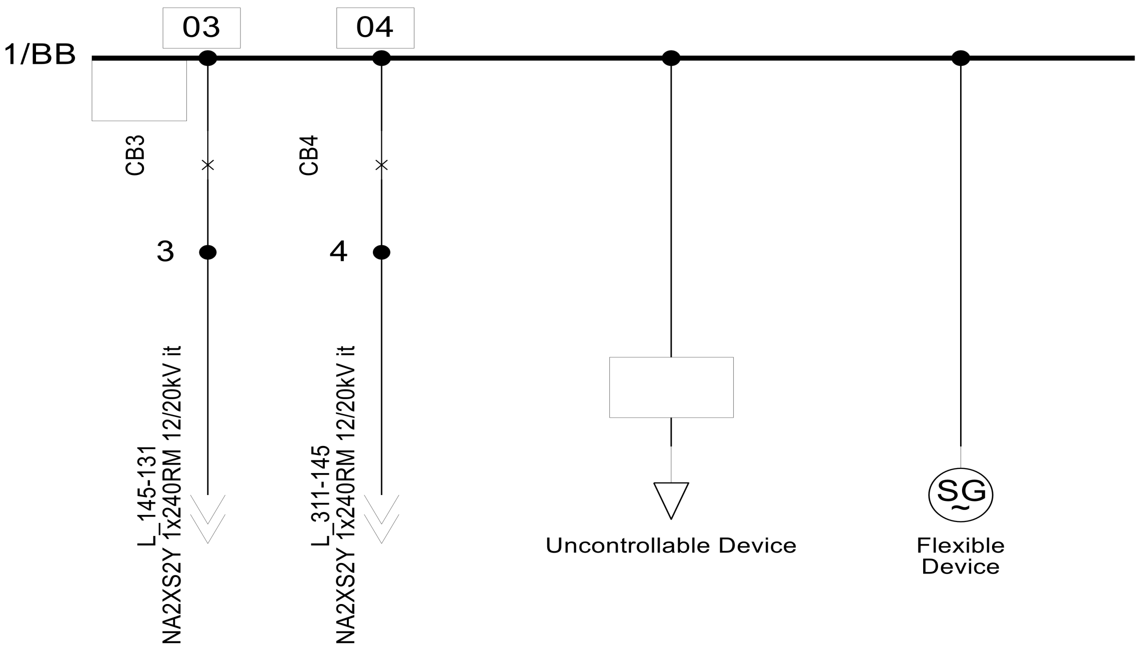

FIGURE 6.14: One MG with an uncontrollable element and a controllable one.

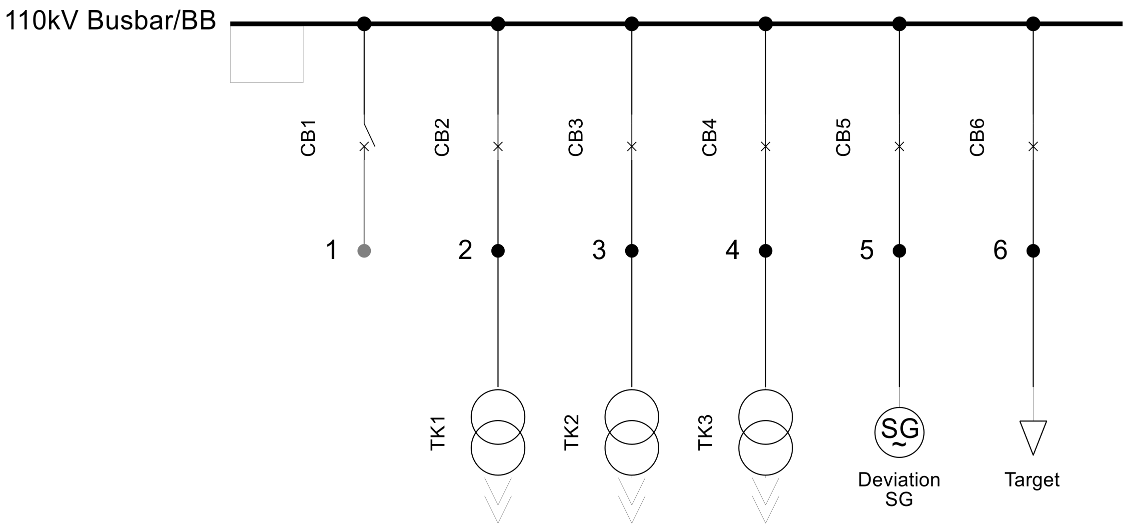

FIGURE 6.15: ULG with the target $\hat{p}_{g0}$ (load) and the deviation Δp_{g0} (Synchronous Generator).

6.2.3 Application of the Algorithm

The algorithm described in 6.1 is applied in this case with five nodes. Notice that, as stated in 4.2, the interest is only in active power flow and the constraints for the lines are calculated with worst case of transmission of reactive power and power losses. Mathematically, the rated active power of the line is calculated as

$$p_{bi} = \sqrt{3} \cdot \overline{V} \cdot I_r \cdot \cos(\phi) \tag{6.5}$$

where $\overline{V}$ is a worst case voltage equal to 98% of the nominal voltage V_r. This choice is based on simulations showing that 90% of V_r is a value excessively conservative. The term $cos(\phi)$ is the power factor and is chosen equal to 0.9. This value is selected to consider approximately half of the total capacity of the line occupied by reactive power, that is compensated by the ULG. The resulting power constraint is

$$p_{bi} = \sqrt{3} \cdot 0.98 \cdot 20000 \cdot 419 \cdot 0.9 = 13 \text{ MW} \tag{6.6}$$

Furthermore, to maximize the transmission of reactive power through the lines it is set an inductive load ($\cos(\phi) = 0.9$) in nodes 1,3 and a capacitive load ($\cos(\phi) = -0.9$) in nodes 2,4.
The optimization problem (5.2) is formulated using the constraints showed in Table 6.5 in MW.

TABLE 6.5: Inequality constraints of the NMG with five nodes.

MG_1 (MW)	MG_2 (MW)	MG_3 (MW)	MG_4 (MW)
$-13 \leq p_{b1} \leq 13$	$-13 \leq p_{b2} \leq 13$	$-13 \leq p_{b3} \leq 13$	$-13 \leq p_{b4} \leq 13$
$-4 \leq p_{s1} \leq 0$	$-1.5 \leq p_{s2} \leq 1.5$	$-1.5 \leq p_{s3} \leq 1.5$	$-4 \leq p_{s4} \leq 0$
$-30 \leq p_{l1} \leq 30$	$-30 \leq p_{l2} \leq 30$	$-30 \leq p_{l3} \leq 30$	$-30 \leq p_{l4} \leq 30$
ULG (MW)			
$-50 \leq \hat{p}_{g0} \leq 50$			

The upper and lower bounds of the uncontrollable elements are selected based on the limits of the lines. Solving the MPP gives 69 polytopes constituting Δ that cannot be displayed because the dimensions of the problem are more than three. A function of MPT verifies the convexity of the union of polytopes with null solution for the deviation Δp_{g0}, i.e. of the set Γ. Once the convexity has been verified the matrix A and the vector $\mathbf{b}$, showed in Appendix A, of the convex hull of Γ are calculated and the constraints are set for the optimization problem (5.14).

Two tests are performed to validate the algorithm in this real case: an admissibility test that consists in choosing values inside and outside the admissible set Γ to show that the constraints are not violated and no deviations from the target $\hat{p}_{g0}$ are necessary for values within Γ. On the contrary, when values outside the set are selected there could be a violation of constraints or a non-null value of Δp_{g0}. Secondly, a maximum-I'_{g0} test consists in considering CDFs for the four uncontrollable devices and in taking four intervals and four probabilities from these CDFs. Thus, the set Φ' and the associated probability π' are calculated as shown in Section 6.1. Based on the set Φ' the maximization problem (5.14) is solved giving the maximum interval I'_{g0}. Then, it is verified if every possible combination of the parameters p_{li} with $i = \{1, \dots, 4\}$ is satisfied with every possible values of the target within the interval I'_{g0}.

Admissibility Test Results

The values inside and outside Γ are chosen based on the result of a function of MPT that allow to calculate for each of the facet of a polyhedron a random point in the relative interior of the facet. The function gives 30 random points shown in Appendix A and the test is carried out with two of these points and three external points. The results are shown in Table 6.6, where when the internal points are considered the optimal power flow in PowerFactory gives a null deviation $\Delta p_{g0} = 0$ MW. On the contrary, with the first external point the optimal power flow cannot be solved while the second external point leads to a deviation from the target by $\Delta p_{g0} = -0.096$ MW. When considering the third external point the optimal power flow cannot be solved. However, this last consideration it is true if the reactive power is compensated only from the ULG. In fact, simulations indicate that if there is a compensation of reactive power in the first MG the line constraints would not be violated and the optimal power flow could be solved. Concluding, this last external point shows that considering the actual flow of reactive power in a network could increase the boundaries of the set Γ.

TABLE 6.6: Admissibility test results.

	p_{l1} (MW)	p_{l2} (MW)	p_{l3} (MW)	p_{l4} (MW)	$\hat{p}_{g0}$ (MW)	Δp_{g0} (MW)
Int. point 1	30	24.105	-13.2574	-11.6954	24.7574	0
Int. point 2	-4.5433	-3.71	2.04	4.6233	1.41	0
Ext. point 1	30	25.105	-14.2574	-15.6954	24.7574	-
Ext. point 2	-4.6433	-3.71	2.04	4.6233	1.41	-0.091
Ext. point 3	5.8353	-0.7057	13.9527	-12.0823	1.7120	-

Maximum-I'_{g0} test results

This test is performed similarly to the part of the tutorial in 6.1.1. Assume the CDFs for the output power of the four uncontrollable devices shown in Figure 6.16-6.17. Furthermore, consider, for example, intervals and the associated probability used to calculate the set Φ' and the probability π' shown in Table 6.7.

TABLE 6.7: Ranges and associated probabilities of P_{li} with $i = \{1, \ldots, 4\}$.

Ranges	$-0.75 \leq p_{l1} \leq 3$	$-1.25 \leq p_{l2} \leq 1$	$-1.25 \leq p_{l3} \leq 1$	$-0.75 \leq p_{l4} \leq 3$
Probability	92.72%	88.64%	88.64%	92.72%

Once that the set Φ' and the associated probability $\pi' = 67.5\%$ are calculated, the optimization problem (5.14) gives the maximum interval $\mathrm{I}'_{g0} = [-1; -3]$. Table 6.8 shows combinations of the parameters $\boldsymbol{\theta}_{P-1}$ tested with values of the target $\hat{p}_{g0}$ within the interval I'_{g0} and outside it.

The column relative to the deviations Δp_{g0} shows that when values of $\hat{p}_{g0}$ are chosen within I'_{g0} the result is null or almost zero. Very small deviations like $\Delta p_{g0} = 0.0001$ MW and $\Delta p_{g0} = 0.0078$ MW represent the losses on the lines that are not considered in the algorithm developed in this thesis. Furthermore, it is worth mentioning that the losses in those two cases are compensated by Δp_{g0} because the flexible

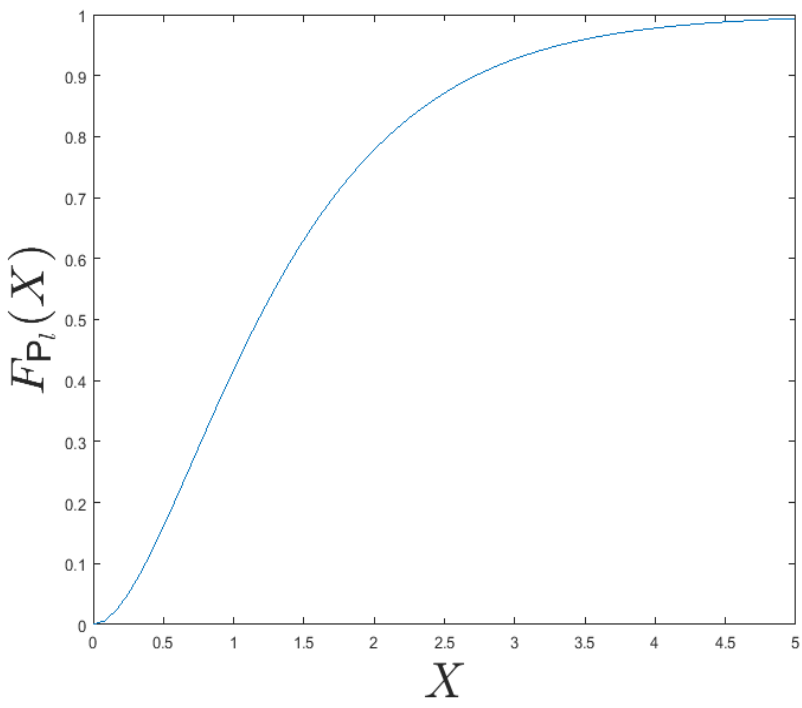

FIGURE 6.16: Gamma CDF of P_{l1} and P_{l4} with $a = 2$ and $b = 0.7$.

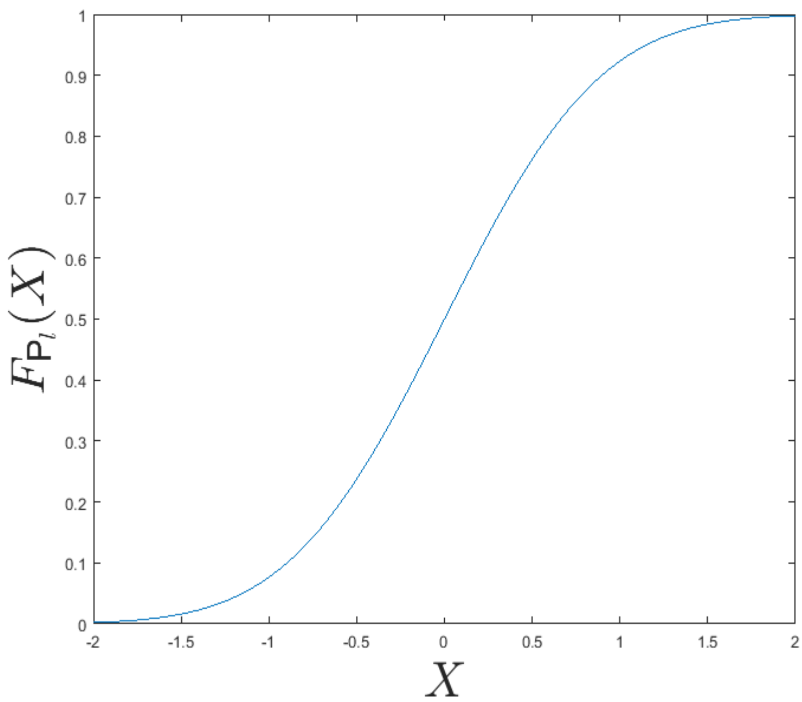

FIGURE 6.17: Normal CDF of P_{l2} and P_{l3} with $\mu = 0$ and $\sigma = 0.7$.

TABLE 6.8: Maximum-I'_{g0} Test Results.

p_{l1} (MW)	p_{l2} (MW)	p_{l3} (MW)	p_{l4} (MW)	$\hat{p}_{g0}$ (MW)	Δp_{g0} (MW)
3	1	1	3	-1	0
-0.75	-1.25	-1.25	-0.75	-1	0.0001
1	-0.25	-0.25	1	-1	0
3	1	1	3	-2	0
-0.75	-1.25	-1.25	-0.75	-2	0
1	-0.25	-0.25	1	-2	0
3	1	1	3	-3	0.0078
-0.75	-1.25	-1.25	-0.75	-3	0
1	-0.25	-0.25	1	-3	0
3	1	1	3	-3.1	0.103
-0.75	-1.25	-1.25	-0.75	-0.9	0.0985

devices are pushed to their power limits. On the other hand, when extreme values of Φ' are selected and the target is chosen outside of the interval I'_{g0}, the deviation is non-null. For example, the last row of the table shows that the lower bounds of the parameters $\boldsymbol{\theta}_{P-1}$ combined with a values of the target $\hat{p}_{g0} = -0.9$ MW lead to a deviation $\Delta p_{g0} = 0.0985$ MW. The latter is almost the value of the distance between the interval I'_{g0} and the chosen target, validating the accuracy of the result of the algorithm.

To verify the reliability of the interval I'_{g0} a Monte Carlo simulation, with 10000 scenarios of the uncertain powers $\mathbf{p}_l$, is performed in MATLAB and employing matpower for the calculation of the optimal power flow. The results of this simulation identifies the frequency of the deviations Δp_{g0} defined as

$$f_{\Delta p_{g0}} = \frac{\#\{\Delta p_{g0} = a\}}{\#scenarios}, \tag{6.7}$$

where $a \in \mathbb{R}$ is one possible value of this deviation. The interest is in the frequency of the deviation equal to zero because it defines the number of times, with respect to the total number of scenarios, in which the combination of parameters is into the admissible set. The frequency obtained for the interval I'_{g0} is displayed in Figure 6.18 and is almost 99% that is largely greater than the estimated reliability $\pi' = 67.5\%$. This simulation is aligned with the theoretical expectations of the method, even though the expected reliability of the interval I'_{g0} is excessively conservative. The results of a similar simulation performed over a different range $\mathrm{I}'_{g0} = [-7; -9]$ are presented in Figure 6.19. As it can be observed, the frequency of null deviations for this second interval is less than the expected probability $\pi' = 67.5\%$. Thus, the reliability π' could be not guaranteed for intervals different from the one computed via the proposed algorithm.

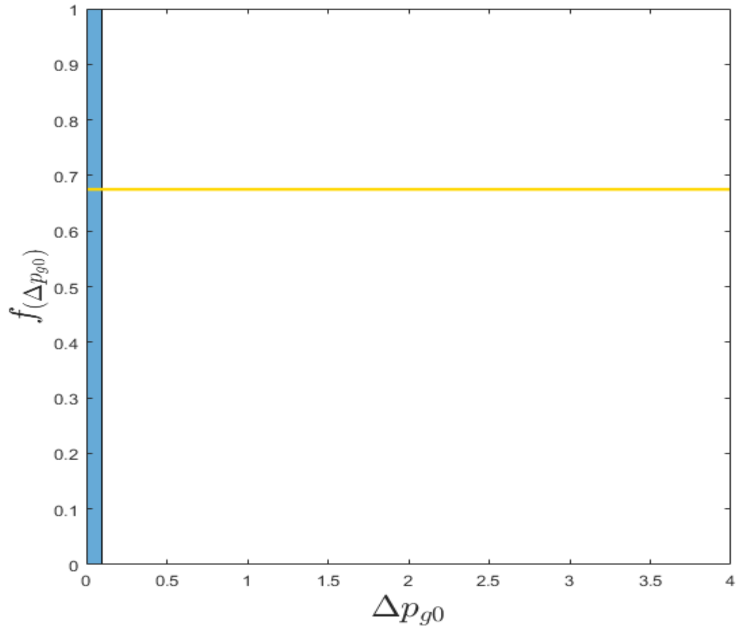

FIGURE 6.18: Frequncy of Δp_{g0} for $\mathrm{I'}_{g0} = [-1; -3]$ with π'= 67.5% (orange).

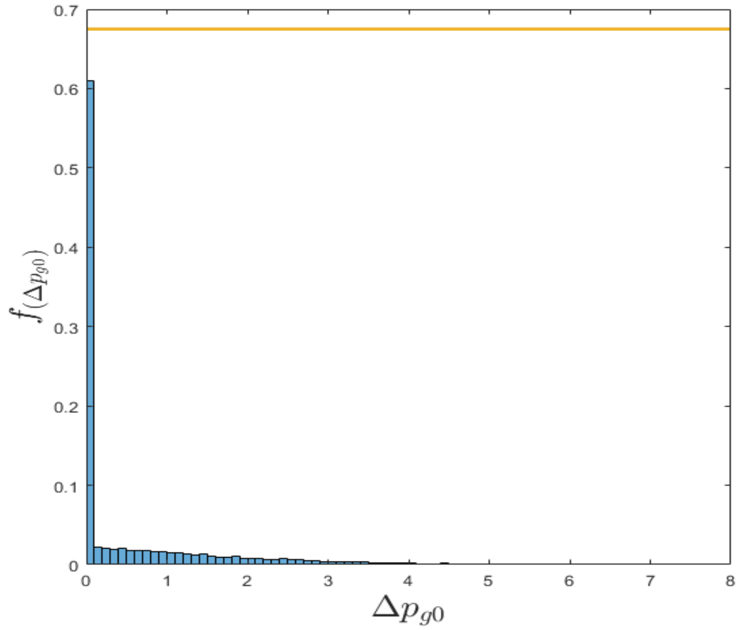

FIGURE 6.19: Frequncy of Δp_{g0} for $\mathrm{I'}_{g0} = [-7; -9]$ with π'= 67.5% (orange).

6.2.4 Conclusion of the Chapter

The examples shown in this chapter aim to validate the algorithm outlined in Chapter 5. The example with three nodes has been chosen in such a way to illustrate the procedure and the steps of the algorithm in a convex case. In fact, the parameters vector has three components therefore the problem is 3-dimensional and all the plots can be displayed.

The real case study points out that the algorithm can provide a reliable variability range to the ULG even when the number of nodes is increased up to five. Moreover, this analysis demonstrates that the set Γ delimits the only regions where every combination of the parameters do not violate the constraints and do not require deviation from the target parameter $\hat{p}_{g0}$. However, this set is delimited using a specific assumption over the amount of power that can flow through the lines. Meaning that the result could be excessively conservative if the real amount of reactive power is not considered.

To sum up, even though the strategy and the algorithm described in this thesis performs as theoretically expected, further analyse are required to include real considerations of losses and reactive power within the constraints of the optimization problem (5.2). Avoiding a conservative restriction of the the set Γ could lead to a most accurate result in the determination of the interval I'_{g0} and the associated probability.

Chapter 7

Conclusions and Future Work

The increase in the total renewable rnergy sources installed capacity leads to problems in the field of power system stability and reliability. As conventional providers of ancillary services, like synchronous generators, are being replaced with intermittent generation, many problems have been occurring in the field of system flexibility. In this scenario the interest for services provided by aggregated distributed energy resources is increasing.

Based on these considerations this thesis mainly focuses on developing a strategy to manage the resources of a group of Micro Grids (MGs) with high penetration of distributed energy resources, in grid-connected mode and able to exchange power with each other. The goal is the computation of reliable variability ranges of power from a Network of Micro Grids (NMG) to participate in the provision of AS. After an intensive literature review on this topic it has been decided to avoid any assumption over the probability distribution of the output power of the uncontrollable devices of each MG and to consider power constraints for the lines of the NMG. To this end it is formulated an optimization problem based on multi parametric programming. The solution of a multi parametric problem consists of a piecewise affine function for each decision variable defined in function of the parameters of the problem. Furthermore, this function is defined over the parametric space partitioned in polytopes. Based on this solution an analysis has been carried out and an algorithm is developed.

The algorithm allows to locate the polytopes with a certain property of one decision variable. The union of these polytopes defines an admissible parametric set. The admissible points of this set are such that even if some constraints of the NMG are activated the parameters can be combined to obtain a desired target. In particular, it can be determined how to allocate the power output of the uncontrollable devices to obtain a range of power output to the ULG. Furthermore, the algorithm allows the opposite procedure in which starting from a fixed range of output power it can be exactly defined how to allocate the resources of the NMG to satisfy that interval of power.
The algorithm has been further improved to include an analysis for convex and nonconvex parametric sets. A combination of this algorithm with information from probabilistic forecasts of the output power of the uncontrollable sources allows to assign a certain probability to the mentioned range identifying the reliability of the

interval.

This algorithm has been implemented in MATLAB and initially tested on a three nodes NMG and later on a real distribution network. The first test has been performed to graphically demonstrate the steps of the algorithm. The real case test instead shows the validity of this strategy in a five nodes NMG simulated in DigSILENT PowerFactory. In particular, an admissibility simulation shows the accuracy of the admissible set. On the other hand, a maximum interval simulation coupled to a Monte Carlo simulation show the reliability of a variability range of the target.
The main limitation in this second test lies in the assumptions about the reactive power and the losses leading to an exact but conservative result.
Furthermore, other tests that have been conducted over networks with more nodes show an issue of multi parametric problem known as the curse of dimensionality. Simulations that are not reported in the thesis, in fact, point out very long computational time (up to 9 hours) when 11 nodes are evaluated. Moreover, with more than 12 nodes MATLAB reach the limits of memory giving the error "out of memory". These problems depend on the formulation of the constraints of the MPP that influence the time of computation and on the number of parameters that can lead to issue of maximum memory.

To conclude, the approach presented in this thesis requires further studies before an application in a real NMG. However, this document outlines and tests an innovative way of looking at one of the debated problem in these years: how to manage distributed energy resources in order to provide ancillary services and therefore more flexibility to the power grid.

7.1 Future Work

In this final section, the tasks of utmost interest for future work are presented. Given tasks may be interesting to further analyse and explore in order to gain even better understanding of the given topic and/or obtain more precise results.

The tasks are listed as follows:

- Include the losses and constraints for the reactive power to obtain more accurate variability ranges.
- Consider energy constraints for flexible devices such as ESS including the time correlation among the power production in different instants.
- Consider a cost analysis to investigate the economical benefit of the approach proposed in this thesis.

Appendix A

This appendix report the matrix A and the vector **b** of the convex hull of the real test case. Moreover is shown the matrix with random point in each facet of this convex hull.

$$A = \begin{bmatrix} -0.0000 & 0.0000 & 0.0000 & -1.0000 & -0.0000 \\ 1.0000 & 0.0000 & 1.0000 & -0.0000 & -1.0000 \\ 1.0000 & 0.0000 & 0.0000 & -0.0000 & -1.0000 \\ -0.0000 & 0.0000 & 1.0000 & -0.0000 & -0.0000 \\ -0.0000 & 1.0000 & 0.0000 & -0.0000 & -1.0000 \\ -0.0000 & 1.0000 & -0.0000 & 1.0000 & -1.0000 \\ -1.0000 & -0.0000 & 0.0000 & -0.0000 & 1.0000 \\ -1.0000 & 0.0000 & -1.0000 & -0.0000 & -0.0000 \\ -1.0000 & -1.0000 & 0.0000 & -1.0000 & 1.0000 \\ -1.0000 & -1.0000 & 0.0000 & 0 & 1.0000 \\ -1.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & -1.0000 \\ -1.0000 & -1.0000 & -1.0000 & 0.0000 & 1.0000 \\ -1.0000 & -1.0000 & -1.0000 & -1.0000 & 1.0000 \\ -1.0000 & -0.0000 & -1.0000 & -0.0000 & 1.0000 \\ -0.0000 & -1.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0000 & -1.0000 & 0.0000 & -1.0000 & 0.0000 \\ 0.0000 & -1.0000 & 0.0000 & -0.0000 & 1.0000 \\ -0.0000 & -0.0000 & -0.0000 & 1.0000 & 0.0000 \\ 0.0000 & -1.0000 & 0.0000 & -1.0000 & 1.0000 \\ 0.0000 & 0.0000 & -1.0000 & -0.0000 & 0.0000 \\ -0.0000 & 1.0000 & 0.0000 & 1.0000 & 0.0000 \\ 1.0000 & 1.0000 & 0.0000 & 1.0000 & -1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & -1.0000 \\ 1.0000 & 1.0000 & 1.0000 & -0.0000 & -1.0000 \\ 1.0000 & 0.0000 & 1.0000 & -0.0000 & 0.0000 \\ 1.0000 & 1.0000 & 0.0000 & -0.0000 & -1.0000 \\ -0.0000 & 1.0000 & 0.0000 & -0.0000 & 0.0000 \\ -0.0000 & -0.0000 & 0.0000 & -0.0000 & 1.0000 \\ 1.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 13.0000 \\ 18.5000 \\ 30.0000 \\ 14.5000 \\ 27.5000 \\ 18.5000 \\ 26.0000 \\ 14.5000 \\ 14.5000 \\ 27.5000 \\ 26.0000 \\ 26.0000 \\ 16.0000 \\ 3.0000 \\ 14.5000 \\ 27.5000 \\ 14.5000 \\ 27.5000 \\ 17.0000 \\ 14.5000 \\ 14.5000 \\ 18.5000 \\ 22.5000 \\ 11.0000 \\ 20.0000 \\ 18.5000 \\ 31.5000 \\ 27.5000 \\ 26.0000 \\ 30.0000 \end{bmatrix}$$

$$
FacetPoints = \begin{bmatrix}
24.8224 & 1.8385 & -12.1325 & -13.0000 & -0.7503 \\
-18.1647 & -2.3353 & 12.5823 & -9.4527 & -24.0823 \\
5.0981 & -1.0452 & -13.4019 & -11.9019 & -24.9019 \\
0.3276 & 0.8042 & 14.5000 & -9.6130 & 0.8253 \\
-24.5359 & 26.0359 & 13.0359 & -11.5359 & -1.4641 \\
-24.0823 & 22.0760 & 12.2943 & -6.2880 & -2.7120 \\
-24.9019 & 25.5981 & 13.4019 & -11.9019 & 1.0981 \\
-1.7641 & 21.9498 & -12.7359 & -9.5607 & -3.0554 \\
-25.0729 & 22.2619 & 13.5729 & -12.0729 & -0.3840 \\
-25.0000 & -2.9142 & 13.5000 & 15.0000 & -0.4142 \\
-26.0000 & 23.9853 & 13.2574 & -11.7574 & -1.7574 \\
0.8536 & -1.6464 & -12.2071 & -9.7071 & -26.0000 \\
-22.1110 & -3.6409 & 9.3591 & 15.7639 & -0.3929 \\
-4.5433 & -3.7100 & 2.0400 & 4.6233 & 1.4100 \\
0.7943 & 25.5823 & -12.5823 & -9.7943 & 2.7120 \\
0.5266 & -27.5000 & 12.7605 & 15.3431 & -2.3431 \\
0.4727 & -3.2641 & 11.9164 & -11.2359 & -3.0554 \\
2.5359 & -2.9641 & 13.0359 & 15.5359 & 24.5359 \\
0.7503 & -1.8385 & 12.1325 & 17.0000 & 22.3230 \\
3.8353 & -0.7057 & 11.9527 & -11.0823 & 2.7120 \\
27.3088 & 0.6747 & -14.5000 & -9.6958 & 0.9812 \\
0.7307 & 25.7359 & 12.0607 & -7.2359 & 24.2359 \\
5.6057 & 24.5833 & -13.5729 & -7.3943 & 0.2947 \\
-1.7433 & -0.9100 & 4.8400 & 7.4233 & -1.3900 \\
-19.0159 & 26.2639 & 13.2639 & -11.7639 & 0.5120 \\
28.2359 & 22.2078 & -9.7359 & -9.4164 & 24.2359 \\
5.4142 & 26.5000 & -13.5000 & -12.0000 & 0.4142 \\
0.6863 & 27.5000 & 12.8431 & -11.3431 & 24.3431 \\
27.6287 & 25.2751 & -12.2751 & -9.9216 & 26.0000 \\
30.0000 & 24.1050 & -13.2574 & -11.6954 & 24.7574
\end{bmatrix}
$$

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